4-25-2012

Essays on Financial Structure, Managerial Compensation and the Product Market

Hae Won Jung

Follow this and additional works at: http://scholarworks.gsu.edu/rmi_diss

Recommended Citation
http://scholarworks.gsu.edu/rmi_diss/27

This Dissertation is brought to you for free and open access by the Department of Risk Management and Insurance at ScholarWorks @ Georgia State University. It has been accepted for inclusion in Risk Management and Insurance Dissertations by an authorized administrator of ScholarWorks @ Georgia State University. For more information, please contact scholarworks@gsu.edu.
PERMISSION TO BORROW

In presenting this dissertation as a partial fulfillment of the requirements for an advanced degree from Georgia State University, I agree that the Library of the University shall make it available for inspection and circulation in accordance with its regulations governing materials of this type. I agree that permission to quote from, to copy from, or publish this dissertation may be granted by the author or, in his/her absence, the professor under whose direction it was written or, in his absence, by the Dean of the Robinson College of Business. Such quoting, copying, or publishing must be solely for the scholarly purposes and does not involve potential financial gain. It is understood that any copying from or publication of this dissertation which involves potential gain will not be allowed without written permission of the author.

Hae Won Jung
NOTICE TO BORROWERS

All dissertations deposited in the Georgia State University Library must be used only in accordance with the stipulations prescribed by the author in the preceding statement.

The author of this dissertation is:

Hae Won Jung
35 Broad Street NW, 11th Floor, Atlanta, GA 30303

The director of this dissertation is:

Ajay Subramanian
Department of Risk Management and Insurance
35 Broad Street NW, 11th Floor, Atlanta, GA 30303
Essays on Financial Structure, Managerial Compensation and the Product Market

BY

Hae Won Jung

A Dissertation Submitted in Partial Fulfillment of the Requirements for the Degree

Of

Doctor of Philosophy

In the Robinson College of Business

Of

Georgia State University

GEORGIA STATE UNIVERSITY

ROBINSON COLLEGE OF BUSINESS

2012
ACCEPTANCE

This dissertation was prepared under the direction of the Hae Won Jung Dissertation Committee. It has been approved and accepted by all members of that committee, and it has been accepted in partial fulfillment of the requirements for the degree of Doctoral of Philosophy in Business Administration in the J. Mack Robinson College of Business of Georgia State University.

H. Fenwick Huss, Dean

DISSERTATION COMMITTEE

Ajay Subramanian
Conrad S. Ciccotello
Martin F. Grace
Lixin Huang
Richard D. Phillips
ABSTRACT

Essays on Financial Structure, Managerial Compensation and the Product Market

BY

Hae Won Jung

April 25, 2012

Committee Chair: Ajay Subramanian

Major Academic Unit: Department of Risk Management and Insurance

This thesis consists of three chapters on financial structure, managerial compensation, and product markets. The unifying theme of these chapters is to examine how the financial decisions of firms are affected by market imperfections. Chapter 1 places emphasis on the impact of internal imperfections arising from asymmetric beliefs (or behavioral biases) and agency conflicts by examining how these internal imperfections affect managerial compensation and corporate financial structure. On the other hand, Chapters 2 and 3 incorporate external market imperfections especially arising from imperfect product market competition. More specifically, these two chapters develop market equilibrium frameworks to examine how the matching market for CEOs and firms interacts with the product market to affect the distributions of CEO compensation and firm size.

In Chapter 1, we develop a dynamic model to examine the effects of asymmetric beliefs of a firm’s manager and blockholders regarding the profitability of the firm’s projects, and differing attitudes towards their risk, on its capital structure. The firm’s capital structure reflects the tradeoff between the positive incentive effects of managerial optimism that increases the manager’s output and blockholders’ private benefits against the negative effects of risk-sharing costs. We provide several testable implications for the effects of the degree of managerial optimism as well as permanent and transitory components of the firm’s risk on different components of capital structure. In our calibration of the model, performed
separately for different industries, we show that while optimism and risk have *qualitatively* similar effects on capital structure in different industries, their *quantitative* effects are significantly different. The interactive effects of asymmetric beliefs and agency conflicts could potentially explain a significant portion of the substantial inter-industry variation in capital structure.

Chapter 2 studies how the distributions of CEO talent and compensation vary across industries, and how product market characteristics affect these distributions. We develop a market equilibrium model that incorporates the competitive assignment of CEOs to firms in a framework in which firms engage in imperfect product market—specifically, monopolistic—competition. Using the distributions of CEO pay and firm value in each of twelve Fama-French industries, we calibrate the parameters of our structural model, and indirectly infer the unobserved distributions of CEO talent and firm quality that together determine firm output. We then conduct several counterfactual experiments using the calibrated models corresponding to each of the industries. We find that the distribution of CEO talent does, indeed, vary dramatically across industries. More importantly, contrary to the conclusions of earlier studies that abstract away from the effects of the product market (Terviö, 2008 and Gabaix and Landier, 2008), the impact of CEO talent on firm value appears to be quite significant. Our estimates of the effect of CEO talent on firm value for the industries in our sample are two orders of magnitude higher than those obtained by the aforementioned studies. Further, our estimates suggest that the compensation of CEOs is quantitatively in line with their contributions to firms. Broadly, our study shows that it is important to incorporate the product market environment in which firms operate when assessing the contributions of CEOs to firms.

Chapter 3 builds a market equilibrium framework in which the CEO-firm matching process is affected by the product market. We show that under reasonable assumptions there is a unique equilibrium in which only managers with ability above a unique cutoff level are matched to firms. This very simple screening process endogenizes the distribution of active managers who match with firms. Our calibration of the model using a parametric approach, which is in contrast with the empirical analysis performed in Chapter 2, strongly supports the principle arguments on the importance of CEO talent and appropriate CEO talent levels (on average) in Chapter 2. In addition, due to the law of demand and supply, which is a key feature of the extended model, we obtain somewhat different influence of some of product market characteristics on CEO pay. Furthermore, our parametric approach allows us to draw some implications for the effects of CEO talent distribution on the market equilibrium.
DEDICATION

To the memory of my mom and grandma
I have been fortunate enough to have inspirational and supportive teachers, colleagues, and my loved ones throughout the whole process of writing this dissertation and earning my PhD. I owe them a great deal of gratitude and wish to express my sincere thanks.

First and foremost, I would like to thank my advisor, Ajay Subramanian, for his insightful research direction and constant support and encouragement during my PhD studies. He has always been available for guidance and advice and pushed me to be a better researcher in every aspect. I am truly indebted to him for everything which has made it possible for me to continue to pursue an academic career as I hope to become an excellent scholar and advisor like him in the future.

I would also like to thank my other committee members, Conrad S. Ciccotello, Martin F. Grace, Lixin Huang and Richard D. Phillips for their valuable suggestions and insights throughout this research work. They have helped me greatly to look critically at my work and given me thoughtful support and inspirational instruction from start to finish. I especially appreciate Richard D. Phillips, the Chair of the Risk Management and Insurance Department, for providing doctoral students with the most supportive research environment.

I would like to express my sincere thanks to S. Hun Seog, my graduate advisor at KAIST, who sparked my passion for research and inspired me to undertake a PhD. I would also like to thank Adam Speight, Shinichi Nishiyama, Zhiyong Liu, Daniel Bauer and Harley (Chip) E. Ryan for giving me a truly challenging and enlightening educational experience which has been a solid foundation for my fruitful research agenda.

My thanks also must go to my classmates, Fan Liu, Jimmy Martinez, Thorsten Moenig, Xue Qi and Nan Zhu, who have shared the joys and frustrations of studying in a PhD program. I have been truly blessed to have such a great group of people because it would have been much more difficult to finish my PhD without their friendly and helpful relationship. I also greatly thank Jeungbo Shim who has constantly given me kind advice since I started my PhD program. Moreover, I am grateful to my friends, Lauren Waters and Sandi L. Kam, who have enriched my experience in Atlanta by providing me with a diverse range of thinking, experiences, and viewpoints. I would like to thank Yoon Sung Chung too for all of her help and strong encouragement.

Last but not least, my special thanks must go to my family members. My farther has always encouraged me to aim for higher goals in life, yet at the same time appreciate what I am currently given. I am grateful too to my brother and sisters for their support and best wishes. And most of all, I would like to express my deepest gratitude and love to my best friend and husband, Hyeon Seong Yun. He has been here with me throughout the whole process and believed in me more than I have believed in myself. I would like to take this opportunity to show how much his continued love and support have meant to me. Thank you for everything.
# TABLE OF CONTENTS

DEDICATION ........................................ viii

ACKNOWLEDGEMENTS ............................... ix

LIST OF FIGURES ................................... xiv

LIST OF TABLES .................................... xv

Chapter 1  ASYMMETRIC BELIEFS, LEARNING AND CAPITAL STRUCTURE ............................ xvii

1.1 Introduction ........................................ xvii

1.2 Related Literature ............................... xxiv

1.3 The Model .......................................... xxvii
   1.3.1 Total Earnings Flow ........................... xxviii
   1.3.2 Capital Structure ............................... xxxi
   1.3.3 Contracting and Bankruptcy ..................... xxxiii
   1.3.4 Incentive Feasible Contracts ................. xxxvi
   1.3.5 Financing and Contract Choices ............... xxxvii

1.4 Managerial Contract and Capital Structure ........................................ xl
   1.4.1 Optimal Contract for a Given Long-Term Debt Structure ................ xli
   1.4.2 Bankruptcy Time ................................ xlv
Chapter 2    CEO TALENT, CEO COMPENSATION AND PRODUCT MARKET COMPETITION    lxxviii

2.1 Introduction .................................................. lxxviii

2.2 Related Literature .......................................... lxxxv

2.3 The Model .................................................... lxxxviii

2.3.1 Preferences, Market Demand and Production ............ lxxxix
Chapter 3  CEO-FIRM MATCHING AND PRODUCT MARKETS  cxxxiv

3.1 Introduction  cxxxiv

3.2 The Model  cxxxv

3.3 Equilibrium  cxxxix

3.4 Empirical Analysis  cxliv

3.4.1 Model Calibration  cxliv

3.4.2 Implications for CEO Talent  cxlix

3.4.3 Implications for Industry Characteristics  cli
3.5 Conclusion

REFERENCES
LIST OF FIGURES

Figure 1.1 Effects of Managerial Optimism  lxiii
Figure 1.2 Effects of Transient Risk  lxiii
Figure 1.3 Effects of Intrinsic Risk  lxiv
Figure 1.4 Effects of Expected Profitability  lxiv
Figure 2.1 Inferred Distributions of Firm Quality and Managerial Talent  cxii
Figure 2.2 Inferred Distributions of Firm Quality and Managerial Talent (CEO Cash Compensation)  cxvii
Figure 2.3 Inferred Distributions of Firm Quality and Managerial Talent (Non-CEO Total Compensation)  cxvii
Figure 2.4 Shifts in CEO Pay Distributions due to Changes in $\sigma$ (Business Equipment and Telecom)  cxxv
Figure 3.1 Distribution of Firm Quality  cxlviii
Figure 3.2 Ex-ante and Ex-post Distributions of Managerial Talent  cxlviii
# LIST OF TABLES

Table 1.1    Parameters of the Model                         liv
Table 1.2    Observed and Predicted Statistics for the Food, Software, and Entertainment Industries lvii
Table 1.3    Baseline Parameter Values for the Food, Software, and Entertainment Industries lviii
Table 2.1    Cross-Industry Summary Statistics            cvi
Table 2.2    Parameter Estimates, Relative Factor Values, and Price-Cost Margin cxii
Table 2.3    Impact of CEO Talent at the Median-Sized Firm cxx
Table 2.4    Effects of Hypothetical Talent Distributions cxvii
Table 2.5    Effects of Product Substitutability $\sigma$ cxxv
Table 2.6    Effects of Discount Rate $\delta$             cxxviii
Table 2.7    Effects of Entry Cost $f_e$                 cxxviii
Table 2.8    Effects of Market Size $R$                  cxxix
Table 3.1    Baseline Parameter Values                   cxlvii
Table 3.2    Impact of CEO talent at the Median-Sized Firm (ex-post dist.) cxlix
Table 3.3 Impact of CEO talent at the Median-Sized Firm (ex-ante dist.)

Table 3.4 Effects of Market Size $R$

Table 3.5 Effects of Entry Cost $f_e$

Table 3.6 Effects of Product Substitutability $\sigma$

Table 3.7 Effects of Total Mass $M$ of Potential Managers

Table 3.8 Effects of CEO Talent Distribution ($y_0$)

Table 3.9 Effects of CEO Talent Distribution ($\beta$)

Table 3.10 Effects of CEO Talent Distribution ($B$)

Table 3.11 Effects of Firm Quality Distribution ($x_0$)

Table 3.12 Effects of Firm Quality Distribution ($\alpha$)

Table 3.13 Effects of Firm Quality Distribution ($A$)
Chapter 1

ASYMMETRIC BELIEFS, LEARNING AND CAPITAL STRUCTURE

1.1 Introduction

There is growing evidence to suggest that managers and investors have differing beliefs about the profitabilities of firms’ projects in addition to having asymmetric attitudes towards their risks (Malmendier and Tate (2005), Baker et al. (2007)). We develop a dynamic model to show how asymmetric beliefs and costs of risk-sharing interact to affect capital structure. We derive novel, testable implications that link the degree of asymmetry in beliefs about a firm’s profitability as well as permanent and transitory components of its risk to different components of its capital structure: long-term debt, short-term debt, inside equity, and outside equity. Managerial optimism has sharply contrasting effects on long-term debt and short-term debt. While long-term debt declines with optimism, short-term debt increases. Permanent and transitory components of a firm’s risk have differing effects on its debt structure. Long-term debt increases with a firm’s intrinsic risk, but varies in a U-shaped manner with its transient risk. Short-term debt decreases with intrinsic risk and increases with transient risk. We calibrate the model to match the distributions of capital structure in different industries. While the qualitative effects of optimism and risk on capital structure are similar across industries, their quantitative effects are significantly different. Our results suggest that asymmetric beliefs are an important determinant of firms’ financial policies,
and the interactive effects of managerial optimism and agency conflicts could potentially rationalize a significant portion of the observed inter-industry variation in firms’ capital structures.

In our discrete-time, finite horizon framework, an all-equity firm approaches the capital markets to finance a positive NPV project. The firm’s shareholders comprise of “large” shareholders or blockholders and “small” atomistic, dispersed shareholders. The firm finances the project through equity and long-term debt that is non-callable and completely amortized, and then hires a manager to operate the firm. The firm’s small shareholders (hereafter “shareholders”) are competitive, while blockholders derive pecuniary private benefits in addition to the payout flows they receive from their equity stake in the firm (see Chapter V of Shleifer and Vishny (1997), Dyck and Zingales (2004)). Apart from equity and long-term debt, the firm’s capital structure also consists of non-discretionary short-term (single-period) debt associated with the financing of the firm’s working capital requirements.\(^1\) The firm’s earnings in each period are distributed among its stakeholders—the manager, blockholders, shareholders, debtholders, and the government (through corporate taxes). In reality, major strategic decisions such as financing and investment decisions require the approval of corporate boards. Boards are significantly influenced by large shareholders, such as venture capitalists and angel investors before a firm goes public, and institutional investors as well as other significant blockholders after a firm goes public (e.g. see Shleifer and Vishny (1997), Tirole (2006)). Accordingly, we assume that all decisions—financing, investment,

\(^1\)The modeling of working capital does not affect the model’s qualitative implications, but is important for its calibration because the financing of working capital forms a significant component of a firm’s short-term debt in the data.
and managerial contracts—are made in the interests of blockholders subject to ensuring that competitive shareholders receive fair returns on their investments.

The firm’s total gross earnings in each period consist of two components: the normally distributed base earnings, and the deterministic discretionary earnings that are generated by the firm’s incremental capital investments and the manager’s effort. All agents have imperfect information about the true mean of the project’s base earnings that we refer to as its intrinsic quality or profitability. While the firm’s investors—blockholders, shareholders, and debtholders—share a common normally distributed prior about the project’s profitability, the manager could have a different prior. The manager’s and investors’ priors have the same variance, but possibly different means. Investors and the manager “agree to disagree” about their respective mean assessments, the difference of which is the degree of managerial optimism. The true variance of the project’s earnings in each period is its intrinsic risk, which is invariant through time. The common variance of investors’ and the manager’s assessments of the project’s base earnings is its transient risk. The transient risk is resolved over time as agents learn about the project’s profitability through observations of the project’s earnings.

Investors are risk-neutral, while the manager is risk-averse with multiplicative CARA preferences. The manager receives dynamic incentives through a sequence of explicit single-period contracts contingent on the firm’s earnings. The contracts must be incentive compatible for the manager, and meet her participation constraints, that is, her expected utility at each date must be at least as great as her expected utility if she were to receive her constant reservation wage in each period. Because the firm’s dispersed shareholders are competitive, the surplus generated by the manager—the discretionary earnings—are shared by blockhold-
ers and the manager, that is, only the firm’s base earnings flow is pledgeable to the firm’s dispersed and atomistic shareholders. In addition to receiving their stake in the firm’s base earnings, blockholders also obtain a portion of the firm’s discretionary earnings that represent their private benefits, that is, pecuniary benefits that blockholders enjoy and that do not accrue to shareholders. The market or “outside” value of the firm’s equity—hereafter, the firm’s equity value—at any date is the net present value of the total payout flows to equityholders—shareholders and blockholders—excluding the private benefits that accrue to blockholders. Blockholder value equals the market or “outside” value of blockholders’ equity stake plus the net present value of blockholders’ private benefits.

As in Leland (1994), debt is serviced entirely in financial distress by the additional issuance of equity and bankruptcy is declared when the equity value falls to zero. For simplicity, we assume that the firm is liquidated if it goes bankrupt after deadweight bankruptcy costs are incurred and the absolute priority of debt is enforced. Blockholders choose the firm’s capital structure and the manager’s contracts to maximize their value subject to ensuring that shareholders receive fair (risk-adjusted) returns on their investments.

We first derive the manager’s explicit contracts for a given long-term debt structure. We implement the risky component of the manager’s compensation in each period through an inside equity stake in the firm, and the performance-invariant or “cash” component through a cash reserve. The cash reserve offsets the short-term debt financing of the firm’s working capital. In our implementation of the manager’s contracts, therefore, the firm’s capital structure consists of inside equity, outside equity, long-term debt, and dynamic short-term
borrowing or lending that reflects the financing of working capital requirements and the
manager’s cash compensation.

The manager’s inside equity stake is a deterministic function of time, but her cash compensation depends on the project’s earnings history through its effect on the players’ posterior assessments of the project’s profitability. The manager’s inside equity stake in each period increases with the initial degree of managerial optimism. When the manager is optimistic, she overvalues the project’s future earnings relative to investors. The optimal contract exploits the manager’s optimism by providing the manager with more powerful incentives, that is, by increasing her equity compensation relative to her cash compensation.

Blockholders choose the firm’s long-term debt structure at date zero to maximize their value that is the sum of two components. The first component is the market or “outside” value of the blockholders’ equity stake that includes the net surplus from financing the positive NPV project. The second component is the net present value of blockholders’ private benefits. Long-term debt has conflicting effects on the two components of blockholder value. Because debt interest payments are shielded from corporate taxes, blockholders can potentially increase the market value of their stake by choosing greater long-term debt. Choosing greater long-term debt, however, makes bankruptcy more likely, which lowers the second component of blockholder value; the value of their future private benefits.

We analytically show that long-term debt declines with the degree of managerial optimism. As discussed earlier, an increase in the manager’s optimism increases the manager’s inside equity stake in each period. The discretionary earnings the manager generates and, therefore, the blockholders’ private benefits increase with managerial optimism. Because
the manager’s beliefs and actions affect the firm’s discretionary earnings, but not its base earnings, managerial optimism has a much bigger impact on blockholders’ private benefits than on the market value of their stake. Consequently, in choosing the firm’s long-term debt, blockholders assign relatively more weight at the margin to their private benefits. As managerial optimism increases, therefore, blockholders choose lower long-term debt to decrease the likelihood of bankruptcy and, thereby, increase their private benefits.

To obtain additional implications of the model, we calibrate it to data. The key parameters of the model—the expected profitability, the degree of managerial optimism, the intrinsic and transient risks, and the technology parameters—are likely to vary significantly across different industries. Consequently, we calibrate the model to capital structure data “industry by industry.” For brevity, we report the results for three representative industries where the level and effects of asymmetric beliefs are likely to be quite different: the food industry, the software industry, and the entertainment industry. We calibrate the model’s parameters to match quantiles of the distribution of firms’ capital structures—the ratios of long-term debt, short-term debt, and firm market value relative to asset value—in each of the industries.

For these industries, the permanent and transitory components of risk—the intrinsic and transient risks—have sharply contrasting effects on long-term debt. Long-term debt increases with intrinsic risk, but varies non-monotonically in a U-shaped manner with transient risk. The intuition for the contrasting effects of the project’s intrinsic and transient risks on long-term debt hinges on their differing effects on the probability of bankruptcy. An increase in the transient risk increases the project’s “signal to noise ratio,” that is, intermediate observations
of the project’s earnings are more informative about its profitability. Consequently, the standard deviation of the evolution of posterior assessments of the project’s quality increases. As a result, the “option value” of continuing to service long-term debt interest payments increases, which delays bankruptcy and, thereby, has a positive impact on the present value of blockholders’ private benefits. An increase in the transient risk, however, also increases the costs of risk-sharing and lowers the discretionary earnings the manager generates in each period, which negatively affects the blockholders’ private benefits. At low transient risk levels, the latter effect dominates so that blockholders choose lower long-term debt to increase the value of their private benefits. At high transient risk levels, the costs of risk-sharing are so high that the value of blockholders’ private benefits is very low. Consequently, they increase long-term debt to exploit its tax advantages. In contrast with the effects of the transient risk, an increase in the intrinsic risk decreases the “signal to noise ratio and the option value of delaying bankruptcy. Further, an increase in the intrinsic risk increases the costs of risk-sharing between the manager and investors, which weakens the manager’s incentives and the output she generates in each period. As a result, an increase in the intrinsic risk unambiguously lowers the value of blockholders’ private benefits and its marginal impact on their long-term debt choice. They, therefore, choose greater long-term debt to exploit its tax advantages.

Next, we explore the effects of project characteristics on short-term debt. In sharp contrast with the negative effect of managerial optimism on long-term debt, short-term debt increases with optimism. As the manager’s degree of optimism increases, she receives more powerful incentives so that her cash compensation declines relative to her equity compen-
sation. Because cash offsets the firm’s short-term debt (see DeMarzo and Fishman (2007)), the firm’s short-term debt increases with managerial optimism. As in the case of the firm’s long-term debt, the project’s intrinsic and transient risks also have conflicting effects on short-term debt. Short-term debt increases with the project’s transient risk, but declines with its intrinsic risk.

Our results for the different industries suggest that, while the qualitative effects of optimism and risk on capital structure are similar for the three industries, the quantitative effects are quite different. Long-term debt decreases only slightly with managerial optimism in the software industry, but much more substantially in the other two industries. The intuition for the contrasting variations is that, unlike the food and the entertainment industries, the software industry is characterized by relatively high levels of managerial optimism and very low levels of long-term debt financing so that the quantitative effects of managerial optimism on long-term debt are small. The sharply contrasting quantitative effects of optimism and risk on capital structure in different industries suggests that the presence of differing levels of asymmetric beliefs could play an important role in reconciling the significant variation in firms’ capital structure distribution across industries.

1.2 Related Literature

We contribute to the literature by showing how asymmetric beliefs affect different components of capital structure—inside equity, outside equity, long-term debt and short-term debt—in a dynamic setting. Landier and Thesmar (2009) present a two-period framework in which contracts are exogenously restricted to be debt contracts. They show that optimistic
entrepreneurs choose short-term debt, whereas realists opt for long-term debt. In their setup, optimistic entrepreneurs prefer short-term debt because they believe that they can refinance it at more favorable terms in the second period by taking a bet on the project’s success. Dittmar and Thakor (2007) develop a three-period model and show that a manager chooses equity financing if there is greater agreement between the manager and investors.

We complement the above studies by considering a framework in which capital structure reflects the dynamic tradeoff between the positive incentive effects of managerial optimism and the negative effects of risk-sharing costs on managerial output and blockholders’ payoffs. The mechanisms that drive the effects of managerial optimism on capital structure are quite different from the above studies. As discussed earlier, a higher degree of optimism offsets the costs of risk-sharing and increases the power of incentives that can be provided to the manager. Consequently, the discretionary earnings generated by the manager increase that, in turn, increase the blockholders’ private benefits. Blockholders, therefore, assign relatively greater weight to their private benefits than the market value of their stake in choosing the firm’s long-term debt. At the same time, the tradeoff between the benefits of optimism and risk-sharing costs makes it optimal for a more optimistic manager to hold a higher inside equity stake in the firm and receive lower cash compensation, which has a positive effect on short-term debt.

The interplay between asymmetric beliefs and costs of risk-sharing among blockholders and managers, therefore, plays a central role in generating the differing impacts of optimism on long-term and short-term debt. Further, we derive implications for how permanent and transitory components of risk affect capital structure. To the best of our knowledge, the
contrasting effects of the intrinsic and transient risks on long-term and short-term debt, which again arise from the interaction between asymmetric beliefs and agency conflicts, are novel predictions of our study. Gorbenko and Strebulaev (2010) show that different components of risk have contrasting effects on capital structure in a very different setting with no imperfect information or asymmetric beliefs.

Adrian and Westerfield (2009) and Giat et al. (2010) develop general dynamic, principal-agent models to study the impact of asymmetric beliefs on optimal dynamic contracts. They abstract away from capital structure choices. Hackbarth (2008) incorporates managerial traits, such as growth and risk perception biases, into a dynamic tradeoff model of capital structure (e.g. see Fischer et al. (1989), Leland and Toft (1996), Goldstein et al. (2001), Hennessy and Whited (2005), Strebulaev (2007)). In his framework, which abstracts away from the effects of Bayesian learning and contracting and where all debt is long-term, a more optimistic manager chooses higher (rather than lower) long-term debt. In a contemporaneous study, Yang (2010), also builds a dynamic tradeoff model of capital structure with asymmetric beliefs. He shows that the presence of asymmetric beliefs can explain debt conservatism and market timing. He too abstracts away from the effects of contracting. We complement the above studies by building a dynamic tradeoff model in which asymmetric beliefs, Bayesian learning, and agency conflicts interact to affect different components of capital structure.
1.3 The Model

We consider a finite horizon framework with equally spaced dates, 0, 1, ..., T. At date 0, an all-equity firm has access to a positive net present value project that requires an initial investment outlay, $K$. The firm has “large” shareholders or blockholders who hold a combined equity stake $g$ and “small” or “atomistic” dispersed shareholders who hold an equity stake $1 - g$. The firm raises the initial investment $K$ for the project from public debt and equity markets, and hires a manager to operate the project. For simplicity, we assume that blockholders continue to hold an equity stake $g$ in the firm after date zero.\(^2\)

Small shareholders are competitive, while blockholders derive pecuniary private benefits in addition to the payout flows they receive from their equity stake in the firm, that is, blockholders obtain additional payoffs that do not accrue to small shareholders (see Chapter V of Shleifer and Vishny (1997)). All decisions—financing, investment, and managerial contracts—are made in the interests of blockholders subject to ensuring that competitive small shareholders receive fair returns on their investments. In reality, major strategic decisions such as financing and investment decisions require the approval of corporate boards. Boards are significantly influenced by large shareholders, such as venture capitalists and angel investors before a firm goes public, and institutional investors as well as other significant blockholders after a firm goes public (e.g. Shleifer and Vishny (1997), Tirole (2006)).

\(^2\)Note that we do not require or assume that the identities of blockholders remains the same through time, that is, a blockholder can sell its stake to another blockholder. We can use arguments analogous to those in Bolton and von Thadden (1998) (see Section II.A of their paper) to show that, as long as trading is non-anonymous, it is sub-optimal for a blockholder to sell its ownership stake to several dispersed buyers, that is, the blockholders either hold onto their shares or sell them as a block. Consequently, the combined equity stake of blockholders persists through time. Since these considerations fall well outside the scope of this paper, we simply assume here that blockholders’ ownership stake is $g$ through time.
Because blockholders play a key role in driving firms’ decisions in reality, the incorporation of blockholders in the model is also important for its calibration to data. For simplicity, we ignore strategic behaviors among different blockholders, that is, they behave as a monolithic unit in their collective interest. We hereafter refer to the firm’s group of blockholders by a single “representative” blockholder, and its small shareholders as simply its “shareholders.” The equity stake held by the representative blockholder is the firm’s block equity while the equity held by shareholders is the firm’s public equity.

The total earnings from the project are distributed among all the firm’s claimants - the manager, the blockholder, shareholders, debt-holders, and the government (through corporate taxes). We ignore personal taxes for simplicity and assume that the corporate tax rate is a constant, $\tau \in (0, 1)$. Security issuance costs are negligible, and the risk-free interest rate, $r$, is a constant and the same for all market participants.

### 1.3.1 Total Earnings Flow

In any period $[i, i + 1]$, the total earnings from the project are affected by physical capital investments by the blockholder and shareholders as well as human capital investments (effort) by the manager. The total earnings flow in any period has two components: the base earnings—a stochastic component that is unaffected by the physical and human capital investments, and the discretionary earnings—a deterministic component that depends on the incremental capital investment by equity holders as well as the manager’s effort. Specifically, if the capital investment and effort over period $[i, i + 1]$ are $k_i$ and $\eta_i$, respectively, the total
earnings flow is

\[ E_{i+1} = \Theta + N_{i+1} + \sum_{\alpha} \sum_{\beta} A_{\alpha \beta} \eta_i \]

(1.1)

The first component of the base earnings, \( \Theta \), represents the project’s *intrinsic quality* or *profitability*. The manager and the firm’s investors—the blockholder, shareholders and debtholders—have imperfect information and possibly asymmetric beliefs about \( \Theta \). Their respective beliefs are, however, common knowledge, that is, they agree to disagree (see Morris (1995), Allen and Gale (1999)). Their respective priors on \( \Theta \) at date zero are normally distributed as follows:

\[ \Theta \sim N(\mu_0^S, \sigma_0^2); \text{ Investors’ Prior,} \]

(1.2)

\[ \Theta \sim N(\mu_0^M, \sigma_0^2); \text{ Manager’s Prior.} \]

Note that all investors—the blockholder, shareholders, and debtholders—share the same beliefs about \( \Theta \). We make no assumptions about the *true* project quality distribution because the equilibrium only depends on how the manager’s and investors’ assessments of project quality relate to each other, and not on the true project quality distribution. Although we could consider the general scenario in which the manager is optimistic or pessimistic relative to investors, we simplify the exposition by considering the (empirically relevant) scenario in which the manager is optimistic. It is important to mention here that, when we calibrate the model to data in Section 1.6, we do not assume *a priori* that the manager is optimistic, and indirectly infer the average level of managerial optimism (or pessimism) implied by the
data. We define
\[ \Delta_0 := \mu^M_0 - \mu^S_0 \] (1.3)
as the degree of managerial optimism at date zero.

The second component of the base earnings in (1.1), \( N_{i+1} \), is a normal random variable with mean 0 and variance \( s^2 \). The random variables \( \{N_{i+1}, i \geq 0\} \) are independent of each other, and are also independent of \( \Theta \). The parameter \( s^2 \) is the intrinsic risk of the firm’s earnings because it is present even when there is perfect information about \( \Theta \). The intrinsic risk is invariant through time.

The discretionary earnings are described by a Cobb-Douglas production function, in which the total factor productivity, \( A \), and the parameters, \( \alpha, \beta \in (0, 1) \), are known constants. The discretionary earnings are observable but non-verifiable, and, therefore, non-contractible. As we discuss later, the discretionary earnings are the source of pecuniary private benefits for the blockholder.

All agents—the manager and investors—update their prior beliefs of the project’s profitability \( \Theta \) over time based on intermediate observations of the project’s earnings. Define the random variable
\[ \xi_{i+1} := E_{i+1} - A k_i^\alpha \eta_i^\beta = \Theta + N_{i+1}, \quad i = 0, 1, \ldots \] (1.4)
The posterior distribution on \( \Theta \) for each date \( i \geq 1 \) is normally distributed under the beliefs of the manager (denoted by \( N(\mu^M_i, \sigma^2_i) \)) and investors (denoted by \( N(\mu^S_i, \sigma^2_i) \)), where
Note that \( \sigma_i \) tends to zero as \( i \to \infty \). The parameter, \( \sigma_i^2 \), is the project’s \textit{transient risk} because it represents the degree of uncertainty about the project’s quality that is resolved through time due to Bayesian learning. Define

\[
\Delta_i := \mu_i^M - \mu_i^S = \frac{s^2 \Delta_0}{s^2 + i \sigma_0^2} = \frac{\sigma_i^2}{\sigma_0^2} \Delta_0, \quad i = 0, 1, \ldots
\]  \( \tag{1.7} \)

We refer to \( \Delta_i \) as the degree of managerial optimism at date \( i \). By (1.7), the degree of managerial optimism declines deterministically over time as the manager and investors update their priors on \( \Theta \) in a Bayesian manner based on observations of the project’s earnings.

