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Investigating Secondary School Students' Experience of Learning Statistics

Kimberly D. Gardner

ACCEPTANCE

This dissertation, SECONDARY SCHOOL STUDENTS' EXPERIENCE OF LEARNING STATISTICS, by KIMBERLY D. GARDNER, was prepared under the direction of the candidate's Dissertation Advisory Committee. It is accepted by the committee members in partial fulfillment of the requirements for the degree Doctor of Philosophy in the College of Education, Georgia State University.

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ABSTRACT

INVESTIGATING SECONDARY SCHOOL STUDENTS' EXPERIENCE
OF LEARNING STATISTICS
by
Kimberly D. Gardner

Although more students are taking courses in statistics before leaving high school, the research base on teaching and learning statistics at the high school level has not accumulated as rapidly (Garfield & Chance, 2000). Very little is known about how secondary school students learn statistics, how the misconceptions they bring to the subject impede their learning, and what should be taught or assessed (Watson & Callingham, 2003). Studies that have investigated these issues tend to focus on the K-5, undergraduate, and graduate levels of education (Groth, 2003). Therefore, more research is needed at the secondary level (Garfield & Chance, 2000).

The purpose of this qualitative investigation is to examine how secondary school students' approaches to learning relate to how they assign meaning to statistics. Phenomenography (Marton & Booth, 1997) is the theoretical orientation that frames the study, and it examines the role human experience plays in learning, by reporting variations in the ways participants experience a phenomenon (Dall'Alba & Hasselgreen, 1996). The research questions for the study were: 1) What are the different ways high school students define statistics? 2) What are the different ways high school students learn statistics? 3) What are the different ways students experience learning statistics?

The nine participants in the study were high school graduates who completed a course in Statistics or Advanced Placement Statistics while enrolled in high school in a suburban area in the southeast. Data sources were semi-structured interviews and journaling. Using phenomenographic methodology, students' descriptions of the experience of learning were analyzed and coded. An outcome space of the collective experiences was constructed. A hierarchical relationship between students' approach to learning and their learning strategies was found. Also, a hierarchical relationship between students' approaches to learning and the meaning they assigned to statistics was found.

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by
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CHAPTER 1

INTRODUCTION

Students do not enter our classrooms as blank slates. They come with prior know-ledge, past experiences, preformed attitudes and beliefs that all have an influence on how they learn and what they learn (Gal & Ginsburg, 1994). With these and countless other variations in each student, it is unrealistic to assume that all students will understand course content in the same way, even if they tend to answer questions correctly and solve problems successfully. We can infer that students exposed to the same instruction process the content differently, and hence they experience the course differently (Marton & Pang, 1999; Prosser & Trigwell, 1999). Allowing a student to describe the experience of learning renders some explanation as to how he or she assembles prior knowledge, past experiences, attitudes and beliefs in order to develop an understanding of what is taught. By investigating the role of human experience in knowledge acquisition, categories of description (a collection of each person's descriptive account of their experiences) will be formulated using phenomenographic methodologies. Phenomenography aims to explore how people learn and why some people learn better than others. The presented study used a phenomenographic approach to examine the various ways secondary school graduates describe the experience of learning statistics and how they currently apply statistical concepts in daily situations.

The study was conceived from my interest in the observable variations between understanding statistics and in performing statistical tasks by my students. Intended learning goals of instruction are often lost in construction of knowledge by many students. In spite of the noble efforts of instructors to effectively teach course content, students still misinterpret and develop misconceptions about the subject matter (Garfield & Chance, 2000). According to Marton and Booth (1997), “teachers teach according to their ways of understanding the subject as a whole” (p. 177). Combining the differences among the students with the variations among teachers, it is obvious how the experience of learning a subject can be radically different for each student. Due to the numerous variations in delivering information to students, it is more feasible to focus on the differing experiences of learning from one perspective. Runesson and Marton (2002) explain,

> It is in no way self-evident what it takes ‘to focus on the enacted object of learning’... [As] there are always an infinite number of ways of describing anything... we have to describe the object of learning in a way which is relevant for accounting for differences in pupil achievement... [T]he teacher has something in mind, something she desires that the pupil will learn. This is the intended object of learning... Our intention is to capture what the students can possibly learn from the point of view of the intended object of learning. (pp. 19-20).

The focus of the study is solely on the students' descriptions of their experience of learning about the phenomena of learning and defining statistics. As Marton (1986) states, a careful account of the different ways of experiencing something may help uncover conditions that facilitate the transition from one way of thinking to a qualitatively better perception of reality. Therefore, the purpose of this phenomeno-graphic study is to investigate how secondary school students’ experience of learning statistics relates to their conceptual development of statistics. By asking my former statistics students to discuss their conceptions of statistics and by having them describe their strategies for learning statistics, a framework for statistical thinking among high school students emerged.

Problem Statement

The first Advanced Placement (AP) Statistics Examination, administered by the College Board in 1997, was taken by approximately 7,600 high school students (Garfield & Chance, 2000). By the year 2005, the number of high school students taking the examination increased to more than 74,761 (College Board, 2005). Although more students are taking courses in elementary statistics before leaving high school, the research base in teaching and learning statistics at the high school level has not accumulated as rapidly (Garfield & Chance, 2000). Very little is known about how secondary school students learn statistics, how the misconceptions they bring to the subject impede their learning, and what should be taught or assessed (Watson & Callingham, 2003) mainly due to the increased rate of course enrollment in such a short period of time. Studies that have investigated these issues tend to focus on the K-5, undergraduate, and graduate levels of education (Groth, 2003), therefore more research is needed at the secondary school level.

Research by Groth (2003), Jones, Thornton, Langrall, Mooney, Perry, and Putt (2000), and Mooney (2002) used Biggs and Collis' (1982) Neo-Piagetian cognitive model of development to build frameworks that describe students' statistical thinking skills. Statistical reasoning is defined by Garfield (2002, para. 1) as, "the way in which people reason with statistical ideas and make sense of statistical information. This involves making interpretations based on data, graphical representations, and statistical summaries". These current statistical thinking frameworks describe the stages of statistical thinking development, based on Biggs and Collis' (1991) five modes of cognitive development (sensorimotor, ikonic, concrete symbolic, formal, post-formal). From this cognitive psychology perspective, a student's statistical thinking skills can be explained by the student's level of modal functioning, suggesting that cognitive processes are independent of the student's emotional domain. These frameworks do not account for the student's conceptual development of statistics through the student's own learning experience, which includes the student's attitudes and beliefs towards mathematics.

Very little research has been conducted on the various ways high school students experience or conceptualize statistics by taking into account the student's affective domain (feelings, beliefs, goals, and attitudes). Research by Gordon (2004) showed that students in an undergraduate psychology program who had negative attitudes towards taking a required service learning statistics course, defined statistics as "procedures with no meaning". Gordon found a connection between the students' feelings about learning statistics and their interpretations of statistics. This study however, was conducted with undergraduate psychology majors whose motives for learning statistics were likely related to the development of a professional identity. High school students take a statistics course to meet a general graduation requirement so their motivation for learning occurs in a slightly different context. In this study, the variations in high school students' experience of learning statistics was investigated, there by addressing the problem of under-coverage within statistics education at the secondary level.

Rationale

Despite the lack of research in statistics education at the secondary school level, new curricula, such as Quantitative Literacy Series by Landwehr & Watkins (1997), Data-Driven Mathematics Series by Burrill & Hopfensperger (1997), and Connected Mathematics Project by Lappan, Fey, Fitzgerald, Friel, & Phillips (2002), emphasizing statistics strands have become available for use at the K-12 levels in the United States of America. While basic concepts of statistics have been integrated into algebra and geometry courses, very few instructional materials exist for high school statistics courses. AP Statistics courses use undergraduate introductory statistics course materials. The presented study is such that any meaningful results will be valuable to practitioners, by offering guidance for curriculum development in secondary school statistics. Since the National Council of Teachers of Mathematics (NCTM, 2000) recommends that all students take a course in statistics despite their career choice, the development of an age appropriate high school statistics course and course materials are needed. The study offers guidance for curriculum development of a rigorous statistics course that is accountable to the professional statistics community, yet appropriate for high school learners.

The study contributes to the body of research on developing a framework for statistical thinking for secondary school students. Traditionally, knowledge based arenas such as medicine, industry, and government agencies have adopted rigorous methods to gather, organize, analyze, and communicate results about data, for the purpose of informing decision making (Groth, 2003). Clearly, statistical thinking has become an integral part of the way society uses and makes sense of data (Gordon, 2004). By recognizing the importance of statistical thinking, learning statistics has been deemed an essential component of the educational foundation for all school students (Watson & Callingham, 2003). By exploring the phenomena of learning statistics through the described experiences of high school students, the study contributes to the development of a statistical thinking framework for secondary level learners.

Focus Questions

Secondary school students, despite a lack of interest in answering challenging questions, can offer unique perspectives on how they understand their processes for acquiring knowledge about the world around them. By establishing a research design that allowed me to capture students' ontology and epistemology, the foundational questions of phenomenography (how do people learn and why are some people better at learning than others) were examined.

According to Schwandt (2001) ontology answers questions about how an individual defines reality and how truth is extracted from this definition; epistemology answers questions about how an individual gains knowledge about the world. By recognizing the existence of the essence of learning and a structure for knowledge validation, phenomenologists describe a dual aspect to learning: (a) *how* an individual approaches learning, and (b) *what* meaning the individual assigns to the object of learning (Marton & Booth, 1997). The how and what aspects of learning vary among students, but these details will be discussed further in the “Theoretical Framework” section. To emphasize the researcher's awareness of these variations in learners, the term *qualitative difference* (Marton & Booth, 1997) is used in phenomenography. A qualitative difference refers to the level or quality of a description, which informs the depth of perception or understanding about a phenomenon. Hence, a qualitative difference in learning refers to the variations in the meaning of the object of learning, as well as variations in approaches to learning among students. From these two aspects of learning, rooted in the phenomenographic perspective, the focus questions for the study are:

1. What are the different ways high school students define statistics? ("What" aspect of learning)
2. What are the different ways high school students learn statistics? ("How" aspect of learning)
    i. How do students go about learning statistics?
    ii. What skills and capabilities do students gain by learning statistics?
3. What are the different ways students experience learning statistics? (Relationship between "what" and "how")

Exploring these questions illuminates the role a student's conception of statistics plays in how they formulate goals and techniques to learn about the phenomena. By asking secondary school students to describe their processes for acquiring knowledge about statistics, the essence of what it means to learn statistics and to think statistically are further documented.

According to Pramling, people innately seek to understand their environment (1996). Students' various interpretations of reality and their perception of the information they receive from it lead to a multitude of ways of learning about phenomena. Students also devise rationales for validating observed truths and, they formulate principles to generalize their findings (Pramling, 1996), although not always in an academic setting. The focus questions posed for the research examine the unique perspectives of high school students. The variations in these experiences are used to explain the development of statistical thinking in the study.

Theoretical Framework

The Postmodern Paradigm

The paradigm in which the study is positioned is postmodernism. By its own principles, it would be inappropriate to state a definition of postmodernism. Unfortunately, some of these principles must be broken to effectively communicate the tenets of the paradigm that frames the study. Postmodernism is an ideology that denies the possibility of acquiring totalizing knowledge about the world (Freebody, 2003; Larrain, 1994). Postmodernism is a counter-ideology to modernism. For example, modernism seeks linearity, but postmodernism seeks plurality. Modernists seek monolithic universals, while postmodernist encourage fractured, fluid, multiple perspectives (Larrain, 1994). To provide an accurate description of postmodernism, perhaps it is best to contrast it with modernism.

Modernism is a philosophy that supports a totalizing theory of the world. It is optimistic of human capabilities to theorize, and it supports the absolute integrity of the knower and what is known (Smith, 1992). This authoritative knowledge is reverberated through discourse that makes sense of everything, and it is often referred to as the meta-narrative or grand narrative (St. Pierre, 2002). Empiricism and reason are privileged because of their absolutist traditions. It is assumed that when all members of society adhere to the grand narrative, an ideal society emerges.

The flawed assumption of this principle is that all people are willing or even capable of following such a grand theory. In this sense, modernism disregards diversity with respect to alternative interpretations of reality from various orientations such as gender, race, ethnicity, culture, etcetera. Modernism is foundational, linear, and steeped in epistemological certainty. On the other hand, postmodernism opposes the rigidity of modernistic views of the world.

Postmodernism is a reaction against the principles and practices of modernism. Postmodernism rejects all boundaries constructed by dominant discourses. It focuses on de-centering humanity, and it holds to the principle that there are limits on the ability of humans to measure and describe the universe in any absolute, universal way (Freebody, 2003). Fluid, multiple representations of reality are acceptable, as localized knowledge is preferred over institutionalized knowledge. Common to the postmodernist perspective is doubt in the integrity of the knower and what is known (Smith, 1992). Postmodernists question definitions and truths because they are viewed as the institutionalized knowledge set forth in meta-narratives or regimes of power. Postmodern discourse is typically self-reflective. An authoritative claim presenting an independent, structured reality does not exist. Instead, its discourse narrates subjective fiction.

Postmodern Mathematics Pedagogy

The postmodern movement has also impacted mathematics pedagogy. Traditionally, a philosophy of mathematics contended that mathematics was the absolute, certain, universal language for describing the world, and that it was independent of human knowledge (Ernest, 1998a). For example, logicism is the view that mathematics is a branch of logic (Barrow, 1991). Formalism constitutes the idea that any branch of mathematics can be axiomized in formal language (by a set of symbols and pre-determined rules); platonism is a school of philosophical thought that believes that mathematics is static and based on transcendental entities which are perfect, eternal, unchangeable, and independent of humanity (Barrow, 1991). Since the postmodern movement however, schools of conceptual change have emerged.

Postmodern mathematics prioritizes representation over abstraction and contextual relevance over absolutist axioms. From the postmodern perspective, mathematics is a social-cultural-historical product of human experience. Mathematics is a function of culture and context (Ernest, 1998b). Certainty is not absolute because mathematical truths are understood with respect to their relevance within a given system, e.g. 2+2=4 in the real number system, but 2+2=1 in the base three modular arithmetic system (Ernest, 1998b). This implies, according to Ernest, that from a pedagogical perspective of mathematics, student knowledge must be characterized by its utility, meaning students learn things to use them, not simply to know them. Furthermore, contradictions and paradoxes are welcomed to aid in the validation of knowledge.

The emergence of the postmodern perspective of mathematics pedagogy has redirected the focus of research in mathematics education. Students were once portrayed as cognitive subjects around which all learning and for whom all teaching happens (Ernest, 1998a). In the postmodern era, factors such as culture, gender, ethnicity, social status, etcetera are included as part of the mathematics education research domain. Also, environment and student interactions with the teacher and other learners have become the focus of some research pertaining to how students learn mathematics. Postmodern mathematics pedagogy has shifted the traditionally rigid definition of what it means to know mathematics to a more encompassing position viewing mathematics as a cultural tool which influences how mathematics is learned by the student.

Learning Mathematics

Because of the influence postmodern mathematics pedagogy has had on mathematics education research, it is worth discussing what it means to learn mathematics, as defined by the professional community of mathematics educators. Since the publication of *Curriculum and Evaluation Standards for Schools* (1989) and *Principles and Standards for School Mathematics* (2000) by NCTM, teaching and learning mathematics has become more than just studying rules, procedures and algorithms. Learning mathematics

> involves the usable heuristics and metacognitive strategies to solve problems, the use of various representations to make sense of information, and a search for mathematical connections or applications in different contexts. It becomes important that students develop a mathematical disposition, and a set of principles and attitudes consistent with the practice of doing mathematics (Santos-Trigo, 1998, p. 632).

While this definition is descriptive, it does not expose a framework for how students achieve these goals. The problem, according to Ernest (1998b), is for mathematics education to provide a theory of learning mathematics that facilitates interventions in the processes of its teaching and learning. Mathematics learning theories are mainly reconceptualized adaptations of prevalent epistemological theories. For example, Piaget's scheme theory and other research on conceptual development underpin the radical constructivist theory of learning mathematics. Glaserfeld (1990) states that the basic principles of radical constructivism emerge from a coherent theory of Piaget's writings:

> 1. Knowledge is not passively received either through the senses or by way of communication; knowledge is actively built up by the cognizing subject.
> 2. The function of cognition is adaptive, in a biological sense of the term, tending towards fit and viability; cognition serves the subject's organization of the experiential world, not discovery of an objective ontological reality (p. 4).

