

Georgia State University

ScholarWorks @ Georgia State University

Accountancy Faculty Publications

School of Accountancy

1998

Using Analysts' Forecasts to Measure Properties of Analysts' Information Environment

Orie E. Barron

Pennsylvania State University - Main Campus, oeb1@psu.edu

Oliver Kim

University of Maryland at College Park, okim@rhsmith.umd.edu

Steve C. Lim

Texas Christian University, s.lim@tcu.edu

Douglas E. Stevens

Georgia State University, dstevens11@gsu.edu

Follow this and additional works at: https://scholarworks.gsu.edu/accountancy_facpub



Part of the [Accounting Commons](#)

Recommended Citation

Barron, Orie E., Oliver Kim, Steve C. Lim, and Douglas E. Stevens. 1998. "Using Analysts' Forecasts to Measure Properties of Analysts' Information Environment". *The Accounting Review* 73 (4). American Accounting Association: 421–33.

This Article is brought to you for free and open access by the School of Accountancy at ScholarWorks @ Georgia State University. It has been accepted for inclusion in Accountancy Faculty Publications by an authorized administrator of ScholarWorks @ Georgia State University. For more information, please contact scholarworks@gsu.edu.

Using Analysts' Forecasts to Measure Properties of Analysts' Information Environment

Orie E. Barron

Pennsylvania State University

Oliver Kim

University of Maryland

Steve C. Lim

Long Island University—C. W. Post

Douglas E. Stevens

University of New Hampshire and University of Arizona

ABSTRACT: This paper presents a model that relates properties of the analysts' information environment to the properties of their forecasts. First, we express forecast dispersion and error in the mean forecast in terms of analyst uncertainty and consensus (that is, the degree to which analysts share a common belief). Second, we reverse the relations to show how uncertainty and consensus can be measured by combining forecast dispersion, error in the mean forecast, and the number of forecasts. Third, we show that the quality of common and private information available to analysts can be measured using these same observable variables. The relations we present are intuitive and easily applied in empirical studies.

Key Words: *Analysts' forecasts, Uncertainty, Consensus, The quality of common and private information.*

I. INTRODUCTION

Empirical accounting research frequently relies on analyst forecasts to construct proxies for variables of interest. For example, the error in the mean forecast is used as a proxy for earnings surprise (e.g., Brown et al. 1987; Bamber and Cheon 1995;

This paper is a merger of three prior working papers by Barron and Kim, Barron and Stevens, and Lim. We gratefully acknowledge the help of Terry O'Keefe and the comments of our anonymous reviewers. We also acknowledge helpful comments from Jeff Abarbanell, Steve Baginski, Linda Bamber, Franklin Fant, Charles Kile, Mary Harris and Pamela Stuerke, as well as from workshop participants at Indiana University, the University of Nebraska, the University of New Hampshire and Temple University.

Submitted December 1997.

Accepted June 1998.

Wiedman 1996; Bamber et al. 1997). In addition, forecast dispersion (that is, cross-sectional variance) and error in the mean forecast is used to proxy for the uncertainty or the degree of consensus among analysts or market participants (e.g., Daley et al. 1988; Ziebart 1990; Imhoff and Lobo 1992; Lang and Lundholm 1996; Barron and Stuerke 1997). However, the relation between these commonly used empirical proxies and such theoretical properties of the information environment as uncertainty and consensus has yet to be rigorously specified. This article investigates, from an analytical perspective, what the forecasts of analysts reveal about the properties of their information environment. We base our analysis on a model of expectations in which each analyst observes two signals about future earnings, one public (common across all analysts) and one private (idiosyncratic). The model demonstrates how these two different types of information result in forecast errors and dispersion and how the underlying unobservable characteristics of the analysts' information environment are revealed by expressions involving observable constructs.

The intuition underlying our main results stems from the fact that forecast dispersion and error relate in different ways to the common and idiosyncratic components of error in analysts' forecasts. The common error component arises from error in the public information analysts rely upon and the idiosyncratic error component arises from error in the private information analysts rely upon. We find that forecast dispersion reflects only idiosyncratic error. By contrast, error in individual forecasts reflects both common and idiosyncratic error and error in the mean forecast reflects primarily common error but may also reflect idiosyncratic error that is incompletely diversified away when only a limited number of forecasts exist.