1.3.2 Capital Structure

All long-term debt issued at date zero matures at date \( T \) and is non-callable. Further, if the time horizon \( T \) is sufficiently long, we can follow Leland (1994) by assuming that long-term debt is completely amortized so that long-term debtholders (hereafter, \textit{bondholders}) receive a constant coupon payment, \( d \), in each period as long as the firm remains solvent. The debt coupon payment \( d \), which determines the firm’s long-term debt structure, is later determined endogenously.
In addition to long-term debt, the firm also has non-discretionary working capital requirements such as inventories, accounts payable and receivable, employee wages, etc. that are financed through single-period, risk-free short-term debt. Although it does not qualitatively alter the main implications of our study, the incorporation of working capital requirements is necessary for the calibration of the model because the financing of working capital is an important component of a firm’s short-term debt in the data. Consistent with empirical evidence (e.g. Fazzari and Petersen (1993)), working capital requirements increase with the firm’s total earnings. For simplicity, we assume that working capital requirements are a constant proportion $\nu$ of the firm’s total earnings flow, where $\nu$ is observable. Hence the firm’s total earnings net of working capital requirements in period $[i, i + 1]$ is

$$Q_{i+1} = (1 - \nu)E_{i+1}.$$ \hspace{1cm} (1.8)

We hereafter refer to the process $Q_{i+1}$ as the EBITM (earnings before interest, taxes, and the manager’s compensation) because it will be distributed among bondholders, the government (through taxes), the manager, the blockholder, and shareholders.

For now, the firm’s capital structure consists of block equity, public equity, long-term debt, and short-term debt financing of working capital. In Section 1.4.4, we implement the manager’s contract through an inside equity stake and a cash reserve that offsets the firm’s short-term debt because cash is effectively negative short-term debt (see DeMarzo and Fishman (2007)). In the calibrated model, therefore, the firm’s capital structure consists of inside equity, outside equity (block + public equity), long-term debt, and short-term debt,
where short-term debt reflects the financing of working capital and the manager’s cash compensation.

1.3.3 Contracting and Bankruptcy

Investors are risk-neutral, while the manager is risk-averse with multiplicatively separable CARA preferences. If the manager’s compensation and her effort in period \([j, j + 1]\) are \(c_j^M\) and \(\eta_j\), respectively, and the firm remains solvent until date \(T\), her total expected utility at date \(i < T\) is

\[
U(c, \eta) = E_i^M \left[ -\exp \left( -\lambda \left( \sum_{j=i}^{T-1} e^{-r(j-i)} (c_j^M - \kappa \eta_j^\gamma) \right) \right) \right].
\] (1.9)

In the above, \(r\) is the risk-free interest rate, \(\lambda\) is the manager’s absolute risk aversion, and \(\kappa \eta_j^\gamma\) (\(\kappa > 0\) is a constant) is the manager’s disutility of effort in period \([j, j + 1]\).

The manager receives dynamic incentives through contracts that could be explicitly contingent on the EBITM process \(Q_{i+1}\) defined in (1.8). We consider an incomplete contracting environment in which only single-period contracts are enforceable. The manager receives a contract in each period, which specifies the division of the firm’s earnings (net of corporate taxes and interest payments) between the manager, the blockholder, and shareholders.

Following Leland (1994), debt payments are serviced entirely as long as the firm is solvent. In financial distress, debt payments are serviced through the additional issuance of public equity. Bankruptcy, therefore, occurs endogenously when the value of public equity falls to zero. We can extend the model to allow for the firm to continue servicing debt even
after the public equity value falls to zero with bankruptcy occurring when it is no longer optimal to continue servicing debt. The implications of the extended model do not differ from those of the simpler model presented here in which bankruptcy occurs when the public equity value falls to zero.

For simplicity, we assume that the firm is liquidated upon bankruptcy. The absolute priority of debt is enforced at bankruptcy. The total payoff to bondholders upon bankruptcy is

\[
\text{Bondholders' Bankruptcy Payoff} = \mathbf{LD}(T_b) = E^{S_{T_b}} \left[ \sum_{i=T_b}^{T-1} e^{-r(i-T_b)} (1 - \tau) (1 - \rho)(\Theta + N_{i+1}) \right],
\]

(1.10)

where the expectation is with respect to investors’ beliefs at the bankruptcy time, \( T_b \). Because the manager leaves the firm at bankruptcy, its “liquidation value” is determined by the net present value of its base earnings flow, that is, the earnings flow in the absence of the manager’s human capital inputs (see (1.1)). As in Leland (1994), the firm incurs bankruptcy costs so that the payoff to bondholders upon bankruptcy is the present value of the base earnings flow, \( \Theta + N_{i+1} \), net of corporate taxes and bankruptcy costs. The parameter \( \rho \in (0, 1) \) in (1.10) represents the firm’s (proportional) bankruptcy costs.

As in traditional principal-agent models with moral hazard (see Laffont and Martimort (2002)), it is convenient to augment the definition of the manager’s contract to also include the manager’s effort choices and the incremental capital investments. We then require that
the manager’s contract be incentive compatible with respect to her effort. Further, we can simplify notation by viewing the sequence of single-period contracts for the manager as a single long-term contract that is implemented by the sequence.

Formally, a contract $\Gamma \equiv [c^M(\cdot), \eta, k]$ is a stochastic process describing the manager’s compensation $c^M_i(\cdot)$, her effort choice $\eta_i$, and the capital investment $k_i$ in each period $[i, i+1]$. If $\mathcal{F}_i$ denotes the information filtration generated by the history of the firm’s base earnings and discretionary earnings, the process $\Gamma$ is $\mathcal{F}_i$-adapted. The bankruptcy time $T_b$ is an $\mathcal{F}_i$-stopping time (recall that the bankruptcy time is determined by the contract).

As in Gibbons and Murphy (1992), the manager has multiplicatively separable CARA preferences as specified in (1.9), and the firm’s earnings flow evolves as a Gaussian process, as specified in (1.1). Following Gibbons and Murphy (1992), therefore, we restrict consideration to contracts in which the manager’s compensation in each period has the affine form

$$c^M_i = a_i + b_i(1 - \tau)(Q_{i+1} - d), \quad i < T_b. \tag{1.11}$$

The component $a_i$ of the manager’s compensation is determined at the beginning of period $[i, i+1]$ and could, therefore, be interpreted as her cash compensation for the period. The second component $b_i(1 - \tau)(Q_{i+1} - d)$ of the manager’s compensation is contingent on the firm’s earnings $Q_{i+1}$ that are realized at the end of the period and could, therefore, be interpreted as her equity compensation. We express the manager’s compensation in terms of the earnings net of interest payments and taxes because it clarifies our subsequent implementation of the manager’s contract through financial securities.
By (1.11), a feasible contract for the manager can be described by the quadruple \((a, b, \eta, k)\) where \(a\) is the manager’s cash compensation process, \(b\) determines her equity compensation over time, \(\eta\) is her effort process, and \(k\) is the capital investment process.

### 1.3.4 Incentive Feasible Contracts

In any period \([i, i + 1]\) for \(i < T_b\), the after-tax earnings flow is

\[
\begin{align*}
c_i^A & = (1 - \tau)Q_{i+1} + \tau d. \tag{1.12}
\end{align*}
\]

The payoff to bondholders during the period is the long-term debt coupon payment

\[
\begin{align*}
c_i^D & = d. \tag{1.13}
\end{align*}
\]

The total payoff to the blockholder and shareholders is the after-tax earnings net of payments to the manager as well as bondholders.

\[
\begin{align*}
c_i^S & = c_i^A - c_i^M - c_i^D. \tag{1.14}
\end{align*}
\]

We now describe the constraints that must be satisfied by incentive feasible contracts. At any date \(i < T_b\), the manager’s conditional expected utility from a contract \(\Gamma = (a, b, \eta, k)\) is

\[
\begin{align*}
M(i) & := E_i^M \left[ - \exp \left( - \lambda \left( \sum_{j=i}^{T_b-1} e^{-r(j-i)} (c_j^M - k\eta_j) \right) \right) \right], \tag{1.15}
\end{align*}
\]

where \(E_i^M\) denotes the expectation with respect to the manager’s beliefs at date \(i\).
The manager’s contract must be incentive compatible, that is, it must be optimal for the manager to exert the effort $\eta$ specified by the contract given her compensation structure. Therefore,

$$\eta = \arg \max_{\eta'} E_i^M \left[ -\exp \left( -\lambda \left( \sum_{j=i}^{T_b-1} e^{-r(j-i)} (c_j^M - \kappa \eta_j^\gamma) \right) \right) \right]. \quad (1.16)$$

The contract must also satisfy the manager’s participation constraint in each period. Let $\phi > 0$ denote the manager’s reservation wage in each period, which could also be interpreted as a measure of the manager’s bargaining power vis-a-vis the blockholder. The contract $(a, b, \eta, k)$ is feasible if and only if the manager’s continuation expected utility at each date $i$ is at least as great as her continuation expected utility if she received the payoff $\phi$ in each period, that is,

$$E_i^M \left[ -\exp \left( -\lambda \left( \sum_{j=i}^{T_b-1} e^{-r(j-i)} (c_j^M - \kappa \eta_j^\gamma) \right) \right) \right] \geq E_i^M \left[ -\exp \left( -\lambda \left( \sum_{j=i}^{T_b-1} e^{-r(j-i)} \phi \right) \right) \right], \quad \forall i < T_b. \quad (1.17)$$

### 1.3.5 Financing and Contract Choices

The firm’s shareholders are competitive, while the blockholder enjoys significant bargaining power vis-a-vis the manager that allows it to extract additional pecuniary private benefits that are not enjoyed by shareholders (Shleifer and Vishny (1997)). As mentioned earlier, major decisions such as capital structure and managerial contract choices must be approved by a firm’s board of directors. Due to their significant equity stakes, large shareholders have substantial bargaining power and representation on corporate boards (Shleifer...
and Vishny (1997)). Accordingly, the firm’s capital structure and the manager’s contract are
chosen to maximize the blockholder’s value, while ensuring that the competitive shareholders
receive fair returns on their capital investments.

We now describe the division of earnings between the blockholder and shareholders. The total after-tax payout flow to the blockholder and shareholders in any period \([i, i+1]\) is \(c_i^S\) given by (1.14). Let \(c_i^B\) and \(c_i^E\) denote the payout flows to the blockholder and the shareholders, respectively, over the period so that

\[
e_i^S = c_i^B + c_i^E. \tag{1.18}
\]

By (1.1), the firm’s total earnings flow in the absence of any actions by the manager
is simply the base earnings flow. By (1.8) and (1.12), therefore, the after-tax payout flow
without any actions by the manager over period \([i, i+1]\) is \((1−\tau)(1−\nu)(\Theta+N_{i+1})+\tau d\). After
the long-term debt coupon payment is made, the after-tax payout flow to the blockholder
and shareholders without any actions by the manager is \((1−\tau)\left((1−\nu)(\Theta+N_{i+1})−d\right)\). Since
shareholders are competitive, the blockholder and the manager share the surplus generated
by the manager, that is, the firm’s discretionary earnings. In other words, only the base
earnings flow is “pledgeable” to shareholders. Given that shareholders hold an equity stake
\(1−g\), while the blockholder holds a stake \(g\), the payout flows to shareholders must satisfy
the following constraints at each date \(i\):

\[
E_i^S \left[ \sum_{j=1}^{T_h-1} e^{-r(j-i)} c_j^E \right] \geq (1−g) E_i^S \left[ \sum_{j=1}^{T_h-1} e^{-r(j-i)} \left( k_j + (1−\tau)(1−\nu)(\Theta+N_{j+1})−d\right) \right].
\]
The above can be rewritten as follows:

\[
E_i^S \left[ \sum_{j=i}^{T_b-1} e^{-r(j-i)} \left( c_j^E - (1 - g)k_j \right) \right] \geq (1-g)E_i^S \left[ \sum_{j=i}^{T_b-1} e^{-r(j-i)} \left( (1 - \tau)(\Theta + N_{j+1}) - d \right) \right].
\]

(1.19)

As indicated above, the payout flow must compensate shareholders for their share \((1 - g)k_j\) of the capital investment, \(k_j\). By (1.18), the blockholder obtains the firm’s remaining earnings net of the manager’s contractual compensation payments. Because it holds an equity stake \(g\), the blockholder obtains (in expectation) a proportion \(g\) of the base earnings flow plus a portion of the discretionary earnings. The latter component of the payout flow constitutes the private benefits that accrue to the blockholder, which cause block equity to trade at a premium relative to public equity (see Shleifer and Vishny (1997)).

The blockholder chooses the firm’s long-term debt at date zero and the manager’s contract to maximize its value. The long-term debt value, \(LD(0)\), blockholder value, \(B(0)\), and public equity value, \(E(0)\), at date zero are respectively given by

\[
\text{Long-Term Debt Value} = LD(0) = E_0^S \left[ \sum_{i=0}^{T_b-1} e^{-ri}c_i^D + e^{-rT_b}LD(T_b) \right].
\]

(1.20)

\[
\text{Blockholder Value} = B(0) = \left\{ \begin{array}{l}
g\text{share of proceeds from long-term debt issuance} \\
\text{share of initial investment} \\
\text{NPV of payout flows net of periodic investments} \\
\end{array} \right\} - gK + E_0^S \left[ \sum_{i=0}^{T_b-1} e^{-ri}(c_i^B - gk_i) \right].
\]

(1.21)
Public Equity Value  =  \mathbf{E}(0) \\
\quad = \frac{\text{share of proceeds from long-term debt issuance}}{(1 - g)\text{LD}(0)} \quad - \quad \frac{\text{share of initial investment}}{(1 - g)K} \\
\quad + \quad \text{NPV of payout flows net of periodic investments} \\
\quad + \quad E^S_0 \left[ \sum_{i=0}^{T_b-1} e^{-ri} (c^E_i - (1 - g)k_i) \right] \\
(1.22)

The block and public equity values include their respective shares of the surplus generated from financing the project at date zero. The right hand sides of the expressions (1.21) and (1.22) for the block equity and public equity values, respectively, incorporate their respective shares of the initial and subsequent capital investments.

The optimal long-term debt coupon, \( d^* \), and the manager’s contract, \( \Gamma^* \), maximize the block equity value:

\[(d^*, \Gamma^*) = \arg \max_{(d, \Gamma)} B(0). \]  
(1.23)

### 1.4 Managerial Contract and Capital Structure

We proceed in two steps. In step one, for a given long-term debt structure \( d \), we derive the manager’s optimal contract, which specifies the manager’s effort choices, the incremental capital investments by equity holders, payoffs to both parties, and the bankruptcy time. In step two, we derive the optimal choice of long-term debt \( d^* \).
1.4.1 Optimal Contract for a Given Long-Term Debt Structure

The following proposition characterizes the manager’s optimal contract for a given long-term debt structure \(d\).

**Proposition 1 (Optimal Contract)**

In any period \([i, i + 1]\) for \(i < T_b\), the manager’s optimal contract for a given long-term debt structure \(d\) is characterized as follows:

(a) The manager’s pay-performance sensitivity, \(b^*_i\), solves

\[
\max_{b_i \geq 0} F_i(b_i),
\]

where

\[
F_i(b_i) = (1 - \tau)(1 - \nu)\Delta_i b_i - 0.5\lambda(1 - \tau)^2(1 - \nu)^2(\sigma_i^2 + s^2)b_i^2 + \frac{(1 - \alpha)\gamma - \beta}{\alpha\gamma} k(b_i).
\]

(b) The incremental capital investment is

\[
k^*_i = k(b^*_i) = \left[ \frac{\alpha\gamma}{\gamma - \beta} \psi(b^*_i) \right]^{\frac{\alpha - \beta}{\alpha\gamma - \beta}},
\]

where

\[
\psi(b^*_i) = (A(1 - \tau)(1 - \nu))^\frac{\gamma - \beta}{\gamma}(\frac{1}{\kappa})^\frac{\beta}{\gamma - \beta} \left( \frac{\beta b^*_i \gamma}{\gamma} \right)^\frac{\beta}{\gamma - \beta} \left( 1 - \frac{\beta b^*_i}{\gamma} \right).
\]
(c) The manager’s effort is

$$\eta_i^* = \eta(b_i^*, k_i^*) = \left[ \frac{A(1 - \tau)(1 - \nu)\beta k_i^\alpha b_i^\gamma}{\gamma K} \right]^{\frac{1}{\gamma - \beta}}. \quad (1.27)$$

(d) The manager’s cash compensation is

$$a_i^* = a(b_i^*, k_i^*, \eta_i^*) = \phi + \kappa(\eta_i^*)^\gamma + 0.5\lambda(1 - \tau)^2(1 - \nu)^2(b_i^*)^2(\sigma_i^2 + s^2) - b_i^*(1 - \tau) \left[ (1 - \nu) \left( \mu_i^M + A(k_i^*)^{\alpha}(\eta_i^*)^\beta \right) - d \right]. \quad (1.28)$$

(e) The endogenous bankruptcy time solves the following optimal stopping problem:

$$T_b = \arg \max_{t \leq T} E_0^S \left[ \sum_{j=0}^{t-1} e^{-rj} \left( (1 - \tau) \left( (1 - \nu) \mu_j^S - d \right) \right) \right], \quad (1.29)$$

where the maximization is over all \( \{F_i\} \)-stopping times \( t \leq T \).

(f) The public equity value and blockholder value are, respectively,

$$E(0) = (1 - g)(LD(0) - K) + (1 - g)E_0^S \left[ \sum_{i=0}^{T_b-1} e^{-ri} \left( (1 - \tau) \left( (1 - \nu) \mu_i^S - d \right) \right) \right], \quad (1.30)$$

$$B(0) = \frac{g}{1 - g}E(0) + E_0^S \left[ \sum_{i=0}^{T_b-1} e^{-ri} (F_i(b_i^*) - \phi) \right]. \quad (1.31)$$

Proof. See Appendix A
Given the manager’s and investors’ beliefs about the project’s quality, \( N(\mu_i^M, \sigma_i^2) \) and \( N(\mu_i^S, \sigma_i^2) \), respectively, and conditional upon the firm’s solvency at the beginning of period \([i, i + 1]\), the optimal contractual parameters for this period, \((a^*_i, b^*_i, \eta^*_i, k^*_i)\), are described in Proposition 1. The equilibrium values for the pay-performance sensitivity, effort, and investment at each point in time are deterministic. The cash component of the manager’s compensation \(a^*_i\) is, however, stochastic and depends on the firm’s earnings history through its effect on the manager’s posterior mean assessment \(\mu_i^M\) of the project’s quality.

Comparing (1.30) and (1.31), we see that the block equity value is the sum of two components. The first component is the present value of the payout flows to the blockholder from its equity stake in the firm. We refer to this component as the “outside” or market value of the blockholder’s equity stake because this is what the blockholder would receive if it were to sell its stake to dispersed atomistic buyers. The second is the present value of the private benefits that accrue to the blockholder, but not to shareholders.

In (1.24) and (1.25), the optimal pay-performance sensitivity in each period maximizes an objective function that has three components; the rents from managerial optimism, the costs of risk-sharing, and the return on investment:

\[
b^*_i = \arg \max_{b_i \geq 0} F_i(b_i) = \left(1 - \tau\right)(1 - \nu) \Delta b_i - 0.5\lambda(1 - \tau)^2(1 - \nu)^2(\sigma_i^2 + s_i^2)b_i^2 + \frac{(1 - \alpha)\gamma - \beta}{\alpha\gamma} k(b_i).
\]

(1.32)

The following proposition describes the effects of optimism and risk on the manager’s optimal pay-performance sensitivity at any date.
Proposition 2 (Optimism, Risk, and Incentives)

(a) The manager’s optimal pay-performance sensitivity $b_i^*$ at each date $i < T_b$ increases with the initial degree of managerial optimism $\Delta_0$.

(b) The manager’s optimal pay-performance sensitivity $b_i^*$ at each date $i < T_b$ declines with the initial transient risk $\sigma_0^2$.

(c) The manager’s optimal pay-performance sensitivity also declines with the intrinsic risk $s^2$, provided that $\Delta_0 \leq 2\lambda(1 - \tau)(1 - \nu)(\sigma_0^2 + s^2)$. Otherwise, the manager’s optimal pay-performance sensitivity could vary non-monotonically with the intrinsic risk.

Proof. See Appendix A

Because the manager is optimistic, she overvalues the firm’s future earnings relative to investors. Consequently, as the degree of managerial optimism increases, the extent to which she overvalues the performance-sensitive component of her compensation increases. The optimal contract exploits this by increasing the performance-sensitive component of the manager’s compensation, that is, her pay-performance sensitivity.

An increase in the initial transient risk or the intrinsic risk increases the costs of risk-sharing; the second component of the objective function in (1.32). It follows from (1.7), however, that the intrinsic and transient risks also affect the degree of managerial optimism, $\Delta_i$, at each date $i$ and, thereby, the rents from managerial optimism; the first component of the objective function in (1.32).

By (1.7), an increase in the initial transient risk lowers the degree of managerial optimism because it increases the “signal to noise ratio” so that the manager learns more quickly. Both the effect of increasing the costs of risk-sharing and that of lowering the rents from managerial
optimism lower the manager’s pay-performance sensitivity. Consequently, the manager’s pay-performance sensitivity declines with the initial transient risk. An increase in the intrinsic risk, however, increases the degree of managerial optimism at each date because it decreases the signal to noise ratio so that the manager learns more slowly. If the degree of managerial optimism is below a threshold, the increase in the costs of risk-sharing with the intrinsic risk dominates so that the manager’s pay-performance sensitivity declines. If the degree of managerial optimism is above the threshold, however, the interplay between the rents from managerial optimism and the costs of risk-sharing causes the pay-performance sensitivity to vary in a complex, non-monotonic manner with the intrinsic risk. In our calibration of the model in Section 1.6, the baseline level of managerial optimism is below the threshold so that the manager’s pay-performance sensitivity also declines with the intrinsic risk.

We later implement the performance-sensitive component of the manager’s compensation through an inside equity stake. In our implementation, the results of Proposition 2 directly translate to the effects of optimism and risk on the manager’s inside equity stake.

1.4.2 Bankruptcy Time

The following proposition shows that bankruptcy occurs in any period if and only if shareholders’ mean posterior assessment of the project’s intrinsic quality falls below an endogenous trigger.

**Proposition 3 (Bankruptcy Time)**

There exists a trigger $\mu_i^*$ for $i = 0, \ldots, T - 1$ such that bankruptcy occurs at date $i$ if and only if $\mu_i^S \leq \mu_i^*$. 
Proof. See Appendix A.

As in Leland (1994), bankruptcy occurs when the public equity value falls to zero. As one would intuitively expect, the public equity value increases with shareholders’ mean posterior assessment of the project’s quality. Consequently, bankruptcy occurs if and only if shareholders’ mean posterior assessment is sufficiently low.

The following proposition describes the effects of project characteristics—shareholders’ initial mean assessment of project quality or the project’s expected profitability, the intrinsic risk, and the transient risk—on the timing of bankruptcy.

**Proposition 4 (Project Characteristics and Bankruptcy)**

The bankruptcy time $T_b$ increases with shareholders’ initial mean assessment of the project’s intrinsic quality $\mu_0^S$, increases with the initial transient risk $\sigma_0^2$, and decreases with the intrinsic risk $s^2$.

Proof. See Appendix A.

The bankruptcy time maximizes the public equity value, which depends on shareholders’ beliefs. An increase in shareholders’ initial mean assessment of the project’s intrinsic quality also increases their posterior assessment in each subsequent period and, therefore, the public equity value at each date. Accordingly, an increase in shareholders’ initial mean assessment reduces the likelihood of bankruptcy.

The two types of risk have opposite effects on the evolution of shareholders’ mean assessments of the project’s intrinsic quality, and, therefore, the bankruptcy time. More precisely, by (1.5) and (1.6), the variance of the evolution of shareholders’ mean assessments
over period \([i, i + 1]\) is

\[
(\sigma_i^2) := \text{Var}_i[\mu_{i+1}^S - \mu_i^S] = \frac{s^2}{[(s/\sigma_0)^2 + i + 1][(s/\sigma_0)^2 + i]}. \tag{1.33}
\]

By (1.33), we see that an increase in the initial transient risk increases the variance of the evolution of the mean assessments of the project’s intrinsic quality. Roughly, an increase in the initial transient risk increases the “signal to noise ratio” so that intermediate signals are more informative about the project’s quality. Since the public equity value is convex in shareholders’ mean assessments of the project’s quality, an increase in the initial transient risk increases the “option value” of continuing to service debt payments and delaying bankruptcy.

On the other hand, we also see by (1.33) that the intrinsic risk decreases the variance of the evolution of mean assessments of project quality because it decreases the signal to noise ratio so that intermediate signals are less informative. Hence an increase in the intrinsic risk decreases the option value of delaying the firm’s bankruptcy.

As we show later, the differing effects of the intrinsic and transient risks on the likelihood of bankruptcy cause them to have sharply contrasting effects on the firm’s capital structure.

### 1.4.3 Optimal Long-Term Debt Structure

The optimal long-term debt structure \(d^*\) solves (1.23), where the manager’s contractual parameters for a given debt structure are described by Proposition 1. As a closed-form analytical characterization of the long-term debt structure is not available, we numerically derive it in Section 1.6.
As shown by (1.23), the optimal long-term debt structure maximizes the blockholder value at date zero. The first component of the blockholder value—the market value of the blockholder’s stake—depends only on the public equity value, \(E(0)\). The second is the present value of the blockholder’s private benefits flow. By (1.29), the bankruptcy time does not depend on the manager’s beliefs or actions. For a given long-term debt structure \(d\), the market value of debt at date zero—the net present value of coupon payments to debtholders plus the bankruptcy payoff—is also unaffected by the manager’s beliefs or actions because the bankruptcy time and bondholders’ bankruptcy payoff only depend on the firm’s base earnings flow and the beliefs of investors (see (1.10) and Proposition 3). Consequently, it follows from (1.30) that the public equity value at date zero does not depend on the manager’s actions. The manager’s actions only affect the NPV of the blockholder’s private benefits through the term \(F_i(b_i^d)\) (see (1.25)).

As in standard dynamic tradeoff models of capital structure, the firm has an incentive to issue long-term debt because of the presence of tax shields on debt coupon payments. By (1.30), debt tax shields only affect the market value of the blockholder’s stake. The NPV of the blockholder’s private benefits is \textit{negatively} affected by long-term debt because of the possibility of bankruptcy. In choosing the firm’s long-term debt, the blockholder trades off the potentially positive effect of long-term debt on the market value of its stake due to debt tax shields against its negative effect on the NPV of its private benefits flow due to the possibility of bankruptcy. If the blockholder’s equity stake \(g\) is significantly less than one (as is typically the case in publicly traded corporations), the incentive to issue long-term debt is much lower because the blockholder does not internalize the positive effects of debt tax
shields on all shareholders. In contrast, in traditional dynamic tradeoff models (e.g. Leland (1994)), long-term debt maximizes total shareholder value. The fact that the blockholder drives capital structure decisions, therefore, leads to significantly lower long-term debt levels that are consistent with those observed in reality. The incorporation of blockholders in the model, therefore, plays an important role in its calibration to data.

1.4.4 Implementation of Manager’s Contract and Dynamic Capital Structure

As in studies such as DeMarzo and Fishman (2007) and Bhagat et al. (2011), we implement the manager’s contract through financial securities. By (1.8), (1.13), and Proposition 1, we can rewrite the manager’s compensation payment in any pre-bankruptcy period $[i, i+1]$, (1.11), as

$$c_i^M = b_i^* \left[ c_i^{TOT} - c_i^D - c_i^{SD} \right],$$

(1.34)

where

$$c_i^{TOT} = (1 - \tau)E_{i+1} + \tau d^*, \quad c_i^D = d^*,$$

$$c_i^{SD} = (1 - \tau)\nu E_{i+1} - \bar{a}_i, \quad \text{and} \quad \bar{a}_i = a_i^*/b_i^*.$$

(1.35)

In the above, $E_{i+1}$ is the firm’s total earnings flow over the period described by (1.1). In (1.34), $c_i^{TOT}$ is the firm’s total after-tax earnings from the project, $c_i^D$ is the long-term debt coupon payment, and $c_i^{SD}$ represents the firm’s total short-term debt payments over the period. Note that, by (1.35), the total payout flow to short-term debt $c_i^{SD}$ reflects the
financing of the firm’s working capital and the manager’s cash compensation. In other words, consistent with what we observe in reality, short-term debt reflects the financing of working capital requirements (inventories, accounts receivable and payable, employee wages, etc.) and the cash compensation of the manager (more generally, insiders).

By the above discussion, the expression $c_i^{TOT} - c_i^D - c_i^{SD}$ in (1.34) represents the total payout flow to all equity holders—the blockholder and shareholders—over the period. Since the manager’s compensation is a proportion $b_i^*$ of the total payout flow to equity in (1.34), her compensation contract is implemented through an inside equity stake, $b_i^*$ (that evolves over time), and dynamic short-term lending or borrowing represented by the short-term debt payments $c_i^{SD}$. In this implementation, the values of long-term debt, short-term debt and outside equity (the value of total equity less the manager’s stake) at any date $i < T_b$ are as follows:

Long-Term Debt Value, $LD(i) = E_i^S \left[ \sum_{j=i}^{T_b-1} e^{-r(j-i)} c_j^D + e^{-r(T_b-i)} LD(T_b) \right]$, 

Short-Term Debt Value, $SD(i) = E_i^S \left[ c_i^{SD} \right]$, 

Outside Equity Value, $S(i) = \widehat{B}(i) + \widehat{E}(i)$ 

$$= E_i^S \left[ \sum_{j=i}^{T_b-1} e^{-r(j-i)} (1 - b_j^*) \left( c_j^{TOT} - c_j^D - c_j^{SD} \right) \right] (1.36)$$

As indicated above, the equity value above is the sum of the blockholder and public equity values. The above-described implementation of the manager’s contract through financial securities leads to a dynamic capital structure for the firm that comprise of inside
equity, outside equity, long-term debt, and dynamic short-term (risk-free) debt that reflects the financing of the firm’s working capital and the manager’s cash compensation. As in DeMarzo and Fishman (2007), the implementation of the manager’s contract is not unique. Similar to their study, the above implementation is intuitive and also captures the main components of financial structure in the real world: long-term debt, short-term debt, inside equity and outside equity, where outside equity includes block equity and public, dispersed equity held by small shareholders.

1.5 Asymmetric Beliefs, Risk-Sharing, and Capital Structure

In this section, we analytically explore some properties of the firm’s capital structure that is described in Section 1.4.4. Given that the focus of this paper is on the impact of asymmetric beliefs on capital structure, the effects of key underlying parameters—the degree of managerial optimism, intrinsic risk, and transient risk—on the manager’s cash and equity compensation are specifically pertinent to our analysis. By (1.34), the manager’s inside equity stake at any date is her pay-performance sensitivity. Proposition 2, therefore, describes the effects of optimism and project risks on the equity component of the manager’s compensation. The manager’s cash compensation (and, therefore, the firm’s short-term debt) described in (1.28) cannot be unambiguously characterized analytically because conflicting effects could cause it to vary non-monotonically (in general) with the underlying parameters. We calibrate the model to data in Section 1.6 to obtain quantitative assessments of these potentially conflicting forces. Our numerical analysis of the calibrated model yields
clear predictions for the variations of the manager’s cash compensation with the underlying parameters.

The following proposition analytically describes the effects of managerial optimism on long-term debt.

**Proposition 5 (Optimism and Long-Term Debt)**

Long-term debt declines with the initial degree of managerial optimism $\Delta_0$.

*Proof.* See Appendix A

As discussed in Section 1.4.3, the manager’s beliefs and actions only affect the second component of blockholder value in (1.31); the NPV of its private benefits. As shown by Proposition 2, an increase in managerial optimism increases the power of incentives that can be provided to the manager and, therefore, the discretionary earnings she generates in each period. The blockholder’s private benefits increase with discretionary earnings. *At the margin*, therefore, as managerial optimism increases, the blockholder assigns relatively more weight to the value of its private benefits than the market value of its equity stake in choosing the firm’s long-term debt. Consequently, an increase in managerial optimism induces the blockholder to choose *lower* long-term debt to lower the likelihood of bankruptcy and, thereby, increase the value of its private benefits.

The effects of intrinsic and transient risks on long-term debt are ambiguous for general parameter values. As shown in Proposition 4, these risks have opposing effects on the *option value* of continuing to service long-term debt coupon payments and, therefore, the bankruptcy time. Moreover, these risks also affect the manager’s incentives, the discretionary earnings she generates in each period and, therefore, the blockholder’s private benefits. Be-
cause the interactions between these forces are complex, an analytical characterization of their net effects for general parameter values cannot be obtained. We numerically explore the effects of the intrinsic and transient risks after calibrating the model to data in the next section.