Ernest's (1998b) criticism of radical constructivism is that it strongly prioritizes the individual aspects of learning mathematics, while de-emphasizing the equally important role social interaction plays in teaching and learning mathematics. The social constructivist theory of learning mathematics is predominately influenced by the social-cultural theories of Vygotsky. Social constructivism emphasizes the importance of culture and context in learning mathematics; meaningful learning occurs when students are engaged in social activity (McMahon, 1997). It looks at how groups work to construct their perceived reality, which accounts for social phenomena such as rules, institutions, and traditions. By prioritizing the social and cultural aspects of learning, the problem for a social constructivist theory of learning mathematics is that it insufficiently articulates the individual's role in learning and constructing mathematics (Ernest, 1998b).

While these epistemological theories both acknowledge the roles of the individual and culture in learning mathematics, their position to prioritize one over the other is inconsistent with my personal epistemology (of which both are equally important). For this reason, the theoretical framework for learning mathematics used in this study will be adapted from the theoretical framework of phenomenography. Phenomenography accounts for the individual's conceptual development as a result of interactions with situations presented by the environment. The social, historical, and cultural aspect of learning is not explicitly stated in phenomenography, but I intend to adapt its framework to include these aspects. By adapting the framework to include the role of the individual and the role of social interaction in learning, both will maintain equal focal points throughout the investigation. This theoretical "20-20" view of learning mathematics is indeed a postmodern endeavor in mathematics education research.

Learning from a Phenomenographic Perspective

Phenomenography is a qualitative orientation to research with its own theoretical framework, and it is often used to describe the experience of learning. Learning is studied by collecting the various ways the learner-phenomenon relationship is experienced (Francis, 1996). Learning implies perceiving, conceptualizing, or understanding something in a new way, by discerning it from and relating it to a context (Pramling, 1996).

Phenomenographers look at learning from the perspective of the learner and they base learning achievement on the quality of descriptions. Learning involves two aspects: (a) how one goes about learning (approach), and (b) what is to be learned (the object of learning) (Uljens, 1996). Figure 1 illustrates the basic structure of learning as defined in phenomenography. The *what* aspect of learning refers to the content being learned (the direct object). The *how* aspect of learning has two parts: (a) the type of capabilities the learner is trying to master (the indirect object of learning), and (b) the quality of the act of learning (Marton & Booth, 1997).

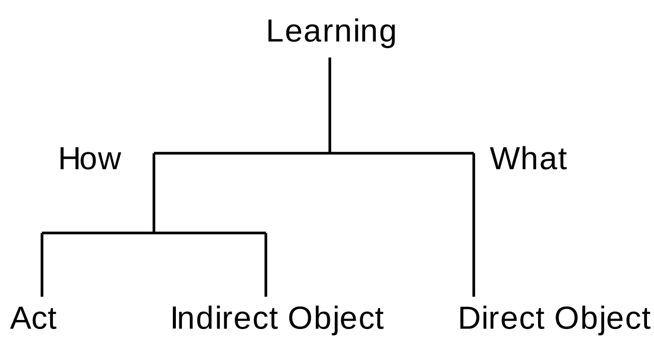

Figure 1. The basic structure of learning (Marton & Booth, 1997, p.85).

The *how* and *what* aspects of learning vary among students. To emphasize the researcher's awareness of these variations in learners, a hierarchical structure of qualitative differences is formed, called the *outcome space* (Marton, 1986).

The *how* aspect of learning in phenomenography is reflected in the student's personal meaning of learning which ultimately frames the student's overall approach to learning (Pramling, 1996; Uljens, 1996). In general, approaches to learning are dichotomous, ranging from *surface* level learning to *deep* learning (Marton & Booth, 1997). A surface approach to learning is when a student's overall strategy for learning merely focuses his or her attention on signs or signals during instruction to gauge what should elicit a reaction. This approach to learning focuses on cues from symbols, procedures, phrases, or text to prompt a set of predetermined behaviors to complete the task with minimal cognitive activity or conceptual understanding of the phenomenon. A deep approach to learning is when the student moves far beyond the surface level approach to more interpretive, comprehensive processes. The student not only identifies symbols, procedures or text, but also feels task accomplishment is reached when connections and meaning can be fully articulated about the phenomenon. The student's overall approach to learning involves using prior knowledge in more advanced and elaborate ways.

Other approaches to learning that fall between these two extremes emerge during data collection and analysis. Each approach shows improvement in the quality of the act of learning, advances in the types of skills the learner is trying to master, and a more elaborate meaning assigned to the object of learning.

The *what* aspect of learning in phenomenography is investigated by deriving qualitative differences in the way students assign meaning to the direct object of learning. The content being learned is experienced differently by each student (Pramling, 1996).

The variations in understanding are hierarchical (Bowden, 1996), and they inform the level of learning achieved by the student.

A student's experience of direct object is explained by the *nature of awareness* (Marton & Booth, 1997). The basic components of awareness are *appresentation*, *discernment*, and *simultaneity* (Marton, 1986; Uljens, 1996). Appresentation means consciousness of a perceptual or sensual experience applied to both concrete and abstract entities (Uljens, 1996). Discernment in the nature of awareness involves recognizing the figure-ground structure of a phenomena (Marton & Booth, 1997). Simultaneity involves knowing how the discerned parts are related to the whole (Marton, 1996). The aspects of the phenomenon (appresentation) and the relationships between them that are discerned and simultaneously present in the individual's focal awareness define the individual's way of experiencing the phenomenon (Marton & Booth, 1997). The limited capacity of simultaneous focal awareness is what accounts for the qualitative differences in both the approach to learning and the object of learning.

The way a student understands content or a phenomenon implies the structure of the student's awareness (Francis, 1996). For the student, some aspects of the content come into focus while others recede (Uljens, 1996). The student's awareness of these aspects suggests that an underlying strategy or procedure exists which involves awareness of other aspects. Phenomenographers suggest that we look at how a student experiences a situation and how making sense of the experience is connected to the phenomenon.

A *situation* refers to the wholeness of what we experience within a socio-spatio-temporal location - a context, a time, and a place (Marton & Booth, 1997). Phenomena are experienced as abstractions, independent of a socio-spatio-temporal location but linked to other situations that lend meaning to them. Our understanding of a situation cannot be separated from our understanding of the phenomena.

> Not only is the situation understood in terms of the phenomena involved, but we are aware of the phenomena from the point of view of the particular situation. Furthermore, not only is our experience of the situation molded by the phenomena as we experience them, but our experience of the phenomena is modified, transformed, and developed through the situations we experience them in (Marton & Booth, 1997, p. 83).

Because the phenomena and situation are inseparable in this non-dualistic ontology, and they are connected by experience, an explanatory framework of experience is needed as illustrated in Figure 2.

An experience is defined as a way of discerning something from its environment and then assigning a meaning to it. Therefore, experience involves the discernment of structure and the assignment of meaning, which are referred to as the structural aspect and the referential aspect of an experience, respectively (Uljens, 1996). The referential aspect of an experience is the meaning assigned to it. The structural aspect of experiencing something is twofold: "discernment of the whole from the context...and discernment of the parts and their relationship within the whole" (Marton & Booth, 1997, p. 87). The former distinction is called the phenomenon's external horizon and the latter is its internal horizon.

Collectively the way of experiencing framework placed within the basic structure of learning defines the structure of the experience of learning as it was used in the study. The *what* aspect of learning illustrates the way the object of learning is experienced and understood. The *how* aspect depicts variations in the experience of developing learning capabilities and the experiences that constitute the act of learning.

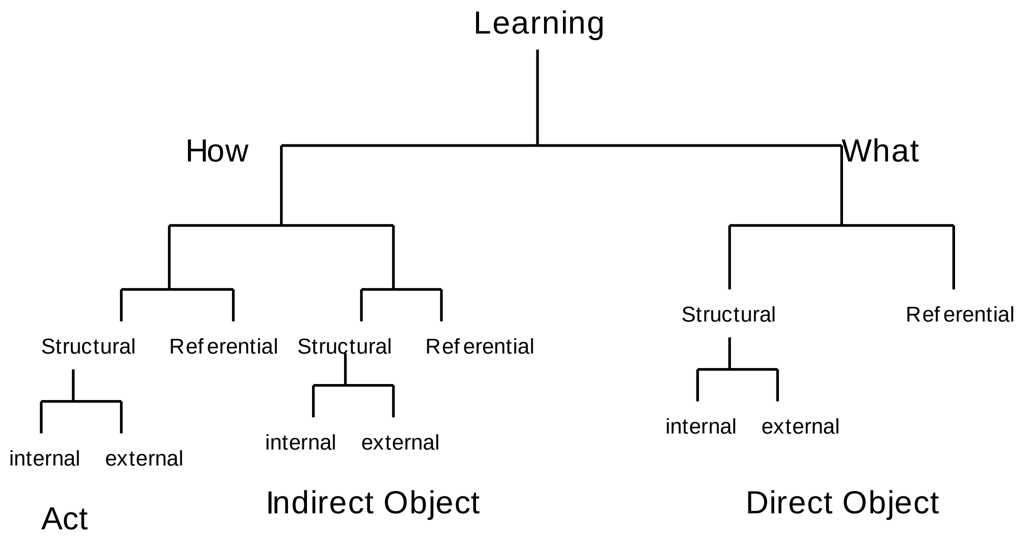

Figure 2. The experience of learning. (Marton & Booth, 1997, p. 91)

Conceptual Framework

In my view of learning mathematics, the internal processes used by the student and external social factors acting on the student are equally important to the student's formulation of understanding. Factors internal to the student include the student's attitudes, beliefs and existing mathematics knowledge. Social factors are in the realm of culture, history, and society (CHS). The cultural, social, and historical component represents norms, rules, and groups that make up the student’s environment, and in the case of this study, that environment is the mathematical society. The mathematical society, or environment, includes the philosophy of mathematics, mathematics pedagogy, mathematics educators, other groups influencing education, the curriculum, and other students. The concept map [Figure 3] illustrates the cyclic interaction between phenomena, student, and society in the conceptual development of the student.

In the beginning, the student and the object exist simultaneously, but the student is unaware of the object's existence. The object is presented in context, as a problem or a situation, requiring some action on the part of the student. In my view, all learning bears some degree of intentionality on the part of the learner. Learning begins when the student recognizes the need to create tools, skills, or procedures to solve a problem or assign meaning to a situation. The student has prior knowledge of mathematics and uses this underlying structure of awareness to develop an initial learning experience about the phenomenon [the experience of learning framework, Figure 2 is the structure implied here for the student]. In another time or place, CHS has encountered the situation and has established cultural, historical and social procedures for learning and validating knowledge about the situation [Figure 2 is implied here for the society]. In some form of activity, the student and CHS negotiate procedures and meaning. These interactions can be summarized using a few terms borrowed from activity theory. The adoption of activity theory to my conceptual framework is supported by Uljens' (1996) criticism that empirical analysis neutralizes how phenomenographers reason about context and subject. Uljens suggests a hermenutic mode of reasoning:

> In hermeneutic phenomenography one would be interested in the essential features of manifestations of experience without forgetting the social, cultural and historical dimensions within which experience is embedded . . . the meaning of human experience is possible to determine only as a relation between its content and context (social, cultural, historical) . . . it is not reasonable to understand [experience] without taking into account [context]. (p.127)

Activity theory is a form of socio-cultural-historical analysis that contends " all human activity is embedded in a social matrix composed of people and artifacts, [and] ... all human activity evolves over time and is distributed among individuals and their cultures" (Jonassen, 2000, p.97). The interactions between the student and CHS are of activity between and within the following subsystems of an activity system: (a) production, (b) consumption, (c) exchange, or (d) distribution. An illustration of an activity system and explanation of these terms are provided in the Appendix A. Interactions within and between these subsystems constitute a goal directed hierarchy of actions that are used to accomplish the task.

The outcome of the activity system manifests itself as the transformed object, which when tied back to phenomenography, represents a new level of awareness that reconstitutes the essence of the phenomenon for the student and CHS. When other situations arise, the re-conceptualized phenomena are applied within the activity system. If the student does not encounter the object in multiple contexts or, if the student is no longer capable of discerning aspects of the object, the ways of experiencing are finalized. The finality of the set of attributes the student can discern accounts for the finite number of ways an individual can describe learning and the object of learning.

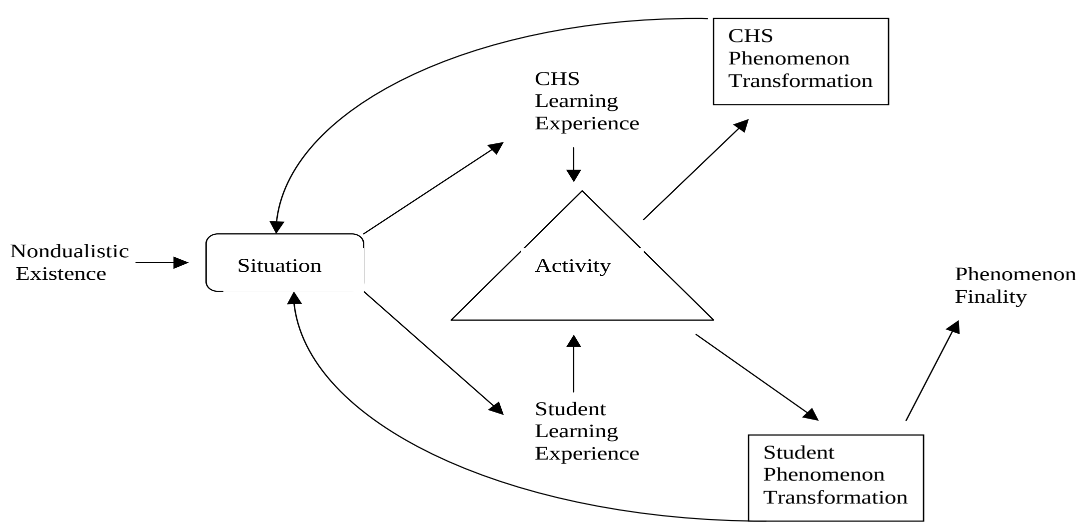

Figure 3. Conceptual framework for the structure of the student's learning experience. This adaptation includes social interaction through activity as an integral part of the students' experience of learning.

The conceptual framework presented in this section will underpin my explanation for the roles of internal and external interactions between student and CHS in the formulation of the student's experience of learning.

Background

Statistical thinking is the way people reason with statistical ideas, and the way they make sense of statistical information (Garfield, 2002), and it has become an integral part in the way society uses and makes sense of data. By recognizing the importance of statistical thinking, learning statistics has been deemed an essential component of the educational foundation for all school students.

Statistical literacy, understanding the basic language and fundamental ideas of statistics is embedded in the mathematics education reform movement (NCTM, 2000). The mathematics education reform movement aspires to eradicate problems associated with innumeracy, which is an inability to deal comfortably with fundamental notions of number and chance (NCTM; Paulos, 1988). The mathematics education community understands that widespread mathematical ignorance and the persistent use of unchallenged mathematical fallacies are just as dangerous as illiteracy in a data driven society (Groth, 2003; NCTM, 1989). The goal then is for society to attain an acceptable level of quantitative literacy or numeracy, the ability to use basic mathematics skills effectively to meet the general demands of life, work, and citizenship (Watson & Callingham, 2003).

The mathematics education community sought to address the innumeracy problem by emphasizing quantitative literacy in mathematics curriculum for all students (NCTM, (1989); Groth, 2003). As a result, in the United States the term *quantitative literacy* emerged in significant reports on the need to improve mathematical and statistical understanding of the general public (Watson & Callingham, 2003). This call to action prompted the mathematics education reform movement in the United States during the late 1980s. The NCTM introduced reform standards for mathematics education in 1989, ultimately situating statistics into the curricula under three major subheadings: probability, data handling, and statistical inference. The NCTM (2000) standards emphasize these ideas in the Data Analysis and Probability Strand:

> Instructional programs from pre-kindergarten through grade 12 should enable all students to:
>
> 1. Formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them;
> 2. Select and use appropriate statistical methods to analyze data;
> 3. Develop and evaluate inference and predictions that are based on data;
> 4. Understand and apply basic concepts of probability (p. 48).

The mathematics education reform movement established probability and statistics as integral topics within the pre-college mathematics curriculum, defining new goals for students (Garfield & Chance, 2000).

The NCTM also enlisted the services of the professional mathematics, statistics, and scientific community to define learning goals. The American Statistical Association (1991) and the American Association for the Advancement of Science (1993) recommended that high school students engage in data exploration, using techniques of formal inference, planning studies, and analyzing how statistics are used in society. The National Science Foundation (NSF) funded reform-based secondary level curricula integrating statistics into traditional topics such as algebra and geometry in the late 1990s (Groth, 2003). By 1996 however, 79.2 % of all twelfth grade students indicated that they had never taken probability or statistics, according to a student questionnaire given with the 1996 National Assessment of Educational Progress (NAEP). By 1997, the first AP

Statistics Examination was offered by the College Board in which approximately 7,600 high school students took the exam (Garfield & Chance, 2000). The number of students taking the exam had increased to more than 74,761 in 2005 (College Board, 2005).

