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An Investigation of Conceptual Knowledge: Urban African American Middle School Students' Use of Fraction Representations and Fraction Computations in Performance-Based Tasks

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ACCEPTANCE

This dissertation, AN INVESTIGATION OF CONCEPTUAL KNOWLEDGE: URBAN AFRICAN AMERICAN MIDDLE SCHOOL STUDENTS' USE OF FRACTION REPRESENTATIONS AND COMPUTATIONS IN PERFORMANCE-BASED TASKS, by SANDRA ANN CANTERBURY, was prepared under the direction of the candidate's Dissertation Advisory Committee. It is accepted by the committee members in partial fulfillment of the requirements for the degree Doctor of Philosophy in the College of Education, Georgia State University.

The dissertation Advisory Committee and the student's Department Chair, as representatives of the faculty, certify that this dissertation has met all standards of excellence and scholarship as determined by the faculty. The Dean of the college of Education concurs.

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ABSTRACT

AN INVESTIGATION OF CONCEPTUAL KNOWLEDGE:
URBAN AFRICAN AMERICAN MIDDLE SCHOOL STUDENTS' USE OF FRACTION REPRESENTATIONS AND COMPUTATIONS IN PERFORMANCE-BASED TASKS
by
SANDRA A. CANTERBURY

A relatively large number of 8th-grade public middle school students in the United States, particularly in urban communities, are not performing at acceptable levels in mathematics. One concept that poses significant difficulty for these students and negatively affects their overall mathematics achievement is fractions. Many researchers have attributed these difficulties primarily to traditional fraction instruction that emphasizes procedural rather than conceptual knowledge. Therefore this study was designed to investigate how students use their computational and conceptual knowledge and fraction representations to solve fraction-related performance-based mathematical tasks. Social constructivism was used as the theoretical framework in examining conceptual knowledge related to learning fractions.

This qualitative study was implemented in an urban middle school in the southeast. It involved an initial sample of 37, 8th-grade, African American pre-algebra students who completed a fraction interest questionnaire and two fraction pretests. During the implementation period, 34 students in the researcher's pre-algebra class completed three performance-based tasks, three reflection logs, and participated in an interview after completing each task. Of the 34 students who completed all tasks, three were

purposefully selected as the informants for the study. In addition, observations, field notes, and artifacts (student work) were utilized to facilitate triangulation of the data.

The findings of the study indicated the informants could compute fractions with an average of 85% of mastery but could conceptualize fractions only to a small extent. This validated prior findings and led to the conclusion that student deficiency with fractions results primarily from their level of conceptual knowledge. In the investigation of the ways in which 8th-grade students use fraction representations, this study found the informants used representations to develop a visual map of their mathematical thinking and reasoning and to check the accuracy of their computations. Therefore, this study suggests, when students' mathematical learning experiences relative to fractions have not emphasized the use of representations to develop conceptual knowledge, they may not be comfortable with the accuracy of the solutions demonstrated in their fractions models.

AN INVESTIGATION OF CONCEPTUAL KNOWLEDGE: URBAN AFRICAN AMERICAN MIDDLE SCHOOL STUDENTS' USE OF FRACTION REPRESENTATIONS AND COMPUTATIONS IN PERFORMANCE-BASED TASKS

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ABBREVIATIONS

ATFI	Attitude Toward Fractions Inventory
CAF	Confidence About Fractions
CRCT	Criterion-Reference Competency Test
EWF	Enjoyment With Fractions
MAF	Motivation About Fractions
NAEP	National Assessment of Educational Progress
NCES	National Center of Educational Statistics
NCLB	No Child Left Behind (Act)
NCTM	National Council of Teachers of Mathematics
NRC	National Research Council
TIMSS	Trends in International Mathematics and Science Study
VFM	Value of Fractions to Mathematics
VMI	View of Mathematics Inventory

CHAPTER 1

INTRODUCTION

Almost three decades ago, the leading national mathematics organization, the National Council of Teachers of Mathematics (NCTM, 1980) called for reform in the way mathematics is taught in order to improve student performance in public schools in the United States. In addition, the NCTM also outlined a 10-year reform program that challenged mathematics educators to shift the focus of instruction and content beyond basic skills objectives to objectives that facilitate more of a problem-solving conception of mathematics.

In 1986, ton continue its push for a better quality of school mathematics and to provide guidelines to help educators to accomplishing this, the Board of Directors of the NCTM established the Commission on the Standards for School Mathematics. The commission's responsibility was to create a coherent vision of what it meant to be mathematically literate and a set of standards to guide the revision of the school mathematics curriculum. Three years later, the NCTM (1989) and the National Research Council (NRC; 1989) called for a closer look at the way students think about and learn mathematics and at mathematics instruction and helped to revitalize mathematics education reform. As a direct result, many public school systems across the United States responded by implementing a variety of reform programs. These programs aimed at improving mathematics instruction in general, instruction for difficult mathematics concept (such as fractions), and ultimately student achievement levels.

However, a perusal of the literature pertaining to mathematics achievement, and in particular the achievement of urban African American students attending public schools in the United States, indicates that during the past several decades, these students have, in general, continuously performed at unsatisfactory levels (Oakes, 1990a, 1990b, 1999; Secada, 1992; Secada & Meyer, 1991) in comparison to their peers in countries abroad (Ferrini-Mundy & Schmidt, 2005; Sheldon & Epstein, 2005). In addition, the literature also indicates that one area of mathematics which has proved to be difficult for many of these students to learn and understand is fractions. This negatively affects their overall performance in mathematics (Test & Ellis, 2005; Wearne & Kouba, 2000) and results in subpar achievement levels in this subject.

This dissertation presents the results of a qualitative study which focused on student performance with fractions through the analysis and assessment of the participants' thought processes, work, and explanations as they sought to find solutions to three performance-based mathematical tasks. I used a case study strategy to determine (a) the extent to which three urban African American 8th-grade students could conceptualize and compute fractions and (b) the ways in which these students used fraction representations (symbolic expressions, physical models, and diagrams that are used to replace fractions) to organize and communicate their mathematical thinking and reasoning when they worked on the three fraction-related performance-based tasks. A performance task is an instructional tool that the NCTM has identified as an important component of the performance-based and standards-based mathematics curricula that have been implemented in states across the United States as a part of mathematics education reform. These tasks are categorized as either small or large depending on their

purpose. They are used as an assessment or an instructional tool, and they can be scored in a nontraditional way via scoring rubrics, which facilitate more effective analysis of the students' thought processes (Danielson, 1997).

Statement of the Problem

In general, in the United States, a large number of public school students and particularly urban African American students are not performing at acceptable levels in mathematics (NCTM, 2005; National Center of Educational Statistics [NCES], 2004; Oakes, 1999; Secada, 1992; Secada & Meyer, 1991). Data obtained from a recent administration of the National Assessment of Educational Progress (NAEP) indicated that African American students performed at a level significantly below expectations and below that of their White peers, and that the achievement gap was wider between White and Black students (NCES). These findings were similar to those reported earlier by Oakes (1990a, 1990b, 1999) and provided evidence that traditional mathematics instruction was still proving to be ineffective for a relatively large percentage of 8th-grade public middle school students and in particular, urban African American middle school students.

Research has indicated that one contributing factor to many students' unsatisfactory performance levels in mathematics is their difficulty with learning and mastering the concept of fractions. This difficulty affects negatively the overall performance and achievement of a large number of middle school students in this subject (Saxe, Taylor, McIntosh, & Gearhart, 2005; Test & Ellis, 2005). Further, a significant number of researchers who have done extensive studies in this area of mathematics have attributed student difficulty with learning fractions to traditional fraction instruction that

emphasizes the memorization and application of rules and procedures for performing primarily computations rather than that which emphasizes conceptual knowledge – the *why* of mathematics and the mathematical knowledge that integrates the concepts underlying the procedures (Groff, 1994, 1996; Lamon, 1996; Moss & Case, 1999; Post, Cramer, Behr, Lesh, & Harel, 1992; Wearne & Kouba, 2000). The information in the next section offers an insight into the factors that influenced the study.

Purpose of the Study

The purpose of this study was two-fold. I sought to determine (a) the extent to which urban African American 8th-grade middle school students could compute and conceptualize fractions and (b) the ways in which these students use fraction representations to organize and communicate their mathematical thinking and reasoning when finding solutions to fraction-related, performance-based tasks. As a means of accomplishing this, two research questions guided this study:

1. To what extent do urban African American middle school 8th-grade students compute and conceptualize fractions when working on fraction-related, performance-based tasks?
2. In what ways do urban African American middle school 8th-grade students use fraction representations to organize and communicate their mathematical thinking and reasoning when working on fraction-related performance-based tasks?

Rationale of the Study

To describe the rationale for this study, I focus on three issues concerning public middle school students' achievement in mathematics in the United States and in

particular, 8th-grade African American middle school students. These are (1) unsatisfactory student performance from an international and national perspective, (2) student difficulties with learning fractions and the impact that this has on their overall mathematics performance, and (3) the Black-White achievement gap in mathematics and the need for reform-based mathematics instruction that meets the needs of these urban African American students.

Unsatisfactory Student Performance in Mathematics in the United States

Students in schools across the United States are taught and expected to learn mathematics, beginning with number recognition in kindergarten. However, international and national studies and achievement assessments continue to suggest that by the middle grades students in the United States perform mathematics at an unsatisfactory level and know less mathematics than their peers in Asian and European Countries (Sheldon & Epstein, 2005). Results from the most recent Trends in International Mathematics and Science Study (TIMSS) comparative achievement study highlighted these deficiencies. Despite the fact that for the first time 8th-grade middle school students from the United States performed above the level of the international average in all content areas, their performances nevertheless placed them at an unsatisfactory achievement level and below that of their peers from abroad (Ferrin-Mundy & Schmidt. 2005; Sheldon & Epstein). On the national front, results from studies done by the National Center of Educational Statistics indicated that though the performance of 8th-grade middle school students who participated in the two most recent NAEP mathematics assessments showed some improvement over their performance in previous assessments, their overall achievement has not yet reached expected satisfactory levels (NCES, 2003; NCTM,

2005). These findings indicate that there is still a definite need for research studies that can shed light on what should be done to improve or rectify this situation, particularly as it pertains to more effective mathematics instructional approaches for African American students attending urban public middle schools.

This study, viewed through the lens of constructivism, facilitated a close examination of (a) the social environment of the classroom in an urban middle school environment, (b) the connections between what is transpiring in the classroom and the constructivist view of how students learn, and (c) the participants' use of fraction representations as they work with the fractions embedded in performance-based tasks. These facets of the study are aligned with the suggestions made by Simon (1994), who stated that educators should use meaningful theories of mathematics learning and should plan and implement various types of instructional approaches that are aligned with the needs of their students. Therefore, as educators implement mathematics reform programs in their middle schools, it is important that they identify the instructional modalities that are best suited to their students' needs, and factors which positively affect student learning, especially the learning of difficult mathematics concepts such as fractions.

Fraction Difficulties and Middle School Students

As far back as the late 1970s, researchers led by Hasemann (1981) conducted studies in the area of fraction teaching and learning and documented their findings on the difficulties that this area of mathematics posed for students. In the ensuing years, other researchers have conducted similar studies with comparable results. Currently, a large number of students continue to experience difficulties with learning fraction concepts. The results from the 2003 and 2005 NAEP mathematics assessments indicated that many

students have not yet mastered fraction concepts (NCES, 2003; NCTM, 2005). These concepts continue to be troublesome for public school students from all levels of schooling and backgrounds but more so for urban African American middle school students who attend public schools (Test & Ellis, 2005; Wearne & Kouba, 2000).

Therefore, it is imperative that educators continue to seek answers to student problems with learning fractions through research studies such as this one and to investigate the impact of mathematics reform programs that facilitate change in instructional methods and approaches and are implemented to aid student improvement and achievement levels in this area of mathematics. This study can allow educators to obtain a better understanding relative to what needs to be done to help African American middle school students to improve their performance in mathematics and subsequently to close the Black-White mathematics achievement gap.

Black-White Mathematics Achievement Gap

Public school students' mathematics achievement levels are not as expected for students across grade levels, levels of schooling, race, and even nationalities. These difficulties continue to persist, despite the implementation of mathematics education reform in many school systems in the United States. The situation is further compounded by the fact that many urban African American middle school students do not have an adequate number of opportunities to benefit significantly from high-level mathematics education programs and/or the instructional changes which result from reform efforts undertaken by school systems (Aksu, 1997; Durodoye & Hildredth, 1995; Martin, 2000; Oakes, Joseph, & Muir, 2004; Secada 1992, 1995). As such, they take fewer higher-level mathematics courses than do their White peers and subsequently perform at lower levels

on national mathematics assessments, thus widening the achievement gap and causing it to persist (Martin; Hall, Davis, Bolen, & Chia, 1999; Mitchell, Hawkins, Jakwerth, Stancavage, & Dossey, 1999; Tate, 2000; Thompson, 2003). Moreover, according to Sheldon and Epstein (2005), problems associated with African American students' proficiency in mathematics are particularly serious in poor schools and school systems.

Despite the fact that concerns about status, quality, and equity in mathematics education in the United States have fueled numerous mathematics reform initiatives at the national, state, and local levels, typically, these programs have not addressed the specific needs (i.e., the cultural needs and learning styles) of socioeconomically disadvantaged students (Durodoye & Hildredth, 1995; Oakes et al., 2004). This contributes to their unsatisfactory performance in mathematics and to the persistent and widening mathematics achievement gap between White and Black students and between suburban and urban middle school students (NCES, 2004; Perie, Grigg, & Dion, 2005). As such, the NCTM's goal of equity – high expectations and strong support for all students – in mathematics education has not been accomplished through the implementation of these reform programs (Sheldon & Epstein, 2005). It is therefore necessary that educators engage in research to obtain a better understanding of the dynamics pertaining to unsatisfactory achievement in mathematics and the difficulties with learning mathematics concepts such as fractions.

Significance of the Study

The results of my study enhance the literature on fractions in the middle school mathematics curriculum, relative to reform-based classroom practices and on the teaching and learning of fractions and the role that fraction representation could play in helping

students to better understand fraction concepts. It also contributes to understandings of fraction learning and the impact of using performance-based tasks and of issues of equity and African American students' mathematics achievement levels.

Fractions in the Middle School Mathematics Curriculum

Researchers have conducted many studies to investigate a variety of issues relative to the concept of fractions. Some of these studies (e.g., Behr, Lesh, Post & Silver, 1983; Bezuk & Cramer, 1989; Bigalke & Hasemann 1978; Hart & Kerslake, 1983; Hasemann, 1981; Hunting, 1983) date as far back as several decades, an indication of the historical importance of this concept in the mathematics curriculum at the elementary and middle school levels and in high school through related and connected algebraic concepts.

In speaking of the importance of fractions in the mathematics curriculum, Behr et al. (1983) stated, "Rational number concepts are among the most complex ideas children encounter during their pre secondary school years" (p. 91). Similarly, Post et al. (1992) posited that fractions have a significant place in the development and structuring of mathematics because the formal teaching of this concept sets the stage for far more sophisticated levels of problem solving. They suggested that because fractions will simply not "disappear" from the mathematics curriculum, educators should consider more effective approaches for teaching this concept than those that are used currently, especially in middle school mathematics classrooms.

Further, the NCTM (2000) has affirmed that the domain of fractions is an integral component of the middle school mathematics curriculum. Therefore, student learning and mastery of these concepts at the middle school level is critical to their success with learning higher-level mathematics and related algebraic concepts at the high school level

(NCTM). However, researchers who advocate instructional changes relative to the teaching of fractions have suggested that because the traditional fraction instructional approach (which emphasizes computations using rules and procedures) has not proved to be very effective for many students, instruction that emphasizes the nontraditional approach (where students are taught to understand the meaning of fractions) should be the norm rather than the exception in mathematics classrooms. The classroom activities that were used in this study were aligned with the NCTM's recommendations that fraction instruction should be conceptually orientated and reform- based. Therefore, this study is significant in that it will augment the literature on fraction learning as it relates to the use of nontraditional fraction instructional strategies and activities in the middle school mathematics classroom that focuses on conceptual knowledge and understanding. This approach fosters conceptual knowledge through the use of symbolic representations of fractions as opposed to focusing on procedural knowledge, which emphasizes primarily the application of rules and algorithms for performing computations.

Fraction Representation and Fraction Learning

My findings contribute to the body of literature on the teaching and learning of fractions relative to the role that the use of fraction representation can play in helping students to understand these concepts better. Groff (1996) suggested that traditional methods for teaching fractions are often ineffective because, though many middle school students learn how to do simple fraction computations using memorized procedures, rules, and definitions, there is not much depth to their understanding: They are unable to explain why the rules and procedures work and have little if any conceptual knowledge of fractions. The NCTM (2000) has therefore recommended that, for middle school students

in particular, fraction instruction should (a) emphasize conceptual understanding, (b) involve the use of fraction representations, and (c) provide students with opportunities to solve the problems in meaningful and real-life contexts.

The NCTM (2000) also suggested that when teaching fraction concepts, teachers in middle school mathematics classrooms should provide their students with a variety of visual images and models of fractions. In addition, the NCTM added that teachers should allow their students to use these fraction representations in conjunction with meaningful mathematics problems and problem-solving activities, such as performance tasks.

Performance-Based Tasks and Fraction Learning

School systems in states across the United States (e.g., Georgia, New York, New Jersey) have implemented mathematics education reform programs to aid in the improvement in student performance in mathematics. Many of these reform-based curricula emphasize the use of performance-based tasks as a means of engaging students in meaningful problem-solving activities and of assessing them through nontraditional, authentic classroom tasks and assessments tools. Therefore, this study is significant from the perspective that its findings can be of value to educators and researchers relative to the use of performance-based tasks as an assessment tool or a classroom activity.

Further, the NCTM (2000) recommended that fraction learning should be situated in meaningful contexts to which students can relate and make connections and from which calculations should arise naturally. Performance tasks provide such situated and meaningful contexts. As such, the use of these tasks in this study is in keeping with the NCTM's suggestions regarding more effective classroom strategies for teaching fractions.

Equity and Urban African American Students' Mathematics Achievement Levels

The school from which the study's participants were selected is located in an urban community of primarily African American residents. African American students have continued to lag behind their White peers relative to their mathematics performance levels (Martin, 2000; Tate, 2000; Thompson, 2003). In discussing its *Equity Principle*, the NCTM stated that mathematics instruction should facilitate equity for all students. However, the data from both the 2003 and 2005 NAEP assessments (NCES, 2003; NCTM, 2005; Perie et al., 2005) pointed to the fact that African American (Black) students continue to lag behind their White peers. There was a 33-point difference between their mean mathematics scores (Perie et al.).

According to Phillips (1997), minority students are often at a disadvantage in terms of the quality of the education that they receive because of the existing social-economic problems in urban schools. These include (a) insufficient instructional materials and school supplies, (b) large class sizes and resulting poor teacher-to-student ratios, (c) teacher shortages, (d) low-teacher salaries, and (e) dilapidated physical facilities. The result is poor student achievement, and these students' performances often lag behind those of their peers who attend more affluent suburban schools. This study is therefore significant in that through the close examination of the data obtained from each of its three cases, it can add to the literature on the problem solving approaches of African American students. In addition, this study can also inform the educational arena relative to mathematics and fraction instructional approaches that may be used to facilitate the needs of urban, African American learners in the middle school mathematics classroom.

Theoretical Framework

In a general sense, this study draws from the constructivist view of how students learn and acquire mathematical knowledge. But more specifically it draws from social constructivism.

Constructivism

Constructivism, a theory of how individuals learn, is based on the work of Piaget (1937), who stressed that knowing is adaptive and therefore that knowledge should be thought of as a short but complete summary of concepts and actions which have proved to be successful (von Glasersfeld, 1995). First, constructivists posit that learning requires self-regulation and the building of conceptual structures through reflection and abstraction (Clements, 1997). They also contend that problems should not be solved solely by the retrieval of answers, but from the perspective that the problem is a personal one and presents an obstacle towards progress and towards a goal. These characteristics of constructivism were aligned with the goals of this study and the instructional approaches I used, which facilitated (a) listening to the participants as they engaged in discussions with each other, (b) focusing on their thought processes and, (c) interpretating their actions in order to build a model of how they used their conceptualizations skills to find solutions to the performance tasks that they worked on.

Constructivism and Knowledge Acquisition

In his discussion on constructivism and its connection to knowledge acquisition, Wood (1995) stated that knowledge is not received passively, either through the senses or by way of communication, but through the active construction of the cognizing individual. A reflection of this constructivist characteristic in the study was the participants’

involvement in meaningful classroom activities in the form of performance-based tasks that provided them with opportunities to participate actively in their learning. They planned and organized their work and shared their ideas regarding the strategies, methods, and fraction representations that they used to arrive at practical and correct solutions to the tasks. This type of learning environment was especially conducive to the three participants' and their classmates engaging in teamwork as they worked with the fractions embedded in the performance tasks and engaged in their learning as a search for meaning of these concepts, constructed mental and physical models of fractions, and reflected on multiple representations (fractions) and of reality (Clements, 1997). In addition to working in groups to find solutions to the three performance tasks, the participants were required to work on the tasks on an individual basis at first. This allowed them to bring their own perspectives about the problem to the learning environment as they actively created, interpreted, and organized their knowledge in individual ways (Hiebert & Carpenter, 1992; Windschitl, 1999) and to make sense of the subject matter (Windschitl).

Social Constructivism

This study was based specifically on the tenets of social constructivism. Social constructivism recognizes that the individual constructs knowledge and that knowledge is also concurrently socially constructed. Social constructivism is based on the philosophy of Vygotsky (1978), who believed that within a group an individual learns from social interactions first and from individual experiences, later. He also believed that the purpose of education was to develop the personalities of the students and that this would in turn influence the discovery and expression of their creative potential. As such, Vygotsky

posited that methods of teaching and learning should (a) relate to the development of students as individuals and as members of a group in which they actively participate and (b) facilitate communication and collaboration between teachers and students and between the students themselves. Within the groups, the social and individual processes are interactive, with the result that groups act to construct knowledge and to resolve the differences in the meanings of individual group members (Simon & Shifter, 1991). My using social constructivism as a framework for the study facilitated the participants' engagement in social interaction and communication with their peers as they discussed and decided on solutions to the performance tasks. According to the constructivist view of learning mathematics, these small-group interactions allowed the participants to collaborate and exchange the ideas as they engaged in problem-based learning, inquiry-related activities, and dialogue.

Constructivism was an appropriate theoretical framework for the study. This constructivist lens allowed me to obtain a better perspective of how the students' learning styles, cultural backgrounds and social characteristics played a role in their construction of knowledge and their social interaction in the classroom (Gergen, 1994). There was a natural and logical connection between constructivism and social constructivism and what I was attempting to accomplish from this study relative to the social and mathematical needs of the participants. Further, my using social constructivism as the primary framework for the study provided opportunities for the participants to work in a learning environment that was better suited to their learning styles and to their social and educational needs in the mathematics classroom.

Summary

The purpose of this study was to determine the extent to which urban, African American middle school 8th grade students compute and conceptualize fractions when working on fraction-related, performance-based tasks and the ways in which these students used representations of fractions to organize and communicate their mathematical thinking and reasoning as they worked on finding solutions to these tasks. The focus of the methodology of this qualitative study, which used a case study method of inquiry, was on analyzing the data obtained from (a) a nontraditional assessment tool (performance-based tasks), (b) one-on-one interviews, (c) individual and group written reflections, and (d) participant-observer notes.

The fraction instructional approaches that were used in this study were in direct contrast to those used for traditional fraction instruction, which is centered on the teacher's demonstration of procedures for doing fraction computations involving the four basic operations. The computations are based on the application of specific rules for doing each type of computation followed by student practice of these procedures and application of related rules, using worksheet or textbook examples. Research on the teaching and learning of fractions has indicated that this instructional method, which promotes rote learning and the memorization of rules and procedures, results in most students' acquiring primarily procedural knowledge, but lacking conceptual knowledge. Subsequently, these students forget the rules or apply them incorrectly. As a result, traditional fraction instruction has proved to be ineffective for a relatively large number of students at all levels of schooling but more so for urban African American middle school students.

Based on these findings, researchers and mathematics education reform writers such as those from the NCTM (2000), have suggested that when teaching fraction concepts educators should employ instructional approaches that focus on the students' gaining primarily conceptual rather than procedural knowledge. They also stated that students should be provided with opportunities to work on meaningful mathematics tasks (e.g., performance-based tasks) that (a) involve fractions, (b) exist in contexts to which the students can relate, and (c) allow students to work with a variety of representations of fractions to stimulate their creative thinking and mathematical thought processes.

In this study, the participants worked on performance-based, fraction related tasks individually and then in groups by sharing their solutions and communicating their ideas and thoughts in order to decide on correct and practical solutions for the tasks. The purpose was to determine how well they compute and conceptualize fractions and the ways in which they use fraction representations to communicate their mathematical thoughts and reasoning after 3 weeks of traditional fraction instruction. In essence, a critical question was "What do the participants' abilities to compute and conceptualize fractions and to use fraction representations for finding solutions to fraction-related performance-based tasks indicate about their readiness in these areas to meet the challenges of their state's new performance-based mathematics standards and curricula that will de-emphasize traditional mathematics instruction?"

The chapter that follows presents the literature review. It will provide a discussion on (a) the literature generated from studies on the teaching and learning of fractions, (b) the history of student difficulties when learning fractions at the elementary and middle school levels, (c) the Black-White mathematics achievement gap, and (d) the

difficulties that teachers experience when teaching fractions. This study also highlights the use of performance-based tasks, a component of an alternative approach to traditional fraction instruction and assessment that can help to alleviate the difficulties that many middle school students experience when learning fractions via traditional instructional approaches and methods.

CHAPTER 2

REVIEW OF THE LITERATURE

In the literature on the teaching and learning of fractions, a large body of data exists on the difficulties that students at all levels experience when learning fractions. Throughout the research, the most frequently cited source of student difficulty with learning and understanding fraction concepts is traditional fraction instruction (Lamon, 1996; Moss & Case, 1999; Wearne & Kouba, 2000). In addition, researchers have also advanced theories and cited reasons for this, and they, along with educators, have offered suggestions for student improvement in this area. For example, the National Council of Teachers of Mathematics (2000) has recommended alternatives to traditional fraction instruction and assessment aimed at facilitating student improvement and achievement in this area. These recommendations and research studies, along with the students' deficits in knowledge of fraction concepts, provide a basis for this research study.

In this chapter, I highlight the literature generated from studies on the history of student difficulties with learning fractions in a broad sense. This includes difficulties that, in particular, elementary and middle school students experience when learning this form of rational number, the instructional challenges that some teachers face when teaching fraction concepts, and alternative approaches to traditional fraction instruction and assessment. The chapter is organized into several sections to facilitate the discussion of the review. First, even though the focus of this study is on fractions, an area of mathematics that poses many difficulties to a large number of students attending public

schools in the U.S., in general these difficulties affect student achievement levels in mathematics. As such, I begin with a discussion of the unsatisfactory level of public school students' performance and achievement in mathematics in general in the United States. Second, because the participants in the study are urban, Africa American middle school students, their unsatisfactory mathematics achievement levels and the resulting Black-White achievement gap. This is followed by a brief description on the overarching theoretical framework for this study, constructivism, and by a more detailed discussion of social constructivism which lends itself to the study. This theoretical framework provides the lens for closely examining the literature and for conducting the study.

The next section summarizes the literature on traditional fraction instruction relative to (a) the history of student difficulties with fractions, (b) fraction difficulty at the elementary school level, and (c) fraction difficulty at the middle school level. Although the focus of this study is on urban middle school students, the literature regarding elementary students' difficulties with learning fractions (Lamon, 1996; Wearne & Kouba, 2000) is also discussed because, from all indications, the problems that middle school students experience when learning fractions begin at the elementary level and manifest themselves more intensely in the middle grades. In addition, the connection between these two levels of schooling and student difficulty with fractions is also discussed because, according to constructivists, researchers should trace the development of fraction concepts in children and use this information in constructing the curriculum so that it reflects the students' natural development (Pitkethly & Hunting, 1996).

In the next section, I discuss some of the alternatives to traditional fraction instruction methods, that is, nontraditional instructional and assessment approaches and

tools. This includes the use of (a) performance-based tasks, (b) fraction representations, and (c) scoring rubrics. Use of these approaches and materials is aligned with the new mathematics curriculum and performance standards that have been or will be soon implemented in school systems in the state where this study originated.

Mathematics Education in the United States: 1950s to Present

In the 1950s, the primarily algorithmic mode of traditional mathematics instruction required little discussion or input from the students, did not afford them opportunities to participate in classroom activities which required a higher level of cognitive processes such as critical thinking, and viewed the student as the receiver of knowledge transmitted from the teacher and the textbook. This approach to teaching mathematics emphasized computation because the traditional view of mathematics held that students needed to master computational skills before application and thinking skills could be engaged (Stiff, 2002). However, by the late 1950s, educators realized that this form of instruction did not adequately provide students with an acceptable level of mathematics literacy, nor did it give them opportunities to participate in the learning process from a highly cognitive standpoint. There was need for change and an improvement in mathematics instruction if students were to be adequately prepared for a changing world.

The launching of Sputnik in 1957 initiated the implementation of mathematics education reform in school systems in the United States. The 1960s led to a transition to a formalist approach with the introduction of “new math,” which unfortunately did not address the needs of many students, primarily those who were economically dis-advantaged (Butler-Kahle, 1999). According to Herscovics (1996), this led to the “back

to the basics" trend in the 1970s, then to the early beginnings of constructivism in the 1980s. This approach, unlike others, focused on the learner as opposed to the teacher. Educators of each era sought a more effective method of mathematics instruction, one that would (a) allow students to more actively participate in their learning, (b) use curricula that were more reflective of the societal changes such as the demographics of diverse student populations, and (c) allow students to think at higher and more critical levels.

As the need for an improvement in mathematics education became more evident, the NCTM (1980) called for reform in the way mathematics was taught in public schools in the United States to improve student performance. The NCTM also outlined a 10-year reform program that challenged mathematics educators to move the focus of mathematics instruction and content beyond basic-skills objectives to a more problem-solving conception of mathematics content and instruction. Subsequently, the NCTM (1989) and the NRC (1989) called for a closer look at the way students think about and learn mathematics and at mathematics instruction and helped to revitalize mathematics education reform.

As a result, during the 1990s, there was a resurgence of efforts at mathematics reform. These efforts were influenced by the findings of data from the National Council of Education Statistics (1998) via its National Assessment of Educational Progress mathematics assessments administered to 4th-, 8th-, and 12th-grade students attending schools in the United States. The data indicated that these students were not performing at satisfactory levels in mathematics. In the ensuing years, and in response to the NCTM, other educational organizations, including the Trends in International Mathematics and

Science Study (TIMSS, 1995, 1999), increased their efforts to find answers to the problem of substandard mathematics performance by a relatively large percentage of students attending public schools in the United States, especially those at the middle school level. These organizations administered mathematic assessments, conducted research studies at the national and international levels, and published their findings.

Further, during the more than two decades since the NCTM suggested that educators make efforts to improve mathematics instruction and student performance in this subject, educators in many states have implemented mathematics reform programs in public schools and in particular in public middle schools. Nevertheless, by the early 2000s, despite these efforts at improving student achievement in mathematics education, the situation had not improved significantly: Student performance in mathematics was still at low levels.

Currently, the situation regarding student achievement and performance in mathematics is somewhat more hopeful than in previous years. The results from the most recent TIMSS comparative achievement study and recent NAEP assessments were somewhat more promising. The findings of the TIMSS international comparative study of mathematics achievement indicated that 8th-grade middle school students from the United States performed above the international average in all content areas, thus showing a slight improvement over their average performance in the 1999 study (Ferrini-Mundy & Schmidt, 2005). On the two most recent NAEP assessments, the data indicated that, in general, these students have shown improvement in their mathematics performance. However, they still have difficulties with mastering some mathematics concepts such as fractions (NCES, 2003; NCTM, 2005), and their overall mean

performance in mathematics has not yet reached the expected satisfactory levels. On the other hand, the data obtained from these assessments relative to subgroups were not as promising. Yet again the findings indicated that the achievement gap in mathematics between White and Black students and between suburban and urban middle school students continued to persist (NCES, 2004; Perie et al., 2005).

Mathematics Performance and the Achievement Gap: Equity for All?

The unsatisfactory mathematics performance and achievement levels of students who attend public schools in the United States have been a concern for educators and members of the public for many years. They have subsequently fueled the implementation of a number of mathematics education reforms, primarily at the middle grades level. However, despite the proliferation of these reforms, beginning in the late 1980s, the data from international and national assessments, such as TIMSS and NAEP have continuously indicated that public school students in the United States were not performing at satisfactory levels in mathematics. And even more disheartening is the fact that some groups of students were not benefiting from these reforms. For example, African American students, particularly those who attend urban public schools, were still not performing at satisfactory levels in mathematics and are therefore outperformed by their White peers (Tate, 2000; Thompson, 2003; NCES, 2004; NCTM, 2005).

Mathematics Achievement Levels

Data from the TIMSS (1995) comparative study indicated that middle school students from the United States were outperformed by their peers abroad (Schmidt, 1996). Although later TIMSS (1999, 2003) results indicate that U.S. students have improved

their performance over that of the 1999 administration, their performance was still not at desired levels.

The findings from the 1995 NAEP mathematics assessment indicated that the performance of African American students continued to lag behind that of their White peers (Martin, 2000; Secada, 1995). The NAEP has monitored the progress of students in mathematics (and reading) since the early 1970s. Recent data on student achievement in mathematics (National Assessment Governing Board, 2003; NCES, 2002, 2003, 2004) have revealed that, even after more than three decades of these reform programs, an unsatisfactory number of 8th-grade, middle school students who attend public schools, are performing below the “Proficient” level in mathematics, though in general their performances have begun to show some improvement. The “Proficient” level requires that the students are able to apply the mathematics they have learned to different, often unfamiliar situations and to set up and solve the problems that they are given. Although these students had begun to show improvement in their mathematics performance, only 22% of the almost 160,000 tested nationwide obtained scores which placed them at the exigent “Proficient” level, an indication that they are competent in challenging subject matter and analysis and that they can make applications to real-world situations (Perie et al., 2005). In addition, while about 40% of White and Asian students read at or above the *Proficient* level, only 14% of African American students read at this level. Because the ability to read and to comprehend what is read is vital to students being able to success at other subjects, incompetence in reading affects negatively their performance in mathematics and this contributes to the continued widening of the Black-White performance gap in mathematics.

The Black-White Achievement Gap

The most recent NAEP (Perie et al., 2005) results have indicated that, in general, while student performance in mathematics has improved, the performance gap between 8th-grade Black students and White students remains intact (Perie et al., 2005). In addition, the data obtained from studies of reform-oriented mathematics classrooms indicated that in urban schools whose population consists of primarily African American minority students, mathematics education reform programs have been ineffective. For the most part, the vast majority of these students still struggle with learning mathematics (Martin, 2000; Thompson, 2003). This was attributed to the fact that, though these students are of concern to their teachers, they are rarely the focus of mainstream mathematics education research (Baxter, Woodward, & Olson, 2001). Several years prior, Aksu (1997) came to a similar conclusion and added that only a few reform programs focused specifically on helping African American and other minority students to become more proficient at mathematics. It is therefore not surprising that the NAEP results (NCES, 2003; Perie et al.) indicate that the Black-White performance gap still exists. The results also showed that the mean score for 10 urban school districts that participated in this assessment was 259 out of a possible 500 and that the performance gap in mathematics achievement between urban and suburban students continues to persist.

In reflecting upon the various attempts at school reform and the plight of students who attend low-performing urban schools, Tate (2000) stated that for many students who attend urban schools, particularly African American students, the late 1950s and 1960s could be best characterized as the era of “benign neglect” with respect to mathematics reform. He added that unfortunately, some 50 years later, many students, and in

particularly minority students in attendance at urban public schools, are still not receiving either a level of education or educational opportunities that are equitable to those of students attending similar schools in suburban school systems. The NCTM (2000) suggested that one reason for this is the societal belief in North America that only some students are capable of learning mathematics. Therefore, irrespective of the numbers of years of mathematics education reform, many ethnic minority students will continue to find the application of mathematical knowledge at even basic levels very difficult (Bruning, Schraw, & Ronning, 1999). However, the NCTM (2000) requires equity in the form of high expectations and strong support for all students. To that end, it recommended that middle grades mathematics learning should provide a rich experience that prepares these students to use their mathematical knowledge to deal effectively with real-life situations in and out of the school environment and for their continued study of mathematics in high school. However, as indicated in the national and international data, African American students are not benefiting sufficiently from either reform programs or from the NCTM's recommendations relative to equity for all students in mathematics.

Educational Equity and African American Students

Some scholars and researchers have posited that the system of education in public schools in the United States is differentially effective for students (in several academic subjects, including mathematics) depending on their social demographic characteristics such as race, ethnicity, social class, and gender (Secada, 1992, 1995; Tate, 2000). Hilliard (1995), in speaking to the issue of ethnicity, posited that one of its ramifications relative to the academic achievement of African American students is that these students do not learn in the same manner or at the same pace as their peers in other majority groups. He

stated that often mathematics instruction does not match the learning styles of African American students and as a result, it negatively affects their academic performance. He suggested that educators must match their teaching styles to their students' learning styles and cultural needs because this is critical for delivering instruction to minority students especially. He added that it is important that teachers not only understand the factors that influence the way in which this group of students learn but also know how to make the connection between these factors and improving instruction for these students.

Durodoye and Hildredth (1995) concurred with Hilliard and added that focusing on how African American students learn and connecting this knowledge to efforts aimed at facilitating their academic achievement are crucial to these students' success in learning and mastering mathematics concepts and in narrowing or eliminating the mathematics achievement gap which exists between African American students and White students. They added that when instruction does not facilitate the learning styles of a group, this might result in conflict and this ultimately negatively affects student learning and performance.

Although the NAEP (NCES, 2003, 2004; Perie et al., 2005) results have indicated that our nation's urban African American middle school students are not yet performing at satisfactory levels in mathematics, some educators believe that the current reform efforts may be a way by which the performance gap in mathematics that exists between Black and White students and between students who attend urban and suburban middle schools could be closed. They also view these mathematics education reform programs as critical to the improvement in the performance of urban African American students in mathematics, particularly those programs which deemphasize the traditional methods of

instruction that employs telling, stating information and demonstrating procedures (Darling-Hammond, 1997; Lobato, Clarke, & Burns-Ellis, 2005) and embrace instruction that focuses on conceptual understanding and is grounded in the constructivist perspective (Smith, 1996; Wood, 1995; Wood, Cobb, & Yackel, 1995). Moreover, according to Secada (1992, 1995), African American students who have not experienced the same success in mathematics as their White counterparts are more likely to be tracked into remedial mathematic classes, particularly because as the proportion of African American students in a school increases, the relative proportion of college preparatory or advanced sections of mathematic classes decreases Oakes (1990b).

In discussing this situation Darling-Hammond and Green (1994) made a pertinent and poignant point when they stated that the students in the greatest need of the best teaching (e.g., African American students who attend public schools that are in socio-economically disadvantaged urban communities) are the least likely to get it. The result is that this inequality of education is reflected in their performance on national assessments such as those administered by the NCES. As a means of improving the situation for the high proportion of minority students from poor urban communities, who attend schools in urban school districts where mathematics education is often challenged by the conditions of teaching, Darling-Hammond (1997) suggested that teachers should use a nontraditional approach to teaching mathematics. Additionally, Phillips (1997) stated that when schools offer demanding curricula and employ teachers who have high expectations for all of their students, these become contributing factors to their students' performance in mathematics. And, since capability in mathematics has become an increasingly important asset in typical employment settings, student success and proficiency in high

level mathematics classes is often viewed as the gateway to economic enfranchisement (Rivera-Batiz, 1992; Schoenfeld, 2002). Therefore, when students of particular groups do not have the opportunities to take advanced mathematics courses and are not succeeding in this subject, they are not only being denied quality mathematics education but also the opportunity to experience economic freedom and choice (Miller, 1995).

After years of mathematics reform and continued unsatisfactory results regarding middle school student performance in mathematics, the most recent NAEP (Perie et al., 2005) results offered new hope and indicated that the reforms have begun to make a difference: Student performance increased for most groups of students. According to the NAEP results, 8th-grade public school students assessed in 2005 improved their performance in mathematics by one point over the performance of 8th-graders in the 2003 assessment and that higher percentages of students performed at or above the basic level than on any previous assessment (Perie et al., 2005). In seven states, these students obtained higher average mathematics scores than those from 2003. Irrespective of these gains, there is still room for improvement in both student achievement in mathematics and mathematics instruction in public schools in the United States. This implies that educators should take the necessary steps to make mathematics instruction more effective for students and, in particular, for African American students, many of whom attend urban, public, middle schools, and do not (in many cases) receive a comparable quality of education as do their White peers and their peers who reside in the suburbs. The result is that they perform at lower levels than do their White peers.

The No Child Left Behind Act and Closing the Achievement Gap

Though education reform is not new and government officials have implemented mathematics reform programs in public school systems across the United States for nearly four decades, none of these efforts has succeeded in significantly narrowing the achievement gap, closing it or improving substantially the academic disparities that exist between White and Black students and between urban and suburban students, with respect to their mathematics achievement levels (Cook & Ludwig, 1998; Thompson, 2003). Therefore, in January 2002, President George W. Bush signed the No Child Left Behind Act (NCLB; U.S. Department of Education, 2001) into law. This act was designed to help to close the persistent achievement gap between White upper- and middle-class public school students, poor children and children of color, through a multifaceted and comprehensive approach. The law promises to (a) achieve excellence through high standards and accountability, (b) make literacy a priority, (c) improve teacher quality, (d) improve mathematics and science instruction, and (e) move students with limited English proficiency to English fluency.

According to Thompson (2003), one of the most controversial aspects of the NCLB plan is the legislation that promises to promote parental options. This option gives parents and guardians of economically disadvantaged students who attend public schools the right to choose their children’s school in seeking the best possible education for them. Throughout the nation, these parents now have the option of moving their children from low- to high-performing schools (U.S. Department of Education, 2001). Thompson further stated that, for the most part, the majority of students attending sub-standard public schools tend to be poor children and children of color, and in most large, urban

cities these students are African American and Hispanic minority students. She posited that these students are more likely than others to have the least qualified teachers (Quality Counts, 2000), to be subjected to low expectations (Drew, 1996; Oakes, 1999; Thompson, 2002), and less likely to have access to the courses (Darling-Hammond & Green, 1994; Dupuis, 1999) and quality of education that would prepare them adequately for college (Phillips, 1997; Thompson). Therefore, the widening mathematics achievement gap that exists between White students and African American students reflects the inequitable distribution of educational opportunities and its resulting negative impact on the achievement levels of African American students. The opportunities afforded to the parents of the latter group of African American students may serve to aid in closing the achievement gap as their children attend schools in which they are less deprived of educational opportunities. This could only serve to improve their educational standing.

