The General Equilibrium Incidence of Environmental Mandates

Garth Heutel  
*Georgia State University, gheutel@gsu.edu*

Don Fullerton  
*University of Illinois at Urbana-Champaign, dfullert@illinois.edu*

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The General Equilibrium Incidence of Environmental Mandates

By Don Fullerton and Garth Heutel*

Pollution regulations affect factor demands, relative returns, production, and output prices. In our model, one sector includes pollution as an input that can be a complement or substitute for labor or capital. For each type of mandate, we find conditions where more burden is on labor or on capital. Stricter regulation does not always place less burden on the better substitute for pollution. Also, restrictions on pollution per unit output create an "output-subsidy effect" on factor prices that can reverse the usual output and substitution effects. We find analogous effects for a restriction on pollution per unit capital. (JEL H23, Q53, Q58)

Much literature compares the efficiency properties of environmental policies, generally finding that incentives like taxes or permits are more cost-effective than mandates—at least in the case where firms are heterogeneous and government cannot tailor mandates to each firm. In contrast, the literature on distributional effects is limited. Some papers identify demographic characteristics or locations of households that are differentially affected by environmental protection, while others look at the burdens on households that buy products made more expensive. All of these papers ignore effects of environmental policies on the wage rate and the return to capital—both of which also affect real incomes. Yet, restrictive command and control (CAC) regulations can simultaneously affect both product prices and factor prices.

Of course, the public economics literature since Arnold C. Harberger (1962) is replete with analytical general equilibrium studies of tax incidence. A few papers look at incidence of environmental taxes, where the question is about how the burden of collecting the revenue is distributed. Mandates do not have revenue for which the...

* Fullerton: Finance Department and Institute of Government and Public Affairs, University of Illinois, 4030 BIF Box #30 (MC520), 515 East Gregory Drive, Champaign, IL 61820 (e-mail: dfullert@illinois.edu); Heutel: Department of Economics, University of North Carolina at Greensboro, PO Box 26165, Greensboro, NC 27402-6170 (e-mail: gaheutel@uncg.edu). We are grateful for funding from the University of Texas, the National Science Foundation (NSF), and Japan’s Economic and Social Research Institute (ESRI). For helpful suggestions, we thank Spencer Banzhaf, Larry Goulder, Carol McAusland, Hilary Sigman, Kerry Smith, Rob Williams, and many seminar participants.

† To comment on this article in the online discussion forum, or to view additional materials, visit the articles page at http://www.aeaweb.org/articles.php?doi=10.1257/pol.2.3.64.


2 The incidence of a pollution tax is studied by, e.g., Gary Yohe (1979), A. Lans Bovenberg and Lawrence H. Goulder (1997), Swee Chua (2003), and Fullerton and Heutel (2007).
burden can be distributed. Yet CAC mandates clearly interfere with firms’ decisions about use of labor, use of capital, the amount to produce, and the price to charge. We therefore find it surprising that we cannot find any analytical general equilibrium model of the incidence of nonrevenue-raising environmental regulations, with simultaneous effects on the sources side (relative factor prices) and the uses side (product prices).\(^3\) Compuetable general equilibrium (CGE) models calculate effects of regulations on factor prices as well as goods prices.\(^4\) These CGE studies can provide elaborate simulations with numerical magnitudes, and some include simple analytical models. But none provides a general analytical model with closed form solutions for any parameter values that show effects of mandates on the wage, return to capital, and output prices.

To begin such a literature, this paper starts with rudimentary models in the style of Harberger (1962), with two competitive sectors and constant returns to scale, but we add the important complication that the “dirty” sector uses three inputs: labor, capital, and pollution. Thus, any two inputs can be complements or substitutes. The “clean” sector uses only labor and capital, which are in fixed supply, but perfectly mobile between sectors. We then solve models representing multiple types of nonrevenue-raising policies, including a “relative” standard on pollution per unit of output, and a relative standard on pollution per unit of an input (such as capital).

In this simple model with perfect certainty, a mandate to restrict the quantity of pollution raises the pollution price in a way that is fundamentally similar to a pollution tax. Since the tax is analyzed in Fullerton and Heutel (2007), we relegate to the Appendix our analyses of quantity restrictions (permits or quotas). The key intuition for these quantity restrictions, however, is that their incidence on labor or capital may be understood using the same two effects already identified by Peter M. Mieszkowski (1967). First, the “substitution effect” raises the return to whichever factor is a better substitute for pollution (and injures the relative complement to pollution). Second, the “output effect” reduces the return to whichever factor is used intensively in the regulated sector.

We then extend the simple model to consider relative standards. Often regulators limit the ratio of emissions to output, a restriction that firms can meet partly by reducing emissions in the numerator and partly by increasing output in the denominator. We derive closed form solutions for each price change, and we identify an additional term we call the “output-subsidy effect.” This effect helps whichever factor is intensively used in the dirty sector, and it dominates the usual output effect under plausible conditions we identify. The important point is that government-imposed costs on the dirty sector may place less burden on the factor intensively used there. Finally, we model a restriction on the ratio of emissions to capital, where solutions are used to identify another new term we call the “capital-subsidy effect.” Since this restriction can be satisfied partly by increased use of capital, this effect

\(^3\) In an analytical trade model with fixed output prices, Monica Das and Sandwip K. Das (2007) show the effect on factor prices from one type of mandate (a pollution quota). Here, we find both factor prices and output prices in a closed economy for multiple types of mandates. Yet results would differ in open economy.

\(^4\) Examples include Michael Hazilla and Raymond J. Kopp (1990); Dale W. Jorgenson and Peter J. Wilcoxen (1990); Goulder, Ian W. H. Parry, and Dallas Burtraw (1997); Goulder et al. (1999); Burtraw et al. (2001); V. Kerry Smith et al. (2004); Carolyn Fischer and Alan K. Fox (2007); and Jared C. Carbone and Smith (2008).
tends to raise the relative return to capital (or, more generally, the return to any input in the denominator of the restricted ratio). We also find effects on the output prices to show burdens on the uses side of income.

Thus, we see standard principles of tax incidence at play, many of which have been known since Harberger (1962), but we introduce other effects specific to mandates. And this analysis can be applied beyond environmental policy to any restriction on use of inputs. For example, our three inputs could be interpreted as labor, capital, and land. Then agricultural policy may restrict use of land per unit output, or urban zoning rules may restrict building heights (capital per unit land).\footnote{Some have studied general equilibrium impacts of land use regulations (e.g., Robert E. Lucas, Jr. and Esteban Rossi-Hansberg 2002) and the effect of zoning constraints on the ratio of land to structure (e.g., Richard Arnott 2005, Alain Bertaud and Jan K. Brueckner 2005). These papers consider some forms of redistribution, such as effects on land prices across locations, but they do not consider distributional effects on returns to capital and labor.}

Section I reviews mandates in actual policymaking, and it reviews some of the literature analyzing them. Section II then introduces our model. Section III considers a restriction on pollution per unit of output, and Section IV restricts pollution per unit input. Section V provides numerical magnitudes, and Section VI offers concluding remarks.

I. Review of Environmental Mandates and Modeling

Since the beginning, environmental regulations have most often been command-and-control (CAC) mandates rather than incentives like taxes or tradable permits. Those mandates, though, can take many forms for different industries. Historically, the Water Quality Act of 1965 was the first policy requiring that states set water quality standards, determine maximum discharge limits, and allocate nontradable quotas. Thus, it is an absolute quantity restriction. Then the Federal Water Pollution Control Act of 1972 authorized the EPA to set effluent standards based on technological factors. Each facility faced restrictions based on the type of facility and perhaps on capital use or output.

The 1970 Clean Air Act Amendments (CAAA) set national ambient air quality standards. States that do not meet them are forced to create specific implementation plans. These plans differ greatly from each other. The Clean Air Act is hard to model simply, except perhaps for new facilities under the New Source Performance Standards of the 1970 CAAA. Like the water standards, these air pollution regulations are technology-based, that is, determined by the current state of abatement technology. Recently, tradable emissions permits are becoming more popular, including the 1990 CAAA US market for sulfur dioxide (SO$_2$) and the market in the northeast for nitrogen oxide (NO$_X$).

While quotas and permits set the absolute quantity of emissions, other mandates set emissions per unit of output or per unit of some input. This approach may be seen as more reasonable than absolute limits, especially when firm sizes differ. A regulator would not expect a large firm and small firm to have the same emissions. By enacting a relative policy, the regulator can avoid deciding on a specific allocation of emissions. Because of the variety of mandates under different state plans,
it is difficult to say what policies have this relative form. In a survey of regulators, however, 97 percent of air pollution agencies and 100 percent of water agencies said they use limits on emissions per unit of some input, and 70 percent of air and 50 percent of water agencies said they limit emissions per unit output (Clifford S. Russell, Winston Harrington, and William J. Vaughan 1986, 19). Such large proportions suggest modeling some policies as limits on ratios of pollution to output or to an input.

Current regulations are complex. Table 1 summarizes how various environmental policies fall into three categories: limits on emissions quantities, standards on emissions per unit output, and standards on emissions per unit input. Permit systems appear in the first column as limits on quantity. In the second column, an example is for producers of sulfuric acid, whose emissions of SO$_2$ are limited to 2 kg per metric ton of acid produced. In the third column, Texas and federal limits on electricity generation apply to emissions of particulate matter per unit heat input (e.g., 43 nanograms per joule of heat).