When statistics was added to mathematics curricula, very little was known about students' prior knowledge of statistics, how they learn it, or what misconceptions they held that would complicate learning; furthermore, very little is known about what should be taught or assessed (Watson & Callingham, 2003). Statistics education research was very young area of research as compared to the numerous decades of research in areas of mathematics education such as algebra and geometry. However, research activity in statistics education focusing on teaching and learning began to bustle and vastly accumulate within the past decade (Shaugnessy, Garfield & Greer, 1996). The components emphasized in the Guidelines for Assessment and Instruction in Statistics Education (GAISE, 2005) report (data representation, designing studies, chance, variation, association, statistical inference, statistical misconceptions, use of technology, and assessment) are typically the focus of most studies in this area. Another aspect of statistics education that is not rigorously researched is student development of statistical thinking skills, which is what this research investigates. Explicitly defining learning goals for statistics instruction is important to guide what is taught, learned, and assessed (delMas, Garfield, & Chance, 2002).

Overview of Methods

The study was conducted at a high school located in a suburban city in the southeastern United States. The nine participants who volunteered for the study are graduates from the high school who completed a semester of Statistics or AP Statistics between 2004 and 2006, their junior or senior year. Qualitative methodologies oriented in phenomenography were used as the research design. The students' descriptions of their experiences of learning statistics were analyzed. Since the object of research in the study is human experience, the phenomenographic approach is appropriate because it is an encompassing methodology used to categorize the variations in descriptions among the participants about a specific phenomenon (Uljens, 1996). Furthermore, phenomenography is rooted in the post-modern position regarding the non-existence of totalizing theory, and the assumption that knowledge and meaning are political constructions steeped in personal agendas. Phenomenographers believe that people experience phenomena in different ways, thus nothing can be completely known or completely understood about a phenomenon; research should aim to expose the various ways people experience a phenomenon, in an effort to holistically present the essence of the object of research (Francis, 1996). Since the study aims to illuminate the participants' varying levels of awareness about the experiences of learning statistics, the phenomenographic method, based on the principles of qualitative research, is being used appropriately.

When conducting a phenomenographic study, the researcher orients herself towards the participant’s ideas or experiences of the world, and suspends her own perceptions of reality (Francis, 1996; Hasselgren, 1996). By suspending her own assumptions, the researcher actually learns about the phenomenon from the participant’s perspective. The participant is the philosopher and the researcher takes the role of the apprentice, experiencing the phenomenon and asking questions to learn more deeply about the phenomenon through the participant (Bowden, 1996; Hasselgreen, 1996). This orientation is common in qualitative research.

Also common to qualitative research is the notion of sample size. As stated by Marton & Booth (1997), “a phenomenographic study always derives its descriptions from a smallish number of people chosen from a particular population” (p. 113). Although the participants in the study were limited in expressing experience and awareness, a sample size of nine did result in saturation of information.

Data Collection

When using the phenomenographic approach to research, data collection relies heavily on semi-structured interviews. The researcher formulates a few questions to focus the interview around the research, but subsequent questions emerge throughout the interview. In particular, questions like, “What do you mean by X?”, “How did you go about understanding X?” or “How did it feel?” are asked to get deeper descriptions from the participant (Bowden, 1996). The participant eventually describes the singular essence of the phenomenon through levels of awareness of his or her capability. Marton and Booth (1997) describe this as reflecting from a state of “meta-awareness in which the participant is aware of his awareness of something" (p. 129). By collecting data from the participants using reflection journals and interviews, coding the transcriptions gave rise to the outcome space. Triangulation and member-checking were used to validate results.

Data Analysis

Data analysis occurs during data collection and as well as after it is complete (Creswell, 2003). Transcribed interviews and journals were analyzed using coding methods from grounded theory defined by Glaser & Strauss (1967) and to help capture meanings conveyed by the participants (Creswell, 2003) in the development of the categories of description. Once these categories were reduced they are decomposed into their referential and structural aspects, constituting the outcome space.

The outcome space represents the hierarchical structure of the categories of description. The more complex the description, in regards to how it includes components of the phenomenon and their relationships, the higher it is ranked (Bowden, 1996). Three criteria must be established as the outcome space is formed:

1. Each category of description should stand in clear relation to the phenomenon under investigations, and the category should reveal something distinct about the phenomenon
2. Categories should be coherent and cohesive
3. The system should capture critical variations in the data with as few categories as possible (Marton & Booth, 1997).

Once the outcome space has been established, a model relating the components of the experience of learning was conceptualized (Francis, 1996).

The common methods of data collection and data analysis for the phenomenographic approach to research are within the tradition of qualitative research. The distinction is the theoretical orientation of phenomenography, and its object of research. If the research is conducted properly, and the data has been analyzed through the lens of phenomenography, a “collective anatomy of awareness” should be revealed in regards to the phenomenon under investigation (Marton & Booth, 1997, p. 136). In summary, a glimpse of the essence of the phenomenon is captured through the collective and collaborative efforts of numerous participants.

CHAPTER 2

REVIEW OF LITERATURE

Introduction

Because of the increased reliance on data driven results to inform decisions in today's age of information, there exists a growing demand for research on improving statistics education (Garfield & Chance, 2000; Holcomb & Ruffer, 2000). Several research articles have suggested reform in pedagogy (Garfield & Ahlgren, 1988; Jones, Langrall, Thornton, & Mooney, 2002), the use of technology (Nicaise & Barnes, 1996; Velleman & Moore, 1996) , content (Holcomb & Ruffer, 2000), and assessment (Gal & Garfield, 1997; Chance, 2002). These articles along with numerous others have been instrumental in transforming the structure of many statistics courses, as well as contributing to formulation of theories on the development of statistical thinking.

An Overview of Statistics Education Research

Research on student learning in statistics has occurred in three different areas: psychological research, statistics education, and mathematics education. Psychological research typically focuses on general patterns of thinking and reasoning about statistics. Interestingly, adults misinterpret statistics, despite a correct technical understanding and accurate calculations (Kahneman, Slovic, and Tversky, 1982). Misconceptions in topics such as correlation and causation, representativeness in sampling, and outcome orientation have lead people to make errors in decision making. The conclusion of this body of research is that inappropriate reasoning about statistical ideas is widespread and persistent, similar at all age levels, and difficult to change (Garfield & Ahlgren, 1998).

Statistics educators focus on how statistics is learned, as opposed to general thinking patterns and fallacies. For example, research has shown (Shaughnessy, 1997) that activity based courses and use of small groups help students overcome misconceptions of probability and enhance student learning of statistics concepts. Also, use of computer simulations lead students to give more correct answers to a variety of probability problems (Garfield & delMas, 1991). Basic principles of learning statistics are deeply rooted in constructivism. According to Garfield (1994) some of these learning principles are: a) students construct knowledge, b) students learn through active involvement in learning activities, c) learning is enhanced by students confronting their misconceptions, and d) calculators and computers should be used to help students visualize and explore data, not just to follow algorithms to predetermined ends. Problems needing further research in statistics education are similar to those found in psychological studies. An issue receiving increased attention is changing student misconceptions. Students tend to respond correctly to items on tests because they anticipate the answers, yet they still have misconceptions about statistics that are difficult to change (Konald, 1989).

Mathematics education research focuses on improving students' general mathematics competencies, which is relevant to teaching statistics. These studies help support research findings on statistical learning. Garfield (2002) summarizes some relevant findings in mathematics education that reinforce statistical learning: (a) increasing time spent on developing understanding increases student performance on problem solving tests, (b) using small groups leads to better group productivity, improved attitudes, and sometimes increase achievement, and (c) students learn more from working open ended problems than from goal-specific problems where there is one right answer.

When developing assessment tools for statistical reasoning skills, one can see how mathematics education research supports statistics education. Because this study focused on statistical learning experiences, research on the course content and assessment practices the participants were exposed to are presented.

Research on Course Development and Assessment

The GAISE (2005) project established six recommendations for statistics curriculum: emphasize statistical literacy and develop statistical thinking, use real data, stress conceptual understanding rather than mere knowledge of procedures, foster active learning, use technology to develop conceptual understanding and to analyze data, and use assessments to improve and evaluate learning. Assessing statistical learning has been supported by findings in mathematics education as well. Since students tend to value what will be assessed, assessment should match learning goals and should provide useful and timely feedback (GAISE; Chance, 2002; Garfield, 1994). Garfield and Chance (2000) suggest homework, quizzes, exams, projects, activities, presentations, lab reports, minute papers, case studies, and article critiques as possible assignments to determine student understanding of statistical concepts.

Research on pedagogy for statistics courses, students' understanding of general statistics concepts, the role assessment plays in the development of statistical abilities provide a view of the more saturated areas of research on statistics education. Narrowing the research domain and positioning the study, interest is given to research that focuses on theories regarding the development of and factors that influence statistical thinking.

Statistical Thinking Models

In recent years, statisticians and educators have begun to model the intellectual activity within the statistics classroom and within the statistics discipline through observation and by attempting to understand patterns of thought (Pfannukugh & Wild, 2002). Cognitive models on statistical thinking have as their aim to either illustrate the stages of development or classify types of thought. With the understanding that all models are considered useful within the parameters of their origination, models on statistical thinking serve as reasoning tools to explain mental processes, and they benefit education by providing vocabulary and classification systems (Fischbein, 1987; Brown, 1998).

The Ben-Zvi and Friedlander model originates from statistics education research , and its purpose is to define and characterize the levels of thinking that were observed for 13 to 15 year olds in a statistics teaching experiment. The model consists of four stages (Mode 0: Uncritical thinking, Mode 1: Meaningful use of representation, Mode 2: Meaningful handling of multiple representations/developing metacognitive abilities, Mode 3: Creative thinking) in which the categories emerged from students' work on data analysis and statistical investigations of their own choice. Whether the developmental stages identified were hierarchical and whether or not the students were going through the stages linearly was not established in the study (Ben-Zvi & Friedlander, 1997), but the presented study addressed the issue, as discussed in Chapter Five.

The Pfannkuch and Wild model arises from in depth interviews with practicing statisticians and the historical empirical inquiry cycle. The model provides a comprehensive description of the processes involved in statistical thinking, from problem formulation to conclusion. The model has four separate dimensions (the investigative cycle, types of thinking, the interrogative cycle, dispositions) and claims that people think within each dimension simultaneously (Pfannkuch & Wild, 2002). Also, the model is not hierarchical because the intent of the model is to show patterns of thought described by statisticians in their approach to solving problems and not progressions of thought. The model is an initial attempt at providing a global overview of statistical thinking, from which refinements or new models can emerge (Wild & Pfannkuch 2002). While the participants in the study did not describe such elaborate schemes of thinking, the model affirms variation in phenomenon experience among professional statisticians as well.

The two models are explanations of empirical observations based on the thought processes of students (Ben-Zvi & Friedlander) and statisticians (Pfannkuch & Wild). Their sole purposes are to communicate an encapsulation of ideas in the form of emergent models not based on any particular theoretical stance (Wild & Pfannkuch, 2002). The Ben-Zvi & Friedlander model focus on the ongoing search for critical interpretation and recognizes that the empirical cycle provides the connections for students to seek deeper meaning. The Wild & Pfannkuch model focuses on integrating the statistical with the contextual to promote a way of viewing the world similar to Ben-Zvi, but from the community of practicing statisticians. From these two models, a sense of what statistical thinking is understood to mean among educators and statisticians is made. Narrowing the focus of the research even further, the conceptions of statistics from phenomenographic research, aim to reveal the variations in understanding based on experience.

Conceptions of Statistics

A phenomenographic study by Petocz and Reid (2002) supports the models and contributes to statistical thinking theory by providing five variations in ways undergraduate students assign meaning to statistics, provided in Table 1.

In a follow up study, Petocz and Reid (2003) investigated the link between students' conceptions of teaching with those of learning statistics. The results suggest that the students viewed learning and teaching as a set of coherent activities. Furthermore, affective components (enthusiasm, interest, and motivation) contributed to the differences in conceptions of learning and teaching, as these were expressed as important components to teaching by the students. The researchers concluded, variations in the conceptions of teaching and learning play an important role in the way students approach their learning and their expectations of the instructor's role (Petocz & Ried, 2003). The also suggest that the variations in categories be acknowledged in developing a total learning environment that helps students develop their thinking about learning and teaching statistics toward the most inclusive level.

Gordon (2004) conducted similar research on undergraduate students in a psychology program taking a service learning statistics course. The study investigated the link between students' attitude towards statistics with the meaning they assigned to the subject. Gordon's research unveiled five conceptions of statistics described by students (No meaning; Processes or algorithms; Mastery of statistical concepts and methods; a Tool for getting results in real life; Critical thinking). The results showed that

Table 1

Describing Students' Conceptions of Learning Statistics (Petocz & Reid, 2002)

Conception	Description
Doing	Learning in statistics is doing required activities in order to pass or do well in assessment or exams.
Collecting	Learning in statistics is collecting methods and information for later use.
Applying	Learning in statistics is about applying statistical methods in order to understand Statistics.
Linking	Learning statistics is linking statistical theory and practice in order to understand Statistics.
Expanding	Learning in statistics is using statistical concepts in order to understand areas beyond Statistics.

students in the program who had negative attitudes towards taking the required course defined statistics as "procedures with no meaning". Gordon established a connection between the students' feelings about learning statistics and their interpretations of statistics.

Gordon's (2004) five categories are consistent with the description of concepts reported by Petocz and Reid (2003). The significance of these phenomenographic inquiries is that they establish a relationship between the meaning assigned to statistics with attitudes toward statistics (Gordon), and conceptions of teaching (Petocz & Reid).

The proposed study is positioned in the field of research on statistical thinking by investigating secondary school students' conceptions of statistics after a period of time, supporting and contributing to the outcome space on learning statistics constructed by Gordon (2004) and Petocz & Reid (2003). The study will produce an outcome space on conceptions of statistics, as described by the participants. Under investigation are the links between learning strategies and conceptions of learning and the link between conception of learning and conception of statistics. Furthermore, the outcome space for the study aims to contribute to the vocabulary of the field and to provide a system of classification for the variations in statistical learning experiences among students.

Additional research has focused on affective characteristics of statistics learners to investigate indicators for student success. In anticipation of the affective components revealed by Gordon (2004) and Petocz & Reid (2002), research providing descriptions of what capabilities can be expected from statistics students, and how these are linked to experience, beliefs, and attitudes are discussed to position the study in the affective domain as well.

Affective Factors on Statistical Thinking

Links between achievement in a statistics course and factors such as self efficacy, statistical anxiety, and motivation acknowledge that affective components are just as important to the development of statistical thinking as pedagogy (Roberts & Bilderback, 1980; Wise, 1985; Gal & Ginsburg, 1994). A recent study by Williamson and Mattiske (2002) found that when students expressed positive perceptions of their mathematics capabilities a negative relationship with anxiety towards learning statistics existed. The researchers did not find a significant difference in course achievement outcomes between students with low expectations and those with high expectations. Further research was recommended on the link between threat and challenge appraisals in relation to the link established in the study with self-efficacy to statistics anxiety and achievement.

Tempelaar (2006) investigated the relationship between students' prior statistical reasoning ability when entering an introductory statistics course and personal background variables such as attitudes towards statistics and approaches to learning. Tempelaar's structural model found that the only attitude that significantly contributed to the development of statistical reasoning was planned effort. Furthermore, students who approached learning to achieve only a surface level understanding demonstrated low statistical reasoning skills, supporting the negative relationship found between surface learning and statistical reasoning ability. Students preferring surface level approaches over deep learning approaches were still able to pass the course, showing that statistical reasoning did not appear to be heavily dependent on students' background characteristics as much as their approach to learning. Tempelaar suggests a need for the development of more tools that address general learning approaches, statistical reasoning, and attitudes.

Lee and Meletiou (2002) analyzed students' motivation in an introductory statistics course and compared it to course performance. Motivation was classified into five types based on existing theories. The study suggests that individual learning goals, instructor's expectation of students, and corrective feedback tended to be components for increasing students' motivation to learn statistics. While the belief systems students entered the course with had some influence on motivation and course performance, the structure of the course and instruction were just as influential. The study supports findings in previous research (Gal & Ginsburg, (1994); Seipel & Apigain, 2005) about the role affective components play in how students approach learning statistics.

Sowey (2006) showed that the meaning students place on statistics is influenced by their motivation to learn and how the students approached learning. In a statistics service learning course, students were encouraged to strengthen their understanding by asking and answering challenging questions. Sowey states that this is an affective approach for students to discover that statistics is a discipline that is resilient to there own challenging questions because of its empirical cycle. Sowey concluded that students who took an investigative, inquiry based approach to learning statistics demonstrated greater statistical thinking skills than students who only focused on content.