Constructivism and Social Constructivism

Constructivism, is a major research paradigm in the field of mathematics education (Ernest, 1996; Sharp & Adams, 2002; Simon, 1995), is the overarching theoretical framework for this research study. However, the study is specifically aligned with the tenets of one of the philosophical positions of constructivism, social constructivism. Constructivism, constructivism and mathematics education, and social constructivism are discussed in the three sections that follow.

Constructivism

Constructivism, a form of cognitivism, is a theory of learning that is based on the work of Piaget who stressed that knowing is adaptive and as such knowledge should be thought of as a short but complete summary of concepts and actions which have proved

to be successful (von Glasersfeld, 1990, 1995). Further, knowledge is also not passively received either through the senses or by way of communication, but it is actively constructed by the cognizing individual (Wood, 1995).

Another important characteristic of constructivism is that students are encouraged and allowed to bring the informal knowledge about their past experiences with mathematics to the classroom. According to Nobles (1990), this facilitates new learning and helps the students in general to master tasks. He posited that this is particularly important when considering the diverse cultural experiences that students bring to the mathematics classroom. Nobles added that this holds true for African American students, especially if mathematics tasks are embedded in a culturally familiar context and are aligned with the characteristics of their learning styles. Therefore, mathematics instruction, which reflects the constructivist perspective, gives students opportunities to play an active role in their learning as they construct their own knowledge and allows the teacher to focus on and examine the students' thought processes by listening to them and interpreting their actions. It also facilitates their analysis and understanding of how the students conceptualize mathematics concepts and problems.

Further, constructivists posit that learning requires self-regulation and the building of conceptual structures through reflection and abstraction (Clements, 1997). Problems therefore should not be solved by only the retrieval of answers, but also from the perspective that the problem is a personal one and presents an obstacle towards progress and towards a goal. As a result, the teacher is able to focus on the students' thought processes and interpret their actions in order to build a model of their levels of conceptual knowledge. These characteristics are especially relative to this study and to the teaching

and learning of fractions. They are also descriptive of the students' engagement in their learning as a search for meaning, as they construct mental and physical models and reflect on multiple presentations of reality (Clements) after the completion of each performance task. The activities in which the participants were involved during the study are aligned with the previously discussed characteristics of the constructivist position relative to what a mathematics-learning environment should look like when students engaged in the process of learning mathematics.

Constructivism and Mathematics Education

As stated previously, a relatively large number of students and in particular urban African American middle school students are not performing at expected levels in mathematics and experience significant difficulty when learning some mathematics concepts such as fractions. Researchers attribute these difficulties to a number of causes, including interacting classroom factors, some of which are addressed via constructivism. According to Sharp and Adams (2002), by the mid-1980s mathematics educators began viewing mathematics learning from a constructivist perspective in an effort to place the focus of instruction on the experiences of the learner rather than on the teacher's (Bruner, 1986; Carpenter & Moser, 1982; Cobb, Yackel, & Wood, 1991; Shoenfeld, 1985; von Glasersfeld, 1990). Sharp and Adams added that this transition away from the behaviorist view paved the way for a shift from the traditional lecture-oriented classrooms to classrooms in which students were engaged actively in their learning. It also gave researchers and classroom teachers a new understanding of how children learn mathematics and reasons to reconsider how students learn fractional ideas (Kieren, 1988; Mack, 1995; Streefland, 1978, 1991, 1993). The constructivist perspective of how

students learn has now evolved into a major research paradigm in the field of mathematics education.

In his discussion of constructivism in relation to the field of mathematics education, Simon (1995) stated that the constructivist perspective of how students learn has been central to a large portion of empirical studies and theoretical work in this field (Steffe, & Gale, 1995; von Glasersfeld, 1991). It has also contributed to shaping the mathematics reform recommendations of the NCTM (1989, 1991, 1995) and the resulting reform programs which public school systems across the United States have implemented (especially in their middle schools) to aid in improving student performance and achievement (Simon).

According to von Glasersfeld (1995), constructivism has played the key role of introducing a new perspective from which educators could approach the dilemma regarding the substandard level of mathematics education and the resulting underachievement and unsatisfactory student performance in this area. In discussing constructivism's promise for mathematics learning in the United States, von Glasersfeld pointed to the fact that there was a growing awareness in the educational arena that educators need to emphasize their students' conceptual development. He posited that this would provide them with a corresponding theory of knowledge (i.e., an epistemology) to guide their efforts as well as facilitate an improvement in their students' mathematics learning and performance, and in general, their education.

The classroom activities that are characteristic of constructivist learning environment are aligned with those that were used in this study to provide the participants with opportunities to develop their conceptualization skills as they worked on finding

solutions to the given fraction-related performance-based tasks. In addition, the study's activities and approaches also reflected the social constructivist perspective on knowledge acquisition.

Social Constructivism

Social constructivism, a branch of constructivism, espouses that learning is a collaborative process by which learners are integrated into a knowledge community and recognizes that though the individual constructs knowledge, it is also concurrently socially constructed. These individual and social processes are also interactive with the result that groups act to construct knowledge and to resolve differences in the meanings of individual group members (Saxe, 1995; Simon & Schifter, 1991). Further, social constructivism implies that social interaction is crucial to the process of constructing knowledge, which resides with whichever group holds more power at any given time (Cobb, Wood, & Yackel, 1990; Saxe). Learners, therefore, construct novel understandings as they attempt to accomplish goals, which are rooted in both their prior understanding and in socially organized activities (Saxe).

Most social constructivists have been influenced by the philosophy of Vygotsky (1978), a social constructivist who believed that cognitive functions originate within and are products of social interactions. Therefore, learning is naturally collaborative and must be described as products of these interactions. In addition, Vygotsky believed that every function in a child's cultural development (e.g., the formation of concepts, logical memory and voluntary attention) emerges on the social level first (interpsychological) and then from within the child (intrapsychological). Also, all higher functions originate from relationships between individuals (Vygotsky).

With regard to education, Vygotsky (1978) posited that the purpose of education was to develop the personalities of the students and that this rested upon the discovery and expression of their creative potential. Therefore, methods of teaching and learning should (a) relate to the development of students as individuals and as members of a group in which they actively participate and (b) facilitate communication and collaboration between teachers and students and between the students themselves. In addition, the teacher's role becomes one of guiding and directing the activities in which individual students are engaged without force or without imposing their ideas on the students.

According to von Glasersfeld (1995), social constructivism emphasizes the social and cultural dimensions of development and the generation of knowledge. Also, because social processes more than individual processes spark ideas of reality, knowledge is constructed through social discourse rather than obtained by objective means (Gergen, 1985, 1994). Additionally, the social constructivist learning environment can provide students with opportunities to develop their mathematical knowledge through their culturally shaped notions, particularly because these cultural and social dimensions are intrinsic to learning mathematics in comparison to child-centered perspectives that stress the autonomy of the learner (Richards, 1996).

In summary, constructivism provides a theoretical basis for and is consistent with the genres of learning experiences that should take place in the classroom and that the NCTM (2000) described in its most recent *Standards*. Simon and Schifter (1991) posited that although constructivism does not prescribe explicit instructional strategies regarding the teaching of mathematics or in general, mathematics education, the constructivist perspective of learning can nevertheless benefit both students and mathematics educators.

Therefore, using the constructivist paradigm, and in particular, social constructivism as a basis for this study and incorporating this from an instructional perspective improves the study and benefits my students as I use classroom learning activities that reflect the constructivist position on how students learn. The ontological, epistemological and methodological perspectives facilitate making sense of the data obtained as a result of the participants' active participation in the learning environment as they construct knowledge.

Fractions: A History of Difficulties

The topic of fractions is a core component of the Number and Operations strand of the mathematics curriculum found in public schools in the United States. Its importance is reflected in the fact that the teaching of fractions is fully ensconced in the elementary and middle school mathematics curricula, beginning in grade three. Although elementary students experience some difficulty when learning fractions, the level of difficulty increases when they get to middle school. From all accounts, teachers in many middle school classrooms are spending a relatively greater amount of time teaching fraction concepts compared to other mathematics concepts, though with limited success (Groff, 1994; Oppenheimer & Hunting, 1999).

Difficulties with Fractions: Public School Students in the United States

Fraction learning has posed significant difficulties for students at all levels of schooling at many public schools in the United States for a very long time, and, as the research indicates, this continues to be the case. The findings from studies on mathematics education continue to indicate that many public school students, particularly at the middle school level, experience difficulty when learning and understanding the mathematics concept of fractions (Groff, 1996; Kouba, Zawojewski, & Struchens, 1997;

Moss & Case, 1999; Test & Ellis, 2005). These researchers contend that the main reason for this difficulty is the fact that most educators use primarily traditional fraction instruction that emphasizes the memorization and application of fraction rules and procedures. Therefore, as Moss and Case posited, though students eventually learn the specific algorithms for computing fractions, they remain relatively deficient in terms of their conceptual knowledge and understanding of this topic. This subsequently affects their overall performance in mathematics and compounds the problem of unsatisfactory student achievement levels in public schools in the United States.

Prevalence of Fractions in the Mathematics Curriculum in the United States

Over the past decade, a large percentage of mathematics teachers have increased the instructional time for teaching fractions in keeping with the suggestions and recommendations of the NCTM (1989). The council suggested that teachers should find and devote extra time and attention to the teaching of fractions and that they should also teach this concept in a systematic and direct way (Groff, 1994). Fractions are an integral part of the elementary and middle schools curriculum and they play an important role in the high school mathematics curricula. Fractions are related to variety of algebraic concepts such as slope, functions, and rationals (Bransford, Brown & Cocking, 1999; NCTM, 2000; Perie et al., 2005; Post et al., 1992). Van de Walle (2004) stated that it is important that elementary and middle school students are able to work with and compute fractions because they can be used to do and are connected to other mathematics concepts, such as estimation and understanding higher level calculations that are done with technology. In spite of the importance of fractions in the mathematics curriculum and the

increased time that teachers spend teaching this concept, it continues to be a difficult area in mathematics for students to learn and understand.

Fractions Studies: 1970s and Beyond

The prevalent aspects of student difficulty with learning fractions and the sources of common errors, which are made when students compute fractions and which contribute to their difficulties, have been documented and date back as far as three decades. The National Assessment of Educational Progress (NAEP, 1973) reported that only 42% of middle school students (13-year-olds) and 60% of high school students (17-year- olds) in the sample could correctly add a unit fraction with different denominators. Carpenter, Coburn, Reys, and Wilson (1976) also found these factors to be prevalent when they conducted a similar study. Bigalke and Hasemann (1978) closely examined the data obtained from a study that they conducted with middle school students and identified the following to be the most prevalent contributors to student difficulty with learning fractions: (a) fractions are used less often in daily life and are less easily described than are natural numbers; (b) the written form of fractions is comparatively complicated; (c) it is not easy to put the fractions in order of size on the number line; and (d) the many rules, which are required for computing fractions, are more complicated than those for natural numbers, and if these rules are introduced too early, there is a danger that they will be used mechanically and without thought.

Hasemann (1981) conducted a study in which he used a sample of low-performing middle school students (aged 12-15) and tested them on the topic of fractions by means of diagrams, word problems and computational questions. Hasemann analyzed the students' responses to identify the specific difficulties and deficiencies that they had

with fractions. He found that the majority of the students were only able to apply memorized rules to find a solution to the problem, but they were unable to understand whether the rule worked. Hasemann therefore concluded that the students' understanding was at the most "instrumental," not "relational." He stated that his findings indicated that students often only use fraction rules correctly when the situation is very clear to them and when it was not substantially new. Additionally, he found that most students lacked a clear understanding of an *idea* of fractions. As a result, the concept of fractions is a difficult topic in mathematics to teach and for students to learn, primarily because many of these students do not understand the relational aspect or the idea of fractions (Hasemann).

To substantiate his conclusion, Hasemann cited Cohors-Fresenberg (1979), who stated in his discussion on algorithms and control processes used in connection with problem-solving, that it is necessary for students who use a rule to understand it clearly and to know when to apply it to an appropriate situation. If this is not the case, the students will find it difficult to know when and how to apply and/or use the rule correctly. Specifically, with regard to fraction learning, Cohors-Fresenberg also stated that, in a broader sense, the most critical contributor to student difficulty with learning fractions could be attributed to the lack of a conceptual knowledge of fractions.

Peck and Jencks (1981) corroborated Hasemann's conclusion in their report that less than 10% of the hundreds of 6th-grade students whom they studied were able to conceptualize fractions adequately. In addition, their examination of the data from prior fraction studies and assessments pointed to the fact that there had still been no significant improvement on fractional computation for 13-year-olds (middle graders) on the NAEP

since 1978. These researchers interviewed hundreds of students to determine the extent of their conceptual understandings of fractions. Though their study involved a sample of 20 students chosen at random from a typical sixth-grade, their findings reflected data from the hundreds of students that they had interviewed. Each student was interviewed on an individual basis for a 45-minute period in which the interviewer asked questions (e.g., how to compare and to add simple fractions and to explain why his or her efforts produced the correct results). The student responded using physical materials. During the first 25 minutes of the interview, the student used only physical materials to help him or her to relax and to understand what he or she would be doing during the activity. This also gave the interviewer an opportunity to make a judgment as to each student's level of conceptualization. This activity was followed by a discussion of fractions. Peck and Jencks found that all of the students were capable of conceptualizing fractions, but when they were questioned about fractions, they lacked the conceptualization skills that were necessary to make sense of these concepts. For example, many of the children could identify examples of specific fractions but they had not developed a generalized concept of a fraction and most of those who had, could not extend their ideas to operations on fractions. The researchers concluded that the difficulties that students have with fractions are conceptual.

Another group of researchers, who also summarized the results from several NAEP assessments done during the late 1970s and early 1980s, was the team of Lindquist, Carpenter, Silver, and Matthews (1983). They found that only 40% of 17-year-olds students had mastered fraction concepts, and by 1986, when the NAEP conducted yet another study, the pattern was the same. They also found that high school students'

performance with fraction items (decimals and percents) reflected the serious gaps in their knowledge of basic fraction concepts (NCTM, 1988). Yet another NAEP study done in 1990 indicated that fewer than half of the 12th-grade student participants could demonstrate a consistent grasp of fractions, and in addition, in any state in the United States no more than half of 8th-grade students at the most could solve problems involving fractions, based on the 7th-grade level of difficulty. This prompted Mullis, Owen, and Philips (1990) to examine the data from the NAEP study. They verified the fact that fractions are exceedingly difficult for children at all levels to master and concluded that students do not learn much about fractions as they move from one grade level to another and forget what little that they learned the previous year.

Hiebert and Wearne (1985) conducted a study with 670 students in grades 5-9. They used a model, which consisted of symbol manipulation rules, which they believed students acquire, store, and use to compute with rationals (decimals). Their hypothesis was that, by the time students reached the upper elementary grades, their behavior on mathematics tasks could be described in syntactic rather than semantic terms. Based on their analysis of written and oral interviews and their comparisons of the students' responses to decimal computations to their predictions, they found that the conceptual foundation on which rules are built plays no essential role in building modules. They found that student difficulty with learning decimals (and fractions) resulted from their lack of understanding of the underlying concepts. They concluded that this difficulty becomes obvious when students rely solely on syntactic rules for computing fractions and are therefore misled when the symbolic configuration of a problem is similar to those that they learned earlier. The result is that they use and/or apply the rules inappropriately.

Hiebert and Wearne also concluded that most computation-related fraction errors are not made because of the students' incorrect use of these rules but because they choose and use the wrong rule for a particular situation. These findings support Hasemann's (1981) conclusion that students learn mathematical operations involving fractions with great difficulty, often because they lack understanding of the underlying concepts.

Difficulties with Fractions: Foreign Students

Interestingly enough, research findings also indicate that student difficulties with fraction concepts do not only pertain to students in the United States. For example, Groff (1994) in his analysis of relevant literature on fraction learning and the difficulties associated with it found that even fifth grade Japanese and Chinese students have difficulties when learning fractions. Further, Stigler, Fernandez, and Yoshida (1996) conducted a study in which they collected tapes of 20 Japanese and 20 U.S. fifth-grade teachers as they taught a lesson on equivalent fractions. The purpose of the study was to characterize the differences between the two contrasting traditions of classroom mathematics instruction. They found that the Japanese students computed fractions at a 31% rate of success, but for non-fraction mathematics items, their success rate was 74%. However, in the same study, their counterparts in the United States, computed fractions at a 14% rate of success, and non-fraction mathematic items, at a rate of 50%.

The findings of Sowder et al. (1998) summarize the points discussed in this section of the literature review. In discussing middle-grades mathematics related to multiplicative structures they make four recommendations for teachers, which deal with different but related forms of reasoning including reasoning with rational numbers. These researchers put the history of student difficulty with fractions in an interesting perspective

in discussing the known and widely held beliefs about the teaching and learning of fractions and the difficulties that many students encounter while learning these concepts. They stated that fractions are the most troublesome form of the rational numbers for students to learn and for teachers to teach.

Teachers' Difficulties with Fractions

From all indications, working with fractions poses problems not only for students but also for teachers. Some researchers believe that, as is the case with students, a small percentage of public school teachers also experience difficulties understanding and teaching fractions and that this is a contributing factor to their students' low level of achievement with fractions. Bracey (1985) and Harling and Tessa (1988) expressed the view that female teachers in elementary schools may have well-established feelings of inferiority about teaching fractions. The focus of Bracey's discussion was a group of elementary student teachers who were considered the best in their class (experts) and who were teaching fraction concepts to fourth grade students (novices) who had obtained good standardized test scores over a 5-year period. The researchers used card-sorting techniques, interviews, and the analysis obtained from protocols to determine how the participants in the study changed as they moved from novice to expert and the system of knowledge that the teachers used to teach equivalence of fractions. The findings of the study indicated that in general the teachers did not understand what they were trying to teach, that there was a great disparity between the teachers' ability to express an algorithm and their lack of underlying mathematics concepts. Bracey concluded that inadequate teacher preparation for teaching fractions negatively affects student performance in this area.

On the other hand, Romberg and Carpenter (1986) posited that there was no substantial empirical verification that there is a correlation between exceedingly low achievement in fractions and ineffective teaching or that this concept is taught less efficiently than other mathematical topics. As such, teachers should not be faulted for their students' poor performance with fractions, especially given the fact that studies related to the teaching of mathematics have not provided educators with a list of tested behaviors that make them competent in all aspects of mathematics teaching. Groff (1994) reported that, through informal discussions with elementary school teachers whom he observed, he discovered that they found it very difficult to teach some aspects of fractions. For example, they could not create genuine problems that could only be solved through the manipulation of fractions, such as 13/18, 7/9, 2/7, and 5/13. Nonetheless, Groff (1996) disagreed with the idea that the quality of fraction instruction has played a major role in students' not being able to comprehend these concepts. He cautioned that identifying inadequate teacher preparation as being a factor which has negatively affected student learning of fractions is only an assumption because of the lack of an adequate amount of empirical evidence to validate this claim. However, the later work of Jones (1995), who evaluated the beliefs and the content and conceptual knowledge about fractions of preservice teachers enrolled in a state university's teacher education program, found that all of the teachers in the study had difficulty answering content and conceptual knowledge questions about fractions. Her findings substantiated Harling and Tessa's (1988) and Bracey's (1985) findings relative to some teachers' limited knowledge of and ineffectiveness with teaching fractions.

Difficulties with Fractions: Elementary School Students in the United States

Between 1980 and the year 2000, *The Journal for the Research in Mathematics Education* published a multiplicity of studies on fraction learning. A majority of the researchers (e.g., Behr, Wachsmuth, & Post, 1985; Bezuk & Cramer, 1989; Corwin, Russell & Tierney, 1991; Empson, 1999; Lamon, 1996; Moss & Case, 1999; Pothier & Sawada, 1983) who conducted these studies focused on the teaching and learning of fractions at the elementary level. Their consensus was that that most of these students do well while learning *some* fraction concepts, but they experience many difficulties when learning other concepts, such as partitioning and fraction equivalence and, in general, when conceptualizing fractions. Also, some students who are successful when working with fractions possess effective schemes for solving partition problems with definite numbers of discrete objects as units, though they may not consistently use a scheme involving units of different sizes in interpreting fractions. A discussion of several of these studies follows.

Hunting (1983) investigated a 9-year-old participant's knowledge of units, partitions, and fractions and the relationship between them. This student's actions when working with fractions highlighted the constructivists' view of learning, which espouses that a child should actively construct his or her own mathematical learning and knowledge. This knowledge, according to Hunting, is acquired through the development and reorganization of cognitive structures that allow children to interpret and control quantitative aspects of their environment, a necessary skill that aids the child with solving a great variety of quantitative problems. As a result of observing the student as he worked with fractions, Hunting concluded that some students' failure to solve many of the given fraction problems was not due to a lack of schemes involving units and the relations

among them, but to the fact that they had not used these schemes to develop a general conception of fractions. In addition, based on his interpretation of the data obtained from the case study, Hunting also concluded that elementary school children have considerable potential for solving fraction problems using self-developed schemes for dealing with units and unit relationships.

A positive outcome of the studies done with elementary-level students was that these students bring to instruction a rich store of informal knowledge about fraction concepts and can do well when working with them. A case in point is the work of Empson (1995), who wanted to determine if her first grade students, whom she realized could compute whole numbers based on intuition, had the same level of intuitive knowledge about fractions and could learn to compute them in similar ways. In her class, the students spent most of their time solving problems involving whole numbers and discussing suitable problem-solving strategies. Empson used a mathematics curriculum that was centered on the students' creating story problems that incorporated their prior knowledge and on encouraging them to use their own problem solving strategies to come up with a solution. She discovered that she did not always need to show the students how to solve a problem; they were able to use their informal knowledge to find a solution on their own. Subsequently, to be convinced that her students could also do the same with fractions, Empson did a case study with a class of 17 first-grade students and found that, as with whole numbers, these students could add and subtract fractions without prior instruction. The key was that they did not have to try to remember procedures for computing fractions but used conceptual knowledge to devise their own solutions.

Hunting, Davis, and Pearn (1996) arrived at a similar conclusion as Empson (1995) when they stated that elementary students' whole number knowledge could be tapped in a positive way to teach basic fraction concepts. These researchers conducted a 2-year experiment involving two elementary school students, an 8-year-old and a 9-year-old. The purpose of the study was to investigate fraction learning and the role that the students' whole number knowledge might have played in it. During the investigation, the two students solved fraction comparison problems using an operator-like computer program called "Copycat." To analyze the data obtained from the study in the form of student responses, the researchers created and identified three cognitive schemes for the students' use: (a) equal outputs schemes (involving unit fraction comparisons); (b) equal inputs schemes (for activating strategies for determining common multiples); and (c) scaling schemes (for scaling fractions up or down using ratios). The activities for each of the session were based on the researchers' observations of the participants and their interpretation of the children's behavior in prior sessions. Hunting et al. found that rational tasks in operator settings can help to stimulate and extend children's whole number knowledge. They concluded that there was interdependence between the development of rational number knowledge and whole number knowledge and that facility with whole number relationships enables students to solve fraction computation problems.

In addition to the findings just discussed, Empson's (1995) findings also paralleled those of another earlier researcher (Leinhardt, 1988), who conducted a study to determine how the students' intuitive, concrete, computational and conceptual knowledge evolved as they moved through a series of fractions lessons and how the participants

could successfully perform operations on fractions by drawing on informal knowledge especially when the problems presented to them were in the context of real-life situations. The study involved 11 fourth-grade students who were observed and interviewed before, during, and after the unit on fractions. Leinhardt found that the students' intuitive grasp of fractions was inhibited by their algorithmic knowledge.

Mack (1995) conducted a similar study but with slightly older participants. She examined the development and understanding of fractions concepts during instruction of a group of third and fourth grade students who possessed limited prior knowledge of fraction symbols. The focus of the study was on their ability to build on their own informal knowledge to conceptualize and give meanings to fraction symbols and procedures during instruction. Mack found that, as the students attempted to construct meaning for the symbolic representations of fractions, they over-generalized the meanings of symbolic representations for whole numbers to fractions and did the same for fractions to whole numbers. She also found that, while many students perform operations on symbolic representations with little understanding of the meanings underlying the representations, others often draw on the informal knowledge which they bring to the classroom to give meaning to the symbolic and the rich conceptual knowledge reflected in concepts related to the part-whole interpretation of fractions. Mack concluded that the influence of students' prior knowledge could be very beneficial when they are studying strong content domains such as fractions, where the symbolic representations result from combining and reinterpreting representations for whole numbers. Ironically, these findings contradicted those of a previous study, which Mack (1990) had conducted with elementary level students. In that study, Mack examined the

development of the eight 6th-grade students' understandings of fractions. The students received individualized instruction on addition and subtraction of fractions on a one-to-one basis during a 6-week period. Mack found that, although the students possessed informal knowledge of fractions, they could not readily relate symbolic representations to this knowledge. Their knowledge of rote procedures for computing fractions interfered with their attempts to build on their informal knowledge.

Gray (1993) examined third-grade elementary students' fraction difficulties from a different angle. He wanted to analyze these students' difficulties when transitioning from whole number arithmetic to the arithmetic of fractions. He observed 9-year-old, 3rd-grade children as they named the parts of a rectangle that had been partitioned into eight equal parts. The children counted the parts using ones instead of eights then named their total eights. These children gave similar meaning to fractions and to whole numbers. This negatively impacted their understanding of fractions as a process and as a concept. Gray obtained evidence from his study that emphasized the differences which exist between elementary school children, who treat number symbolism flexibly as process and concept, and children who view fractions in terms of counting procedures by using whole numbers rather than fractions to count and identify fractional parts. He concluded that the students who failed to understand the flexible nature of fractions would do the same when studying more complex forms of rational numbers.

As stated earlier, one tenet of constructivism is that students construct knowledge through interacting in learning situations. This usually occurs as they participate in mathematics learning and are engaged in discourse with their peers or with their teachers in the learning environment. Empson's (2003) recent study, which focused on teaching

fractions concepts for understanding to low-performing first-grade students, highlighted the fact that, even though the two 1st-grade participants in the study knew the least about fractions (compared to their classmates) when the study began, by end of its 5-week duration, they were able to benefit from the instruction because of the effective way in which their teacher orchestrated their participation in solving and discussing problems. At the end of the study, Empson analyzed the data obtained from the case study of the two male first-grade participants after a 5-week, 15-lesson unit on fractions which was organized around eliciting and building on the participants' informal knowledge of equal sharing situations.

In the Empson (2003) study, the teacher's mathematics instruction on fractions revolved around posing story problems for children to solve using their own strategies. The discussion of the problems was directed to facilitate the understanding of children's thinking, comparing strategies, and resolving disagreements or ambiguous mathematical claims. This instructional approach was based on evidence that this kind of instruction leads to greater mathematical understanding and problem-solving achievement. Empson combined constructs from interactional sociolinguistics and developmental task analysis to investigate the nature of the students' participation in classroom discourse about fractions, despite the students' lack of cognitive and social skills, by using interviews and documenting the learning and analysis of classroom interactions.

Empson (2003) proposed that three main factors accounted for the two 1st-grade (low-performing) participants' success: (a) the use of tasks that elicited the students' prior understanding, (b) creation of a variety of participants' frameworks in which the students were treated as mathematically competent, and (c) the frequency of opportunities for

identity-enhancing interactions. She found that, though the two participants made gains in their understanding of the fraction concepts that they worked on during the study, their relative standing with respect to the other children in the class remained unchanged. However, each participant could solve some fraction problems, such as subtracting a fractional quantity from a whole-number quantity or partitioning whole-number quantities into fractional quantities, the types of problems that first-graders typically do not solve. Empson concluded that the participants demonstrated an adequate understanding of fractions for students at the first-grade level and that the explanations for student success and failure fundamentally depended on the dynamics of instructional interactions and their benefits of consequences for students.

Saxe et al. (2005) conducted a study involving 384 elementary school students from 19 classes. The students were 4th-graders, 5th-graders, and 6th-graders. The study was based on the premise that one contributing factor to student difficulty with mastering fraction concepts is their difficulty in acquiring flexible use and understanding of written notations for fractions. The researchers investigated the developmental relationship between the students' uses of fraction notations and their understanding of part/whole relationships with the aim of analyzing the role of fraction instruction in students' use of notation to represent parts of an area. To organize their study Saxe et al. introduced a framework that distinguished between two aspects of the students' written representations of fractional parts of areas, *notation* (marks or symbols for fractions, such as the numerator, denominator, and separation line) and *reference* (the conceptual work of using a notational form to point to or index physical objects or mathematical ideas, such as part-whole, part-part, or other kinds of relations). Tests were administered before and

after fraction instruction. During implementation of the study, the researchers videotaped key lessons and recorded field notes. The students' work was coded based on their use of fraction notation and on the concepts captured by the notation. The lessons were rated with respect to their alignment with the principles supported by reform frameworks in mathematics education, such as providing students with opportunities to build their understanding of fraction concepts. Saxe et al. found that students acquired notation and reference somewhat independently and that classroom instructional practices that focused on student conceptualization were more likely to support shifts toward normative uses of notation.

The data from the studies discussed above indicate that, though some elementary students experience significant difficulty with learning certain fraction concepts, in general they do well on others. For example, the students could successfully partition shapes and perform basic operations, such as adding and subtracting. This tends to occur when they are able to draw upon their knowledge of whole numbers and upon other informal knowledge that they bring to the classroom. However, these students did not perform well when working with fractions because of several factors, including (a) a lack of conceptual understanding of fractions, (b) the extent and level of their previous knowledge of whole numbers, and (c) ineffective teacher preparation, which resulted in ineffective instruction.

Even though the dates of some of these studies may be categorized (for research purposes) as *outdated,* I felt compelled to use this information because the findings from these studies have contributed significantly to the research on the domain of fractions. In particular, the initial findings of Hasemann (1981), who conducted several studies in the

late 1970s and 1980s involving middle school students (between the ages of 12-15 yrs), are critical to the review. These studies provide verification that the domain of fractions has been a source of interest to researchers, of concern for educators, and of difficulty for many students for a very long time. More important, Hasemann's findings have influenced many years of subsequent similar research, as educators sought answers to similar questions about student difficulty with learning fractions, as well as questions regarding more effective strategies for teaching this form of rational numbers.

Difficulties with Fractions: Middle School Students in the United States

Fractions have always represented a considerable challenge for students, even at the middle school level (Groff, 1994, 1996; Moss & Case, 1999; Test & Ellis, 2005; Wearne & Kouba, 2000). A little more than a decade ago, data from national mathematics assessments and achievement studies (NCES, 2002) indicated that student performance with fractions tends to lag the most at the middle school level. Results from several NAEPs have consistently shown that middle school students have a very weak understanding of fraction concepts (Wearne & Kouba). In the international mathematics arena, the data obtained from several TIMSS comparative studies conducted with U.S. middle school students and their peers in other countries indicated that the domain of fractions was one of the areas of mathematics in which U.S. middle school students did not perform as well as their foreign peers (TIMSS, 1995, 1997, 1999). These findings are significant and pertinent to this study because they point to the difficulties experienced in teaching and learning fractions in middle schools throughout the United States.

The NCTM (2000) recommends that students in grades 6-8 should continue to refine their understandings of the four basic operations with whole numbers as they use

them with fractions. They also suggested that, in order for students to develop fraction concepts and computational skills, they should be able (a) to work flexibly with fractions to solve problems, (b) to compare and order fractions efficiently, (c) to find the location of fractions on the number line, (d) to understand the meaning and effects of arithmetic operations with fractions, (e) to use the associative and commutative properties of addition and the distributive property of multiplication over addition to simplify computations with fractions, (f) to select appropriate methods and tools for computing with fractions, (g) to develop and analyze algorithms for computing with fractions, and (h) to develop and use strategies to estimate the results of rational number computations. Despite these recommendations for middle school students, for many years, researchers and educators have been aware of the difficulties that middle school students experience in reaching these expectations when learning fractions, as is evidenced by their subsequent poor performance with these concepts. Surprisingly, given this awareness, over the past two decades, there have been a relatively smaller number of studies conducted at the middle school level in comparison to the number of fraction studies conducted at the elementary level.

Hart and Kerslake (1983) interviewed a group of twenty-three 13- and 14-year-old students as part of a study on ratio and proportion and the meanings the students gave to fractions in the form of (a/b). First, the researchers interviewed the students individually about their reactions to tasks involving fractions and then administered a pretest. The next phase involved an instructional unit where 59 students learned fraction concepts. The researchers found that most of the students avoided using fractions on the pretests, and about one-third of the 59 students, who participated in the instructional unit,

still avoided fractions when responding to the test questions and refused to acknowledge the existence of fractions. They concluded that, even after years of fraction instruction, middle school students are reluctant to use fractions when solving mathematical problems and when answering multiple-choice type questions. Instead, they choose answers that are in remainder form.

More than a decade later, Groff (1996) concurred with Hart and Kerslake's (1983) findings that middle school students dislike working with fractions and are reluctant to work with them. As a middle grades mathematics teacher, Groff realized that the domain of fractions was integral to the mathematics curriculum, particularly because the NCTM had recommended that fractions should be taught in a systematic and direct way and that teachers should give increased attention to this concept. He also sensed that his students harbored a dislike for the study of fractions and felt that learning to compute them was a dead-end activity, a waste of time and irrelevant to the purpose of mathematics in anyone's daily life. As a result of these personal experiences, Groff (1996) conducted an investigation to determine what the experimental research did or did not conclude about the issue of fraction instruction in the middle grades. His investigation included analyzing the fraction content in all of the mathematics textbooks that were used during a 10-year period and conducting a survey using the literature found in mathematical journals and other relevant writings regarding fraction instruction. Groff's findings confirmed that many middle school students (a) dislike learning about fractions, (b) have a stronger dislike for learning fractions than that of their peers at other grade levels, and (c) do not do well during fraction instruction and perform poorly on standardized test-questions pertaining to fractions.

More recently, Test and Ellis (2005) conducted a study that used a multiple probe across participants to evaluate the effectiveness of a mnemonic strategy for teaching addition and subtraction of unlike fractions to six 8th-grade middle school students. Using the findings of earlier studies which indicated that mnemonic devices were used successfully to teach mathematics concepts, they created a unit where the students used a LAP (L - Look at the denominators and the sign of the fraction; A – Ask yourself: Can the smallest denominator be divided into the largest an even number of times?; and P – Pick your fraction type) fraction strategy. The students worked in pairs and using the cue cards they took turns questioning each other and checking to make sure that they used the correct procedures. After 6 weeks of instruction using the mnemonic strategy the participants were able to achieve mastery of the fraction skills that they were taught. Test and Ellis concluded that, though fractions are one of the hardest mathematics concepts for students to learn, the use of nontraditional mathematics approaches and instructional materials can facilitate success with fraction concepts for students despite their ability levels. Test and Ellis's study is a direct parallel to this research study in that the participants used a nontraditional approach via performance-based tasks and fractions representations rather than only computations with symbolic representations of fractions and rules and procedures, to work with the fractions in the task.

In addition to the studies done by the researchers discussed in the preceding sections, organizations including the National Center of Education Statistics (2001, 2002, 2004) through its mathematics assessments of U.S. students (NAEP) and through the administration of several comparative mathematics studies (TIMSS, 1995, 1997, 1999, 2003), also provided data regarding fractions and student difficulties with learning them.

This data indicated that middle school students experience significant difficulties with learning fractions. The implication was that educators needed to make efforts to facilitate more effective mathematics instruction, student learning, and student achievement. Following is a discussion of some of the results from several of these NAEP assessments and TIMSS comparative mathematics studies.

Student Difficulties: NAEP Mathematics Assessment Data

The NAEP, also known as the Nation's Report Card, is the only nationally representative and continuing assessment of what U.S. 4th-grade students and 8th-grade students know and can do in various subjects, including mathematics. In the 1988 assessment, less than one-third of the 13-year-old, 8th-grade, middle school students tested could correctly choose the largest fraction among 3/4, 9/16, 5/8, and 2/3 (Kouba, Carpenter, & Swafford, 1989). Despite a lapse of many years and advances in the teaching of fractions, researchers examining the data from another NAEP assessment found that this type of student difficulty with comparing fractions still existed (Kouba, Zawojewski, & Structhens, 1997). In the 2003 NAEP assessment, only 27% of the 8th-grade students could determine which term in a pattern of fractions would have a specified decimal value, and only 64% could locate 3/4 on a number line, even though they were given 1/2 as a guide. These findings are a clear indication that middle school 8th-grade middle school students do not do well with fraction items on standardized tests and reflect the findings relative to the difficulties that they experience when learning fractions.

Student Difficulties: TIMSS Mathematics Assessment Data

In 1995, the first TIMSS was conducted to determine the performance and achievement levels of elementary and middle school students in mathematics and science among 4th-grade students and 8th-grade students in 42 countries, including the United States. The results were disappointing; 8th-grade students in 11 of the 42 countries outperformed their U.S. peers in mathematics (Schmidt, 1996). Four years later, TIMSS conducted its second study. This time U.S. 8th-grade students fared even worse. Their performance in mathematics placed them in the 19^{th} position in the list of 38 countries that participated. Their average score was 5 points above the international average and 117 points below the highest average (TIMSS, 1999).

Further, as a means of closely examining the data from its 1999 comparative mathematics assessment, TIMSS focused on mathematics curricula and other factors that negatively affected student performance in this subject. This revealed that at the middle school level in public schools in the United States, the 8th-grade curriculum was less advanced than that taught at the same level in countries, such as Japan, where their students had outperformed their U.S. peers. Further, TIMSS found that, in middle schools in the United States, teachers repeat a large percentage of the content already covered in the elementary grades and that the curricula covered many topics but lacked depth (Stigler & Hiebert, 1999). The TIMSS researchers concluded that there was a weakening in instruction at the middle grades, while Porter (2002) in a subsequent assessment of the data inferred that the U.S. students' achievement in mathematics is low relative to other countries. He reasoned that this was because U.S. students do not study a focused and manageable set of content that they are able to master within the time constraints of schooling.

The review of the literature and the discussions in the prior three sections focused on research data that examined student difficulties with learning and understanding fraction concepts, the unsatisfactory mathematics performance of urban middle school students and the Black-White achievement gap that exists in mathematics. From all indications, there is a need for the use of alternative approaches to fraction learning and comprehension in particular at the middle school level, where students experience many difficulties with learning these concepts and in general performing at satisfactory levels in mathematics. In the next section, the discussion is centered on the alternative, reform-based approaches that were used in the study in order to determine the extent to which the participants compute and conceptualize fractions and the ways in which they use fraction representation to work with fractions embedded in performance-based tasks.

Alternatives to Traditional Fraction Instructional Approaches

The discussion in this section begins with an explanation of conceptual and procedural knowledge, two opposing issues that are pertinent to fraction teaching and learning.

Conceptual Knowledge

In the NCTM's (1995) *Standards*, mathematics is represented as a discipline of conceptual inquiry and mathematics learning is considered a conceptual endeavor (Gearhart et al., 1999). In addition, according to Eisenhart et al. (1993), conceptual knowledge is the *why* of mathematics, and mathematical knowledge integrates the concepts underlying the procedures, the relationships between concepts, and how the concepts and procedures are used in various types of problem-solving situations. Aksu (1997) defined conceptual knowledge as ability to understand relationships that are

integrated with or connected to other mathematical ideas and concepts. Further, he stated that a variety of nontraditional fraction instructional approaches and activities could be used to help students to learn and understand fractions from a conceptual perspective and to de-emphasize the rules and procedures characteristic of the traditional approach. For example, teachers can help to improve student performance and minimize their difficulties with learning fractions by using classroom activities such as those that involve models, manipulative materials, and situations involving chance (Booker, 1996) and must plan lessons which facilitate tapping into their students' everyday worlds to relate fractional concepts to what they already know (Sharp & Adams, 2002). Using realistic experiences helps students to better conceptualize fractions and contributes to the development of theory regarding fraction instruction (Streefland, 1991, 1993). Also, teachers can then develop the emerging knowledge by openly discussing students' solutions for these real-world problems (Kamii & Warrington, 1995; NCTM, 1991; Streefland, 1991).

Cramer, Post, Lesh, and Behr (1998) stated that fraction activities which can enhance conceptual learning are a means of helping students to understand that fraction quantities, fractions relations, or fraction operations can each be represented using a variety of representational media (such as written symbols, spoken or written language, concrete models [manipulatives], diagrams, or experienced-based metaphors). This is because conceptual knowledge comes from the students' understanding of representational fluency. Therefore, if a quantity, relation, or operation is described using one medium, the student should be able to produce the same information using another medium. Also, if changes or comparisons are acted out using one medium, students

should also be able to act out the changes or comparisons using another medium. For example, students who can calculate 1/4 + 1/3 should also be able use concrete materials, diagrams, or metaphors to do the same. Further, they should be able to use mental images of fractions to make judgments of their relative size and to use that understanding to estimate reasonable answers to fractional operational tasks. With the acquisition of conceptual knowledge and understanding of the fraction concepts they are learning, these students will be able to provide evidence that they can recognize, label, and generate examples of concepts, use and interrelate models, diagrams, manipulatives, and varied representations of concepts, and identify and apply the principles which are involved.

According to the NCTM (2000), fraction instruction in the middle grades curriculum should emphasize conceptual understanding of fractions and the other forms of rational numbers to facilitate proficiency with using them to solve problems in context. Further, when students have opportunities to solve problems in context and to use a variety of visual images and physical models of fractions, these experiences in the classroom increase their flexibility and facility with working with rational numbers and enhance and deepen their knowledge and understanding of these and other related mathematics concepts, such as proportionality, functions, integers, ratios and slope (NCTM). More than a decade earlier, Kieren (1988) had made a similar point, and Behr, Harel, Post and Lesh (1992) later reinforced it when they stated that a full understanding of fraction ideas would seem to require exposure to numerous rational number concepts.

Oppenheimer and Hunting (1999) conducted a clinical interview study involving 49 sixth-grade students in which the students were given a variety of problem-solving tasks involving fractions and decimals. The researchers wanted to determine how

students understood the connections between fractions and decimals through their findings, and the effectiveness of using follow-up questions and individual interviews to help to make the connections between different but related forms of rational numbers such as fractions and decimals. According to Oppenheimer and Hunting, their findings revealed that most middle school students had not yet accomplished this task, could not perform the task of converting between fractional and decimal representations, and lacked a clear understanding of these concepts. They, therefore, concluded that teachers are obligated to comply with the recommendations of the *Standards* and the expectations of the NCTM regarding improving instruction. This would help to ensure that the mathematics which they teach reflects a standards-based curriculum and that their students are more adequately prepared to take the annual, state-mandated standardized tests, which in some instances devote a minimum of 20% of content to fractions and fraction related concepts.

