Exploring First Graders' Mathematical Explanations

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EXPLORING FIRST GRADERS’ MATHEMATICAL EXPLANATIONS

by

NICOLE VENUTO GEARING

Under the Direction of Dr. Lynn Hart

ABSTRACT

Communicating their thinking in mathematics is challenging for young children. This research studied the change in first-grade students’ oral and written solution explanations before and after six problem-based mathematics lessons that focused on developing conceptual understanding of adding or subtracting a 2-digit number and a multiple of ten. A pre/post quasi-experimental design was used. Participants were assigned to a comparison group or an intervention group based on the classroom in which they are assigned. All students completed a pre-and post-assessment. Both groups received the same problem-based lessons. To encourage growth in their communication skills, students in both groups were asked to talk about their strategies, while the intervention group was asked to both talk and write about their strategies during each lesson. Oral and written pre-and post-assessments were scored using a rubric adapted from the Project M3 curriculum (Gavin et al., 2006-2008) and interrater reliability was established. T-test analyses were conducted to determine if a significant difference exists between first-graders oral and written mathematical explanations within discourse modes (comparing pre/post writing or pre/post talking) and between discourse modes (comparing talking and writing) for the intervention and comparison groups. A significant difference between discourse modes was found on the pre-assessments but not the post-assessments, suggesting that increasing oral discourse decreased the gap between the children in both groups ability to talk and write about their thinking. A significant difference was found within discourse modes for the intervention group, but not the comparison group, suggesting that adding written discourse to problem-based lessons further increased the children in the intervention group’s ability not only to write about their solutions, but also to talk about their thinking. ANCOVA analyses were conducted to determine if there was a difference in the oral and written
explanations between the comparison and intervention groups. ANCOVA analyses found a significant difference between the comparison group and intervention group’s oral and written explanations at the completion of the study, suggesting that adding written discourse to problem-based lessons increased the intervention children’s ability to both talk and write about their thinking.

INDEX WORDS: Oral Discourse, Written Discourse, Problem-Based Lessons, Sociomathematical Norms, Social Constructivism
EXPLORING FIRST GRADERS’ MATHEMATICAL EXPLANATIONS

by

Nicole Venuto Gearing

A Dissertation

Presented in Partial Fulfillment of Requirements for the

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in

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in

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in

the College of Education and Human Development

Georgia State University

Atlanta, GA
2018
DEDICATION

This research is dedicated to the children who taught me the importance of asking them questions, listening to their answers, and understanding their mathematical thinking.
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1 THE PROBLEM

Traditionally the focus in young children’s mathematics classrooms has been arithmetic (National Research Council, 2001). However, many children learn to compute without making sense of the procedures they are using (Burns, 2004), leading to an inability to be problem solvers. As technology improves and becomes more readily available, being able to explain mathematical reasoning has become vital (Boaler, 2016). Calculators and computer software can now quickly and accurately compute complex mathematics. However, solving problems using technology requires knowing what to input; making being able to reason quantitatively increasingly more important. Further, working collaboratively on projects is much more common than working independently, increasing the need for individuals to be able to communicate their reasoning. In recognition of the need to help children develop their ability to communicate their mathematical thinking, the Common Core State Standards for Mathematics (2010) place emphasis on developing mathematical reasoning and justification. Many of the standards specifically state students explain the reasoning used to solve problems. Further, the third Standard for Mathematical Practice, within the Common Core Standards, specifically states that students should be able to construct viable arguments and critique the reasoning of others (CCSSM, 2010).

With standards come assessment, and if constructing viable arguments and explaining reasoning is a standard, then it must be assessed. Classroom teachers assess their student's oral reasoning and explanation informally in the classroom, but this can be a challenge for teachers who often have over twenty students to assess. Further, assessing mathematical reasoning and explanation on standardized assessment proves challenging through traditional multiple-choice items, pushing standardized assessment developers to create constructed response questions.
Approximately 21% of Third Grade Mathematics Summative Assessment requires students to “express grade/course-level appropriate mathematical reasoning by constructing viable arguments, critiquing the reasoning of others, and/or attending to precision when making mathematical statements” (PARCC, 2016). While some of these responses require only a numerical response, others require a written response, creating a need to also develop student’s ability to explain their mathematical reasoning through writing.