1.6 Numerical Analysis

The key parameters of the model—the degree of managerial optimism, the expected profitability, the intrinsic risk, the transient risk, and the technology parameters—are comparable across firms in the same industry, but are likely to vary significantly across industries. Consequently, it is more appropriate to calibrate the model “industry by industry.” Although we have calibrated the model to several industries, we report the results for three representative industries—food, software, and entertainment—where the model parameters are likely to vary significantly.

1.6.1 Model Calibration

Basic Economic Parameter Values: We set the risk-free rate $r$ to 4.65% and the effective corporate tax rate $\tau$ to 0.15, which is consistent with the estimates of Graham (2000). The time horizon $T$ is set to 10 years and the length of each period is one year.

Calibration Strategy: Table 1.1 lists the remaining parameters of the model that we group into “technology,” “belief,” “preference,” and “other” categories. We calibrate the baseline values of these parameters by matching key relevant moments predicted by the model to

---

3An “effective” corporate tax rate of 0.15 also incorporates the effects of personal taxes that are not explicitly modeled in our framework.
Table 1.1. Parameters of the Model

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Technology Parameters</strong></td>
<td></td>
</tr>
<tr>
<td>$A$</td>
<td>total factor productivity</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>the share of capital in production</td>
</tr>
<tr>
<td>$s$</td>
<td>the intrinsic risk</td>
</tr>
<tr>
<td><strong>Belief Parameters</strong></td>
<td></td>
</tr>
<tr>
<td>$\mu_0^2$</td>
<td>investors’ initial mean assessment of project quality</td>
</tr>
<tr>
<td>$\Delta_0$</td>
<td>the degree of asymmetric beliefs (managerial optimism)</td>
</tr>
<tr>
<td>$\sigma_0$</td>
<td>the initial transient risk</td>
</tr>
<tr>
<td><strong>Preference Parameters</strong></td>
<td></td>
</tr>
<tr>
<td>$\lambda$</td>
<td>the manager’s risk aversion</td>
</tr>
<tr>
<td>$\kappa; \gamma/\beta$</td>
<td>the manager’s disutility of effort parameters</td>
</tr>
<tr>
<td>$\phi$</td>
<td>the manager’s reservation wage</td>
</tr>
<tr>
<td><strong>Other Parameters</strong></td>
<td></td>
</tr>
<tr>
<td>$\nu$</td>
<td>the sensitivity of working capital to total earnings</td>
</tr>
<tr>
<td>$\rho$</td>
<td>the proportion of bankruptcy cost</td>
</tr>
<tr>
<td>$g$</td>
<td>the blockholder’s equity stake</td>
</tr>
</tbody>
</table>

Their corresponding values in the data. Appendix B describes the numerical implementation of the model and the calibration procedure in detail.

It is likely that “new” or “young” firms are more likely to be characterized by asymmetric beliefs about their prospects. Accordingly, we calibrate the model using data for a sample of firms within three years of their initial public offerings (IPOs). We construct separate empirical samples for the three representative industries—food, software, and entertainment—and calibrate our model separately to each sample.

Given the focus of our study, we calibrate the model to match moments of the capital structure distribution of young firms in each industry. Specifically, for a candidate set of parameter values, we simulate the distributions of (i) the ratio of long-term debt value to asset value, (ii) the ratio of short-term debt value to asset value, and (iii) the ratio of firm value to asset value. The moments that we aim to match are the quintiles of the distributions of the long-term debt to asset value ratio, short-term debt to asset value ratio, and the firm value.
to asset value ratio. The baseline parameter values are those that minimize the “distance”
between the model-predicted and empirical moments (see Appendix B).

**Definitions of the Statistics:** We now define the model proxies for the statistics that we
use in our calibration. Our proxy for the asset value is the value of the hypothetically un-
levered (all-equity) firm in the absence of the manager’s human capital inputs. Consequently,
the asset value at any date \( i \), \( AV(i) \), is the present value (with respect to investors’ beliefs)
of the stream of the firm’s base earnings net of corporate taxes, that is,

\[
AV(i) = E_i^S \left[ \sum_{j=i}^{T-1} e^{-r(j-i)}(1 - \tau)(\Theta + N_{j+1}) \right]. \tag{1.37}
\]

We set the initial investment outlay \( K \) to the asset value at date zero.

We define the long-term debt and short-term debt values at any date \( i \) by the first and
second equations in (1.36). We define the firm value at any date \( i < T_b \), \( FV(i) \), as the sum of
the present value of the stream of the total after-tax earnings from the project \( c_i^{TOT} \), which
is specified in (1.35), and the present value of the bankruptcy payoff to bondholders, that is,

\[
FV(i) = E_i^S \left[ \left( \sum_{j=i}^{T_b-1} e^{-r(j-i)}c_j^{TOT} \right) + e^{-r(T_b-i)}LD(T_b) \right]. \tag{1.38}
\]

By (1.10), the total payoff to bondholders upon bankruptcy \( LD(T_b) \) is equal to a fraction
\( (1 - \rho) \) of the asset value at that date, \( AV(T_b) \).

**Data Description:** We obtain firm-year observations over the period 1992-2009 from the
Standard and Poor’s Compustat Fundamentals Annual database except for those with miss-
ing variables or with nonpositive assets or investment. As mentioned earlier, we construct
separate samples of firms within three years of their IPOs in the food, software, and entertainment industries.\(^4\)

We use Compustat item \(DLTT\) and \(CAPXV\) for long-term debt and investment and calculate short-term debt as debt in current liabilities (item \(DLC\)) minus debt due in one year (item \(DD1\)) minus cash (item \(CHE\)). We measure firm value by the sum of equity value, which is the closing stock price (item \(PRCC_F\)) multiplied by the number of common shares outstanding (item \(CSHO\)), and total debt value, which is long-term debt (item \(DLTT\)) plus debt in current liabilities (item \(DLC\)) minus cash (item \(CHE\)). These variables are all divided by asset value (item \(AT\)) to eliminate scale differences between firms and then winsorized at the respective 2nd and 98th percentiles. Table 1.2 shows that there are substantial differences in the values of the statistics across the three industries. In particular, software firms typically have lower debt ratios and higher firm value ratios than firms in the other two industries.

**Baseline Parameter Values:** We calibrate the baseline values of the model parameters listed in Table 1.1 by matching the observed and model-predicted moments as closely as possible. Table 1.2 (Panels A, B and C) compares the observed values with the predicted values of the statistics for the different industries. The model is able to match the observed statistics reasonably well. We report the baseline values of the parameters in Table 1.3.

The calibrated parameters suggest that managers are significantly optimistic relative to investors in the three industries. It is worth noting that the relative level of managerial optimism (compared to the expected profitability \(\mu_0^S\) by outside investors) is higher in the software and entertainment industries as one would expect from anecdotal evidence.

\(^4\)We use the Fama-French 48-industry classification to obtain the separate samples.
Table 1.2. Observed and Predicted Statistics for the Food, Software, and Entertainment Industries

The table displays our calibration results for three samples of firms that belong to the food, software, and entertainment industries, respectively. Each sample consists of the observations within 3 years of their IPO from the Compustat database. Their long-term and short-term debt values and firm value are scaled by asset value and winsorized at the 2nd and 98th percentiles. We use the quintiles of these ratios, denoted by Q1, Q2, Q3, Q4, and Q5, as the moments to be matched in the calibration.

Panel A: Food Industry

<table>
<thead>
<tr>
<th>Q1</th>
<th>Long-term Debt Ratio</th>
<th>Q2</th>
<th>Q3</th>
<th>Q4</th>
<th>Q5</th>
<th>Short-term Debt Ratio</th>
<th>Q2</th>
<th>Q3</th>
<th>Q4</th>
<th>Q5</th>
<th>Firm Value Ratio</th>
<th>Q2</th>
<th>Q3</th>
<th>Q4</th>
<th>Q5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Observed</td>
<td>0.0264</td>
<td>0.1456</td>
<td>0.2394</td>
<td>0.3913</td>
<td>0.7172</td>
<td>-0.1770</td>
<td>-0.0333</td>
<td>-0.0072</td>
<td>0.0228</td>
<td>0.2101</td>
<td>0.7288</td>
<td>0.9443</td>
<td>1.3261</td>
<td>1.7970</td>
<td>5.4598</td>
</tr>
<tr>
<td>Predicted</td>
<td>0.0222</td>
<td>0.0432</td>
<td>0.3744</td>
<td>0.4477</td>
<td>0.6658</td>
<td>-0.1497</td>
<td>-0.0433</td>
<td>-0.0067</td>
<td>0</td>
<td>0</td>
<td>0.4477</td>
<td>1.1700</td>
<td>1.2992</td>
<td>1.6611</td>
<td>6.0539</td>
</tr>
</tbody>
</table>

Panel B: Software Industry

<table>
<thead>
<tr>
<th>Q1</th>
<th>Long-term Debt Ratio</th>
<th>Q2</th>
<th>Q3</th>
<th>Q4</th>
<th>Q5</th>
<th>Short-term Debt Ratio</th>
<th>Q2</th>
<th>Q3</th>
<th>Q4</th>
<th>Q5</th>
<th>Firm Value Ratio</th>
<th>Q2</th>
<th>Q3</th>
<th>Q4</th>
<th>Q5</th>
<th>Q5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Observed</td>
<td>0</td>
<td>0</td>
<td>0.0043</td>
<td>0.0384</td>
<td>0.6238</td>
<td>-0.6972</td>
<td>-0.5357</td>
<td>-0.3633</td>
<td>-0.1592</td>
<td>0.2581</td>
<td>0.6131</td>
<td>1.3749</td>
<td>2.4884</td>
<td>4.8699</td>
<td>24.0403</td>
<td></td>
</tr>
<tr>
<td>Predicted</td>
<td>0.0012</td>
<td>0.0018</td>
<td>0.0034</td>
<td>0.0522</td>
<td>0.3401</td>
<td>-0.9254</td>
<td>-0.4213</td>
<td>-0.2576</td>
<td>-0.1687</td>
<td>0</td>
<td>1.4817</td>
<td>1.7028</td>
<td>2.0724</td>
<td>3.2538</td>
<td>23.7778</td>
<td></td>
</tr>
</tbody>
</table>

Panel C: Entertainment Industry

<table>
<thead>
<tr>
<th>Q1</th>
<th>Long-term Debt Ratio</th>
<th>Q2</th>
<th>Q3</th>
<th>Q4</th>
<th>Q5</th>
<th>Short-term Debt Ratio</th>
<th>Q2</th>
<th>Q3</th>
<th>Q4</th>
<th>Q5</th>
<th>Firm Value Ratio</th>
<th>Q2</th>
<th>Q3</th>
<th>Q4</th>
<th>Q5</th>
<th>Q5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Observed</td>
<td>0.0066</td>
<td>0.0705</td>
<td>0.2571</td>
<td>0.4751</td>
<td>0.9710</td>
<td>-0.2731</td>
<td>-0.1145</td>
<td>-0.0445</td>
<td>-0.0145</td>
<td>0.7308</td>
<td>0.7212</td>
<td>1.0730</td>
<td>1.6107</td>
<td>2.9274</td>
<td>11.1078</td>
<td></td>
</tr>
<tr>
<td>Predicted</td>
<td>0.0070</td>
<td>0.0144</td>
<td>0.2576</td>
<td>0.5528</td>
<td>0.5782</td>
<td>-0.2740</td>
<td>-0.0931</td>
<td>-0.0353</td>
<td>0</td>
<td>0</td>
<td>0.5528</td>
<td>1.1932</td>
<td>1.3548</td>
<td>1.8095</td>
<td>9.06105</td>
<td></td>
</tr>
</tbody>
</table>
Table 1.3. Baseline Parameter Values for the Food, Software, and Entertainment Industries

<table>
<thead>
<tr>
<th>Panel A: Food Industry</th>
<th>Panel B: Software Industry</th>
<th>Panel C: Entertainment Industry</th>
</tr>
</thead>
<tbody>
<tr>
<td>Technology Parameters</td>
<td>Belief Parameters</td>
<td>Preference Parameters</td>
</tr>
<tr>
<td>$A$</td>
<td>$\alpha$</td>
<td>$s$</td>
</tr>
<tr>
<td>85.552</td>
<td>0.358</td>
<td>66.209</td>
</tr>
<tr>
<td>569.190</td>
<td>0.132</td>
<td>51.537</td>
</tr>
<tr>
<td>85.720</td>
<td>0.358</td>
<td>70.144</td>
</tr>
</tbody>
</table>
To compare the relative levels of managerial optimism across the three industries, we compute the “market to book ratio” under the manager’s beliefs and under the investors’ beliefs, respectively, for each of the industries. More precisely, we compute the firm value at date zero, as specified in (1.38), under the manager’s and investors’ beliefs, respectively. We then divide each by the initial investment outlay which is assumed to be the asset value under investors’ beliefs at date zero. We observe that the average manager in the sample of young firms in the food industry overestimates the “market to book ratio” by 159% relative to investors. In contrast, the average managers in the sample of young firms in the software and the entertainment industries overestimate the market to book ratio by 175% and by 211%, respectively. We also find that the ratios of the baseline value of the initial transient risk to that of the intrinsic risk for the food and the entertainment industries are 2.45 and 1.70 respectively, while the ratio is 4.58 for the software industry, which suggests that there is more significant uncertainty about project quality for software firms. Further, as one would expect from anecdotal evidence, the total factor productivity and the reservation wage of managers are higher for software firms.

1.6.2 Sensitivity Analysis

We now explore the effects of managerial optimism $\Delta_0$, the transient risk $\sigma^2_0$, the intrinsic risk $s^2$, and investors’ initial mean assessments $\mu^S_0$ (or the expected profitability of the project) on capital structure by varying these parameters about their baseline values for each of the industries. We investigate the effects of varying these parameters on the median
values of the long-term debt ratio, short-term debt ratio, and the inside equity stake that we calculate by simulating the model over 3 years as described in Appendix B.

**Effects of Managerial Optimism:** Figure 1.1 (Panels A, B and C) displays the variations of the median values of the long-term debt ratio and the short-term debt ratio with the initial degree of managerial optimism $\Delta_0$ for the three industries. Consistent with Proposition 5, long-term debt declines with the degree of optimism. By contrast, the short-term debt ratio increases with managerial optimism. Interestingly, while the *qualitative* effects of managerial optimism are similar for the two industries, the *quantitative* effects are very different. Long-term debt declines only slightly with managerial optimism in the software industry, but declines significantly in the food and entertainment industries.

To understand the effects of optimism on short-term debt, consider the definition of short-term debt in (1.35). By Proposition 2, an increase in managerial optimism increases the manager’s inside equity stake in each period that, in turn, lowers her cash compensation relative to her equity compensation. Second, because the manager receives more powerful incentives and thus exerts greater effort, the firm’s discretionary earnings increase so that the short-term debt payments associated with its working capital requirements increase. Consequently, short-term debt increases with managerial optimism. In unreported results, we also find that outside equity declines with managerial optimism, while the leverage ratio increases.

Our results are broadly consistent with empirical evidence. Landier and Thesmar (2009) empirically show that more optimistic managers prefer short-term debt over long-term debt. Using various measures of agreement about project payoffs between managers and investors,
Dittmar and Thakor (2007) find that managers tend to use debt rather than equity in the presence of disagreement.

**Effects of Transient Risk and Intrinsic Risk:** Figures 1.2 and 1.3 show the effects of the initial transient risk and the intrinsic risk on capital structure in the two industries. The figures show that the two risks have contrasting effects on long-term debt and short-term debt. Long-term debt varies non-monotonically in a U-shaped manner with transient risk, but increases with intrinsic risk. Short-term debt increases with the transient risk, but decreases with the intrinsic risk.

The intuition for the contrasting effects of the project’s intrinsic and transient risks on long-term debt hinges on their differing effects on the probability of bankruptcy. As shown by Proposition 4, the transient risk lowers the probability of bankruptcy and, thereby, has a positive impact on the present value of blockholders’ private benefits. An increase in the transient risk, however, also increases the costs of risk-sharing and lowers the discretionary earnings the manager generates in each period, which negatively affects the blockholders’ private benefits. At low transient risk levels, the “bankruptcy” effect dominates the “risk-sharing” effect so that blockholders choose lower long-term debt to increase the value of their private benefits. At high transient risk levels, the costs of risk-sharing are so high that the value of blockholders’ private benefits is very low. Consequently, they increase long-term debt to exploit its tax advantages. An increase in the intrinsic risk, however, increases the costs of risk-sharing and also hastens bankruptcy by Proposition 4. Both of these effects work in the same direction to lower the value of blockholders’ private benefits. Consequently,
the marginal impact of blockholders’ private benefits on their long-term choice is lowered so that they choose greater long-term debt to exploit debt tax shields.

The intuition for the contrasting effects of the transient and intrinsic risks on short-term debt is as follows. An increase in the intrinsic risk increases the probability of bankruptcy. It also increases the costs of risk-sharing and, therefore, the manager’s cash compensation relative to her equity compensation. Both forces have a negative impact on the short-term debt value. An increase in transient risk, however, lowers the probability of bankruptcy that has a positive impact on the short-term debt value. Consequently, short-term debt increases with the transient risk.

To the best of our knowledge, the contrasting effects of the intrinsic and transient risks on long-term debt and short-term debt are novel implications of our study that have not been explored by previous studies that examine the effects of asymmetric beliefs on capital structure.

**Effects of Expected Profitability:** Figure 1.4 shows the effects of investors’ initial mean assessment of the project’s profitability $\mu_0$ on the firm’s capital structure. We see from the figure that long-term debt increases with the expected profitability, but the effect on short-term debt varies by industry. Further, the impact on long-term debt is relatively more pronounced than that on short-term debt.

The effects of the expected profitability on blockholders’ long-term debt choice are subtle because it affects both components of blockholder value in (1.31). An increase in the expected profitability increases the public equity value and, therefore, the first component of blockholder value. However, an increase in the expected profitability also delays bankruptcy
Figure 1.1. Effects of Managerial Optimism

Figure 1.2. Effects of Transient Risk
Figure 1.3. Effects of Intrinsic Risk

Figure 1.4. Effects of Expected Profitability
by Proposition 4 and, therefore, also increases the second component. For the calibrated model, the former effect dominates the latter. Blockholders, therefore, chooses greater long-term debt to increase the market value of their equity stake.

An increase in the expected profitability increases the firm’s expected base earnings and, therefore, the firm’s working capital requirements. As mentioned above, a higher expected profitability also lowers the probability of bankruptcy. However, since short-term debt is also negatively affected by the long-term debt coupon payment, the effect of slightly decreasing the firm’s short-term debt is observed for the food industry.

1.7 Conclusion

We examine the effects of heterogeneous beliefs on capital structure in a dynamic framework. The manager of a firm that has large and small shareholders receives dynamic incentives through explicit contracts. Managers and investors have imperfect information and differing beliefs about the project’s profitability. We derive the manager’s contracts and implement them through financial securities, which leads to a dynamic capital structure that reflects the effects of external imperfections arising from taxes and bankruptcy costs as well as internal imperfections arising from asymmetric beliefs and agency conflicts. The theoretical and numerical analyses of the model generate novel testable implications that link project characteristics—the degree of managerial optimism, intrinsic risk, transient risk, and expected profitability—to different components of capital structure. In particular, our results show that managerial optimism has contrasting effects on long-term debt and short-term debt. The intrinsic and transient risks have conflicting effects on debt structure. The
interplay among imperfect information, asymmetric beliefs, and agency conflicts plays a central role in generating these predictions.

We calibrate the model parameters to different industries. While the qualitative effects of optimism and risk on capital structure are similar across industries, their quantitative effects are significantly different. Broadly, our study shows that asymmetric beliefs are important determinants of firms’ financial policies and could potentially reconcile the substantial inter-industry variation in capital structure observed in the data.

1.8 Appendix A: Proofs

1.8.1 Proof of Proposition 1

We use backward induction to solve the manager’s optimal contracting problem for a given long-term debt structure $d$. We normalize the interest rate to zero to simplify notation in this proof as well as subsequent proofs.

We first derive the optimal contractual parameters in period $[T-1, T]$. Suppose that the firm is solvent as of date $i = T - 1$. The manager’s conditional expected utility derived from her compensation including her disutility of effort over this period is

$$M(i) := E_i^M \left[ - \exp \left( - \lambda \left( c_i^M - \kappa \eta_i^\gamma \right) \right) \right] = - \exp \left( - \lambda \Lambda (a_i, b_i, k_i, \eta_i) \right), \quad (1.39)$$

where

$$\Lambda(a_i, b_i, k_i, \eta_i) = a_i + (1-\tau)(1-\nu) \left( \mu_i^M + Ak_i^\alpha \eta_i^\beta \right) b_i - \kappa \eta_i^\gamma - (1-\tau)db_i - 0.5\lambda(1-\tau)^2(1-\nu)^2(\sigma_i^2 + s^2)b_i^2. \quad (1.40)$$

Equations (1.39) and (1.40) are followed from (1.1), (1.8), (1.11), and the fact that $\Theta + N_{i+1} \sim N(\mu_i^M, \sigma_i^2 + s^2)$ under the manager’s beliefs.

A feasible contract must be incentive compatible, that is, it is optimal for the manager to exert the specified effort in the contract given the other contractual variables $(a_i, b_i, k_i)$. We assume that

$$(1 - \alpha)\gamma / \beta > 2, \quad (1.41)$$

which guarantees a unique optimal effort choice. The manager’s optimal effort $\eta_i$ as a function of the other variables is given by
\[ \eta_i = \eta(b_i, k_i) = \left[ \frac{A(1 - \tau)(1 - \nu)\beta k_i^\alpha b_i}{\gamma \kappa} \right]^\frac{1}{\gamma - \beta} \]. \tag{1.42}

In addition, shareholders’ participation constraint (1.19) must be satisfied by the remaining contractual variables \((a_i, b_i, k_i)\):

\[ CV_i = E_i^S \left[ c_i^E - (1 - g)k_i \right] \geq (1 - g)E_i^S \left[ (1 - \tau)(1 - \nu)(\Theta + N_{i+1}) - d \right]. \tag{1.43} \]

It is evident that the participation constraint (1.43) and the manager’s participation constraint (1.17) must be satisfied with equality to maximize the blockholder value. By the equality of (1.17), the contractual parameter \(a_i\) is derived as a function of the other variables:

\[ a_i = a(b_i, k_i, \eta_i) = \phi + \kappa \eta_i^\gamma + 0.5\lambda(1 - \tau)^2(1 - \nu)^2b_i^2(\sigma_i^2 + s^2) - b_i(1 - \tau) \left[ (1 - \nu) \left( \mu_i^M + Ak_i^\alpha \eta_i^{\beta} \right) - d \right]. \tag{1.44} \]

By (1.18), (1.43), which must be satisfied with equality, and (1.44), the blockholder’s objective function to determine the manager’s contract at date \(i\) is

\[ \Pi(b_i, k_i) = E_i^S \left[ c_i^B - gk_i \right] = g(1 - \tau) \left[ (1 - \nu)\mu_i^S - d \right] - \phi + (1 - \tau)(1 - \nu)\Delta_i b_i + \psi(b_i)k_i^\frac{\alpha\gamma}{\gamma - \beta} - k_i - 0.5\lambda(1 - \tau)^2(1 - \nu)^2(\sigma_i^2 + s^2)b_i^2, \tag{1.45} \]

where

\[ \psi(b_i) := \left( A(1 - \tau)(1 - \nu) \right)^\frac{\gamma}{\gamma - \beta} \left( \frac{1}{\kappa} \right)^\frac{\beta}{\gamma - \beta} \left( \frac{\beta b_i}{\gamma} \right)^\frac{\beta}{\gamma - \beta} \left( 1 - \frac{\beta b_i}{\gamma} \right). \tag{1.46} \]

It now remains to determine the incremental capital investment and the manager’s pay-performance sensitivity that maximize (1.45). For \(b_i \geq \gamma/\beta\), it follows from (1.45) that the optimal investment is zero. For \(b_i \in [0, \gamma/\beta)\), the optimal capital investment as a function of the manager’s pay-performance sensitivity is

\[ k(b_i) = \left[ \frac{\alpha \gamma}{\gamma - \beta} \psi(b_i) \right]^\frac{\gamma - \beta}{(1 - \alpha)\gamma - \beta}. \tag{1.47} \]

By (1.47), the optimal contract choice problem reduces to the optimal choice of the manager’s pay-performance sensitivity

\[ b_i^* = \arg \max_{b_i \geq 0} \Pi(b_i, k(b_i)) = \arg \max_{b_i \geq 0} F_i(b_i), \tag{1.48} \]
where
\[ F_i(b_i) = (1 - \tau)(1 - \nu)\Delta_i b_i + Bk(b_i) - 0.5\lambda (1 - \tau)^2(1 - \nu)^2(\sigma_i^2 + s^2)b_i^2 \] (1.49)
and \[ B = \frac{(1-\alpha)\gamma - \beta}{\alpha}. \]

In sum, the optimal contracting problem for period \([T - 1, T]\), conditional on the firm’s solvency at the beginning of this period, is completely specified by \(b_i^*\). The optimal capital investment, \(k_i^*\), is \(k(b_i^*)\), the manager’s optimal effort, \(\eta_i^*\), is \(\eta(b_i^*, k_i^*)\), and her cash compensation, \(a_i^*\), is \(a(b_i^*, k_i^*, \eta_i^*)\). Note that the contract exists in period \([T - 1, T]\) if and only if the firm is solvent at date \(i = T - 1\). If the public equity value at this date is non-positive, bankruptcy is declared and the relationship is terminated, that is, \(T_b = T - 1\).

We now set \(i = T - 2\) and consider the optimal contracting problem for period \([i, i + 1]\). Suppose that the firm is still solvent at date \(i\). By the law of iterated expectations, the manager’s conditional expected utility can be written as
\[
M(i) = E_i^M \left[ -\exp \left( -\lambda \left( \sum_{j=i}^{T_b-1} (c_j^M - \kappa \eta_j^i) \right) \right) \right]
= -\exp \left( -\lambda \Lambda (a_i, b_i, k_i, \eta_i) \right) \left( -\hat{M}^*(i + 1) \right),
\] (1.50)
where \(\Lambda (a_i, b_i, k_i, \eta_i)\) is given by (1.40). The last term \(-\hat{M}^*(i + 1)\) equals one if bankruptcy occurs at date \(i + 1\) because the manager’s future payoffs will be zero. If bankruptcy is not declared at date \(i + 1\), the term is the conditional expected utility at date \(i + 1\), which was determined by the contracting problem for period \([T - 1, T]\) above. In either case, the term is unaffected by the contractual variables for the current period.

Consequently, as in the analysis of period \([T - 1, T]\), the incentive compatibility of the manager’s contract requires that her effort maximize \(\Lambda (a_i, b_i, k_i, \eta_i)\) for given \((a_i, b_i, k_i)\). Moreover, the dynamic participation constraint (1.19) for shareholders is now given by
\[
CV_i = E_i^S \left[ c^F - (1 - g)k_i + \max \left\{ CV_{i+1}, 0 \right\} \right]
\geq (1 - g)E_i^S \left[ \sum_{j=i}^{T_b-1} (1 - \tau) \left( (1 - \nu)(\Theta + N_{j+1}) - d \right) \right].
\] (1.51)

If \(CV_{i+1} > 0\) so that \(T_b > i + 1\), then it follows from the analysis of period \([T - 1, T]\) that
\[
CV_{i+1} = (1 - g)E_{i+1}^S \left[ \sum_{j=i+1}^{T_b-1} (1 - \tau) \left( (1 - \nu)(\Theta + N_{j+1}) - d \right) \right].
\]
It immediately follows from the above that, regardless of whether bankruptcy is declared at date \(i + 1\), shareholders’ participation constraint (1.51) is equivalent to the following:
\[ E^S_i \left[ c^E_i - (1-g)k_i \right] \geq (1-g)E^S_i \left[ (1-\tau) \left( (1-\nu)(\Theta + N_{i+1}) - d \right) \right], \]

which is identical in form to (1.43). Using similar arguments, the manager's participation constraint (1.17) is also identical in form to the participation constraint in the last period. Consequently, the optimization problem is identical to the one for period \([T-1,T]\). Hence optimal contractual variables for period \([T-2,T-1]\) are determined in the same manner as in the contracting problem for period \([T-1,T]\). This argument can clearly be extended by backward induction to any period \([i,i+1]\) for \(i < T-2\). Q.E.D.

1.8.2 Proof of Proposition 2

By (1.48) and (1.49),

\[ b^*_i = \arg \max_{b_i \geq 0} F_i(b_i) = (1-\tau)(1-\nu)\Delta_i b_i - 0.5\lambda(1-\tau)^2(1-\nu)^2(\sigma_i^2 + s^2)b_i^2 + \frac{(1-\alpha)\gamma - \beta}{\alpha \gamma} k(b_i). \]

We first establish some properties of the "optimal investment function" \(k(b_i)\) since they determine the properties of the optimal contract. By (1.46) and (1.47),

\[ k'(b) \propto \left( \frac{1}{k} \right)^{\frac{1}{t-1}} b^t(\gamma/\beta - b)^u(1-b), \]

where

\[ t := \frac{2 - (1-\alpha)(\gamma/\beta)}{(1-\alpha)(\gamma/\beta) - 1} and \quad u := \frac{\alpha \gamma}{(1-\alpha)\gamma - \beta}, \]

and where the symbol \(\propto\) means "equal up to a positive multiplicative constant." Under Assumption (1.41), the parameter \(t\) is negative and the parameter \(u\) is positive. Since \(\gamma/\beta > 1\) (Assumption (1.41)), it follows from (1.53) that the function \(k(\cdot)\) is strictly quasi-concave. Since \(k(0) = k(\gamma/\beta) = 0\) and \(k'(0) = +\infty\), it also follows from (1.53) that \(k(\cdot)\) achieves its maximum at \(b = 1\).

Next, we observe that

\[ k''(b) \propto b^{t-1}(\gamma/\beta - b)^{u-1}[t(\gamma/\beta - b)(1-b) - ub(1-b) - b(\gamma/\beta - b)]. \]

The expression inside the brackets is a strictly convex quadratic function whose value at 1 is negative, whose value at \(\gamma/\beta > 1\) is positive, and whose value at 0 is negative, since \(t < 0\). Consequently, there is exactly one root \(b^M\) of the quadratic in the interval \((1, \gamma/\beta)\) such that \(k''(b^M) = 0\). At \(b^M\) the marginal investment is at its minimum. Moreover, since \(k''(\cdot)\) is negative on \([0,b^M]\) and is positive on \((b^M, \gamma/\beta)\), the optimal investment function is strictly concave on \([0,b^M]\) and strictly convex on \([b^M, \gamma/\beta]\).

We henceforth assume that

\[ \Delta_0 \leq \frac{\lambda(1-\tau)(1-\nu)(\sigma_0^2 + s^2)}{b^M}. \]
(a) We write $F_i(b; \Delta_i)$ to make explicit the functional dependence of $F_i$ on the parameter $\Delta_i$, and write $F'_i(\cdot; \Delta_i)$ to denote its derivative with respect to $b$. Let $b(\Delta_i)$ denote the optimal pay-performance sensitivity when the degree of managerial optimism is $\Delta_i$.

We first show that

$$b(\Delta_i) \leq \max \left( \frac{\Delta_i}{p_i}, 1 \right).$$

where

$$p_i = \lambda(1-\tau)(1-\nu)(\sigma_i^2 + s^2).$$

Obviously the result immediately follows if $b(\Delta_i) \leq 1$, so suppose that $b(\Delta_i) > 1$. It remains to show that $b(\Delta_i) \leq \Delta_i/p_i$. Since by assumption $b(\Delta_i) \geq 1$, it follows from the properties of the optimal investment function established earlier that $b'(b(\Delta_i)) \leq 0$. Since $F'_i(b(\Delta_i)) = 0$, it follows from (1.52) that $\Delta_i - p_i b(\Delta_i) \geq 0$, which establishes that $b(\Delta_i) \leq \Delta_i/p_i$, as required.

Next, observe that

$$\frac{\Delta_i}{p_i} = \frac{\Delta_0}{\lambda(1-\tau)(1-\nu)(\sigma_i^2 + s^2)} \leq \frac{\Delta_0}{\rho_0} \leq b^M$$

by Assumption (1.55). Hence,

$$b(\Delta_i) \leq b^M.$$  

Pick two values $\Delta_1^i$ and $\Delta_2^i$ such that $\Delta_1^i < \Delta_2^i$. We must show that $b(\Delta_1^i) < b(\Delta_2^i)$. Since $b(\Delta_1^i)$ maximizes $F_i(\cdot; \Delta_1^i)$, and since $\Delta_1^i < \Delta_2^i$, it follows that

$$0 = (1-\tau)(1-\nu)\Delta_1^i - \lambda(1-\tau)^2(1-\nu)^2(\sigma_i^2 + s^2) b(\Delta_1^i) + \frac{(1-\alpha)\gamma - \beta}{\alpha\gamma} k'(b(\Delta_1^i))$$

$$< (1-\tau)(1-\nu)\Delta_2^i - \lambda(1-\tau)^2(1-\nu)^2(\sigma_i^2 + s^2) b(\Delta_1^i) + \frac{(1-\alpha)\gamma - \beta}{\alpha\gamma} k'(b(\Delta_1^i)) = F'_i(b(\Delta_1^i); \Delta_2^i).$$

It follows from (1.59) and the properties of the optimal investment function that $F_i(\cdot, \Delta_2^i)$ is strictly concave over the range of possible values of $b(\Delta_1^i)$. Consequently, its derivative can only be positive at values below the unique maximizer $b(\Delta_2^i)$, which proves that $b(\Delta_1^i) < b(\Delta_2^i)$, as required.

