

Georgia State University

ScholarWorks @ Georgia State University

---

ECON Honors Theses

Department of Economics

---

5-10-2014

## Against Indifference: Popper's Assumption of Distribution Preference

Brett Mullins  
*Georgia State University*

Follow this and additional works at: [https://scholarworks.gsu.edu/econ\\_hontheses](https://scholarworks.gsu.edu/econ_hontheses)

---

### Recommended Citation

Mullins, Brett, "Against Indifference: Popper's Assumption of Distribution Preference." Thesis, Georgia State University, 2014.

doi: <https://doi.org/10.31922/0NDQ-1S17>

This Thesis is brought to you for free and open access by the Department of Economics at ScholarWorks @ Georgia State University. It has been accepted for inclusion in ECON Honors Theses by an authorized administrator of ScholarWorks @ Georgia State University. For more information, please contact [scholarworks@gsu.edu](mailto:scholarworks@gsu.edu).

AGAINST INDIFFERENCE: POPPER'S ASSUMPTION OF  
DISTRIBUTION PREFERENCE

An Honors Thesis

Submitted in Partial Fulfillment of the  
Requirements for Graduation with  
Undergraduate Research Honors

Georgia State University

2014

By

Brett Cameron Mullins

Committee:

---

Dr. Yongsheng Xu, Honors Thesis Director

---

Date

AGAINST INDIFFERENCE: POPPER'S ASSUMPTION OF  
DISTRIBUTION PREFERENCE

By

BRETT CAMERON MULLINS

Under the Direction of Dr. Yongsheng Xu

ABSTRACT

As a central tenet of falsificationism, Karl Popper holds that all possible scientific theories individually have a probability equal to zero. Popper's position rests upon the Principle of Indifference, the equiprobability of mutually exclusive outcomes, to derive this zero probability. In this paper, I will illustrate that the Principle of Indifference fails to compute objective probabilities in cases in which an epistemic agent faces ignorance. Prior to experience, there is no sufficient reason to prefer any probability distribution to any other; yet, the Principle of Indifference implies a preference for a uniform probability distribution. Distribution preference is determined by the relevant experience and rational expectations of epistemic agents. Relevant experience is defined by observations and other sense experience regarding the relevant trial. Rational expectations represents the non-arbitrariness of distribution preference. Without rational expectations, the distribution preference is arbitrary even when informed by experience. If an agent lacks relevant experience, then any distribution preference is arbitrary; however, if an agent possesses relevant experience, then the Principle of Indifference does not apply. A rejection of the Principle of Indifference undermines the necessity of zero probabilities for scientific theories in which case Popper's conclusions of falsificationism do not follow. Objective probability, then, understood within the logical interpretation, is a problematic notion.

AGAINST INDIFFERENCE: POPPER'S ASSUMPTION OF  
DISTRIBUTION PREFERENCE

By

Brett Cameron Mullins

An Honors Thesis Submitted in Partial Fulfillment  
of the Requirements for the Degrees of

Bachelor of Arts in Philosophy

In the College of Arts and Sciences

Bachelor of Science in Economics

In the Andrew Young School of Policy Studies

Georgia State University

2014

## ACKNOWLEDGMENTS

I am grateful to Yongsheng Xu, my thesis advisor, for both patience and insight through various drafts of this paper. Without his assistance, I could not have pursued the topics explored within the paper with the same amount of rigor and precision. I am grateful to Ed Cox for both allowing me to initially pursue this topic within the classroom and continuing to provide support and discussion afterward. I additionally thank both the Georgia Philosophical Society and the University of Georgia Chapter of Phi Sigma Tau for allowing me to present drafts of this paper. It goes without saying that there are numerous others who have contributed to this project who are not recognized here. Finally, the completion of this project would not have been possible without the persistent support, understanding, and encouragement of S.B. and P.B. to both of whom I am quite indebted.

# Table of Contents

Introduction .....	1
First Proof .....	2
Second Proof .....	5
Falsificationism as a Methodology of Science .....	6
Principle of Indifference .....	7
Distribution Preference .....	9
Other Conjectures and Refutations of PI .....	12
Bet Making.....	12
Survey of Objections.....	14
Maximum Entropy Principle.....	16
Falsificationism Reconsidered .....	19
Objective Probability .....	22
Alternative Approaches to Probability.....	24
Conclusion .....	27
Bibliography.....	30

# Introduction

As a central claim of falsificationism, Karl Popper holds that all possible scientific theories individually have a probability equal to zero. No amount of evidence can increase the likelihood of a theory to obtain; evidence can only disagree with predictions and implications and, thus, falsify a theory. Popper's position rests upon the Principle of Indifference, the equiprobability of mutually exclusive outcomes, to derive this zero probability. In this paper, I will provide an original argument which demonstrates that the Principle of Indifference is inconsistent insofar as it fails to compute objective probabilities in cases in which the relevant epistemic agent faces ignorance. Prior to experience, there is no sufficient reason to prefer any probability distribution to any other, including a uniform distribution implied by the Principle of Indifference.

In light of this rejection of the Principle of Indifference, two objections are considered that attempt to provide additional foundations: the first models distribution preference through bet making, and the second is an objective Bayesian perspective, which employs the Maximum Entropy Principle. It will be argued that neither account satisfactorily resolves the inconsistency demonstrated as a consequence of the Principle of Indifference. To complement the main result of this paper, three historical objections to the Principle of Indifference will be surveyed. With the loss of this central feature to falsificationism, Popper can no longer hold that all possible scientific theories are logically false. To conclude, consequences for the possibility of objective logical probability will be critically considered and alternative approaches to objectivity in probability will be surveyed.

Popper posits that there is no positive solution to the problem of induction, understood as “the question of how to establish the truth of universal statements which are based on experience.”<sup>1</sup> A universal statement, e.g., 'all As are Bs,' is not merely a conjunction of various singular instances; it is a

---

<sup>1</sup> Karl Popper, *The Logic of Scientific Discovery* (New York: Harper and Row, Publishers, 1968), 28.

claim about an unlimited number of instances.<sup>2</sup> A theory is represented as a universal statement where the relation between the properties present is not definitional. More precisely, theories are of the form of non-tautological law-like generalizations (henceforth NLGs), expressed as  $(\forall x)(Px \rightarrow Qx)$ . The above singular instances are captured by basic statements which assert “that an observable event is occurring in a certain individual region of space and time.”<sup>3</sup> This is to say that basic statements represent one singular fact and not a complete cross section of the universe at time T.

## First Proof

Consider the set of all possible basic statements,  $\Omega$ , which represents “all possible empirical worlds.”<sup>4</sup> For some statement  $S$ , it is either the case that  $S$  is consistent with  $\Omega$ ,  $S$  is inconsistent with  $\Omega$ , or  $S$  is consistent with basic statement  $x \in \Omega$  but inconsistent with  $y \in \Omega$ .  $S$  is consistent with  $\Omega$  if and only if there does not exist an  $x \in \Omega$  such that the conjunction of  $S$  and  $x$  results in contradiction. Likewise,  $S$  is inconsistent with  $\Omega$  just in case there exists an  $x \in \Omega$  such that the conjunction of  $S$  and  $x$  results in contradiction. In the first case above,  $S$  is a tautology; in the second,  $S$  is a logical contradiction.<sup>5</sup> In the third case,  $S$  partitions  $\Omega$  into two subsets: basic statements that are consistent with  $S$  and basic statements that are inconsistent with  $S$ . The former subset is defined as range, while the latter is identified as empirical content.<sup>6</sup>

The empirical content of  $S$  is the set of basic statements that must be false if  $S$  obtains; hence, if any basic statement within the empirical content of  $S$  obtains, then  $S$  is false. As the empirical content of a statement increases, both the amount of potential falsifiers and the degree of testability increase.<sup>7</sup> The union of range and empirical content is equivalent to  $\Omega$ . In terms of range and empirical content, an empirical statement  $S$  is consistent with  $\Omega$  only when it is not the case that a member of its empirical

---

<sup>2</sup> Ibid., 62-4.