While there is an immediate need to develop children’s ability to express their mathematical reasoning through written expression for the purpose of assessment, the benefits of writing about mathematical thinking go beyond preparation for assessment. NCTM’s Principles and Standards for School Mathematics (2000) stresses the importance of facilitating meaningful mathematical discourse, beginning with kindergarten students. The use of discourse helps students develop mathematical language so that they can express their mathematical thinking to others (NCTM, 2015). Further, the use of writing in the mathematics classroom supports learning because it requires children to organize and consolidate their thinking (NCTM, 2000). Writing also requires writers to reflect on their ideas, increasing metacognitive awareness (Pugalee, 2004).

Communicating mathematical thinking orally is difficult for children (Moyer, 2000). Writing about mathematical thinking can be even more challenging (Lee, 2006) because written production is more difficult than oral production (Bourdin & Fayol, 1994), particularly for children who have not yet mastered the ability to translate their thoughts into writing (Berninger et al., 1992). Casa (2015) suggests teachers begin to build a foundation for written discourse by connecting written discourse and oral discourse in the early years of elementary school.

Discourse is a term that can have many meanings. For this study, mathematical discourse is
defined by Lee (2006) as when teachers and students “talk about mathematical ideas, negotiate meanings, and discuss ideas and strategies in their own mathematical language” (p.1).

**Theoretical Framework**

The main focus of this study is communication in the mathematics classroom. In this study, communication is referred to as the way children talk and write about how they reason mathematically. For this study, communication is situated within theories typically associated with literacy, therefore, theoretical frameworks in literacy and mathematics were used to frame this study. Within the context of the mathematics instruction, social constructivism was employed to develop students’ mathematics content and ability to communicate their thinking. Children’s ability to communicate their thinking using oral and written language is at the foundation of this study. Therefore, theories in writing processes and the relationships between oral and written language were also used to frame the study.

**Social Constructivism**

The most commonly accepted theoretical perspective for learning mathematics is constructivism, which claims that learners must construct their own knowledge. This study grounds its social constructivism thinking in the work of Vygotsky. While traditional constructivism agrees with the importance of developing knowledge through inquiry (Wells, 2007), Vygotsky (1981) took that thinking further. He felt that knowledge develops through collaboration and discussion arguing that, “all higher mental functions are internalized social relationships...Even in their own private sphere, human beings retain the functions of social interactions” (p. 146). For Vygotsky (1987), speech is an essential component of social interactions within a learning community such as a classroom, claiming, “Speech does not merely serve as an expression of developed thought. Thought is restructured as it is transformed
into speech” (p. 251). This restructuring of thought through the process of producing speech is when learning occurs. Unfortunately, many classrooms lack opportunity for open-ended discussion of ideas and are often structured in a teacher lecture followed by seat work structure (Tharp & Gallimore, 1988). This is particularly true in mathematics classrooms.

Mathematics is most often communicated through symbols such as numerals on paper (Ernest, 1998). As a result, conversation is frequently overlooked in mathematics classrooms. However, conversation is a key component of intellectual growth (Tharp and Gallimore, 1991). Ernest (2008), who grounds his thinking in the philosophy of Wittgenstein (1921) and the psychology of Vygotsky (1978), puts forth the argument that mathematics is not only constructed but constructed socially. “Truth is not to be found inside the head of an individual person, it is born between people collectively searching for truth, in the process of their dialogic interaction” (Ernest, 2008, p. 164). Understanding the language of mathematics and being able to communicate mathematical reasoning becomes essential to constructing mathematical knowledge.

Mathematics is learned through participating in and sharing the language of mathematics. Through discourse learners “generate, test, correct and validate mathematical performances, with the aim of ensuring that the learner has appropriated the collective mathematical knowledge and competencies, and not some partial or distorted version” (Ernest, 1998, p. 221). Therefore, language should be at the center of mathematics education. Mathematical conversation, referred to as discourse throughout this document, is conversation about key words used in mathematical ways, visual mediators, and distinctive mathematical routines (Sfard, 2012). While mathematics is based on a language of rules, children must move beyond the rules and be able to express their thinking in the language of mathematics (Schoenfeld, 1992).
Constructing the Language of Mathematics.