(b) We note that

$$F'_i(b, \sigma_0) = \frac{(1-\tau)(1-\nu)\Delta_0 s^2}{s^2 + i\sigma_0^2} - \lambda(1-\tau)^2(1-\nu)^2bs^2 \frac{s^2 + (i+1)\sigma_0^2}{s^2 + i\sigma_0^2} + \frac{(1-\alpha)\gamma - \beta}{\alpha\gamma} k'(b),$$

which is decreasing in $\sigma_0$. $b_i(\sigma_0)$ denote the optimal pay-performance sensitivity when the initial transient risk is $\sigma_0$. Consider two values $\sigma_1^i < \sigma_2^i$ of $\sigma_0$. By definition,

$$0 = F'_i(b_i(\sigma_2^i), \sigma_2^i) = F'_i(b_i(\sigma_1^i), \sigma_1^i) > F'_i(b_i(\sigma_1^i), \sigma_2^i),$$

which immediately implies $b_i(\sigma_1^i) > b_i(\sigma_2^i)$ by the strict quasi-concavity of $F_i(\cdot)$. 

(c) We note that
\[ F'_i(b, s) = \frac{(1 - \tau)(1 - \nu)\Delta_0 s^2}{s^2 + i\sigma_0^2} - \lambda(1 - \tau)^2(1 - \nu)^2s^2 + (i + 1)\sigma_0^2 + \frac{(1 - \alpha)\gamma - \beta}{\alpha\gamma}k'(b). \]

By arguments similar to those used in parts (a) and (b), it suffices to show that \( F'_i(b, s) \) is decreasing in \( s \).

Clearly, \( F'_0(b, s) \) is decreasing in \( s \). Now suppose \( i \geq 1 \). The sign of the derivative of \( F'_i(b, s) \) with respect to \( s^2 \) coincides with the sign of
\[ -\left( b\lambda(1 - \tau)(1 - \nu)s^4 + i\sigma_0^2 [b\lambda(1 - \tau)(2s^2 + (i + 1)\sigma_0^2) - \Delta_0] \right), \]
and therefore the result will follow if we can establish that
\[ b^*_i\lambda(1 - \tau)(1 - \nu)(2s^2 + (i + 1)\sigma_0^2) \geq \Delta_0 \]
and \( b^*_i > 0 \). To this end, let
\[ \hat{b}_i := \frac{\Delta_0}{\lambda(1 - \tau)(1 - \nu)(2s^2 + (i + 1)\sigma_0^2)}. \]

Since \( \Delta_0 \leq 2p_0 \) by the assumption in part (c) of the proposition, \( \hat{b}_i \leq 1 \), which implies \( k'(\hat{b}_i) \) is nonnegative. Therefore,
\[ F'_i(\hat{b}_i, s^2) = \frac{(1 - \alpha)\gamma - \beta}{\alpha\gamma}k'(\hat{b}_i) + \frac{(1 - \tau)(1 - \nu)\Delta_0 s^2}{s^2 + i\sigma_0^2} \left[ 1 - \frac{s^2 + (i + 1)\sigma_0^2}{2s^2 + (i + 1)\sigma_0^2} \right] \geq 0. \]

Hence, we may conclude that \( b^*_i \geq \hat{b}_i > 0 \) since \( F_i(\cdot) \) is strictly quasi-concave and \( b^*_i \in [0, b^M] \).

Thus,
\[ b^*_i\lambda(1 - \tau)(1 - \nu)(2s^2 + (i + 1)\sigma_0^2) \geq \hat{b}_i\lambda(1 - \tau)(1 - \nu)(2s^2 + (i + 1)\sigma_0^2) = \Delta_0 \]
as required. Q.E.D.

1.8.3 Proof of Proposition 3

As mentioned earlier, the bankruptcy time is endogenously determined by the optimal stopping problem (1.29). The continuation value of shareholders at date \( i \leq T - 1 \) is given by (1.51), which must be satisfied with equality. We denote the continuation value as \( CV_i(\mu_i^S) \) to make explicit its dependence on shareholders’ mean posterior assessment of project quality at each date \( i \). Let \( Z \) denote a standard normal random variable. By (1.4), (1.5), and (1.6), the continuation value can be expressed as
\[ CV_i(\mu_i^S) = (1 - \tau)(1 - \nu)\mu_i^S - d + E^S_i \left[ \max \left\{ CV_{i+1}(\mu_i^S + \sigma_i^u Z), 0 \right\} \right], \] (1.60)
where we replace \( \mu_{i+1}^S \) with \( \mu_i^S + \sigma_i^\mu Z \), and \( \sigma_i^\mu \) is defined in (1.33). We first need to show that the continuation value \( CV_i(\cdot) \) is a continuous, non-decreasing function of \( \mu_i^S \) for \( 0 \leq i \leq T - 1 \).

To do so, we use backward induction. For the proof of its continuity, we show that there exist positive constants \( \kappa_1^i, \kappa_2^i \) such that

\[
CV_i(\mu_i^S) \leq \kappa_1^i + \kappa_2^i \max\{\mu_i^S, 0\}.
\]  

(1.61)

The assertions of monotonicity and continuity for date \( T - 1 \) immediately follow because \( CV_{T-1}(\mu_{T-1}^S) = (1 - \tau)((1 - \nu)\mu_{T-1}^S - d) \). Suppose the assertions are true for dates \( i + 1, \ldots, T - 2 \). Then we only need to demonstrate that the assertions also hold for date \( i \).

The monotonicity of \( CV_i(\cdot) \) is a direct consequence on the fact that the expectation on the right-hand side of (1.60) is taken with respect to the standard normal density, and the monotonicity of \( CV_{i+1}(\cdot) \) by the inductive assumption.

The proof of continuity of \( CV_i(\cdot) \) will be accomplished by (1.60) if the limit and expectation operators can be interchanged because \( CV_{i+1}(\cdot) \) is continuous in \( \mu_{i+1}^S \) by the inductive assumption and, therefore, in \( \mu_i^S \). Due to the assumed continuity for date \( i + 1 \), the function \( CV_{i+1}(\cdot) \) is bounded by a positive function whose expectation,

\[
E_i^{\mu_i^S}[\kappa_{i+1}^1 + \kappa_{i+1}^2 \max\{\mu_{i+1}^S, 0\}] = \kappa_{i+1}^1 + \kappa_{i+1}^2 \left[ \frac{\sigma_i^\mu}{\sqrt{2\pi}} \exp \left[ -\frac{1}{2} \left( \frac{\mu_i^S}{\sigma_i^\mu} \right)^2 \right] + \mu_i^S P(Z > -\frac{\mu_i^S}{\sigma_i^\mu}) \right],
\]  

(1.62)

is finite, and thus the interchange is justified by the dominated convergence theorem. Further, by (1.62), we can see that the expectation on the right-hand side of (1.60) is bounded above by

\[
\left( \kappa_{i+1}^1 + \kappa_{i+1}^2 \frac{\sigma_i^\mu}{\sqrt{2\pi}} \right) + \kappa_{i+1}^2 \max\{\mu_i^S, 0\}.
\]  

(1.63)

Hence it is possible to define positive constants \( \kappa_1^i \) and \( \kappa_2^i \) for which (1.61) holds for date \( i \), as required.

So far, we have proved that each function \( CV_i(\cdot) \) is continuous and non-decreasing. By (1.60), each \( CV_i(\cdot) \) is negative for sufficiently small \( \mu_i^S \). Since each \( CV_i(\cdot) \) is obviously positive for sufficiently high \( \mu_i^S \), there exists a unique value \( \mu_i^\ast \) for which \( CV_i(\mu_i^\ast) = 0 \). By the dynamic programming principle of optimality, bankruptcy is declared at date \( i \) if and only if \( \mu_i^S \leq \mu_i^\ast \). Q.E.D.

### 1.8.4 Proof of Proposition 4

From the proof of Proposition 3, we know that \( CV_i(\cdot) \) is a continuous, non-decreasing function of \( \mu_i^S \) and, therefore, of \( \mu_i^S \) because of its linear relation with \( \mu_i^S \) as shown in (1.6). We now demonstrate that \( CV_i(\cdot) \) increases with the initial transient risk \( \sigma_0^2 \), but decreases with the intrinsic risk \( S^2 \). By backward induction, we first show that each \( CV_i(\cdot) \) is convex in \( \mu_i^S \). It obviously holds for date \( T - 1 \) because it is a linear function of \( \mu_{T-1}^S \). Suppose the assertion is true for dates \( i + 1, \ldots, T - 2 \). Based on the assumption, we need to establish
the assertion for date $i$. By the definition of a convex function, we need to show that, for any two points $x$ and $y$ in the domain of $CV_i(\cdot)$ and any $t \in [0, 1]$,

$$CV_i(tx + (1-t)y) \leq tCV_i(x) + (1-t)CV_i(y).$$

(1.64)

Using (1.60) and the inductive assumption that $CV_{i+1}(\cdot)$ is convex in $\mu_{i+1}^S$, we can see that the left hand side of (1.64) is less than or equal to $(1 - \tau)((1 - \nu)(tx + (1-t)y) - d) + E_i^S \left[ \max \{ tCV_{i+1}(x + \sigma_{i}^gZ) + (1-t)CV_{i+1}(y + \sigma_{i}^gZ), 0 \} \right]$, which is also less than or equal to the right hand side of (1.64). Hence the inequality is satisfied for date $i$, which completes the proof of the convexity of $CV_i(\cdot)$.

By (1.60), we notice that the risk parameters, $\sigma_0^2$ and $s^2$, affect each $CV_i(\cdot)$ only through $\sigma_i^g$. It is straightforward to show that $\sigma_i^g$ defined in (1.33) increases with $\sigma_0^2$ but declines with $s^2$. Let $\tilde{\mu}_{i+1}^S$ denote a normal random variable with the same mean $\mu_i^S$ but a greater variance $(\bar{\sigma}_i^g)^2$ attributed to an increase in $\sigma_0^2$ or a decrease in $s^2$. Then the distribution of $\mu_{i+1}^S$ second-order stochastically dominates that of $\tilde{\mu}_{i+1}^S$. By the convexity of $\max \{CV_{i+1}(\cdot), 0\}$, it follows that

$$E_i^S \left[ \max \{ CV_{i+1}(\tilde{\mu}_{i+1}^S), 0 \} \right] \geq E_i^S \left[ \max \{ CV_{i+1}(\mu_{i+1}^S), 0 \} \right],$$

(1.65)

which implies that each $CV_i(\cdot)$ increases with $\sigma_0^2$, but declines with $s^2$.

As shown by Proposition 3, the optimal bankruptcy trigger $\mu^*_0$ is uniquely determined by the condition $CV_i(\mu^*_0) = 0$. Since $CV_i(\cdot)$ increases with $\mu_0^S$ by (1.60), increases with $\sigma_0^2$, and decreases with $s^2$, the bankruptcy time increases with $\mu^S_0$, increases with $\sigma_0^2$, and decreases with $s^2$. Q.E.D.

**1.8.5 Proof of Proposition 5**

By (1.23), (30) and (31), the optimization program for the determination of the long-term debt coupon payment is

$$G(d^*(\Delta_0), \Delta_0) := \max_d g(LD(0) - K) + gE_0^S \left[ \sum_{i=0}^{T_{i-1}} e^{-ri} \left( (1 - \tau)((1 - \nu)\mu_i^S - d) \right) \right] + E_0^S \left[ \sum_{i=0}^{T_{i-1}} e^{-ri}(F_i(b_i^*) - \phi) \right].$$

(1.66)

We denote the indirect objective function by $G$ and indicate the dependence of $G$ on the degree of managerial optimism $\Delta_0$. Due to the implicit function theorem, we have

$$\frac{\partial d^*}{\partial \Delta_0} = -\frac{\partial^2 G/\partial dd\Delta_0}{\partial^2 G/\partial d^2} \bigg|_{d=d^*}.$$
right hand side of (1.66) do not depend on $\Delta_0$. Consequently,

$$\frac{\partial^2 G}{\partial d \partial \Delta_0} = \frac{\partial^2 \text{PB}(0)}{\partial d \partial \Delta_0},$$

(1.68)

where

$$\text{PB}(0) = E_0^S \left[ \sum_{i=0}^{T_b-1} e^{-ri}(F_i(b^*_i) - \phi) \right],$$

(1.69)

is the net present value of the blockholder’s future private benefits at date zero.

By Proposition 1, $\text{PB}(0)$ depends on $d$ only through the bankruptcy time $T_b$ that decreases monotonically with $d$. Further, it can be easily shown by (1.46), (1.47), and (1.49) that $F_i(b^*_i)$ increases with $\Delta_0$. Consequently, the sign of (1.68) is non-positive. By (1.67), $\frac{\partial d^*}{\partial \Delta_0} \leq 0$; that is, long-term debt decreases monotonically with the initial degree of managerial optimism. Q.E.D.

1.9 Appendix B: Numerical Implementation and Calibration Procedure

In this appendix, we describe our numerical implementation and calibration of the model. The parameters that we need to calibrate are given by

$$\pi = (A, \alpha, s, \mu_0^S, \Delta_0, \sigma_0, \lambda, \kappa, \gamma/\beta, \phi, \nu, \rho, g).$$

(1.70)

Table 1.1 describes these parameters. The parameters $\beta$ and $\gamma$ cannot be separately identified because all economic variables in the model only depend on their ratio, $\gamma/\beta$. For a given candidate parameter vector $\pi$, we determine the optimal level $d^*$ of the long-term debt coupon payment. We then simulate the distributions of long-term debt and short-term debt values, capital investment, inside equity stake, asset value, and firm value. Using these simulated distributions, we obtain the vector $V(\pi)$ of model-predicted values of the statistics reported in Table 1.2. The vector $\pi^*$ of baseline parameter values solves

$$\pi^* = \arg \min_{\pi} (V(\pi) - \hat{V})^T W (V(\pi) - \hat{V}),$$

(1.71)

where $\hat{V}$ is the vector of observed values of the corresponding statistics from the empirical data and $W$ is a diagonal matrix whose respective entries are the reciprocals of the observed values.

We now describe how we determine the vector $V(\pi)$ of model-predicted values of the statistics for a given candidate parameter vector $\pi$. By the results of Section 2.4, all output statistics can be expressed as functions of the date, $i$, and investors’ current mean assessment of project quality, $\mu_i^S$. In other words, the state vector is

$$\text{State Vector} \equiv (i, \mu_i^S).$$

(1.72)

We compute the vector $V(\pi)$ of model-predicted values of the calibration statistics by simulating the evolution of $\mu_i^S$ using (1.4), (1.5) and (1.6).
We compute the continuation values at earlier dates if we determine the optimal bankruptcy trigger at each date $i$. We use the discrete lattice to determine the optimal bankruptcy trigger at each date $i$ for a given long-term debt structure $d$ as described by Proposition 3. In the second stage, we determine the optimal long-term debt structure by numerically solving (1.23) using Monte-Carlo simulation. In the third stage, we estimate the model-predicted statistics again using Monte Carlo simulation starting from each node of the discrete lattice that we build in the first stage. We now describe each step of our numerical analysis in more detail.

**Construction of discrete-state stochastic process:** We first approximate the evolution of $\mu^S_i$ using a discrete lattice. At date 0, investors’ mean assessment of the project’s profitability is $\mu^S_0$. Let $n(i)$ denote the number of nodes on the lattice at date $i > 0$. We set $n(i) = Mi$ with $M = 30$. Let $\mu^S_{i,j}$ denote investors’ mean assessment in the $j^{th}$ state at date $i$ where $j = 1, \ldots, n(i)$. We design the lattice such that the minimal and maximal states at date $i$, $\mu^S_{i,1}$ and $\mu^S_{i,n(i)}$, respectively, are 2.5 standard deviations below and above the minimal and maximal states at date $i - 1$. The values for the remaining $n(i) - 2$ states are equally spaced between the minimum and maximum states.

Let $p_{i,j}^{i+1,k}$ denote the transition probability that investors’ mean assessment transitions from state $\mu^S_{i,j}$ at date $i$ to state $\mu^S_{i+1,k}$ at date $i + 1$. Using (1.4), (1.5) and (1.6), we approximate the transition probability as

$$p_{i,j}^{i+1,k} = \Phi \left[ \left( \frac{1}{2}(\mu^S_{i+1,k} + \mu^S_{i+1,k+1}) - \mu^S_{i,j} \right) \frac{1}{\sigma^S_i} \right] - \Phi \left[ \left( \frac{1}{2}(\mu^S_{i+1,k} + \mu^S_{i+1,k-1}) - \mu^S_{i,j} \right) \frac{1}{\sigma^S_i} \right],$$

if $\mu^S_{i+1,k}$ is within $\pm 2.5\sigma^S_i$ from $\mu^S_{i,j}$. In the above, $\Phi(\cdot)$ denotes the cdf of the standard normal distribution. If $|\mu^S_{i+1,k} - \mu^S_{i,j}| > 2.5\sigma^S_i$, we set the transition probability to zero.

We now describe the determination of the bankruptcy triggers for a given long-term debt structure $d$. By Proposition 3, bankruptcy occurs at date $i$ if and only if investors’ mean assessment at date $i$ is less than a trigger $\mu^*_i(d)$, where the argument explicitly denotes the fact that the bankruptcy trigger depends on the long-term debt coupon. To determine $\mu^*_i(d)$, we compute the continuation value $CV_{i,j}$ of shareholders at state $\mu^S_{i,j}$. We obtain the continuation values on the nodes of the lattice by backward induction. To do so, we first define a terminal condition that specifies the firm’s going concern value at date $T$ as follows:

$$CV_{T,j} = \frac{(1 - \tau)\mu^S_{T,j}}{(1 - e^{-\tau})}, \text{ if } \mu^S_{T,j} > 0,$$

$$CV_{T,j} = 0, \text{ otherwise.}$$

We compute the continuation values at earlier dates $i < T$ as described in (1.60), that is,

$$CV_{i,j} = (1 - \tau) \left( (1 - \nu)\mu^S_{i,j} - d \right) + e^{-\tau} \sum_{k=1}^{n(i+1)} p_{i,j}^{i+1,k} \max\{CV_{i+1,k}, 0\},$$
where $p_{i+1}^{i+1,k}$ is the transition probability described in (1.73). Starting from $i = T - 1$ and working backwards through time, we compute the continuation values for all states and dates. Since the true continuation value function is continuous and non-decreasing as shown in the proof of Proposition 3, we complete the approximation to $CV_i(\cdot)$ by linear interpolation. We then determine the optimal trigger $\mu_i^*(d)$ at date $i$ that solves $CV_i(\mu_i^*(d)) = 0$.

**Determination of Optimal Long-Term Debt Structure:** Next, we determine the blockholder’s optimal long-term debt choice, $d^*$, at date zero for the candidate vector $\pi$ of parameter values. To compute the long-term debt choice, we simulate the total earnings flow described in (1.1) $N = 10,000$ times starting from date zero. By (1.1), the total earnings in each period is the sum of the base earnings and the discretionary earnings. From Proposition 1, the discretionary earnings, $Ak^s_i \eta_i^b$, is deterministic and, therefore, can be pre-computed for each date $i$. The base earnings, $\Theta + N_{i+1}$, is the sum of two normal random variables that are drawn from $N(\mu^S_0, \sigma_0)$ and $N(0, s)$, respectively. We generate a sample path identified by a vector of $T + 1$ independent standard normal random variables, $(\hat{\theta}, \hat{N}_1, \hat{N}_2, \ldots, \hat{N}_T)$, so that the base earnings at date $i$ is given by $\mu^S_0 + \sigma_0 \hat{\theta} + s \hat{N}_i$. We use (1.6) to compute investors’ mean posterior assessments $\mu_i^S$ of $\Theta$ as the earnings are realized over time along the sample path. We then determine the optimal bankruptcy time $T_b$ using the bankruptcy trigger $\mu_i^*$ that we obtained from the discrete lattice as described earlier. We carry out the above procedure for each of the $N = 10,000$ sample paths.

By (1.23), (1.21), and (1.18), the blockholder’s objective function for a given long-term debt structure $d$ can be rewritten as

$$B(0) = g \left( E_S^0 \left[ \sum_{i=0}^{T_b-1} e^{-ri} (1 - \tau)(1 - \nu)(\Theta + N_{i+1}) + \tau d + e^{-rT_b}LD(T_b) \right] - K \right)$$

$$+ E_S^0 \left[ \sum_{i=0}^{T_b-1} e^{-ri} (F_i(b_i) - \phi) \right] .$$

(1.74)

In the above, we set $K$ to the asset value $AV(0)$ at date zero, which is given by

$$AV(0) = E_S^0 \left[ \sum_{i=0}^{T-1} e^{-ri}(1 - \tau)(\Theta + N_{i+1}) + e^{-rT}(1 - \tau) \max\{0, \Theta\} / (1 - e^{-\tau}) \right] ,$$

(1.75)

where the last term represents the firm’s going concern value at date $T$ as mentioned earlier. The expectation terms in (1.74) can be computed for a given $d$ using the set of sample paths simulated at date zero. Especially, $E_S^0 \left[ e^{-rT_b}LD(T_b) \right]$ can be calculated by the law of iterated expectations. We evaluate the blockholder’s value for different values of $d$. We use Matlab’s Nelder-Mead optimization algorithm to determine the value $d^*$ that maximizes blockholder value.

**Determination of the model-predicted statistics:** In the final stage of the implementation, we use the optimal solution $d^*$ obtained in the previous step to compute the asset
value, long-term and short-term debt values, and firm value at each node of the discrete lattice for dates 0, 1, 2, 3. We use these values to construct the simulated distributions of the calibration statistics.

We use Monte-Carlo simulation starting from each node \((i,j); i \leq 3\) of the discrete lattice to compute the statistics at the node. We simulate a set of \(N = 10,000\) sample paths over \(T - i\) periods from date \(i\) to date \(T\). By (1.4), (1.5) and (1.6), the project’s profitability, \(\Theta\), is drawn from a normal distribution with mean \(\mu_S^{i,j}(\mu_S^0\text{ at date 0})\) and standard deviation \(\sigma_i\). We estimate the asset value \(AV(i,j)\), long-term debt value \(LD(i,j)\), short-term debt value \(SD(i,j)\), and firm value \(FV(i,j)\) at node \((i,j)\) as follows:

\[
AV(i,j) = E_{i,j}^S \left[ \sum_{t=i}^{T-1} e^{-r(t-i)}(1 - \tau)(\Theta + N_{t+1}) + e^{-r(T-i)}(1 - \tau)\max\{0, \Theta\}/(1 - e^{-r}) \right],
\]

\[
LD(i,j) = E_{i,j}^S \left[ \sum_{t=i}^{T_b-1} e^{-r(t-i)}d^* + e^{-r(T_b-i)}LD(T_b) \right],
\]

\[
SD(i,j) = E_{i,j}^S \left[ (1 - \tau)\nu(\Theta + N_{i+1} + Ak_i^\alpha \eta_i^\beta - a_i/b_i) \right],
\]

\[
FV(i,j) = E_{i,j}^S \left[ \sum_{t=i}^{T_b-1} e^{-r(t-i)}\left( (1 - \tau)(\Theta + N_{t+1} + Ak_t^\alpha \eta_t^\beta) + \tau d^* \right) + e^{-r(T_b-i)}LD(T_b) \right].
\]

After collecting the simulated values of long-term debt and short-term debt ratios and firm value ratios over the first three periods, we obtain the quartiles of these ratios as the model-predicted statistics. In addition, we also calculate the median ratio of the incremental capital investment \(k^*_i\) to asset value and the median value of the manager’s equity stake \(b^*_i\). Using the vector, \(V(\pi)\), of model-predicted statistics, and the vector of observed values, \(\hat{V}\), we determine the baseline parameter values by numerically solving (1.71). We use the Nelder-Meade optimization routine in MATLAB to perform the minimization. To avoid a local minimum solution, we repeat the optimization routine multiple times by randomizing the initial seed.
2.1 Introduction

In 2005, the average (median) CEOs in the energy and telecom industries earned $10.22 (4.72) million and $7.48 (3.86) million, respectively, whereas the average (median) CEO in the consumer durable goods industry earned only $3.24 (2.68) million (based on data on S&P 1500 firms). The difference between the maximum and minimum CEO pay levels within an industry ranges from $12.63 million in the consumer durable goods industry to $92.20 million in the business equipment industry. Why do the levels and distributions of CEO pay vary so dramatically across industries? Are variations in industry characteristics largely responsible for the variations in the distributions of CEO pay or do inter-industry variations in the distributions of managerial ability/talent play an important role in explaining these findings? How important is managerial talent when industry characteristics and, more generally, the product market environment in which firms operate are considered? To what extent do product market characteristics affect the levels and distributions of CEO compensation and firm value across industries?

We address these questions by developing a market equilibrium model in which the competitive assignment of CEOs to firms and imperfect product market competition among
firms interact to determine the distributions of firm value and managerial compensation. Managers of different talent levels are matched to firms of different qualities, and their match quality determines firm productivity. There exists a unique, stationary equilibrium of the model in which CEO-firm matches and the distributions of firm market value and managerial compensation are endogenously determined. We calibrate the structural model using the distributions of firm value and CEO pay in each of twelve Fama-French industries. In addition to the key structural parameters of the model, we also indirectly infer the unobserved distributions of CEO talent and firm quality in each of the industries. We then conduct counterfactual experiments using the calibrated models corresponding to each of the industries to explore the quantitative effects of managerial talent and product market characteristics.

First, we show that there is substantial variation not only in the distributions of firm quality and managerial talent across industries, but also in their relative contributions to firm value. The dispersions of these attributes are much larger in high-tech industries—including business equipment, health care, and telecom—relative to traditional manufacturing industries. Second, in contrast with the strikingly small estimate obtained by Gabaix and Landier (2008), we find that, when product market characteristics are taken into account, CEO talent has a much more significant impact on firm value. Our estimates of the impact of CEO talent are roughly two orders of magnitude higher than that of Gabaix and Landier (2008). Our results show that the incorporation of product market characteristics and intra-industry competition among firms plays a central role in generating the significantly different estimates. Third, again in contrast with the findings of Gabaix and Landier (2008) and Terviö
(2008), our estimates of the impact of CEO talent on firm value are of the same order of magnitude as the ratio of CEO compensation to firm value. The compensation of CEOs is, therefore, quantitatively in line with their marginal productivity. Fourth, we analytically derive a number of novel implications for the effects of product market characteristics on the number of active firms in an industry as well as the distributions of managerial compensation and firm value.

We build a discrete time, infinite horizon model of an industry in which there is a continuum of heterogeneous firms engaging in imperfect product market—specifically, monopolistic—competition (Dixit and Stiglitz, 1977). We explicitly incorporate heterogeneity in manager and firm attributes as well as the endogenous matching of CEOs to firms. Managers are characterized by a variable referred to as talent, and firms are also characterized by a single variable, namely firm quality. Firms are established by entrepreneurs who make an initial sunk investment. Firm quality, which is a random variable drawn from a known distribution, is realized after entry. Consequently, firms are identical ex ante (i.e. prior to entry), but differentiated ex post. Each firm then hires a manager. Firm qualities and managerial talents are observable to market participants. Similar to Terviö (2008), the match quality is a multiplicative function of CEO talent and firm quality. The match quality determines the firm’s productivity in each period. Firms are monopolistically competitive in that they take the aggregate price index—the weighted average of the prices charged by all firms in the market—as given when they make their output and pricing decisions. A firm produces in each period unless it faces an exogenous shock that forces it to exit the market (Melitz, 2003).
We derive the unique, *stationary equilibrium with free entry* in which the matching of CEOs to firms as well as the distributions of firm value and CEO pay are endogenously determined. The equilibrium satisfies several conditions. First, the free entry condition implies that the value of an entering firm—which rationally anticipates the outcome of the matching process between firms and managers—equals the entry cost. Second, in the stationary equilibrium, exiting firms are exactly replaced by new entrants so that the mass of firms remains constant through time. Third, the market clearing condition is satisfied in equilibrium, that is, the aggregate revenue of firms equals the aggregate payoffs to entrepreneurs, managers and workers. The equilibrium of the model depends on product market characteristics—the entry cost, the elasticity of substitution between products, the market size, and the exit probability of firms—as well as the distributions of managerial talent and firm quality.

Our structural approach requires the identification of the distributions of firm quality and managerial talent (that are unobservable to the econometrician) as well as the estimation of unknown model parameters such as the exit probability and the elasticity of substitution between products that determine the product market structure. We treat a sample of firm-CEO observations that belong to an industry as a market equilibrium outcome. We choose quantiles of the observed distributions of firm value and CEO pay as the moments to be matched to the corresponding model-predicted statistics. In addition, we use the observed distributions of firm value and CEO pay to identify the unobserved distributions of firm quality and CEO talent—the *factor distributions*—as in the analysis of Terviö (2003, 2008).

There is significant variation in the inferred distributions of firm quality and managerial talent across industries. Consistent with what casual empiricism might suggest, the disper-
sions of CEO talent and firm quality are much larger in high-tech industries, such as business equipment, health care, and telecom, relative to traditional manufacturing industries. Second, in line with anecdotal and empirical evidence (e.g., Daines, Nair, and Kornhauser, 2005; Pan, 2010; Falato, Li and Milbourn, 2011), we show that managerial talent is, indeed, an important factor in the production process as differences in managerial talent could make a significant difference to firm value. Following Gabaix and Landier (2008), we measure the impact of talent as the benefit the median firm could obtain from the replacement of the current CEO with the best one in the same industry. Our estimates for different industries in our sample, which are obtained by explicitly incorporating product market influences, are largely two orders of magnitude higher than the estimate of Gabaix and Landier (2008), who abstract away from product market effects. For example, the median firm in the business equipment industry could obtain an about 2.9% increase in its market value if its CEO were replaced with the best CEO. It should also be noted that, while the ratio of extra compensation payments to be incurred if the firm had to provide the best CEO with his current level of compensation is larger than the CEO’s impact on the firm’s value, it is not orders of magnitude higher as in Gabaix and Landier (2008). For instance, for the median firm in the business equipment industry, the ratio of additional future compensation payments to firm value is 10.7%. For other industries too, the costs are about three or four times greater than the benefits. This finding suggests that the remunerations of CEOs are roughly in line with their relative contributions to firms. Moreover, we find that the impact of CEO talent varies dramatically across industries. The median firm in the business equipment and health care industries could increase its market value by about 2.9% and 2.5%, respectively, whereas the
sizes of the impact in the chemical, consumer durable goods, and manufacturing industries are only 0.85%, 1.15% and 1.16%, respectively.

Finally, we analytically derive the effects of product market characteristics on firm value, CEO compensation, and the mass of active firms. A decline in the entry cost and/or the exit probability leads to more firms in the market and, therefore, tougher price competition, which in turn lowers firm value and CEO pay. While the effects of a marginal increase in product substitutability are ambiguous in general because they depend on the values of other product market parameters, our analysis of the calibrated model yields clear predictions. An increase in the product substitutability induces less firms to enter the market, which in turn lessens price competition. Since the marginal return to managerial talent decreases (on average) with the product substitutability, so does the average managerial compensation. Furthermore, the effects of product market characteristics vary quantitatively across industries. Taken together, our results suggest that industry-related factors, including those linked to the CEO labor market and those linked to the product market, are very critical determinants of the levels and distributions of CEO compensation.

Our work revisits the fundamental question of how important managerial talent is if competition for CEO talent in an efficient labor market exists, which is raised and explored by the seminal studies of Terviö (2003, 2008) and Gabaix and Landier (2008). Both studies conclude that differences in CEO talent are very small and have little influence on shareholder value that is largely driven by firm-specific factors. Recent empirical studies, however, show that managers matter to firm performance (Bertrand and Schoar, 2003; Daines, Nair, and Kornhauser, 2005; Bennedsen, Perez-Gonzalez, and Wolfenzon, 2006; Graham, Li, and Qiu,
These studies find both statistically and economically significant impact of CEO characteristics, which are especially narrowed down to CEO talent for some studies, on CEO pay and firm performance, controlling for firm characteristics.

