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A Comparison of Learning Subjective and Traditional Probability in Middle Grades

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ACCEPTANCE

This dissertation, A COMPARISON OF LEARNING SUBJECTIVE AND TRADITIONAL PROBABILITY IN MIDDLE GRADES, by JEANNE DORTCH RAST, was prepared under the direction of the candidate's Dissertation Advisory Committee. It is accepted by the committee members in partial fulfillment of the requirements for the degree Doctor of Philosophy in the College of Education, Georgia State University.

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ABSTRACT
by
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The emphasis given to probability and statistics in the K-12 mathematics curriculum has brought attention to the various approaches to probability and statistics concepts, as well as how to teach these concepts. Teachers from fourth, fifth, and sixth grades from a small suburban Catholic school engaged their students ($n=87$) in a study to compare learning traditional probability concepts to learning traditional and subjective probability concepts. The control group ($n=44$) received instruction in traditional probability, while the experimental group ($n=43$) received instruction in traditional and subjective probability. A Multivariate Analysis of Variance and a Bayesian t-test were used to analyze pretest and posttest scores from the Making Decisions about Chance Questionnaire (MDCQ). Researcher observational notes, teacher journal entries, student activity worksheet explanations, pre- and post-test answers, and student interviews were coded for themes.

All groups showed significant improvement on the post-MDCQ ($p < .01$). There was a disordinal interaction between the combined fifth- and sixth-grade experimental group ($n=28$) and the control group ($n=28$), however the mean difference in performance on the pre-MDCQ and post-MDCQ was not significant ($p=.096$). A Bayesian t-test indicated that there is reasonable evidence to believe that the mean of the experimental group exceeded the mean of the control group. Qualitative data showed that while students have beliefs about probabilistic situations based on their past experiences and

prior knowledge, and often use this information to make probability judgments, they find traditional probability problems easier than subjective probability. Further research with different grade levels, larger sample sizes or different activities would develop learning theory in this area and may provide insight about probability judgments previously labeled as misconceptions by researchers.

A COMPARISON OF LEARNING SUBJECTIVE
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ABBREVIATIONS

NCTM	National Council of Teachers of Mathematics
TIMSS	Third International Mathematics and Science Study
NAEP	National Assessment of Educational Progress
MDCQ	Making Decisions about Chance Questionnaire
MANOVA	Multivariate Analysis of Variance

CHAPTER 1

INTRODUCTION

Background

The National Council of Teachers of Mathematics (NCTM) *Principles and Standards for School Mathematics* emphasizes data analysis and probability (2000). The topics of probability and statistics have become increasingly important and gained attention from business and government (Shaughnessy & Zawojewski, 1999). Probability is used to make weather predictions, describe uncertainty in medical tests and procedures, describe risks involved in business, and describe likelihood in games.

Students have difficulty understanding probability and statistics (Cosmides & Tooby, 1996; Fast, 1999; Fischbein & Gazit, 1984; 1997; Garfield & Ahlgren, 1988; Shaughnessy, 1977). In the Third International Math and Science Study (TIMSS), the largest, most comprehensive international study of schools, United States students scored only average in Data Representation, Analysis and Probability (National Center for Educational Statistics, 2005). The National Assessment of Educational Progress (NAEP) data showed that students couldn't always apply probability and statistics in problem-solving situations (Shaughnessy & Zawojewski, 1999). Nearly 80% of the graduating secondary school students in the NAEP sample reported little experience in probability or statistics, despite having presence in the elementary, middle, and high school curricula (Shaughnessy, 2003).

Traditional school curricula of the last decade reveal two definitions of probability, theoretical and experimental. A third type of probability, subjective probability, is not at all obvious in the current curricula. Before discussing this study of probability and middle grade students, it is necessary to define what is generally understood by the range of terms that scholars use concerning probability.

Theories of Probability

This study of probability in the middle grades commanded attention to the definitions and descriptions of probability throughout the literature. This section will attempt to show these terms and definitions in relation to one another. The terms are summarized in Table 1 and an explanation of each follows.

Probability is the measure of how likely it is that an event will occur (Ford, 2000). Theoretical, or classical, probability is the ratio of the number of ways an event can occur to the number of possible outcomes of the event. The probability is obtained by making an assumption that the possible outcomes are equally likely (Hawkins & Kapadia, 1984). Frequentist, or objective, probability is the ratio of the results of repeated trials of an experiment to the number of trials (Albert, 2003). Because frequentist probability is experiment-based, it is also called experimental. The majority of the researchers on the subject use the term frequentist probability instead of experimental probability, therefore frequentist is the term that will be used in this study (Albert, 2003; Cosmides & Tooby, 1996; Gigerenzer, 1994; Hoffrage, Gigerenzer, Krauss, & Martignon, 2002; Kyburg, 1964). This definition assumes that a random experiment can be repeated many times under the same conditions.

Table 1

Three Theories of Probability

Theory	Definition
Theoretical / Classical	Ratio based on possible outcomes
Frequentist / Experimental / Objective	Ratio based on experimentation
Subjective	Ratio based on beliefs

Note. Traditional is the combination of theoretical and frequentist

For example, suppose you would like to know the probability a head occurs when a coin is tossed. The theoretical probability of heads is ½ since there are two possible outcomes and one of them is heads. The frequentist probability, computed from an experiment, is the ratio of the number of heads that landed to the number of tosses. The ratio obtained would be an estimate of the theoretical probability. As noted in Table 1, this study will refer to the combination of theoretical and frequentist probability as traditional, because these are the viewpoints currently included in the curricula.

In contrast to traditional probability, the subjective definition of probability is the degree of belief that a person holds about the occurrence of an event (De Finetti, 1974). Subjective probability reflects a person's opinion about the likelihood of an event. For example, when tossing a coin, the probability of obtaining a head could be influenced by a person's prior experience, or knowledge, or beliefs.

The subjective theory, discovered by Ramsey (1926) and de Finetti (1937), abandons the assumption of consensus because different individuals, all reasonable and having the same evidence, may have different degrees of belief about the probability of an event. De Finetti (1937) argues that we should speak of each individual's probability. Others besides de Finetti and Ramsey, such as economist Fisher (1906) had expressed the subjectivist viewpoint of probability, however they could not derive mathematical expressions for probabilities from personal beliefs. The Ramsey-de Finetti view was axiomatized and developed into full theory by Savage (1954) in his *Foundations of Statistics.*

The definitions in Table 1 show that all three theories of probability express probability as a ratio. Like all probabilities, subjective probability is conventionally expressed on a scale from zero to one. A rare event has a subjective probability close to zero. A very common event has a subjective probability close to one. Just as traditional probability leads to statistical inference, the subjective theory of probability is the basis for the Bayesian theory of statistics. Bayes rule is a formula used to compute the probability that an event occurs under a certain condition. In Bayesian statistics this condition is determined either by one's beliefs, by a data-based hypothesis, or by a hunch. Bayesian statistical inference is a model of scientific knowledge with a subjective element of probability (Austin, 2002).

According to the Ramsey-de Finetti theory, people make choices in uncertain situations based on their personal beliefs about the outcome. Thus, subjective probabilities can be inferred from the observation of people's actions. The action a person takes in an uncertain situation is based on the information and knowledge they have at that time. When new data is acquired about a situation, a person can update the belief in light of the new data. In subjective theory, probability is a quantitative measure of a belief about how likely it is an event will occur (Ramsey, 1926).

DeFinetti (1974) argues that the frequentist assumption of repeating an experiment many times under the same circumstances is unrealistic and impossible. Many factors influencing the experiment would have changed such as environmental conditions or the person engaged in the experiment, so the supposition is idealistic. Thus the subjective probability of an event might change according to the circumstances surrounding it.

Theoretical Framework and Statement of the Problem

Much of the research on learning probability and statistics in the last 30 years has been based on a theory developed by Kahneman and Tversky (1972) that is anchored in heuristics and misconceptions about probabilistic reasoning. Kahneman and Tversky argue that people use certain heuristics, which have been categorized and labeled, that lead to wrong answers. These heuristics are procedures that are intuitive ways of solving a problem that are in conflict with algorithms or procedures involving rules. The purpose of Kahneman and Tversky's theoretical framework, known as "heuristics and biases", is to understand the cognitive processes that lead to valid or invalid judgments.

These heuristics and consequently the "misconceptions" that arise from their use are rooted in the traditional definition of probability. Kahneman and Tversky's theory of heuristics is not compatible with this present study in subjective probability, however it is the theory that has been the framework for mathematical educators and is relevant. More recent conceptual theory from cognitive psychology suggests that there is a mismatch between human reasoning and traditional probability theory. Hawkins and Kapadia (1984) suspected that traditional probability runs counter to children's intuitions because subjective probability is closer to the way children think. Gigerenzer (1996) believes that judgments that deviate from the "narrow norms" for evaluating reasoning have been incorrectly labeled cognitive illusions.

While subjective probability is excluded from classrooms, students have experiences about chance that are subjective, as well as hunches and beliefs about probability (Shaughnessy, 2003). Fischbein (1987) refers to these beliefs as primary intuitions and concludes they are often in conflict with formal mathematical rules. Subjective probability might explain how children think about probabilistic situations and perhaps shed light on "misconceptions". Wang (1994) argues that traditional probability theory is accepted as the norm because it is historically well developed, whereas the Bayesian approach and subjective probability is not. There does not seem to be a clear theory that considers subjective judgments in conjunction with the heuristics people use to make decisions in the face of uncertainty. Perhaps a new theoretical framework is needed that considers intuition, heuristics, and subjective probability.

Purpose and Research Questions

Currently, theoretical and frequentist probabilities are being taught in schools. Subjective probability, which recognizes students' beliefs about probabilistic situations, may account for what are considered students' misconceptions about probability. Therefore the purpose of this study was to come to an understanding of children's reasoning when subjective probability is taught by including subjective probability in the curriculum.

In order to explore student performance and reasoning about probability I chose two research questions for this study. The first question was "Is there a mean difference in performance in applying probability between students who received instruction in traditional probability and those who received instruction in both traditional and subjective probability?" The second question was "What are the salient themes that emerge from students' explanations about situations involving chance?"

Brief Overview of the Study

To provide answers to the research questions, I used a mixed methods design. An integrated use of method allowed for conclusions based on statistical data for the first question and conclusions based on themes for the second research question.

The study was conducted in a small, Catholic school in Atlanta. The participants were a total of 87 fourth-, fifth-, and sixth-grade students, three teachers per grade, and myself. Prior to lessons with the students, I conducted professional development for the teachers so they could learn the basic concepts of subjective probability.

Students were randomly assigned by grade level to experimental and control groups. After completing a pretest, called the Making Decisions about Chance Questionnaire (MDCQ), the control group engaged in five lessons on traditional probability which lasted 45 minutes each. The experimental group received instruction on the same lessons, in the same amount of time, on the same day as the control group, but with parallel concepts in subjective probability. The five lessons were: (a) more, less, equally likely; (b) sample space; (c) finding probability; (d) additive probability; and (e) using data to find probability. The lessons were activity-based, with a worksheet and manipulatives. The three teachers taught their respective classes.

Throughout the study, teachers kept reflective journals. I recorded student and teacher comments made during the lessons. The five lessons were taught over seven school days. At the conclusion of the lessons, students took the MDCQ again. I then selected six students for interviews, two students from each grade level, one from the control group and one from the treatment group. I choose these students because they showed an increase in score from the pre-MDCQ to post-MDCQ, and had answers on the activities that were interesting and required more explanation.

Data consisted of pre- and post-MDCQ scores, activity sheet responses, teacher journal entries, written responses on the MDCQ, researcher notes, and student interview responses. All narrative data were coded and constant comparative analysis was used to look for themes. A Multivariate Analysis of Variance (MANOVA) and a Bayesian t-test were used to analyze the pre- and post-MDCQ scores.

In summary, the purpose of this study was to experiment with the inclusion of subjective probability in the curriculum, as well as to describe student reasoning patterns about probabilistic situations. The next chapter will present a review of the literature that has influenced this study.

CHAPTER 2

REVIEW OF THE LITERATURE

The primary literature that shaped this study encompasses three particular areas. The first body of research explores students' difficulties in learning probability and statistics, especially students' intuitive "misconceptions" about probabilistic events. The second area is strategies that aid students in overcoming these difficulties and influencing how people think. The third area of research examines subjective probability theory and its current place in mathematics education.

"Misconceptions" in Probability

Research in the 1970s and 1980s focused on heuristics that caused people to make errors in judgment under uncertainty. Kahneman and Tversky (1972) formed the theory that people make judgments about probabilistic events using heuristics that have been labeled representativeness, the conjunction fallacy, gamblers' fallacy, availability, and base rate neglect, to name a few. Categorizing and labeling these heuristics formed the theory researchers (Crawford, 1997; Fischbein, 1987; Fischbein & Schnarch, 1997; Shaughnessy, 1977; Vahey, 1999) used for examining difficulties in learning probability and statistics. While the Kahneman and Tversky theory is not the basis of the present study, it is beneficial to examine these well known findings that have greatly impacted the teaching and learning of probability and statistics over the last 30 years.

Some of the heuristics that have historically been studied will now be explained. Representativeness refers to making a decision based on how similar the event is to its population. For example, outcomes that preserve a majority-minority relation to the population are judged to be more representative of the population and are therefore more probable. The birth sequences GBGBBG and BGBBBB are equally likely; however, most people agree that they are not equally representative. Kahneman and Tversky (1972) found that 75 of the 92 subjects in their study believed that the sequence GBGBBG was more likely to occur because the number of boys and girls is closer to equal. This was the basis for the samples and events used as questions on the survey in the landmark study.

Another widely studied heuristic is the conjunction fallacy, which refers to the idea that the probability of an event appears to be higher than the intersection of the same event with another. An example of this misconception is the Dan problem examined by Fischbein and Schnarch (1997):

> Dan dreams of becoming a doctor. He likes to help people. When he was in high school he volunteered for the Red Cross organization. He accomplished his studies with high performance and served in the army as a medical attendant. After ending his army service, Dan registered at the university. Which seems to you to be more likely?
> a. Dan is a student of the medical school.
> b. Dan is a student.

People often answer with the choice "Dan is a student of the medical school." Traditional probability theorists claim that the correct answer is choice *b* because there are more students than there are medical students. Choosing *a* is an example of what has been labeled a misconception using the conjunction fallacy. However this answer could be interpreted as making a decision based on the given information and therefore considered correct by subjective theory. In the Fischbein and Schnarch (1997) study, the conjunction fallacy was prevalent until ninth grade, but lessened in eleventh grade and college as about half as many students held the conjunction fallacy at this age. On the Dan question the participants answered with choice a as follows: 5^{th} grade -85%, 7th-70% 9th-80% 11th-40%, College students-44%. Thus it appears that the conjunction fallacy is very prevalent in middle and high school children, but by adulthood disappears in about half of the people. I included a question similar to the “Dan” question on the MDCQ in order to examine the students’ explanations for their answers (see Appendix C, # 19).

The gambler’s fallacy heuristic, or the negative recency effect, is a manifestation of the belief in representativeness. For example, if a coin is tossed and has landed on heads three times in a row, people might believe it is more probable that the coin will land on tails on the fourth toss. This is contrary to the theoretical probability of ½ on each toss. Negative recency effect is also a manifestation of the belief that an event should reflect a process of randomness. The belief is that if a ratio in a population is preserved in a short sequence of events, then in a long sequence of the same event, one outcome must eventually be followed by another to restore balance. Fischbein and Schnarch (1997) found that like the conjunction fallacy, negative recency effect decreased with age. There was a question on the MDCQ which considers the toss of a coin after having obtained heads many times. Again, I wanted to examine the students’ explanation for their answers and consider their judgments in light of subjective probability (see Appendix C, # 17).

The heuristic of availability is the belief that outcomes more easily brought to mind are more likely to occur. For example, in selecting two members from a group of 10, the possibilities are more easily brought to mind then selecting eight members from a group of 10. The number of ways to perform each of these tasks is equal, however many people believe there are less ways to select two members because it is easier to think of the ways to do so.

The heuristic called base rate neglect refers to the mistake of ignoring the base rate frequency and misusing other information in the problem. The following example was put to 60 students and staff at Harvard Medical School (Fenton, 2002):

> A particular heart disease has a prevalence of 1/1000 people. A test to detect this disease has a false positive rate of 5%. Assume that the test correctly diagnoses every person who has the disease. What is the chance that a randomly selected person found to have a positive result actually has the disease?

Almost half of the subjects tested gave the answer 95% and the average answer was 56%. The correct answer is about 2%. When people give a high answer like 95% they are ignoring the fact that only .1% of those tested actually have the disease. Some researchers believe that base rate neglect is more prevalent when the events described are familiar to people and when descriptions of events fit into certain stereotypes (Fenton, 2002).