Mathematics is a language of symbols. When put together, these symbols form a language used to create, record, and justify mathematical reasoning (Ernest, 2008, p. 169). Therefore, to begin to solve problems, children must first construct an understanding of the symbols of mathematics and what they represent. Vygotsky (1978) explains that written language consists of signs and symbols that designate sounds and words in spoken language. Signs refer to letters and sounds while symbols refer to objects. However, in mathematics, signs and symbols are often used interchangeably. For example, the symbol for equivalence is referred to as an equal sign. When learning the signs and symbols of written language, spoken language acts as a link between the two forms of language. When writing about mathematical thinking, children must also consider the signs and symbols that represent mathematical concepts such as numerals and operation symbols such as the plus and minus sign. Learning to communicate mathematical reasoning through writing requires children to learn the symbols of mathematics and combine them with the symbols of everyday language.

Writing Theories

While the focus of this study is developing young children’s ability to write about their mathematical thinking, considering the theories behind the writing process is warranted. Further, for young children, the ability to talk is developed prior to writing (Berninger, 2000), therefore, how the two processes are connected will be considered in this section.

The writing theories that frame this work are cognitive writing process and sociocultural writing theory. As pioneers in the writing field, Hayes and Flower (1980) developed a theory regarding the cognitive processes of writing. They found that the writing consists of three major processes, planning, translating, and reviewing. In the planning process, writers use information from their environment and long-term memory to establish a plan. This process requires the
writer to retrieve relevant information and organize it with the information found within the environment. In the translating process, the writer, under the guidance of the plan developed, produces language that corresponds to the information in the writer’s memory. In the reviewing process, the quality of the text produced in the translating process is improved through revision and editing.

While Hayes and Flower were among the first to consider the writing process, others rejected their processes of writing as being too simplistic and lacking acknowledgement of environment. In an effort to further understand the internal and external factors of the writing process, Sociocultural theories of writing were explored. Sociocultural theory sees “writing as chains of short- and long-term production, representation, reception, and distribution” (Prior, 2006, p. 57). These processes are situated in previous experiences and serve as more than a mean of communication, but as a mean of action (Prior, 2006). Acknowledging the importance of environment, Hayes (2006) extends the writing process to indicate a more reciprocal process between the cognitive processes of writing outlined above and the task environment, such as social, audience, other texts and physical components. Hayes also notes the reflective practice of writing, describing the processes as follows,

Writers rely on general problem-solving and decision-making skills to devise a sequence of steps to reach their writing goals, drawing inferences about audience, possible writing content, and so forth as they engage in these reflective processes. Cues from the writer’s plan or text produced so far act to guide the retrieval of possible ideas for text. A suitable idea is then held in working memory, as the writer expresses it vocally or sub vocally as sentence parts, evaluating what to keep and modify as text is produced. Through the writing process, the writer engages in reading to define the writing task, obtain writing content, or evaluate text produced so far (p. 26).

In this study, writing is serving as a tool to develop not only communication but conceptual understanding. The cognitive processes that occur during the planning stage of the writing process require children to consolidate their thinking (NCTM, 2000) in order to plan for
the language they will use when writing to communicate their thinking. Writing also gives children the opportunity to share their mathematics experience with their teacher and peers, serving as a record of mathematical thought. This creates a more equitable mathematics classroom as teachers have access to the ideas of all students, not only the students who have the opportunity to talk to about their solution strategies.

First Graders as Writers

In this study, writing is serving as a mediating tool for developing mathematical communication as well as conceptual understanding base-ten addition and subtraction. Therefore, this section explores how first-grade children engage in writing.

Bereiter and Scardamalia (1987) consider the writing process of novice writers, finding that they often convert writing to a task of telling the reader what they know about a given topic. Their writing generally features three components, forming a mental representation of the assignment, drawing on discourse knowledge from long-term memory, and drawing on discourse knowledge to tell about the topic.

Graves (1983) considers the five areas in which children’s consciousness problem solves during the writing process, spelling, motor aesthetic, convention, topic information, and revision. He notes that young children function in all five categories as they begin to write. However, spelling is often the first problem children must reconcile as they begin to write, followed by aesthetic and convention. Children in classrooms that emphasize these three problems often struggle to take ownership of their writing and overcome these problems.