The fact that errors in common vs. private information influence forecast dispersion and error differently yields insights into how certain theoretical properties of the *analysts'* information environment are reflected in empirical forecast measures *on average*. One can understand these insights best by referring to two theoretical constructs: uncertainty and consensus. "Uncertainty" refers to the expected squared error in individual forecasts aggregated (or averaged) across analysts. "Consensus" refers to the degree to which analysts share a common belief.¹ As we demonstrate, the expected dispersion in forecasts is an increasing function of uncertainty but a decreasing function of consensus, while the expected squared error in the mean forecast is an increasing function of both uncertainty and consensus. These findings provide guidance on how to construct valid measures of uncertainty and consensus from widely available earnings forecast (and realization) data.

This article is related to prior studies focused on the use of analyst forecasts to infer characteristics of investors' information environment (e.g., Barry and Jennings 1992; Abarbanell et al. 1995). Abarbanell, Lanen and Verrecchia (1995) (hereafter ALV) modeled the relation between analyst forecasts and investor beliefs and analyzed its implications for price and volume reactions to earnings surprises.² Our study extends ALV by modeling the information analysts use to make forecasts. While the earnings forecasts in ALV's model are exogenously determined, the forecasts in our model are endogenously determined by the common and idiosyncratic information analysts use. This extension allows us to demonstrate how analyst forecasts reveal underlying characteristics of their information such as the precision (or quality) of common and private information.²

We have organized the rest of this article as follows. In section II, we outline our model and describe two properties of analysts' information environment—namely, the level of

¹ This consensus construct is a generalized version of the consensus construct Holthausen and Verrecchia (1990, 1999) introduced.

² A more detailed description of the difference between our model and the models in ALV (1995) and Barry and Jennings (1992) appears in section II.

uncertainty and the degree of consensus among analysts. We show how expected forecast dispersion and error in the mean forecast are functions of uncertainty, consensus and the number of forecasts. We then reverse these relations and express uncertainty and consensus as functions of forecast dispersion, error, and the number of forecasts. We also show how one can express the precision (or quality) of analysts' common or private information as a function of these same observable measures. In section III, we discuss important issues related to interpreting and applying the model. Section IV includes some concluding remarks.

II. THE MODEL

This section presents a model of how financial analysts' earnings forecasts are related to their information environment—that is, to the general properties of their information and beliefs. We consider a firm for which N financial analysts forecast earnings. We denote the earnings that analysts forecast by y .³ It is assumed that y is normally distributed with mean \bar{y} and precision (inverse of variance) h . Each analyst's available information about earnings consists of common (public) and idiosyncratic (private) information. The common information is represented by the common prior (with mean \bar{y} and precision h). The private information of analyst i is represented by a signal $z_i = y + \epsilon_i$. This information is private in the sense that z_i is observed only by analyst i and that ϵ_i is distributed independently of all other variables. We assume that ϵ_i is normally distributed with mean zero and precision s_i . We allow s_i s to differ among analysts. That is, analysts' private information can be of differential quality.

The information environment of the N analysts described above is quite general under the assumption of normally distributed and additive errors. For example, consider two analysts with any finite number of differing signals (with one additive error term each) about y . The error terms of the signals can be correlated in any way. Still, one can show that each analyst's belief can be represented by one common signal (the prior) and one idiosyncratic signal as in our model. The only strong assumption is that common information between any two analysts is common among all analysts.

We assume that analyst i 's earnings forecast is his or her best estimate of earnings based on available information.⁴ As is well known, the conditional expectation is a weighted average of the common and private information available at the time of forming the expectation, with the precision of each as the weight. That is, analyst i 's forecast, denoted by μ_i , is defined by:

$$\mu_i \equiv E[y|z_i] = \frac{h\bar{y} + s_i z_i}{h + s_i}. \quad (1)$$

We define the mean forecast, denoted by μ , as:

$$\mu \equiv \frac{1}{N} \sum_{i=1}^N \mu_i = \frac{1}{N} \sum_{i=1}^N \frac{h\bar{y} + s_i z_i}{h + s_i}. \quad (2)$$

³ The primitive random variable in our model is earnings, under the assumption that analysts' earnings forecasts are strictly forecasts of *earnings*. Thus, information in our model is information about earnings to be disclosed in the near future, and not directly about the firm's future prospects (liquidating dividends).

⁴ The implications of this assumption, in light of analysts' tendencies to issue optimistic earnings forecasts are discussed in section III.