To reconcile the results from the competitive assignment models with the empirical evidence, we conjecture that industry structures matter in determining the level of CEO pay and in identifying the importance of CEO talent. There are three channels through which industry structures could affect the assignment process of CEOs to firms and, thereby, the distribution of CEO pay. First, the fundamental economics of an industry, that is, the nature of the product market, varies across industries. Specifically, different product market structures could imply different marginal returns to CEO talent and, therefore, CEO pay. Second, different industries might be characterized by different degrees of firm heterogeneity, and the sources of firm heterogeneity could also be different across industries. Third, to the extent that markets for CEO talent are segmented by industry, industry-level talent distributions might differ, which suggests that the CEO talent distribution and the effects of CEO talent on firm value should be estimated at the industry level rather than at the entire economy level.

Terviö (2008) and Gabaix and Landier (2008) abstract away from industry and product market effects in their models. They estimate their models using a full sample of the largest firms in different industries, that is, they aggregate firms across industries in their estimation exercises. By contrast, given the discussion above, we develop a single market equilibrium framework in which the competitive assignment of CEOs to firms is incorporated and then
estimate the structural model “industry by industry.” In the literature on CEO turnover, Parrino (1997) and Cremers and Grinstein (2010) report that a dominant portion of new CEOs are insiders of hiring firms or come from other firms in the same industry and that most of those from outside their industry still have some relevant industry experience such as business relationships. The latter study further documents cross-industry differences in CEO selection practices and their explanatory power when examining different CEO compensation practices, thereby providing the evidence of fragmented CEO talent pools across industries. These studies in part support our premise that there are CEO labor markets composed of senior managers within or outside firms in the same industry who have industry-specific skills.

The plan for the paper is as follows. Section 2 reviews the related literature in more detail. In Section 3, we present the model. In Section 4, we characterize the equilibrium and analytically derive implications of the model for the qualitative effects of product market characteristics on firm value and CEO pay. In Section 5, we describe our data and estimation procedure. Section 6 presents the results of the model calibration, including the structural parameter estimates and the factor distributions implied by the data. Section 7 contains counterfactual exercises using the calibrated models. Section 8 summarizes and concludes.

2.2 Related Literature

As discussed earlier, our work contributes to the recent literature initiated by Terviö (2003) that studies CEO pay levels in a competitive assignment framework. In Terviö (2003, 2008) and Gabaix and Landier (2008), the underlying idea is that, in a competitive and fric-
tionless labor market for CEO talent, CEOs with different talents are competitively matched to heterogeneous firms at different pay levels. Both studies mainly argue that while talent differences between CEOs are very small, significant differences in firm quality, which is complementary to CEO talent and thus affects the marginal return to talent, can explain large pay dispersions for such small talent differences. In this study, we argue that when product market/industry characteristics are taken into account, differences in CEO talent are much more significant and can justify a substantial portion of the difference in CEO pay levels.

Our research is related to the literature that addresses how the distributions of firm size and CEO compensation change in response to economic conditions. The literature includes the studies of Raith (2003), Falato and Kadyrzhanova (2007), Baranchuk, MacDonald, and Yang (2010), Subramanian, Plehn-Dujowich and Li (2011), and Wu (2011). These studies mainly look at the effects of product market characteristics on optimal managerial incentives by incorporating agency conflicts arising from moral hazard in a market equilibrium framework. As in our study, Wu (2011) employs a standard monopolistic competition framework with Dixit-Stiglitz preferences to address the role of the product market. However, he bases his analysis on the early job assignment model, which considers the allocation of heterogeneous managers across ex ante identical firms, so that ex post firm size and CEO pay differences across firms are solely attributed to the heterogeneity in managerial skills. By changing the degree of complementarity between firm and managerial attributes in a business cycle model, rather than stressing product market effects, Alder (2009) shows that managerial attributes actually play an important role in the determination of firm size and CEO pay.
Recently, several studies have empirically examined the association between managerial characteristics—especially, managerial talent—and CEO pay and firm performance. Falato, Li, and Milbourn (2011) study the effects of CEO talent on firm performance using a media-based measure of CEO talent, the age of the executive when he took his first CEO job, and the selectivity of his undergraduate college. They document that replacing the CEO of median talent with the most talented CEO in their sample would improve firm operating performance by between 1.3% and 2.3%, which is two orders of magnitude greater than the estimate of Gabaix and Landier (2008) but largely consistent with our estimates for the industries in our sample. Daines, Nair, and Kornhauser (2005) define CEO skill as the persistence of positive performance and the reversal of poor performance and find a positive link between CEO skill and pay especially when pay is performance based and when there are large shareholders. Also, the link between skill and pay appears to be stronger in industries where pay dispersion is large, which supports our conjecture that CEO talent may matter more for firms in some industries than those in other industries. Focusing on the U.S. property-liability insurance industry, Leverty and Grace (2010) use several firm efficiency variables obtained from the Data Envelopment Analysis (DEA) as proxies for managerial ability and find that managerial ability plays an important role in reducing the duration of regulatory scrutiny, the likelihood of failure, and the cost of failure. Finally, Pan (2010) estimates an executive-firm matching model incorporating three matching dimensions, one of which is the usual complementarity between firm size and managerial talent, and finds that higher matching quality is associated with better subsequent firm performance.
2.3 The Model

We develop a discrete-time, infinite horizon model of an industry with dates \( t = 0, 1, 2, \ldots \). The industry consists of a continuum of heterogeneous operating firms, heterogeneous managers, and identical production workers. The firms engage in Dixit-Stiglitz monopolistic competition with a constant elasticity of product substitution. Production requires raw labor supplied by production workers and specialized human capital provided by managers.

There are three stages of the model as follows:

• Stage 1: (Entry) A group of (identical) entrepreneurs drawn from the pool of workers establish a firm at date \( t \) by making an initial sunk investment. Subsequent to entry, the firm’s quality is realized. Firm quality is a random variable that is drawn from a known distribution and then remains constant through time. Firms are, therefore, identical ex ante, but differentiated ex post.

• Stage 2: (Manager-Firm Matching) The owners (entrepreneurs) of each firm hire a manager from a continuum of potential managers of various talent levels in a competitive executive labor market. Managerial talent is observable and is constant through time.

• Stage 3: (Production and Exit) In each period, each firm produces its good, generates profit, and pays its manager. It continues over an infinite time horizon unless it faces an exogenous negative shock that forces it to exit the market (Melitz, 2003).
Since each firm’s quality is realized ex post after entry, there is \textit{two-sided heterogeneity} in the assignment process between firms and managers. We focus on a stationary equilibrium with free entry in which exiting firms are exactly replaced by new entrants. Consequently, the equilibrium distributions of managerial talent and firm quality among active firms, and the equilibrium distributions of firm market value and CEO pay are stationary.

We solve the model by backward induction. We first analyze an active firm’s (that is, a firm that successfully matches with a manager) profit maximization problem in each period. Next, we study the competitive assignment process between heterogeneous firms and managers. Finally, we examine firms’ entry decisions into the market.

2.3.1 Preferences, Market Demand and Production

The representative consumer has preferences for consumption of a continuum of goods in each period that are described by the utility function

\[ U = \left[ \int_{\Omega} q(\omega)^\rho d\omega \right]^{\frac{1}{\rho}} ; \quad 0 < \rho < 1, \]

(2.1)

where \( \Omega \) is the set of available goods and \( \omega \) is a finite measure on the Borel \( \sigma \)-algebra of \( \Omega \). If \( p(\omega) \) is the price of good \( \omega \), the consumer’s demand \( q(\omega) \) for good \( \omega \) solves the following utility maximization problem:

\[
\max_{q(\omega)} \quad U = \left[ \int_{\Omega} q(\omega)^\rho d\omega \right]^{\frac{1}{\rho}}
\]

subject to \( \int_{\Omega} p(\omega)q(\omega)d\omega = R. \)

(2.2)
In the above, $R$ represents the total expenditure of the representative consumer on goods produced by the industry. It is natural to interpret $R$ as the *size* of the industry.

As shown by Dixit and Stiglitz (1977), the optimal consumption of each good is

$$q(\omega) = U \left[ \frac{P}{p(\omega)} \right]^{1/(1-\rho)}, \quad (2.3)$$

where $P$ is the *aggregate price index* that is given by

$$P = \left[ \int_\Omega p(\omega) \frac{\rho}{1-\rho} d\omega \right]^{\rho/(\rho-1)} . \quad (2.4)$$

Further,

$$R = PU. \quad (2.5)$$

The optimal consumption (2.3) of each good represents the market demand that the firm producing this particular good faces in the market. From (2.3), we see that

$$\frac{q(\omega)}{q(\omega')} = \left[ \frac{p(\omega')}{p(\omega)} \right]^{1/(1-\rho)}, \quad (2.6)$$

which implies that any two products in the market are substitutes, and the elasticity of substitution between these products is

$$\sigma = \frac{1}{1-\rho} > 1. \quad (2.7)$$

This is also the constant price elasticity of demand for each good as seen in (2.3).
We now turn to the production decision of each firm for any period after it has matched with a manager. The assignment process of managers to firms is described in the next section. Production is driven by production labor supplied by production workers. We denote the labor wage rate by $w$. The firm’s quality and the manager’s talent together determine the firm’s productivity.

More precisely, suppose that the firm has quality $x \in \mathbb{R}_+$ and its manager has talent $y \in \mathbb{R}_+$. As in Tervio (2008), there is complementarity between firm quality and managerial talent. Specifically, the match quality $\theta$ takes the multiplicative form

$$\theta(x, y) = xy.$$  \hspace{1cm} (2.8)

The firm’s productivity equals the match quality $\theta(x, y)$, that is, the inverse of the match quality is the firm’s marginal cost of production measured in units of labor. As discussed by Tervio (2008), the key, substantive property of (2.8) is the complementarity of firm quality and managerial talent. There is actually little loss of generality in assuming the particular form (2.8). We could alternatively assume that $\theta(x, y) = f(x)g(y)$ for strictly increasing and nonnegative functions $f(.)$ and $g(.)$. In this case, we could simply “redefine” firm quality as $x' = f(x)$ and managerial talent as $y' = g(y)$.

At the beginning of each period, the firm chooses its price in order to maximize its net profit, that is, revenue net of variable production costs and managerial compensation. Suppose the manager’s compensation is $u$ (note that this is endogenously determined as the outcome of the matching process between firms and managers). The firm’s net profit is
\[ \pi(x, y, u) = \max_p pq(p) - w \cdot \frac{q(p)}{\theta(x, y)} - u, \quad (2.9) \]

where the market demand curve, \( q(p) \), is given by (2.3) (We omit the argument \( \omega \) to simplify the notation). Since there is a continuum of firms, each firm takes the aggregate variables \( U \) and \( P \) as given when it chooses its price and output quantity. The second term on the right hand side of (2.9) represents total labor wages received by production workers who are employed by the firm. The profit maximization condition equates the marginal revenue with the marginal cost of production, thereby yielding the firm’s optimal price,

\[ p(x, y) = \frac{w}{\rho xy}. \quad (2.10) \]

Consequently, the firm produces the following level of its good,

\[ q(x, y) = q(p(x, y)) = RP^{\sigma - 1} \left( \frac{\rho xy}{w} \right)^{\sigma}, \quad (2.11) \]

and its resulting revenue is given by

\[ r(x, y) = p(x, y)q(x, y) = R((P/w)\rho xy)^{\sigma - 1}. \quad (2.12) \]

The firm’s gross profit (profit inclusive of managerial compensation) is

\[ \Pi(x, y) = p(x, y)q(x, y) - \frac{q(x, y)}{\theta(x, y)} = \frac{R((P/w)\rho xy)^{\sigma - 1}}{\sigma}. \quad (2.13) \]
The firm’s net profit—the gross profit less managerial compensation—is

\[ \pi(x, y, u) = \Pi(x, y) - u = \frac{R((P/w)\rho xy)^{\gamma - 1}}{\sigma} - u. \]  

(2.14)

### 2.3.2 Manager-Firm Matching

We now describe the assignment of managers of different abilities to firms of different qualities and the determination of managerial compensation. Managerial abilities and firm qualities are constant through time as in Gabaix and Landier (2008) and Tervio (2008). Since the distributions of CEO talent and firm quality are stationary, each firm continues to hire the same type of manager and pays the same remuneration. Hence, its productivity level remains constant over time, and so does its net profit. More specifically, if a firm of quality \( x \) hires a manager of talent \( y \) whose compensation is \( u \), the firm will earn net profit \( \pi(x, y, u) \), given by (2.14), in each period until it has to exit the market for some exogenous reasons. Denoting the probability of staying in the market for another period by \( \delta \), the firm’s market value—the present value of future earnings net of payoffs to the manager—is

\[ \phi(x, y, u) = \sum_{t=0}^{\infty} \delta^t \pi(x, y, u) = \frac{\pi(x, y, u)}{1 - \delta} = \frac{1}{1 - \delta} \left[ \frac{R((P/w)\rho xy)^{\gamma - 1}}{\sigma} - u \right]. \]  

(2.15)

Following Legros and Newman (2007b), we refer to the function \( \phi(x, y, u) \) as the bargaining frontier for the firm, which implies its maximum payoff when it pays the manager \( u \), and denote the quasi-inverse function of \( \phi(x, y, u) \) by \( \psi(x, y, v) : \phi(x, y, \psi(x, y, v)) = v \), where \( \psi(x, y, v) \) is the bargaining frontier for the manager. Let \( S(x, y) \) be the total surplus
generated by this firm-manager pair, which is the first term of \( \phi(x, y, u) \),

\[
S(x, y) = \frac{R((P/w)\rho xy)^{\sigma-1}}{(1 - \delta)\sigma}.
\]  

(2.16)

We easily see that \( S(x, y) \) satisfies the *supermodularity* condition

\[
\frac{\partial^2 S}{\partial x \partial y} > 0.
\]  

(2.17)

It follows from the results of Legros and Newman (2007b) that the matching equilibrium is unique, and is characterized by positive assortative matching (*PAM*), that is, higher quality firms are matched to managers with higher talents.

We now derive the equilibrium payoffs to firms and managers. Let \( F_X(\cdot) \) and \( F_Y(\cdot) \) be the cumulative distribution functions of firm quality and managerial talent, respectively. As in Terviö (2008), we work with the quantiles of the distribution functions. Specifically, for each \( i \in [0, 1] \), define

\[
x[i] = x \quad \text{if} \quad F_X(x) = i,
\]

\[
y[i] = y \quad \text{if} \quad F_Y(y) = i.
\]  

(2.18)

Consequently, \( x'[i] > 0 \) and \( y'[i] > 0 \) where \( i \in [0, 1] \), that is, higher \( i \) denotes a higher quality firm and a more talented manager. By *PAM*, we can restrict attention to matches where firm \( i \) is matched with manager \( i \). In other words, we can use the index \( i \) to denote a matched manager-firm pair.
The total surplus, \( S(x[i], y[i]) \) in (2.16), must be apportioned to the manager and the firm in a way that ensures the stability of the matching correspondence. Let \( u[i] \) be the equilibrium compensation of manager \( i \) for each period and \( v[i] \) be the equilibrium payoff to firm \( i \), that is, its market value. To begin with, we consider the participation constraints for both parties. The payoff to each party must be on its frontier given its partner’s payoff, \( \phi(x, y, \cdot) \) and \( \psi(x, y, \cdot) \), respectively, and can never be less than their outside payoffs, \( v_0 \) and \( u_0 \), which are assumed to be identical for all types, that is,

\[
\begin{align*}
v[i] &= \phi(x[i], y[i], u[i]); \quad v[i] \geq v_0, \\
u[i] &= \psi(x[i], y[i], v[i]); \quad u[i] \geq u_0.
\end{align*}
\]

(2.19) (2.20)

Note that, by (2.15), (2.16), and (2.19), the relation between \( v[i] \) and \( u[i] \) is given by

\[
S(x[i], y[i]) = v[i] + \frac{u[i]}{1 - \delta} = \frac{R((P/w)\rho x[i]y[i])^{\sigma-1}}{(1 - \delta)\sigma}, \tag{2.21}
\]

and that the outside payoffs, by definition, imply the payoffs of the lowest active firm-manager pair in the market:

\[
S(x[0], y[0]) = v_0 + \frac{u_0}{1 - \delta} = \frac{R((P/w)\rho x[0]y[0])^{\sigma-1}}{(1 - \delta)\sigma}. \tag{2.22}
\]

The next set of constraints to be considered are the incentive compatibility constraints. Each party chooses its best matching partner. If \( m(i) \) denote firm \( i \)'s choice of its manager
and \( n(i) \) denotes manager \( i \)'s choice of his firm, then the incentive constraints are

\[
v[i] = \max_{m(i)} \phi(x[i], y[m(i)], u[m(i)]),
\]

\[
u[i] = \max_{n(i)} \psi(x[n(i)], y[i], v[n(i)]).
\]

As in a usual screening problem, a “single crossing” property holds in the framework analyzed here so that the set of incentive constraints above is equivalent to the following two sets of constraints: (i) Monotonicity and (ii) Local incentive compatibility (Bolton and Dewatripont, 2005). We describe the procedure for the firm’s constraint (2.23) because it is analogous to the procedure for the manager’s constraint (2.24). Firm \( i \) faces a set of choices that can be described as \((m, u[m])\) and needs to choose the best one. From (2.15) and (2.23), it is easy to check the single crossing property that the indifference curve for a higher type has a greater slope, that is, the marginal payoff of partner type \( m \) relative to that of payment \( u \) rises with firm type \( i \):

\[
\frac{\partial}{\partial i} \left[ -\frac{\partial v/\partial m}{\partial v/\partial u} \right] > 0.
\]

(2.25)

As a consequence, the set of incentive constraints can be replaced by the monotonicity condition,

\[
\frac{dm(i)}{di} \geq 0,
\]

(2.26)

and the local incentive condition,

\[
\left. \frac{d\phi(x[i], y[m(i)], u[m(i)])}{dm} \right|_{m(i)=i} = 0.
\]

(2.27)
Since the monotonicity condition holds due to \( P.AM \), the firm’s global incentive compatibility is equivalent to the local incentive condition above. The same argument applies to the incentive compatibility for the manager. From the local incentive conditions, we derive the following differential equations in \( u[i] \) and \( v[i] \):

\[
\begin{align*}
    u'(i) &= -\frac{\phi_2}{\phi_3} y'[i] = R(P/w)^{\sigma-1} \rho^\sigma x[i]^{\sigma-1} y[i]^{\sigma-2} y'[i], \\
    v'(i) &= -\frac{\psi_1}{\psi_3} x'[i] = R(P/w)^{\sigma-1} \rho^\sigma x[i]^{\sigma-2} y[i]^{\sigma-1} x'[i],
\end{align*}
\]

(2.28)

where the second equation can also be obtained from (2.21). Integrating the above, we obtain

\[
\begin{align*}
    u(i) &= u_0 + \int_0^i R(P/w)^{\sigma-1} \rho^\sigma x[j]^{\sigma-1} y[j]^{\sigma-2} y'[j]dj, \\
    v(i) &= v_0 + \int_0^i R(P/w)^{\sigma-1} \rho^\sigma x[j]^{\sigma-2} y[j]^{\sigma-1} x'[j]dj.
\end{align*}
\]

(2.30)

(2.31)

### 2.3.3 Market Entry

Given the distributions of managerial talent and firm quality, we have derived the distributions of firm value and managerial compensation for active firms in the market. These distributions are rationally anticipated by prospective entrants (a group of entrepreneurs) into the market. Entry into the market requires a fixed sunk investment of \( f_e > 0 \). Since the quality \( x \) of a newly established firm is determined only after it enters the market, the quality of the firm is an unknown random variable with cdf \( X \) at the “market entry” stage. By (2.18), it can be shown that its rank, compared with incumbent firms, is also a random
variable. Free entry ensures that the expected firm value equals the entry cost, that is,

$$E[v[i]] = \int_0^1 v[i]di = f_e,$$

(2.32)

where $v[i]$ is given by (2.31).

### 2.4 Equilibrium

A stationary equilibrium is characterized by a mass $N^*$ of active firms, a distribution $x[i]$ of firm quality, a distribution $y[i]$ of managerial talent, an aggregate price index $P^*$, and payoff profiles—managerial compensation $u[i]$ and firm value $v[i]$—such that the following conditions hold:

1. **Firm Profit Maximization**: In any period, each active firm $i$ produces $q(i)$ units of its good at price $p(i)$ per unit to maximize its net profit as described in (2.9), where

$$p(i) = \frac{w}{\rho x[i]y[i]}; \quad q(i) = R(P^*/w)^{\sigma-1}(\rho x[i]y[i])^\sigma.$$  

(2.33)

2. **Manager-Firm Matching and Payoffs**: A manager ranked $i$ is assigned to the equally ranked firm $i$. The equilibrium payoff profiles satisfy

$$v[i] = \frac{1}{1-\delta} \left[ R((P^*/w)\rho x[i]y[i])^{\sigma-1} - u[i] \right],$$

(2.34)

$$u[i] = u_0 + \int_0^i \left( R(P^*/w)^{\sigma-1} \rho^{\sigma-1} x[j]y[j]^{\sigma-2} y'[j] \right) dj.$$
3. Free Entry of Firms and Aggregate Price Index: The free entry condition determines the aggregate price index $P^*$ as follows:

$$R(P^*/w)^{\sigma-1} \left[ \frac{(\rho x[0]y[0])^{\sigma-1}}{\sigma} + \rho^{\sigma} \int_0^1 \left[ \int_0^i x[j]^{\sigma-2}y[j]^{\sigma-1}y'[j]dj \right] di \right] = u_0 + (1 - \delta)f_e.$$  

(2.36)

4. Market Clearing and Mass of Firms: In any period, the aggregate revenue of active firms must equal the aggregate expenditure $R$ by the representative consumer, that is,

$$R = \int_0^1 r(x[i], y[i]) N^* di \quad \Rightarrow N^* = \left[ \int_0^1 ((P^*/w)\rho x[i]y[i])^{\sigma-1} di \right]^{-1},$$  

(2.37)

which is derived from the revenue function given by (2.12) and determines the equilibrium mass $N^*$ of producing firms.

It immediately follows from the above that there exists a unique, stationary equilibrium in which the aggregate price index $P^*$ and the mass $N^*$ of active firms are determined by (2.36) and (2.37), respectively. Further, they remain constant over time so that the distributions of firm value and managerial compensation are also invariant over time.

Using the above equilibrium characterization, we analytically explore the effects of product market characteristics—the entry cost, the likelihood of exit, the market size, and the elasticity of substitution—on the mass of active firms, firm value and CEO compensation. We begin by deriving the effects of the product market characteristics on the aggregate price index because the other equilibrium variables depend on it.
Proposition 6 (Product Market Characteristics and Aggregate Price Index)

- The aggregate price index $P^*$ increases with the entry cost $f_e$ and the likelihood of exit $1 - \delta$, while it decreases with the market size $R$.

- There exist threshold levels $\bar{f}_e(\sigma)$ and $1 - \bar{\delta}(\sigma)$ of the entry cost and the exit probability, respectively, such that the aggregate price index increases with a marginal increase in the elasticity of substitution $\sigma$ if the entry cost $f_e$ or the likelihood of exit $1 - \delta$ is below its threshold, and decreases if it is above.

Before discussing the intuition for the above proposition, we derive the effects of product market characteristics on the mass of firms.

Proposition 7 (Product Market Characteristics and the Mass of Firms)

- The mass of active firms declines with the entry cost $f_e$ and the likelihood of exit $1 - \delta$, but increases with the market size $R$.

- The mass of active firms may decrease or increase with a marginal increase in the elasticity of substitution $\sigma$.

The intuitions for Propositions 6 and 7 are as follows. An increase in either the entry cost $f_e$ or the exit probability $1 - \delta$ deters potential firms from entering the market because the expected payoff from entry declines. Accordingly, there are fewer firms operating in the market. Few active firms reduces competition so that the aggregate price index increases. An increase in the market size $R$ attracts more entrants due to the expectation of a higher profitability. The larger mass of active firms intensifies competition so that the aggregate price index declines. Contrary to the entry cost, the exit probability and the market size,
the effects of the elasticity of substitution $\sigma$ are more complex. For a fixed aggregate price index, one can show that the left-hand side of (2.36) increases with a marginal increase in $\sigma$ when the price index is above a threshold $\overline{P}(\sigma)$ and decreases otherwise. This observation in turn implies that, to satisfy the free entry condition, the aggregate price index must decrease with a marginal increase in $\sigma$ in the former case, but increase in the latter case. Since the aggregate price index increases with the entry cost $f_e$ and the exit probability $1 - \delta$, there exist threshold levels of the entry cost and the exit probability, respectively, such that the aggregate price index is greater than $\overline{P}(\sigma)$ if the entry cost or the exit probability exceeds their respective thresholds, and is less than $\overline{P}(\sigma)$ otherwise. Consequently, the variation of the aggregate price index with the product elasticity depends on the entry cost and the exit probability. By (2.37), the differential effects of the product elasticity on the equilibrium aggregate price cause the former to have an ambiguous effect on the mass of active firms.

**Proposition 8 (Product Market Characteristics, Firm Value, and CEO Pay)**

- Managerial compensation increases with the entry cost $f_e$ and the likelihood of exit $1 - \delta$, but is not affected by the market size $R$.

- Firm value increases with the entry cost $f_e$, but does not change with the market size $R$. However, the effect of the exit probability $1 - \delta$ is ambiguous.

- There exists a trigger $\bar{i}$ such that managerial compensation increases with a marginal increase in the elasticity of substitution $\sigma$ if $i > \bar{i}$, but decreases if $i < \bar{i}$.

An increase in the market size $R$ decreases the aggregate price index. While the former has a positive effect on firm value and CEO pay, the latter has a negative effect. To ensure
that the free entry condition (2.36) is satisfied, these effects exactly cancel out so that an individual firm’s market value and its manager’s compensation do not change in response to the increase in the market size. An increase in the entry cost or the exit probability increases the aggregate price index so that CEO pay increases. The effect of the exit probability on firm value is, however, ambiguous. This is because the increase in the aggregate price index with the exit probability has a positive effect on firm value, whereas an increase in the exit probability also has a direct, negative effect on the present value of firm profits by (2.34).

The effects of product substitutability on CEO pay depend on the rank of the firm in the industry. A more elastic product market has differential effects on firms with larger firms benefiting more in terms of greater gross earnings but smaller firms experiencing a decline. Consequently, CEO pay increases with product substitutability if the rank is above a trigger, and decreases if it is below. Because firm value depends on the firm’s net earnings (gross earnings less CEO pay), the effects of product substitutability on firm value are ambiguous, in general. As we see in the next section, some of the ambiguous effects of product market characteristics on firm value and CEO pay can be pinned down when we calibrate the model to data.

2.5 Empirical Analysis

To quantitatively investigate the extent to which managerial talent and product market characteristics affect firm size and managerial compensation, we calibrate the model to U.S. data. Because the model is one of competing firms in the same industry, we calibrate the model “industry by industry.” We then compare the calibrated models across industries.
2.5.1 Data

Our sample includes S&P 1500 firms from the ExecuComp database. We obtain firm-specific variables from the Compustat annual database, and the Fama-French “twelve industry” classification as well as industry portfolio returns from Kenneth French’s website. We collect firm-CEO observations over the period 1992-2009 except for those in which sales or book value of equity are nonpositive. We partition the entire sample into different industry sectors. Among the twelve sectors, we exclude financial firms (SIC = 6000-6999) because the model is strictly applicable to conventional firms producing goods and services and selling them to consumers instead of financial intermediaries. In addition, because our model is one of unregulated firms, we do not include regulated firms (SIC = 481 and 4900-4949) based on Loughran and Ritter (1997). Further, we do not consider firms in miscellaneous industries that are classified as “Other” in the Fama-French classification. Accordingly, the final sample consists of 2,049 publicly traded U.S. companies and 20,635 firm-year observations, which are grouped into nine different industry sectors labeled as Consumer Nondurables, Consumer Durables, Manufacturing, Energy, Chemicals, Business Equipment, Telecom, Shops, and Health Care.¹

We now describe the variables used in the analysis. We compute total firm value (debt plus equity) as the market value of equity plus the book value of total assets minus the book

value of equity. Our measure of CEO compensation is “total compensation,” which includes salary, bonus, other annual, restricted stock grants, stock options (using the Black-Scholes formula), and long-term incentives. For the purpose of comparison, we also carry out tests using only CEO “cash compensation” (the sum of salary and annual bonus). In our analysis, we indirectly infer the product substitutability of each industry by calibrating the model to match moments of the distributions of CEO compensation and firm value in the data. To validate and evaluate our indirect estimates of the industry product substitutabilities, we also compute the price-cost margin for each industry. The “negative” price-cost margin is often used as an empirical proxy for product substitutability (see Nevo, 2001). We compute the price-cost margin as industry sales divided by industry operating costs. In particular, a firm’s operating costs include costs of goods sold, selling, general and administrative expenses, and depreciation, depletion, and amortization. We measure all variables in 2005 U.S. dollars using the GDP deflator provided by the Bureau of Economic Analysis.

Since the model’s implications are cross-sectional, we take the averages of the variables for each firm over the time period during which it was operating. Importantly, rather than using the actual CEO pay variables, we employ a Lowess (locally weighted regression scatter plot smoothing) curve to capture a smoothed pattern between CEO pay and firm value from the actual noisy relation. As Terviö (2008) points out, it facilitates the calibration of the assignment model that involves a monotone matching correspondence. For each industry, we rank firms by firm value and then perform a Lowess smoothing (bandwidth, 0.7) of

---

2Terviö (2003, 2008) uses market capitalization that is common shares outstanding multiplied by the share price (Compustat item MKVALT) as firm market value.
the relation between CEO compensation levels and firm ranks. Hereafter, the observed
distribution of CEO compensation refers to the smoothed levels.

Table 2.1 provides the cross-industry summary statistics for the variables in our analysis.
There is wide variation across industries. First, the business equipment industry contains the
largest number of firms. Given that this measure counts the number of firms that appeared
in the sample even for a short time period, it could be because many new dot-com companies
went public especially around the dot-com bubble period even though many of them failed
shortly. Second, the mean value of CEO total compensation is much higher in the energy,
business equipment, and telecom industries, relative to the manufacturing industry. Third,
differences in CEO pay levels across industries do not necessarily correspond to firm size
differences across industries. For instance, the total market value of the average firm in the
business equipment industry is $5.65 billion and that in the consumer durable goods industry
is $12.10 billion, whereas the average CEO earned $4.88 million in the former industry but
earned only $3.33 million in the latter industry. These preliminary findings suggest that CEO
pay does not increase with firm size across industries, and that product market/industry
characteristics are key determinants of CEO pay and firm size. Our subsequent analysis, in
which we calibrate our model to “industry by industry” data on the distributions of CEO
compensation and firm value, provides further support for these preliminary findings.
Table 2.1. Cross-Industry Summary Statistics

We extract firm-specific variables and CEO compensation variables for 1992-2009 from the Compustat Fundamentals Annual database and ExecuComp database, respectively. CEO Total Compensation is ExecuComp item $TDC$, which represents the total compensation comprised of salary, bonus, other annual, total value of restricted stock granted, total value of stock-options granted (using Black-Scholes), long-term incentive payouts, and all other total, whereas CEO Cash Compensation (ExecuComp item $TOTAL\_CURR$) is comprised of only salary and bonus. Total Firm Value (market value of common equity plus book value of debt) is computed as common stock price (item $PRCC\_F$) times shares outstanding at the end of fiscal year (item $CSHO$) plus total assets (item $AT$) minus book value of equity, which is computed as common equity (item $CEQ$) plus balance sheet deferred taxes (item $TXDB$). Total Assets is item $AT$, and Net Sales is item $SALE$. Operating Costs contains costs of goods sold (item $COGS$), selling, general and administrative expenses (item $XGAS$), and depreciation, depletion, and amortization (item $DP$). All nominal variables are converted to 2005 U.S. dollars (in millions) using the GDP deflator provided by the BEA. We take the averages of all of the above variables for each firm over the time period during which it was operating to construct a cross-sectional sample, and then group them into Fama-French’s 12 industries (except for finance, regulated, and other miscellaneous groups). CEO total compensation variables below are not the actual variables, but smoothed ones obtained by performing a Lowess (bandwidth, 0.7) on the relation between the actual compensation levels and the ranks of the firms in terms of total firm value. In this table, We report the means and standard deviations (in parentheses) of the variables by industry.

