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Statistical Genetic Interval-Valued Type-2 Fuzzy System and its Application

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STATISTICAL GENETIC INTERVAL-VALUED TYPE-2 FUZZY SYSTEM

AND ITS APPLICATION

by

YU QIU

Under the Direction of Yanqing Zhang

ABSTRACT

In recent years, the type-2 fuzzy sets theory has been used to model and minimize the effects of uncertainties in rule-base fuzzy logic system. In order to make the type-2 fuzzy logic system reasonable and reliable, a new simple and novel statistical method to decide interval-valued fuzzy membership functions and a new probability type reduced reasoning method for the interval-valued fuzzy logic system are proposed in this thesis. In order to optimize this particle system's performance, we adopt genetic algorithm (GA) to adjust parameters. The applications for the new system are performed and results have shown that the developed method is more accurate and robust to design a reliable fuzzy logic system than type-1 method and the computation of our proposed method is more efficient.

INDEX WORDS: Interval-valued fuzzy logic, type-2 fuzzy logic, statistical interval-valued fuzzy reasoning, fuzzy control, genetic algorithm

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CHAPTER 1

INTRODUCTION

The fuzzy logic systems based on fuzzy sets are frequently used as Computational Intelligence (CI) tools and techniques after Zadeh first introduced fuzzy sets in 1965 [1]. In the real-world field like financial, engineering, life science, psychology etc. many systems' information comes from two sources: "one source is human experts who describe their knowledge about system in natural languages; the other is sensory measurement and mathematical models that are derived according to physical laws." [18]. The first sources are vague, ambiguous, imprecise and confused. The classical two-valued modeling (Boolean logic system) needs to be defined precisely. It can't well solve these problems. Fuzzy logic systems provide more freedom to handle uncertainty and impreciseness. It incorporates human knowledge and thinking with numerical information. With the rapid development of information technology, a wide variety of products or system in the area including decision-making, prediction, control, and pattern recognition employed the fuzzy system to perform a task.

Boolean logic defines a value either one or zero to each individual in a universal set. This can distinguish individual between members and nonmembers of a set. A fuzzy logic system is different with Boolean logic system. In order to indicate the membership grade of each element, Fuzzy set assigns a value of specified range [0, 1] to it in the universal set. In other word, the fuzzy set maps elements of a given universal set U into real numbers in [0, 1] by each membership function (MF). For example, 25 degree C temperature belongs to "hot" or "not hot" in Boolean system and it can't be described as "little bit hot", but in fuzzy logic system, we may use a degree to describe it. Fuzzy linguistic values of fuzzy linguistic variables can be used to represent uncertainty in human natural language. For instance, “very cold”, “cold”, “mild”, “hot” can describe temperature of a day. The type-1 fuzzy membership functions are used by the type-1 fuzzy logic systems (FLS).

Fuzzy logic system involves many human thinking which is so vague. Type-1 FLSs are unable to completely handle the vagueness. Their membership functions are often overly precise. Zadeh extended the concept of type-1 fuzzy set to type-2 fuzzy sets. Type-2 fuzzy sets introduce an additional dimension that represents grades of membership that are themselves fuzzy [3, 4, 5, 10, 11, 12].

Type-2 fuzzy logic systems that use antecedent or consequent membership functions are Type-2 fuzzy sets which were introduced by [3, 4] to deal with rule uncertainties. Liang and Mendel [6] present the theory to design interval type-2 FLSs for different kinds of fuzzifiers and applications. The interval-valued type-2 FLSs are the simplest type-2 FLSs. However, constructing a reasonable membership functions still is a very difficult problem, and another challenging problem is how to find an efficient inference method. Many researchers change different type-1 fuzzy membership function’s parameters to obtain interval-valued type-2 fuzzy sets [3, 6, 9]. These methods just consider unique probability for each element in an interval. In a real world, each element in an interval usually has different probabilities. For example, there are 6 out of 10 people who agree to assign same degree to 60 year-old people. And 4 of them agree to assign other degree. This thesis aims to develop a novel theory to consider various probabilities and design a system that is based on interval-valued type-2 FLS to involve human activity and thinking.

The first thrust of this thesis will be to develop a reasonable method to construct interval-valued type-2 FLS membership functions after surveying different experts. Two procedures that apply linear regression and non-linear regression have been proposed for such construction.

The second thrust of this thesis will be to develop new statistical interval-valued fuzzy inference method by combining probability theory and fuzzy logic to make more reliable fuzzy reasoning after generating statistical membership function. The inference engine is an important part of FLS. For an interval-valued linguistic variable, we consider as many as values to perform reasoning.

The third thrust of this thesis will be to incorporate genetic algorithm (GA) and least square error into our methodology. These methods will be used to optimize parameters and improve the system performance.

The thesis is organized as follows. In Chapter 2 we focus on basic concepts and background of FLS. Then, compare type-1 and type-2 FLS and introduce related work. In Chapter 3, we introduce a new statistical method to generate membership function. Chapter 4 illustrates the new probability-based fuzzy inference method. Chapter 5 describes the parameter optimization using genetic algorithm. In Chapter 6 we apply the techniques that we developed to do some simulation study. In Chapter 7 we summarize conclusions.

CHAPTER 2

FUZZY LOGIC SYSTEMS

Since Zadeh first introduced fuzzy sets theory in 1965, the research and application for FLSs has been increasing, especially after it was successfully used in Japanese consumer products. In this chapter we give the review of fuzzy logic system. In Section 2.1, we introduce the basic concept for Type-1 FLSs. The extension of type-1 FLSs – type-2 FLS and one specific type-2 FLS are described in section 2.2 and section 2.3, respectively.

2.1 Type-1 Fuzzy Logic Systems

What are fuzzy systems? “Fuzzy systems are knowledge-based or rule-based systems” [18]. The membership functions of a Type-1 FLS (some time call Type-1 Fuzzy logic controller) are used by a type-1 fuzzy set. A type-1 fuzzy set defines different degrees of membership over the range of [0, 1] to represent uncertainty. So we can use type-1 fuzzy sets to describe complex systems with linguistic terms used in real life. The type-1 fuzzy membership functions are two-dimensional. They map an element of a given universal set into a crisp real number in [0, 1] – which represents from complete non-membership to complete membership. The following is the most commonly used notation to denote MF (μ_A) of a fuzzy set A:

$$\mu_A : X \to [0,1]. \tag{1}$$

The mapping is followed by membership function of A.

A type-1 Fuzzy Logic System consists of four parts: the fuzzifier, the rule base, the inference engine, and the defuzzifier. Figure 1 shows a basic structure of the type-1 FLS.

The fuzzifier converts a crisp input to membership values for relevant fuzzy sets. A fuzzy rule base is a set of IF-THEN rules with a linguistic model of human control actions directly based on human thinking and decision. In the inference engine, a fuzzy reasoning method is used to derive fuzzy conclusions (fuzzy sets). Finally, the defuzzifier converts output fuzzy sets into a crisp value.

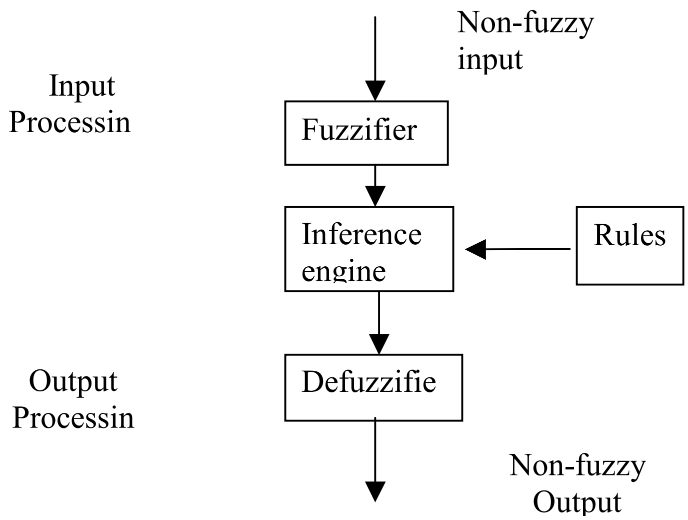

Figure 1. A type-1 fuzzy logic system

There are various methods to define antecedent or consequent membership functions. For example, there are linear membership functions: triangular functions, trapezoidal functions, etc. and nonlinear membership functions: bell shape functions, Gaussian functions, etc. Figure 2 shows a typical example for triangular type-1 membership function.

The important component of a fuzzy system is inference method that is based on a set of IF-THEN rules and membership functions. There are two popular inference models: Mamdani-Assilian model, and TSK model [17, 21]. They have different rule structures. For

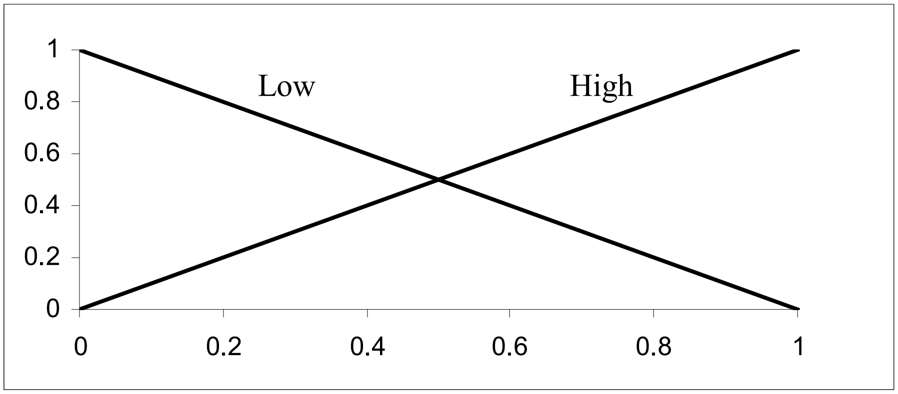

Figure 2. An example of type-1 membership function

example, the following is a rule defined by Mamdani-Assilan model [17]:

IF Pressure is medium

and Temperature is high

THEN speed is high

In the above rule, input variables are Pressure and Temperature, output variable is speed. {medium, high, high} are linguistic values for these variables. TSK model has different rule structure. The THEN part of TSK model uses a simple mathematical formula instead using words in nature languages. The previous example rule will become as following [21]:

IF Pressure x is medium

and Temperature y is high

THEN speed s is $s = p_0 + p_1*x + p_2*y$

where, p_0, p_1, p_2 are constant for the linear function.