Therefore, even though traditional fraction instruction does not encourage meaningful performance from most students (Lamon, 2001) and middle school students in particular experience significant difficulties when learning fractions, the research indicates that fractions are an integral part of the mathematics middle grades curriculum. As a result, more educators are beginning to use nontraditional ways of teaching fraction concepts so that these students are allowed opportunities in the classroom that will facilitate the development of their conceptualization skills. This is not to say that procedural knowledge should not continue to play a role in fraction learning. Carpenter (1986) suggested that teachers must recognize the importance of conceptual knowledge and understanding in their efforts to help students to build procedural knowledge.

Establishing conceptual understanding first results in children inventing highly sophisticated strategies for all algorithms with whole numbers (Carroll & Porter, 1997; Kamii, 1985) as well as with fractions (Huinker, 1988; Sharp, Garofalo, & Adams, 2002; Warrington, 1997).

Procedural Knowledge.

Fraction instruction, particularly at the middle grades level, is based on the teacher's demonstrating to the students procedures for performing basic fraction computations by applying specific rules for each type of computation. Students are required, therefore, to memorize, learn, and apply these rules and procedures when performing fraction computations, such as adding, subtracting, multiplying, and dividing. As such, traditional fraction instruction is based on primarily a cycle of teacher demonstration followed by student engagement in skill and drill practice using textbooks or worksheets. This approach to teaching fraction concepts facilitates mainly procedural knowledge.

According to several researchers and organizations, such as the NCTM (1991), instructional approaches based primarily on rote memorization do not enhance student knowledge. Their contention is that though the students memorize the rules and procedures and learn to use and apply them for performing computations, they do not understand the material from a conceptual perspective (Hiebert & Carpenter, 1992; NCTM, 1989, 1991, 2000; NRC, 1989). Furthermore, this type of instruction does not provide opportunities for the students to engage in activities that require them to think at high levels of cognition, communicate and share their mathematical ideas and thoughts with each other in the classroom, or develop strong critical thinking, reasoning, or

analytic skills. Similarly, Streefland (1991) reasoned that approaching teaching of fractions from a primarily mechanistic perspective detaches it from reality and focuses instead on rigid application of rules. As a result, in many cases the students forget the rules, apply them incorrectly, and make procedural errors when computing fractions. In general, many of them also do not understand the underlying concepts, a crucial part of fraction learning and instruction. Streefland stated that, in addition, educators' extreme underestimation of the complexity of this area of learning for children contributes to student difficulty with fractions and their subsequent poor performance on fraction assessment and comparison tests. Behr et al. (1992) concurred with Streefland and attributed student difficulty with learning fractions to the fact that these concepts are taught mainly via computations and memorization of rules and procedures but without context, thus implying to the learner that algorithms are an ungrounded code only mastered through memorization. They added that this ineffective method of instruction results in many students' being unable to develop connected knowledge about fractions relative to number sense, operations sense, and algorithmic skills, leading them to experience many difficulties as they learn these and other related concepts.

Carraher's (1996) contribution to the discussion on the ineffectiveness of traditional fraction instruction centered on the mechanistic pattern of using rules and procedures to perform meaningless computations, which, in his opinion, most students do not remember nor understand. To explain further his position on the shortcomings of the traditional instructional approach to teaching fractions, Carraher created and identified several categories in which he placed the disadvantages of traditional fraction instruction. These are (a) the part-whole fixation, which resulted from students' being taught to

associate fractions to wholes and parts, which impedes their transfer of knowledge to other cases; (b) the cardinal sin, in which students were misled to focus on cardinal number counting via counting and matching tasks and as such ignore the ratio meaning of fractions; (c) missing links, in which students were not taught to link fractions to integer multiplication and division, ratio, proportion, functions, and other concepts; and (d) no challenge, the providing of exercises that lacked authenticity and were unchallenging, computational tasks. Carraher also posited that these disadvantages of the traditional instructional approach to teaching fractions are reasons for seeking and using alternative approaches, particularly those that emphasize conceptual knowledge.

Given the fact that the mathematics middle school curriculum requires students to know and understand how to compute and problem solve with fractions, student acquisition of procedural knowledge of these concepts is a necessary part of fraction learning, but it should not be the central focus. Instead, students should be able to understand underlying concepts and the meaning or the *idea* of fractions (Hasemann, 1981) and the why of fractions (Eisenhart et al., 1993). Students can derive this meaning of fractions if the focus of instruction is on conceptual knowledge, a theme that permeates the previous sections of this review on student difficulty with fraction learning and their understanding of fractions. As discussed above, several researchers who have done extensive work on fraction teaching and learning contend that many students who find it difficult to learn and understand fractions lack conceptualization skills and that this results from traditional fraction instruction that emphasizes procedural knowledge. The indication is that fraction instruction that emphasizes conceptual instead of procedural knowledge and the use of a variety of alternate instructional techniques and tools such as

fraction representations can serve to enhance student learning and lessen their difficulty with learning fraction concepts.

Representation

A major focus of this research study is on how the participants use fraction representations and not merely symbolic representations, procedures, and calculations as they work with the fractions embedded in three performance tasks to find practical, but not necessarily specific solutions. This is a critical aspect of the study because it requires the participants to move away from familiar ground – calculations-based responses – to conceptual oriented ones via the use of fraction representations.

In the field of mathematics a fraction is commonly represented by a number in the form (a/b) and is used with a set of well-defined and well-known operations and properties. Therefore, learning about fractions requires the learner to be aware and knowledgeable about the special relations between numbers and quantities and to express these relations in *diverse* ways (Carraher, 1996). Fractions, like relations, can be expressed in several ways, and these representations can be used in lieu of the standard symbolic (a/b) form. On a broader scope, researchers such as Goldin (1982, 1987, 1990), Goldin and Herscovics, (1991a, 1991b), and Kaput (1992, 1993) have been developing the concept of representation in the psychology of mathematical learning and problem solving. What then is representation as it pertains to mathematical learning and fraction instruction?

Representation, one of the ten standards of the recent *Principles and Standards for School Mathematics* (NCTM, 2000), is comprised of two major forms – external and internal (Goldin & Shteingold, 2001; Miura, 2001). The NCTM defined "representation"

as a process or product that is used in the act of capturing a mathematical concept or relationship in some form, as well as the form itself; these forms include diagrams, graphical displays, and symbolic expressions. Goldin and Shteingold defined “representation” as a sign or configuration of signs, characters, or objects that can stand for, depict, represent, or symbolize something other than itself. The NCTM’s goals for this standard, which covers prekindergarten through grade 12, seek to enable these students to (a) create and use representation to organize, record, and communicate mathematical ideas; (b) select, apply, and translate among mathematical representations to solve problems; and (c) use representations to model or interpret physical, social and mathematical phenomena.

In its general discussion on the need for educators to use representations in the mathematics classroom, the NCTM (2001) stated that the way in which mathematical ideas are represented is fundamental to how people understand and use them. The NCTM also pointed to the fact that data from research on the issue indicate that students at all levels experience difficulties with developing understanding of the complex ideas of conventional representations. Further, the NCTM stated that, though representations of fractions (and other conventional forms of representations) may be difficult to develop and understand, they are nevertheless effective tools for teaching mathematics, allowing the students opportunities to represent their mathematical ideas in ways that make sense to them to communicate their mathematical ideas and help them better to learn and understand mathematic concepts. The NCTM was supported in this view of the usefulness of representation in the mathematics classroom by several researchers who stated that encouraging children to generate their own visual representations of

mathematical patterns and relationships has many advantages and benefits. These include challenging the students to organize and display significant mathematical features of an experience, thereby allowing them to use their insights and inventiveness (Atkinson, 1992; Folkson, 1996; Groves & Stacey, 2001; Whitin, 1997; Whitin & Whitin; 2001).

The NCTM (2000) recommended, therefore, that mathematics students have opportunities to learn these forms of conventional representations and to construct, refine, and use their own representations as tools to support learning and doing mathematics. It also suggested that teachers help their students to use representations flexibly and appropriately by (a) encouraging them to create and use representations to support and communicate their mathematical thinking, (b) helping them to develop facility with representations by listening, questioning, and trying to understand what they are trying to communicate via their own representations, and (c) helping them to develop meaning for important forms of representation. To accomplish this, teachers should provide students with classroom experiences using a wide range of visual representations and must introduce them to new forms that are suitable for problems solving in a variety of contexts and related to a variety of mathematics topics and concepts. Teachers should also be mindful of the fact that students' early algorithms with fractions are generally diagrams and mental reasoning (Huinker, 1998). Because children can develop overwhelming trust in this visual knowledge rather than operational knowledge, Kamii and Clark (1995) cautioned that careful attention must be given to the facilitation of connections between informal, pictorial representations and the corresponding knowledge, which eventually must be expressed with mathematical symbols. More importantly, teachers must help students to use representations meaningfully.

Subsequently, teachers can use this information to gain insight into how the students think about and interpret mathematics and to bridge the gap between the students' own constructions of representations and conventional ones.

Further, having taken the stance that representation is central to the study of mathematics, the NCTM (2000) noted that students can develop and deepen their understanding of mathematical concepts and relationships as they create, compare, and use various representations, such as physical objects, drawings, charts, graphs, and symbols, to communicate their mathematical thinking (NCTM, 2000). This reasoning implies that middle school students can begin their experiences with representations by creating and using them to work with topics, such as rational numbers, rates or linear relationships, which require them to think more abstractly. These representations can help students to solve problems, model, clarify, or explain a mathematical idea or real-world relationship. They can also help students to improve their recognition and understanding of different forms symbolic representations. For example, representations that may appear to be or look different may represent the same phenomenon.

Goldin and Shteingold (2001) posited that the concept of representation and its relevance to the teaching and learning of mathematics has been the focus of many researchers for a long time, though it has evolved considerably in recent years. They stated that an important aspect of representation is its two-way nature: The representing relation (depicting, encoding, or symbolization) often can go in either direction. For example, a Cartesian graph, which is a representation of data, could represent an equation in two variables, and conversely an equation relating x and y can represent an algebraic symbolization of a Cartesian graph. Whatever is represented can vary according to the

context or the use of the representation. Other characteristics of representations are that they (a) can be *external* – things that students can produce (such as graphs) and teachers can identify in the classroom and use in a discussion, (b) are a part of a system - the numeral 5 is a part of the conventional symbol system of arithmetic, (c) are static in the sense that they provide rules or frameworks for creating *fixed* external formulas, equations, graphs or diagrams, and (d) dynamic – that is an external system that can be changed and linked to each other through the use of technology. However, Goldin and Shteingold cautioned that, despite the usefulness of external representational systems, their usage does present limitations. They do not necessarily facilitate student understanding of mathematical meaning, recognition of structures, or the ability to interpret results. They enable students to perform skillful arithmetic computations, manipulate mathematical expressions well, learn and follow mechanically oriented procedures, and memorize definitions. Yet all of this occurs without the students developing conceptual knowledge and understanding of the concepts that they are learning.

Conversely, Goldin and Shteingold (2001) purported that representations can also be categorized as internal or *mental* representations which are used as a framework to characterize the complex cognitions that can occur when a student tries to understand mathematical concepts. These internal systems of representation can be of several different kinds and include (a) *verbal/syntactic,* those that describe the students' natural language capabilities (mathematical and nonmathematical vocabulary and the use of grammar and syntax); (b) *imagistic*, those that include kinesthetic coding (related to actual or imagined hand gestures or body movements important in terms of capturing the

students' *feel* of mathematics), visual and spatial cognitive configurations, or "mental images" that contribute greatly to mathematical understanding and insight, and auditory and rhythmic internal constructs, such as hand clapping and counting sequences; (c) *formal notational,* those that allow students mentally to manipulate numerals, perform arithmetic operations, or visualize the symbolic steps, such as those involved in solving an equation; (d) *strategic and heuristic,* those that involve processes for solving mathematical problems and require students to develop and to use mentally organized methods, such as trial and error, establishing subgoals, or working backwards; and (e) *affective,* those concerned with students' changing their emotions, attitudes, beliefs and values about mathematics, and therefore possibly enhancing or impeding their mathematical understanding.

Goldin and Shteingold (2001) added that because a researcher cannot observe anyone's internal representations directly, he or she has to make inferences about these representations based on their interaction with, discourse about, or production of external representations. However, they cautioned that these internal representations do not simply encode or represent what is external but can refer to each other in complex ways, thus allowing educators to use them to characterize the individuals' conceptual understanding.

To focus on representations from a philosophical perspective, Goldin and Shteingold (2001) addressed issues relating to the two broad perspectives which have influenced classroom use of representations in the teaching of mathematics, behaviorism and constructivism, with the former explaining learning through the lens of external, observable variables and the latter through the internal processes through which knowledge is formulated (Ernest, 1991). They alluded to the fact that their work on

representations aimed at bridging the gap between the two groups of believers of the two distinct philosophies about teaching and public education that have influenced mathematics education and instruction. They posit that educators in favor of basic mathematic skills, correct answers through correct reasoning, individual drill and practice, more direct models of instruction and measures of achievement via objective tests, tend to prefer the behaviorists' characterization of skills and the philosophy of how students learn.

Conversely, according to Goldin and Shteingold (2001), those who favor children's discovering their own learning in mathematics, the use of open-ended questions, different conceptualization by different children, less use of teacher-centered instructional models, groups and individual problem-solving activities, and alternative assignments, regard constructivism as the preferred philosophy. These researchers stated that their work focused on *both* the external and internal forms of representations but strongly emphasized the interplay between them. They explained that when students interact with structured external representations in the learning environment, this facilitates the development of their internal representational systems and subsequently generates new external systems. Their conceptual understanding is reflected, therefore, in the power and flexibility of the internal representations, including the richness of the relationships among the different kinds of representation. They concluded that their contribution to the research on representation synthesizes the behaviorists and constructivists' perspectives on the subject and lends itself to a more inclusive philosophy that sees the benefits of both schools of thought, without seeing them as contradictory. This blending lends itself to making a positive contribution to the goal of high

achievement in mathematics for a majority of students through a variety of different representational approaches (Goldin & Shteingold).

Also, in speaking specifically about the difficulties which middle school students experience when learning the fraction form of the rational numbers, the NCTM (2000) pointed to the fact that though students may learn to represent fractions by way of partitioning circles, rectangles, or other commonly used shapes and can use them to interpret or understand the meaning of fractions, these forms of representations do not provide meaning of or interpretations of fractions, such as ratio, indicated division, or the fraction as a number. According to the NCTM, this situation can be corrected and students can become deeply knowledgeable about fractions and other concepts if they are allowed to use a variety of fraction representations to support their understanding. This is true because “different representations often illuminate different aspects of a complex concept or relationship” (p. 69). However, the NCTM cautioned that teachers should not introduce representations before students are able to use them in a meaningful manner.

Performance Assessment and Performance-Based Tasks

Classroom teachers have been encouraged to use a variety of tools to aid students’ mathematical learning (Stephan, Cobb, Gravemeijer, & Estes, 2001). The NCTM (1991) emphasized the point that teachers must value and encourage the use of a variety of tools when teaching mathematics in order to promote discourse to focus more on mathematical ideas than on observable calculations and methods. Because of current and ongoing mathematics reform efforts in many states and the need to address the issues related to the limitations of traditional testing such as validity, design, and their influence on instruction, educators have begun to place more emphasis on the use of nontraditional

instructional and assessment tools than on traditional forms of assessments to enhance student instruction, student learning and student achievement (Danielson, 1997).

Performance assessment. According to Danielson (1997), a performance assessment is a nontraditional means of assessing student learning that requires evaluation of the students' writing, physical products, or behavior. This includes all types of assessments with the exception of multiple choice, matching, true/false testing, or problems with a single correct answer. Written products are anything written by students but not under testing conditions, while physical products are three dimensional creations such as a diorama, a science construction, a mathematics project, or a sculpture. While participating in a performance assessment students demonstrate their knowledge or skill through their behavior and this behavior can be captured, stored, and then evaluated. Classroom-based assessment occurs in the classroom and allows each student to show what he or she can do by using his or her own strategies and approaches to finding a solution rather than the teacher's. The teacher, as distinct from large-scale, statewide performance testing, then evaluates the students' solutions. Educators who use criterion-referenced, performance-based assessment are concerned with the degree to which students can demonstrate knowledge and skill in a certain field as well as demonstrate competence.

Performance-based tasks. One type of performance assessment is performance-based (or performance) tasks. Danielson (1997) described performance tasks as classroom tasks that are designed to assess learning or to be used as an instructional activity based on clearly defined instructional goals. She identified the benefits of performance tasks, which include (a) clarity regarding criteria and standards, (b)

professional dialogue about criteria and standards, (c) improvement in student work, and (d) improved communication with parents. Further, Danielson stated that performance tasks are comprised of problems which require the students to solve primarily open-ended questions. These questions describe a task structure that allows students to determine their own approach when solving problems and typically include a request for either the student's approach or reasoning process. Open-ended performance tasks have an added advantage of providing detailed information concerning the students' mathematical reasoning, knowledge, and understanding (Moskal & Magone, 2002).

Further, Danielson (1997) categorized performance tasks as being small or large, depending on the purpose. The size of a performance task is determined by its purpose, that is, if it is used as an immediate or culminating assessment or as an instructional activity, and by the time constraints and experience of the teacher. Small performance tasks are designed for purely assessment purposes and require the students to solve a relatively small problem, explain their thinking, and show their work. For example, if after completing an instructional unit a teacher wants to determine if the students have understood the concept, then a small performance task is desirable. However, this type of task in itself will not contain activities to be completed as part of the task. In addition, if the purpose of the task is to assess a small part of the curriculum, small tasks are better suited for this because they can be administered frequently and the results may be used for adjusting the instructional approach. On the other hand, Danielson described large performance tasks as those that are more suited to instructional and assessment purposes, such as culminating assessments that are spread over a number of days, tap a number of different skills, and involve many subactivities. For example, when the goal is to teach

new content, a large task that is spread over a number of days and that involves many subactivities will be more suitable than a small task. Preparing and administering large tasks have the disadvantage of requiring many hours of preparation time on the part of the teacher because of the length of days over which it may be extended.

In concluding her discussion, Danielson (1997) cautioned that the initial efforts to use performance assessment should involve using small tasks that allow for opportunities to experiment with a new methodology in a way that carries low stakes for success for both the students and the teacher. She identified and described the characteristics of a good performance task as one that (a) is engaging – it interests the students, (b) is authentic – students can relate it to their *real life*, (c) elicits desired knowledge and skill – it is aligned with instructional goals, (d) enables assessments of individuals – individual learning is evaluated, and (e) contains clear directions for students - they are not in doubt as to what they are expected to do. Similarly, Herman, Aschbacher, and Winters (1992) suggested that characteristics of the best performance tasks should (a) match the learning goals and desired outcomes, (b) require students to use critical thinking skills, (c) be worthwhile of instructional time, (d) use engaging tasks from the *real world*, (e) facilitate measuring several outcomes, (f) be fair and free from bias, (g) be credible, that is they are meaningful, challenging, and appropriate, (g) feasible, and (h) clearly defined. The next section discusses scoring rubrics, the instrument used to score student work resulting from performance-based tasks.

Scoring Rubrics

Bush and Leinwand (2000) stated that assessment involves gathering evidence and making judgments about what students know and are able to do, with the two most

common methods of making judgments about student work being scoring and grading. They defined scoring as the act of comparing the students' work to standards which are designed to communicate the educator's expectations for the students' work and to provide a structure for reliably and accurately evaluating the work. On the other hand grading is what is done with the set of scores to summarize or compare students' performance and communicate it to others, while evaluation is the process of making judgments or placing values on the evidence gathered through assessment.

A scoring rubric is one type of assessment tool that is used to evaluate student work. In general, according to Bush and Leinwand (2000), a rubric represents a hierarchy of standards used to assign scores to student' work, most often on a 4, 5, or 6-point scale, and facilitates keeping the focus of the assessment process on performance rather than on the performer. They added that a rubric also facilitates (a) providing quality feed back to students, (b) the monitoring of student progress, (c) increasing student skills in mathematical reasoning and meta-cognition, and (d) defining specific performance expectations. A well-designed rubric allows students to see descriptions of the requirements for their performance. Danielson (1997) defined a rubric as a guide for evaluating performance.

Further, Bush and Leinwand (2000) identified the two main types of scoring rubrics: holistic and analytic rubrics. First, holistic rubrics describe the qualities of performance for each performance level and are usually general, in that they may be used to score any mathematics task. The score that the students receive through holistic rubrics depends on the level of performance that they have achieved. Students receive one numerical score for one task. By assigning one score to the students' work, the evaluator

judges the work on its overall quality. Bush and Leinwand identified the advantages of holistic rubrics by stating that (a) student work is judged by its overall quality, (b) all processes are given equal weight, and (c) they stress thinking processes and mathematical communication. Danielson (1997) stated that the teacher's use of holistic rubrics facilitates analysis of student work relative to its strengths and weaknesses.

The second type of scoring rubric discussed by Bush and Leinwand (2000) and Danielson (1997) was the analytic rubric. They described this type of rubric as one in which the teacher assigns scores to the components of a task and assigns specific points for completion of each component. These points are then added to obtain an overall score for each task. The advantages of an analytic rubric include the fact that it (a) stresses the different steps in solving a task, (b) gives some processes more weight or emphasis, (c) allows for partial credit, and (d) is easier to apply. In addition, analytic scoring rubrics are used to describe the qualities of performance for each performance level as well as to score any mathematical task (Bush & Leinwind).

Summary

The contents of this chapter represented a review of the literature on the teaching and learning of fractions and the difficulties, which primarily middle (and elementary) school students experience with understanding and learning these concepts. The review began with a discussion on the history of U.S. public school students' (and to a lesser degree teachers') difficulties with fractions and moved to fraction learning at the elementary and middle grades level, its importance in the curriculum, and the difficulties which these students' experience in the classroom and on achievement and standardized tests. The next section of the review centered on the literature regarding alternatives to

traditional fraction instruction. This included fraction instruction that focuses on teaching for conceptual (as opposed to procedural) knowledge, the use of fraction representations, performance assessments and performance-based tasks, and scoring rubrics. One theme, which permeated the literature was that a large number of middle school students lack fraction conceptualization skills. Consequently, these students do not understand the meaning of fractions and the underlying concepts. Any students' lack of understanding and conceptualization skills is a major, contributing factor to his or her difficulties with learning fraction concepts and other related concepts. From a broader perspective, these difficulties can contribute to the student's unsatisfactory levels of performance and achievement in mathematics.

CHAPTER 3

METHODOLOGY

This chapter provides a description of the design and methodology that were used to conduct this research study, giving special emphasis to the analysis of data. I used qualitative research paradigm (in conjunction with a case study strategy of inquiry) was used in the study. The characteristics of a qualitative research design facilitated the use of multiple data collection sources, methods, and analyses to facilitate the triangulation of data and confirm emerging themes (Bogdan & Biklen, 1998). The study's three participants represent individual cases.

Purpose of the Study

The purpose of this study was to determine the extent to which urban 8th-grade, middle school students could compute and conceptualize fractions when working on fraction-related performance-based tasks and the ways in which these students use representations of fractions to organize and communicate their mathematical thinking and reasoning when finding solutions to fraction-related, performance-based tasks.

Research Questions

Two research questions guided this study:

1. To what extent do urban, middle school, 8th-grade students compute and conceptualize fractions when working on fraction-related performance-based tasks?

2. In what ways do urban, middle school, 8th-grade students use fraction representations to organize and communicate their mathematical thinking and reasoning when working on fraction-related performance-based tasks?

Rationale for Using a Qualitative Research Paradigm

A major characteristic of qualitative research is the propensity for generating large amounts of textual data which can then be used to facilitate the discovery of meaningful patterns that describe the research problem or answer the questions which guide the study (Fraenkel & Wallen, 1990; Locke, Spirudoso, & Silverman, 1987; Marshall & Rossman, 1995; Merriam, 1988). Likewise, Auerbach and Silverstein (2003) posited that qualitative research involves analyzing and interpreting texts and interviews to discover meaningful patterns that are descriptive of a particular phenomenon.

Lincoln and Guba (1985) and Merriam (1998) defined qualitative research as an umbrella that covers several forms of inquiry that help researchers to understand and explain the meaning of social phenomena with as little disruption of the natural setting as possible. Fraenkel and Wallen (1990), Merriam (1988), and Patton (1990) posited that a qualitative research design allows the researcher to (a) observe the type of processes and strategies that the students employ, (b) understand what is happening in the classroom, and (c) observe the participants in their natural setting without the added task of trying to apply a treatment or predict what may happen in the future. Geertz (1973) posited that a major goal of qualitative research is to generate “thick description,” the explication and detailed description of the world of the people who live out the phenomenon that is being investigated. This descriptive material is analyzed subsequently to yield sensitizing concepts that articulate categories by which people understand their own world.

The goals of this research study were aligned with the characteristics of a qualitative research design, particularly relative to (a) describing and understanding educational phenomena, (b) emphasizing the acquisition of a large quantity of descriptive data, (c) generating answers to the research questions during interviews and observations, and, (d) documenting the experiences of the participants in the learning environment.

More importantly, a qualitative design facilitated using a variety of data gathering techniques such as interviews, artifacts, and field notes to obtain data regarding the participant's competencies, their thought processes, and their discoveries while they worked on performance-based tasks individually and in groups. For example, observing them while they worked and communicated in small groups provided opportunities to hear their comments, questions, explanations and opinions regarding the strategies and approaches that they used to find solutions to the performance-based tasks.

Therefore, using this form of research was both logical and practical, and it was consistent with the point that a qualitative design facilitates a better understanding and description of students' learning by providing researchers with opportunities to observe, investigate, and document the environment in which the learning occurs (Creswell, 1994; Glesne & Peshkin, 1992; Guba & Lincoln, 1989; Lincoln & Guba, 1985).

Rationale for Using a Case Study Strategy of Inquiry

Merriam (1998) lists ethnography, phenomenology, grounded theory, and case study as the four main research methodologies under the umbrella of qualitative research. In her discussion on case studies, she states that the three main characteristics of a case study are its particularistic, descriptive, and heuristic nature. Particularistic refers to the fact that case studies focus on a particular event or phenomenon. The focus of this case

study is urban, 8th-grade, middle school students' level of ability to compute and conceptualize fractions and the ways in which they use representations of fractions when finding solutions to fraction-related performance-based tasks.

The second characteristic of a case study is its descriptive nature, which, in the field of education, allows the researcher to focus on individual or groups of students and problems of practice (Stake, 1995). In her discussion on this characteristic, Merriam (1998) stated that the case study, as a research methodology, facilitates (a) intensive descriptions and analysis of data resulting from interviews, observations and field notes, (b) an in-depth understanding of the problem being studied, (c) direct observations of participants in their natural setting and, (d) opportunities for the researcher to monitor the participants and to get immediate feedback to questions. Merriam also added that this form of inquiry allows for a "thick description" of the data that is a complete, literal description of the incident or entity that is investigated.

The third characteristic of a case study, according to Merriam (1998), is its heuristic nature, which allows the reader to gain insight into the phenomenon being studied. In terms of this study, this facilitated my understanding of the extent to which middle school, 8th-grade students compute and conceptualize fractions and the ways in which they use fraction representations when working on performance-based tasks to communicate their mathematic thinking and understanding.

All of the characteristics described above were consistent with the goals of this research study and provided me with opportunities to gather data, describe, make and present judgments about the participants' feelings, thinking processes, and opinions about using representations of fractions when working on fraction-related, performance-based

tasks, and about their level of ability to conceptualize and compute fractions. In a broader sense, the case study strategy of inquiry was an appropriate means of accomplishing the broader goal of generating a thick description of the experiences, thoughts, feelings, discoveries, and the general dynamics of the learning environment of the students who participated in the study.

Research Paradigm

My own belief system accepts multiple realities and the constructivist philosophy about how individuals learn. Mertens (2005) and Schwandt (2000) stated that the basic assumptions guiding the constructivist paradigm are that knowledge is socially constructed by the individuals active in the research process and that researchers should attempt to understand the complex world of lived experiences from the point of view of those who live it. I used the constructivist paradigm as the theoretical underpinning for this study because it is consistent with my beliefs and with the nature of the study and because I believed that the research questions were situated within the context of this paradigm. Further, I was influenced to ground this study within the context of the constructivist paradigm because it (a) uses a primarily qualitative study, (b) emphasizes giving the participants opportunities to play an integral role in their learning, and (c) allows for the existence of socially constructed realities. According to Guba (1991), the methodology used in constructivism is “hermeneutic in the sense that individual constructions are elicited and refined through interactions between and among investigator and respondents” (p. 5). Guba also defined the constructivist paradigm in terms of its ontology, epistemology, and methodology and added that for constructivism, reality is “socially based” in that, multiple realities exist in the form of mental

construction, and findings are created by the interaction of the inquirer and the participants. Lincoln and Guba (2000) identified three questions that help define a paradigm relative to its ontology, epistemology and methodology: (a) What is the nature of reality? (b) What is the nature of knowledge and the relationship between the knower and the would-be known? (c) How can the knower go about obtaining the desired knowledge and understandings?

One major perspective of constructivism that greatly influenced the instructional approach and activities used in this study was social constructivism. The philosophy of Vygotsky (1978), a social constructivist who believed that in a group the individual learns from social interactions first and from individual experiences later, influenced most modern day social constructivists. Wertsch and Toma (1990) also shared this point of view. According to these researchers, the group acts to construct knowledge and to resolve the differences in meanings of individual group members; therefore, knowledge is constructed interpersonally and then intrapersonally.

The tenets of constructivism and social constructivism parallel my beliefs about the role that students should play in the classroom. I concur that mathematics instruction should provide learners with opportunities to participate in the lesson, to share their ideas and thoughts, and to make connections between the knowledge from within, the informal knowledge which they bring to the classroom, and that which they learn from socially interacting and communicating with their peers and with the teacher.

The purpose of this study was to determine the extent to which the participants could compute and conceptualize fractions when finding solutions to performance-based tasks and the ways in which they use fraction representations to organize and

communicate their mathematical thinking and reasoning as they worked these tasks. A case study approach involving three participants was used (each being a case), and their work was analyzed to find answers to the research questions. This warranted individual work followed by group discussion. After working on the tasks individually, each of the three participants got together in groups to discuss their ideas and thoughts about the tasks, the approaches and strategies that they used, their choices of fraction representations, and their solutions. The students then decided on one solution that they thought was the correct one for the tasks and used input from each other to do so. In addition, the participants discussed the questions given in the form of prompts on their group reflection logs and collaborated regarding the answers. All of these group activities gave them the opportunities to interact socially. By working in this socially interactive classroom environment the participants shared the prior knowledge, both formal and informal, that they brought to the classroom and the knowledge they generated by working together in the classroom. I believe that grounding this research study in the constructivist paradigm best facilitated these classroom learning activities and social interactions.

Research Setting

The study was conducted in a public, middle school located in the southeast United States in an inner city community of primarily African American, retired, and middle-class residents. As such, the vast majority of the student population of 11-14 year-olds did not live in the immediate environs of the school. These students were bussed from nearby communities within the school's attendance zone. Other students who resided outside of the attendance zone obtained special permission (for a variety of

reasons) to attend the school. Almost all of the school's students were African American while a very small number of Hispanic students represented the rest of the student body. Approximately 70% of the total student population came from families with a low, socioeconomic status. The school is one of twelve middle schools in one of the state's largest school districts. It has three levels (grades 6-8) and an approximate total student population of 900. The students were housed in classrooms located in two sections of the premises, in a main building and in classrooms in two rows of eight portable buildings (trailers) located directly behind the main building.

All of the students in the school were given diagnostic tests in reading and mathematics at the beginning of the 2004-2005 school year and were subsequently grouped by ability within teams, based on their performance. Each team of students comprised of four groups of approximately 30 students and had four teachers, each of whom taught one of the four core subjects: language arts, mathematics, social studies, and science. In addition, each group of 30 students was assigned to a designated homeroom and homeroom teacher. This teacher was responsible for doing clerical tasks, such as attendance and preparing grade and other academic reports. All of the students assigned to a homeroom were required to report to their homeroom teacher's class every morning for a 10-15 minute homeroom period before moving to their other core subject classes. Within the core classes the students were grouped either by individual homerooms or by a combination of the four homerooms on their team. Each group of students reported to one of their four core classes for a 75-minute block of instruction and rotated among the four teachers who taught the core subjects of mathematics, language arts, social studies, and science.

Unlike the rest of the student body, the team to which I was assigned represented the only two-teacher team in the school. This occurred because there were not enough students to form a four-teacher team. Each of the two teachers was responsible, therefore, for teaching two core subjects instead of one. I taught mathematics and social studies; therefore, the students who were involved in the research study attended these classes in my portable classroom. There was a total of 37 students on my team at the beginning of the school year. These students were assigned to either of two groups – Group I (23 students) or Group II (14 students). The criteria for placement in Group I was a minimum passing score of 70 or more (out of 100 points) on each of two diagnostics tests – mathematics and reading. Students who did not meet these criteria were placed in Group II. (If a student needed remedial help in both subjects, the reading/language arts teacher made the final decision as to where to place the student.) For the most part, if a student needed more help with reading and language arts than with mathematics, he or she was assigned to Group II. The instructional objectives for mathematics were the same for both groups of students, but I varied the strategies, pacing and difficulty level of the content to meet the needs of the students in Group II, most of whom performed below grade level in both mathematics and reading. Twenty (54%) of the initial 37 students on my team did not meet state standards in mathematics on the state-mandated Criterion Reference Competency Test (CRCT) when they took it as 7th-grade students the previous spring. Further, despite the fact that officially these students were assigned to a prealgebra class, the objectives of the state's curriculum pertained to 8th-grade general mathematics primarily rather than to prealgebra concepts. In addition, the contents of the state-

mandated examination that these students were required to take at the end of the school year comprised of primarily mathematics, rather than prealgebra concepts.

Selection of Participants

A primary characteristic of qualitative research methodology is to produce information that is highly descriptive and more conducive to understanding the human perspective as it examines phenomena, in this case, in the classroom-learning environment. An added advantage of the qualitative methodology is the ability to select population samples with the assumption that the data obtained will be garnered in a reliable context (Lincoln & Guba, 1985).

I used the purposeful form of a nonprobabilistic sampling strategy to select the participants for the study. Patton (1990) stated that purposeful sampling is based on the assumption that the investigator wants to discover, understand, and gain insight and therefore must select a sample from which the most can be learned. He also argued "the logic and power of purposeful sampling lies in selecting information-rich cases for in-depth study. Information-rich cases are those from which one can learn a great deal about the issues of central importance to the purpose of the research" (p. 169). Similarly, Merriam (2001) described purposeful sampling as being based on the assumption that if the investigator wants to discover, understand, and gain insight, a sample must be selected from which the most can be learned.

Because the above descriptions were aligned with the goals of this study, using this form of nonprobabilistic sampling was logical and practical as well as relevant. Further, because Merriam (2001) also suggested that the researcher begins the process of purposeful sampling by first determining the essential criteria for selecting the sample, I

decided that the participants would be selected from the 8th-grade, middle school students attending my school, and in particular, from among the students to whom I would teach mathematics.

Initially, all of the 37 students in the two, prealgebra classes that I taught participated in the study by completing two fraction pretests and a fraction interest questionnaire, which I administered to them prior to the start of the study. I intended to select the study's three participants from both classes (Group I and Group II), if possible, based on the fact that even though the second group comprised of students who needed more remedial help in reading and language arts, at least three of them were relatively strong in mathematics and could inform the study if they were selected.

I established two criteria to select the three participants who would each represent a case. The first criterion was a score of 75 or higher out of a possible 100 points on the fraction computation and on the fraction concepts pretests. I based this decision on current standards for average and above average passing grades that are used in my school system. The second criterion was that the students obtained mean scores of 25 or more out of a possible total score of 30 on four of the five attitudinal categories of the scoring key that was used to score the Attitude Toward Fractions Inventory (ATFI) questionnaire. The four-attitudinal categories were (a) Value of Fractions to Mathematics (VFM), (b) Confidence About Fractions (CAF), (c) Enjoyment With Fractions (EWF), and (d) Motivation About Fractions (MAF). It should be noted that only four of the five-attitudinal categories (of the ATFI scoring key) were used for scoring the questionnaires because these categories reflected the highest possible level of positive attitudes

regarding fraction learning in the students' responses to the statements of the questionnaire.

However, after scoring the two pretests and the questionnaire, and analyzing the data obtained from both of these data collection sources, I decided to use only the 23 students in my Group I prealgebra class as the sample from which I would select the final three who would be the study's participants and whose work would be used for the data analysis phase of the study. I based my decision on the fact that none of the students in Group II met either of the two selection criteria and, in general, the students in Group I obtained higher individual and mean scores on the two fraction pretests and on the questionnaires compared to the students in Group II.

With regard to the computation pretest, 67% (representing 14 out of the 21 students who took the test) of the students in Group I satisfied the selection criterion of obtaining a score of 75% or more. For this group, the mean score was 76.6%, the median 83%, the highest 100%, and the lowest score was 32%. In Group II only 17% (representing 2 out of the 12 students who took the test) of the students satisfied this criterion. The mean score for Group II was 44%, the median 50%, the highest 77%, and the lowest score was 13%. With regard to the concepts pretest, none of the 37 students (all students were present) in the two groups who took the tests obtained a passing score. For group I, the mean score was 44%, the median 50%, the highest 62% and the lowest 12%, while for Group II, the mean was 26%, the median 33%, the highest 46%, and the lowest 12%. The indications were that the students in Group I (compared to their peers in Group II) represented a sample that would facilitate selecting the best three students to be

the participants for the study, thus allowing me to obtain the most informative and descriptive research data.

In terms of the second criteria (25 or more out of 30 points on each of the selected four attitudinal categories of the scoring key for the questionnaire) only a small percentage of the 37 students in both prealgebra groups who completed the fraction, interest questionnaire, met the criterion. With regard to the VFM attitudinal category, 5% of the students satisfied the criterion. For the CAF category 14% met the criterion. For the EWF category, 3% met the criterion. Finally, for the MAF category, 5% met the criterion. Again, the responses that garnered all but one (5%) of the scores just previously mentioned came from students in Group I. Therefore, based on the fact that none of the 23 students in the selection sample met both of the two established selection criteria, I adjusted the criteria and selected the three students (each representing a case) who (a) obtained the highest scores on the fraction computation and concepts pretests, (b) had consistently demonstrated their enthusiasm about learning and doing mathematics during class, (c) communicated fairly well, orally and in writing, and (d) were likely to agree to participate in the interviews and answer the questions posed in an honest manner. These students' pre-test scores and questionnaire responses indicated that as participants, they would provide me with the most informative and descriptive research data. The following is a description of each of the study's three participants.

First Participant: Liza

The first participant, Liza, scored 87% and 56% respectively on the fraction computation and fraction concepts pretests and satisfied the other selection criteria. Despite not obtaining a score on the fraction concepts pretests that satisfied the selection

criterion, Liza's overall pretest scores placed her among the top three performances of all of the students in my two prealgebra classes.

Liza, who is an only child, was somewhat shy and did not usually talk in class unless I asked her a question or if she volunteered to answer. Even though she rarely spoke above a whisper, she always participated in class. She loved mathematics and was always enthusiastic about learning the subject. Liza had perfect attendance for the school year, was always on time to class, paid attention, came to class prepared to work every day, and did not miss any of her mathematics assignments. She frequently answered the most difficult and challenging mathematics questions during class discussions.

Unlike her mathematics skills, Lisa's language and communication skills were not indicative of the level of ability for an 8th-grade student, although she obtained above grade-level scores on the state-mandated, standardized language arts test which she took at the end of the 2003-2004 school year when she was a seventh grader. During her school year with me, Liza did not do as well in her two reading-related subjects, language arts and social studies. I taught Liza both mathematics and social studies, and she was a joy to have in class. She willingly agreed to participate in the study when I told her that I considered selecting her.

Second Participant: Allan

The second participant, Allan, obtained the highest score (100%) on the computations pretest, thus partially meeting the first criterion. His score of 58% on the fractions concepts test, though not a qualifying one, was nevertheless the second highest among the students' scores in both groups. Allan satisfied all of the other criteria for selection.

Allan is the younger of two siblings. His passion is playing the saxophone, and he is a member of the school's band. He is 13 years old, had a very pleasant personality and a constant smile. He loved mathematics and was very strong in performing mathematics computations. He liked to work on challenging mathematics problems and did not like to give up until he found a solution to whatever problem on which he was working. Allan was a very logical thinker, and he reasoned very well.

When Allan came to class, he was always enthusiastic about learning mathematics and participated fully in class. However, his handwriting and his work were usually very untidy, and he did not like to follow strict procedures when solving a problem. Instead, he liked to take short cuts or to find his own way of doing the problem. Although he frequently had the correct answers, his work was usually unorganized and hard to read. At times, he was very self-opinionated, even when it was clear to him that another option may be better. When Allan took the state-mandated standardized tests at the end of the 2003-2004 scores, his scores indicated that he performed above state standards in both reading and mathematics. However, during his year with me, he did not do well in any of his reading subjects, either failing or barely obtaining passing grades during both semesters. He also lacked strong writing skills.

Third Participant: Ray

Ray was the third participant whom I selected for study. Ray's fraction computation pretest score of 75% was the lowest of the three participants, but his score of 62% on the fraction concepts pretests was the highest among all of the scores. Ray satisfied all of the other criteria, particularly his love for and enthusiasm for learning mathematics.

Ray was 13 years old, but very big for his age. Most of his classmates believed that he was much older. In class Ray was not the best of students in terms of his classroom behavior and motivation about doing his schoolwork. He was often inattentive and, at times, disruptive in class. He found it difficult to remain focused on his work during class and rarely did his homework assignments unless they were projects or involved research. At the beginning of the school year, Ray's performance on the mathematics diagnostic test indicated that he was capable of working either at or above grade level, compared to nearly all of his class and teammates. I checked his standardized test scores from previous administrations and this confirmed my belief. Allan had obtained some of the highest scores on each of the content areas on the most recent CRCT, which he took in the 7th grade. However, his grades at school were no indication of his true ability.

Nevertheless, Ray was a very interesting student. He loved to draw and did so at every opportunity, even during instruction. Even though he was often absent from school, when he did attend, he provided me with answers to the most difficult questions, particularly those which required more than average critical thinking skills. Because he loved mathematics, when he was at his best, he was very enthusiastic about learning the subject, was very intuitive, and usually found unorthodox ways of solving open-ended type mathematics questions. He was very meticulous and communicated his ideas very well. I often praised his level of intelligence and encouraged him to come to school more frequently.