<sup>3</sup> Ibid., 103.

<sup>4</sup> Ibid., 90.

<sup>5</sup> Ibid., 117-8.

<sup>6</sup> Ibid., 124.

<sup>7</sup> Ibid., 112-3.



content obtains.

Range is the converse or complement of empirical content; range specifies the free play or degree of freedom that  $S$  allows to reality.<sup>8</sup> As  $S$ 's range increases, the statement says less about the world insofar as it prohibits a decreasing amount of basic statements. As a measure of range as a proportion of  $\Omega$ , assuming range can be measured as a proportion of  $\Omega$ , a tautology is assigned 1 and a logical contradiction is assigned 0.<sup>9</sup> The former is always consistent with any  $x \in \Omega$ , while the latter is never consistent with any  $x \in \Omega$ . Empirical statements, the third case above, are assigned a fraction, bound exclusively between 1 and 0, corresponding to the size of the range relative to  $\Omega$ . Given two comparable statements such that, regarding ranges,  $S_1 > S_2$ , the less falsifiable statement,  $S_1$ , is the more logically probable statement.<sup>10</sup> Likewise, the better testable statement,  $S_2$ , is the less likely statement in virtue of its logical form. It must be noted that this notion of probability is distinct from that which describes the likelihood or chance of events; however, both notions adhere to the probability calculus.<sup>11</sup>

Following the example above, suppose that the logical probabilities of  $S_1$  and  $S_2$  are .7 and .4, respectively. These probabilities are interpreted such that, in the case of  $S_1$ , the statement will be consistent with an arbitrary  $x \in \Omega$  in 70% of trials and inconsistent in 30% of trials.<sup>12</sup> Suppose, however, that the relative proportions of range and empirical content are unknown for  $S_3$ . For some  $x \in \Omega$ , it is either the case that  $S_3$  will be consistent with  $x$  or that  $S_3$  will be inconsistent with  $x$ ; that is,  $x$  will belong either to  $S_3$ 's range or empirical content but the membership is presently unknown. Thus, by the Principle of Indifference (henceforth PI), the rule "that equal probabilities must be assigned to each

---

<sup>8</sup> Ibid., 124.

<sup>9</sup> This assignment is not wholly arbitrary. Since the union of range and empirical content equals  $\Omega$  and a tautology lacks empirical content, a measure of range for such a statement as a fraction is  $\frac{\Omega}{\Omega} = \frac{1}{1} = 1$ . Likewise, a logical contradiction lacks range; hence, a measure of its range is  $\frac{0}{\Omega} = 0$ .

<sup>10</sup> Ibid., 116-8.

<sup>11</sup> It is not relevant for the purposes of this paper whether or not logical probability can be reconciled or equated with so-called physical probability. Furthermore, this paper assumes the soundness of the Kolmogorovian Probability Axiomatization.

<sup>12</sup> This is not an appeal to the frequency interpretation of probability. To say that  $S_1$  has a probability of .7 is to say that, for some single arbitrary but particular basic statement  $x$ , the likelihood that  $x$  is consistent with  $S_1$  is 70%. A characterization of the frequency interpretation will be developed below in "Alternative Approaches to Probability."

of several arguments, if there is an absence of positive ground for assigning unequal ones,” the probability that  $S_3$  is consistent with  $x$  is  $\frac{1}{2}$ .<sup>13</sup> Assuming independence, this result is generalized for  $n$  basic statements belonging to  $\Omega$ :  $\frac{1}{2^n}$ .<sup>14</sup>

Since NLGs are claims regarding an unlimited number of trials, the probability that an NLG  $h$  obtains is

$$\lim_{n \rightarrow \infty} \frac{1}{2^n} = \frac{1}{2^\infty} = \frac{1}{\infty} = 0.$$

In the case in which  $p(h)$  is supposed to take any value between 1 and 0, exclusively, at the limit as the number of trials tends to infinity,  $p(h)$  approaches 0.<sup>15</sup> Thus, a priori, the probability that any possible NLG obtains is 0.<sup>16</sup> This result is significant insofar as it asserts the falsity of all possible universal empirical statements; additionally, the result substantiates the oft-cited idea: “[Scientists] have to choose between high probability and high informative content, since for logical reasons they cannot have both.”<sup>17</sup>

In computing the limit, it is supposed that the world is “not bounded in time.”<sup>18</sup> This is to say that NLGs cannot be reduced to a conjunction of singular instances; even if it is the case that the world is existentially restricted in time, an NLG will retain the form of a universal conditional statement. Additionally, Popper claims, empirical content and, hence,  $\Omega$  are sets with infinite members.<sup>19</sup> NLGs are not constrained by either scope or depth; yet, the proffered measurement of range supposes that the consistency of an NLG with basic statement  $x \in \Omega$  is tested sequentially. This formulation does not entail temporal differentiation between basic statements. Consider the following: let  $\Gamma$  be a set of

---

<sup>13</sup> John Maynard Keynes, *A Treatise on Probability* (London: Macmillan and Co., 1948), 42.

<sup>14</sup> Popper, *The Logic of Scientific Discovery*, 364-5.

<sup>15</sup> ' $p(h)$ ' is to be read 'the probability that  $h$  obtains.'

<sup>16</sup> *Ibid.*, 363-6.

<sup>17</sup> *Ibid.*, 363.

<sup>18</sup> *Ibid.*, 62-3.

<sup>19</sup> *Ibid.*, 113-5.

singular facts, represented by basic statements, occurring at time  $T$  such that  $\Gamma \subset \Omega$ .<sup>20</sup>  $\Gamma$  represents a cross-section of occurrences at time  $T$ . If  $\Gamma$  is an infinite set, then the probability that  $S_3$  is consistent with  $\Gamma$  is equivalent to the zero probability result above.

## Second Proof

A second proof is offered for the zero probability of NLGs that is largely independent of the first result. Consider a set of basic statements  $E$  that have been observed such that  $E \subset \Omega$ . To explain the occurrence of  $E$ , an NLG will be employed; however, for any finite set of basic statements, there exist infinitely many NLGs that are consistent with the observations.<sup>21</sup> This is to say that  $E$ , the available evidence, underdetermines the resulting explanatory theory.<sup>22</sup> This underdetermination necessarily holds; for any set of evidence with  $n$  members, there exist infinitely many NLGs that are consistent with  $n+1$  many basic statements. For the purposes herein,  $E$  will be assumed to be a finite set.

Consider the following example of evidence underdetermining an explanatory theory: “if all I know is that you spent \$10 on apples and oranges and that apples cost \$1 while oranges cost \$2, then I know that you did not buy six oranges, but I do not know whether you bought one orange and eight apples, two oranges and six apples, and so on.”<sup>23</sup> Since apples and oranges are discrete variables, there are only six possible, mutually exclusive outcomes; however, the evidence does not suggest any one bundle of apples and oranges over the others.<sup>24</sup> If, instead, apples and oranges are continuous variables, then there exist infinitely many possible, mutually exclusive bundles consistent with the empirical evidence. Just as with the discrete case, the evidence does not suggest any one bundle over the others. Hence, Popper concludes, in the latter case, the probability that any particular theory  $h'$  obtains is

---

<sup>20</sup> ' $\Gamma \subset \Omega$ ' is to be read ' $\Gamma$  is a proper subset of  $\Omega$ '

<sup>21</sup> Ibid., 372.

<sup>22</sup> 'NLG' and 'theory' are used interchangeably given their respective appropriate contexts.

<sup>23</sup> Kyle Stanford, “Underdetermination of Scientific Theory” *The Stanford Encyclopedia of Philosophy*, (Winter 2009).

