2007

What is Mathematics?: Teachers Exploring the Philosophy of Mathematics

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Abstract:

Background

Visions of effective mathematics teaching and learning are going through tremendous change (e.g., see Kilpatrick, Martin, & Schifter, 2003). The National Council of Teachers of Mathematics (NCTM) calls for changes to curriculum and evaluation (1989), teaching (1991), and assessment (1995). An accumulated summary of these changes is provided within the pages of the NCTM's *Principles and Standards for School Mathematics* (2000). The *Principles and Standards* describes a vision of mathematics education that is highly ambitious—a mathematics education that "requires solid mathematics curricula, competent and knowledgeable teachers who can integrate instruction with assessment, education policies that enhance and support learning, classrooms with ready access to technology, and a commitment to both equity and excellence" (p. 3). In general, constructivist learning, student-centered classrooms, worthwhile tasks, and reflective teaching are all a part of NCTM's vision of mathematics education in the 21st century (e.g., see Stein, Smith, Henningsen, & Silver, 2000).

To reform mathematics teaching and learning, mathematics educators need to look beyond the traditional view of mathematics as fixed and rigid, a subject of absolute truths, what Lerman (1990) termed an *absolutist* view of mathematics. In other words, constructivist teaching and inquiry-based learning demands a new view of mathematics, the *fallibilist* view, which "focuses attention on the context and meaning of mathematics for the individual, and on problem-solving processes…[and positions] mathematical knowledge…[as a] library of accumulated experience, to be drawn upon and used by those who have access to it" (Lerman, 1990, p. 56). Recent studies have explored teacher change, examining teacher beliefs and mathematical reform (e.g. see Wilson & Goldberg, 1998). This study, however, unlike many previous studies (e.g., see Fennema & Nelson, 1997), examined teachers' philosophies regarding mathematics, not simply their beliefs; it sought to understand how teachers philosophically understand mathematics and how this understanding affects their philosophies of mathematics teaching and learning and their pedagogical practices.

Purpose and Research Questions

The changes within mathematics education recommended by the NCTM are ambitious. How can teachers teach children mathematics in ways that are radically different from the ways that they

were taught—in student-centered classrooms, using investigative problem-solving approaches, employing rich mathematical discourse (see Hiebert, 2003, for a discussion of "traditional" curricula and pedagogy)? How can teachers challenge traditional beliefs about mathematics as a competitive, individualistic subject that they transmit to a "chosen" few students, into a subject that is explored with all students (Stinson, 2004)? In general, do teachers ever question the philosophical basis of the subject they teach?

Mathematicians and philosophers, over the past few decades, have challenged society's perception and philosophy of mathematics (e.g., see Davis & Hersh, 1981; Ernest, 1998; Hersh, 1997; Tymoczko, 1998). These challenges have positioned mathematics as a human activity, an activity not based on rote rules and procedures, but guided by intuition, exploration, and investigation (Dossey, 1992). But this philosophy is not held consistently among mathematicians (e.g. compare Russell, 1919/1993, to Lakatos, 1976), let alone among mathematics teachers and teacher educators. The lack of a common philosophy of mathematics has serious ramifications for both the practice and teaching of mathematics; it often silences even a discussion of differing philosophies (Dossey, 1992). But, without discussing philosophy, can reform truly take hold in school mathematics? Ernest (2004) asks mathematics educators to explore five essential questions about their subject: What is mathematics? How does mathematics relate to society? What is learning mathematics? What is teaching mathematics? What is the status of mathematics education as a field of knowledge? These questions challenge educators to not only reflect on their pedagogical practices, but also to question their own beliefs about mathematics and mathematical teaching and learning.

This study examined the philosophies of mathematics held among practicing mathematics teachers (P-K–College) and describes how those philosophies develop as they participated in a graduate-level, seminar that explored the philosophical development of mathematics. Three questions guided the study: How can teachers develop an understanding of their personal philosophies of mathematics? How do these philosophies change (or not) as teachers explore the writings of various philosophers from various traditions of Western mathematics? How do these philosophies impact (or not) their philosophies of mathematics teaching and learning and pedagogical practices?

Methodology

The participants of the study were 15 graduate students enrolled in an elective, 3-credit hour, graduate-level seminar offered at Georgia State University, a large urban research university in Atlanta, GA. The 6-week seminar (summer semester 2007) was reading intensive, engaging the students in a number of philosophical writings: Davis and Hersh's (1981) *The Mathematical Experience*, Russell's (1919/1993) *Introduction to Mathematical Philosophy*, Lakatos' (1976) *Proof and Refutations: The Logic of Mathematical Discovery*, Tymoczko's (1998) *New Directions in the Philosophy of Mathematics*, and Hersh's (1997) *What is Mathematics, Really?* The focus of the seminar was to challenge teachers' conceptions of mathematics and to assist them in exploring new and different philosophies of Western mathematics and, consequently, mathematics teaching and learning. Class sessions were student centered; in that, students

summarized and presented the seminar readings and facilitated the class discussions.
The participants include students who were earning a Specialist or Doctor of Philosophy degree in mathematics education. Among the participants were elementary, middle, high school, and college and university mathematics teachers, school mathematics coaches, and full-time graduate students.
[Note: The study is ongoing.]

Data collection included an initial, 3–5 page, reflective essay regarding each participant's personal philosophy of mathematics; reading journals from each participant that include written summaries of each reading during the seminar, participant-selected significant quotations from each reading, and comments regarding the participant's struggles with each reading and how it might (or might not) assist in her or his teaching (and research); a final 8–10 page, reflective, academic essay, outlining each participant's philosophy of mathematics and positioning her or his pedagogical practices within that philosophy; and scribed notes taken during each class discussion (the scribe notes were a rotating responsibility completed by the students).
In addition, three participants were selected for two individual, in-depth interviews. The three participants were selected purposively (Silverman, 2000) among the students who were secondary mathematics teachers. The first interview was a face-to-face, semi-structured, traditional, question-and-answer interview (Hollway & Jefferson, 2000), conducted at the end of the seminar (last week of July 2007); the second interview was a narrative interview (Hollway & Jefferson, 2000), conducted 3 months after the end of the seminar. This interview asked the participants to explain how the seminar changed (or not) their pedagogical practices. The interviews were recorded and transcribed; narrative analysis (Riessman, 1993) served as the data analysis methodology.

Findings and Conclusions

[Note: We will have initial findings and conclusions to report by August 24, 2007.]

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