By the end of the first semester, Ray's attendance and behavior had improved, though he still did not complete his class work or homework assignments. In addition, he

often had no materials for writing but had a huge gym bag with his gym clothes and art supplies. His grades at the end of the first semester of the school year indicated that he had flunked all of his core classes (mathematics, language arts, science and social studies) but passed his Connection classes, which included physical education, art and technology. I used the fact that the two core subjects that he loved, mathematics and social studies, were the ones that I taught him to my and his advantage. I solicited his help with classroom tasks, such as setting up the computer when we did mathematics lessons that involved technology, allowed him to make posters for the class, and made him the class' materials and technology manager.

Assigning Ray responsibilities encouraged him to attend school more frequently and to stay focused in class. In addition, because I used a variety of hands-on activities in my mathematics class, this also played a role in helping Ray to stay focused during my classes. When I saw that he was making an effort to improve, I decided that during the second semester, I would try to work closely with him. I requested and had a conference with his parent and found out that Ray was not living with his biological mother, but with a foster family, and as a result, he was dealing with related social and family issues.

As the school year progressed, Ray made efforts to stay more focused in class and to be less disruptive. He improved in these areas and in mathematics and social studies. However, because he did not do his homework or other assignments, his grades did not change much in any class. I offered to have him make up assignments, during after-school tutorial sessions, but he could not stay because he had no means of getting home. Even when the school provided a late bus to take home the students who stayed for after-

school tutoring, Ray did not take advantage of this. He said that he did not need to attend tutorial sessions.

Initially, I did not plan to include Ray in the study, fearing that he would not stay focused during the administration of the tasks. However, one day, during a mathematics hands-on activity, I was so impressed with his level of participation and the lead role that he played in his group and in the follow-up discussions that I decided that I would give him a shot at being a participant. By the middle of the second semester, Ray had improved his behavior to a level where his frequent disruptions had lessened considerably. On a whim, one day I told him that I was very pleased with his improvement and that I would consider involving him when I was ready to work on my research project. He seemed very surprised, promised to improve even more, and said that he would be willing to participate. As the implementation date approached, I was convinced that I could learn a lot from involving Ray in the study.

Data Sources

The data collected in this study came from a variety of sources to achieve triangulation. Creswell (2002) defined triangulation as the process of corroborating evidence from different individuals, types of data, or methods of collection and ensuring the accuracy and credibility of the researcher's findings thus contributing to the trustworthiness of the data (Denzin, 1988; Glesne & Peshkin, 1992; Merriam, 2001). Triangulation allows the researcher to regard his or her own material critically, test it, identify its weaknesses, and make decisions about changes which might become necessary as the study progresses (Fielding & Fielding, 1986).

To achieve triangulation of the data collected in this study, I used the following data sources: (a) ATFI questionnaire, (b) fraction pretests, (c) artifacts in the form of student work from performance-based tasks, (d) one-on-one interviews, (e) field notes from participant observations, and (f) individual and group student reflection logs.

Instrumentation

Several instruments and recording processes were used in the data collection process. These included (a) performance-based tasks (Appendixes A, B, C), (b) scoring rubrics (Appendixes E, F, G), (c) a fraction interest inventory questionnaire (Appendix H), (d) a fraction computations pretest (Appendix J), (e) a fraction concepts pretest (Appendix K), (f) interview guides I, II, and III (Appendixes L, M, N), (g) reflection logs (Appendixes O, R), and (h) participant-observer's log (Appendix Q).

Data Collection

Data collection began during the last week of April, 1 week after administration of the CRCT ended, and it lasted for approximately 3 weeks. I chose this time-period to begin data collection, because there is usually some flexibility in the day-to-day school schedule as preparations begin for the 8th-grade spring and end-of-year activities. This facilitates extending class periods if it became necessary to do so. Though during this time the students are usually involved in those activities, they are motivated to continue to work hard and to focus on their work as they prepare for the end of year final exams.

The data collected during the study comprised of qualitative and quantitative types so that I could elicit answers to the research questions. The following is a description of each data source and the procedures for the data collection process.

Questionnaire.

A modified version of a mathematic questionnaire, the View of Mathematics Inventory (VMI), in the form of a fraction questionnaire, the Attitude Toward Fractions Inventory (see Appendix H), was administered to each of the 34 students in my two prealgebra classes during their 75-minute mathematics block in two different class periods. The reasons for administering this questionnaire were (a) to get a feel for the students' thoughts and feelings about having to learn about and work with fractions and (b) to use the resulting data to aid in the selection of the study's three participants whose work would be analyzed during the data analysis unit of the study.

Downes and Thompson developed the original VMI in 1974, and Downes and Drummond later modified it. The original VMI questionnaire was designed to investigate the underlying dimensions of attitudes toward mathematics and to address factors reported to be important in mathematics research. The constructed dimensions assessed confidence, anxiety, value, enjoyment, motivation and perceptions of the mathematics teacher. Since its development in 1974, the VMI has been used in modified versions in numerous doctoral studies, evaluated by several researchers, and identified as a reliable data collection instrument. Each of its six subscales had a reliability factor ranging from 0.76 to 0.86.

In modifying the VMI, I structured the ATFI questionnaire in a similar manner to one used by Paschal (1994). For example, I replaced the word "mathematics" with "fractions," and "teach" with "learn" to make the statements relevant to the concept of fractions and to the students who are learning fractions rather than to teachers. I also made changes to the statements as I saw fit in order to align them with my students'

reading and comprehension levels and omitted irrelevant ones such as "All college students will have graphing calculators by 1998."

I administered the ATFI questionnaire at the beginning of the 75-minute mathematics block to my students who attend my class in the morning session and to the second group of students during the third block, which began at noon. I began by distributing a questionnaire to each student and then read the directions as they read silently to ensure that they were clear as to what they were required to do. Next, I informed the students that there were no right or wrong answers for the statements and that they should choose to respond to each one as they wished. The questionnaire (Appendix H) consisted of 30 statements relating to the concept of fractions and fraction learning. Each statement related to one of five attitudinal categories: attitude toward fractions, value of fractions, confidence about fractions, enjoyment with fractions, and motivation about fractions.

The participants responded to each statement by choosing and circling a numerical value in the range of 1 (*strongly agree*) to 5 (*strongly disagree*) from the Likert-type scale on the questionnaire and took approximately 25 minutes to complete it. I scored the questionnaires using a teacher-made scoring key (see Appendix I) based on the original version. I completed this within two days of data collection. I then tabulated the data, and stored and secured them in a computer file using the Microsoft Excel spreadsheet program.

Fraction Pretests

I administered the two fraction pretests to the students over the course of two days, following the administration of the ATFI questionnaire. The first test was a two-part, 53-

item, fraction computation test (see Appendix J), which took approximately 75 minutes to complete. The second pretest was a 26-item fraction Concepts test (see Appendix K), which required approximately one hour to complete. The main purposes of administering the pretests were (a) to determine my students' level of ability to compute and conceptualize fractions after a 3-week instructional unit on fractions (and a subsequent lapse of approximately 5 weeks) prior to the implementation of the study, and (b) to use the results as parts of the criteria for selecting the three students for the data analysis phase of the study. The following are descriptions of each of these two pretests.

Fraction computation pretest. The computation pretest comprised of typical fraction items found in most 8th-grade mathematics textbooks. To complete these items the students had to use the four basic operations for computing fractions. The items reflected the domains related to fraction computations described in the state's curriculum and in the *Curriculum and Evaluation Standards for School Mathematics* proposed by the National Council of Teachers of Mathematics (NCTM, 1989). The fraction items on the test were connected to Standard 1 – Number and Operations. The domains tested included creating equivalent fractions, reducing fractions, changing mixed numbers to improper fractions and improper fractions to mixed numbers, addition and subtraction of like and unlike fractions, multiplication and division of proper fractions and mixed numbers, and word problems involving fractions.

The creator of the test is unknown, but it was used in an unpublished thesis done by a Georgia State University student as a part of an instructional unit on teaching advanced mathematics skills to at-risk middle school students. This test was selected, because it was well structured, the items were interrelated and sequential, and in general,

its contents covered all of the domains of fraction learning required at the 8th-grade level. Based on my many years as an experienced mathematics teacher and educator, I felt that the test was an excellent one and would provide an effective means of determining the students' fraction computational skills.

Fraction concepts pretest. Ideas for the questions on the fraction concepts pretest came from a variety of 8th-grade level mathematics textbooks and mathematics resources. Some of the questions (used in modified versions) came from Rational Number Project curriculum written by Cramer, Behr, Post, and Lesh (1997a, 1997b). These items, used with permission from the authors, assessed a variety of fraction conceptual skills, such as partitioning, identifying and naming fractional parts, drawing models for simple fractions and mixed numbers, estimating fractions using benchmarks, and finding solutions to fraction problems, which require the use of conceptualization skills. The purpose of administering the fraction concepts test was to determine the students' ability to conceptualize fractions and to answer non-routine, non-computational fraction questions.

The pretests were administered to 34 students during the morning and afternoon mathematics blocks, respectively. The students worked quietly at their seats on an individual basis and took 75 minutes (all of the block time) to complete the computations test and approximately 60 minutes to complete the concepts tests. The students were required to show their work for each question of the computation test, if necessary, and to record their answers on a slot next to each question. For the concepts test, the students used physical models of fraction representations and any necessary strategy to find solutions to the mostly open-ended questions.

Each test was scored using a teacher-made key and the results were used as one of the criteria for selecting the three participants in the study. Potential study participants were initially required to score 75 or more out of a possible 100 points on each test, however, this requirement was modified because no student met the criterion. In addition, I examined the students' answers to the questions on the fraction concepts pretest with much more scrutiny to determine the extent to which answers reflected their ability to conceptualize fractions. This was done to facilitate selecting the participants who would best provide answers to the research questions, especially to the question related to student ability to conceptualize fractions, which according to the literature, is a major area of weakness for most middle school students when learning fractions.

Interviews

Merriam (2001) stated that interviewing is the best data collection technique for conducting intensive case studies of a few selected individuals. I used interview guides containing approximately 10-15 questions (see Appendixes L, M, N) and a combination of a semi-structured and informal format to interview each of three participants during the study. I also, on occasion, chose to use an informal conversational format because, according to Merriam, individual respondents define the world in unique ways, and the informal conversational format can provide a better way to witness these viewpoints that individual students may share. The interviews were used mainly to collect data regarding the participants' feelings, thoughts, reactions and experiences relative to (a) working with a nontraditional form of instructional-assessment tool (performance-based tasks), (b) using fraction representations when finding solutions to the tasks, and (c) working in

small groups and discussing and sharing their ideas and suggestions for finding solutions to the tasks.

The one-on-one interviews contained some open-ended and some exploratory questions to allow the participants to answer freely in their own terms (Gall, Gall & Borg, 2005), as well as a few interpretive questions. I used the latter type of questions during the course of the interview when this became necessary to check with the participant as to whether I correctly understood what he or she said and also to allow the participant to offer additional information on his or her opinions, thoughts and feelings.

I conducted the majority of the nine interviews immediately following the administration of each performance task. This was done on an individual basis in the rear of my classroom while the rest of the students in the class completed their performance-task, their written reflections, or other assignments. Prior to beginning the actual interview, I exchanged pleasantries with the participant to make him or her feel comfortable, then read aloud a paragraph which contained the directions and a reminder to the effect that there were no right or wrong answers to the questions and that the participants were free to answer however they chose.

I used the interview guides to conduct each interview, but when necessary, I used other questions as I probed for more detailed explanations from the participant. I also assumed the role of an active listener and encourager as the participant elaborated and explained his or her answer more fully. When necessary, I asked open-ended questions so that I could obtain as much data as possible on the students' ways of representing fractions, of communicating their mathematics ideas, on their impressions and thoughts

about the mathematics tasks and concepts they were working on, and their reasoning, decision-making, strategies and discoveries.

I took notes as copiously as I could during each interview, then typed and saved them into a computer file. On one occasion, because of a change in the day's schedule, I conducted one interview during my one-hour planning time and another two immediately after school ended. Each interview lasted for approximately 30-45 minutes.

Classroom Observations

During the study, I played the dual role of researcher/participant-observer as the students worked in the classroom on performance-based tasks. Though I monitored the class as a whole, I also focused on the three participants whose work I used in the data analysis unit. My role as a participant-observer allowed me to obtain first-hand information about what was transpiring in the classroom as the students worked in the natural setting of their learning environment – the classroom. For example, I observed one of the participants as he struggled to find a solution to the task on which he worked. I was also able to observe the students as they (a) formulated their ideas and transferred them to their paper, (b) organized their approach to the task and applied strategies, (c) displayed persistence in finding answers, (d) discussed their strategies and approaches, and (e) displayed evidence of fraction representations such as diagrams, pictures or symbols, and calculations.

In addition, my role as a participant-observer provided me with the opportunity to obtain a deeper understanding of the participants' (particularly the three main participants) actions, interactions with each other, and their thought processes as they

worked on the performance-based, fraction-related tasks in the classroom and socially interacted in their small groups while engaged in the tasks.

Field Notes

To establish trustworthiness, Lincoln and Guba (1985) recommended that the researcher record his or her procedures, decisions, and observations during the research process. I used the recommendations of Merriam (2001) as a guide for effectively taking and recording field notes. She suggested that field notes contain (a) a verbal description of the setting and the students being observed, (b) direct quotations or the substance of what was said or done, (c) comments that were written in the correct format, and (d) the researcher's reflections regarding the feelings, ideas, hunches, impressions, or problems encountered in the classroom. In keeping with these recommendations, I recorded notes from my classroom observations on a teacher-made observation log and sometimes on cards. I created the observation log based on given guidelines and an example found in *Mathematics Assessment: A Practical Handbook* (Bush & Leinwand, 2000). When time or what was transpiring in the class did not facilitate note taking, I made mental notes of my observations and recorded them as soon as I could after the class ended, as advised by Merriam. I rewrote and then typed these notes usually within a day of the observation.

Performance-Based Tasks

Danielson (1997) defined a performance task as a form of assessment of student learning that results in a written or physical product. Products provide a rich source of information for educators seeking to understand their students' knowledge and ability levels. A written product is anything written by students not under testing conditions, while physical products are three-dimension things that take up space (Danielson).

For this study, I used each of three performance-based tasks as an individual and group assignment and as an assessment tool to assess my students' proficiency with fractions. The performance-based tasks, therefore, served dual purposes. First, they provided the students with opportunities to work in their natural setting on non-routine tasks containing some open-ended questions, which required them to think about, reason, and communicate their mathematical ideas individually and with their peers in small groups. Second, the performance-based tasks facilitated assessing the students' fraction and mathematical skills in a nontraditional way, and through their explanations, evaluating their understanding of more complex mathematical topics and skills. The alternative was to use a traditional paper-and-pencil test, which assesses primarily procedural knowledge.

According to the NCTM (2000), "assessment should support the learning of important mathematics and furnish useful information to both teachers and students [and] assessment should reflect the mathematics that students should know and be able to do" (p. 11). These recommendations and the higher level of expectations for instruction and student learning, as reflected in my school system's new curriculum standards and consistent with current mathematics education reform, influenced my decision to use performance tasks in the study. The information obtained from the participants' written work – the product - represented primarily quantitative data and some qualitative data in the form of their written explanations given as a part of their answers. During the study, the participants worked on three small, fraction-related, performance-based tasks. According to Danielson (1997), small performance tasks are primarily suitable for

assessment purposes, require students to solve a problem, explain their thinking, and show their work, in one class period. Small tasks do not contain subactivities.

Scoring Rubrics

Bush and Leinwand (2000) stated that, in general, rubrics represent a hierarchy of standards used to score a student's work and help to keep the focus of the assessment process on performance rather than on the performer. They added that a rubric is an assessment tool that facilitates (a) quality feedback to students, (b) the monitoring of student progress, (c) increasing student skills in mathematical reasoning and meta-cognition, and (d) defining specific performance expectations. They also stated that holistic rubrics describe the qualities of performance for each performance level and may be used to score any mathematical task (Bush & Leinwand, 2000).

The above characteristics of rubrics and the advantages of using them influenced my decision to use this assessment tool to evaluate the participants' work on performance-based task relative to the phenomenon being studied – fraction representations. I, therefore, created a holistic, 4-point scale rubric (see Appendixes E, F, G) to evaluate each of the three performance-based task that were administered to the participants in the study by using the information and the examples given by Bush and Leinwand (2000).

Implementation of Study

The study was implemented during the spring term of the school year. Implementation began during the last week of April, lasted approximately 3 weeks, and ended during the 3rd week of May, about 2 weeks before the end of the school year. The three participants and the remaining 20 students in their prealgebra class worked on three

different performance-based, fraction-related, mathematics tasks (see Appendixes A, B, C) during their mathematics class over a 10-day period, with a 2-day interval between each task. The procedures for administering each task were very similar. All of the students sat at individual desks in prearranged groups of four. I selected group members based on their learning styles, level of written and oral skill comprehensions, the ability to get along with each other, and attitude about their work. I placed the three participants whose work would be analyzed in the data analysis unit of the study in different groups (a) to academically balance the groups academically, (b) to prevent the participants from collaborating or influencing the other group members' responses, and (c) to prevent bias in the study.

Before beginning each task, I placed all necessary materials such as pens (blue, black), pencils, rulers, small circular objects, safety compasses, paper (scrap, plain, graph), and individual and group reflection logs on a table which was accessible to all of the students. I began by reminding the students that they were to work individually first, then as a group after they completed the task and their individual reflection logs. Next, I distributed a sheet containing the task (and directions for completing it) to each student. I allowed them to read it to themselves first. I then asked them to follow along with me as I read aloud the tasks and the directions. Next, I allowed the students an opportunity to ask questions, and I answered them and made clarifications if these were necessary. Each of the three participants and their 20 classmates then worked on the given performance task individually, and upon completion I collected each student's work, including their rough drafts. If they completed their tasks and reflection logs before the end of the 75-minute class period, the three participants got into their different, prearranged groups of four and

collaborated, discussed and shared their solutions to the task as well as the methods, strategies and representations (if any) that they used to find their solutions. If the participants and their classmates were unable to complete all of the task-related activities by the end of period, these activities were completed when the students returned during their second class of the day with me, social studies. Each of the three participants was placed in different groups so that each group was relatively well balanced academically and to allow each student to make a contribution to the group, regardless of whether he or she was perceived as being smart, so that they could agree on a solution to the task.

Each person in the group was responsible for either recording the notes during their discussions, writing the answers to the prompts for the reflection, or managing the group so that everyone stayed on task and participated in discussion, getting the materials for the group, or reporting their findings to the class. The students discussed what they did from memory and decided on a solution, then reported this to the class. I collected all of the students' work and written reflections at the end of the period.

In general, each of the three performance tasks assessed the students' ability (a) to use representations of fractions to communicate their mathematical knowledge and understanding, (b) to apply their problem solving skills, (c) to conceptualize and compute fractions, and (d) to determine suitable approaches and strategies for finding a solution to a non-routine task. The following is a brief description of each task.

Task # 1: Pizza!

This adapted and modified version of a similar task reflects how mathematics is used in and can be applied to real-life situations outside of the classroom and describes a context to which the students can relate (see Appendix A). The skills required to

complete the task involved using their choice of fraction representations, analyzing and understanding the problem, reasoning, developing strategies to solve it, and using procedural and conceptual understanding and knowledge of fractions to find a practical answer to the question. The students were also required to use their written communication skills to explain the strategies and thought processes that they used to arrive at a solution.

Task# 2: *The Mangoes Problem*

The problem given in this task assessed the students' ability to (a) solve an open-ended problem by applying and adapting a variety of appropriate strategies, including "working backward," (b) identify and use an applicable type fraction representation, and (c) apply their conceptual and computational fraction skills to find a solution to the given problem and communicate their mathematical thinking coherently to each other (see Appendix B). The lesson plan for this activity was written by Jerry Stonewater and appeared in the November-December 1994 edition of, *Mathematics Teaching in the Middle School*. The lesson plan was an NCTM Publication-Based Lesson Plan adapted from the NCTM's journals.

Task # 3: Science Fair

This task required the students to analyze and reason about fractions, to use spatial and numerical reasoning to identify fractional parts of a rectangle, and to demonstrate their ability to use fractions, decimals, and percents interchangeably (see Appendix C). Students were given a rectangular shape that represented an auditorium where a science fair would be held. No measurements were given for the dimensions of the rectangle. Instead, they were given information regarding the number of schools and

students that would participate in the science fair. The students were required to apply their conceptual and procedural knowledge and understanding of fractions to divide the rectangle based on the information regarding each school. Other questions required the students to provide answers to questions regarding fractions, percents, and money. This task was one of the *Exemplary Mathematics Assessment Tasks for the Middle Grades,* released by *the New Standards*. The *New Standards* assessment system includes performance standards with performance descriptions, student work samples and commentaries, on-demand examinations, and a portfolio system.

How the Data Answer the Research Questions

The data sources used in this study were designed to provide answers to its two research questions:

1. To what extent do urban African American 8th-grade middle school students compute and conceptualize fractions when working on fraction-related performance-based tasks?
2. In what ways do urban African American 8th-grade middle school students use fraction representations to organize and communicate their mathematical thinking and reasoning when working on fraction-related performance-based tasks?

Research question #1 was concerned with the extent to which the participants could compute and conceptualize the fractions in the three fraction-related performance-based tasks. The focus was on their procedural and conceptual knowledge and skills relative to fractions. This question was answered through the analysis of the data from the

(a) fraction computation and concepts pretests, (b) performance-based tasks (and scoring rubrics), (c) interviews, and (d) field notes.

Research question #2 was concerned with how the participants used fraction representations when finding solutions to the fraction-related, performance-based tasks. These tasks represented a nontraditional assessment tool and a means of assessing the participants on their computational and conceptual fraction skills and their problem solving skills. This research question was answered via the analysis of the data obtained from the (a) performance-based tasks, (b) interviews, (c) individual and group reflection logs, and (d) observer/researcher's field notes.

Researcher's Role

In this study, I used qualitative techniques to gather data, and qualitative methodology uses the researcher as an instrument (Creswell, 1994). As the researcher-teacher, I played the role of human-as-instrument because one of the characteristics of all forms of qualitative research is that the researcher is the primary instrument for data collection and analysis (Lincoln & Guba, 1985; Merriam, 1998). Further, as Creswell (1994) suggested, "Data are mediated through this human instrument, rather than through inventories, questionnaires or machines" (p. 145). Therefore, as the primary data collector, I administered fraction pretests and a questionnaire in the form of a fraction interest inventory; conducted one-on-one interviews with the participants; collected artifacts in the form of samples of student work; and recorded field notes as a participant-observer in the study.

As a result of my involvement in various aspects and levels of research projects, I have been educated on how to collect and analyze data for qualitative and combined

methodological studies, based on the methods described by scholars such as Creswell (1994, 2002), Geertz (1973), Glesne and Peshkin (1992), Lincoln and Guba (1985), and Merriam (1998, 2001). Further, as a classroom teacher for many years, I have engaged in (informal) data collection activities such as interviewing students, observing them as they interact in the classroom, and collecting and analyzing data and documents in the form of student work. Therefore, as the researcher and the person responsible for collecting the data for this study, I think that I am adequately competent to collect and process the data that will be collected during its implementation and to do so in a professional manner.

Assumptions and Biases

One of the questions that influenced this research study was that which asked why so many middle school students find it very difficult to understand and learn fraction concepts. It is a fact that, traditionally, elementary and middle school teachers are not required to have a strong mathematics background, nor are they trained while in college teacher preparation programs to teach a high level of mathematics content. One can also safely assume that most of these teachers choose to work at these levels specifically for these reasons. As such, in some cases these teachers may “skim the surface” or teach only what is necessary when teaching difficult mathematics concepts. This can adversely affect student learning and performance. This is especially the case when the elementary students move on to the middle school level and need to learn a higher level of mathematics, to think more critically, abstractly, more conceptually, and when learning concepts, such as fractions and other challenging mathematics concepts. The result is that these students can and do become frustrated, unmotivated and even turned off from mathematics in general, and this is reflected in their below-level performance and poor

attitudes toward learning the subject. Ultimately, this leads to the assumption that ineffective instruction at the elementary level is a mitigating factor in relation to student difficulty with learning and understanding mathematics, in general, and concepts such as fractions. This assumption can then be categorized as a bias on the part of the researcher.

Data Analysis

This qualitative study used several data collection sources. In keeping with the suggestions of researchers such as Bogdon and Biklen (1998), Merriam (2001), and Miles and Huberman (1994), I collected and analyzed the data concurrently; however, an in-depth analysis of each source was not done until data collection ended. I used several data analysis strategies to do so.

The initial data obtained from the study resulted from the fraction interest inventory questionnaire that I administered to each of the 37 students on my team, then from the two fraction pretests administered to 34 of these students, just prior to implementing the study. The primary purpose of administering the fraction interest inventory questionnaire and the two fraction pretests was to obtain data that would help me to identify the three students who, based on their responses to both of these instruments, met the established criteria and who would be selected, therefore, as the three participants whose work would be analyzed during the data analysis phase of the study.

First, I analyzed the data from the fraction computation pretest (see Appendix J) by closely examining them and looking for domains in which the students did and did not perform well. For those domains where the students' work demonstrated a higher level of difficulty, I looked for patterns of consistent student errors and tried to determine if they

were due to the students' deficiencies relative to performing computations using fraction rules and procedures or to conceptual understanding. I carefully examined each student's work and decided on the reason an answer was incorrect and created a list of 24 error codes (see Appendix D). For example, code 1 represented a basic arithmetic/careless error; code 8 represented an addition across numerators and/or denominators error, and code 17 represented a procedure and operation mismatch error.

I also created a spreadsheet to store the data. I used the list of error codes and a master list of students' responses and cross-checked the data to determine where the errors were concentrated and the type of errors the students made most frequently. This also helped me to determine my students' level of ability to compute and conceptualize fractions after having 3 prior weeks of fraction instruction, approximately 5 weeks before the start of the study.

I present and discuss in detail the information from the analyses of the pretest data in a subsequent chapter. However, I was able to identify the domains of fractions with which the students had the most difficulty, the types of errors they made, and determine if the errors were procedural or conceptual. The domains from which most of the errors came included (a) comparing fractions, (b) multiplying fractions when the problem was written with word "of" rather than with a multiplication sign (e.g., 3/4 of 8/9), and (c) adding, subtracting, multiplying, and dividing mixed numbers.

Next, I analyzed the data obtained from the fraction concepts pretest (Appendix K). The strategy for doing so was similar to the one described above, though I looked for student errors and difficulties regarding their conceptual skills, or lack thereof, as I closely examined each student's response to the 26 individual items. AgainMy

preliminary analysis of this data indicated that nearly all of the 37 students who took the test could not conceptualize fractions very well and did not understand the true meaning of fractions. The items with which the participants had the most difficulty came from the following domains of fractions: (a) modeling fractions, (b) identifying and placing a fraction on a number line, (c) partitioning and fair sharing, and (d) comparing and estimating fractions, using fraction models.

The primary source of qualitative data collection for this study was the set of 9 one-on-one interviews that I conducted with the three participants. During each interview and sometimes after, I shared at times portions of it with each participant and asked that he or she comment on its accuracy. This act of performing a “member check” helped to ensure that I did not misrepresent the information that they shared with me.

At the end of the data collection phase, I typed all of the information that I obtained from the nine interviews and from the students’ individual and group reflection logs. I created a unique code name for each participant and was the only one who knew and could access this information. I reread each set of typed data several times to familiarize myself with them and to get a sense of the “totality of the data” (Bogdan & Biklen, 1992, p. 176).

To begin the in-depth analysis, I read the data several times, bearing in mind the research questions. I also used “observer’s comments” regarding the ideas that came to my mind as I analyzed the data, such as (a) many of the students regarded a performance task as being just a long word problem; (b) to many of the study’s participants, diagrams, charts, and tables were representations of fractions; and (c) a *regular* fraction problem had no words and no steps. Next, I developed a list of analytical questions: (1) How did

the participants feel about having to use fraction representations? (2) What were the participant's thoughts regarding the usefulness of fraction representations? (3) What do the participants think about working on performance-based tasks? I kept these questions in mind as I closely examined the data.

In-depth Data Analysis and Coding the Data

As I continued the in-depth analysis of the data, I wrote down the words and phrases that stood out or that the participants used repeatedly to answer the questions posed to them during the interview or to express their opinions or thoughts in the reflection logs. These represented repeating ideas that Auerbach and Silverstein (2003) defined as ideas that are often repeated by two or more research participants in the relevant text. The data generated an initial list of 42 repeating ideas. In continuing the data analysis process, I used these repeating ideas and identified four broad topics that this data covered, based on the emerging patterns. These were (a) fractions, (b) performance-based tasks, (c) fraction representations, and (d) the importance of fractions to mathematics.

After closely examining the data (repeating ideas) further, I developed a preliminary list of codes (Bogdan & Biklen, 1992, 1998; Auerbach & Silverstein, 2003), and continued this process in order to let other codes emerge. Bogdan and Biklen (1992) stated that major codes are more general and are developed first, while subcodes are developed after rereading the data items. They also stated that the first attempt to assign codes is really a test of the codes, and as such, they should be expanded or collapsed until workable codes have been developed. They recommended the use of 30 to 50 codes in all. They also described 10 kinds of codes that can be used to categorize information, and

to do so at different levels, though not all of the codes will be used in every qualitative study. A description of each of these codes follows.

The first coding family is “Setting/Context Codes” (Bogdan & Biklen, 1992, p. 167). This is reserved for descriptive literature, pamphlets about the setting, general descriptive statements that people make or quantitative data that describe the context.

“Definition of the situation codes” (Bogdan & Biklen, 1992, p. 169) provides information about how the informants describe the setting, including their worldview, what is important to them and what they hope to accomplish.

The next family of codes is the “Perspectives held by [informants]” (Bogdan & Biklen, 1992, p. 168). These are less general than the previous family of codes, and include the ways of thinking that some or all of the informants share. They are frequently typified by a word, a phrase, or an idiom that one or more of the informants use to characterize a feeling or situation.

Next is the “Ways of thinking about people and objects” coding family (Bogdan & Biklen, 1992, p. 168). This family of codes deals with the participants’ understandings of each other, outsiders, and the objects that make up the world.

Another coding family is the “Process codes” (Bogdan & Biklen, 1992, p. 169). This family of codes is reserved for word or phrases that identify passages of time or that categorize sequences of events and are often used in life histories.

Bogdan and Biklen (1992, p. 170) identified yet another of its codes as the “Activity codes” family of codes. These are used for regularly occurring activities (formal or informal) that are a part of an experience. For example, “connections” activities in a school, such as band, home economics or art, are a part of a school day.

Another coding family is the "Event codes" (Bogdan & Biklen, 1992, p. 170). Activities that fall into this family of codes pertain to specific activities that occur in the lives of the informants. An example of this would be "graduating from college."

According to Bogdan and Biklen (1992, p. 171), "Strategy codes" refer to the tactics, ways, techniques, maneuvers, ploys, and other conscious ways by which people accomplish things. They cautioned that it is important to not assign motives to people's behavior or strategies and to understand what you are doing, if you do.

Bogdan and Biklen (1992, p. 171) described their "Relationship and social structure codes" as the family of codes in which regular, but unofficial, patterns of behavior among people are placed. These include friendships, cliques, romances, and coalitions.

The tenth and final family of codes refers to methods and isolates any material that is relevant to research. Bodgan and Biklen stated that this family of codes is adequate for most research studies.

The coded data from this study was broken down into the following major coding categories: (a) Personal Attributes (a setting/context code); (b) Knowledge about Fractions (a strategy code); (c) Views about Working with Fractions (a ways of thinking code), (d) Views about Fraction Representation (a ways of thinking code), (e) Views about Working with Performance-based Tasks (a ways of thinking code), and (f) Perspectives of the Participants (a ways of thinking code).

On several occasions, during the interviews, one or more of the three participants stated, "It (the task) was very easy" when responding to my question of, "how would you describe this performance-based task?" This data item was in the coding family for,

"views about working with performance-based tasks." Other examples of the data that fell into this family of codes included (a) "performance tasks are fun," (b) "this task was very hard for me to do," and (c) "this performance task was just a long word problem."

Another major code was "views about fraction representation," which fell into the "ways of thinking about people or objects" coding family, and one in which the participants use a word, phrase, or idiom to characterize a feeling or situation. An example of this was that, during the interviews, the participants said, "fraction representation means graphs, diagrams, tables, pictures or charts." Other data examples that were categorized under this major code included (a) "fraction representations are helpful," (b) "I use fraction representations to help me to organize my work," and (c) "if I know how to do the calculations, fraction representation just wastes my time."

Further, another major code that emerged from the data was the "Views about working with fractions." An example of this that emerged from the data was the fact that several of the participants said, "When I think of fractions, I think of adding, subtracting, multiplying and dividing." Other data items that fell under this major code included (a) "fractions are sometimes difficult for me," (b) "when I see certain words in a fraction problem, I know that I need to add, subtract, multiply, or divide," and (c) "fractions mean I must do calculations."

In conjunction with coding the data, I also used the constant comparative method as part of the data analysis process. Bogdan and Biklen (1992) described this as a research design that is used with multiple data sources and is similar to analytic induction in that the formal analysis begins early in the study and is nearly completed by the end of

data collection (p. 72). Merriam (1998) stated that the basic strategy for the constant comparative method is to compare constantly aspects found within the study where:

> The researcher begins with a particular incident from an interview, field notes, or document, and compares it with another incident in the same set of data or in another set. These comparisons lead to tentative categories that are then compared to each other and to other instances. (p. 159)

The constant comparative method involves (a) collecting data; (b) looking for key issues, recurrent events, or activities that become categories of focus; (c) looking for diversity of the dimensions under the categories that you see resulting from the data; (d) writing about and describing the categories; (e) searching for new incidents that relate to the categories; (f) looking for emerging themes, social processes, and relationships; and (g) continuously engaging in sampling, coding, and writing while using the categories as the point of focus.

Identifying Emergent Themes

During the data analysis phase of this study, I consistently examined the data, carefully searching for themes as I created codes, discovered categories, and made comparisons to discover other categories, "realizing always that the developed categories are abstractions derived from the data, not the data themselves" (Merriam, 1998, p. 181). These categories reflected several emerging, overriding themes. Auerbach and Silverstein (2003) described a theme as an implicit idea or topic that a group of repeating ideas have in common. Engaging in continued data analysis and comparison allowed me to break down the data further and identify the following six overriding themes: (a) efficiency of fraction representations; (b) recognizing the connection between key words and fraction procedures; (c) level of students' confidence about their ability to compute fractions; (d) students' views on the value of using fraction representations; (f) influences on the

participants' decisions to use fraction representation; and (g) students' ratings of tasks versus the scores obtained for the tasks.

The focus of my final analysis of the data was on relating and connecting the data from the various sources that I used to the research questions and connecting the six themes that emerged from the codes that I developed as a result of the process of coding the data. Although there were some variations in the participants' responses to the questionnaire, interview questions, and reflection prompts, the similarities in the data were strong enough to provide me with a realistic picture regarding the participants' abilities (a) to compute and conceptualize fractions and (b) to use and fraction representations to organize and communicate their mathematical knowledge and understanding when working on performance-based tasks. In addition, I was able to get a holistic view of the learning environment and what was transpiring as the participants engaged in mathematics learning and development. I also became cognizant of things about the participants that I could not observe, such as their feelings, thoughts, and interpretation of what they experienced in the classroom during the study as suggested by Merriam (2001).

Trustworthiness of the Data

In a qualitative study, validity refers to the correctness or credibility of a description or interpretation. Threats to validity may include inaccurate or incomplete data, the imposition of the researcher's own framework or meaning, and failure to pay attention to discrepant data or to consider alternate explanations (Maxwell, 1996). To avoid such threats, I took copious notes while interviewing the participants and recorded detailed descriptive notes during classroom observations. I also used multiple sources of

data, including interviews, classroom observations, reflection logs, samples of student work, and questionnaires to facilitate triangulation of the data by providing corroborating evidence.

It was imperative that I considered the very important aspect of the trustworthiness of the data in this primarily qualitative design. According to Lincoln and Guba (1985), in a study grounded in the constructivist paradigm, the criterion of trustworthiness is used to evaluate its *goodness* and parallels the *normal* criteria of validity and reliability in a conventional study. Lincoln and Guba delineated four criteria that are essential for ensuring that a qualitative study is trustworthy. These are (1) credibility, (2) transferability, (3) dependability and, (4) confirmability. To establish trustworthiness of the study, I addressed these four criteria as they relate to this study.

Credibility

The term credibility refers to research actions taken during the collection of data that provide greater assurance that the findings are derived from the data collected and account for the interpretation of the researcher. Guba (1981) defines credibility as the researchers' ability to take into account all of the complexities that present themselves in the study and to deal with patterns that are not easily explained. He suggests that the researcher engages in (a) prolonged participation, (b) persistent observation, (c) peer debriefing, (d) triangulation, and (e) negative case analysis.

Even though this was not a prolonged study, as the participant-observer, I persistently observed the participants during the class period as they worked on the performance-based tasks. I did this every time the participants were engaged in working on a performance task. I wanted to identify qualities or patterns that were continuously

present and/or atypical characteristics. I also listened as the participants discussed their opinions or thoughts about processes or approaches for finding solution to the tasks and when they were engaged in doing group reflections on the tasks. Next, I used triangulation to cross-check the data from the interviews, field notes from observations, and student reflections. To facilitate credibility, I paid special attention to data that were consistent and data that were different from the majority of data to satisfy the characteristic of negative case analysis.

To facilitate credibility further, I used peer debriefing by conferring with a very experienced mathematics teacher at the study site, who uses instructional approaches and assessment techniques similar to those that I used in the study. We discussed the performance-tasks, possible solutions, as well as any other pertinent information relevant to tasks and fractions in the study.

I engaged in another form of credibility analysis by frequently conducting member checks with each participant during and after the interviews and soliciting their responses about the data that they shared with me, so that my conclusions did not misrepresent their thoughts. I also maintained all versions of the data collected in their original forms.

Dependability

Lincoln and Guba (1985) defined dependability as the stability of the data, and suggested that the researcher uses (a) an overlap of methods, (b) triangulation of methods, and (c) an audit trail to facilitate the data being dependable and to reduce the presence of bias as might be the case when only one data source is used.

Ensuring the ability to replicate the methodology used for the study is very much dependent on the ability to document procedural parameters. Therefore, I ensured reliability of this study's methodology by employing protocols from Creswell (1994) for a *dominant/less-dominant* design of a combined model, in conjunction with a case study method of inquiry and interview standards from Merriam (2001). This served the purpose of triangulating the methodological approach used for the study and effectively situating it in a reliable and accepted theoretical framework. Further, I showed dependability of the data by triangulation and leaving an audit trail in the form of copies of all of the instruments used in the study such as the questionnaire, interview guides, and student reflections, and left each in its original form. This information was available to someone external to the study so that he or she could determine that I carefully analyzed the data and could examine the data collection and analysis processes that I used to conduct this research study.

Confirmability

Confirmability is the neutrality or objectivity of the data and the criterion that shows that the findings of the study are grounded in the data, and that inferences were made logically. To establish the criterion of confirmability, and therefore, trustworthiness, the researcher should (a) practice triangulation, (b) practice reflexivity, (c) keep a journal, and (d) use an audit trail (Lincoln & Guba, 1985).

I focused on this confirmability primarily during the data analysis phase of the study. I identified my biases in the context of the study to remove my influence from the data that I collected. In addition, all of the data that were collected are available to readers, and documents that I collected represented real sources and were not fabricated. I

engaged in journaling and writing notes regularly to record my reflections on things that I observed during the study to facilitate reflexivity because the characteristics of triangulation and reflexivity are important aspects of confirmability.

Transferability

Transferability refers to making similarity judgments based on the information found in the study. The trustworthiness criterion is the equivalent of external validity in positivist research. Lincoln and Guba (1985) defined transferability as the qualitative researchers' beliefs that everything that they study is context-bound and that the goal of their work is not to develop "truth" statements that can be generalized to larger groups of people. The authors suggest that the researcher should (a) collect detailed descriptive data, (b) develop detailed descriptions of the context (thick description), and (c) maintain all data in original form.

In an effort to establish transferability, I used thick description (Geertz, 1973). By my including the complete description, the reader of the study can determine if the findings can be transferred based on similarity of circumstances. This detailed description also provided the reader with a rich portrait of the school (the context of the study) and the participants, thereby increasing his or her ability to determine the transferability of the findings relative to his or her specific context and to provide an opportunity for making judgments about other possible contexts that might be compatible. It also allowed others to "see" what transpired in the context of the study. To enhance further the thick description, I included a section in the discussion of each case that described each participant in detail.

I also maintained transferability by keeping all portions of the study in the original form in an effort to allow for the potential replication of the study elsewhere. I made copies of the original documents and used these to make comments. The participants' identities were kept in confidence and stored in a safe place. As the researcher, I believe that maintaining these protocols facilitates transference of the findings to other contexts with a high level of surety.

Summary

I used several qualitative techniques for collecting data in conjunction with a case study strategy of inquiry. I collected data primarily through semi-structured, one-on-one interviews along with a variety of other sources, including pretests, a questionnaire, student artifacts, and field notes from classroom observations, and written student reflections. This triangulation of data collection sources and methods facilitated dependability, a criterion of trustworthiness of the data. I analyzed the data through a process of detailed coding and thick, descriptive reporting of the participants' thoughts and feelings. The goal of the study was to determine the extent of the participants' ability to compute and conceptualize fractions embedded in performance-based tasks and the ways in which they use representations of fractions to organize and communicate their mathematical thinking and reasoning as they found solutions to fraction-related, performance-based tasks. Data analysis techniques facilitated the emergence of patterns and themes. The constructivist approach to learning was the theoretical construct that undergirded the study.

CHAPTER 4

RESULTS

As stated in Chapter 1, the purpose of the qualitative study reported here was to determine the extent to which urban, 8th-grade, middle school students could compute and conceptualize fractions when working on fraction-related, performance-based tasks and the ways in which these students use fraction representations to organize and communicate their mathematical thinking when working on the tasks. This chapter contains three sections. The first section is a discussion of the themes that emerged from the data obtained from the nine interviews with the study's three participants, Liza, Allan, and Ray (pseudonyms). Samples of data items are used to support the discussion. The second section contains a case study of each participant that discusses his or her (a) responses to the fraction interest inventory questionnaire, (b) performance on the Fraction Computation and Fraction Concepts pretests, (c) work and performance on each task, (d) responses to the interview questions, (e) reflections regarding each performance-based task, and (f) field notes. In the third section of the chapter, I compare the three participants' results and summarize the chapter.