<sup>24</sup> The six possible outcomes are expressed as ordered pairs of the form (apples, oranges): (0,5), (2,4), (4,3), (6,2), (8,1), (10,0).

zero.<sup>25</sup> This is to say that the probability that  $h'$  obtains is given by

$$\lim_{n \rightarrow \infty} \frac{1}{n} = \frac{1}{\infty} = 0$$

where  $n$  is the number of NLGs consistent with  $E$ .

Unlike the first proof, the proof by underdetermination does not require an arbitrarily large, indefinite, or infinite number of basic statements in the universe. All that is required is a finite set  $E$  that is consistent with some NLG. Given that  $E$  is finite, if it is consistent with a single NLG, then it will be consistent with infinitely many NLGs. With this being said, it is necessary either that all NLGs consistent with  $E$  are mutually exclusive or that there are, at least, infinitely many mutually exclusive NLGs that are consistent with  $E$ . This necessity results from considering the generality of theories. Regarding the apples and oranges example, the theories 'one bought 2 apples and 4 oranges' and 'one bought apples and oranges' are consistent; the latter is simply less precise than is the former.<sup>26</sup>

## Falsificationism as a Methodology of Science

The zero probability of NLGs renders null the possibility of confirming the statement through the consideration of empirical evidence. The intuition of confirmation is illustrated through Bayes'

Theorem:  $p(h|e) = \frac{p(e|h)p(h)}{p(e)}$ .<sup>27</sup>  $p(h)$  is the probability of  $h$  prior to the consideration of evidence.

$p(e|h)$  is the degree to which  $h$  predicts evidence  $e$ .  $p(e)$  is the probability that the evidence occurs.

$\frac{p(e|h)}{p(e)}$  illustrates the magnitude of which  $e$  supports  $h$ .  $p(h|e)$  is the posterior probability; it is the

probability of  $h$  conditional on  $e$ .<sup>28</sup> Through the process of rational updating, one considers a series of evidence such that the posterior probability of the first update becomes the prior probability in the second update and so forth.

---

<sup>25</sup> Popper, *The Logic of Scientific Discovery*, 372.

<sup>26</sup> Ibid., 121-3.

<sup>27</sup> A formal derivation from Kolmogorov's Axioms is found in *Howson and Urbach*, 26-7.

<sup>28</sup> Timothy Childers, *Philosophy & Probability* (Oxford, UK: Oxford University Press, 2013), 62-3.

Since both NLGs have a probability of zero in virtue of their logical form and the relation between the meta-variables of Bayes' Theorem is multiplicative, the resulting posterior probability for an NLG is always zero irrespective of the strength of the evidence considered.<sup>29</sup> Science, thus, cannot proceed by the method of confirmation; however, theories can be shown as deductively false through the modus tollens of classical logic.<sup>30</sup> This is to say that  $h$  implies the falsity of the membership of its empirical content; thus, if one such member obtains, then  $h$  is falsified. This methodology is not exclusive to NLGs; it can be generalized to any empirical statement.<sup>31</sup>

Consider an example from the history of science: Galen's Systems Theory. Galen was a Roman physician and philosopher whose theory dominated physiology for nearly 1500 years. He held that the body consists of "three connected systems the brain and nerves, which were responsible for sensation and thought; the heart and arteries, which were responsible for life-giving energy or "vital spirit"; and the liver and veins, which were responsible for nutrition and growth."<sup>32</sup> Galen's system theory, then, represents an NLG. William Harvey and others challenged this view by way of an experiment. Harvey measured the volume of blood in various animals and compared it to the size of the organism's arteries exiting the heart. For Galen, blood left the heart never to return; yet, Harvey's measurements suggested that an organism would empty of blood in under an hour.<sup>33</sup> The observation that an organism does not replenish their blood supply each hour represents a set of basic statements prohibited by Galen's theory; hence, logically speaking, the Systems Theory is falsified by the observations and cannot obtain.

## Principle of Indifference

PI distributes probability equally among possible outcomes. For example, the probability of a fair coin landing 'heads up' is  $\frac{1}{2}$ , while the probability of the same coin landing 'tails up' is  $\frac{1}{2}$ . The total

---

<sup>29</sup> Popper, *The Logic of Scientific Discovery*, 364.

<sup>30</sup> *Ibid.*, 75-7.

<sup>31</sup> Childers, *Philosophy & Probability*, 67.

<sup>32</sup> Stanley Schultz, "William Harvey and the Circulation of the Blood: The Birth of a Scientific Revolution and Modern Physiology." *Physiology* 17, no. 5 (October 2002), 175-180.

<sup>33</sup> *Ibid.*, 177-9.

probability of a trial is the summation of the probabilities of all possible mutually exclusive outcomes. Regarding the coin flip,  $p(\text{heads}) + p(\text{tails}) = 1$ . In general, total probability is represented as

$$\sum_{i=1}^n p(h_i) = p(h_1) + p(h_2) + \dots + p(h_n) = 1$$

where  $h_i$  is a metavariable representing any possible mutually exclusive outcome. The summation extends from the first to the  $n$ th possible outcome to equal a total probability of 1. This indicates that since these are the only possible outcomes, it is necessary that one outcome among this set must obtain.

Hitherto, in the first proof, PI has been assumed only to extend the scope of zero probabilities from the set of NLGs with presently known probability values to all possible NLGs. In developing falsificationism as a methodology of science, Popper treats  $\Omega$  as a sample space, the set of outcomes over which probability is defined.<sup>34</sup> To say that  $p(S_3) = .6$  is to say that the range of  $S_3$  occupies 60% of the area or space span by  $\Omega$ . To define such a spatial relation, each point in the space corresponds to an  $x \in \Omega$ . The spatial relation is consistent only if the area occupied by the range is proportional to the probability that that area represents. This is to say that the distribution of probability over  $\Omega$  is such that each  $x$  must be equiprobable; hence, PI is assumed to compute the probabilities of each  $x \in \Omega$ . For NLGs, to say that their logical probability tends to zero is to say that the corresponding empirical content grows infinitely close to  $\Omega$ .

In the second proof, without the assumption of PI, it is not clear that the probability of each explanatory NLG will equal zero. Though the evidence may underdetermine an NLG, this does not necessarily entail that the set of explanatory NLGs is equiprobable. The only property that underdetermination confers to this set is that at least one member among this set must obtain and that that member is presently unknown.<sup>35</sup> Without the equiprobability assumption by way of PI, no probability distribution can be defined over this set of explanatory NLGs and, resultantly, the zero

---

<sup>34</sup> Childers, *Philosophy & Probability*, 5.

<sup>35</sup> Stanford, "Underdetermination of a Scientific Theory."

probability result does not follow.<sup>36</sup>

The objection advanced in this paper concerns the use of PI to compute objective prior probabilities. Without an appeal to PI, Popper's claim that all theories of this sort are equally unlikely is invalid; specifically, the identity between logical probability and range will be severed. If it is the case that for any NLG  $p(h) \neq 0$ , then the methods prescribed by falsificationism do not follow. Hereafter, the properties of PI will be critically evaluated.

## Distribution Preference

Since PI divides probability equally among the possible outcomes, it features a uniform distribution.<sup>37</sup> This is in contrast to other probability distributions, such as the Normal distribution, in which some outcomes are weighted more heavily than others in terms of likelihood.<sup>38</sup> Distribution preference is determined by the relevant experience and rational expectations of epistemic agents. Relevant experience is defined as observations and other sense experience regarding both the relevant trial and the possible distributions to describe its outcomes. Rational expectations represent the non-arbitrariness of distribution preference; there must exist a sufficient reason to prefer one distribution to another. When flipping a coin, experience provides information that coins land 'heads up' and 'tails up' each nearly half of the time while landing on the edge is a rare occurrence. Rational expectations provide a distribution such that the probability for both a 'heads up' outcome and a 'tails up' outcome each approach  $\frac{1}{2}$ , while the probability of landing on the edge approaches 0.

