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Operational Risk Capital Provisions for Banks and Insurance Companies

Edoh Fofo Afambo

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OPERATIONAL RISK CAPITAL PROVISIONS FOR BANKS AND INSURANCE COMPANIES

By

Edoh Fofo Afambo

A Dissertation Submitted in Partial Fulfillment of the Requirements for the Degree of Doctor of Philosophy in the Robinson College of Business
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Acceptance

This dissertation was prepared under the direction of Edoh Fofo Afambo's Dissertation Committee. It has been approved and accepted by all members of that committee, and it has been accepted in partial fulfillment of the requirements for the degree of Doctor of Philosophy in the Robinson College of Business of Georgia State University.

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ABSTRACT

OPERATIONAL RISK CAPITAL PROVISIONS FOR BANKS AND INSURANCE COMPANIES

BY

Edoh Fofo Afambo

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Committee chair: Samuel H. Cox

Major Academic Unit: Risk Management and Insurance

This dissertation investigates the implications of using the Advanced Measurement Approaches (AMA) as a method to assess operational risk capital charges for banks and insurance companies within Basel II paradigms and with regard to U.S. regulations. Operational risk has become recognized as a major risk class because of huge operational losses experienced by many financial firms over the last past decade. Unlike market risk, credit risk, and insurance risk, for which firms and scholars have designed efficient methodologies, there are few tools to help analyze and quantify operational risk. The New Basel Revised Framework for International Convergence of Capital Measurement and Capital Standards (Basel II) gives substantial flexibility to internationally active banks to set up their own risk assessment models in the context of the Advanced Measurement Approaches. The AMA developed in this thesis uses actuarial loss models complemented by the extreme value theory to determine the empirical probability distribution function of the overall capital charge in terms of various classes of copulas. Publicly available operational risk loss data set is used for the empirical exercise.

DEDICATION

To my wife Peace, my children Rose and Nitya Cedric, and my parents.

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CHAPTER 1

INTRODUCTION

1.1 Motivation

A look inside the banking industry over the last decade clearly reveals two stylized facts. On the one hand, increasing complexity of financial technology combined with deregulation and globalization trends have made banking practices more sophisticated and challenging. As a result, the industry faced new multifaceted risks envisioned as part of 'other risks' and as such, different from market and credit risk. These include system security and fraud risks arising from the expansion of e-commerce, system failure risks on account of the use of highly automated technology, and many other significant risks resulting from the increased use of outsourcing arrangements and new risk mitigation techniques such as credit derivatives, swaps, and asset securitization (BCBS, 2003c). On the other hand, the banking industry all over the world has witnessed a growing number of insolvencies and experienced high-profile 'other risks' losses. In 1998, the press reported more than US$20 billion of 'other risk' losses in financial service firms, including the insurance industry. These combined facts brought supervisors as well as banking and insurance executives to view the management of these 'other risks' as a comprehensive practice comparable to the management of credit and market risk (BCBS, 2003c).

In the quest for solutions to issues raised by these challenging 'other risks' faced by the banking industry, the Basel Committee on Banking Supervision (the Committee) set up, in its June 1999 First Consultative Package, the principle of developing a Pillar One explicit capital charge for 'other risks', such as operational risk. Subsequent to the consultation process and its own analysis, the Committee adopted a definition of operational risk in its January 2001 Second Consultative Package and decided that only this specific risk should be subject to capital charges under Pillar One of the Framework (Minimum Regulatory Capital Requirements). Additional components of other risks such as interest rate risk and liquidity risk will be addressed only through Pillar Two (Supervisory Review Process) and Pillar Three (Market Discipline)[1].

The definition of operational risk, formulated by the British Bankers' Association (BBA) was refined in the September 2001 Working Paper on the Regulatory Treatment of Operational Risk, as follows: "the risk of loss resulting from inadequate or failed internal processes, people and systems, or from external events". The Committee specified that the aforementioned definition encompasses legal risk but excludes systemic, strategic, and reputational risks for the purpose of a minimum regulatory operational risk capital requirement.

There exist four computational methodologies to determine the regulatory capital requirements for financial institutions. These include fixed ratios, risk-based capital, scenario-based approaches[2], and probabilistic approaches (IAIS, 2000). In many views

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1 The Committee believes that, taken together, these three elements (Minimum Regulatory Capital Requirements, Supervisory Review, and Market Discipline) are the essential pillars of an effective capital framework (BCBS, 1999).

2 Under the fixed ratio method, the capital requirement is expressed as a fixed proportion of a proxy for exposure to risk often an item from the insurer's balance sheet or profit and loss account.
Under the risk-based capital model, results are determined by applying factors to exposure proxies such as invested assets risks, reserving risks, just like in the fixed ratio model.

(see for example IAIS, 2000; KMPG, 2002), probabilistic approaches such as the Advanced Measurement Approach (AMA), provide the preferred greatest framework for a meaningful capital requirement characterization. These methodologies use simulations to determine the full probability distribution of possible outcomes from which the capital requirement is determined using ruin-probability, expected policyholder approaches (Butsic, 1994) or other risk measures. As such, probabilistic methodologies are the most complex of the four approaches to assessing regulatory capital charges in terms of consistency, codification, and data requirements. Their complexity is also reflected in large costs associated with their application (KMPG, 2002).

As to the operational risk, it has been assumed that this specific risk will be more accurately captured under the AMA[3] and, therefore, incentives in terms of a lower capital charge granted to AMA applicant banks that refine and develop sound operational risk methodologies (Fitch, 2004). However, due to the specificity of this major risk class, there is no clear idea about the actual implications for using the AMA as a method to assess operational risk capital charges and, importantly, how its implementation would ultimately result in a lower capital charge for financial institutions that adopt it. According to a survey carried out by Fitch in 2004, forty-two large banks around the world believe that the AMA may generate capital charges that are not lower than those under the standardized or basic indicator approaches (Fitch, 2004).

As of today, there is a small body of literature that focuses on how the AMA should be effectively implemented in financial institutions. Literature on AMA can be

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KMPG (2002) describes scenario-based model as a methodology that explores the impact of specific risk variables to company specific exposure.

[3] The two other approaches include the Basic Indicator Approach (BIA) set according to the fixed ratio methodology and the Standardized Approach (TSA) established according to the risk-based capital approach.

traced only as far back as 2001 when the Committee published its document "Working Paper on the Regulatory Treatment of Operational Risk" in September 2001. With regard to the AMA-related academic literature, Embrechts et al. (2003), Chavez-Demoulin et al. (2004b), Embrechts et al. (2004), and Neslehova et al. (2006) question the ability of the standard actuarial model[4] as well as extreme value theory[5] to adequately address AMA issues because the assumptions behind these models are barely in line with the actual characteristics of operational risk losses. The authors consider models that include the particular case of the Cramer-Lundberg model[6], and general risk processes where the underlying intensity model follows a finite state Markov chain, allowing the modeling of underlying changes in the economy. In line with Embrechts et al., Chernobai and Rachev (2004) advocate for the use of the compound Cox model[7] or the alpha-stable distribution model[8] instead of the simple compound Poisson process[9].

As it appears, nearly all of these models suggest approaches which are more appropriate for large data sets without significant reporting bias. As a result, there is a need for more formal empirical research about operational risk capital requirements, taking into account various constraints in terms of data availability, data collection costs, limited computational resources, and limited decision time.

On the practitioners' side, Frachot et al. (2001), Frachot et al. (2002), and Baud et al. (2002) describe the Loss Distribution Approach (LDA) for operational loss and provide a methodology that allows banks to pool internal data with external data to

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4 Klugman et al. (2004).
5 Embrects et al. (1997).
6 See Embrects et al. (1997).
7 See Cox et al. (1980).
8 Zolotarev (1994), Embrechts et al., (1997), Rachev, S. Mittnik, S. (2000), Nolan (2001).
9 See Rolski et al. (1998).

estimate operational risk capital charge. Fontnouvelle et al. (2003) use the aforementioned methodology to provide preliminary empirical evidence on how publicly available operational loss data could be used to calibrate large loss severity probability distribution functions and capital charges. In their model, the random truncation point used to account for the reporting biases that plague this specific dataset is assumed to be logistically distributed. This assumption highly impacts the underlying loss severity distribution function and even though it is computationally convenient, it has been criticized on the account that it is not grounded on empirical evidence (Leandri, 2003). In addition, the dependency across risk categories is not accounted for. Di Clemente et al (2003) develop a model that considers a dependence structure based on the Student's t-copula and historical rank correlations. The empirical exercise, however, is carried out using catastrophe insurance loss data of three different lines – namely, hurricane, wind-storm, and flood. As such, the authors do not consider actual operational risk loss data issues. The next subsection presents my main contributions and the structure of the thesis.

1.2 Contributions and Organization of the Dissertation

My research objective is to investigate the implications of using the AMA to address operational risk capital charge modeling issues with regard to Basel II and US Regulations. More specifically, this dissertation examines the extent to which the four key elements of the AMA[10] could be incorporated into a model that has the potential to capture the relationship between the adequacy of capital and the quality of risk

[10] These elements include internal data, external data, scenario analysis, and business environment and control factor.

management (Caruana, 2003). For the empirical exercise, my work extends the methodology proposed in Fontnouvelle et al. (2003) that derives operational loss severity probability distribution function for large losses as well as the required capital charges. As mentioned in the preceding section, these authors use publicly available operational losses to design a model based on Extreme Value Theory (EVT) and random truncation point to estimate the loss severity probability distribution function that could be relevant to internationally active banks. The above-cited model requires using a convolution of distribution functions, and as such, is computationally intensive. In view of the foregoing concern, my research produced an efficient symbolic computational paradigm that facilitates the calibration of the parameters of the model. Within this symbolic framework, I provide answers to a key issue that involves calibrating the parameters of the loss severity to reflect specific firm size, rating, internal control environment as well as market-related factors. Finally, my work lead to a methodology based on upper tail dependence properties of elliptical copulas, and finite mixture distribution analysis to determine the empirical probability distribution function of the overall capital charge by means of Monte Carlo simulation runs. As a result, this latter paradigm also accounts for the quality of firm internal control environment through the weight assigned to each type of copula.

Using the wording of Fitch (2003), one may claim that the proposed approach provides incentives in terms of a lower capital charge to AMA applicant banks and insurance companies that refine and develop sound operational risk methodologies. Indeed, this dissertation clearly reveals that operational risk losses are driven by two loss event types - namely, Clients, Products and Businesses Practices on the one hand, and

Internal Fraud on the other hand. As a result, the quality of firm internal control environment appears as a key factor that significantly impacts the loss severity distribution and thereby the capital charge. Overall, my framework actually provides preliminary answers to some major concerns raised by regulators, scholars and practitioners as summarized in Fitch (2003). A crucial future consideration is to set up a paradigm that could accurately quantify the quality of firm internal control environment with regard to Pillar 2, Pillar 3, Sarbanes-Oxley Act[11], and Risk-Focused Surveillance Framework[12].

The structure of the dissertation is as follows. After this general introduction that explains why operational risk has become a major risk class, chapter 2 reviews the BCBS literature relevant to the topic as well as the literature from scholars and practitioners. Chapter 3 describes the model used to perform the simulation runs while chapter 4 presents the empirical procedure and the key results. Finally chapter 5 gives the concluding remarks.

Chapter 2 begins by concisely exploring the rationale for banking and insurance solvency regulation. Next, section 2.1 sketches out the evolution of the Basel Accord from 1988 to 2004 and explains the rationale for the move from the Cooke ratio that addresses credit risk capital issues toward the current McDonough ratio that addresses market, credit and operational risk capital requirements. Subsequently, the regulatory framework for the U.S. insurance undertaking is explored. Section 2.2 reviews the literature devoted to the computational aspects of the AMA. Specifically, subsection 2.2.1 describes the three approaches set forth by BCBS to address AMA issues.

[11] See Sarbanes-Oxley (2002).
[12] See NAIC (2004).

Subsection 2.2.2 examines the relevant literature that deals with operational risk emerging practices and related issues from both practitioners and scholars.

Chapter 3 concentrates on the model used to determine the empirical probability distribution function of the overall capital charge. Section 3.2 mathematically describes the capital charge formulation under the BCBS requirements while section 3.3 is dedicated to the Loss Distribution Approach. With regard to this latter model, the basic framework –namely, the Cramer-Lundberger model is first described, and next the point process methodology envisioned as a promising extension of the basic model is covered, notably, the new results from Chavez-Demoulin et al. (2005) and Pfeifer and Neslehova (2004). Subsection 3.3.2 focuses on the methodology proposed by Fontnouvelle et al. (2003) to extract both the observed and underlying loss severities from the publicly available operational loss data. Subsection 3.3.2.1 presents the above-cited approach, its symbolic implementation is described in subsection 3.3.2.2, and finally an extension of the model to account for specific firm size, rating, internal control environment as well as market-related factors is set forth in subsection 3.3.2.3. Subsection 3.3.3 considers the loss frequency distribution modeling. First, the calibration of this distribution is tackled through a risk assessment exercise as mentioned in Fontnouvelle et al. (2003). Next a basic common Poisson shock model suggested by Powojowski et al. (2002) and mentioned in Chavez-Demoulin et al. (2005) is explored.

Subsection 3.3.4 discusses some dependence structure issues that are considered when it comes to determine the overall capital charge. Indeed, within the BCBS framework, banks are required to estimate the capital charge for each of the 56 business line/event type and use a dependence structure model to aggregate these values. Since copulas provide more significant information on dependence structure than the conventional Pearson correlation, various families of copulas are examined with particular emphasis on copulas of extreme dependence and elliptical copulas. Finally, Subsection 3.3.5 uses upper tail dependence properties of elliptical copulas as well as finite mixture distribution analysis to express the overall capital charge as a mixing weighted capital charges.

Chapter 4 deals with the empirical investigation in the context of publicly available operational losses. Section 4.2 highlights some key descriptive statistics and motivates the risk classification schemes within which loss severities are calibrated and capital charges determined. Section 4.3 discusses the calibration of the parameters of the loss severity distribution using three distributional assumptions for the random truncation point. Finally, section 4.4 presents the descriptive statistics as well as the histogram of the empirical probability distribution function (that accounts for the randomness of the Monte Carlo simulation runs) of the overall capital charge.

CHAPTER 2

LITERATURE REVIEW

As a starting point for the literature review, we will briefly explore the rationale for banking and insurance solvency regulation and outline the development of the Agreed Framework of the Committee as well as the U.S. Risk Based Capital framework. Subsequently, diverse strands of the literature will be surveyed and discussed.

2.1 The Regulatory Framework

2.1.1 Rationale for Banking and Insurance Solvency Regulation

The three alternative views of bank supervision, as mentioned in Barth et al. (2003), include the "helping hand" view, the "grabbing hand" vision (Becker and Stigler, 1974; Shleifer and Vishny, 1998), and the "private monitoring" view (Haber, 2003). Specifically, the helping hand view emphasizes market failures while the grabbing hand view highlights political failures - politicians may take advantage of powerful supervisory agencies to compel banks to lend to privileged borrowers. The "private monitoring" view is a compromise that encompasses the "grabbing hand" view as well as "the helping hand" (Barth et al., 2003). In the sequel, we elaborate on the "the helping hand" vision which is commonly referred to as the public interest theory of regulation.

According to the public interest theory of regulation (see for example Stigler, 1971 and Posner, 1974), governments enforce regulation when free markets fail to allocate resource efficiently. For financial institutions, externalities and government guarantees[13] have the potential to create market failure, and thereby provide a rationale for government intervention. With regard to the banking industry, externalities in terms of systemic risk are defined by the Bank for International Settlements (BIS) as "the risk that the failure of a participant to meet its contractual obligations may in turn cause other participants to default with a chain reaction leading to broader financial difficulties". Systemic events like bank runs[14] can inflict significant social costs on the affected economies, by disrupting inter-bank and foreign credit relations. As a result, identification and close monitoring of systemic risks are higher priority on the policy agenda of central bankers.

Government guarantees for bank deposits partly eliminate the rationale for bank runs and protect small depositors who cannot effectively monitor their bank because of the high costs involved. Early on, public policymakers, as well as scholars[15], focused on issues involved in this specific guarantee. Recent empirical studies revealed a strong and robust link between the generosity of the deposit insurance system and bank fragility[16].

Generous deposit insurance schemes lessen market discipline enforcement and create a moral hazard issue, since there is a potential incentive for banks to engage in

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[13] Government guarantees include the deposit insurance for the banking industry and the guaranty fund for the insurance industry.

[14] By definition, bank runs is caused by depositors trying to withdraw their assets from the bank to avoid a loss of capital (Dionne , 2003).

[15] See for example Merton (1977).

[16] Demirguc-Kunt and Detragiache (2003), Barth et al. (2003).

higher-risk activities because the cost of the deposit insurance premium is not related to their risk taking-activity[17].

Thus, the rationale for controlling a bank's risk-taking activities stems from an asymmetric information issue, due to the existence of deposit insurance. Banking supervisors accomplish this control by performing solvency assessments that require, among other things, banks to carry minimum levels of capital that act as a cushion to protect the insurance fund. In this regard, the 1988 Basle Accord framed under the chairmanship of W.P. Cooke provided the first decisive step toward efficient banking regulation for internationally active banks.

With regard to the U.S. banking industry, in 1989, the three banking agencies[18] (the Board of Governors of the Federal Reserve System (FRB), the Office of the Comptroller of the Currency (OCC), and the Federal Deposit Insurance Corporation (FDIC)) and the Office of Thrift Supervision (OTS) adopted a common regulatory framework that establishes minimum capital adequacy ratios for commercial banks, in line with the 1988 Basle Accord.

As to the insurance industry, Cummins et al. (1995) mention that the rationale for insurance solvency regulation includes the difficulty for the insured to really monitor the insurer solvency due to the complexity of the insurance activity and the likelihood that insurers could increase risk following policy issuance, particularly, if the interests of the new policyholders and existing policyholders diverge. Also included is the existence of

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17 See Cull et al. (2004), Barth et al. (2003).

18 At the federal level, the Federal Reserve has primary supervisory responsibility for state-chartered banks that are members of the Federal Reserve System, as well as for all bank holding companies and certain operations of foreign banking organizations. The FDIC has primary responsibility for state nonmember banks and FDIC supervised savings banks. National banks are supervised by the OCC. The OTS has primary responsibility for savings and loan associations (The Federal Reserve Board (1999)).

non-risk-rated guaranty fund[19] coverage that provides insurers with the possibility to increase the value of owners' equity by engaging in riskier activities without being penalized by the market (Cummins, 1988).

Just as banks hold capital to protect the insurance fund, so do insurers carry minimum levels of capital to protect the guaranty fund. However, as pointed out by Medova et al. (2004), "an optimal balance must be struck between holding economic capital to ensure solvency and its cost, in order to provide a decent return on equity for shareholders"[20].

2.1.2 Basel Committee on Banking Supervision Framework: From the Cooke Ratio to the McDonough Ratio

In July 1988, the Basel Committee on Banking Supervision released its report "International Convergence of Capital Measurement and Capital Standards". In many regards, the paper that became known as the 1988 Basel Capital Accord was a milestone (Caruana, 2003). Indeed, for the first time, the Group of Ten (G-10)[21] banking supervisory authorities set up an agreed framework for establishing minimum levels of capital, in relation to credit risk, for internationally active banks. Essentially, the 1988 Accord aims at reinforcing the soundness and stability of the international banking system and "diminishing an existing source of competitive inequality among international banks" (BCBS, 1988). Typically, within this new framework, a weighted risk ratio

[19] State guaranty funds require solvent insurers to pay losses of insolvent firms. Guaranty fund coverage weakens the market incentive for insureds to monitor insurers' solvency (Cummins et al., 1995).

[20] See also Merton and Perold (1993), Froot and Stein (1998), Perold (2001).

[21] The Basle Committee on Banking Supervision comprises representatives of the central banks and supervisory authorities of the Group of Ten countries (Belgium, Canada, France, Germany, Italy, Japan, Netherlands, Sweden, Switzerland, United Kingdom, United States) and Luxembourg. The Committee meets at the Bank for International Settlements, Basle, Switzerland.

known as the Cooke ratio or risk asset ratio (RAR), requires international banks to hold capital at least equal to 8 percent of their reported risk-weighted assets.

The Committee set up the components of the regulatory capital in two equally-weighted tiers. These include tier 1 - the core capital, comprised of equity capital and disclosed reserves, tier 2 - the supplementary capital, consisted of other elements of capital such as undisclosed reserves, reevaluation reserves, general provisions and general loss reserves, hybrid debt capital instruments and subordinated term debt.

With regard to the ratios that could appropriately assess bank capital adequacy, the Committee expressed the view that the weighted risk ratio in which capital is tied to different classes of asset and off-balance sheet exposure, weighted according to some risk categories, was the best method for making such an assessment. Three facts underpinned the Committee choice over a more regular gearing ratio methodology. Firstly, risk ratio facilitates comparisons between heterogeneous international banking systems in that it offers a more adequate basis for assessment. Secondly, this specific ratio permits off-balance-sheet exposures to be included more smoothly into the capital measurement framework. Thirdly, the risk ratio does not prevent banks from holding assets deemed to be low risk such as cash or claims on central governments and central banks. The Committee kept as simple as possible the weighting structure by allowing only five weights i.e. 0, 10, 20, 50 and 100 percent. The weighting structure for on-balance-sheet assets items is set out in the Appendix 1, Table A1.

As to the level of the weighted risk ratio, in light of consultations and pilot testing of the framework, the Committee agreed that the proposed ratio of capital to weighted risk assets should be set at 8 percent. International active banks in member countries had been expected to achieve the target ratio by the end of 1992.

Regarding capital requirements for the U.S. banking and thrift agencies, CAS (1992) mentions that, in 1986, US banking regulatory agencies issued a risk-based capital proposal that had been criticized on the grounds that without a comparable framework for foreign players, U.S. banks would suffer from a competitive disadvantage. In view of the foregoing concerns, in 1987, the US banking regulatory agencies and the Bank of England examined various issues involved in such approaches and set out a joint proposal. Afterward, in 1988, the Basle Committee refined this proposal and expanded it to incorporate the G-10 member countries. In the U.S., the FRB, the OCC, the OTS and the FDIC implemented risk-based capital standards that were in line with the 1988 Basle Capital Accord.

On many accounts, the 1988 Basel Capital Accord was a great success story (Caruana, 2003). It has been adopted in over 100 countries and the Cooke ratio has come to be recognized as a popular yardstick to quantify a bank's capital adequacy. The rationale for its success was perhaps its simplicity that arose from the fact that the G-10 governments as well as the financial communities were eager to release a straightforward applicable framework that could revert the downward trend in international active bank capitalization (Caruana, 2003). Indeed, during the mid-80's, the capital of the world's major banks had become dangerously low after continual erosion through competition (BCBS, 2001a). Another reason for its success might stem from the fact that the Accord has been set out in the aftermath of serious disturbances in equity markets: on Monday, October 19, 1987 US Stock collapsed by 23 percent, wiping out US$ 1 trillion in capital (Hong, 2003).

In the years since 1988, the weaknesses of the simple approach to capital regulation have come to light. As a matter of fact, the 1988 Accord has suffered from many criticisms, the most obvious being that it did not adequately address off-balance sheet exposures on market risk in particular risks associated with the bank's positions in derivatives. In addition, it did not account for portfolio diversification effects, netting effects and the borrower's credit rating. Indeed, taking into account correlations between risk categories of the portfolio may lower total portfolio risk while matching lenders and borrowers may decrease bank's net exposure (Jorion, 1997). Another shortcoming of the 1988 Accord was that it tended to support transactions and investments whose sole benefits were regulatory arbitrage. Investments in costly but better risk management without tangible regulatory capital relief were not fully realized. CAS (1992) mentions that studies in the Wall Street Journal indicate that the new capital requirement, soon after its implementation in the U.S., triggered many banks to change their investment policies by moving assets out of corporate loans (which carry the maximum 100 percent risk weight) into government securities (which require 0 percent risk weight). As a result, the market witnessed a decrease in bank lending and the FRB discussed the possibility of facilitating some of the capital requirements. In this regard, under the risk-based capital guidelines, the FRB may modify the rules in order to reflect significant changes in the economy, financial markets, banking practices, etc.

Over the past decade, the state-of-the-art in measuring and managing risk has made tremendous progress in ways the builders of the 1988 Accord could not have foreseen (Caruana, 2004). The explosive growth in the markets for credit derivatives and for securitized assets and liabilities has provided banks with new ways to manage and transfer credit risk. Advances in technology and telecommunications have changed the way that banks process data on their exposures. In response to these aforementioned criticisms and challenging innovations, the Committee issued a series of amendments to provide banks and their supervisors with sound measures of the actual risks they face.

In the quest for addressing risks other than credit risk, in January 1996, the Committee issued its paper "Amendment to the Capital Accord to incorporate Market Risks". Through this amendment, the Committee set up capital requirements for market risks arising from banks' open positions in foreign exchange, traded debt securities, equities, commodities and options. Two alternative approaches to the measurement of this specific risk have been proposed, namely the standardized method and the internal models approach. The capital charge under the standardized measurement method has been set out as an arithmetic sum of specific measures of the five risk categories addressed by the Committee.

Through internal models that reflected practices in leading financial institutions, the Committee has allowed banks to use their own qualified risk assessment models to determine their capital charge. In many regards, this approach has been a ground-breaking step forward in banking regulation.

Using a models-based methodology requires banks to satisfy minimum qualitative and quantitative standards. Qualitative standards include, among other things, the design and implementation of the bank's risk management system by an independent risk control unit, as well as a periodic independent review of the risk measurement system. The minimum quantitative standards involve, among other things, the computation of value-at-risk (VaR) as a risk metric, to quantify bank's exposure according to the following:

- Value-at-risk are computed with a holding period of 10 trading days or two calendar weeks, at the 99th percentile, with an historical observation period constrained to a minimum length of one year.
- Empirical correlations within risk categories as well as across risk categories may be recognized.
- Each bank has to meet, on a daily basis, a capital requirement expressed as the maximum between its previous day's value-at-risk and an average of the daily value-at-risk measures on each of the preceding sixty business days, multiplied by a factor. The multiplication factor (set to a minimum value of 3) has to be determined by the supervisor and adjusted according to back-testing results.

The rationale of this practice is to give incentives to banks to improve the predictive accuracy of their model.

In addition to the above minimum qualitative and quantitative standards, the Committee set out three other requirements for the use of internal models. These were:

- The selection of a suitable set of market risk factors, that is the market prices and rates that impact the value of the bank's on-and off-balance-sheet trading positions.
- The use of stress testing scenarios as stress testing to capture events that could significantly impact banks is a major exercise in assessing bank's capital position.

- An external validation of models' accuracy by external auditors and/or supervisory authorities.

To calculate the total capital-adequacy requirements, the credit-risk charge is added to the market-risk charge. The eligible capital to cover market risks includes tier 1 capital and tier 2 capital as defined in the 1988 Accord. Also included (at the discretion of the national authority) is tier 3 capital consisting of short-term subordinated debt. Tier 3 capital is restricted to approximately 70 percent of an institution's measure for market risk.

In a quest to push further forward its responsiveness to financial innovation and developments in risk management practices, the Committee released in July 1999 its consultative paper "A New Capital Adequacy Framework". According to William J. McDonough, previous chairman of the Basel Committee, the key objective of the New Accord is to strengthen the stability of the global financial system. To achieve this objective, the New Basel Accord implements goals that consist, among other things, of "capturing the relationship between the adequacy of capital and the quality of risk management by relying on three mutually reinforcing pillars" (Caruana, 2003). These include, minimum capital requirements, supervisory review, and market discipline. In the sequel, we review the position of Jaime Caruana (the current chairman of the Basel Committee) on the objective of the three pillars.

Essentially, the first pillar aims at matching capital requirements more closely with actual risks banks incur. For instance, the Advanced Measurement Approaches to operational risk build on the bank's internal loss data and allows banks to rely on their own assessments of risk to calculate how much capital to hold. As a result, economic incentives in terms of lower capital charges are granted to banks that appropriately assess their exposures and develop better techniques for managing their risks. In addition to performing that match, pillar 1 seeks to achieve convergence between economic capital and regulatory capital.

Through the second pillar, supervisors will be accountable for evaluating the internal processes banks employ to determine their need for capital. By engaging managers in a discussion about the risks they incur and the controls they have adopted to address them, supervisors create incentives for managers to act prudently.

The third pillar uses the market itself to provide discipline to banks to make sure that they are not holding low levels of capital. This is achieved by making the banks' public reporting of their risks as well as measures taken to control such risks, available to investors and customers. This generates a strong incentive for bank management to enhance their handling of those risks.

As a result, these three pillars, taken together, aim at providing incentives to banks to get a more accurate recognition of the risks they incur and take preemptive actions to protect themselves against those risks by means of their control structures and their holdings of capital (Caruana 2003). To make certain that the risks within an entire banking group are considered, the New Accord is extended on a consolidated basis to holding companies of banking groups.

After pilot testing of the proposed new Framework and large consultations with the financial community worldwide, the Committee issued on June 26, 2004, its final report "International Convergence of Capital Measurement and Capital Standards: A Revised Framework". The Central Bank Governors as well as the Heads of Banking

Supervision of the G-10 countries have endorsed the Framework and the Standard it contains.

Under the Agreed Framework (also referred to as Revised Framework or Basel II), Pillar 1, covers regulatory capital requirements for market, credit and operational risk. Three key elements characterize the minimum capital. These are, the definition of regulatory capital, the risk weighted assets and the minimum ratio of capital to risk weighted assets. With regard to the definition of the eligible regulatory capital, it remains the same as the one set out in the 1988 Accord, and refined in the 27 October 1998 press release on instruments eligible for inclusion in Tier 1 capital. The total risk weighted assets is calculated by multiplying the capital requirements for market risk and operational risk by 12.5 (i.e. the reciprocal of the minimum capital ratio of 8 percent) and adding the resulting figures to the sum of risk-weighted assets calculated for credit risk. The capital ratio (also referred to as the McDonough ratio) is simply the ratio of the regulatory capital to the total risk weighted assets. The ratio must be no lower than 8 percent for total capital. Tier 2 capital will continue to be limited to 100 percent of Tier 1 capital. Tier 3 capital remains restricted to approximately 70 percent of an institution's measure for market risk.

To enhance risk sensitivity, the Committee offers a variety of options for addressing both credit and operational risk. As to credit risk, the range of options includes the standardized approach, a foundation internal ratings-based approach (IRB), and an advanced IRB approach. Regarding operational risk minimum capital requirements, the Committee recommends several approaches that reflect those of credit and market risk. These include three methods in the continuum of increasing sophistication and risk sensitivity (BCBS, 2001): the Basic Indicator Approach (BIA), the Standardized Approach (TSA) and the Advanced Measurement Approaches (AMA).

Banks adopting the Basic Indicator Approach are required to hold capital for operational risk that amounts to the average over the previous three years of a fixed percentage (termed alpha) of positive annual gross income. The level of the parameter alpha has been set to 15 percent following the Quantitative Impact Survey (QIS) data analysis. Gross income is defined as net interest income plus net non-interest income. The Committee has not set any special requirement for use of the Basic Indicator.

The Standardized Approach is based on a three-stage calculation. The starting point consists of allocating the bank's previous three-year gross income into eight standard business lines. Then, for each business line, operational risk capital requirement is calculated as in the BIA case but with the business line specific factor (beta factor). The computation ends with calculating the total operational risk capital requirement as the sum of the individual business line operational risk capital requirements. The business lines with their respective beta factors are reported in Appendix 1 Table A2.

If a bank is primarily active in Retail or Commercial Banking business lines, the Committee authorizes the use of the Alternative Standardized Approach (ASA) instead of the TSA. For these two specific business lines, the relevant exposure indicator is the three-year average of the total nominal amount of loans and advances for each business line multiplied by 0.035.

The qualifying criteria for the use of the Standardized Approach as well as the Alternative Standardized Approach include, among other things, the involvement of the board of directors and senior management in the oversight of the operational risk management process.

As to the AMA, the operational risk regulatory capital requirement under this approach is drawn from the bank's own internal operational risk measurement system. The use of the AMA is subject to regulatory approval and some specific qualitative and quantitative standards need to be met.

Qualitative standards essentially require that an AMA applicant bank must integrate its internal operational risk measurement framework into its day-to-day risk management systems (FSA, 2005), and create an independent risk management function for operational risk.

Quantitative standards involve the following: first, banks can base the minimum regulatory capital requirement on unexpected losses alone if they succeed in demonstrating to the satisfaction of the national supervisor that their expected loss exposure is measured and accounted for; second, the operational risk measure should meet a soundness standard comparable to the IRB approach for Credit Risk, that is, one-year holding period and 99.9 percent confidence; third, operational risk measurement must include four key elements - namely, internal data, external data, scenario analysis and business environment and control factors.

The Committee envisions internal data as essential for relating risk estimates to loss experience. This is carried out by using internal loss as the basis of empirical risk estimates.

A bank's internal loss collection processes must meet sound standards in order to qualify for regulatory capital purposes. These include the mapping of its historical internal loss data into the supervisory categories of business lines and loss event types, the design of a comprehensive dataset accounting for different types of exposures, and geographic locations. Also included are the choice of an appropriate threshold, dates of events, any recoveries of gross loss amounts, as well as descriptive information about drivers or causes of loss events. Banks must design specific criteria for allocating loss data stemming from events or activities that extend over many business lines, as well as from related events over time.

The Committee requires AMA applicants to use a minimum five-year observation period of internal loss data to generate risk measures for regulatory capital purposes. A three-year historical data window is sufficient for the first application of the AMA.

To address internal loss data intrinsic weaknesses such as data gaps and backwards-looking measure of exposure, the Committee requires the use of external data, scenario analysis, business environment, and internal control factors.