Themes

I used Bogdan and Biklen's (1992) recommendations for coding and analyzing the data obtained from the one-on-one, semi-structured interviews, which contained both open-ended and exploratory questions. I also used the constant comparative method to continuously compare the data from the various sources that I used. The in-depth

analyses of the data resulted in the emergence of the following themes: (a) efficiency of using fraction representations, (b) recognizing connections between key words and fraction procedures, (c) participants' confidence about their ability to compute fractions, (d) the value of using fraction representation, (e) things that influenced the participants' decision to use fraction representation, and (f) participants' ratings of tasks versus the scores obtained for the tasks. The data, which facilitated the emergence of these themes, also provided answers to the study's two guiding questions:

1. To what extent do urban African American 8th-grade middle school students compute and conceptualize fractions when working on fraction-related performance-based tasks?
2. In what ways do urban African American 8th-grade middle school students use fraction representations to organize and communicate their mathematical thinking and reasoning when working on fraction-related performance-based tasks.

The first three of the above themes reflected answers to the first research question, and the second three themes reflected answers regarding the second research question. The following section presents each of these themes relative to each participant.

Efficiency of Using Fraction Representations

The three participants, via their interviews and in their reflection logs, indicated that after reading each task they decided whether to use computations (involving standard fraction notation and rules and procedures) or fraction representations (e.g., shapes, sets, number lines) to find solutions to the fraction problems in the task. They based their decisions on which of the two methods allowed them to find a solution and complete the

task in the least amount of time. For example, during my interview with Liza, after she completed the *Pizza!* task, she told me that it was quicker to do the calculations for a problem such as 5/8 + 1/2 than it was to figure out how to do it by using fraction representation. According to Liza, she did not think of doing the problem by drawing any diagrams of fraction representation because she already knew which procedures and computations to use to add these two fractions, and she did not want to waste her time. Therefore, to her, this was task required her to use her procedural knowledge to find a solution.

Allan described using a graphical fraction representation as a waste of his time because he already knew how to do the fraction computations. During my interview with him and in response to my question regarding this issue, Allan said, "I will always try to solve a fraction problem with computations first, unless the directions tell me that I must use representation. 'Cause if you know the computations, it might help you to do it faster, and the representations make it longer, and it seems like a waste of time."

On the other hand, in our interview for *The Mangoes Problem,* Ray stated that the rules for doing computations confuse and "mix" him up, so it is faster and easier for him to draw a diagram to solve a fraction problem. He consistently shared this same viewpoint in the interviews for each of the three tasks and in his reflection logs. In addition, as I observed him working on the tasks, I noticed that he had always drawn diagrams to organize and do his work.

Recognizing the Connection between Key Words and Fraction Procedures

Liza stated that she kept doing calculations to find an answer to *The Mangoes Problem* task but could not get the right answer. She stated that she decided to read the

problem again carefully; when she read the sentence that said, “the third Prince ate 1/2 of what was left, leaving only three mangoes,” she realized what she had to do. She recognized that she could use the fraction that she had, which was 1/6, and multiply the 6 by 3, to find out how many mangoes were in the bowl before the third Prince ate 1/2 of it. Further, Liza stated that when she was trying to compute the fractions in the *Pizza!* task and she read that there were only 9 out of 12 slices of pizza in the second box of pizza, she knew that the words “out of” meant that she had use 9 and 12 to make a fraction. However, she said that she was still somewhat confused because she was not sure if this was what she should do, or if she should use 9 as a whole number and find 1/2 of 9, as this was the number of slices of pizza that were in the box before Naquan and Derasha ate from it. She decided to make a fraction with 9 and 12.

In trying to find a solution to *The Mangoes Problem* task, Allan stated that he recognized that the word *of* in the sentence, “Being hungry, he took 1/6 of the mangoes,” meant that he should multiply, and so he did the opposite – divided – to do the calculations for finding an answer to the question in the task. In the *Pizza*! task, he also recognized that he should multiply when he saw the word *of* in the sentence, “Lorenzo ate 1/3 of the pizza,” but he said that he was confused because he thought he should subtract instead of multiply to find out how much of the pizza was left after Lorenzo ate 1/3. He reasoned that eating the pizza represented subtraction.

Ray said that when he read the *Pizza!* task and saw the word *of* in the sentence, “Naquan and Derasha ate a total of 1/2 of their meat pizza” that was in the second box, he subtracted to find out how many of the nine slices they ate, even though he recognized that the word *of* meant multiplication. However, Ray said that when he worked on *The*

Mangoes Problem task and saw the word *of*, he multiplied the fractions that were given by the number of mangoes to find out how many mangoes each person had eaten.

Participants' Confidence About Their Abilities to Compute Fractions

In each of the three interviews that I conducted with Liza, she stated that she was very good at computing fractions so she only used fraction representations to check to make sure that her answers were all correct. For example, with regard to the *Science Fair* task, Liza stated, "I would rate the task a 2 because it was easy for me to do. It was easy to change fractions to decimals and to percents." She had no doubt that all of her answers were correct and was very confident about her ability to compute fractions and only used fraction representations to check the accuracy of her computations and to make sure that her answers were correct.

From our conversation during the interview for *The Mangoes Problem* task, Allan stated that he did not attempt to use any fraction representations to help him to find a solution because he felt that he could work the problem out correctly by doing fraction computations. However, Allan said that though he did not find the correct answer for the task, he did not attempt to use any diagrams or pictures to see if this would help him to find a solution.

During our interview for the *Pizza!* task, Allan said that he began trying to find an answer by doing "straight computations," because he felt confident that he could find the answer by doing so. However, this did not work. He decided, therefore, to use a fraction representations and drew a circle. He found that this made the problem easier for him to understand because, as he said, "I drew it out and I was able to see it."

Further, Allan also indicated to me that he saw no need to use fraction representations for the *Science Fair* task other than to divide the rectangle that was given, because he knew how to do all of the computations, such as changing a fraction to a decimal and to a percent. He stated that he was confident that his answers were all correct because his computations were correct.

Of the three participants, Ray was not confident that his fraction computations were correct. He said that the rules, such as knowing when to add, subtract, multiply, divide, or how to change fractions to percents and decimals, confused him and he often forgot how to do them. Therefore, he preferred to draw a model or a picture because this made it easier for him to do the problem. For example, during the interview for the *Pizza!* task Ray said, “My dislike is that I got mixed up and confused on some parts like multiplying and subtracting fractions.”

The Value of Using Fraction Representations

To Liza, a fraction representation, such as a shape, is valuable because she uses it to check her work, and this helps her to make sure that her answers are correct. For example, Liza said that when she worked on *The Mangoes Problem,* task, she did the computations first, then used fraction representations in the form of a set because as she said, “it made it clear because all I had to do was to cross out 1/6 (of the circles in the set) for every one that they had left over.” During the *Pizza!* task, in response to my asking why she thought we use fraction representations, Liza stated, “so that you can see the problem clearer because you can actually see the problem.” She added that she learns better when she could see what the problem actually looks like.

Allan thought that using fraction representations is valuable, especially when he could not find an answer to a problem by using fraction computations or when he does not understand a problem, though at times he also described using fraction representations as a waste of his time. For example, during our *Pizza!* interview, he said, “we use representations like shapes and drawings of sets to help us understand the problem better, ‘cause if you only do computations, you are just learning how to work it out, but if you use other representations you can see a picture and work the problem out both ways and understand what you are doing better.” He added, “the picture that you draw puts the problem right there in front of you because what you see helps you to understand the problem better.”

Ray values opportunities to use fraction representations. In our interview for *The Mangoes Problem* task he said, “I used a diagram, sets. It helped me to solve the problem because looking at the problem, I could figure it out and see what goes where, and what to take away for each person.” During the interview for the *Pizza!* task, Ray said, “I like doing this type of problem that involves drawing diagrams because if you are seeing what you are doing, it can help you to understand better than just knowing some numbers and having to divide and stuff, but if you can draw it out and do it, it is much easier to understand, period.”

Influences on Participants’ Decisions to Use Fraction Representations

Liza was only influenced to use fraction representations when (a) she needed to check the answer that she found by doing computations, (b) she wanted to see a picture of what the problem looked like, or (c) the problem was a difficult word problem. Allan’s decision to use fraction representations to solve a fraction problem was influenced by

several things: (a) fraction word problems, (b) directions which state that diagrams or pictures may be used, (c) problems that are difficult for him to understand, and (d) obtaining incorrect answers when doing fraction computations. Ray was influenced to use fraction representation when solving a fraction problem by his (a) confusion with fraction rules and procedures, (b) failure to remember when to use a particular rule or computation procedure, (c) love for drawing, and (d) preference to see a problem in the form of a picture, chart, or diagram rather than just numbers.

Participants' Ratings of Tasks Versus the Scores Obtained for the Tasks

Liza

The data from the interview with Liza indicated that her rating of the *Pizza!* task was a 5 or 6 (out of 10) in terms of its difficulty level. She stated, "the part with how many slices Naquan and Derasha ate had kind of threw me off." On her reflection log, she rated it a 7 (out of 10) and repeated the comment that the second part of the problem dealing with the second box of pizza threw her off. Her score for this task was a 3.6 (out of 4.0).

For *The Mangoes Problem* task Liza indicated in her interview that she would rate this task as having a difficulty level of a 4 or a 5 (out of 10) because as she stated, "it took me a while to work it out to get my answer." On her reflection log, she gave it the same rating stating, "I would rate it a 4 or 5 (4.5) because I worked the problem from the front at first, then worked it from the back after realizing my mistake." Liza's score for this task was a 3.6 (out of 4.0).

As part of our interview conversation, Liza said that she rated the *Science Fair* task as having a difficulty level of 2 (out of 10) because it was very easy for her to do. In

her reflection log, she rated it a 3 and also stated that it was a very easy task. Her score for this task was a 3.8 (out of 4.0).

Allan

During the interview, Allan rated the *Pizza!* task a 6 (out of 10) in terms of its level of difficulty. He stated that he gave the task this rating because “it seems hard but it really wasn’t.” On his reflection log he rated it a 9, saying, “I didn’t really understand it to a point where I could work the problem to get a final answer that I could agree with.” Allan’s score on this task was a 3.8 (out 4.0).

During the interview for *The Mangoes Problem* task, Allan said that he rated the difficulty level of this task a 4 or 5 (out of 10) because, “it took me a while to understand the problem and work it out to get my answer.” On his reflection log, he rated the task a 5, stating, “I thought it was easy, but it wasn’t. That is why I chose the middle.” Allan’s score on this task was a 1.0 (out of 4.0).

The data from the *Science Fair* interview indicated that Allan rated the level of difficulty of this task a 3 (out of 10). His comment about its difficulty level was, “it wasn’t so hard, but it took me a while to understand the problem and to work it out.” On his reflection log, Allan said, “I felt I did a good job, and that I got it right. I believe the problem I did was correct. I was satisfied with my answer.” He continued, “I easily understood the problem and it took me a while to complete.” He rated the task a 2. Allan’s score on this task was a 3.8 (out of 4.0).

Ray

The data from Ray’s interview with me regarding the *Pizza!* task indicated that he rated its difficulty level a 4 (out of 10). He stated, “I would rate it a 4 because it was kind

of difficult because I was counting the slices when I should have been counting the parts of the pizza they ate." On his reflection log he rated this task a 5 (out of 10), saying, "I had half of the problem right but I messed up with the fractions and the slices of pizza that were left." Ray's score on this task was a 2.0 (out of 4.0).

With regard to *The Mangoes Problem* task Ray said that he would rate it a 2 (out of 10) in terms of its level of difficulty because it took him only about 5 minutes to complete the task and because nothing about the task was confusing. On his reflection log, he rated the task a 4 (out of 10). He stated, "I knew what to do when I first got the problem. I knew that I had to work backward to come up with the answer of the total number of mangoes." Ray scored a 3.6 (out of 4.0) on this task.

In rating the *Science Fair* task a 7 (out of 10) for its level of difficulty, Ray said, "I knew what I was doing on the problem to get my answer right, but I couldn't seem to put my data in the chart correctly after I had finished." He rated it the same way in his reflection log and used the same comments. His score on this task was a 3.8 (out of 4.0).

Liza

Scores on the Attitude Toward Fraction Inventory Questionnaire

As can be seen in Table 1, Liza satisfied only partially the criteria for selection relative to the ATFI questionnaire, which was a score of 25 or higher in each of the four categories. Liza's score regarding her confidence about working with fractions was the highest of her scores and the only one for which she met the criterion of 25 or higher out of 30. She regarded fractions as being somewhat valuable, was not very motivated about learning fractions, and found very little enjoyment in doing so. Despite the fact that Liza did not meet the criteria in all of the four categories, I selected her based on my

Table 1

Attitude Toward Fractions Inventory Questionnaire: Liza's Scores

Attitudinal Category	Score (Out of 30)
Value of Fractions	21
Confidence about Fractions	26
Enjoyment of Fractions	7
Motivation about Learning Fractions	18

experience with her as one of my mathematics students who was always willing to learn mathematics, to work hard, to participate in class, and to try even when the problem seemed overly difficult. As was demonstrated by her scores, even though she said that she got very little enjoyment from working with fractions, she was confident that she could do well when working with fractions.

Results of Fraction Computation and Fraction Concepts Pretests

As can be seen in Table 2, Liza obtained a grade of 87% on the fraction computation pretest. The error codes that I created for analyzing this data indicated that five of her six incorrect answers were careless calculation errors. In addition, Liza made each of these errors when she worked on fractions that contained mixed numbers. It was difficult to identify the type of error that she made on the seventh item.

Liza's grade on the 26-item Fraction Concepts pretest was 54%. She demonstrated strong partitioning skills, could identify a given fraction in a model, but she had great difficulty estimating fractions, and answering correctly, questions that required her to think at a higher conceptual level. For example, though she recognized 3/2 as being

Table 2

Fraction Computation and Fraction Concepts Pretests: Liza's Scores

Pretest	Raw Score	Percentage
Fraction Computation	46	87%
Fraction Concepts	14	54%

Note. Possible maximum raw score for Fraction Computation Pretest is 53. Possible maximum raw score for Fraction Concepts is 26.

1 and 1/2, she could not conceptualize that the improper fraction 3/2 represented three halves and that she could represent this with a fraction model that contained three halves rather than one whole and one half. Although Liza did not score 75% or higher on the fraction concepts pretest, I selected her to be one of the study's participants primarily because of her strong performance on the Fraction Computation pretest and her very high level of enthusiasm about learning mathematics.

Results Regarding Performance Task 1

Table 3 summarizes Liza's scores on each of the three performance tasks. As can be seen in the table, Liza obtained a total score of 3.6 for this task. This task (*Pizza!*) assessed the participants' ability to select and use the appropriate fraction representation to model and partition correctly, two large-sized pizzas based on given information and to use their analytical, reasoning, problem-solving, conceptual and computational fraction skills to find a solution to the open-ended question in the task (see Appendix A). The participants also had to demonstrate their ability to select and use an appropriate strategy, as well as use their written communication skills to explain their thought processes as they worked on finding a solution to the task.

Table 3

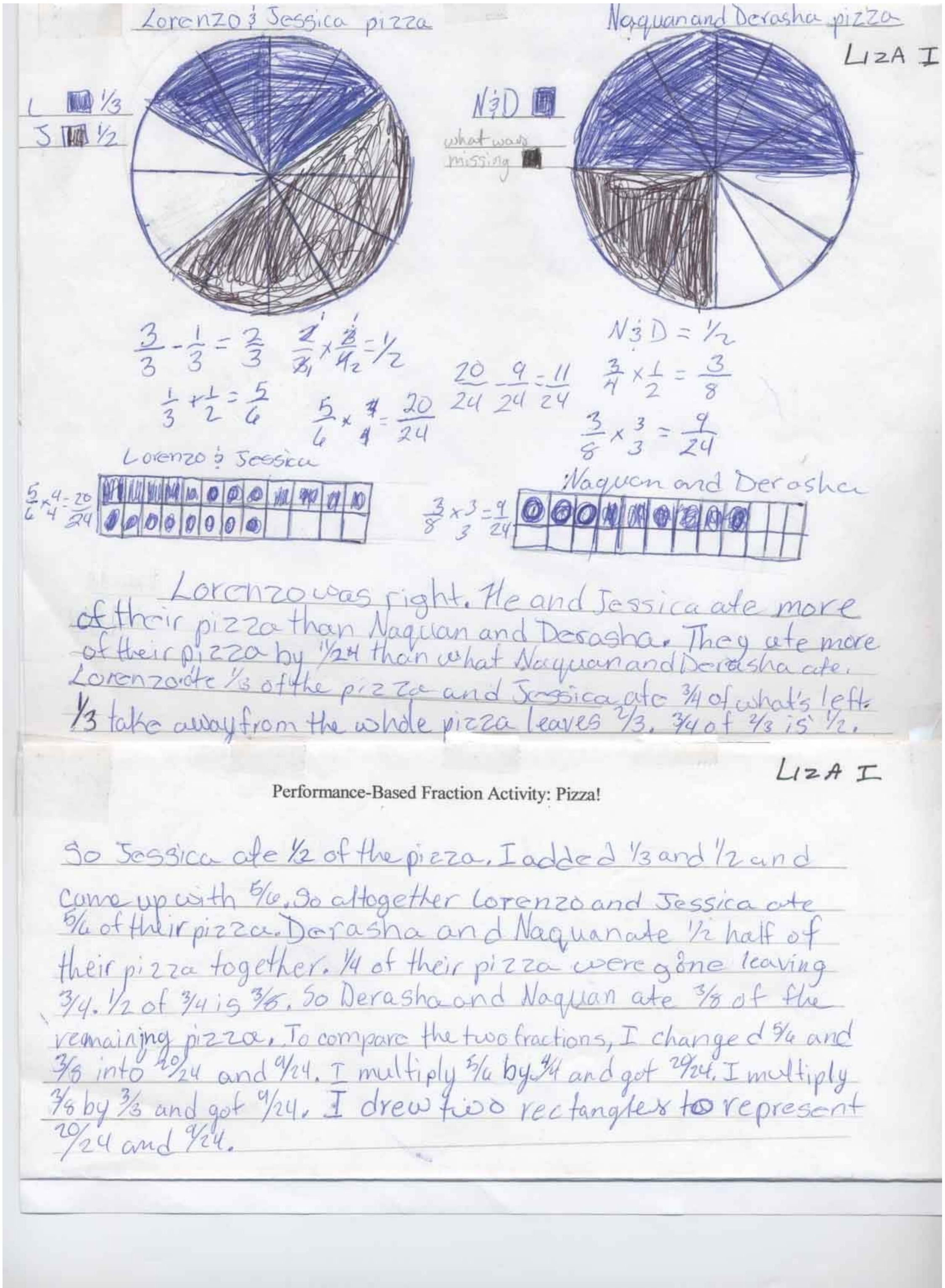
Performance Tasks Results: Liza

Task Name	Math Concepts	Math Reasoning	Math Errors	Explana-tion	Represen-tation	Strategy & Procedure	Score
Pizza!	4	4	4	4	2	-	3.6
Mangoes	4	4	-	3	3	4	3.6
Science Fair	4	4	4	-	4	4	4.0

Note. Possible maximum score for each component of the rubric was 4.0. Mathematical concepts, mathematical reasoning, mathematical errors, explanation, representation, and strategy & procedure are the components of the rubric. Not every component may be used for every task.

Liza used two circles of similar sizes to represent the two pizzas that the two sets of friends shared. Liza drew and partitioned one circle to represent the pizza that Lorenzo and Jessica shared. She divided it into 11 equal-sized parts, rather than into 12 parts. However, she shaded the correct number of parts (4) to represent and show the fraction (1/3) of the pizza that Lorenzo ate. Then, to represent and show the fraction of the pizza that Jessica ate, Liza shaded one-half (1/2) of the whole pizza based on her calculations of 2/3 x 3/4. She did the calculations correctly, indicating that she fully understood what she needed to do to come up with the answers by using fraction computations and procedures (see Figure 1).

Despite the fact that Liza did not divide the circle into the correct number of segments to begin with, her representation of the fraction of the pizza that Lorenzo and Jessica ate seemed to be correctly partitioned at first glance because she added a line to make the 12th segment after she did the calculations. She did this to show that the



Lorenzo & Jessica pizza

Naquan and Derasha pizza

LIZA I

L 1/3

J 1/2

N&D

what was missing

$\frac{3}{3} - \frac{1}{3} = \frac{2}{3}$ $\frac{2}{3} \times \frac{3}{4} = 1/2$

$\frac{1}{3} + \frac{1}{2} = \frac{5}{6}$ $\frac{5}{6} \times \frac{4}{4} = \frac{20}{24}$

N & D = 1/2

$\frac{20}{24} - \frac{9}{24} = \frac{11}{24}$ $\frac{3}{4} \times \frac{1}{2} = \frac{3}{8}$

$\frac{3}{8} \times \frac{3}{3} = \frac{9}{24}$

Lorenzo & Jessica

$\frac{5}{6} \times \frac{4}{4} = \frac{20}{24}$

Naquan and Derasha

$\frac{3}{8} \times \frac{3}{3} = \frac{9}{24}$

Lorenzo was right. He and Jessica ate more of their pizza than Naquan and Derasha. They ate more of their pizza by 11/24 than what Naquan and Derasha ate. Lorenzo ate 1/3 of the pizza and Jessica ate 3/4 of what's left. 1/3 take away from the whole pizza leaves 2/3. 3/4 of 2/3 is 1/2.

LIZA I

Performance-Based Fraction Activity: Pizza!

So Jessica ate 1/2 of the pizza. I added 1/3 and 1/2 and come up with 5/6. So altogether Lorenzo and Jessica ate 5/6 of their pizza. Derasha and Naquan ate 1/2 half of their pizza together. 1/4 of their pizza were gone leaving 3/4. 1/2 of 3/4 is 3/8. So Derasha and Naquan ate 3/8 of the remaining pizza. To compare the two fractions, I changed 5/6 and 3/8 into 20/24 and 9/24. I multiply 5/6 by 4/4 and got 20/24. I multiply 3/8 by 3/3 and got 9/24. I drew two rectangles to represent 20/24 and 9/24.

Figure 1. Liza's work on *Pizza!*

fraction of pizza that Jessica ate was 1/2, the fraction that Lorenzo and Jessica ate together was 5/6, (1/3 + 1/2), and that two slices (1/6) of the pizza remained.

To determine the fraction of pizza that Naquan and Derasha ate, Liza first divided the circle, which she used to represent the second pizza, into the correct number of parts (12) to show that it was a 12-slice pizza. She then shaded 3 parts (1/4) to represent the 3 missing slices because the problem stated that the second box of pizza contained nine of its original twelve slices. To represent and show the fractional part of the pizza (3/4) that Naquan and Derasha ate, Liza shaded 1/2 of the whole pizza (6 segments) rather than 1/2 of 3/4, which was the fraction of the pizza in the box before these two students began to eat. Therefore, although Liza's calculations were correct (she multiplied 3/4 by 1/2) and her answer, 3/8, was correct, these did not match any of the fraction representations shown by the segments that she shaded.

Next, to compare the fractions that represented the amount of pizza that the two pairs of students ate and to see which pair ate more, Liza performed the correct calculations and came up with the correct answers. She then used fraction representations in the form of two rectangles that she divided into 24 equal parts. Using one rectangle, she marked 20 of the 24 parts (20/24 or 5/6) to show what fraction of the pepperoni pizza Lorenzo and Jessica ate, and in the other rectangle, she marked 9 of the 24 parts (9/24 or 3/8) to show what fraction Naquan and Derasha ate. She also answered correctly the question which asked how much more pizza, the winning pair of students ate (11/24) and did the supporting calculations, which included renaming the fractions by finding the least common denominator. She labeled both rectangles correctly as well as the two

circles. Her explanation for her answer was in the form of the steps and the calculations that she used to find the answers to each part of the task.

Liza's approach to finding a solution to this task involved performing primarily computations. Each of the fraction representations that she used to show the portions of the two pizzas that the students ate as well as those use to compare the two fractions (5/6 & 3/8) of pizza that the two pairs of students ate were drawn based on these calculations.

Performance Task 1 interview. Liza was the first to complete this task and so was the first participant that I interviewed. The interview took place in a quiet corner of my classroom. Because Liza seemed very nervous, I made a light comment and told her that she could relax for a minute. When she was ready, I began the interview by reading a brief opening statement which stated that there was no right or wrong answer to any of the questions and that she should answer in whatever manner she chose. The first few questions were general fraction questions. Liza answered each question in a low tone of voice. On several occasions, I had to signal to her that she needed to speak a little louder so that I could hear what she was saying. She identified a fraction as "a part of something" and added that the parts must be equal. She knew what the term *fraction representation* meant and defined it as "something that is used to show something in the place of another." However, Liza said that she would only use drawings of fractions when the given problem is a word problem; she would not use these types of representations to do a problem such as (1/2 + 5/8). To her, it was quicker to perform the calculation because she knew how to do them.

Liza said that she used fraction computations to find the answers to the fraction questions when she worked on the *Pizza!* task but then used fraction representations in

the form of shapes (a circle and a rectangle) to check to see if her calculations were correct. She used a circle to represent the two pizzas and the rectangles to compare the fractions that represented the amount of pizza that the two pairs of friends ate from each box. Liza said that doing this helped her to see the problem clearer because, she was a visual learner. She thought that the *Pizza!* task was easy at first but was confused because the second pizza had missing slices to begin with, and she was not sure if she should count the number of slices of the pizza or figure out the fraction of the pizza that remained in the box. She recalled that, at first, she counted the slices of pizza in the second box rather than use a fraction that represented what was in the box. She decided to use circular diagrams to double-check her answer after she used fraction computations. However, Liza stated that the diagrams (representations) helped her to "see" her answers and to compare the fractions for the two pizzas that the two sets of students shared. She did not have to use any computations to compare the fraction of pizza remaining in the two boxes.

In terms of the level of difficulty of the task, Liza rated it between a 5 and a 6 (out of 10) because, as she said, "some parts were easy and some parts were hard." She liked the task, because it was challenging for 8th-grade mathematics. To find her solution, which she thought was correct, she used several fraction skills: adding, subtracting, multiply, and comparing fractions. She felt that she had computed the fractions correctly and was able to select appropriate fraction representations in the form of shapes (a circle and a rectangle) to represent the fractions she was working with and to check her answers that she found by doing mostly computations.

Liza's reflections on Performance Task 1. Liza stated that she was "thrown off" initially because the second box of pizza was short by 3 slices before the students shared it. She described the task as challenging and one that pushed to her fullest. She also said that though she did not use fraction representations (the circles and the rectangles) to find the answers to the questions in the task; she used the circles and rectangles to help her to make the problem clearer, and this made it easier to find the answers. She rated the task a 7 out of 10 because of the second part of the problem, where the box of pizza was not full to start with, and this made the problem harder. She liked working in the group because the students had different points of views and shared them with each other.

My field notes for Performance Task 1. When I observed Liza as she worked on this task, she did not seem to be very relaxed at first. I could see that she was doing quite a lot of calculations, and there was no evidence that she was using any fraction representations. At one point she seemed frustrated. When I walked around to her desk and closely observed her work, I realized that she was doing primarily calculations and had done so several times and started over. I noticed that for the second part of the problem she used nine slices to represent the pizza in the second box, rather than the fraction 3/4 to indicate that this represented (1 – 1/4), that is, *one* whole box of pizza minus one-fourth (3 out of 12 slices).

Results Regarding Performance Task 2

Liza's score on this task was a 3.6 (see Table 3) . The problem given in this task assessed the students' ability to (a) engage in problem solving by using a strategy of their choice, including "working backward," (b) identify and use an applicable type of fraction

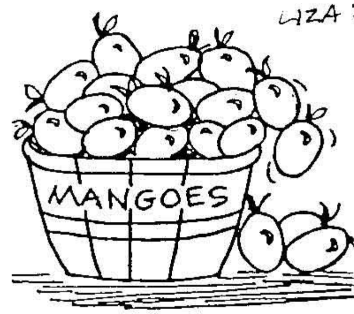
representation, (c) recognize and use a pattern, and (d) apply their conceptual and computational fraction skills to find a solution to the given problem (see Appendix B).

To verify her calculations and her answer Liza used fraction representations in the form of sets. Her calculations and her answers were all correct. She found her answer by performing calculations and using a pattern. She used a chart as a means of organizing her work to show the different parts of her calculations relative to each of the six different personalities stated in the problem. For example, to begin the problem, she used 1 to represent the set of mangoes in the bowl and then multiplied this by 1/6 to show the fraction of the set of mangoes that the King ate (See Figure 2). She then subtracted 1/6 from 1 and got 5/6. This represented the fraction of mangoes that remained after the King ate his share. She continued this pattern of calculation for the Queen, and each of their three sons.

Liza also used a set of 6 circles to represent the set of mangoes in the bowl to begin with. For each person who ate, she filled in one circle and used the remaining circles to make the fraction that represented the mangoes remaining. As an example, one shaded-in circle represented 1/6 of the whole (one) set of mangoes, and (6/6 - 1/6 = 5/6). Therefore 5/6 of the set remained. She continued this process until 1/6 remained for the servants, and this was equal to the three mangoes remaining in the bowl after everyone had eaten. She used this to find her answer, which was 18. Then Liza calculated 1/6 of 18 and got 3 to verify that 18 was the correct answer.

Liza’s work was well organized, and her mathematical reasoning somewhat clear, but her explanation of the pattern that she found and used was not very well communicated, that is, not very clear to me. Her calculations indicated that she could

The "Mangoes Problem"



LIZA 2

One night the King couldn't sleep, so he went down into the Royal kitchen, where he found a bowl full of mangoes. Being hungry, he took 1/6 of the mangoes.

Later that same night, the Queen was hungry and couldn't sleep. She, too, found the mangoes and took 1/5 of what the King had left.

Still later, the first Prince awoke, went to the kitchen, and ate 1/4 of the remaining mangoes.

Even later, his brother, the second Prince, ate 1/3 of what was then left. Finally, the third Prince ate 1/2 of what was left, leaving only three mangoes for the servants.

How many mangoes were originally in the bowl?

King
$1 \times \frac{1}{6} = \frac{1}{1} \times \frac{1}{6} = \frac{1}{6}$
$1 - \frac{1}{6} = \frac{5}{6}$
$\frac{5}{6}$ is left

Queen
$\frac{5}{6} \times \frac{1}{5} = \frac{1}{6}$
$\frac{5}{6} - \frac{1}{6} = \frac{4}{6}$
$\frac{4}{6}$ is left

Prince 1
$\frac{4}{6} \times \frac{1}{4} = \frac{1}{6}$
$\frac{4}{6} - \frac{1}{6} = \frac{3}{6}$
$\frac{3}{6}$ is left

Prince 2
$\frac{3}{6} \times \frac{1}{3} = \frac{1}{6}$
$\frac{3}{6} - \frac{1}{6} = \frac{2}{6}$
$\frac{2}{6}$ is left

Prince 3
$\frac{2}{6} \times \frac{1}{2} = \frac{1}{6}$
$\frac{2}{6} - \frac{1}{6} = \frac{1}{6}$
$\frac{1}{6}$ is left

Servants
$\frac{1}{4}$ is left
$\frac{1}{4} = 3$

$3 \times 6 = 18 \quad \frac{1}{6} \times \frac{18}{1} = 3$

King
$1 - \frac{1}{6} = \frac{5}{6}$

Queen
$\frac{5}{6} - \frac{1}{6} = \frac{4}{6}$

Prince 1
$\frac{4}{6} - \frac{1}{6} = \frac{3}{6}$

Prince 2
$\frac{3}{6} - \frac{1}{6} = \frac{2}{6}$

Prince 3
$\frac{2}{6} - \frac{1}{6} = \frac{1}{6}$

$\frac{1}{6} = 3$

Originally there was 18 mangoes. When I worked out the problem, I had found a pattern. The numerator of how many mangoes are left is the same as the denominator and when I multiply the two fractions I alway get 1/6. I added up what fraction of the mangoes that the five royalities ate and got 5/6. The servants ate 3 mangoes. So 1/4 is equal to 3 and I multiply 6 by 3 and got 18. When I checked the problem, I multiply 1/6 y 18 and got 3. So my answer was correct.

Figure 2. Liza's work on *The Mangoes Problem.*

correctly compute fractions involving multiplying and subtracting simple fractions. Her work also indicated that she had a high level of conceptualization of fractions and computational skills.

Performance Task 2 interview. After I read the paragraph that reminded Liza that she should relax and remember that there was no right or wrong answer, she was ready to begin the interview. She did not seem as nervous as she was during the first interview. In response to the first question about any ideas that came to her mind as to how she would solve the problem, Liza said that she thought of drawing a set because everyone had eaten a different amount of mangoes, and it would be easier for her to see who ate what if she used this form of representation. To solve this closed-ended problem, the answer to which could be found in a variety of ways, Liza first used computations and then used fraction representations in the form of a set to check to see if the answer she found by doing the fraction computations were correct.

Liza said that because this problem did not give all of the information that was needed to begin the problem, this made it difficult at first, but she realized that she had to use the working backward strategy to find the answer to the problem in the task. This was after she saw a pattern in which the numerator of the fraction, which represented the fraction of mangoes that each person ate, was the same as the denominator of the fraction that represented what remained, after someone ate his or her share of the mangoes. Liza said that she also noticed that for every calculation that she did to find out the fraction of the mangoes that each person ate, the answer was the same, 1/6. Therefore, she was able to use this information to find the correct answer to the problem. For fraction representation, she used a set to represent the bowl of mangoes and the fraction that each person ate. She felt that this helped her to find the answer more quickly than if she had used trial-and-error. She said that she used several types of fraction computations and rules including adding, subtracting, multiplying, and reducing fractions to find her answer.

Liza said that she did not use any manipulatives because when she used a set for the fraction representation, the problem became clear enough and this was all that she needed to solve the problem. In our discussion of manipulatives, Liza said that she was a kinesthetic learner. She said that she rated the difficulty level of the problem a 4 or a 5 (out of 10) because it took her a while to understand the problem and to work it out to get her answer. I asked her why she rated the task with a 4 or 5 if she thought that it were difficult and that it caused her to spend a long time trying to figure it out. She said that she would rate a problem a 10 only if it had different numbers, parts were missing, and she had to find all of the missing information to get her answer.

According to Liza, this task was different from the pizza task because she had to work backward to find the answer. She said that it took her about 15 minutes to realize this because she began trying to find an answer by doing calculations and did them incorrectly at first. After she used a set to represent the fractions in the task, this made the problem clearer and it became easier to figure out the problem this way.

Liza's reflections on Performance Task 2. In answering the prompts on the reflection log for this task, Liza stated that she used fraction representation in the form of a set but only to check her calculations and the final answer. She said that she began the problem by working forward first but thought that this was a mistake because her answers did not seem right. She said that she changed her strategy and worked backward instead and found the correct answer to the problem. Her first impression of the task was that it was more of a calculation problem rather than one for which she could use fraction representation to find the answer. She also said that this task made her realize that she

could solve mathematical problems by other ways than by using computations or calculations.

My field notes for Performance Task 2. From my observation of Liza as she worked on this task, I could see that she began the task immediately and seemed much more relaxed than she was when working on the previous task. I noticed that she was doing many calculations and seemed to have started over several times. There was no visual indication that she was using any sort of fraction representation while I had observed her. She finished the task within 45 minutes and could be seen working on her final draft.

Results Regarding Performance Task 3

As was seen in Table 3, Liza's scored a 4.0 on this task, which required the participants to analyze and reason about fractions, to use spatial and numerical reasoning to identify fractional parts of a rectangle, and to demonstrate their ability to use fractions, decimals, and percents, interchangeably (see Appendix C). Liza partitioned correctly the given rectangle (see Appendix S) using the information given regarding the number of students that each of the three schools would bring to the science fair. She calculated the percentages for each school correctly based on their students. For example, she put 1,000 over 2,000 (total number of students) and reduced this to 1/2. She did this for each of the three schools, using the number that represented their students, as the numerator and the total number of students as the denominator. Each of her reduced fractions was correct, as was the calculated percentage for each school, though she did not show the calculations for these percentages.

Liza also calculated correctly the answers of $150 (1/2 of the total), $90 (3/10 of the total), and $60 (1/5 of the total). These values represented the amount that each school should pay of the total cost of $300 for the science fair based on the number of their students who would be attending. She did not show the exact calculations, but explained that she figured the costs based on the percentages for each school. All of her answers and calculations were correct, and she did a good job of justifying her answers, though she did not show how she arrived at the amounts for all of the questions. For example, she stated that the school with 1,000 students should pay $150 because it was bringing 1,000 students, which was 1/2 of all of the students who were going to attend the science fair.

Performance Task 3 interview. Liza said that as she read the information regarding the task, she thought of how to divide the rectangle to give the correct space to each school. She added that she did not clearly understand what the task was asking her to do at first, so she had to read it several times. Also, as she thought of how she would go about solving the problem, she knew that she had to make sure that the fractions that she used were in the same fraction family so that she could see which sections were bigger when she divided the rectangle.

I asked Liza if she thought of using fraction representation as she read the task. She stated that she decided that she would use the blank rectangle to fill it with three smaller shapes that would represent the fraction of the space that each school would get; this would be her fraction representation. She used the smaller rectangles and the fraction symbol to represent the fractions that she was working with. Liza said that she found the

answers to all of the questions in the task and was very certain that her answers were correct because she checked them when she was finished.

Liza rated the difficulty level of the task a 2 out of 10 because it was very easy for her to do. She said that in order to solve the problem and work with fractions, she had to know how to change a fraction to a decimal and to a percent and to add fractions. The easiest part of the questions was changing fractions and decimals to percents. She also liked the fact that she was given a fraction representation in the form of a rectangle and that she had to only figure out each part for each school and show this on the big rectangle. Relative to using fraction representation to work on tasks that had fractions, Liza said that she liked using it because it helped her to figure out the answers to the questions and to check her work with a picture if she thought that her answer was not correct.

During the next phase of the interview, I focused my questions on Liza’s thoughts about using fraction representations in the future. First, I asked her if she is given, in the future, fraction problems, would she think of automatically using fraction representations such as shapes to solve these problems. She said that this depends, that she would only use representations if she were not sure of her answer and wanted to check it to make sure that it was right. She added that having a picture to actually look at helps to make the problem clearer. I asked Liza if she would use fraction representation to add 1/2 and 2/3. Her reply was, “No. I will always do the computations first. However, if I want to check my answer I will use representation.” I then asked, “If you are given a word problem, will you automatically use fraction representation?” Liza said, “No. I will do the computations

first; if I do not understand the problem and I want it to be clearer, I will draw a picture or some other representation to actually see the problem."

Liza's reflections on Performance Task 3. Liza's opinion of this task was that it was very easy because she understood the problem entirely. She liked the fact that the blank rectangle allowed her to create and to use fractions to solve the problems and answer the questions. She said that she used fraction representation to find the answer when she divided the rectangle into three parts, and that the space for each school represented the fraction of the total space that the three schools shared. She rated the problem a 1 or 2 because it was very easy and she knew how to do it.

My field notes on Performance Task 3. Liza seemed relaxed as she worked on this task. I observed that she was performing computations. I saw her examine closely the diagram of the rectangle on the sheet with the task. On another occasion, I saw that she had completed the partitioning of the rectangular diagram on her sheet. She completed the task ahead of time.

Allan

Scores on the Attitude Toward Fraction Inventory Questionnaire

As can be seen in Table 4, Allan's responses to statements in the CAF category of the ATFI questionnaire totaled 29 out of a possible 30 points and indicated that he is very confident about his ability to learn fractions. His score in this category was the only one that satisfied the selection criteria of 25 or higher out of 30. His score of 21 for the VFM category indicated that he thinks that fractions are a valuable part of mathematics, but he does not enjoy working with fractions and is not very motivated about learning these concepts. I chose to include Allan as a participant in the study because of his very high

Table 4

Attitude Toward Fractions Inventory Questionnaire: Allan's Scores

Attitudinal Category	Score (Out of 30)
Value of Fractions	21
Confidence about Fractions	29
Enjoyment of Fractions	17
Motivation about Learning Fractions	16

level of proficiency with computing and problem solving with fractions, his enthusiasm for learning mathematics, and his love for working on challenging mathematics problems.

Results of Fraction Computation and Fraction Concepts Pretests

Table 5 below summarizes Allan's scores on the Fraction Computation and Fraction Concepts pretests. As can be seen, Allan obtained a perfect score on the computation test but only a score of 58% on the concepts pretest. On the fraction concepts pretest, Allan could identify and draw models of simple fractions, but he could not draw correctly a model for the improper fraction 3/2. In his model he shaded a whole circle and a half of another circle, but he did not recognize that he could shade one half of each of three circles to represent 3/2. He had incorrect answers for almost all of the questions involving estimation of fractions. For example, he estimated that 7/8 + 12/13 was approximately 1. He stated that 7/8 and 12/13 were both close to 1/2 so their sum was 1. On a similar question, he stated incorrectly that the product of 3/5 and 6/5 is less

Table 5

Fraction Computation and Fraction Concepts Pretests: Allan's Scores

Pretest	Raw Score	Percentage
Fraction Computation	53	100%
Fraction Concepts	15	58%

Note. Possible maximum raw score for Fraction Computation Pretest is 53. Possible maximum raw score for Fraction Concepts is 26.

than 3/5 and 6/5, though he correctly stated that the sum of 3/8 and 4/9 is less than 1. The following is another question that required Allan to use estimation.

> Mr. Kingley ordered several large pizzas for the students in his math class. Jamie, Joylyn and Jared each shared slices from a different kind of pizza. Jamie ate 1/3 of a pepperoni pizza, Joylyn ate 4/8 of a veggie pizza, and Jared ate 5/6 of a cheese pizza. Who ate the most pizza? Show and explain your work.

Allan showed his strong procedural and computational knowledge by converting each fraction to a decimal and then comparing each one to find his answer, rather than estimating his answer by looking at the fractions. He said, "you can also change the denominators to the same thing to find the answer." Even though Allan's score on the concepts did not satisfy the selection criterion of a score of 75% or higher, I selected him because his score of 58% was the highest of the three participants, and he answered some of the harder questions correctly.