Thus, we define observed dispersion in forecasts, denoted by d , as the sample variance of the μ_i s, i.e.,⁵

$$d \equiv \frac{1}{N-1} \sum_{i=1}^N (\mu_i - \mu)^2. \quad (3)$$

The sample variance, d , is itself a random variable prior to observing forecasts. Our non-random dispersion measure, denoted by D , is simply the unconditional expectation of d , i.e.,

$$D \equiv E[d] = \frac{1}{N-1} \sum_{i=1}^N \text{Var}(\mu_i - \mu), \quad (4)$$

where the equality holds because the mean of $\mu_i - \mu$ is zero.

The observed error in the mean forecast, denoted by e , is the actual earnings minus the mean forecast, i.e.,

$$e \equiv y - \mu. \quad (5)$$

Our measure of error in the mean forecast, denoted by SE , is the expectation of the squared error in the mean forecast, i.e.,

$$SE \equiv E[e^2] = E[(y - \mu)^2]. \quad (6)$$

The variables D , SE and N represent the three observable variables in this model.⁶

We can now introduce two general properties of the analysts' information environment. Later in this section we show that, along with the number of forecasts, these two properties determine forecast dispersion and error in the mean forecast. The first property is the level of residual uncertainty. We first define the level of uncertainty for analyst i , denoted by V_i , as:

$$V_i \equiv E[(y - \mu_i)^2] = \frac{1}{h + s_i}, \quad (7)$$

which is analyst i 's expected variance of y , conditional on his or her available information. We define the overall uncertainty level, V , as simply individual uncertainty averaged over N analysts, i.e.,

$$V \equiv \frac{1}{N} \sum_{i=1}^N V_i = \frac{1}{N} \sum_{i=1}^N \frac{1}{h + s_i}. \quad (8)$$

The second measure we introduce is the degree of consensus among analysts. Our

⁵ Dispersion in forecasts is sometimes viewed as a population variance rather than a sample variance. In this case the denominator in equations (3) and (4) would be N rather than $N - 1$. If this alternative definition of D in equation (4) is labeled D' then our later formulas can be converted to this alternative view by replacing D with $D'N/(N - 1)$.

⁶ In section III we discuss the implication of treating expected dispersion and error in the mean forecast (D and SE) as observable when applying our model in empirical studies.

consensus measure, denoted by ρ , is the same as that introduced by Holthausen and Verrecchia (1990, 1999) except that it is generalized to our model with differential precision (that is, with different s_i s among analysts). We first define the average pair-wise covariance among analysts' beliefs, denoted by C , as:

$$C \equiv \frac{1}{N} \sum_{i=1}^N C_i = \frac{1}{N} \sum_{i=1}^N \left[\frac{1}{N-1} \sum_{j \neq i}^N \text{Cov}(y - \mu_i, y - \mu_j) \right], \quad (9)$$

where C_i is the average covariance between the beliefs of analyst i and the beliefs of the rest of the analysts. One can interpret the average covariance, C , as common uncertainty—that is, uncertainty shared by all analysts attributable to their reliance on imprecise common information. Consensus, ρ , is defined by:

$$\rho \equiv \frac{C}{V}. \quad (10)$$

Consensus is the degree to which the analysts' beliefs covary relative to the overall level of uncertainty, or the ratio of common uncertainty to the overall uncertainty. Stated differently, consensus measures how much the average (mean) belief reflects common vs. private information. When all available information is public, all analysts' beliefs are identical and $\rho = 1$. As h approaches zero, common information becomes so imprecise that analysts do not rely on it at all and their individual beliefs diverge more from the average belief as ρ approaches zero. In appendix B we show that our consensus measure ρ effectively summarizes two sources of information asymmetry: the ratio of private to total information and the degree to which individual uncertainty differs among analysts.

In the following lemma we first state relations among four variance-covariance measures defined earlier—namely, forecast dispersion and the (expected squared) error in the mean forecast, D and SE , uncertainty, V and average covariance, C . The proof appears in appendix A.

Lemma 1

$$V = C + D \quad (11)$$

and

$$SE = C + \frac{D}{N}. \quad (12)$$

Understanding lemma 1 is critical because our main results (Propositions 1 and 2) follow from lemma 1. Equation (11) shows that overall uncertainty, V , is simply the sum of the average covariance, C and expected dispersion, D . C is the uncertainty common to all analysts, so equation (11) implies that expected dispersion, D , reflects only the idiosyncratic uncertainty among analysts.