Chanza and Ocaya (2006) investigated the role that students' perceptions play in how statistics is understood in a first year undergraduate course. They found a relationship between pre-conceptions of statistics and readiness to learn. Factors (applicability, perceived workload, computing ability, feedback on learning, statistical packages, availability of computers) that students perceived to be detrimental or useful to their learning of statistics were established which were used to re-structure the course.

Students' experiences contribute heavily to the beliefs and attitudes they bring to statistics courses (Seipel & Apigian, 2005). Research has shown consistently that due to these experiences, variations in how students understand statistics is vast, yet classifiable (Gordon, 2004; Neuman, 1998). Prosser and Trigwell (1999) reported an internal relationship between students' perception of learning and teaching with course expectations. Supporting the findings, studies by Gordon (2004) and Petocz & Reid (2003) reported the variety of different ways that statistics students understand learning statistics, and how undergraduate students' epistemological beliefs about learning are related to a series of learning strategies and intended outcomes. Langrall and Nisbet (2006) showed how students' statistical knowledge shares an internal relationship with their contextual knowledge when analyzing data. Based on factors such as students' field experience, statistical or mathematical knowledge needed for the task, and the meaningful role statistics plays in understanding the data, students constructed models of understanding in which they shifted back and forth between the use of contextual knowledge and statistical knowledge. As a result of the study, the researchers state that the findings emphasize the importance of providing students with opportunities to engage in investigations that encourage them to integrate contextual and statistical information in order to develop statistical literacy.

The role experience plays in developing conceptions of statistics is key to the proposed study. Although specific affective factors are not under investigation, research on the matter was discussed to bring into view theories on how students experience statistics. With a focus on experience, the study is also positioned in the phenomeno-graphic tradition, which may be unfamiliar to some educators. Research studies using phenomenographic investigation are discussed to show the parallel between implications of and contributions to educational research of these studies with the anticipated findings of the proposed study.

Phenomenographic Methodolgy in Educational Research

Because of its nature to explain knowledge acquisition through human experience, phenomenography is relevant to educational research and curriculum development. Phenomenographic research can reveal critical educational differences in the ways students understand concepts in various subjects. In regards to educators, just as the learners experience the world in distinctly different ways, teachers do as well. Phenomenographic studies have been conducted to address the variations in instruction and to develop pedagogy.

Determining how students make sense of an educational task is a valuable asset to instructors and curriculum developers. A model of the outcome space of such a study would reveal the varying levels of awareness to be anticipated by evaluators and demonstrated by students. Furthermore, the link from one level to the next is provided, diagramming the natural progression of development in terms of awareness, as will be demonstrated by this study in Chapter Four. From this, a connection can be made between the students' level of awareness of the phenomenon and the level of skill mastery associated with the educational goals (proposed follow up research to the current study). For example, Neuman (1998) studied qualitatively different ways seven year-old Swedish students experienced numbers. The outcome space contained four categories of description: numbers as names (focus on ordinal aspects), numbers as extents (focus on cardinal aspects), finger numbers (focus on ordinal and cardinal simultaneously), and abstract numbers (reflects complete simultaneous experience with ordinal, cardinal, part-whole) (Neuman, 1998, p. 71). The students who did not move beyond the "numbers as names" category did not seem to develop mental calculation skills, and they had more difficulty than other students learning subtraction (Neuman, 1998). Other phenomenographic studies of this type can provide competency maps to help assess the progress of students.

The phenomenographic approach to research is also beneficial to those who educate teachers. Studies done by Saljo (1981) and Marton (1986) suggests that manipulative changes to learning tasks do not improve learning, they simply change the focus of the learning task (Marton & Booth, 1997). The students focused on answering the question presented to them, but they did not voluntarily seek deeper meaning of the phenomenon. Even when the questions intended to invoke the students' analytical capabilities to find deeper understanding, most of the students answered the question at hand without much elaboration. So the focus should simultaneously be on the quality of the instruction and the educator.

According to Marton & Booth (1997, p.177), "teachers teach according to their ways of understanding the subject as a whole." Combining the qualitative differences among the students with the qualitative variations among teachers, it is obvious how radically different the content of learning can be for students. A study by Prosser and Trigwell (1999) reveal that teachers typically focus more on content (factual content or what is thought of as content) than on the learner. If teachers are made aware of the qualitatively different ways students experience the content, the instruction could be redirected to address these levels of awareness. This way, even the differences among teachers in the same content area can be directed to a common, well-documented outcome space.

As Marton and Pang (1999) states, a careful account of the different ways of experiencing something may help uncover conditions that facilitate the transition from one way of thinking to a qualitatively better perception of the phenomenon. This could aid teachers in helping students experience or understand a phenomenon from a given perspective. Finally, the phenomenographic approach would make students conscious of their own contradictions in reasoning and they become more open to alternative ideas as they reflect on their own perceptions and understandings of the world around them (Marton, 1986).

The most common critique of the phenomenographic approach to research concerns reliability, dependability, and validity. Marton (1986) admits that it is possible for two independent researchers to discover different categories of description for the same data. The issue of reliability is addressed by how well the categories are developed, described and deemed useful by the research community. It is similar to a botanist developing a category for a new species; once the category is developed and described, it becomes useful to others who use the results of the study (Marton, 1986). Neuman (1998) suggests ways to address the validity of the outcome space that emerges from a phenomenographic study. By putting the results of the study into practice, pragmatic validity, as described by Kvale (1989) is established.

Conclusion

Reviews of research in statistics education and phenomenography that underpin and support the purpose and significance of the study were presented in this chapter. By discussing research on curriculum development, course development, and assessment, the instructional design of the statistics course in the study has been framed. By reviewing research on how students learn statistics, the need for further research on a framework of statistical thinking for secondary school students supports the significance of the study. Finally, discussing the use of phenomenography in educational research supports the choice of methodology for the study and how it will guide the study.

CHAPTER 3

METHODOLOGY

Use of Qualitatitive Methods

The study bears the distinctive characteristics of qualitative research as described by Creswell (2003):

- the study is exploratory and descriptive
- the research will take place in the participants' natural setting
- multiple methods will be used to collect data and these methods are inquisitive, interactive and humanistic (sensitive to the role of the participant)
- the research is emergent and iterative in that, as data are collected and analyzed the problem may be reformulated and the research questions may be refined.
- the role of the researcher is that of an interpreter who is required to suppress her beliefs about the phenomenon under study to draw conclusions about the meaning of themes or categories (p. 191-195).

The phenomenographic approach supports the qualitative nature of this study. Phenomenography is exploratory and descriptive because it maps the qualitatively different ways the participants experience, conceptualize, perceive, and understand various aspects of a phenomenon in the world around them (Bowden, 1996). In this study, a deeper understanding of how the participant experienced learning the content of a course in statistics and how the participant experienced learning itself are explored through descriptions of the phenomenon.

Description of the Setting

The study was conducted by correspondence with graduates from a suburban secondary school located in the southeastern United States. The secondary school in which the participants graduated from is the only high school in a small school district in the city. Enrollment during 2006, the year the study is conducted was about 813 students, and the racial make-up was 48% White, 49% African-American, 2% Multiracial, and 1% other (Hispanic, African, Asian). The school is on a 4-by-4 block schedule, in which students attend 4 90-minute classes per day per semester.

All of the participants had graduated from the high school within the previous 2 years, and each participant had completed a course in AP Statistics or College Preparatory Statistics while enrolled as students. The students were enrolled in the course to meet a requirement for graduation (four mathematics courses), or they were taking an additional mathematics course as an elective in the college preparatory program of study. The course is one choice among other advanced mathematics courses offered by the mathematics department, in which the minimum requirement is that the student received credit for classes in advanced algebra and trigonometry. The school practices open enrollment, meaning the student has the freedom to choose any level of a course (applied, college preparatory, honors, or advanced placement) without faculty approval or a grade point average requirement, contingent upon parent or guardian approval. Therefore, teacher placement was not a mitigating factor for the student's enrolled in AP Statistics or Statistics.

Role of Researcher

In phenomenographic research, the researcher is both the data collection instrument and the interpreter of the data. As the instrument of data collection, I conducted and transcribed participant interviews, analyzed documents, and corresponded with the participants as they engage in tasks for the study.

As a veteran mathematics educator (15 years in the classroom), I realize the value of differentiated instruction and differentiated assessment to address the diverse needs of learners. The classroom and instruction should be student-centered, and I believe the responsibility for learning is ultimately placed on the learner. Although all students are capable of learning and ultimately responsible for learning, I think the course content should be presented in numerous ways to help students conceptualize and understand what is to be learned. Clearly stating what is to be learned, diversifying instruction, appropriately using technology, allowing the student some input into how he or she will be assessed, and clearly defining how grades will be determined are just a few my common practices. I also believe that students express varying levels of understanding for a given topic. Thus, assessment must allow the student to express their understanding in numerously different ways while adhering to the standards and benchmarks required by the school system. Concepts should be integrated within the context of their use and not taught in isolation nor practiced only rotely. In an AP Statistics class for example, students should collect data, analyze real data, conduct research, and practice statistics in situations similar to those of professionals who use statistics. This will help the student conceptualize what it means to know statistics, as it is defined by the community of practicing statisticians.

I view learning as a change in the level of awareness of the attributes a particular phenomenon possesses. If a student can report and discuss characteristics or meanings of a phenomenon that he or she was not previously aware of, then learning has occurred, with the understanding that all students will not attain the same level of awareness. In a constructivist sense, the student uses what was previously known about the phenomenon to extract or discover other present but unseen attributes. I believe this can be expected to occur when the student reconsiders or reconceptualizes the phenomenon in a different context, from a different perspective, or through multiple representations of the phenomenon. Of course, these constructions and deductions must be validated by the student before a proclamation of learning can be declared.

This discussion of my beliefs is a common characteristic of qualitative research. Even though an attempt will be made to avoid filtering the data through the lens of my personal experience, I have acknowledged the role my beliefs may have in this inquiry.

Data Collection

The participants volunteered to be in the study, representing a convenient sample as described by Merriam (1998). A mailing list of 114 graduates from the classes of 2005 and 2006 who completed Statistics or AP Statistics and were 18 years of age or older was drawn from school rosters. Because of relocation, the possibility of incorrect information, timing of the study when many were returning to college, and other factors, I received 31 letters declining the invitation to participate and 9 letters of acceptance. Despite the sample size, the participants were found to represent information rich cases.

The site is unique for two main reasons. First, the secondary school offered its first Statistics course fall semester 2004, and the first AP Statistics course was offered in fall 2005. The courses are still relatively new to the students and to the school. The students and I, as the teacher, entered the course as clean slates; thus, a vast range of experiences were expected. Also, for myself, the themes that emerged from this study were used to evaluate the course and to develop better practices for future Statistics and AP Statistics courses taught at the school. The second unique feature of the site is the school's open enrollment practices. The students in the Statistics and AP Statistics courses can be considered a good cross section of the varying levels of statistical understanding.

Every reasonable effort was made to protect the rights of the participants in the study. Institutional Review Board approval for the study was obtained on August 8, 2006. To preserve confidentiality, each participant was asked to make up a pseudonym for identification purposes. Furthermore, all contact information provided by the school was used solely to locate former statistics students. Students declining participation or those who did not respond within 4 weeks of the mailing had their contact information deleted from the list. Students who volunteered to participate were asked to provide alternative contact information, such as an electronic mail address or a mobile phone number, and this information was stored in a locked file cabinet. Also, the volunteers were mailed a consent form, questionnaire, and journal questions.

Because the object of the study was students' experience of learning statistics, I used phenomenographic research methods to find patterns concerning the phenomenon. A questionnaire, interviews, and reflective journals were used to collect data, and copies of the forms can be found in Appendixes B, C, and D.

Participants returned consent forms and questionnaires by mail. The reflection journals and all other correspondence were collected by electronic mail. The participants were asked to complete the journal in 2 weeks, taking about 2 days to reflect on each question. Once a completed journal was submitted and follow-up questions were complete, initial coding took place and interview questions were generated.

Semi-structured interviews along with the reflective journals served as the primary methods of data collection. Francis (1996) states," the aim of the [phenomenographic] interview is to have the interviewee thematise the phenomenon of interest and make the thinking explicit. Awareness of experience is tapped through reflection and report" (p. 38). For the study, the focus of the interviews was on how the participant experiences and ultimately understands the phenomenon, learning statistics. The questions posed for the interview came from the reflective journals submitted by the participant as well as preconstructed questions suggested by Marton and Booth (1997) about the meaning assigned to the phenomenon and strategies for how one learns about it. Students had the choice of being interviewed face-to-face, by electronic mail, or by telephone. All participants corresponded by electronic mail. Printed copies of electronic mail served as transcripts for analysis. All data collection was completed by January 9, 2006, based on the saturation and reduction of data.

Data Analysis

Explicit details on how the data were analyzed are discussed in chapter 4. Creswell (2003) suggests a generic process for data analysis and interpretation:

1. Organize and prepare the data for analysis (transcriptions, field notes, sorting, optical scanning)
2. Read through all the data
3. Begin detailed analysis with a coding process

4. Use the coding process to generate a description of the setting or people as well as categories or themes for analysis
5. Advance how the description and themes will be represented in the qualitative narrative.
6. Make an interpretation of meaning of the data, detailing lessons learned and capturing the essence of the phenomenon (pp. 191-195).

Because qualitative research methodologies are broad and diverse, Creswell acknowledges that the procedures used to analyze data could never be reduced to simple algorithm. The methods used to analyze the data should ultimately allow the researcher to make an accurate interpretation of the meaning of the data. Phenomenographic data analysis approaches to organizing and interpreting information are aligned with Creswell's suggested process, as describe in the analysis below.

I organized and prepared the data by downloading or printing transcriptions of interviews and journals. All journals, transcribed interviews, and questionnaires were saved to my secure home computer and to a storage diskette. I read and reread transcripts to conceptualize a broad spectrum of themes and categories, remaining open-minded each time to gain different perspectives needed to clarify student meaning. Categories were drafted based on quotes of interest that seemed to address the meaning of the learning statistics. Subsequent readings were to redefine and narrow the categories by allocating similar quotes to their respective categories.

This iterative process, described by Creswell (2003) above and in the phenomenographic tradition, focuses on the individuals' descriptions being categorized to deduce meaning about the whole phenomenon. As Neuman (1998) explains, this iterative process is never completed, so I concluded this phase of analysis when the narrowed categories of description were deemed relatively stable.

The final phase of data analysis was to interpret "the pool of meaning" (Marton, 1986, p. 42) to which each quotation belongs. I defined the categories in terms of their core meaning, then I reflected on the similarities and differences between them, and lastly I established connections between the core meaning of the phenomenon and the core meaning of the categories of description.

Trustworthiness of Data

Trustworthiness of data was achieved using Lincoln and Guba’s (1985) four criteria: credibility, transferability, dependability, and confirmability. To ensure credibility, triangulation, prolonged engagement with participants, persistent observation, and member checking were used. Thick, rich description was used to convey findings, promoting transferability. An audit trail, preserving and securing records and data collected from the study, ensured dependability and confirmability.

The criteria for authenticating phenomenographic research is comparable to that of all qualitative methodologies in which reliability, validity, and dependability (Lincoln & Guba, 1985) must be confirmed. According to Neuman (1998), the researcher's interpretive awareness is the criterion for reliability in phenomenography. Sandberg (1996) explains,

> in order to be as faithful as possible to the individuals' conceptions of reality, the researcher must demonstrate how he/she has controlled and checked his/her interpretations throughout the research process: from formulating the research question, selecting individuals to be investigated, obtaining data from those individuals, analyzing the data obtained , and reporting results. Hence . . . establishing reliability of the researcher's interpretation is crucial. (p. 137)

The measures I have taken to suspend my beliefs in each phase of the study are as follows. As a teacher at the study site, my prolonged time spent with the students lends credibility to the description of the site and the participants. My extensive career in education aids in developing a comprehensive understanding of the phenomenon under study, which is learning statistics. In terms of verifying the credibility of the data collected, an iterative process along with triangulation of data sources were used to build a coherent justification of categories of description (Creswell, 2003). As for member checking, the participants were asked follow up questions to clarify discrepancies in meaning and to verify their responses. The categories of description and the quotes associated with them were peer reviewed to enhance accuracy and to dispel any suspicions about my interpretations. Thick, rich description was used to convey results. Finally, an external auditor provided an assessment of the entire project (Lincoln & Guba, 1985).