While many milestones in aural and oral language are reached during preschool years, reading and writing milestones are often not reached until middle childhood. Further, few milestones within the writing system, except letter production, spelling, and beginning
composition are achieved prior to adolescence (Berninger, 2000). Since first-grade children are beginning writers, transcription, handwriting and spelling, and translation, thoughts transformed into written language, exert demands on working memory if they are not yet automatic, interfering with the child’s ability to write (Puranik et al., 2012). While many first-grade children have achieved letter formation milestones, they are still exploring spelling and beginning composition. Beginning composers in first grade typically produce between a single clause and a few related sentences (Traweek & Berninger, 1997). As children begin to develop phonological awareness, they use their knowledge of the alphabetic system to combine letters to spell words, often referred to as invented spelling (Steffler, Varnhagen, Treimen & Friesen, 1998). Eventually, this process becomes automatized, lessening the demand on working memory.

While communicating mathematically is essential, it is often difficult for young children (Moyer, 2000), particularly the act of writing about mathematical thinking. When children encounter a problem that is slightly difficult, they use speech as a tool to work through a solution (Vygotsky, 1978). Very young children use speech to plan a solution for a problem, connecting the two processes (speech and thinking) into one psychological process. Further, Vygotsky (1978) states, “The more complex the action demanded by the situation and the less direct its solution, the greater importance played by speech as a whole. Sometimes speech becomes of such vital importance, that if not permitted to use it, young children cannot accomplish the given task” (p. 25-26). Since problem-solving and writing about thinking are two challenging tasks for young children, the use of speech acts as a tool to mediate the writing process as children begin to write about their mathematical thinking (Vygotsky, 1978).

Since first-grade children have more experience talking than writing, their use of oral language builds a foundation for children to begin to write about their mathematical thinking.
Graves (1983) and Calkins (1994) refer to the use of rehearsal as a tool to mediate the writing process for beginning writers. Graves (1983) refers to rehearsal as preparing to compose, including “daydreaming, sketching, doodling, making lists of words, outlining, reading, conversing, or even writing lines” (p. 221). In mathematics, children often draw pictures as a tool for problem solving. These pictures are representations of their mathematical thinking, becoming a tool in the planning stage of the writing process. The use of speech in explaining the problem-solving process then acts as a tool to mediate the translating process.

As Graves (1983) suggests, rehearsal helps children become aware of what they are doing, allowing them to more easily communicate their thinking. Further, these processes help children consolidate their thinking (NCTM, 2000), increasing metacognitive awareness (Pugalee, 2001).

Writing about mathematics is not a common experience for many children (Silver, 1999), leading to a lack of discourse knowledge. Defined as “what one knows about how to write” (Olinghouse & Graham, 2009, p. 433), discourse knowledge is an important component of the writing process. This can include knowledge regarding various forms of writing, such as genre, as well as linguistic knowledge such as letter formation, spelling, grammar, and usage (Olinghouse & Graham, 2009). Without the experience of writing about mathematics or exposure to the discourse knowledge of the genre, the decision-making process becomes difficult for children, leaving them unsure of where to begin their writing process.

Connection Between Oral and Written Language

In this study, the relationship between oral and written language was explored in two ways. The first is whether or not the use of increased oral discourse during problem-based lessons is an effective strategy for introducing writing as a tool for mediating mathematical communication.
and conceptual understanding. Second, this study explores the impact oral and written discourse have on one another.

When children begin elementary school, oral language is more developed than written language (Berninger, 2000), making it easier for children to express their thinking orally than in writing. Therefore, Casa (2015) suggests teachers begin building a foundation for writing in mathematics by connecting oral discourse with written discourse. This study utilizes this thinking to help children begin to use written discourse to communicate their mathematical thinking. While written language develops later than oral language (Berninger, 2000; Vygotsky, 1978), they appear to develop concurrently in early and middle childhood (Harrell, 1957; Shanahan, 2006). Shanahan (2010) explains that while children begin elementary school with more developed oral language, as sub processes such as handwriting automatize, the difference between oral and written language decreases (Shanahan, 2010). Frequently, teachers recognize that young children are able to communicate using talk, and therefore, do not focus on developing oral language skills (Shanahan, 2010). However, several studies with young children found that oral language continues to develop as children learn to write (Berninger & Abbot, 2010). Further, there are correlations between the development of oral and written language (Kim et al., 2014; Berninger & Abbot, 2010; Shanahan, 2010).