Equation (12) shows that the error in the mean forecast reflects common uncertainty, C , and a fraction of idiosyncratic uncertainty. Equation (12) also indicates that idiosyncratic uncertainty is diversified from the mean forecast. In other words, error contained in the

analysts' forecasts attributable to idiosyncratic information tends to cancel out in the mean forecast. When $N = 1$, no diversification occurs and the expected forecast error fully reflects overall uncertainty and coincides with V . As N grows large, the expected magnitude of error in the mean forecast reflects only common uncertainty.

In the following proposition, we express forecast dispersion and error as functions of uncertainty and consensus. The proof follows from lemma 1.

Proposition 1: Expected forecast dispersion, D and expected squared error in the mean forecast, SE , can be expressed as:

$$D = V - C = V(1 - \rho) \quad (13)$$

and

$$SE = \frac{V + (N - 1)C}{N} = V \left(\rho + \frac{1 - \rho}{N} \right) \quad (14)$$

where V , C and ρ are measures of overall uncertainty, common uncertainty and consensus, respectively, and N is the number of forecasts.

Proposition 1 shows that forecast dispersion and error in the mean forecast reflect uncertainty and consensus in a simple way. From equation (13), dispersion is an increasing function of uncertainty, V and a decreasing function of consensus, ρ . This result is consistent with the intuition of studies that use forecast dispersion as a proxy for uncertainty or lack of consensus. From equation (14), error in the mean forecast is an increasing function of both uncertainty and consensus. Proposition 1 leads to Proposition 2, which shows how one can use analyst forecasts to measure uncertainty and consensus more precisely than in previous studies.

Before moving to Proposition 2, we should discuss two analytical papers closely related to Proposition 1. In a similar setting, Barry and Jennings (1992), showed forecast dispersion to be affected by both uncertainty and information asymmetry as in equation (13). Barry and Jennings (1992) did not, however, examine error in the mean forecast in conjunction with forecast dispersion, which is a critical feature of our model. ALV analyzed a model in which forecasts were exogenously specified and arrived at some conclusions qualitatively similar to ours. However, the exogenous nature of the forecasts in their model creates subtle but important differences between their results and ours. For example, ALV suggested that forecast dispersion indicates a lack of precision in the mean forecast. Equation (13) suggests, however, that forecast dispersion can be caused by low levels of consensus, which equation (14) suggests can be associated with a *decrease* in the error in the mean forecast.⁷

The distinguishing feature of our model lies in the next proposition, in which we reverse the relations in Proposition 1 and express uncertainty, V , and consensus, ρ , in terms of D and SE .

Proposition 2: Uncertainty and consensus can be expressed as:

⁷ We present a more formal discussion of the difference between ALV (1995) and our model at the end of section II.

$$V = \left(1 - \frac{1}{N}\right) D + SE \quad (15)$$

and

$$\rho = \frac{SE - \frac{D}{N}}{\left(1 - \frac{1}{N}\right) D + SE}. \quad (16)$$

Proposition 2, our main result, shows how one can infer uncertainty and consensus from observable forecast dispersion, error in the mean forecast and the number of forecasts. In a situation where the assumptions of our model reasonably apply, Proposition 2 provides concrete measures of total uncertainty and consensus among analysts. The simultaneous examination of dual measures, V and ρ , of the information environment is important in situations where information consists of both common and idiosyncratic elements.

We now examine the special case in which all analysts' private information is of equal precision—that is, $s_i = s$ for all i . This special case is easy to apply and is appropriate if financial analysts are all relatively well informed, with little difference existing in the quality of information available to them.⁸

Proposition 3: When analysts' private information is of equal precision, the following relations hold in addition to those of Proposition 2:

$$V = \frac{1}{h + s}, \quad \rho = \frac{h}{h + s} \quad (17, 18)$$

and

$$D = \frac{s}{(h + s)^2}, \quad SE = \frac{h + \frac{s}{N}}{(h + s)^2}. \quad (19, 20)$$

Proposition 3 shows how the precision of public and private information relates to information characteristics and to forecast measures in this special case. Note from equation (18) that consensus is the ratio of the precision of common information to the sum of the precisions of common and private information. This means that the relative degree of consensus among analysts depends upon the relative precision of the common information available to them.⁹ From this result, one can express the primitive information parameters, h and s , directly in terms of D and SE .