The size of the group chosen for the study brings into question the issue of generalizability. Although generalizability is not of great concern in qualitative inquiry (Creswell, 2003), the phenomenographic nature of the study makes a case for the generalization of the findings. The main objective of phenomenographic research is to investigate variations in personal experiences. So, as Akerlind (2004) explains, the group chosen for the study should be heterogeneous rather than representative in terms of demographics

> to the extent that the *variation* within the sample reflects the *variation* within the desired population, it is expected that the *range* of meanings within the sample will be representative of the *range* of meanings within the population. In this sense, the results of a phenomenographic study should be generalizable to other groups of people from a similar population, in that the *range* of ways of experiencing constituted in relation to a particular group should be common to other groups with a similar spread of characteristics (and presumably ways of experiencing). Even with less similar groups of people, the meanings and dimensions of variation that

> emerge from the sample group should still be relevant, but are likely to constitute a less complete representation of the range. (para. 15)

The students in the AP Statistics and Statistics classes volunteered for the course, as the school selected for the study practiced open enrollment, and statistics was a choice among a set of classes. Furthermore, because the courses had been offered only a few times at the school, students were eager to take the class, and they were encouraged to take the course despite any difficulties they experienced in previous mathematics classes. Finally, a grade point average limit or testing requirement was not a factor in the students' enrollment in the course. For these reasons, the variation of learners within the group constitutes the variation of learners within the school who would volunteer to take a statistics course. Hence the range of ways of experiencing learning and statistics as described by the group of students in the study, although not a complete set, are generalized to similar classes of AP Statistics and Statistics at the school.

The findings of the study are conceivably replicable. A researcher using the same set of data could generate a similar yet more expansive set of descriptions because a single phenomenon cannot be completely described by one individual (Marton, 1986). This evolving collection of meanings was expected in the study because the qualitatively different ways of experiencing statistics delineated by my interpretation of the data do not constitute a complete set of descriptions. The varying perspectives of another researcher will and should rightfully add to the set of existing categories.

Finally, the findings of the phenomenographic study will be put into practice by the researcher. Kvale (1996) refers to this way of defending the outcomes of qualitative research as pragmatic validity. As explained further by Akerlind (2004),

> qualitative research validity includes the extent to which the research outcomes are seen as useful . . . and the extent to which they are meaning-ful to their intended audience . . . the research aim becomes to provide useful 'knowledge', where knowledge is defined as the ability to perform effective actions . . . research outcomes may then be judged in terms of the insight they provide into more effective ways of operating in the world. (para. 26)

In terms of pragmatic validity, I believe the findings in this study are useful because my instructional practices will be affected by this inquiry. I intend to develop and implement instructional strategies supported by the research to use in future AP Statistics and Statistics classes taught at the site.

CHAPTER 4

RESULTS

Reflections from the Participants

Using the words of the participants, as recorded in the reflection journal, a glimpse into their personalities and experiences are reported. A reference to course performance and numerical grades were not used because most of the participants expressed discomfort in such information being revealed. It is also my strong preference to omit such information to avoid bias in the interpretation of the responses.

Selected questions from each participant's journal are presented to illustrate the variations in responses, and to provide foundations from which the categories of descriptions emerged. However, entire journals are not presented to avoid redundancy.

Cara

Cara is a 19-year-old sophomore at a small women's college in the southeastern United States. Her anticipated college major is accounting, and she is interested in becoming a sports agent. She took an introductory course in statistics during the Fall semester of 2005, her senior year of high school. A few reflections from Cara's journal give insight to her experience of learning and using statistics.

> *What was learning statistics like?*
> I enjoyed learning statistics in high school. It was easy for me to grasp the concepts [because] it [was] interesting for me to learn.

How did you go about interpreting data before you took a statistics course?
I just use to read over the statistics and think about what I thought they meant.

How do you go about interpreting data now?
Now I know what I am looking for in statistical data and what all the specific terms mean. Like for college, I am better able to take and record data for projects. [When] I read through different articles and see statistical data, I know how to read and interpret statistical information.

Do you consider yourself statistically literate?
I think that I am more statistically literate than most people that I know. I was taught how to read and fully understand the statistics that I come across in life or those I want to [collect] in order to get research.

Aimee

Aimee is an 18-year-old freshman at a state university in the south. She is undecided about her college major, but she wants to pursue a career in fashion design. She took an introductory statistics course during her senior year of high school, the Spring semester of 2006.

What was learning statistics like?
Learning statistics was like looking at things through a magnifying glass. When you see something from far away, it has a certain look. It often gives the appearance of being simple. Based on this simple appearance, you might decide you basically know all there is to know about it. Before taking statistics, I really only know what I had always seen. A 75% chance of rain. Forty-five percent of teens prefer A to B and so forth. It wasn't hard to imagine there was little more behind statistics than what was on the surface. Counts and predictions. Taking statistics has been like seeing things up close because in the class, you find out there is so much more involved. There are certain ways you design surveys and several equations in data organization. There are so many things I would have never known about if I'd never taken statistics and the thing is, I would have never realized it.

How did you go about interpreting data before taking a statistics course?
I use to take statistics at face value. I figured the sources had to be reliable so I would believe statistics I heard without question. If I heard that 75% of cows give more milk when they're on a cow merry-go-round I would probably have believed it. I didn't think about where the information was coming from or what [population] it was taken from.

> *How do you interpret statistics now?*
> After taking [a statistics course], I no longer go without questioning statistics when I hear them. Statistics vary depending on the location and the sample so if one doesn't have all the information needed, it is probably not a good idea to put one's total trust in whatever statistics are being given.

Sheldon

Sheldon, 19-years-old, is a business major with a concentration in accounting. He is undecided about a particular occupation in the field of business, but he expresses a very strong interest and dedication to the field. Sheldon attends a large university in the southeastern United States. He took an introductory statistics course during Fall semester of his senior year in 2004.

> *What was learning statistics like?*
> Stat[istic]s was one of those classes that seemed like something you should know by common sense, but you just had to be shown arithmetically. Not that everything made perfect sense at first or that it was like second nature, but it seemed to all be logical after having learned it. The thing is, before stat[istic]s is learned for the first time, we are all exposed to it to a certain extent every single day. Learning the subject just made it all come together, and it was easy to apply to everything we consider on a day to day basis. It was also somewhat rewarding to learn. Because we are exposed to stat[istic]s so frequently, once it was understood on an introductory level, it seemed that I could apply it constantly outside of class without any effort. Unlike a more abstract (or more boring class) I felt like I could immediately take what I learned from stat[istic]s and use it in other contexts. This could be done by analyzing basic stat[istic]s functions in my head while, say watching sports or the weather, or even just considering how a set of values would look once analyzed fully with every application and formula. The fact that my stat[istic]s projects [in the class] were open to basically any topic desired, this furthered the widespread nature of the subject. We were able to choose pretty much any topic within reason, and this really showed me that it could be applied to anything I was interested in. This aspect of the class definitely opened me up to learning the concepts that were reinforced by these projects.

> *How do you see yourself using statistics in the future?*
> I see myself using statistics in many, many ways and many facets in the future. I am considering a career in the business field, and could see myself learning considerably more about statistics. More specifically, I

> might consider a career in financial accounting, and I would have to learn how to organize and present transactions. No matter where I go with this career, I would have to learn much about presenting monetary gains and losses and the financial possibilities [of] an individual or firm. In college courses, I would probably use statistics for performing studies, making presentations, and pretty much so I can decipher business-related concepts in the majority of my classes…Hopefully, I am not one of many who knows so little about statistics that I will have to re-teach myself basic concepts in order to survive financially.

Mary

Mary took introductory statistics in Spring 2005 of her high school senior year. She is a sales and marketing major at a medium sized university in the south. Mary is 19 years old, and she is undecided about a specific occupation, but she has an interest in business administration. Mary completed the journal reflections, but she did not respond to follow up questions, nor did she respond to requests for an interview.

> *What was learning statistics like?*
> Statistics was fun. I learned a lot about why people should be cautious when surveying people or considering doing a statistical study.
>
> *How did you go about interpreting data before you took a statistics course?*
> Before I took the statistics course, I really didn't care about possible outcomes that would happen to me on a daily basis.
>
> *How do you go about interpreting data and statistics on a daily basis now?*
> After the statistics course, I had a little more of an understanding on why it is important to be careful with the decisions that a person is faced [with] on a daily basis because it could change the way your day goes. Like if you ask yourself several questions before your day start, you always have an alternate choice but, you usually know the chances that you will take the easy way out or not do what is important for you to do on that day.
>
> *How do you see yourself using statistics in the future?*
> I take a statistics course now and because I took statistics in high school it makes it a little more easy because I am knowledgeable to the subject. Also, I am a business/sales and marketing major, so therefore I will use statistics in just about everything once I graduate from college and go into the workforce.

Angela

Angela is 18 years old, and she attends a small college in the south. Although her college major is undecided, she is interested in a career in public relationships or international language translation. Angela took statistics during Fall semester of her high school senior year in 2005.

> *What was learning statistics like?*
> At times statistics was intriguing, frustrating, and satisfying. I say intriguing because I like Algebra and in many ways, Statistics is like Algebra for me. You have an equation, you have data, and you can use both to come up with an answer. It's the answer and what the answer signifies that was the most intriguing part to me. When I was being taught statistics, whenever a new formula was put on the board, I had the urge to figure out what was going to happen with the data that I held.
>
> The most frustrating part of learning statistics was that if I didn't understand a particular idea, or my teacher could not convey to me how to solve or read a problem, there were no other math teachers to explain it to me in a different light. Statistics is fairly new in high schools. It's also very specialized, and no other teacher had any understanding of it.
>
> I think that despite all frustrations, however, that over all learning statistics was very satisfying to learn. At the end of the class, I had a problem solved, I had worked through any type of misunderstandings I had, and also the knowledge that I had been able to expand on what was once only some data and an equation.

> *How did you go about interpreting data before you took a statistics course?*
> Before taking statistics, I probably would not have understood how to interpret data. More than likely, if pressed for an answer, I would take the data presented to me at face value. I would not know how to question how statistical[ly] significant the data was, or how to read a graph correctly. Like most people, the only part of statistics that I was familiar with were percentages. I did not realize the amount of collecting, computing, placing, and application that is involved in creating a statistics.

> *How do you go about interpreting data and statistics on a daily basis now?*
> For the most part, I now find myself questioning any statistic thrown my way. Sometimes I just accept what I'm told, but a majority of the time, I question what is presented to me. In one of my [college] classes, the teacher throws all sorts of information, percentages, graphs, etc. at us students. He does not question the information he provides, nor does he

> ask us to question it. I find myself, though, wondering how the information was gathered, how statistical[ly] significant a piece of information actually is, and whether not the writer manipulated the information in his favor.

Rebecca

Rebecca is an 18-year-old freshman at a large university in the southern United States. Her major is art with a minor in dance. She is undecided about an occupation or career field after college. Rebecca took AP Statistics during Fall semester of her high school senior year in 2005.

> *What was learning statistics like?*
> Learning statistics introduced me to many methods of how to present information from a non-biased perspective. Statistics organizes facts and/or observations into various categories that serve as a representation for a larger group. I found that learning statistics could be difficult, demanding, and time-consuming.
>
> *Do you consider yourself statistically literate?*
> I do consider myself statistically literate because if one were to present me with a statistical study I would be able to read through the research and comprehend the conclusion. However, I do not believe that I am statistically knowledgeable because I often find myself blindly agreeing to someone who says "oh yeah, there are statistics that back it up", without questions [like] which statistics, when they were taken, who they were taken by, and where they were found.
>
> *How do you see yourself using statistics in the future?*
> [Presently]…statistics are part of my everyday life. If I am interested in pursuing an opportunity, I will often check the statistics of how many people get in to such before applying myself. When I register for classes, I often check the student "key" to find out the statistics on certain professors or particular courses. These statistics help to give me a general idea of a large, more complicated subject. I have no doubt hat after college, when I pursue my career options, I will take statistical success rates into consideration.

Tina

Tina took introductory statistics during Fall semester 2004 of her senior year in high school. She is 19 years old and attends a large urban university in the southern

United States. Tina is undecided on a college major, but she has a very strong interest in a career in the medical field. On occasion, she has mentioned her aspirations of becoming a nurse.

> *What was learning statistics like?*
> Learning statistics was very interesting. It gave me a better understanding of how we use statistics in our daily lives. It was definitely on of the more useful courses to take in high school.
>
> *How do you see yourself using statistics in the future?*
> Statistics is used in every profession. Pursuing a career in the medical field requires a lot of work dealing with data. Along with calculated formulas, surveys and testing patients is one of the things that has made cures and other health benefits a success.

Andrew

Andrew took introductory statistics during Fall semester 2004 of his senior year in high school. He is 19 years old and attends a large southern university and is majoring in building science. He wants a career as a program manager or contractor. Andrew completed and responded to a follow-up question for the reflection journal, but he was unavailable for the interview segment of the study.

> *What was learning statistics like?*
> Understanding everyday number data.
>
> *Do you consider yourself statistically literate? Explain.*
> Yes. My friends who are taking statistics now in college can talk about statistics and I understand.
>
> *How did you go about interpreting data and statistics on a daily basis before you took a course in statistics? How do you go about interpreting data and statistics on a daily basis now?*
> I believed all the given statistics in magazine articles, rather than understanding the context. I know now what these stat[istic]s really mean.
>
> *How do you see yourself using statistics in the future?*
> I will definitely use all the tools we learned about on *Excel* for work and I will continue to understand stat[istic]s in published articles.

Sally

Sally is an 18-year-old freshman at a large university in the southeastern United States. Her major is chemistry and she wants to be a physician. Sally took AP Statistics during Fall semester of 2005, her senior year of high school.

> *What was learning statistics like?*
> Learning statistics was definitely different than any other math[ematics] class I've taken…I do remember some concepts being difficult, but a different kind of difficult. Instead of "oh man this is really hard! I'll feel so good about myself if I can solve it!" (the way I was in classes like calculus) it was "I know I can do this problem, but it's kind of hard and will take a long time, and I really don't feel like it." It's like statistics problems were really annoying to me. That seems strange, since statistics apply to real life and calculus doesn't, but I guess I'm just not all that interested in statistics…It's kind of like statistics was just a mundane, necessary task…like something boring you have to learn in order to do research and stuff that you really care about. Statistics in itself is not interesting, but what you do with it is…Some activities were fun, especially when we got to eat M&Ms and stuff!
>
> *How did you go about interpreting data and statistics on a daily basis before you took a course in statistics? How do you go about interpreting data and statistics on a daily basis now?*
> I wouldn't say I interpret data and statistics on a daily basis…but that's mostly because I don't read the newspaper everyday. However, when I do go on MSN or something, and it's saying "80% of French men wan to be pregnant!" I wonder how statistically valid that is. I wonder what kind of question they asked to get that response…probably something like "if you had the opportunity [to] take the burden of childbearing from your wife, would you do it?" (What kind of jerk would say no to that?) Probably before I took statistics, I would take that pregnancy statement totally serious. I have definitely learned to question data you read in the newspaper and on the internet.

The journal entries of the participants depict the range of responses encountered throughout the study. Journal entries and interviews were read through several times. After each reading, statements were grouped by similarity into categories of description for further analysis. Once data could no further be reduced from this iterative process, the outcome space of data was constructed. The analysis of the categories and the structure of learning statistics are discussed in reference to the two components of learning as described by Marton and Booth (1997)

The "What" Aspect of Learning

By analyzing the direct object of learning, the first focus question of the study (what are the different ways participants define statistics) is investigated. In reference to Figure 3 from chapter 2, the structural and referential aspects of the meaning of statistics are presented in Table 2. The term background is used instead of external horizon and foreground replaces internal horizon. The three categories that emerged from the data accompany Table 2.

Category 1. Statistics as Basic Mathematics Skills.

The concept of statistics as maintaining basic mathematics skills was generally described by some participants as learning ways to manipulate data with equations, and using numerical and visual ways to represent data for the purposes of completing tasks to do well in the course. Excerpts from Andrew's journal illustrate the attributes of statistics brought to the foreground in this category.

Journal Question:	What is your personal definition of statistics?
Andrew:	A graphical and numerical way to represent numbers. Just like any other math[ematics] class, you basically deal with equations and graphs.
Follow-up:	What does your statement about "dealing with equations and graphs" mean?
Andrew:	In statistics, you put numbers into formulas, solve equations, and learn how to make different graphs for different kinds of data.

Andrew's focus on equations, formulas, and graphs are presented as discrete components of statistics. While suggesting the usefulness of statistics with respect to representing numbers, a context for statistical application is not mentioned. The perception of how relationships among equations, formulas, and graphs contribute holistically to statistics is not explicitly stated. The discerned parts from the statistics curriculum background that emerge are unrelated tasks that involve applying algorithms, manipulating formulas, and displaying data for the purpose of passing the course.