In order to fire the strength for one rule, the intersection operator of two fuzzy sets A and B (usually referred to as T-norm [17]) is very important operator. Usually FLS use *t-norm* to perform the reasoning [1, 2, 5]. Frequently used *t-norm* operators are minimum and algebraic products. The minimum *t-norm* performs the intersection of the two sets. If there is more than one antecedent, the weight is

$$w_m = \min(w_{1m}, \ldots, w_{km}), \tag{2}$$

where $k = 1, 2, \ldots, n$ is antecedent, m is the m th rule.

The minimum *t-norm* just considers one of the antecedent weights and loses some important weights. To take care of all weights in different antecedent, the algebraic product *t-norm* operator is used:

$$w_m = w_{1m} * w_{2m} * \ldots * w_{km}, \tag{3}$$

where $k = 1, 2, \ldots, n$ antecedent, m is the m th rule.

Many fuzzy systems are constructed using Mamdani-Assilan model which is also used in this thesis. Since the IF-THEN rule THEN part for this model is a fuzzy set, Defuzzifer strategy must be used to convert a fuzzy set to a crisp value. There are five frequently used defuzzifer methods: centroid of area, bisector of area, mean of maximum, smallest of maximum, largest of maximum. In the thesis we apply the most widely adopted strategy called centroid of area (COA):

$$Z_{COA} = \frac{\int_z \mu_A(z) z dz}{\int_z \mu_A(z) dz}, \tag{4}$$

where $\mu_A(z)$ is the aggregated output MF.

2.2 Type-2 Fuzzy Logic Systems

Type-1 FLSs are not effective to completely handle fuzziness for complex application systems (especially human related systems). Their membership functions are often overly precise. However, for some concept and context in which they are applied, we may only be able to identify approximate MFs, i.e. the knowledge that is used to construct MFs and rules which are uncertain [8]. For example:

1) When you go shopping for a digital camera, the linguistic expression “good” will have different meaning for different people. Some people may consider high megapixel, some people may like high X optical lens, some people may think picture quality is more important, and others may agree that price is more important.
2) The experts that design the fuzzy logic system may have different knowledge for this system. So if we survey a group of experts, they may give different MFs with possible interval values.
3) When we collect data, some factor may affect the measurements. So data themselves are noisy in real applications.

Once the MFs for type-1 FLS have been assigned, they become precise and no longer can handle more uncertainty. In order to consider different types of uncertainty, type-2 fuzzy sets and type-2 FLS are introduced. As mentioned before, type-2 fuzzy sets are extension of type-1 fuzzy sets by introducing an additional dimension that represents grades of membership. According to [6], “A type-2 membership grade can be any subset in [0, 1] - the primary membership; and corresponding to each primary membership, there is a secondary membership (which can also be in [0, 1]) that defines the possibilities for the primary membership”. The secondary membership in type-2 FLSs gives us more freedom to describe and handle uncertainties. In many applications, if it is difficult to determine the exact MF for a fuzzy set, the type-2 FLSs are suitable for the applications.

As shown in Figure 3, a structure of a type-2 FLS is similar to a type-1 FLS. The type-2 FLS includes a fuzzifier, a fuzzy rule base, an inference engine, and an output processor. It has antecedent or consequent type-2 fuzzy sets. Especially, the output process includes one more step—type-reduce to generate a crisp number.

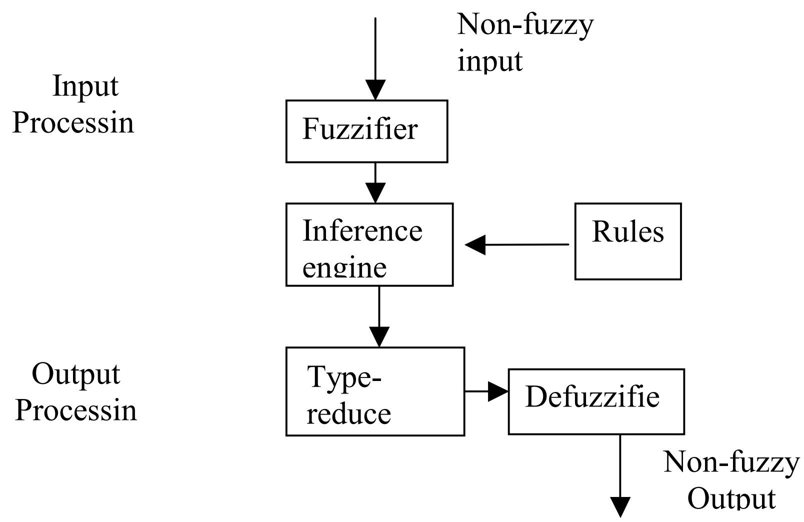

Figure 3. A type-2 fuzzy logic system

In [10], the theoretical operations like union, intersection, complement and some properties of type-2 fuzzy sets have been studied. They [11] also examined the operations of algebraic product and algebraic sum of type-2 sets. Karnik and Mendel [3] developed a type-reduction operation and study set operation on type-2 fuzzy sets, type-2 relations and their compositions, defuzzification. Duboid and Prade [15] discussed sup-star composition of type-2 relations and a general formula for extended sup-star composition of type-2 relation which is given by [4].

2.3 Interval Type-2 Fuzzy Logic System

Although additional dimension gives the system more freedom to deal with vagueness, type-2 fuzzy sets are hard to understand and design since it is very difficult to draw three-dimension function and the computation for type-2 fuzzy set is too complicated.

When we define the secondary membership either zero or one the primary membership function will be interval set and the problem is simplified. Figure 4a shows an example of

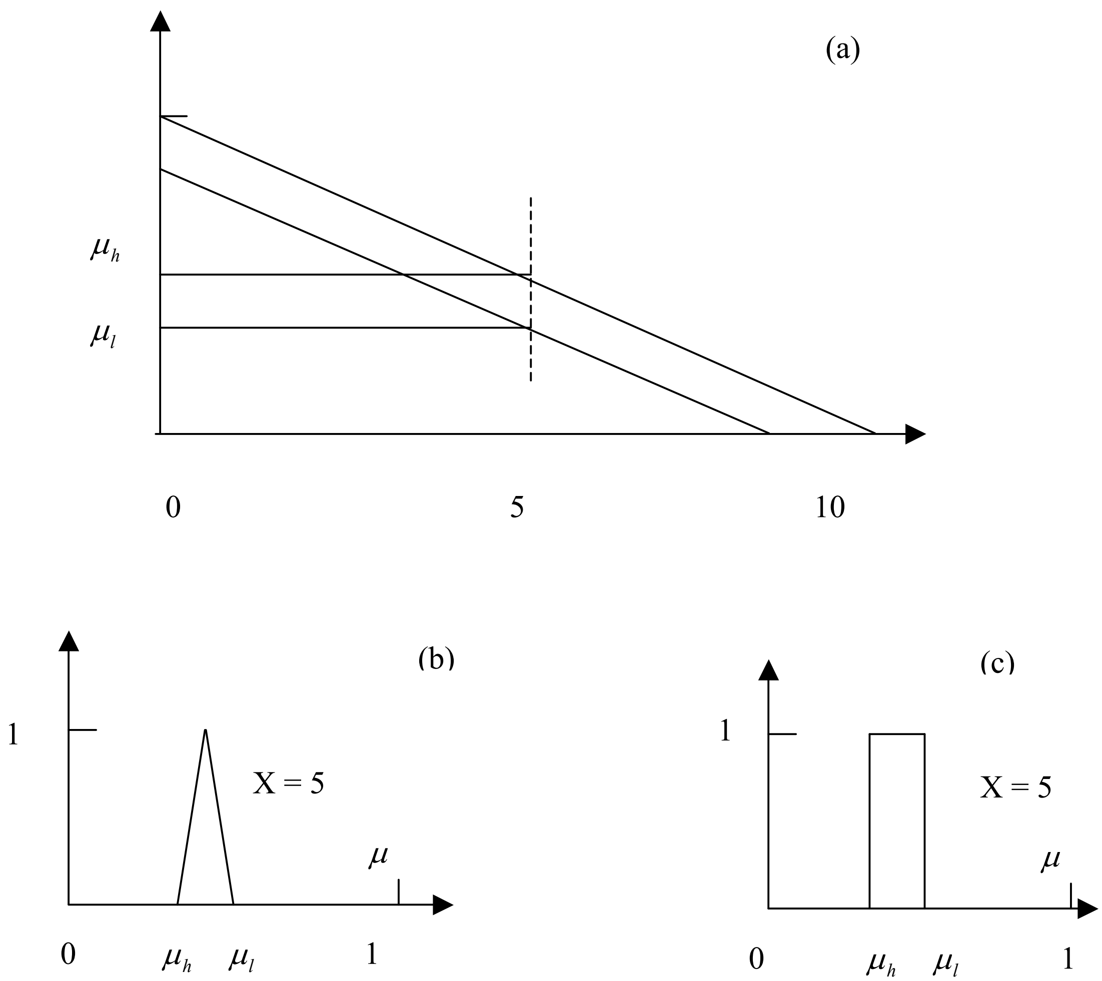

Figure 4. Primary and secondary membership functions of a regular and interval type-2 fuzzy sets. (a) Primary membership function (b) Secondary membership function for regular type-2 (c) Secondary membership function for interval type-2

primary membership function of type-2 fuzzy sets. μ can be any value between μ_h and μ_l for a given value x. Figure 4b presents the secondary membership function for type-2 fuzzy set. The secondary membership value between μ_h and μ_l can be different grade. If the secondary degrees become same value between μ_h and μ_l, in other words, they all equal to 1, the shape of secondary membership function will be rectangular. Then regular type-2 fuzzy sets become interval type-2 fuzzy sets. Figure 4c shows the secondary membership function of interval type-2 sets with same primary membership function.

Liang and Mendel [6] studied how to construct membership function for interval type-2 fuzzy sets, proposed a method to compute the operations and inference method for interval type-2 FLSs. They also use Gaussian primary MFs to illustrate their theory. They change the parameters in the primary MF to form interval type-2 fuzzy sets MF. In their method, the probability of linguistic value in an interval is equal. If we can consider probability in the process to develop a type-2 FLS, the result will be more reliable and robust. For example, according to Ozen and Garibaldi [9], the six clinicians tend to agree each other in the middle of range and have less agreement in the other ranges. There is a distribution along with desired result. The thesis aims to design a reliable and robust interval-valued fuzzy logic system from survey result.

CHAPTER 3

STATISTICAL GENERATION METHOD

Suppose we ask a group of expert to develop the same control FLS. Since they have different knowledge about that system, they may not agree to use the same shape of membership function, or same shape but different interval. It is very hard to ask them to completely agree with each other. In order to solve this problem, we develop a new method using statistical technology. The proposed method using linear regression tool to generate membership function of interval-valued type-2 fuzzy logic system are presented in Section 3.1. In order to improve the performance, we propose non-linear regression model at Section 3.2.