⁸ Most empirical tests for differences in the magnitude of forecast error across analysts have failed to find significant differences. For example, O'Brien (1987), Coggin and Hunter (1989), O'Brien (1990) and Butler and Lang (1991) failed to find statistically significant differences in forecast accuracy across analysts. Sinha et al. (1997) did find statistically significant differences in accuracy after controlling for forecast recency. However, expected differences in the magnitude of forecast error across analysts (that is, differential uncertainty) may be economically insignificant, because these differences have been relatively difficult to detect.

⁹ See appendix B for the general relation between consensus and the common-to-total information ratio when differences in the quality of analysts' information exist.

Corollary 1: When analysts' private information is of equal precision, the following relations hold:

$$h = \frac{SE - \frac{D}{N}}{\left[\left(1 - \frac{1}{N} \right) D + SE \right]^2}, \quad (21)$$

and

$$s = \frac{D}{\left[\left(1 - \frac{1}{N} \right) D + SE \right]^2}. \quad (22)$$

Corollary 1 allows a researcher to obtain separate estimates of the precision of the common and private information incorporated into analysts' forecasts.

At this point the difference between our model and ALV can be described in more detail. The major difference in the two models is that ALV assume an exogenous individual forecast with two error terms, one common and one idiosyncratic (that is, they defined each forecast as $f_i = y + \delta + \eta_i$), while we derive an individual forecast from information signals, one common and one private. To emphasize this difference in the two models, assume that no differences exist in the precision of private information among analysts. Then, an individual forecast in our model is written in a form similar to that ALV used, $\mu_i = (h\bar{y} + sz_i)/(h + s) = y + h(\bar{y} - y)/(h + s) + se_i/(h + s)$. Here the terms $h(\bar{y} - y)/(h + s)$ and $se_i/(h + s)$ correspond to the terms δ and η_i , respectively. In ALV the variance of these two error terms, g^{-1} and m^{-1} , can take any values because they are exogenous, while in our model the two corresponding terms have variances $h/(h + s)^2$ and $s/(h + s)^2$, respectively, and are related to each other. This difference arises from the exogenous nature of forecasts in ALV and leads the two models to have different implications. For example, note that as the number of analysts, N , approaches infinity as in ALV, the two variances $h/(h + s)^2$ and $s/(h + s)^2$ are SE and D , respectively, from Proposition 3. In ALV forecast dispersion, $s/(h + s)^2$, is independent of the variance of the common error term, $h/(h + s)^2$, because they are assumed to be independent exogenous variables. In our model, however, the link between forecast dispersion and error in the mean forecast ($s/(h + s)^2$ and $h/(h + s)^2$ in this special case) through h or s is central to the results we obtain. Therefore, our model and its implications can be viewed as an important extension of ALV that endogenizes analysts' forecasts.

III. ISSUES RELATED TO INTERPRETING AND APPLYING THE MODEL

In this section we address some issues regarding both the interpretation and application of our model. These issues are particularly important for researchers wanting to use the model in empirical studies.

In the model, it is assumed that forecasts reflect the information analysts possess in an unbiased way. Prior research suggests, however, that analysts' incentives and strategic concerns can influence their forecasts (see for example, Dugar and Nathan 1995; McNichols and O'Brien 1997). The tendency of analysts to issue optimistic forecasts has the potential to confound the construct validity of some of the measures derived in the previous section.

Because bias inflates the expected magnitude of error in forecasts, it inflates estimates of uncertainty unless the bias is measured and removed. Ignoring bias could also inflate or deflate consensus estimates, depending on the extent to which bias is commonly shared or varies across analysts. Whether a researcher should attempt to obtain estimates of sample-wide bias or firm-specific bias in any particular application based upon our model is likely to depend upon context. We do note, however, that recent research suggests that bias in analysts' forecasts has decreased markedly in recent years, with optimistic bias in quarterly earnings forecasts having disappeared as of 1993 for S&P 500 firms (Brown 1996, 1997).

The tendency of analysts to "herd" (Trueman 1994) may also have an impact on the construct validity of the empirical forecast measures derived from our model. Herding occurs when analysts observe certain actions (for example, forecast revisions) of other analysts and then publish new forecasts influenced primarily by these observations. The distinction between primitive public and private information sources is clouded if some analysts use other analysts' forecasts as an information source. The impact of herding behavior on the construct validity of the measures developed in our model is an open issue.