This distribution for coin flipping approximates a uniform distribution for a fair coin. This need not be the case for all instances of distribution preference, e.g., observations randomly sampled from a

---

<sup>36</sup> Darrell Rowbottom, *Popper's Critical Rationalism: A Philosophical Investigation* (New York: Routledge, 2011), 43-4.

<sup>37</sup> One may contend that PI implies the employment of a different distribution or, instead, a set of distributions; however, it is only relevant that it, as a rule, implies *at least* one such distribution.

<sup>38</sup> Colin Howson and Peter Urbach, *Scientific Reasoning: The Bayesian Approach* (La Salle, IL: Open Court Publishing Company, 1989), 29-31.

large data set usually approximate the Normal distribution.<sup>39</sup> Without relevant experience, the possibility of the coin landing on its edge cannot be discounted, and one need not assume that one side is not favored more highly than the other. Without rational expectations, distribution preference is arbitrary even when informed by relevant experience.<sup>40</sup>

An epistemic agent's relevant experience is a subset of  $\Omega$ . This is to say that relevant experience is expressed as a collection of basic statements. Rational expectations is a relation or transformation from an agent's relevant experience to the set of possible probability distributions. To say that relevant experience and rational expectations determine distribution preference is to say that rational expectations map relevant experience onto the preferred distribution. To prefer one probability distribution to others is to possess the relevant experience, which corresponds to that distribution.<sup>41</sup>

There are two distinct ways of characterizing objectivity regarding probabilities and distributions. The first is a description of seemingly indeterminate events as they are in the world. This view of probability is potentially the ideal eventual posterior probability for any given set of trials.<sup>42</sup> This interpretation is not within the present scope; it will be considered below. The second characterization is the foundational prior probability. Recall that in Bayes' Theorem the posterior probability is the product of both the support of the evidence considered and the prior probability. As new evidence is encountered, the previous posterior probability becomes the new prior probability and so forth. When considering additional evidence, this process of rational updating is forward looking; however, to obtain the foundational prior probability, this process can also function as backward looking. This is to say that by removing the support of all considered evidence, one will obtain the

---

<sup>39</sup> Childers, *Philosophy & Probability*, 171-4.

<sup>40</sup> A practical application of the model for distribution preference is expressed in David Kreps, *A Course in Microeconomic Theory* (Princeton, NJ: Princeton University Press, 1990), 7. Kreps offers the following characterization regarding models: "Based on personal experience and intuition about how things are, does this make sense?" To rephrase his question 'is the model in question consistent with one's preferred distribution?'

<sup>41</sup> Describing rational expectations as a relation does not suggest that each possible distribution corresponds to a unique subset of  $\Omega$ , but, conversely, it does suggest that an agent's relevant evidence determines a single probability distribution.

<sup>42</sup> Childers, *Philosophy & Probability*, 92-5.



foundational prior. This is the objective probability that PI supposedly computes under the condition of ignorance.

A second distinction is necessary: preferences over possible distributions and preferences over possible outcomes. To say that an epistemic agent prefers one outcome to another is to say that the agent has reason to believe that the former outcome is more likely than the latter.<sup>43</sup> In the coin flipping example, for instance, one generally prefers a ‘heads up’ outcome to an ‘on its edge’ outcome since one judges one outcome to be more likely than the other. Preferences over a complete set of outcomes from a single trial can be expressed as a probability distribution.

While PI entails a preference for a uniform distribution, it does not imply a preference over possible outcomes; one is *indifferent* to the possible outcomes of a trial. To say that an epistemic agent prefers one distribution to another is to say that the agent has reason to believe that one set of preferences over outcomes of some trial better represents their relevant experience than another set of preferences over outcomes of the relevant trial. In this sense, a preference over distributions is a metapreference over outcomes. It is this metapreference that PI implies. It must be noted that metapreference is foundational to preference. Rational expectations maps an agent’s relevant experience onto a distribution for some trial. This distribution entails individual likelihoods for each possible mutually exclusive outcome of the trial. Preferences over outcomes are derived, then, from this result.

In order to determine the appropriate probability distribution to describe the possible outcomes of a trial, one must possess both relevant experience and rational expectations. In cases of ignorance, however, epistemic agents lack relevant experience. Resultantly, in cases of ignorance, epistemic agents must lack a preference regarding probability distributions.<sup>44</sup> This suggests that without relevant

---

<sup>43</sup> In general, outcome  $x$  is preferred to outcome  $y$  if and only if  $p(x) > p(y)$ , given by an agent’s relevant experience.

<sup>44</sup> This is to say that the empty set does not map onto the set of possible distributions. Rational expectations does not include the state of ignorance as a member of its domain.

experience there can be no objective prior probabilities. To the contrary, PI purports to compute objective prior probabilities in the face of ignorance; yet, it does so only by giving preference for a uniform distribution over all other possible distributions to describe the trial. In cases of ignorance, one lacks a sufficient reason to prefer one distribution to another; thus, PI cannot obtain in the sense that it describes objective prior probabilities. If it instead describes probabilities that both are not objective, in the relevant sense, and are informed by experience, then PI is not relevant since the epistemic agent is not in a state of ignorance. Either way, PI fails to adequately account for objective prior probabilities in the face of ignorance and, thus, for this purpose, must be rejected entirely.

## Other Conjectures and Refutations of PI

Hitherto, Popper's methodology of science has been developed, and its reliance upon PI has been illustrated. Both the consistency and the relevance of PI have been challenged from an analysis of distribution preference. In the following section, a response will be considered to the objection proffered against PI in this paper. This response will model preferences over outcomes of some trial as placing bets on the likelihood that an outcome obtains. After this response is critically considered, three historically relevant objections to PI will be surveyed: language dependency, the so-called 'Color Paradox,' and Bertrand's Paradox. A recent reformulation of PI as the Maximum Entropy Principle will then be formulated and evaluated critically.

### Bet Making

Consider a situation in which it is mandatory that one place bets on possible disjoint outcomes of a trial. Resulting from relevant experience and rational expectations, the best gambling strategy is to place higher bets on outcomes that one believes are more likely. This yields a subjective probability distribution that is of no consequence to the present evaluation of PI. If a condition of ignorance of the trial is placed on the epistemic agent, then the best gambling strategy is one in which bets are placed

equally on each of the outcomes in order to minimize risk.<sup>45</sup> Any other gambling strategy is arbitrary and is not permitted by rational expectations.

In the case of bet making, the strategy of risk minimization is a sufficient reason for preferring a uniform distribution to other distributions when the agent is ignorant of the trial. This potentially resolves the above worries regarding the lack of distribution preference. This strategy is objective insofar as its implied distribution preference is not relative to a specific trial and is not affected by an agent's prior experience. If one is informed only that there are five possible disjoint outcomes regarding a trial, one of which must obtain, and that they must wager on these outcomes, then the only non-arbitrary betting strategy is to spread the risk of loss and bet equally on each of the five outcomes. As information regarding the nature of the trial is encountered, the betting strategy will be updated to maximize returns. This is analogous to and functions as a potential model for rational belief formation and deliberation. By appealing to bet making, probability distributions of both objective and subjective varieties are derivable. If this method is consistent, then the above worries are resolved.

With this attempt to disentangle the conflict between ignorance and distribution preference, the justification is pragmatic rather than epistemological. The gambling case takes as an assumption that epistemic agents come to decisions *as if* they are placing bets on the possible outcomes of a trial. Packaged with this assumption is the necessity of bet placing; without which, one need not place bets under varying uncertainty. Epistemic justification of distribution preference is determined by relevant experience and rational expectations. The justification in this instance is pragmatic in the sense that it imposes an additional assumption that has implications for the determination of preferences. Since PI seemingly obtains with the gambling case but not without, PI is conditional upon the soundness of this additional assumption.