Concerning external data, the Committee recommends the use of relevant external data to supplement internal data, each time an AMA applicant bank is exposed to infrequent, yet potentially severe losses. Requirements for the use of external data include information on the scale of business operations where the event occurred, information on the causes and circumstances of the loss events as well as information that would help in assessing the significance of the loss event for other banks.

The conditions and practices for external data use, notably the pooling methodologies must be well-documented and subject to periodic independent review.

The Committee requires an AMA applicant bank to employ scenario analysis of expert opinion associated with external data to evaluate the bank's exposure to high severity events. Typically, scenario analysis uses the knowledge of experienced business managers and risk management experts to derive consistent assessments of plausible severe losses. Examples of uses of this methodology include assessment of potential losses occurring from simultaneous operational risk loss events as well as assignment of values to parameters of assumed statistical loss distributions. A specificity of this approach is that banks need to constantly ensure the accuracy of assessments with respect to actual loss experience.

The Committee requires banks to include into their risk assessment methodology key business environment and internal control factors that have an impact on their operational risk profile. Control factors strengthen bank's risk assessments in many ways. Besides being more forward-looking, they make capital assessments reflect risk management objectives, and above all, they account for enhancements and declines in operational risk profiles quickly. The use of these factors in a bank's risk measurement framework is subject to some standards. First, banks have to motivate the choice of each factor as a significant driver of risk. Second, the link between bank's risk estimates and factors and the relative weighting of the various factors should be well conceptualized. Third, over time, banks need to contrast the assessment estimates with actual internal loss experience, external data, and make suitable amendments.

Internally determined correlations and dependencies in operational risk losses are recognized for the use of the AMA. With regard to risk mitigation issues, AMA applicant banks will be allowed to take into account the risk mitigating impact of insurance in operational risk measurement up to 20 percent of the total operational risk capital charge calculated under the AMA.

With regard to U.S. banking regulations, recent legislation that aims to infuse a great deal of discipline within the banking system and financial markets include (1) the Gramm-Leach-Bliley Act of 1999 (GLBA), (2) the Federal Deposit Insurance Corporation Improvement Act of 1991 (FDICIA), and (3) the Sarbanes-Oxley Act of 2002 (SOX)[22]. Now, we review the capital requirement literature in connection with the U.S. insurance industry.

2.1.3 Regulatory Capital Framework for the U.S. Insurance Undertakings

In the United States, state insurance regulators came together in 1871 to create the National Association of Insurance Commissioners to address the need to coordinate regulation of multistate insurers (NAIC, 2005). In many regards[23], the development of insurance risk-based capital (RBC) requirements stemmed from the report "Failed Promises: Insurance Company Insolvencies" issued in February, 1990, by the Subcommittee on Oversight and Investigations of the U.S. House Committee on Energy and Commerce - the Dingell Committee (U.S. House of Representatives, 1990). Essentially, this Committee investigated U.S. insurer insolvencies during the mid-1980 as well as various deficiencies in the existing solvency regulatory system. Reflecting the Dingell Committee‘s work and proposals, in December 1992, the NAIC adopted a life-health insurer risk-based capital framework and model law that entered into force with the 1993 annual statement filed in March 1994. Similarly, a property-liability insurer

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22 FDICIA was passed in the wake of the Savings & Loan disastrous decline, SOX was enacted in reaction to pervasive corporate malfeasance, such as events at Enron, Tyco, and WorldCom. GLBA was written to modernize financial regulation and also to fight personal identity theft (BITS Operational Risk Management Working Group, 2005).

23 See Cummins et al. (1995), Insurance Information Institute (2005)

risk-based capital and model law adopted by the NAIC in December 1993 entered into force with the 1994 annual statement filed in March 1995.

Lewis (1998) recognizes that RBC rules help supervisors in taking prompt regulatory steps against insurers without court action. Cummins et al. (1995) question risk-based capital requirements on the grounds that insolvency risk is hard to quantify in that it encompasses various non-tangible factors that are difficult to assess. In addition, the insurance market is characterized by many different players in terms of company size and organizational form, types of business written and customer specificities. As a result, it is inappropriate to specify the correct amount of capital for most insurers through a formula. Cummins et al. (1995) carry out an empirical exercise, investigating the relationship between the industry insolvency experience and the ratio of actual capital to RBC for property-liability insurers from 1989 through 1991. Results indicate that more than half of the companies that later failed had RBC ratios outside the mandated ranges for regulatory and company action. In addition, RBC models usually fail in predicting large firm insolvencies so that accounting for firm size and organizational form improves the predictive accuracy of the insolvency risk models.

SOA (2002) admits that certain risks such as liquidity risk, operational risk, and the risk of fraud are difficult to assess through a RBC formula, and as a result, are mostly not accounted for in the formula. Brender (2004) assimilates business risk to operational risk for life insurers and points out that "this is the area most in need of future development in considering insurance required capital. The emphasis will be different from that in banking, since the insurance business is not as transaction based as is banking."

In terms of operational risk quantification, for the whole insurance industry, business risk approximately accounts for 13 percent of the total regulatory capital (Rochette, 2005). In the UK, a recent illustrative case developed in GIRO Working Party (2004) reveals that operational risk capital charge could approximately amount to 2 percent of net premiums on average. However, the authors recognize that further work is needed to quantify the real impact of operational risk.

As of today, the trend in the management and measurement of the operational risk within the insurance industry is toward a Basel-based framework. In this regard, the U.S. regulating authorities are currently reshaping their methodology for a more efficient insurance solvency regulation. In that sense, the new Framework, in tune with Basel II, offers a more robust methodology to supervise and assess the solvency of insurers on an ongoing basis. Part of the Framework's focus is on the Risk Assessment Matrix "intended to be an all-encompassing tool incorporating risk assessment, examination procedures and results" (NAIC, 2004). Nine types of risk classes including operational risk[24] are determined and input in phase two of the Risk Assessment Matrix. Now we are in a position to review the relevant literature that discusses the computational aspects of the AMA.

2.2 The Computational Aspects of the AMA

The literature on the AMA started in 2001 when the Committee published its document in September 2001 "Working Paper on the Regulatory Treatment of Operational Risk".

[24] NAIC (2004)'s definition of operational risk. "Operational problems such as inadequate information systems, breaches in internal controls, fraud, or unforeseen catastrophes will result in a disruption in business and financial loss."

In fact, a range of operational risk internal approaches initiated by practitioners emerged since 1998 when the Committee issued its report "Operational Risk Management". This report presented the outcome of a working group of thirty major banks from the member countries on the management of operational risk. Reflecting these developments, the Committee adopted the concept of the AMA through which a bank's internal mechanism for quantifying operational risk may be accepted by the supervisor, subject to a number of requirements.

As of today, there are three strands in the AMA literature. The first strand is provided by the Basel Committee itself. The second strand arises from academia (see for example Embrechts et al., 2003; Neslehova et al., 2006) and the third strand stems from practitioners in the banking industry (see for example Frachot et al., 2001; Shi et al., 2000; Fontnouvelle et al. 2003). These three strands will be examined in the sequel.

2.2.1 BCBS Literature

In September 2001, the Basel Committee set out three approaches to addressing AMA issues (BCBS, 2001c). These include Internal Measurement Approaches (IMA), Loss Distribution Approaches (LDA), and Scorecard Approaches (SA). Now, we review the literature with regard to these approaches.

Internal Measurement Approaches are based on a two-stage calculation process. First, banks derive expected and unexpected losses from estimates of loss frequency and severity for various business line/event type combinations, based on internal and external loss data. Second, operational risk capital is computed by business line/event type as a product of expected losses and factor (referred as to gamma factor) defined by banks but subject to regulatory approval. This methodology assumes a fixed and stable relationship between expected losses (the mean of the loss distribution) and unexpected losses (the tail of the loss distribution). A non-linear relationship can also be assumed. The total capital charge is then calculated as the sum of the capital charge for individual business line/event type cell.

Loss Distribution Approaches involve a four-step calculation.

- For each business line/risk type cell, AMA applicant banks estimate the shape of the distributions of the severity of individual events. This is obtained either by imposing specific distributional assumptions as lognormal or by using empirical methods as bootstrap and Monte Carlo simulation.
- The distribution of the number of losses for one-year horizon is derived for each business line/risk type cell in order to compute the aggregate loss and its distribution for the considered period.
- The capital charges (for each individual business line/event type) resulting from the aggregate loss distribution is computed based on a high percentile of the loss distribution.
- The overall capital charge is computed by assuming either perfect positive correlation of losses across these cells, or by using other aggregation methods that recognize the risk-reducing impact of less-than-full correlation. In the former case, the total capital charge is calculated as the sum of the capital charges for individual business line/event type cell.

Loss Distribution Approaches differ from Internal Measurement Approaches in that they use disaggregated data and tend to assess unexpected losses directly rather than via an assumption about the relationship between expected loss and unexpected loss on aggregated data basis. As of today, several kinds of Loss Distribution Approach methods are being developed and no industry standard has yet emerged.

The Scorecard Approaches can be conceptualized in two stages. First, banks have to determine an initial level of operational risk capital at the firm or business line level. Second, this initial level is modified over time by use of scorecards to reflect the underlying risk profile and risk control environment of the different business lines. Typically, scorecards bring, on a qualitative basis, a forward-looking dimension to the capital calculations, by reflecting improvements in the risk control environment that will decrease both the frequency and severity of future operational risk losses.

In terms of methodology, Scorecard Approaches may be rooted in initial estimation methods that are identical to those used in Internal Measurement or Loss Distribution Approaches. In some cases, it can be based on identification of a number of indicators used as proxies for particular risk types within business units/lines. Whatever methodology is selected, emphasis on a robust quantitative basis is needed in order to get the Scorecard Approaches qualified for the AMA. In addition, the overall size of the capital charge has to be based on an accurate analysis of internal and external loss data.

AMA is still evolving in the banking industry. Prior to implementation of the Revised Framework by the end of 2007, the Committee will review leading industry practices regarding credible and consistent estimates of potential operational losses as well as the level of capital requirements estimated by the AMA. We now turn to the survey of the practitioner and academic literature on the AMA.

2.2.2 Practitioner and Academic Literature

As a starting point for reviewing these strands, we will explore emerging practices in operational risk measurement as set out by practitioners from leading banks as discussed in the Industry Technical Working Group –ITWG (2003). Subsequently, we will examine academics' claims regarding these practices. Next, following Frachot et al. (2003) who discuss these methodologies in five steps, the literature review will survey various approaches related to each of these steps. These approaches include loss severity estimation, loss frequency estimation, capital charge computations, confidence interval, and scenario analysis. Finally in line with Cruz (2002), the operational risk back-testing analysis will be explored.

2.2.2.1 Emerging Practices and Related Issues

Relevant literature on operational risk emerging practices includes Frachot et al. (2001), Baud et al. (2002), Frachot et al. (2003), ITWG (2003), Embrechts et al. (2003), Chavez-Demoulin et al. (2004b), Embrechts et al. (2004), Chavez-Demoulin et al. (2005), and Neslehova et al. (2006). Essentially, the academic literature investigates the prerequisites under which practitioners emerging practices hold.

As mentioned earlier, the AMA consists of three approaches – namely, IMA, LDA, and SA. IMA is generally not considered in the literature. As to the SA, Currie (2004) points out that there is no conclusive evidence that this model is actually functional and has predictive properties. Holmes (2003) questions its ability to provide reliable information about bank risk over time. Concerning the LDA, ITWG (2003) provides some insights into the way leading banks envision its implementation. ITWG was created in 2000 by operational risk practitioners from leading financial institutions around the world[25]. ITWG's key objective is to develop and share practical new ideas for the quantification of operational risk. Its agreed core approach for operational risk measurement is essentially based on the actuarial modeling of operational risk losses, notably the LDA. The Bayesian method and the causal modeling are considered as well.

Specifically, ITWG assumes that operational risk loss data is the most significant risk indicator currently available that reflects the specific operational risk profile of each bank. A well-managed bank, because of its effective operational risk management processes and tools, will be less exposed to operational risk losses- both expected and unexpected. On the other hand, a bank without a robust operational risk management control is likely to experience higher losses. As a result, loss data should be used as initial input for implementing loss distribution, which will, in turn, be the underlying driver of the AMA. ITWG also considers that its basic assumption still allows banks to employ various components of the AMA with specific weights when assessing the overall AMA. For instance, emphasis could be put on scenario analysis and business environment and control factors in the risk assessment of business lines with a heavy-tailed loss distribution and a small number of observed losses. Furthermore, the weight attached to each element within the overall AMA will be adjusted over time as banks collect more reliable data and expand their knowledge in operational risk management.

[25] These are: ABN AMRO, Banca Intesa, BNP Paribas, BMO Financial Group, Crédit Lyonnais, Citigroup, Deutsche Bank, ING, JP Morgan Chase, RBC Financial Group, Royal Bank of Scotland, San Paolo IMI and Sumitomo Mitsui BC.

Embrechts et al. (2003) question the ability of LDA to address adequately operational risk loss modeling issues by analyzing the impact of LDA on operational risk loss classified in two categories - namely, repetitive and stationary, non repetitive and non- stationary. First, they discuss a series of prerequisites under which standard actuarial methods supplemented with extreme value theory are appropriate for banks when dealing with capital charge issues, in the context of operational risk. For standard actuarial methods, these preconditions include independently and identically distributed (iid) random variables assumptions (which implies data stationarity[26]), and repetitiveness of observations. For the standard extreme value theory, notably the Peak-Over-Threshold approach (POT)[27], these requirements encompass, among other things, the abundance of data over high thresholds, the number of exceedances as a homogeneous Poisson process[28]. Embrechts et al. (2003) point out typical features displayed by operational risk losses, notably the data paucity for certain loss event types, irregularities in the occurrence times, and the existence of extremes. In addition, certain risks generate non-repetitive losses (Crouhy et al., 2000). The authors contend that the observed irregularities generally appear to go beyond ordinary randomness similar to that of a homogeneous Poisson process. As to the non-stationarity, the authors argue that it might stem from a sample selection bias like survivorship bias in that the discipline of collecting operational risk losses in financial institutions is quite recent. As such, many historical losses have not been kept in banks' datasets. Alternatively, non-stationarity can originate from business cycles, economic cycles, management interactions and regulation.

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26 See Rolski et al. (1998)
27 See Embrechts et al. (1997) ; McNeil and Siladin, (1997)
28 See Rolski et al. (1998)

Reflecting these non repetitiveness and serial dependence, Embrechts et al. (2003) argue that banks need to carry out initial clarifying analysis to find out which business line/loss event types are sure candidates for actuarial techniques and POT methods as well. They point out that actuarial methods and their enhancements can be used for repetitive and stationary losses to assess capital charges. In contrast, for non-repetitive and non-stationary losses that significantly endanger the existence of banks, Basel's pillar 1 is inefficient so that pillar 2 and 3 should be considered and enforced.

Moving forward the theoretical framework of operational risk measurement, Chavez-Demoulin and Embrechts (2004b) discuss some of the more recent extreme-value theory approach that may be effective in modeling certain types of operational risk loss data. The authors describe an adapted extreme-value method that accounts for non-stationarity (time dependence) and covariates (different types of losses). However, as of today, the model is not fully applicable due to lack of large datasets.

More specifically, Chavez-Demoulin and Embrechts (2004b) extend the POT method by allowing the three parameters of the distribution -namely, the Poisson parameter and the Generalized Pareto Distribution (GPD) parameters (the shape and the scale parameters) to be dependent on time and explanatory variables so as to account for the non-stationatity. Following the non-parametric approach in Chavez-Demoulin and Embrechts (2004a), they fit different models for the three parameters allowing for functional dependence on time and on type of loss data (three types). A discontinuity parameter that models the regime switching effect is added as well. Through the model, two risk measures, the Value-at-Risk and the Expected-Shortfall[29] (ES) both dependent on time and covariates are estimated. For illustration purpose, 99%VaR and 99% ES for year 2002 are computed.

The results indicate that the model reasonably estimates the POT parameters. As for the risk measures, the authors find out that the risk measures of loss type 3 is significantly smaller than those of loss type 1 and 2. This result highlights the importance of using all the provided information about the data by including loss types as covariates instead of mixing data and getting a unique risk measure. However due to lack of large historical data, the authors cannot ensure, through back-testing, that the methodology correctly estimates the risk measures.

The next chapter examines the marked point process methodology sets forth in Chavez-Demoulin et al. (2005) and Pfeifer and Neslehova (2004) to model the dependence structure across risk categories in terms of aggregate losses.

2.2.2.2 Loss Severity Modeling

The related literature dealing with estimating operational risk loss severity distribution, especially in the tail, comprises different convergent approaches. Each paper offers a discussion of various biases that afflict external loss data - most notably the data scalability and paucity, the reporting bias - and provides a methodology to circumvent these isssues. Significant contributions include Frachot and Roncalli (2002), Baud et al. (2002), Frachot et al. (2003), Baud et al. (2003), Fontnouvelle et al. (2003), Fontnouvelle et al. (2004), Moscadelli (2004).

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[29] ES at a specified level α was introduced in both Acerbi et al. (2001) and Rockafellar and Uryasev (2001). It is defined by the former authors as “the average loss in the worst 100 α % cases.

Frachot et al. (2003) consider that operational risk loss data used to derive severity distribution come from diverse sources that include business units within a bank and external providers. As a result, prior to assessing loss severity distribution calibration, one should reasonably address various issues raised by the mixture of such apparently disparate data sources. To that aim, the authors set out two hypotheses concerning the nature of data sources and develop arguments in favor of the one that could lead to pragmatic and acceptable results, given the current resources devoted to operational risk.

Hypothesis 1: The diverse sources of data are assumed to be homogeneous in the sense that they originate from the same primary probability distribution even though each source reports loss data according to its specific threshold.

Hypothesis 2: The diverse sources of data are presumed to be heterogeneous in the sense that they derive from different probability distributions and as such, they need to be re-scaled. Furthermore, each source may provide loss data in line with its particular threshold.

As a matter of fact, Frachot et al. (2003) recognize that hypothesis 2 is not only the broadest but also the most accurate. But what are the actual cost and benefit tradeoffs associated with using alternative modes of accounting for data-pooling issues? Given the current state of knowledge of the operational risk community on this topic and given the resources dedicated to that class of risk, the authors advocate that hypothesis 2 is extremely complicated to be adequately addressed. They argue that the scaling formula attempts to derive a mathematical expression that links the internal severity distribution to the external one (See Shih et al., 2000). Furthermore, under hypothesis 2, the scaling formula involves large sets of data drawn from various internal and external sources and it is not even guaranteed that this scaling adjustment is practicable for all loss event types. In the near-future, as soon as loss datasets become larger, further inspection may lead to more insights into the issue. Therefore, as of today, it is untimely to think of the derivation of consistent scaling functions. Frachot et al. (2003) also maintain that under hypothesis 2, specific scaling formulas need to be estimated to account for differences between business lines within a bank. This practice would require a large amount of work to be carried out. Finally, the authors cite the empirical work performed by Fontnouelle et al. (2003) in which severity distributions are derived from two different external loss datasets. The results indicate that both datasets display great similarities once the reporting bias has been properly accounted for.

In the same spirit as Frachot et al. (2003), Baud et al. (2003) mention that the discipline of collecting and recording operational risk loss data within financial institutions has been set out only lately. As a result, banks do not have access to sufficient and adequate data. To mitigate the paucity of data issues, internal loss data must be supplemented by external data from public and/or pooled industry databases (Baud et al., 2002). This practice, however, pose some challenging problems. As pointed out by the authors, the data generating processes that underlie the data collection exercise exhibit some specific features that challenge the estimation of the loss severity distribution. In effect, all loss data are subject to a truncation process by which data are recorded only when individual losses exceed some threshold. Baud et al. (2003) argue that mandatory thresholds either related to internal loss or set for industry-pooled data cannot always be made enforceable within entities that collect data. As for public databases, Baud et al.

(2003) point out that they generally comprise of large losses that are released and recorded in an informal way without any specific threshold. In light of the aforementioned facts, they conclude that for all types of datasets, stated thresholds should be regarded as unknown parameters which need to be assessed as well. For industry-pooled data, the threshold (also referred to as truncation point) should be treated as a finite, discrete random variable while, for public datasets, it should be modeled as a continuous random variable. In addition, effective thresholds used to report losses, will tend to exceed stated thresholds so that industry-pooled and especially public data are highly likely to be biased toward extreme losses leading to over-estimated capital charges. Reflecting these conclusions, Baud et al. (2003) consider that the discipline of using data in the context of operational risk losses should be in a way that the main source of data heterogeneity should be accounted for through modeling the threshold distribution along with the severity distribution. Furthermore, internal data must be employed for estimating the main body of the severity distribution, (expected losses) whereas external data should be used to assess the tail of the distribution (unexpected losses).

Fontnouvelle et al. (2003) provide preliminary empirical evidence on how publicly available databases could be used to assess operational risk capital charges for internationally active banks. Essentially, the authors use the theoretical framework suggested by Baud et al. (2002) and extreme value theory to derive both the sample (or observed) loss severity and the true (or underlying) loss severity distribution functions for publicly available operational risk losses. Assuming that these categories of losses are representative of the risks to which internationally active banks are actually exposed to, the authors compute various capital charges depending on different levels of control environments, sizes and business line's riskiness of banks. The tests performed to validate the aforementioned assumption reveal that there is no evidence of any significant time trend in the tail of the loss severity distribution and most importantly, there is “no statistically significant relationship between the size of a bank and the value of the tail thickness” (see also Shih et al., 2000). However, the authors argue that the tail of the severity distribution at a specific bank could reflect the quality of its control environment. The results from the study indicate that the log-logit-exponential distribution function accounts for the reporting bias appropriately and therefore provides a good estimate for the sample and the true loss severity distribution function, which in turn substantially reduces the required capital charge. The results also reveal that the sample loss severity distribution varies by business line. However, the authors cannot conclude whether the underlying loss distribution actually varies across business lines. On this account, capital charges have been computed as if the underlying loss severity distribution, in the context of large operational risk losses, did not vary across business lines.

Specifically, Fontnouvelle et al. (2003) use data provided by two vendors of publicly available operational loss data, OpRisk Analytics and OpVantage. A preliminary analysis based on descriptive statistics reveals a difference in terms of data collection processes and underlying loss distributions between U.S. losses and non-U.S. losses. In addition, more than 66 percent of the reported losses occur in U.S. As a result, the authors focus only on U.S. losses in the estimation of the loss severity and the capital charge.

The true loss severity distribution is derived, based on authors’ assumptions and results from EVT:

- Assumption 1: The threshold beyond which nominal losses are reported can be considered as sufficiently high to apply EVT results.
- Assumption 2: Operational risk loss distributions belong either to the maximum domain of attraction of the Frechet distribution or to that of the Gumbel distribution so that conditional excesses loss distributions can be approximated by the GPD (Pickands et al. 1974).
- The maximum domain of attraction of the Frechet distribution can be embedded into that of the Gumbel distribution[30].
- The maximum domain of attraction of the Gumbel distribution is closed under logarithm transformations[31] .

From these results, Fontnouvelle et al. (2003) conclude that the conditional excess distribution of the logarithm of large operational risk losses can be approximated by the GPD distribution with shape parameter equal to 0 (that is, the exponential distribution). Both the normal and the logistic distribution functions are investigated to model the distribution of the logarithm of the random truncation point. Fontnouvelle et al. (2003) argue that capturing an operational loss in public disclosures depends on many random factors. These are, among other things, the location of the bank, "personal idiosyncrasies of the executives and other individuals involved in the disclosure decisions". As a result, according to the authors, a central limit reasoning suggests that the logarithm of the random threshold should be normally distributed. However, this specific distribution raises computational issues in terms of non convergence of the maximum likelihood optimization function. The logistic distribution, on the other hand, provides a convenient

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30 Embrechts et al. (1997) Examples 3.3.33 Page 148
31 Embrechts et al. (1997) Examples 3.3.34 Page 148

framework to estimate the values of the parameters of the sample loss severity distribution which turns out to be log-logit-exponential distribution function. Leandri (2003) questions the logistic distribution assumed for the random truncation point on the grounds that the choice of this specific distribution is not rooted in empirical evidence. Because this assumption greatly influences the results, he concludes that further investigation is needed to measure the capital charge's sensitivity to various types of distributions. The author also maintains that there is no clear evidence of fit robustness of the logit-exponential function to the loss data since the tail Q-Q plot fit test deteriorates toward the tail of the loss distribution. As to the main conclusion of the Fontnouvelle et al (2003)'s paper, that is, the true distribution of operational risk large losses does not vary across business lines, the author contrasts with two evidences – namely, the raw data that differs across business lines and the median of the empirical distribution that also differs across business lines.

In recent study, Fontnouvelle et al. (2004) carry out an empirical exercise consisting of modeling operational risk using only internal operational loss data from 2002 Loss Data Collection Exercise (LDCE)[32]. Essentially, the study aims at understanding modeling issues faced by banks that start collecting operational loss data. For the largest losses, results show that severity ranking of event types is similar across banks and heavy-tailed distributions reasonably fit the data. In addition, the tail parameter estimates for the loss severity distribution are quite similar to the ones based on publicly

[32] The 2002 LDCE asked participating banking organizations to report the amount of individual operational losses during 2001, internal capital allocation for operational risk, expected operational losses, and a number of exposure indicators related to specific business lines. (BCBS, 2003).

available losses (Fontnouvelle et al. 2003). The authors investigate whether a full-data[33] approach might work for certain loss event types or business lines. Results indicate that for a typical bank, this is not a reasonable way to model operational risk. We now turn to the review of the second step of the AMA implementation, that is, the estimation of the loss frequency distribution.

2.2.2.3 Loss Frequency Modeling

Existing literature on operational risk loss frequency modeling includes Cruz (2002), Fontnouvelle et al. (2003), Frachot and Roncalli (2002), Fontnouvelle et al. (2004), Moscadelli (2004), Chavez-Demoulin at al. (2005) and Neslehova et al. (2006).

Cruz (2002) indicates that the negative binomial distribution is probably the most popular loss frequency distribution in operational risk after the Poisson distribution. It is a two-parameter distributions and as such, is more flexible in shape that the Poisson distribution. Results from Fontnouvelle et al. (2004) and Moscadelli (2004) show that the negative binomial distribution provides a good fit to the frequency of operational risk losses. On the other hand, Fontnouvelle et al. (2003) and notably, Frachot and Roncalli (2002) develop a model in which Poisson distribution plays a critical role.

Frachot and Roncalli (2002) argue that a bank's internal loss frequency data only provide partial information about the bank's specific riskiness and the effectiveness of its risk management practices. As a result, bank's average historical loss frequencies need to be adjusted while assessing capital requirement. The authors use credibility theory to handle this specific issue. Typically, they assume that for a bank, the number of loss

[33] This methodology involves fitting parametric loss severity (heavy-tail or light-tailed) distributions over the entire range of loss amounts.

events for a business line/loss event type is Poisson distributed with its parameter equal to the unobserved riskiness multiplied by the gross income used as the exposure indicator. The unobserved riskiness is assumed to be gamma distributed with two parameters. They focus on the random variable represented by the number of loss events conditional to historical loss frequencies and compute its expected value as well as its distribution.

Frachot et al. (2003) point out that the aforementioned method as well as approach that uses a square-root pattern to scale external frequencies require large data sets that are not currently available. As a result, such computational approaches cannot be implemented and the use of internal loss frequencies validated by the bank's expert should be recommended.

Fontnouvelle et al. (2003) model the frequency of publicly available operational losses by using the standard assumptions behind the GPD model which require the frequency of large losses to be Poisson distributed. The Poisson parameter is calibrated using results of the 2002 LDCE and the fact that a typical internationally active bank experiences an average of 50 to 80 losses beyond one million dollar per year, depending on its size, control environment and riskiness of its business lines.

Recently, Chavez-Demoulin et al (2005) advocate the use of point process methodology for an advanced loss frequency modeling. They maintain that the issue of dependence can be elegantly analyzed through this approach. This will be investigated in more details in the chapter 3. We now review the modeling of the capital charge.

2.3.2.1 Capital Charge Modeling

In operational risk modeling, the regulatory capital requirement is generally defined in three different ways. The first definition considers the capital requirement as the 99.9

percentile of the aggregate loss distribution which means that both expected loss (EL) and unexpected loss (UL) account for it (BCBS, 2004). The second definition calls for unexpected loss only according to BCBS (2004) requirements. Alternatively, Frachot et al (2003) determine the capital charge as the 99.9 percentile of the aggregate loss distribution where only above-the-threshold individual losses are taken into account. In all cases, the focus is on the estimation of the aggregate loss distribution for a given time horizon to determine the Value-at-risk at a specific rating target greater or equal to 99.9 percent.

Within the standard LDA, the distribution of the aggregate loss is commonly estimated either by means of Panjer's recursion formula or through Monte Carlo simulation scheme. The latter approach is based on a two-stage calculation. The first step simulates a value of the counting process and the second stage generates several severities (depending on the simulated value of the counting process) and aggregates them. Capital charge accuracy is sensitive to the number of simulations in the Monte Carlo scheme and to the number of grid points in the Panjer algorithm (Frachot et al, 2001).

The overall capital charge for the firm is determined by aggregating the capital charge as computed above, taking into account correlations or more specifically dependence structure across business line/loss event type cells.

Frachot et al (2003) recognize that the capital charge is driven by two sources of randomness and therefore by two sources of correlation – namely, frequency and severity. However, the authors argue that within the standard actuarial model "it is conceptually difficult to assume simultaneously severity-independence within each class of risk and severity-correlation between two classes". As a result, they believe that the correlation between aggregate losses by event type is essentially driven by the underlying correlation between frequencies

Frachot et al. (2004) carry out an empirical exercise related to the estimation of correlation between aggregate losses. They restrict the investigation to the case where only frequencies are correlated. Essentially, the authors compute the formula of the covariance between two aggregate losses associated to two classes and find out that the aggregate loss correlation is always lower than the frequency correlation. In addition, their results indicate that for high severity risk types, aggregate loss correlations may be very small even if the frequency is high. On the other hand, for high frequency– low severity risk, aggregate loss correlations approximate the frequency correlation. However, as evidenced in McNeil et al. (2005) page 205 and emphasized in Chavez-Demoulin et al. (2005), low correlations do not necessary implied weak dependence.

2.2.2.4 Confidence Interval

In many views (see Frachot et al. 2003; Embrects et al. 2003), the data paucity that characterizes operational risk category has the potential to lead to unstable estimates of distribution parameters. Frachot et al (2003) provide a three-stage process to determine the capital charge's empirical distribution as well as its confidence interval. The first stage derives the distribution of the underlying estimators of the parameters of the counting process and the loss severity. The second step generates from these distributions a large set of simulated values. Lastly, for each path, the third step computes the capital charge to get its empirical distribution. In the sequel we examine the first and the third step in detail.

Two different methodologies can be employed to obtain the distribution of the estimators of the parameters – namely, the bootstrap method and the Gaussian approximation from the maximum likelihood theory. The authors recommend the Gaussian approximation due to its relative easiness. Results in this setting indicate that increasing the size of the loss data set improves the accuracy of the severity estimates while expanding the number of recorded years enhances the precision of the frequency estimate.

Specifically, Frachot et al (2003) address the accuracy of the capital charge estimate by defining and computing a coefficient denoted c, obtained from the following expression.

$$\Pr\{\widehat{\mathrm{VaR}} \geq (1\text{-c}) \times \mathrm{VaR}\} = \alpha$$

Where $\widehat{\mathrm{VaR}}$ denotes the capital charge estimate, VaR its true value and α the level of confidence. The authors contend that such a coefficient is appropriate for supervisory purposes since regulators are interested in assessing the risk of under-estimating the capital charge. To illustrate the case, they find out that for 1000 losses, the capital charge may be undervalued by less than 15 percent for a 95 percent confidence level. The authors also use their confidence interval framework to derive the analytical expression of the minimum number of observations needed to achieve a specific accuracy for the capital charge.

2.2.2.5 Self-Assessment and Scenario Analysis

Scenario analysis is performed by banks' experienced managers to adjust the level of riskiness conveyed by the bank's historical loss data. This is particularly the case for business lines/loss event types for which historical loss data are infrequent. Frachot et al (2003) mention a methodology by which useful information from expert's scenarios can be extracted and plugged into a standard LDA. More specifically, the approach is to embed the scenarios into constraints on the parameters of the counting process and the loss severity distribution. As such, the parameters are calibrated by means of a constrained maximum likelihood optimization procedure that uses the loss data along with the scenario restrictions.

2.2.2.6 Back-Testing

Back-testing a risk model, as mentioned in Marshall (2001) involves measuring the performance of the model by examining any divergence between realized losses and historical estimates from the model. There is a compelling need to back-test risk model periodically and also following any major event. This helps evaluate the accuracy of the model and determine whether some major structural changes have occurred. Cruz (2002) presents four tests that provide multiple sources of information on the accuracy of the model. These include the clustering of the violations (that reveals whether the model was incapable of protection against unexpected losses), the frequency of the violations and the size of the violations that necessitates a specification of loss boundaries for the acceptance of the model. Also included is the size of the over/under allocation which quantifies the difference between the average operational losses and the operational risk capital on a daily basis for example.

Cruz (2002) also indicates that operational risk back-testing consists of two stages - the basic analysis and the statistical analysis. The basic analysis aims at reporting a summary of the findings to the analyst that checks whether the model fits well. If this test is conclusive, then the statistical analysis is performed. Three types of statistical tests are mentioned in Cruz (2002). Besides the author’s specific model which is based on extremal index and appropriate for operational risk, the two other tests include Kupiec test, and the Crnkovic-Drachman test[34]. The two latter tests are already in use for market risk models.