Numerous studies (Fischbein, 1987; Garfield, 1988; Kohler, 1996; Maher, 1998) explored intuitions that people hold concerning probability and the specific “misconceptions” commonly associated with these intuitions. The questions in Fischbein and Schnarch’s (1997) study pertained to the misconceptions of representativeness, negative and positive recency effects, simple and compound events, and the conjunction fallacy. The results are reported according to each misconception. The findings from Fischbein and Schnarch’s study include the following statements that are related to age of the participants. The misconception of representativeness decreased with age. Negative recency effect decreased with age, but positive recency effect was negligible. The conjunction fallacy was strong until ninth grade, but lessened in eleventh grade and college as about half as many students held the conjunction fallacy at this age. The effect of sample size fallacy, basing a conclusion or generalization on a sample size too small, actually increased with age, as did the misconception of availability. Compound events, when two or more events occur simultaneously, was the only misconception in the Fischbein and Schnarch (1997) study that was stable across ages.

Fischbein and Gazit (1984) also found that students showed little improvement on items involving compound events, as did Maher (1998). The concept of sample space is directly related to compound events and is difficult for students (Jones, Langrall, Thornton, & Mogill, 1999). Fischbein and Schnarch (1997) offer an interpretation that might have influenced the students' decision making. The student intuitively accepts a general intellectual schema that molds a solution whether it is meaningful to the probabilistic situation or not. There was a question on the MDCQ which involved compound events (Appendix C, # 14) and activities in lesson two (Appendix D, # 3).

To summarize, Tversky and Kahneman (1972), Fischbein and Gazit (1984), Fischbein and Schnarch (1997), and Maher (1998) identified and labeled "misconceptions" commonly occurring in probability and asserted that an explanation for the errors is a processing error inside the mind. The studies were quantitative in nature, using surveys involving theoretical probability problems and situations. These studies provided a framework for researchers in the field of probability and statistics. However, the heuristics mentioned are classified as "misconceptions" according to the traditional definition of probability. When considering the subjective theory of probability, these "misconceptions" might be personal beliefs based on available knowledge or opinion and not misconceptions at all. This idea will be considered as the literature concerning strategies for overcoming misconceptions is discussed.

Strategies for Overcoming Misconceptions

In addition to the literature that describes "misconceptions" about probability and statistics there are studies on what can practically be done to change them. Some of the methods for overcoming misconceptions in learning probability and statistics are hands-on experimentation, games, data gathering, and computer simulations. I will discuss the research results of studies involving these four strategies.

Many studies designed on overcoming "misconceptions" concern using hands-on experiences (Crawford, 1997; Edwards & Hensien, 2000; Gainey & Kloosterman, 1993; Shaughnessy, 1977). Misconceptions are attributed to intuitions which will not disappear just because they may be contrary to formal mathematical reasoning. The source of probabilistic intuition is experience. If experience were a main factor in producing intuition, then practice or new experiences would alter intuition (Fischbein, 1987). Therefore, to teach probability successfully it is not sufficient to present mathematical rules and facts. Students must experience probabilistic situations with dice, coins and marbles. Students must be a part of gathering data about probabilistic events and witness unpredictable outcomes (Fischbein, 1987). While these ideas were formed by researchers in traditional probability theory, they appear to mimic subjective probability as a belief which changes as new information is acquired.

Shaughnessy (1977) studied whether students can overcome misconceptions by using an activity-based approach to elementary probability. The "misconceptions" investigated in this study are the ones that arise from the use of representativeness and availability. The participants were college freshman with little or no previous formal experience with probability and who demonstrated "misconceptions" on a pre-test.

Students in the control group in Shaughnessy's study were taught by lecture only. Students in the experimental groups performed nine activities in probability using expected value and combinatorics. The classes worked in small, cooperative groups of four or five members. The group members were changed throughout the course. The mathematics content of both the control and experimental courses was similar. Shaughnessy's hypothesis was that the students from the experimental groups would overcome their misconceptions if they experienced probability as a process rather than as a collection of rules and techniques.

Shaughnessy concluded that the experimental group relied less on the heuristics. The experimental group showed a significant difference at the $p =. 005$ level in overcoming the representativeness misconception and were successful at overcoming availability, though only at $p < .19$. From his daily observations, Shaughnessy concluded that college students could learn models and formulas on their own. The results from the posttest indicated that the manner in which students learned probability did affect student learning. From a subjective viewpoint, this could mean that students update their beliefs as they acquire new information from hands-on experimentation. All of the lessons in my study involve hands-on activities.

Another instructional technique for overcoming "misconceptions" about probability is using games. Researchers from Brazil, Israel, and the United States conducted a cross-cultural investigation using dice games. Students formed mathematical representations and models of sample spaces (Maher, 1998). Since games are an informal way of acquiring new information about probabilistic situations, this study could also be interpreted as support for the idea that the students' personal beliefs were changed through experience.

Edwards and Hensien (2000) provided students with opportunities to compare theoretical and frequentist probability by physically gathering data. Their goal was to determine how probability and statistics relate to each other in the instructional environment. Data generated from the experiments and computation of statistics from the data supported the theoretical probability concepts. A mathematics exploration involving this type of activity fosters discourse among students and aids them in understanding abstract concepts. Garfield and Ahlgren (1988) recommend an exploration of how ideas of statistical inference can be taught independently of correct probabilistic thinking.

Computers have been used as tools to simulate probabilistic events, especially those involving many trials. Bright (1985), however, found that computers were not very effective at promoting the learning of probability. Non-computer games had proved effective so this result raised the possibility that students may not process information presented in a computer environment in the same way as with non-computer games.

Technology is easily incorporated into the probability and statistics curriculum. Use of computer simulations or using a graphing calculator to randomly generate events is a technique that teachers need to experience and be comfortable with. In my study, a graphing calculator is used to generate a lottery situation in Lesson Five (Appendixes D and E). However, Shaughnessy (1992) cautions teachers in using computer simulations exclusively. It seems to be necessary for students to physically gather their own data with experiments. Vahey (1999) created a Probability Inquiry Environment in which students investigated games of chance. The primary research questions in Vahey's study did not concern use of the computer, however he did conclude that students in a computer setting showed a greater understanding of probabilistic situations.

Fast (1999) conducted an experiment involving a different approach to overcoming difficulties that was designed to help students reconstruct their misconceptions concerning probability. The theoretical basis for this study was constructivist learning theory, which asserts that students only truly achieve relational understanding when they are actively engaged in constructing their own knowledge. The teacher's role was seen as a facilitator, assisting students in coming to their own conclusions by providing appropriate learning activities.

At the elementary level this occurs through activities with concrete manipulatives. At the secondary level it may require more intervention because students may have prior conceptions that are contrary to accepted theory. Some research shows that "misconceptions" about probabilistic events are extremely resistant to change (Fischbein, 1987). Therefore it is necessary to find methods for reconstructing students' prior knowledge. Fast's (1999) approach with high school students was to use analogies.

In Fast's study, two versions of a multiple-choice test involving probabilistic situations were given to students. Each question in Version B was a situation analogous to the question in Version A. The Version B questions used one of three techniques to prompt students to realize their misconceptions from Version A. Those techniques were: (a) present a simpler case in which the cues activating the misconception are removed, (b) present an extreme situation to illustrate the correct concept, and (c) present the situation from a different perspective.

The overall success rate of .72 indicated the students were able to use the new questions to correct their previous thinking. If this idea of reconstructing prior knowledge is examined from a subjectivist perspective, it could be argued that students were updating prior beliefs with new knowledge.

Another approach for overcoming difficulties with probability is a research-based framework that includes a description of students' probabilistic thinking (Jones et al., 1999). There has been little research on the development and evaluation of instructional programs in probability. Jones advocates the use of a general instructional model in which research based knowledge of students' thinking is used to inform classroom instruction.

Research based knowledge of students' thinking is increasingly being identified as a component of instruction because it is useful to teachers as they plan and implement instruction. Ongoing experiences with experimental activities seemed to be successful in enabling the majority of students to recognize that no one outcome was certain (Jones et al., 1999).

In summary, during the 1970s and 1980s researchers established and categorized "misconceptions" that people possess in probability and statistics. During this time and into the 1990s, researchers explored ways to overcome these "misconceptions". A mixed methodology was dominant in the majority of these studies, consisting primarily of pre-test/posttest designs, but incorporating interviews, observations and case studies. Given the importance of probability, the consequences of various approaches to this material are desirable. These studies were based on the traditional approach to probability. If interpreted using the subjective theory of probability, overcoming the "misconceptions" could be thought of as updating one's beliefs in light of new knowledge and not really be misconceptions after all.

Subjective Probability

Since the late 1990s many researchers have turned their attention from studying the heuristics and difficulties people have with probability to how people reason probabilistically (Maher, 1998). Some attention has also shifted to subjective reasoning in probability and also to Bayesian statistics, which is the ability to factor in hunches as well as hard data.

The traditional approach to statistical inference is called frequentist because of the way it interprets probability. When a random event is repeated many times under identical conditions the probability of the event is determined by its relative frequency. In contrast to this approach the Bayesian model of statistical inference interprets probability subjectively, so that different people could have different degrees of belief in the likelihood of a specific event (Austin, 2002).

The subjectivist viewpoint of probability has been expressed by mathematicians as early as LaPlace, who in 1812 stated that one could not find the probability of heads on a toss of a coin because one cannot know the weight of the coin, the strength of the tosser, or other conditions. To say the probability is ½ for this event really measures a lack of knowledge about the conditions. However there was difficulty with the viewpoint until Ramsey (1926) and de Finetti (1974) fully developed the theory of subjective probability when they independently derived mathematical axioms and laws for probability defined as a degree of belief.

Ramsey states that it is not enough to measure probability. In order to correctly apportion our belief to the probability, we must be able to measure that belief. Some beliefs can be measured more easily then others. The measurement of beliefs is an ambiguous process that leads to a variable answer depending on how exactly the measurement is conducted. Numbers must be assigned to our degree of belief in some intelligible manner. Full belief can be denoted by 1 and full belief in the contradictory by 0. However, it is more difficult to say what is meant by a belief of 2/3. Our judgment about the strength of our belief can be determined by the extent upon which we act on that belief. Ramsey defines an individual's degree of belief in a proposition *p* to be *m/n* if the individual had to repeat it exactly *n* times then his action is such as he would choose it to be *m* times. By this definition a probability of 2/3 would be assigned by an individual if he would choose the same action 2 out of 3 times when an event occurs. Ramsey defines probability terms and proves mathematical laws based this definition of probability.

De Finetti proposed a similar theory of subjective probability in a 1931 essay and fully developed that theory in the "Theory of Probability" (De Finetti, 1974). The conceptual theory of probability proposed by de Finetti is that only subjective probabilities exist. The degree of belief in the occurrence of an event attributed by a person at a given time with a given set of information is the subjective probability. This definition is in contrast to probability involving events that can be repeated under the same conditions. The interest is only to understand what one means according to one's own conception and in one's own language. De Finetti gives the statements summarized in Table 2, as examples of the distinctions between what he terms subjectivists and frequentists.

For the subjectivist, the evaluation of an individual's probability, as a degree of belief, is based on whether or not the probability is coherent. This means studying the opinion and saying whether or not it is free of or affected by intrinsic contradictions. Coherence is an important component of subjective theory (Hawkins & Kapadia, 1984). In

Table 2

Sample Definitions for Traditional and Subjective Probability

Probability Term	Frequentist Interpretation	Subjective Interpretation
Repeatable trials	Two events of the same type in identical conditions	Two events are never the same and depend upon a person or information
Independent events	If the occurrence of one does not influence the occurrence of the other	Independent for a person if knowledge of the outcome of one does not change his evaluation of the other

my study, the students were asked provide explanations for their answers in an effort to judge whether or not their answers were coherent and rational.

The subject matter to which the concepts of subjective probability refer is irrelevant. De Finetti (1974) provides examples such as election of a public official, winning the lottery, winning a game of chance, results of a criminal trial, gender of a child at birth, and the state of the weather. In all of these cases we express ourselves in numerical quantities. He asserts that in none of these examples is it possible to describe a situation in which the conditions are always the same. Considering the toss of a coin, a description of the circumstances would have to include how a person tosses, the air movement, the peculiarity of the ground, and so on. By changing any circumstance we obtain other events. Based on these concepts, De Finetti proposed laws of subjective probability and formed a theory very similar to that proposed by Ramsey. Savage (1954) fully developed subjective probability theory in his revolutionary *Foundations of Statistics*.

The subjectivist theory of probability and its statistical counterpart, Bayesian inference, have had little to no place in mathematics education, especially at the elementary and secondary level. Traditionally, statistical analysis in research has been carried out from a frequentist perspective. However, there has been recent interest in Bayesian methods (Austin, 2002; Malakoff, 1999) and thus in its foundation of subjective probability. Albert (2003) summarizes what he sees as the three views of probability as classical, frequentist, and subjective. The classical interpretation assumes that one can represent the sample space of an event as a collection of equally likely outcomes and define the probability as the ratio of the number of favorable outcomes to the total number of outcomes in the sample space. The frequentist believes that one can repeat a random experiment many times under similar conditions and defines the probability as the estimate of the relative frequency of the event in the collection of the experiment results. This is an extension of the classical viewpoint to situations where the outcomes are not equally likely. The subjectivist defines probability as a numerical measure of a person's opinion of the likelihood of an event.

While the philosophical debate amongst the frequentists and subjectivists continues, the subjective theory of probability is slowly having a presence in mathematics education. Albert (2003) recommended using the interpretation of probability that is determined by the nature of the task that students are investigating. College students in an introductory statistics class were given nine probability problems and asked to make an intelligent guess at the probability and explain how it was obtained. The 75 students were presented with three classical type problems, three frequency type problems and three subjective type problems. The classical problems were the easiest for the students to solve. However, students tried to use some type of computation for a probability even if it did not make sense. There was also reluctance on the part of students to use a personal belief to state a probability. On the basis of the study Albert (2003) advocates spending less time on classical probability and more time discussing frequentist and subjective viewpoints.

While the Ramsey-de Finetti theory of subjective probability involves assigning a numerical quantity to a personal degree of belief, Huber and Huber (1987) recommend using comparative probability. Comparative probability means labeling events as more or less likely than other events rather than assigning a ratio to the event. Huber and Huber contend that people without a background in probability theory spontaneously use comparative probability.

In this study, 144 subjects from age eight to nineteen, engaged in gambling and sports tasks. The subjects were asked to compare the tasks using phrases such as “more probable” and “equally probable.” According to Huber and Huber, the main result of the experiment showed that comparative probability provides a much better theoretical framework for children than ratio-based probability. In my study, Lesson One concerns the use of comparative probability (see Appendices D and E).

There have been studies in recent years designed to investigate teaching Bayesian inference. Psychologists Sedlmeir and Gigerenzer (2001) designed an instructional program to teach Bayesian reasoning to college students. Four groups of participants took part in the study. One group worked with a frequency grid similar to a 2x2 table, one group with the frequency tree, one group with Bayes rule training, and a control group with no training. An issue in the teaching of Bayesian reasoning and in teaching probability in general, is the representation of the probabilities involved in the problems (Shaughnessy, 2003).

This debate (Goldin, 2003; Hoffrage et al., 2002; Lewis & Keren, 1999) is not pertinent to my study, but further descriptions of the formats used to present the problems can be found in a study by Sedlmeir and Gigerenzer (2001). Participants in the study were given 10 problems as a baseline. The 56 participants showed substantial improvement after training. The median performance for the rules training increased to 60%, the frequency grid performance increased to 75%, and the frequency tree performance to 90%. This type of instructional program lasts only one to two hours and could be implemented in a high school curriculum to teach students how to evaluate diagnostic testing (Sedlmeier & Gigerenzer, 2001).

What about Bayesian reasoning with children? Zhu and Gigerenzer (2001) contend that they show, for the first time, that Bayesian reasoning can be educed in children. The researchers constructed seven Bayesian problems that were all presented in a frequency format. Frequency format means to express a probability as "_____ out of _____." Gigerenzer and Hoffrage (1999) claim that using this format elicits correct reasoning for Bayesian problems. This is the format I choose to use for probabilities in my study.

The participants in Zhu and Gigerenzer's study (2001) were Chinese children in grades four, five, and six. I used these same grade levels in my study. Results indicated that the transition age for children using Bayesian reasoning is around 10 or 11. The children in fourth grade gave answers that indicated correct Bayesian reasoning in 17% of the problems, the fifth graders applied correct reasoning in 25% of the problems, and the sixth graders reasoned correctly 70% of the time. Critics insist that the method elicits correct answers in only particular types of problems using a patterned approach and that Bayesian reasoning is not really occurring (Lewis & Keren, 1999). In spite of the criticism, the study shows that with the right kind of instruction children can learn to solve problems based on Bayesian reasoning.