While developing a foundation for writing mathematical explanations for assessment is necessary, there are other valid reasons for helping children develop their ability to write about their mathematical reasoning. While building a foundation for writing about mathematical thinking, the use of drawings and oral discourse serve as a mediating tool to help children develop their ability to engage in writing about their mathematical reasoning. Once children have built a foundation for writing about their mathematical reasoning, writing becomes a mediating
tool for problem solving (Bicer, Capararo, Capraro, 2013; Pugalee, 2004). Young children often use internal speech to work through challenging problems (Vygotsky, 1978). As children develop their ability to write, the use of writing can be used as a tool to work through difficult problems (Vygotsky, 1978), while also developing metacognitive awareness about their solution process. Further, the use of written discourse also helps teachers know more about a child’s conceptual understanding of the mathematics (Pugalee, 2001). When teachers can assess what children understand about mathematical concepts, they can help children further develop their mathematical reasoning.

Although first-grade children are still developing writers, writing can be a useful tool in the mathematics classroom. Before a writer begins to put words on the paper, they must develop a plan (Hayes & Flower, 1980). Engaging in this planning process requires the writer to reconsider the steps used to solve the mathematics problem, developing metacognitive awareness. Further, when children engage in writing about their mathematical reasoning and have opportunities to share their writing with an audience, they begin to develop discourse knowledge (Olinghouse & Graham, 2009). These experiences, coupled with an understanding of the transcription and translation abilities of first-grade children, demonstrate that they are able to effectively communicate their solution process through writing.

**Conceptual Framework: Problem-Based Lessons**

The focus of mathematics classrooms should be problem-solving (Carpenter et al., 2015; Lambdin, 2003). Children should frequently have the opportunity to think about and solve problems using strategies they create (Carpenter et al., 2015). Further, children should have opportunities to share their solution strategies with peers. Sharing solution strategies with peers increases access and equity in mathematics as it gives children the opportunity to see various solution strategies and pushes children to develop more sophisticated strategies (Stein & Smith,
This section explores the use of problem-based lessons to elicit oral and written discourse in the mathematics classroom.

Communicating mathematically is difficult for children (Moyer, 2000), making eliciting meaningful talk and writing in the elementary mathematics classroom challenging. As instruction continues to shift from the traditional model to an inquiry approach, teachers must have an understanding of the ways children solve problems, and how children share their thinking and solution strategies (Yackel, 1995). The types of tasks and cognitive demand of those tasks have an impact on mathematical discourse. Many traditional classrooms rely on tasks or assignments that require low levels of cognitive demand, referred to by Stein, Smith, Henningsen and Silver (2009) as *memorization* and *procedures without connections*. These types of tasks involve reproducing learned facts, or applying an algorithm, and require little or no explanation. Tasks requiring higher levels of cognitive demand are referred to by Stein et. al. (2009) as *procedures with connections* and *doing mathematics*. Tasks requiring higher levels of cognitive demand are more complex than memorization or procedure tasks. These tasks can have a variety of solution strategies, and those solution strategies are not always clear. These tasks also elicit mathematical discourse. The varying solution strategies students may use act as a platform to begin the facilitation of discussion about the task. Teachers who can facilitate meaningful mathematical discourse through high cognitive demand tasks can gain more information about what students understand about mathematical concepts (Yackel, 1995). Meaningful discourse allows the teacher to capitalize on mathematical opportunities that emerge from classroom discourse, enabling the teacher to push his or her pedagogical goals (McClain & Cobb, 2001; NCTM, 2015).
The conceptual framework for this study is grounded in the work of Stein, Engle, Smith and Hughes (2008) who integrate practices suggested by other researchers into a framework for mathematical discourse. The five practices are:

1. anticipating likely student responses to cognitively demanding mathematical tasks,
2. monitoring students’ responses to the tasks during the explore phase,
3. selecting particular students to present their mathematical responses during the discuss-and-summarize phase,
4. purposefully sequencing the student responses that will be displayed, and
5. helping the class make mathematical connections between different students’ responses and between students’ responses and the key ideas (p. 322).

The practice of anticipation includes solving the task using as many strategies as possible to anticipate how students may attack the problem, and difficulties they may have throughout their work period. Once students begin working on the task, Stein and colleagues explain the importance of monitoring students as they work. Through this practice, the teacher asks students questions about their work, guiding them, without telling them, towards a solution strategy.