We cannot incorporate all of these different types of mandates in a single model with clear analytical results. We can, however, model a few types of mandates and compare results, to see their differential impacts. For example, we model technology mandates and per facility standards as limits on the amount of pollution per unit capital. Our model can also be applied to other policies such as the limit on fertilizer per hectare, by reinterpreting our input of “capital” as input of “land.”

The economics literature often uses a tax on pollution to summarize the effects of all environmental policies. Some papers look at quotas or permits that restrict absolute amounts of pollution. Yet actual policy rarely employs a pollution tax, and mandates typically restrict emissions per unit output or per unit of an input. The few
studies of relative standards are focused on economic efficiency, not distributional effects. The most exhaustive theoretical analysis of different environmental mandates is in Gloria Helfand (1991). Her output uses a “dirty” input and a “clean” input. The mandates considered are: a limit on emissions, a limit on output, an upper limit on the dirty input, a lower limit on the clean input, and limits on the ratio of emissions to output or to an input. She compares their effects on output, each input, and firm profits. She finds that the output restriction most reduces input and output levels. The restriction on pollution itself yields the highest profits. In most cases, the signs of these changes depend on the form of production. One counterintuitive result is that a standard per unit output may actually increase total emissions; the same result may occur from a limit on total output.

While Helfand (1991) provides a number of valuable insights regarding the differences among various mandates, she does not address incidence. In fact, her input supply curves are horizontal, so no policy can affect factor prices. Furthermore, the two inputs are a clean and dirty input. Even as these two input prices change, the implications are unclear for returns to labor and capital. Here, we model production as using capital, labor, and pollution. These inputs have endogenous prices, so we capture the differential effects of environmental standards on their relative returns.

II. The Basic Model

Our model is similar to that in Fullerton and Heutel (2007), where we analyze a tax on emissions. For ease of exposition, we start here with the simplest version where emissions, \( Z \), have a price, \( p_Z \), which directly applies to a policy of tradable permits. The following two sections then explain changes necessary to model each mandate. In this model, as in many papers since Harberger (1962), we assume a closed economy with many identical firms, perfect competition, fixed factor supplies, perfect mobility, and no uncertainty. We compare two equilibria rather than

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6 Ethan Hochman and David Zilberman (1978) model standards as limits on emissions per unit output or per unit input. Jon D. Harford and Gordon Karp (1983) compare the two policies and find that a standard per unit output is more efficient than a standard per unit input. Similarly, Vinod Thomas (1980) compares the welfare costs of different policies. Fullerton and Metcalf (2001) model a technology restriction as a limit per unit output. Per G. Fredriksson, Herman R. J. Vollebergh, and Elbert Dijkgraaf (2004) model environmental policy as a limit on the energy-capital ratio, citing the Corporate Average Fuel Economy Standards. None of these studies investigate distributional impacts.


8 Helfand and Brett W. House (1995) empirically estimate the costs of different environmental policies for lettuce growers in California’s Salinas Valley. They find that mandates reduce farm profits less than do taxes.

9 The assumption of homogeneous firms within a sector means that marginal compliance costs are equated across firms (as would policies with tradable credit provisions). Differing marginal compliance costs under non-tradable mandates would raise overall burdens, but not necessarily have much effect on the distribution of burdens. Factor mobility rules out irreversible investment, which could matter for incidence. Such capital is essentially a fixed factor that can bear specific burdens, such as on stockholders in that industry.
the transition period between them. If capital has time to move between sectors, then why does it not have time to grow? The best interpretation is not that we move from one equilibrium to the other, but that we compare two different states of the world—one that has always faced a particular mandate and the other with the same capital that has always faced a different mandate.

The model has two sectors. One sector produces a clean good $X$, with price $p_X$, in a constant returns to scale (CRS) function using capital and labor, $X = X(K_X, L_X)$. Totally differentiate this production function and the zero-profits condition, to get

1. $\hat{X} = \theta_{XX} \hat{K}_X + \theta_{XL} \hat{L}_X$
2. $\hat{p}_X + \hat{X} = \theta_{XX}(\hat{r} + \hat{K}_X) + \theta_{XL}(\hat{w} + \hat{L}_X)$,

where a hat over any variable represents a proportional change (e.g., $\hat{X} \equiv dX/X$). Also, $r$ is the rate of return, $w$ is the wage, and $\theta_{Xi}$ is the share of production for factor $i$ in sector $X$ (e.g., $\theta_{XX} \equiv rK_X/Xp_X$). Then we use the definition of $\sigma_X$, the elasticity of substitution in production between capital and labor to get

3. $\hat{K}_X - \hat{L}_X = \sigma_X(\hat{w} - \hat{r})$.

The other sector produces a dirty good $Y$, with price $p_Y$, in a CRS production function, $Y = Y(K_Y, L_Y, Z)$. Here, we use input demand equations for each of the three inputs based on all three input prices ($r$, $w$, and $p_Z$). An existing restriction on pollution implies an initial $p_Z > 0$. Differentiate the three input demand equations and use the fact that only two of the three are independent to get

4. $\hat{K}_Y = a_{KK} \hat{r} + a_{KL} \hat{w} + a_{KZ} \hat{p}_Z + \hat{Y}$
5. $\hat{L}_Y = a_{LK} \hat{r} + a_{LL} \hat{w} + a_{LZ} \hat{p}_Z + \hat{Y}$,

where $a_{ij}$ is the elasticity of demand for factor $i$ with respect to the price of factor $j$. R. G. D. Allen (1938) shows that $a_{ij} = \theta_{Yi} e_{ij}$, where $e_{ij}$ is the Allen elasticity of

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10 Using a CGE model, Goulder and Lawrence Summers (1989) analyze transitions due to imperfect factor mobility. Adding imperfect mobility or adjustment costs can alter the equilibrium outcome, not just the transition. Also, a dynamic model would allow new policies to apply only to new sources or vintages of investment.

11 Aggregation to two sectors is not meant to be “realistic,” but to aid intuition and interpretation, as in the original Harberger model. For a disaggregated CGE model of US taxes, Tyler Fox and Fullerton (1991) show that key results are almost identical when aggregated to two key sectors. Their focus is on numerical results. Here, we focus on analytical solutions and on identifying the primary effects of mandates on factor prices, so the need for interpretable results dominates the need for realistic quantitative results.

12 Our model does not distinguish between goods that cause pollution when produced (e.g., electricity) or when consumed (e.g., gasoline). This distinction may matter in some models, but not in our case. Our dirty sector’s output can be interpreted as the aggregation of electricity and vehicle miles traveled (VMT), where the latter is in effect “produced” by the driver using inputs of vehicle ($K$), time ($L$), and pollution ($Z$).

13 See Mieszkowski (1972) for an early use of this method. Stability conditions for the input demand system are satisfied so long as the own-price elasticities $a_{ii}$ are negative, as assumed here.
substitution between inputs $i$ and $j$. For this sector, differentiation of production and of the zero-profits condition yield

$$\dot{Y} = \theta_{YK}\dot{K}_Y + \theta_{YL}\dot{L}_Y + \theta_{YZ}\dot{Z},$$

$$\hat{p}_Y + \dot{Y} = \theta_{YK}(\hat{r} + \dot{K}_Y) + \theta_{YL}(\hat{w} + \dot{L}_Y) + \theta_{YZ}(\hat{p}_Z + \dot{Z}).$$

The resource constraint for capital is $K_Y + K_Y = \bar{K}$, where $\bar{K}$ is the fixed total capital stock. Differentiation of that and the analogous labor constraint yields

$$\dot{K}_X\lambda_{KX} + \dot{K}_Y\lambda_{KY} = 0$$

$$\dot{L}_X\lambda_{LX} + L_Y\lambda_{LY} = 0,$$

where $\lambda_{ij}$ is sector $j$’s share of input $i$ (e.g., $\lambda_{XX} \equiv K_X/\bar{K}$). We define $\gamma_K \equiv K_Y/K_X$ and $\gamma_L \equiv L_Y/L_X$, so the dirty sector is capital-intensive whenever $(\gamma_K - \gamma_L) > 0$.

Finally, preferences are modeled using the definition of $\sigma_u$, the elasticity of substitution in utility:

$$X - \dot{Y} = \sigma_u(\hat{p}_Y - \hat{p}_X).$$

The clean good is chosen as numeraire, so $\hat{p}_X$ is fixed at zero, and we have 10 equations for 11 unknown changes: $\dot{K}_X, \dot{K}_Y, \dot{L}_X, \dot{L}_Y, \hat{w}, \hat{r}, \dot{X}, \hat{p}_Y, \dot{Y}, \hat{p}_Z, \dot{Z}$. For each policy below, we specify one more equation or exogenous policy change. For example, authorities can reduce allowed pollution by a small amount (e.g., $\dot{Z} = -0.1$). The system (1)–(10) can then be used to solve for the ten remaining unknowns, by successive substitution. The steps are omitted but may be requested from the authors. Here, we report only the solutions for $\hat{r}, \hat{w}, \hat{p}_Z$, and $\hat{p}_Y$. The first three of these determine the sources-side incidence of the policy, and the last determines the uses-side.