Instead of define membership functions, we ask experts only assign a grade of membership for each discrete input value for each linguistic term of linguistic variable, e.g. the 60 years old has grade sets {1.0, 0.8, 0.9, 0.7} in the age "old" people by four experts. The result is a set of data.

In this thesis, we have the following notations:

1). Let $X_1, X_2, ..., X_n$ denote linguistic variables in domain $U_1, U_2, ..., U_n$.

2). There are n terms for X_1, i.e. $X_{11}, X_{12}, ..., X_{1n}$.

Definition 1: A statistical interval-valued fuzzy data set D_{ij} for linguistic term X_{ij} is defined by

$$D_{ij} = \{(x_1, d_{11}), (x_1, d_{12}), \cdots (x_1, d_{1k}), \cdots (x_n, d_{n1}), \cdots (x_n, d_{nk})\}, \tag{5}$$

where x_n are distinguish crisp value of X_{ij}, d_{nk} is membership grade for x_n and k indicates k experts.

Definition 2: A statistical interval-valued fuzzy data set D_i for linguistic variable X_i is defined by

$$D_i = \sum_{j=1}^{n} D_{ij} \tag{6}$$

We can plot D_i as shown in Figure 5.

Definition 3: An interval-valued fuzzy set S is defined by

$$S = \int_R [\underline{\mu}_S(x), \overline{\mu}_S(x)] / x, \tag{7}$$

where $\underline{\mu}_S(x)$ is a lower bound membership function of linguistic term X_{ij} of x and $\overline{\mu}_S(x)$ is an upper bound membership function of linguistic term X_{ij} of x for $x \in U_i$ (U_i is an universe of discourse for linguistic variable X_i).

3.1 Generate linear MFs for interval-valued FLS

After collecting all the survey data from experts, we obtain the data set D_i. Then we can design fuzzy linguistic variable membership function of interval-valued FLSs using statistical method.

3.1.1 Generate lower bound membership function

Step 1: create data set of lower bound.

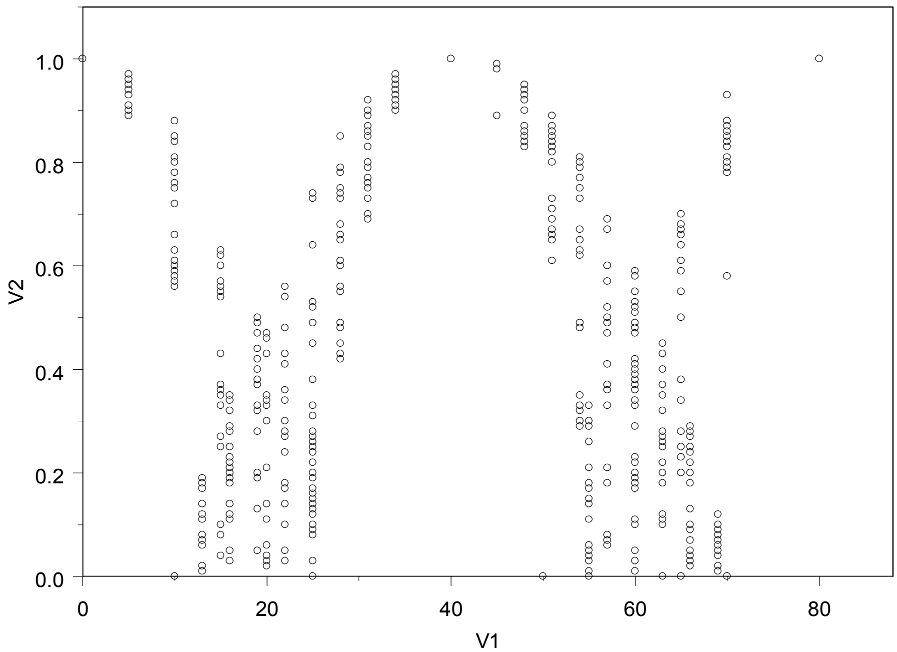

Figure 5. Plot for three different linguistic terms and 20 experts

Obtain minimal grade value for discrete input value x_i from data set D_{ij} by

$$\underline{d}_i = \min_{x_i} (d_{i1}, d_{i2}, \cdots, d_{ik}). \tag{8}$$

Then data set of lower bound for linguistic term X_{ij} is defined by

$$\underline{D}_{ij} = \{(x_1, \underline{d}_1), (x_2, \underline{d}_2), ..., (x_n, \underline{d}_n)\}, \tag{9}$$

$x_1, x_2, ..., x_n$ are crisp values in domain U_i for linguistic term X_{ij}, and then data set of lower bound for linguistic variable X_i is

$$\underline{D}_i = \sum_{j=1}^{n} \underline{D}_{ij} . \tag{10}$$

Step 2: Obtain membership function of lower bound value for each linguistic term X_{ij}.

There are a lot of statistical models which can be used to generate membership function from data set $\underline{D}_{ij}$. We can choose one model from linear regression, non-linear regression, spline-smoothing model etc. which depends on different conditions. The simplest way is linear regression model. First, we hypothesize a linear probability model to relate grade, $\mu(x)$ to the linguistic term X_{ij} by

$$\mu_{X_{ij}}(x) = \beta_0 + \beta_1 x + \varepsilon \quad (x \in U_i)\,. \tag{11}$$

Next, we enter data in set $\underline{D}_{ij}$ into this model and get the least squares estimate of the slope $\widehat{\beta}_1$, intercept $\hat{\beta}_0$ and the least squares equation for $\hat{\mu}_{X_{ij}}(x)$.

Then we use the probability distribution of random error component to estimate standard deviation for error ε.

Fourth, we do hypothesis test to check correctness of the hypothesized model. If there is no linear relationship between μ and x, then the null hypothesis (β_1 is 0) test will be true. In order to test alternative hypothesis, we can obtain p-value. The confidence interval and coefficient of determination are used to test the strong linear relationship between μ and x.

After finishing this step, lower bound MF ($\mu^l_{X_{ij}}(x)$) of the linear probability model for X_{ij} is obtained:

$$\mu^l_{X_{ij}}(x) = \alpha^l_0 + \alpha^l_1 x \ \ (x \in U_i)\,, \tag{12}$$

where α^l_0 and α^l_1 are regression constants.

Repeat these steps until we get all the lower bound MF for each X_{ij}.

3.1.2 Generate upper bound MF

Step 1: create upper bound data set.

Obtain maximal grade value for discrete input value x_i from data set D_{ij} by

$$\overline{d}_i = \max_{x_i}(d_{i1}, d_{i2}, \cdots, d_{ik}), \tag{13}$$

and upper bound data set for linguistic term X_{ij} is defined by

$$\overline{D}_{ij} = \{(x_1, \overline{d}_1), (x_2, \overline{d}_2), \ldots, (x_n, \overline{d}_n)\}, \tag{14}$$

where $x_1, x_2, \ldots, x_n$ are crisp values in domain U_i for linguistic term X_{ij}. Then, upper bound data set for linguistic variable X_i is

$$\overline{D}_i = \sum_{j=1}^{n} \overline{D}_{ij} . \tag{15}$$

Step 2: Obtain membership function of upper bound value for each linguistic term X_{ij}.

In order to generate upper bound MF ($\mu^u_{X_{ij}}$) from set $\overline{D}_{ij}$ for each linguistic term X_{ij}, use the same method of 3.1.1 step2 and get a linear upper bound MF:

$$\mu^u_{X_{ij}}(x) = \alpha^u_0 + \alpha^u_1 x \ (x \in U_i) \tag{16}$$

where α^u_0 and α^u_1 are regression constants.

3.1.3 Generate expected value MFs

Step 1: create data set of expected value.

There is a probability theory called strong *law of large numbers*. The law states when the sample size is large enough, the average of values based on these samples is close to the expected value. In order to calculate the expect grade value in one crisp input, we apply the strong law of the large numbers and get expect grade value for discrete input value x_i from data set D_{ij} by

$$d_i^{Ex} = \frac{\sum_{l=1}^{n} b_{il}}{n}, \tag{17}$$

and the data set of expected value for linguistic term X_{ij} is defined by

$$D_{ij}^{Ex} = \{(x_1, d_1^{Ex}), (x_2, d_2^{Ex}), ..., (x_n, d_n^{Ex})\}, \tag{18}$$

where $x_1, x_2, ..., x_n$ are crisp values in domain U_i for linguistic term X_{ij}. Then data set of the expected value for linguistic variable X_i is

$$D_i^{Ex} = \sum_{j=1}^{n} D_{ij}^{Ex}. \tag{19}$$

Step 2: obtain membership function of expected value for each linguistic term X_{ij}.

By using the same method as step 2 of Section 3.1.1 we generate a linear MF ($\mu_{X_{ij}}^{Ex}$) for expected value from set D_{ij}^{Ex} for each linguistic term X_{ij}.

$$\mu_{X_{ij}}^{Ex}(x) = \alpha_0^{Ex} + \alpha_1^{Ex} x \ (x \in U_i), \tag{20}$$

where α_0^{Ex} and α_1^{Ex} are regression constants.

3.1.4 Generate probability function for expected value

In each discrete input value, we calculate sampling distribution from a sample of n measurements. Using this method, we derive the probability for expected grade value from the distribution for discrete input.

Step 1: create probability data set generated from expected value.

Probability data set of expected grade value for linguistic term X_{ij} is defined by

$$P_{ij}^{Ex} = \{(x_1, p(d_1^{Ex})), \cdots, (x_n, p(d_n^{Ex}))\}, \tag{21}$$

where $x_1, x_2, ..., x_n$ are crisp values in domain U_i for linguistic term X_{ij}. Then probability data set of the expected value for linguistic variable X_i is

$$P_i^{Ex} = \sum_{j=1}^{n} P_{ij}^{Ex}. \tag{22}$$

Step 2: get the function defined by the probability of expected value for each linguistic term X_{ij}.

To generate probability function $f_{X_{ij}}^{Ex}(x)$ of expected value from set P_{ij}^{Ex} for each linguistic term X_{ij}, we use the same method as step of Section 3.1.1 and get a probability function of expected value:

$$f_{X_{ij}}^{Ex}(x) = \gamma_0^{Ex} + \gamma_1^{Ex} x + \gamma_2^{Ex} x^2 \ (x \in U_i), \tag{23}$$

where $\gamma_0^{Ex}, \gamma_1^{Ex}$ and γ_2^{Ex} are constants.