There exists a conflation between the argument against distribution preference and the bet

---

<sup>45</sup> Childers, *Philosophy & Probability*, 61.

making response. Both approaches specify that a condition of ignorance is assumed to obtain objective prior probabilities; however, there are subtle differences between these specifications. The argument against distribution preference imposes ignorance such that an epistemic agent lacks the relevant experience necessary to have a sufficient reason for a distribution preference.<sup>46</sup> With the gambling case, the agent lacks relevant experience but is aware of both their ignorance and that they are required to place bets. The former formulation is a much stronger condition than is the latter. In the context of gambling under ignorance, the agent lacks experience relevant to the trial; yet, the awareness of ignorance and the betting mandate yields a preference for a uniform distribution when utilizing rational expectations. PI only holds in this case of ignorance because of weakened conditions and additional assumptions on belief and justification. Since the ignorance relevant to the validity of PI is the stronger condition, the bet making response is not applicable to the present consideration.

### Survey of Objections

Several objections and challenges have historically been advanced against PI. Three instances are surveyed below: language dependence, the Color Paradox, and Bertrand's Paradox. For PI, the probability assigned to each possible mutually exclusive outcome of a trial is a function of the number or amount of corresponding outcomes. These possible outcomes are expressed as sentences in a particular language.<sup>47</sup> Consider two languages  $L$  and  $L'$  such that the latter is more expansive than the former. Regarding some trial, there are  $n$  many possible mutually exclusive outcomes as expressed in  $L$ . In  $L'$ , however, there are  $2n$  many possible mutually exclusive outcomes. The probability that an outcome obtains computed by PI in  $L$  is  $\frac{1}{n}$ ; whereas, in  $L'$ , the corresponding probability is  $\frac{1}{2n}$ . This is to say that "linguistic divisions rather obviously need not reflect real divisions."<sup>48</sup> If the result of PI is an objective probability, then there exists only a single likelihood that an outcome obtains, irrespective of

---

<sup>46</sup> Recall that only the empty set, i.e., ignorance, is excluded from the domain of rational expectations.

<sup>47</sup> Keynes, *A Treatise on Probability*, 65.

<sup>48</sup> Childers, *Philosophy & Probability*, 125.

the language employed. This suggests that further criteria are necessary to establish the objectivity of probabilities computed by PI.

The 'Color Paradox' violates the condition, discussed above, that the probabilities of all mutually exclusive outcomes of a trial sum to one. Even more concerning is that this employment of PI potentially yields inconsistent results. Consider a library in which the cover of each individual book is either all red, all blue, or all black.<sup>49</sup> Since there are three mutually exclusive possible outcomes, by PI, the probability that the cover of a random book chosen from a shelf is red is  $\frac{1}{3}$ . Applied to both 'blue' and 'black,' each outcome, likewise, results in a probability of  $\frac{1}{3}$ . Consider, instead, the probability that the cover of a book chosen at random is red such that the possible alternative outcome is that the cover is not red. In this case, PI yields a probability of  $\frac{1}{2}$  for 'red.' Applied to both 'blue' and 'black,' each outcome results in a probability of  $\frac{1}{2}$ .<sup>50</sup>

It should first be noted that the results of the first and second applications are obviously inconsistent:  $\frac{1}{3} \neq \frac{1}{2}$ . This is problematic since if the probabilities computed by PI are objective then either one or both of the applications of PI are in error or PI attributes multiple probabilities to the same outcome. Difficulties exist with the first disjunct; to determine the correct application of PI, one must ensure that the set of possible mutually exclusive outcomes is the set that corresponds to the trial. These difficulties are related to the language dependence objection above. The second disjunct illustrates that PI yields inconsistent results. Either way, this result is problematic.

The second application of PI to the color of book covers violates the summation condition of the probability calculus.<sup>51</sup> This is to say that the summation of the probability of the possible mutually exclusive outcomes exceeds one, the size of the sample space of the trial. If this is the case, then the

---

<sup>49</sup> Ibid., 119-20.

<sup>50</sup> Keynes, *A Treatise on Probability*, 42-4

<sup>51</sup> Childers, *Philosophy & Probability*, 120.

number computed by PI is not a probability measure. In sum, the Color Paradox offers a substantive case against the validity of employing PI to compute objective probabilities.

Bertrand's Paradox refers to an instance in which using PI to compute probabilities regarding continuous measurements of geometric objects results in inconsistency.<sup>52</sup> Consider a manufacturing plant that produces cubes with random side lengths  $l$  such that  $0 \leq l \leq 1$ , in units of feet. This is to say that the probability that  $l$  is less than or equal to 1 or  $p(l \leq 1) = 1$  and, resultantly,  $p(l > 1) = 0$ . By PI, the probability that a cube will be produced such that  $l$  is .5 or less is  $\frac{1}{2}$ . The probability that a cube will be produced such that the surface area,  $l^2$ , is .25 or less is  $\frac{1}{4}$ . The probability that a cube will be produced such that the volume,  $l^3$ , is .125 or less is  $\frac{1}{8}$ .

Rather than particular outcomes, this case considers the probabilities of sets of geometric objects. The largest cube,  $l = 1$ , is the upper bound for all possible cubes produced with a length, surface area, and volume of 1. The range of measurements in each of the three instances above are equivalent; they describe the same set of cubes.<sup>53</sup> With this being said, PI computes three distinct probabilities to describe the same class of objects; hence, the employment of PI produces inconsistent results. These three objections, language dependence, the Color Paradox, and Bertrand's Paradox, present PI with potentially insurmountable difficulties and are offered to supplement the above argument regarding distribution preference.

### Maximum Entropy Principle

PI is a rule for assigning probabilities to empirical statements in which an agent's relevant experience affords no preferences over outcomes. The purpose of this rule is to preserve the partial entailment of the premises to the conclusion; yet, there is no foundational justification present for PI.<sup>54</sup>

---

<sup>52</sup> Note that the following case is a variant of Bertrand's Paradox. This example is chosen for the purpose of illustration. While subtleties exist between the two cases in approach, the result is equivalent.

<sup>53</sup> Ibid., 121-3.

<sup>54</sup> Keynes, *A Treatise on Probability*, 111-2.

A reformulation of PI as the Maximum Entropy Principle (henceforth MEP) is an attempt to provide a foundational justification for a uniform distribution under ignorance. This principle states, “in making inferences on the basis of partial information we must use that probability distribution which has maximum entropy subject to whatever is known.”<sup>55</sup> Entropy is to be understood as uncertainty in the context of information theory; to maximize entropy is to attribute probabilities to the possible outcomes of a trial such that the informative content of each outcome is minimized relative to an agent’s relevant experience.<sup>56</sup>

There exists a tradeoff between informative content and probability with regard to an empirical statement.<sup>57</sup> The truth of a statement that has a probability of 1 does not contain informative content; it is a tautology. Likewise, the truth of a logical contradiction provides boundless information. Between these limiting cases, a statement with greater probability will possess lower informative content and a statement with lower probability will possess greater informative content.<sup>58</sup> This relationship between information and probability is similar to Popper’s deconstruction of empirical statements. Recall that empirical statements can be expressed in terms of basic statements, empirical content, and range. In this case, empirical content is identified with informative content, and, following Popper, range is identified with probability.

MEP provides justification for distribution preference; however, an additional principle is required that relates an agent’s relevant experience with a particular distribution: the Invariance Principle (henceforth IN). This is to say that IN transforms a set of information or relevant experience onto a particular probability distribution for some trial.<sup>59</sup> MEP, in turn, determines the preferred distribution. The conjunction of MEP and IN (denoted by MEP\*) functions approximately identically

---

<sup>55</sup> E. T. Jaynes, “Information Theory and Statistical Mechanics,” *The Physical Review* 106, no. 4, 623.