Because $X$ is produced with no excess profit using only labor and capital, and its output price is fixed by assumption, $r$ and $w$ cannot both move in the same direction. If $\hat{r} = \hat{w} = 0$, the implication is not that factors bear no burdens. Rather, since $p_Y$ may rise, $\hat{r} = \hat{w} = 0$ means that labor and capital bear real burdens in proportion to their shares of national income. Hence, a positive value for $\hat{r}$ just means that capital bears a burden that is proportionally less than that of labor.

The solution for a quantity mandate is presented in the Appendix, from which we take the following key intuition (based on Mieszkowski 1967). The “substitution effect” includes the term $(e_{KZ} - e_{LZ})$. It reduces $r/w$ if labor is better than capital as a substitute for pollution. The “output effect” includes $\sigma_u(\gamma_K - \gamma_L)$; the mandate

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14 Since total capital and labor are fixed, our model cannot comment on how “green” policies might affect total investment or employment. It still sheds light on the “green jobs” debate, however, since we find effects on the economy-wide wage (from the overall demand for labor). The claim that environmental policy can favor green jobs is more valid if labor is a better substitute for pollution than is capital.

15 Capital’s burden could be small or zero if its quantity could react via long-run savings or via international mobility (Jane G. Gravelle and Kent A. Smetters 2006). The big looming problem of climate change may induce many or all nations to act, however, reducing capital’s ability to escape the burden.
raises costs, raises the equilibrium output price, and reduces output in a way that depends on consumer preferences $\sigma_u$. It reduces $r/w$ if the dirty industry is capital-intensive, $(\gamma_K - \gamma_L) > 0$.

III. “Performance Standard”: Emissions per unit Output

An alternative form of environmental policy is to limit the ratio of emissions to output, a policy we call a “performance standard.” With heterogeneous firm sizes, at least some consideration of this ratio seems necessary for a plausible policy. A large producer cannot reasonably be expected to achieve the same limit on emissions as a small firm. Considerations like these are also taken into account in other policies, such as a fixed number of tradable permits that are initially allocated according to market share. If firm-specific emission limits are tied to the firm’s output level, even implicitly, then the policy may have no absolute limit on total emissions. Instead, total emissions vary with total output in a way that affects incentives and prices.\textsuperscript{16}

This model uses the same production functions as the basic model. Behavior in the clean sector is unchanged, with equations (1), (2), and (3). Total capital and labor are still fixed by equations (8) and (9), and consumer preferences are unchanged in (10). The only changes involve incentives facing firms in the dirty sector. Their behavior is

$$\max_{K_Y, L_Y, Z} p_Y(Y, L_Y, Z) - r K_Y - w L_Y,$$

subject to the constraint $Z/Y \leq \delta$. The firms pay no explicit price for the $Z$ input.\textsuperscript{17} Instead, their use of that input is limited by their output. The constraint must bind, since the production function is monotone increasing in all inputs. Solving the firms’ first-order conditions and rearranging terms yields $r = p_Y Y_K / (1 - \delta Y_Z)$ and $w = p_Y Y_L / (1 - \delta Y_Z)$, where subscripts on $Y$ denote marginal products. The firm does not set the marginal value of an input equal to the input price, as it would without the performance standard, because of the denominator in these two equations. This denominator is less than one, so the marginal value of the factor is set lower than its input price. In other words, the firm wants to proceed further down its factor demand curves, using more labor and capital in order to increase output and qualify for an increase in valuable emission rights.

Tighter regulation means a decrease in $\delta$. Totally differentiating the production function yields an equation analogous to (6) in the previous model:

$$\dot{Y} = \theta_Y (1 - \nu) \dot{K}_Y + \theta_L (1 - \nu) \dot{L}_Y + \nu \dot{Z},$$

\textsuperscript{16} We consider policies one at a time. If a quantity mandate is imposed on top of another policy, then the combination may be different from the sum of the parts. If a permit policy already reduces emissions per unit energy, for example, then a mandate to do so may be irrelevant (or may merely affect permit price).

\textsuperscript{17} Firms do pay for fuel, but this input is not the same as “emissions,” as when scrubber technology can reduce emissions such as SO$_2$ per unit of fossil fuel. Emissions of CO$_2$ are proportional to fuel use, but absent a carbon tax, firms still do not pay for emissions of CO$_2$. We discuss the unpriced input, emissions. To capture all these relationships, we would need a model with an additional input: $K, L, Z,$ and fuel.
where $\nu \equiv \delta Y_Z = Y_Z / Y$. In the basic model with (6), an increase in a factor would raise output in proportion to its factor share. Since the marginal product of each factor is reduced by $(1 - \nu)$, its marginal contribution to output is reduced by $(1 - \nu)$. An increase in emissions $Z$ raises output in proportion to $\nu = \delta Y_Z$, to reflect its marginal product $Y_Z$ and its factor share $\delta = Z / Y$. Emission rights are valuable, of course, but firms do not pay for them through an explicit price. Instead, they pay for emission rights by paying factors more than their marginal products.

The assumptions of perfect competition and free entry/exit lead to a zero profit condition in the basic model. This condition remains under the policy specified here, though it takes a different form. Since costs no longer include the price of emission permits, the final term in equation (7) is dropped. The zero profit condition thus implies

\[(7') \quad \hat{p}_Y + \hat{Y} = \theta_{yk}(\hat{r} + \hat{K}_y) + \theta_{yl}(\hat{w} + \hat{L}_y).\]

The constraint may impose a “shadow price” on the factor $Z$, but since no explicit price is paid for that input, it is not included in the profits equation.18

Finally, we must replace equations (4) and (5) with their counterparts under the new policy. Input demand equations can no longer be functions of output and three explicit input prices $(r, w, p_Z)$. Instead, we write input demand equations as functions of $r, w, \delta, and Y$. Then totally differentiate these equations to get:

\[
\hat{K}_Y = b_{kk} \hat{r} + b_{kl} \hat{w} + b_{xz} \hat{\delta} + \hat{Y},
\]
\[
\hat{L}_Y = b_{lk} \hat{r} + b_{lw} \hat{w} + b_{lz} \hat{\delta} + \hat{Y},
\]
\[
\hat{Z} = b_{zk} \hat{r} + b_{zl} \hat{w} + b_{zz} \hat{\delta} + \hat{Y}.
\]

Notice that the $b_{ij}$ appear in a form similar to the $a_{ij}$ parameters in (4) and (5). They both represent input demand elasticities. For example, either $a_{KL}$ or $b_{KL}$ is the percent change in capital for a 1 percent change in the wage. However, $a_{KL}$ is that response in the first model holding $p_Z$ and $Y$ constant (so $Z$ can change), while $b_{KL}$ is that response in the model holding $\delta$ and $Y$ constant (so $Z = \delta Y$ cannot change).19

The third equation giving the input demand for $Z$ can be simplified greatly. We know that the constraint binds, so $Z = \delta Y$. Then total differentiation yields

\[(5') \quad \hat{Z} = \hat{\delta} + \hat{Y},\]

18 This alters the dynamic of firm entry and exit, but since our concern is general equilibrium effects and not the transition periods leading up to them, it does not affect our results.

19 The $b_{ij}$ elasticities (and the $c_{ij}$ elasticities from the following section) are conceptually similar to the direct and indirect substitution effects of Masao Ogaki (1990).
which implies that $b_{ZK} = b_{ZL} = 0$, and $b_{ZZ} = 1$. Since only two of the three equations are independent, we subtract the second equation from the first and get

$$(4') \quad \dot{K} - \dot{L} = b_r \dot{\bar{r}} + b_w \dot{\bar{w}} + b_\delta \dot{\delta},$$

where $b_r \equiv b_{KK} - b_{LK}$, $b_w \equiv b_{KL} - b_{LL}$, and $b_\delta \equiv b_{KZ} - b_{LZ}$. In the Web Appendix (also available in the NBER working paper version of this paper), we derive expressions for the $b_{ij}$ elasticities in terms of $e_{ij}$ and other parameters. Both $b_{KK}$ and $b_{LL}$ are negative, since increasing the price of a factor decreases its demand, even with the constraint on $\delta = Z/Y$. The Web Appendix also shows that the cross-price $b_{ij}$ are positive $(i,j = K, L)$, whether or not capital and labor are substitutes as defined by the sign of the Allen cross-price elasticity.\(^{20}\) Thus, in $(4')$, a higher wage increases the capital/labor ratio $(b_w > 0)$, and higher price of capital reduces it $(b_r < 0)$. In fact, that Appendix shows that $b_r = -b_w$. Finally, it shows that $b_\delta = b_{KZ} - b_{LZ}$ has the opposite sign of $e_{KZ} - e_{LZ}$. A tighter regulation means $\delta$ is decreased, and less pollution is allowed per unit output. If capital is a better substitute for pollution than is labor, that is, if $e_{KZ} > e_{LZ}$, then more capital must be used relative to labor $(b_{KZ} < b_{LZ})$, and hence $b_\delta$ is negative.