Eliciting classroom mathematical discourse requires selected students to share their solution strategies. Stein and colleagues encourage teachers to select students who used various solution strategies. Sharing a variety of strategies elicits more mathematical discourse as students can discuss the similarities, differences, and efficiency of the strategies. Once the sharers are selected, the teacher sequences the sharers, beginning with the least sophisticated sharer. Stein and colleagues suggest the least sophisticated sharers explain their strategy first because students who did not have access to the task, or did not fully understand the task, are likely to understand the least sophisticated strategy and then have access to the more sophisticated strategies. The final practice is connecting. During the mathematical discourse, it is important to make connections to mathematical concepts.
Purpose

While writing across the curriculum is not required by the Common Core Standards for English Language Arts, and formal assessment of written mathematical explanations do not begin on assessments such as the PARCC until third grade (CCCSM, 2010; PARCC, 2016), younger children should be given opportunities to explore and explain their thinking through oral and written discourse. This study occurred within a Problem-Based Lesson format, in which young students were asked to either talk or talk and write about mathematics problems in the number and operation domain. Currently, there are three First Grade Common Core Standards (2010) that specifically ask first-grade students to explain their reasoning. These three standards were the focus of instruction. They are,

**CCSS.MATH.CONTENT.1.NBT.C.4**
Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used (emphasis added). Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten.

**CCSS.MATH.CONTENT.1.NBT.C.5** Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used (emphasis added)

**CCSS.MATH.CONTENT.1.NBT.C.6** Subtract multiples of 10 in the range 10-90 from multiples of 10 in the range 10-90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. (emphasis added).

Overview of the Study

A quasi-experimental pre/post design was used in this study. The study involved two groups of first-grade students in a Title I school in a large metropolitan school district in the southeastern United States. The students in both groups participated in six 45-minute problem-
based mathematics lessons in the numbers and operations domain. The students in the intervention group engaged in oral and written discourse during each of the six lessons, while the students in the comparison group only engaged in oral discourse. Pre-and post-assessments included oral and written explanations of the strategies used to solve a simple story problem. The assessments were independently scored by two graduate students using a rubric adapted from Gavin et al. (2006-2007) by the researcher. The teachers of the four first grade classrooms were informally interviewed prior to the problem-based unit and at the conclusion of the unit to consider their perceptions of their students’ ability to explain their mathematical reasoning before and after the unit.

The research questions were

(1) Is there a significant difference between children’s oral and written mathematical explanations before and after the problem-based lessons?

(2) Is there a significant difference in the mean score of the oral and written mathematical explanations of the intervention and comparison groups?

**Significance of the Study**

Discourse is the primary medium through which instruction is delivered (Wagner, Herbel-Eisenmann & Choppin, 2012). Typically discourse encountered in elementary classrooms involves the teacher doing most of the talking about their own mathematical thinking. However, engaging children in discourse gives learners the opportunity to share their ideas, as well as ask their peers questions about their solution strategies (Carpenter et al., 2015), affording children the opportunity to engage in the social construction of their mathematical thinking (Ernest, 1998). Further, when students share their solution strategies and reasoning, the gap between the
students who did not understand the mathematical concept and those who did is reduced (Boaler, 2016).

Writing in the mathematics classroom helps students reflect on and consolidate their reasoning about mathematical concepts, while also clarifying and deepening their mathematical thinking (NCTM, 2000). Written explanations of mathematical reasoning serve as a record of mathematical thought (Lee, 2006), providing teachers with information about how children learn and think about mathematics (Pugalee, 2001). The early years of elementary school serve as a foundation for mathematical learning and future growth, therefore, investigating the relationship between oral and written discourse during these formative years is warranted (Cohen et al., 2015).

Assumptions and Limitations

This study assumes that the participants in the study have developed enough fluency in text translation so that translation does not interfere with their ability to translate their thinking into text (Graham & Weintraub, 1996). The study also assumes that the students in the four classes have received similar mathematics instruction up to the point of this study. While they all participate in the same curriculum, teachers vary in instructional delivery (Borko & Livingston, 1989). Therefore, experiences children have had and exposure to discourse knowledge among the students may vary. Further, the lessons were taught four times to four different classes. While the instruction for the 2 interventions and two comparison classrooms was done by the same instructor and caution was taken to keep instruction the same across all four classrooms, this is not possible, especially given the nature of problem-based instruction.