<table>
<thead>
<tr>
<th>Industry Sector</th>
<th>Number of Firms</th>
<th>CEO Total Compensation Mean</th>
<th>CEO Total Compensation SD</th>
<th>Total Firm Value Mean</th>
<th>Total Firm Value SD</th>
<th>Total Assets Mean</th>
<th>Total Assets SD</th>
<th>Net Sales Mean</th>
<th>Net Sales SD</th>
<th>Operating Costs Mean</th>
<th>Operating Costs SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>All</td>
<td>2,049</td>
<td>4.158</td>
<td>(3.183)</td>
<td>6,656.87</td>
<td>(2,078.71)</td>
<td>3,568.24</td>
<td>(12,479.35)</td>
<td>3,523.28</td>
<td>(11,341.15)</td>
<td>3,184.76</td>
<td>(10,410.12)</td>
</tr>
<tr>
<td>Consumer Nondurables</td>
<td>179</td>
<td>4.519</td>
<td>(3.277)</td>
<td>7,733.62</td>
<td>(19,706.46)</td>
<td>3,826.24</td>
<td>(8,719.88)</td>
<td>3,797.87</td>
<td>(7,107.49)</td>
<td>3,270.09</td>
<td>(5,864.65)</td>
</tr>
<tr>
<td>Consumer Durables</td>
<td>79</td>
<td>3.332</td>
<td>(2.143)</td>
<td>12,101.22</td>
<td>(48,959.69)</td>
<td>10,560.74</td>
<td>(46,190.00)</td>
<td>7,955.60</td>
<td>(28,532.63)</td>
<td>7,463.66</td>
<td>(26,899.93)</td>
</tr>
<tr>
<td>Manufacturing</td>
<td>344</td>
<td>3.247</td>
<td>(2.120)</td>
<td>4,800.97</td>
<td>(9,333.92)</td>
<td>3,093.20</td>
<td>(5,633.12)</td>
<td>3,011.03</td>
<td>(5,439.08)</td>
<td>2,724.85</td>
<td>(4,961.03)</td>
</tr>
<tr>
<td>Energy</td>
<td>129</td>
<td>4.901</td>
<td>(2.130)</td>
<td>11,112.22</td>
<td>(9,333.92)</td>
<td>7,308.58</td>
<td>(5,633.12)</td>
<td>7,307.09</td>
<td>(5,439.08)</td>
<td>6,571.67</td>
<td>(4,961.03)</td>
</tr>
<tr>
<td>Chemicals</td>
<td>81</td>
<td>3.803</td>
<td>(2.330)</td>
<td>7,811.80</td>
<td>(19,369.27)</td>
<td>4,139.14</td>
<td>(8,998.54)</td>
<td>3,871.18</td>
<td>(7,518.10)</td>
<td>4,300.54</td>
<td>(6,447.83)</td>
</tr>
<tr>
<td>Business Equipment</td>
<td>570</td>
<td>4.878</td>
<td>(4.197)</td>
<td>5,650.21</td>
<td>(19,557.22)</td>
<td>2,190.35</td>
<td>(6,704.69)</td>
<td>1,825.21</td>
<td>(5,951.96)</td>
<td>1,627.05</td>
<td>(5,276.70)</td>
</tr>
<tr>
<td>Telecom</td>
<td>47</td>
<td>9.427</td>
<td>(8.435)</td>
<td>19,090.18</td>
<td>(22,914.43)</td>
<td>13,465.62</td>
<td>(18,590.19)</td>
<td>4,712.52</td>
<td>(7,080.78)</td>
<td>4,042.46</td>
<td>(6,028.28)</td>
</tr>
<tr>
<td>Shops</td>
<td>351</td>
<td>3.355</td>
<td>(2.195)</td>
<td>4,858.26</td>
<td>(14,937.68)</td>
<td>2,667.92</td>
<td>(6,774.00)</td>
<td>5,423.12</td>
<td>(14,390.24)</td>
<td>5,146.80</td>
<td>(13,699.57)</td>
</tr>
<tr>
<td>Health Care</td>
<td>269</td>
<td>3.674</td>
<td>(2.758)</td>
<td>6,538.02</td>
<td>(19,934.36)</td>
<td>2,350.04</td>
<td>(4,300.51)</td>
<td>1,686.00</td>
<td>(3,291.66)</td>
<td>1,361.08</td>
<td>(3,901.99)</td>
</tr>
</tbody>
</table>
2.5.2 Calibration

We estimate the structural parameters of the model by matching moments of the distributions of CEO pay and firm value in a given industry. The calibration strategy is as follows:

1. By (2.34) and (2.35), the distributions of firm value and CEO pay depend on the distributions of firm quality, $x[i]$, and CEO talent, $y[i]$, that are *unobserved* by the econometrician. We hereafter refer to the latter distributions as *factor distributions*. For a given candidate vector of model parameter values—that is, the “deep” structural parameters of the model—we indirectly infer the factor distributions from the observed distributions of firm value and CEO pay in the industry using (2.28) and (2.29).

2. In the second step, we employ the indirectly inferred factor distributions obtained from step 1 to generate the model-predicted distributions of CEO pay and firm value using (2.34) and (2.35).

3. In the third step, we compare some statistics from the model-predicted distributions of CEO pay and firm value with the corresponding observed statistics.

4. We repeat steps 1-3 until we obtain the baseline set of model parameters as those that minimize the distance (in a suitable metric) between the observed moments and the predicted moments.

We now describe the calibration procedure in more detail. The set of parameters to be calibrated is
\[ \Delta = \{ \delta, R, \sigma, u_0 \}. \]  

(2.38)

Each of the above parameters is determined as follows. As Melitz (2003) notes, \( \delta \) can be viewed either as the likelihood that each operating firm will stay in the market for another period or as the time discount rate for the industry. The industry discount rate is usually calculated using the industry cost of capital, \( r \), assuming annual compounding:

\[ \delta = \frac{1}{1 + r}. \]  

(2.39)

The cost of capital \( r \) is computed as the weighted average of the cost of equity and after-tax cost of debt, weighted by the market values of equity and debt (see the Data Page of Damodaran Online for details). Because \( R \) is the aggregate revenue of all operating firms in the market, we set it equal to industry sales, that is, the sum of net sales for firms operating in the industry.

We determine the remaining two parameters—the elasticity of product substitution \( \sigma \) and the wage of the lowest ability manager, \( u_0 \)—by matching several moments from the model-predicted distributions of firm value and managerial compensation with the corresponding observed moments. More specifically, we match mean values, minimum and maximum values, and deciles of the distributions of firm value and CEO pay. Let \( \text{Obs}_i \) and \( \text{Pred}_i \) be the observed and predicted values of each selected statistic; then the baseline values of \( \sigma \) and \( u_0 \) solve

\[ (\sigma, u_0) = \arg \min_{\tilde{\sigma}, \tilde{u}_0} \sum_i \left( \frac{\text{Pred}_i(\tilde{\sigma}, \tilde{u}_0) - \text{Obs}_i}{\text{Obs}_i} \right)^2. \]  

(2.40)
As indicated above, the baseline values of $\sigma$ and $u_0$ minimize the sum of the squared percentage deviations of the predicted statistics from the observed ones.

We now describe how we generate the model-predicted distributions of firm value and CEO pay for a candidate parameter vector $\Delta$, that is, steps 1 and 2 of our calibration strategy described earlier. We first identify the unobserved factor distributions following Terviö (2003, 2008). Recall that the slopes of the payoff functions must follow (2.28) and (2.29), which guarantees matching stability. Dividing these slopes by the equation for the total surplus (2.21), respectively, yields the following equations for the rates of increase of the factors.

\[
\frac{x'[i]}{x[i]} = \frac{v'(i)}{(\sigma - 1)[v(i) + u(i)/(1 - \delta)]},
\]

(2.41)

\[
\frac{y'[i]}{y[i]} = \frac{u'(i)}{(1 - \delta)(\sigma - 1)[v(i) + u(i)/(1 - \delta)]}.
\]

(2.42)

We can then obtain the profiles of the unobserved factors relative to the lowest type by integrating these equations, respectively.

\[
\frac{x[i]}{x[0]} = \exp \left\{ \int_0^i \frac{x'[j]}{x[j]} \, dj \right\} = \exp \left\{ \frac{1}{\sigma - 1} \int_0^i \frac{v'(j)}{v(j) + u(j)/(1 - \delta)} \, dj \right\},
\]

(2.43)

\[
\frac{y[i]}{y[0]} = \exp \left\{ \int_0^i \frac{y'[j]}{y[j]} \, dj \right\} = \exp \left\{ \frac{1}{(1 - \delta)(\sigma - 1)} \int_0^i \frac{u'(j)}{v(j) + u(j)/(1 - \delta)} \, dj \right\}.
\]

(2.44)

We calculate the relative factor values by numerically integrating the right-hand sides of equations (2.43) and (2.44). As in Terviö (2003, 2008), the constants $x[0]$ and $y[0]$ cannot be identified and are not required for our calibration, that is, we only need the relative factor distributions, $x[i]/x[0]$ and $y[i]/y[0]$. 
Next, we use the indirectly inferred relative factor distributions to generate the model-predicted distributions of firm value and CEO pay from (2.34) and (2.35). Note that the equilibrium aggregate price index $P^*$ enters both (2.34) and (2.35). We determine the equilibrium aggregate price index as follows. Treating the observed number of firms in the industry as the equilibrium mass $N^*$ of firms in the model, the equilibrium condition (2.37) can be rewritten as

$$\left(\frac{P^*}{w}\right) = \frac{1}{\rho} \left[ \int_0^1 (x[i]y[i])^{\sigma-1} N^* di \right]^{\frac{1}{1-\sigma}}. \tag{2.45}$$

Since $x[0]$ and $y[0]$ are undetermined, we can only compute the relative aggregate price index, $P_0$, which is defined as

$$P_0 = (P^*/w)\rho x[0]y[0] = \left[ \int_0^1 \left( \frac{x[i]y[i]}{x[0]y[0]} \right)^{\sigma-1} N^* di \right]^{\frac{1}{1-\sigma}}. \tag{2.46}$$

Third, we generate the predicted distributions of firm value and CEO pay by plugging the given parameter values, the inferred relative factor values $x[i]/x[0]$ and $y[i]/y[0]$, and the relative aggregate price index $P_0$ into (2.34) and (2.35).

We compare the selected statistics from the predicted distributions with their observed counterparts. We minimize the sum of squared percentage differences between the model-predicted and observed values of the statistics as described in (2.40) to obtain the baseline values of $\sigma$ and $u_0$. 

2.5.3 Calibration Results

Through the calibration procedure illustrated in the previous section, we obtain the baseline parameter values and the factor distributions for each of the nine industries in our sample. Table 2.2 reports the estimates of the parameters in the first four columns. The first two parameters $\delta$ and $R$ are obtained directly from the data as described earlier. The other two parameter values—$\sigma$ and $u_0$—are obtained by matching moments of the predicted distributions of firm value and CEO compensation to their observed values. There are significant differences in the calibrated values of the product substitutability $\sigma$ across industries. The product substitutability is lower in the business equipment, health care, and telecom industries, whereas it is higher in the industries of consumer durable goods and shops. This result is intuitive given that, in high-technology-intensity sectors, product innovation plays a critical role and products tend to display a high degree of product differentiation (Ioannidis and Schreyer, 1997; Anderton, 1999). Moreover, because $\sigma$ can also be interpreted as the price elasticity for each good, the finding of Tellis (1988) also supports our result. By reviewing econometric studies that estimate price elasticities of different brands and markets, he documents that price elasticity for pharmaceutical products is lower than all other product categories and the difference is particularly significant and large for detergents and durable goods in comparison with pharmaceutical products.\(^3\)

Previous empirical literature uses the negative price-cost margin of the industry as a proxy for product substitutability (e.g. Nevo, 2001; Karuna, 2007). The underlying

\(^3\)The very high value of $\sigma$ for the consumer durable goods industry is, in fact, consistent with a general consensus in the literature on industrial organization that in any durable good industry, competition will be more intense than in a non-durable good industry (e.g., Coase, 1972).
Table 2.2. Parameter Estimates, Relative Factor Values, and Price-Cost Margin

<table>
<thead>
<tr>
<th>Industry Sector</th>
<th>$\delta$</th>
<th>$R$</th>
<th>$\sigma$</th>
<th>$u_0$</th>
<th>$x_1/x_0$</th>
<th>$y_1/y_0$</th>
<th>PCM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumer Nondurables</td>
<td>0.9301</td>
<td>679,818.9</td>
<td>7.6894</td>
<td>0.9473</td>
<td>2.5822</td>
<td>1.0105</td>
<td>1.1614</td>
</tr>
<tr>
<td>Consumer Durables</td>
<td>0.9295</td>
<td>628,492.7</td>
<td>15.5667</td>
<td>0.6806</td>
<td>1.5280</td>
<td>1.0028</td>
<td>1.0659</td>
</tr>
<tr>
<td>Manufacturing</td>
<td>0.9125</td>
<td>1,035,794</td>
<td>11.8989</td>
<td>1.4094</td>
<td>1.8379</td>
<td>1.0033</td>
<td>1.1050</td>
</tr>
<tr>
<td>Energy</td>
<td>0.9265</td>
<td>942,614.6</td>
<td>10.3864</td>
<td>2.1119</td>
<td>1.9014</td>
<td>1.0017</td>
<td>1.1119</td>
</tr>
<tr>
<td>Chemicals</td>
<td>0.9190</td>
<td>313,565.5</td>
<td>7.7124</td>
<td>0.6533</td>
<td>2.3079</td>
<td>1.0059</td>
<td>1.1384</td>
</tr>
<tr>
<td>Business Equipment</td>
<td>0.9010</td>
<td>1,040,368</td>
<td>3.5215</td>
<td>0.8246</td>
<td>22.5727</td>
<td>1.0282</td>
<td>1.1218</td>
</tr>
<tr>
<td>Telecom</td>
<td>0.9450</td>
<td>221,488.3</td>
<td>4.9714</td>
<td>0.9281</td>
<td>4.1659</td>
<td>1.0200</td>
<td>1.1658</td>
</tr>
<tr>
<td>Shops</td>
<td>0.9254</td>
<td>1,903,514</td>
<td>14.6211</td>
<td>0.8163</td>
<td>1.6698</td>
<td>1.0039</td>
<td>1.0537</td>
</tr>
<tr>
<td>Health Care</td>
<td>0.9207</td>
<td>453,534.7</td>
<td>3.6899</td>
<td>0.8390</td>
<td>13.7848</td>
<td>1.0244</td>
<td>1.2387</td>
</tr>
</tbody>
</table>

Figure 2.1. Inferred Distributions of Firm Quality and Managerial Talent
motivation for this proxy is that the higher is the extent of product substitutability in an industry, the greater is the price elasticity of demand, and the lower is the price-cost margin. To further validate our estimates of the parameter $\sigma$, we compare the ranking of industries with respect to our estimates of $\sigma$ with the corresponding ranking obtained by using the industry negative price-cost margin estimates.

We calculate the industry price cost margin as industry sales divided by industry operating costs. We show the estimates of the measure in the last column of the table shows the estimates of the measure. Note that the numbers should only be interpreted in an ordinal sense, that is, the ranking of the industries according to the measure is more meaningful for our purposes than the actual numbers themselves. Note that the higher the product substitutability of an industry, the lower is the price-cost margin. We observe that the health care and telecom industries have relatively high price-cost margins, whereas shops and consumer durable goods industries have low values. Except for the business equipment industry, the ranking of industries by our estimated values of $\sigma$ are consistent with the ranking by the negative price-cost margin. Because our estimates of $\sigma$ are indirectly inferred from data on the distributions of firm value and CEO pay, the close correspondence between the two rankings provides additional support for the model.

In addition to the model parameter values, we indirectly infer the unobserved factor distributions in our calibration exercise. The two middle columns of Table 2.2 report the highest values of firm quality and managerial talent relative to their lowest ones. For any industry, it is obvious that differences in firm quality between the highest and lowest ranking firms are greater than those in managerial talent, which implies that the relative impact of
firm quality on the resulting payoffs is higher than that of managerial talent as concluded by Gabaix and Landier (2008) and Terviö (2003, 2008). However, more importantly, *intra-industry dispersions* of the factors vary significantly across industries. Compared to other industries, the business equipment, health care, and telecom industries have higher relative values of firm quality and managerial talent.

In Figure 2.1, we report the entire factor distributions for the business equipment, health care, telecom, and manufacturing industries. The first two industries have much more widely dispersed firm and managerial characteristics across firms. In particular, the distributions of firm quality tend to be highly skewed to the right (convex), whereas those of managerial talent tend to be monotonically increasing (concave). The telecom industry shows a similar pattern (albeit higher) of firm heterogeneity to that of the manufacturing industry, whereas its talent distribution is closer to the first two industries even though there is little dispersion at the top of the distribution. Finally, firms and managers are very homogeneous in the manufacturing industry, which is also true for the other industries not reported in the graph.

Although our main focus is on the implications of managerial talent, we first discuss how to interpret the exogenous firm quality given that there are significant inter-industry variations in the inferred distributions of firm quality. The three industries with higher firm heterogeneity (business equipment, health care, and telecom industries) are usually referred to as high-tech industries (Loughran and Ritter, 2004). Moreover, in the first two industries, there are a small mass of firms that are much more significantly differentiated from other firms in the same industry. Terviö (2008) broadly interprets the dimension of
firm heterogeneity that is complementary to CEO talent as the natural scale of a firm, that is, all exogenous determinants of the scope (niches) of a firm’s operations that are linked to technology and consumer preferences. The strategy literature also attributes intra-industry firm heterogeneity to the establishment of unique product market positions (e.g., Caves and Porter, 1977). Such an interpretation is enhanced by our finding that variations in the firm-side dimension appear to be much greater in high-tech industries that are characterized by a greater variety of business ideas and technological innovations (Andersson et al., 2009).

We now discuss the implications of the inferred distributions of managerial talent. In Figure 2.1, compared to the manufacturing and other unreported industries, we observe greater intra-industry differences in CEO talent in high-tech industries. This finding, which is obtained by conducting the same procedure for each industry without any a priori assumption, suggests differences in CEO talent pools across industries. Although our analysis cannot compare the absolute levels of managerial talent between industries because the lowest levels are undetermined, a higher degree of CEO talent dispersion within an industry implies higher competition for CEO talent among firms. In other words, firms put greater emphasis on CEO human capital in those industries with greater heterogeneity in managerial talent. It is intuitive in the sense that the success of high-tech firms, which need to continuously develop new products and manage technological innovation in a highly competitive environment with very low barriers to entry and very high risk, is very closely tied to the talents of the workforce (Andersson et al., 2009). Further, “managerial rents” models in the management literature argue that managerial human capital is more emphasized in industries in the early stages of the product-life cycle (e.g., early biotechnology companies),
relative to that in more mature industries, and in industries with characteristics that allow
greater managerial discretion than industries with less latitude for managerial discretion
(Castanias and Helfat, 2001). Given that industries producing a differentiable product or
service and high-growth industries tend to provide more managerial discretion (Hambrick
and Abrahamson, 1995), our finding of larger talent dispersions in high-tech industries is in
line with these arguments.

One may argue that this finding might mainly be induced by the prevalence of equity-
based compensation of CEOs in high-tech industries. To examine that possibility, we perform
the same analysis using CEO cash compensation, containing only salary and annual bonus.
Figure 2.2 shows that greater differences in CEO talent are still observed in high-tech indus-
tries, relative to other traditional industries, even though the relative talent levels are overall
lower than those are in Figure 2.1. As another robustness check, we also test an extension
in which the impact of management team (top senior executives) is taken into account. As
a caveat on firm-CEO matching models, it is often pointed out that, upon CEO turnover,
the top management team usually tends to be replaced together. An easier way to deal
with this extension would be to assume that the quality of the management team is simply
characterized by a one-dimensional variable and keep the current framework as it is except
that the observed CEO compensation distribution should be replaced by the distribution of
the average compensation of non-CEO executives. In Figure 2.3, one can observe similar
patterns of firm quality and managerial talent distributions.
Figure 2.2. Inferred Distributions of Firm Quality and Managerial Talent (CEO Cash Compensation)

Figure 2.3. Inferred Distributions of Firm Quality and Managerial Talent (Non-CEO Total Compensation)
2.5.4 Counterfactual Experiments

We now conduct counterfactual experiments using the respective calibrated models that consist of the estimated baseline parameter values and the implied distributions of firm quality and CEO talent. First, we examine the quantitative impact of CEO talent in each industry. Second, we examine how product market characteristics affect firm value and CEO pay.

Impact of CEO Talent

We consider the experiment of Gabaix and Landier (2008) that examines the impact of CEO talent at the median-sized firm among the largest 500 firms. Suppose that the reference firm indexed by $i = 1/2$ could replace its manager by the best CEO indexed by $i = 1$ in the same industry. We assume that the aggregate market structure, such as the aggregate price index $P^*$ and the equilibrium mass $N^*$ of firms, remains unchanged with this event associated with only one firm. To begin with, using (2.21), we calculate the rate of increase in the total surplus $S$ at this reference firm as

$$\frac{\Delta S}{S[1/2]} = \frac{S(x[1/2],y[1]) - S(x[1/2],y[1/2])}{S(x[1/2],y[1/2])} = \left(\frac{y[1]/y[0]}{y[1/2]/y[0]}\right)^{\sigma-1} - 1. \quad (2.47)$$

Next, we estimate how much the firm’s market value $v$ would change due to this event by using (2.15). In fact, this measure captures the gross benefit from hiring the best CEO, that is, the present value of additional future gross earnings relative to the current market value as shown below:
\[
\frac{\Delta v}{v[1/2]} = \frac{\phi(x[1/2], y[1/2], u[1/2]) - \phi(x[1/2], y[1/2], u[1/2])}{\phi(x[1/2], y[1/2], u[1/2])} = \frac{R(P^*\rho x[1/2])^{\sigma-1} (y[1]^{\sigma-1} - y[1/2]^{\sigma-1})}{v[1/2]}. \tag{2.48}
\]

For the purpose of comparison, we also consider the cost to be incurred if the firm was required to pay the best CEO his current compensation at the largest firm. We compute this as the ratio of the present value of future additional compensation payments relative to the current market value, that is,}

\[
\frac{\Delta u/(1 - \delta)}{v[1/2]} = \frac{(u[1] - u[1/2])/(1 - \delta)}{v[1/2]}. \tag{2.49}
\]

Table 2.3 shows the results of this counterfactual experiment. Note that, since CEO compensation is relatively small compared to the firm’s gross profit, changes in surplus and those in firm value are very similar. The percentage changes in firm value in the second column should be compared to the result of Gabaix and Landier (2008), whose sample is the largest 500 firms among S&P 1500 firms in different industries. In their estimation, replacing the median CEO by the number one CEO at no extra compensation payment increases firm value by only 0.016%. Our estimates, by contrast, are almost two orders of magnitude greater than their estimate in most sectors.

More importantly, there is considerable variation across industries. The impact of better CEOs is, indeed, much more quantitatively significant in the business equipment and health care industries (about 2.9% and 2.5%, respectively), whereas the size of the impact in the
Table 2.3. Impact of CEO Talent at the Median-Sized Firm

<table>
<thead>
<tr>
<th>Industry Sector</th>
<th>$\frac{\Delta S}{S^{[1/2]}}$ (%)</th>
<th>$\frac{\Delta \nu}{\nu^{[1/2]}}$ (%)</th>
<th>$\frac{\Delta \nu/(1-\delta)}{\nu^{[1/2]}}$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumer Nondurables</td>
<td>1.723</td>
<td>1.762</td>
<td>6.945</td>
</tr>
<tr>
<td>Consumer Durables</td>
<td>1.134</td>
<td>1.151</td>
<td>4.017</td>
</tr>
<tr>
<td>Manufacturing</td>
<td>1.134</td>
<td>1.163</td>
<td>3.944</td>
</tr>
<tr>
<td>Energy</td>
<td>1.229</td>
<td>1.246</td>
<td>3.147</td>
</tr>
<tr>
<td>Chemicals</td>
<td>0.841</td>
<td>0.852</td>
<td>2.157</td>
</tr>
<tr>
<td>Business Equipment</td>
<td>2.841</td>
<td>2.909</td>
<td>10.719</td>
</tr>
<tr>
<td>Telecom</td>
<td>1.179</td>
<td>1.195</td>
<td>3.985</td>
</tr>
<tr>
<td>Shops</td>
<td>1.575</td>
<td>1.608</td>
<td>5.353</td>
</tr>
<tr>
<td>Health Care</td>
<td>2.465</td>
<td>2.532</td>
<td>8.693</td>
</tr>
</tbody>
</table>

The chemical industry (0.85%) is the lowest. In particular, notice that industries with lower impact of managerial talent are the chemical, consumer durable goods, and manufacturing industries which are often viewed as old economy industries in contrast with new economy or high-tech industries. Since there is no much difference in talent between the highest and the median ranking CEOs in the telecom industry as shown in Figure 2.1, this industry shows a somewhat low impact of managerial talent, which might be different if the replacement of the lowest ranking CEO with the best one was considered.

Further, these ratios of the benefit from hiring the best CEO should be compared with the ratios of additional compensation payments relative to firm market value that we report in the last column. Higher costs than benefits is a natural result of the competitive assignment process because otherwise the matching of the median firm-manager pair would not be sustained. As one can expect, the size of the additional cost is higher in the business equipment and health care industries (10.72% and 8.69%, respectively) in which the marginal returns to managerial talent are higher than in other industries. However, it is
worth emphasizing that, for any industry, the size of the cost is roughly of the same order of magnitude as the size of the benefit. More precisely, the cost is about three or four times greater than the benefit.

This result also contrasts sharply with the findings of Gabaix and Landier (2008). They document that the talent difference resulting in a mere 0.016% increase in firm market value implies 530% difference in CEO pay, which might be mainly attributed to the huge difference in firm size between the highest and median ranking firms, possibly, from different industries. For the purpose of comparison, we compute the cost measure from their calibration results,

\[ 4 \]

and obtain the estimate of 1.77% if the discount rate is 0.9 (based on the estimated values of \( \delta \) in our analysis). This cost measure is two orders of magnitude higher than their estimate of the rate of increase in firm market value, 0.016%. Overall, our results show that when different industries are characterized by different structures of CEO talent and product markets, the impact of managerial talent is not negligible at all, and the compensation of CEOs is quantitatively in line with their contributions to firms.

In addition to the effects of the hypothetical employment only at a reference firm, we now estimate the effects of counterfactual distributions, which is similar to the approach used by Terviö (2008). More specifically, we look at three cases in which all managers hypothetically have the same level of talent \( y[I] \) with \( I = 0, 1/2, \) and 1, respectively, while the existing distribution of firm quality is kept in place. Since there is no heterogeneity on the side of managers, the current levels of compensation (2.35) from the competitive assignment process cannot be applied. Following Terviö (2008), we assume that all the managers would

\[ 4 \] According to their notation, it is measured as the present value of additional future compensation payments, \( w(1) - w(n_*) \), divided by \( S(n_*) \), where \( n_* \) is 250.
Table 2.4. Effects of Hypothetical Talent Distributions

<table>
<thead>
<tr>
<th>Industry Sector</th>
<th>I=0</th>
<th>I=1/2</th>
<th>I=1</th>
<th>I=0</th>
<th>I=1/2</th>
<th>I=1</th>
<th>I=0</th>
<th>I=1/2</th>
<th>I=1</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>I=0</td>
<td>I=1/2</td>
<td>I=1</td>
<td>I=0</td>
<td>I=1/2</td>
<td>I=1</td>
<td>I=0</td>
<td>I=1/2</td>
<td>I=1</td>
</tr>
<tr>
<td>Industry Sector</td>
<td>∆U^I (%)</td>
<td>∆N^I (%)</td>
<td>∆E^S^I (%)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Consumer Nondurables</td>
<td>-0.866</td>
<td>-0.154</td>
<td>-0.200</td>
<td>0.728</td>
<td>0.240</td>
<td>-1.765</td>
<td>-0.720</td>
<td>-0.239</td>
<td>1.815</td>
</tr>
<tr>
<td>Consumer Durables</td>
<td>-0.228</td>
<td>-0.046</td>
<td>-0.058</td>
<td>0.507</td>
<td>0.142</td>
<td>-1.149</td>
<td>-0.505</td>
<td>-0.142</td>
<td>1.227</td>
</tr>
<tr>
<td>Manufacturing</td>
<td>-0.251</td>
<td>-0.059</td>
<td>-0.082</td>
<td>0.504</td>
<td>0.155</td>
<td>-1.223</td>
<td>-0.500</td>
<td>-0.154</td>
<td>1.243</td>
</tr>
<tr>
<td>Energy</td>
<td>-0.096</td>
<td>-0.080</td>
<td>-0.076</td>
<td>0.396</td>
<td>0.174</td>
<td>-1.004</td>
<td>-0.380</td>
<td>-0.173</td>
<td>1.040</td>
</tr>
<tr>
<td>Chemicals</td>
<td>-0.440</td>
<td>-0.060</td>
<td>-0.103</td>
<td>0.623</td>
<td>0.094</td>
<td>-1.024</td>
<td>-0.598</td>
<td>-0.094</td>
<td>1.078</td>
</tr>
<tr>
<td>Business Equipment</td>
<td>-2.224</td>
<td>-0.743</td>
<td>-0.661</td>
<td>0.784</td>
<td>0.354</td>
<td>-2.216</td>
<td>-0.769</td>
<td>-0.348</td>
<td>2.297</td>
</tr>
<tr>
<td>Telecom</td>
<td>-1.662</td>
<td>-0.119</td>
<td>-0.286</td>
<td>0.777</td>
<td>0.265</td>
<td>-1.561</td>
<td>-0.771</td>
<td>-0.264</td>
<td>1.581</td>
</tr>
<tr>
<td>Shops</td>
<td>-0.305</td>
<td>-0.068</td>
<td>-0.087</td>
<td>0.693</td>
<td>0.193</td>
<td>-1.616</td>
<td>-0.682</td>
<td>-0.190</td>
<td>1.668</td>
</tr>
<tr>
<td>Health Care</td>
<td>-2.009</td>
<td>-0.663</td>
<td>-0.432</td>
<td>0.627</td>
<td>0.241</td>
<td>-1.558</td>
<td>-0.611</td>
<td>-0.234</td>
<td>1.609</td>
</tr>
</tbody>
</table>

earn the same level of compensation that manager I receives in the original equilibrium with managerial heterogeneity, u[I]. All the product market characteristics are assumed to be the same, and we set the fixed entry cost f_e to the value of E[v[i]] in the current equilibrium, following the free entry condition (2.32). We derive a new equilibrium under this structure, that is, a new set of the relative aggregate price index, P_0^I, and the mass of firms, N^I, using (2.36) and (2.37):

\[
\frac{R}{\sigma} (P_0^I)^{\sigma-1} \int_0^1 \left( \frac{x[i]}{x[0]} \frac{y[I]}{y[0]} \right)^{\sigma-1} di = u[I] + (1 - \delta)f_e, \tag{2.50}
\]

\[
N^I = \frac{R}{\sigma} \frac{u[I] + (1 - \delta)f_e}{R}, \tag{2.51}
\]

\[
S([x[i], y[I]]) = \frac{R}{(1 - \delta)\sigma} \left( P_0^I \frac{x[i]}{x[0]} \frac{y[I]}{y[0]} \right)^{\sigma-1}, \tag{2.52}
\]

and then compare the results of each counterfactual with the original equilibrium outcome.

Table 2.4 displays the percentage differences in consumer welfare, the mass of firms, and the mean value of the surplus. Note that consumer welfare is measured by the equilibrium
utility level of the representative consumer, $U^* = R/P^*$, by (2.5), which is inversely related to the aggregate price index.

First, consumer welfare in each counterfactual is worse than in the original economy. The integral on the left-hand side of (2.50) represents the industry-wide productivity and has a greater value if $I$ is bigger. $u[I]$ on the right-hand side is also larger if $I$ is bigger. When the hypothetical talent level of all managers is the lowest one, $y[0]$, the former effect is larger than the latter effect, resulting in a higher aggregate price and thus negatively affecting consumer welfare. Interestingly, even when all managers are of the highest type with $y[1]$, that is, the industry-wide productivity in this counterfactual is higher than in the original equilibrium, consumer welfare is still worse than the original equilibrium because of the high value of $u[1]$. Note that the mass of firms would decrease in this counterfactual. In other words, the high managerial compensation might attract less firms in the market and dampen market competition, thereby causing a deterioration in consumer welfare. In particular, industries with higher levels of CEO compensation show greater consumer welfare losses.

**Impact of Product Market Characteristics**

We now examine the effects of different product market characteristics on the equilibrium outcome. In the monopolistically competitive product market, there are several dimensions influencing the market structure: the elasticity of product substitution $\sigma$, the exit probability $1 - \delta$, the entry cost $f_e$, and the market size $R$. We explore how each of these dimensions alters the equilibrium outcome, including consumer welfare (due to a change in
the aggregate price index), the mass of firms, and the levels and distributions of CEO pay and firm market value, by varying that specific parameter over its plausible range.