3.1.5 Generate probability function for upper bound value

Step 1: create probability data set of upper bound value.

Upper bound probability data set for linguistic term X_{ij} is defined by

$$\overline{P}_{ij} = \{(x_1, p(\overline{d}_1)), \cdots, (x_n, p(\overline{d}_n))\}, \tag{24}$$

where $x_1, x_2, ..., x_n$ are crisp values in domain U_i for linguistic term X_{ij}. Then upper bound probability data set for linguistic variable X_i is

$$\overline{P}_i = \sum_{j=1}^{n} \overline{P}_{ij}. \tag{25}$$

Step 2: obtain probability function of upper bound value for each linguistic term X_{ij}.

To generate probability function $f_{X_{ij}}^{u}(x)$ of upper bound from set $\overline{P}_{ij}$ for each linguistic term X_{ij}, we use the method of step 2 of Section 3.1.1 and get a probability function for the upper bound value:

$$f_{X_{ij}}^{u}(x) = \gamma_0^u + \gamma_1^u x + \gamma_2^u x^2 \ (x \in U_i), \tag{26}$$

where γ_0^u, γ_1^u and γ_2^u are constants.

3.1.6 Generate probability function for lower bound value

Step 1: create probability data set for lower bound value.

Probability data set of lower bound for linguistic term X_{ij} is defined by

$$\underline{P}_{ij} = \{(x_1, p(\underline{d}_1)), \cdots, (x_n, p(\underline{d}_n))\}, \tag{27}$$

where $x_1, x_2, ..., x_n$ are crisp values in domain U_i for linguistic term X_{ij}. Then probability data set of lower bound for linguistic variable X_i is

$$\underline{P}_i = \sum_{j=1}^{n} \underline{P}_{ij} \tag{28}$$

Step 2: get probability function of lower bound for each linguistic term X_{ij}.

To generate probability function for lower bound $f_{X_{ij}}^{l}(x)$ from set $\underline{P}_{ij}$ for each linguistic term X_{ij}, use the same method of step 2 in Section 3.1.1 and get a probability function for lower bound value:

$$f_{X_{ij}}^{l}(x) = \gamma_0^l + \gamma_1^l x + \gamma_2^l x^2 \ (x \in U_i), \tag{29}$$

where γ_0^l, γ_1^l and γ_2^l are constants.

Figure 6 shows an example of interval-valued fuzzy logic membership function of upper bound, lower bound and expected value.

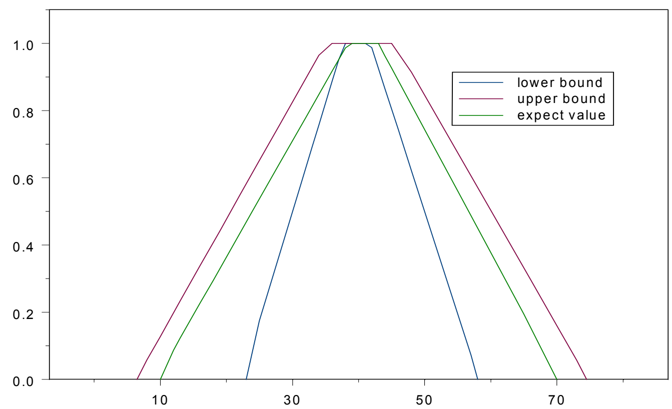

Figure 6. An example of interval-valued fuzzy logic membership function

3.2 Generate polynomial MFs for interval-valued FLS

As we mentioned in Section 3.1, there are a lot of statistical models can be used to generate membership function from data set $\underline{D}_{ij}$. The linear regression model is the simplest way. Considering more reliable and weight of all data, we extend linear regression to non-linear regression model. Many researchers prefer to use Gaussian primary MF [6, 8, 9]. Because the Gaussian primary MF is more close to 0 in two sides but never equal to 0. In the thesis, we choose the polynomial MF. Following the same procedure introduced in Section 3.1, we first generate lower bound membership function by hypothesizing a polynomial probability model to relate grade, $\mu(x)$ to the linguistic term X_{ij}

$$\mu_{X_{ij}}(x) = \beta_0 + \beta_1 x + \beta_2 x^2 + \varepsilon \ (x \in U_i)\,. \qquad (30)$$

When the regression step is finished, a polynomial probability model based lower bound MF ($\mu^l_{X_{ij}}(x)$) for X_{ij} is obtained:

$$\mu_{X_{ij}}^{l}(x) = \alpha_0^l + \alpha_1^l x + \alpha_2^l x^2 \ (x \in U_i), \tag{31}$$

where α_0^l, α_1^l and α_2^l are regression constants. Second, the membership function for upper bound and expected value for non-linear method can be obtained using same strategy:

$$\mu_{X_{ij}}^{u}(x) = \alpha_0^u + \alpha_1^u x + \alpha_2^u x^2 \ (x \in U_i), \tag{32}$$

$$\mu_{X_{ij}}^{Ex}(x) = \alpha_0^{Ex} + \alpha_1^{Ex} x + \alpha_2^{Ex} x^2 \ (x \in U_i), \tag{33}$$

where α_0^u, α_1^u, α_2^u, α_0^{Ex}, α_1^{Ex} and α_2^{Ex} are regression constants. Third, generate probability function for expected value, upper bound and lower bound as equation (23), (26), and (29), respectively.

CHAPTER 4

STATISTICAL INTERVAL-VALUED FUZZY LOGIC SYSTEMS

In this chapter we describe the structure of statistical interval-valued fuzzy logic systems. We use the approach developed at Chapter 3 to generate membership functions for each linguistic variable of a FLS. In Section 4.1, a novel inference and type-reduce method is developed for statistical fuzzy logic system in order to consider different probability in interval. We present defuzzification and get final output method in Section 4.2.

4.1 Fuzzy reasoning and type reduce

Suppose we ask a group of experts to develop an n-input-1-output FLS. For each discrete input value, they will assign a grade of membership for each linguistic term of linguistic variable. Then we can apply one of the methods (either linear or non-linear method) presented in Chapter 3 to obtain all the upper bound, lower bound and expected value membership functions for each value of linguistic variables and the probability function for them.

Liang and Mendel [6] discuss the operators for interval type-2 FLSs. They use extension of sup-star method. For example, there is an interval type-2 FLS with two-input variables A and B, one-output variable C. The fired weight of upper and lower bound for variables A and B are $w_{ua}, w_{la}, w_{ub}, w_{lb}$, respectively. After fuzzification with product t-norm operator we get output weight as follows:

$$w_u = w_{ua} * w_{ub} \tag{34}$$

and

$$w_l = w_{la} * w_{lb}. \tag{35}$$

In this typical interval type-2 fuzzy logic system, there are only upper and lower bound membership functions for each linguistic value. So the fuzzy reasoning method for the firing strength is not suitable for statistical interval-valued type-2 FLSs. The critical factor is to find a reasonable approach to do fuzzy reasoning for our proposed FLS.

If there are m n-input-1-out interval-valued fuzzy IF-THEN rules for a particular fuzzy logic system:

IF x_1 is X_{1r} and … and x_n is X_{nr} THEN y is Z_r.

After constructing MF and probability functions of the expect value, the upper lower bound value for statistical FLS, we can map any numerical input (a practical value x) to this rule and use equation (12), (16), (20) for linear model or equation (31), (32), (33) for non-linear model to find firing strength $\mu_{X_{ij}}(x)$ which is an interval value $[\underline{\mu}_{X_{ij}}(x), \overline{\mu}_{X_{ij}}(x)]$, expected value $\mu_{X_{ij}}^{Ex}(x)$. By using equation (23), (26), and (29) we get the probability for expect value ($f_{X_{ij}}^{Ex}(x)$), upper bound value ($\overline{f}_{X_{ij}}(x)$) and lower bound value ($\underline{f}_{X_{ij}}(x)$).

In order to find firing strength, we randomly choose two grade values y_1, y_2 between upper bound $\overline{\mu}_{X_{ij}}(x)$ and expected value $\mu_{X_{ij}}^{Ex}(x)$, another two grade values y_3, y_4 between expected value $\mu_{X_{ij}}^{Ex}(x)$ and lower bound $\underline{\mu}_{X_{ij}}(x)$. The weight (grade value) for practical antecedent i and practical rule m is calculated as:

$$\begin{aligned} w_{im} &= \mu_{X_{ij}}^{Ex}(x) * f_{X_{ij}}^{Ex}(x) + \overline{\mu}_{X_{ij}}(x) * \overline{f}_{X_{ij}}(x) + \underline{\mu}_{X_{ij}}(x) * \underline{f}_{X_{ij}}(x) \\ &+ \frac{y_1 + y_2 + y_3 + y_4}{4} * (1 - f_{X_{ij}}^{Ex}(x) - \overline{f}_{X_{ij}}(x) - \underline{f}_{X_{ij}}(x)). \end{aligned} \tag{36}$$

In our model, we not only consider probability for upper, lower, expected value, but also include other probability of randomly chosen four values. Here, we briefly provide the fuzzy reasoning and type-reduce procedure for statistical interval-valued type-2 FLS.

1) Obtain weights for each antecedent i and each rule m using (36). If there is only one antecedent (i.e. one linguistic variable), then we directly go to step 3). If there are more than one linguistic variable, we go to step 2).

2) Determine the firing strength for each rule with algebra product t-norm (3) or minimal t-norm (2).

3) Applying Mamdani model, we obtain the result of consequent for the m th rule

$$\mu_{c'}(z) = w_m \wedge \mu_c(z), \tag{37}$$

where $\wedge$ denotes interaction between fuzzy sets.

4.2 Defuzzification for statistical interval-valued type-2 FLS

We get the result of consequent for each involved rule. The fire strength is one number instead of an interval value. If the consequent membership function is type-1, we can directly use centroid of area to calculate final crisp output value via (4).

If the consequent membership function is type-2 fuzzy logic membership function, we have different centroids of area. First, do the type-reduce by

$$Z_{COA} = \overline{Z}_{COA} * 0.1 + Z_{COA}^{Ex} * 0.8 + \underline{Z}_{COA} * 0.1 , \tag{38}$$

where Z_{COA} denotes the final result, $\overline{Z}_{COA}$ is a centroid of area from upper bound membership function, $\underline{Z}_{COA}$ is a centroid of area from lower bound membership function and Z_{COA}^{Ex} is a centroid of area from expected value membership function.

The different membership functions are obtained by using the method that is described in Chapter 3. The constant in (38) indicates that weight for each MF. From statistical result, the most value (around 80%) will be close to the expected value, and probability of upper bound and lower bound is very small, each is only equal to 0.1.