Our main results depict unconditional expectations of forecast dispersion and error in the mean forecast (D and SE). Using our model in empirical contexts, therefore, will require the pooling of observations to measure dispersion and error in the mean forecast. Averaging over multiple observations will reduce measurement error and lead to measures with greater construct validity.

Despite some limitations and caveats, our model can be used in a variety of empirical studies. For example, Byard (1998) used the measures of common and private information precision described in this article to show that private information contributes 56 percent of the total precision of information available to analysts immediately prior to annual earnings announcements, up from 20 percent one year prior to the announcement. Furthermore, Byard (1998) found that large firms exhibit lower pre-announcement uncertainty and lower consensus than do small firms. Botosan and Harris (1997) used the consensus measure described in this article to examine the impact of managerial decisions to increase segment disclosure frequency. They hypothesized and found evidence to suggest that managers are more likely to provide quarterly segment data when consensus among analysts is low. In addition, firms initiating quarterly segment reporting experience an immediate and sustained increase in consensus among analysts.¹⁰ These papers suggest that the measures developed in this paper can open new avenues to understanding the joint roles of common and private information in capital markets.

IV. CONCLUSION

In this article we show how empirical researchers can use observable forecast dispersion and error in the mean forecast to understand important properties of analysts' information environment. We base our analysis on a simple model in which published forecasts are determined by common information (common to all analysts) and idiosyncratic information. Nevertheless, we believe that this study can serve as a practical guide for future research.

A significant gap exists between the information environment constructs found in theory and the empirical forecast-based proxies used to test theory. We narrow this gap by suggesting empirical forecast measures that more accurately reflect underlying theoretical constructs. For research using forecast data to improve, theoretical and empirical researchers

¹⁰ See Barron et al. (1998) for another example of a study that uses our model to study the quality of financial disclosures.

must strive for a common vocabulary. For example, we recommend that the term “consensus” refer only to the degree to which the average belief is commonly shared. The average forecast is often referred to as the “consensus forecast,” regardless of whether it is commonly shared. Researchers have also used the term “consensus” in reference to forecast dispersion, even though dispersion reflects both uncertainty and a lack of consensus.

One can use the model described here to better understand the usefulness of financial reports to analysts, and to the extent that analysts’ earnings forecasts reflect investors’ beliefs, we may also achieve a clearer understanding of the usefulness of these reports to investors. This “forecast approach” for assessing the utility or impact of financial reports can be used as an alternative to or in conjunction with the traditional “market approach.” Since the two approaches are subject to different limitations, together they can improve our understanding of the usefulness of accounting information and the determinants of the market reaction to the release of accounting information.

APPENDIX A

Proof of Lemma 1:

Let $e_i \equiv y - \mu_t$ for $i = 1, \dots, N$. Then:

$$\begin{aligned}
 D &= \frac{1}{N-1} \sum_{i=1}^N \text{Var} \left[e_i - \frac{1}{N} \sum_{j=1}^N e_j \right] \\
 &= \frac{1}{(N-1)N^2} \sum_{i=1}^N \text{Var} \left[Ne_i - \sum_{j=1}^N e_j \right] \\
 &= \frac{1}{(N-1)N^2} \sum_{i=1}^N \text{Var} \left[\sum_{j \neq i}^N (e_i - e_j) \right] \\
 &= \frac{1}{(N-1)N^2} \sum_{i=1}^N \left[(N-1)^2 V_i + \sum_{j \neq i}^N V_j - 2(N-1) \sum_{j \neq i}^N \text{Cov}(e_i, e_j) + \sum_{j \neq i}^N \sum_{k \neq i, j}^N \text{Cov}(e_j, e_k) \right] \\
 &= \frac{1}{(N-1)N^2} \sum_{i=1}^N \left[\{(N-1)^2 + (N-1)\} V_i + \{-2(N-1) + (N-2)\} \sum_{j \neq i}^N \text{Cov}(e_i, e_j) \right] \\
 &= \frac{1}{N} \sum_{i=1}^N V_i - \frac{1}{N} \sum_{i=1}^N \frac{1}{N-1} \sum_{j \neq i}^N \text{Cov}(e_i, e_j) \\
 &= V - C.
 \end{aligned}$$