For this model, we now have ten equations: (1), (2), (3), (4'), (5'), (6'), (7'), (8), (9), and (10). As before, we set $\dot{\bar{r}} = 0$ and solve for changes in returns to capital and labor attributable to a change in the policy $(\delta)$. The solutions are presented in Table 2, where the denominator $D$ is positive-definite. Expressions for $\dot{\bar{r}}$ and $\dot{\bar{w}}$ can be decomposed into three terms, each corresponding to a single effect. The last term is the “substitution effect,” since it involves $b_\delta \equiv b_{KZ} - b_{LZ}$ (with sign opposite of $e_{KZ} - e_{LZ}$). A reduction in $\delta = Z/Y$ raises $r$ if capital is better than labor as a substitute for emissions $(e_{KZ} > e_{LZ})$. The second term is the “output effect” including $\sigma_u(\gamma_K - \gamma_L)$. This effect hurts capital if $Y$ is capital intensive. Here, however, the first term is a new effect we call an “output-subsidy effect”: since the policy mandates a lower ratio of pollution to output, it can be satisfied partially by increasing output—which helps the factor used intensively.\(^{21}\)

Note that the first two terms have opposite signs and differ only by the appearance of $\sigma_u$ in the output effect. An intuition is that consumer preferences $(\sigma_u)$ affect the way in which the higher price reduces demand for the output, but not the firm’s incentive to produce more output. Thus, the usual output effect dominates only when $\sigma_u > 1$, which raises an important question about the likely size of $\sigma_u$. This parameter has never been estimated for this particular aggregation, where the “dirty good”

\(^{20}\) Why is complementarity ruled out in this case? The Allen elasticities are defined for the input demand functions where all inputs are allowed to vary. Raising the price of labor $w$ may then decrease the demand for capital, if the two inputs are complements, but the firm would be forced to increase its other input, pollution. Here, however, the third input demand equation $(\dot{Z} = \dot{\delta} + \dot{Y})$ indicates that a change in $w$, with no change in $\delta$ or $Y$, cannot change $Z$. Only labor and capital can vary, so they must be substitutes.

\(^{21}\) In fact, the restriction on $Z/Y$ could be modeled as a combination of a tax on emissions plus subsidy to output, which might be more consistent with the usual studies of incentive policies. We choose to model mandates directly as quantity constraints, however, for several reasons. First, they are quantity constraints. Second, these mandates do not raise revenue, so the equivalent incentive policy combination would require a complicated calculation of the endogenous output subsidy rate necessary to return all revenue from the emissions tax. Third, we want specifically to see what insights can be obtained by tackling the problem of quantity constraints in a way that is different from the usual model of incentive policies.
could represent a composite of gasoline, heating fuel, electricity, and all goods that make intensive use of fossil fuels. Yet all of these goods are usually found to have relatively low demand elasticities, which would imply that $\sigma_u$ is less than one (see Section V calibration below). If so, then the new “output-subsidy effect” found here dominates the usual output effect, and tighter environmental policy places less burden on the factor that is used intensively in the dirty sector. We have identified a new effect that likely reverses the usual output effect, and it indicates the importance of further research to estimate $\sigma_u$.

In the equation for $\hat{p}_y$ in Table 2, the last term is a “direct effect” that raises the price of the dirty good. The rest is an ambiguous “indirect effect.” It is complicated by the fact that producers have incentive to sell more of this good, to qualify for more pollution rights. The ambiguity can be resolved in special cases.

A. Equal Factor Intensities

Since the output effect and output-subsidy effect operate through differential factor intensities, the assumption $\gamma_K = \gamma_L = \gamma$ makes them both disappear. Then only the third term for the substitution effect remains in $\hat{r}$ and $\hat{w}$. The solutions reduce to

$$\hat{r} = \frac{\theta_{XL} \gamma b_\delta}{\sigma_X + \gamma b_w} \hat{\delta}$$
$$\hat{w} = \frac{-\theta_{XK} \gamma b_\delta}{\sigma_X + \gamma b_w} \hat{\delta}$$
$$\hat{p}_y = \frac{-\nu}{1 - \nu} \hat{\delta}.$$

In this case, the factor that is a relative substitute for pollution is burdened less by a strengthening of environmental policy ($\hat{\delta} < 0$). If labor is the better substitute for
pollution \((b_\delta > 0)\), then the wage rises and the return to capital falls. This case also provides unambiguous results for incidence on the uses side of income. Only the “direct effect” remains in the expression for \(\hat{p}_y\). A tightening of environmental policy increases the price of the dirty good, hurting those who buy more than average amounts of it.

**B. No Substitution Effect in Dirty Sector**

We can isolate the effect of factor intensities by assuming away differential substitution. Instead of setting all cross-price Allen elasticities equal to zero, we set only \(b_\delta\) to zero. Hence, the substitution effect is eliminated. Then solutions reduce to

\[
\hat{r} = -\frac{1}{D} \theta_{XL} \nu(1 - \sigma_u)(\gamma_K - \gamma_L) \delta
\]

\[
\hat{w} = \frac{1}{D} \theta_{XX} \nu(1 - \sigma_u)(\gamma_K - \gamma_L) \delta
\]

\[
\hat{p}_y = \left[ - (\theta_{YK} \theta_{XL} - \theta_{YL} \theta_{XX}) \nu(1 - \sigma_u)(\gamma_K - \gamma_L) \frac{1}{D} - \frac{\nu}{1 - \nu} \right] \delta.
\]

In the first two expressions, we combine the “output effect” and the “output-subsidy effect” from Table 2. If the dirty sector is capital intensive, so that \((\gamma_K - \gamma_L) > 0\), then capital is hurt more than labor only if \(\sigma_u\) is greater than one. This special case does not remove the ambiguity in \(\hat{p}_y\), however.

All of the effects in this paper are summarized in Table 3. For the policy in each row, the sign of the term in each cell indicates the sign of that column’s effect on the rate of return to capital, \(r\). Because all of these policies restrict \(Z\) in some fashion, the substitution effect in the first column always raises \(r\) if capital is better than labor as a substitute for emissions \((e_{KZ} > e_{LZ})\). The output effect always raises \(r\) if the impacted sector is labor intensive \((\gamma_L > \gamma_K)\). Those two effects follow Harberger (1962) and Mieszkowski (1967), and they still pertain to all mandates here. Yet this section has analyzed a restriction in emissions per unit of output \((Z/Y)\) and found an “output-subsidy effect.” It raises \(r\) if the impacted sector is capital intensive. The next section analyzes a restriction of emissions per unit of capital \((Z/K_y)\), and it identifies another new effect.

**IV. “Technology Mandate:” Emissions per Unit Input**

Whereas the previous section examines a limit on emissions per unit output, we now examine a regulation that limits emissions per unit of an input. Such limits are common, as described in our first section above. We have only two clean inputs in our model, so we capture the nature of a limit on emissions per unit input by

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22 This intuition could fail with permits, but here the denominator cannot be negative (see the Appendix).

23 We cannot set all \(b_\delta\) to zero, since the Web Appendix shows that some of them are of definite sign.
modeling a limit on emissions per unit of capital. We refer to this policy as a “technology mandate,” since forcing the adoption of a particular technology in production may effectively fix the emissions/capital ratio. Capital and labor are each in fixed supply and mobile between sectors, so they are perfectly symmetric in this model. Thus, the results for a limit per unit labor can be obtained directly from results below by interchanging every $K$ and $L$ (as well as every $w$ and $r$).

As with the earlier policy, the equations that describe the behavior of consumers and of producers of the clean good do not change here. Equations (1), (2), (3), (8), (9), and (10) fall into this category and are applicable to this section. The only aspect of the model that requires revision is the behavior of producers of the dirty good. Consider their maximization problem. As in the previous policy, firms pay no explicit price for the pollution input. Instead, they face an exogenous ceiling on their ratio of emissions to capital. Formally, this problem is

$$\max_{K_y, L_y, Z} p_y Y(K_y, L_y, Z) - r K_y - w L_y$$

subject to the constraint $Z/K_y \leq \zeta$. A tightening of environmental policy is defined as a decrease in $\zeta$. It is clear that the policy constraint binds. Since firms pay no price per unit of pollution, and this input is productive, they will employ as much of it as possible, an amount $Z = \zeta K_y$. Thus, we use the fact that $\partial Z/\partial K_y = \zeta$. The first-order conditions for the maximization problem are

$$r = p_y (Y + \zeta Z)$$

$$w = p_y Y_L.$$

The second of these equations is identical to the first-order condition in the basic model where firms face a price for all three inputs and no other constraint: the marginal value of labor is equal to the wage. The first equation differs from the standard condition. For the choice of capital input demanded, the marginal value

\[24\] Of course, actual technology mandates are more complicated. They may require a particular technology and thus inhibit innovation, or they may encourage or require innovation (e.g., required fuel efficiency targets for which the technology is not yet available). Our paper is not about the uncertain effects on technological change, but about distributional incidence. If “ability to innovate” is a skill, modeled as a factor of production, then either type of technology mandate may reward or punish such a factor.
of capital $Y_K$ is lower than the rental rate (since $\zeta Y_Z$ is positive). The intuition here is that each unit of capital employed gives value to the firm in two ways. First, it increases their output directly (since $Y_K > 0$). Second, it allows more pollution, which also increases output. The second term represents this effect, since $Y_Z$ is the marginal product of pollution, and $\zeta = \partial Z / \partial K_Y$ is the pollution increase made possible by the increased capital. The value of investing in a marginal unit of capital is composed of these two terms and at the optimum is set equal to the cost of that investment, the rental rate $r$.