<sup>56</sup> Childers, *Philosophy & Probability*, 137-9.

<sup>57</sup> This tradeoff is represented as a measure of informative content:  $I(x_i) = -\log_2 p(x_i)$ .  $p(x_i)$  is a probability measure for empirical statement  $x_i$ .

<sup>58</sup> *Ibid.*, 136-7.

<sup>59</sup> E. T. Jaynes, “Where Do We Stand on Maximum Entropy?” in *The Maximum Entropy Formalism* (Cambridge, MA: The MIT Press, 1979), 88.

to rational expectations; however, one significant difference remains. Whereas MEP\* requires that two agents with identical relevant experience sets will possess identical distribution preference, rational expectations requires only that a particular relevant experience set determines a unique preferred distribution. This difference is significant insofar as, with MEP\*, all probability attributions are objective; if all agents possess identical information sets then there would exist universal agreement regarding the likelihood of outcomes of the relevant trial. With rational expectations, recall that only foundational prior probabilities are objective. As experience is considered, objectivity is forfeit.

There are two distinct cases in which MEP\* may determine a uniform distribution over the outcomes of some trial. First, an epistemic agent may possess relevant experience that supports each of the possible outcomes equally. Consider a six-sided die with which the agent has rolled several times. To summarize one's relevant experience: each of the six sides have landed facing up an equal number of times. This experience set justifies the attribution of a uniform distribution over these possible disjoint outcomes, since this distribution minimizes the informative content of the trial.<sup>60</sup> Second, an epistemic agent may lack relevant experience altogether; the agent is thus in a state of ignorance with regard to the trial. The uniform distribution minimizes the informative content with regard to this relevant experience set; hence, by MEP\*, the uniform distribution is the preferred distribution under the condition of ignorance.

Recall that, regarding PI, an epistemic agent must possess both relevant experience and rational expectations to determine distribution preference. If the agent's relevant experience set is non-empty, then PI does not yield an objective probability. If, instead, the agent is ignorant of the relevant trial, then the agent lacks relevant experience and, hence, lacks a distribution preference. MEP\* is a stronger condition than rational expectations; it requires both that an agent prefers one distribution to others and that the preferred distribution possesses the greatest entropy between outcomes, given the relevant

---

<sup>60</sup> Childers, *Philosophy & Probability*, 138.



experience set. This is to say that MEP\* is an instance of rational expectations such that MEP is the rule that determines distribution preference. Given that there are infinite possible distributions, some better than others and some worse than others, there exist other possible instances of rational expectations.<sup>61</sup>

Since MEP\* is an instance of rational expectations, it is subject to the limitations of rational expectations. In the determination of distribution preference, rational expectations maps an epistemic agent's relevant experience set onto a particular distribution; both features are necessary to determine distribution preference. Contrary to proponents of MEP\*, a state of ignorance does not entail distribution preference. This is to say that by reducing MEP\* to an instance of rational expectations it is subject to the same argument posed against PI: lacking relevant experience does not justify a meta-preference over outcomes of some trial. If it is the case that MEP\* yields better preferred distributions, in a normative sense, than other possible instances of rational expectations, as proponents claim, then it remains still not the case that an empty relevant experience set entails a distribution preference.

## Falsificationism Reconsidered

Revisiting Popper's methodology of science, the rejection of PI is detrimental to a central claim of falsificationism: the probability of any known or possible NLG is zero. The equiprobability of basic statements and the resulting zero probability of NLGs depends upon PI to compute objective prior probabilities. Since PI has been demonstrated to be inconsistent, in the sense that it cannot yield objective probabilities under ignorance, the set of all possible basic statements cannot be identified as a sample space. This is to say that the identity between range and logical probability is largely severed thereby eliminating the measure of range as probability. This, likewise, eliminates the measure of

---

<sup>61</sup> 'Better or worse' is defined in terms of relative entailment by the relevant experience set. Consider that distribution A may represent the relevant experience set better than distribution B. Let the relevant experience set regard the rolling of a six-sided die such that outcomes 2, 3, 4, 5, have each been observed once. In this case, a uniform distribution is better entailed relative to the normal distribution. With this being said, there is no distinction of relative entailment between a uniform distribution and a distribution that is uniform except zero is assigned to outcome 6. This is reflective of the problem of underdetermination discussed above.

empirical content, computed as the complementary set to range, i.e., the union of range and empirical content is equivalent to  $\Omega$ , the set of all possible basic statements.

While NLGs can still be reduced or deconstructed to basic statements, range, and empirical content, the number of valid derivations regarding the probability of NLGs and, hence, the significance of this model is reduced substantially. Since a uniform distribution cannot be justifiably assumed to obtain over  $\Omega$ , the possibility exists, for some proper subset of  $\Omega$ ,  $Z$ , that either  $p(Z) = 1$  or  $p(Z) = 0$ .<sup>62</sup> This is to say that the probability of  $Z$  depends upon the distribution that obtains over  $\Omega$ . If it is the case that the summation of the probabilities of all basic statements  $x \in Z$  equals 1, then  $Z$  is logically true. If it is the case, instead, that the summation of the probabilities of all  $x \in Z$  equals 0, then  $Z$  is logically false. If both  $x \in Z$  and  $y \notin Z$  have non-zero probabilities, then  $p(Z)$  is bound, exclusively, between 1 and 0; however, lacking a measure of range, the analysis cannot be extended to compute a precise value.

Importantly, the consequence of the distribution over  $\Omega$  determining the probability of  $Z$  disallows the necessity for the zero probability of all possible NLGs. Given that  $Z$  can have a probability of 1, it is not the case that an arbitrary basic statement  $x \in \Omega$  either belongs to the range or the empirical content for some NLG. The possibility exists that  $Z$  is either a subset of range or a subset of empirical content. In the case in which  $p(Z) = 1$  and  $Z \subset Range$ , it is not possible for a basic statement to obtain such that both  $x \in \Omega$  and  $x \notin Z$ . This is to say that for any basic statement that obtains  $x \in Z$ . Without specifying the distribution of basic statements *ex ante*, it cannot be claimed that an arbitrary basic statement will belong to either range or empirical content, since either subset may have a zero probability. This result illustrates that Popper's first proof for the zero probability of NLGs does not obtain with necessity.

The proof for the zero probability of NLGs from underdetermination relies more directly upon

---

<sup>62</sup>  $Z$  must be a proper subset in the sense that  $Z \neq \Omega$ ; otherwise, the possibility exists for  $Z$  to be equivalent to  $\Omega$  such that, necessarily,  $p(Z) = 1$ , irrespective of the probability distribution that obtains over  $\Omega$ .

PI. The uniform distribution is applied to the set of all possible NLGs that explain or are consistent with the available evidence set  $E$ . Given that this set of explanatory theories is infinite, the equiprobability of individual NLGs results in individual probabilities that each approach zero. In the absence of PI, it is not clear which probability distribution is appropriate to describe this infinite set. The possibility exists that, for an arbitrary NLG  $h'$ ,  $p(h') = .7$  such that all other members of the set have a combined probability of  $.3$ . As with the above result, this result effectively undermines the necessity of the zero probability argument.

With this being said, it remains the case that if a member of the empirical content obtains, then the NLG under consideration is falsified.<sup>63</sup> The methodology is deprived, however, of a foundational objection to both confirmation and inductivism regarding universal statements. The impossibility of confirmation through Bayesian rational updating is no longer the case. This is only to say that, contra Popper, all possible NLGs need not be logically false. A positive case for confirmation is still required, though, if rational updating through Bayes' Theorem is to succeed as an alternative methodology of science to Popper's falsificationism.