From Category 1, participants who experienced statistics as a course in which they maintained basic mathematics skills placed statistics in the context of a traditional mathematics curriculum. The parts of the statistics curriculum they focused on were isolated algorithms in which they memorized procedures that did not have connections to applications. The meaning assigned to statistics based on this experience is: statistics is a traditional mathematics course in which algorithms are applied to problems such as evaluating expressions, solving equations, and displaying data.

Category 2. Statistics as Procedures for Data.

In this category, specific tasks of statistics are brought into focus. Statistics as a mathematical discipline that involves using techniques for collecting, analyzing and interpreting data to understand the real world, is the description that generalized the conception of statistics as Procedures for handling data. The participants discussed statistics as methods for studying data that pertained to a specific population. This is a step up from Category 1, because it applies the procedures to a context. As recorded in the journal entries of Sheldon, Mary, and Rebecca reference to real world applications or specific subject matter are mentioned.

Journal Question:	What is your personal definition of statistics?
Sheldon:	My definition would be the use of analytical math concepts as well as techniques involving the presentation of findings in order to fully understand data in a succinct and detailed manner.
Mary:	The process of gathering information on a specific topic and looking at the possible outcomes.
Rebecca:	Statistics [are] a group of data that is gathered, combined, and analyzed in such a way that it can objectively represent the aspects of a particular population.

The acts of collecting, displaying, analyzing and interpreting data about specific populations are the discernable parts of statistics brought into view in Category 2. While statistics is recognized as having relevant value to the real world, the participants do not discuss anything beyond the interpretive and analytical characteristics of statistics. It is not seen as a tool of inference, nor are cautions given to indicate that procedures or conclusions may not be valid. In this category, statistics is described as procedures for handling data, which is a part of the statistics curriculum framework. With the curriculum as a background, recognizing the need for procedures to make sense of data are placed in the foreground by the participants.

Participants experiencing statistics in this way focused on specific tasks within the statistics curriculum, which included ways of collecting, displaying, and analyzing data. The procedures brought into the forefront were well established processes that served specific predetermined purposes. For these participants however, principles and procedures were not discrete entities, but seen as part of an overall approach to doing and understanding statistics. Understanding the research process and determining appropriate procedures are a part of the statistics curriculum as well (Garfield, 2002). As a result, when statistics is experienced in this way, its meaning is a course in which established procedures for collecting, displaying, and interpreting data are learned to summarize results within a specific subject matter.

Category 3. Statistics as Standards for Research.

The attributes of statistics described in Category 3 are inclusive of all the discernable parts in the two previous categories, along with the additions. Statistics as Standards for research was described as the study of testing and developing procedures to gather, analyze, and interpret data to depict tendencies in real world situations with the responsibility to report honestly and conduct studies ethically, with the goal of improving the world. Excerpts for some the journals of Angela, Sally, Aimee, and Tina are presented to support the description.

Journal Question:	What is your personal definition of statistics?
Angela:	My personal definition of statistics is the collection of data, whether quantitative or qualitative, the usage of that data within an equation, and most importantly, the application of both data and equation into real world situations with the responsibility to report honestly.
Sally:	I define statistics as the discipline of interpreting data and determining its relevance to our lives. People do studies to find out about our world, and statistics helps make sure that the studies are accurate.
Aimee:	My personal definition would have to be the numbering and categorizing of the world to understand it more…We want to know why

things are the way they are or gain an advantage…we learn about patterns in people whether it's attitudes, emotions, preferences, or tendencies knowing that if patterns can be found in an area, they can possibly be connected to patterns in another area. In the case of making predictions statistics can be used to foresee patterns that occur in nature and allow people to plan accordingly.

Tina: Statistics is a level of math[ematics] that is learned or developed to provide suggestions about important subjects. By using collected data and analyzing the data, different [things] can be proven. You can…present the data to prove or state inferences.

Although the participants only took an introductory course in statistics, the attributes they discerned represent statistics holistically, and are more aligned with the practices of the community of statisticians. By describing the most inclusive experience, the descriptions provided suggest that participants experiencing statistics in this way are close to *statistical enculturation*. The process of enculturation refers to entering a community (or practice) and picking up the community's point of view (Resnick, 1988; Schoenfeld, 1992). *Statistical enculturation* regards the statistical thinking of statisticians, with its own values and belief systems and its habits of questioning, representing, concluding, and communicating about data (Ben-Zvi, 2004).

The parts of the statistics curriculum the participants tended to focus on in Category 3 are methods, knowledge acquisition, validity, and ethics. Employing valid procedures to collect and analyze data for the purposes of acquiring knowledge about populations and making informed decisions were both very strong themes. By the descriptions of this experience, statistics is defined as the professional practices of statisticians which include the norms and standards for quality research as well as the rules and standards for making inferences and decisions for the purposes of improving processes and seeing the world in a different way. Table 2 summarizes the structural and referential parts of the presented categories.

From the "what" aspect of learning, three definitions of what statistics is understood to be or mean emerged from the group. A second aspect of learning must now be considered. The "how" branch of learning in the anatomy of awareness looks at the act of learning and the indirect object of learning (motive or capability sought). The interview and journal questions that addressed this aspect of learning were used to investigate the second focus question of the study: what are the different ways high school students approach learning and what capabilities do they expect to gain by learning statistics?

The "How" Aspect of Learning

Capabilities Attained by Learning Statistics

The indirect object of learning refers to the capabilities participants attained or sought to attain after completing the statistics course. During interviews, participants were asked to explain how they knew when they had learned something. They responded by describing skills, tasks or actions they were able to perform that revealed to them that something had been learned. The categories are in order according to their inclusiveness. For example, a participant stating that learning statistics meant being able to understand its concepts included memory and recall as pathways to attaining the goal of under-standing. The descriptions of the categories were analyzed to come up with ways of

Table 2

Ways of Defining Statistics

	Experience	Structural Aspect		Referential Aspect
		Background	Foreground	
1.	Maintaining basic mathematics skills	Statistics curriculum	Algorithms	Evaluating expressions, solving equations, and graphing
2.	Procedures for handling data	Statistics curriculum	Techniques for collecting, representing, and analyzing data	Using established procedures to collect and interpret data
3.	Professional standards of research	Statistics curriculum	developing research methods, knowledge acquisition, quality of life, and ethics	A set of norms for quality research and a set of evolving rules to inform decision making

experiencing the indirect object of learning. The structural and referential components of skills or capabilities gained after learning statistics are summarized below.

Category 1. Learning Statistics as Being Able to Recall Facts and Procedures.

This conception of learning statistics was described as being able to memorize and recall basic terms, principles, formulas, and procedures. Category 1 was mainly identified as a pathway to attaining more sophisticated capabilities more so than an end goal of learning. Another interesting comment made by the participants was that learning had occurred when concepts could be recalled after a long period of time. So for this capability, longevity was of great importance as opposed to memorizing or recalling concepts for more immediate response such as on a test or quiz. Others described the speed at which a concept could be recalled or task performed as qualifiers for justifying learning. A portion of Sheldon's response was used in developing this category because he clearly distinguishes memorization and recall as a low level capability of learning. His complete response is given in Category 4.

Interviewer: How do you know when you have learned something?
Sheldon: …when I can recall it a substantial time after it has been taught to me or, in the case of something learned at school, after I have been tested on it…I know when I have memorized something will enough to regurgitate it…

Interviewer: How do you know when you have learned something?
Aimee: I can bring it back from memory a long time after learning it.
Interviewer: It's been about a year since you took statistics. What tells you you've learned it?
Aimee: …now when I read statistics and see certain words…I know what they mean and I question where [the results] came from. I don't put my total trust in articles anymore because I remember what to question. This class has made me more aware of the inner workings of statistics.
Interviewer: So finish this statement for me, learning statistics means being able to…
Aimee: If you know statistics you can read and understand the terms on a pretty basic level…and like if you have a school assignment that involves statistics you remember the steps and equations and how to do surveys.

Interviewer: How do you know when you've learned something?
Rebecca: …I have truly learned something…if I can perform this new activity without hesitation.
Interviewer: By activity do you mean a problem or process?
Rebecca: Yeah. If I still have to pause and think about the steps or whatever it is I just learned, I know I don't actually know it yet.

From interview and journal statements grouped into Category 1, learning statistics results in being able to remember or have a basic understanding of terms and procedures after a long period of time. To reiterate, however, the statements when placed back into their original context do not indicate that these capabilities are the only ones sought by the participants. In fact, the skills sought by the participants for learning statistics coincide with their personal goals for being successful in the course.

As a result of Category 1, when a student's goal of learning is to recite information and duplicate procedures, then after learning statistics the student will be able to state statistical terms and principles as well as perform routine statistical tasks. Information or tasks deemed important by the student are held into view against the backdrop of the course curriculum.

Category 2. Learning statistics as being able to understand when, how and why concepts are used.

Understanding the meaning of statistical terms and principles, as well as understanding when, how, and why statistical formulas and procedures are used summarizes the description of Category 2. Describing the capabilities attained after learning statistics in this category focused not only on remembering concepts but having a deeper understanding of their function, relationships, and meanings. While this could have been implied by the participants in Category 1, the depth of elaboration in the description of understanding were more explicit in Category 2. Understanding, in Category 2, not only implies recalling or knowing meaning, but requires more awareness such as recognizing the context for which formulas are applied and a technical awareness of how the formulas work.

Interviewer:	How do you know when you've learned something?
Cara:	…when I fully understand how the conclusion was brought about and can reproduce the techniques used.
Interviewer:	How do you "fully understand" something?
Cara	…I know I paid attention to it and…can re-use methods to understand how they are used.

Interviewer:	How do you know when you've learned something?
Sally:	When I've learned something, I can remember it for an extended period of time…If it's a task, I can repeat it. When looking over my notes, I am simply reviewing and not having to re-learn the information.
Interviewer:	So after a person takes a statistics class, what kind of things should they be able to do if they actually learned it?
Sally:	[They] can probably read better than they can create…[they] have a basic understanding but need review on doing their own research or statistical tests. But if totally learning it means being able to teach someone else how to manipulate data and such, I guess I didn't really learn it.
Interviewer:	I think you learned quite a bit. Like, let's see. Well, what kind of statistical things can you do?
Sally:	…we read a lot of articles in college and most of them are very complicated…they can say they used a two-tailed t-test to compare the mean of the control to experiment, and I know basically what that means…when a study says that some results are not significant, I know that they actually tested it…statistics is mostly about what method to use and knowing how to use it.

Cara emphasizes the importance of understanding not only what the concepts are (meaning gained by paying attention), but also by knowing how methods are used and how conclusions are drawn from the results of statistical procedures. In Sally's statements, understanding implies knowing meaning and not just recall, as well as knowing what method was used, and why it was used. For both participants, the focus of learning is on assigning meaning and developing a technical understanding of procedures and formulas.

When the aim of learning is to understand information and procedures, as depicted in Category 2, then a student who has learned statistics will be able to establish relationships among statistical concepts in order to assign meaning as well as develop an understanding of how and why routine tasks are performed. Once again the framework from which the focus is drawn is learning the course curriculum, but unlike the first experience, the student's intention for learning is to find connections among the objectives.

Category 3. Learning statistics as being able to relate statistics concepts to life experiences.

Being able to relate statistical concepts was described as being able to apply and relate concepts within the context they were learned as well as to personal life experiences. Another attribute described in the category was being able to consolidate existing knowledge, but there was no mention of using that knowledge to formulate new theories. Category 3 was derived for the purpose of addressing a particular statement in Tina's interview. While her collective record of data could be appropriately placed in Category 4, Tina's discussion about concepts of statistics being connected like building blocks, illustrate her conception that statistics is an integrated process leading to a way of thinking. This new way of thinking is the goal of learning she seeks. Sally, Sheldon, and Angela describe similar capabilities in their journals, but they do not mention the re-structuring of existing knowledge, as Tina does. Being able to apply and relate statistical concepts to life encounters were discussed in the journal excerpts presented earlier in the chapter. These statements were also used to develop a description of Category 3.

Interviewer:	How do you know when you've learned something?
Tina:	I base how well I understand something on how I use it.
Interviewer:	So how do you use statistics?
Tina:	Everyday I unconsciously use statistics to prove hypothesis, tests, and judgments…while registering for classes I used the [statistics] of

	each…professor to determine which courses I would take. I would have just chosen by word of mouth if I hadn't learned [statistics].
Interviewer:	So learning something in statistics means you are able to use it elsewhere?
Tina:	…you should be able to apply everything you learn. Each step is a component of what is to follow. Everything you learn in statistics should be stored for [later] use.

While Tina's identification of a technical understanding of formulas is not explicitly stated, knowing the meaning of terms and concepts are necessary in being able to apply what is learned. Tina acknowledges her heavy reliance on existing knowledge to gauge if learning has occurred. Her method of self regulation helps organize statistical concepts in relation to situations that arise. The capability Tina seeks from learning statistics if being able to continually build, disassemble, and rebuild her structured blocks of understanding. By experiencing learning statistics as an integrated process of thinking, Tina not only organizes her knowledge, but she consolidates her existing knowledge and relates what she knows to her life.

When the goal of learning statistics is being able to relate and integrate concepts with real life situations, finding meaningful applications are held in focus as the motivation for learning. Learning statistics means being able to authenticate concepts of statistics by relating them to real life for the purpose of refining an existing structure of knowledge.

Category 4. Being able to uncover attributes of other things using statistics.

The conception of learning statistics as being able to uncover attributes using statistics was described as adapting and generalizing concepts of statistics with situations outside of the context in which they were learned. Another attribute of the category was being able to create new knowledge and formulate new theories as a result of statistical thinking. Similar to Category 3, gauging learning by how concepts are used outside of the setting in which they were learned was still a strong theme that emerged in Category 4. However, the subtle difference is the extra lengths taken by the participant to adapt statistical concepts to situations with higher complexities to those that were described in previous sections. While the structure of the participant's knowledge is continually restructured to consolidate knowledge, other capabilities evolved from this way of thinking. For example, applying statistics concepts to choose a course professor, as Tina did, means statistics is used for decision making. Using statistics principles within an unfamiliar context as a tool for critical thinking or to uncover characteristics of an unfamiliar phenomenon are skills that can develop from thinking for decision making. Portions of Angela's and Sheldon's interviews support this point.

> Interviewer: How do you know when you've learned something?
> Angela: …when I can apply it outside of class or the learning experience.
> Interviewer: …what do you mean by that?
> Angela: If I can use any information I have been taught and apply it to a different situation with some ease or without reviewing, then I know I've learned something.
> Interviewer: So what kind of things should you be able to do if you learned statistics?
> Angela: The part that I will take away with me from statistics is the critical thinking that was enforced during my class. I think that…the most important thing I was taught was to not just accept information and that thought can be applied to whatever I do in the future to make sense of stuff I don't understand.
>
> Interviewer: How do you know when you have learned something?

Sheldon:	…when I can recall it a substantial time after it has been taught to me or, in the case of something learned at school, after I have been tested on it…just passing a test doesn't mean you actually learned something, though…I know when I have memorized something will enough to regurgitate it…A true test of what I really and fully understand is when [I] am reminded of something outside of the classroom that was learned at school…and I remember the details.
Interviewer:	How would you say you use statistics outside the classroom?
Sheldon:	One thing I've especially noticed on a day to day level is the precautions taken on officially reported [statistics]. I am more likely to notice how many people are surveyed, or…I might question precautions that are not taken…I notice more in depth details about things from the statistics of others.

Angela describes her capabilities as critical thinking skills that help her analyze and understand research or statistical statements. Similarly, Sheldon mentions skills in which he is able to uncover attributes of a phenomenon that are not so apparent articles or research he reads. In addition, Sheldon provides his perception of a hierarchy of skills by contrasting memorization with being able to apply what he learned. Both Angela and Sheldon include the capabilities attained from learning statistics as mentioned in previous categories, with the additional skills of adapting their thinking to unfamiliar situations for the purpose of making inferences and forming theories.

Based on descriptions from the fourth category, when the goal for learning is to develop analytical or critical thinking techniques, then after learning statistics, the student should be able to adapt and synthesize concepts of statistics with objects experienced in their surroundings to uncover attributes and to formulate theories. This is the most evolved description of seeing the object or statistics in a new way because more explorative, adaptive, investigative aspects are held in focus within the environment. The structural and referential component of each category is listed in Table 3.