Hawkins and Kapadia (1984) suggest that classroom instruction should be based on subjective rather than theoretical probability. They propose that to build a good framework for children developing probabilistic reasoning, subjective as well as theoretical and frequentist approaches should be utilized. Hawkins and Kapadia recommend research on teaching techniques that take into account children's intuitive notions of probability while developing formal knowledge of probability. According to Gigerenzer (2002) "the time is ripe for an educational campaign aimed at teaching schoolchildren, undergraduate and graduate students, ordinary citizens, and professionals how to reckon with risk" (p .230). Shaughnessy (1992) states:

> As we encounter new stochastic challenges, either mathematical or educational, our current set of stochastic models proves inadequate; a new paradigm for thinking about probability will have to evolve (p. 494).

Perhaps subjective probability should be incorporated into the new paradigm. When teaching probability and statistics to children, we do not encourage them to explore judgment under uncertainty and make good decisions rather we offer them a static definition of probability. Is our curriculum making our students good decision-makers in the face of uncertainty? Exploring probability from the subjective theory might produce new insights into teaching and learning in this area of mathematics.

Conclusion

For the last 30 years, a large amount of the research on learning probability is a consequence of the theory that people employ certain heuristics which produce specific "misconceptions" about probabilistic events. The studies were based on the traditional approach to probability and include examining "misconceptions" as well as testing strategies to overcome them. Strategies for overcoming these misconceptions suggest the use of hands-on experimentation, games, computer simulations, and analogies.

The theory of subjective probability has not been considered in these studies about misconceptions and overcoming them. The idea of probability as a degree of belief was discovered by de Finetti and Ramsey and has not been a part of traditional school mathematics. Recent research in subjective probability suggests it might be closer to the way children think. Hawkins and Kapadia (1984) argue that children can have probabilistic intuition, which is subjective, from the time they are very small. These authors also state that schools are often responsible for discouraging these emerging probabilistic conceptions by applying incorrect cognitive strategies that are in conflict with subjective interpretation. All of this research impacted my study in various ways such as the structure and content of the MDCQ, the grade levels involved, the lesson objectives, and the instructional methods of the lessons. This leads to the purpose of my study, which is to experiment with the inclusion of subjective probability in the mathematics curriculum.

CHAPTER 3

METHODOLOGY

The purpose of this study was to experiment with the inclusion of subjective probability in the curriculum, as well as to describe student reasoning patterns about probabilistic situations. The two research questions were: "Is there a mean difference in performance in applying probability between students who received instruction in traditional probability and those who received instruction in both traditional and subjective probability?" and "What are the salient themes that emerge from students' explanations about situations involving chance?"

In this chapter I will discuss the choice of a mixed methods design, describe the characteristics of data sources and provide the professional development plan. This chapter also contains an explanation of the intervention and instrumentation, details of the procedures and timeframe, and the particulars of the data analysis.

Research Design

The research design for this study was mixed methods which extended the breadth of the research by confirming findings from different data sources. An integrated use of method allowed for conclusions based on themes and statistical data. The quantitative and qualitative data were collected concurrently and had equal priority (Creswell, 2003).

According to Johnson and Onwuegbuzie (2004) the five major purposes for conducting mixed methods research are (a) triangulation, (b) complementarity, (c) initiation, (d) development, and (e) expansion. Of these, triangulation and development were vital to this study. Following, I will describe how each of these was used.

Triangulation is a technique that looks for convergence of results from different methods. A coding scheme was used to explore reasoning strategies children used when making decisions in uncertain situations that were subjective in nature. The data were emergent and descriptive as I looked for themes in students' reasoning from class observations, interviews, teacher journals, and test item responses.

Development used the findings from one method to inform the other (Johnson & Onwuegbuzie, 2004). A Multivariate Analysis of Variance (MANOVA) and a Bayesian t-test were used to compare learning outcomes from both subjective and traditional probability to learning outcomes in only traditional probability. Student written explanations to the multiple choice items on the pretest/posttest and student responses on lesson activities were used to substantiate these quantitative data.

Integration of the two types of data occurred at two stages through "mixing" (Creswell, 2003). During data collection the multiple choice answers on the pretest / posttest were scored for analysis for the MANOVA, but the reasons provided were coded for themes. The codes were transformed into numbers and compared as well.

A mixed methods designed was used for this study because the research problem incorporated the need to both explore and explain students' reasoning in light of subjective probability. It was not sufficient to only test the students understanding of probability, but was also necessary to investigate their reasoning patterns through their explanations.

Data Sources

The data sources for this study were students from across the fourth, fifth, and sixth grades, their teachers and myself as researcher. I will describe the characteristics of the participants in each of these groups in the following sections.

Students

The student participants were fourth, fifth and sixth graders attending a Catholic pre-kindergarten through eighth-grade school in the Southeast. The school was a National School of Excellence with 278 students who came from diverse ethnic backgrounds. There was one section of each grade in the school. The ethnicity and gender of the students are provided in Table 3.

Teachers

The lessons were taught by the fourth-, fifth- and sixth-grade teachers who were all Caucasian females. Each teacher conducted the study with the experimental and control groups from their homeroom. The fifth- and sixth-grade teachers taught mathematics every day, however the fourth-grade teacher did not teach mathematics. The 51 year old fourth-grade teacher had been teaching for 21 years and had a pre-kindergarten through eighth grade certification. The fifth-grade teacher, 59 years old, had 20 years of teaching experience and a middle grades certification with a concentration in math and science. The 47 year old sixth-grade teacher had 19 years of teaching experience and a certification in middle school science.

I was the 47 year old female researcher and was a teacher at the same school as the students and the homeroom teachers. A doctoral student with 26 years of teaching experience, I was certified in middle and secondary mathematics.

Table 3

Percent of Students across Ethnicity and Gender

Grade	n	Black	Caucasian	Asian	Hispanic	Female	Male
Four	28	71	18	11	0	39	61
Five	28	54	25	14	7	71	29
Six	31	39	39	16	6	42	58
Total	87	54	28	10	8	51	49

In summary, the fourth-, fifth- and sixth-grade students were data sources for the first research question that compared performance between those students who received instruction in traditional probability and those who received instruction in both subjective and traditional probability. The teachers per grade, me as researcher, and all 87 students were the data sources for the second research question concerning the themes of the student explanations.

Professional Development

I conducted professional development with the teachers in the study to teach the teachers the lessons and curriculum concepts before they taught the students. The details of the professional development program are outlined in Table 4. The teachers involved in the study had no previous experience with teaching or learning subjective probability. The fifth-grade and sixth-grade teachers had taught traditional probability, but the fourth-grade

Table 4

Professional Development for Teachers

Purpose	To prepare the middle grades teachers to teach both traditional and subjective probability.
Learner Outcomes	Understand the different philosophies of probability. Summarize educational research in the area of probability. Become comfortable with the lesson plans.
Required Reading	Albert, J.H. (2003). College Students' Conceptions of Probability. *The American Statistician*, 57(1), 37-45.
Session Activities	Review lesson plans for the research study. Observe researcher teaching a probability lesson. Discuss required reading.
Timeframe	These activities began five weeks prior to the start of the sessions with the students. They occurred once a week and lasted one hour per session.

teacher had not. Activities included discussion of readings, observing me teach the seventh-grade students, and solving sample problems.

Intervention

A random number generator assigned the students in each of the three grades to the control or experimental group. The control groups received instruction using the traditional curriculum objectives for probability in fourth, fifth, and sixth grades (NCTM, 2000). The lesson plans and activities for all the grade levels were the same. The experimental group received instruction in traditional probability concepts but was also taught a parallel concept in subjective probability. The amount of time spent on the lessons was equal for both groups, therefore the experimental group received less examples and activities on traditional concepts since they were also engaged in subjective probability.

Lessons were based on objectives from the local school curriculum and the NCTM Principles and Standards (2000). The probability objectives for the five lessons were (a) more likely, less likely, equally likely; (b) sample space; (c) finding probability; (d) finding the probability of additive events; and (e) using data to predict probability. There were other objectives for the sixth grade which were not included in this study so that the same lessons could be taught to all three grade levels. Appendix A contains the lesson plans for the control group and Appendix B contains the plans for the experimental group.

The teacher began each lesson with a whole class discussion of the concept. During the professional development the teachers received an outline for the class discussions that included definitions necessary to understand the concepts, example problems, and questions to prompt student discussion. This discussion was typically 10 minutes. An example of the class discussion for Lesson 1B, the subjective lesson, is as follows:

> "If I were to come into the room and pick a student to run an errand, is it more likely that I would choose a girl or a boy?" Let the students respond. If they make comments such as, "You would pick a girl, because you usually pick Caitlin". Allow the discussion to follow this path. Explain that sometimes there is a situation where a person's knowledge or past instances of similar situations influence what that person believes is the probability that an event will occur.

Both control and experimental groups received instruction in traditional probability using coins, spinners and dice. The problems, activities and discussions were very objective in nature. The worksheets for the activities for the control group can be found in Appendix D. For example, an activity for Lesson 1:

> Look at the spinner with the areas marked 1, 2, 3, 4, 5, 6 and circle the choice you think is correct for each of the following. On the line below the statement, explain your answer.

> Getting a 4 on a spin is (more likely) (equally likely) (less likely) than getting a 6.
> Getting a 5 on a spin is (more likely) (equally likely) (less likely) than getting a 3.

The subjective component of the study for the experimental group involved assigning probability to events based opinion, prior experience, or knowledge about the event involved. The activities were about weather, betting that an event might occur, and assigning probability based on the information provided. The worksheets for the activities for the experimental group can be found in Appendix E. A sample activity from Lesson 1B for the experimental group is:

> Look at the weather map. Use the key and the symbols on the map to help you decide which phrase is the best choice for each statement then explain why you choose your answer.
> (a) It is (more likely) (less likely) to rain in Seattle than in Arizona.
> (b) It is (unlikely) (very possible) that the high in Boston will be 80 degrees Fahrenheit today.
> (c) It is (even-chance) (very likely) that it will snow in Maine today.

The experimental group activities were composed of one-half of the same traditional activities as the control group, as well as additional activities involving subjective probability. The intention of the intervention was to test whether or not the inclusion of subjective probability in the elementary school curriculum would produce a difference in performance between the groups, as well as to look for themes in student explanations about situations involving chance.

Instrumentation

Data were gathered from four instruments: (a) pretest and posttest, (b) researcher observations from the lessons, (c) teacher journals, and (d) researcher interviews with students. The use of multiple instruments strengthened reliability and internal validity, and provided answers to the two research questions. The data analyzed from the pretest-posttest, which is called the Making Decisions about Chance Questionnaire (MDCQ), were used to answer the first research question and the data from researcher observations, teacher journals, student interviews, and the MDCQ explanations was used to answer the second research question.

Making Decisions about Chance Questionnaire

The pre- and post-test was an instrument called pre- and post-MDCQ, which can be found in Appendix C. The structure of the MDCQ was similar to an instrument used by Albert (2003) in a study that suggested mathematicians use the interpretation of probability that is determined by the nature of the task. In Albert's study, college students in an introductory statistics class were asked to make an intelligent guess to answer nine probability problems and explain how the probability was obtained. The 75 students were presented with probability problems that Albert labeled classical type problems, frequency type problems, and subjective type problems. The MDCQ questions are different from Albert's and the frequency and classical problems are combined in a category called traditional. The instrument used in my study had 20 questions, 10 were labeled subjective and 10 were labeled traditional.

The MDCQ was designed to evaluate students on five probability objectives that were common to the fourth-, fifth-, and sixth-grade curricula. There were four questions on more likely / less likely, four questions on sample space, four questions on finding the probability of an event, four questions on finding the probability of additive events, and four questions on using data to find probability. Of the four questions in each lesson, two questions involved traditional probability situations and two questions involved subjective situations. All 20 questions were multiple choice requiring a written explanation. Three of the subjective questions involved betting, since one component of measuring the coherence of subjective probability is to determine how much a person is willing to risk.

Observational Notes

During the lessons I made descriptive and analytical notes on the teachers' introduction to the lesson and the students' comments as they participated in the activities. I was an observer, however since the lessons were part of the students' curriculum, I addressed any inconsistencies or incorrect statements by the teachers in terms of probability theory. Therefore, I also served as a facilitator of the lessons. This was necessary because the teachers were expected to be able to teach the lesson objectives so that the students had a chance to learn.

I made recorded notes about the teachers' introduction to the lesson, student answers to the teacher's introductory questions, and reactions of the teacher to the student answers. As the students carried out the activities, I recorded group comments and questions. My notes were coded and organized categorically as themes emerged.

Teacher Journals

The teachers were asked to keep a journal containing their thoughts on the professional development, lesson plans, lessons, and activities. The teacher journals were descriptive notes with no standard format or content. The teachers were asked to record any comments that they made to students in their own classrooms concerning the probability activities when I was not present. The journals served as an opportunity for the teachers to express their feelings, as well as to note any significant student comments and reactions made during the lessons.

Student Interviews

I conducted individual interviews with two participants from each grade level, one from the experimental group and one from the control group. Interviews were held after the post-MDCQ. These students were selected based on their preliminary scores from the MDCQ. The students chosen were those who showed improvement in reasoning or gave interesting reasons for their answers. They were not necessarily the students with the best or worst score differences on the MDCQ. During the interviews, the students were asked to explain their answers in greater detail. The purpose of these data was to determine the characteristics of the reasoning skills of students.

In summary each of the four instruments was used to answer one of the two research questions. The data analyzed from the MDCQ were used to answer the research question "Is there a mean difference in performance in applying probability between students who received instruction in traditional probability and those who received instruction in both traditional and subjective probability?" The data from researcher observations, teacher journals, student interviews and the MDCQ written answers were used to answer the question "What are the salient themes that emerge from students' explanations about situations involving chance?"

Procedures/Timeframe

The first activity was professional development I conducted for the teachers. The professional development sessions occurred one hour a week for five weeks. The day after the conclusion of the professional development, the teachers gave all students the pretest in their homerooms. There was no time limit for completing the pretest. There were five 45-minute lessons taught over a seven day period for both the experimental and control groups. Students were assigned to the experimental or control group using a random number generator. The teachers taught the control and experimental groups of their own grade level. They taught the control group in the morning period and the experimental group in the afternoon period. Due to the nature of the school schedule it was necessary that the time periods for the groups were consistent.

The lessons were taught in the mathematics and science lab where each student sat with a partner at a table. The lessons were introduced by the homeroom teachers and sample problems were done on the board with the whole group. The students then worked with partners on the activity based lessons. The pairs of students were provided with a worksheet, dice, coins, cups and chips, a weather map, and a spinner. The teacher and I circulated among the pairs of students, facilitating the activities and insuring that they followed directions. The five lessons were presented sequentially within a seven day period. There were two school days when the classes did not meet. At the completion of all five lessons, on a separate day, the homeroom teachers administered the MDCQ as a posttest to the students. I interviewed two students from each grade level, one from the experimental group and one from the control group. The procedures and timeframe are summarized in Table 5.

Data and Data Analysis

Using a mixed method design for this study incorporated the strengths of both quantitative and qualitative methodologies. The research questions called for a need to

Table 5

Procedures and Timeframe

Activity	Participants	Timeframe
Teacher journal entries	Homeroom teachers	First day of professional development Week 1 Day 1
Five professional development	Taught by researcher To homeroom teachers	Once a week for five weeks One hour each session Weeks 1-5
Pretest	Homeroom teachers administer To their students	First day of research with students Unlimited amount of time Week 6 Day 1
Division of each grade into control and experimental groups	Random number assignment By researcher To students by grade level	Assigned upon completion of pretest Week 6 Day 1
Five Student Lessons	Taught by homeroom teachers To control and experimental groups of their classes Researcher observes	45 minutes for control group in a.m. 45 minutes for experimental group in p.m. 5 out 7 consecutive school days Week 6 Day 2,3,4 Week 7 Day 2,3
Posttest	Homeroom teachers administer To their students	Day after lesson five Unlimited amount of time Week 7 Day 4
Student interviews	By researcher Two students from each grade	Day after posttest Week 7 Day 5
Teacher journal completion	By homeroom teachers	Within a week after posttest Week 8 day 5

experiment on learning subjective probability and explore student's explanations for their answers. I will discuss the quantitative and qualitative data and analysis separately.

Quantitative Data

Quantitative data were comprised of the MDCQ scores. The pre- and post-MDCQ were scored and coded blindly. A scoring rubric for the MDCQ had two parts. A question received one point for a correct answer and one point for a coherent, reasonable explanation. Incorrect answers and incoherent reasoning for a question received a score of zero. Therefore the total score for a question could be zero, one, or two. The MDCQ can be found in Appendix C.