Results Regarding Performance Task 1

Table 6 summarizes Allan's performance on each of the three performance-based tasks. For the *Pizza!* problem, he scored a perfect 4 on three components of the scoring rubric. Allan drew two similar-sized circles to represent each of the two pizzas that the friends were sharing, but he did not divide them into the correct number of sections (12)

Table 6

Performance Tasks Results: Allan

Task Name	Math Concepts	Math Reasoning	Math Errors	Explana-tion	Represen-tation	Strategy & Procedure	Score
Pizza!	4	4	4	3	2	-	3.4
Mangoes	1	1	-	1	1	1	1.0
Science Fair	4	4	3	-	4	4	3.4

Note. Possible maximum score for each component of the rubric was 4.0. Mathematical concepts, mathematical reasoning, mathematical errors, explanation, representation, and strategy & procedure are the components of the rubric. Not every component may be used for every task.

to indicate that each box held a 12-slice pizza. Instead, he divided each pizza into three unequal sections. However, below the diagram of this pizza, he wrote, “12 pieces in total – 1 whole pizza” (see Figure 3).

For his work on the first pizza that Lorenzo and Jessica shared, Allan explained that he divided the circle into thirds and then shaded in 1/3 to show what fraction of the pizza Lorenzo ate. He then explained that he divided what was left of this pizza into fourths “for Jessica, and shaded in 3 parts” to show the fraction of the pizza Jessica ate, which was 3/4. He did not show any calculations for this part of the problem, but he stated, “Looking at the shaded part I could see that I shaded in 5/6 of the total box.” He did not do any calculations to show how he arrived at 5/6.

For the second part of the problem, Allan labeled 1/4 of the whole pizza as missing. This represented the 3 (out of 12) pieces that were missing from the box before Naquan and Derasha ate their portion of the meat pizza. Allan then multiplied (9/12 by

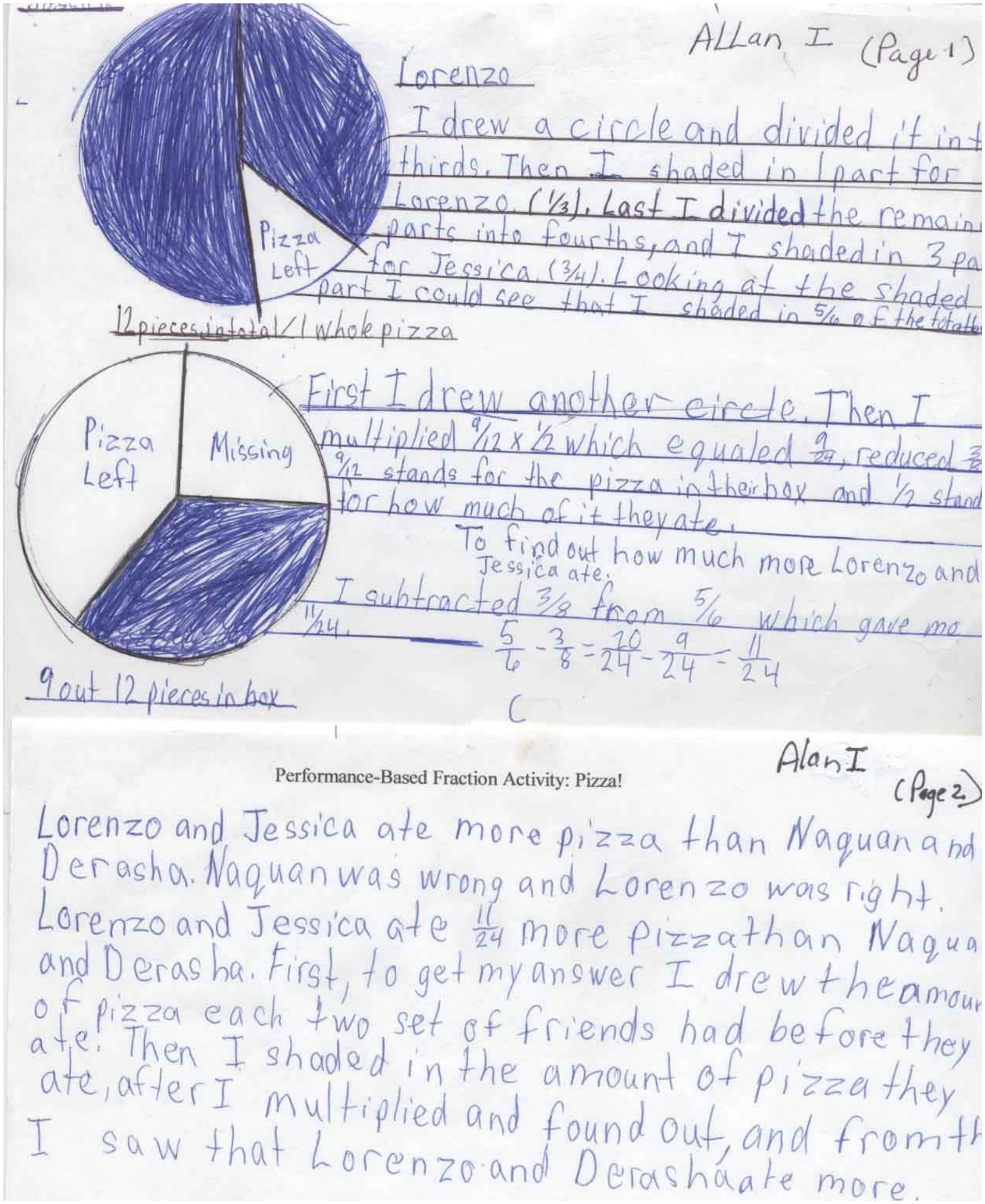


Figure 3. Allan's work on *Pizza!*

1/2) to find out what fraction of the 3/4 of the pizza in the box the second pair of friends, Naquan and Derasha ate. His answer was 3/8.

Last, Allan explained that, to find out how much more pizza Lorenzo and Jessica ate, he subtracted the fraction of pizza that Naquan and Derasha ate from that which Lorenzo and Jessica ate (5/6 – 3/8) by using a denominator of 24 and renaming the two fractions, to get an answer of 11/24.

Performance Task 1 interview. When asked to explain to his understanding of what the word “fraction” meant, Allan said, “it is a total divided into parts.” I then asked him if there was anything special about the parts, and he said that all of them must be equal. He stated that when he has to solve a fraction problem, the first thing that he thinks of is, “can I work it regularly or am I going to have to use fraction representation to help me.” His understanding of the word “regularly” is “with computations.” I then asked him if he meant that for one type of fraction problem he might use computations but for another type of problem, he might use representations instead. He answered in the affirmative. He stated that he would most likely use fraction representations when he did not understand the problem, or when he could not do it with computation. My next question was, “so does this mean that when you have to solve a problem if it is easy you will use computations, and when it is hard you will use other representations?” Allan responded by saying that he will always try to solve a fraction problem by first using computations unless it tells him that he must use fraction representations.

Next, I asked Allan to name some of the ways in which he could represent fractions. His answer was “a set, and using fraction parts for separating a circle, square or a rectangle into parts.” He also said that we use visual fraction representations to help us to understand the problem better because if we only do computations we are just learning how to work out a problem, but if we use representations we can work it out both ways

and understand what we are doing better. He explained this by saying, “the picture that you draw puts the problem right there in front of you, and what you see helps you to understand better.”

Next, we talked specifically about the task, *Pizza!* Allan said that to solve the problems given in the task, he thought of a way of doing this with computations but wondered if he would have to use representations to help him to figure out the problem and understand it better. He said that it was hard to find a way to come up with an answer because of the part of the problem that said that Lorenzo ate 1/3 of the pizza, and that his friend Jessica ate 3/4 of what remained. He felt that this made the problem harder. I asked him if there were any clue words in the statement “Jessica ate 3/4 of what was left” that could have given him an idea as to what he should do or the type of operation he should perform. He said that he knew that the word “of” meant that he should multiply, but that he did not realize this the first time that he read the problem. However, when he read it the second time after trying to work the problem out, he realized that he needed to multiply, and he did.

In our discussion on fraction representations, I asked Allan if he thought of using representations of fractions immediately to solve the problem. He said that he did not. I asked him to explain why he did not think of using the representations right away, if he thought that doing this helps him to understand the problem better. He replied, “cause if you know how to do the computations, it might help you to do it faster and the representations will make it longer, and it seems like a waste of time.” I then asked, “seeing that at first you thought that using the calculations would be easier, but it turned out that the problem was not as easy, and you decided to try using representations and

that worked, how do you feel about always trying to use representations first?" He said that just to get the work done, he would do what he had to do.

I wanted to find out if the fact that the directions given in the problems which stated that diagrams and pictures could be used to solve the problem influenced Allan's decision to use fraction representations. Allan said that this did not because he would only use representations if he needed help with the problem or did not understand it. He said that after he decided to use another type of fraction representation to try to solve the problem, it helped him to understand the problem better. He stated, "because when they asked who ate more pizza, I drew it out and it helped me to understand the problem because I was able to see it." In terms of the fraction skills that he needed to know to be able to do this problem, Allan said that he had to know to multiply, add, subtract, and compare fractions.

To find his solution, Allan said that he first tried doing straight computations. However, when he saw that doing this did not work, he drew a circle to represent the pizza. He explained, "I divided it into the parts, like thirds and fourths that was needed." He said that he worked on Lorenzo and Jessica's pizza first and found the answer, but when he worked on the part with Naquan and Derasha, he was confused because they did not have a whole pizza to start with; three pieces of pizza were missing from their box, and this made the problem harder. He added, "because it wasn't a whole pizza, first you had to find out the fraction of how much pizza was in their box." To find this out, he put the number of pieces that was there, 9, over 12 and multiplied it by 1/2. When he calculated this and found the answer, he compared this fraction with the first one for Lorenzo and Jessica to see which two students ate more. He did this by using the pictures

that he drew and did not use the numbers. He said that he just looked at the pictures to see who ate a larger portion. Then he subtracted the smaller fraction of the pizza from the larger one to see how much more pizza they ate.

We then talked about the difficulty level of the problem. Allan stated that he would rate the problem a 6 (out of 10), because even though the problem seemed hard, it really was not. Nevertheless, Allan said that the problem was suitable for an 8th-grade level of math and that it was very challenging.

Allan's reflections on Performance Task 1. Allan said that he had a hard time when he first started doing the problem. He also said he was not feeling well and did not think that he did very well on this task. However, he stated that he likes mathematics and so he liked working on this task because it was challenging and made him think. He did not get the problem at first and struggled. He also felt discouraged at first because the problem seemed hard to do, and the more he read it, the harder it seemed. He decided to use another kind of fraction representation, and this helped him to understand the problem, which seemed simpler. He rated the task a 9 (out of 10), though he felt that his answers were correct.

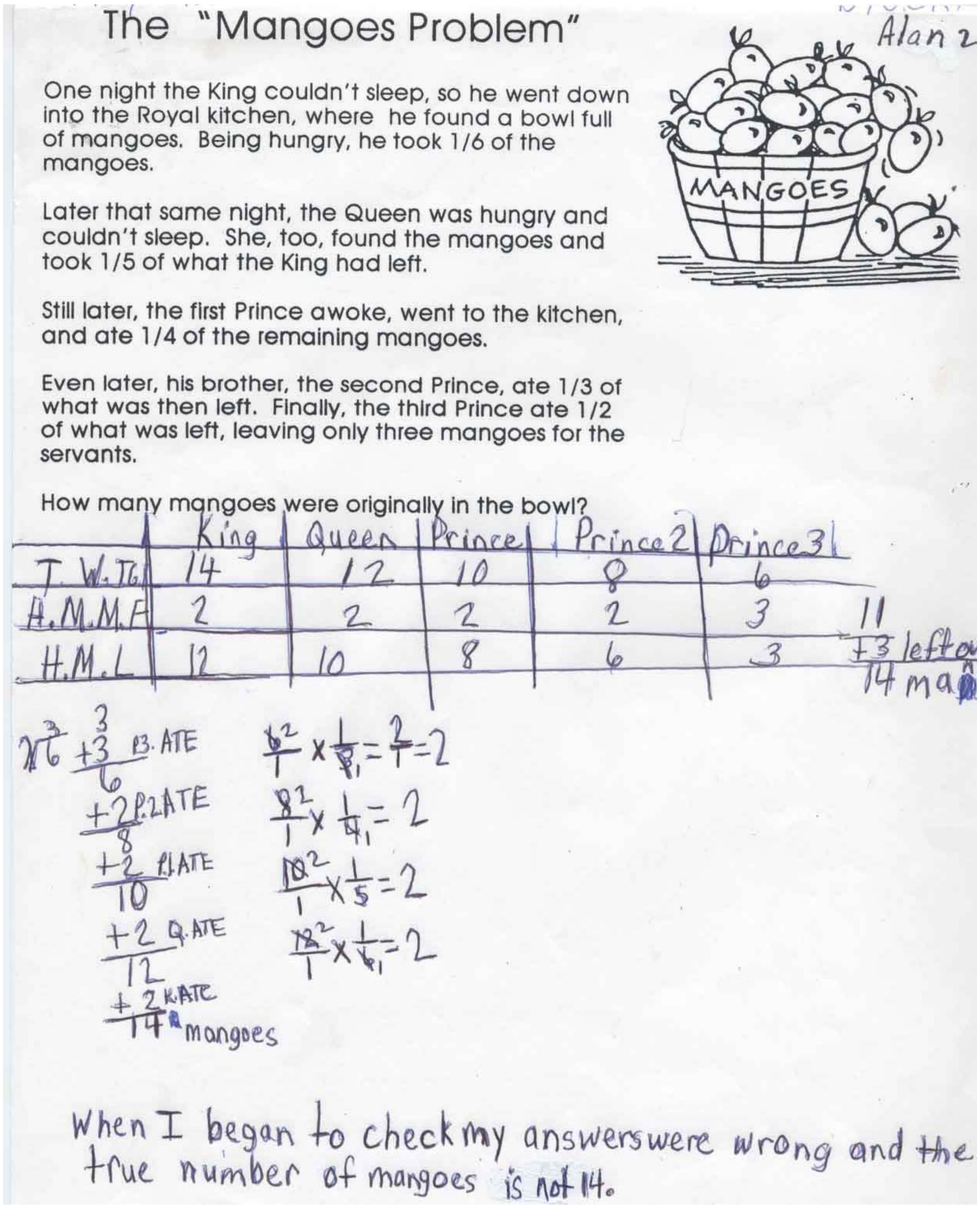
My field notes on Performance Task 1. I observed that Allan did not begin the task immediately. He seemed to be reading the problem and pondering what to do next. His expression indicated that he was confused or not certain as to what to do. After about 10 minutes, he had still not written down anything on his paper. He looked worried. When he eventually began working on the problem, I saw him drawing circles for the two pizzas and doing computations. Allan seemed nervous throughout the period that he spent working on the task.

Results Regarding Performance Task 2

As is seen in Table 6, Allan did not do very well on this task. Allan used a grid with six columns to solve the problem (see Figure 4). He attempted to work backward; beginning with 14, which it seems represents the number of mangoes in the bowl to begin with. The column labeled "Queen" has 12, then 10, 8, and 6 is in each of the other three columns, respectively. There is a 2 in each of the cells in the second row, with the exception of the last one that has a 3. Allan subtracted these numbers from the cells in the first row (14, 12, 10, 8, and 6). The cells in the last row contain the differences. I was unable to follow clearly what Allan attempted to do or to recognize any sound mathematical reasoning on his part, especially because he did not write any explanations about what he did in trying to find a solution to this task.

Allan used the "guess and check" method in trying to find an answer to the question posed in the task. In the accompanying calculations Allan did as follows: (6 x 1/3), (8 x 1/4), (10 x 1/5), and (12 x 1/6). Again, I could not follow or determine what Allan attempted to do except to say that he attempted to work backward, beginning with an arbitrary number, but this did not work. There was no evidence of a clear, logical reasoning in his work. He stated that when he checked his work, he saw that 14 did not represent the number of mangoes in the bowl to begin with. Allan did not use any fraction representations to try to find a solution to the question in this task.

Performance Task 2 interview. The first question that I asked Allan during our interview on *The Mangoes Problem, was,* "what ideas, if any, came to mind as to how you would go about solving the problem related to this task?" Allan said that he thought of working backwards, because he knew that, "they didn't give you something to start



The "Mangoes Problem"

Alan 2

One night the King couldn't sleep, so he went down into the Royal kitchen, where he found a bowl full of mangoes. Being hungry, he took 1/6 of the mangoes.

Later that same night, the Queen was hungry and couldn't sleep. She, too, found the mangoes and took 1/5 of what the King had left.

Still later, the first Prince awoke, went to the kitchen, and ate 1/4 of the remaining mangoes.

Even later, his brother, the second Prince, ate 1/3 of what was then left. Finally, the third Prince ate 1/2 of what was left, leaving only three mangoes for the servants.

How many mangoes were originally in the bowl?

	King	Queen	Prince	Prince 2	Prince 3
T. W. T6.	14	12	10	8	6
A. M. M. A	2	2	2	2	3
H. M. L	12	10	8	6	3

11
+3 left ov
14 ma

3
+3 P3. ATE
6
+2 P.2 ATE
8
+2 P.1 ATE
10
+2 Q. ATE
12
+2 K. ATE
14 mangoes

$\frac{6^2}{1} \times \frac{1}{3_1} = \frac{2}{1} = 2$

$\frac{8^2}{1} \times \frac{1}{4_1} = 2$

$\frac{10^2}{1} \times \frac{1}{5} = 2$

$\frac{12^2}{1} \times \frac{1}{6_1} = 2$

When I began to check my answerswere wrong and the true number of mangoes is not 14.

Figure 4. Allan's work on *The Mangoes Problem*.

with; they just gave you a specific number at the end of the problem." I then asked him if he used any fraction representation in trying to solve the problem, and he said that he did

not. I asked him to explain why he did not, and his reply was, “I felt that it was one of those type of problems that I would be able to figure out with computations.” I then asked, “When you were doing the computation and it seemed that you were not finding a solution, did you think that you should switch to using representations to see if that may help you?” Allan said that he did not because he felt that the answer was right. He added that when he checked it, however, it did not work out.

My next question to Allan related to the fractions given in the problem. I began by asking him if he could describe anything that stood out regarding the fractions in the problem, that is, was there anything that seemed special about the fractions. Allan indicated that the fractions were all unit fractions and was able to explain that a unit fraction has a one (1) in the numerator. I then asked him if the fact that they were unit fractions helped him to solve the problem in any way. He explained that this made it easier to solve the problem. I then decided to probe further and asked if there was anything else that stood out regarding the fractions. His answer was, “they were all, they weren’t from . . ., they were all like, when you start from the top there were all 6, I mean 1/6, 1/5, and 1/4, 1/3, and 1/2.” He then explained that the denominators were consecutive, in decreasing order. I asked him if he thought that the fractions were given for any particular reason. He said that they were given to make the problem easier. I then asked him if this turned out to be as easy as he thought it would be. He replied, “not really. It seemed easy at first, but when I started working on it, I started seeing that the numbers seemed to be too large and I checked it when I finished the first time. I tried it and I saw that it was way off.”

Allan and I then talked about the strategies that he used to try to find a solution to the problem. I reminded him that he said that he worked backwards and asked him to tell me exactly what he did. I said, “for example, you said that you tried numbers when you first tried to do the problem; what numbers did you use?” Allan’s reply was, “I started with three since I thought that they multiplied to get the numbers going down, so I divided. I did the opposite.” I did not understand what Allan was trying to say, so I asked him to explain this to me. He did not seem to know what to say, so I asked, “What do you mean by saying that you started with three?” Allan replied, “in the problem they said that it was three mangoes left after everybody ate theirs and they said that the last person ate one half, so I just multiplied the three by two to get 6, and three is one half of six.” I then asked, “And what did you do with the 6 and the 1/3?” He stated that he divided 6 by 1/3. I then asked, “Did you divide 6 by 1/3 or did you multiply 6 by 1/3?” He said that he divided. When I asked him to explain further his reason for dividing, he said that he wanted to do the opposite of what he thought they did in the problem, and that this was a part of the working backward strategy that he was using to try to find the solution to the problem. When I probed further, Allan explained that he did this because when they used the word “of” such as, “being hungry, the King took 1/6 of the mangoes,” this told him to multiply, and that was the reason he decided to do the opposite and divide. He said that he did this because the problem did not give him any specific number to start with and added, “When I kept on reading, they said that there were 3 mangoes left so I knew that I had to start there.”

I then focused on the topic of fraction representations and began by asking Allan if he had used any fraction models, diagrams, charts, tables, or pictures to try to find a

solution to the problem. Allan said that he used a table to organize the information after he did the calculations. He said that when he did this, he noticed that when he got the first answer, the total was the same as the one that was right before it. I asked him if he saw any connections, or if this made any sense. He indicated that the connection he made was the number of mangoes which remained and the first person that he calculated the mangoes for was the same. I asked him to explain this to me. He said, "If there were two people, the number of mangoes that was there for the second person, was the same as what the first person left." I asked him if this told him anything special, or if it made him think of anything. He replied, "Yes. I knew that they were back to the same number. Each person was eating the same amount of mangoes even though they were using a different fraction for each person, and when you reduced the unit fraction the answers were all equal." His use of the words, *reduced the unit fraction*, prompted me to ask, "What do you mean by reduce? Can you reduce a unit fraction if it always has a 1 in the numerator?" His response was, "no; I meant when you use it to divide and you find the reciprocal, and then multiply, you can reduce the answers. And they were all the same thing."

I continued the interview by asking Allan to identify the fraction skills that he needed to have to solve this fraction problem. He said that he had to know how to multiply and divide fractions. I then asked him if he needed to know any fraction rules and procedures to do the problem. He said that he had to know how to find the reciprocal and use it to divide fractions and to reduce fractions.

I moved on by discussing the difficulty level of the task and asked Allan to rate it. He rated it a 3 or 4 (out of 10). Being somewhat surprised at his rating, which did not

support the effort that he displayed in his work, and based on the fact that Allan had told me that he had a lot of problems trying to find a solution to the problem, I asked him if he thought that he had the right answer to the problem. He said that he did not have the right answer. I then asked him to explain why he rated the task the way he did. He said that this was because he believed that he could have found the answer if he had enough time. I then asked, “are you making your judgment about how easy or difficult the problem was on whether or not you found the correct answer, or on what you had to do to get it and how long it took?” he replied, “what I had to do to get the answer, and how long it took.” I then asked Allan, “If you think that the problem is that easy, don’t you think that you should have finished it in a shorter time and should not have had to keep doing it over and over?” He said, “Yes.” I asked, “Then, why do you think that you could not come up with a solution, or a strategy that would have helped you to find the answer in the given time?” He did not answer. I moved on and asked, “do you think that maybe if you had another problem like this you would try to see (even though you may think that it is easy at first) if perhaps using a diagram or a picture or some representation of fraction might help to make the problem a little less difficult?” Allan answered, “yes.” Next, I asked Allan, if on looking back, he should have thought of trying to use some sort of fraction representation, a picture or a diagram that may have helped him to solve the problem. His answer was, “not really.”

I tried to get Allan to tell me a little more as to why he thought he did not find a solution to the problem in the task, especially because at the beginning of the interview he said when he read the problem he knew right away that he had to work backwards.

However, Allan just shrugged his shoulders. I sensed that he did not want to continue the interview so I thanked him for speaking with me and brought it to a close.

Allan's reflection on Performance Task 2. Allan said that when he first read the task, he thought that it was easy and that his first answer was right, but he stated that when he checked, it was incorrect. He did not use any fraction representation and he felt that this made the problem harder. He did not think that it was necessary to use fraction representation for this task. He stated that he felt crushed that he did not find an answer to the question in the task.

My field notes for Performance Task 2. Allan seemed to be much more relaxed than he was when he was completing the *Pizza*! task. He worked at a consistent pace. I saw him doing calculations but was not able to see the actual content because he always tried to cover his work with his hand whenever I passed by him. I saw no signs of his using fraction representation.

Results Regarding Performance Task 3

Allan's work on the Science Fair task (see Appendix T) earned him a score of 3.8 out of 4.0, as can be seen in Table 6. He partitioned correctly the rectangle, which represented the amount of space in an auditorium that three schools would use for their science fair. The fractions, which Allan said represented the portion of the auditorium allotted to each school based on the number of students they would bring and their equivalent percentages, were correct. All of his calculations and answers to three of the four given questions were correct, and he justified and explained his answers when asked to do so. Allan lost one point from the *Mathematic Error* component of the scoring rubric when he used *percents* in his answer to a question, which asked. "What *fraction* of the

space should each school get on the basis of the number of students? Show your mathematical reasoning."

Performance Task 3 interview. Allan said that as he read the details of the task he thought about how he would do it because he was clear about what the problem was asking him to do. This was to divide a rectangle to show the amount of space each school would get based on the number of their students who were coming to the science fair. The rectangle represented the auditorium where the schools would have the science fair. Allan said that he decided to use fractions to divide the rectangle and to reduce them to percents.

Next, I asked Allan about the approach that he used to do the task. My first question pertained to how he made the decision about dividing the rectangle. Allan said that he just looked at the numbers for the students and this told him which school would have the largest, middle, and smallest space. For example, because Bret Harte Middle School was bringing 1,000 students then that would represent the largest space. However, Allan also said that he was going to divide the rectangle into thirds but corrected this when I pointed this out to him. Allan also could not explain what about the numbers (1,000, 600, and 400) caused him to divide the rectangle the way he did. But on further questioning he said, "I knew that Kennedy and Malcolm X together would equal Brett Harte, so I knew that Brett Harte had to be half and Kennedy and Malcolm X should get the other half." When I asked him how he arrived at the actual fractions that he used, he said, "I knew that MX would get a little more than half of what was left from the other half, so I knew to move the middle line that was dividing the rectangle in half, over some more." I replied, "Okay, this was the drawing, but how did you find the fraction that 600

students represented?" Allan's response was, "I put 600 over the total number of 2,000 students and then reduced the fraction."

I moved the focus of the interview from Allan's work on this problem to the use of fraction representations for doing the task and asked Allan if he found it easy to think of using fraction representations for this task. He answered that he knew that one way to find a fraction was to put the 600 over the 2,000. I then asked, "Do you see this as fraction representation?" He said, "yes." I also asked him if he could have solved the problem without using fraction representation, given that he had to divide the rectangle. He explained that this could not be done without fraction representation because the first question asked about showing the space.

Next, I asked Allan about the fraction representations with which he was familiar. He said, "models and shapes." I asked him to tell me about the models he was referring to and he said, "like shapes, sets, and fraction symbols like 1/2." My next question to him was, "other than the fraction symbol were there any other representations that you could have used to do this problem or to represent those fractions that you created?" Allan said that he could have used sets and explained that he could have drawn 2,000 circles and would have shaded and used different colors to show 1,000, 600, and 400 for the students coming from each of the three schools. I asked him if that would have allowed him to find/see the fractions for each school, and he answered in the affirmative.

My next set of questions pertained to the type of fraction skills that Allan needed to have to answer the questions given in the task. He identified these as reducing fractions, converting a fraction to a decimal, and a decimal into a percent. I questioned his reasoning for changing the fraction into a decimal first, in order to change it to a percent.

He said, "if you know how to do it you could do it, but it was not necessary." However, he chose to do this step as a part of his calculations.

In closing, I asked Allan to rate the task in terms of its difficulty. He rated it a 3 (out of 10). I asked him if he had a reason for rating the task a three, and he stated that it was not very hard, but it took him a while to understand the problem and to work it out. Nonetheless, he was satisfied that this was a fair rating.

Allan's reflection on Performance Task 3. Allan thinks that his answers to the questions on all parts of the task are correct. He liked getting a chance to work with fraction representations again because he believed that using them made the problem clearer. He used fraction representations for this task because it was required. He said that he would rate the task a 3 because it was easy to understand, though he took a while to complete it. At first, Allan thought that the task was an easy one. He also summarized his experience with doing the task by saying that it was too easy.

My field notes on Performance Task 3. Allan seemed relaxed and confident as he worked on this task. I saw him doing primarily computations and noted that he finished the task ahead of time.

Ray

Scores on Attitude Toward Fraction Inventory Questionnaire

Table 7 summarizes Ray's questionnaire scores. Like the other two participants, Ray's scores on the ATFI questionnaire did not satisfy the selection criteria of a score of 25 or higher in all categories. His two highest scores were 22 in the VFM category and 22 in the CAF category. Ray's scores in the other two categories indicated that he somewhat

enjoyed learning about fractions and was also somewhat motivated about learning fraction concepts.

Results of Fraction Computation and Fraction Concepts Pretests

Table 8 presents the results of Ray's fractions pretests. Ray completed only 21 items on the Fraction Concepts pretest because the last page of the test which contained five items, was missing. None of Ray's errors indicated that he had major computational problems, though of the 8 items that he did incorrectly, seven of them involved mixed numbers. Further, the majority of his errors seemed to be due to carelessness. For example, on one item he divided two fractions when he should have subtracted, and on another, he wrote the incorrect answer to a subtraction problem (5 5/8 – 2 4/8 = 4 1/8). The correct answer was 3 1/8. The only concept that he seemed to have done incorrectly, because of a lack of knowledge of this skill was using the inequality signs to compare pairs of fractions. He used the wrong sign for three items on the test.

Results Regarding Performance Task # 1: *Pizza!* (Ray)

Ray did not do very well on this task, as can be seen in Table 9. Ray's score for this task was a 2.0. He earned a score of 1 on the *Mathematics Errors* component of the scoring rubric because of the serious calculation errors that he made. In addition, he partitioned incorrectly the two circles that he drew to represent the two pizzas and again earned only a score of 2 on the *Representation* component. The information in the task stated that both of the boxes contained 12-slice pizzas, and as such, he should have divided the circles into 12 sections to represent each of the 12 slices, rather than 9 sections. The first circle was neatly drawn and correctly labeled. Ray identified the fraction of the pizza that

Table 7

Attitude Toward Fractions Inventory Questionnaire: Ray's Scores

Attitudinal Category	Score (Out of 30)
Value of Fractions	22
Confidence about Fractions	22
Enjoyment of Fractions	13
Motivation about Learning Fractions	15

Table 8

Fraction Computation and Fraction Concepts Pretests: Ray's Scores

Pretest	Raw Score	Percentage
Fraction Computation	40	75%
Fraction Concepts	13	62%

Note. Possible maximum raw score for Fraction Computation Pretest is 53. Possible maximum raw score for Fraction Concepts is 26.

Lorenzo ate as 1/3 and shaded three segments to demonstrate this. However, the segments were neither proportionately nor mathematically correct.

Ray calculated correctly the fraction of the pizza that Lorenzo ate, but he stated that Lorenzo ate 2/3 rather than 2/3 as the fraction of pizza that remained after Lorenzo ate his share (see Figure 5). This appeared to be a careless error because when he did the calculations to determine what fraction of the pizza Jessica ate, he used 2/3 as the fraction of pizza that remained after Lorenzo ate. The source of Ray's second major calculation

Table 9

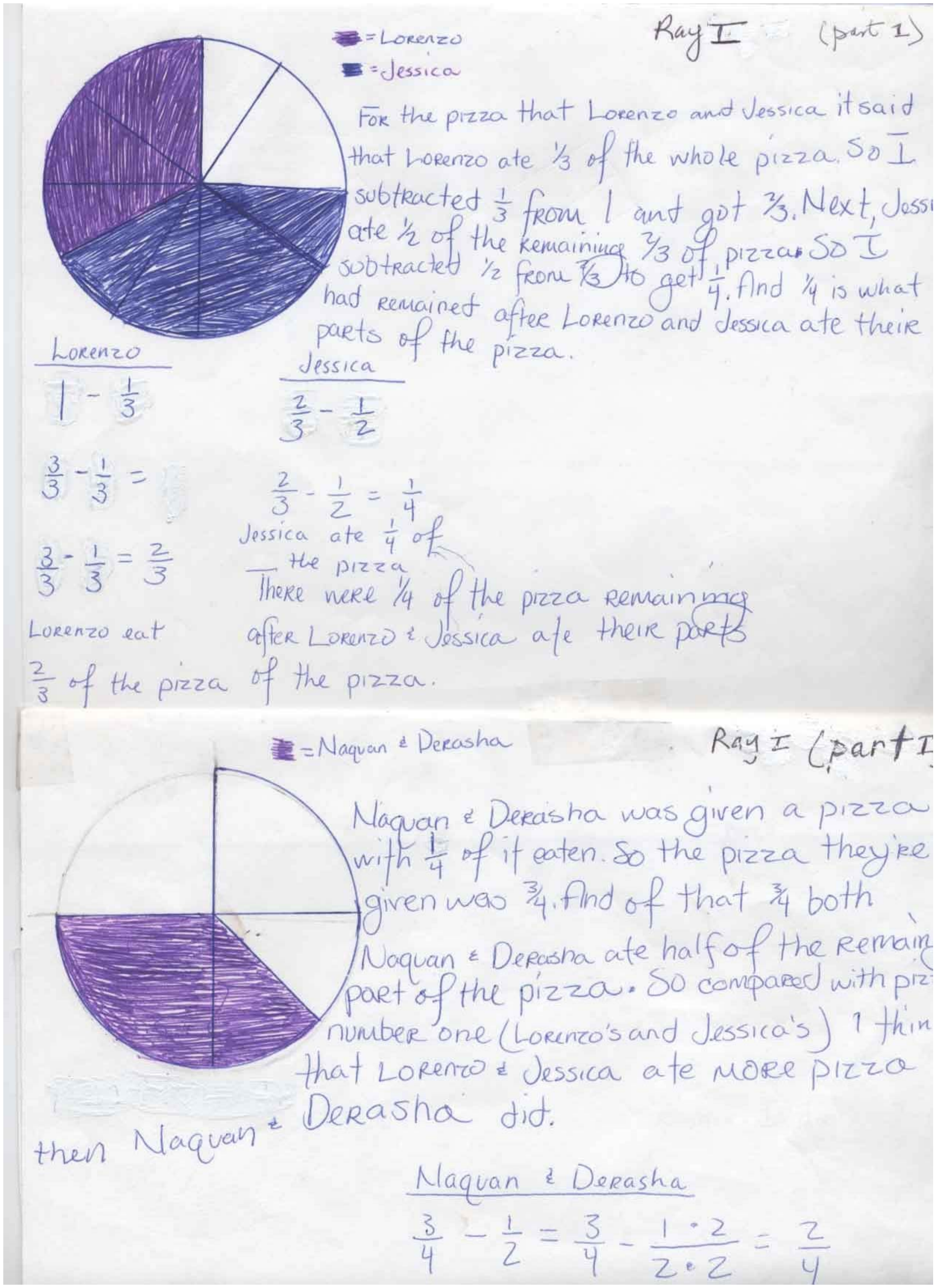
Performance Tasks Results: Ray

Task Name	Math Concepts	Math Reasoning	Math Errors	Explana-tion	Represen-tation	Strategy & Procedure	Score
Pizza!	3	2	1	2	2	-	2.0
Mangoes	2	2	-	2	2	2	2.0
Science Fair	4	4	4	-	3	4	3.8

Note. Possible maximum score for each component of the rubric was 4.0. Mathematical concepts, mathematical reasoning, mathematical errors, explanation, representation, and strategy & procedure are the components of the rubric. Not every component may be used for every task.

error was his use of subtraction rather than multiplication to determine the fraction of pizza that Jessica ate. He reasoned and stated in his explanations that because Jessica ate 1/2 of the remaining pizza (2/3), he subtracted 1/2 from 2/3, totally ignoring the fact that the word *of* means multiplication in most cases. In addition, his answer of 1/4 for this subtraction was incorrect. Again, he made the same mistake of incorrectly identifying the fraction of pizza that remained after Jessica ate as the fraction that she ate.

In figuring out the fraction of pizza that the second pair of friends ate from the second box, Ray drew a very neat circle and erased 1/4 of it to indicate that this represented the three slices that were missing from the original 12-slice box. This was correct, but again he did not divide the pizza into 12 sections to show this is what the box contained *before* someone ate three slices. However, Ray shaded the correct segment to show that Naquan and Derasha ate together 1/2 of the 3/4 box of the meat pizza. His circle was very neatly partitioned and labeled correctly, though his calculations did not match the fraction representation seen in the diagram. In addition, Ray repeated the same process of



= Lorenzo

= Jessica

Ray I (part 1)

For the pizza that Lorenzo and Jessica it said that Lorenzo ate 1/3 of the whole pizza. So I subtracted $\frac{1}{3}$ from 1 ant got 2/3. Next, Jessi ate 1/2 of the remaining 2/3 of pizza. So I subtracted 1/2 from 2/3 to get $\frac{1}{4}$. And 1/4 is what had remained after Lorenzo and Jessica ate their parts of the pizza.

Lorenzo

$1 - \frac{1}{3}$

$\frac{3}{3} - \frac{1}{3} =$

$\frac{3}{3} - \frac{1}{3} = \frac{2}{3}$

Lorenzo eat $\frac{2}{3}$ of the pizza

Jessica

$\frac{2}{3} - \frac{1}{2}$

$\frac{2}{3} - \frac{1}{2} = \frac{1}{4}$

Jessica ate $\frac{1}{4}$ of the pizza

There were 1/4 of the pizza remaining after Lorenzo & Jessica ate their parts of the pizza.

= Naquan & Derasha

Ray I (part I

Naquan & Derasha was given a pizza with $\frac{1}{4}$ of it eaten. So the pizza they're given was 3/4. And of that 3/4 both Naquan & Derasha ate half of the remain part of the pizza. So compared with piz number one (Lorenzo's and Jessica's) I thin that Lorenzo & Jessica ate more pizza then Naquan & Derasha did.

Naquan & Derasha

$\frac{3}{4} - \frac{1}{2} = \frac{3}{4} - \frac{1 \cdot 2}{2 \cdot 2} = \frac{2}{4}$

Figure 5. Ray's work on *Pizza!*

subtracting the two fractions (3/4 – 1/2 = 1/4) rather than multiplying them (3/4 x 1/2) to get the correct answer of 3/8.

In his final calculations, Ray subtracted 1/2 from 3/4 (3/4 – 1/2) and got a correct answer of 1/4 after renaming 1/2 and getting 1/4 for his answer. His calculations (though not necessary) were correct, but he used the wrong fractions and the wrong operation. Ray stated that Lorenzo and Jessica ate more pizza, the main question on the task, but he did not figure out how much more they ate.

Performance Task 1 interview. Ray described fractions as parts of a whole. He stated that the first thing that comes to his mind when he has to solve a fraction problem is adding, subtracting, multiplying, dividing, and simplifying fractions. I asked Ray if he would know what I was speaking about if I said *fraction representation*. He said fraction representation is something that you would use to show your answer using circles, number lines or sets.

I then asked Ray when he would most likely use fraction representations, and he stated that he would do so when he wanted to show how to divide a certain number or a shape or sets, or to add two fractions. He said that he would use it to figure out the problem and see what he was dealing with. Ray further stated that fraction representation helps him to understand what he is doing with the problem and how to divide shapes into equal parts. He added, "fraction representation helps because when you look at the problem it gives you a better understanding of what you are doing than when you see it on paper with the numbers. You will understand it better if you see it in a picture or if you have to diagram it."

I shifted the focus of my questioning to the pizza task. Ray was clear as to what he was required to do, which was "to find what was left of the pizza that was eaten, and who ate more." I then asked him what thoughts came to mind as he read the task. He said, "I thought that if I used a circle and divide it into parts, I could find the answer better than working it out using numbers." He thought that the task was easy after he drew the circle and divided it because he could see what he had to do and how to get his answer. Ray's response led me to ask him if he thought immediately of using fraction representation to do the task. He said that he did and that the type of fraction representation that he thought of using was a circle because the friends were eating a pizza. I then asked him if he would still use fraction representation when the directions do not say that he could use diagrams and pictures. He said that he would not use fraction representation if the directions did not state this.

Next, I asked Ray if he had used a particular strategy to solve the problem and he said that he did not; he said that he just read the problem and followed the directions. After we discussed the computations that Ray did to come up with his answers, he said that he realized that he had made major errors when figuring out his answer. For example, the problem stated that Lorenzo ate 1/3 of the full 12-slice box of pizza and that Jessica ate 3/4 of what remained. Ray said that instead of multiplying 3/4 by 2/3 to find out what fraction of the pizza Jessica ate, he used the wrong fraction (1/2), subtracted it from 2/3 and got a wrong answer. He also realized that he confused the fraction of pizza that remained (2/3) after Lorenzo ate 1/3 of it, with what Lorenzo ate, and wrote that Lorenzo ate 2/3 of the pizza.

I asked Ray to name the fraction skills that he had to know to do this problem. He stated that he had to know how to divide the pizza into the given parts and to subtract and multiply fractions. In our discussion about some of the other computations that Ray performed to find answers to the questions in the task, he said that he was confused because he did not know if he had to subtract or multiply when the problem stated that Naquan and Derasha ate half of the pizza in their box. I asked him several probing questions and he again realized that because *of* means multiply, and he knew this, he should have done so, rather than subtract.

Next, I asked Ray to rate the task. He rated it a 4 (out of 10). When I asked, why he chose to rate the task this way, he said, “Because in some parts it was kind of difficult because I was counting the slices when I should have been counting the parts of the pizza that they ate.” I then said, “A four means that it was not that hard. Are you saying that it was somewhat easy but you did not realize that you made errors and so did the problem incorrectly? Is that so?” Ray answered in the affirmative.

I asked Ray for his opinion about what he liked or did not like about having to do performance tasks. He stated that he liked doing these types of problems because he is able to use diagrams to help him figure out his answers better by seeing how to divide things into parts and this helps him a lot. He did not like that some of the information in the problem caused him to be confused about what he was supposed to do to find his answers. Nevertheless, he stated that when the problem is stated as it was in the problem, it made him think and that this was better for him in mathematics. We ended the interview on Ray’s point that fraction representation helps him to see the problem as he is doing it and that drawing the models helps him to understand the problem better.

Ray's reflections on Performance Task 1. Ray thought that he did very well because the task was very easy and because he was good at using representation for figuring out his answers. He thought that his answer was correct because he used representation, and this helped him to find his answer. Ray rated the task a 5 (out of 10) but said that the task was a "good" one. A subsequent analysis and scoring of Ray's work on this task indicated that his positive reflection on how he thought he did relative to finding a correct solution was not warranted. Ray scored only a 2.0 on this task.

My field notes on Performance Task 1. When I observed Ray as he worked on this task (see Figure 5), he was very involved in what he was doing – drawing a circle. I also observed that he was doing a fair amount of computations. He seemed relaxed as he worked, but he did not finish in the allotted time. He completed his task when he returned for his second class of the day with me during the third instructional block.

Results Regarding Performance-Based Task 2

The result of Ray's work on this task (see Table 9) earned him a score of 2.0. Even though he had the correct answer to the problem (see Appendix U), it was difficult for me to follow his train of thought or to understand his explanations of how he arrived at the answer. He explained that he used sets for representing fractions and drew a diagram which showed a rectangular figure divided into 6 sections, and there were 3 circles in each section. However, it was not clear as to what the fractions or the diagram represented.