Effects of Product Substitutability:

Table 2.5 shows the effects of the elasticity of product substitution across industries. To begin with, it is observed that, for any industry, the aggregate price index increases and the mass of firms declines as product substitutability increases. According to Proposition 6, this observation implies that the entry cost and the exit probability are below their respective thresholds. In this case, at the current equilibrium aggregate price, a marginal increase in product substitutability implies more intense price competition and therefore lowers firms’ gross and net profits, which, in turn, adjusts the aggregate price upward to a new level for the free entry condition (2.36) to be met again. While the percentage change in the mass of firms seems to be similar across industries, the change in the aggregate price index varies significantly across industries. In particular, the aggregate price index is more sensitive to the elasticity of product substitution especially in the business equipment and health care industries, for which the baseline value of the parameter is relatively lower as shown in Table 2.2. Hence, the observation implies that these industries are not only heterogeneous in terms of product substitutability but also more vulnerable to some exogenous factors that would affect the degree of product substitutability. In contrast, the consumer durable goods industry has the least sensitivity of the aggregate price index. In fact, its least change in the consumer durable goods industry is also observed when any of other product market characteristics changes. This result might be explained by the argument of the price rigidity in this industry (Domowitz, Hubbard, and Petersen, 1986, 1988; Leith and Malley, 2007).
Table 2.5. Effects of Product Substitutability $\sigma$

<table>
<thead>
<tr>
<th>Industry Sector</th>
<th>$\Delta U (%)$</th>
<th>$\Delta N (%)$</th>
<th>$\Delta E_u (%)$</th>
<th>$\Delta E_v (%)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumer Nondurables</td>
<td>10.63</td>
<td>4.77</td>
<td>-4.01</td>
<td>-7.37</td>
</tr>
<tr>
<td>Consumer Durables</td>
<td>3.69</td>
<td>1.68</td>
<td>-1.44</td>
<td>-2.66</td>
</tr>
<tr>
<td>Manufacturing</td>
<td>7.01</td>
<td>3.20</td>
<td>-2.75</td>
<td>-5.09</td>
</tr>
<tr>
<td>Energy</td>
<td>6.91</td>
<td>3.15</td>
<td>-2.66</td>
<td>-4.93</td>
</tr>
<tr>
<td>Chemicals</td>
<td>9.60</td>
<td>4.33</td>
<td>-3.64</td>
<td>-6.70</td>
</tr>
<tr>
<td>Business Equipment</td>
<td>47.11</td>
<td>19.15</td>
<td>-13.68</td>
<td>-23.77</td>
</tr>
<tr>
<td>Telecom</td>
<td>16.07</td>
<td>7.09</td>
<td>-5.84</td>
<td>-10.61</td>
</tr>
<tr>
<td>Shops</td>
<td>5.43</td>
<td>2.49</td>
<td>-2.14</td>
<td>-3.98</td>
</tr>
<tr>
<td>Health Care</td>
<td>35.95</td>
<td>14.95</td>
<td>-11.06</td>
<td>-19.45</td>
</tr>
</tbody>
</table>

Figure 2.4. Shifts in CEO Pay Distributions due to Changes in $\sigma$ (Business Equipment and Telecom)
Next, we examine the effects of product substitutability for the equilibrium distributions of CEO pay and firm market value. CEO pay levels, measured by their mean values, decline with this dimension of market competition. In particular, CEO pay levels in the health care and business equipment industries are affected most by a change in this parameter. Roughly speaking, the average CEOs in these industries would face an about 8% pay cut in response to a 5% increase in product substitutability in the market.

In addition to the mean values of CEO pay, we further examine the shifts in CEO pay distributions in response to a 10% increase and a 10% decrease in product substitutability for the business equipment and telecom industries. Figure 2.4 displays the shifts in CEO pay distribution for these industries. As noted in the comparison of the mean values of CEO pay, the shifts in CEO pay distributions in the former industry are more noticeable than those in the latter industry. More importantly, the figure confirms the analytical result presented in Proposition 8 that an increase in the elasticity of product substitution affects managerial compensation differently across firms. That is, there is a certain rank such that managers below the rank get paid less than currently, whereas managers above that rank get paid more. While the trigger rank in the former industry is almost the highest one, it is lower ($\bar{\tau} = 0.9$) in the latter industry. This result implies that a change in product substitutability may induce a larger CEO pay dispersion within an industry.

Finally, while firms would face a more price-elastic demand in the market in response to an increase in product substitutability, they could also reduce CEO pay levels. It is shown in Table 2.5 that, due to these contrasting effects, product substitutability has a less significant quantitative effect on firm market value than it does on CEO pay.
Effects of the Exit Probability and Entry Cost:

The exit probability $1 - \delta$ and the entry cost $f_e$ affect the equilibrium outcome mainly through the aggregate price index determined by the free entry condition (2.36). As either of them increases, the free entry condition implies that the aggregate price index also must increase. The other equilibrium condition (2.37) implies that this market change lowers the mass of firms in the market. The intuition is that an increase in the exit probability or the entry cost induces fewer firms to enter the market and, therefore, reduces price competition, resulting in a higher aggregate price index. Consequently, the marginal returns to talent, the integrand in (2.35), increase so that managers would get paid more.

Table 2.6 displays the effects of the industry discount rate $\delta$ instead of the exit probability $1 - \delta$, and the effects of the fixed entry cost $f_e$ are reported in Table 2.7. Notice that we consider smaller percentage changes in $\delta$ because of its upper limit, $0 < \delta < 1$. The exit probability has a much more significant quantitative impact on the equilibrium outcome than any other parameters do. Since the parameter can be viewed as being negatively associated with industry risk that active firms face in the market, this observation suggests the important role of risk in determining the market equilibrium outcome. In particular, the telecom industry is overall most sensitive to this factor. Specifically, a 5% increase in risk leads to a 65.3% increase in the mean value of CEO pay in the telecom industry. Moreover, one can observe that the changes in firm value are relatively smaller than those in CEO pay and that their signs vary across industries, which in fact confirms the ambiguous effect of the discount rate on firm market value in Proposition 8. In contrast with the influence of the exit probability, that of the entry cost seems to be uniform across industries. Nonetheless,
### Table 2.6. Effects of Discount Rate $\delta$

<table>
<thead>
<tr>
<th>Industry Sector</th>
<th>$\frac{\Delta U}{E}$ (%)</th>
<th>$\Delta N$ (%)</th>
<th>$\frac{\Delta E[u]}{E[u]}$ (%)</th>
<th>$\frac{\Delta E[v]}{E[v]}$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumer Nondurables</td>
<td>-6.71</td>
<td>-3.81</td>
<td>5.27</td>
<td>13.94</td>
</tr>
<tr>
<td>Consumer Durables</td>
<td>-2.38</td>
<td>-1.29</td>
<td>1.62</td>
<td>3.81</td>
</tr>
<tr>
<td>Manufacturing</td>
<td>-3.21</td>
<td>-1.76</td>
<td>2.20</td>
<td>5.19</td>
</tr>
<tr>
<td>Energy</td>
<td>-4.47</td>
<td>-2.50</td>
<td>3.45</td>
<td>8.71</td>
</tr>
<tr>
<td>Chemicals</td>
<td>-5.80</td>
<td>-3.23</td>
<td>4.28</td>
<td>10.56</td>
</tr>
<tr>
<td>Business Equipment</td>
<td>-13.05</td>
<td>-7.35</td>
<td>9.87</td>
<td>24.20</td>
</tr>
<tr>
<td>Telecom</td>
<td>-12.37</td>
<td>-7.17</td>
<td>11.12</td>
<td>34.00</td>
</tr>
<tr>
<td>Shops</td>
<td>-2.95</td>
<td>-1.63</td>
<td>2.16</td>
<td>5.30</td>
</tr>
<tr>
<td>Health Care</td>
<td>-15.13</td>
<td>-8.62</td>
<td>12.55</td>
<td>33.78</td>
</tr>
</tbody>
</table>

### Table 2.7. Effects of Entry Cost $f_e$

<table>
<thead>
<tr>
<th>Industry Sector</th>
<th>$\frac{\Delta U}{E}$ (%)</th>
<th>$\Delta N$ (%)</th>
<th>$\frac{\Delta E[u]}{E[u]}$ (%)</th>
<th>$\frac{\Delta E[v]}{E[v]}$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumer Nondurables</td>
<td>1.56</td>
<td>0.74</td>
<td>-0.76</td>
<td>-1.47</td>
</tr>
<tr>
<td>Consumer Durables</td>
<td>0.72</td>
<td>0.34</td>
<td>-0.34</td>
<td>-0.65</td>
</tr>
<tr>
<td>Manufacturing</td>
<td>0.95</td>
<td>0.46</td>
<td>-0.45</td>
<td>-0.87</td>
</tr>
<tr>
<td>Energy</td>
<td>1.14</td>
<td>0.55</td>
<td>-0.52</td>
<td>-1.01</td>
</tr>
<tr>
<td>Chemicals</td>
<td>1.58</td>
<td>0.76</td>
<td>-0.74</td>
<td>-1.43</td>
</tr>
<tr>
<td>Business Equipment</td>
<td>4.30</td>
<td>2.07</td>
<td>-1.94</td>
<td>-3.76</td>
</tr>
<tr>
<td>Telecom</td>
<td>2.61</td>
<td>1.24</td>
<td>-1.27</td>
<td>-2.40</td>
</tr>
<tr>
<td>Shops</td>
<td>0.78</td>
<td>0.38</td>
<td>-0.36</td>
<td>-0.70</td>
</tr>
<tr>
<td>Health Care</td>
<td>4.05</td>
<td>1.94</td>
<td>-1.84</td>
<td>-3.55</td>
</tr>
</tbody>
</table>
the price rigidity of the consumer durable goods industry still holds in response to a change in the exit probability and the entry cost.

**Effects of the Market Size:**

Table 2.8 confirms the effects of the market size $R$ discussed in Section 2.4. At the current aggregate price, as the market size increases, the market demand each firm faces increases, and so does its profitability, which attracts more firms to the market. A greater mass of firms in the market induces more intense price competition, thereby driving the aggregate price index down. As discussed in Proposition 8, we also empirically observe that a firm’s market value and managerial compensation are not affected by the market size $R$ (therefore, unreported).

### 2.6 Conclusion

We study how the distributions of CEO talent and compensation vary across industries, and how product market characteristics affect these distributions. We develop a market equi-
librium model that incorporates the competitive assignment of CEOs to firms in a framework in which firms engage in imperfect product market—specifically, monopolistic—competition. The model enables the simultaneous analysis of the effects of managerial talent and product market characteristics on the determination of firm value and CEO pay distributions. We characterize the unique, stationary equilibrium of the model and then calibrate the model to a sample of firm-CEO observations in each of the nine industry sectors based on the twelve Fama-French industry classification. Using the respective calibrated models for different industries, we perform several counterfactual experiments to investigate the quantitative effects of managerial talent and those of product market characteristics.

There are several main results obtained from the analysis of the paper. First, we find that there is much variation in the distributions of firm quality and managerial talent across industries. As compared with other industries, high-tech industries are characterized by higher heterogeneity both in firm quality and in managerial talent. Second, contrary to the conclusions of Terviö (2008) and Gabaix and Landier (2008), the impact of CEO talent on shareholder value is, indeed, significant, and it is roughly of the same order of magnitude as CEO pay. The explicit incorporation of the product market environment in which firms operate plays a key role in generating these findings. Third, the contribution of CEO talent varies significantly across industries. As one may expect from the inferred talent distributions, managerial talent is more important to firm value in high-tech industries so that more intense competition for CEO talent in those industries leads to higher pay dispersions. Fourth, we analytically derive the effects of different product market characteristics on firm value and CEO pay. In particular, either the entry cost or the exit probability shifts the
entire CEO pay distribution upward or downward, whereas the elasticity of product substitution may affect large and small firms differently, which leads to higher pay differences between CEOs in the same industry. Overall, our study shows that industry structures associated with CEO labor markets and product markets help explain the variations in the levels and distributions of CEO pay across industries.

In this paper, we abstract away from asymmetric information, risk and incentive provisions as in Gabaix and Landier (2008) and Tervio (2008). However, since a large body of research on CEO compensation is based upon agency problems, a natural next step would be to introduce asymmetric information stemming from moral hazard. Such an analysis could explore the importance of risk and moral hazard in the endogenous matching of CEOs to firms and the determination of CEO compensation levels and incentives. We could estimate agency costs arising from moral hazard across industries and obtain qualitative as well as quantitative implications for the effects of product market characteristics on managerial incentives and the inefficiencies arising from agency problems.

2.7 Appendix: Proofs

2.7.1 Proof of Proposition 6

As the entry cost $f_e$ or the likelihood of exit $1 - \delta$ increases, the right-hand side of (2.36) increases. Since the left-hand side of (2.36) is an increasing function of the aggregate price index, it follows that the equilibrium aggregate price must increase with $f_e$ or $1 - \delta$ to satisfy the equilibrium condition (2.36). The market size $R$ has an opposite effect because $R$ is on the left-hand side of (2.36).

The result above is used to show the second result about the effect of a marginal increase in $\sigma$. Here, we only prove the result with the threshold of the entry cost, $\bar{f}_e(\sigma)$ because the result with the threshold of the exit probability, $1 - \bar{\delta}(\sigma)$ can be similarly shown. First,
define from (2.36)
\[
f(\sigma, P) = \frac{RP^\sigma - 1}{1 - \delta} \left[ \frac{(\rho x[0]y[0])^{\sigma - 1}}{\sigma} + \rho^\sigma \int_0^1 \left[ \int_0^i x[j]^{\sigma - 2} y[j]^{\sigma - 1} x'[j]dj \right] di \right] - \frac{u_0}{1 - \delta}. \tag{2.53}
\]

If \( P^*(\sigma) \) denotes the equilibrium aggregate price index when the elasticity of substitution is \( \sigma \),
\[
f(\sigma, P^*(\sigma)) = f_e. \tag{2.54}
\]
By taking the derivative of \( f \) with respect to \( \sigma \), one can observe that \( \frac{\partial f}{\partial \sigma} \) is greater than zero if \( P \) exceeds a threshold \( \bar{P}(\sigma) \) and is less than zero otherwise. In addition, note that \( \frac{\partial f}{\partial P} > 0 \). By (2.54) and the implicit function theorem, we can write
\[
\frac{dP^*(\sigma)}{d\sigma} = -\frac{\partial f/\partial \sigma}{\partial f/\partial P} \bigg|_{P = P^*(\sigma)}. \tag{2.55}
\]
In the proof of the first result of this proposition, the aggregate price index \( P^*(\sigma) \) has been shown to increase with the entry cost \( f_e \). It then follows that there exists a threshold level \( \bar{f}_e(\sigma) \) of the entry cost such that \( P^*(\sigma) > \bar{P}(\sigma) \) if \( f_e > \bar{f}_e(\sigma) \) and \( P^*(\sigma) < \bar{P}(\sigma) \) if \( f_e < \bar{f}_e(\sigma) \), which determines the sign of \( \partial f/\partial \sigma \). Taken together, \( \frac{dP^*(\sigma)}{d\sigma} < 0 \) if \( f_e > \bar{f}_e(\sigma) \) and \( \frac{dP^*(\sigma)}{d\sigma} > 0 \) if \( f_e < \bar{f}_e(\sigma) \).

**2.7.2 Proof of Proposition 7**

In the proof of Proosition 6, we have showed that the equilibrium aggregate price increases with the entry cost \( f_e \) or the exit probability \( 1 - \delta \), whereas it decreases with \( R \). By the observation and (2.37), we immediately have the first result of this proposition. On the other hand, the impact of a marginal increase in \( \sigma \) cannot be unambiguously determined because the mass of firms, given by (2.37), depends on the factor distributions as well as other product market characteristics. Hence, we empirically explore the effect of \( \sigma \) after calibrating the model to data. Q.E.D.

**2.7.3 Proof of Proposition 8**

Equation (2.36) can be rewritten as an equation for \( RP^\sigma - 1 \). Plugging that equation into (2.35) shows that CEO compensation increases with the the entry cost \( f_e \) and the likelihood of exit \( 1 - \delta \), whereas it does not change with the market size \( R \). To prove the effects on firm market value, we plug the equation for \( RP^\sigma - 1 \) and (2.35) into (2.34) and perform a partial integration, which provides the following equation:
\[
v[\bar{z}] = Qf_e + \frac{(Q - 1)u_0}{1 - \delta}, \tag{2.56}
\]
where \( Q = \frac{\int_0^1 x^{\sigma-2} y [j]^{\sigma-1} x'[j] dz}{\int_0^{1/\sigma} x^{\sigma-2} y [j]^{\sigma-1} x'[j] dz} \) > 0. In the above, one can see that firm market value increases with the entry cost \( f_e \), but is not affected by the market size \( R \). It is also shown that firm market value increases with the exit probability \( 1 - \delta \) if \( Q < 1 \), but decreases if \( Q > 1 \). However, since the value of \( Q \) depends on the factor distributions, the effect of the exit probability should be empirically tested after the factor distributions are implied by data.

Finally, we show the effect of product substitutability on CEO pay. We first differentiate equation (2.35) with respect to \( \sigma \) as follows:

\[
\frac{\partial u[i]}{\partial \sigma} = \int_0^i \left( \frac{\partial h(j, \sigma)}{\partial \sigma} \right) \, dj = \int_0^i \left( h(j, \sigma) \frac{\partial \ln h(j, \sigma)}{\partial \sigma} \right) \, dj, \tag{2.57}
\]

where \( h(j, \sigma) = R P^*(\sigma)^{\sigma-1} \rho^\sigma x[j]^{\sigma-1} y[j]^{\sigma-2} y'[j] \) and \( P^*(\sigma) \) is the equilibrium aggregate price index when the elasticity of substitution is \( \sigma \). Taking the derivative of \( \ln h(j, \sigma) \), we obtain

\[
\frac{\partial}{\partial \sigma} \ln h(j, \sigma) = \ln P^*(\sigma) + (\sigma - 1) \frac{\partial}{\partial \sigma} \ln P^*(\sigma) + \ln \rho + \frac{1}{\sigma - 1} + \ln(x[j]y[j]). \tag{2.58}
\]

It then follows that there exists a trigger level \( \tilde{j} \) of firm rank such that \( \frac{\partial \ln h(j, \sigma)}{\partial \sigma} > 0 \) for \( j > \tilde{j} \) and \( \frac{\partial \ln h(j, \sigma)}{\partial \sigma} < 0 \) for \( j < \tilde{j} \). Since \( h(j, \sigma) \) is positive, the integrand in (2.57) has the same sign as that of \( \frac{\partial \ln h(j, \sigma)}{\partial \sigma} \). Accordingly, it is evident that the right-hand side of (2.57) is negative unless \( i \) is sufficiently high. Note that the threshold for \( \tilde{i} \), to be denoted by \( \bar{i} \), is different from \( \tilde{j} \) at which \( \frac{\partial \ln h(j, \sigma)}{\partial \sigma} = 0 \). Therefore, CEO pay level increases (decreases) with \( \sigma \) when the rank of a firm is above (below) \( \bar{i} \). Q.E.D.
3.1 Introduction

In this chapter, we generalize the model developed in chapter 2 by allowing for the possibility that the mass of potential managers exceeds the mass of managers who successfully match with active firms.\(^1\) We hope to achieve two broad objectives in this chapter. First, we provide a market equilibrium framework in which the CEO-firm matching process is affected by the product market. In the previous chapter, we take an agnostic view towards the underlying mechanism that endogenizes the equilibrium distributions of firm quality and managerial talent, but attempt to empirically infer those distributions. In this chapter, on the other hand, we show that, under some reasonable assumptions, there exists a unique equilibrium cutoff level of talent such that only managers with ability above the level match with firms. The distribution of active managers who succeed in matching with firms is affected by the cutoff level. Because the cutoff level is determined by the market equilibrium conditions, not only do product market characteristics affect the endogenous determination of the distribution of active managers, but the total mass of potential managers and their distribution affect the market equilibrium outcome.

\(^{1}\)We thank Volker Nocke for this suggestion.
Second, we pursue a parametric approach in the empirical analysis that assumes specific forms for the distributions of the factors. Although this methodology is necessary to fully implement our extended model, it is also worthwhile to compare the results from a parametric approach with those of Gabaix and Landier (2008) who evaluate differences in talent by assuming talent follows the extreme value distribution. In particular, since our calibration considers the ex-ante distribution of potential managers in addition to the ex-post distribution of active managers, we can perform counterfactual experiments using both distributions.

3.2 The Model

We consider a discrete-time, infinite horizon framework of a single industry. The economy has a continuum of people who are divided into two groups depending on whether or not the agent possesses specialized human capital (or managerial talent). In each period, there is a mass $M$ of potential managers who have specialized human capital and an unbounded mass of people who do not have managerial skills but only supply one unit of labor each inelastically. Managerial talent $y$ is drawn from the cumulative distribution function $G(y)$ over $[y_{min}, y_{max}]$ which represents the \textit{ex-ante} talent distribution. A group of agents in the latter group—hereafter, “entrepreneurs,” establish a firm by supplying “investment” labor. Since we assume an inelastic supply of labor, there is an unbounded pool of prospective entrants (firms) into the industry. Firms are identical prior to entry, whereas their quality is randomly realized upon entry from the cumulative distribution $F_X(x)$ over $[x_{min}, x_{max}]$. 
Once realized, firm qualities and managerial talents are observable and remain constant over time.

We are particularly interested in the case in which insufficiently talented managers cannot be hired by a firm, that is, the mass of active managers is less than the total mass $M$ of potential managers. In what follows, we show that, under certain reasonable conditions, there exists a unique cutoff talent level such that all managers with ability above the cutoff level match with firms. Especially, as we show later, the cutoff level is endogenously determined by the equilibrium conditions.

Let $\bar{y}$ be the endogenous cutoff talent level. We can then define a cumulative distribution function for a given $\bar{y}$ that represents the talent distribution only for active managers, that is, the ex-post talent distribution of the sector, as follows:

$$F_Y(y|\bar{y}) = Pr\{Y \leq y|Y \geq \bar{y}\} = \frac{G(y) - G(\bar{y})}{1 - G(\bar{y})}, \quad \text{for } y \geq \bar{y}, \quad (3.1)$$

which has support $[0, 1]$. We then index those active managers on the unit interval using this cumulative distribution function

$$y[i|\bar{y}] = y \quad s.t. \quad F_Y(y|\bar{y}) = i, \quad (3.2)$$

where we explicitly denote its dependence on $\bar{y}$. The profile of firm quality is also similarly defined as

$$x[i] = x \quad s.t. \quad F_X(x) = i. \quad (3.3)$$
The product market environment and the matching process between firms and managers are identical to those developed in Section 2.3 except that only managers with talent above the cutoff level are matched to firms. We consider a Dixit-Stiglitz type framework of monopolistic competition and a two-sided matching mechanism between firms and managers in the executive labor market. As in Melitz (2003), we assume that a producing firm exits the market in any period for exogenous reasons with probability \( \delta \in (0, 1) \) which can be viewed as a common time discount factor.

In the stationary equilibrium, exiting firms are exactly replaced by new entrants in each period so that the mass \( N \) of producing firms remain the same. Once they enter the market and learn their qualities and, therefore, their ranks indexed over the unit interval, the mass \( \delta N \) of new entrants participate in the matching process in the executive market with the mass \( \delta M \) of potential managers. The solution of the matching problem is exactly the same as before. Complementarity between firm quality and managerial talent ensures positive assortative matching between managers and firms so that manager ranked \( i \) is assigned to the equally ranked firm \( i \). The total surplus \( S \) that each pair \( i \) generates under monopolistic competition in the product market, which is given by (2.21), is apportioned into both parities in a way that neither of them can become strictly better off by matching with a new type of partner.

As in the previous chapter, the equilibrium payoff profiles to active firms and managers are given by

\[
v[i] = \frac{1}{1 - \delta} \left[ \frac{R((P/w)\rho x[i]y[i|\bar{y}]^{\sigma-1}}{\sigma - 1 u[i]} \right], \quad (3.4)
\]
\[ u[i] = u_0 + \int_0^1 (R(P/w)^{\sigma-1} \rho^\sigma x[j]^{\sigma-1} y[j|\hat{y}]^{\sigma-2} y'[j|\hat{y}]) \, dj, \quad (3.5) \]

where \( P \) represents the aggregate price index which is taken as given by each individual firm and \( u_0 \) represents the pay level of the manager ranked the lowest among active managers, who has talent equal to the cutoff level, from her outside opportunity.

We now describe the market equilibrium conditions. First, the equilibrium mass \( N \) of active firms must be equal to that of active managers due to the one-to-one matching process, that is,

\[ N = M(1 - G(\hat{y})), \quad (3.6) \]

where \( 1 - G(\hat{y}) \) represents the fraction of managers with ability above \( \hat{y} \). Second, the product market clearing condition can be rewritten as an equation for the aggregate price index \( P \)

\[ R = N \int_0^1 r(x[i], y[i|\hat{y}]) \, di \quad \Rightarrow \quad (P/w)^{\sigma-1} = \left[ N \int_0^1 (\rho x[i] y[i|\hat{y}])^{\sigma-1} \, di \right]^{-1}, \quad (3.7) \]

where the second equation follows from equation (2.12). The last condition is the free entry condition which is given by (2.32). Using equations (3.4) and (3.5), we can have

\[ E[v[i]|\hat{y}] = \frac{R(P(\hat{y})/w)^{\sigma-1}}{(1 - \delta)} \left[ \frac{\rho x[0] y[0|\hat{y}]}{\sigma} \right]^{\sigma-1} + \rho^\sigma \int_0^1 \left[ \int_0^i x[j]^{\sigma-2} y[j|\hat{y}]^{\sigma-1} x'[j|\hat{y}] \, dj \right] \, di - \frac{u[0|\hat{y}]}{1 - \delta} \]

\[ = \frac{R(P(\hat{y})/w)^{\sigma-1}}{(1 - \delta)} \left[ \frac{\rho x[0] y[0|\hat{y}]}{\sigma} \right]^{\sigma-1} + \rho^\sigma \int_0^1 \left[ \int_0^i x[j]^{\sigma-2} y[j|\hat{y}]^{\sigma-1} x'[j|\hat{y}] \, dj \right] \, di - \frac{u_0}{1 - \delta}, \quad (3.9) \]
where the augment of the aggregate price index explicitly indicates its dependence on the cutoff talent level $\bar{y}$ as shown in equations (3.6) and (3.8). The free entry condition ensures that the expected firm value equals the entry cost $f_e$, which endogenously determines the equilibrium cutoff talent level.

Let us now examine how the expected firm value varies with $\bar{y}$. First, the term $(P(\bar{y})/w)^{\sigma-1}$ varies non-monotonically with $\bar{y}$ in general. By (3.6), on the one hand, $N$ decreases with $\bar{y}$ because the cumulative distribution function $G(\bar{y})$ increases. On the other hand, it can be easily shown by its definition (3.2) that the term $y[i|\bar{y}]$ in the integrand increases with $\bar{y}$. The combination of these terms that increase or decrease with the cutoff talent level makes its effect on the aggregate price index ambiguous. Second, the term in the bracket of (3.9) increases with $\bar{y}$ again because $y[i|\bar{y}]$ in the integrand increases. As a result, the uniqueness of the equilibrium is not guaranteed in general.

### 3.3 Equilibrium

To ensure that, in equilibrium, there is a unique cutoff level such that there is a positive mass of managers who cannot succeed in matching with firms due to their talent below the cutoff level, we impose the following assumptions on the ex-ante talent distribution $G(y)$ and product market characteristics.

1. The hazard rate of the talent distribution function

$$h(y) \equiv \frac{g(y)}{1 - G(y)} \quad (3.10)$$
is increasing in $y$.

2. The elasticity of product substitution satisfies the following condition

$$ (\sigma - 1) \leq g(y_{\text{min}}) y_{\text{min}}. $$

(3.11)

3. The fixed entry cost $f_e$ is sufficiently large so that

$$ E[v[i]|\bar{y} = y_{\text{min}}] < f_e. $$

(3.12)

The first two conditions guarantee that the aggregate price index, and, therefore, the expected firm value $E[v[i]|\bar{y}]$ monotonically increase with the cutoff talent level $\bar{y}$. A non-decreasing hazard rate is satisfied by frequently used distributions and commonly used in the literature especially as a sufficient condition for the monotonicity constraint in standard screening models (See Chapter 2 of Bolton and Dewatripont (2004)). The last condition says that the expected firm value is less than the entry cost when the entire mass of potential managers match with firms, that is, the mass of firms equals $M$. The following lemma establishes the existence and uniqueness of equilibrium.
Lemma 1 (Existence and Uniqueness of Equilibrium)

Under the assumptions stated above, there exists a unique equilibrium in which a unique cutoff level $\bar{y}^* \in (y_{min}, y_{max})$ of talent satisfies the following equation

$$ R\left(\frac{P(\bar{y}^*)}{w}\right)^{\sigma-1} \left[ \frac{(\rho x[0]\bar{y}^*)^{\sigma-1}}{\sigma} + \rho^\sigma \int_0^1 \left[ \int_0^i x[j]^{\sigma-2} y[j]\bar{y}^*]^{\sigma-1} x'[j]dj \right] di \right] - \frac{u_0}{1-\delta} = f_e, $$

(3.13)

where $P(\bar{y}^*)$ is given by (3.6) and (3.8).

The equilibrium aggregate price index and mass of firms, denoted by $P^*$ and $N^*$, respectively, are also uniquely determined by plugging $\bar{y}^*$ into equations (3.6) and (3.8). Then, the equilibrium payoff profiles of the matched pairs, firm value $v[i]$ and managerial compensation $u[i]$ at each rank $i$, are also obtained by plugging the equilibrium aggregate price index $P^*$ and cutoff level $\bar{y}^*$ into equations (3.4) and (3.5). Using this characterization of the equilibrium, we perform a comparative static analysis. We begin by deriving the effects of different industry characteristics on the cutoff talent level which is determined by equation (3.13).

Proposition 9 (Industry Characteristics and Cutoff Talent Level)

- **The cutoff talent level $\bar{y}^*$ increases with the entry cost $f_e$, while it decreases with the market size $R$.**

- **The cutoff talent level $\bar{y}^*$ increases with the total mass $M$ of potential managers.**

- **There exist a threshold level $\bar{f}_e(\sigma)$ of the entry cost such that the cutoff talent level increases with a marginal increase in the elasticity of substitution $\sigma$ if the entry cost $f_e$ is below its threshold, and decreases if it is above.**
The first argument in Proposition 9 is easily obtained from equation (3.13). Specifically, because the left-hand side of (3.13) increases with $\bar{y}$ as discussed above, an increase in each of the market characteristics affects the new equilibrium cutoff level in a way that satisfies the condition again. The intuition behind this first argument needs to be discussed along with the endogenous mass of producing firms. An increase in the entry cost reduces the incentives of potential entrants to enter the market because the expected firm value, given by the left hand side of (3.13), decreases. It is then obvious that the mass of managers who can be matched to firms must be smaller, thereby increasing the cutoff talent level. The second argument in this proposition follows from (3.13), (3.6), and (3.8). At the current cutoff level, an increase in $M$ implies more supply of managers than demand which must be adjusted by a new cutoff level. The effect of the elasticity of substitution can be shown in a manner similar to that used to prove its effect on the aggregate price index in Proposition 6. The following proposition examines the effects of different industry characteristics on the aggregate price index.

**Proposition 10 (Industry Characteristics and Aggregate Price Index)**

- The aggregate price index $P^*$ increases with the entry cost $f_e$, while it decreases with the market size $R$.

- The aggregate price index $P^*$ decreases with the total mass $M$ of potential managers.

- There exist a threshold level $\tilde{f}_e(\sigma)$ of the entry cost such that the cutoff talent level increases with a marginal increase in the elasticity of substitution $\sigma$ if the entry cost $f_e$ is below its threshold, and decreases if it is above.
As noted before, there are two conflicting effects of \( \bar{y} \) on the aggregate price index given by (3.8). An increase in \( \bar{y} \) implies a decline in the mass \( N \) of firms, which dampens market competition and thus increases the aggregate price index. However, an increase in \( \bar{y} \) implies a market-wise production efficiency improvement as \( y[i|\bar{y}] \) for \( \forall i \) increases in \( \bar{y} \), thereby lowering the aggregate price index. Under mild restrictions stated in Section 3.3, the former effect dominates the latter one, which explains the first observation in Proposition 10. The total mass of potential managers lowers the aggregate price index because the mass of active firms remain the same but the resulting increase in \( \bar{y} \) improves the overall production efficiency.

Next, we examine the effects of industry characteristics on individual CEO compensation. By (3.5), we can see that there are three terms that are affected by \( \bar{y} \). The term \( (P/w)^{\sigma-1} \) increases with \( \bar{y} \) under our assumptions. While the ex-post talent profile \( y[j|\bar{y}] \) in the integrand also increases with \( \bar{y} \), the derivative of the talent profile \( y'[j|\bar{y}] \) decreases with \( \bar{y} \) because, by (3.2), the slope of the talent profile is given by

\[
y'[i|\bar{y}] = \frac{1 - G(\bar{y})}{g(y)} = \frac{(N/M)}{g(y)}, \quad (3.14)
\]

where the second equation follows from (3.6). Note that the slope of the equilibrium talent profile depends on the ratio of the mass of active firms looking for a manager to the total mass of potential managers and the ex-ante talent density function. If the total mass of potential managers becomes larger in the sector, then the marginal talent at each \( i \) gets smaller, which implies that the marginal returns to talent below \( i \) in the ranking decline,
lowering the surplus earned by each manager. In addition, if potential managers are more likely to have a particular talent level $y[i]$, the marginal change in the rankings gives a small variation in talent. Due to these conflicting effects of $\bar{y}$, the effects of industry characteristics on CEO compensation are ambiguous for general parameter values. We thus numerically explore these ambiguous effects after calibrating the model to data in the next section.