According to Cruz (2002), the basic idea in the extremal index test is to check the clustering of extreme events and investigate whether the model errors are correlated. The Kupiec test (or K test) attempts to verify whether the violation ratio of the model is in line with some specific confidence level. Finally, the Crnkovic-Drachman test (or Q test) as mentioned in Cruz (2002) focuses on analyzing the difference between the probability distribution function of the prediction with the uniform distribution, and thereby assesses the fitness of the predictions.

Contrary to the market VaR models which are validated against the P&L on a daily basis, Operational VaR models require to be tested against the losses themselves, taking into account the time lag between the event and its effect on the earnings (Cruz , 2002).

[34] See Kupiec (1995) and Crnkovic and Drachman, J. (1996)

CHAPTER 3

THE MODEL

3.1 Introduction

As mentioned earlier, the Committee suggests three approaches to calculating operational risk capital charges in a "continuum" of increasing sophistication and risk sensitivity (BCBS, 2001c). These are (i) the Basic Indicator Approach; (ii) the Standardized Approach and (iii) the Advanced Measurement Approaches. The discussion to follow formulates the mathematical expression of each approach, and subsequently investigates the implications for modeling the Advanced Measurement Approaches, especially the various components of the Loss Distribution Approach. These include the loss severity distribution, the loss frequency distribution, the risk dependence structure, and lastly, the overall capital charge. Specifically, this chapter presents a promising mathematical framework based on point process methodology (see Pfeifer and Neslehova 2004 and Chavez-Demoulin et al. 2005), within which aggregate losses are efficiently formulated and risk dependence engineered and simulated. This dissertation uses the aforementioned framework as well as copulas and finite mixture distribution framework to propose a methodology that achieves the estimation of the empirical probability distribution function of the capital charge. As to the loss severity probability distribution function, publicly available operational loss data set is used for the empirical exercise. An efficient symbolic computational framework to estimate the parameters of the loss severity according to the model suggested in Fontnouvelle et al. (2003) is provided. An extension of the model to calibrate loss severities of specific firms is proposed as well.

3.2 BCBS Models for the Capital Charge

3.2.1 Basic Indicator Approach

The capital charge K_{BIA} under this specific approach is expressed as:

$$K_{BIA} = \alpha \times GI$$

where GI denotes the average annual gross income over the previous three years and α a coefficient set by the Committee to 15 percent.

3.2.2 Standardized Approach

Under the Standardized Approach, the capital charge K_{SA} is equal to:

$$K_{SA} = \sum_{i=1}^{8} \beta(i) GI(i)$$

where $GI(i)$ stands for the average annual gross income over the previous three years for business i and $\beta(i) \in [12\%, 18\%], i = 1, 2, ..., 8$ set by the Committee

3.2.3 Advanced Measurement Approaches

The Advanced Measurement Approaches encompass three approaches described in the following lines.

3.2.3.1 Internal Measurement Approach

The capital charge K_{IMA} is expressed as:

$$K_{IMA} = \sum_{i=1}^{8}\sum_{j=1}^{7} \gamma(i,j)e(i,j)$$

where for business line/loss event type (i,j) cell, $e(i,j)$ and $\gamma(i,j)$ represents the expected loss and a scaling factor, respectively.

3.2.3.2 Scorecard Approach

The capital charge K_{SCA} is equal to

$$K_{SCA} = \sum_{i=1}^{8}\sum_{j=1}^{7} GI(i,j)\times\omega(i,j)\times RS(i,j)$$

where for business line/loss event type (i,j) cell, $GI(i,j)$ denotes the average annual income over the previous three years, $\omega(i,j)$ a scaling factor, and $RS(i,j)$ a risk score.

3.2.3.3 Loss Distribution Approach

The notation used to describe this approach is in line with Furrer (2004) page 14.

One represents the operational loss data set as follows:

$$\begin{cases} L_k^t(i,j), t \in \{T-m+1,\dots,T-1,T\} \text{ } (m \text{ years}), \\ \qquad i \in \{1,2,\dots,8\} \text{ (business line)}, \\ \qquad j \in \{1,2,\dots,7\} \text{ (loss event type)}, \\ \qquad k \in \{1,2,\dots,N^t(i,j)\} \text{ (number of losses for the period [t,t+1])} \end{cases}$$

where $L_k^t(i,j)$ denotes an individual loss.

For each business line/loss event type (i, j) cell , let $AggL^{T+1}(i, j)$ denote its aggregate loss over the period $[T, T+1]$. Then, $AggL^{T+1}(i, j) = \sum_{k=1}^{N^{T+1}(i,j)} L_k^{T+1}(i, j)$

For the sake of simplicity regarding the notation, the following rules are adopted through out the text:

(1) Within a business line/loss event type (i, j) cell, when there is no need to address (i, j), one uses $(L_k)_{k \in \mathbb{N}}$, $(N_t)_{t \geq 0}$, $(AggL_t)_{t \geq 0}$

(2) To address a specific business line or loss event type (i) for a time period equal to 1 year, we have:

$L(i)$, $(L_k(i))_{k \in \mathbb{N}}$, $N(i)$, $AggL(i)$

Now, the capital charge $K(i, j)$ for a cell (i, j) is expressed as the 99.9 percentile of the aggregate loss distribution.

$$K(i, j) = F^{-1}_{AggL^{T+1}}(99.9\%) = VaR_{99.9\%}(AggL^{T+1})$$

where $F^{-1}_{AggL^{T+1}}$ denotes the quantile function of the aggregate loss function $AggL^{T+1}(i, j)$, and *VaR* stands for the Value at Risk.

For the whole institution, the Committee first suggests expressing the capital charge as follows:

$$K_{LDA} = \sum_{i=1}^{8} \sum_{j=1}^{7} K(i, j)$$

which is achieved under the assumption of comonotonicity of the vector $\left(K(1,1), ..., K(8,7)\right)$. As formulated in McNeil et al. (2005) page 199, $K(1,1), ..., K(8,7)$

are comonotonic if and only if $\left(K(1,1),...,K(8,7)\right)\stackrel{d}{=}\left(v_1(z),...,v_{56}(z)\right)$ for some random variable z and increasing functions $v_1,...,v_{56}$.

Coherent risk measures such as Conditional Tail Expectation and Wang Transform Measure[35] could be used to calculate the capital charges at some specific rating target. This consideration is left to future research. The next subsection presents the framework and assumptions behind the Loss Distribution Approach.

3.3 Loss Distribution Approach

3.3.1 The Framework

In the sequel, two contexts in which the Loss Distribution Approach could be modeled are presented. These include the standard Cramer-Lundberg model and the point process methodology envisioned as an extension of the first model.

3.3.1.1 The Cramer-Lundberg Model

This presentation is close in spirit to Embrechts et al. (1997) page 22. The classical Cramer-Lundberg model underlying the Loss Distribution Approach assumes the following regarding the four stochastic processes described by the model.

1- The claim or loss sizes $(L_k)_{k\in N}$ are positive iid random variables with finite mean and variance. Notice that the terms claim and loss are used interchangeably.

[35] See Wang (2002)

2- The inter-arrival claim times $Y_1 = T_1, \quad Y_k = T_k - T_{k-1}, \quad k = 2,3,...$ where $(T_n)_{n\geq 1}$ is a sequence of random variables such that $0 < T_1 < T_2 < ....$ a.s. are iid exponentially distributed with finite mean $E(Y_1) = \frac{1}{\lambda}$

3- The claim counting defined as the number of claims in the interval $[0,t]$ is expressed as $N_t = \sup\{n \geq 1 : T_n \leq t\}, t \geq 0$

The counting process $(N_t)_{t\geq 0}$ is required to satisfy the following three conditions. For all $t, h \geq 0$

$$\begin{aligned} &N_0 = 0 \\ &N_t \in \mathbb{N} \\ &N_t \leq N_{t+h} \end{aligned}$$

4- The total claim amount or the aggregate losses $(AggL_t)_{t\geq 0}$ is modeled by random sums and defined as $AggL_t = \begin{cases} 0 & , \quad N_t = 0 \\ \sum_{i=1}^{N_t} L_i, & N_t > 0 \end{cases}$

5- In addition, the sequences $(L_k)_{k\in N}$ and $(Y_k)_{k\in N}$ are independent of each other.

As a consequence of this set of assumptions, it follows that:

(1) The process $(N_t)_{t\geq 0}$ is a homogeneous Poisson process with intensity λ, that is

$$P(N_t = k) = e^{-\lambda t}\frac{(\lambda t)^k}{k!}, \quad k = 0,1,2...$$

(2) The processes $(L_k)_{k\in N}$ and $(N_t)_{t\geq 0}$ are independent and the process $(AggL_t)_{t\geq 0}$ is a compound Poisson process

Notice that in the case where the inter-arrival claim times are iid with arbitrary distribution, the counting process is defined as a renewal process. The expression renewal process arises from a special type of random process, where the events represent replacement of an item.

3.3.1.2 The Point Process Methodology

Pfeifer and Neslehova (2004) show that point process methodology is as an appropriate approach to modeling dependent loss processes in insurance and finance. The authors develop two models that aim at engineering dependent risk in this specific context. This subsection defines and summarizes the point process methodology useful to formulate the correlation between aggregate losses of operational risk categories. Next, it depicts the two above-cited models.

Definitions

A point process is a distinct class of stochastic process, for which the time points of the occurrence of events are random. Specifically, a point process encompasses a counting process as well as an inter-arrival time process. The simplest point process is the homogeneous Poisson process while the marked point processes as stated by Rolski et al. (1998) page 493 extend standard point processes by incorporating other information about the claims like their size or type. Other constructions are superposition and clustering of point processes that generate new point processes. Pfeifer and Neslehova (2004) page 352 define finite point processes as follows:

Definition 1: (finite point processes)

Let N stand for a non-negative integer-valued random variable and $(\boldsymbol{T}_i)_{i\in N}$ be a family of iid random vectors with values in $[0,\Delta]^d$ independent of N, for some fixed dimension $d \in \mathbb{N}$. All components of $\boldsymbol{T}_i$ are assumed to be in the interval $[0,\Delta]$. In this specific case, Δ is one year. Then the random measure[36] $\xi := \sum_{i=1}^{N} I_{\boldsymbol{T}_i}$ is referred to as a (finite) point process with counting variable N and multiple event points $(\boldsymbol{T}_i)_{i \in N}$. In this setting, $I_{\boldsymbol{T}}$ stands for the Dirac measure concentrated in the point $\boldsymbol{T} \in [0, \Delta]^d$, that is

$$I_T(A) = \begin{cases} 1, & T \in A \\ 0, & T \notin A \end{cases} \text{ for all sets } A \subseteq [0, \Delta]^d$$

Note that this definition assumes a common counting random variable N for each component of the iid random vectors of the family $(T_i)_{i \in N}$.

<u>Definition 2</u>:

A point process ξ is called a finite Poisson point process, if the common counting variable N is Poisson-distributed.

The following result in Chavez-Demoulin et al. (2005), page 18, is useful for modeling and simulating dependent Poisson process triggered by a common effect.

Let $\xi := \sum_{i=1}^{N} I_{T_i}$ denote a finite Poisson point process with *d*-dimensional event points $\boldsymbol{T}_i = (T_i(1), ..., T_i(d))$. Each of the marginal processes,

$$\xi(k) = \sum_{i=1}^{N} I_{T_i(k),} \quad k = 1, ..., d,$$

is therefore a one-dimensional Poisson point process with intensity $E(N)F_k(.)$ where $F_k(.)$ represents the k-th margin of the joint distribution F of the $\boldsymbol{T}_i$.

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[36] Simply stated, a measurable function is such that the inverse image of a "nice" set is "nice". A random measure is a measurable function ξ defined on some probability space taking values almost surely on a space with some "nice" properties (see Kallenberg, 1983). Well known examples of random measures include the empirical process $\mu_n = n^{-1} \sum_{i=1}^{n} \delta_{X_i}$ where the X_i's are iid random variables and point processes.

Conversely, if $\xi(k) = \sum_{i=1}^{N} I_{T_i(k),} \quad k = 1,...,d$, are one-dimensional Poisson processes, then $\xi := \sum_{i=1}^{N} I_{\boldsymbol{T}_i}$ with $\boldsymbol{T}_i = (T_i(1),...,T_i(d))$ is a d-dimensional Poisson point processes with intensity measure $E\xi(.) = E(N)F(.)$ where F stands for the joint distribution of $\boldsymbol{T}_i$.

Point process dependence engineering

Chavez-Demoulin et al. (2005) page 23 point out that a suitable theory of dependence for processes do not really exist. However, if the process is Lévy, the recent concept of Lévy copulas[37] provides a fruitful approach. Alternatively, Griffiths et al. (1979) define the correlation between two one-dimensional point processes $\xi(1)$ and $\xi(2)$ as the correlation coefficient $\rho(\xi(1)(A), \xi(2)(B))$ between the random variables $\xi(1)(A)$ and $\xi(2)(A)$ for some Borel sets[38] $A, B \subset \mathbb{R}$. Chavez-Demoulin et al. (2005) use this specific background as well as approaches suggested in Pfeifer and Neslehova (2004) to model dependent aggregate losses assuming stationary and independent loss amounts. They retrieve key results from Frachot et al. (2004) and Powojowski et al. (2002). The two approaches set forth in Pfeifer and Neslehova (2004) include:

Approach 1:

This methodology yields Poisson point processes equipped with a common random number N of events. Let $\xi := \sum_{i=1}^{N} I_{T_i}$ be a Poisson point process with iid d-dimensional event-time points $\boldsymbol{T}_i = (T_i(1),...,T_i(d))$ whose joint distributions for each i are given

[37] See Tankov (2004), Cont and Tankov (2004), Kallsen and Tankov (2004)

[38] A Borel set is an element of a family of sets with some "nice" properties (sigma-field generated by all intervals).

through a copula[39] function $C_{\boldsymbol{T}}$. Then the marginal processes $\xi(k)=\sum_{i=1}^{N} I_{T_i(k)},\quad k=1,...,d$ are Poisson, but dependent. For all Borel sets $A,B\subset\mathbb{R}$, the correlation between two processes $\xi(j)$ and $\xi(k)$ is expressed as

$$\rho(\xi(j)(A),\xi(k)(B))=\frac{F_{j,k}(A\times B)}{\sqrt{F_j(A)F_k(A)}}\quad j,k\in\{1,...,d\}$$

where $F_{j,k}$ stands for the joint distribution of $T_i(j)$ and $T_i(k)$ and F_j and F_k denote the marginal distributions of $T_i(j)$ and $T_i(k)$, respectively. As pointed out in Pfeifer and Neslehova (2004), the marginal processes have a common counting random variable N and as a result, only positively correlated Poisson distribution can be achieved through this methodology.

Approach 2:

This approach is based on two steps. First, different dependent Poisson random variables $N(1),..,,N(d)$ governed by a specific copula C_N are generated. Then, the occurrence time points $T_i(k)$ that are possibly dependent, are produced as margins of a d-dimensional event-time points $\boldsymbol{T}_i=(T_i(1),...,T_i(d))$. As a result, d dependent processes $\xi(k)=\sum_{i=1}^{N(k)} I_{T_i(k)},\quad k=1,...,d$, governed by a copula C_T are constructed.

If the $T_i(k)$ are mutually independent, the corresponding correlation can be specified as

$$\rho(\xi(j)(A),\xi(k)(B))=\rho(N(j),N(k))\sqrt{F_j(A)F_k(A)}\quad j,k\in\{1,...,d\}$$

[39] Typically, a copula function links univariate marginal distributions to their joint distribution.

Two facts are worth mentioning here. First, the joint d-dimensional point process is not in general a Poisson process. Second, negative correlations are attainable because the sign of $\rho(N(j), N(k))$ could be negative.

As it is stated in Chavez-Demoulin et al. (2005) page 25, a wide range of models can be constructed from these approaches to gain insight into the dependence structure of operational risk types. For instance, thinning of specific processes or superposition of independent homogenous Poisson point processes with homogeneous positively dependent Poisson point process is achievable.

In the next paragraph, we proceed along the lines of the two abovementioned methodologies to present three key findings related to the correlation between aggregate losses of operational risk types.

<u>Dependent Aggregate Losses</u>

This presentation closely follows Chavez-Demoulin (2005) Page 26. The basic idea is to incorporate the loss amounts into the point process modeling. Let $AggL(1)$ and $AggL(2)$ denote two aggregate losses related to two operational risk types and let $[0, \Delta]$ stands for some period of time. The loss amounts are assumed to be stationary and independent. In addition, the loss occurrence times of each risk type establish a Poisson point process $\xi(k) = \sum_{i=1}^{N(k)} I_{T_i(k),}\ k = 1,2$. Let $L_i(1)$ and $L_i(2)$ represent the severities related to $T_i(1)$ and $T_i(2)$, respectively. The severities are each iid and $L_i(1)$ and $L_j(2)$ independent of one another for $i \neq j$. Using the marked point processes framework, it is then possible to describe the entire risk process as point processes as indicated by $\xi^*(k)=\sum_{i=1}^{N(k)} I_{(T_i(k),L_i(k)),}$ $k=1,2$

The resulting aggregate losses are specified as follows:

$$AggL(1)=\sum_{k=1}^{N(1)} L_k(1) \text{ and } AggL(2)=\sum_{k=1}^{N(2)} L_k(2)$$

The correlation between $AggL(1)$ and $AggL(2)$ is now expressed in terms of various types of dependence between the underlying loss occurrences processes $\xi(1)$ and $\xi(2)$.

Case1: Modeling $\xi(1)$ and $\xi(2)$ according to Approach 1 gives rise to the correlation coefficient suggested in Pfeifer and Neslehova (2004).

$$\rho\left(AggL(1), AggL(2)\right)=\frac{E\left(L_1(1)L_1(2)\right)}{\sqrt{E\left(L_1(1)^2\right)E\left(L_1(2)^2\right)}}$$

provided that $E\left(L_1(1)^2\right)<\infty$ and $E\left(L_1(2)^2\right)<\infty$

Case 2: In case $L_i(1)$ and $L_i(2)$ are independent for any i, Approach 2 yields:

$$\rho\left(AggL(1), AggL(2)\right)=\rho(N(1),N(2))\frac{E\left(L_1(1)\right)E\left(L_1(2)\right)}{\sqrt{E\left(L_1(1)^2\right)E\left(L_1(2)^2\right)}}$$

as described in Frachot et al. (2004).

A simple example of superposition (common Poisson shock) is constructed when the processes $\xi(1)$ and $\xi(2)$ are set as the sum of independent homogeneous Poisson point processes ξ_k with intensities λ_k, $k=1,2,3$, that is $\xi(1)=\xi_1+\xi_3$ and $\xi(2)=\xi_2+\xi_3$.

It follows that

$$\rho(N(1),N(2))=\frac{\lambda_3}{\sqrt{(\lambda_1+\lambda_3)(\lambda_2+\lambda_3)}}$$

and therefore

$$\rho\left(AggL(1), AggL(2)\right) = \left(\frac{\lambda_3}{\sqrt{(\lambda_1+\lambda_3)(\lambda_2+\lambda_3)}}\right)\frac{E\left(L_1(1)\right)E\left(L_1(2)\right)}{\sqrt{E\left(L_1(1)^2\right)E\left(L_1(2)^2\right)}}$$

This example appears in Powojowski et al. (2002). The subsection devoted to the loss frequency distribution discusses it in more details. The next subsection considers the calibration of the loss severity distribution using the assumptions mention earlier, that is, loss severities are stationary and independent.

3.3.2 Loss Severity Distribution Models

3.3.2.1 Publicly Available Operational Loss Modeling

This subsection focuses on the calibration of the loss severity distribution, using publicly available operational loss data set. As noted earlier, this specific data set is plagued by many biases that impede one's ability to uncover the true underlying loss severity distribution. The following lines build on Fontnouvelle et al (2003) and suggest a symbolic computational approach that makes such a calibration easier.

Frachot et al. (2003) and Fontnouvelle et al. (2003) assume that contributors' operational loss data are sampled from the same probability distribution[40], and as such are not different from each other. However, losses are captured according to some unobserved random truncation point that needs to be accounted for. One way to proceed is to jointly estimate the parameters of the loss distribution as well as those of the random truncation point. The random truncation point modeling is described as follows.

Let us consider two independent random variables X and H and let $f_{X|H}(x \mid h < x)$ denote the probability density function of the observed values of X (that is, X is observed when it exceeds the unobserved truncation point H).

[40] See also Okunev (2005).

$f_{X|H}(x \mid h < x)$ is expressed as[41]:

$$f_{X|H}(x \mid h < x) = \frac{f_X(x)F_H(x)}{\int_{\mathbb{R}} f_X(t)F_H(t)dt}$$

where $f_X(x)$ and $F_H(x)$ represent the probability density function and the cumulative distribution function of X and H, respectively. Random truncation modeling is generally used in economics, reliability, and astronomy. In this latter field, it is known as the Malmquist bias in the study of galaxies.

Now let L denote the random variable representing the operational loss amount, u the nominal threshold ($1 million) and $X = \log(L) - \log(u) \mid L > u$ the conditional excess loss. Notice that $X = \log(L) \mid L > 1$ since $u = 1$. If one assumes that the distribution of operational losses of a specific business line/event type cell belong to the maximum domain of attraction of either the Frechet distribution or the Gumbel distribution and if one considers u as a sufficient high threshold, results from EVT (Embrechts et al, 1997 page 148), indicate that the distribution function of X may be approximated by $G_{0,\beta}(x) = 1 - \exp\left(-\frac{x}{\beta}\right)$ which is the exponential distribution with density $g_{0,\beta}(x) = \frac{1}{\beta}\exp\left(-\frac{x}{\beta}\right)$.

As to the random truncation point H^*, in addition to the known and constant case, this study assumes two other distributions for $H = \log(H^*)$, namely the logistic distribution as

[41] See Fontnouvelle et al. (2003) for the proof.

in Fontnouvelle (2003) and the normal distribution[42]. A fourth case, worth mentioning is that of the alpha stable non Gaussian exponentially truncated distributions. This class of distributions is gaining importance in empirical finance in that it provides better fit of the tails of distribution than normal distributions. As pointed out by Nolan (2001), these distributions are now more computationally tractable and should be part of quantitative risk managers' toolkit. This will be examined in future work.

Let (X,H) denote the random vector representing the conditional excess log losses and the log random truncation point, let $(H < X)$ denote the event that characterizes publicly available operational risk loss data. For the random truncation point distribution, let σ and μ denote the scale and location parameters respectively.

For the normal distribution,

$$F_H(h) = \frac{1}{\sqrt{2\pi}\sigma}\int_{-\infty}^{h} \exp\left(-\frac{1}{2}\left(\frac{t-\mu}{\sigma}\right)^2\right)dt = \Phi\left(\frac{h-\mu}{\sigma}\right)$$

where Φ denotes the standard normal cumulative distribution function.

For the logistic distribution,

$$F_H(h) = \frac{1}{1+\exp\left(-\frac{h-\mu}{\sigma}\right)}$$

The expression of the loss severity pdf is then described as follows:

[42] See Fontnouvelle et al. (2003) page 12 for a discussion on factors that impact public disclosures of operational losses.

$$
f_{X|H}(x \mid h < x) = \begin{cases} \dfrac{\exp\left(-\dfrac{x}{\beta}\right)\Phi\left(\dfrac{x-\mu}{\sigma}\right)}{\int_{u_x}^{+\infty} \exp\left(-\dfrac{t}{\beta}\right)\Phi\left(\dfrac{t-\mu}{\sigma}\right)dt}, & \text{for the normal case} \\ \dfrac{\dfrac{\exp\left(-\dfrac{x}{\beta}\right)}{1+\exp\left(-\dfrac{x-\mu}{\sigma}\right)}}{\int_{u_x}^{+\infty} \dfrac{\exp\left(-\dfrac{t}{\beta}\right)}{1+\exp\left(-\dfrac{t-\mu}{\sigma}\right)}dt}, & \text{for the logistic case} \end{cases}
$$

where u_x denotes a value related to the date of occurrence of the loss. Its expression is determined in the sequel.

Since the calibration of the loss severity pdf is performed at the end of 2003, one needs to express all individual loss amounts in real terms, using various levels of the Consumer Price Index (CPI). This removes the effect of inflation and allows better comparison of the individual loss amount across years. The lower bound of the domain of integration u_x is adjusted accordingly. So, in real terms, a loss with nominal value $L_0 \geq u$ that occurs in year $k \leq 2003$, amounts to $L = L_0 \times \dfrac{CPI_{2003}}{CPI_k}$ in 2003, where CPI_k denotes the year k Consumer Price Index. As a result, the lower bound of the set containing all conditional excess losses $x = \log\left(L_0 \times \dfrac{CPI_{2003}}{CPI_k}\right) - \log(u)$ (from losses $L_0 \geq u$ that occur in year $k \leq 2003$) is expressed as

$$
\begin{aligned}
x_k^{lb} &= \log\left(u \times \frac{CPI_{2003}}{CPI_k}\right) - \log(u) \\
&= \log\left(\frac{CPI_{2003}}{CPI_k}\right)
\end{aligned}
$$

Thus,

$$f_{X|H}(x \mid h < x) = \begin{cases} \dfrac{\exp\left(-\dfrac{x}{\beta}\right)\Phi\left(\dfrac{x-\mu}{\sigma}\right)}{\displaystyle\int_{x_k^{lb}} \exp\left(-\dfrac{t}{\beta}\right)\Phi\left(\dfrac{t-\mu}{\sigma}\right)dt}, & \text{for the normal case} \\[2ex] \dfrac{\dfrac{\exp\left(-\dfrac{x}{\beta}\right)}{1+\exp\left(-\dfrac{x-\mu}{\sigma}\right)}}{\displaystyle\int_{x_k^{lb}} \dfrac{\exp\left(-\dfrac{t}{\beta}\right)}{1+\exp\left(-\dfrac{t-\mu}{\sigma}\right)}dt}, & \text{for the logistic case} \end{cases}$$

This means that the support of the probability density function $f_{X|H}(x \mid h < x)$ describing the observed losses is the interval $\left[\log\left(\dfrac{CPI_{2003}}{CPI_k}\right), +\infty\right)$

The location parameter μ represents the magnitude of loss that has a 50 percent chance of being captured, while the scale parameter σ reflects the rate at which the reporting probability changes as the loss size varies (Fontnouvelle et al., 2003)

3.3.2.2 Symbolic Computational Model

The following subsection describes a symbolic computational approach to estimating the parameters $\Theta = (\beta, \mu, \sigma)$ of the observed loss severity distribution. Specifically, for this illustration, the log of the random truncation point is assumed to be logistically distributed.

Suppose that one is interested in estimating the parameters of the severity for a specific business unit/event type cell. The publicly available operational risk data loss consists of n losses beyond \$1 million over m years, that is $\left\{\left\{T_{k_1}, L_1\right\}, ..., \left\{T_{k_n}, L_n\right\}\right\}$ where T_{k_i} denotes the year of occurrence of loss L_i with $1 \leq i \leq n$ and $1 \leq k_i \leq m$. Since the study period ranges from 1960 to 2003 $T_{k_i} \in \{1960, ..., 2003\}$ and $m = 2003 - 1960 + 1$ $= 34$ years.

$\left\{\left\{T_{k_1}, L_1\right\}, ..., \left\{T_{k_n}, L_n\right\}\right\}$ is then transformed into $\left\{\left\{x_1, x_{k_1}^{lb}\right\}, ..., \left\{x_n, x_{k_n}^{lb}\right\}\right\}$ where $x_{k_i}^{lb} = \log\left(\frac{CPI_m}{CPI_{k_i}}\right)$ (CPI_{k_i} for year T_{k_i}) and x_i the conditional excess loss ($x_i = \log(L_i) - \log(u) \mid L_i > u$)

The maximum likelihood function based on the set of data $\left\{\left\{x_1, x_{k_1}^{lb}\right\}, ..., \left\{x_n, x_{k_n}^{lb}\right\}\right\}$ is given by

$$L(\Theta \mid X) = \prod_{i=1}^{n} \frac{f_X(x_i \mid \beta) \times F_H(x_i \mid \mu, \sigma)}{\int_{x_{k_i}^{lb}}^{\infty} f_X(t \mid \beta) \times F_H(t \mid \mu, \sigma) dt}$$

Replacing $f_X(x_i \mid \beta)$ and $F_H(x_i \mid \mu, \sigma)$ by their respective expressions, one gets:

$$L(\Theta \mid X) = \prod_{i=1}^{n} \frac{\dfrac{\exp\left(-\dfrac{x_i}{\beta}\right)}{1 + \exp\left(-\dfrac{x_i - \mu}{\sigma}\right)}}{\displaystyle\int_{x_{k_i}^{lb}}^{\infty} \frac{\exp\left(-\dfrac{t}{\beta}\right)}{1 + \exp\left(-\dfrac{t - \mu}{\sigma}\right)} dt}$$

It turns out that this maximum likelihood function is computationally intensive[43] because of the convolution of distributions functions appearing in the denominator. To easy this calculation, a symbolic-numeric approach is adopted.

Since

$$x_{k_i}^{lb} \in \left\{ \log\left(\frac{CPI_m}{CPI_1}\right), \ldots, \log\left(\frac{CPI_m}{CPI_m}\right) \right\}$$

and $CPI_1 \leq \ldots \leq CPI_m$ do not depend on the conditional excess loss amount x_i it is feasible to symbolically compute the vector of integrals $\left(I_{k_1}, \ldots, I_{k_m}\right)$ beforehand.

$$I_{k_i} = \int_{x_{k_i}^{lb}}^{\infty} \frac{\exp\left(-\frac{t}{\beta}\right)}{1+\exp\left(-\frac{t-\mu}{\sigma}\right)} dt$$

Now, one can express the maximum likelihood function as

$$L(\Theta \mid X) = \prod_{i=1}^{n} \frac{\exp\left(-\frac{x_i}{\beta}\right)}{y_i \times \left(1+\exp\left(-\frac{x_i-\mu}{\sigma}\right)\right)}$$

where $y_i = I_{k_i}$ for some k_i.

The computational process is now based on the set of data $\{\{x_1, y_1\}, \ldots, \{x_n, y_n\}\}$.

Specifically, the code of the above algorithm can be implemented in *Mathematica* as follows:

Step 1

Define a vector containing $x_{k_i}^{lb} = \log\left(\frac{CPI_m}{CPI_{k_i}}\right)$ as cpiVector

43 See Baud et al, 2002, Fontnouvelle et al, 2003, Frachot et al 2003

Step 2

Compute the vector of integrals symbolically

$$\text{integralVector=Table}[\int_{\text{cpiVector[[i]]}}^{\infty} \frac{\exp\left[-\frac{t}{\beta}\right]}{1+\exp\left[-\frac{t-\mu}{\sigma}\right]} dt, \{\text{i},1,\text{Length[cpi]}\}$$

This vector is computed once and saved on disk for future use. For example, the expression of I_{k_1}, the first component of integralVector, is expressed in *Mathematica* numerics as follows:

$$\frac{1}{-1+\frac{\beta}{\sigma}}\left(e^{-\mu\left(\frac{1}{\beta}+\frac{1}{\sigma}\right)}\sigma\left(e^{\mu/\sigma}\pi\left(-1+\frac{\beta}{\sigma}\right)\textbf{Csc}\left[\frac{\pi\sigma}{\beta}\right]-\frac{e^{\frac{\mu}{\beta}}\beta\,\textbf{Hypergeometric2F1}\left[1,1-\frac{\sigma}{\beta},2-\frac{\sigma}{\beta},-e^{-\frac{\mu}{\sigma}}\right]}{\sigma}\right)\right)$$

Step 3

Define the probability density function as

$$f=\frac{\exp\left(-\frac{x}{\beta}\right)}{y\times\left(1+\exp\left(-\frac{x-\mu}{\sigma}\right)\right)}$$

Step 4

Express the maximum likelihood function as

$$\text{LogL=Log}\left[\prod_{\text{i}=1}^{\text{n}}\left(f\,/.\{x\to x_i, y\to y_i\}\right)\right]$$

Step 5

Compute the observed log-likelihood from a matrix dataMat, containing the individual losses with their ages.

$\text{sampleLik=LogL/.}\{\text{n->Length[dataMatrix]}\}, x_{i\_} :\rightarrow \text{dataMat[[i,1]]}$
$y_{i\_} :\rightarrow \text{integralVector[dataMat[[i,2]]]}\}$

Step 6

Maximize the objective function sampleLik with an optimization program to get the estimates of the targeted parameters $\beta, \mu,$ and σ . These values are relevant to large internationally active banks.

The aforementioned approach is easy to implement, significantly reduces the computing time and as a result, facilitates the calibration of the loss severity which is a major issue in operational risk capital modeling. For a specific organization, additional constraints on the targeted parameters $\beta, \mu\, \sigma$ imposed through a risk assessment framework could be easily plugged into this maximum likelihood paradigm to get the estimates of the parameters. Setting these constraints is examined in the sequel.

3.3.2.3 Calibration for Specific Organizations

The parameters of the loss severity as well as those of the random truncation point were jointly estimated assuming that all losses from the data set were incurred by a typical large internationally active bank (Fontnouvelle et al, 2003 page 3). In the rest of this dissertation such a large internationally active bank will be simply referred to as an "industry-wide organization". Furthermore, it is possible to envision different categories of industry-wide organizations, each with a specific yearly loss frequency distribution. This section of the dissertation investigates the extent to which Fitch data set could be used to calibrate the severity of a specific bank. In other words, if all these losses, drawn from the same probability distribution, were incurred by a specific firm, how could one account for the positive correlation that exists between the loss amount and the probability of its disclosure[44]? This is an important question left for future research in Fontnouvelle et al. (2003) page 22.

The following subsection proposes an approach that uses the concept of Probable Maximum Loss (PML) to account for firm size, rating, quality of internal control environment, and market-related factors to estimate the parameters of the distribution of the random truncation point independently of the maximum likelihood framework described previously.