Ray also did calculations that were not clear. He began with 6, subtracted 1, and was left with 5. Next he subtracted 1 from 5, and then continued to subtract 1 from the

difference until the difference was 1. In addition, Ray began with 18 and used the same procedure to subtract 3 each time, until the difference was 3.

Performance Task 2 interview. Ray said that, before beginning the problem, he read it carefully and realized that he had to work backwards on the problem to come up with his answer. This was because at the beginning of the problem they did not tell him the total number of mangoes that were in the bowl, but that at the end they said how many mangoes were left. Ray said that this was where he started, to find how many mangoes were in the bowl in the beginning. Ray also said that he used a diagram, but not a circle, for fraction representation. Instead, he used sets; he used 6 boxes and put 3 mangoes in each box. I asked Ray if using this fraction representation helped him to solve the problem. His response was, “Yes, looking at the problem, I could figure it out and see what goes where, and take away for each person.” He also used manipulative blocks and put them in groups. This helped him to see how the diagram was going to work, and it was easier for him to put it on the paper.

In addition, Ray said that he had to multiply, divide, and subtract fractions, and he was able to find out how many mangoes the King, Queen, and their sons ate. Ray explained that to do this he kept working backward. For example, he took the number of mangoes remaining and multiplied it by the denominator of the fraction that was given to get a number for his answer. This helped him to discover that even though the fraction that represented the number of mangoes that each person ate was different, they all ate the same amount of mangoes. I asked Ray if he could explain why this was so. He said, “You could see a pattern. Every time you calculated the number of mangoes that a person ate, even though you used a different fraction you came up with the same number for the

number of mangoes that each person ate." Ray felt that he had found the correct solution for the task.

Ray rated the difficulty level of the problem a 2 (out of 10) because he felt that it was not difficult. He stated that he did not find the correct answer immediately because he saw the fractions and got anxious and just did calculations. He explained, "I kept messing up because at first I saw the fractions and rushed to do the problem, and kept getting high numbers and I knew that the king could not eat that many mangoes." For example, he said that he got as far as 360 mangoes, and so stopped and did the problem over. He then decided to use the fraction representation, by using sets and computation, and was able to find the answer to the problem.

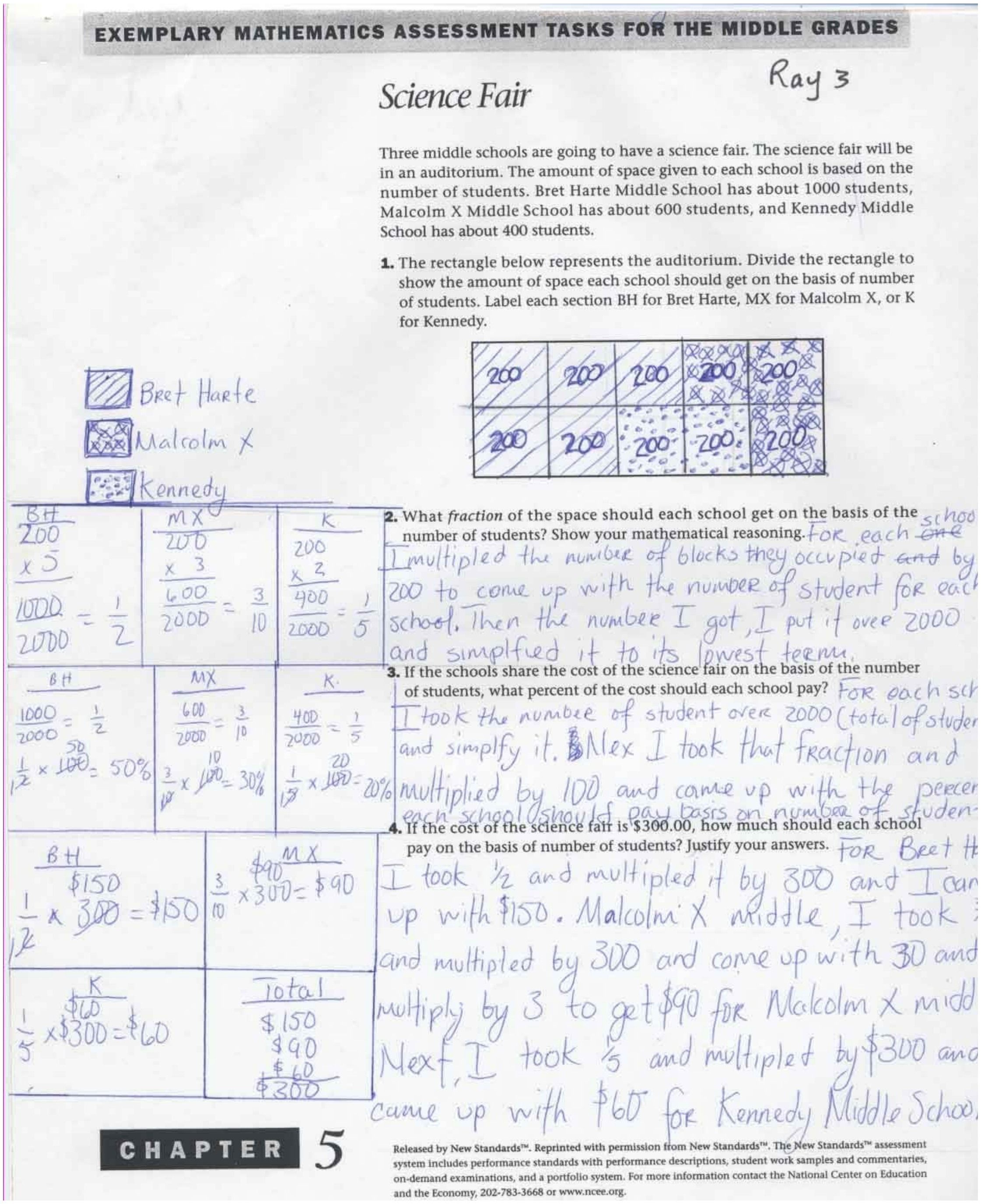
Ray's reflection on Performance Task 2. Ray's reflection notes indicated that he thought that he did very well on the task because it was very easy. He also felt confident about his work on this task because he saw himself as being good at using representations for figuring out his answers. Further, he stated that he felt that using representation helped him to find the correct answer. Ray rated the task a 6 (out of 10). He liked working and participating in the group discussion because he could discuss and compare his answer with the other students in his group.

My field notes on Performance Task 2. From my observation of Ray as he worked on this task, I could see that he was using only calculations. There was no visual sign of a diagram or any form of fraction representation. He scrapped his work at least twice and started over. He completed the task in the allotted time.

Results Regarding Performance Task 3

Ray partitioned correctly the diagram of the rectangle which represented the auditorium where the science fair would be held (see Figure 6). He divided the rectangle into 10 equal sections and wrote 200 in each of the sections. Ray explained in his interview that the 200 represented a fraction (1/10) of the total 2,000 students who would attend the science fair, and 1/10 of the whole rectangle. Ray then used the 200 to calculate the number of 200-sections that each school would have, based on the number of students that they would bring to the science fair and the fraction of the total students that the students in each school represented. All of Ray's answers were correct, and he explained and justified each, as was required. He used procedures and computations to arrive at his answers; his diagram was labeled to identify each school, and he also made a key to serve the same purpose. As is shown in Table 9, Ray scored 3.8 on this task.

Performance Task 3 interview. Ray stated that as he read the task he decided to divide the rectangle to give each school their equal amount of space, but he wondered how he would do this. He knew that the task required that he use the rectangle that represented the shape of the auditorium, and the information about the number of students that each school would bring to determine the space that each school would get. To do this, Ray said that he took an even number that could be divided into 2,000 evenly, 200. He stated that he divided 200 into 2,000, which was the total number of students who would go to the fair, and got 10. This 10 represented the number of sections in the rectangle, and each section had 200 students. Ray explained, "based on the information that was given to me, I shaded each box using the number of students that was given for each school and labeled it." He added that, for example, because Malcolm X Middle



EXEMPLARY MATHEMATICS ASSESSMENT TASKS FOR THE MIDDLE GRADES

Ray 3

Science Fair

Three middle schools are going to have a science fair. The science fair will be in an auditorium. The amount of space given to each school is based on the number of students. Bret Harte Middle School has about 1000 students, Malcolm X Middle School has about 600 students, and Kennedy Middle School has about 400 students.

1. The rectangle below represents the auditorium. Divide the rectangle to show the amount of space each school should get on the basis of number of students. Label each section BH for Bret Harte, MX for Malcolm X, or K for Kennedy.

200	200	200	200	200
200	200	200	200	200

Bret Harte
Malcolm X
Kennedy

BH	MX	K
200 × 5	200 × 3	200 × 2
$\frac{1000}{2000} = \frac{1}{2}$	$\frac{600}{2000} = \frac{3}{10}$	$\frac{400}{2000} = \frac{1}{5}$

2. What *fraction* of the space should each school get on the basis of the number of students? Show your mathematical reasoning. For each school I multipled the number of blocks they occupied by 200 to come up with the number of student for each school. Then the number I got, I put it over 2000 and simplfied it to its lowest term.

BH	MX	K.
$\frac{1000}{2000} = \frac{1}{2}$	$\frac{600}{2000} = \frac{3}{10}$	$\frac{400}{2000} = \frac{1}{5}$
$\frac{1}{2} \times 100 = 50\%$	$\frac{3}{10} \times 100 = 30\%$	$\frac{1}{5} \times 100 = 20\%$

3. If the schools share the cost of the science fair on the basis of the number of students, what percent of the cost should each school pay? For each school I took the number of student over 2000 (total of students) and simplfy it. Next I took that fraction and multiplied by 100 and came up with the percent each school should pay basis on number of students.

4. If the cost of the science fair is $300.00, how much should each school pay on the basis of number of students? Justify your answers. For Bret Harte I took ½ and multipled it by 300 and I came up with $150. Malcolm X Middle, I took 3/10 and multipled by 300 and come up with 30 and multiply by 3 to get $90 for Malcolm X middle. Next, I took 1/5 and multiplet by $300 and came up with $60 for Kennedy Middle School.

BH	MX
$\frac{1}{2} \times 300 = \$150$	$\frac{3}{10} \times 300 = \$90$

K	Total
$\frac{1}{5} \times \$300 = \$60$	$150 + $90 + $60 = $300

CHAPTER 5

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Figure 6. Ray’s work on *Science Fair*.

School was bringing 600 students, he shaded in three boxes of 200 students. He did this for the two other schools, using the information on the number of students that they would bring to the science fair.

I asked Ray to explain why he used 200, as there were several other even numbers that could also divide into 2,000 evenly, such as 100. He said, “Because I had to pick a number that was not too small and not too big.” My next question was, “Too small or too big compared to what?” Ray said that this was compared to the shape of the rectangle that he was using. I then said, “Let’s say you picked 100. One hundred goes into 2,000, twenty times. Could you have used 100 and have 20 boxes instead of 10?” Ray said, “Yes.” I then asked, “Then what made you pick 200 instead of another factor of 2,000?” Ray said that he did not know.

Next, I asked Ray to explain to me what he did to answer the question, which asked, “What fraction of the space did each school get?” He replied, “I took 200 which was the number of students for each box and I multiplied it by the number of boxes that I shaded for each school to come up with the numerator of the fraction, and I put it over 2000. After getting my fraction, I simplified it to its lowest terms.” He said that, for example, 600 over 2000, reduced to 3/10.

The focus of my questioning shifted to the use of fraction representation, and I began by asking Ray what, if any, fraction representation he used in doing this task. He said that he used shapes in the form of squares to represent the fraction of the space in the rectangle that each school would get. He used these shapes and the fractions that they represented to find the fraction of space for each school and to calculate the amount of money that each school had to pay for space that they occupied in the auditorium.

Next, Ray and I discussed what he liked about the task. He stated that he liked having to divide the rectangle to show how much space each school would get, because seeing a diagram of what he was doing helped him to solve the problem. He added that

working on this type of task could be easy or difficult, depending on how you look at it, but that they are all challenging to him, and they helped him to understand fractions better than he did in the past. I asked Ray to explain what he meant by this. He explained, "It helps me because using fraction representaions in the problem to come up with my answer and looking at it and the diagrams help me to know what to do."

Next, I asked Ray how he thought that he would approach a fraction problem in the future. He said that because he would be used to doing it as he does it in class, he would do it the same way. I then asked, "So, if I were to say multiply 1/2 by 3/4 do you think that you would use fraction representation to do this type of problem?" Ray answered in the affirmative. I questioned him as to why he would want to do so. His reply was, "because I think that if I can see it, I can do the problem better." To end the interview, I asked Ray to rate the task in terms of its level of difficulty. He rated it a 7 (out of 10) saying that though he knew how to do it, at first, he did not know how to put the data in the diagram, but after he found a way to do it, he was comfortable that his answers were correct.

Ray's reflections on Performance Task 3. Ray indicated via his reflection that the *Science Fair* task was harder than the others were. He stated that he liked using different types of fraction representations to find his answers, and that this helped him to find his answers quicker. Ray also said that this task helped him to learn more about fractions and fraction representations. He also liked the fact that when he discussed the task with his group members, each person had a different answer, so they all shared their ideas and opinions about how to do the task, and what was the correct answer. Ray rated the task 7

(out of 10) for its level of difficulty because he had problems putting the data together in the problem.

My field notes on Performance Task 3. I watched Ray, as he partitioned the rectangle and scrapped his work several times. He seemed to be very occupied with partitioning the rectangle to perfection. As the class progressed, I also observed that he was doing computations. As he worked on his final draft, he had meticulously completed the process of partitioning the rectangle and identifying each section.

Comparison of the Three Participants

Ability to Compute and Conceptualize Fractions

As was seen in the results, Allan's work for all of the related data sources indicated that he computes fractions with a high measure of ability. Liza's work indicated that she also computes fractions with a high measure of ability, although on the pretests she made careless errors and did not obtain as high a grade as did Allan. Ray's work from all of the data sources indicated that he computes fractions with an average measure of ability and that his level of confidence about working with fractions is higher than his actual performance. The majority of his mistakes on the fraction computation pretest were also attributed to the careless errors that he made, although most of them originated from his work with mixed numbers. He indicated that fraction rules and procedures confuse him, and he does not remember when to apply certain rules and procedures.

In their work on the performance tasks, Allan and Liza computed all fractions correctly and chose and applied suitable procedures for performing fraction computations. This was particularly evident when they worked on the first performance-based task, *Pizza!* Ray, on the other hand, performed very basic fraction computations to find

answers to the questions on this task, some of which involved the incorrect application of fraction rules, which resulted in incorrect answers. In *The Mangoes Problem*, all of the fractions with which the participants had to work were unit fractions. Allan and Liza performed correct computations with these fractions, though Allan did not find an answer to the question in the task. However, Ray's computations for this task involved whole numbers rather than fractions. For the third task, *Science Fair,* each of the three participants performed correctly computations involving fractions and percents and found the correct answers.

In terms of conceptualizing fractions, all three of the participants indicated through their work that they do not conceptualize fractions as well as they compute them. Beginning with their work on the Fraction Concepts pretest, Allan, Liza, and Ray all failed to indicate a satisfactory measure of ability to conceptualize fractions. None of them obtained a passing score or met the criterion of 75% or more. Further, their measure of ability to conceptualize fractions varied according to the various performance tasks that they completed.

First, in terms of the three participants' work done on the *Pizza!* task, Liza's work indicated the highest measure of ability to conceptualize fractions, but this was still not done at a very high level. Allan's measure of ability to conceptualize fractions on this task was slightly less than Liza's, whereas Ray's work on this task indicated that he demonstrated a very small measure of ability to conceptualize fractions. In terms of *The Mangoes Problem*, Liza's work indicated a high measure of fraction conceptualization, Allan's indicated a very low measure of ability, and Ray's could not be accurately judged because in his work and in his explanations, he did not use any fractions, using instead

the denominators of the given fractions to multiply them as whole numbers. On the *Science Fair* task, each participant's work indicated a high measure of ability to conceptualize fractions.

Ways of Using Fraction Representation

When working on the *Pizza!* task, each participant recognized that the most appropriate fraction representation for modeling a pizza and its fractional parts was a circular shape and used this to work on the task. However, none of the three used this representation primarily to arrive at an answer for the question posed in the problem. Allan and Liza used calculations to find their answers, although Allan stated that to compare the fractions that were represented by the parts in the circle, he looked at it and realized what his answer should be. Further, none of the three participants partitioned correctly the circles that he or she drew to represent the first pizza that the Lorenzo and Jessica shared. However, Ray's perfectly partitioned circle indicated that that box was missing three slices and that the friends would share 1/2 of 3/4 of the box, rather than 1/2 of the whole box. Liza, on the other hand, used a rectangle that she partitioned into 24 equal parts to compare 20/24 to 9/24 and to indicate that she could use this fraction representation to compare the two fractions rather than to apply the rules and computations for doing so. In doing so, Liza demonstrated that she could use fraction representations in many ways to work on the task.

For *The Mangoes Problem,* Allan did not demonstrate his ability to use fraction representation, while Liza did much better by using representation in the form of a set. In addition, Ray also indicated that he used a set, but it was hard to follow or determine if the diagram that he drew represented a set and if he used it to find an answer to the

question or drew it after he found his answer via computations/calculations. For the *Science Fair* task, all three of the participants used the given diagram to create representations for fractions and did so correctly. However, Ray's use of squares to create fractional sections of the given rectangle was unique and demonstrated his ability to use fraction representation in a variety of ways. In addition, all three of the participants also correctly used decimal fractions and percents to represent the fractions and to answer the questions in the task.

Summary

In this chapter, I presented the results of the study based on the data obtained from a variety of data sources. The chapter began with a discussion of the themes, which emerged from the data analysis, and this was followed by a description of each participant. The next section presented a case study on each of the three participants and discussed the results of the pretests, questionnaire, performance tasks, and interviews, as well as the data from the participants' individual reflections logs, and participant-observer's field notes. In the last section, the results regarding the three participants were compared for similarities and differences based on their ability to compute and conceptualize fractions, and on the ways in which they use fraction representations to find solutions to the three performance-based tasks. Chapter 5 presents my conclusions and discussion of the findings of the study.

CHAPTER 5

CONCLUSIONS AND RECOMMENDATIONS

This final chapter of the dissertation, in general, restates the research problem and reviews the major methods used in the study. In addition, the major sections of this chapter summarize the results and discuss their implications.

Statement of the Problem

In general, in the United States, a large number of public school students and particularly urban African American students who attend public middle schools are not performing at acceptable levels in mathematics. One contributing factor to these students' unsatisfactory performance levels in mathematics is their difficulty with learning and mastering fractions. The literature on the teaching and learning of mathematics has indicated that the concept of fractions is an area of mathematics that poses significant difficulty for a large number of middle school students and affects negatively their overall performance and achievement in mathematics (Saxe et al., 2005; Test & Ellis, 2005). The literature on the teaching and learning of fractions has attributed student difficulty with this concept to traditional fraction instruction that emphasizes the memorization and application of rules and procedures for primarily performing computations rather than emphasizing conceptual knowledge and understanding (Groff, 1994, 1996; Lamon, 1996; Moss & Case, 1999; Post et al., 1992; Wearne & Kouba, 2000). The information in the next section offers an insight into the factors that influenced the study.

Based on the suggestions of concerned citizens, politicians, and educators and in an attempt to facilitate student improvement in mathematics in the United States, national mathematics organizations such as the National Council of Teachers of Mathematics (1989, 1995, 2000) and the National Center of Educational Statistics (TIMSS, 1999) have called on educators nationwide to implement mathematics education reform programs in their school districts. Members of these organizations believed that if instructional adjustments were made to facilitate more effective mathematics instruction, particularly for difficult mathematic concepts such as fractions, this would ultimately facilitate improvement in student performance and achievement in this subject at all levels.

Review of the Methodology

As was explained in Chapter 3, this qualitative study, which used a case study strategy of inquiry, took place in an urban middle school in a southeastern U.S. state. It initially involved 37, 8th-grade, prealgebra students who completed a fraction interest questionnaire, a fraction computation pretest, and a fraction concepts pretest before the start of the study. Three of these students were selected as the participants (each representing a case) whose work would be analyzed during the data analysis phase of the study.

The purpose of the study was to determine the extent to which the participants could compute and conceptualize fractions and the ways in which they use fraction representations to communicate their mathematical thinking and reasoning as they worked on finding solutions to three fraction-related, performance-based tasks. Scoring rubrics were used to assess the students’ solutions to the performance tasks.

The study was implemented in the spring, during the last week of April for approximately 3 weeks. During this time, the participants completed three fraction-related, performance-based tasks, participated in 3 one-on-one interviews, and completed individual and group reflection logs as a part of the data collection process. Several other types of qualitative data collection sources were used (participant observations, field notes from observations, and artifacts in the form of student work) to facilitate triangulation of the data.

Using performance-based tasks to assess the extent to which the participants' could compute and conceptualize fractions and the ways in which they used fraction representations to organize and communicate their mathematical thinking was a nontraditional form of assessment and instructional tool. This was consistent with the state's soon-to-be-implemented new mathematics curriculum and performance standards that stress the use of performance-based tasks as an instructional and assessment tool for enhancing student learning, improvement, and achievement in mathematics in the middle grades.

Summary of the Results

The first of the two research questions that guided this study sought to determine the extent to which each of the participants could compute and conceptualize fractions. Each of the three participants engaged in performing computations with fractions from several domains as they found solutions to each of the three performance tasks.

To use a numerical value that could be translated to a verbal phrase that would best describe the extent to which each participant could compute fractions, I used the school system's grading scale as a guide and decided on the following ranges of percents:

(a) 90 – 100%, which represented a very large extent; (b) 70 – 89%, which represented a large extent; and (c) 0 – 69%, which represented a small extent. These percentages were computed based on the number of fraction computation skills that each participant performed correctly relative to the total number of skills that each of the performance tasks assessed. Extra points were also added for partially correct answers or computations.

To decide on the extent to which each participant could conceptualize fractions I closely examined instances in each task where the participants could demonstrate and use their conceptual knowledge based on their reasoning, logic and explanations of fraction concepts. The extent to which each participant could compute and conceptualize fractions, along with the relevant details relative to their demonstrated skills in these areas, are as follows:

1. Liza can compute fractions to a very large extent (96%), and can conceptualize fractions to a small extent.
2. Allan can compute fractions to a large extent (72%) and can conceptualize fractions to a small extent.
3. Ray can compute fractions to a large extent (87%) and can conceptualize fractions to small extent.

The fraction computation skills that Performance Task 1 (*Pizza!*) assessed were (a) partitioning (equal-sized), (b) adding and subtracting simple, like and unlike fractions, (c) multiplying simple fractions, (d) creating equivalent fractions, (e) reducing fractions, and (f) comparing fractions. The skills that Performance Task 2 (*The Mangoes Problem*) assessed were (a) adding and subtracting simple, like fractions, (b) multiplying a fraction by a whole number, and (c) reducing fractions. For Performance Task 3 (*Science Fair*),

the skills assessed were (a) partitioning (unequal-sized), (b) multiplying a fraction by a whole number, (c) reducing fractions, (d) comparing fractions, and (e) changing fractions to percents.

Extent of the Participants' Computation Skills

Performance Task 1

Liza. Liza's work on this task indicated that she performed the fraction computations in the task with an 89% level of success. Relative to Research Question 1, Liza could compute fractions to a large extent. Out of the six skills assessed, Liza's work indicated that she has mastered five skills and had a partially correct answer relative to the sixth skill. She was unable to demonstrate mastery of the equal-sized partitioning skill relative to one of the two circles that represented the two pizzas that the friends shared. Liza's decision as to the question regarding which pair of friends ate more pizza (and how much more) and all of the accompanying calculations were correct.

Allan. Allan's work on this task indicated that he performed the fraction computations with an 83% level of success. Therefore, with regard to Research Question 1, Allan could compute fractions to a large extent. Allan demonstrated that he was able to master five of the six skills assessed, but his partitioning of the two circles that represented the pizzas was incorrect, and he also did not reduce the fraction (9/12) that represented the part of the second pizza before he used it in his computations. Nevertheless, Allan's final answer, based on his decision regarding which of the two pairs of friends ate more pizza and how much more, was correct.

Ray. Ray's work on this task indicated that he performed the fraction computations with a 60% level of success. Therefore, with regard to Research Question 1,

Ray could compute fractions only to a small extent. His major weaknesses with computing fractions were related to partitioning one of the two circles that represented a pizza and selecting and using an incorrect procedure to perform computations.

Performance Task 2

Liza. Liza's work on *The Mangoes Problem* indicated that she computed the fractions related to this task with a 100% level of success. With regard to Research Question 1, this shows that she could compute fractions to a very large extent. All of Liza's computations were correct, and this led to a correct answer to the problem given in the task.

Allan. For this task, Allan performed only one type of fraction computation (multiplication a fraction by a whole number) and did this correctly. However, this did not lead to his finding the correct solution for the task and resulted in a 33% level of success with computing the fractions in this task. Therefore, with regard to Research Question 1, this indicates that Allan could compute fractions to only a small extent.

Ray. In his attempt to find a solution for this task, Ray used an approach that did not require the use of primarily fraction computations, but the procedures and the computations that he used led to his answer being correct relative to the question posed in the task. Therefore, with respect to Research Question 1, Ray indicated that he could compute fractions to a very large extent.

Performance Task 3

Liza. For the *Science Fair* task, Liza performed correctly each of the fraction computations that she did to find her answers to the questions in the task. This 100%

level of mastery with the skills that this task assessed indicates that with regard to Research Question 1, Liza could compute fractions to a very large extent.

Allan. For this task, Allan performed correctly each of the fraction computations that he did to find the answers to the questions in the task. Allan's 100% mastery of the skills which this task assessed indicates that regard to Research Question 1, he could compute fractions to a very large extent.

Ray. For this task, Ray's computations relative to the skills that this task assessed, earned him a score of 100% for finding the correct answers to all of the questions posed in the task. This indicated that with regard to Research Question 1, he could compute fractions to a very large extent.

Extent of the Participants' Conceptualization Skills

Performance Task 1

With regard to their ability to conceptualize fractions, each of the participants demonstrated their ability to do so with varying levels of success, which ranged from a small extent to a very large extent, relative to each of the three performance-based tasks.

Liza. Liza's work on this task indicated that she relied on her strong fraction computation skills to solve this task and therefore demonstrated her ability to conceptualize fractions only to a small extent. For example, Liza did not seem to be able to use her understanding of the concept of fractions to visualize from the fraction representations that she drew that she had shaded a larger portion of the circle to represent the total portion of the pizza that the first two students ate and that this represented the larger of the two fractions. Rather, Liza determined which fraction was larger by renaming and comparing them. Also, Liza did not seem to have recognized that

she could have used the numerators of the two fractions to compare them based on the fact that they were similar fractions.

Allan. Allan's work on this task demonstrated that he could conceptualize fractions to a small extent based on the fact that to find a solution to the task he used primarily his knowledge of computing fractions. Though the problem stated that the two pizzas were 12-slice ones, Allan did not use this information or his understanding of what it meant to begin the process of finding an answer to the question posed in the task. On the other hand, Allan used the visual representation of the model that he created to decide and offer a correct explanation as to which pair of students ate more pizza.

Ray. Ray demonstrated his conceptualization skills to a large extent when he worked on this task. For example, he recognized that he could begin his work on the second part of the task by "removing" 1/4 of the pizza from the model of the second pizza to show that three slices were missing. His work also indicated that he used his understanding of equal-sized partitioning to draw a realistic model of the fraction of the pizza that the second pair of friends shared.

Performance Task 2

Liza. Liza's work, reasoning, and explanations relative to this task, particularly the fact that she identified a pattern and used this to find the correct solution to the task indicated that she applied her conceptualization skills to a large extent.

Allan. Allan's effort at finding a solution to this task was based on primarily a strategy of guess-and-check in conjunction with fraction computations, but neither resulted in his finding a correct solution to the task. This indicated that Allan did not use any conceptualization skills in his attempt to find the solution to the task.

Ray. For this task, Ray's correct solution indicated that he had a firm understanding of the problem and what it was asking him to do. However, though he used the fractions given in the problem to find the correct answer, his work, reasoning and explanations were hard to follow and showed no visual proof of how he applied his knowledge and understanding of fractions to solve it. Therefore he demonstrated that he could only conceptualize fractions to a very small extent.

Performance Task 3

Liza. Liza's work for this task and in particular her partitioning of the rectangle which represented the gymnasium where the science fair would be held, indicated that she applied successfully her knowledge and understanding of fractions and their relation to percents to find the answers to all of the questions posed in the task. Therefore, she demonstrated that she could conceptualize fractions to a very large extent.

Allan. Allan's work on this task indicated that he had a clear understanding of the fraction concepts that were involved in this task. For example he used his understanding of fractions to partition the rectangle correctly to show the amount of space that should be allocated to each school based on the information given and to support his decisions. Allan's work on this task indicated that he could conceptualize fractions to a very large extent.

Ray. Ray's strategy and approach for partitioning the rectangle to allocate space for each of the three schools that would attend the science fair indicated that he clearly understood the fraction concepts associated with this part of the task. As such, he demonstrated to a very large extent that he could conceptualize fractions.

Participants' Use of Fraction Representations to Solve Tasks

The data pertaining to the ways in which the participants used fraction representations to communicate and organize their mathematical thinking and reasoning, indicated that they used them to (a) organize/set up their work, (b) obtain a visual of the task, (c) check the accuracy of their work, and (d) get a clear picture of task. Further, the participants' decisions to use fraction representations to solve the tasks were based on their judgments regarding the level of difficulty and the clarity of the task and whether or not they had an adequate amount of time to do so.

Liza

The data obtained from Liza's work showed that she used fraction representations primarily to check the accuracy of her computations, which she did first when solving each task. In addition, Liza chose to use representations when the problem was not very clear and was difficult to understand. In each instance, Liza chose an appropriate type of fraction representation based on the type of fraction problem that was embedded in the task. Liza used circles to represent the two pizzas in the first task and then sets to compare two fractions. She also used sets for *The Mangoes Problem* but only to provide a visual in support of her explanation regarding how she arrived at her answer. For the third task, *Science Fair*, Liza partitioned the given rectangle correctly to show the fractional representations of the space that would be allotted to each of the three schools based on the number of their students who would attend.

Allan

Allan's work indicated that he only used fraction representations if the directions indicated that pictures or diagrams could be used or if the task were a difficult one. In

addition, he used fraction representations as an alternative to doing computations only when the latter did not result in an answer. Further, he regarded using fraction representations as a waste of time if the fraction problem was easy to understand and solve using computations. Allan's use of fraction representations was very limited when he worked on the tasks. For the *Pizza!* task, Allan chose the appropriate representation of two circles to represent the two pizzas that the friends shared. He did not use any fraction representations when he worked on *The Mangoes Problem*, despite the fact that he was unable to find a correct solution to the task via his performing computations. For the *Science Fair* task, Allan recognized that the partitions that he created using the rectangle which represented the gymnasium and the given information about the students who would attend the science fair were representations of fractions.

Ray

Ray preferred to use different types of fraction representations for each task and did so to obtain a visual representation of the task and a better, clearer understanding of the details of the problem in the task. He also used fraction representations to set up and organize his work. Ray's consistent use of fraction representations to organize his work indicated that he was very proficient with using them and could select those that were appropriate for the task, which he was trying to solve. He also used fraction representations as an extension of his artistic talent and used this to his advantage to find creative ways of "illustrating" the details of the fraction problems in the tasks.

Discussion

The Participants' Extents of Computing and Conceptualizing Fractions

On the basis of the data obtained from this study alone, it is difficult to be certain about all of the factors which contribute to middle school students' difficulties with learning, understanding, and working with fractions. However, I can pinpoint certain factors which played a role in the participants' difficulties with working with fractions and performance-based tasks during the study. These factors include their (a) lack of conceptual knowledge and understanding of fractions, (b) reliance on rules and procedures for performing computations, and (c) reluctance to focus on the use of fraction representations when working with fractions. In general, the indication is that there is more room for improvement relative to proficiency with fractions in the urban, 8th-grade, middle school student population. The participants in the study could work with and problem solve with fractions by performing, primarily, basic fraction computations. However, they did so without fully understanding the underlying concepts, why the procedures that they used worked, and when to use and apply the appropriate procedures and computations.

These findings validate those of several researchers, including (a) Wearne and Kouba (2000), who studied the data from several NAEPs and found that middle school students have a weak conceptual understanding of fraction concepts; (b) Mullis et al. (1990), who concluded, also based on NAEP data from several studies, that fractions are difficult for students to master and that students do not learn much about these concepts as they move from one grade level to the next; (c) Test and Ellis (2005), who studied 8th-grade, middle school students and concluded that one of the hardest mathematical skills

for students to learn is fractions, and (d) Peck and Jencks (1981) and Hasemann (1981), whose studies on the teaching and learning of fractions led to the conclusion that middle school students' lack of conceptual knowledge of fractions was the primary source of their difficulties with learning this concept.

Furthermore, my findings indicate not only that participants used computations based on rules and procedures, but also that they did so even when this was not necessary. For example, in the first part of *Pizza!*, partitioning the circle correctly and shading the fractional parts using the information given to determine the fraction of the pizza that the first two students ate and applying a conceptual knowledge and understanding of the concept of partitioning would have resulted in the answer without having to do any computations. Instead, the participants did not realize this and therefore used computations to find their answers. This approach resulted in their making calculation errors and supported the statement made by the NCTM (2000) that, in the middle grades, students need to be able to conceptualize fractions as quantities (i.e., parts of a whole) and to understand how to use them as rates, ratios, and operators.

Another finding of the study was that the participants did not possess a satisfactory level of conceptualization skills. For example, when they worked on the *Pizza!* task, only Ray displayed through his diagrams to a satisfactory extent that he understood what the concept of equal-sized partitioning (fair sharing) meant. Liza and Allan disregarded the meaning of this concept and partitioned the two circles that represented the two pizzas in ways that suited what they were trying to do through computations rather than by applying their knowledge of the concept of equal-sized partitioning and ensuring that the circles were divided into twelve equal-sized parts.

Further, Allan partitioned one of the circles into 3 unequal sectors but still named each part as thirds. Liza partitioned one circle into 11 (for the most part, unequal) sectors instead of 12 but named each sector twelfths. Both participants did not seem to have examined these fractions or connected their sizes to their names, but used them to perform computations to find their answers even though their representations of these fractional parts and their calculations did not match.

Ironically, despite the participants' unsatisfactory level of conceptual knowledge and understanding of fractions and the difficulties that they experienced when working with fractions, they described working with fractions in the task to be easy and the related tasks the same. This was the case even when they performed incorrect computations which resulted in incorrect solutions to the tasks and reflects one of the themes which emerged from the data analysis process, *participants' ratings of the tasks versus the scores obtained for the tasks*.

Also, in order to perform computations, one of the participants, Ray, disregarded the fact that he knew that the word *of* indicated that he should multiply but subtracted instead. He used the information which stated that Jessica ate 3/4 of what was left (2/3) and subtracted to come up with an incorrect answer. He did not seem to have realized that he had applied the incorrect procedure. However, in the subsequent interview that was related to this task, Ray stated that he knew that *of* meant that he should multiply but was confused by the problem. This reflects another of the themes that emerged from the analysis of the data, *recognizing connections between keywords* (such as "of") *and fraction procedures*. The indication is that Ray's confusion may have resulted because he was used to doing primarily fraction computations without context when working with

fractions. Because the fraction computations he was working on in the task were a part of a more complex problem which required him not only to compute fractions but also to apply his conceptual knowledge of fractions and to problem solve simultaneously, working out of context was problematic for him.

Additionally, none of the participants demonstrated in their work that 1/2 of something means the same as dividing by 2 and that this was an alternative to multiplying by 1/2. The information in the second part of the pizza problem stated, "Naquan and Derasha ate a total of 1/2 of their meat pizza." Using this information may have proved to be easier for visualizing this, and finding the answer, for at least one participant, Ray, who instead of multiplying by 1/2 or dividing by 2, erroneously subtracted 1/2. This participant's conceptual knowledge and understanding of fractions should have played a role in his recognizing this fact and thus using this information to draw a representation of (1/2 of 9) or at least selecting the correct procedure for finding the answer.

Another indication of the participants' lack of conceptual understanding of fractions is the fact that two of them performed computations to subtract 1/3 from 1 (one whole pizza). This should have required no computations but the application of their knowledge and understanding of the concept of creating fractions such as, "there are three thirds in one whole," and "taking away one third, leaves two thirds." As far back as two decades, Cohors-Fresenburg (1979) and Bigalke & Haseman (1978) reported similar findings in studies on fraction learning and suggested that student difficulty in this area was often tied to their lack of conceptual knowledge of fractions. Unfortunately, the findings of this study indicate that this problem still exists, despite the ensuing years,

numerous subsequent studies, and mathematics reform which included adjustments in fraction instruction.

Another finding of the study was that the participants viewed a performance task as a “long” word problem and assumed that it was difficult based on this preconceived notion about word problems. Therefore, initially they became somewhat intimidated, and this resulted in their initial difficulties with working with the fractions embedded in the tasks even though they could perform the necessary computations, had demonstrated this on the pretests, and had stated in the questionnaire and repeated in the interviews that they were very confident about their ability to work with fractions, a theme that emerged from the data.

Further and interestingly, during the implementation of each task the participants frequently rated the tasks as being easy. More often than not they based their judgments on their confidence to compute correctly the fractions in the task and to do so efficiently as opposed to using fraction representation. This was despite the fact that at times their solutions were totally or partially incorrect. This reflects two themes that emerged from the data via the process of analysis, *participants' rating of the tasks versus the scores that they obtained for the tasks* and *efficiency of doing fraction computations versus using representations.*

Ways in Which the Participants Used Fraction Representations

Each of the three participants used fraction representation in different ways and held varying perspectives regarding their use and usefulness, a reflection of two emergent themes: *the value of representations* and *things that influenced the participants' decision to use fraction representation*. However, because, for the most part, all of the participants

were confident about their ability to work with fractions, they did not realize that working with fractions in a successful manner included being able not only to perform computations but also to conceptualize fractions, such as having a clear understanding of what the idea of fractions and the underlying concepts mean why the procedures they used worked. As a result, when they worked with the fractions in the task, they did not automatically think of using fraction representations but focused on using the standard notation and doing computations, a reflection of one of the emergent themes, *efficiency of doing computations versus using fraction representations*.

In general, the participants saw the use of fraction representations as beneficial but only when their efforts to determine an answer using computations did not yield the intended results or when finding a solution to the task proved to be more difficult than they anticipated. This reflects the emergent theme, *things that influenced the participants' decision to use fraction representations*. Though the participants seemed to understand clearly that there was more than one way of representing a fraction and that using these representations could help them to work with, and successfully solve fraction problems or fraction-related tasks, they were not confident that using representations could help them to find their answers. A case in point is participant Allan, who can compute fractions very well and is very confident about his ability to do so. When he was experiencing difficulties with finding the correct solution to *The Mangoes Problem,* he did not even consider trying to find another way of representing the fractions in the problem to lessen his difficulties. Instead he continued to do computations in the hope that he would find the correct answer. This is a reflection of the emergent theme, *participants' confidence about working with fractions*. In several instances during the

interview related to this task and on his reflection log, Allan expressed varying opinions about using fraction representations. He stated that they (a) were beneficial, (b) helped to make the problem clearer, and (c) were a waste of time.

In contrast, participant Liza attempted to find solutions to all of the tasks by performing computations first, then using fraction representations to check the accuracy of her answers. Liza was very confident about her ability to compute fractions (a reflection of emergent theme, *efficiency of doing computations versus using fraction representations*) but still felt the need to double-check her work with fraction models such as shapes and sets. This was interesting because though Liza knew the value of these representations (a reflection of an emergent theme, *value of representations*), she did not use them to begin the process of finding a solution to any of the tasks. On the other hand, only participant Ray who, in general, experienced difficulties with computing fractions and stated that the numerous rules and procedures confused him consistently used fraction representations to visualize the task, organize his work and work with the fractions in the tasks. He relied on these representations to find solutions to the tasks, but he used computations when he found it necessary to do so. However, his computations were frequently incorrect.

Theory Revisited

In a broad sense, constructivism was used to undergird this study. This paradigm assumes that knowledge and reality are socially constructed by people active in the research process and that researchers should attempt the complex world of lived experience from the point of view of those who live in it (Schwandt, 2000). This

paradigm also emphasizes that research is a product of the values of researchers and cannot be independent of them (Mertens, 2005).

Constructivist thinking is defined by its ontology, epistemology and methodology and each of these tenets of this paradigm was reflected in this study. One of my goals of this study was to understand the multiple social constructions of the meaning and knowledge which resulted from the participants' work relative to fraction learning, such as their constructs of the difficulty levels of the tasks and their ability to work with the fractions in the tasks. From an ontological perspective, this was in keeping with the notion that reality is objective and can be known (Mertens, 2005; Schwandt, 2000).

From an epistemological perspective, my role as a participant-observer in the study facilitated my personal interaction with the participants in the natural setting of the classroom as I engaged in data collection. I interacted with the participants while administering the performance-based tasks, during the individual interviews, while they worked on the tasks individually and as they discussed their strategies and solutions in small groups. Additionally, at the completion of the first task, *Pizza!,* the participants were uncertain about what to write for the reflection portion of the data collection process and shared these concerns with me. Therefore, I used this information to create a reflection log that contained prompts, and this made it much easier for the participants to complete their individual and group reflection logs. Further, when I presented the results of the study, I provided excerpts of direct quotations from the participants to support the meanings that I drew from the various sources of data.

From a methodological perspective, the methods that I employed in this qualitative study is in line with the constructivist assumption that the social construction

of reality in research study can be conducted only through interaction between and among the investigator and the respondents in a hermeneutical and dialectical manner (Lincoln & Guba, 2000). As such, I made efforts to obtain multiple perspectives in order to provide me with better interpretations of meanings of the data which I collected from multiple sources and which I constantly compared to facilitate triangulation. This helped me to construct the "reality" (Mertens, 2005) based on my interpretations of the data in conjunction with the participants in the study who provided the data.

In a more specific sense, this study was grounded on the tenets of social constructivism, a branch of constructivism, which recognizes that knowledge is constructed by the individual and is also concurrently socially constructed. Social constructivism is based on the philosophy of Vygotsky (1978), a social constructivist who believed that in a group the individual learns from social interactions first, and from individual experiences, later, and that the purpose of education was to develop the personalities of the students which rests on the discovery and expression of their creative potential.

