TITLE: Dynamics of traveling waves in neural networks in presence of period inhomogeneities

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Background: The study of traveling waves of activity in neural tissue can provide deep insight into the functions of the brain during normal physiologic sensory processing and pathologic states such as epilepsy, migraine headache and hallucination. The experimental models widely used to study these phenomena are brain slices where spontaneous or induced propagation of neural activity is monitored using a variety of optical imaging techniques, such as calcium imaging and optical imaging dyes. Computational models of the neural tissue are typically simulated as a vast interconnected network of simplified neuronal units that interact strongest with the nearest neighbors. These assumptions can be used to formulate a set of integral-differential equations for the neural network, a system which support the propagation of constant-speed traveling wave fronts. We are interested in examining how local inhomogeneities in synaptic connections, likely to exist in the brain tissue, modulate the propagation of activity over longer spatial scales, an issue that has received less attention in computational models.

Methods: Previous work from Osan lab indicates that for neural network comprised by homogenous populations of integrate-and-fire neurons connected by exponential kernel excitation, the single-spike activity propagation the local acceleration of the traveling wave depends only on the instantaneous speed: 

$$a = \frac{1}{\tau_1 \tau_2} \left( \frac{g}{2V_r} \tau_2 c - (1 + \frac{c \tau_1}{\sigma})(1 + \frac{c \tau_2}{\sigma}) \right).$$

We extend these results to non-homogenous excitatory kernel functions: 

$$J(x, y) = J(|x - y|, x) = \frac{e^{\frac{|x - y|}{\sigma}}}{2\sigma} + K(x)$$

where $K(x)$ is a period function, for example: $K(x) = e^{-\epsilon \cdot \cos(\omega \cdot x)}$, with $\epsilon$ typically being a small parameter.

Results: The change in the connectivity function affects the dynamics of traveling waves as follows:

$$a = \frac{1}{\tau_1 \tau_2} \left( \frac{g}{2V_r} \tau_2 c(1 + \epsilon \cos(wx)) - \left(1 + \frac{c \tau_1}{\sigma}\right)\left(1 + \frac{c \tau_2}{\sigma}\right)\sigma \right).$$

Using the assumption that speed is a constant plus a periodic component: $c = c_2 + \epsilon \cdot C \cdot \cos(wx + \phi)$, valid for small $\epsilon$ we compute the magnitude $C$ and phase $\phi$ of the perturbation. These results are in perfect agreement with results from numerical simulations (shown in the figure to the right).

Conclusions: Our results are a first step in understanding how inhomogeneities affect the dynamics of traveling waves in neural tissue. These results can contribute toward building a framework to understand the activity propagation from experimental preparation that does not have a linear, constant-speed behavior.

Future directions: We plan to extend our results to higher order terms in $\epsilon$, in order to obtain series of approximations that describe the propagation dynamics more and more accurately. These results can then be used to understand more difficult aspects of these dynamics, such as propagation failure.