The MDCQ scores were analyzed using a two-way Multivariate Analysis of Variance (MANOVA) to test the difference of the means of the pre-MDCQ and the pos-MDCQ. The research hypothesis was:

> H0: There are no significant differences in performance in applying probability between students who received instruction in traditional probability and those who received instruction in traditional probability and subjective probability.

The within-subject effect tested was time. The between-subject effects tested were group and grade. Descriptive measures for groups as well as grade levels were used to provide additional information about the data.

A Bayesian t-test was conducted on the differences in the means of the experimental and control groups for the pre-MDCQ and the post-MDCQ. Two different versions were of the t-test were run using macros for Minitab. One program used an approximation for the posterior distribution of the means (Berry, 1996). The other program was based on simulating from the actual posterior distribution, assuming noninformative priors on parameters. Since the basis for this study was subjective probability, using Bayesian methods for data analysis was appropriate.

Qualitative Data

The qualitative data for the study consisted of my notes, teacher journal entries, MDCQ responses, and student interviews. The qualitative data were analyzed at the same time they were collected. I took random notes, recorded "snap shots" of student conversations, and looked for themes between grade levels and lessons. The teachers recorded their evaluations, feelings, and observations about the lessons at the conclusion of each day in a journal. There was no specific structure for the journal entries. The journals were kept electronically and I manually analyzed the entries for themes. Student interview responses were also coded for themes.

The MDCQ was used as both quantitative and qualitative data. The open-ended written responses were coded for themes, but also assigned a point value for explanations. The qualitative coding scheme for the MDCQ involved a code for whether or not the answer was correct based on the reason given, as well as a code for whether the explanation used was traditional or subjective. The coding scheme is summarized in Table 6. If the reason was based on the traditional ratio definition of probability it was coded as traditional. If the reason for the answer was an opinion based on prior knowledge or experience, then it was coded as subjective. The answer for a subjective reason could be considered correct even if it would not be considered correct by traditional theory.

As an example, the second question on the MDCQ is as follows:

You toss a fair penny one time, are you
a) Equally likely to get heads as tails?
b) More likely to get heads than tails?
c) More likely to get tails than heads?

The traditionally correct answer would be choice a, because there are two sides to the coin and it is fair. However, if students answered with choice b, explaining that they

Table 6

Qualitative Coding Scheme

Code	Answer	Reason
AT	Correct	Traditional
AS	Correct	Subjective
IT	Incorrect	Traditional
IS	Incorrect	Subjective
I	Incorrect	No reason
U	Incorrect	Left Blank
N	Correct	No reason

had tossed coins and in their experience the coin lands on heads more than tails, then subjectively it would be considered correct. Basing an answer on past experiences is a valid reason for a subjective answer. The codes in Table 6 were used for the MDCQ answers, activity worksheets, class discussion answers, and student interview responses. In addition to the codes, I looked for patterns in the student explanations for their answers on the MDCQ, student activity worksheets, and the student interview responses. I recorded the explanations which were common among students.

In summary, the qualitative data analysis consisted of coding the student interview responses, teacher journal entries, researcher observations, student activity worksheets, and the MDCQ for emerging themes. Quantitative data analysis consisted of a repeated measures MANOVA test for significance, descriptive measures of the pre- and post-MDCQ, and a Bayesian t-test.

Summary

The data sources were fourth-, fifth-, sixth-grade students, teachers per grade, and myself. The treatment, or intervention, for the study were lessons that contained only concepts in traditional probability for the control group and lessons that contained both subjective and traditional probability for the experimental group. The data were the MDCQ scores, MDCQ written responses, teacher journal entries, researcher observational notes, and student interview responses.

I conducted five professional development sessions with teachers. All students then took the MDCQ as a pretest. Over the next two week period, teachers conducted five 45-minute classes with each group, within grade level. I made observational notes during the lessons. The teachers kept a journal throughout the study. At the completion of the teaching unit, the students took the MDCQ as a posttest.

The MDCQ scores were analyzed using an MANOVA and a Bayesian t-test. The MDCQ written responses were coded for themes. Constant comparative analysis was used throughout the study to look for themes as I observed the classes, read the teacher journals, read student explanations on the MDCQ, and conducted student interviews.

CHAPTER 4

RESULTS AND DISCUSSION

This was a mixed methodology study. The qualitative data, which informed the quantitative, were analyzed when collected. Therefore in the first section of this chapter the qualitative data will be presented and discussed simultaneously. The quantitative data were collected and then analyzed later. In the second section of this chapter I will share the quantitative data then talk about those results separately.

Qualitative Data

The qualitative data were student responses to Making Decisions about Chance Questionnaire (MDCQ), researcher observations of lesson implementation, teacher journal entries, and student interview responses. These data provided insight into the research question, “What are the salient themes that emerge from students’ explanations about situations involving chance?” These data also supported the quantitative findings for the research question “Is there a mean difference in performance in applying probability between students who received instruction in traditional probability and those who received instruction in both traditional and subjective probability?”

Analysis of the qualitative data produced four themes: (a) fourth-grade students have difficulty with concepts of probability, (b) traditional questions are easier to answer then subjective questions, (c) students have subjective thoughts about chance events, and (d) misconceptions commonly labeled in research appear to be subjective judgments.

Each of these themes emerged from various data sources. Excerpts from teacher journals as well as researcher observational notes indicated that fourth grade students had significant difficulties with probability concepts. The finding that traditional questions are easier than subjective questions was supported by coded responses from the MDCQ, teacher journal responses and researcher observations of lessons. Student responses to lesson worksheets, researcher observational notes, student interview responses, and coded student explanations from the MDCQ were used to determine that students come in with subjective thoughts. Coded responses from the MDCQ suggested that commonly labeled misconceptions appear to be subjective judgments. Each of these four themes and the data that supports them is discussed in the following sections.

Fourth-Grade Students and Probability

The quantitative data suggested that there was a confusion effect among the fourth-grade students. These data will be discussed at length later in the chapter. The fourth grade was the only grade where the experimental group did not perform better than the control group on the post-MDCQ. The fourth-grade teacher made comments in her journal concerning the make-up of the experimental group and her interaction with them in the classroom. Some of those comments are summarized below.

> I think Group B (experimental group), even though they were randomly picked, had more trouble than Group A (control group) with oral directions, and in the lab had more difficulty following procedures, rules, and struggled with the activities. I don't know if the make-up of Group A is sharper or more attentive than Group B (experimental), or if the time of day that they are taking part in the research had any bearing on it. (Group A morning / Group B afternoon).This group had a lot more trouble when it came to the questions on the data collected. They couldn't seem to understand what the questions were asking and how to give the answer. They had a lot of trouble applying any of the data from today. Also, the probability information from previous days didn't help them.

Concerning her own command of the subject matter, the fourth-grade teacher also commented on her reservations about the material. The first teaching day of the study she wrote in her journal:

> I was not nervous, but felt a little inadequate because probability is foreign to me. The students, however, felt comfortable with my explanation that led into the activity sheet. They worked in pairs and all of them completed the entire sheet.

I noted in my observations that the teacher insufficiently described probability the first day. Students were not clear on what a unit on probability might cover. One student said it meant to estimate something and another offered that it meant to sort things out. The teacher replied that probability was making a deduction based on facts, such as a logic problem. I added that it involved reasoning but that probability dealt with the chance that something would occur.

As the lessons progressed, I helped facilitate the fourth-grade lessons with the teacher rather than just observe. Although this was a research project, it was also a teaching unit for the students and it was necessary that they comprehend, at least, the traditional concepts that were required of the curriculum. The teachers and I met daily to debrief and discuss the next day's lesson. After Lesson 3 for the experimental group, the fourth-grade teacher noted in her journal:

> I personally, felt a better understanding of this project, but some of the concepts are still challenging for me to totally understand. I do feel that teaching it makes it easier to understand.

Given these circumstances with the fourth grade, there is support for removing the fourth-grade data and considering the data of the fifth and sixth grades separately. The teacher's inexperience with the subject matter, the make-up of the groups, the students' lack of previous exposure to probability, as well as their age, are variables that could have influenced the outcomes of this study. Quantitative data related to this theme will be discussed in the next section.

Findings in other research studies corroborate that fourth-grade students are not prepared for certain abstract probability concepts. Zhu and Gigerenzer note that fourth graders applied Bayesian reasoning correctly only 17% of the time, as compared to 25% for fifth graders and 70% for sixth graders. Fischbein and Gazit (1984) reported that some of the concepts introduced during their study were too hard for even fifth graders. The study performed by Fischbein and Schnarch (1997) did not include fourth grade students, the youngest were fifth graders.

Traditional Probability

Comparing the students' performance on the subjective questions to the traditional questions shows that they consistently did better on the traditional questions and found them easier. Albert (2003) had a similar finding on a survey he gave to college students. He found that students performed better on the theoretical probability questions than either the subjective or experimental questions. The results from the MDCQ in my study, student answers on activity worksheets, and entries from the teacher journals supported this finding.

On the fifth-grade post-MDCQ only one student, from control group A did as well on the subjective questions as that student did on the traditional questions. On the sixth-grade post-MDCQ, four students did as well or better on the subjective items as they did on the traditional items. Two of these students answered the same number of each type correctly, one from group A and one from group B. The other two students, one from A and one from B, answered more of the subjective questions correctly than the traditional. The remainder of the students answered more traditional questions correctly.

The data in Table 7 show that the traditional questions on the MDCQ were answered correctly more often then the subjective questions on both the pre- and post-MDCQ. The fifth-grade teacher stated in her journal that the use of a pattern or formula "seems to reassure the students." While students came into the study with subjective thoughts about chance situations, questions requiring coherent, rational subjective answers were more difficult for them to answer. Traditional questions were more familiar and easier to get right, especially since the fifth-and sixth-grade students had been exposed to traditional probability in previous years.

Students from the experimental group sometimes tried to apply the ratio definition of probability in subjective situations where it did not make sense. For example, during Lesson 3 (see Appendix E), the students were asked the following question:

> Consider the statements:
> Bet 1: You get $100 if it rains on July 4th this year at your home and nothing if it does not.
> Bet 2: You get $100 if you get HH on the coins in #3 and nothing if you do not.
> Which bet do you take and why?
> What probability do you assign to the situation in Bet 1? ______ out of ______
> Why?

The majority of the students assigned a probability of 1 out of 31 to the situation in Bet 1 because there are 31 days in July and July 4th is only one day. In spite of the class discussion for assigning subjective probabilities based on experience and information, students incorrectly applied a traditional ratio definition. This type of response indicates an unwillingness to be subjective after learning traditional probability. Similarly, Albert (2003) found that students sometimes believe that a probability was not valid unless found by using computation.

Table 7

Fifth- and Sixth-Grade Answers by Type

Question Type	Number Correct Pre-MDCQ	Number Correct Post-MDCQ
Subjective	212	327
Traditional	450	507

The teachers in the study found subjective probability more challenging than traditional, especially since they had no previous experience with the topic. The fifth-grade teacher noted in her journal that

> There is a student activity that is written-up that is intended to be helpful for contrasting the classical and subjective interpretations of probability. Sorry to say I did not understand it, therefore, I did not find it helpful. I was able to mark all group A unit tests but due to the subjective answers of Group B tests on the last four questions I turned them over to the researcher for her evaluation.

The fourth-grade teacher stated that kids have trouble thinking “outside the box”. She thought that students wanted to have a simple method for every activity and did not want to express answers in written form to explain them. This also indicated that a traditional ratio answer was easier for the students.

According to the Ramsey-DeFinetti theory of subjective probability, one can measure the degree of belief a person has about a probabilistic situation by how much they are willing to risk. Therefore betting is often used as a measure of risk in subjective probability. However, the written explanations that students made on the MDCQ as well as responses during the activities indicated that children are not willing to bet money. On the MDCQ only 7 of the 87 students were willing to bet on questions about Marcus Giles getting a hit, or choosing two people with the same birthday (see Appendix C).

Explanations for not betting during the class lessons included: (a) betting is not nice, (b) my parents do not let me bet, (c) I only bet when I am sure, and (d) I don't want to lose my money.

During an interview with a student from the experimental group, the fifth-grade student said it would depend on how much to bet and that $5 was too much. A fourth-grade student stated that he would only bet if he was100% sure of the outcome. Therefore it was impossible to measure the students' personal belief about a situation using a bet involving money. This aspect of subjective probability was also more difficult than traditional probability.

Coming in with Subjective Reasoning

The observational notes on the class discussions, student explanations on the MDCQ, and student comments on the lesson worksheets indicated that students have subjective thoughts about probabilistic situations based on their past experiences or some intuition. Students at all grade levels in both the control and experimental groups continuously made comments about probability judgments that did not pertain to traditional mathematics, but instead to their personal beliefs. Some examples from the lessons follow.

The fourth-grade students had no previous experience with probability in mathematics class. Therefore, each lesson was a first-time exposure to the concepts. The fourth-grade teacher introduced the first lesson to the control group by asking the class, "If I came into the room and choose a student to run an errand, would I be more likely to pick a boy or a girl?" Student responses were:

> You would pick a boy because boys are stronger
> You would pick a boy because boys are faster
> You would pick a boy because I am a boy

You would pick a boy because the girls already have jobs

It was obvious that students had opinions about the likelihood of the event occurring that were not based on logical reasoning but instead on their past experience in situations where a student was picked to run an errand. The fourth-grade responses from the treatment group were similar:

A boy because the girl's hair might get messed up
A girl-no reason
A girl because boys don't usually behave well
It could be anybody

Only the last response indicated that personalities of boys and girls would not influence who was picked for the errand. The fifth-grade class had some instruction in probability in the fourth grade. However, most of their responses to the question about choosing a boy or girl from the class were similar to those of the fourth-grade students:

A girl because there are more girls in the class
A girl because girls are more responsible
Maybe someone near the front
A girl may pick a girl
A random person could pick anybody

The one response that referenced randomness implied that a person unfamiliar with the class might pick anybody because that person would not have knowledge of the differences among students. The response of "someone near the front" suggests environmental conditions were also considered a factor in making decisions. The comment that referenced the number of students was clearly a traditional answer. However, the majority of the comments were based on opinions.

The sixth-grade students had lessons in traditional probability in both fourth and fifth grades, as well as hands-on lab experience with probability experiments. The sixth-grade teacher began the Lesson One discussion by asking for the definition of probability.

A student responded that it is the amount of getting something out of a certain number, obviously the traditional ratio definition. However, another student stated that while probability made sense, since you can pick something your way it is never really random. This statement implies that the student knew that the traditionally correct answer was to use an approach of randomness and a traditional ratio for the answer. Yet, he did not believe that was the way the situation would occur in reality.

All of the teachers directed the students in the control group away from the subjective viewpoint by telling them not to base their answer on anything from their past experience. The students in the experimental group were encouraged to express their beliefs about the situations and base their decisions on these beliefs.

Continuing with the class discussion for Lesson One each teacher asked the class "If you were to toss a coin one time would you be more likely to get a head or a tail?" The intention of this question was to see if student responses would be less subjective in a situation that did not involve personalities. In the fourth-grade, student responses were:

> The chances are equal
> It depends on how hard you flip it
> You could flip it at an angle so the chances are not equal
> There is an equal chance for heads and tails
> I usually start the coin on tails and then flip it so it falls on heads
> The heads side weighs more so it is more likely to be heads

In the fifth grade, the class agreed that the outcomes of heads and tails were equal. However, during the interview, a fifth-grade student from the control group said that if you toss a fair coin it would more likely come up heads because when she and her brother flip a coin, she usually got heads. Even after a unit in traditional probability this was still a belief for some students.

If a situation was more familiar to a student, meaning they had personal experience with the events involved, they tended to make judgments that were subjective. For example, when asked who the teacher would pick from the class to run an errand, one fifth-grade student commented, “She would pick Caitlin because she knows she is responsible.”

During the introduction to Lesson Three, teachers asked the students a question concerning the probability of whether the Dodgers or the Braves would win a playoff game in baseball. All of the student responses were subjective in nature, assigning probability based on who was pitching, which player had a good batting average, and especially which team had historically won in the series. No student in any grade or group suggested that the probability was 1 out of 2 because there are two outcomes, win or lose.

Students’ responses on the MDCQ also indicated that having more information about a situation influences students’ judgments. Following is a question from the MDCQ, which is requires students to use the information provided to make a decision:

> Linda is 31 years old, single, outspoken and very bright. She loves mathematics. A survey of 100 people who love mathematics there are 24 accountants, 26 secretaries, 48 engineers and 2 unemployed people. You have to take a bet on what Linda’s job is. Do you
>
> a) bet $20 that Linda is an accountant,
> b) bet $20 that Linda is a secretary,
> c) bet $20 that Linda is an engineer, or
> d) refuse to bet?