Totally differentiate the production function and substitute in these first-order conditions. After dividing through by $Y$, we have

$$(6'') \quad \hat{Y} = (\theta_{YK} - \nu) \hat{K}_Y + \theta_{YL} \hat{L}_Y + \nu \hat{Z}. $$

The constant $\nu$ is still equal to $Y_Z Z / Y$, as in the previous section. Also, as before, an increase in any one input does not generally increase output by a proportion equal to its factor share. This condition does hold for labor in $(6'')$, since that input choice is not distorted, but the constraint does distort the choice of capital. Yet, from $(6'')$, we see that a 1 percent increase in all three inputs yields a 1 percent increase in output, from the assumption of constant returns to scale. The zero profit condition still holds as well, even though firms do not pay for pollution, because entry and exit are still allowed. Thus, equation $(7')$ from the prior model also applies to this one.

Finally, the dirty sector’s chosen amount of each input ($K_Y, L_Y$, and $Z$) depends on input prices, the policy parameter, and output ($r, w, \zeta$, and $Y$). We totally differentiate these input demand equations to get

$$\hat{K}_Y = c_{KK} \hat{r} + c_{KL} \hat{w} + c_{KZ} \hat{\zeta} + \hat{Y}$$

$$\hat{L}_Y = c_{LK} \hat{r} + c_{LL} \hat{w} + c_{LZ} \hat{\zeta} + \hat{Y}$$

$$\hat{Z} = c_{ZK} \hat{r} + c_{ZL} \hat{w} + c_{ZZ} \hat{\zeta} + \hat{Y}.$$

The elasticity of demand for input $i$ with respect to price $j$ is defined here as $c_{ij}$ (but this response depends on the nature of the constraint, so the $c_{ij}$ elasticities are not the same as the $a_{ij}$ or $b_{ij}$ elasticities). Only two of these equations are independent of each other, so we subtract each of the bottom two equations from the top one to get two equations to use in our solution. The first of these equations is

$$(4'') \quad \hat{K}_Y - \hat{L}_Y = c_r \hat{r} + c_w \hat{w} + c_\zeta \hat{\zeta},$$

where $c_r \equiv c_{KK} - c_{LK}$, $c_w \equiv c_{KL} - c_{LL}$, and $c_\zeta \equiv c_{KZ} - c_{LZ}$. The second resulting equation can be simplified using the policy constraint $Z / K_Y = \zeta$, since total differentiation gives

$$(5'') \quad \hat{K}_Y - \hat{Z} = -\hat{\zeta}.$$
Substituting this into the equations above implies that $c_{KK} - c_{ZK} = 0$, $c_{KL} - c_{ZL} = 0$, and $c_{KZ} - c_{ZZ} = -1$. These relationships are verified in the Web Appendix.

Also in that Appendix, we evaluate the elasticities of input demand. An important condition for their signs relates to the relative complementarity of capital and pollution:

"Condition 1": $e_{KZ} > (e_{KK} + e_{ZZ})/2$.

The right-hand side of this inequality must be negative, since all own-price elasticities are negative. This condition always holds, then, when capital and pollution are substitutes ($e_{KZ} > 0$). It also holds when capital and pollution are not "too complementary." With this condition, the Web Appendix shows that $c_r < 0$ and $c_w > 0$. That is, an increase in the capital rental rate must reduce the ratio $K_y/L_y$ demanded, and an increase in the wage must increase it. The ratio of $Z$ to $K_y$ is fixed, and so producers really have only two inputs between which they can substitute; once they choose $K_y$ and $L_y$, then $Z$ is given by the constraint. With only two inputs $K_y$ and $L_y$, they must be substitutes.

The system of equations containing $(1)$, $(2)$, $(3)$, $(4''')$, $(5'')$, $(6''')$, $(7')$, $(8)$, $(9)$, and $(10)$ are ten equations in ten unknowns. In Table 4, these equations are solved for the proportional change in each price from an exogenous change in $\zeta$. When condition 1 holds, the denominator $D$ must be positive.

These equations are strikingly similar to their counterparts for the previous policy. As before, the second term in either factor price equation is an "output effect:" this policy impinges on the dirty sector, which tends to raise the output price and discourage purchases. By itself, this effect would hurt capital if the sector is capital intensive. Again, the first term is an "output-subsidy effect" with the opposite sign. Again, it is larger than the output effect if $\sigma_u < 1$. In the prior case, however, the output-subsidy effect arises because the mandate to reduce $Z/Y$ provides an implicit subsidy to output. Why is output subsidized here? As we show in a moment, this mandate to reduce $Z/K_y$ provides an implicit subsidy to the use of capital $K_y$, but this subsidy itself also reduces the cost of production, and therefore it also creates an output-subsidy effect.

The third term in these factor price equations depends on $c_\zeta$, which the Web Appendix shows can itself be subdivided. In particular, it shows that $c_\zeta$ can be written:

$$c_\zeta = M + H(e_{LZ} - e_{KZ}) + H(e_{KK} - e_{KL}),$$

where $M$ and $H$ are defined in that Appendix and are both positive under condition 1. In this expression, the last term is the promised "capital-subsidy effect." It includes

25 If condition 1 fails, then $e_{KZ} < (e_{KK} + e_{ZZ})/2 < 0$, so $c_r > 0$ and $c_w < 0$. Counterintuitively, an increase in $r/w$ then raises the desired $K_y/L_y$ ratio. Because capital and pollution are highly complementary, the increase in $r$ makes firms want less $K$ and less $Z$. Wanting less $Z$ reduces the pressure of the constraint ($Z/K_y \leq \zeta$), which reduces the shadow price on $Z$ (i.e., the right to emit is not so valuable). The reduced shadow price on $Z$, by itself, would mean more demand for $Z$ and more $K_y$, since they are complements. If they are sufficiently complementary, then the result is a net increase in capital relative to labor.
which is always negative, so the policy $(\hat{\eta} < 0)$ has a positive effect on the use of capital and its return $r$. It also includes $-e_{KL}$, which operates in the same direction if labor and capital are substitutes $(e_{KL} > 0)$. The extent to which dirty firms can substitute away from labor and into capital helps drive up demand for $K$, and thus the return $r$. As shown in Table 3, the capital-subsidy effect raises $r$ if $(e_{KK} - e_{KL}) < 0$.

The second term in the $c_\zeta$ expression represents the usual “substitution effect:” the mandate to reduce the ratio $\zeta$ can also be satisfied partly by reducing $Z$ in the numerator, which means substituting from $Z$ into other inputs ($K$ or $L$). If labor is a better substitute for pollution than capital $(e_{LZ} > e_{KZ})$, then this policy induces more demand for labor than capital from the substitution effect. Then this term is positive, so multiplication by $\hat{\zeta} < 0$ means that it decreases $r$ and increases $w$.

In summary, the forced reduction in $Z/K_Y$ can be satisfied partly by reducing emissions in the numerator (the substitution effect), but also partly by increasing capital in the denominator (the capital-subsidy effect). This implicit subsidy itself reduces the cost of production (the output-subsidy effect), which offsets the usual way in which regulations raise costs (the output effect). All four effects appear in Table 3.

### A. Equal Factor Intensities

The assumption $\gamma_K = \gamma_L = \gamma$ eliminates the output and output-subsidy effects in Table 4. The factor price equations do not need to be repeated here, as they then contain only the third term with $c_\zeta$ (including both the substitution effect and the capital-subsidy effect). However, the output price equation reduces to

$$\hat{p_Y} = -\nu \hat{\zeta},$$

On the uses side, incidence is unambiguous. A tighter environmental policy must increase the price of the dirty good relative to the price of the clean good, due to the direct effect of the policy on the cost of production in the $Y$ sector only.
B. No Substitution Effect in Dirty Sector

Here, we assume that \( c_r = c_w = c_\zeta = 0 \), which is not quite as strong as saying that the dirty sector cannot substitute at all.\(^{26}\) Instead, this assumption eliminates the capital-subsidy effect and the substitution effect. The remaining factor price equations are straight from Table 4, but without the last term \((c_\zeta)\). Thus, they still include the output effect and the output-subsidy effect.

The output price equation also looks much like the one in Table 4, but without the \( c_\zeta \) term. The last term \((-\nu)\) is definitely negative, so this “direct effect” raises the cost of production and thus raises the breakeven price \( p_Y \). The long first term is an indirect effect. Since \( \gamma_K - \gamma_L \) has the same sign as \((\theta_{yK}\theta_{XL} - \theta_{yL}\theta_{XK})\), this term has the opposite sign of \((1 - \sigma_u)\). When \( \sigma_u \) is smaller than one, then a tighter mandate must increase the price of good \( Y \). When \( \sigma_u \) is large, however, the indirect effect can dominate the direct effect, so that a tighter mandate decreases the price of the dirty good.\(^{27}\)

V. Numerical Magnitudes

We now employ plausible parameter values to quantify effects identified above. We stress that our main contribution is the analytical model, but some quantification can help complement those results. Because the model abstracts from a number of important features of the economy and of environmental policy, we do not expect these calculations to provide accurate point estimates of the size of general equilibrium effects of mandates. Instead, we only hope to provide some general idea about relative magnitudes. Note that the regulation of a single industry would likely have small general equilibrium impacts. Therefore, the best interpretation of our analysis is not about any one regulation, but about all environmental regulation of dirty industries taken together.