The concept of Probable Maximum Loss stems from fire insurance where it has been noticed that total losses were very infrequent in categories where there are public fire protection and fire-resistive structures. Bennett (1992) defines the PML as "the largest possible loss that may occur, in regard to a particular risk, given the worst combination of circumstances". Wilkinson (1992) and Kremer (1990, 1994) suggest expressing the PML as either $(1-\theta)E[M_n]$ or $E[M_n]+\theta\sqrt{VarM_n}$ where $M_n = \max(L_1,\dots,L_n)$ is the maximum of n claims and θ a safety loading coefficient. Cebrian et al (2004) obtain the PML by solving the following equation

$$P[M_n \le PML_\varepsilon] = 1-\varepsilon,$$

for some $\varepsilon > 0$. In other words, the PML can be considered as a high quantile of the maximum of a random sample of size n, that is

$$\mathrm{PML}_\varepsilon = F_{M_n}^{-1}(1-\varepsilon).$$

This latter formula can be estimated using two different methodologies. Wilkinson (1992) advocates the use of order statistics, while Kremer (1990, 1994) and

44 See Fontnouvelle et al (2003). The probability of disclosure is also referred to as the reporting probability.

Cebrian et al (2004) suggest a methodology rooted in extreme value theory. Now, we are in a position to describe the proposed model.

It is assumed that for a specific organization, each business line/event type cell has an explicit random truncation point that is logistically distributed. It is further assumed that through an appropriate risk assessment exercise or a computational paradigm as the one described earlier, a PML is assigned to each business line/event type cell, and this PML reflects the size, the rating, the quality of internal control of the firm as well as market-related factors. Thus, to derive the scale and location parameters of the probability distribution function of the truncation point, it suffices to match percentiles at two different losses.

For a specific firm, let us consider a business line/event type cell endowed with its PML. For this cell, let F_S denote the probability distribution function of the log random truncation point $\log[H_S]$ of this specific organization. Similarly, let F_i stand for the probability distribution function of the log random truncation point $Log[H_i]$ of an industry-wide organization. It is worth noting that the parameters of F_i are jointly estimated with that of the loss severity pdf using the industry-wide operational losses.

Now, let $F_S(\log(PML))$ denote the probability of the event $\{H_S \leq PLM\}$ that characterizes the disclosure of the specific organization' PML (the reporting probability of the PML). Likewise, let $F_i(\log(PML))$ stand for the probability of disclosure of the industry-wide organization' PML. In passing, note that $F_S(\log(PML)) = \Pr[\log(H_s) \leq \log(PML)] = \Pr[H_s \leq PML]$.

If a risk assessment exercise sets the value of $F_S(\log(PML))$ to a specific level, (depending on the firm's size, rating, internal control environment, and market-related factors), and if, for example, it is further assumed that the median of the two distributions matches, then one is in a position to derive the parameters of the distribution of the random truncation point of the specific firm and, thereby, estimate the underlying loss severity parameter using the maximum likelihood estimation approach. The key finding is that if $F_S(\log(PML)) \geq F_i(\log(PML))$, then the underlying loss severity parameter of the specific firm is lower than that of the industry-wide organization which means that the specific organization could experience less severe operational losses than the industry-wide organization. The converse holds true if $F_S(\log(PML)) \leq F_i(\log(PML))$.

It is worth noticing that this model still assumes that all losses of the data set could be experienced by the specific firm. However, the random truncation function that accounts for firm size, rating, internal control environment as well as market-related factors provides a system of weights that impact the observed losses and thereby the underling loss severity distribution.

The next paragraph examines different ways to assess the loss frequency distribution.

3.3.3 Loss Frequency Distribution.

In the standard POT model, it is well known that the number of exceedances of a high threshold follows a Poisson process[45]. This result underpins the loss frequency

[45] See Embrecht et al. (1997) page 366.

distribution modeling in this study. To calibrate the distribution, one uses the fact that large international active banks incur an average of 50 to 80 losses above $1 million each year, depending on their sizes, control environments and riskiness of their business lines (Fontnouvelle et al. 2003). As to small-size organizations, expert judgment is also used to extract the loss frequency distribution function parameters. Besides this base model, this study presents a simple common Poisson shock framework suggested by Powojowski et al. (2002) and mentioned in Chavez-Demoulin et al. (2005). Impact of this model on the capital charge is investigated. Details of the suggested methodology are described as follows:

The intent of the approach described in Powojowski et al. (2002) is to account for the correlation of loss frequency across operational loss risk class or unit by means of an underlying common shock methodology. More specifically, the model is based on the idea that a set of m independent underlying loss processes, each characterized by a one-dimensional Poisson process, (that is $\xi^*(i) = \sum_{q=1}^{N^*(i)} I_{T_q(i)}$, where the counting variable $N^*(i)$ is Poisson-distributed with intensity $\lambda(i)$ $i = 1,...,m$) can be constructed to generate the dependence structure of n observed operational loss processes, each characterized by one-dimensional point process $\xi(j) = \sum_{q=1}^{N(j)} I_{T_q(j),}$ $j = 1,...,n$. Each of these underlying loss processes can be ascribed to one or more of the observed operational loss processes. It turns out that the counting variable $N(j)$ of the observed loss process can be expressed as:

$$N(j) = \sum_{i=1}^{m} \delta(i, j) N^*(i)$$

where $\delta(i, j)$ denotes the indicator variable.

$N(j)$ is therefore Poisson-distributed with intensity

$$\tau(j) = \sum_{i=1}^{m} \delta(i, j)\lambda(i)$$

Intuitively, this model presupposes the existence of common shocks which affect more than one operational risk class. The covariance and correlation coefficients between $N(j)$ and $N(k)$ are as follows:

$$Cov(N(j), N(k)) = \sum_{i=1}^{m} \delta(i, j)\lambda(i)\delta_{ik}$$

$$\rho(j,k) = \frac{\sum_{i=1}^{m} \delta(i, j)\lambda(i)\delta(i,k)}{\sqrt{\sum_{i=1}^{m} \delta(i,k)\lambda(i) \times \sum_{i=1}^{m} \delta(i,k)\lambda(i)}}$$

Under this model, only positive correlation is permissible. A simple case that assumes a single enterprise-wide source of loss is such that $m = n+1$ and $N(j) = N^*(j) + N^*(m)$, $\tau(j) = \lambda(j) + \lambda(m)$ for $j = 1,...n$

Thus,

$$Cov(N(j), N(k)) = \lambda(m)$$

$$\rho(j,k) = \frac{\lambda(m)}{\sqrt{\left(\lambda(j) + \lambda(m)\right) \times \left(\lambda(k) + \lambda(m)\right)}}$$

for $j,k = 1,...n$ and $j \neq k$

This case is investigated through the empirical exercise conducted in this dissertation.

3.3.4 Modeling Dependence Structure

As a tool to model joint effects of multiple risks, the concept of copula has recently attracted extensive attention from the financial community as it conveys more meaningful information about dependence structure than the conventional Pearson correlation. A typical example is the key concept of tail dependence that will be examined in the sequel. This subject is relevant to operational risk practitioners since within the BCBS framework, banks are required to calculate the capital charge for each of the 56 business line/event type cells and use a dependence structure model to aggregate these values. As pointed out in Frachot et al. (2003), the dependence structure envisioned by the Committee is that of aggregate losses since it is this latter dependence structure which is considered when it comes to aggregating capital charges.

Simply stated, a copula function links univariate marginal distributions to their joint distribution. A theorem due to Sklar (1959) states that if $X = (X_1,...,X_d)$ is a random variable with joint distribution function F, then there exists a copula function C such that

$$F(x_1,...,x_d) = C(F_1(x_1),...,F_n(x_d))$$

where F_i is the ith marginal distribution function, for $i = 1,2,...,d$.

Conversely, any given copula C can be used to link any collection of univariate marginal distribution functions $F_1,...,F_d$ to create a joint distribution function F that satisfies the aforementioned relation. It should be noted that the latter statement constitutes the rationale for the methodology I implement to derive the aggregated capital charges using various families of copulas. This statement also underpins the concept of meta distributions as described in McNeil et al. (2005) page 192. Notice also that most of the results related to copulas are stated under the assumption of absolute continuity of the univariate marginals. Indeed, under this requirement, there is a unique copula C such that for $u=(u_1,...,u_d)\in[0,1]^d$

$$C(u_1,...,u_d)=F(F_1^{-1}(u_1),...,F_d^{-1}(u_d))$$

where

$$F_i^{-1}(u_i)=\inf\{x:F_i(x)>u_i\},\quad i=1,...,d$$

are the marginal quantile functions.

When the marginals are not continuous as it is in the case of discrete distributions, the underlying copula is not unique. Typically, the copula framework becomes more complicated to tackle and the determination of the dependence structure may involve the marginals (Neslehova, 2004).

Recent developments on copulas can be found in Marshall (1996) for the discrete case, Joe (1997), Embrechts et al (1999), Nelsen (1999), Neslehova (2004), and McNeil (2005). Following is a brief presentation of some useful families of copulas that are considered in this study.

Tang et al. (2004) describe three classes of copulas that are generally used in finance and insurance. These are the copulas of extreme dependence, the Archimedean copulas and the elliptical copulas. The copulas of extreme dependence include the independence copula, the Frechet lower bound for copulas and the Frechet upper bound for copulas. The independence copula or product copula $\Pi(u)$ is expressed as:

$$\Pi(u)=u_1...u_d\,.$$

while, the Frechet bounds for copulas are

$$M(u) = \min(u_1, ..., u_d)$$

and

$$W(u) = max(u_1 + ... + u_d - d + 1, 0)$$

with

$$W(u) \le C(u) \le M(u)$$

Note that for $d \ge 2$, $M(u)$ defines a copula, called the comonotonic copula that describes a perfect positive dependence structure, while for $d > 2$, $W(u)$ is no longer a copula. Archimedean copulas or explicit copulas constitute the second class of copulas. They are based on one generator function, and as such, have simple closed forms (Aas, 2004). This class of copulas allows for asymmetry, and as a result, exhibits greater dependence in the negative tail or in the positive tail. However these copulas generally fail to account for multivariate dependence structure as they have one single parameter to describe the dependence. Examples of Archimedean copulas include the Clayton copula and the Gumbel copula. Elliptical copulas or implicit copulas comprise the third class of copulas. Typically, elliptical copulas are copulas implied by elliptical distributions. Well-known examples of elliptical distributions include multivariate normal, t-student, and logistic distributions. Elliptical copulas allow for joint extreme events, but fail to account for asymmetries. In addition, they do not have a simple close form. Regardless of these shortcomings, they are becoming more and more popular for empirical exercises as they are remarkably easy to simulate. Tang et al (2004) also acknowledge the flexibility of this family of copulas to account for differences in pair-wise dependence structure by using a variance-covariance framework.

The expressions of the aforementioned copulas are as follows:

For the normal copula:

$$C(u) = \Phi_R^d\left(\Phi^{-1}(u_1),...,\Phi^{-1}(u_d)\right)$$

where Φ_R^d denotes the joint distribution function of the *d*-dimensional multivariate standard normal distribution function with linear correlation matrix R.

In the bivariate case, the copula expression is:

$$C_\rho(u_1,u_2) = \int_{-\infty}^{\Phi^{-1}(u_1)}\int_{-\infty}^{\Phi^{-1}(u_2)} \frac{1}{2\pi(1-\rho^2)}\exp\left\{-\frac{s^2-2\rho st+y^2}{2(1-\rho^2)}\right\}dsdt$$

where ρ denotes the parameter of the copula.

The expression of the Student's t-copula is

$$C(u) = t_{\nu,R}^d\left(t_\nu^{-1}(u_1),...,t_\nu^{-1}(u_d)\right)$$

where $t_{\nu,R}^d$ denotes the joint distribution function of the *d*-dimensional multivariate Student's t-distribution function with linear correlation matrix R and ν degrees of freedom.

In the bivariate case, the copula expression is:

$$C_{\rho,\nu}(u_1,u_2) = \int_{-\infty}^{t_\nu^{-1}(u_1)}\int_{-\infty}^{t_\nu^{-1}(u_2)} \frac{1}{2\pi(1-\rho^2)^{1/2}}\left\{1+\frac{s^2-2\rho st+y^2}{\nu(1-\rho^2)}\right\}^{-(\nu+2)/2} dsdt$$

where ρ denotes the parameter of the copula.

The Clayton copula in the bivariate case has the following expression:

$$C_\delta(u_1,u_2) = (u_1^{-\delta}+u_2^{-\delta}-1)^{-1/\delta}$$

where $0<\delta<\infty$ denotes a parameter controlling the degree of dependence.

The Gumbel copula in the bivariate case can be expressed as:

$$C_{\delta}(u_1,u_2) = \exp\left(-((-\log u_1)^{\delta} + (-\log u_2)^{\delta})^{1/\delta}\right)$$

where $1 \le \delta < \infty$ denotes a parameter controlling the degree of dependence.

This study presents the capital charges in the context of comonotonic, independence, and elliptical copulas. For elliptical copulas, we make use of the converse statement in Sklar's theorem that gives rise to various meta distributions (meta- Gaussian distribution, meta-t_{ν} distribution, see McNeil et al. (2005)). The case of Archimedean copulas will be investigated in future work.

The following is a summary of the algorithm that simulates a vector of dependent aggregate losses $(AggL(1),...,AggL(d))$ with marginal $F_{AggL(1)},...,F_{AggL(1)}$ and the associated elliptical copula C.

For the normal copula, the ith simulated aggregate loss is

$$AggL(i) = F^{-1}_{AggL(i)}\left(\Phi\left(A_i(\Phi^{-1}(u_1),...,\Phi^{-1}(u_d)\right)\right)$$

where $\Phi(u)$ denotes the standard normal cumulative distribution, A the lower triangular matrix obtained from the Choleski decomposition of the linear correlation matrix of the specified copula, and u_i for $1 \le i \le d$, are d independent standard uniform variables.

Notice that the normal copula transformation gives rise to the simulation of the transform of the aggregate losses $AggL(i)$, that is $AggL^{*}(i)$, under the Wang Transform. Indeed, by setting

$$A_i(\Phi^{-1}(u_1),...,\Phi^{-1}(u_d)) = \Phi^{-1}(u_i) + \lambda$$

where $\lambda = \Phi^{-1}(\alpha)$, with α denoting the specified rating target or confidence level.

one gets

$$AggL^*(i) = F_{AggL(i)}^{-1}\left(\Phi\left(\Phi^{-1}(u_i)+\lambda\right)\right)$$

As to the Student's t-copula, the ith simulated aggregate loss is

$$AggL(i) = F_{AggL(i)}^{-1}\left(t_\nu\left(\sqrt{\frac{\nu}{S}}\left(A_i(\Phi^{-1}(u_1),...,\Phi^{-1}(u_d))\right)\right)\right)$$

where t_ν is the Student's t cumulative distribution function with ν degrees of freedom, A the lower triangular matrix obtained from the Choleski decomposition of the linear correlation matrix of the specified copula, S a random number generated from the chi-square distribution random variable $\chi^2(\nu)$ independent from each of the standard normal variables $\Phi^{-1}(u_i)$.

We have previously mentioned that elliptical copulas allow for joint extreme events. To clarify this statement, we need to elaborate on the concept of upper tail dependence.

Let $X \sim F_X$ and $Y \sim F_Y$ denote a pair of random variables. By definition[46], the upper tail dependence coefficient is formulated as

$$\lambda_u = \lim_{\alpha \to 1^-} P(Y > F_Y^{-1}(\alpha) \mid X > F_X^{-1}(\alpha))$$

This expression measures the probability of observing a large Y, assuming that X is large. The interpretation of this coefficient is that if $\lambda_u > 0$, extreme events tend to occur concurrently while in the case where $\lambda_u = 0$, there is no tail dependence and the random variables X and Y are said to be asymptotically independent.

As to the normal copula characterized by its parameter ρ, the coefficient of the upper tail dependence is expressed as

46 See McNeil et al. (2005).

$$\lambda_u = 2 \lim_{x \to -\infty} \Phi\left(x \sqrt{\frac{1-\rho}{1+\rho}} \right) = 0$$

For the Student's t-copula characterized with its parameters ν and ρ, it comes that

$$\lambda_u = 2t_{\nu+1}\left(-\sqrt{\frac{(\nu+1)(1-\rho)}{1+\rho}} \right)$$

where $t_{\nu+1}$ denotes the distribution function of the univariate Student's t-distribution with $\nu+1$ degrees of freedom ρ the Pearson correlation coefficient between X and Y. The aforementioned formula expresses the idea that lower degrees of freedom give rise to heavy tail dependence for the Student's t-copula. As a result, the copula theory predicts that the lower the degrees of freedom, the higher the capital charges since the simultaneous occurrence of extreme events will adversely impact the resulting aggregate loss distribution from which the capital charge is derived. Figure 6.1 plots upper tail values in function of the correlation coefficient for 3 degrees of freedom.

For the empirical exercise, this study uses the empirical rank correlations from which linear correlations are derived and adjusted by expert judgment[47]. In addition, the approach developed in Powojowski et al. (2002) is investigated whenever all second moments of loss severities are finite. Specifically, as shown previously, for two aggregate losses, we have

$$\rho\left(AggL(i), AggL(j)\right) = \rho(N(i), N(j)) \frac{E\left(L_1(i)\right)E\left(L_1(j)\right)}{\sqrt{E\left(L_1(j)^2\right)E\left(L_1(j)^2\right)}}$$

with $\rho(N(i), N(j)) = \dfrac{\lambda_m}{\sqrt{\left(\lambda_i + \lambda_m\right)\left(\lambda_j + \lambda_m\right)}}$

[47] See Tang et al. (2004) for similar adjustment.

provided that $E\left(L_1(i)^2\right) < \infty$ and $E\left(L_1(j)^2\right) < \infty$. This holds whenever all underlying loss severity distributions (which are log-exponential or Pareto type I) have parameters β strictly less than 0.5.

3.3.5 Capital Charge Modeling

This subsection proposes an approach towards deriving the overall capital charge, taking into account a whole set of dependence structures. As a matter of fact, one may ultimately select the copula that minimizes the distance to the empirical copula of the data (Romano et al., 2002; Deheuvels, 1979) or extract the matrix of rank correlations (Kendall's tau) as well as the degrees of freedom from the data set (see Mashal and Zeevi, 2002). However, in the context of publicly available operational losses, it is argued that for each business line/loss event type cell, an accurate estimation of the rank correlation matrix, the degrees of freedom as well as of the empirical copulas cannot be obtained due to the lack of sufficient data and the presence of reporting biases in the data set. Therefore, one way to settle this issue is to account for a family of copulas to get the empirical distribution of the overall capital charge. In this study, a set of 13 dependences structures is explored. These include the comonotonic dependence, the Student's t-copula with the degrees of freedom ranging from 1 to 10, the normal copula and the independence copula.

The copula theory, through the upper tail dependence properties predicts that $Cap_{com} \geq Cap_{t_1} \geq ....Cap_{t_{10}} \geq Cap_{normal} \geq Cap_{indep}$ where Cap_{com} and Cap_{t_i} for $i = 1,...,10$ denote the capital charge under the comonotonic dependence and the student's t copula, respectively. These will be examined in the empirical exercise.

Now, we are in a position to describe the approach to estimating the overall capital charge.

The framework is that of the finite mixture distribution, especially the component-mix distribution in which the overall capital charge is expressed as a mixing weighted capital charges. Recent literature dealing with distributions formed from component-mixes can be found in Rose et al. (2002) and Titterington et al. (1985).

Specifically, component mix distributions are generated from linear combinations of distributions. Following Rose et al (2002), in the case of a discrete random variable X_i, let $f_i(x) = P(X_i = x)$ for $i = 1,...,n,$ denote the probability mass function and let π_i denote a parameter such that $0 \leq \pi_i \leq 1$ and $\sum_{i=1}^{n} \pi_i = 1$. Then, the n-component-mix random variable is defined as

$$X \sim \pi_1 X_1 + ... + \pi_n X_n$$

and its probability mass function is expressed as

$$f(x) = \sum_{i=1}^{n} \pi_i f_i(x)$$

The parameters π_i for $i = 1,...,n,$ are defined as the mixing weights and the functions f_i for $i = 1,...,n,$ are called the component densities.

The abovementioned formula applied to our framework calls for three remarks. First, a weight is attached to each capital charge that reflects a specific dependence structure. Specifically, π_1 is attached to f_1 the distribution of the capital charge obtained under the comonotonic copula. Second, it is assumed that these mixing weights could be determined through a risk assessment exercise so as to reflect organizations' quality of internal control environment. For example for firms with improved internal control environment, the first weight π_1 assigned to the comonotonic copula, could be set close to zero. Third, the empirical distribution of the overall capital charge is generated by Monte Carlo simulation runs from the f_i and the mixing weights π_i for $i = 1,...,n.$ Consequently, key descriptive statistics regarding the overall capital charge can be provided.

In light of these clarifications, the proposed approach can be seen as an efficient framework that provides not only bounds for the overall capital charge, but also incentives for banks and insurers to improve their handling of operational risk.

It is worth noticing at this stage that the mathematical formulation of this approach is particularly simple. Its computational implementation, by contrast, is quite complex due to the rating target set by the Committee (99.9%), which may require a very large number of simulation runs to get consistency in the results according the upper tail dependence properties.

The next section presents the empirical analysis. Key descriptive statistics are given and various results related to the estimation of the loss severity distribution for industry-wide banks and insurers as well as for specific firms are analyzed. The sensitivity of the capital charge to the choice of copulas is investigated and finally, the empirical probability distribution function of the overall capital charge is described.

CHAPTER 4

EMPIRICAL ANALYSIS

4.1 Introduction

This study examines the implications of using the AMA as a method to assess operational risk capital charges for banks and insurance companies and analyzes the extent to which the four key elements of the AMA, that is, internal data, external data, scenario analysis, and business environment and control factor could be encompassed in a model. The theoretical model, presented in chapter 3 provides the mathematical background within which Monte Carlo simulations are carried out to determine the empirical distribution of the overall capital charge. This chapter describes the empirical investigation using publicly available operational losses. Section 4.2 describes key descriptive statistics and motivates the risk classification schemes within which loss severities are calibrated and capital charges determined. Section 4.3 discusses the calibration of the parameters of the loss severity distribution using three distributional assumptions for the random truncation point. Lastly, section 4.4 provides the descriptive statistics as well as the histogram of the empirical distribution of the overall capital charge.

4.2 The Data Set

The empirical exercise uses publicly available operational losses provided by Fitch Risk Management. This firm captures financial and non-financial operational risk losses that are in excess of $1 million from public sources such as court filings and news reports. In addition to individual losses, the data set contains various organizations' exposure indicators such as number of employees, gross income, assets, physical assets, compensation, and deposits. Essentially, these large operational losses are used to supplement banks' internal loss data in calibrating the tail of the loss severity distribution.

In the sequel, key descriptive statistics related to contributors of losses and individual losses (that occurred in the United States) are provided. Contributors of losses are referred to as bank and insurance organizations in the US market that incurred the losses captured by Fitch.

For the period ranging from 1980 to 2002, Table 1.1 indicates that operational losses were captured from 1244 bank organizations grouped in 998 parent banks and 381 insurers grouped in 302 parent insurance organizations. The total losses incurred by these organizations amount to $58,552 million for banks, and $22,535 million for insurers. In terms of total number of losses per contributor, Table 1.1 also shows that Fitch has captured only one loss in excess of 1$ million from nearly 80% of contributors. This is an important fact that impacts the calibration of the underlying loss distribution.

This subsection analyzes the distribution of contributors' truncation point above which Fitch captures operational losses. Fitch is supposed to capture and report all losses in excess of a threshold set to $1 million. The focus here is to investigate the actual distribution of the truncation point by contributor (See Fontnouvelle et al. (2003) page 10 for more discussion on threshold and truncation point). For US banks, Table 1.2 indicates that the contributor's truncation point ranges from $1 million to $1980 million. Among business lines, Retail Banking has the highest number of contributors, i.e. 599 and the highest contributor's truncation point i.e. $1980 million while Payment and Settlement has the lowest number of contributors, i.e. 21, and at the same time, the lowest contributor's truncation point, i.e. $209 million. As to loss event types- Table 1.3, CPBP has the highest number of contributors, i.e. 598 and the highest contributor's truncation point, i.e. $1980 million. Internal Fraud ranks second in terms of both number of contributors and contribution's truncation point.

With regard to the insurance industry-Table 1.4, CPBP has the highest number of contributors, i.e. 264 and the highest contributor's truncation point, i.e. $1094 million.

Both bank and insurer contributors' truncation point are significantly skewed to the right. According to Table 1.2, 1.3 and 1.4, the coefficient of skewness is 15 for banks and 5 for insurers. A log scale is thus used to represent the distribution of contributors' truncation point.

Figures 1.1 and 1.2 show the histogram of the contributor's log-truncation-point for US banks and insurers. For the first category, according to Table 1.5, the contributor' truncation point at the 25^{th}, 50^{th}, 75^{th}, and 95^{th} percentiles are $2 million, $4 million, $12 million, and $80 million, respectively. For the insurers, these percentiles are $2 million, $5 million, $19 million and $120 million.

The results of these preliminary analyses and considerations visibly suggest that it would not be appropriate to treat the contributor's truncation point as constant and known i.e. $1 million.

As to the size of these contributors, table 2.1 provides summary statistics for banks and insurers' exposure proxied by their total revenue. It is noticed that more than 50% of both organizations have no exposure reported. The revenue is clustered according to the euclidean distance into 3 categories based on the size of the organizations, i.e. small size, medium size and large size. According to this classification scheme, within the US bank contributors, 26 contributors could be considered as large banks while within the US insurer contributors, 10 could be deemed as large insurers. Table 2.2 shows that the median of the total revenue amounts to $7,793 million for banks and $9,241 million for insurers. The two aforementioned classifications are used to calibrate the loss severity distribution according to organization size.

Figures 3.1 and 3.2 display the yearly aggregate losses for US banks from 1980 to 2002. One notices the existence of a cycle with peaks in 1984, 1988, 1994, 1998 and 2002. The length of the cycle is approximately four years. The first figure splits the total yearly aggregate losses into the standard BCBS eight business lines. In 1988 and since 2000, retail banking has become a major business line in terms of yearly aggregate losses. Trading and Sales ranks second. Figure 3.2 breaks the total yearly aggregate losses into the seven event types. Clearly, CPBP is the main risk driver of operational risk losses for US banks. Internal Fraud also accounts for an important part of the total yearly aggregate losses. Figure 3.3 analyzes CPBP losses by splitting them into various components defined by Fitch. Deceptive Sales Practices and Concealment followed by Failure to Disclose appear to be the main risk drivers of CPBP.

As to US insurers, Figure 3.4 indicates that insurers' operational losses started increasing from 1992 and that CPBP is also the main risk driver. It may be the case that insurers' operational losses are subject to more disclosure from 1992. Similar to US banks, Figure 3.5 shows that Deceptive Sales Practices and Concealment most account for insurance CPBP losses.

Figure 3.6 compares the US bank and insurer yearly aggregate losses and clearly indicates that banks incurred more operational losses than insurers.

As to loss occurrences, Figure 3.7 displays the US bank yearly loss occurrences and indicates an upward trend. The same result holds true for the US insurer yearly loss occurrences as shown by Figure 3.8.

Tables 4.1 and 4.2 show total loss amounts and occurrences incurred by the US banks from 1960 to 2003. Total loss amounts are split into BCBS eight business lines and seven event types. Retail Banking followed by Trading and Sales is the leading business line while CPBP and Internal Fraud are the two major loss event types.

As to loss occurrences, Retail Banking has the highest number of individual losses both overall and specifically for CPBP. Again CPBP among the seven event types shows the highest number of individual losses. Internal Fraud ranks second. Likewise, for the US insurers, Table 4.3 indicates that CPBP is the main risk.

One notices that some business lines and event types such as Agency Service, Payment and Settlement, Damage to Physical Assets and Business Disruption & System Failure have few observations or no observations.

In view of these results, it seems appropriate to conduct the calibration of the loss severity as well as the calculation of the capital charges by dividing banks' activities as follows:

1- All business lines – CPBP (or relationship risk class according to Fitch classification).
2- All business lines – Internal Fraud and Employment Practices and Workplace Safety (or people risk class according to Fitch).

3- All business lines – Other event types.

Alternatively, for comparison purposes, a classification by business units instead of business lines is employed.

For the insurance industry, the following classification is used.

1- CPBP

2- Other event types

These calculations are developed and explained in the sequel.

4.3 Loss Severity Distribution Function

The period of study ranges from 1960 to 2002 and is conducted according to the abovementioned classification. For each business unit, Table 5.1 and Figure 5.1 give the parameters of the observed loss severity distribution that include the underlying loss severity distribution and the random truncation point distribution. The results indicate that the constant and known assumption regarding the truncation point yields the highest level of the loss severity parameter while the logistic assumption gives rise to the lowest level. Specifically, the constant and known assumption does not account for reporting bias and assigns a uniform weight to all losses. Further developments (Table 5.7) show that this line of reasoning leads to a higher level of capital charges and to the belief that operational risk is extremely risky.

The most risky business unit is Investment Banking that comprises two business lines, namely Corporate Finance and Trading and Sales. The underlying loss severity parameters (also referred to as tail parameters) are 2.550, 1.1199 and 1.232 for the constant and known assumption, the logistic assumption, and the normal assumption, respectively. The most risky event type is CPBP, especially for Investment Banking.

As to insurers, tail parameters except for the constant and known assumption are less than 0.6. This range of tails leads to the conclusion that insurers may be less exposed to operational risk than banks.

It is to be noted that for a specific business line/loss event type cell i, the following relationship between the tail parameter $\beta(i)$ and the first and second moment of the loss severity distribution holds:

(1) If $\beta(i) \geq 1$ then $E(L(i)) = \infty$

(2) If $05 \leq \beta(i) < 1$ then $E(L(i)) < \infty$ and $E(L(i)^2) = \infty$

(3) If $\beta(i) < 0.5$ then $E(L(i)) < \infty$ and $E(L(i)^2) < \infty$

The log likelihood of the three models suggests that the logistic distributional assumption most accounts for the reporting bias. But since the log likelihood yields a bias in comparing different distributions, the Akaike information Criterion (AIC) is computed and the likelihood ratio test is performed to acknowledge the fit of the logistic distribution (Werneman, 2005). The AIC is defined as follows:

$$AIC = -2\ln L + 2q$$

where $\ln L$ is the log-likelihood function and q is the number of parameters of the distribution fitted. The smaller the AIC, the better the model fits the data.

Table 5.2 and Figure 5.2 provide the loss severity and the truncation point distribution parameters analyzed by business lines. Trading and Sales appears to be the most risky business line, followed by Agency Services.

Tables 5.3 and 5.4 provide the results of the loss severity calibration by firm size. They indicate that small firms, or firms with revenue below the median, have the highest level of tail parameter. These results are in line with those obtained by Shih et al (2001), that is the size of an operational loss is weakly related to firm size.

The severity parameter of a specific organization is calibrated using the methodology previously described. Table 5.5 provides the results of this calibration. The PML along with its reporting probability is set to $1000 and 0.99, respectively. The resulting tail parameter is 0.472 when all business lines and event types are combined.

For the most prominent business lines and event types, Figures 5.3 to 5.12 show the Quantile-Quantile plots, the graph of the observed severity distribution and the underlying severity distribution. CPBP QQ-plot shows a slight decline in fit towards the tail of the distribution, while retail banking display a substantial decline in fit. As to insurers, the QQ-plots cannot be displayed since the acceptance-rejection algorithm used to simulate the observed loss severities fails to converge[48]. The Kolmogorov-Smirnov and Anderson-Darling tests have not been performed because these tests are not appropriate for distributions of excesses over some thresholds (Moscadelli, 2004 page 43).

Figures 5.13 and 5.14 present the graph of the distribution function of the random truncation point for both the specific organization and the industry-wide organization.

4.4 Capital Charges

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[48] Simulating new data from the initial data set using bootstrap technique may help to get the convergence. This is left for future research. See Moscadelli (2004) page 18 for similar application in operational risk.

Value at Risk at 99.9% rating target is the risk measure required by BCBS. This paper aims at deriving the empirical distribution of the aggregated capital charge so as to reflect the distribution of estimates of the underlying parameters and randomness of the Monte Carlo simulations. Specifically, 1 million of aggregate marginal losses and 150 000 aggregate dependent losses are simulated. Aggregate loss empirical rank correlations are computed from historical data and linear correlations derived from these rank correlations are adjusted according to expert judgment. Typically, when aggregate loss empirical correlations are negative, they are adjusted to 4% and when they are greater than 10%, they are lowered to 10% (see Frachot et al. (2004) and Tang et al. (2004) for similar adjustment). Table 5.6 shows the sample aggregate loss correlations with their adjustments. For the base scenario, the estimates are assumed to be non-random. Other scenarios reflecting estimate and correlation uncertainty as well the risk mitigating impact of insurance will be examined in future work. The computer program has been designed accordingly[49]. Figures 5.15, 5.16, and 5.17 present the distribution of the capital charges for three event types, while Figure 5.18 gives the distribution of the aggregated capital charge under the Student's t-copula with one degree of freedom. In all cases, distributions are approximately normal. Figure 5.19 plots the aggregated capital charge in terms of the degrees of freedom for elliptical copulas. It is noticed that the level of capital charge is inversely related to the number of degrees of freedom. Within this specific family of copulas, the Cauchy copula gives rise to the highest aggregated capital charge, while the normal copula yields the lowest aggregated capital charge. This result is in line with the upper tail dependence property for elliptical copulas. For banks and insurers,

[49] A simulated data set accompanied by the Mathematica and C# programs will be made available upon request to the author.