Table 3

The Indirect Object of Learning Statistics

	Experience	Structural Aspect		Referential Aspect
		Background	Foreground	
1.	Recall information and duplicate procedures	Learning statistics	Information or tasks regarded as important to instructor	State terms and perform routine task.
2.	Understand information and procedures	Learning statistics	Finding connections among course objectives	Establish relationships among course objectives to assign meaning to concepts and to develop an understanding of how and why routine tasks are performed
3.	Relate and integrate information and procedures	Learning statistics	Reflecting on how concepts of statistics have been encountered or how the concepts can be applied to objects of interest to validate or restructure understanding	Authenticate concepts of statistics by relating and integrating them with personal interest or real life to refine the existing structure of knowledge
4.	Adapt and empirically analyze surroundings	Learning statistics	Exploring how concepts of statistics can be applied and adapted to phenomenon encountered in surroundings	Adapt and synthesize concepts of statistics with objects experienced to uncover attributes, see the object in a new way, and formulate theories

In this section, four goals of learning were summarized to address the research question 2A. These conceptions of learning are consistent with those found Petocz and Reid (2002), as discussed in chapter 2.

The Approach to Learning Statistics

When the question of how learning takes place arises, it is not enough to simply respond to what the learner should be able to do, but it is equally important to address what activities or actions the student deliberately engaged in to gain their sought after skills. So in a sense, the goal of learning (capabilities sought) will influence the actions taken to learn. All of the participants named several activities or actions they undertook to learn statistics. Similar activities or actions were grouped, and are shown in Table 4. Responses from Rebecca, Cara, and Sally are shared to demonstrate the variety of responses as well as the commonalities among them.

Interviewer:	What kind of things help you learn?
Rebecca:	Usually visual demonstrations help me learn because I am very visually oriented….teaching someone else helps me learn because it forces me to break the subject matter down and look at it from various angles.
Cara:	Visual aids and in depth explanations…what I liked about the statistics class I took was the group work. That helped me learn because you have to hear and understand what other people have to say.
Sally:	Visual models help me learn…When information is interesting, I learn it much better. When the teacher gives good examples that relate to the material, I learn it well…writing down the information also helps me learn it.
Interviewer:	If a good friend of yours was taking statistics for the first time, what advice would you give them about being successful in the class?
Rebecca:	Like any other math class, I believe statistics takes plenty of practice. Also, I found that it was easiest to understand statistics when it was specific to my interest…for example the final project where I picked my subject to study helped me make sense of a lot of what we did in class.

Cara:	Pay attention, do all the homework, do the group work, go to tutoring,…understand how the techniques work so you can do good on your projects.
Sally:	…pay attention in class and try to relate everything to real life…practice the techniques over and over again.

The categories of description for the approach to learning statistics are summarized in Table 6. In the first approach to learning, Observation, the participants' descriptions revealed a structural aspect in which techniques for inspecting and recording events were held in view against the strategies for learning, which is the background of the experience. Observational techniques such as watching activities and interactions, seeing numerous examples, taking field notes, and listening to instructions helped to focus on what needed to be learned. By this structure, learning by observation is interpreted as using observational techniques such as listening to instructions, recording information, and watching events within the classroom to direct attention to presented concepts.

The participants also described approaches to learning they had customized to suit their personal preferences for reaching their goals. For the most part, these practices occurred outside of the classroom. The second approach to learning, study skills, uses as its foundation the student or individual. Introspectively, the individual tends to focus on aspects of their academic history, which includes study habits or routines that have lead to successful or desirable academic experiences in the past. Therefore in this context, learning by using study skills means using personalized strategies for learning and review outside of the classroom setting that have proven successful in reaching academic goals.

Table 4

Grouped Learning Strategies

Type		Description
1.	Isolated practices	A) Practice, taking notes, studying, reading articles or the textbook, doing challenging problems, doing homework
		B) visual demonstrations, seeing various relevant examples, paying attention
		C) Making connections between concepts and things of personal interest or things in the real world, individual projects in which the topic is chosen by the student
2.	Social Interaction	D) Tutoring or teaching a peer
		E) Being tutored or taught by a peer
		F) class discussions, group work, group projects in which the topic is chosen by the group

Note. Strategies under Type 1 are isolated approaches to learning the participants described using when working alone. Strategies in which the participants described working with a peer or group were classified as Social interactions.

Another approach to learning that emerged from the categories is Reflection. Similar to study skills, reflection is an individual preference. However, with Reflection the individual focuses on making connections between the presented content and personal experiences. When reflecting, the learner compares content with experience to find relationships that help build conceptual understanding, with the intent of authenticating learning. Therefore, when learning through Reflection, the individual will pay close attention to data that is presented in context, the multiple representations of concepts, and examples that are relevant to personal interests. Learning by Reflection is thus taken to mean seeking connections between personal experiences and course content to build a deeper understanding of concepts.

Learning by Self regulation is another individual preference for learning. What surfaces from within the individual is an awareness of the need for personal practices to evaluate and assess conceptual understanding. Therefore, to approach learning through self regulation means to learn by formulating strategies that assess and evaluate personal understanding.

Approaching learning by One-way exchanges of information occurs when the student interacts with one peer or the instructor. The focus of learning is on either the conceptual understandings of the individual or that of a peer. Only one is held into focus at a time, depending on whether or not the individual is providing academic support or receiving support. For example, in the strategy for learning, Tutoring or being tutored, disseminating or receiving information was seen as one way, mainly because the interaction between two students is such that the weaker student is given additional instruction by being paired with the stronger student. For the most part, only one student is learning, while the other provides academic support. The individual tends to prefer student centered activities for individualized instruction such as teaching someone else or being tutored by someone else. Thus, the one way exchange is an approach to learning in which the individual learns by helping someone else or by receiving help from someone else.

The final approach to learning that emerged from the data is Collaboration. The social interactions for learning may occur in the classroom or outside of the classroom with two or more peers. What tends to resonate from this activity of learning are student centered activities involving group learning in which evidence of the depth of understanding is determined by a product of work produced by the group. The focus then is on the quality and meaningfulness of social interactions as well as the group's goal for learning in order to complete the task at hand. Ideally the individual benefits from teaching and learning from peers. Therefore, to learn by Collaboration means working with others to investigate or uncover attributes of concepts and to produce a product of work to demonstrate conceptual understanding. The referential and structural components of the approaches to learning described by the participants are listed in Table 5.

The focus question regarding the various ways in which statistics is learned has been addressed by analyzing the skills attained and by analyzing the approaches to learning. These parts are combined with the meaning of statistics to construct the outcome space of the experience, learning statistics. By doing so, the final focus question of the study is investigated.

The Outcome Space Learning Statistics

The final focus question of the study, how can the experience of learning statistics be described, is answered phenomenographically by reviewing the structure of the experience and finding relationships within the outcome space. Figure 4 illustrates the structure of the experience, learning statistics, which identifies each conception along with its structural and referential aspect discussed earlier in this chapter. From the phenomenographic perspective, it is important to remember that every conception is not accounted for in the structure, and Figure 4 only represents the collective experiences of group in the study.

Table 5

Anatomy of the Approach to Learning Statistics

	Experience	Structural Aspect		Referential Aspect
		Background	Foreground	
1.	Observation	Learning strategy	Observational research techniques	Listening to instruction, watching events, and recording information in the classroom
2.	Study Skills	Learning strategy	Individual academic history of successful study habits or routines	Using personalized study skills outside of the classroom that have proven successful in the past
3.	Reflection	Learning strategy	Data presented in context, multiple representations of concepts, examples relevant to personal interest	Considering how personal interest and experience relate to concepts and skills being taught
4.	Self regulation	Learning strategy	Individual inventory knowledge, test depth of understanding	Personal assessment and evaluation of knowledge
5.	One-way information exchange	Learning strategy	Individualized delivery or receipt of instruction	Assisting a classmate or receiving help from a classmate
6.	Collaboration	Learning strategy	meaningful social interaction, productive collaboration, group learning goals for product construction	Working in collaborative groups to investigate concepts or produce a product of work; assisting and being assisted by peers and teacher.

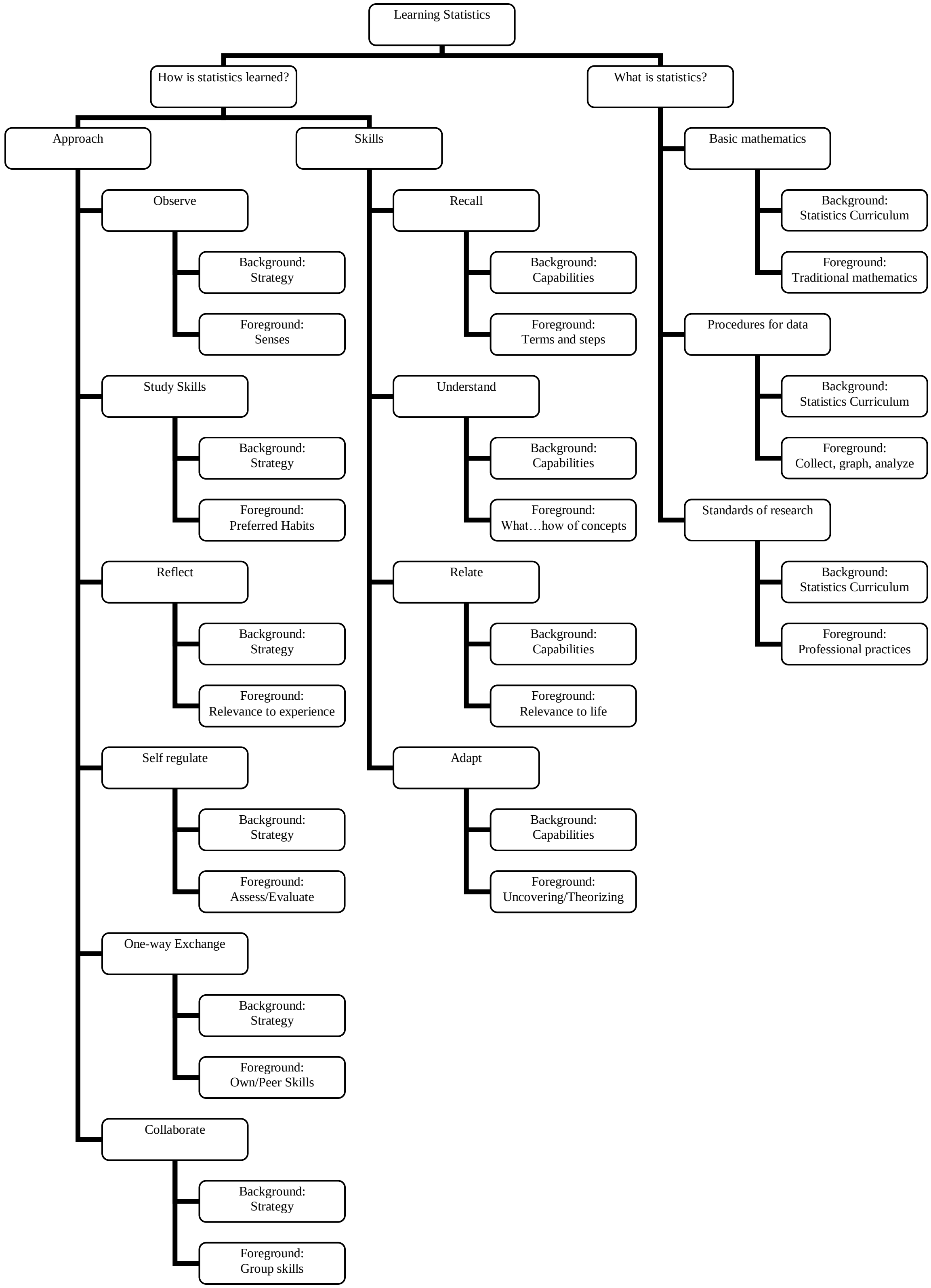

Figure 4. Structure of the Outcome Space, the Experience of Learning Statistics.

The relationships among the ways of experiencing statistics will vary. For example, ideally a student who learns statistics through a collection of learning strategies, say study skills, reflection, and collaboration, will be able to analyze their surrounding empirically because to them statistics is seen as professional standards of research. Or along the least inclusive realm, a student who learns statistics by mere observation will be able to state and perform tasks because to this student statistics is seen as a course in which basic mathematics skills are maintained.

The outcome space is not so rigid, however as to suggest a lock and step form of navigation. Therefore, a model is needed to explain the relationships among the components of the learning experience [see Figure 5]. It is conceivable that a student using fewer strategies for learning will attain a surface level understanding and define statistics based on one of the less inclusive categories. Take for example Andrew's collective data. Andrew described study habits and observation as strategies for learning, which were classified as isolated approaches. When asked to provide a personal definition for statistics, Andrew described statistics as a course in basic mathematics skills, constituting a surface level approach to learning.

In contrast, it is expected that a student incorporating multiple strategies of learning will attain a deeper understanding and define statistics in more inclusive ways. The depth and sophistication of Sheldon's descriptions support this notion. Sheldon described numerous learning strategies to help him reach an adaptive level of learning. He defined statistics as procedures for handling data.

A similar analysis was conducted on all the participants, in which their learning strategies, conception of learning, and meaning of statistics were identified and grouped for comparison. Although variations occurred, an overall pattern was discernable, and Figure 5 shows the relationships among the components of the learning experience. While the Approaches to Learning were presented as a hierarchical structure in the outcome space, in Figure 5 an emphasis is not placed on inclusiveness, but on the incorporation of multiple strategies. As students incorporate multiple strategies, and as those strategies require more complex thinking, higher levels of learning and a more inclusive definition of statistics are anticipated. More research is needed in regards to which combinations of strategies tend to produce certain levels of learning and conceptions of statistics. Another point for future research is to determine the breadth of each conception of learning. For instance, if a student's description of learning falls in the Relate category, what is the expected range of their description for the definition of statistics? Hypothetically, the variation in definitions covers all of the descriptions, but perhaps by empirical investigation the lowest category could be eliminated. This would indicate that if instructors wanted students' understanding of statistics to fall between Procedures for handling data and Standards for research, then multiple modes of instruction and differentiated learning opportunities for students must be included when structuring the course.

Conclusion

The analysis of data collected in the study has been presented to justify and validate the results. The nine participants were given the opportunity to verify their responses, my interpretations of their responses, and to provide feedback, for the purpose of member checking. Journals and transcribed interviews were coded and categorized using an iterative process. The categories of description and the outcome space were

Conceptions of Learning		Conceptions of Statistics		
		Basic skills	Procedures	Standards
Approaches to Learning	Observation			
		Recall		
	Study Skills			
			Understand	
	Reflect Regulate			
				Relate
	One-way			
	Collaboration			**Adapt**

Figure 5. Relationship between the "How" and "What" aspects of learning statistics.

derived using phenomenographic methodologies outlined by Marton and Booth (1997). Relationships among the components of the experience, learning statistics, were established using codes based on participants' responses.

Briefly summarizing the results, based on the descriptions of the participants in the study, the outcome space contains a multitude of combinations that constitute the aspects surrounding an individual's experience of learning statistics. The model [Figure 5], based on the data analysis, is representative of the various ways the collective group in the study experienced learning statistics. What is also being suggested in the model is that a student who experiences statistics as professional standards of research has delineated more discernable attributes of the subject than students experiencing it in less descriptive ways, and that perhaps this student incorporated some of the more sophisticated approaches to learning and sought to attain more complex skills. As with all phenomenographic studies, the outcome space is by no means a complete set of attributes of the experience. Similar studies are expected to support and contribute to the outcome space that emerged. Further validation of the study in reference to how it relates to the literature in the field is provided in the final chapter.

CHAPTER 5

CONCLUSION

The goal of the study was to investigate secondary school students' experience of learning statistics by addressing five focus questions:

1. What are the different ways high school students define statistics? ("What" aspect of learning)
2. What are the different ways high school students learn statistics? ("How" aspect of learning)
   i. How do students go about learning statistics?
   ii. What skills and capabilities do students gain by learning statistics?
3. What are the qualitatively different ways students experience learning statistics? (Relationship between "what" and "how")

Evidence for how the methodology and results of the study addressed all of the focus questions were presented in detail in chapter 4. In concluding the research, I will summarize the study and findings, provide a critique of the study, and make suggestions for future research.

Summary of the Study

Nine of my former statistics students volunteered to participate in the study, which investigated their learning experience in the course. Taking into account the phenomenographic perspective of learning (Marton & Booth, 1997) and conceptions of statistics (Gordon, 2004; Petocz & Reid, 2002), I used journaling and an interview protocol to collect data on students' definitions of statistics, their definitions of learning, and their strategies for learning. By analyzing the responses of the participants through an iterative process, I identified categories of description for learning, meaning of statistics, and actions for learning which created a hierarchical structure or outcome space. I further analyzed descriptions within the outcome space to establish links between strategies for learning and conceptions of learning as well as links between conceptions of learning and conceptions of statistics. The links between variations and levels of sophistication established in the study formed a framework for the role that the experience of learning plays in the attainable levels of statistical thinking among secondary school students.