This is not to say that the falsificationist project need be abandoned altogether. There may exist reasons for either adopting falsificationism nevertheless or, at least, adopting particular aspects of the methodology.<sup>64</sup> Relying only on results that are demonstrable or reproducible will preserve a notion of objectivity, since practitioners are free to test the respective NLG themselves.<sup>65</sup> Falsification provides an effective, if not complete, distinction between science and non-science or pseudoscience; at the least, the methodology captures many of the features commonly attributed to science as a practice.<sup>66</sup> To both retain the favorable features and account for problematic elements, one may adopt falsificationism as a heuristic rather than a methodology of science. This is to say that one may employ falsificationism to

---

<sup>63</sup> Childers, *Philosophy & Probability*, 73.

<sup>64</sup> For portions of this section, I am indebted to a comment on an early presentation of this paper from Neil Van Leeuwen.

<sup>65</sup> Sergio L. de C. Fernandes, *Foundations of Objective Knowledge: The Relations of Popper's Theory of Knowledge to that of Kant* (New York: Springer, 1985), 166.

<sup>66</sup> Popper, *The Logic of Scientific Discovery*, 40-2.

guide action in the practice of science as opposed to a logical justification of the method of scientific knowledge attainment.

Irrespective of the features of falsificationism retained or adopted, the rejection of PI largely undermines falsificationism as a methodology of science. In the absence of PI, both proofs for the zero probability of NLGs proffered by Popper fail to follow from his analysis. In the first proof, PI is necessary to define probability as a measure of range. In the second proof, PI assigns likelihoods to the set of infinite explanatory theories. The resulting picture of Popper's methodology of science lacks a foundational argument; it is no longer the case that the logical confirmation of NLGs is impossible. Without the necessity of the zero probability of universal statements, falsificationism cannot proceed as an effective methodology of science.

## Objective Probability

In the preceding section, the consequences for falsificationism of rejecting PI were examined. In the present section, the prospect of objective logical probability will be considered and rejected. Without a justified principle of the form of PI, there is no potential for an objective theory of logical confirmation. Insofar as logical probabilities accurately describe probability attributions in the world, those attributions must be subjective. Rather than proffering a theory of subjective confirmation, beyond Bayesian rational updating, it will be argued that such a process can only proceed in the subjective variety.

Recall that logical probabilities are determined by the relevant experience and rational expectations of epistemic agents. If a probability results from a non-empty relevant experience set, then that probability is not objective; it is informed by the agent's, i.e., the subject's, experience. The only case in which a probability can be objective is when the relevant experience set is empty. In the case of ignorance, however, the epistemic agent lacks a distribution preference and, resultantly, no probability for an outcome is derivable. This is to say that in the instance in which the requisites for objective

logical probability are met no such probabilities exist.

Put into the context of Bayesian rational updating, a non-arbitrary, foundational prior probability is necessary for further results to be justified; otherwise, the possibility of the attribution of a prior probability of either 1 or 0 could not be excluded.<sup>67</sup> In cases in which an agent has relevant experience regarding the trial, a foundational prior is determined through rational expectations. As it is informed by experience, this foundational prior is subjective. It is only in the case of ignorance that the foundational prior is equivalent to an objective prior; however, when the relevant experience set is empty, no corresponding probability exists. It follows, then, that there can be no objective foundational prior probabilities.

Without objective foundational prior probabilities, confirmation cannot proceed objectively; yet, the possibility for a theory of subjective confirmation exists. Rather than develop a theory of subjective confirmation, which is outside of the scope of this paper, it will be suggested that foundational prior probability attributions that describe the probability of events in the world must be subjective. Consider the backward looking function of rational updating: the effect on the likelihood of an outcome resulting from encountered evidence is removed, and the resulting probability of an outcome approaches its foundational prior probability. The quantitative application of this method computes a number bound, exclusively, between 1 and 0.<sup>68</sup> As this numerical quantity, the foundational prior probability, both exists and is a logical probability, it can only be a subjective probability. This subjectivity results from the informed rational expectation of an epistemic agent.

---

<sup>67</sup> Childers, *Philosophy & Probability*, 93-4. Two cases are problematic for the arbitrary selection of prior probabilities: 1 and 0. When  $p(h) = 1$ ,  $p(e|h) = p(e)$  since  $h$  obtains irrespective of  $e$ . Hence, the ratio illustrating the extent to which the evidence confirms the hypothesis  $\frac{p(e|h)}{p(e)} = 1$ . By this result,  $p(h|e) = 1$ . Since the posterior probability can never diverge from 1, arbitrarily choosing this prior probability disallows the possibility of any change in the posterior probability. Likewise, with a prior probability of 0, the resulting posterior probability equals 0, irrespective of the evidence considered. This latter prior illustrates the strength of Popper's claim for NLGs. The possibility of either of these prior probabilities is problematic for the arbitrary selection of priors.

<sup>68</sup> For the purpose of computation, consider the inverse relation of Bayes' Theorem:  $p(h) = \frac{p(h|e)p(e)}{p(e|h)}$ .

To denote a probability as subjective is not to diminish the value of the claim; rather, it is to recognize two features of the attribution. First, the foundational prior probability that results in the present posterior probability is not arbitrarily assigned. It is determined by the relevant experience and rational expectations of an epistemic agent. Second, this probability is either the result of the rational expectation by an epistemic agent or the posterior probability of rational updating. This is to say that subjective logical probabilities represent reality insofar as the evidence employed in their determination is representative of reality. Subjective probability attributions need not be relegated to psychologistic statements, as Popper, for instance, claims.<sup>69</sup> Put another way, denying the possibility of objective logical probabilities is not either to deny the existence of truth, however construed, or to embrace a radical notion of subjectivity.

In this section, it has been demonstrated that inasmuch as logical probability is an accurate interpretation of probability attributions those attributions can only be subjective. This is not to say that all possible statements or interpretations of probability must, therefore, be subjective. Moreover, subjectivity is not intended as a placeholder for epistemological skepticism; logical probability statements must conform to the analysis hitherto detailed. In the section that follows, possible candidates for objective probability will be surveyed.

## Alternative Approaches to Probability

To complement the discussion of objective probability above, two alternative approaches to probability, distinct from the logical interpretation developed hitherto, will be considered in this section. The frequency and propensity interpretations of probability describe probability attributions as objective, physical states of the world, not relative to the epistemic properties of an agent. The frequency account to be considered is that of Richard von Mises, and the propensity theory is that of

---

<sup>69</sup> Popper, *The Logic of Scientific Discovery*, 31-2.

Karl Popper.

The frequency interpretation treats probability claims as not “an assertion about the next [trial]; rather, it is an assertion about a whole class of [trials] of which the next [trial] is merely an element.”<sup>70</sup> This is to say that probability attributions claim that, regarding a collection of mass phenomena, there exists a rate at which the relevant outcome obtains relative to all other possible outcomes; given a sufficiently large set, this rate is equivalent to the probability of the outcome.<sup>71</sup> Consider the case of coin flipping. Given a sufficiently large collection of coin flips, the probability that ‘the coin lands heads up’ is the relative frequency of heads up outcomes to total outcomes. This probability says nothing about the outcome of the next coin flip; rather, given a sufficiently large collection of coin flips, the frequency of heads up to total outcomes is expected to obtain.

To define a numerical probability on a collection of mass phenomena, two conditions must hold for the collective. First, the axiom of convergence holds that as the number of trials belonging to the collective increases the relative frequency of relevant outcomes to total outcomes tends toward a definite limit. By obtaining frequencies for each possible outcome, one obtains a distribution.<sup>72</sup> In the case of a prototypical coin, the rate at which the coin lands heads up relative to total outcomes tends toward 0.5. Second, the axiom of randomness holds that individual outcomes must be unpredictable, despite the existence of a limiting frequency for the collective. Purchasing patterns for goods that exhibit seasonality, such as with both firewood and air conditioners, fail to meet the condition of randomness and, hence, cannot formally be a collective.<sup>73</sup> Though these two axioms are potentially contentious, they will be assumed to obtain in this presentation.