Tables 5.10 and 5.11 give the aggregated capital charge along with the capital saving for both an industry-wide organization and a specific organization. The yearly loss frequency is assumed to be equal to 50. The highest capital saving is achieved through the independence copula case. In terms of percentage, the saving ranges from 6% to 11% for banks, and from 2% to 3% for large insurers. For the specific insurer it ranges from 5% to 10%. For large organizations, the capital savings are less significant for insurers since, due to lack of sufficient data, two event type subclasses was considered compared to three for the banks. This result was expected since the diversification benefit increases with the number of business line/event types used. These levels of capital charge need to be compared with those obtained by combining all business lines/event types. Table 5.9 allows such a comparison. For a typical large bank, when all business lines and event types are combined, the capital charge amounts to $3,460 million. In the case where bank's activities are divided into three lines, the capital charge for the normal copula amounts to $6,324 million.

Table 5.12 provides the descriptive statistics for the distribution of the aggregated capital charge. The amount obtained under the Cauchy copula ranks first for most locations, scales, and percentile measures. The skewness and kurtosis excess coefficients are close to those of normal distribution.

The overall capital is then calculated as a mixing weighted capital. The illustrative case assumes that the weight assigned to each dependence structure is 8% except for the comonotonic dependence. As to this latter case, the weight is 4%. Table 5.13 provides these weights. To get the empirical distribution of the weighted capital charges, 1 million of n-component mix random variables are simulated (n=13). Figure 5.20 and 5.21 show the histograms of the mixing weighted capital charge for both the industry-wide organization and the specific organization. Table 5.14 provides the descriptive statistics. It shows that for the industry-wide bank, the mixing weighted capital as measured by the mean of the distribution is $6,433 million while for the specific organization, it amounts to $443 million.

The last table (Table 5.15) uses the model set in Powojowski (2002) to show the variation of the capital charge as the common shock intensity increases from 1 to 2. This variation is $2 million for the specific bank (from $73 million to $75 million) and $1 million for the specific insurer (from $59 million to $60 million).

CHAPTER 5

CONCLUSION

This dissertation investigates the implications of using the AMA-LDA to model operational risk capital provisions for banks and insurance companies. My study clearly reveals that operational risk is a major risk class, as evidenced by the level of capital charges that banks and insurers are required to hold. My results suggest that the level of operational risk capital charge could exceed US$6 billion for large internationally active banks, and US$600 million for large insurers. These amounts are in line with those disclosed by these institutions, that is, US$2 billion to US$7 billion for banks and 2 percent of net premium on average for insurers. They are also consistent with the amounts estimated in Fontnouvelle et al. (2003) for banks.

More specifically, this dissertation develops an approach based on the methodology set forth in Fontnouvelle et al. (2003) to calibrate the tail of operational loss severity distribution using publicly available operational loss data and accounting for firm's specificities through the level assigned to the PML. My study also proposes a model that expresses the distribution of the overall capital charge as a finite mixture distribution, accounting for quality of risk management by means of the weight attached to each component distribution that reflects a specific dependence structure.

Consistent estimates of capital charges and loss severity distribution parameters are obtained by modeling the contributor's truncation point as an unobserved random variable. In addition, my study makes use of extreme value theory, assumes a homogeneous Poisson distribution for loss frequencies, and accounts for dependence structure across risk types through copulas.

My findings also indicate that operational risk losses are driven by CPBP and internal fraud. As a result, quality of internal control environment of a firm is a key factor that highly impacts firm's loss severity distribution and thereby its capital charge distribution. Consequently, a natural extension of my research will be a formal quantification of quality of internal risk control environments. The frameworks set by NAIC[50], GLBA, FDICIA, SOX, and BCBS[51] will provide an appropriate background for this exercise.

A further extension of my study would be the quantification of the capital charge by means of coherent risk measures such as CTE and Wang Transform measure at 95% confidence level currently used in the insurance industry. Rescaling individual loss amounts based on firm's exposure remains a fruitful area for future research as well. Currently, the scaling formulas that have been proposed in the literature are still in their infancy due to a lack of adequate loss data to test for their robustness.

Another noteworthy finding lies in the fact that the capital charge is significantly driven by the number of risk types set out in the bank's risk classification scheme. As a result, BCBS needs to provide incentives for banks that use a more granular classification scheme.

As of today, operational risk accounts for at least 25% of the overall economic capital of a firm. Consequently, this specific risk class should be envisioned as a key

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50 Via the Risk-Focused Surveillance Framework.

51 Through the second pillar (supervisory review) and the third pillar (market discipline).

component of an enterprise-risk management strategy and failure to account for its capital requirement significantly distorts decisions based on risk-adjusted return on capital (RAROC).

Table 1. 1 --US Bank and Insurers - Number of Losses per Contributor

Number of Losses Per contributor	Banks				Insurers			
	Parent Organization		Organization		Parent Organization		Organization	
	Number	%	Number	%	Number	%	Number	%
1	791	79	1026	82	217	72	305	80
2 - 9	187	19	202	16	81	27	76	20
>9	20	2	16	1	4	1	0	0
Total	998	100	1244	100	302	100	381	100

Table 1. 2 --US Bank Contributors' Truncation Point ($ M)
Descriptive Statistics by Business Lines

	COFI	TRSA	REBA	COBA	PASE	AGSE	ASMA	REBR	All
Number of Contributors	47	96	599	237	21	52	121	242	1,244
Minimum	1	1	1	1	1	1	1	1	1
Maximum	213	1,899	1,980	453	209	536	417	254	1,980
Mean	16	114	18	24	20	24	32	10	23
Standard Deviation	6	18	9	7	7	9	8	5	10
Skewness	4	5	18	5	3	6	4	7	15
Excess Kurtosis	21	21	378	29	10	38	16	59	259

COFI: Corporate Finance-TRSA: Trading & Sales- REBA Retail Banking- COBA: Commercial Banking- PASE: Payment & Settlement AGSE: Agency Services – ASMA: Asset management- REBR Retail Brokerage

**Table 1. 3 --US Bank Contributors' Truncation Point ($ M)
Descriptive Statistics by Event Types**

	DAPA	EXFR	EPWS	INFR	EDPM	CPBP	BDSF
Number of Contributors	6	272	53	436	79	598	7
Minimum	1	1	1	1	1	1	1
Maximum	89	242	52	1,899	417	1,980	363
Mean	23	13	9	27	15	30	61
Standard Deviation	6	5	3	12	7	11	12
Skewness	2	5	2	12	7	12	2
Excess Kurtosis	1	32	6	153	49	175	2

DAPA: Damage to Physical Asset- EXFR: External Fraud- EPWS: Employment Practices & Workplace Safety- INFR: Internal Fraud-
EDPM: Execution, Delivery & Process Management - CPBP: Clients, Products & Business Practice BDSF: Business Disruption & System Failure

**Table 1. 4 --US Insurer Contributors' Truncation Point ($ M)
Descriptive Statistics by Event Types**

	DAPA	EXFR	EPWS	INFR	EDPM	CPBP	BDSF	ALL
Number of Contributors	1	19	17	71	53	264	1	381
Minimum	208	1	1	1	1	1	341	1
Maximum	208	295	94	420	92	1,094	341	599
Mean	208	21	21	21	8	38	341	25
Standard Deviation		8	5	8	4	10		8
Skewness		4	2	5	4	7		5
Excess Kurtosis		14	1	31	14	69		37

Table 1. 5 --US Bank & Insurer Contributors' Truncation Point ($ M) by Percentiles

Percentile	25%	50%	75%	95%
US Banks	2	4	12	80
US Insurers	2	6	19	120

Figure 1.1 --US Banks - Histogram of Contributor's Log-Truncation-Point. – All Business Lines and All Event Types

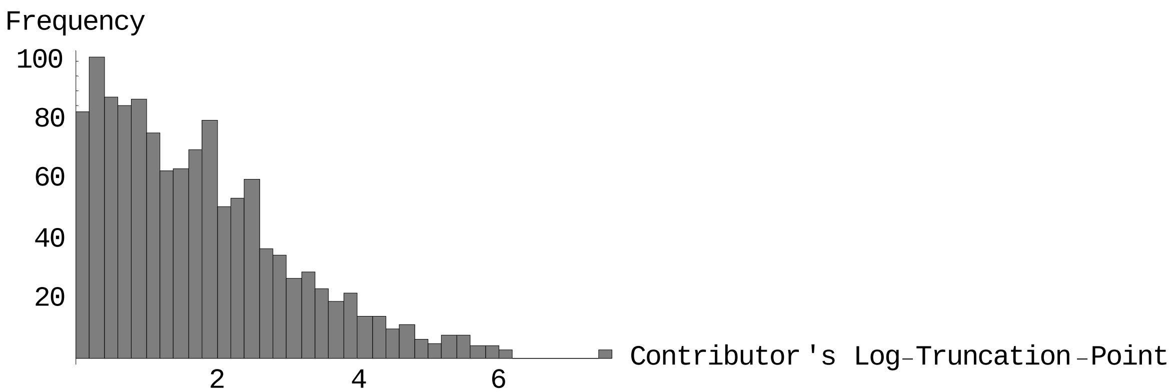


Figure 1.2 --US Insurers - Histogram of Contributor's Log-Truncation-Point – All Business Lines All Event Types

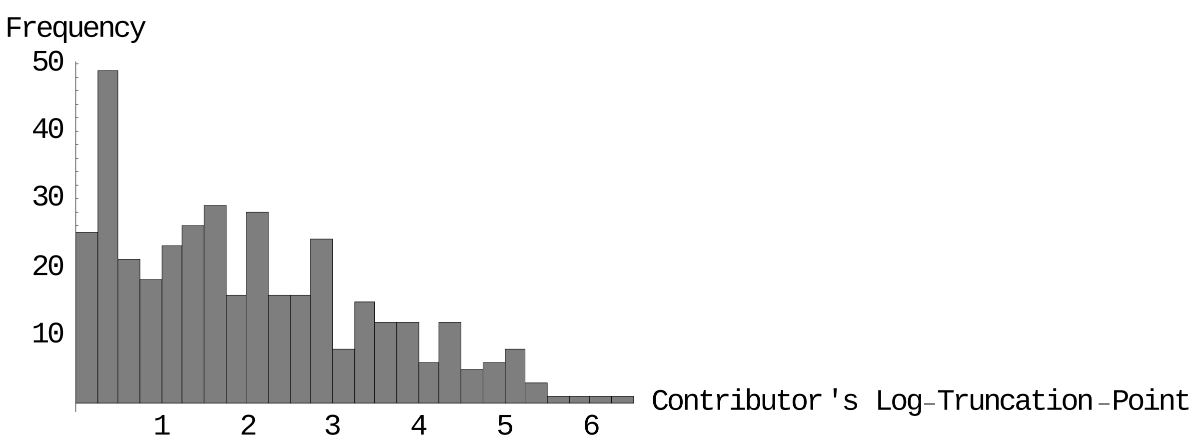


Table 2. 1 -- US Banks and Insurers' Total Revenue
Descriptive Statistics by Size

	US Banks				US Insurers			
	No Exposure Reported	Small Size	Medium Size	Large Size	No Exposure Reported	Small Size	Medium Size	Large Size
Number of Contributors	723	383	113	26	213	109	50	10
Mean		3,458	33,118	109,991		3,995	25,610	81,698
Min		1	18,631	72,772		17	14,978	60,391
Max		18,342	65,601	192,390		13,958	49,221	116,729
Std		4,547	12,749	33,603		3,907	7,774	18,824
Skewness		1	1	1		1	1	1
Excess Kurtosis		1	0	1		0	2	-1

Table 2. 2 --US Banks and Insurers' Total Revenue by Percentile

	Total Number of Losses	Total With Revenue Reported	Min	25% Percentile	50% Percentile	75% Percentile	Max
US Banks	1989	891	1	927	7,793	24,695	192,390
US Insurers	530	250	17	3,055	9,241	26,158	116,729

Figure 3.1 --US Banks Yearly Aggregate Losses By Business Lines & Settlement Year

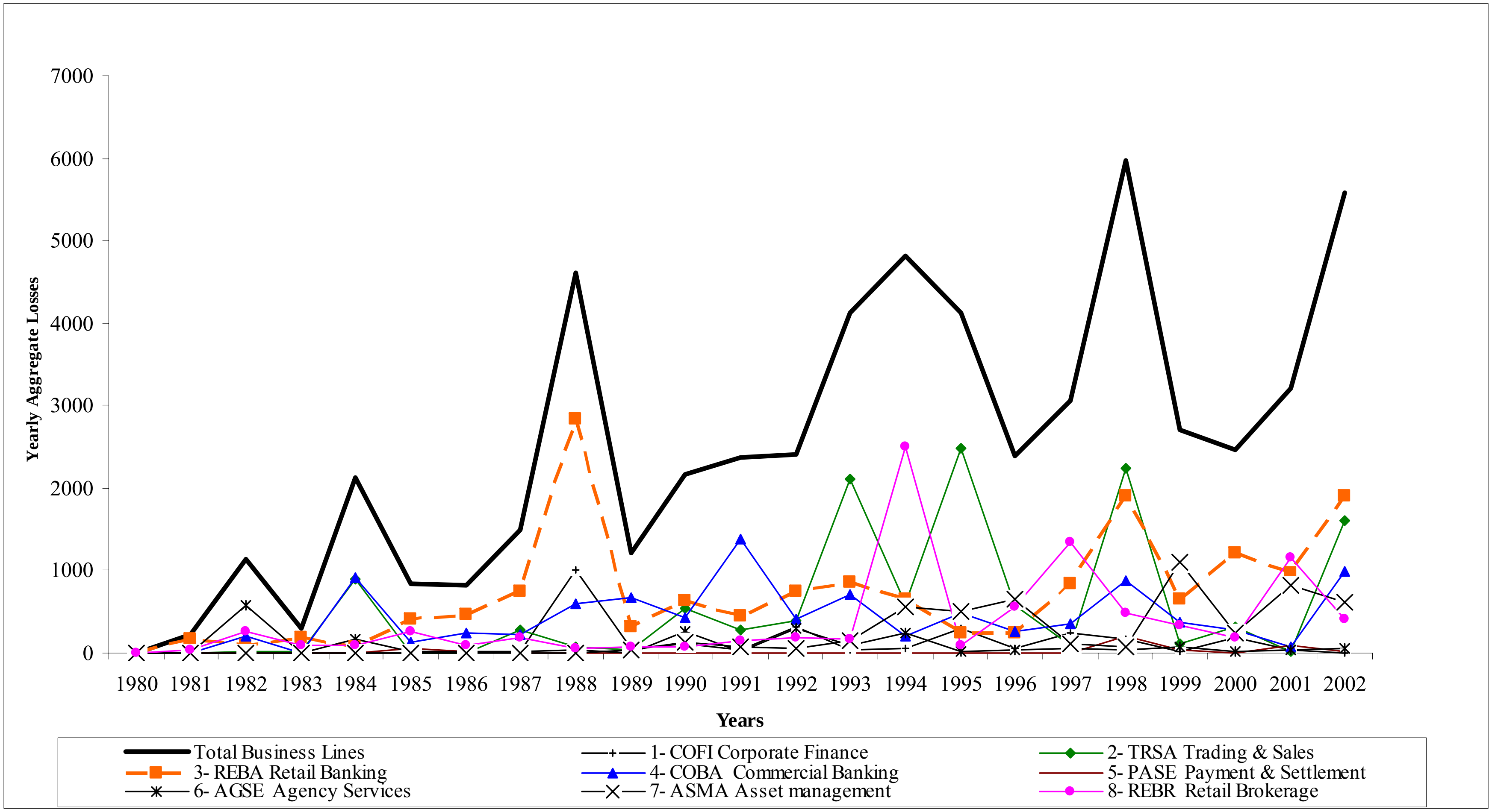


Figure 3.2 --US Banks - Yearly Aggregate Losses By Event Types & Settlement Year

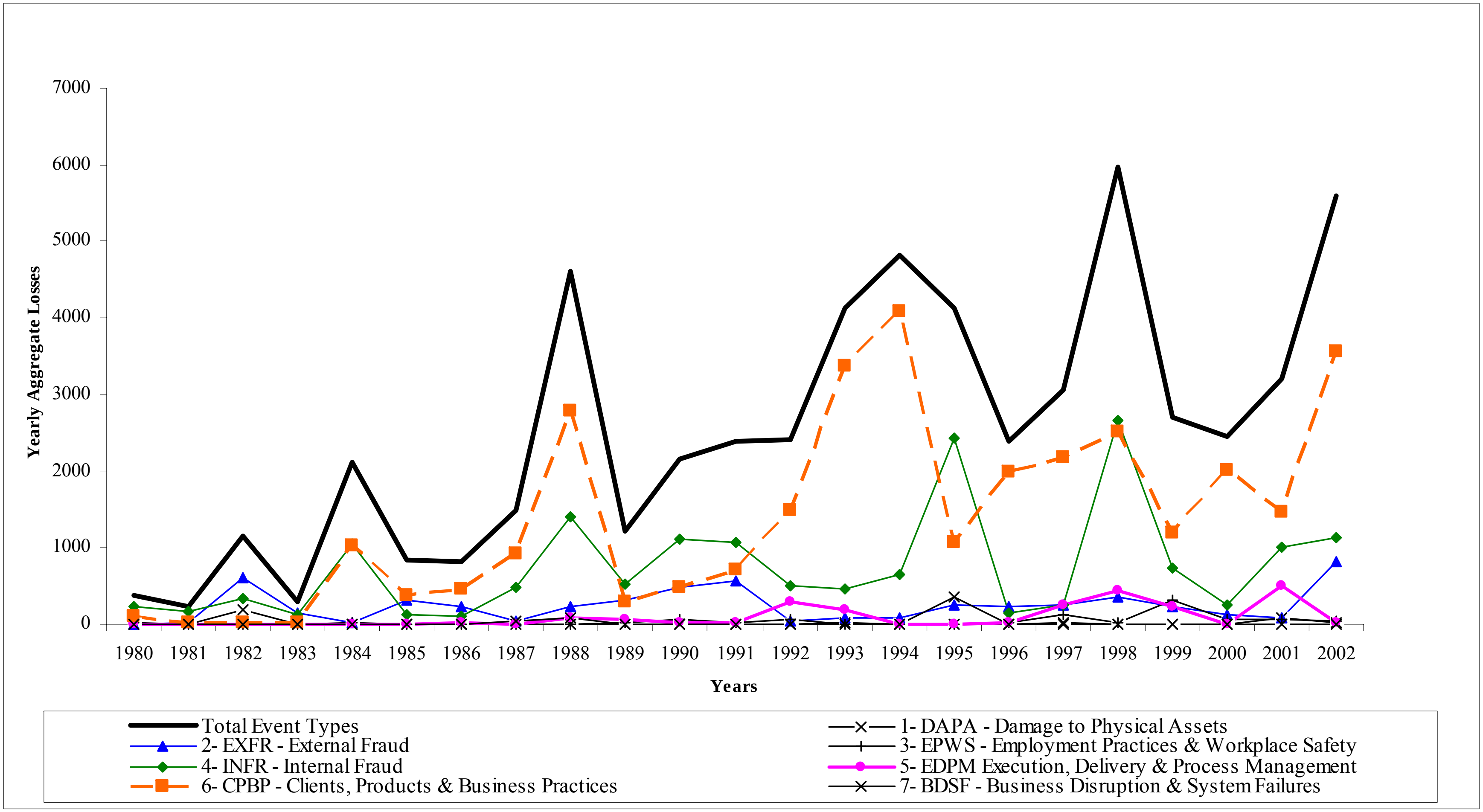


Figure 3.3 --US Banks - Yearly Aggregate Losses By CPBP Sub Event Types & Settlement Year

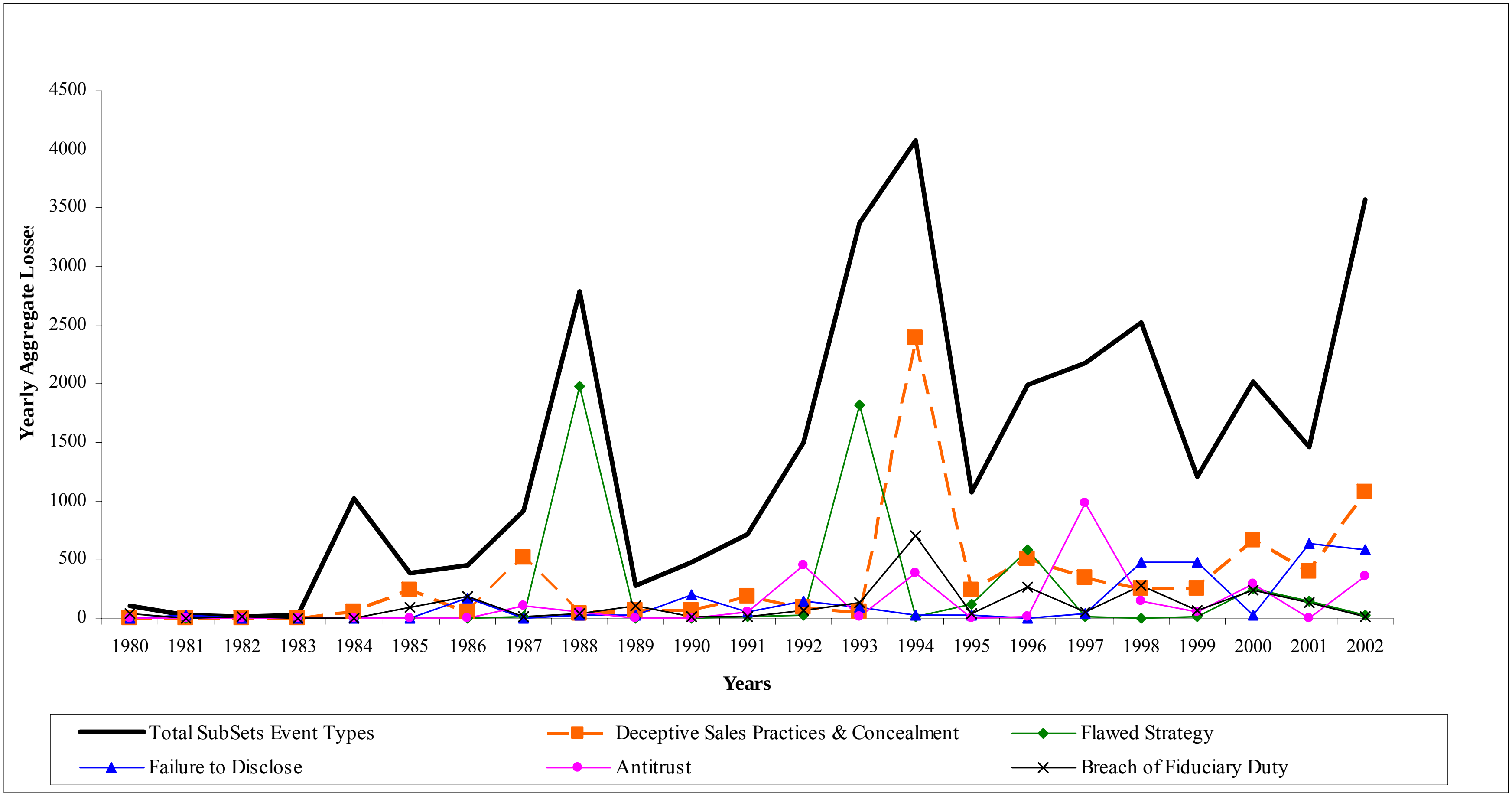


Figure 3.4 --US Insurers - Yearly Aggregate Losses By Event Types & Settlement Year

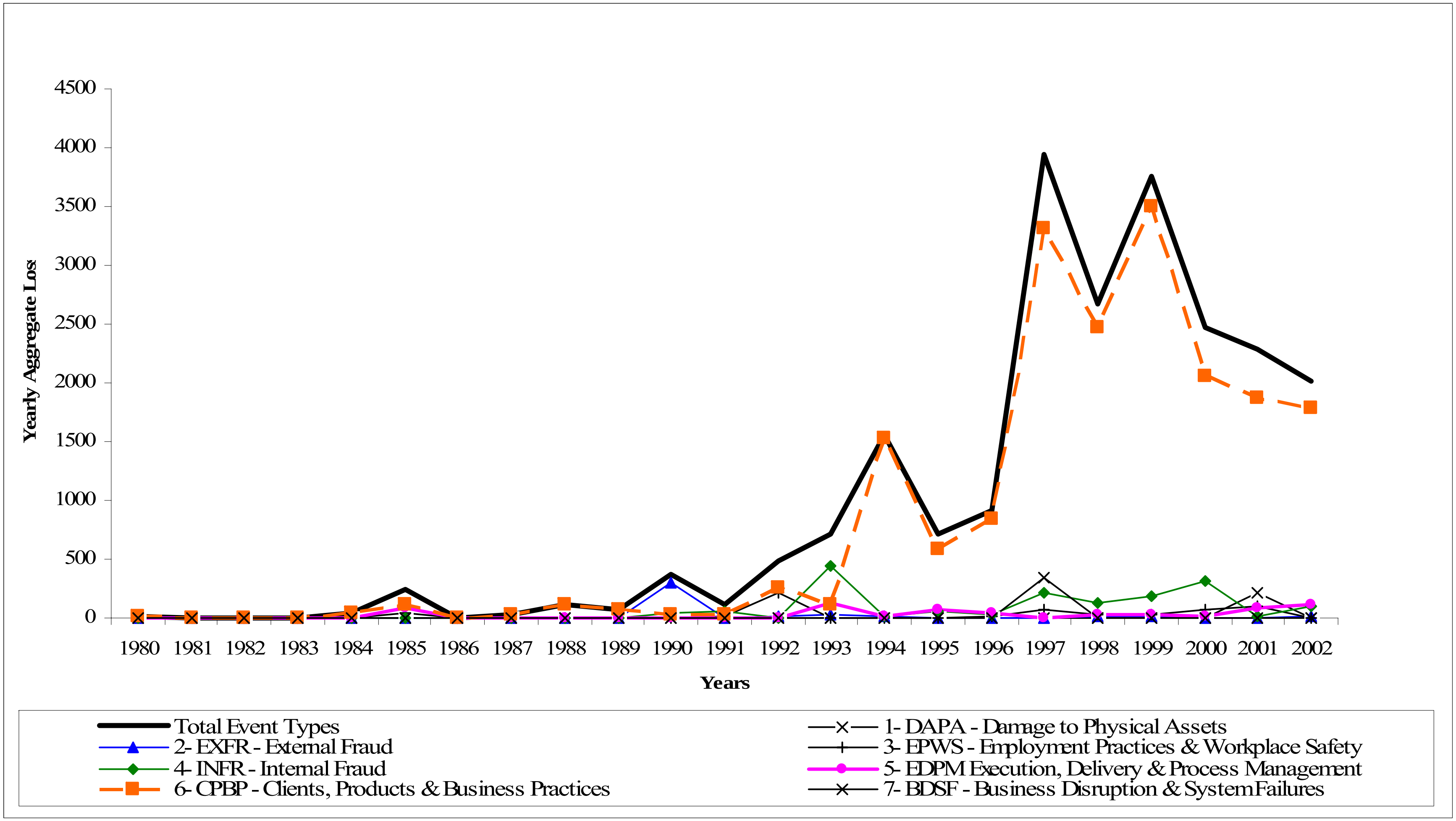


Figure 3.5 --US Insurers - Yearly Aggregate Losses By CPBP Sub Event Types & Settlement Year

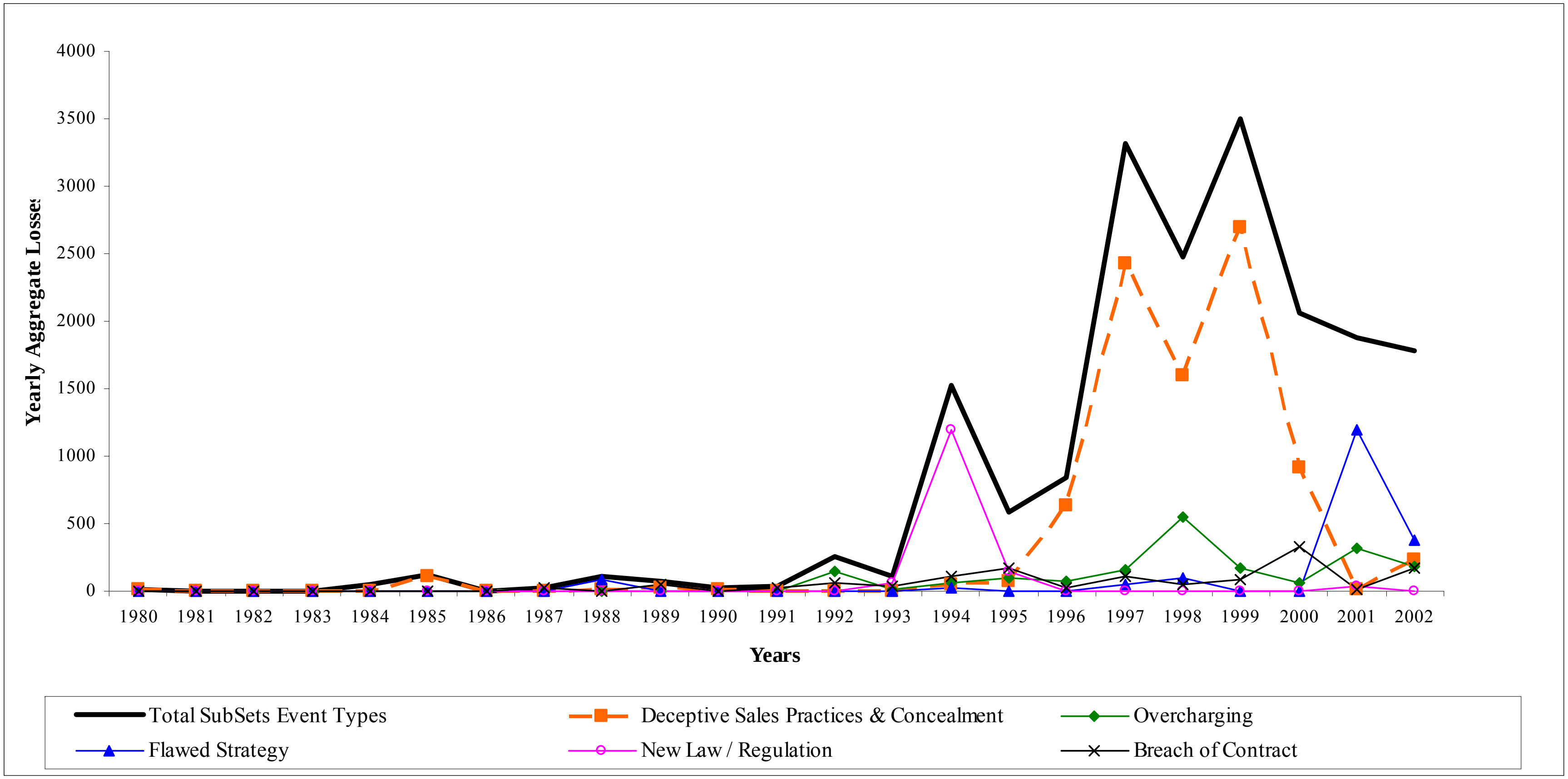


Figure 3.6 --US Banks & Insurers - Yearly Aggregate Loss by Event Types & Settlement Year

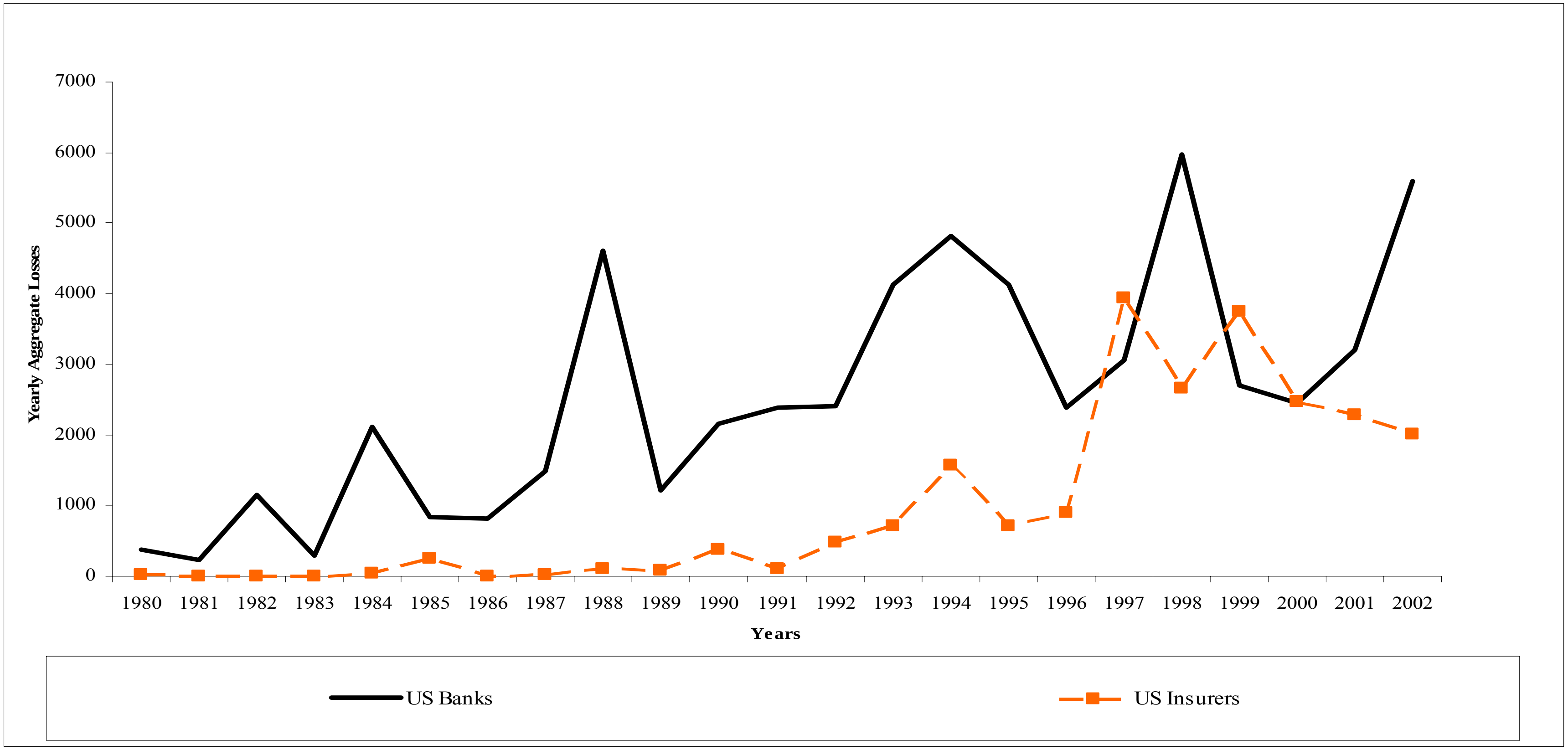


Figure 3.7 --US Banks - Yearly Aggregate Loss Amounts & Occurrences by Settlement Year