4.3 An example of statistical interval-valued type-2 FLS via linear regression

In this section, we apply our proposed approach to a sample example: a learned function of pyramid. The two inputs will be x and y coordinate. To simplify this problem, we assume the same three linguistic values and function for input variables: low, middle, and high. The output will be z coordinate. There are two linguistic values in output variable: low and high. Now we define output as a type-1 fuzzy set.

We use data generation to simulate 30 experts to define different linguistic variables and terms for x and y coordinates. The upper, lower, expected value membership functions for each linguistic value of x and y coordinates are obtained by applying the method as that in Chapter 3. Figure 7 shows the membership functions of x and y coordinates generated by our method and Figure 8 shows the output – z coordinate membership function used in this simulation.

In order to generate the probability function, we add some error to it and get an interval value:

$$\mu^{Ex'} = \mu^{Ex} \pm (\overline{\mu} - \underline{\mu}) * 10\%, \qquad (39)$$

where μ^{Ex}, $\overline{\mu}$ and $\underline{\mu}$ are expected value degree, upper bound and lower bound degrees, respectively. Then derive the probability of expected grade value

$$p(x) = \frac{m}{N}, \tag{40}$$

where m is the number of values in $\mu^{Ex'}$ and N is the number of experts. Then, following the regression model, we produce a probability function. We do not add error when calculating probability of upper bound and lower bound.

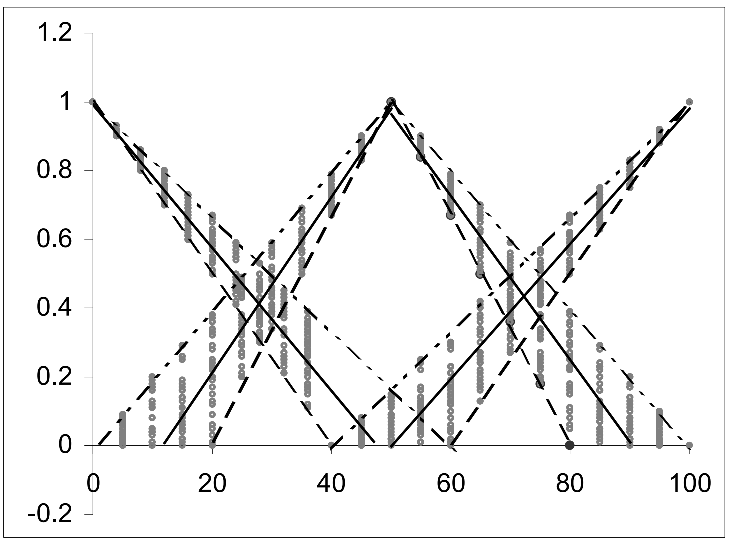

Figure 7. The Type-2 membership function for x-coordinate and y-coordinate that is generated by statistical method. Solid lines indicate expected MFs. Two doted lines are upper bound MFs. Doted lines are lower bound MFs.

In this simulation, we use the fuzzy rules that are from [16]:

If y = low. Then f (x, y) low,

If x = low. Then f (x, y) low,

If x = med and y = med. Then f (x, y) high,

If y = high. Then f (x, y) low,

If x = high. Then f (x, y) low.

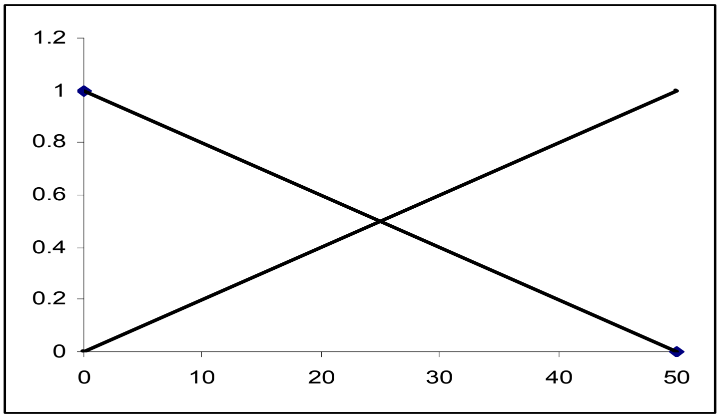

Figure 8. Membership function for output F (x, y).

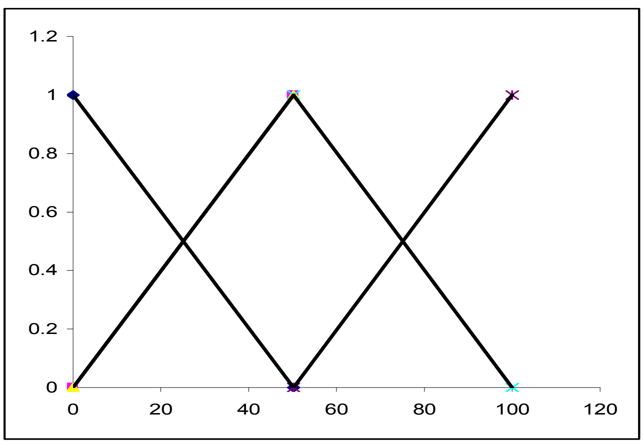

Figure 9. Type-1 member function for x and y coordinates.

The Equation (36) and the algebra product t-norm operator have been used to do the fuzzy reasoning. Since the output membership functions are defined as type-1 fuzzy sets, we use the same defuzzification method with type-1 FLS. We compare our approach with type-1 FLS. The type-1 input x and y coordinate MFs are defined as that in Figure 9. The results that are obtained by the type-1 and statistical interval-valued fuzzy logic system are presented in Figure

10. From the diagram, the interval-valued FLS captures more data; its shape is more symmetric and has a much better fit to the learned function of pyramid than Type-1 FLS.

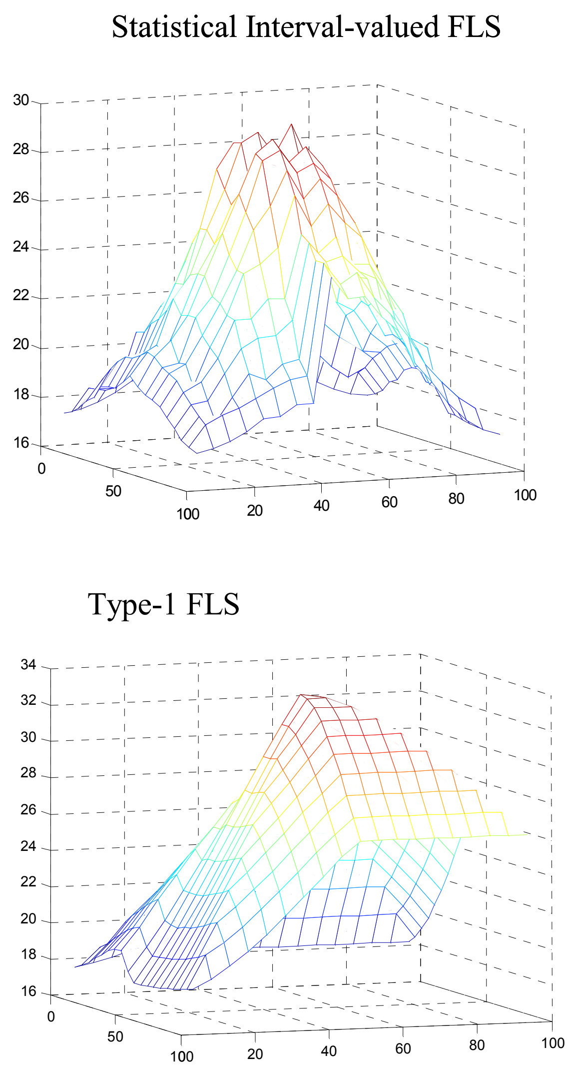

Figure 10. Approximation of pyramid function using statistical interval-valued and type-1 FLS.

4.4 An example FLS using non-linear regression

A traditional washing machine has a number of programs for user to select the type of wash (whites, colors, etc.), size of load, material of cloth (silks, nylons, polyesters, cotton, etc.), temperature and so on. The machine then follows the pre-set routine irrespective of what is actually happening to the clothes inside. At the end of the cycle it stops. But if the clothes were clean half way through the cycle, stopping early would save energy as well as reduce wear and tear on the clothes. On the other hand, the clothes may still be dirty and need more washing.

Applying fuzzy logic system, we can build an intelligence washing machine. In this ‘fuzzy’ washing machine no program selection is required. Sensors are used to determine the size of washcloth and type of fabric and so control the required amount of water and the temperature of the wash. In order to simplify the problem, we use two input and one output Mamdani fuzzy model. Through the input and output variable we use interval-valued fuzzy set to simulate the process of fuzzy washing machine. The simulation system allows users to make selection of fabric type (F), washcloth size (S) (Just same as sensors determined) and the system will provide an interval output for washing temperature (T).

4.4.1 Membership function

We use data generation to simulate the 20 experts to define different linguistic variables and terms.

1. Membership functions for washing cloth size (S)

There are three linguistic values for washing cloth size: low, middle, and high. We are following new method that is developed in Section 3.2 to generate the upper bound, lower bound and expected value membership functions for these three values. Figure 11 shows the result of non-linear MFs.

2. Membership functions for fabric type (F)

In our system, a number between 0 and 10 is assigned to different cloth fabric. The rules are listed as Table 1. There are three linguistic values for fabric: thin, middle, thick. Using the same method for the size of cloth, we get membership function for fabric three linguistic values. Figure 11 shows these non-linear functions.

3. Membership functions for temperature (T)

There are three linguistic values for temperature: low, middle, and high. Figure 11 shows the result from the analysis of 20 different experts' definition.

4. Probability functions

In each discrete input value, we use method presented in Section 3.1 to obtain the expected probability functions for two inputs and one output.

4.4.2 Fuzzy Rules

The fuzzy rules that are based on our knowledge are listed in Table 2. The user can change it if he thinks it is not reasonable.

4.4.3 Output process

Since we need generate an interval valued water temperature for cloth washing, we skip type reduce and directly go to defuzzification step.

4.4.4 Implementation

We implement simulated machine in java script. After user inputs the washing cloth size and material and chooses "type-1" or "type-2" button, the system will generate the washing temperature. Type-1 result is a crisp value. Interval-valued result is interval between lower bound and upper bound temperature. The interface is shown in Figure 12.