While the frequency theory of probability presents a seemingly objective and elegant account of

---

<sup>70</sup> Popper, *The Logic of Scientific Discovery*, 149.

<sup>71</sup> Childers, *Philosophy & Philosophy*, 2-4.

<sup>72</sup> Popper, *The Logic of Scientific Discovery*, 153.

<sup>73</sup> Childers, *Philosophy & Probability*, 7.

probability, it is not without potential worries; two will be presented below. First, probability is limited to instances in which indefinitely many, homogenous, in the relevant respect, trials can potentially occur. Von Mises offers three categories of application: games of chance, mass social phenomena, and mechanical physical interaction.<sup>74</sup> The probability that a particular idiosyncratic event would occur is, thus, not defined under this interpretation. Second, frequencies are relative to the set of specified mutually exclusive outcomes of the trial; specifying more or less outcomes of a trial potentially changes the probability obtained.<sup>75</sup> If specified outcomes of the trial are not both complete and accurate, then the resulting relative frequency will not objectively reflect the event.<sup>76</sup>

The propensity interpretation of probability purports to offer an alternative objective physical theory to the frequency interpretation. The motivation for this approach is to offer an account of physical probability for finite collectives that is independent of the epistemic states of agents.<sup>77</sup> If this probability applies to finite collectives, then a collective may be specified as a single, idiosyncratic event. Probabilities are taken to describe both deterministic and indeterministic features of the world. This is to say that if an outcome has a probability of either 1 or 0 then the generative conditions of the trial determine the outcome. If the probability is bound, exclusively, between 1 and 0, then the outcome of the trial is indeterminate.<sup>78</sup>

Despite allowing for the possibility of probabilities for single-case phenomena, the propensity interpretation both inherits the worries of the frequency approach and uniquely generates additional difficulties.<sup>79</sup> This position is seemingly committed to an indeterministic view of the world, a position that is potentially problematic; however, depending on how the reference class of generative conditions

---

<sup>74</sup> Richard von Mises, *Probability, Statistics and Truth* (New York: The Macmillan Company, 1957), 9-10.

<sup>75</sup> Note that this worry is equally applicable to PI.

<sup>76</sup> Childers, *Philosophy & Probability*, 36-8.

<sup>77</sup> Popper, *The Logic of Scientific Discovery*, 155.

<sup>78</sup> Childers, *Philosophy & Probability*, 34-6.

<sup>79</sup> Rowbottom, *Popper's Critical Rationalism*, 75-6.



is specified, any propensity can be either deterministic or indeterministic. This places an epistemological burden on the agent of making such a specification, and resultantly, challenges the notion of the objectivity of propensity attributions.<sup>80</sup>

This survey of two alternative interpretations of objective probability illustrates that although other prospects for objectivity exist those prospects are, themselves, problematic. The frequency and propensity interpretations provide a stark contrast with logical probability insofar as the former theories describe physical events while the latter theory describes the partial entailment or the degree of justified rational belief of propositions.<sup>81</sup> With this being said, each interpretation faces difficulties with regard to defining and maintaining objectivity independent of the epistemic states of an agent. The frequency and propensity interpretations are both relative to the reference class specified by the epistemic agent; although, the propensity theory's case is more severe. For the logical interpretation, a measure is not defined for the case in which probability would be objective.

## Conclusion

It has hitherto been illustrated in this paper that the Principle of Indifference is not a consistent method to compute objective logical probabilities. Distribution preference is determined by the relevant experience and rational expectations of epistemic agents. If agents possess relevant experience, then the resulting probability distribution preference is subjective. In the absence of relevant experience, ignorance, distribution preferences are undefined; hence, any preference under the condition of ignorance is arbitrary. From the rejection of the Principle of Indifference, several corollary results are considered.

The derivation of the above result is framed within an analysis of Karl Popper's methodology of science. Central to falsificationism is the claim that all scientific theories, expressed as universal

---

<sup>80</sup> Childers, *Philosophy & Probability*, 39.

<sup>81</sup> Popper, *The Logic of Scientific Discovery*, 148-9.

statements and denoted as non-tautological law-like generalizations, have a logical probability of zero. Two proofs are considered for this result. First, given that universal statements apply to an unlimited number of instances and, for any empirical scientific theory, an arbitrary fact about the world is either consistent or inconsistent with the theory, the limit as the number of instances considered grows arbitrarily large tends toward zero. This is to say the resulting logical probability is zero. Second, for any finite set of evidence, infinitely many scientific theories are consistent; hence, for an arbitrary theory, the resulting probability at the limit tends toward zero. Each of these proofs rely foundationally upon the Principle of Indifference; as the Principle of Indifference has been demonstrated to be inconsistent to compute objective probability, the zero probability result does not follow.

Two objections to the main result are considered. First, by modeling distribution preference as betting strategies under the conditions of ignorance and mandatory bet making, the only non-arbitrary distribution is the uniform distribution; hence, the Principle of Indifference is potentially justified. This response employs a weaker condition of ignorance by assuming mandatory bet making and is not relevant to the evaluation of the computation of objective probabilities. Second, the combination of the Invariance Principle and the Maximum Entropy Principle yields a preference for distributions that maximize uncertainty relative to an epistemic agent's relevant experience set. Given the condition of ignorance, the distribution that maximizes the uncertainty is the uniform distribution, hence, a second potential justification for the Principle of Indifference. The Invariance Principle is a stronger instance of rational expectations and, resultantly, is subject to the properties of rational expectations; therefore, with an empty relevant experience set, distribution preference is undefined.

To strengthen the case against the consistency of computing objective probabilities with the Principle of Indifference, three historical objections are considered: language dependence, the Color Paradox, and Bertrand's Paradox. In light of the arguments posed against the Principle of Indifference, the possibility for objective logical probability is rejected. Since objectivity can only arise under the condition of ignorance, any distribution preference will, thus, be arbitrary and not permitted by rational

expectations. Insofar as the logical interpretation accurately describes probability attributions, those attributions can only be subjective. The frequency and propensity interpretations of probability face similar difficulties in maintaining objectivity.

## Bibliography

- Childers, Timothy. *Philosophy & Probability*. Oxford, UK: Oxford University Press, 2013.
- Howson, Colin, and Peter Urbach. *Scientific Reasoning: The Bayesian Approach*. La Salle, IL: Open Court Publishing Company, 1989.
- Jaynes, E. T. "Information Theory and Statistical Mechanics," *The Physical Review* 106.4, 620-30.
- Jaynes, E. T. "Where Do We Stand on Maximum Entropy?" in *The Maximum Entropy Formalism*. Cambridge, MA: The MIT Press, 1979.
- Keynes, John Maynard. *A Treatise on Probability*. London: Macmillan and Co., 1948.
- Kreps, David. *A Course in Microeconomic Theory*. Princeton, NJ: Princeton University Press, 1990.
- L. de C. Fernandes, Sergio. *Foundations of Objective Knowledge: The Relations of Popper's Theory of Knowledge to that of Kant*. New York: Springer, 1985.
- Popper, Karl. *The Logic of Scientific Discovery*. New York: Harper and Row, Publishers, 1968.
- Rowbottom, Darrell. *Popper's Critical Rationalism: A Philosophical Investigation*. New York: Routledge, 2011.
- Schultz, Stanley. "William Harvey and the Circulation of the Blood: The Birth of a Scientific Revolution and Modern Physiology." *Physiology* 17.5, October 2002, 175-80.
- Stanford, Kyle. "Underdetermination of Scientific Theory." *The Stanford Encyclopedia of Philosophy*, edited by Edward N. Zalta, Winter 2009.
- von Mises, Richard. *Probability, Statistics and Truth*. New York: The Macmillan Company, 1957.