Our calibration uses parameter values from Fullerton and Heutel (2007). As described in more detail there, we divide the US economy into “clean” and “dirty” sectors based on industry-level emissions reported to the EPA’s 2002 Toxic Release Inventory (TRI).\(^{28}\) Other industry-level data on capital and labor inputs imply that the dirty sector accounts for 20 percent of factor income, and that the \( K/L \) ratio in both sectors is 2/3. When we set total labor and capital income to 1.0, and pollution’s share of the value of output in the dirty industry \((\theta_{yZ})\) to 0.25, we calculate the initial factor allocations shown in \[\text{Table 5}\]. That table also presents the assumed

\(^{26}\) In fact, all \( c_j \) cannot be zero, since we showed earlier that \( c_{iZ} - c_{iZ} = -1 \).

\(^{27}\) In the \( p_Y \) equation, for a large indirect effect, suppose \( \sigma_u \) and \((\gamma_K - \gamma_L)\) are large. The sector is highly capital intensive. The output effect dominates the output-subsidy effect, so the tighter mandate means less demand for capital. Thus, \( r \) falls. As seen in the \( r \) equation, large \( \sigma_u \) and \((\gamma_K - \gamma_L)\) mean \( r \) falls a lot. The dirty sector is highly capital intensive, so its cost of production and \( p_Y \) fall.

\(^{28}\) The TRI database includes emissions of toxic chemicals from industry, not carbon dioxide from industry nor any emissions from consumer use of fossil fuel. Yet, the dirty sector output could be defined to include vehicle miles traveled, which would require refinements of these data. Of course, the magnitudes used here are only meant to be suggestive in any case, and we vary the numbers in sensitivity analysis below.
elasticity of substitution in the clean sector $\sigma_X$, the elasticity of substitution in utility $\sigma_u$, and the cross-price Allen elasticities.\footnote{These elasticities have not been estimated, but some studies suggest that capital may be a better substitute for pollution than is labor (Ruud A. DeMooij and Bovenberg 1998; Timothy Considine and Donald Larson 2006).}

Using the definition of $\sigma_u$ and log-linearization of the consumer’s budget, we can show that the price elasticity of demand for $Y$ is $-\alpha - \sigma_u (1 - \alpha)$, where $\alpha$ is its expenditure share. Our implied $\alpha$ is 0.25, so $\sigma_u = 0.8$ corresponds to $-0.85$ for the price elasticity.\footnote{West and Williams (2004) estimate gasoline price elasticities in the range of $-0.18$ to $-0.73$. Kenneth Small and Kurt Van Dender’s (2007) estimates are between $-0.4$ and $-0.6$. In a meta-analysis of long-run price elasticity estimates for electricity demand, James A. Espey and Molly Espey (2004) find “a range from $-2.25$ to $-0.04$ with a mean of $-0.85$ and a median of $-0.81”$ (p. 66). We vary this parameter in sensitivity analysis.}

The values in Table 5 are used to set all other “base case” parameters.\footnote{For example, $\theta_{XK} \equiv K_K/(K_K + L_X) = 0.32/(0.32 + 0.48) = 0.40$. Also, $\theta_{YK}e_{KL} + \theta_{YL}e_{LZ} + \theta_{YZ}e_{ZL} = 0$ (for $i = K, L, Z$), as shown by Allen (1938), so the cross-price elasticities can then be used to set own-price elasticities, $e_{ii}$.}

Then the Allen elasticities determine input demand elasticities $b_{ij}$ for the performance standard and $c_{ij}$ for the technology mandate.\footnote{As shown in the Web Appendix, the $b_{ij}$ and $c_{ij}$ input demand elasticities are functions of factor shares (the $\theta$’s), prices (all normalized to one), the three first derivatives of production ($Y_K, Y_L,$ and $Y_Z$), and six second derivatives. Because each marginal product matches its factor price, set at one, the first derivatives also equal one. The definitions of the six Allen elasticities are $e_{ij} \equiv [(p_j Y) / (i_j y_j)] (F_{ij} / F)$, where $F$ is the determinant of the bordered Hessian of the production function, and $F_{ij}$ is the cofactor of element $i, j$. These six equations are functions of the six unknown second derivatives of production and are solved numerically for the second derivatives, which are then substituted into the definitions for the $b_{ij}$ and $c_{ij}$ elasticities.}

Using all those parameters in equations above, we calculate the effect of each policy on the wage rate and the capital rental rate.

The initial ratio of pollution to output ($\delta$) is smaller than its ratio to capital ($\zeta$), so the same 10 percent reduction in both ratios does not have the same scale of impacts on the economy. Instead, for comparability, we calculate the change in each ratio that achieves the same 10 percent cut in pollution ($\hat{Z}$). For base case parameters, this same stringency is achieved by a 7.8 percent reduction in $\delta$ or 13.6 percent reduction in $\zeta$. For different parameter values used in Table 6, though, it is achieved by a

### Table 5—Base Case Parameter Values

<table>
<thead>
<tr>
<th>Factor intensity parameters</th>
<th>$K_Y$</th>
<th>0.08</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$L_Y$</td>
<td>0.12</td>
</tr>
<tr>
<td></td>
<td>$K_X$</td>
<td>0.32</td>
</tr>
<tr>
<td></td>
<td>$L_X$</td>
<td>0.48</td>
</tr>
<tr>
<td>Elasticity of substitution in $X$</td>
<td>$\sigma_X$</td>
<td>1.0</td>
</tr>
<tr>
<td>Elasticity of substitution in utility</td>
<td>$\sigma_u$</td>
<td>0.8</td>
</tr>
<tr>
<td>Allen elasticities in the dirty sector</td>
<td>$e_{KL}$</td>
<td>0.5</td>
</tr>
<tr>
<td></td>
<td>$e_{KZ}$</td>
<td>0.5</td>
</tr>
<tr>
<td></td>
<td>$e_{LZ}$</td>
<td>0.3</td>
</tr>
</tbody>
</table>

The Allen elasticities determine input demand elasticities $b_{ij}$ for the performance standard and $c_{ij}$ for the technology mandate. Using all those parameters in equations above, we calculate the effect of each policy on the wage rate and the capital rental rate.

The initial ratio of pollution to output ($\delta$) is smaller than its ratio to capital ($\zeta$), so the same 10 percent reduction in both ratios does not have the same scale of impacts on the economy. Instead, for comparability, we calculate the change in each ratio that achieves the same 10 percent cut in pollution ($\hat{Z}$). For base case parameters, this same stringency is achieved by a 7.8 percent reduction in $\delta$ or 13.6 percent reduction in $\zeta$. For different parameter values used in Table 6, though, it is achieved by a
reduction in $\delta$ ranging from 7.5 percent to 8.1 percent, or a reduction in $\zeta$ ranging from 9.5 percent to 16.7 percent. For the base case, where $\gamma_K - \gamma_L = 0$, neither sector is more capital-intensive. Thus, both the output effect and output-subsidy effect are zero. For the performance standard, the only effect remaining is the substitution effect, which reduces the wage/rental ratio because $e_{KZ} > e_{LZ}$ (capital is a better substitute for pollution than is labor). Thus, the wage falls by 0.12 percent, and the return to capital rises 0.18 percent. For the technology mandate, the substitution effect is augmented by the capital-subsidy effect. Both raise the return to capital (by 0.73 percent) relative to the wage (which falls by 0.49 percent). These factor price changes might seem small, for a pollution reduction of 10 percent, but note that pollution is only 25 percent of the value of $Y$, which itself is only 20 percent of GDP. This 10 percent pollution mandate in one small sector has nearly 1 percent effects on all labor and capital.

Rows 2–4 vary only the relative cross-price elasticities. When $e_{KZ}$ is much greater than $e_{LZ}$, as in row 2, the substitution effect is pronounced, and the increase in the rental rate $\hat{r}$ is even higher than in the base case, for both policies. In row 3, the elasticities are reversed ($e_{KZ} < e_{LZ}$), so labor is a better substitute for pollution. Now, the substitution effect goes in the opposite direction, and the wage increases. Row 4 presents the case where $e_{KZ} = e_{LZ}$, so the substitution effect disappears. The output effect and output-subsidy effect are already zero (since $\gamma_K - \gamma_L = 0$), so the performance standard has no effect whatsoever on the wage or the rental rate. For the technology mandate, the only remaining effect is the capital-subsidy effect, which increases the rental rate (because the demand for capital increases to help reduce the ratio $Z/K_Y$).