As with all research, there are limitations to this study. The sample size in this study is small, producing small to medium effect sizes. This study is focusing only on the base-ten
numbers and operations domain in first grade, limiting the results of the study to this grade and domain only. Further, assessments with different problem types or written explanations in different mathematical domains may have different findings.
2 REVIEW OF THE LITERATURE

Communication is essential in mathematics education (NCTM, 2000). Students who have the opportunity to discuss and justify their solutions gain a better understanding of mathematical concepts (NCTM, 2000). Meaningful opportunity for mathematical growth occurs when children have the opportunity to make sense of the mathematical explanations of their peers and compare others’ solutions with their own (Yackel & Cobb, 1996). Communication within the mathematics classroom is referred to as discourse. Discourse is a term that can have many meanings. For this study, mathematical discourse is defined by Lee (2006) as when teachers and students “talk about mathematical ideas, negotiate meanings, and discuss ideas and strategies in their own mathematical language” (p.1). The use of discourse in the classroom helps students learn mathematics, as well as learn to communicate mathematically (Moyer, 2000). Students may communicate new concepts by acting out situations, drawing, using objects, giving oral accounts, creating diagrams, writing, or through the use of mathematical symbols (NCTM, 2015).

The purpose of this research study is to explore first-grade children’s ability to talk and write about their mathematical solutions within the context of problem-based instruction. A review of the literature in the areas of oral and written discourse in mathematics, first-graders as communicators, problem-based instruction, and the development of sociomathematical norms was conducted. Studies were chosen for inclusion in this review based on their relevance to the current study.

Oral Discourse in Mathematics

Many elementary teachers use oral discourse in their classroom each day, as discourse is a medium through which teachers deliver instruction (Wagner, Herbel-Eisenmann, Choppin, 2012; Moyer, 2000). The use of oral discourse within the mathematics classroom also promotes
Students need to do more than observe mathematics; they need to be a part of mathematics (Wagner et al., 2012). Discourse in the mathematics classroom gives students the opportunity to share their ideas, but it also gives other students the opportunity to ask their peers questions about their solution strategies (Carpenter, Fennema, Franke, Levi, Empson, 2015). Engaging in mathematics through speaking creates a space where students begin to take ownership in what they are learning and sharing with their peers. When students share their solution strategies and reasoning, the gap between students who understand the concept and who do not understand the concept is reduced (Boaler, 2016). The studies discussed below support the benefits of incorporating oral discourse in the mathematics classroom. Due to a paucity in research, studies from across grade levels are included in this section.

Cobb, Boufi, McClain, and Whitenack (1997) conducted a qualitative study in a first-grade classroom. They observed two mathematical discussions between the students and their teacher. In the first discussion, children were considering all the ways to split five monkeys between two trees. In their discussion, they were trying to identify all possible combinations. However, the group was unable to explain if they had found all possible combinations beyond saying, “Because (that’s) all the ways that they can be” (p. 263). They lacked the oral discourse skills to communicate their thinking. Over the course of the next few weeks, the teacher continued to work with students on the idea of partitioning and producing more organized findings through shared discourse. In the second discussion observed by the researchers, students were attempting to find all combinations for packing 43 pieces of candy into rolls of ten. The teacher drew the rolls of candy for tens and pieces for ones to record the students’ suggestions on the board. The students’ suggestions included 0 tens and 43 ones, 1 ten and 33 ones, 2 tens and 23 ones, 3 tens and 12 ones and 4 tens and 3 ones. Karen, a student in the class explained, “Well, see, we’ve
done all the ways. We have 43 pieces…and, see, we had 43 pieces (points to 43 p) and right here we have none rolls, and right here we have one roll (points to 1r33p). The teacher then numbers all of the pictures with 1 roll, 2 rolls, 3 rolls, and 4 rolls to clarify Karen’s justification for finding all the possible combinations. In this episode, the students began to shift to their own ability to justify their reasoning for finding all possible combinations. This study demonstrates the effectiveness of using oral discourse to further children’s thinking through sharing ideas and strategies with one another.