As previously mentioned, the choices were about betting because measuring a person’s subjective probability can be based on how much they are willing to risk. On this particular question, more students were willing to bet than on the questions involving Marcus Giles batting or on choosing two people with the same birthday (see Appendix C).

Perhaps the increase in number of students willing to bet on the "Linda" question is due to the amount of information provided about the situation.

The noteworthy data for the "Linda" problem were the reasons students wrote for their choices. The traditionally correct answer would be that Linda is an engineer because there are more engineers in the survey. Of the 56 students in the combined fifth and sixth grades, only 10 gave this answer and correctly explained the reason. There were, however, 25 students who provided a subjective answer that was rational and coherent and could be considered correct in subjective theory. It seems that many students knew that accountants were involved with mathematics and choose answer *a* for that reason. Some students choose answer *b* due to Linda's personality and noted that secretaries needed to be bright and outspoken. No student mentioned that engineers need to be good at mathematics or choose c for this reason. Accepting answers *a* and *b* as correct does not adhere to traditional probability theory, yet the students were making a judgment based on the information provided and their knowledge of the events.

Analyzing the data from the following question on the MDCQ indicates that in particular situations students overwhelming use traditional reasoning:

2. You toss a fair penny one time are you

a) Equally likely to get heads as tails?
b) More likely to get heads than tails?
c) More likely to get tails than heads?

Explanation: ____

On the post-MDCQ, 67 of the 87 students choose *a* and explained that there were only two outcomes and they were equally likely. There were a total of nine students who used subjective reasoning and choose either *b* or *c* based on their experience. A student from the experimental group choose a, but then explained that it actually depended on which side you started the coin on. Even though that student knew the correct traditional answer, a personal belief was used as a reason.

By traditional theory, one would answer the question about the Braves in the same manner as one would answer the coin question since there are two possible outcomes. The majority of the students, no matter which group they were in, answered the Braves question using their beliefs and the coin question using a traditional ratio. This seemed to be because they had knowledge about the baseball situation.

Students not only came with subjective thoughts, they were able to learn the basic concepts of subjective probability even though the traditional questions seemed easier. The combined fifth- and sixth-grade experimental group showed an increase of 68% from the pre-MDCQ to the post-MDCQ on the subjective items. These students learned both subjective and traditional probability in the same amount of time that the control group learned only traditional.

In summary, it appears that students think subjectively based on their past experiences with chance. These thoughts include opinions about environmental conditions, luck, and past experiences. While we traditionally teach only a traditional ratio method of making judgments, students have personal beliefs in situations of uncertainty and often use them to make decisions. Students tend to use subjective reasoning more in situations where they have experience or more information.

Commonly Labeled Research Misconceptions are often Subjective Judgments

Garfield and Ahlgren (1988) made the following statement that encapsulates the thoughts of many scholars concerning student learning of probability:

> Students' intuitive ideas, presumably formed through their experience, may be reasonable in many of the contexts in which students use them but, can be distressingly inconsistent with the statistics concepts that we would like to teach them (p. 238).

However, the “concepts we would like to teach them” refers to traditional probability and statistics concepts. In this section I will examine student answers that are considered misconceptions in traditional probability, but are acceptable in subjective theory.

The following question from the post-MDCQ can be answered correctly in two ways depending on the definition of probability being used.

> Suppose that you toss a coin 20 times and get 19 heads and one tail. If you toss the coin one more time, do you think are
> a) more likely to get heads
> b) more likely to get tails
> c) equally likely to get heads as tails

A traditional answer would be c since there are two outcomes and they are assumed to be equally likely. However 23 of the 48 students from the combined fifth- and sixth-grades choose *a* and explained that this choice was made based on the information provided, that the coin was turning heads more than tails. They believed that if the coin had been coming up heads, it was more likely to continue to do so. In subjective probability this would be a rational, coherent answer and would be considered correct.

The question from the MDCQ previously discussed about Linda’s profession is also an example of a situation where an answer determined using an opinion would be considered wrong in traditional probability. Yet, students repeatedly based their decision on what they knew, which was that accountants need to be good in mathematics and secretaries need to be outgoing. These are coherent answers that are correct under subjective reasoning.

Giving students experience with probability, especially hands-on activities, allows them to build their knowledge and update their beliefs. DeFinetti (1974) says all probability is subjective, even theoretical and frequentist probability. The belief used to make a decision could be based on mathematical knowledge acquired from formulas or experimentation. Just because a student has not had enough experience with a situation, misinterprets a question, or has a subjective opinion, does not mean they are operating under a misconception.

Data from the question 12 on the MDCQ supports several of the themes described in this chapter:

> What is the probability that the Atlanta Braves will win a baseball game against the New York Yankees? _______ out of ______

This question can be answered correctly from a traditional viewpoint with the answer of 1 out of 2 by reasoning that there are two possible outcomes, win or lose. However, the question also elicits subjective reasoning based on a person's knowledge about the teams and players involved, the teams' current records, or the history of the teams who are playing. Because the students in the study live in the city that is home to one of these teams, the question was put on the MDCQ with the intention of provoking a subjective answer.

Of the 87 students in the study, 39 provided the traditionally correct answer of 1 out of 2 on the post-MDCQ with the explanation that the Braves could win or lose. However, 32 of the 87 gave a ratio answer that was rational and coherent because it was based on past knowledge of the teams. Most interesting was the fact that in the sixth grade, 13 students gave traditionally correct answers and 17 gave subjectively correct answers. Therefore, more of the sixth graders used subjective reasoning in this situation regardless of which group they were in.

Out of the 59 students in the combined 5th and 6th grades, there were 30 who changed their answers on this question from the pre-MDCQ to the post-MDCQ. The majority of the changes, 18 of 30, were made by students from the experimental group. Only four of the 18 changed to correct traditional reasoning by answering “1 out of 2”. The other 14 students from the experimental group changed from either correct traditional reasoning or an incorrect answer to correct subjective reasoning. The change from correct traditional to correct subjective reasoning is an indication that students who learned subjective probability felt that using it was more appropriate for this question than a theoretical probability answer. A summary of this data can be found in Table 8.

The 12 students from the control group who changed their answers on this question from the pre-MDCQ to the post-MDCQ did so in a variety of ways. There were six students who answered correctly with “1 out of 2” and two students who gave a correct subjective answer. However, there were four students who changed their correct subjective answers on the pretest to incorrect traditional ratios on the posttest. This change suggests that once traditional probability has been learned, students are reluctant to employ the subjective ideas which they brought to the study. This also indicates that they used a traditional ratio incorrectly rather than give a subjective probability answer.

In summary, I have described four themes that emerged from the explanations students provided about their reasoning in probabilistic situations. First, the subjective probability concepts were too abstract and confusing for most of the fourth-grade students. Second, traditional questions were easier to answer then subjective questions for the

Table 8

Changes to Question 12 from Pre-MDCQ to Post-MDCQ

Group	Change to Subjective	Change to Traditional	Change to Incorrect	Total Changes
Control	2	6	4	12
Experimental	14	4	0	18
Total	16	10	4	30

students. Third, students bring subjective thoughts based on experiences and opinions to probability situations. Finally, some probability "misconceptions" appear to be subjective judgments. These findings provide insight into the research question "What are the salient themes that emerge from students' explanations about situations involving chance?"

Quantitative Data

Quantitative Results

The quantitative results were based upon the MDCQ scores of the 87 students. The MDCQ was used as a pretest and posttest and contained 20 questions that were curriculum concepts in probability at the fourth-, fifth-, and sixth-grade levels. The five lessons were on (a) equally likely, (b) sample space, (c) probability, (d) additive probability, and (e) making decisions based on data. The MDCQ was composed of two traditional questions and two subjective questions from each of these five areas, for a total of twenty questions. In this section I will provide the quantitative data results for all students, for students by grade level, and for the combined fifth- and sixth-grade results.

The specific findings for this section will be done with the following hypothesis:

> H0: There are no significant differences in performance in applying probability between students who received instruction in traditional probability and those who received instruction in traditional probability and subjective probability.

The hypothesis was tested using a repeated measure, two-way MANOVA to test the means of the control and treatment groups on the pre- and post-MDCQ, as well as the means of the grade levels. The significance level was p =.05 for all tests. The within-subject effect of time showed that the difference from pretest to posttest was highly significant for all subjects. See Table 9 for the MANOVA results. This result indicates that all students, regardless of group or grade, performed significantly better on the post-MDCQ after the instructional program.

The between-subjects factors of group and grade were also tested. The effect of grade was highly significant (see Table 9). The data showed that sixth-grade students performed significantly better than the fifth-grade students who performed significantly better than the fourth-grade students.

A between-subjects effect tested the research hypothesis that there was a difference between the means of pre- and post-MDCQ for the students in the control group and the experimental group due to the treatment. The effect of group was not significant, p=.27 (see Table 9). The average score of the experimental group did increase more than the control group. Table 10 gives the mean and the per cent increase of both groups.

Looking at the results by grade-level fourth was the only grade in which the experimental group subjects did worse than the control group. The MANOVA shows that there is no difference due to treatment. Table 11 provides the results

When comparing the mean MDCQ scores of the fourth grade, both groups start the same, however the treatment group does not do as well as the control group on the post-MDCQ. Table 12 provides this information

Table 9

Multivariate Analysis of Variance Fourth-Sixth Grade

Source	df	F	p
		Within Subjects	
Time	1	107.438	.000*
Time*grade	2	000.463	.631
Time*group	1	000.259	.612
Time*group*grade	2	000.932	.398
		Between Subjects	
Grade	2	15.764	.000*
Group	1	1.235	.270
Grade*group	2	0.196	.823

Note. N=87
* value rounded to three digits

Table 10

Means Fourth-Sixth Grades

Group	Pretest Mean	Posttest Mean	Difference	% Increase
Control	21.51	28.61	7.10	33
Experimental	19.70	28.02	8.32	42

Note N=87

Table 11

Multivariate Analysis of Variance for Fourth Grade

Source	df	F	p
		Within Subjects	
Time	1	20.067	.000*
Time x Group	1	.303	.587
		Between Subjects	
Group		.475	.524

Note N=28
* value rounded to three digits

Table 12

Mean for Fourth-Grade

Group	Pretest Mean	Posttest Mean	Difference	% Increase
Control	17.142	26.286	9.144	53
Experimental	16.929	24.071	7.142	42

Note N=28

. In the fifth grade, the treatment group starts lower and finishes almost even, however the difference is not significant. Table 13 contains the MANOVA results for the fifth grade. Examining the mean values and per cent increase, the experimental group for the fifth grade did show a greater increase than the control group in the fifth grade (see Table 14). The fifth grade had three subjects whose scores were identified as outliers due to a pretest score of eight or below. If these outliers are removed, then it can be seen that the experimental group began lower than the control group, but finished higher (see Figure 1).

Table 13

Multivariate Analysis of Variance for Fifth Grade

Source	df	F	p
		Within Subjects	
Time	1	44.459	.000
Time x Group	1	1.821	.190
		Between Subjects	
Group	1	. 44.308	.428

Note. N=28

Table 14

Means for Fifth-Grade

Group	Pretest Mean	Posttest Mean	Difference	% Increase
Control	22.308	27.462	5.154	23
Experimental	19.154	26.923	7.769	41

Note. N=28

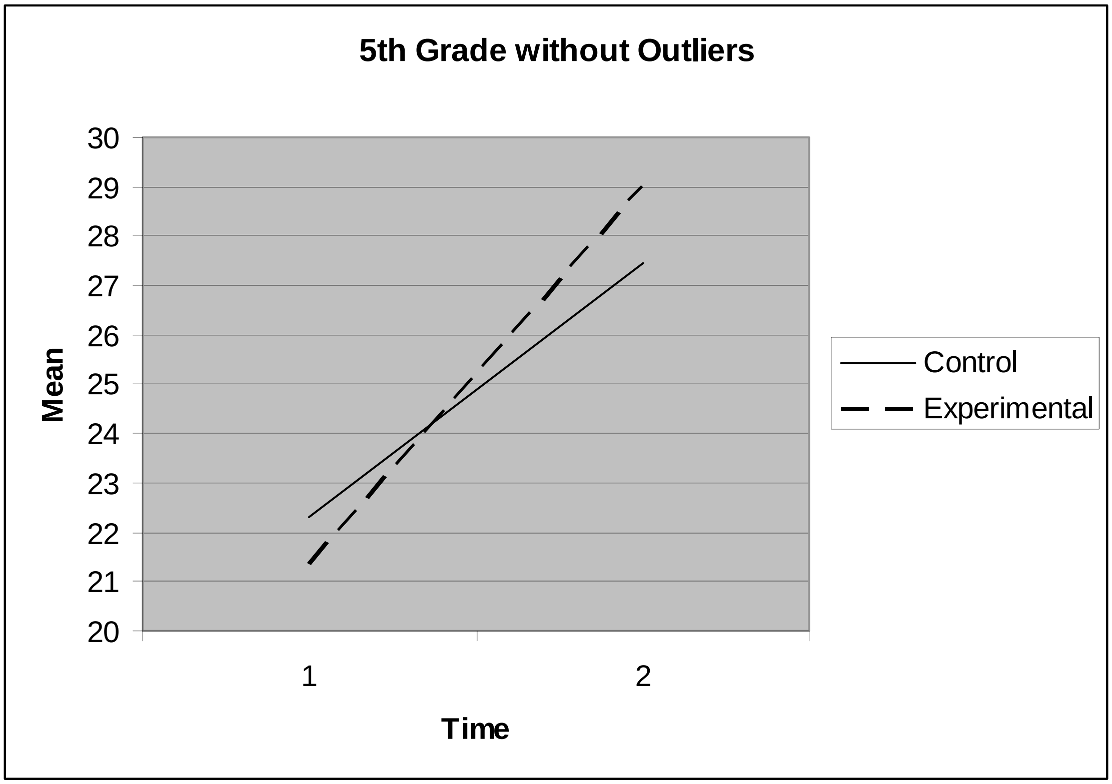

Figure 1. Means for Fifth-Grade

In the sixth grade, the treatment group started lower, but finished higher than the control group. The MANOVA results show that the difference was not significant, p=.265 (see Table 15).

Examining the mean values of the experimental and control groups for the sixth grade shows that the experimental group had the greatest increase from pre to post-MDCQ of all grade levels and groups in the study. These results are provided in Table 16. A graph of the mean values for sixth grade illustrates the difference in the performance of the control and experimental group which can be found in Figure 2. The experimental group had a lower average on the pre-MDCQ, but finished with a higher average on the post-MDCQ.

Table 15

Multivariate Analysis of Variance for Sixth Grade

Source	df	F	p
		Within Subjects	
Time	1	122.385	.000
Timex Group	1	1.289	.265
		Between Subjects	
Group		0.811	.058

Note N=31

Table 16

Means for Sixth-Grade

Group	Pretest Mean	Posttest Mean	Difference	% Increase
Control	23.333	31.000	7.667	33
Experimental	22.688	32.938	10.25	45

Note N=31

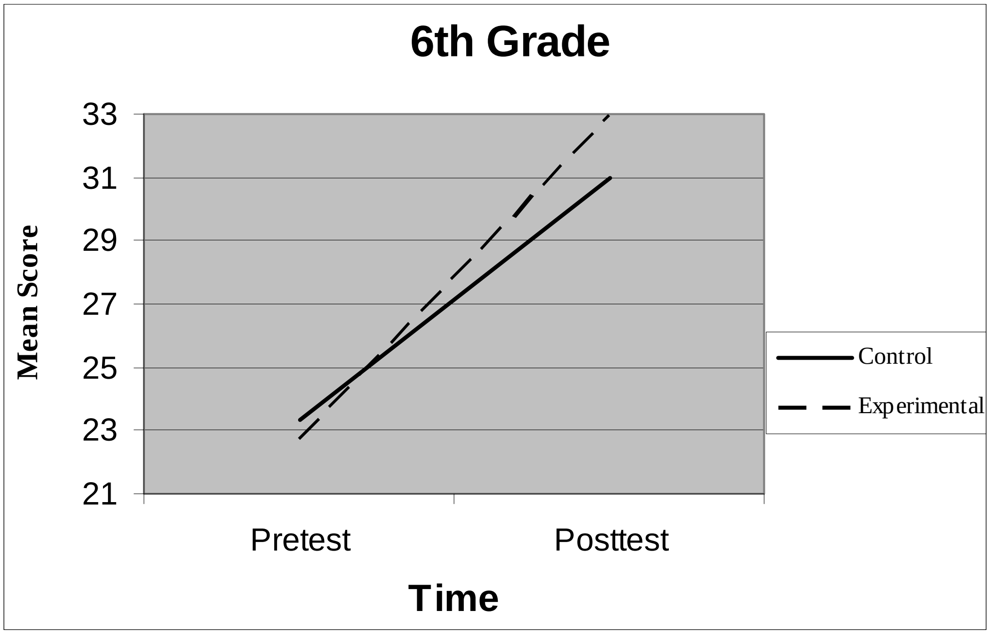

Figure 2. Means for Sixth Grade

If only the fifth- and sixth-grade subjects are considered, and three outliers are removed from the data set, the difference in the means is not significant, p=.096. In this case, the treatment group started at a lower average score on the pre-MDCQ and actually finished higher than the control group on the post-MDCQ. The MANOVA results are provided in Table 17.