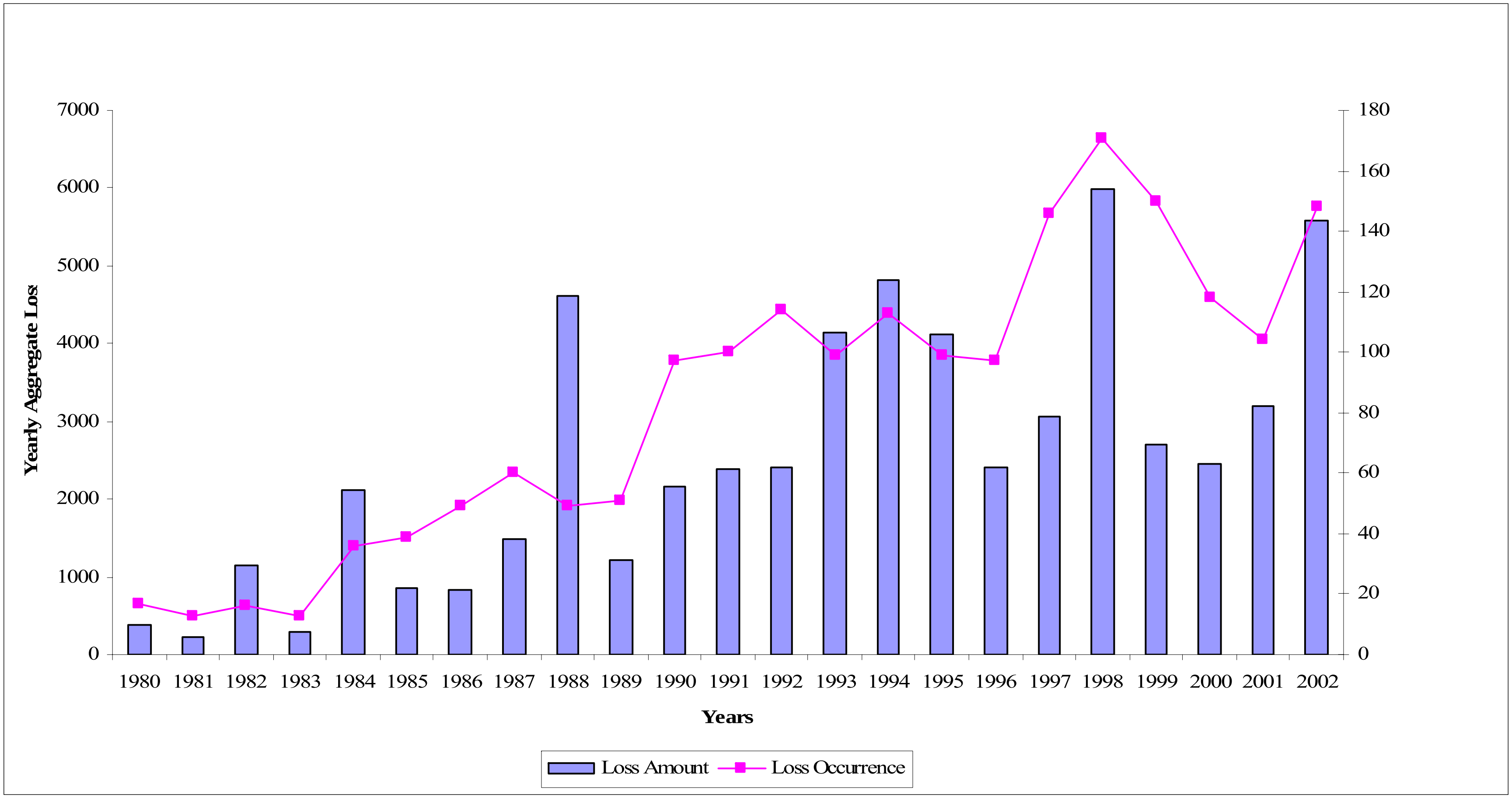


Figure 3.8 --US Insurers - Yearly Aggregate Loss Amounts & Occurrences by Settlement Year

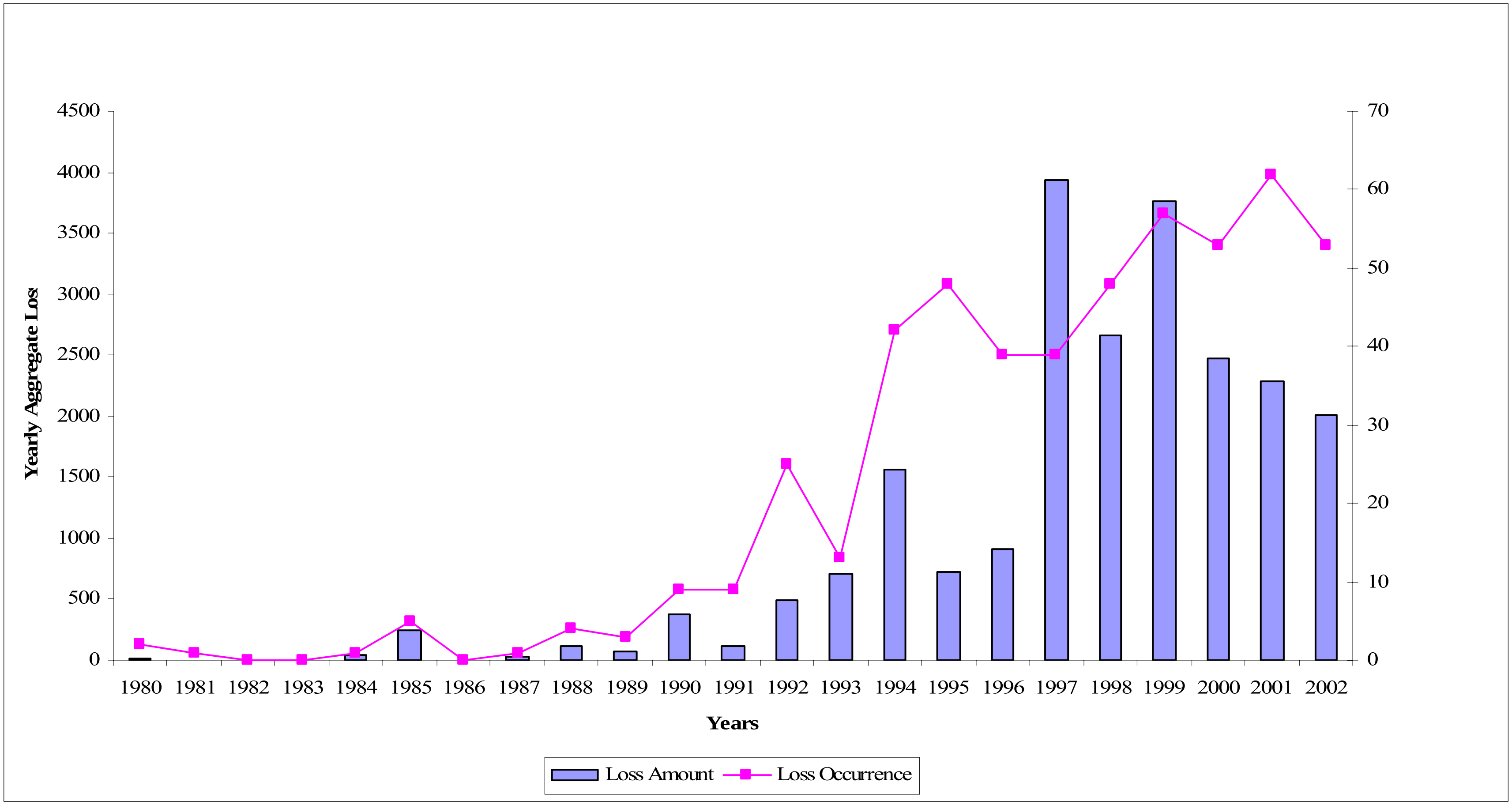


Table 4. 1 --US Banks - Total Loss Amount by Business Lines & Event Types

	Internal Fraud	External Fraud	Employment Practices & Workplace Safety	Clients, Products & Business Practices	Damage to Physical Assets	Business Disruption & System Failures	Execution, Delivery & Process Management	Total
Corporate Finance	1,426	0	8	1,214	0	0	4	2,652
Trading & Sales	6,670	0	5	7,232	0	363	223	14,494
Retail Banking	3,623	1,830	292	13,409	22	3	990	20,169
Commercial Banking	3,605	2,843	327	3,491	213	128	42	10,649
Payment & Settlement	61	8	0	304	89	8	4	474
Agency Services	123	758	3	1,296	0	0	362	2,542
Asset management	2,046	204	111	3,249	0	0	532	6,143
Retail Brokerage	1,072	52	214	7,383	0	16	54	8,791
Total	18,626	5,695	961	37,579	324	519	2,212	65,915

Table 4. 2 --US Banks - Loss Occurrences by Business Lines & Event Types

	Internal Fraud	External Fraud	Employment Practices & Workplace Safety	Clients, Products & Business Practices	Damage to Physical Assets	Business Disruption & System Failures	Execution, Delivery & Process Management	Total
Corporate Finance	12	0	1	62	0	0	1	76
Trading & Sales	48	0	2	60	0	1	8	119
Retail Banking	272	191	20	271	3	1	51	809
Commercial Banking	74	127	14	101	3	2	8	329
Payment & Settlement	6	2	0	13	1	1	1	24
Agency Services	13	3	1	44	0	0	4	65
Asset management	40	9	3	82	0	0	5	139
Retail Brokerage	69	12	33	293	0	3	18	428
Total	534	344	74	926	7	8	96	1989

Table 4. 3 -- US Insurers - Loss Occurrences by Business Lines & Event Types

	Internal Fraud	External Fraud	Employment Practices & Workplace Safety	Clients, Products & Business Practices	Damage to Physical Assets	Business Disruption & System Failures	Execution, Delivery & Process Management	Total
Total Loss Amount	1,616	411	573	19,214	208	341	648	23,011
Total Loss Occurrence	74	21	20	344	1	1	68	529

Table 5. 1 --US Banks & Insurers –Observed Loss Severity Distribution Parameters by Business Units/Event Types and Random Truncation Distributional Assumptions

Business Units	Event Types	Tail beta			Scale sigma		Location mu		LogLikelihood			AIC			# Loss	Max
		Constant	Logistic	Normal	Logistic	Normal	Logistic	Normal	Constant	Logistic	Normal	Constant	Logistic	Normal		Loss
US Banks																
All Business Units	All Event Types	1.826	0.750	0.886	0.934	2.177	4.481	4.481	-3187	-3107	-3116	6376	6220	6235	1989	2243
		(0.052)	(0.086)	(0.040)	(0.104)	(0.108)	(0.417)	-								
	CPBP	2.031	0.848	0.935	0.890	1.835	3.807	3.807	-1582	-1518	-1522	3166	3042	3048	926	2243
		(0.091)	(0.111)	(0.050)	(0.082)	(0.102)	(0.513)	-								
	Internal Fraud - EPWS	1.648	0.778	0.908	1.054	2.429	4.581	4.581	-936	-919	-921	1873	1845	1848	608	1899
		(0.083)	(0.185)	(0.080)	(0.257)	(0.266)	(1.040)									
	Other Event Types	1.560	0.352	0.805	0.432	2.621	5.636	5.636	-657	-645	-650	1316	1298	1304	455	535.8
		(0.085)	(0.194)	(0.118)	(0.287)	(0.446)	(0.357)	-								
Investment Banking	All Event Types	2.550	1.199	1.232	0.887	1.611	3.105	3.105	-378	-356	-356	757	717	716	195	1899
		(0.276)	(0.288)	(0.112)	(0.116)	(0.165)	(1.018)	-								
	CPBP	2.535	1.041	1.099	0.763	1.464	3.204	3.204	-235	-217	-218	473	440	439	122	1825
		(0.369)	(0.277)	(0.126)	(0.106)	(0.160)	(0.944)	-								
Banking	All Event Types	1.755	0.665	0.838	0.841	2.176	4.682	4.682	-1917	-1869	-1875	3836	3743	3755	1227	2000
		(0.062)	(0.104)	(0.052)	(0.140)	(0.144)	(0.444)	-								
	CPBP	2.079	0.978	1.018	0.887	1.648	3.128	3.128	-743	-710	-710	1488	1425	1425	429	2000
		(0.140)	(0.171)	(0.069)	(0.088)	(0.128)	(0.750)	-								
Other Business Lines	All Event Types	1.733	0.674	0.840	0.848	2.146	4.455	4.455	-879	-857	-860	1759	1720	1724	567	2243
		(0.091)	(0.136)	(0.075)	(0.175)	(0.210)	(0.666)	-								
	CPBP	1.813	0.634	0.831	0.757	2.037	4.468	4.468	-598	-579	-582	1198	1164	1168	375	2243
		(0.121)	(0.134)	(0.088)	(0.159)	(0.219)	(0.617)	-								
US Insurers																
	All Event Types	2.184	0.479	0.896	0.535	2.237	5.471	5.471	-942	-900	-912	1887	1806	1828	529	2272
		(0.129)	(0.084)	(0.091)	(0.098)	(0.200)	(0.294)	-								
	CPBP	2.540	0.598	0.857	0.591	1.743	4.862	4.862	-665	-609	-616	1331	1225	1236	344	2272
		(0.220)	(0.101)	(0.079)	(0.088)	(0.118)	(0.331)	-								
	Other Event Types	1.522	0.127	1.064	0.138	4.639	6.293	6.293	-263	-259	-262	527	524	529	185	420
		(0.119)	-	(0.426)	-	(3.893)	-	-								

Investment Banking includes two business lines: Corporate Finance and Trading and Sales. Banking includes: Retail Banking, Commercial Banking, Payment & Settlement and Agency Services. Other business lines include Asset Management and Retail brokerage. EPWS: Employment Practices and Workplace Safety

Table 5. 2 --US Banks–Observed Loss Severity Distribution Parameters by Business Lines/Event Types and Random Truncation Distributional Assumptions

Business Lines	Event Types	Tail beta			Scale sigma		Location mu		LogLikelihood			AIC			# Loss	Max
		Constant	Logistic	Normal	Logistic	Normal	Logistic	Normal	Constant	Logistic	Normal	Constant	Logistic	Normal		Loss
Corporate Finance	All Event Types	2.106	0.930	0.961	0.644	1.191	2.507	2.507	-133	-122	-122	267	250	248	76	990
		(0.378)	(0.249)	(0.130)	(0.125)	(0.166)	(0.857)	-								
	CPBP	1.994	0.656	0.722	0.550	1.156	3.054	3.054	-105	-94	-94	212	194	192	62	299
		(0.425)	(0.237)	(0.127)	(0.127)	(0.164)	(0.802)	-								
Trading & Sales	All Event Types	2.834	0.982	1.084	0.914	1.930	4.813	4.813	-243	-227	-227	488	461	459	119	1899
		(0.407)	(0.374)	(0.154)	(0.243)	(0.232)	(1.213)	-								
	CPBP	3.092	1.040	1.149	0.760	1.580	4.048	4.048	-128	-114	-115	257	235	234	60	1825
		(0.716)	(0.422)	(0.196)	(0.165)	(0.206)	(1.123)	-								
Retail Banking	All Event Types	1.592	0.755	0.866	1.046	2.339	4.253	4.253	-1185	-1166	-1169	2372	2339	2342	809	2000
		(0.066)	(0.149)	(0.066)	(0.210)	(0.230)	(0.910)	-								
	CPBP	2.034	1.005	1.056	1.055	1.985	3.459	3.459	-463	-450	-450	929	906	905	271	2000
		(0.162)	(0.264)	(0.099)	(0.165)	(0.234)	(1.388)	-								
Commercial Banking	All Event Types	2.103	0.540	0.788	0.579	1.779	4.741	4.741	-574	-539	-544	1149	1084	1092	329	766
		(0.167)	(0.144)	(0.086)	(0.150)	(0.158)	(0.419)	-								
	CPBP	2.251	0.810	0.860	0.672	1.337	3.455	3.455	-183	-167	-166	368	339	337	101	415
		(0.365)	(0.305)	(0.121)	(0.140)	(0.161)	(0.985)	-								
Payment & Settlement	All Event Types	-	-	-	-	-	-	-							24	209
Agency Services	All Event Types	2.073	0.962	1.004	0.893	1.671	3.285	3.285	-112	-107	-107	227	221	219	65	536
		(0.358)	(0.536)	(0.182)	(0.253)	(0.340)	(2.292)	-								
Asset Management	All Event Types	2.248	0.782	0.918	0.829	1.911	4.490	4.490	-252	-239	-240	505	485	484	139	967
		(0.274)	(0.349)	(0.138)	(0.310)	(0.258)	(1.220)	-								
	CPBP	2.291	0.670	0.850	0.699	1.799	4.655	4.655	-150	-141	-142				82	440
		(0.380)	(0.424)	(0.173)	(0.394)	(0.300)	(1.234)	-								
Retail Brokerage	All Event Types	1.565	0.573	0.778	0.734	2.110	4.365	4.365	-620	-605	-608	1242	1217	1221	428	2243
		(0.092)	(0.107)	(0.085)	(0.150)	(0.257)	(0.584)	-								
	CPBP	1.679	0.590	0.805	0.723	2.063	4.369	4.369	-445	-432	-435	892	871	874	293	2243
		(0.123)	(0.119)	(0.099)	(0.151)	(0.270)	(0.646)	-								

Figure 5.1 - US Banks Underlying Loss Severity Distribution Parameter by Business Units and Random Truncation Point Distributional Assumption

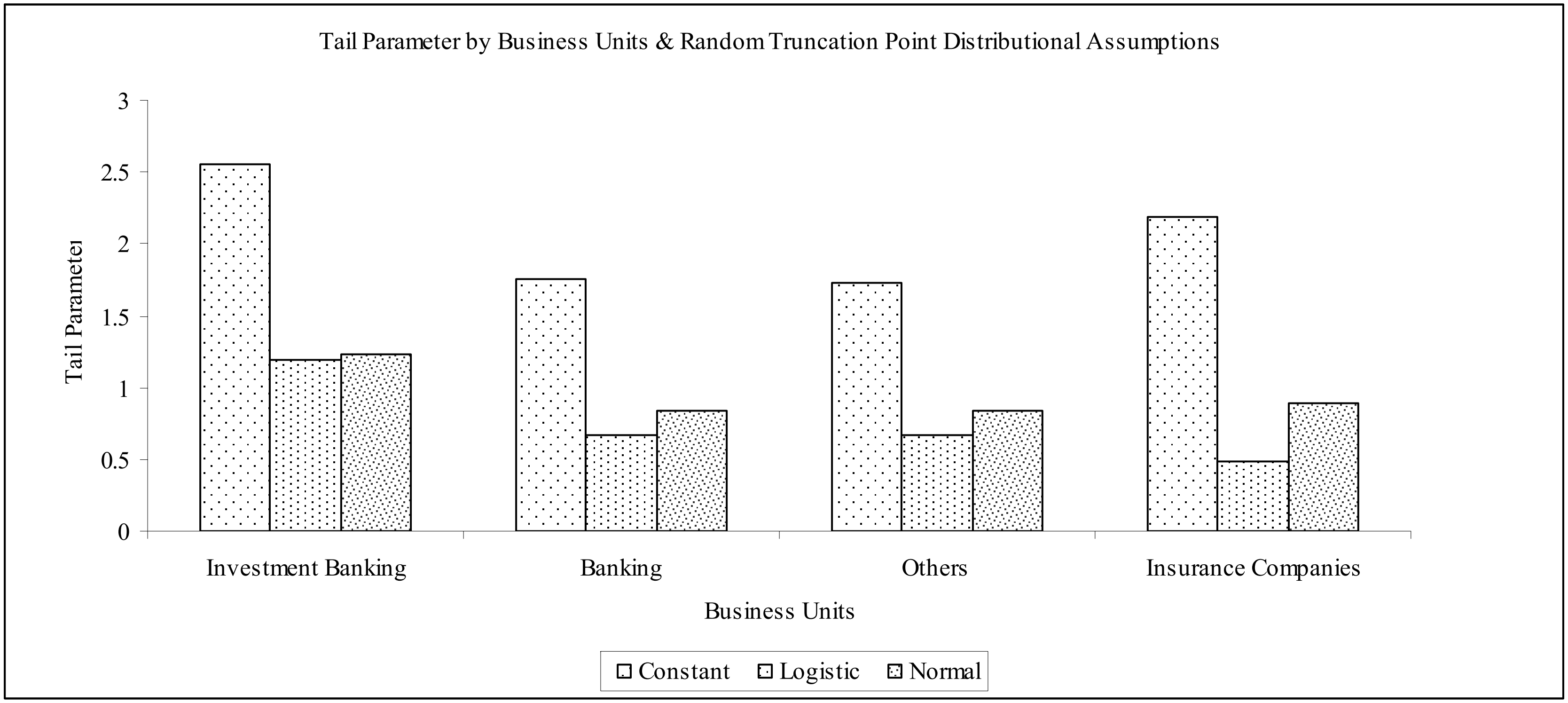


Figure 5.2 - US Banks Underlying Loss Severity Distribution Parameter by Business Lines and Random Truncation Point Distributional Assumption

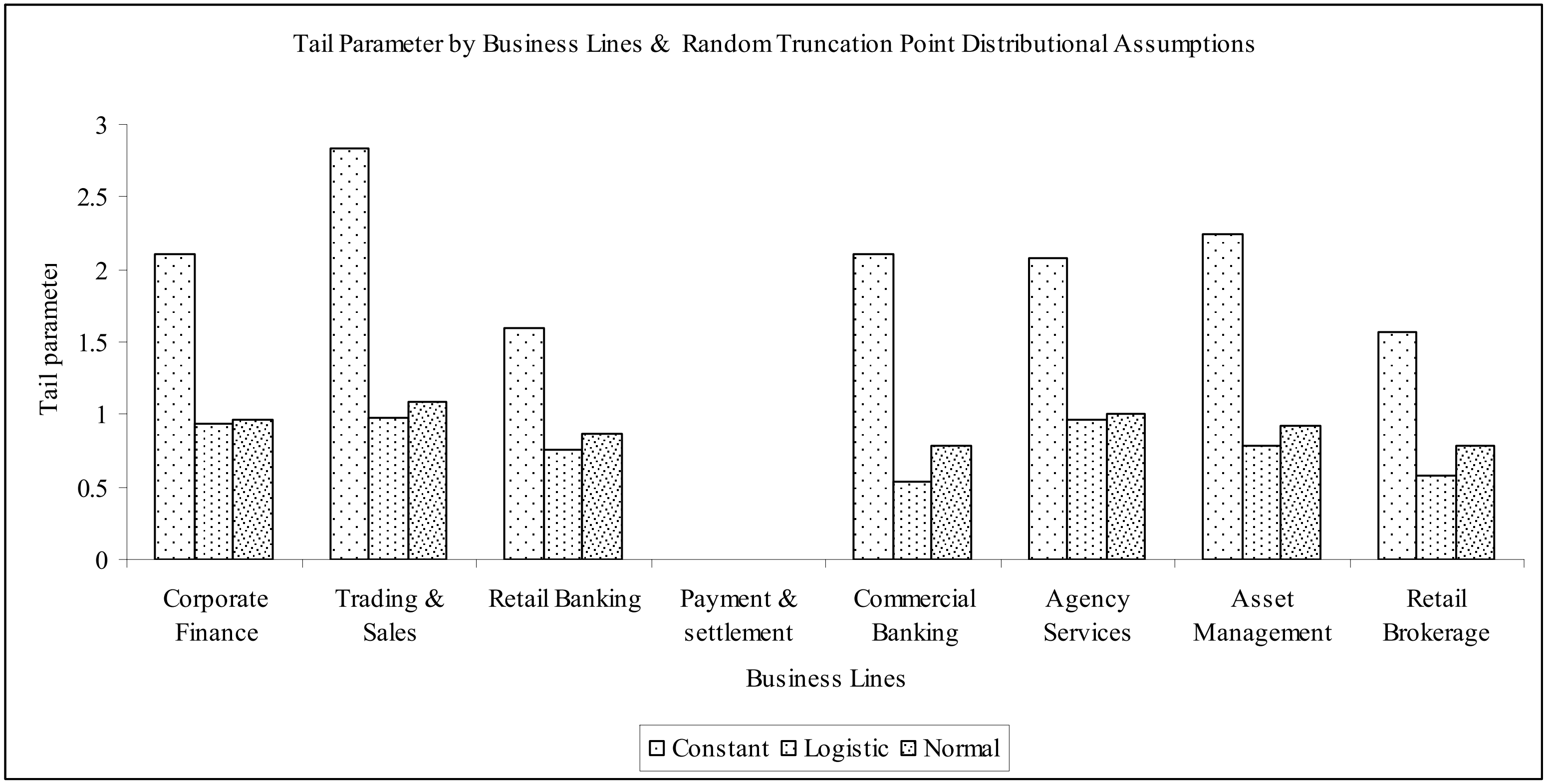


Figure 5.3 --US Banks - Quantile-Quantile Plot All Business Lines All Event Types

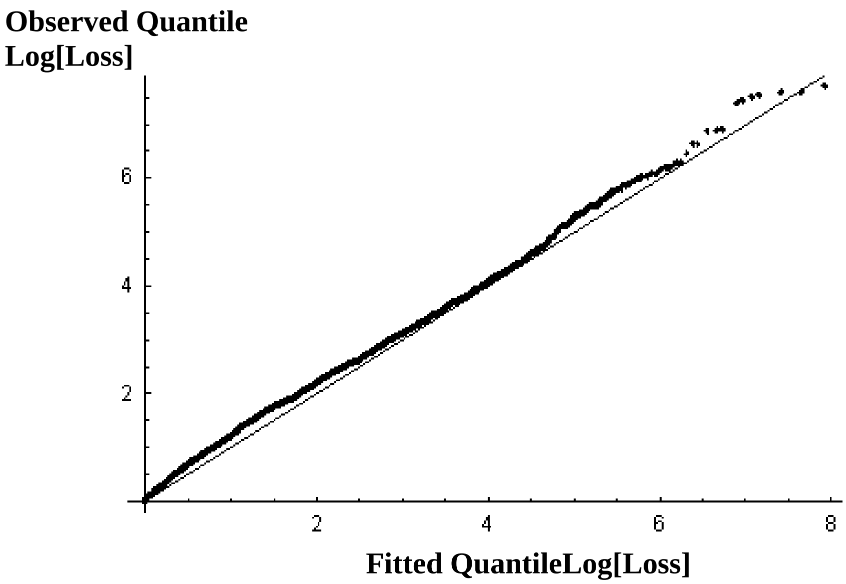


Figure 5.4 --US Banks - Observed Severity Distribution and Underlying Severity Distribution. All Business Lines All Event Types.

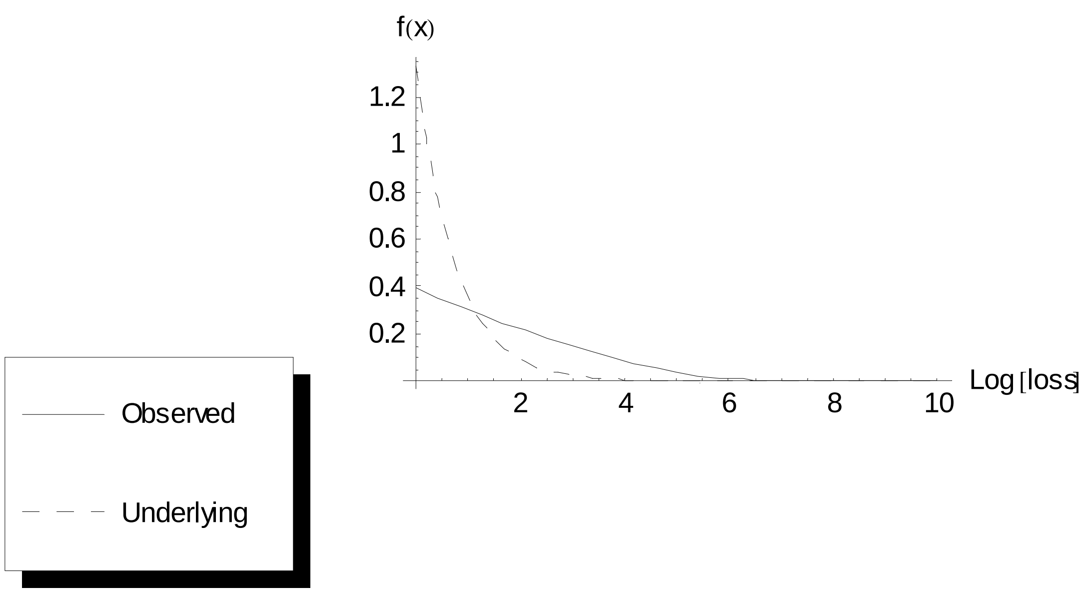


Figure 5.5 --US Banks - QQ Plot CPBP

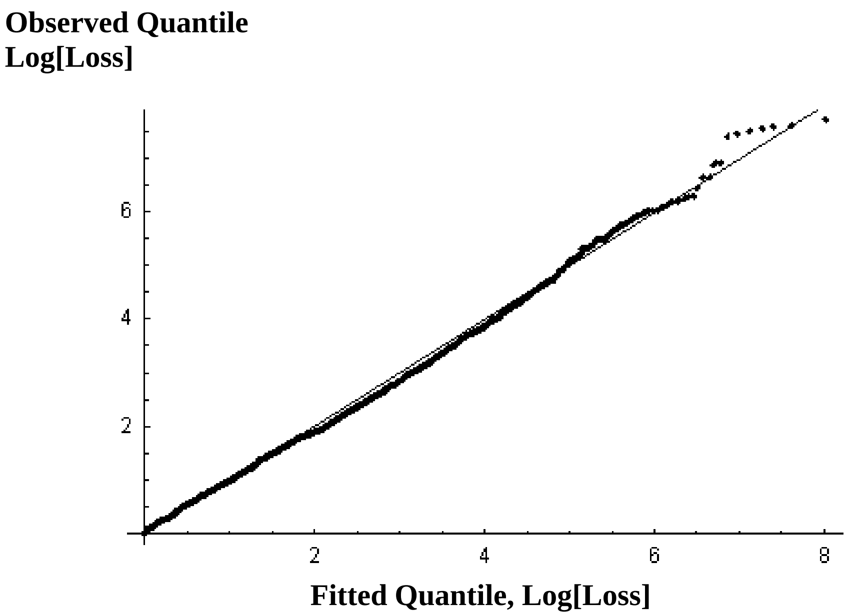


Figure 5.6 --US Banks - Observed Severity Distribution and Underlying Severity Distribution. CPBP.

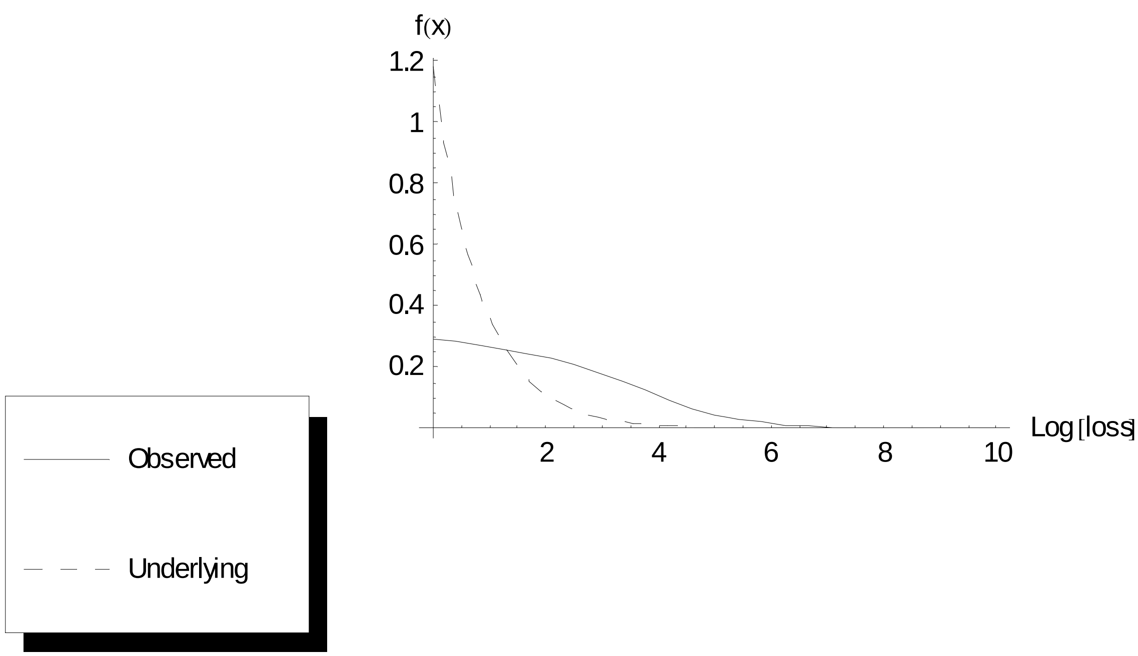
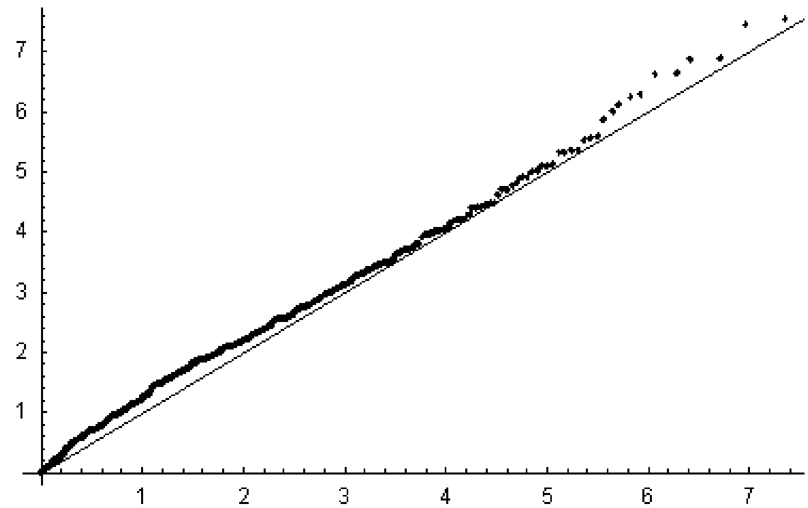


Figure 5.7 US Banks - QQ Plot Internal Fraud-EPWS

Observed Quantile
Log[Loss]



Fitted Quantile, Log[Loss]

Figure 5.8 US Banks - Observed Severity Distribution and Underlying Severity Distribution. Internal Fraud - EPWS.