Next, we vary the elasticity of substitution in utility, $\sigma_u$, which determines the relative strength of the output effect (because it is always multiplied by $\gamma_K - \gamma_L$ in the equations). But this output effect is zero in the first four rows, where $\gamma_K - \gamma_L = 0$, so rows 5 and 6 set $\gamma_K - \gamma_L = 0.1$ (the dirty sector is capital-intensive). The output effect serves to decrease the rental rate and increase the wage, while the output-subsidy effect works in the opposite direction. When $\sigma_u < 1$, we show above that the output-subsidy dominates, so the net of these two effects raises the rental rate. Thus, $r$ rises more in row 5 with $\sigma_u = 0.6$ than in row 6, where $\sigma_u = 1$. Indeed, these two

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Table 6—Sensitivity Analysis, Where Each Policy Achieves 10 Percent Less Pollution (all other parameters the same as in the base case)

<table>
<thead>
<tr>
<th>Row</th>
<th>$\gamma_K - \gamma_L$</th>
<th>$e_{KZ}$</th>
<th>$e_{LZ}$</th>
<th>$\sigma_u$</th>
<th>Performance standard</th>
<th>Technology mandate</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$\hat{w}$ (%)</td>
<td>$\hat{r}$ (%)</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
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<td>0.3</td>
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<td>-0.12</td>
<td>0.18</td>
</tr>
<tr>
<td>2</td>
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<td>0.8</td>
<td>0.8</td>
<td>-0.59</td>
<td>0.89</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>0</td>
<td>1.0</td>
<td>0.8</td>
<td>0.40</td>
<td>-0.59</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>0.5</td>
<td>0.5</td>
<td>0.8</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>5</td>
<td>0.1</td>
<td>0.5</td>
<td>0.3</td>
<td>0.6</td>
<td>-0.16</td>
<td>0.26</td>
</tr>
<tr>
<td>6</td>
<td>0.1</td>
<td>0.5</td>
<td>0.3</td>
<td>1.0</td>
<td>-0.11</td>
<td>0.18</td>
</tr>
<tr>
<td>7</td>
<td>-0.2</td>
<td>0.5</td>
<td>0.3</td>
<td>0.8</td>
<td>-0.07</td>
<td>0.09</td>
</tr>
<tr>
<td>8</td>
<td>-0.1</td>
<td>0.5</td>
<td>0.3</td>
<td>0.8</td>
<td>-0.10</td>
<td>0.14</td>
</tr>
<tr>
<td>9</td>
<td>0.1</td>
<td>0.5</td>
<td>0.3</td>
<td>0.8</td>
<td>-0.13</td>
<td>0.22</td>
</tr>
<tr>
<td>10</td>
<td>0.2</td>
<td>0.5</td>
<td>0.3</td>
<td>0.8</td>
<td>-0.14</td>
<td>0.25</td>
</tr>
</tbody>
</table>
effects cancel out when \( \sigma_u = 1 \), leaving only the substitution effect and the capital-subsidy effect.

Rows 7–10 raise the value of \( \gamma_K - \gamma_L \) in steps from –0.2 to +0.2, so the dirty sector changes from labor-intensive to capital-intensive (keeping fixed the overall size of \( X, Y, \bar{K}, \) and \( \bar{L} \)). These rows all employ \( \sigma_u = 0.8 \), which is less than one, so the output-subsidy effect dominates the output effect. Therefore, as the dirty sector becomes more capital-intensive, we expect more increase in the relative rental rate (higher \( \hat{r} - \hat{w} \)). This result certainly holds for the performance standard, in Table 6, where \( w \) falls more and \( r \) rises more (from row 7 to row 10). For the technology mandate, however, \( \hat{r} - \hat{w} \) becomes smaller from row 8 to row 10. The reason is that the output effect and the output-subsidy effect in this case are joined by the capital-subsidy effect. As we show next, this capital-subsidy effect becomes smaller as the dirty sector becomes more capital-intensive, offsetting the other three effects.

To understand these effects separately, Table 7 shows the breakdown of effects on \( \hat{r} \) for each policy. The first two rows show the base case, where \( \gamma_K - \gamma_L = 0 \), so neither policy has an output effect or an output-subsidy effect. For the performance standard, only the substitution effect remains to explain the finding above that \( \hat{r} = 0.18 \) percent. For the technology mandate, Table 6 shows \( \hat{r} = 0.73 \) percent in the base case, which Table 7 reveals is composed of both a substitution effect (0.11 percent) and capital-subsidy effect (0.61 percent).

The rest of Table 7 varies \( \gamma_K - \gamma_L \) from –0.2 to +0.2 (as in rows 7 and 10 of Table 6). When \( \gamma_K - \gamma_L = -0.2 \), the dirty sector is labor-intensive, so the output effect helps capital, but the output-subsidy effect more than offsets (because \( \sigma_u < 1 \)). For the performance standard, the net negative contribution of these two effects is dominated by the positive substitution effect, so that the overall net effect is positive (0.09 percent). For the technology mandate, those three terms add to approximately zero, and the positive capital-subsidy effect accounts for the bulk of the net effect. Finally, in the last two rows, where \( \gamma_K - \gamma_L = 0.2 \), the dirty sector is capital intensive. Thus, the output effect on \( r \) is negative, while the output-subsidy effect is positive (and larger).

### Table 7—Numerical Decomposition of Effects on the Return to Capital

<table>
<thead>
<tr>
<th>( \gamma_K - \gamma_L = 0 ) (base case)</th>
<th>Subst. effect</th>
<th>Output effect</th>
<th>Output-subsidy effect</th>
<th>Capital-subsidy effect</th>
<th>Net effect</th>
</tr>
</thead>
<tbody>
<tr>
<td>Perf. standard</td>
<td>0.18</td>
<td>0</td>
<td>0</td>
<td>—</td>
<td>0.18</td>
</tr>
<tr>
<td>Tech. mandate</td>
<td>0.11</td>
<td>0</td>
<td>0</td>
<td>0.61</td>
<td>0.73</td>
</tr>
<tr>
<td>( \gamma_K - \gamma_L = -0.2 )</td>
<td>Perf. standard</td>
<td>0.14</td>
<td>0.21</td>
<td>-0.26</td>
<td>—</td>
</tr>
<tr>
<td>Tech. mandate</td>
<td>0.08</td>
<td>0.35</td>
<td>-0.44</td>
<td>0.69</td>
<td>0.68</td>
</tr>
<tr>
<td>( \gamma_K - \gamma_L = +0.2 )</td>
<td>Perf. standard</td>
<td>0.19</td>
<td>-0.23</td>
<td>0.29</td>
<td>—</td>
</tr>
<tr>
<td>Tech. mandate</td>
<td>0.13</td>
<td>-0.28</td>
<td>0.35</td>
<td>0.48</td>
<td>0.67</td>
</tr>
</tbody>
</table>

VI. Conclusion

Just like taxes, regulations that restrict emissions affect producer decisions about use of labor and capital, and they thus affect relative factor prices, total production, and output prices. Existing models analyze the distribution of burdens from taxes,
but this paper points out that nonrevenue-raising restrictions also have burdens on the sources side of income through changes in factor prices as well as burdens on the uses side through changes in output prices. Our model is based on the standard two-sector tax incidence model, but with two important modifications. First, we allow one sector to include pollution as a factor of production that can be a complement or substitute for labor or for capital. Second, we look not at taxes, but at different types of mandates.

The model in this paper can be applied beyond environmental policy to analyze any regulation that restricts use of inputs. Alternatively, the model could be extended in any of the many ways that the Harberger model has been extended, for example to consider more realistic market structures, asymmetric information about costs and benefits of pollution abatement, irreversible investments, or adjustment costs. Future research in a dynamic model could consider capital formation, endogenous technology, vintage-differentiated regulation, and uncertainty. Also important is the interaction of environmental mandates with other types of regulation, especially in the highly regulated electric utility sector. Finally, we focus on the incidence of the costs of environmental mandates; a large parallel literature instead examines the incidence of benefits (e.g., policy may reduce pollution in poorer neighborhoods more than in richer neighborhoods).

With any of those extensions, the model would become more complicated, and the price change equations might have more terms. But the effects we have uncovered here would still pertain. With no existing research on these effects yet, we believe that this simple model is the right place to begin. And even in this simple model, we get some interesting results. First, a mandate may hurt consumers of the clean good more than consumers of the dirty good. Second, we show how a mandate may burden either the factor that is a better substitute for pollution or the factor that is a relative complement to pollution. Third, restrictions on the absolute level of emissions differ from restrictions on emissions per unit output or per unit of an input. A restriction on pollution per unit of output has not only an “output effect” that burdens any factor used intensively in production, but also an “output-subsidy effect” that encourages output to help satisfy the mandated ratio. Similarly, a restriction on pollution per unit capital creates a “capital-subsidy effect” that increases demand for capital and thus raises the rental rate.