Table 1. Rule for assigning number to different fabric

1	Acrylic Acetate	6	Cotton
2	Silk Nylon	7	Heavy cotton
3	Polyester	8	Linen
4	Rayon	9	Jeans
5	Wool	10	Quilt

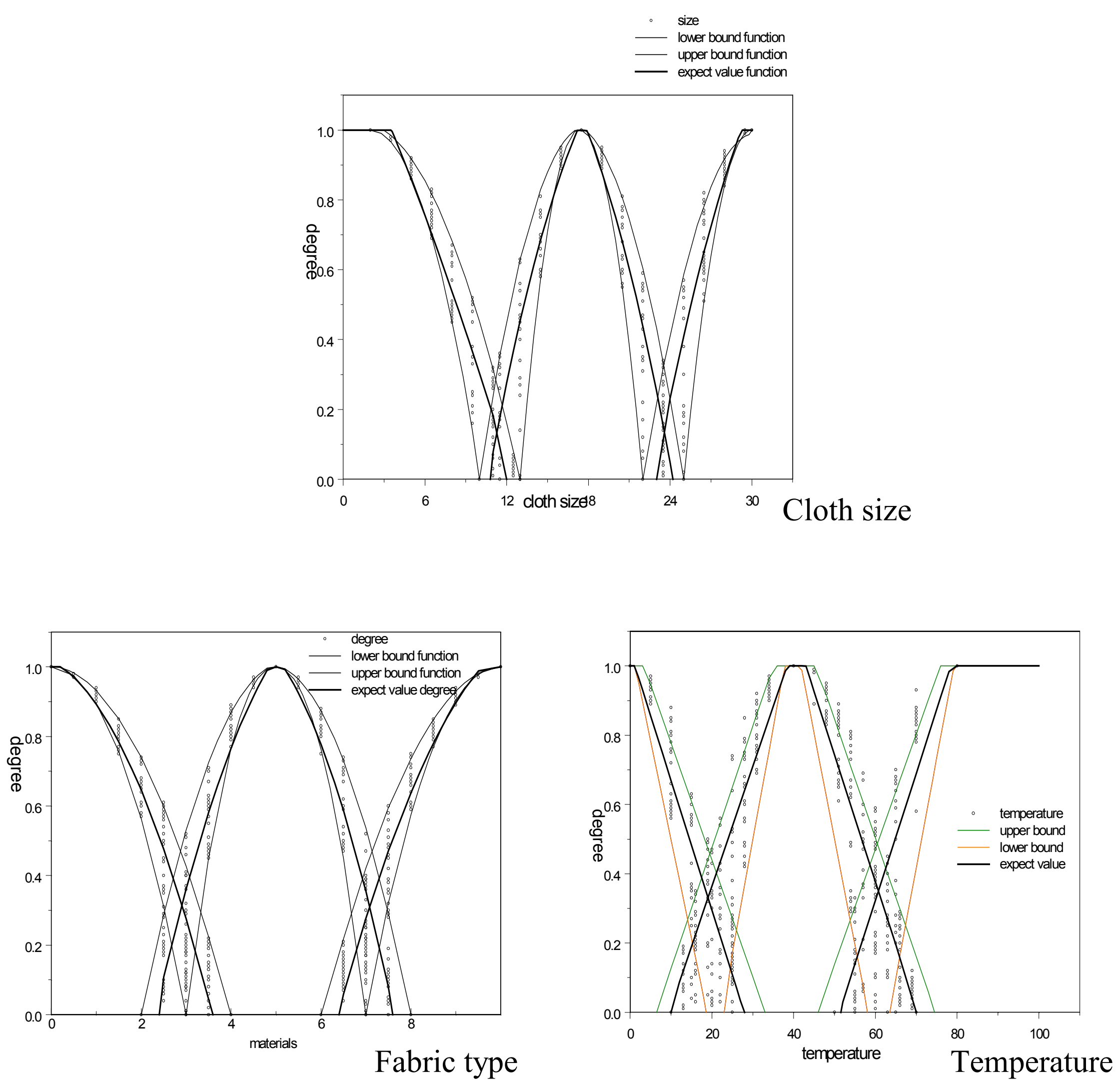

Figure 11. The membership functions for cloth size, fabric type and temperature.

Table 2. Fuzzy Rules for intelligence washing machine

	Input		Output
Rule number	Fabric of cloth	Size of cloth	Washing temperature
Rule 1	Thin	Low	Low
Rule 2	Thin	Middle	Low
Rule 3	Thin	High	Middle
Rule 4	Middle	Low	Low
Rule 5	Middle	Middle	Middle
Rule 6	Middle	High	Middle
Rule 7	Thick	Low	Middle
Rule 8	Thick	Middle	High
Rule 9	Thick	High	High

We run different data combinations: some are one-rule based firing; some are two-rule firing and others are four-rule based firing. We obtain 24 data sets. We compare the type-1 output with interval-valued output. In this implementation, we calculate output by

$$\text{Temperature} = (\text{upper bound} + \text{lower bound})/2. \tag{41}$$

Figure 13 is the comparison of the temperature that is obtained by the type-1 and interval-valued fuzzy logic system.

From the diagram, the interval-valued fuzzy logic system need lower temperature than type-1 fuzzy logic system when the washing clothes size is more than 10 and no matter what a material is. This can save a lot of energy.

Figure 12. The interface of an intelligence washing machine

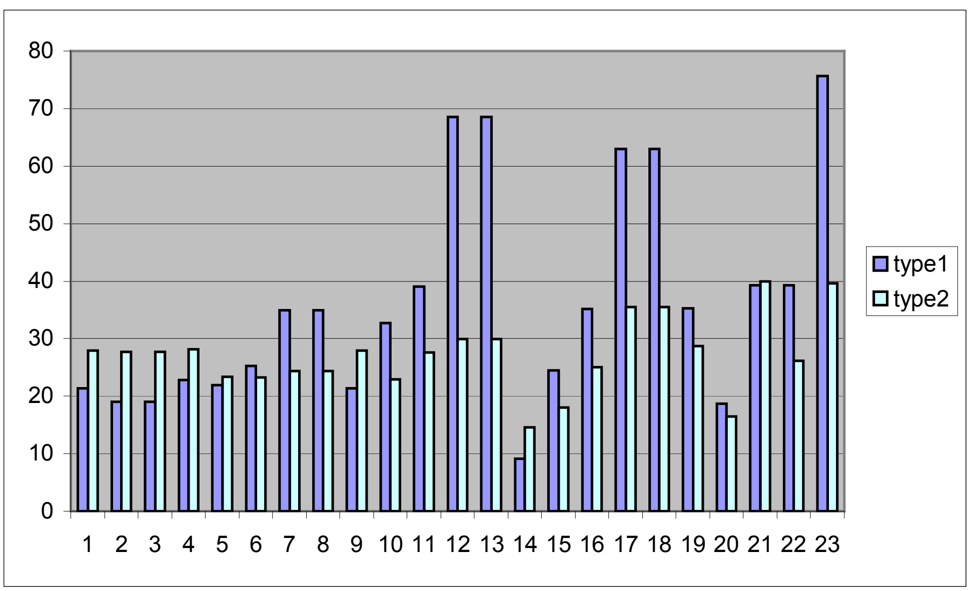

Figure 13. The result of type-1 and statistical interval-valued intelligence washing machine

CHAPTER 5

PARAMETER OPTIMIZATION USING GENETIC ALGORITHM

How to design a fuzzy logic system in order to get result with best performance? There are several important components. The first one is to define the reasonable membership function of the linguistic variables. Different procedures have been studied. Our new procedure is presented at Chapter 3. The second one is to construct the fuzzy rule base. There are some heuristic and iterative methods to optimizing rules. The third one is an efficient fuzzy reasoning method. In this thesis we provide reasoning method in Section 4.1. We can develop an FLS without experimental input-output data following the procedure as described in Chapters 3 and 4. Assume there are real input-output data for particle system that needs to be constructed. To obtain the better performance of an FLS, the parameter of the membership function and probability function need to be optimized. Genetic algorithm is powerful tool to perform the optimization. In this chapter, we introduce the parameter optimization of fuzzy system with genetic algorithm.

5.1 Overview of genetic algorithm

"Genetic algorithms (GAs) are derivative-free stochastic optimization methods based loosely on the concepts of natural selection and evolutionary processes." [17]. As a general-purpose optimization tool, GAs are receiving an increasing attention and application in many fields: GAs can be used in parallel processing machines to increase their operation speed; GAs can be used in both continuous and discrete optimization problems and they are applied to both structure and parameter optimization. In particular, GAs are robust and simple search technique and very powerful optimization tool for complex function.

The important components of GAs include encoding schemes, fitness evaluations, and operators.

Encoding schemes: in this component, the parameter or solution will be transformed to a chromosome which is a binary bit string. The chromosome is composed by genes. For instance, a point (4,5) in a two-dimensional parameter space can be encoded to a binary string

01000101

where 0100 and 0101 are two genes in this chromosome.

Fitness evaluation: the calculation of fitness value of each member in the population will be performed in this component. The GA will be a success with a right selected fitness function.

Operators: GA has three main operators including selection, crossover and mutation. After each evaluation, GA need determine which parents will be selected to produce the next generation. The selection operator means the strategy to do this selection. The crossover operator will select pairs of parents to generate new chromosome. The common mutation operator is to flip a bit.

5.2 Parameter optimization

To build a statistical interval-valued type-2 FLS with experimental data with better performance, parameters for different membership functions and probability functions need to be optimized with genetic algorithm after we apply the approach in previous chapter.

Since the genetic algorithm is a popular optimization tool, many literal and code can be obtained in [25]. The critical task is to define a robust and suitable fitness function for particular problem. The optimized parameter can be obtained through minimizing the error between the observed outputs (target values or real values) and fuzzy system outputs. In this thesis, the following equation is used as fitness function:

$$Error = \sqrt{\sum_{i=1}^{N} (z_i - \overline{z_i})^2}\ , \tag{42}$$

where z_i is the output from the building fuzzy system for each pair of input, the $\overline{z_i}$ is the observed value from the experiments. The error is calculated by using least-squares. For a given input-output data set, we can optimize the parameter using the following general training algorithm:

Begin

1) Read training data
2) Select membership functions' constants as tuning parameters
3) Initialize tuning parameters and population
4) Find the rules for input training data
5) Calculate the fuzzy training output using method which is presented in Chapters 3 and 4
6) Calculate the error using Equation (42)
7) Perform selection, crossover and mutation operators to adjust the tuning parameter and got next generation
8) Repeat step 4 to 7 until the minimum error is found or population size is reached
9) Output: final tuning results

End

5.3 A simple example of statistical genetic interval-valued type-2 FLS

In this section, we use the genetic statistical interval-valued type-2 FLS to simulate the function

$$Z = \sqrt{x^2 + y^2} \ . \qquad (43)$$

We limit the input x and y in [-5, 5]. We define three linguistic values for each input variable: low, middle, and high. There are two output linguistic values: low and high.