Also, the (expected squared) error in the mean forecast, SE, can be written as:

$$\begin{aligned}
 \text{SE} &= \text{Var} \left[\frac{1}{N} \sum_{i=1}^N e_i \right] \\
 &= \frac{1}{N^2} \left[\sum_{i=1}^N V_i + \sum_{i=1}^N \sum_{j \neq i}^N \text{Cov}(e_i, e_j) \right] \\
 &= \frac{V}{N} + \frac{(N-1)C}{N} \\
 &= C + \frac{V-C}{N} \\
 &= C + \frac{D}{N}.
 \end{aligned}$$

APPENDIX B

Consider two sources of information asymmetry among analysts. First, analysts' beliefs differ according to the presence of private information, a situation captured by (1 minus) the average common-to-total information ratio α , defined by:

$$\alpha \equiv \frac{1}{N} \sum_{i=1}^N \alpha_i \equiv \frac{1}{N} \sum_{i=1}^N \frac{h}{h + s_i} = hV,$$

where α_i is analyst i 's individual common-to-total information ratio. When all available information is common, $\alpha = 1$. As h approaches zero—that is, as analysts share less and less common information— α approaches 0.

Another source of information asymmetry is differential uncertainty, which may exist because of differential quality of private information across analysts. Our measure of differential uncertainty, denoted by γ , measures relative differences across analysts' individual uncertainty. This differential uncertainty is defined by:

$$\gamma \equiv \frac{1}{N} \sum_{i=1}^N \gamma_i^2 \equiv \frac{1}{N} \sum_{i=1}^N \left(\frac{V_i - V}{V} \right)^2 = \frac{1}{V^2} \left[\frac{1}{N} \sum_{i=1}^N \left(\frac{1}{h + s_i} - \frac{1}{N} \sum_{i=1}^N \frac{1}{h + s_i} \right)^2 \right],$$

where γ_i is the relative deviation of analyst i 's individual uncertainty from the average. In other words, differential uncertainty γ is the average squared relative deviation of individual uncertainty from the average over N analysts.

The average covariance can now be written as:

$$\begin{aligned} C &\equiv \frac{1}{N} \sum_{i=1}^N \left[\frac{1}{N-1} \sum_{j \neq i}^N \text{Cov}(y - \mu_i, y - \mu_j) \right] \\ &= \frac{1}{N} \sum_{i=1}^N \left[\frac{1}{N-1} \sum_{j \neq i}^N \text{Cov} \left(\frac{h(y - \bar{y}) - s_i \varepsilon_i}{h + s_i}, \frac{h(y - \bar{y}) - s_j \varepsilon_j}{h + s_j} \right) \right] \\ &= \frac{1}{N} \sum_{i=1}^N \left[\frac{1}{N-1} \sum_{j \neq i}^N \frac{h}{(h + s_i)(h + s_j)} \right] \\ &= \frac{h}{N(N-1)} \sum_{i=1}^N \left[\frac{1}{h + s_i} \sum_{j \neq i}^N \frac{1}{h + s_j} \right] \\ &= \frac{h}{N(N-1)} \left[\left(\sum_{i=1}^N \frac{1}{h + s_i} \right)^2 - \sum_{i=1}^N \frac{1}{(h + s_i)^2} \right] \\ &= \frac{-h}{N-1} \left[\frac{1}{N} \sum_{i=1}^N \left(\frac{1}{h + s_i} \right)^2 - \left(\frac{1}{N} \sum_{i=1}^N \frac{1}{h + s_i} \right)^2 - (N-1) \left(\frac{1}{N} \sum_{i=1}^N \frac{1}{h + s_i} \right)^2 \right] \\ &= \left(\frac{-1}{N-1} \right) \left(\frac{\alpha}{V} \right) [V^2 \gamma - (N-1)V^2] \\ &= V\alpha \left(1 - \frac{\gamma}{N-1} \right). \end{aligned}$$

Therefore:

$$\rho \equiv \frac{C}{V} = \alpha \left(1 - \frac{\gamma}{N - 1} \right).$$

This relation shows that $\rho = \alpha$ if either $\gamma = 0$ or as N approaches infinity. When differential uncertainty exists (because of differential quality of private information across analysts) and N is finite, the average ratio of common information to total information (α) across analysts does not exactly translate into the ratio of common uncertainty to total uncertainty (ρ). Instead, consensus summarizes the effect of the two sources of information asymmetry on analysts' beliefs—namely, the relative presence of private information and differential uncertainty.