An implication is that researchers need to be careful about the nature of an environmental restriction before concluding that it injures the factor used intensively or the factor that is a better substitute for pollution. Those usual effects can be completely offset by other effects we identify in this paper.
Appendix: Tradable Pollution Permits

Table A1—Incidence of Absolute Quantity Restriction

\[ \hat{r} = \frac{\theta_{yz}\theta_{yz}}{D} \{ \sigma_u(\gamma_K - \gamma_L) - e_{xy} \gamma_L(1 + \gamma_L) + e_{xy} \gamma_L(1 + \gamma_K) \} \hat{Z}, \]

\[ \hat{w} = \frac{\theta_{yz}\theta_{yz}}{D} \{ - \sigma_u(\gamma_K - \gamma_L) + e_{xy} \gamma_K(1 + \gamma_L) - e_{xy} \gamma_L(1 + \gamma_K) \} \hat{Z}, \]

\[ \hat{p}_z = F^{-1} \left\{ \left( \frac{1}{D} \right) (G\gamma_L(1 + \gamma_K) - F(\gamma_K(1 + \gamma_L)) \left[ \frac{\sigma_x}{A} (C + \beta_L) + \theta_{xy} \theta_{xy} (e_{KL} - e_{XX}) \right] - \gamma_L(1 + \gamma_K) \right) \hat{Z}, \]

\[ \hat{p}_y = \left\{ \left( \frac{\gamma_U(1 + \gamma_K) - F(\gamma_K(1 + \gamma_L)) \left[ \frac{\theta_{yz}}{D} (\gamma_{xy} - \theta_{xy} - \theta_e \theta_{xy}) + \frac{\sigma_x}{A} (C + \beta_L) + A \theta_{yz} \theta_{xy} (e_{KL} - e_{XX}) \right] }{A \theta_e} \right\} \hat{Z}, \]

where \( \gamma_L \equiv \frac{\lambda_{xy}}{L_x} > 0, \quad \gamma_K \equiv \frac{\lambda_{xy}}{K_x} > 0, \quad \beta_K \equiv \theta_{xy} \gamma_L + \theta_{xy} > 0, \quad \beta_L \equiv \theta_{xy} \gamma_L + \theta_{xy} > 0, \quad A \equiv \gamma_L \beta_K + \gamma_K \beta_L > 0, \quad C \equiv \beta_K \theta_{xy} - \beta_L \theta_{xy}. \]

\[ F \equiv \sigma_u \left[ \gamma_K(1 - \theta_{xy}) - \gamma_L \theta_{xy} \right] + e_{xy}, \quad G \equiv \sigma_u \left[ \gamma_K(1 - \theta_{xy}) - \gamma_L \theta_{xy} \right] + e_{xy}, \quad \text{and} \]

\[ D \equiv \sigma_u [\theta_{xy} \theta_{xy} - \theta_{xy} \theta_{xy}] \left[ G(A - \gamma_L(1 + \gamma_K)) - F(A - \gamma_K(1 + \gamma_L)) \right] \]

\[ + \sigma_x [C(1 + \beta_L) + F(\beta_K - C)] + A \theta_{xy} \theta_{xy} [Fe_{KL} - Ge_{KL}] - A \theta_{xy} \theta_{xy} [Fe_{KL} - Ge_{KL}] \]

The permit policy imposes costs by forcing firms to reduce emissions or to buy permits. The mandated overall limit on pollution creates scarcity rents, however, and the distribution of those rents is part of the incidence. If the permits are grandfathered to firms, then their owners capture the scarcity rents. In addition to evaluating changes in returns to capital and labor, our model solves for changes in permit-created scarcity rents. All of these changes contribute to the sources-side incidence.

General solutions are presented in Table A1, where we assume the denominator \( D \) is positive. The factor price equations demonstrate effects first identified by Mieszkowski (1967). The first term in the curly brackets, \( \sigma_u(\gamma_K - \gamma_L) \), is his “output effect.” The policy \( \hat{Z} < 0 \) raises the cost of production and thus reduces output in a way that depends on consumer preferences \( \sigma_u \). Then if \( Y \) is capital intensive,

33 Parry (2004) uses a partial equilibrium model to calculate incidence, including the distribution of scarcity rents created by emissions permits for carbon, SOX, and NOX.

34 The denominator is likely to be positive, except in perverse cases that are not the subject of this paper. In fact, that condition is stronger than necessary. \( D \) is positive unless

"Condition A1": \( e_{KL} < -\sigma_x [\theta_{xy} \sigma_u + (\theta_{xy} \gamma + \theta_{xy}) e_{xy} + (\theta_{xy} \gamma + \theta_{xy}) e_{xy}] - A \theta_{xy} \theta_{xy} (Ge_{KL} + Fe_{KL}) \). \( \frac{A \theta_{xy} \theta_{xy} (F + G)}{F + G} \)

The right side of this inequality must be negative, so Condition A1 says that \( e_{KL} \) is even more negative. Thus, \( D > 0 \) does not require that \( K \) and \( L \) are substitutes, but only that they not be “too complementary.”
(γ_K − γ_L) > 0, the output effect reduces r and raises w. The other terms represent a “substitution effect” involving the Allen elasticities. If capital is a better substitute for pollution than is labor (e_KZ > e_LZ), then the restriction ˆ Z < 0 is more likely to help capital.

Next, in Table A1, the expression for ˆ p_Z seems quite complicated, but the final term inside the curly brackets, γ_L/(1 + γ_K)/A, is unambiguously positive and could be called a “direct effect.” It reflects a downward-sloping demand for emissions permits, so the leftward shift of the vertical supply curve tends to raise the equilibrium permit price. Then the long first term could be called the “indirect effect,” but it need not be positive. If it is sufficiently negative, then a decrease in the total permit allocation may actually decrease the permit price. The conditions under which this counterintuitive effect occurs are quite cumbersome and difficult to interpret, and hence they are not presented here. Yet the effect is analogous to previous findings that an increase in the pollution tax can lead to an increase in emissions.35

Yet, unlike the incidence on labor and capital owners, the incidence on permit holders is not determined solely by the change in their factor price. Labor and capital are in fixed total supply and earn net returns determined by w and r, but the supply of permits has just been restricted by the policy (ˆ Z < 0). The total return to permit holders is p_Z Z, and the proportional change in this product is ˆ p_Z + ˆ Z. Even if the policy raises the price p_Z, then permit holders are still not necessarily better off.

Furthermore, even the uses-side incidence result (ˆ p_Y) is ambiguous. The final term in the curly brackets is a “direct effect” on the cost of production, indicating that a decrease in the number of permits tends to increase the price of the dirty good. However, the long previous term is an “indirect effect” that cannot be signed. It allows for another counterintuitive result: reducing the number of emissions permits may hurt consumers of the clean good more than consumers of the dirty good.

A. Equal Factor Intensities

Suppose that γ_L and γ_K are equal to each other, and let their common value be γ. Note that this condition implies L_Y/L_X = K_Y/K_X. The output effect then disappears, and the substitution effect simplifies. Unfortunately, this special case does little to simplify the long expressions for ˆ p_Z and ˆ p_Y, but for factor prices we have

\[ \hat{r} = -\frac{\theta_{YZ} \theta_{XL}}{D} \gamma (1 + \gamma) (e_{KZ} - e_{LZ}) \hat{Z} \]
\[ \hat{w} = \frac{\theta_{YZ} \theta_{XK}}{D} \gamma (1 + \gamma) (e_{KZ} - e_{LZ}) \hat{Z} \]

35 See DeMooij and Bovenberg (1998) or Fullerton and Heutel (2007). This example is comparable to the “Edgeworth Taxation Paradox” studied in Harold Hotelling (1932), where the imposition of a tax on a good can reduce its price to consumers and increase their purchases. Though Hotelling’s (1932) model is not perfectly analogous to the one here, his inequalities (25) and (26) present conditions when the paradox holds; they similarly involve cross-price demand elasticities.
The denominator in the general solution reduces a bit, but $D > 0$ still requires condition A1. If so, we reach a definitive conclusion about the effect of the regulation on $r$ and $w$. When emissions must be reduced, the dirty sector wants to substitute into both labor and capital, but if labor is a better substitute for pollution ($e_{LZ} > e_{KZ}$), then labor is hurt relatively less (i.e., $\hat{r} < 0$ and $\hat{w} > 0$).

**B. No Substitution in Dirty Sector**

We now let the factor intensities of the two sectors differ but assume the dirty sector cannot substitute among its inputs ($e_{ij} = 0$ for all $i, j$). While this assumption is clearly restrictive, it allows us to isolate the impact of factor intensities. The denominator $D$ then simplifies to $\theta yZ \sigma X \sigma u$, and the substitution effects in $\hat{r}$ and $\hat{w}$ disappear. Again, $\hat{p}_Z$ and $\hat{p}_Y$ are not much simplified, but the changes in factor prices become

$$
\hat{r} = \frac{\theta_{Xl}}{\sigma_X} (\gamma_K - \gamma_L) \hat{Z},
$$

$$
\hat{w} = -\frac{\theta_{Xk}}{\sigma_X} (\gamma_K - \gamma_L) \hat{Z}.
$$

Here, the denominator is always positive. The sources side incidence includes only an output effect, determined by the sign of $\gamma_K - \gamma_L$. If the dirty sector is capital-intensive, this term is positive. Since $\hat{Z} < 0$, the rental rate falls and the wage rises. The magnitude of this effect is mediated by $\sigma_X$. If the clean industry can easily substitute between capital and labor, then these effects on input prices become smaller, since the clean sector can easily accommodate the additional labor or capital.

Finally, instead of tradable permits, consider a policy where each firm faces a restriction on $Z$. Then, pollution has no market clearing price $p_Z$, but each firm with a restriction on $Z$ can be said to face a shadow price $p_Z$. Each firm gets an allocation of permits that are not tradable. In our model with many identical firms, however, the firms cannot gain from trade. With constant returns to scale, each firm’s labor and capital can adjust to its allocation of nontradable permits in a way that is equivalent to the transfer of permits to some other firm using that same labor and capital. In other words, firm-specific restrictions on pollution levels in this model yield the same results as for tradable permits. Equations above can be used for effects on total dirty-industry use of labor and capital and for consequent economy-wide returns.

REFERENCES