Since there are targeted data, we use a table look-up scheme to obtain the fuzzy rule base [18]. First, for each input-output pair (x, y; z), we determine the membership values of x, y and z in their fuzzy sets. For this practical function, that means if x = -2, what is the low, middle and high value's degrees. Second, select largest value of each variable as its degree and obtain the rule as

IF x is {low, middle, high} and y is {low, middle, high} Then z is {low, high}.

Third, to solve the conflict, assign a degree to each rule generated by following step 1 and 2 and keep the maximum degree. The degree is calculated by

$$D(rule_i) = \mu_{xi} * \mu_{yi} * \mu_{zi} \, . \qquad (44)$$

For example, if we generate two rules from input-output pairs:

R_1: If x is low, y is low, then z is low;

R_2: If x is low, y is low, then z is high.

We can get degree for rule 1 and rule 2 by using (44). If rule 1's degree is greater than rule 2, then we just keep rule 1. Otherwise we keep rule 2. Finally, we got linguistic rules from human experts to finish the rule table.

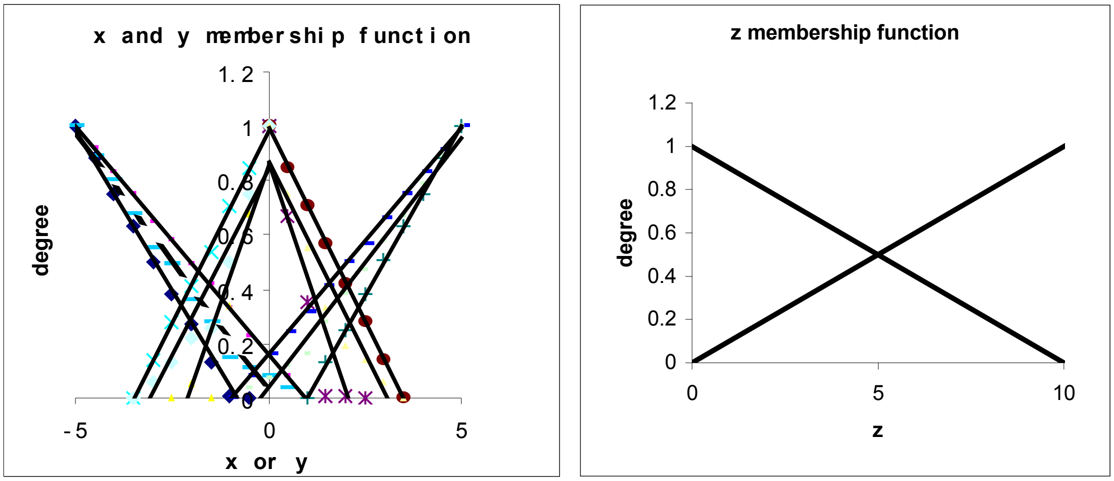

Figure 14. Member ship function for input variable x, y and output variable z

The general steps of building genetic statistical interval-valued type-2 FLS are provided as following:

1) Collect 50 sets of experimental data using equation (43). The x and y can take values--5+ 0.5i, i =0,…, 20.

2) Obtain the upper, lower, expected value membership functions with procedure presented as section 3.1. The input x and y have the same universal and same membership functions. Figure 14 gives the functions for input variables.

3) Obtain the probability function for three linguistic values with the procedure in Section 3.1.

4) Using table look-up scheme that is introduced as before obtain the fuzzy rules. Table 3 shows the result rules for this system. One example rule is: If the x coordinate is Low and y coordinate is Low, then z coordinate is High.

5) Using training algorithm as described in Section 5.2 obtain the optimized parameters. Our proposed fuzzy reasoning and defuzzification method is used to get the output value for Equation (42).

6) Perform the test data using optimized parameters.

Table 3. Fuzzy rules for function (43)

X\Y	Low	Middle	High
Low	High	Low	High
Middle	Low	Low	Low
High	High	Low	High

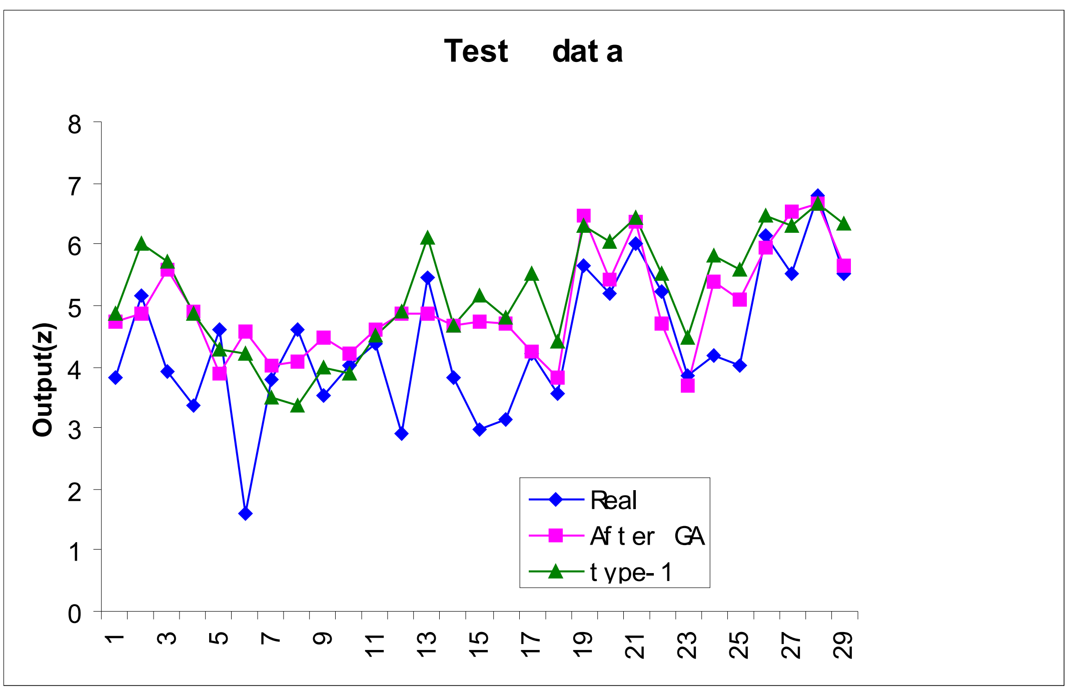

Figure 15. The output for real, genetic statistical interval-valued type 2 fuzzy system and type-1 fuzzy system for function (43)

We select data from 121 pair data as training data, and randomly choose other 28 pair as test data. We also use the same rule, same data to build the type-1 fuzzy system for the function (43). The least-squares error is:

$$E_{type-1} = 6.281347\,,$$

$$E_{gdtype-2} = 5.67884\,.$$

The real value and fuzzy system output are shown in Figure 15. Our system is better than type-1 fuzzy logic system

CHAPTER 6

A PREDICTION FOR CLINICAL TRIAL

In previous chapters, we introduced a novel method to construct a fuzzy logic system and simple example to use our proposed method. In this chapter, we will give a real application for the new method— prediction of the patient survival time in clinical trials.

6.1 Implementation

Krall, Uthoff, and Harley [25] analyzed data from a study on multiple myeloma in which researchers treated 65 patients with alkylating agents. The data set can be found in SAS/STAT User's guide (1999, pp. 2608--2617, 2536--2641). Of those patients, 48 died during the study and 17 survived. In the data set MYELOMA, the variable TIME represents the survival time in months from diagnosis. The variable VSTATUS consists of two values, 0 and 1, indicating whether the patient was alive or dead, respectively, at the of end the study. If the value of VSTATUS is 0, the corresponding value of TIME is censored. The variables thought to be related to survival are LOGBUN (log BUN at diagnosis), HGB (hemoglobin at diagnosis), PLATELET (platelets at diagnosis: 0=abnormal, 1=normal), AGE (age at diagnosis in years), LOGWBC (log WBC at diagnosis), FRAC (fractures at diagnosis: 0=none, 1=present), LOGPBM (log percentage of plasma cells in bone marrow), PROTEIN (proteinuria at diagnosis), and SCALC (serum calcium at diagnosis). Interest lies in identifying important prognostic factors from these nine explanatory variables. In the original data sets, [25] there are total nine attributes. From most research work related this data set, the attributes LOGBUN, HGB are highly related to patient survival time. To simplify the problem, we use the fuzzy logic system which has two inputs: LOGBUN and HGB. These two inputs have the statistical interval-valued type-2 fuzzy membership function to handle the higher-level uncertainties. We choose log (survival time) as response. It has the type-1 fuzzy membership function.

The key of designing fuzzy system is the generation of fuzzy inference model – rule bases as well as membership functions. In most system, the membership function and rule bases are obtained from the experts. To design the genetic statistical interval-valued type-2 fuzzy prediction system, first we need to define linguistic variables and their linguistic values, second we consider membership function for each linguistic value. The third step is obtaining the fuzzy rules. After deciding the reasoning and defuzzification method, the last step is to optimize the parameters. It is well known that LOGBUN and HGB are most significant variables among others. In this example, we only choose these two factors and ignore the other issue to reduce rule complexity. The output is log patient survival time.

From the data set, the value LOGBUN is located in [0.5, 2.5], HGB value is limited in [0, 20]. The response log survival time is limited in [0, 2]. We design three linguistic values to represent the LOGBUN linguistic variable: low, middle, and high. The HGB has two linguistic values: low and high. These two linguistic variables are statistic interval-valued type-2 fuzzy sets. The output survival time also has three linguistic values: short, normal, and long. It is designed as type-1 fuzzy sets.

We simulate 30 experts to design the system, and then use statistical membership generator method which is described in Chapter 3 with the linear regression to create all membership functions and probability function. Figure 16 shows all generated membership function for LOGBUN variable and Figure 17 shows membership function of variable HGB.