3.4 Empirical Analysis

We now move on to our quantitative analysis using the model developed in this chapter. As in Section 2.5, this quantitative analysis includes model calibration and some counterfactual experiments industry by industry to examine the extent to which managerial talent and industry characteristics affect the equilibrium variables. Compared to the model of Chapter 2, this extended model is more complicated to implement. We have an additional equilibrium condition (3.6) for the endogenous mass of firms that involves the total mass of potential managers and the ex-ante distribution function of managerial talent. Because the cutoff talent level that determines the mass of active managers is endogenously determined, it is convenient to use a parametric approach where we assume specific forms for the factor distributions as in Gabaix and Landier (2008) instead of the semi-parametric approach we used in Chapter 2.

3.4.1 Model Calibration

In this subsection, our calibration procedure is described in detail. The separate samples of nine different industry sectors, which include S&P 1500 firm-CEO observations from the
ExecuComp database, are exactly the same as before, so we do not include details here for brevity. Instead of indirectly inferring the profiles of the factors, we directly specify their forms as follows:

\begin{align}
    x[i] &= x_0 + A i^\alpha, \\
    y[i] &= y_0 + B i^\beta, \\
\end{align}

(3.15) (3.16)

where \( y[i] \in [y_0, y_0 + B] \) represents the ex-ante profile of managerial talent defined over the unit interval using the ex-ante talent distribution \( G(y) \) of potential managers. We thus derive the specific form of \( G(y) \) from (3.16) as

\[ G(y) = \left( \frac{y - y_0}{B} \right)^{1/\beta}, \]

(3.17)

from which the ex-post talent distribution \( F_Y(y|\bar{y}) \) is obtained by equation (3.1). We then have the specific form for the ex post profile \( y[i|\bar{y}] \) of managerial talent for active managers who successfully match to firms by equation (3.2) as follows:

\[ y[i|\bar{y}] = y_0 + B \left[ G(\bar{y}) + (1 - G(\bar{y})) i \right]^\beta. \]

(3.18)

Using these specific functional forms, we have a set of parameter values to be estimated as follows:

\[ \Psi = \{ R, \delta, x_0, A, \alpha, y_0, B, \beta, \sigma, u_0, M \}. \]

(3.19)
Among the parameters stated above, $R$ and $\delta$ are directly estimated from the data as described in Section 2.5.2 and the remaining parameter values are obtained by matching several moments (mean values and decile values) from the model-predicted distributions of firm value and managerial compensation with the corresponding observed moments. We now describe how to generate the model-predicted distributions of firm value and CEO pay for a candidate parameter vector $\Psi$.

1. Using the observed number of firms in the industry as the equilibrium mass $N$ of active firms, the equilibrium condition (3.6) derives the cutoff talent level $\bar{y}$.

2. We then compute the equilibrium aggregate price index relative to labor wage $(P/w)^{\sigma-1}$ by (3.8).

3. By plugging the parameter values, the cutoff talent level, the aggregate price index, and the specific profiles of the factors, given by (3.15) and (3.18), into equations (3.4 and (3.5)), we obtain the predicted distributions of firm value and CEO pay.

Table 3.1 report the baseline values of the parameters that are obtained from our calibration. First, the variations in the estimated elasticity of product substitution $\sigma$, the key structural parameter of the model, are largely consistent with those reported in Table 2.2. As discussed in detail in Section 2.5.3, the observation of lower product substitutability in the business equipment, health care, and telecom industries and higher product substitutability in the industries of consumer durable goods and shops is no more or less than the ranking of industries by the negative price-cost margin which is commonly used as a proxy for product
substitutability in the empirical literature. Second, the calibrated values of $\alpha$ and $\beta$ suggest that the profile of firm quality is convex (except for the telecom industry) and ex-ante and ex-post profiles of managerial talent are concave. Since these profiles are defined using their respective cumulative distribution functions, the slopes of the factor profiles are inversely related to the density functions of these variables. A concave talent distribution suggests an increasing density function of managerial talent which is in contrast with a “square root” distribution suggested by Gabaix and Landier (2008).\footnote{Similar results were obtained when we performed model calibration by assuming a Pareto distribution for firm quality and a distribution with a finite upper bound for managerial talent.}

More specifically, Figure 3.1 displays the distributions of firm quality (relative to the lowest level) for the business equipment, health care, manufacturing, and durable goods industries, whereas Figure 3.2 reports the relative talent distributions of potential managers and those of active managers. It is worth noting that the first two industries still turn out to have much more widely dispersed firm and managerial characteristics across firms compared to the other two industries. Compared to Figure 2.1, however, inter-industry variations in

<table>
<thead>
<tr>
<th>Industry Sector</th>
<th>$x_0$</th>
<th>$A$</th>
<th>$\alpha$</th>
<th>$y_0$</th>
<th>$B$</th>
<th>$\beta$</th>
<th>$\sigma$</th>
<th>$u_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumer Nondurables</td>
<td>0.3978</td>
<td>0.2448</td>
<td>1.3697</td>
<td>0.0125</td>
<td>0.0018</td>
<td>0.2153</td>
<td>10.680</td>
<td>0.9299</td>
</tr>
<tr>
<td>Consumer Durables</td>
<td>0.5370</td>
<td>0.0858</td>
<td>1.4462</td>
<td>0.1021</td>
<td>0.0058</td>
<td>0.1654</td>
<td>29.667</td>
<td>0.5780</td>
</tr>
<tr>
<td>Manufacturing</td>
<td>1.7850</td>
<td>1.0731</td>
<td>1.4942</td>
<td>0.0488</td>
<td>0.0039</td>
<td>0.3139</td>
<td>9.523</td>
<td>0.7565</td>
</tr>
<tr>
<td>Energy</td>
<td>0.9678</td>
<td>0.3705</td>
<td>1.2124</td>
<td>0.0916</td>
<td>0.0130</td>
<td>0.1646</td>
<td>14.654</td>
<td>1.7031</td>
</tr>
<tr>
<td>Chemicals</td>
<td>1.6676</td>
<td>0.9094</td>
<td>1.0649</td>
<td>0.0074</td>
<td>0.0011</td>
<td>0.1191</td>
<td>9.874</td>
<td>0.4461</td>
</tr>
<tr>
<td>Business Equipment</td>
<td>0.4882</td>
<td>0.5947</td>
<td>1.7450</td>
<td>0.0335</td>
<td>0.0079</td>
<td>0.4372</td>
<td>6.075</td>
<td>0.7942</td>
</tr>
<tr>
<td>Telecom</td>
<td>3.8350</td>
<td>11.5593</td>
<td>0.8773</td>
<td>0.0015</td>
<td>0.0004</td>
<td>0.2750</td>
<td>5.640</td>
<td>1.1749</td>
</tr>
<tr>
<td>Shops</td>
<td>0.6218</td>
<td>0.1235</td>
<td>1.2800</td>
<td>0.0629</td>
<td>0.0079</td>
<td>0.1009</td>
<td>23.628</td>
<td>0.7575</td>
</tr>
<tr>
<td>Health Care</td>
<td>1.7196</td>
<td>1.6014</td>
<td>1.6443</td>
<td>0.1635</td>
<td>0.0035</td>
<td>0.3116</td>
<td>7.349</td>
<td>0.7540</td>
</tr>
</tbody>
</table>
Figure 3.1. Distribution of Firm Quality

Figure 3.2. Ex-ante and Ex-post Distributions of Managerial Talent
the distributions of firm quality are less significant, which might be attributed to some differences between the indirect inference and the parametric approach.

### 3.4.2 Implications for CEO Talent

Following the experiment of Gabaix and Landier (2008), we now evaluate the impact of CEO talent using the calibrated models. In addition to measuring the impact using the ex-post talent distribution as in the previous chapter, we can also perform the same counterfactual experiment using the ex-ante talent distribution.

We first consider the equilibrium distribution of active managers who are matched to firms. Suppose the median-sized firm could replace its current manager by the highest talent manager in the industry. Table 3.2 shows the rate of increase in total surplus $S$, the rate of increase in firm market value $v$, and the ratio of additional CEO pay to firm value for different industries. The results are very similar to those obtained using the indirectly inferred factor distributions in Chapter 2 (see Table 2.3). Similarly, we observe the significant impact of managerial talent which is in contrast with the result of Gabaix and Landier (2008),

<table>
<thead>
<tr>
<th>Industry Sector</th>
<th>$\Delta S_{S[1/2]}$ (%)</th>
<th>$\Delta v_{v[1/2]}$ (%)</th>
<th>$\Delta u/(1-\beta)_{v[1/2]}$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumer Nondurables</td>
<td>1.8731</td>
<td>1.9175</td>
<td>8.2065</td>
</tr>
<tr>
<td>Consumer Durables</td>
<td>1.6970</td>
<td>1.7379</td>
<td>7.2694</td>
</tr>
<tr>
<td>Manufacturing</td>
<td>1.3010</td>
<td>1.3243</td>
<td>4.9253</td>
</tr>
<tr>
<td>Energy</td>
<td>1.3109</td>
<td>1.3309</td>
<td>5.0353</td>
</tr>
<tr>
<td>Chemicals</td>
<td>1.0183</td>
<td>1.0322</td>
<td>2.7359</td>
</tr>
<tr>
<td>Business Equipment</td>
<td>2.8112</td>
<td>2.8847</td>
<td>11.8834</td>
</tr>
<tr>
<td>Telecom</td>
<td>1.9347</td>
<td>1.9590</td>
<td>6.0721</td>
</tr>
<tr>
<td>Shops</td>
<td>1.7824</td>
<td>1.8283</td>
<td>6.4421</td>
</tr>
<tr>
<td>Health Care</td>
<td>2.4676</td>
<td>2.5380</td>
<td>10.5836</td>
</tr>
</tbody>
</table>
considerable variations in the size of the impact of a better manager across industries, and differences in CEO pay that are comparable to the relative contribution of a better manager to firm value.

We next evaluate the differences in talent using the ex-ante distribution of managers. Suppose that all active managers hypothetically were from the ex-ante distribution with no cutoff level, while the firm quality distribution remains the same. This hypothetical talent distribution, in contrast with the hypothetical employment only at a reference firm, would affect the market variables so that a new market equilibrium must be derived. First, we compute the entry cost \( f_e \), which was not estimated from the calibration procedure, by the free entry condition (3.13) in the current equilibrium, that is, we plug the ex-post talent distribution into the equation. Second, given the entry cost, we use the free entry condition again to obtain the new equilibrium aggregate price index when managers were all from the ex-ante distribution. To do so, we plug the ex-ante talent distribution into the equation. Third, we generate new payoff distributions by (3.4) and (3.5) at the new aggregate price index. Fourth, we measure the changes in total surplus, firm value, and CEO pay due to the replacement of the median manager with the top ranked manager.

As one might expect, the impact of a better manager appears more significant in Table 3.3 because the talent level of the median manager from the ex-ante distribution is lower than that from the ex-post distribution. In sum, this counterfactual experiment suggests that our main argument on the importance of CEO talent which justifies CEO pay is very robust.
Table 3.3. Impact of CEO talent at the Median-Sized Firm (ex-ante dist.)

<table>
<thead>
<tr>
<th>Industry Sector</th>
<th>$\frac{\Delta S}{S[1/2]}$ (%)</th>
<th>$\frac{\Delta v}{v[1/2]}$ (%)</th>
<th>$\frac{\Delta u/(1-\beta)}{v[1/2]}$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumer Nondurables</td>
<td>1.9013</td>
<td>1.9527</td>
<td>8.3439</td>
</tr>
<tr>
<td>Consumer Durables</td>
<td>1.7559</td>
<td>1.8128</td>
<td>7.5535</td>
</tr>
<tr>
<td>Manufacturing</td>
<td>1.3206</td>
<td>1.3471</td>
<td>5.0032</td>
</tr>
<tr>
<td>Energy</td>
<td>2.0831</td>
<td>2.1515</td>
<td>7.8373</td>
</tr>
<tr>
<td>Chemicals</td>
<td>1.0358</td>
<td>1.0534</td>
<td>2.7878</td>
</tr>
<tr>
<td>Business Equipment</td>
<td>3.1018</td>
<td>3.2077</td>
<td>13.1213</td>
</tr>
<tr>
<td>Telecom</td>
<td>2.0436</td>
<td>2.0723</td>
<td>6.3970</td>
</tr>
<tr>
<td>Shops</td>
<td>1.9035</td>
<td>1.9780</td>
<td>6.9212</td>
</tr>
<tr>
<td>Health Care</td>
<td>2.6237</td>
<td>2.7199</td>
<td>11.2795</td>
</tr>
</tbody>
</table>

3.4.3 Implications for Industry Characteristics

In this subsection, we explore using the calibrated models how industry characteristics affect the equilibrium outcome. This quantitative analysis allows us to pin down some of the ambiguous effects of industry characteristics which are discussed in Section 3.3. We derive a new equilibrium by varying the parameter about its baseline value while keeping the estimated factor distributions and other parameter values in place. And then we compare the new equilibrium outcome with the original one and report the percentage changes.

Effects of Product Market Characteristics

Table 3.4 and 3.5 shows the effects of the market size $R$ and those of the entry cost $f_e$. The analytical results presented in Propositions 9 and 10 are confirmed as, for any industry, the equilibrium cutoff talent level and the aggregate price index decrease with $R$, whereas they increase with $f_e$. An increase in $R$ or a decrease in $f_e$ increases the net value of entry, which attracts more firms into the market. Accordingly, more managers are demanded, lowering
the cutoff level, and there is more intense competition due to an increase in the mass of producing firms, lowering the aggregate price index.

It is interesting to see that the mean value of CEO compensation increases with the market size, while it decreases with the entry cost. This observation is in contrast with their effects reported in the previous chapter, that is, no influence of the market size on individual CEO pay and higher CEO pay in response to the entry cost. This is because of the additional equilibrium variable, the cutoff talent level, in this modified model. By (3.14), marginal talent which affects the marginal returns to managerial talent depends on the mass of active firms relative to the total mass of managers, which can be viewed as demand for managers by firms relative to supply. An increase in $R$ or a decrease in $f_e$ increases this ratio, and this effect on the marginal returns to managerial talent dominates other competing effects, so that such a change in the product market increases the mean value of CEO pay. The effects of product substitutability are presented in Table 3.6. An increase in $\sigma$ increases the aggregate price index and lowers the mean value of CEO pay, which is similar to the findings obtained in the previous chapter. In addition, for any industry, product substitutability increases the cutoff talent level, albeit to a small degree.

**Effects of CEO Talent Market**

Table 3.7 presents the effects of the population size $M$ of potential managers. As $M$ increases, the cutoff talent level must go up because the mass of active firms remains the same. However, the increased cutoff level implies an improvement in production efficiency in the market,
### Table 3.4. Effects of Market Size $R$

<table>
<thead>
<tr>
<th>Industry Sector</th>
<th>$\Delta P_P$ (%)</th>
<th>$\Delta \bar{y}_{\bar{y}}$ (%)</th>
<th>$\Delta E_{E[u]}$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumer Nondurables</td>
<td>0.199</td>
<td>-0.093</td>
<td>-0.114</td>
</tr>
<tr>
<td>Consumer Durables</td>
<td>0.067</td>
<td>-0.032</td>
<td>-0.064</td>
</tr>
<tr>
<td>Manufacturing</td>
<td>0.228</td>
<td>-0.109</td>
<td>-0.134</td>
</tr>
<tr>
<td>Energy</td>
<td>0.146</td>
<td>-0.072</td>
<td>-0.143</td>
</tr>
<tr>
<td>Chemicals</td>
<td>0.216</td>
<td>-0.100</td>
<td>-0.137</td>
</tr>
<tr>
<td>Business Equipment</td>
<td>0.384</td>
<td>-0.188</td>
<td>-0.374</td>
</tr>
<tr>
<td>Telecom</td>
<td>0.425</td>
<td>-0.208</td>
<td>-0.414</td>
</tr>
<tr>
<td>Shops</td>
<td>0.085</td>
<td>-0.042</td>
<td>-0.082</td>
</tr>
<tr>
<td>Health Care</td>
<td>0.305</td>
<td>-0.149</td>
<td>-0.296</td>
</tr>
</tbody>
</table>

### Table 3.5. Effects of Entry Cost $f_e$

<table>
<thead>
<tr>
<th>Industry Sector</th>
<th>$\Delta P_P$ (%)</th>
<th>$\Delta \bar{y}_{\bar{y}}$ (%)</th>
<th>$\Delta E_{E[u]}$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumer Nondurables</td>
<td>-0.114</td>
<td>-0.094</td>
<td>0.097</td>
</tr>
<tr>
<td>Consumer Durables</td>
<td>-0.065</td>
<td>-0.033</td>
<td>0.033</td>
</tr>
<tr>
<td>Manufacturing</td>
<td>-0.134</td>
<td>-0.109</td>
<td>0.112</td>
</tr>
<tr>
<td>Energy</td>
<td>-0.145</td>
<td>-0.072</td>
<td>0.072</td>
</tr>
<tr>
<td>Chemicals</td>
<td>-0.137</td>
<td>-0.101</td>
<td>0.105</td>
</tr>
<tr>
<td>Business Equipment</td>
<td>-0.380</td>
<td>-0.190</td>
<td>0.188</td>
</tr>
<tr>
<td>Telecom</td>
<td>-0.421</td>
<td>-0.210</td>
<td>0.209</td>
</tr>
<tr>
<td>Shops</td>
<td>-0.084</td>
<td>-0.042</td>
<td>0.042</td>
</tr>
<tr>
<td>Health Care</td>
<td>-0.301</td>
<td>-0.150</td>
<td>0.150</td>
</tr>
</tbody>
</table>
### Table 3.6. Effects of Product Substitutability $\sigma$

<table>
<thead>
<tr>
<th>Industry Sector</th>
<th>$\Delta P / P$ (%)</th>
<th>$\Delta \bar{y} / \bar{y}$ (%)</th>
<th>$\Delta E_u / E_u$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumer Nondurables</td>
<td>-0.956</td>
<td>-0.510</td>
<td>0.998</td>
</tr>
<tr>
<td>Consumer Durables</td>
<td>-0.269</td>
<td>-0.133</td>
<td>0.131</td>
</tr>
<tr>
<td>Manufacturing</td>
<td>-1.303</td>
<td>-0.688</td>
<td>0.678</td>
</tr>
<tr>
<td>Energy</td>
<td>-0.683</td>
<td>-0.338</td>
<td>0.330</td>
</tr>
<tr>
<td>Chemicals</td>
<td>-0.921</td>
<td>-0.489</td>
<td>0.483</td>
</tr>
<tr>
<td>Business Equipment</td>
<td>-2.723</td>
<td>-1.352</td>
<td>1.334</td>
</tr>
<tr>
<td>Telecom</td>
<td>-1.745</td>
<td>-0.864</td>
<td>0.846</td>
</tr>
<tr>
<td>Shops</td>
<td>-0.493</td>
<td>-0.244</td>
<td>0.239</td>
</tr>
<tr>
<td>Health Care</td>
<td>-1.824</td>
<td>-0.904</td>
<td>0.888</td>
</tr>
</tbody>
</table>

### Table 3.7. Effects of Total Mass $M$ of Potential Managers

<table>
<thead>
<tr>
<th>Industry Sector</th>
<th>$\Delta P / P$ (%)</th>
<th>$\Delta \bar{y} / \bar{y}$ (%)</th>
<th>$\Delta E_u / E_u$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumer Nondurables</td>
<td>0.095</td>
<td>0.010</td>
<td>0.005</td>
</tr>
<tr>
<td>Consumer Durables</td>
<td>0.006</td>
<td>0.002</td>
<td>-0.002</td>
</tr>
<tr>
<td>Manufacturing</td>
<td>0.103</td>
<td>0.008</td>
<td>-0.005</td>
</tr>
<tr>
<td>Energy</td>
<td>0.002</td>
<td>0.001</td>
<td>-0.001</td>
</tr>
<tr>
<td>Chemicals</td>
<td>0.091</td>
<td>0.012</td>
<td>-0.007</td>
</tr>
<tr>
<td>Business Equipment</td>
<td>0.016</td>
<td>0.008</td>
<td>-0.007</td>
</tr>
<tr>
<td>Telecom</td>
<td>0.013</td>
<td>0.006</td>
<td>-0.006</td>
</tr>
<tr>
<td>Shops</td>
<td>0.005</td>
<td>0.002</td>
<td>-0.002</td>
</tr>
<tr>
<td>Health Care</td>
<td>0.016</td>
<td>0.007</td>
<td>-0.007</td>
</tr>
</tbody>
</table>
which lowers the aggregate price index. With similar arguments made above, an increase in 
$M$ lowers the mean value of CEO pay due to the law of demand and supply.

Our parametric approach allows us to examine how the distribution of CEO talent 
affects the market equilibrium as well as CEO pay levels. First, an increase in the minimum 
talent level $y_0$ causes an upward shift of the ex-post distribution of active managers as shown 
in equation (3.18). At the current cutoff level, an increase in $y_0$ lowers the aggregate price 
index given by (3.8) because $N$ increases with $y_0$ and the integral term also increases due 
to an increase in $y[y|\bar{y}]$. Although the second term in the bracket on the left hand side of 
(3.13) increases with $y_0$, the decrease in the aggregate price index causes the left hand side 
of (3.13) to be smaller than the entry cost, which reduces the mass $N$ of operating firms 
and, therefore, increases the cutoff talent level. The smaller mass of active firms lowers the 
ratio $N/M$, and the smaller demand for managers negatively affects CEO pay.

Second, Table 3.9 shows the effects of the exponent $\beta$ of the ex-ante talent distribution 
which determines the shape of the distribution. More specifically, a lower $\beta$, given that it 
is less than 1, implies more rapid increase in talent and flat at the top of the distribution, 
which lowers marginal talent, and, therefore, CEO pay levels as discussed in Section 3.3. 
Although its effects on the aggregate price index and cutoff level are small, the changes in 
the mean value of CEO pay are not insignificant. Lastly, we show the effects of the coefficient 
of the ex-ante distribution of talent in Table 3.10. By (3.18), an increase in $B$ implies an 
increase in the upper bound of CEO talent, which, intuitively, leads to more competition 
among firms for a better manager so that the overall CEO pay level increases.
### Table 3.8. Effects of CEO Talent Distribution ($y_0$)

<table>
<thead>
<tr>
<th>Industry Sector</th>
<th>$\Delta p / p$ (%)</th>
<th>$\Delta y / y$ (%)</th>
<th>$\Delta E_{[u]} / E_{[u]}$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumer Nondurables</td>
<td>5.195</td>
<td>-2.410</td>
<td>-4.970</td>
</tr>
<tr>
<td>Consumer Durables</td>
<td>5.235</td>
<td>-2.427</td>
<td>-4.739</td>
</tr>
<tr>
<td>Manufacturing</td>
<td>5.228</td>
<td>-2.424</td>
<td>-4.733</td>
</tr>
<tr>
<td>Energy</td>
<td>5.190</td>
<td>-2.408</td>
<td>-4.679</td>
</tr>
<tr>
<td>Chemicals</td>
<td>5.190</td>
<td>-2.408</td>
<td>-4.679</td>
</tr>
<tr>
<td>Business Equipment</td>
<td>5.161</td>
<td>-2.395</td>
<td>-4.679</td>
</tr>
<tr>
<td>Telecom</td>
<td>5.145</td>
<td>-2.388</td>
<td>-4.665</td>
</tr>
<tr>
<td>Shops</td>
<td>5.199</td>
<td>-2.412</td>
<td>-4.710</td>
</tr>
<tr>
<td>Health Care</td>
<td>5.165</td>
<td>-2.397</td>
<td>-4.682</td>
</tr>
</tbody>
</table>

### Table 3.9. Effects of CEO Talent Distribution ($\beta$)

<table>
<thead>
<tr>
<th>Industry Sector</th>
<th>$\Delta p / p$ (%)</th>
<th>$\Delta y / y$ (%)</th>
<th>$\Delta E_{[u]} / E_{[u]}$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumer Nondurables</td>
<td>-0.007</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>Consumer Durables</td>
<td>-0.002</td>
<td>0.001</td>
<td>0.002</td>
</tr>
<tr>
<td>Manufacturing</td>
<td>-0.006</td>
<td>0.003</td>
<td>0.005</td>
</tr>
<tr>
<td>Energy</td>
<td>-0.004</td>
<td>0.002</td>
<td>0.004</td>
</tr>
<tr>
<td>Chemicals</td>
<td>-0.006</td>
<td>0.003</td>
<td>0.006</td>
</tr>
<tr>
<td>Business Equipment</td>
<td>-0.018</td>
<td>0.009</td>
<td>0.018</td>
</tr>
<tr>
<td>Telecom</td>
<td>-0.016</td>
<td>0.008</td>
<td>0.016</td>
</tr>
<tr>
<td>Shops</td>
<td>-0.004</td>
<td>0.002</td>
<td>0.004</td>
</tr>
<tr>
<td>Health Care</td>
<td>-0.014</td>
<td>0.007</td>
<td>0.013</td>
</tr>
</tbody>
</table>
### Table 3.10. Effects of CEO Talent Distribution ($B$)

<table>
<thead>
<tr>
<th>Industry Sector</th>
<th>$\frac{\Delta P}{P}$ (%)</th>
<th>$\frac{\Delta y}{y}$ (%)</th>
<th>$\frac{\Delta E[u]}{E[u]}$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumer Nondurables</td>
<td>0.062</td>
<td>-0.031</td>
<td>-0.062</td>
</tr>
<tr>
<td>Consumer Durables</td>
<td>0.025</td>
<td>-0.016</td>
<td>-0.025</td>
</tr>
<tr>
<td>Manufacturing</td>
<td>0.032</td>
<td>-0.016</td>
<td>-0.032</td>
</tr>
<tr>
<td>Energy</td>
<td>0.066</td>
<td>-0.058</td>
<td>-0.066</td>
</tr>
<tr>
<td>Chemicals</td>
<td>0.066</td>
<td>-0.058</td>
<td>-0.066</td>
</tr>
<tr>
<td>Business Equipment</td>
<td>0.092</td>
<td>-0.040</td>
<td>-0.092</td>
</tr>
<tr>
<td>Telecom</td>
<td>0.107</td>
<td>-0.056</td>
<td>-0.107</td>
</tr>
<tr>
<td>Shops</td>
<td>0.058</td>
<td>-0.046</td>
<td>-0.058</td>
</tr>
<tr>
<td>Health Care</td>
<td>0.088</td>
<td>-0.044</td>
<td>-0.088</td>
</tr>
</tbody>
</table>

### Table 3.11. Effects of Firm Quality Distribution ($x_0$)

<table>
<thead>
<tr>
<th>Industry Sector</th>
<th>$\frac{\Delta P}{P}$ (%)</th>
<th>$\frac{\Delta y}{y}$ (%)</th>
<th>$\frac{\Delta E[u]}{E[u]}$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumer Durables</td>
<td>4.674</td>
<td>-4.280</td>
<td>-4.280</td>
</tr>
<tr>
<td>Manufacturing</td>
<td>3.640</td>
<td>-3.404</td>
<td>-3.404</td>
</tr>
<tr>
<td>Energy</td>
<td>4.035</td>
<td>-3.742</td>
<td>-3.742</td>
</tr>
<tr>
<td>Chemicals</td>
<td>3.703</td>
<td>-3.462</td>
<td>-3.462</td>
</tr>
<tr>
<td>Business Equipment</td>
<td>2.842</td>
<td>-2.701</td>
<td>-2.701</td>
</tr>
<tr>
<td>Telecom</td>
<td>1.515</td>
<td>-1.473</td>
<td>-1.473</td>
</tr>
<tr>
<td>Shops</td>
<td>4.550</td>
<td>-4.177</td>
<td>-4.177</td>
</tr>
<tr>
<td>Health Care</td>
<td>3.136</td>
<td>-2.963</td>
<td>-2.963</td>
</tr>
</tbody>
</table>
### Table 3.12. Effects of Firm Quality Distribution (α)

<table>
<thead>
<tr>
<th>Industry Sector</th>
<th>∆P/P (%)</th>
<th>∆y/y (%)</th>
<th>∆E[u]/E[u] (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumer Nondurables</td>
<td>-0.387</td>
<td>-0.192</td>
<td>0.002</td>
</tr>
<tr>
<td>Consumer Durables</td>
<td>-0.129</td>
<td>-0.064</td>
<td>0.001</td>
</tr>
<tr>
<td>Manufacturing</td>
<td>0.001</td>
<td>0.001</td>
<td>0.001</td>
</tr>
<tr>
<td>Energy</td>
<td>0.002</td>
<td>0.001</td>
<td>0.001</td>
</tr>
<tr>
<td>Chemicals</td>
<td>0.002</td>
<td>0.001</td>
<td>0.001</td>
</tr>
<tr>
<td>Business Equipment</td>
<td>0.001</td>
<td>0.001</td>
<td>0.001</td>
</tr>
<tr>
<td>Telecom</td>
<td>0.003</td>
<td>0.001</td>
<td>0.001</td>
</tr>
<tr>
<td>Shops</td>
<td>4.001</td>
<td>2.001</td>
<td>2.001</td>
</tr>
<tr>
<td>Health Care</td>
<td>0.001</td>
<td>0.001</td>
<td>0.001</td>
</tr>
</tbody>
</table>

### Table 3.13. Effects of Firm Quality Distribution (A)

<table>
<thead>
<tr>
<th>Industry Sector</th>
<th>∆P/P (%)</th>
<th>∆y/y (%)</th>
<th>∆E[u]/E[u] (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumer Nondurables</td>
<td>1.589</td>
<td>0.790</td>
<td>0.006</td>
</tr>
<tr>
<td>Consumer Durables</td>
<td>0.531</td>
<td>0.265</td>
<td>0.002</td>
</tr>
<tr>
<td>Manufacturing</td>
<td>1.498</td>
<td>0.745</td>
<td>0.002</td>
</tr>
<tr>
<td>Energy</td>
<td>1.125</td>
<td>0.561</td>
<td>0.002</td>
</tr>
<tr>
<td>Chemicals</td>
<td>1.441</td>
<td>0.716</td>
<td>0.002</td>
</tr>
<tr>
<td>Business Equipment</td>
<td>2.274</td>
<td>1.126</td>
<td>0.002</td>
</tr>
<tr>
<td>Telecom</td>
<td>3.632</td>
<td>1.784</td>
<td>0.001</td>
</tr>
<tr>
<td>Shops</td>
<td>0.645</td>
<td>0.322</td>
<td>0.001</td>
</tr>
<tr>
<td>Health Care</td>
<td>1.985</td>
<td>0.984</td>
<td>0.003</td>
</tr>
</tbody>
</table>
Effects of Firm Quality Distribution

As shown in Tables 3.11, 3.12, and 3.13, the parameters of the firm quality distribution have some contrasting effects, especially, on the mean value of CEO pay. First, an increase in the minimum firm quality level increases the cutoff talent level, lowers the aggregate price index, and increases the mean value of CEO pay. The increase in the mean value of CEO pay is obtained because the marginal returns to CEO talent at each rank increase due to the complementarity between firm quality and managerial talent. Second, an increase in $\alpha$, similar to an increase in $\beta$, lowers the cutoff talent level and increases the aggregate price index, but lowers the mean value of CEO pay. Third, an increase in $A$ lowers the cutoff talent level, the aggregate price index, and the mean value of CEO pay.

3.5 Conclusion

This chapter has presented a market equilibrium framework, which is a simple extension of the basic model in Chapter 2, in which the total mass of potential managers is larger than the mass of firms. In equilibrium, there is a unique cutoff talent level such that only managers with talent higher than the cutoff point can match with firms. And the cutoff level is a joint outcome of product market and labor market characteristics. Accordingly, this extension provides a simple mechanism on how CEO-firm matching process is affected by the product market. The empirical analysis through a parametric approach to implement this extended model shows that the principal implications of the basic model for CEO talent and CEO compensation are robust both to the extension that incorporates the endogenous determination of the distribution of talent and to a different estimation approach. In ad-
dition, through the sensitivity analysis, several implications that explicitly link CEO talent markets and product markets are obtained.

To model the connection between CEO talent markets and product markets more seriously, it would be also interesting to incorporate endogenous industry selection by heterogeneous managers by linking product market characteristics such as market risk and growth opportunities to managerial characteristics such as talent and risk attitude, especially through the effects of product market characteristics on the pay level and structure.
REFERENCES


[29] Falato, Antonio, Dan Li, and Todd Milbourn, 2011, To each according to his ability? The returns to CEO talent, Working paper.


[52] Legros, Patrick, and Andrew Newman, 2007b, Beauty is a beast, frog is a prince: assortative matching with nontransferabilities, *Econometrica* 75 (4), 1073-1102.