REFERENCES

- Abarbanell, J. S., W. N. Lanen, and R. E. Verrecchia. 1995. Analysts' forecasts as proxies for investor beliefs in empirical research. *Journal of Accounting and Economics* 20: 31–60.
- Bamber, L. S., and S. Y. Cheon. 1995. Differential price and volume reactions to accounting earnings announcements. *The Accounting Review* 70 (July): 417–442.
- , O. Barron, and T. Stober. 1997. Trading volume and different aspects of disagreement coincident with earnings announcements. *The Accounting Review* 72 (October): 575–597.
- Barron, O., and P. Stuerke. 1998. Dispersion in analysts' earnings forecasts as a measure of uncertainty. *Journal of Accounting, Auditing, and Finance* 13 (Summer): 243–268.
- , C. Kile, and T. O'Keefe. 1998. MD&A quality as measured by the SEC and analysts' earnings forecasts. *Contemporary Accounting Research* (forthcoming).
- Barry, C., and R. Jennings. 1992. Information and diversity of analyst opinion. *Journal of Financial and Quantitative Analysis* 27: 169–183.
- Botosan, C. A., and M. S. Harris. 1997. Motivations for changes in disclosure frequency and its consequences: An examination of voluntary quarterly segment disclosures. Working paper, Pennsylvania State University.
- Brown, L. D., G. Richardson, and S. Schwager. 1987. An information interpretation of financial analyst superiority in forecasting earnings. *Journal of Accounting Research* 25: 49–67.
- . 1996. Forecasting errors and their implications for security analysis: An alternative perspective. *Financial Analysts Journal* 52 (January/February): 40–47.
- . 1997. Analyst forecasting errors: Additional evidence. Working paper, Georgia State University.
- Butler, K. C., and L. H. P. Lang. 1991. The forecast accuracy of individual analysts: Evidence of systematic optimism and pessimism. *Journal of Accounting Research* 29 (1): 150–156.
- Byard, D. 1998. Pre-disclosure information inferred from analysts' forecast: Quality, consensus and firm size. Working paper, University of Maryland.
- Coggin, T. D., and J. E. Hunter. 1989. Analyst forecasts of EPS and EPS growth: Decomposition of error, relative accuracy and relation to return. Working paper, Michigan State University.
- Daley, L. A., D. W. Senkow, and R. L. Vigeland. 1988. Analysts' forecasts, earnings variability, and option pricing: Empirical evidence. *The Accounting Review* 58 (October): 563–585.
- Dugar, A., and S. Nathan. 1995. The effect of investment banking relationships on financial analysts' earnings forecasts and investment recommendations. *Contemporary Accounting Research* (Fall): 131–160.
- Holthausen, R. W., and R. E. Verrecchia. 1990. The effect of informedness and consensus on price and volume behavior. *The Accounting Review* 65 (January): 191–208.
- Imhoff, E., and G. Lobo. 1992. The effect of *ex ante* earnings uncertainty on earnings response coefficients. *The Accounting Review* 67 (April): 427–439.
- Lang, M., and R. Lundholm. 1996. Corporate disclosure policy and analyst behavior. *The Accounting Review* 71 (October): 467–492.
- McNichols, M., and P. O'Brien. 1997. Self selection and analyst coverage. Working paper, Stanford University.

- O'Brien, P. 1987. Individual forecasting ability. *Managerial Finance* 13 (2): 3–9.
- . 1990. Forecast accuracy of individual analysts in nine industries. *Journal of Accounting Research* 28 (2): 286–304.
- Sinha, P., L. D. Brown, and S. Das. 1997. A re-examination of financial analysts' differential earnings forecast accuracy. *Contemporary Accounting Research* 14 (1): 1–42.
- Trueman, B. 1994. Analyst forecasts and herding behavior. *Review of Financial Studies* 7 (Spring): 97–124.
- Wiedman, C. I. 1996. The relevance of characteristics of the information environment in the selection of a proxy for the market's expectations for earnings: An extension of Brown, Richardson and Schwager (1987). *Journal of Accounting Research* 34: 313–324.
- Ziebart, D. 1990. The association between consensus of beliefs and trading activity surrounding earnings announcements. *The Accounting Review* 65 (April): 477–488.