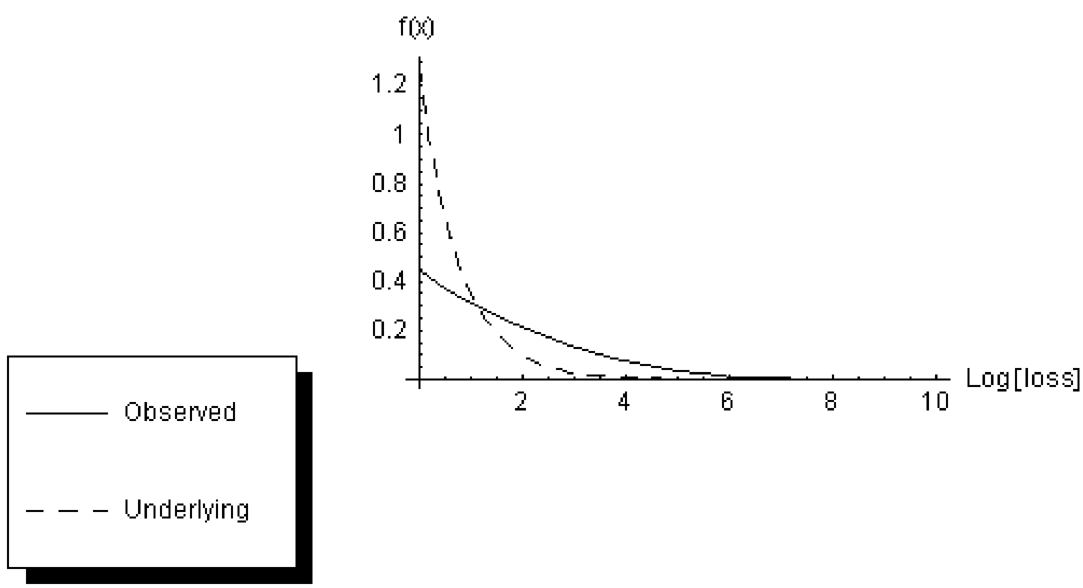


Figure 5.9 US Banks- Observed Severity Distribution and Underlying Severity Distribution. Retail Banking.

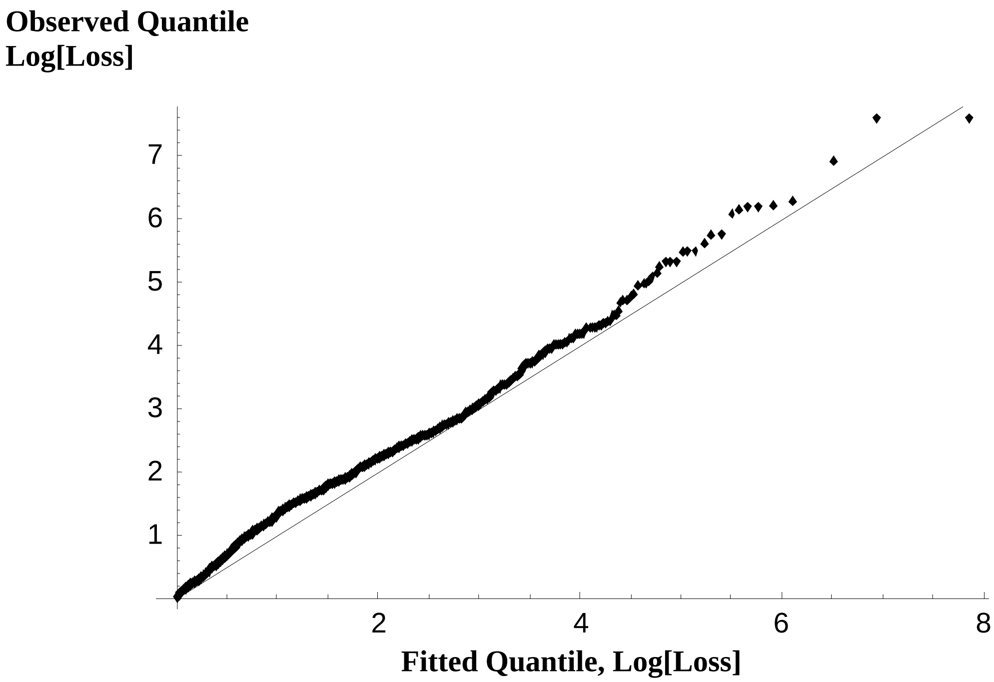


Figure 5.10 US Banks- Observed Severity Distribution and Underlying Severity Distribution Retail Banking.

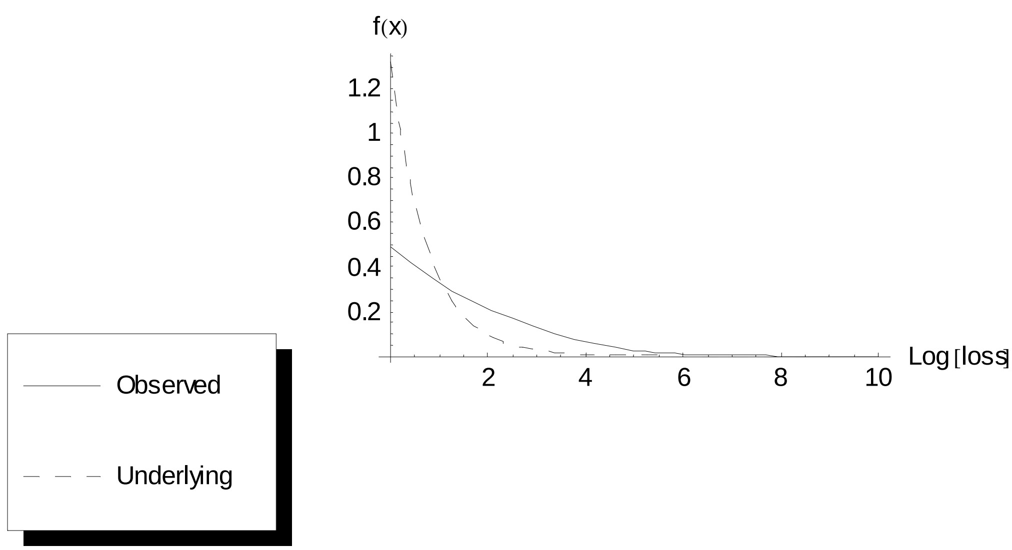


Figure 5.11 US Banks- Observed Severity Distribution and Underlying Severity Distribution. Retail Brokerage

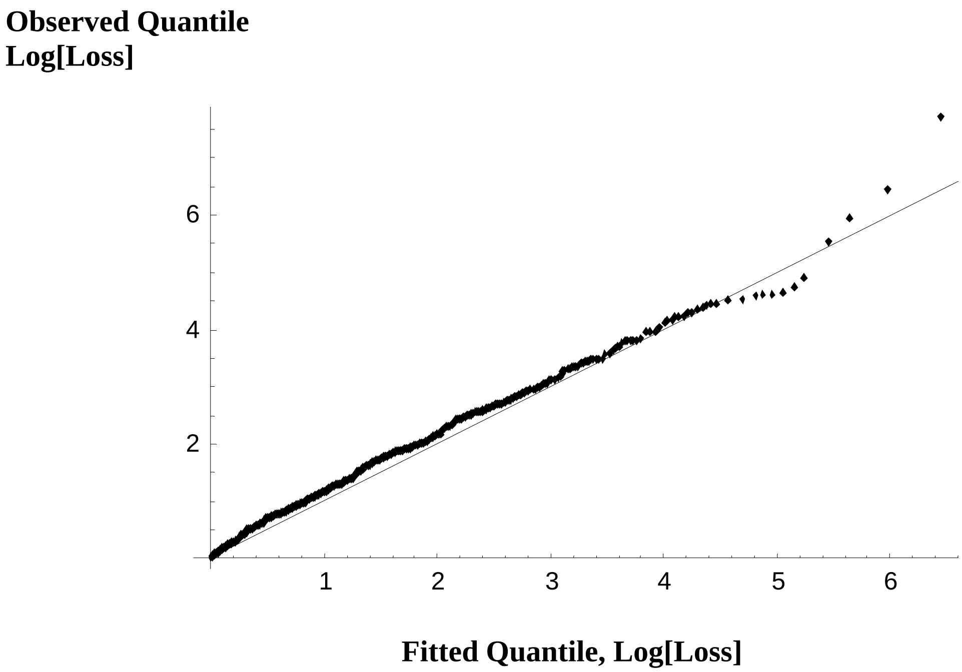


Figure 5.12 US Banks- Observed Severity Distribution and Underlying Severity Distribution Retail Brokerage.

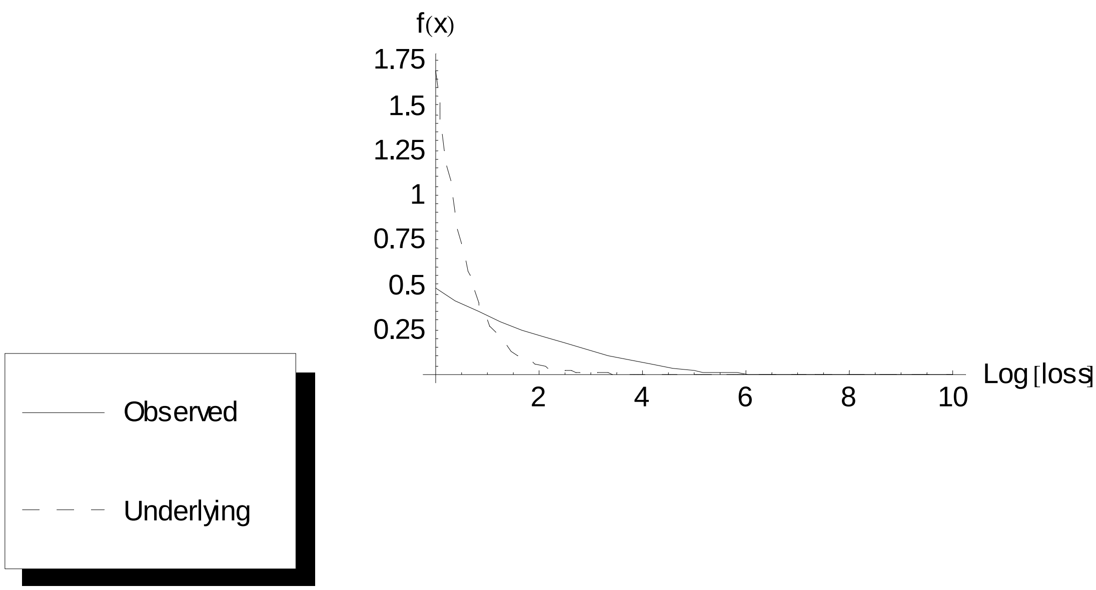


Table 5. 3 --US Banks & Insurers: Underlying Loss Severity by Exposure (Revenue) All Business Lines and Event Types

	Bellow Median Revenue				Above Median Revenue			
	Number of Losses	Severity Parameter	Maximum Loss($M)	99.95% Percentile of the Underlying Severity ($M)	Number of Losses	Severity Parameter	Maximum Loss ($M)	99.95% Percentile of the Underlying Severity ($M)
US Banks	582	0.759	2,243	320	446	0.6794	1,824	175
US Insurers	125	0.6108	2,272	104	125	0.512	1,852	49

Table 5. 4 --US Banks & Insurers: Loss Severity by Exposure (Revenue) All Business Lines and Event Types

	US Banks			US Insurers		
	Small Size	Medium Size	Large Size	Small Size	Medium Size	Large Size
Number of Losses	582	243	64	144	88	11
Severity Parameter	0.878	0.497	0.322	0.571	0.598	
Maximum Loss ($M)	2,243	631	363	2,272	1,852	198
99.99% Percentile of the Underlying Loss Severity ($M)	790	44	12	76	94	

Table 5. 5 – Observed Loss Severity Distribution Parameters Industry-Wide Organization vs Specific Firm

	PML ($M)	Median ($M)	Prob[Truncation Point <= Specific PML]	Severity Distribution Parameter		
				Tail beta	Location mu	Scale sigma
Industry-Wide Organization	2,243	88	0.931	0.750	4.481	0.934
Specific Firm	1,000	88	0.99	0.472	4.481	0.528

Figure 5.13 – Random Truncation Point Distribution CDF
Industry-Wide Organization vs Specific Firm

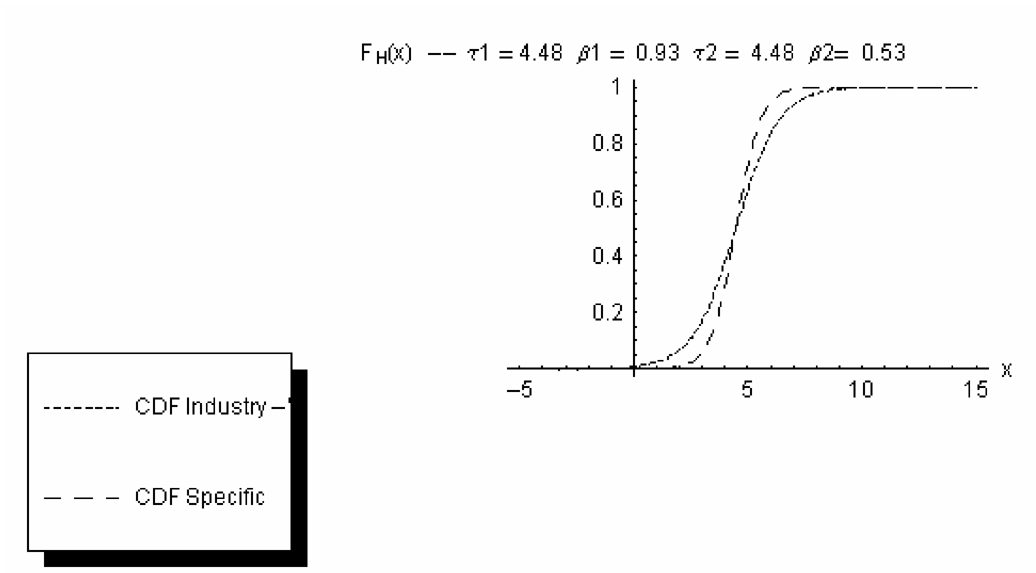


Figure 5.14 – Random Truncation Point Distribution PDF
Industry-Wide Organization vs Specific Firm

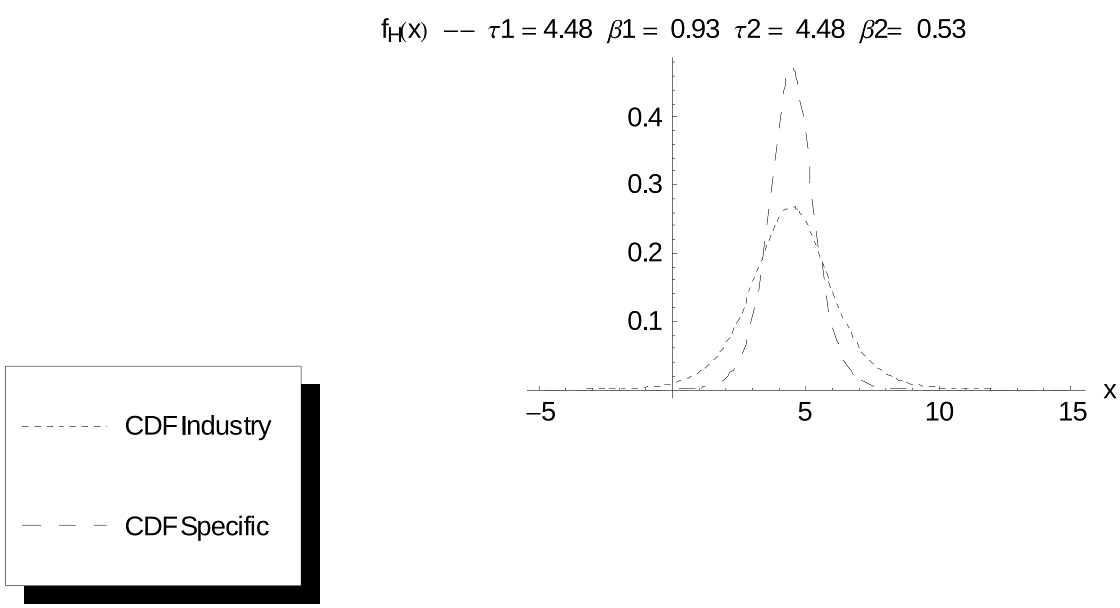


Table 5. 6 US Banks and Insurers: Sample Rank & Linear Correlation by Business Unit/Event types

Class	Business Units	Event Types	Number of Losses	Amount ($M)	Severity Parameter	Weight	Kendall Rank Sample Correlation			Adjusted Sample Linear Correlation			# Yearly Claims
1	All Business Units	CPBP	926	37,579	0.848	0.466	1	0.33		1	0.1		50
	All Business Units	Other Event Types	1063	28,336	0.659	0.534	0.33	1		0.1	1		
2	All Business Units	CPBP	926	37,579	0.848	0.466	1	0.33	0.60	1	0.1	0.1	50
	All Business Units	Internal Fraud	608	19,587	0.778	0.306	0.33	1	0.73	0.1	1	0.1	
	All Business Units	Other Event Types	455	8,749	0.352	0.229	0.60	0.73	1	0.1	0.1	1	
3	Insurance	CPBP	344	19,214	0.598	0.650	1	0.20		1	0.1		50
	Insurance	Other Event Types	185	3,797	0.127	0.350	0.20	1		0.1	1		

* Adjusted Sample Linear Correlation Matrices are Positive Definite

Table 5. 7 --US Banks and Insurers
Capital Charge's Sensitivity to the Truncation Point Distributional Assumption All Business Lines and All Event Types.

	Constant		Logistic		Normal	
	Severity Tail b	VaR ($M)	Severity Tail b	VaR ($M)	Severity Tail b	VaR ($M)
US Banks	1.826	>100,000	0.750	2,089	0.886	8,220
US Insurers	2.184	>100,000	0.479	180	0.896	8,633

Assuming yearly number of loss occurrences exceeding $1M equal to 25

Table 5. 8 – US Banks and Insurers
Capital Charge ($M) Assuming Various Yearly Number of Loss Occurrences

	Severity Tail b	Yearly Number of Loss Occurrences in Excess of $1M				
		5	10	25	50	70
US Banks	0.750	599	1,041	2,106	3,562	4,596
US Insurers	0.479	70	104	179	278	350

Table 5. 9 -- US Banks and Insurers
Capital Charge ($M) for Three Business Line and Event Type Combinations

	All Business Lines & Event Types	CPBP - Other Event Types		CPBP- Internal Fraud- Other Event Types	
	VaR	VaR	VaR Increase %	VaR	VaR Increase %
US Banks	3,460	5,653	63.4	6,324	82.8
US Insurers	285	611	114.6		

Assuming a yearly number of loss occurrences exceeding $1M equal to 50

Figure 5.15 --US Banks
CPBP Capital Charge Distribution ($M)

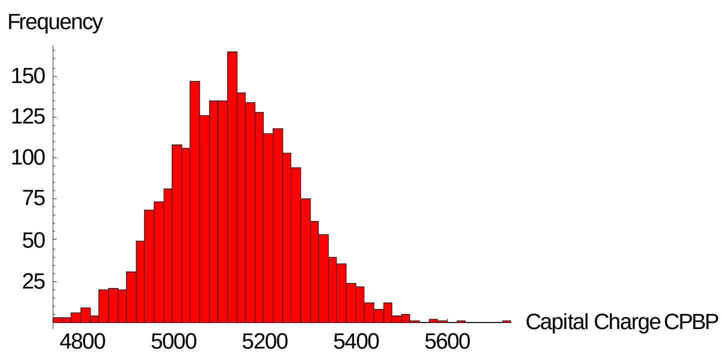


Figure 5.16 --US Banks
Internal Fraud Capital Charge Distribution ($M)

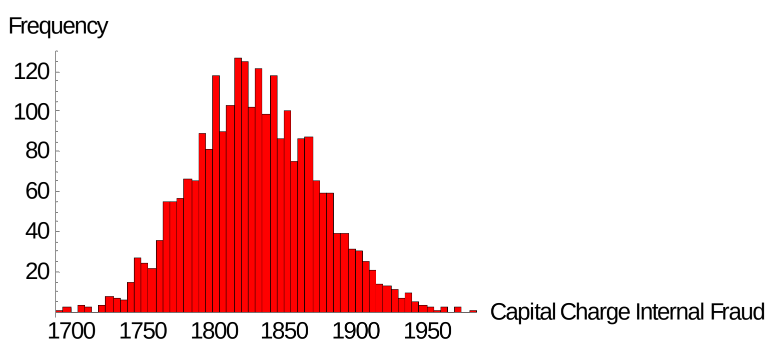


Figure 5.17 --US Banks
Other Event Types Capital Charge Distribution ($M)

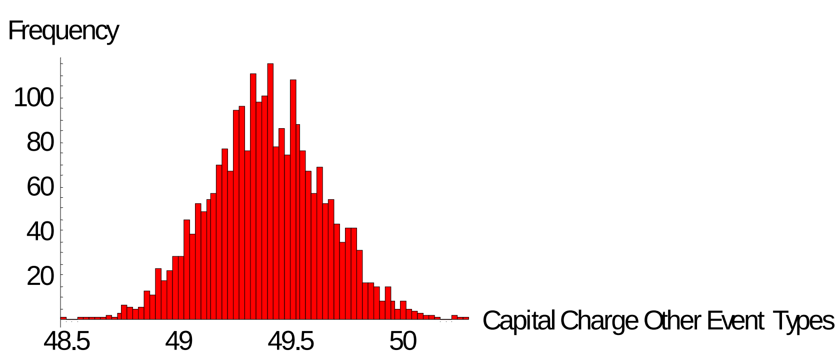


Figure 5.18 --US Banks
Aggregated Capital Charge Distribution ($M) Using Cauchy Copula

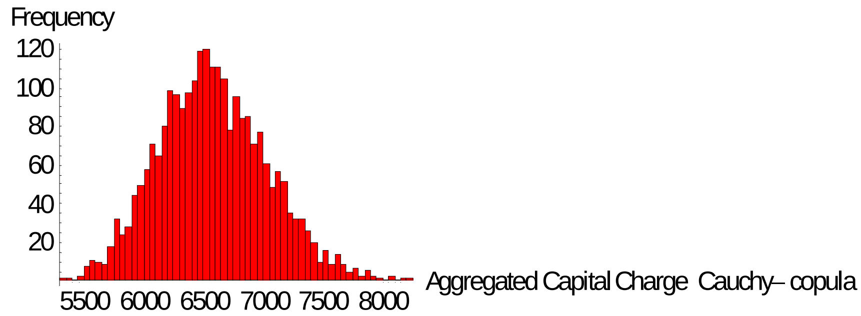


Figure 5.19 -- US Banks
Aggregated Capital Charge ($M) for the Student's t -Copula

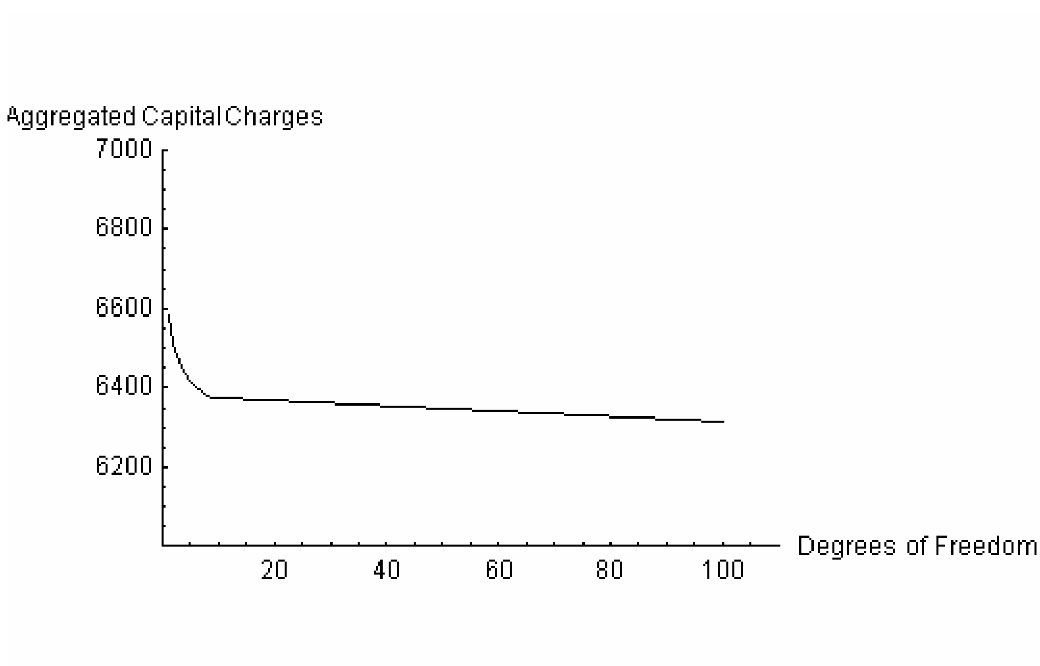


Table 5. 10 -- US Banks
Capital Charges ($M) and Capital Savings ($M) by Types of Copulas.

		Industry-Wide Bank			Specific Bank		
		VaR	Capital Saving		VaR	Capital Saving	
			Amount	%		Amount	%
	Comonotonic	7,015	0	0.00	481	0	0.00
Degrees of freedom	1 - Cauchy	6,582	433	6.58	448	33	7.26
	2	6,509	506	7.77	446	35	7.81
	3	6,468	547	8.45	444	36	8.17
	4	6,435	580	9.01	443	37	8.41
	5	6,416	599	9.33	442	38	8.68
	6	6,400	615	9.61	441	39	8.88
	7	6,391	624	9.77	441	40	9.00
	8	6,381	634	9.93	440	40	9.17
	9	6,374	641	10.06	440	41	9.26
	10	6,373	642	10.07	440	41	9.30
	Infinite- Normal	6,317	698	11.05	437	44	10.02
	Independent	6,290	724	11.52	433	47	10.92

Banks' activities are classified into three event types: CPBP – Internal Fraud & EPWS and Other Event Types.

Table 5. 11 -- US Insurers
Capital Charges ($M) and Capital Saving ($M) by Types of Copulas.

		Industry-Wide Insurer			Specific Insurer		
		VaR	Capital Saving		VaR	Capital Saving	
			Amount	%		Amount	%
	Comonotonic	625	0	0.00	125	0	0.00
Degrees of freedom	1 - Cauchy	612	14	2.24	119	6	4.91
	2	611	15	2.43	118	7	5.92
	3	611	15	2.42	117	8	6.84
	4	610	15	2.50	117	9	7.53
	5	611	15	2.41	116	9	8.04
	6	611	15	2.40	116	10	8.38
	7	611	14	2.37	115	10	8.65
	8	611	15	2.38	115	10	8.84
	9	610	15	2.44	115	10	9.01
	10	610	15	2.44	115	10	9.14
	Infinite- Normal	608	18	2.91	113	12	10.43
	Independent	606	19	3.16	112	13	11.78

Insurers' activities are classified into two event types: CPBP and Other Event Types

Table 5. 12 --US Bank
Descriptive Statistics of the Capital Charge ($M)

		Mean	St-Dev	Median	95-Perc	99-Perc	Min Confidence Interval	Max Confidence Interval	Skewness	Excess Kurtosis
Degrees of freedom	1 - Cauchy	6,582	455	6,560	7,353	7,717	6,552	6,612	0.26	-0.07
	2	6,509	456	6,485	7,284	7,650	6,479	6,539	0.29	0.04
	3	6,468	457	6,436	7,262	7,610	6,438	6,498	0.30	0.09
	4	6,435	454	6,404	7,205	7,584	6,405	6,465	0.30	0.13
	5	6,416	456	6,392	7,204	7,572	6,386	6,446	0.32	0.18
	6	6,400	451	6,376	7,160	7,575	6,370	6,430	0.30	0.23
	7	6,391	451	6,371	7,149	7,576	6,361	6,420	0.31	0.18
	8	6,381	448	6,358	7,142	7,528	6,351	6,410	0.32	0.33
	9	6,374	448	6,354	7,140	7,536	6,344	6,403	0.30	0.15
	10	6,373	453	6,349	7,142	7,516	6,343	6,403	0.34	0.24
	Infinite-Normal	6,317	436	6,298	7,083	7,384	6,288	6,345	0.29	0.22
	Independent	6,290	436	6,279	7,037	7,342	6,262	6,319	0.27	0.16

Table 5. 13 -- Mixing Weights by types of copulas

	Comonotonic	4%
Degrees of freedom	1 - Cauchy	8%
	2	8%
	3	8%
	4	8%
	5	8%
	6	8%
	7	8%
	8	8%
	9	8%
	10	8%
	Infinite- Normal	8%
	Independent	8%

Table 5. 14 --US Industry-Wide Bank & Specific Bank Descriptive Statistics of the Mixing Weighted Capital Charges ($M)

	Industry-Wide Bank	Specific Bank
Minimum	5,892	410
Maximum	7,007	481
Mean	6,433	443
Median	6,431	443
1% Percentile	6,071	422
5% Percentile	6,234	431
95% Percentile	6,639	455
99.9% Percentile	6,830	467
Confidence Interval 1	6,433	443
Confidence Interval 2	6,434	443
Skewness	0.10	0.08
Excess Kurtosis	0.00	0.00

Figure 5-20 US Industry-wide Bank
Base Scenario Histogram of the Mixing Weighted Capital Charges

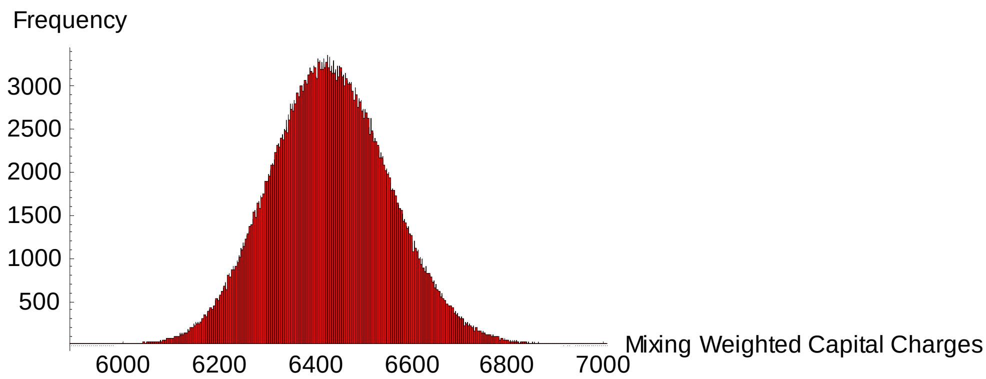


Figure 5-21 US Specific Bank
Base Scenario Histogram of the Mixing Weighted Capital Charges