An inspection of the means for the fifth- and sixth-grade groups reveals that the experimental group has a pre-MDCQ mean that is lower than that of the control group, but they have a post-MDCQ mean that is higher. Table 18 provides this information and Figure 3 shows this in a graph.

Table 17

Multivariate Analysis of Variance for Fifth and Sixth Grades

Source	df	F	p
		Within Subjects	
Time	1	136.123	.000
Time x Group	1	2.87	.096
		Between Subjects	
Group	1	.00	.989

Note. N=56

Table 18

Means for Fifth- and Sixth-grade

Group	Pretest Mean	Posttest Mean	Difference	% Increase
Control	23.537	29.607	6.07	26
Experimental	22.571	30.679	8.108	36

Note. N=56

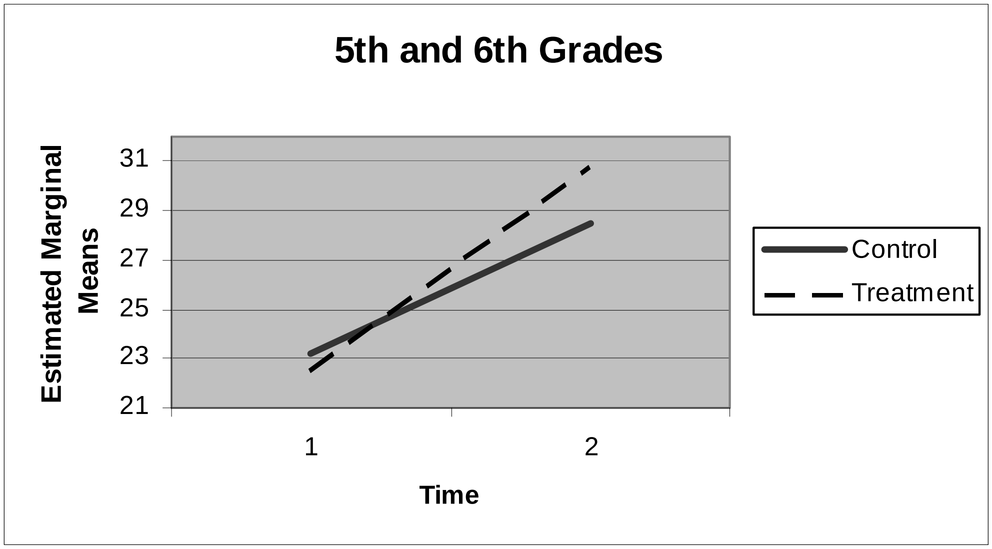

Figure 3. Means for Fifth- and Sixth-grades

A Bayesian t-test was conducted for the data from the combined fifth and sixth grade using Minitab. The results were produced using Minitab and a program called mm_cont that uses Berry's (1996) approximation for the posterior distribution of the difference of the means for the control and experimental groups. The mean difference for the control group was 6.07 and the standard deviation was 4.14, n=28. The mean difference for the experimental group was 7.78 with standard deviation 4.48, n=27. The posterior density for the difference was Normal, with a mean difference of 1.71 and standard deviation of 1.2. The experimental group showed greater improvement. The posterior for the difference of the means of the two groups is approximately normal (1.71, 1.2). The probability that the difference of the means is greater than zero is the probability N (1.71, 1.19) exceeds zero which is .925. Therefore, there is reasonable evidence to believe that mean of the experimental group exceeds the mean of the control group.

Discussion of Quantitative Results

The students in the experimental group received the same amount of instructional time on each lesson, 45 minutes, as the students in the control group. However they received instruction in both traditional and subjective probability. In spite of the fact that the experimental groups learned two objectives for each lesson in the same amount of time that the control group only learned one, they did as well, or better than, the control groups in all grades except fourth.

The fourth-grade groups had similar performance on the pre-MDCQ, however the experimental group did not finish as well. There are several factors that might account for this. Subjective probability is more abstract by nature than traditional probability. Perhaps fourth-grade students are not intellectually prepared for the subjective concepts in a formal sense, although they certainly have subjective ideas about probabilistic situations.

Another factor that might have influenced fourth-grade performance could be the teacher. The fourth-grade teacher is the only teacher in the study who does not teach mathematics each day. As discussed in the qualitative section, there could be a confusion effect for the fourth-grade students due to the nature of the material, the teacher, or because the control group was composed of lower achieving students.

Because of these variables in the fourth grade, these subjects were removed from the data set in order to analyze fifth- and sixth-grade scores. The students in fifth and sixth grades had more experienced teachers, had previous exposure to probability concepts and were, of course, older. There were three outliers with pretest scores of less than eight that were identified in this set and removed. The results were still not significant at $p < .05$, however the statistic p = .096 does suggest that with a larger sample size it is possible that significance could be attained.

The most interesting result of the fifth- and sixth-grade group is the fact that the experimental group began lower and finished higher. The resulting graph (see Figure 3) shows the crossover interaction. This also occurs with the fifth-grade students alone (see Figure 1) and the sixth-grade students alone (see Figure 2). The important point of this is that the lower scoring experimental group overtakes the high achieving control group. According to Cook and Campbell (1979), this type of result is often more interpretable than other outcomes of control group design. This disordinal interaction combined with a p=.096 suggests that additional research is warranted. Given larger sample sizes it is possible that the interaction would be significant. A Bayesian t-test with normal priors also provides reasonable evidence to believe that there was some difference between the groups.

Summary

Analysis of data indicated that students have subjective thoughts about situations involving chance. Some of thoughts were answers that would be correct in subjective probability theory but have been labeled misconceptions by researchers in traditional probability. Students in grades four, five, and six found traditional probability easier than subjective. Subjective probability concepts were too difficult for most children younger than fifth grade. There was no significant difference between the means on the pre- and post-MDCQ for the control and experimental groups. However, a crossover interaction for the fifth and sixth grades suggested that further research with larger sample sizes might provide significance. A Bayesian t-test provided evidence to believe that there is a mean difference in performance in applying probability between students who received instruction in traditional probability and those who received instruction in subjective and traditional probability.

CHAPTER 5

SUMMARY, RECOMMENDATIONS, AND CONCLUSIONS

In this chapter I will summarize the study and discusses the findings in an attempt to evaluate the inclusion of subjective probability in the middle grades mathematics curriculum. The limitations of the study will be discussed in conjunction with the recommendations and conclusions.

Summary

The purpose of this study was to experiment with the inclusion of subjective probability in the curriculum, as well as to describe student reasoning patterns about probabilistic situations. A total of 87 students in grades four, five, and six from a small, suburban Catholic school participated in a teaching experiment conducted by their respective teachers.

The research design was a mixed methods study. The quantitative component of the design involved an experimental-control group with a pretest and posttest. The pretest and posttest were an identical questionnaire called Making Decisions about Chance Questionnaire (MDCQ). The data from the MDCQ were analyzed using a Multivariate Analysis of Variance (MANOVA) and a Bayesian t-test. The qualitative component included data collected from teacher journal entries, researcher notes on the observation of lessons, student interview responses, and student responses on the MDCQ. These data were analyzed and coded for themes using constant comparative analysis.

The first research question was “Is there a mean difference in performance in applying probability between students who received instruction in traditional probability and those who received instruction in both traditional and subjective probability?” This question was answered based on student scores on the pre- and post- MDCQ.

A two- way Multivariate Analysis of Variance showed that the difference in means of the pre- and post-MDCQ scores was significant (p = .00) for the effect of time for all students. The effect of grade was also significant (p = .00). The sixth-grade students performed better than the fifth-grade students, who performed better then the fourth-grade students.

In the fourth grade, the experimental and control groups began at about the same level on the pre-MDCQ, but the control group finished with a higher mean. Due to variables such as lack of confidence of the teacher and the age of the students, I decided to examine the fifth- and sixth-grade results separately from the fourth grade.

The fifth-grade experimental group had a lower mean on the pre-MDCQ than the control group, but finished with a higher mean on the post-MDCQ. A MANOVA showed that the difference was not significant, but there was a slight effect (p = .19). In the sixth grade, the experimental group began with a lower MDCQ score, but had a higher post-MDCQ score, finishing about 5% ahead of the control group. A MANOVA showed that there was no significance for group (p = .27) in the sixth grade.

There were three students in the combined fifth and sixth grades who had scores of 8 or below on the pre-MDCQ and these data were considered outliers. Dropping these scores, the mean difference between the pre- and post-MDCQ was at a p = .096 level (n=56). In this case the experimental group had a lower pre-MDCQ average and finished with a higher average than the control group on the post-MDCQ. The crossover interaction can be seen in Figure 3. This type of result provides evidence that the difference between the control and experimental groups is noteworthy and warrants further research.

A Bayesian t-test indicated that the experimental group showed greater improvement. The posterior for the difference of the means was approximately normal and the probability that the difference is significant is .925. I believe this is reasonable evidence that the treatment was effective. While including subjective probability in the elementary school curriculum after fourth grade did not produce a significant difference in the means, there was evidence that further research is necessary. Given more instructional time, teachers trained in subjective probability theory, or larger sample size, the difference in performance might be significant.

The second research question was “What are the salient themes that emerge from students’ explanations about situations involving chance?” To answer this question I analyzed the student MDCQ responses, researcher observations of lesson implementation, teacher journal entries, and student interview responses. There were four findings: (a) fourth-grade students have difficultly with concepts of probability, (b) traditional questions are easier to answer then subjective questions, (c) students bring subjective thoughts concerning the chances of events to probability situations, and (d) misconceptions commonly labeled in research appear to be subjective judgments. I will summarize each of these themes.

It appears these fourth-grade students were not able to comprehend the abstract nature of subjective probability, or in some cases even traditional probability. Variables in this study with the fourth grade were the inexperience of the teacher and the experimental group makeup. Gigerenzer (2001) also found that the fourth-grade students appeared to be too young to comprehend some concepts of probability.

Traditional probability is easier for students than subjective probability. Both the control and experimental groups performed better on the MCDQ traditional items than the MCDQ subjective items. Teacher journal entries confirmed that applying a traditional ratio was easier for the students during the activities than applying subjective concepts. The teachers themselves found the traditional probability material easier to grasp than the subjective. Students sometimes incorrectly applied traditional ratios to subjective problems. Measuring the degree of belief using a betting situation was not effective as students were unwilling to bet unless they were completely confident that they would not lose money. This was confirmed by written explanations on the activities and the MDCQ, as well as some student interview responses.

Students have subjective thoughts about probability situations based on past experience and personal beliefs. Subjective probability is closer to students' beliefs than traditional probability. Students hold on to these beliefs even after learning traditional concepts and even after answering a question using the traditional ratio method. Students tend to use subjective reasoning if they have knowledge about the situation or are provided with more information, even if the question could be answered using traditional methods.

Educational research in probability and statistics for the last 30 years has been anchored in the theory that students reason incorrectly about probability and have common misconceptions. Evidence from student interviews, MDCQ explanations, and lesson activity responses indicated that reasoning considered "misconceptions" was actually beliefs based on experience with the situation or the information provided. If an answer is based on opinion, knowledge, or experience and is coherent and rational, then in light of subjective probability, it should be considered correct.

Recommendations

Before making suggestions for recommendations, it is important to note the limitations of this study. The teachers involved had no previous experience with subjective probability, therefore the material was challenging for them. The students were from a small school, where there was only one section of each grade and all students in each grade were included in the study. There were 87 students overall and only 56 in the combined fifth and sixth grades resulting in a small sample size for the control and experimental groups. Because of the lack of flexibility in the school schedule, the grade level teachers had to teach the control group in the morning and the experimental group in the afternoon.

I was a teacher at the school and taught the sixth-grade math course, as well as fourth- and fifth-grade labs. Although I was the researcher, I was also familiar with the students and their capabilities. During the interviews the students were somewhat unresponsive, perhaps because I was their teacher. They only wanted to respond with the "correct" answer, rather than explain their responses.

In addition to these limitations, I chose to use what is known in the literature as a frequentist representation of probability. All of the questions on the MDCQ and lessons asked that the probability be stated as "___ out of ____". I chose this representation because students in the middle grades often have difficulty with fractions, ratios, and per cent. As noted in the literature review, there is debate over which representation elicits correct probabilistic reasoning. Gigerenzer (1999) recommends using the representation "___ out of ___", so I consistently used this throughout the study.

With these limitations in mind, there are two different categories of recommendations. The first recommendations are for teaching subjective probability, and the others are recommendations for further research. The following recommendations assume that subjective probability would be a component of the middle and high school curricula beginning in the fifth grade. The curriculum would then contain three viewpoints of probability: (a) theoretical, (b) frequentist, and (c) subjective. Albert (2003) makes this recommendation following his research study with college students.

With regards to the teaching of subjective probability in the middle grades, the first recommendation is that students should be directed to validate subjective answers with reasons that are coherent and rational. Not just any answer is acceptable because the student believes it is correct. The answer must be based on experience and the information that the student possesses about the event in question. The strength of a student's belief must be measured by some wager, however that should not be betting money.

Secondly, teachers must direct students to recognize when a theoretical answer is not appropriate. This idea is tied to sample space. For example, the probability it will rain tomorrow is not 1 out of 2 just because it could rain or not rain. Raining or not raining is not necessarily the sample space for this problem, nor are those two possibilities necessarily equal. Therefore teachers need to provide examples of situations where sample spaces are more subjective and the outcomes are not equally likely. There must also be practice in determining sample space for subjective situations.

Albert (2003) asserts that the type of task should determine which method of probability to use. However, sometimes either a subjective or theoretical answer is appropriate. For example, when tossing a coin one time, the probability of heads is ½ according to theoretical probability. However, on a single toss of a coin, if a person believes that heads occurs more often because that is their past experience, or because they have reason to believe the coin is weighted, or for some other valid reason, then the answer might be different from ½ for subjective reasons.

The subject matter to which subjective probability refers is irrelevant. De Finetti (1974) provides examples such as election of a public official, winning the lottery, winning a game of chance, results of a criminal trial, gender of a child at birth, and the state of the weather. In all of these cases we express ourselves in numerical quantities. He asserts that in none of these examples is it possible to describe a situation in which the conditions are always the same. Considering the toss of a coin, a description of the circumstances would have to include how a person tosses, the air movement, the peculiarity of the ground, and so on. By changing any circumstance we obtain other events. This is, of course, pure subjective theory. By this theory, all tasks should be considered as subjective. However, for elementary and high school students, introducing three types of probability and using each viewpoint for a particular type questions is more practical.

The last recommendation for education from this study involves teachers. In order to teach subjective probability teachers must become familiar with the concepts. Since subjective probability has not been a part of the traditional curriculum, it is probable that few elementary and high school teachers are familiar with the concepts. Therefore, training for preservice teachers and professional development for current teachers is necessary.

While there is an abundance of research in probability and statistics, there is little research in subjective probability and education. There are six recommendations for further research in subjective probability.

1. Conduct a similar study with different age groups. It is clear from this study that fourth grade is too young for subjective probability. Studies in grades 7-12 concerning the teaching of subjective probability would be of interest.
2. Explore misconceptions from the research in light of subjective probability. Test representativeness, the conjunction fallacy, availability, and other heuristics from the subjective viewpoint of probability.
3. Investigate methods of measuring the strength of a belief and assigning probability based on this measure. This study showed that betting money was not an appropriate measure of belief for students, nor were students comfortable in assessing their belief in this manner.
4. Conduct research in the statistical counterpart of subjective probability, which is Bayesian statistics. Although there is some research in this area for adults, there is very little research on Bayesian theory and children.
5. Develop a new theoretical framework for teaching and learning probability that includes subjective probability and considers students intuitions and personal beliefs. Presently, research is focusing on the experimental and theoretical views of probability and how to teach them. Not only does subjective probability need to be included, but a framework encompassing all three views needs to be developed.
6. Explore student instruction in subjective probability in relation to learning theories. Which learning theory applies? As I analyzed data I came to see that students

learned some probability concepts in a context other than a school context. How did this learning occur?