During the study, the participants worked on each task individually first and then in small groups. They shared and discussed their solutions to the tasks and the strategies they used and decided on the most practical solution to the task, as a group. In addition, they discussed the prompts on the groups' reflection log and used the consensus of the group to complete the log. During these activities, I interacted with the participants and their classmates, and answered their questions when necessary. These classroom activities reflected the tenets of social constructivism and in particular the social constructivist assumption that these social and individual processes are interactive with

the result that groups act to construct knowledge and to resolve differences in the meanings of individual group members (Simon & Schifter, 1991).

During the study, my role became one of guiding and directing the activities without force or without imposing my ideas on the participants and the other students This was based on Vygotsky's (1978) position that methods of teaching and learning should (a) relate to the development of students as individuals and as members of a group in which they actively participate and (b) facilitate communication and collaboration between teachers and students and between the students themselves. Further, as a result of being able to participate in the group discussions, each of the participants indicated via their reflection logs and interviews that they liked working with their peers in groups. They stated that they benefited from the sharing of ideas and suggestions relative to the strategies and approaches used for completing the tasks and felt good about being able to offer their personal contributions to the groups. They also indicated that they liked having a responsible role to play, such as "group manager" or "group recorder." More importantly, they indicated that they appreciated learning from each other via the sharing of ideas about how to solve the task, identifying their errors, and learning about the different approaches that their classmates used to solve the tasks.

For example, Ray stated that he enjoyed working with the group and being selected as his group's recorder. This, he said, helped to boost his self-confidence and self-esteem and he was made to feel truly a part of his class. Allan and Liza stated that their groups helped them to see that though they were strong with computing fractions there were other ways of working with fractions such as drawing shapes, diagrams and pictures. The participants' and their classmates' reactions to the group experience when

they worked on the tasks support Boykin's (1986), Hilliard's (1995), and Ladson-Billings's (1997) suggestions about the type of classroom learning environment that is best suited for and meets the needs of African American learners.

Recommendations for Practice

While a single multiple-case study cannot provide a sound basis for generating or suggesting all of the answers to questions regarding the unsatisfactory mathematics achievement levels among urban, 8th-grade, middle school students and specifically their difficulties with learning and understanding fractions, this study (and other studies with similar findings) would suggest that these students can improve in terms of their ability to compute and conceptualize fractions and to use and feel much more confident about the benefits and effectiveness of using fraction representations to work with fractions. To accomplish this, fraction instruction, as researchers have suggested (Aksu, 1997; Moss & Case, 1999; NCTM, 2000; Simon, 1995), should emphasize conceptual knowledge and understanding and use reform-based practices. As has been espoused by supporters of the constructivist perspective on how students learn mathematics, these instructional practices should allow students to be actively involved in their learning by providing opportunities for them to work in small groups, share ideas, engage in discussion and interaction, and reflect on their learning with their classmates. This would facilitate alignment with some of the suggestions regarding meeting the needs of African American students in the classroom such as those proposed by Boykin (1986), Hilliard (1995), and Secada (1995).

I also recommend that fraction instruction in the elementary levels should place emphasis on and consistently reflect conceptual knowledge and understanding so that

these students are prepared adequately by middle school to work with fractions from a more conceptual standpoint. Based on the findings of several researchers (Empson, 1995; Hunting, 1983; Hunting et al., 1993; Leinhardt; 1988; Mack, 1995), elementary students bring a rich store of informal knowledge to the classroom. Therefore, this knowledge can be tapped in a positive way to teach fraction concepts and to help them to give meaning to symbolic representations to facilitate their success with working with fractions. A higher level of readiness and proficiency with fractions (and mathematics in general) at the elementary level than is currently the case can help these students to have a smoother transition to middle school mathematics and beyond as they work with fractions, other forms of rationals, and fraction-related concepts such as rates, slope, and algebraic rationals.

My final recommendation is that mathematics educators introduce their students to a variety of representations when learning mathematics concepts, particularly fractions, and encourage their students to use them on a consistent basis. According to the NCTM (2000), representation is a process or product that is used in the act of capturing a mathematical concept or relationship in some form, as well as the form itself; these forms include diagrams, graphical displays, and symbolic expressions. I believe that student familiarity with using representations in the mathematics classroom will help them to experience their benefits and could lead them to a higher level of mathematics proficiency. And because the NCTM's goals for the representation standard cover prekindergarten through grade 12 and seek to enable these students to (a) create and use representation to organize, record, and communicate mathematical ideas; (b) select, apply, and translate among mathematical representations to solve problems; and (c) use

representations to model or interpret physical, social and mathematical phenomena, I would suggest that representations be introduced to students as soon as possible in the lower grades so that they become more proficient relative to conceptual knowledge and understanding, rather than relative to performing computations using standard-notations, rules, and procedures. This I believe will aid in helping to move students in the right direction (towards improved mathematics achievement and performance) as they work with difficult mathematics concepts such as fractions.

Summary

The purpose of this qualitative study, which used a case study method of inquiry, was to determine the extent to which urban 8th-grade, middle school students could compute and conceptualize fractions when working on fraction-related performance-based tasks and the ways in which these students use fraction representations to organize and communicate their mathematical thinking and reasoning when finding solutions to fraction-related performance-based tasks.

The results and conclusions drawn from this study may inform researchers about urban, 8th-grade, middle school students' performance and difficulties relative to their fraction computation and conceptualization skills and their use of fraction representations when working with fractions in their mathematics classes. Further, the results may also serve to indicate the readiness and preparedness levels of the mathematics students who will be subjected to the rigors of the soon-to-be released curriculum performance standards that will be implemented in the state where this study was implemented as a part of its current mathematics education reform initiatives. These reforms will require educators to emphasize task-oriented instructional activities and to use performance-

based tasks as an instructional and assessment tool for teaching mathematics concepts in classrooms at all levels but particularly in the early middle grades.

The participants in the study were excited about being able to participate in the performance-based tasks during the study. However, because of their difficulties with mathematics and with fractions, their focus was not entirely on finding the most logical solutions to the tasks, but with performing the fraction computations with the fractions in the task, even though this is not always necessary for finding solutions to the tasks. Therefore, this seems to suggest that if middle school mathematics students are consistently exposed to these types of instructional activities and particularly those that allow them to be actively engaged in their mathematics learning, they could benefit more in terms of their conceptual knowledge and understanding of fractions and subsequently improve their achievement and performance in mathematics.

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APPENDIXES

APPENDIX A

Performance-Based Task # 1: *Pizza!*

Four friends, Lorenzo, Jessica, Naquan, and Derasha, held a pizza-eating at contest at lunchtime to see who would eat the most pizza. Lorenzo and Jessica shared one box that contained a whole, 12-slice, pepperoni pizza, while Naquan and Derasha shared another large-sized box of pizza that contained nine of its original twelve slices of meat pizza. Lorenzo ate 1/3 of the box of pepperoni pizza and Jessica ate 3/4 of what remained. Naquan and Derasha ate a total of 1/2 of their meat pizza. After lunch, Naquan said that together, he and Derasha ate much more pizza than what Lorenzo and Jessica ate together. Lorenzo disagreed.

You are the judge. Make a decision about who is right and why, and also determine how much more pizza the winning pair of students ate. Show your work, and use diagrams and pictures to help you to make your decision. You must also explain how you arrived at your answers. Please write your work and explanations neatly and clearly.

APPENDIX B

The Mangoes Problem

One night the King could not sleep, so he went down into the Royal kitchen, where he found a bowl of mangoes. Being hungry, he took 1/6 of the mangoes. Later that same night, the Queen was hungry and could not sleep. She, too, found the mangoes and took 1/5 of what the King had left. Still later, the first Prince awoke, went into the kitchen, and ate 1/4 of the remaining mangoes. Even later, his brother, the second Prince, ate 1/3 of what was left then. Finally, the third Prince ate 1/2 of what was left, leaving only three mangoes for the servants.

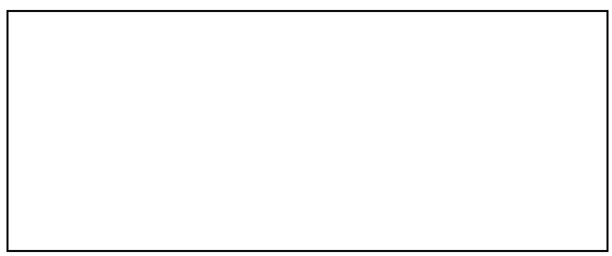
How many mangoes were originally in the bowl?

APPENDIX C

Performance-Task # 3: Science Fair

Three middle schools are going to have a science fair. The science fair will be in an auditorium. The amount of space given to each school is based on the number of students coming to the fair. Bret Harte Middle School may bring about 1,000 students, Malcolm X 600 students, and Kennedy Middle School about 400 students.

1. The rectangle below represents the auditorium. Divide the rectangle to show the amount of space each school should get based on the number of students that each school may bring to the fair. Label one section BH for Bret Harte, MX for Malcolm X, and K for Kennedy.



2. What *fraction* of the space should each school get, based on the number of students? Show your mathematical reasoning.

3. If the schools share the cost of the science fair on the basis of the number of students, what percent of the cost should each school pay?

4. If the cost of the science fair is $300.00, how much should each school pay based on the number of students? Show your work to justify your answer.

APPENDIX D

FRACTION COMPUTATION PRETEST ERROR CODES

Explanation of Error	Error Code	Error Iden.
Calculation error/basic arithmetic/careless error	1	P
Not Reducing/simplifying fraction	2	P
Changing an improper fraction to a mixed number	3	P
Changing a mixed number to an improper fraction	4	P
Conceptualization error	5	P
Division error - not using reciprocal	6	P
Division error - not changing division sign to multiplication	7	P
Subtraction error - incorrect borrowing and carrying	8	C
Addition error - across numerators and denominators	9	C
Ignoring the need to have common denominators (LCD not used)	10	C
Choosing the incorrect inequality sign to compare fractions	11	P
Multiplication Error - using LCD to multiply	12	P
Incorrectly renaming of an equivalent fraction	13	P
Incomplete work	14	P
Use of incorrect Lowest Common Denominator (LCD)	15	P
Inappropriate application of procedure	16	C
Unidentifiable error /reason for error is not clear	17	NC
Procedure and operation mismatch	18	P
Problem not attempted (left blank)	19	NC
Subtracting error - subtracting across numerators and denominators	20	P
Divison error - reciprocal not used/ flipping first fraction	21	P
Mixed numbers - multiplying across numerators and denominators	22	C
Rewriting a horizonal subtraction problem - using the wrong subtrahend	23	P
Incorrect choice of procedure for doing calculations	24	P
Total lack of knowledge of the correct way to do the problem	25	C

Error Identifiers: C: conceptual error; P: procedural error; NC: error type not clear.

APPENDIX E

Performance Task: *Pizza!*

Teacher Name: Mrs. Canterbury Scoring Rubric

Student Name: _________________________________. Total Score: ______

CATEGORY	4	3	2	1
Mathematical Concepts	Explanation shows complete understanding of the mathematical concepts used to solve the problem(s)	Explanation shows substantial understanding of the mathematical used to solve the problem(s)	Explanation shows some understanding of the mathematical concepts needed to solve the problem(s)	Explanation shows very limited understanding of the under-lying concepts needed to solve the problem(s) OR is not written
Mathematical Reasoning	Uses complex and refined mathematical reasoning	Uses effective mathematical reasoning	Some evidence of mathematical reasoning	Little evidence of mathema-tical reasoning
Mathematical Errors	90-100% of the steps and solution have no mathematical errors	Almost all (85-89%) of the steps have no mathematical errors	Most (75-84%) of steps and solutions have no mathematical errors	More than 75% of the steps and solutions have mathema-tical errors
Explanation	Explanation is detailed and clear	Explanation is clear	Explanation is a little difficult to understand but includes critical components	Explanation is difficult to understand & is missing several components OR was not included
Representa-tion	Uses appropriate and accurate representation of fractions to solve the problem, explain the solution and communicate mathematical thinking	Uses appropriate representation of fractions to solve the problem, explain the solution and to communicate mathematical thinking but, they are not totally accurate	Uses very little appropriate or accurate representations of the fractions to solve the problem, explain the solution, or to communicate mathematical thinking	Makes no attempt to use any representa-tions of frac-tions to solve the problem, explain the solution, or to communicate mathematical thinking

APPENDIX F

Performance Task: *The Mangoes Problem*

Teacher Name: Mrs. Canterbury   Scoring Rubric

Student Name: ________________________________. Total Score: ______

CATEGORY	4	3	2	1
Mathematical Concepts	Explanation shows complete understanding of the mathematical concepts used to solve the problem(s)	Explanation shows substantial understanding of the mathematical used to solve the problem(s)	Explanation shows some understanding of the mathematical concepts needed to solve the problem(s)	Explanation shows very limited understanding of the underlying concepts needed to solve the problem(s) OR is not written
Mathematical Reasoning	Uses complex and refined mathematical reasoning	Uses effective mathematical reasoning	Some evidence of mathematical reasoning	Little evidence of mathematical reasoning
Explanation	Explanation is detailed and clear	Explanation is clear	Explanation is a little difficult to understand but includes critical components	Explanation is difficult to understand and is missing several components OR was not included
Strategy/ Procedures	Typically, uses an efficient and effective strategy to solve the problem(s)	Typically, uses an effective strategy to solve the problem (s)	Sometimes uses an effective strategy to solve problems but does not do it consistently	Rarely uses an effective strategy to solve problems
Representatio n	Uses appropriate and accurate representation of fractions to solve the problem, explain the solution and communicate mathematical thinking	Uses appropriate representation of fractions to solve the problem, explain the solution and to communicate mathematical thinking but, they are not totally accurate	Uses very little appropriate or accurate repre- sentations of the fractions to solve the problem, explain the solution, or to communicate mathematical thinking	Makes no attempt to use any representa- tions of fractions to solve the problem, explain the solution, or to communicate mathematical thinking

APPENDIX G

Performance Task: *Science Fair*

Teacher Name: Mrs. Canterbury   Scoring Rubric

Student Name: ________________________________. Total Score: ______

CATEGORY	4	3	2	1
Mathematical Concepts	Explanation shows complete understanding of the mathematical concepts used to solve the problem(s)	Explanation shows substantial understanding of the mathematical used to solve the problem(s)	Explanation shows some understanding of the mathematical concepts needed to solve the problem(s)	Explanation shows very limited under-standing of the underlying concepts needed to solve the problem(s) OR is not written
Mathematical Reasoning	Uses complex and refined mathematical reasoning	Uses effective mathematical reasoning	Some evidence of mathematical reasoning	Little evidence of mathematical reasoning
Mathematical Errors	90-100% of the steps and solution have no mathematical errors	Almost all (85-89%) of the steps have no mathematical errors	Most (75-84%) of steps and solutions have no mathematical errors	More than 75% of the steps and solutions have mathematical errors
Strategy/ Procedures	Typically, uses an efficient and effective strategy to solve the problem(s)	Typically, uses an effective strategy to solve the problem (s)	Sometimes uses an effective strategy to solve problems but does not do it consistently	Rarely uses an effective strategy to solve problems
Representation	Uses appropriate and accurate representation of fractions to solve the problem, explain the solution and communicate mathematical thinking	Uses appropriate representation of fractions to solve problem, explain solution & communicate mathematical thinking but, they are not totally accurate	Uses very little appropriate or accurate representations of the fractions to solve the problem, explain the solution, or to communicate mathematical thinking	Makes no attempt to use representations of fractions to solve the problem, explain the solution, or to communicate mathematical thinking

APPENDIX H

ATITUDE TOWARD FRACTIONS INVENTORY

The following statements are about learning fractions. Please read each one carefully, then select a number from 1 to 5 to show how much you disagree or agree with the statement. There is no right or wrong answer.

SD: Means that you strongly disagree with the statement
D: Means that you disagree
N: Means that you do not disagree or agree
A: Means that you agree
SA: Means that you strongly agree

		SD	D	N	A	SA
1.	I clearly understand what is meant by a fraction.	1	2	3	4	5
2.	I can clearly explain to someone the difference between a fraction and a whole number.	1	2	3	4	5
3.	Fractions are useful for solving problems in everyday life.	1	2	3	4	5
4.	I really enjoy working with fractions.	1	2	3	4	5
5.	I only like working with the easy part of fractions.	1	2	3	4	5
6.	Working with fractions is just as much fun as working with whole numbers.	1	2	3	4	5
7.	Doing well with whole numbers helps me with fractions.	1	2	3	4	5
8.	Working with fractions is a fun part of mathematics.	1	2	3	4	5
9.	I feel relaxed when working with fractions.	1	2	3	4	5
10.	When I hear the word "fractions," I feel annoyed.	1	2	3	4	5
11.	I wish that I did not have to learn fractions.	1	2	3	4	5
12.	Fractions are easy for me to learn.	1	2	3	4	5
13.	Sometimes I do extra work to improve my fraction skills.	1	2	3	4	5
14.	Learning fractions helps me to learn other topics in math.	1	2	3	4	5
15.	Fractions are a part of many things in real life.	1	2	3	4	5
16.	Fractions are helpful in understanding the world of mathematics.	1	2	3	4	5

	SD	D	N	A	SA
17. I don't like learning about fractions.	1	2	3	4	5
18. In class, I usually understand what I am learning about fractions.	1	2	3	4	5
19. Working with fractions helps me to understand decimals.	1	2	3	4	5
20. No matter how hard I try, I can't seem to understand fractions.	1	2	3	4	5
21. I feel tense when my teacher says that we will be learning fractions.	1	2	3	4	5
22. I often think, "I can't learn this," when I am working fractions.	1	2	3	4	5
23. It is important to know fractions in order to get a good job.	1	2	3	4	5
24. It does not scare me to work with fractions.	1	2	3	4	5
25. I enjoy talking to my friends about fractions.	1	2	3	4	5
26. I like to play games that have fractions.	1	2	3	4	5
27. I can get along well in life without knowing fractions.	1	2	3	4	5
28. Working with fractions excites me.	1	2	3	4	5
29. Fractions are an important part of mathematics.	1	2	3	4	5
30. I try harder to learn fractions because math is my favorite subject and I want to do well in math.	1	2	3	4	5

APPENDIX I

Attitude toward Fractions Inventory Scoring Key

	Attitudinal Category	Statement Numbers
1.	Anxiety toward Fractions	10, 11, 17, 20, 21, 22
2.	Value of Fractions to Mathematics	3, 14, 15, 16, 19, 27
3.	Confidence about Fractions	1, 2, 9, 12, 18, 24
4.	Enjoyment with Fractions	4, 6, 8, 25, 26, 30
5.	Motivation about Fractions	5, 7, 13, 23, 28, 29

APPENDIX J

FRACTIONS COMPUTATIONS PRETEST

Name: ____

Complete.

1. 3/5 = ?/15   2. 2/3 = ?/12   3. 5/8 = ?/40   4. 1/3 = ?/15

Write in lowest terms.

5. 16/20 = _____   6. 18/27 = _____   7. 27/36 = _____   8. 24/48 = _____

< OR > ?

9. 4/7 ☐ 5/7   10. 2/3 ☐ 5/6   11. 3/5 ☐ 2/7

Add.

12. 3/5 + 1/5 = _____   13. 1/2 + 1/4 = _____   14. 2/5 + 1/4 = _____

Subtract.

15. 5/9 − 1/9 = _____   16. 5/6 − 2/3 = _____   17. 3/4 − 2/3 = _____

Complete this table.

	18.	19.	20.	21.
Fraction	8/5		25/6	
Mixed Number		1 3/4		4 2/3

Give each sum in simplest form.

22. 3 1/4 + 4 2/4 = ________

23. 2 3/8 + 1 1/4 = ________

24. 6 3/5 + 2 4/5 = ________

25. 6 5/8 + 2 3/4 = ________

Give each difference in simplest form.

26. 5 5/8 + 2 4/8 = ________

27. 6 3/5 + 2 1/3 = ________

28. 4 2/5 + 1 4/5 = ________

29. 4 1/3 + 2 5/6 = ________

APPENDIX K

Name ________________________________ Block: ______________

Fraction Concepts Pretest

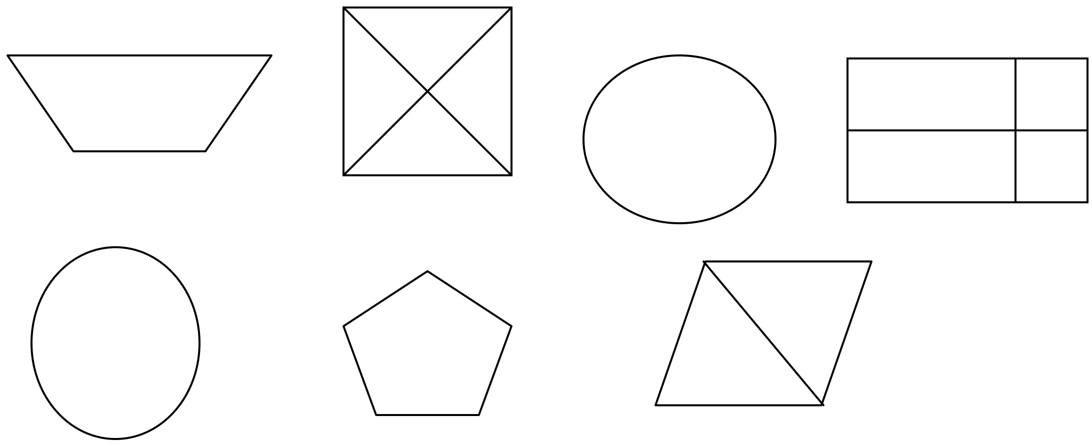
The purpose of this test is to find out what you know about fractions. Read each question carefully. Please answer all questions, show work and write your explanations clearly.

1. Use the space below to draw models of each of these fractions.
   (a) 1/8 (b) 3/4 (c) 1/3 (d) 3/2

2. Is the product of 3/5 and 6/5 more or less than 3/5, or is it more or less than 6/5? Estimate your answer. DO NOT work out the problem. Explain your reasoning.

3. During the last 9 months, the computers in the library were used 2/3 of the time. Use a diagram to show the number of months the computers were used.

4. Circle the fraction model below that shows fourths.



5. **Estimate** the answers to each of the problems below by choosing the whole number that each answer is closest to. Explain how you arrived at your answers.

(i) 7/8 + 12/13 =

(a) 1 (b) 2 (c) 19 (d) 21 (e) I don't know

Explanation:

(ii) 5/9 – 3/8 =

(a) 0 (b) 1/2 (c) 1 (d) 2 (e) I don't know

Explanation:

(iii) 4/9 x 7/8 = ?

(a) less than ½ (b) greater than ½ but less than 1

(c) a little bigger than 1 (d) 28 (e) 72

Explanation:

6. Mira used 2 1/3 oz of beads to make a necklace and 7/8 oz to make a ring. About how much more beads did she use to make the necklace?

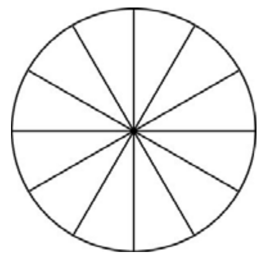
(a) 1 oz (b) 1 1/2 oz (c) 1 1/3 oz (d) 2 1/2 oz

7. Find a fraction that is less than 1/3. Find another fraction that is less than the second fraction. Do this one more time. Write down all four fractions.

8. In a class of 36 students, 3/4 of them used 240 sheets of construction paper for their math projects. How much more paper could the teacher expect the class to use by the time all of the students have finished their projects? You may use a diagram to show and explain your answer.

9. Latoya added 4 /9 and 1/3 and got an answer of 5/12. Is this answer correct or incorrect? If it is incorrect, show and explain how you would find the correct answer.

10. Shade $\frac{1}{4}$ of the fraction circle below, then shade in 1/3 of what is left. What fraction of the whole circle have you shaded altogether? Show how you got your answer.



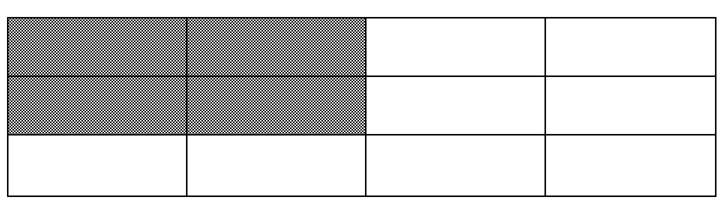
11. A man ordered two medium pizzas. He cut the first one into 5 equal pieces, and the other into 7 equal pieces. He gave his son one piece from each pizza. From which pizza did his son get a bigger slice? Explain your answer.

12. Mr. Kingley ordered several large, pizzas for the students in his math class. Jamie, Joylyn, and Jared each ate slices from a different kind of pizza. Jamie ate 1/3 of a pepperoni pizza, Joylyn ate 4/8 of a veggie pizza, and Jared ate 5/6 of a cheese pizza. Who ate the most pizza? Show and explain your work

13. Are the following sums larger or smaller than 1? Use estimation to find your answer. DO NOT work out the problems. Explain how you know each answer.

    (a) 3/8 + 4/9 ___________ (b) 1/2 + 1/3 ___________

14. Write a fraction that represents the shaded region.



15. Draw a picture that shows two-thirds of 24.

16. Fred has a large bag of jellybeans that weighs 3 and 2/3 pounds. He wants to fill at least 12 smaller bags. Will he be able to do so if each of the smaller bags hold 1/3 of a pound of jellybeans? You may use diagrams or pictures to show and explain how you found your answer.

17. Ms. Mack the sewing teacher has a piece of ribbon that is 1/4 of a yard long. She wants to use the ribbon to make three bows that are the same size. What will be the length of each of the three pieces? Show how you found your answer.

18. Write three fractions that are equivalent to 3/8.

19 Joanne had a piece of wood that measured 9/10 ft. She then cut off a piece that measured 2/5 ft. How long is the piece that remained?

20. Make a number line and show where 3/8 should be, by placing an "X" at the appropriate spot.

APPENDIX L

Interview Guide I (Based on Task # 1), *Pizza!*

Researcher:

> Hello Liza (fictitious name), in this interview, I am going to ask you questions about the performance-based task, named (task name) that you worked on in class. The questions will be mostly about fractions and fraction representations, but I am also very interested in how you came up with your answers, what strategies and procedures you used, and why you decided to use them. It is very important that you tell me what you were thinking and your feelings as you worked on the task. I will not grade this interview, so you do not have to worry about your answer being wrong.

General Fraction Questions:

1. Can you explain to me, your understanding of what the word "fraction" means?
2. Please tell me what is the first thing that comes to mind, when you have to solve a fraction problem.
3. Please explain what I am talking about if I say "fraction representations"
4. When would you most likely use fraction representations to work on a fraction problem?
5. Can you give me some examples of ways in which you can represent fractions?

Task-Related Questions:

6. In this task, what exactly were you asked to do?
7. Can you describe your thoughts as you read the problem?
8. How easy or difficult was it, to decide on a strategy for finding an answer to the questions in the task?
9. Did you immediately think of using representations of fractions to work on finding a solution to the problem? If yes, what kind, and why?
10. Do you think that your approach to finding a solution for the task would have been different if the directions did not state that you might use diagrams or pictures.
11. In what ways, was using fraction representations useful for solving the problem?
12. Explain the steps that you took to find a solution and complete the task.
13. What fraction skills did you have to know, and use to solve the problem?
14. On a scale of 1 to10 (1 being very easy and 10 being very hard) how would you rate this problem? Why?

15. What did you like or not like, about having to do this type of fraction problem?
16. How did you feel about working on this performance task?

APPENDIX M

Interview Guide II (Based on Task #2, The Mangoes Problem)

1. As you read the problem, did any ideas in particular come to mind as to how you would go about solving the problem? If so, please explain.
2. Was there a particular strategy that you thought would work best for solving the problem? Please tell me about this.
3. What fraction representation, if any, did you think of using to work on this task?
4. Can you describe anything that stood out, or that was special about the fractions that are given in the problem?
5. What (if any) information given in the problem helped you to think of a strategy for solving the problem?
6. If you used fraction representation to solve the problem, please explain your reason for choosing the type that you did.
7. What fraction skills did you need to have to find the answer to the problem in the task?
8. Did you need to use any fraction rules and/or procedures to find the answer to the problem in the task? If so, please explain.
9. Can you say if, and what manipulatives you used to find a solution to the problem?
10. How did you use the information about the remaining mangoes to find a solution to the problem?
11. Was it easy or difficult for you to use fraction representations to solve this problem? Why?
12. On a scale of 1 to 10 (1 being very easy, and 10 very hard) how would you rate this problem? Why?

APPENDIX N

Interview Guide III (Based on task # 3): Science Fair

1. What were you thinking as you read the task?
2. Did you clearly understand what you were asked to do to complete the different parts of the task?
3. As you read the task, did you have any ideas about how you would find the answers to the questions in the task?
4. Did fraction representations come to mind as you read the task? Please explain.
5. Did you use any type of fraction representation to answer any of the questions given in the task? Please explain.
6. Did you find answers to all of the questions in the task?
7. How certain are you that your solutions are correct? Why?
8. If you think that any of your answers are incorrect, do you know what you did wrong?
9. What fraction skills did you have to use to complete the task?
10. Was there anything in the task that you found really easy or hard to do? Please explain.
11. Was there anything that you especially liked or disliked about doing this task?
12. How was this task similar or different from the ones you did before?
13. Can you explain if and how, these types of performance tasks help you to understand fractions?
14. Now that you have done several performance tasks, how do you feel about working on this type of task?
15. On a scale of 1 to 10 (with 1 being *very easy* and 10 being *very hard*), how difficult was this task? Why?

APPENDIX O

Student Reflection Log

Performance Task: ________________ Name: ______________ Block: _______

Please complete each of the prompts below to express your feelings, thoughts, ideas and opinions about your work on today's performance-based, fractions task. There is no right or wrong answer, so please relax and write as you wish.

a) Today when I worked on the performance-based task, I …

b) When I first read the task, I felt that …

c) I liked / disliked (circle one) working on this task, because …

d) I used / did not use (circle one) fraction representations in this task, because …

e) The fractions representations that I used helped / did not help (circle one) me to find a solution for the task, because …

f) I would rate the level of difficulty of this task as a _____ (choose: 1, 2, 3, 4, 5, 6, 7, 8, 9, or 10) because …

g) In general, I think that this performance task . . .

APPENDIX P

Group Reflection Log

Performance Task: ___________________ Group #: _________

Please complete each of the prompts below to express your feelings, thoughts, ideas and opinions as a ***group,*** about your work on today's performance-based, fraction-related task. There is no right or wrong answer, so please relax and write as you wish.

a) The group discussion about the performance-based task that we worked on today helped / did not help (circle one) us, because …

b) Our group discussion helped us to realize that …

c) Everyone in the group agreed that this task …

d) Our group agreed / did not agree (circle one) on a solution because …

e) For this task, the students in this group used __________, _____________,

_____________, and ________________ , to represent fractions, because …

Appendix Q

Participant-Observer's Log

Performance Task: ___________________ Participant: _____________

a) Beginning the task:

b) On task/ focused:

c) Signs of progress with task:

d) Work shown:

e) Fraction representations visible?:

f) Participant's demeanor:

g) Task completion/wrap up:

APPENDIX R

Letter of Consent

Letter of Consent

Dear Parent or Guardian,

As a doctoral student in the College of Education at Georgia State University and an Atlanta Public School teacher, I am interested in improving mathematics instruction and student performance. My study investigates my students' ability to compute and conceptualize fractions, and to use fraction representations to organize and communicate their mathematical thinking. I will provide my 8th-grade students at XXXXXXXXX Middle School with opportunities to work on fraction-related, performance-based tasks. They will decide on the appropriate strategies that they can use and apply, to find feasible solutions to three tasks. These engaging and real world mathematics activities present scenarios to which the students can relate. These types of learning activities will soon become an integral part of the Georgia State Department of Education new curriculum and performance mathematics Standards.

All of my students will complete the performance tasks during their pre-algebra class. However, I will be working on a one-on-one basis with three students so that I can analyze their work as a part of my research. As a result, during the three-week study, which will begin in May 200X, I will interview each of these students individually during the school day at times that are convenient for both of us. These interviews will allow me to capture each student's thoughts, feelings and experiences regarding working with fractions and on the performance-based tasks. Please note that your child's participation in the study will not result in any loss of instructional time. In addition, I will handle the information gathered during the study in a manner that will not personally identify him or her, and I will not share it with anyone other than my doctoral committee members and their supervisors. A report of the findings of the study will be prepared for the Middle and Secondary Education and Instructional Technology Department at Georgia State University and for Atlanta Public Schools.

I therefore request that you please give permission for your son or daughter to participate in the study. The names of all students will be held in strict confidence. Please put a check on the bottom of this letter to indicate whether you give permission for your child to participate or not, then sign, cut it, and return it to me. Thank you very much for your time, attention, and a timely response.

(continued on next page)

Respectfully,

....
Sandra A. Canterbury (Graduate Student, Georgia State University)

Approved: …………………………………………. (XXXXXXXXXXX, Principal)

I **DO ___ DO NOT ____** give permission for my child to participate.

Signed: ______________________________ (Parent/Guardian) Date: __________

APPENDIX S

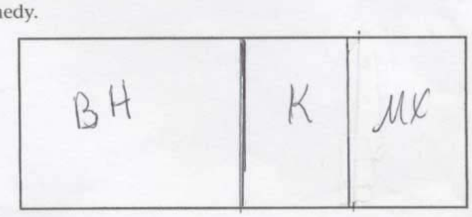
Liza's Work on *Science Fair*

Liza 3

Science Fair

Three middle schools are going to have a science fair. The science fair will be in an auditorium. The amount of space given to each school is based on the number of students. Bret Harte Middle School has about 1000 students, Malcolm X Middle School has about 600 students, and Kennedy Middle School has about 400 students.

1. The rectangle below represents the auditorium. Divide the rectangle to show the amount of space each school should get on the basis of number of students. Label each section BH for Bret Harte, MX for Malcolm X, or K for Kennedy.



2. What *fraction* of the space should each school get on the basis of the number of students? Show your mathematical reasoning.

BH: $\frac{1000}{2000} = \frac{1}{2}$ MX: $600/2000 = 3/10$ K: $400/2000 = 1/5$

Bret Harte middle school would get the most space base on the number of students. Half of the auditorium would go to BH, 3/10 of the space goes to MX & 1/5 would go to K.

3. If the schools share the cost of the science fair on the basis of the number of students, what percent of the cost should each school pay?

BH: 50%
MX: 30%
K: 20%

If the schools shared the cost of the science fair, BH would have to pay 50% of the money because they take up half of the space while MX pays 30% & K 20%.

4. If the cost of the science fair is $300.00, how much should each school pay on the basis of number of students? Justify your answers.

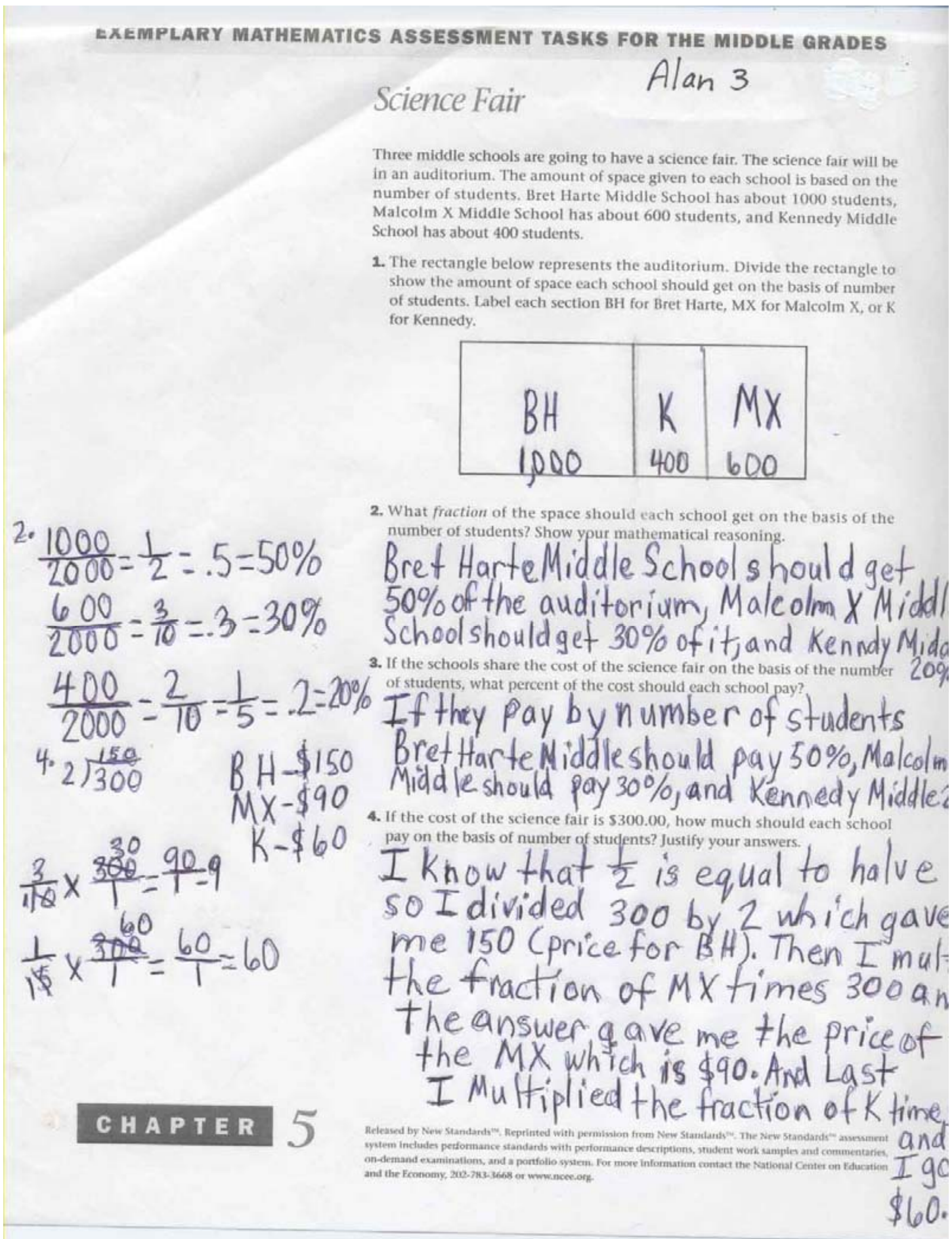
BH: $150
MX: $90
K: $60

150
90
+60
$300 ✓ = $300

BH would pay $150 because they pay 50% of the fair. MX would pay $90 because they pay 30% of the fair. K would pay $60 because they pay 20% of the fair.

APPENDIX T

Allan's Work on *Science Fair*



EXEMPLARY MATHEMATICS ASSESSMENT TASKS FOR THE MIDDLE GRADES

Alan 3

Science Fair

Three middle schools are going to have a science fair. The science fair will be in an auditorium. The amount of space given to each school is based on the number of students. Bret Harte Middle School has about 1000 students, Malcolm X Middle School has about 600 students, and Kennedy Middle School has about 400 students.

1. The rectangle below represents the auditorium. Divide the rectangle to show the amount of space each school should get on the basis of number of students. Label each section BH for Bret Harte, MX for Malcolm X, or K for Kennedy.

BH	K	MX
1000	400	600

2. What *fraction* of the space should each school get on the basis of the number of students? Show your mathematical reasoning.

Bret Harte Middle School should get 50% of the auditorium, Malcolm X Middl[e] School should get 30% of it, and Kenndy Mid[dle] 20%

2. $\frac{1000}{2000} = \frac{1}{2} = .5 = 50\%$

$\frac{600}{2000} = \frac{3}{10} = .3 = 30\%$

$\frac{400}{2000} = \frac{2}{10} = \frac{1}{5} = .2 = 20\%$

3. If the schools share the cost of the science fair on the basis of the number of students, what percent of the cost should each school pay?

If they pay by number of students Bret Harte Middle should pay 50%, Malcolm Middle should pay 30%, and Kennedy Middle[...]

4. If the cost of the science fair is \$300.00, how much should each school pay on the basis of number of students? Justify your answers.

4. $2\overline{)300}$ = 150

BH-\$150
MX-\$90
K-\$60

$\frac{3}{\cancel{10}} \times \frac{\cancel{300}^{30}}{1} = \frac{90}{1} = 90$

$\frac{1}{\cancel{5}} \times \frac{\cancel{300}^{60}}{1} = \frac{60}{1} = 60$

I know that $\frac{1}{2}$ is equal to halve so I divided 300 by 2 which gave me 150 (price for BH). Then I mul[tiplied] the fraction of MX times 300 an[d] the answer gave me the price of the MX which is \$90. And Last I Multiplied the fraction of K time[s] [300] and I go[t] \$60.

CHAPTER 5

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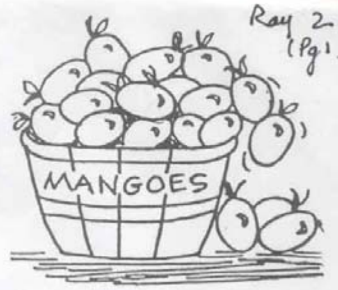
APPENDIX U

Ray's Work on *The Mangoes Problem*

The "Mangoes Problem"

Ray 2 (Pg1.



One night the King couldn't sleep, so he went down into the Royal kitchen, where he found a bowl full of mangoes. Being hungry, he took 1/6 of the mangoes.

Later that same night, the Queen was hungry and couldn't sleep. She, too, found the mangoes and took 1/5 of what the King had left.

Still later, the first Prince awoke, went to the kitchen, and ate 1/4 of the remaining mangoes.

Even later, his brother, the second Prince, ate 1/3 of what was then left. Finally, the third Prince ate 1/2 of what was left, leaving only three mangoes for the servants.

How many mangoes were originally in the bowl? There were originally 18 mangoes in the bowl. First I started working backward. I took the last 3 mangoes and multiplyed by the fractions that were ate by the Royal family. At, first I multiplet 2 by 3 and got 6. And I kee do this by multiping the denominator of the fraction by 3. Next, I drew 6 boxes and drew 3 circles (mangoes) in all 6 of the boxes. After doing that I counted the number of circles (mangoe and come up withe 18 mangoes or circle. After doing all of this I subtracted the whole number by the denominator of the fracti the family ate and after this kuas left with 3 mangoes. So that means that each of the King, Queen, and three princes ate 3 mangoes. To check take 3 and multiply by 6 and it should equ 18 mangoes. Ray 2 (page 1)

Performance-Based Fraction Activity: The "Mangoes Problem"

Ray 2
(Pg 2)

3x6 = 18 $\boxed{OOO}$ = 3 mangoes

For the King 6-1=5/18-3=15

For the Queen 5-1=4/15-3=12

For the First Prince 4-1=3/12-3=9

For the Second Prince 3-1=2/9-3=6

For the Theid Prince 2-1=1/6-3=3

For the servant 3 mangoes