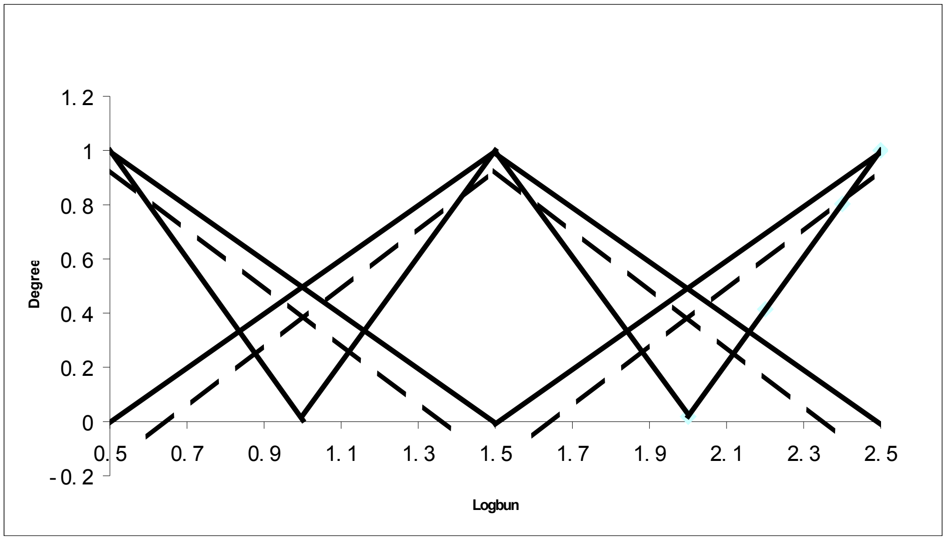

Figure 16. Membership functions for LOGBUN variable. The dashed lines show the expected value MF for low, middle, and high value. Upper bound and lower bounds MF are indicated as solid line

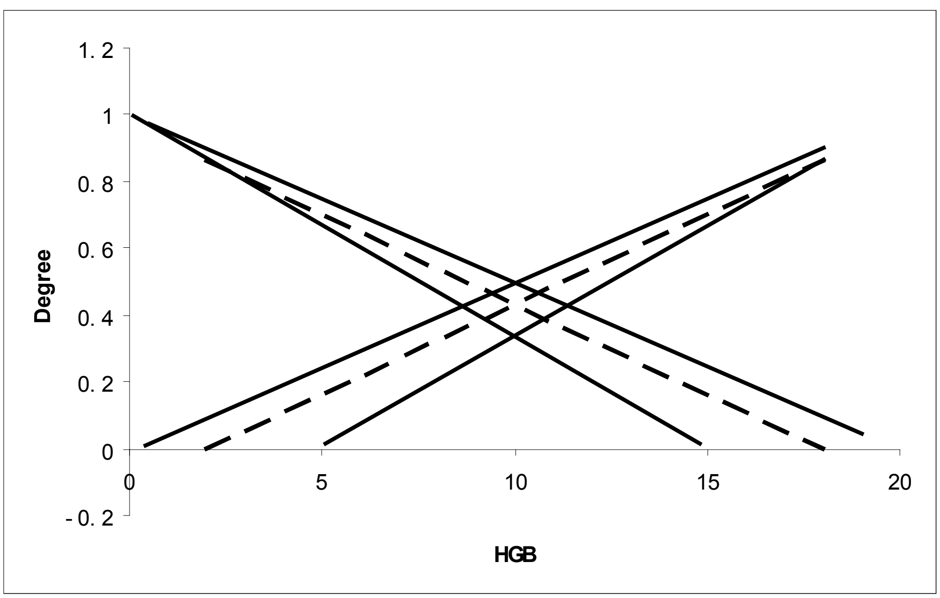

Figure 17. Membership functions for HGB variable. The dashed lines show the expected value MF for low, and high value. Upper bound and lower bound’s MF are indicated as solid line

Since there are two inputs, one has three linguistic values, and anther one has two linguistic values, the rules of the fuzzy system are:

If LOGBUN is {low, middle, high} and HGB is {low, high}, then survival time is {short, normal, long}.

So the number of total rules is calculated by 3*2 that equal to 6. We use the table looking up scheme which is presented in Section 5.3 to find the rule base for this predication. Step by step, we obtain the final rules that are presented in Table 4.

The optimization for parameters is performed by genetic algorithm with least squares error model (see Chapter 5). In this data set, it has total 65 pair data. We randomly select 45 pairs of data set as a training data set and the reminding 20 pairs of data as testing data. The weight for fired rule is calculated as (36) and product t-norm operator (3) has been used. In this example, we just optimize the probability function parameters for each value and we tune the parameter between –1 and 1. The error before tuning that is obtained from the equation (42) is 3.026508, after tuning, the error reduces to 2.715.

6.2 Comparison and Discussion

In Section 6.1, we develop a survival time prediction system based on fuzzy set theory. The proposed genetic statistical interval-valued type-2 fuzzy logic system is employed to handle uncertainties.

In order to compare type-1 and interval type-2 with our system, we design the type-1 and interval value type-2 fuzzy systems by using the same input and output variable. Also the rule base does not change. The sup-star method is used to do interval type-2 fuzzy reasoning. Table 5 shows the test data, expected output and the results with type-1, interval type-2, genetic statistical interval-valued type-2 fuzzy system.

Comparing with type-1 and regular interval type-2 fuzzy systems, the least-squares error is minimum for our system. Table 6 shows the log survival times that are obtained with type-1, interval type-2 and proposed fuzzy logic system for the test data and error for different system. Figure 18 shows prediction for these data using the three methods. Genetic statistical interval-valued fuzzy system provides best performance. In this 20 pairs of test, more than half of the outputs for our system are close to expected value, especially number 10, 11, 12, 15, and 19.

Table 4. Fuzzy rules of the prediction system

LOGBUN\HGB	L	H
L	Normal	Normal
M	Normal	Long
H	Short	Short

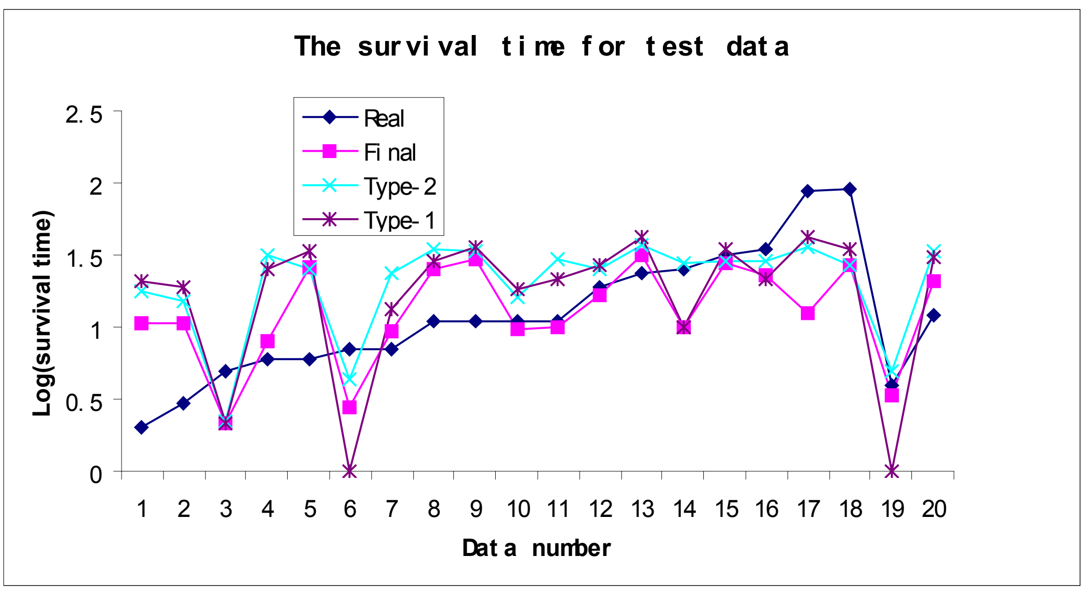

Figure 18. The survival time for different system.

Table 5. Error for different fuzzy system

	LOGBUN	HGB	Expected Logtime	Type-1	Type-2	Proposed
1	1.3010	5.1	0.301030	1.315667	1.243258	1.027297
2	1.5441	6.7	0.477121	1.273346	1.175004	1.021153
3	2.2355	10.1	0.698970	0.331970	0.340499	0.337110
4	1.1139	9.7	0.778151	1.406552	1.497576	0.896477
5	1.4150	10.4	0.778151	1.521937	1.409155	1.419568
6	1.9777	9.5	0.845098	0.000000	0.645378	0.439668
7	1.0414	5.1	0.845098	1.120476	1.370090	0.966816
8	1.1139	14	1.041393	1.454544	1.536541	1.398267
9	1.2304	12	1.041393	1.548984	1.534298	1.467223
10	1.5682	7.5	1.041393	1.263958	1.205480	0.983281
11	1.0792	9.6	1.041393	1.338709	1.479142	0.994134
12	1.2553	7.5	1.278754	1.429658	1.400939	1.220086
13	1.3010	14.6	1.380211	1.622450	1.567620	1.496851
14	1.0000	12.4	1.397940	1.000000	1.444564	1.000000
15	1.3222	10.6	1.505150	1.536283	1.453930	1.437839
16	1.1139	7	1.544068	1.334212	1.454408	1.366922
17	1.3222	14	1.949390	1.619696	1.552790	1.097737
18	1.4314	11	1.963788	1.541808	1.429473	1.425242
19	1.9542	10.2	0.602060	0.000000	0.700283	0.533147
20	1.1461	11.6	1.079181	1.479624	1.532312	1.319015
Error for test data				2.279686	2.040072	1.778261

CHAPTER 7

CONCLUSION

The type-2 or interval-valued fuzzy system can be very efficient to handle the uncertainty in human decision. How to design an optimized interval-valued fuzzy membership function in real world is a very challenging problem. In this thesis, a simple statistical method has been proposed to define a reliable and reasonable membership function using linear regression. This approach considers expected value, upper and lower bound probability to calculate the weight of a fired strength after generating their MFs. In Chapter 4, a simple example to perform learned function of pyramid shows that our approach provides a more useful technique to design a real interval-valued FLS and it is more accurate than type-1 FLS.

To obtain more accurate result, we extend the method with non-linear regression. In this thesis, we generate data and apply new non-linear regression method that is proposed in Section 3.2 to implement a smart washing machine. From simulation, the new method is more reliable and robust than type-1 FLSs.

In order to achieve best performance, the parameters in the membership function generated by our method are optimized by genetic algorithm with least-squares error as fitness function. From the two simulations' result, the performance of genetic statistical interval-valued type-2 fuzzy logic system is better than type-1 and regular interval type-2 FLS. The interval-valued FLS using statistical method can be applied to many areas, such as control systems, shopping systems, decision making system prediction system and so on.

Due to the time limit, there are still a lot of works that need to do in the future. For instance, we plan to try other simulations, try to use a large data set instead of small data set to compare the different fuzzy logic system. We will do a real survey to collect data and perform the comparison and so on.

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