7. This study reinforces research in conceptual theory from cognitive psychology that suggests that there is a mismatch between human reasoning and traditional probability theory (Gigerenzer, 1996; Wang, 1994). A conceptual framework that considers subjective judgments and heuristics people use to make decisions in the face of uncertainty needs to be developed.

Conclusion

People face decisions about uncertain events every day. Subjective assessments of uncertainty are an important element of making good decisions. For some purposes, subjective probabilities are more appropriate than either theoretical or experimental probability. Students are able to comprehend the basic concepts of subjective probability as young as fifth grade. It is time to consider exposure to subjective probability in the elementary, middle, and high school curricula.

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APPENDIXES

APPENDIX A

Lesson Plans for Control Group A

Lesson		1	2
Objective	Pretest	1. Students will use the words *less likely, more likely, equally likely, same chance, more of a chance, less of a chance* to describe situations involving probability. 2. Students will use manipulatives to model probabilistic events	1. Students will determine the sample space for simple probability experiments. 2. Students will list the sample space for compound events
Activities	Students will take the pre-MDCQ administered by the teacher	1**.** The teacher will introduce the lesson with a whole class discussion, and some explanation of the concepts and activities. 2. Working with their partners, students will decide which events are more likely for the spinner, cards, the coins and the chips as they complete the lab sheet.	1**.** The teacher will introduce the lesson with a whole class discussion, and an explanation and examples of sample space. 2. Working with their partners, students will find the sample space of events involving the spinner, chips, coins, and the dice as they complete the lab sheet
Materials	Copy of pretests Pencils	Spinners (equally marked with numbers 1-6) A nickel and a dime A deck of cards Student Lab sheets	Spinners-equally marked with numbers 1-6 A nickel and a dime A deck of cards A red and a green die Student Lab sheets

Lesson	3	4	5	
Objective	1.Students will assign a numerical value to probability events by expressing the probability as " the number of possible ways the event occurs out of the number of all possible outcomes	1. Students will use the additive rule to find the probability of events involving "or".	1. Students will predict the results of a probability experiment.	Posttest
Activities	**1.** The teacher will introduce the lesson with a whole class discussion, and an explanation and examples of finding numerical probability 2.Working with their partners, students will find the probability of events involving the spinner, cards, the coins and the chips as they complete the lab	1. The teacher will introduce the lesson with a whole class discussion, and an explanation and examples of events involving the addition rule. 2. Working with their partners, students will find the sample probability of events involving the spinner, cards, the coins and the chips as they complete the lab sheet.	1. The teacher will introduce the lesson with a whole class discussion of collecting data to predict an outcome of an experiment 2. Working with their partners, students will observe and record data, make predictions about the results of a random generating experiment. .	Students will take the post-MDCQ administered by the teacher
Materials	Spinners-equally marked with numbers 1-6 A nickel and a dime A deck of cards A cup with 2 red and 3 blue chips Student lab sheets	Spinners-equally marked with numbers 1-6 A nickel and a dime A deck of cards A cup with 2 red and 3 blue chips Student lab sheets	Teacher graphing calculator Overhead projector TI-83 overhead connector Student lab sheets	Posttest

APPENDIX B

Lesson Plans for Experimental Group (B)

	Objective	**Activities**	**Materials**
Lesson	Pre-MDCQ	Students will take the pretest administered by the homeroom teacher.	Copy of pretests Pencils
1	1. Students will use the words likely, more likely, equally likely to describe situations involving probability. 2. Students will choose from words used to describe the likeliness of an event and rank their meaning according to likelihood. 3. Students will use manipulatives to model probabilistic situations	Working with their partners, students will: 1. Decide which events are more likely for the spinner, the coins and the weather map 2. Decide which words are appropriate for more likely, equally likely, and less likely events	Spinners- equally marked with numbers 1-6 A penny and a dime Weather maps
2	1. Students will determine the sample space for simple probability experiments. 2. Students will find the sample space for subjective probability situations. 3. Students will find the sample space for compound events.	1. The teacher will introduce the lesson with a whole class discussion, an explanation, and examples of sample space. 2. Working with their partners, students will find the sample space of events involving the spinner, cards and the coins as they complete the lab sheet. 3**.** Students will complete the Lab Sheet questions on sample space for probability situations that are specifically subjective.	Spinners- equally marked with numbers 1-6 A penny Weather maps
3	1**.**.Students will assign a	1**.** The teacher will introduce	Spinners-

	numerical value to probability events by expressing the probability as " the number of possible ways the event occurs out of the number of all possible outcomes 2. Students will assign a numerical value to subjective probability situations that are based on their personal belief. 3. Students will use comparative probability to rank probability items.	the lesson with a whole class discussion, and an explanation finding the probability of events 2. Working with their partners, students will find the probability of events involving the spinner, cards, the coins, and the chips as they complete the lab sheet. 3. Students will complete the Lab Sheet questions for probability situations that are specifically subjective.	equally marked with numbers 1-6 A penny and a dime Weather maps Student lab sheets
4	1. Students will find the probability of events using the additive rule. 2. Students will assign subjective probabilities to events based on information they have.	1. The teacher will introduce the lesson with a whole class discussion, an explanation, and examples of how to find the probability of additive events. 2. The teacher will explain that when judging an event using personal probability, you cannot use the definition of frequentist probability in a nonsense way. She will give examples from their previous worksheets. 3. Students will complete the Lab Sheet questions finding the probability of situations that are specifically subjective.	Spinners- equally marked with numbers 1-6 A penny and a dime Weather maps Student lab sheets
5	1. Students will predict the results of a probability experiment.	1.The teacher will introduce the lesson with a whole class discussion, and an explanation and examples of using an experiment to predict the results 2. Students will observe and record data, make predictions about the experiment for generating random numbers	Teacher graphing calculator Overhead projector TI-83 graphing calculator overhead attachment Student lab sheets
	Posttest		

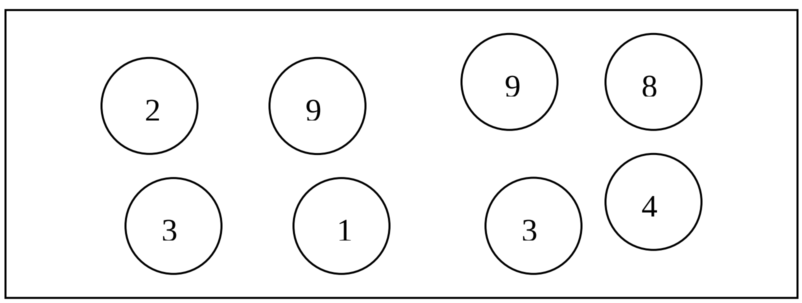
APPENDIX C

Making Decisions about Chance Questionnaire

For each of the following questions, make an intelligent guess for the answer by filling in the blank or circling the answer. Then write an explanation of how you obtained your answer.

More likely, less likely, equally likely

1. If you choose a ball from the box without looking are you more likely to choose a 4 or a 9?



Explanation: ____

____

2. You toss a fair penny one time are you

   a. equally likely to get heads as tails?

   b. more likely to get heads than tails?

   c. more likely to get tails than heads?

Explanation: ____

____

3. The Weather News magazine reports that it sometimes rains in Houston and frequently rains in Seattle. If you are traveling in the United States, which place is it more likely to rain?
   a. Seattle
   b. Houston
   c. Equally likely to rain in both cities

Explanation: ____

____

4. A sports announcer says that there is an even chance that Marcus Giles will get a hit the next time he bats for the Braves. Given this information would you:

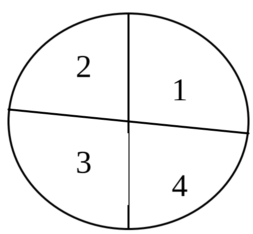
   a. bet $20 that he gets a hit next time?
   b. bet $20 that he does not get a hit next time?
   c. not take a bet about Marcus batting?

Explanation: ____

____

Sample Space

5. List the sample space for the spinner shown below. Assume the areas on the spinner are equal.



Sample Space: ____

Explanation: ____

5. You draw a card from the deck and look at the suit of the card.

Sample Space: ____

Explanation: ____

____

6. You choose a teacher from your school and ask their age.

Sample space: ____

Explanation: ____

____

7. You ask everyone in your class what time they wake up to get to school.

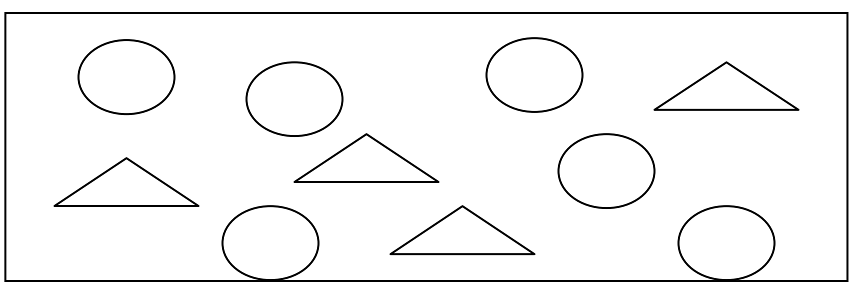
Sample space: ____

Explanation: ____

____ _______________

Assigning Probabilities

8. Suppose you choose an object at random from this box. What is the probability that you choose a triangle? _____ out of ______



Explanation: ____

10. Suppose you have a spinner with 4 equally marked spaces that are colored red, blue, white and black. If you spin the spinner one time, what is the probability that the arrow lands in the blue space?________ out of _______

Explanation: ____

____

11. A baby is born at the local hospital this morning. What is the probability that the baby is a boy? ____________ out of __________

Explanation: ____

____

12. What is the probability that the Atlanta Braves will win a baseball game against the New York Yankees? ________ out of _______

Explanation: ____

____

Additive Rule of Probability

13. A fair die is tossed one time. What is the probability that the face on the die shows a four or a six? __________ out of _________

Explanation: ____

____

14. You toss a penny and a nickel at the same time. What is the probability that at least one of the coins shows heads? ______ out of ______

Explanation: ____

____

15. You are asked to make an intelligent guess about the high temperature in Atlanta tomorrow, April 27. Do you think the probability that the high temperature will be between 70 and 80 is

a. 10 out of 10?

b. 8 out of 10?

c. 5 out 10?

d. none of these

Explanation: ____

____

16. Suppose forty people are in a room. You are asked to bet on whether or not any two people have the same birthday. Are you willing to bet

a. nothing?

b. $5 ?

c. $10 ?

d. $20 ?

Explanation: ____

____

Using Data to estimate probabilities

17. Suppose that you toss a coin 20 times and get 19 heads and one tail. If you toss the coin one more time, do you think are

a. more likely to get heads?

b. more likely to get tails?

c. equally likely to get heads as tails?

Explanation: ____

____

18. There is a bag that contains 10 chips that are either red or blue. You reach in the bag, draw a chip, and then put it back. You repeat this process 20 times. You get 16 blues and 4 reds. How many chips out of the 10 do you think are red?_______

Explanation: ____

____

19. Linda is 31 years old, single, outspoken and very bright. She loves mathematics. A survey of 100 people who love mathematics there are 24 accountants, 26 secretaries, 48 engineers and 2 unemployed people. You have to take a bet on Linda’s job is. Do you

    a. bet $20 that Linda is an accountant?

    b. bet $20 that Linda is a secretary?

    c. bet $20 that Linda is an engineer?

    d. refuse to bet.

Explanation: ____

____

20. In the Summer Softball League, the Hurricanes have beaten the Silver Bullets 4 out of 5 times. It is the championship game and they are playing again. What do you think the probability is that the Hurricanes will win? _____ out of _____

Explanation: ____

____

APPENDIX D

Activities for Control Group

Probability Worksheet Lesson 1A

More Likely, Less Likely, Equally Likely

Lab Rules: Stay in your group ____
Work Cooperatively ____
Stay on Task ____
Follow Directions ____

Activity 1
Take turns with your partner spinning the spinner 12 times each. Keep tally marks in the following chart to show which number is spun.

Number	1	2	3	4	5	6
Tally						

1. Look at the spinner with the areas marked 1, 2,3,4,5,6 and circle the choice you think is correct for each of the following. On the line below the statement, explain your answer.

a. Getting a 4 on a spin is (more likely) (equally likely) (less likely) than getting a 6

____

b. Getting a 5 on a spin is (more likely) (equally likely) (less likely) than getting a 3.

____

Activity 2

Shuffle the deck of cards. Take turns drawing a card from the deck until each person has drawn 10 times. Put an X in the column of the card that is drawn

Card	Club	Diamond	Heart	Spade
Ace				
King				
Queen				
Jack				
Ten				
Nine				
Eight				
Seven				
Six				
Five				
Four				
Three				
Two				

2. Look at the deck of cards. Spread them on your table and circle the answer you think is correct. On the line below the statement, explain your answer.

    a. Drawing a King from the deck of cards is (more likely) (equally likely) (less likely) than drawing a Club.

    ____

    ____

    b. Drawing a 5 from the deck is (more likely) (equally likely) (less likely) than drawing a Jack.

    ____

    ____

    c. Drawing a red card from the deck is (more likely) (equally likely) (less likely) than drawing an Ace.

    ____

Activity 3

Take turns as you and your partner toss both coins at the same time. Put an X in the column of the result the player obtained. Repeat until each player has tossed the coins three times.

Coin Result	Heads on both	Tails on both	Heads on penny Tails on nickel	Heads on nickel Tails on penny
Player 1				
Player 2				

3. Refer to the coins. Circle the answer you think is best if you toss both the penny and the nickel at the same time. Explain your answer on the line below the statement.

   a. Getting heads on both coins has (the same chance) (more of a chance) (less of a chance) as getting tails on both coins.

   ____

   ____

   b. Getting heads on the nickel and tails on the penny has (the same chance) (more of a chance) (less of a chance) as getting tails on both coins.

   ____

   ____

Activity 4

Do not look at the chips in the cup. Take turns drawing one chip out of the cup, then replace it until both people have drawn 5 times. Record your results in the table below.

Player	Draw 1 Color	Draw2 Color	Draw3 Color	Draw4 Color	Draw5 Color
1					
2					

4a. Empty the cup containing the chips and answer the following statements the chips by circling the answer you think is correct. On the line below the statement, explain your answer.

____

____

b. If you put the chips in the cup and choose one without looking, drawing a blue chip from the cup is (more likely) (equally likely) (less likely) than drawing a red chip.

____

____

Probability Worksheet Lesson 2A

Sample Space

Lab Rules: Stay in your group ____
Work Cooperatively ____
Stay on Task ____
Follow Directions ____

Activity 1

Look at the spinner. If you spin it, what are the possible outcomes? Discuss this with your group and make a list of these outcomes below. This is the sample space for the spinner.

Sample space : ____

Number of outcomes in sample space: _____

Activity 2

Look at the deck of cards. Spread them on your table. The sample space can be different depending on what you are interested in. List the sample space if the question you are interested in:

a. The suit of the card ____

   Number of outcomes in sample space: _____

b. The face of the card ____

   ____

   Number of outcomes in sample space: _____

Activity 3

a. Look at the penny. With coins the sample space refers to the side that the coin lands on if it is tossed. What is the sample space when you toss the penny one time?

____

Number of outcomes in the sample space: _____

b. Now refer to the nickel and the penny. The sample space for this experiment involves tossing both the nickel and the penny. This is an example of a **compound** event because two events are involved. Turn the coins, or flip them several times to help you find the sample space.

Nickel

Penny

	H	T
H		
T		

Sample space for tossing two coins:

____

Number of outcomes in sample space: _____

Activity 4

Another example of a compound event is tossing two die. You have red and green dice in your probability box. Turn the faces of the dice to help you find the sample space. Remember that means all the possible outcomes if you toss both dice. Try to make your list organized by thinking through all of the possibilities before you write them below. You can use the table to help you.

Green Die

Red Die		1	2	3	4	5	6
	1						
	2						
	3						
	4						
	5						
	6						

Number of outcomes in sample space: ______

Activity 5

The following examples have no manipulatives to help you. Think through the problems and write the sample space for each.

1. A box contains three blue chips and 2 red chips. What is the sample space if you choose a chip from the box?

____

Number of outcomes in sample space: _____

2. A box contains slips of paper numbered 1,2,3,4. You draw one slip. What is the sample space?

____

Number of outcomes in sample space: _____

3. The box contains slips of paper numbered 1,2,3,4. You put your hand in and draw out two slips at the same time. What is the sample space?

____

Number of outcomes in sample space: _____

Probability Worksheet Lesson 3A

Probability

Lab Rules: Stay in your group ____
Work Cooperatively ____
Stay on Task ____
Follow Directions ____

Probability (A) = Number of favorable out comes (out of) Number of possible outcomes

For each of the following situations, use the definition of probability provided above as well as your Worksheet from Lesson 2 to find the probability of the given event.