Summary of the Findings and the Connections to Literature

To investigate focus questions regarding the "how" aspect of learning, I analyzed journals and interview transcriptions to develop classifications for students' goals for learning and the actions they took to learn. The four conceptions of learning described in the study were Recall, Understand, Relate, and Adapt. These conceptions are capabilities participants said they gained or sought to attain when something had to be learned. Strategies for learning that participants described were Observation, Study skills, Reflection, Self regulation, One-way exchanges of information, and Collaboration. The hierarchical structure of the conceptions of learning coincided with the hierarchical structure of the strategies for learning. For example, the lower surface level goals of learning, Recall and Understand, were described to have incorporated the lowest strategies for learning, such as Observation and Study skills, with some variation of course. The in-depth goals, learning as being able to Relate or Adapt knowledge not only drew upon subordinate goals of learning for attaining these level, but also incorporated more sophisticated and inclusive strategies for learning, such as Self-regulation and Collaboration.

The findings regarding the link within approaches to learning statistics are consistent with learning and knowledge acquisition from the phenomenographic perspective. As I discussed in chapter 1, the theoretical framework underpinning the study defines learning as a change in the awareness of the discernable attributes about an object. The hierarchical structure in the conceptions of learning suggests that a student who sees learning as that described in the Adapt category possesses an elaborate awareness of the attributes of learning and how they are interconnected.

Participants expressed how surface level approaches to learning such as problems requiring simple answers about facts or requiring the use of procedures were best learned by observing and practicing numerous examples, which lead to levels of learning such as Recall and Understand. According to the conceptual framework, the student did not engage (or exhibited low levels of engagement) in challenging situations or experiences that required the learner to re-conceptualize the set of attributes and their relationships about the phenomenon. In contrast, when learning was categorized as Relate or Adapt, the participants contributing to these descriptions stated they encountered and actively engaged in complex learning activities such as challenging cognitive tasks (non-routine problems), projects, investigative group work, and open-ended questions. Therefore, as encounters with the phenomenon grow in complexity, a greater need for flexibility in learning strategies exists in order to understand the interrelatedness of complex concepts thereby reconstituting the phenomenon.

Although the participants were exposed to generally the same instruction and teacher, the final set of discernable attributes for each participant supports the phenomenon finality component in the conceptual model, explaining the finite number of ways an individual can describe learning and the object of learning (Marton, 1986; Uljens, 1996).

After data were analyzed to address the "what" aspect of learning, I identified three conceptions of statistics described by participants (Basic mathematics skills; Procedures for handling data; Professional standards of research). The narrowest interpretation, statistics as basic mathematics skills, focused on the procedures, formulas and algorithms of the course. The broadest interpretation, statistics as professional standards of research, is the most inclusive view. For example, understanding statistics as procedures for handling data revealed a more limited conception or experience than a focus on statistics as standards for professional research, which includes all subordinate levels of awareness and ultimately leads to the enculturation of statistics.

The three conceptions of statistics in the study are consistent with results of other phenomenographic inquiries on definitions of learning (Gordon, 2004; Petocz & Reid, 2002). For the study, understanding statistics to be basic mathematics skills is aligned with Petocz and Reid’s Doing concept and with what Gordon describes as students perceiving no meaning of statistics outside of its mathematical processes or algorithms. The second conception, understanding statistics as processes for handling data, is similar in description to Petocz and Reid's categorizations, Collecting and Applying, and parallels Gordon's classification of statistics being the mastery of statistical concepts and methods. The final and broadest conception, statistics as professional standards of research, include Linking and Expanding (Petocz & Reid), and it surpasses Gordon's two final descriptions, statistics as a tool for getting results in real life and statistics as critical thinking. The participants in the study included codes of ethics, the evolution of procedures, and seeing the world in a new way as part of the description for statistics as professional standards, while neither Petocz and Reid nor Gordon mentioned these notions.

The final focus question had to be addressed by formulating a framework to explain the relationship between the "what" and "how" aspects of learning. The framework is illustrated in Figure 5. With the conceptions of learning advancing along the diagonal, the progression in meaning assigned to statistics is shown horizontally. Similarly, more inclusive conceptions of learning tended to yield more sophisticated strategies for learning, which is shown vertically. Overall, Figure 5 aims to explain the interconnectedness of all three conceptions. Tempelaar's (2006) structural model depicts similar relationships, in that undergraduate students in the study who only sought to attain a surface level understanding of statistics demonstrated lower statistical reasoning skills than those seeking in-depth understanding. The statistical thinking framework of Ben-Zvi and Friedlander (1997) characterized four levels of thinking, but whether or not these stages were hierarchical had not been established. The results from Figure 5 demonstrate and support a hierarchical structure to conception of statistics in which students described their statistical thinking skills.

Critique of the Study

As noted earlier in the study, models are useful within the context from which they are derived, but they also serve as a way to develop a coherent vocabulary within the field and to explain theory by presenting tools or systems of classification (Brown, 1998). A model for the experience of learning statistics is never really complete, leaving room for further investigation and refinement. The study presented contributes to the broad framework of statistical experiences, particularly focusing on those of secondary school students. The model that emerged from the study represents a single case in the larger domain of research on how students learn statistics. Furthermore, the students' descriptions are snapshots of their experience. The conceptions that emerged are limited to the duration of the study, under the assumption that students' perception of their experience will change over time. The framework is generalizable to the extent that similar research on a larger sample of participants may yield similar conceptions as well as expand the outcome space to bring into view a holistic representation of statistical thinking.

Although the purpose of the presented framework is to explain relationships among meaning, learning, and learning strategies, a gauge of the range of each conception is unclear. For example, if a students' conception of learning is categorized Understand, then a variety of learning strategies preferential to the student can be assumed to meet this end, but I did not investigate the number and sophistication of the learning strategies in the study. Furthermore, if the course is structured based on guidelines for assessment and instruction (GAISE, 2005), the same student should at least reach a level in which statistics is seen as procedures for handling data. The range of understanding in procedures for handling data was not investigated in the study because an assessment instrument on statistical thinking and a learning styles inventory was not administered to participants. In future research, I intend to deepen my understanding of learning strategies and their association with the range of capabilities and the range of assigned meanings so that a focused identification of the origins of statistical thinking and reasoning can be made.

Implications for Educational Research

The significant contribution of this research is the opportunity it provides to reflect on the factors affecting ways students experience statistics. Teaching statistical thinking is difficult (Chance, 2002; Garfield & Ahlgren, 1988; Tempelaar, 2006), but researchers emphasize the importance of providing opportunities for students to engage in investigations that encourage them to integrate contextual and statistical information to aid in its development (Langrall & Nisbet, 2006; Pfannkuch & Wild, 2004).

Based on the ways students conceptualize their learning, multiple approaches to instruction and assessment seem to encourage students to incorporate strategies for learning conducive to their learning style. In light of the relationships established in framework, the study re-emphasizes the importance of learning support for students from educators in courses where the content is perceived to be challenging. Participants in the study with the lowest conceptions of statistics or learning were still able to pass the course mainly because of learning activities they found useful, differentiated instruction and assessment, awareness of teacher expectations and their goals for learning.

Although tools of instruction and assessment are recommended based on educational principles of student centered learning environments (Chance, 2002; Moore, 1997), the presented study does not suggest diminishing students' responsibility to organize their own learning processes. In fact, it is recommended that students be more attuned to their learning styles, and that they take responsibility in planning their strategies for reaching their learning goals, aligning them with course expectations. The presented study reiterates that curriculum, course structure, the student, and the educator are integral parts of achievement in statistics (delMas, Garfield, & Chance, 2002).

In all, the study implicates a further need for reflection on ways to help students learn complex statistical concepts. The progression of learning and ways of defining statistics presented in the study demonstrate that students are capable of acquiring knowledge about challenging concepts that are difficult to teach. Based on the participants' descriptions of what they felt were components paramount to their success in statistics, I provide the following suggestions for the classroom:

1. Instructors should clarify their expectations on all assignments by presenting multiple and relevant activities or examples that illustrate their perceived levels of acceptable performance.
2. Instructors should use multiple pedagogical techniques and multiple representations of statistics concepts when teaching a unit.
3. Instructors should consider using statistics learning activities that develop a functional conceptual understanding during class time.
4. Instructors should require that students take responsibility for their own learning but balance the requirement with learning support.
5. Instructors should strive to maintain a student-centered learning environment that emphasizes participatory learning and is managed such that all students maintain a medium to high level of engagement.
6. Assignments and tasks should use real data from research or generated by students.
7. Students should be required to conduct a study on a topic of their interest.

I have adopted these recommendations in my own practices and find them effective in helping students meet my performance expectations in statistics.

Suggestions for Future Research

Because statistical thinking is difficult to teach, more research on its development is needed. A good place to start is perhaps a meta-analysis of research on existing statistical thinking frameworks. By cross-referencing existing systems of classification, a holistic view of statistical thinking and its integral stages can be extracted to aid in pedagogical research as well as research on the development of assessment instruments.

Aside from defining frameworks of statistical thinking, other factors that may enhance or impede its development must be investigated. A holistic model of statistical thinking must also address affective characteristics. Numerous studies on the role attitudes toward statistics play in course achievement have been done in recent years, showing that motivation is a prominent indicator of success (Lee & Meletiou, 2002; Tempelaar, 2006; Williamson & Mattiske, 2002). However, additional research on attributes, such as learning styles, instructional practices, student's expectation of teachers, and the instructor's expectation, are needed to determine whether relationships exist among other mitigating factors of statistical thinking development.

One further suggestion for future research involves comparing student's existing mathematical knowledge and level of mathematical thinking to their statistical understanding. The roles of contextual and mathematical knowledge in the development of statistical knowledge warrant inquiry to help understand the origins of a student's misconceptions and fallacies about statistics. The research would also contribute to determining how students acquire knowledge about complex or abstract concepts.

Final Words

It is my hope that the study will be viewed by the research community as theoretically anchored research on statistics education. I consider the study a significant accomplishment, perhaps the beginning of many contributions I will make to educational research. I encourage others to take up similar research to advance the debate concerning good teaching and learning in statistics, as such research is always in the best interest of students.

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APPENDIXES

APPENDIX A

ACTIVITY THEORY

Activity theory is a form of socio-cultural-historical analysis that contends "[...] all human activity is embedded in a social matrix composed of people and artifacts, [and] ... all human activity evolves over time and is distributed among individuals and their cultures" (Jonassen, 2000, p.97). Figure 1A is an illustration of an activity system.

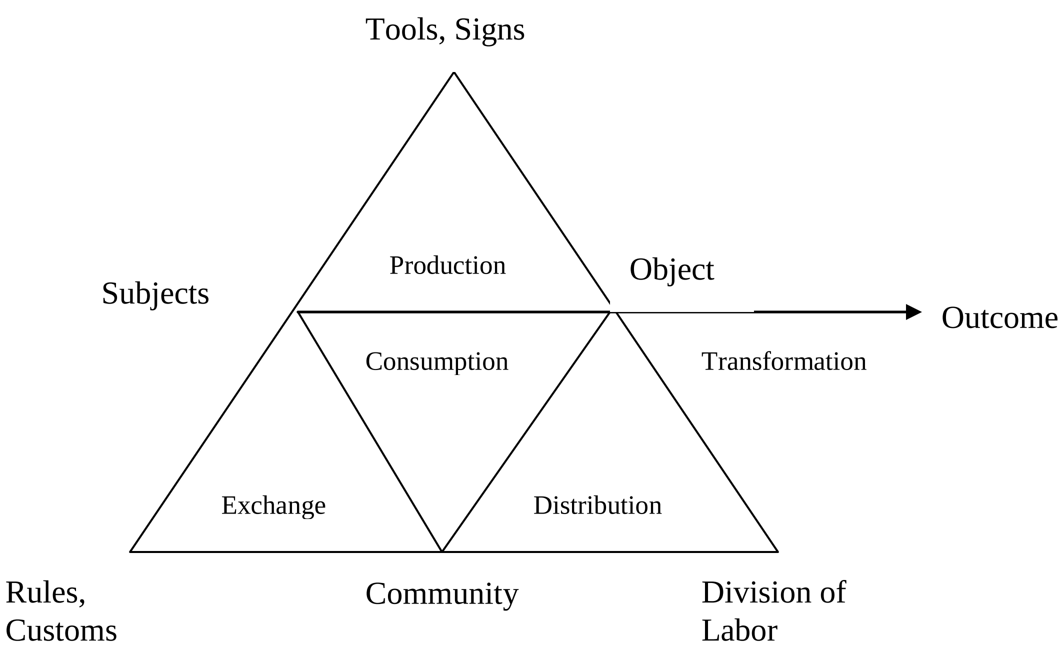

Figure 1A. Activity system (from Johassen, 2000, p.99)

The *subject* of any activity is the individual or group engaged in the activity. The *objects* are artifacts (physical, mental or symbolic article acted on) that are produced by the system. The *transformation* of the object into the outcome represents the purpose or intention of the activity. *Tools* and *signs* are the means the subject uses to act on the object. *Rules* and *customs* refer to the explicit regulations, laws, policies and conventions that constrain the activity as well as the implicit social norms, standards and relationships among members of the community. The *community* represents individuals and groups that focus some amount of effort on the object. The *division of labor* is the division of tasks between cooperating members as well as the division of power and status. The *outcome* is the result of the activity.

The *production* subsystem analyzes the interactions between the subject, tools or signs, and the object of learning. The *consumption* subsystem describes how the student and community collaborate to act on the object. The *exchange* subsystem regulates the interactions of the system in terms of the student's personal needs. It engages the student with the community under the constraint of rules that govern the activity. The *distribution* subsystem ties the object of the situation to the community by defining the division of labor.

APPENDIX B

QUESTIONNAIRE AND CONSENT FORM

Questionnaire

Your Name:____

Pseudonym:____

Age:_____________ Gender:________

Summer Mailing Address:____

____

Home Phone#:______________________ Cell Phone#__________________

E-mail____

How would you prefer to be contacted:(Rank from 1=most preferred to 4=least preferred)
_____ E-mail _____Cell Phone _____ Home Phone _____ Mail

Highest level of education completed:____

Year and Semester you completed a high school statistics course:___________________

Post-secondary plans (circle one):
College Military Technical School Work Undecided

- If you circled College or Technical School, what is your major (or anticipated major)?

- If you circled College or Technical School, what are your career aspirations? If unknown state as such.

- If you circled Military, what occupational skill will you receive (or would like to receive) training in? If unknown state as such.

- If you circled Work, what type of work do you (or will you) do?

APPENDIX C

REFLECTION JOURNAL ENTRIES

Using the format in **Bold** print, write a reflective response to each question (about 15 – 20 minutes per question). You may answer one per day or answer a couple at a time. When your journal is complete, mail or email your responses to K. Gardner (principle researcher). Thank you for your time.

Your Code Name_________________________________ Date___________
Type the question.
Write your reflection underneath each question.

Journal Entry:

Day 1: What was learning statistics like?

Day 2: What is your personal definition of statistics?

Day 3: Do you consider yourself statistically literate? Explain.

Day 4: How did you go about interpreting data before you took a statistics course? How do you go about interpreting data and statistics on a daily basis now?

Day 5: How do you see yourself using statistics in the future (work, college, in everyday life)?

APPENDIX D

INTERVIEW QUESTIONS

How do you know when you have learned something?

What kind of things help you learn?

If a good friend of yours was taking statistics for the first time, what learning and study strategies would you advise them to use to be successful in the course?

In the Reflection Journal, you were asked if you considered yourself to be statistically literate. Actual definitions have been provided below. For each category, rate how well the definition fits your skills and elaborate a little on why you feel this way.

Definitions of Statistical Literacy, Statistical Reasoning, and Statistical Thinking

Statistical literacy
Statistical literacy involves understanding and using the basic language and tools of statistics: knowing what statistical terms mean, understanding the use of statistical symbols, and recognizing and being able to interpret representations of data.

I am not statistically literate	I am average	I am statistically literate

Statistical reasoning
Statistical reasoning is the way people reason with statistical ideas and make sense of statistical information. Statistical reasoning may involve connecting one concept to another (e.g., center and spread) or may combine ideas about data and chance. Reasoning means understanding and being able to explain statistical processes, and being able to fully interpret statistical results.

I do not have statistical reasoning skills	My statistical reasoning skills are average	I have strong statistically reasoning skills

Statistical thinking
Statistical thinking involves an understanding of why and how statistical investigations are conducted. This includes recognizing and understanding the entire investigative process (from question posing to data collection to choosing analyses to testing assumptions, etc.), understanding how models are used to simulate random phenomena, understanding how data are produced to estimate probabilities, recognizing how, when,

and why existing inferential tools can be used, and being able to understand and utilize the context of a problem to plan and evaluate investigations and to draw conclusions.

I am not a statistical thinker	I am an average statistical thinker	I am a strong statistical thinker