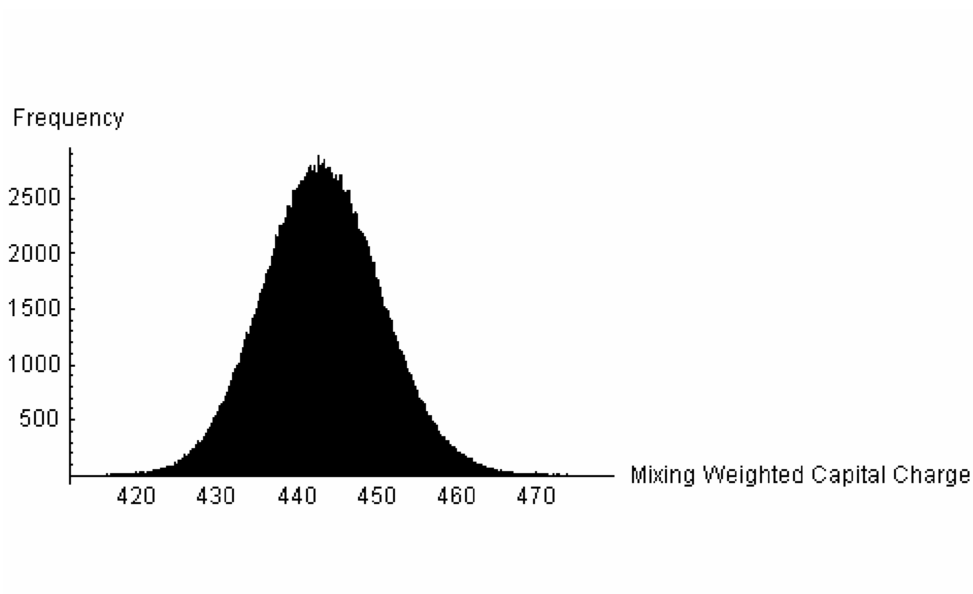


Figure 6.1

Coefficient of Upper Tail Dependence for Student's t-Copula

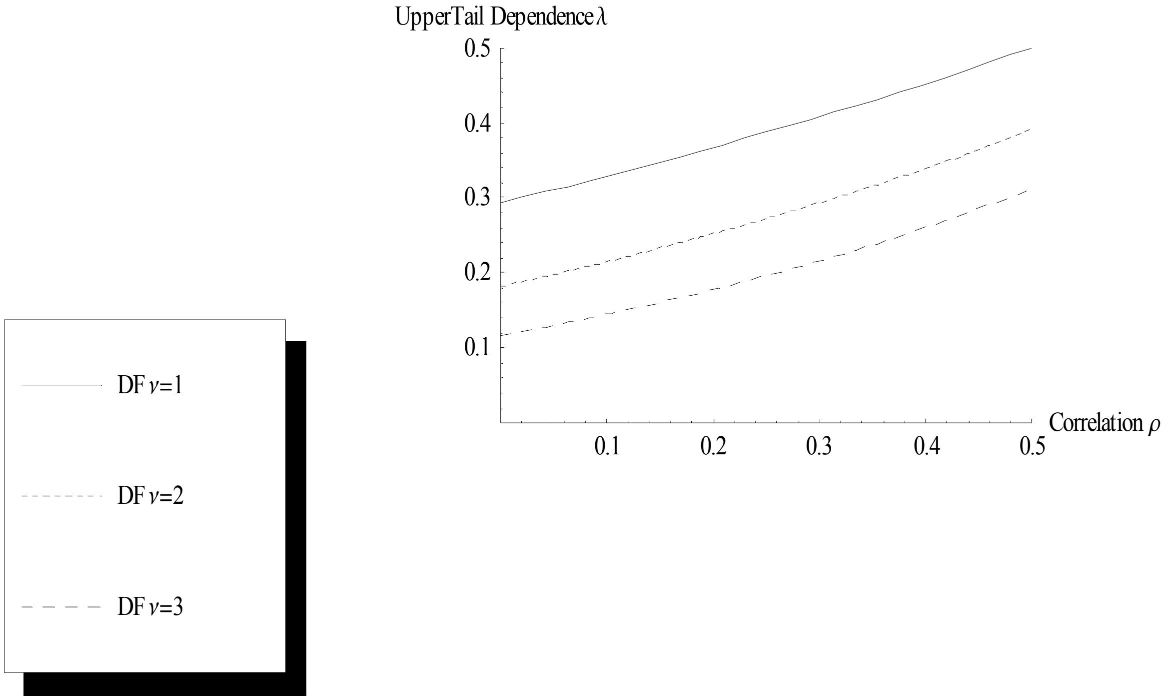


Table 5. 15 -- Capital Charges by Common Shock Intensity

Industry	Business Units	Event Types	Log Loss Severity Tail	Maximal Moment of Loss Severity	Yearly Number of Losses	Common Shock Intensity[52] 1 loss per year									Common Shock Intensity 2 losses per year								
						Correlation						Capital Charges ($M)			Correlation						Capital Charges ($M)		
						Loss Frequency			Aggregate Losses			Min	Mean	Max	Loss Frequency			Aggregate Losses			Min	Mean	Max
Banks												72	73	73							74	75	76
	All	CPBP	0.393	2	9	1	0.14	0.15	1	0.10	0.11				1	0.27	0.30	1	0.20	0.23			
	All	Internal Fraud	0.244	4	6	0.14	1	0.18	0.10	1	0.17				0.27	1	0.37	0.20	1	0.34			
	All	Others	0.080	12	5	0.15	0.18	1	0.11	0.17	1				0.30	0.37	1	0.23	0.34	1			
Insurers												59	59	60							60	60	61
	-	CPBP	0.342	2	13	1	0.10		1	0.09					1	0.21		1	0.18				
	-	Others	0.046	21	7	0.10	1		0.09	1					0.21	1		0.18	1				

[52] Following Powojowski et al. (2002) model, the dependency between loss event type frequency is generated by a single enterprise-wide source of loss that follows a Poisson distribution with intensity λ

APPENDIX I

Table A. 1. Risk Weights by Category of On-Balance-Sheet Asset BCBS (1988)

Category of on-balance-sheet asset	Risk weights
(a) Cash (b) Claims on central governments and central banks denominated in national currency and funded in that currency (c) Other claims on OECD central governments3 and central banks (d) Claims collateralized by cash of OECD central-government securities3 or guaranteed by OECD central governments	0%
(a) Claims on domestic public-sector entities, excluding central government, and loans guaranteed4 by such entities	0, 10, 20 or 50% At national discretion
(a) Claims on multilateral development banks (IBRD, IADB, AsDB, AfDB, EIB[53] and claims guaranteed by, or collateralized by securities issued by such bank (b) Claims on banks incorporated in the OECD and loans guaranteed4 by OECD incorporated banks (c) Claims on banks incorporated in countries outside the OECD with a residual maturity of up to one year and loans with a residual maturity of up to one year guaranteed by banks incorporated in countries outside the OECD (d) Claims on non-domestic OECD public-sector entities, excluding central government, and loans guaranteed4 by such entities (e) Cash items in process of collection	20%
(a) Loans fully secured by mortgage on residential property that is or will be occupied by the borrower or that is rented	50%
(a) Claims on the private sector (b) Claims on banks incorporated outside the OECD with a residual maturity of over one year (c) Claims on central governments outside the OECD (unless denominated in national currency - and funded in that currency - (d) Claims on commercial companies owned by the public sector (e) Premises, plant and equipment and other fixed assets (f) Real estate and other investments (including non-consolidated investment participations in other companies) (g) Capital instruments issued by other banks (unless deducted from capital) (h) all other assets	100%

[53] International Bank for Reconstruction and Development (IBRD), Inter-American Development Bank (IADB), Asian Development Bank (AsDB), African Development Bank (AfDB), European Investment Bank (EIB)

Table A. 2. BCBS Business Lines

Business Lines	Beta Factors
Corporate finance	18%
Trading and sales	18%
Retail banking	12%
Commercial banking	15%
Payment and settlement	18%
Agency services	15%
Asset management	12%
Retail brokerage	12%

REFERENCES

Aas, K. (2004). Modeling the dependence structure of financial assets: A survey of four copulas. Preprint, Norwegian Computing Center, Oslo, http://www.Nr.No/files/samba/bff/samba2204.Pdf.

Acerbi, C., Nordio, C., Sirtori, C. (2001). Expected shortfall as a tool for financial risk management. Working paper. Italian association for financial risk management, http://www.Gloriamundi.Org/var/wps.Html.

Allen, L. and Saunders, A. (1993). Forbearance and valuation of deposit insurance as a callable put. *Journal of Banking and Finance, 17*: 629-643.

Alvarez, G. (2002). Operational risk event classification, http://www.Garp.Com/library/articles/operational%20riskeventclassification.Pdf. *GARP Risk Review*.

American Academy of Actuaries. (2002). Comparison of the NAIC life, P&C and Health RBC Formulas. Summary of differences. Prepared by the Academy Joint Risk Based Capital Task Force, http://www.Actuary.Org/pdf/finreport/jrbc_12feb02.Pdf.

Artzner, P., Delbaen, F., Eber, J.M. and Heath, D. (1999). Coherent measures of risk, http://www.Math.Ethz.Ch/~delbaen/ftp/preprints/coherentmf.Pdf. *Mathematical Finance, 9*: 203-228.

Balkeman, A. and De Haan, L. (1974). Residual lifetime at great age. *Ann.Probab, 2*: 792-804.

Barth, J. R., Caprio, G. Jr., and Levine, R. (2003). Bank supervision and regulation: What works best? http://web6.Duc.Auburn.Edu/~barthjr/papers/mexico.Pdf. *Journal of Financial Intermediation, forthcoming.*

Basel Committee on Banking Supervision. (1988). International Convergence of Capital Measurement and Capital Standards, http://www.Bis.Org/publ/bcbs04a.Pdf.

Basel Committee on Banking Supervision. (1996). Amendment to the capital accord to incorporate market risks, http://www.Bis.Org/publ/bcbs24.

Basel Committee on Banking Supervision. (1998a). Operational risk management, http://www.Bis.Org/publ/bcbs42.Pdf.

Basel Committee on Banking Supervision. (1998b). Instruments eligible for inclusion in tier 1 capital, http://www.Bis.Org/press/p981027.Htm#pgtop.

Basel Committee on Banking Supervision. (2001a). The New Basel Capital Accord: An explanatory note, http://www.Bis.Org/publ/bcbsca01.Pdf.

Basel Committee on Banking Supervision. (2001b). Operational risk consultative document. Supporting document to the New Basel Capital Accord, http://www.Bis.Org/publ/bcbsca07.Pdf.

Basel Committee on Banking Supervision. (2001c). Working paper on the regulatory treatment of operational risk, http://www.Bis.Org/publ/bcbs_wp8.Pdf.

Basel Committee on Banking Supervision. (2001d). Operational risk data - QIS, http://www.Bis.Org/bcbs/qisoprisk.Htm#pgtop.

Basel Committee on Banking Supervision. (2002). Operational risk data collection exercise 2002, http://www.Bis.Org/bcbs/oprdata.Htm#pgtop.

Basel Committee on Banking Supervision. (2003a). Operational risk. Transfer across financial sectors, the Joint Forum, http://www.Bis.Org/publ/joint06.Pdf.

Basel Committee on Banking Supervision. (2003b). Consultative document. The New Basel Capital Accord, http://www.Bis.Org/bcbs/cp3full.Pdf.

Basel Committee on Banking Supervision. (2003c). Sound practices for the management and supervision of operational risk, http://www.Bis.Org/publ/bcbs96.Htm.

Basel Committee on Banking Supervision. (2003d). Trends in risk integration and aggregation, http://www.Bis.Org/publ/joint07.Pdf.

Basel Committee on Banking Supervision. (2004). International Convergence of Capital Measurement and Capital Standards. A Revised Framework, http://www.Bis.Org/publ/bcbsca.Htm.

Baud, N., Frachot, A., and Roncalli, T. (2002). Internal data, external data and consortium data for operational risk measurement: How to pool data properly? Preprint, Crédit Lyonnais, Paris, http://gro.Creditlyonnais.Fr/content/wp/oprisk-data-light-version.Pdf.

Baud, N., Frachot, A., and Roncalli, T. (2003). How to avoid over-estimating capital charge for operational risk? Preprint, Crédit Lyonnais, Paris, http://gro.Creditlyonnais.Fr/content/rd/home_ro.Htm.

Becker, G. and Stigler, G. (1974). Law enforcement, malfeasance, and the compensation of enforcers. *Journal of Legal Studies, 3*: 1-18.

Brender, A. (2004). Risk-based capital requirements for life insurers. Presentation, GARP, http://www.Garp.Com/library/papers/latepapers/garp2004/brenderworkshop. Ppt.

Butsic, R. (1994). Solvency measurement for property- liability risk-based capital applications. *Journal of Risk and Insurance, 17*: 438-477.

Carriere, J. (2000). Bivariate survival models for coupled lives. *The Scandinavian Actuarial Journal, 100*(1): 17-32.

Caruana, J. (2003). The New Basel Capital Accord: Why we need it and where we're at. Central Bank articles and speeches. Fifth meeting of the Asian program of the Institute of International Finance, Beijing, http://www.Bis.Org/review/r031202c.Pdf.

Caruana, J. (2004). The New Accord and where we stand today. Central Bank articles and speeches. Annual Washington Conference of the Institute of International Bankers, Washington, http://www.Bis.Org/review/r040305c.Pdf.

Casualty Actuarial Society. (1992). Property-casualty risk-based capital requirement. A conceptual framework, http://www.Casact.Org/pubs/forum/92spforum/92sp211.Pdf.

Cebria, A., Denuit, M., and Lambert, P. (2004). Generalized Pareto fit to the Society of

Actuaries' large claims database, http://library.Soa.Org/library-pdf/naaj0307_2.Pdf. *North American Actuarial Journal, 7*(3): 18-36.

CenterSpace Software. (2004). Numerical Component Librairies for the.NET.

Chavez-Demoulin, V. and Embrechts, P. (2004a). Smooth extremal models in finance and insurance, http://www.Math.Ethz.Ch/~baltes/ftp/smooth.Pdf. *The Journal of Risk and Insurance, 71*(2): 183-199.

Chavez-Demoulin, V. and Embrechts, P. (2004b). Advanced extremal models for operational risk. Preprint, department of mathematics ETH-Zentrum, http://www.Math.Ethz.Ch/~baltes/ftp/opriskevt.Pdf.

Chavez-Demoulin, V., Embrechts, P., and Neslehova, J. (2005). Quantitative models for operational risk: Extremes, dependence and aggregation**,** http://www.Math.Ethz.Ch/~baltes/ftp/manuscript_cen.Pdf. *Journal of Banking and Finance, forthcoming*.

Chernobai, A. and Rachev, S. (2004). Toward effective risk management: Stable modeling of operational risk. Preprint, University of California, Santa Barbara, http://www.Gloriamundi.Org/picsresources/acsr.Pdf.

Clemente, A., and Romano, C. (2003). A copula-extreme value theory approach for modelling operational risk. Preprint University of Rome "La Sapienza", http://www.Gloriamundi.Org/picsresources/adcror.Pdf.

Cline, K. (2004). De-siloing risk. *Banking Strategies, LXXX Number V*.

Colorado State Banking Board. (2005). State Banking Board Rule TC13 pertaining to Title 11 Article 102, Section 104 Colorado Revised Statues, http://www.Dora.State.Co.Us/banking/publicnotices/tc13.Pdf.

Cont, R. and Tankov, P. (2004). *Financial Modelling with Jump Processes*. Chapman & Hall/CRC, London.

Cox, D. R. and Isham, V. (1980). *Point Processes*. Chapman and Hall, Englewood Cliffs.

Crnkovic, C. and Drachman, J. (1996). Quality control. *Risk, 9*: 139-143.

Crouhy, C. and Mark, R. (2000). *Risk management*. McGraw-Hill, New York.

Cruz, M. (2002). *Modeling, measuring and hedging operational risk*. Wiley, Chichester.

Cull, R., Senbet, L., and Sorge, M. (2004). Deposit insurance and bank intermediation in the long run. Preprint Bank for International Settlements (BIS), http://www.Bis.Org/publ/work156.Pdf.

Cummins, D., Harrington, S., Klein, R. (1995). Insolvency experience, risk-based capital, and prompt corrective action in property-liability insurance, http://knowledge.Wharton.Upenn.Edu/papers/364.Pdf. *Journal of Banking & Finance, 19*(3-4): 511-527.

Cummins, J. D. (1988). Risk-based premiums for insurance guaranty funds. *Journal of Finance (September)*: 823-839.

Currie, C. (2004). Basel II operational risk overview of key concerns. IQPC operational risk forum Carlton Crest Hotel, Sydney, 25th March 2004, http://www.Gloriamundi.Org/picsresources/cvc.Pdf.

Davison, A., and Smith, R. (1990). Models for exceedances over high thresholds (with discussion). *Journal of the Royal Statistical Society, B 52*: 393-442.

Deheuvels, P. (1979). Propriétés d'existence et proprietés topologiques des fonctions de dépendance avec applications a la convergence des types pour des lois multivariées. *C. R. Acad. Sci. Paris Sér. A-B 288*(2): A145-A148.

Demirgüç-Kunt, A. and Detragiache, E. (2003). Does deposit insurance increase banking system stability? An empirical investigation. *Journal of Monetary Economics*: 120-145.

Dionne, G. (2003). The foundations of banks' risk regulation: A review of the literature. Preprint, HEC Montréal, CIRPÉE and CREF, http://www.Gloriamundi.Org/picsresources/mgd.Pdf.

Dreyfus, J. F., Saunders, A., and Allen, L. (1994). Deposit insurance and regulatory forbearance. *Journal of Money,Credit and Banking, 26*(3): 412-438.

Embrechts, P., Furrer, H., and Kaufmann, R. (2003). Quantifying regulatory capital for operational risk, http://www.Math.Ethz.Ch/~baltes/ftp/opriskweb.Pdf. *Derivatives Use, Trading & Regulation, 9*(3): 217-233.

Embrechts, P., Kaufmann, R., and Samorodnitsky, G. (2004). Ruin theory revisited: Stochastic models for operational risk, http://www.Math.Ethz.Ch/~baltes/ftp/ersamo.Pdf. *Risk Management for Central Bank Foreign Reserves*: 243-261.

Embrechts, P., Kluppelberg, C., and Mikosch, T. (1997). *Modelling extremal events for insurance and finance*. Springer, Berlin.

Embrechts, P., Lindskog, F., and McNeil, A. (2001). Modelling dependence with copulas and applications to risk management. In *Handbook of heavy tailed distributions in finance, ed. S. Rachev* (pp. 329-384): Elsevier.

European Commission. (2001). Risk-based capital systems. Note to the solvency subcommittee financial institutions, Brussels, http://europa.Eu.Int/comm/internal_market/insurance/docs/markt-2085/markt-2085-01_en.Pdf.

European Commission. (2003). Solvency 11-reflections on the general outline of a framework directive and mandates for further technical work.

European Commission. (2004). Framework for consultation on solvency, http://europa.Eu.Int/comm/internal_market/insurance/docs/markt-2506-04/framework_en.Pdf.

FDIC. (1995). Revisions to the reports of condition and income (call reports) for 1995. Inactive financial institution letters, http://www.Fdic.Gov/news/news/inactivefinancial/1995/fil9520a.Html. *Inactive Financial Institution Letters*.

Financial Services Authority. (2003). Integrated prudential sourcebook - near-final text on prudential risks systems and controls.

Financial Services Authority. (2005). Strengthening capital standards.Consultation paper, http://www.Occ.Treas.Gov/ftp/release/2005-6a.Pdf.

Fishman, G. (1996). *Monte carlo: Concepts, algorithms and applications.* Springer-Verlag, New York.

Fitch Risk Management. (2004). Fitch sees hitch in Basel operational risk rules. Technical report, Reuters 04 21 04 5 2/ AM ET.

Fontnouvelle, P., DeJesus-Rueff, V., Jordan, J., and Rosengren, E. (2003). Capital and risk: New evidence on implications of large operational losses. Preprint, Federal Reserve Bank of Boston, http://www.Bos.Frb.Org/economic/wp/wp2003/wp035.Pdf.

Fontnouvelle, P., Rosengren, E., and Jordan, J. (2004). Implications of alternative operational risk modeling techniques. Preprint, federal reserve bank of boston, http://papers.Ssrn.Com/sol3/papers.Cfm?Abstract_id=556823.

Frachot, A., Georges, P., and Roncally, T. (2001). Loss Distribution Approach for operational risk. Preprint, Crédit Lyonnais, France, http://gro.Creditlyonnais.Fr/content/wp/lda.Pdf.

Frachot, A., Moudoulaud, O., and Roncalli, T. (2003). Loss Distribution Approach in practice. Preprint, credit lyonnais, France, http://gro.Creditlyonnais.Fr/content/wp/lda-practice.Pdf.

Frachot, A. and Roncally, T. (2002). Mixing internal and external data for managing operational risk. Preprint, Crédit Lyonnais, France, http://gro.Creditlyonnais.Fr/content/wp/mixing-riskop.Pdf.

Frachot, A., Roncally,T., and Salomon, E. (2004). The correlation problem in operational risk. Preprint, credit lyonnais, France, http://gro.Creditlyonnais.Fr/content/wp/lda-correlations.Pdf.

Froot, K. and Stein, J. (1998). Risk Management, capital budgeting, and capital structure policy for financial institutions. *Journal of Financial Economics, 47*: 55-82.

Furrer, H. (2003). Quantifying operational risk: Possibilities and limitations. Preprint, Deutsche Bundesbank Training Centre, Eltville,

http://www.Math.Ethz.Ch/~hjfurrer/deutschebundesbank19-03-2004.Pdf.

Garcia, V. (2004). Strategies for risk and compliance transformation. Newsletter, IBM, Financial Services, http://www.Ibm.Com/industries/financialservices/doc/content/news/newsletter/1120385103.Html.

Giesecke, K. (2003). A simple exponential model for dependent defaults. Preprint, Cornell University, http://www.Orie.Cornell.Edu/~giesecke/paper6.Pdf.

GIRO Working Party. (2004). Quantifying operational risk in general insurance companies. Preprint, institute of actuaries, http://www.Actuaries.Org.Uk/files/pdf/sessional/sm20040322.Pdf.

Griffiths, R., Milne, R. K., and Wood, R. (1979). Aspects of correlation in bivariate poisson distributions and processes. *Australian Journal of Statistics, 21*(3): 238-255.

Hines, K. (2002). Risks considerations for the allfinanz organization. Preprint, Casualty Actuarial Society 2002 dfa call paper program, http://www.Casact.Org/pubs/forum/02sforum/02sf001.Pdf.

Holmes, M. (2003). Measuring operational risk a reality check. *Risk*: 16.

Hong, Y., Cheng, S., and Wang, S. (2003). Extreme risk spillover between chinese stock markets and international stock markets. Preprint, Cornell University, http://www.Vanderbilt.Edu/econ/sempapers/hong2.Pdf.

Insurance Information Institute. (2005). Insolvencies/guaranty funds, http://www.Iii.Org/media/hottopics/insurance/insolvencies/.

International Association of Insurance Supervisors. (2002a). Principe on capital adequacy and solvency, http://www.Iaisweb.Org/02solvency.Pdf.

International Association of Insurance Supervisors. (2002b). Report on solvency, solvency assessment and actuarial issues and some of their practical applications. IAIS SubCommittee on Solvency and Actuarial Issues, http://www.Iaisweb.Org/08151istansolv.Pdf.

Jenkinson, A. (1969). Statistics of extreme. In estimation of maximum flood. Technical Note 98 World Meteorological Organization Geneva.

Joe, H. (1997). *Multivariate models and dependence concepts*. Chapman & Hall, London.

Jorion, P. (1997). *Value at risk: The new benchmark for controlling market risk.* McGraw-Hill, New York.

Kallengerg, O. (1983). *Random measures.* Akademie-Verlag, Berlin.

Kallsen, J. and Tankov, P. (2004). Lévy copulas for general Lévy processes. Preprint, Technische Universitat Munchen.

Klugman, S. A., Panjer, H., and Willmot, G.E. (2004). *Loss models: From data to decisions*. John Wiley, New York.

KPMG. (2002). Commission services study prepared by KPMG on the methodologies to assess the financial position of an insurance undertaking from the perspective of prudential supervision. Preprint, European Commission, http://europa.Eu.Int/comm/internal_market/insurance/docs/solvency/solvency2-study-kpmg_en.Pdf.

Kremer, E. (1990). On the probable maximum loss. *Blatter der Deutschen Gesellschaft fur Versicherungsmathematik*: 201-205.

Kremer, E. (1994). More on the probable maximum loss. *Blatter der Deutschen Gesellschaft fur Versicherungsmathematik*: 319-325.

Kupiec, P. (1995). Techniques for verifying the accuracy of risk management models. *Journal of Derivatives, 7*: 41-52.

Leandri, F. (2003). Discussion on fontnouvelle et al's paper. Preprint, Banking Supervision, Bank of Italy, http://www.Bis.Org/bcbs/events/wkshop0303/disleand.Pdf.

Lewis, R. (1998). Capital from an insurance company perspective. *FRBNY Economic Policy Review*: 183-185.

Lindskog, F. and McNeil, A. (2003). Common poisson shock models: Applications to insurance and credit risk modeling, http://www.Math.Ethz.Ch/~mcneil/ftp/commonpoissonshockmodels.Pdf. *ASTIN Bulletin, 33(2)*: 209-238.

Marshall, A. W. (1996). Copulas, marginals and joint distributions. *In Ruschendorf, L. Schweizer, B., and Taylor, M.D., editors, Distributions with Fixed Marginals and Related Topics*: 213-222.

Marshall, C. (2002). *Measuring and managing operational risks.* Wiley, Singapore.

Mashal, R., Naldi, M., and Zeevi, A. (2002). Extreme events and multi-name credit derivatives. Preprint Columbia University, http://www.Columbia.Edu/~rm586/pub/credit_derivatives.Pdf.

Mashal, R. and Zeevi, A. (2002). Beyond correlation: Extreme co-movements between financial assets. Preprint, Columbia University, http://www2.Gsb.Columbia.Edu/faculty/azeevi/papers/beyondcorrelation.Pdf.

McNeil, A., Frey, R., and Embrechts, P. (2005). *Quantitative risk management: Concepts, techniques and tools.* Princeton University Press, Princeton.

McNeil, A. and Saladin, T. (1997). The peaks over thresholds method for estimating high quantiles of loss distributions. Proceedings of XXVIIth International ASTIN Colloquium.Cairns, Australia, http://www.Math.Ethz.Ch/~mcneil/ftp/cairns.Pdf.

Medova, E., and Kyriacou, M. (2002). Extremes in operational risk management. In risk management: Value at risk and beyond, http://www-cfr.Jims.Cam.Ac.Uk/archive/papers/2001/bookchapter-final.Pdf. *Cambridge University Press.*: 247-273.

Merton, R. (1977). An analytic derivation of the cost of deposit insurance and loan guarantee. *Journal of Banking and Finance, 1*: 3-11.

Merton, R. and Perold, A. (1993). Theory of risk capital in financial firms. *Journal of Applied Corporate Finance, 5*: 16-32.

Moscadelli, M. (2004). The modeling of operational risk: Experience with the analysis of the data collected by the Basel committee. Preprint, Banca d'Italia, http://www.Bancaditalia.It/ricerca/consultazioni/temidi/td04/td517/td_517/tema_517.Pdf.

National Association of Insurance Commissioners. (2004). Risk-focused surveillance framework, http://www.Naic.Org/frs/solvency_regulation/risk_assessment_wg/docs/clean_adopted_framework.Doc.

National Association of Insurance Commissioners. (2005a). NAIC history and background, http://www.Naic.Org/about/background.Htm.

National Association of Insurance Commissioners. (2005b). Risk-based capital general overview. Technical report, Capital Adequacy Task Force, http://www.Naic.Org/frs/rbc/docs/rbcoverview.Pdf.

Neslehova, J. (2004). *Dependence of non-continuous random variables.* Unpublished PhD, Carl von Ossietzky Universitat Oldenburg.

Neslehova, J., Embrechts, P., Chavez-Demoulin, V. (2006). Infinite mean models and the LDA for operational risk, http://www.Math.Ethz.Ch/~baltes/ftp/manuscript.Pdf. *Journal of Operational Risk, forthcoming*.

Nolan, J. (2001). *Stable distributions: Models for heavy-tailed data*. Birkhauser, Boston.

Orros, G. and Howell, J. (2003). Operational risk management for UK insurers. Preprint, the Centre for Future Studies, http://www.Futurestudies.Co.Uk/communications/infocus/231.Pdf.

Panjer, H., and Willmot, G. (1992). *Insurance risk models.* Society of Actuaries, Schaumberg, IL.

Panjer, H. and Willmot, G. (1981). Recursive evaluation of compound distributions. *Astin Bulletin, 12*: 22-26.

Peemoller, F. (2002). Operational risk data pooling. Deutsche bank ag, presentation at cfs forum, frankfurt, http://www.Ifk-cfs.De/papers/20020207peemoeller.Pdf.

Perold, A. (2001). Capital allocation in financial firms. Working Paper, Graduate School of Business Administration, Harvard University.

Pfeifer, D. and Neslehova, J. (2004). Modeling and generating dependent risk processes for irm and dfa, http://www.Gloriamundi.Org/picsresources/dpjn.Pdf. *ASTIN Bulletin, 34*(2): 333-360.

Pickands, J. I. (1975). Statistical inference using extreme order statistics. *Ann, Statist, 3*: 119-131.

Posner, R. (1974). Theories of economic regulation. *The Bell Journal of Economics and Management Science, 5*(2): 335-358.

Powojowski, M. R., Reynolds, D., and Tuenter, H. J. H. (2002). Dependent events and operational risk, http://www.Algorithmics.Com/research/summer2002/dependent_events.Pdf. *Algo Research Quarterly, 5*(2): 68-73.

Rachev, S. (2004). *Handbook of computational and numerical method in finance.* Birkhauser, Boston.

Rachev, S. and Mittnik, S. (2000). *Stable paretian models in finance.* John Wiley, England.

Robertson, J. (1992). The computation of aggregate loss distributions.

Proceedings of the Casualty Actuarial Society, LXXIX: 57-133.

Rochette, M. (2005). Operational risk. *The Actuary. February - March 2005*: 10-13.

Rockafellar, R. T., Uryasev, S. (2001). Conditional value-at-risk for general loss distributions. *Journal of Banking & Finance., 26*(7): 1443-1471.

Rolski, T., Schmidli, H., Schmidt,V., and Teugels, J. (1998). *Stochastic processes for insurance and finance*. Wiley, New York.

Romano, C. (2002). Calibrating and simulating copula functions: An application to the Italian stock market. Working paper n. 12, CIDEM, http://w3.Uniroma1.It/cidem/files/wpromanodicembre02.Pdf.

Rose, C. and Smith, M. (2002). *Mathematical statistics with mathematica.* Springer, New York.

Sarbanes-Oxley. (2002). Financial and Accounting Disclosure Information, http://www.sarbanes-oxley.com/

Semke, R. (2003). Operational risk management. Presentation SOA - ERM Mini-

Seminar, http://rmtf.Soa.Org/erm_miniseminar.Pdf.

Shah, S. and Longley-Cook, A. (2001). Insurance operational risk: The big unknown. Erisk.Com, http://www.Erisk.Com/portal/news/features/news_feature2001-09-07.Pdf.

Silverman, B. (1986). *Density estimation for statistics and data analysis, monographs on statistics and applied probability*. Chapman & Hall, London.

Sklar, A. (1959). Fonctions de répartition à *n* dimensions et leurs marges. *Publ. Inst. Stat. Univ. Paris., 8*: 229 - 231.

Sklar, A. (1996). Random variables, distribution functions, and copulas- a personal look backward and forward. Distributions with fixed marginals and related topics. Institute of Mathematical Statistics, Hayward, CA.: pp. 1-14.

Smith, R. (1987). Estimating tails of probability distributions. *The Annals of Statistics, 15*(3): 1174- 1207.

Smith, R., and Weissman, I. (1994). Estimating the extremal index. *Journal of the Royal Statistical Society. Series B (Methodological), 56*(3): 515-528.

Society of Actuaries. (2002). Risk-based capital. Preprint SoA, http://rmtf.Soa.Org/riskbased_capital.Pdf.

Stigler, G. (1971). The theory of economic regulation. *The Bell Journal of Economics and Management Science., 2*(1): 3-21.

Tang, A. and Valdez, E. (2004). Economic capital and the aggregation of risks using copulas. Preprint, school of actuarial studies faculty of commerce & economics, university of new south wales, sydney, http://wwwdocs.Fce.Unsw.Edu.Au/actuarial/research/papers/2005/econcapital5c.Pdf.

Tankov, P. (2004). *Levy processes in finance: Inverse problems and dependence modelling. Http://www.Math.Jussieu.Fr/~tankov/.* Unpublished PhD, Ecole Polytechnique, Paris.

The Commission of Sponsoring Organizations of the Treadway Commission. (2004). Enterprise risk management - integrated framework. Preprint COSO.

The Federal Reserve Board. (1999). Differences in capital and accounting standards among the federal banking and thrift agencies, http://www.Federalreserve.Gov/boarddocs/rptcongress/differences/.

The Federal Reserve Board. (2003). Differences in capital and accounting standards among the federal banking and thrift agencies, Report to Congress, http://www.Federalreserve.Gov/boarddocs/rptcongress/differences/2003.Htm.

Titterington, D., Smith, A., and Makov, U. (1985). *Statistical analysis of finite mixture distributions*. Wiley, Chichester.

US House of Representatives Subcommittee on Oversight and Investigations of the Committee on Energy and Commerce. (1990). Failed promises - insurance company insolvencies. Report U.S. Government Printing Office.

Wang, S. (2000). A class of distortion operators for pricing financial and insurance risks, http://www.Soa.Org/library/arch/2000-09/arch2000v19.Pdf. *Journal of Risk and*

Insurance., 67(1): 15-36.

Wang, S. (2002). A risk measure that goes beyond coherence. Preprint Scor Reinsurance Co. USA.

Werneman, O. (2005). Pricing lifelong joint annuity insurances and survival annuity insurances using copula modeling of bivariate survival, thesis kth matematik, stockholm, sweden, http://www.Math.Kth.Se/matstat/seminarier/050110.Pdf.

Wilkinson, M. E. (1982). Estimating probable maximum loss with order statistics, http://www.Casact.Org/pubs/dpp/dpp82/82dpp505.Pdf. *Proceedings of the Casualty Actuarial Society, LXIX*: 195-209.

Wirch, L. and Hardy, M. (1999). A synthesis of risk measures for capital adequacy. *Insurance: Mathematics and Economics., 25*: 337-347.

Zolotarev, V. (1994). On the representation of densities of stable laws by special functions. *Theory Probab. Appl, 39*: 354-362.