Activity 1

Look at the spinner. If you spin it, what is the probability that you get:

a. The number 3? __________out of ____________

b. An even number? __________out of ____________

c. A number greater than 4?__________out of ____________

d. The number 10? __________out of ____________

Activity 2

Look at the deck of cards. Spread them on your table. If you shuffle the cards then draw them from the deck, what is the probability you get:

a. A club? __________out of ____________

b. The jack of diamonds? __________out of ____________

c. A face card? __________out of ____________

Activity 3

Refer to the nickel and the penny. Recall the sample space from worksheet 2. You can use the table below for the sample space

Nickel

Penny

	H	T
H		
T		

If you toss both coins, what is the probability you get:

a. Heads on both coins? __________out of ____________

b. A head on the nickel and a tail on the dime? __________out of ____________

Activity 4

Another example of a compound event is tossing two dice. You have red and green dice in your probability box. You can use the table below to help you with the sample space.

Green Die

Red Die

	1	2	3	4	5	6
1						
2						
3						
4						
5						
6						

If you toss both dice, what is the probability that you get:

a. 5 on the red die? __________out of ____________

b. Doubles (the dice match) __________out of __________

c. 7 on the green die? __________out of ____________

d. A sum of 12 on the dice? __________out of _________

Probability Worksheet Lesson 4A

Additive Probability

Lab Rules: Stay in your group ____
Work Cooperatively ____
Stay on Task ____
Follow Directions ____

.

Probability (A) = Number of favorable out comes (out of) Number of possible outcomes

Probability (A or B) =
Number of favorable outcomes of A + Number of favorable outcomes of B
out of Number of possible outcomes in the sample space

For each of the following situations, use the definition of probability provided above to find the probability of the given event.

Activity 1

Look at the spinner. If you spin it, what is the probability that you get:

a. The number 2 or 3? __________out of ____________

b. An even number or an odd number? __________out of ____________

c, A number greater than 3 or a number less than 2?__________out of ____________

Activity 2

Look at the deck of cards. Spread them on your table. If you shuffle the cards then draw them from the deck, what is the probability you get:

a. A club or a space? __________out of ____________

b. The jack of diamonds or an ace of hearts? __________out of ____________

c. A ten or a queen? __________out of ____________

Activity 3

Refer to the nickel and dime. If you toss both coins, what is the sample space?

____

Find the probability you get:

a. Heads on both coins or tails on both coins: __________out of ____________

b. A head on the nickel and a tail on the dime or a head on the dime and a tail on the nickel: __________out of ____________

Activity 4

Refer to the nickel and the cup of chips. Suppose you toss the coin and then draw a chip from the cup. Use the table to help you find the sample space.

Chip

Coin

	Blue	Red
H		
T		

Sample Space____

1. What is the probability of getting a heads and a red chip? _____ out of _____

2. What is the probability of getting a heads and a red chip or a tails and a red chip? _____ out of _____

3. What is the probability of not getting a tails and a red chip?____ out of ____

Activity 5

The following examples have no manipulatives to use to help you. Think through the problems and write the probability for each.

1. A box contains three blue chips and 2 red chips. You draw one chip without looking. What is the probability you draw a blue chip or a red chip?

__________out of ____________

2. A box contains slips of paper numbered 1,2,3,4. You draw one slip. What is the probability you get a 3 or a 4?

__________out of ____________

3. You choose a student from this class to be on your team. What is the probability that it is Alexis or Jordan?

__________out of ____________

Number ________ Date________

Worksheet Lesson 5A

Deriving Probability from Data

The teacher will use the graphing calculator to generate random numbers. Copy the data into the table below.

Draw	N1	N2	N3	N4	N5	N6
1						
2						
3						
4						
5						
6						
7						
8						
9						
10						

1. Look at the data. What do you think the numbers are being randomly picked from?
____

2. Take a guess at the next 6 numbers to be picked, then below your guess write the numbers that occurred.

	N1	N2	N3	N4	N5	N6
Guess						
Actual						

	N1	N2	N3	N4	N5	N6
Guess						
Actual						

	N1	N2	N3	N4	N5	N6
Guess						
Actual						

	N1	N2	N3	N4	N5	N6
Guess						
Actual						

	N1	N2	N3	N4	N5	N6
Guess						
Actual						

	N1	N2	N3	N4	N5	N6
Guess						
Actual						

	N1	N2	N3	N4	N5	N6
Guess						
Actual						

	N1	N2	N3	N4	N5	N6
Guess						
Actual						

	N1	N2	N3	N4	N5	N6
Guess						
Actual						

	N1	N2	N3	N4	N5	N6
Guess						
Actual						

	N1	N2	N3	N4	N5	N6
Guess						
Actual						

3. Look at the data you collected.

a. Not considering the order of the numbers, what is the most numbers you matched?

____

b. Did you have a method of picking your numbers? If so, describe it below:

____

____

4. Watch the numbers as they are generated 50 more times. Write down any data you think is relevant. Answer the questions.

a. Do you think any of the numbers occur more than others? Why or why not?

____

____

b. What do you think would happen if we continued to generate the 6 numbers 1000 times?

____

____

c. What do you think the probability of getting a 15 when a number is drawn is?

____

Why? ____

d. Write anything else here that you observed or any other important mathematical ideas about this experiment.

____

____

____

____

____

APPENDIX E

Activities for Experimental Group

Probability Worksheet Lesson 1B

More Likely, Less Likely, Equally Likely

Lab Rules:	Stay in your group	____
	Work Cooperatively	____
	Stay on Task	____
	Follow Directions	____

Activity 1

1. For each of the pairs of words, circle the word that you believe indicates a **greater** chance of an event happening.

    a. Sometimes Very frequently

    b. Seldom Even-chance

    c. Unlikely Possible

    d. Always Very frequently

    e. Never Sometimes

2. Make a list of the following eight words *sometimes, seldom, possible, unlikely, very frequently, never, always, even-chance* with the most likely word at the top and the least likely word at the bottom.

Activity 2

Look at the weather map. Use the key and the symbols on the map to help you decide which phrase is the best choice for each statement. Explain why you choose your answer.

a. It is (more likely) (less likely) to rain in Seattle than in Arizona

____

____

b. It is (unlikely) (very possible) that the high in Boston will be 80 degrees Fahrenheit today.

____

____

c. It is (even-chance) (very likely) that it will snow in Maine today.

____

____

Activity 3

Shuffle the deck of cards. Take turns drawing a card from the deck until each person has drawn 10 times. Put an X in the column of the card that is drawn

Card	Club	Diamond	Heart	Spade
Ace				
King				
Queen				
Jack				
Ten				
Nine				
Eight				
Seven				
Six				
Five				
Four				
Three				
Two				

4. Look at the deck of cards. Spread them on your table and circle the answer you think is correct. On the line below the statement, explain your answer.

   a. Drawing a King from the deck of cards is (more likely) (equally likely) (less likely) than drawing a Club.

      ____

   b. Drawing a 5 from the deck is (more likely) (equally likely) (less likely) than drawing a Jack.

      ____

   c. Drawing a red card from the deck is (more likely) (equally likely) (less likely) than drawing an Ace.

      ____

Activity 4

Take turns as you and your partner toss both coins at the same time. Put an X in the column of the result the player obtained. Repeat until each player has tossed the coins three times

Coin Result	Heads on both	Tails on both	Heads on penny Tails on nickel	Heads on nickel Tails on penny
Player 1				
Player 2				

5. Refer to the coins. Circle the answer you think is best if you toss both the penny and the nickel at the same time. Explain your answer on the line below the statement.

   a. Getting heads on both coins has (the same chance) (more of a chance) (less of a chance) as getting tails on both coins.

   ____

   ____

   b. Getting heads on the nickel and tails on the penny has (the same chance) (more of a chance) (less of a chance) as getting tails on both coins.

   ____

   ____

Probability Worksheet Lesson 2B

Sample Space

Lab Rules: Stay in your group ____
Work Cooperatively ____
Stay on Task ____
Follow Directions ____

Activity 1

Look at the deck of cards. Spread them on your table. The sample space can be different depending on what you are interested in. List the sample space if the question you are interested in is as follows:

a. The suit of the card ____

Number of outcomes in sample space: _____

b. The face of the card ____

____

Number of outcomes in sample space: _____

Activity 2

The following "experiments" are more subjective in nature. Discuss each situation with your partner and make a list of reasonable outcomes for the sample space.

a. The age of a person in this grade_____________________________________

b. The year when a man will land on the moon again ________________________

c. The age of the teachers in this school ________________________________

d. The time it would take a person to walk a lap around the soccer field

____

Activity 3

Refer to the nickel and penny. With coins the sample space refers to the side that the coin lands on if it is tossed. The sample space for this experiment involves tossing both the nickel and dime. This is an example of a **compound** event because two events are involved. Turn the coins, or flip them several times to help you find the sample space. Use the table to help you.

Nickel

Penny

	H	T
H		
T		

Sample space for tossing two coins: ______________________________

Number of outcomes in sample space: _____

Activity 4

Another example of a compound event is tossing two dice. You have red and green dice in your probability box. Turn the faces of the dice to help you find the sample space. Remember that means all the possible outcomes if you toss both dice. Try to make your list organized by thinking through all of the possibilities before you write them below:

Green Die

		1	2	3	4	5	6
Red Die	1						
	2						
	3						
	4						
	5						
	6						

Number of outcomes in sample space: ______

Probability Worksheet Lesson 3B

Lab Rules: Stay in your group ____
Work Cooperatively ____
Stay on Task ____
Follow Directions ____

Probability Rules: Probability is a number between 0 and 1. The sum of the probabilities of all the outcomes for an event must be 1.

Traditional Probability (A):
Number of favorable out comes (out of)Number of possible outcomes

Personal Probability: degree of belief that an event will occur, measured by the amount you are willing to risk that it will occur

For each of the following situations, use one of the definitions of probability provided above to find the probability of the given event. You may discuss these in your group and use anything you need from the Probability Box.

Activity 1

A bowl contains 5 red and 5 blue chips. You reach in and draw a chip without looking. Find the probability that you:

a. Draw a blue chip: _____ out of _____

b. Draw a red chip: _______ out of ____

Activity 2

Look at the spinner. If you spin it, what is the probability that you get:

a. The number 3? _________out of ___________

b. An even number? _________out of ___________

c. A number greater than 4? _________out of ___________

d. The number 10? _________out of ___________

Coins

Activity 3

Refer to the nickel and penny. If you toss both coins, what are the possible outcomes? Fill in the table to help you decide.

Nickel

Penny

	H	T
H		
T		

Use the possible outcomes the find the probability of:

a. Heads on both coins? __________out of ____________

b. A head on the nickel and a tail on the dime? __________out of ____________

Activity 4

1. Consider the statements:

Bet 1: You get $100 if you draw a blue chip from the bowl in Activity 1 and nothing if you do not get a blue.
Bet 2: You get $100 if you get HH on the coins in Activity 3 and nothing if you do not.

Which bet do you take and why?

____

____

2. Consider the statements:

Bet 1: You get $100 if you draw an Ace of Hearts from the deck of cards and nothing if you do not.

Bet 2: You get $100 if the spinner in Activity 2 lands on the line and nothing if you do not.

Which bet do you take and why?

____

____

3. Consider the statements:

Bet 1: You get $100 if it rains on July 4 this year at your home and nothing if it does not.
Bet 2: You get $100 if you get HH on the coins in Activity 3 and nothing if you do not.

Which bet do you take and why?

____

____

What probability do you assign to the situation in Bet 1? ______ out of ______

Why?____

1. Consider the statements:

Bet 1: You get $100 if it rains on July 4 this year at your home and nothing if it does not.
Bet 2: You get $200 if it snows in December this year at your home and nothing if it does not.

Which bet do you take and why?

____

____

What probability do you assign to the situation in Bet 1? ______ out of ______

Why?
____

What probability do you assign to the situation in Bet 2? ______ out of ______

Why?____

Probability Worksheet Lesson 4B

Additive Probability

Lab Rules: Stay in your group ____
Work Cooperatively ____
Stay on Task ____
Follow Directions ____

Probability Rules: Probability is a number between 0 and 1. The sum of the probabilities of all the outcomes for an event must be 1.

Personal Probability: degree of belief that an event will occur, measured by the amount you are willing to risk that it will occur

Additive Rule: Probability (A or B) =
Number of favorable outcomes of A + Number of favorable outcomes of B
out of Number of possible outcomes in the sample space

Activity 1-Traditional probability

1. A bowl contains 3 red and 5 blue chips. You reach in and draw a chip without looking. Find the probability that you draw a blue chip or a red chip:

_____ out of _____

2. Look at the spinner. If you spin it, what is the probability that you get:

a. A 3 or a 4? __________out of ____________

b. An even number or 5? _________out of ____________

c. A number greater than 4 or a number less than 2?__________out of ____________

3. Refer to the nickel and dime. If you toss both coins, what are the possible outcomes? _________________________

Use the possible outcomes the find the probability of:

a. Heads on both coins or tails on both coins; _________out of ____________

b. A head on the nickel and a tail on the dime or a head on the dime and a tail on the nickel: _______out of ______

Activity 2- Assigning Personal probabilities

Use the knowledge you have about the situations involved to help you assign probabilities.

4. On Friday, the school is holding the Race for Education walk. Each student and each class will be raising money.

a. Rank the classes from 4-8 in the order of how much money you think they will raise, with the class raising the most money listed first.

____

b. Now assign probabilities to the question above based on your ranking. Remember the rules for probability.

4th Grade _____ out of 10
5th Grade _____ out of 10
6th Grade _____ out of 10
7th Grade _____ out of 10
8th Grade _____ out of 10

Explain why you assigned the probabilities as you did:

____

____

5. Consider the following information. You are tossing coins with your best friend. Whoever gets a head wins. You have played 9 times and your friend has won 5 times and you have won 4 times. How much money are you willing the bet that you win on the next toss and why? Circle the amount and explain on the line below.

a) No bet
b) Very little money
c) An average amount of money
d) A lot of money
e) All my money

____

6. Look at the weather map. You have to assign a probability to each of the following situations by using the information on the map. Use the probabilities you assign to each event to help you with the others by comparing the chances.
Write your explanation on the line beneath the question.

a. P (it will rain in Atlanta, Georgia) ____ out of _____

____

____

b. P (it will snow the next day in Idaho) ____ out of ___

____

____

c. P (there will be ice in California) ____ out of ____

____

7. Choose the probability for the following based on your knowledge and prior experience.

a. P (the principal will give students a day off tomorrow) ____ out of _____

____

____

b. P (Atlanta Braves will go to the World Series) ____ out of _____

____

____

c. P (a batter will get a hit if he/she did not get a hit last time he/she was up)

____ out of ____

____

____

Worksheet Lesson 5B

Deriving Probability from Data

The teacher will use the graphing calculator to generate random numbers. Copy the data into the table below.

Draw	N1	N2	N3	N4	N5	N6
1						
2						
3						
4						
5						
6						
7						
8						
9						
10						

1. Look at the data. What do you think the numbers are being randomly picked from?

____

2. Take a guess at the next 6 numbers to be picked, then below your guess write the numbers that occurred.

	N1	N2	N3	N4	N5	N6
Guess						
Actual						

	N1	N2	N3	N4	N5	N6
Guess						
Actual						

	N1	N2	N3	N4	N5	N6
Guess						
Actual						

	N1	N2	N3	N4	N5	N6
Guess						
Actual						

	N1	N2	N3	N4	N5	N6
Guess						
Actual						

	N1	N2	N3	N4	N5	N6
Guess						
Actual						

	N1	N2	N3	N4	N5	N6
Guess						
Actual						

	N1	N2	N3	N4	N5	N6
Guess						
Actual						

	N1	N2	N3	N4	N5	N6
Guess						
Actual						

	N1	N2	N3	N4	N5	N6
Guess						
Actual						

	N1	N2	N3	N4	N5	N6
Guess						
Actual						

3. Look at the data you collected.

a) Not considering the order of the numbers, what is the most numbers you matched?
____

b) Did you have a method of picking your numbers? If so, describe it below:

____

____

4. Watch the numbers as they are generated 50 more times. Write down any data you think is relevant then answer the questions.

a) Do you think any of the numbers occur more than others? Why or why not?

____

____

b) What do you think would happen if we continued to generate the 6 numbers 1000 times?

____

____

5. What do you think the probability of getting a 15 when a number is drawn is? _______

Why? ____

Do have any personal beliefs or feelings about any of the numbers?

____

____

____

How much would you be willing to bet on any 6 numbers that you can choose?

Why?

____

____

____

____

____