Giving students the opportunity to participate in oral discourse provides the teacher insight as to which students have a solid understanding of a mathematical concept and which students have misconceptions (Yackel & Cobb, 1996; Lee, 2006). Understanding children’s’ thought process through discourse can help the teacher evaluate what the child knows or does not know, in turn helping the teacher improve access and equity within the classroom (Yackel, 1995). For example, Levenson (2013) studied a progression of one student’s thinking about numbers in second, fifth and tenth grades. For example, prior to formal multiplication instruction in second grade the student, Sharon, was asked to explain and solve the following problems, 3x2, 2x3, 3x0, and 0x3. The first two problems investigated Sharon’s understanding of the commutative property. The second two investigated Sharon’s understanding of multiplication with zero. In second grade, Sharon first responded that 3x2=12. She then drew objects to explain her thinking and was able to correct herself. She was, however, unable to decide if 3x0=3 or if 3x0=0. In tenth grade, Sharon was able to quickly solve all four problems, but still had a difficult time explaining her thinking. She asked, “What does it mean, ‘to explain’? What am I supposed to say?” In tenth grade, she referred to objects again to demonstrate her thinking, but was still was unsure how to explain why 3x0=0. In another sequence in fifth and tenth grades Sharon was
asked about even and odd numbers, specifically 14, 9, and 0. Sharon assessed that 14 is even and 9 is odd in fifth and tenth grade, but was unable to decide if zero was an odd or even number either time. She attempted to determine if zero were odd or even by dividing it in half. When she still got zero, she did not know what to do with the information, leaving her unable to determine if zero is an odd or even number. Levenson (2013) contributes Sharon’s confusion about the number zero to the way she thinks about the number. Students frequently use the terms zero and nothing interchangeably, in turn thinking about zero as not really being a number. Sharon’s explanations about her mathematical thinking throughout her schooling give a clear picture of what she understands about concepts such as odd and even numbers and zero. Sharon’s misunderstanding about zero may not have been realized without the use of discourse. Further, Sharon, like many other children asks the researchers, “What does it mean to explain,” demonstrating the importance of not only providing children with opportunities to explain their thinking, but developing an understanding of what it means to share your thinking.

An important aspect of discourse in the classroom is that both the teacher and the students must be engaged. Students may have the correct solution to a problem, but their reasoning may be flawed. Without open discourse the teacher is unaware of the misunderstanding. For example, Heng and Sudarshan (2013) examined nine second-grade teachers from Singapore as they learned to use clinical interviews to understand children’s mathematical thinking. They found that the teachers in their study did not engage in discourse with students in their classrooms prior to learning about clinical interviews. Therefore, the teachers did not have a firm understanding of student thinking surrounding mathematics. For example, a high achieving student was asked to explain how he added two numbers in an addition problem that he correctly solved, to the teacher’s satisfaction. However, when the researcher asked him why he added the
numbers, the child responded, “Because the second number is bigger” (p. 471). Further, when asked if the second number were smaller, the child responded, “Smaller number means you have to minus” (p. 471). While he achieved the correct answer to the problem, it was based on incorrect reasoning and could lead to further misconceptions. Often children find the correct solutions to problems, leading teachers to believe they have developed understanding of the concept. However, many children develop misconceptions that may impact their ability to solve problems in the future. These misconceptions often go undetected without the use of discourse.

While sometimes students have a correct solution and incorrect reasoning, sometimes they may have an incorrect solution while having accurate mathematical reasoning. Jorgenson (2012) explores how discourses of mathematics contribute to the learning of working-class and indigenous students, ages 12-13, in Australia. In the study, students were asked to solve the problem “There are 365 students at the sports field. If a bus can hold 50 people, how many buses are needed to transport students back to school?” Jorgenson found that working class students were more practical in their responses, but did not receive credit because they did not interpret the question the same way as the teacher. For example, some children said seven, but some students will stand or sit three to a seat. Others responded seven and some would have to come back with the teachers. These children understood the action in the problem, but did not know what to do with the remainder, so they reasoned. Asking children to explain how they found their solutions helps teachers understand the child’s reasoning and better determine a strategy for furthering their conceptual understanding.

The Common Core State Standards for Mathematics frequently ask children to explain their reasoning. Further, the Standards for Mathematical Practice ask children to construct viable arguments and critique the arguments of others. The use of oral discourse allows children to
begin to make connections among mathematical ideas across several lessons and develop the ability to construct arguments about their thinking. Further, asking children questions about their mathematical thinking helps the teacher understand how children are reasoning about mathematics, therefore improving access and equity within the mathematics curriculum.

Written Discourse in Mathematics

“The place of writing is rather less established” (Morgan, 1998, p.22) than oral discourse within the mathematics classroom. Silver (1999) surveyed 117 New York mathematics teachers regarding their use of writing-to-learn in the mathematics classroom. Forty-three percent of the teachers responded that they had never heard of writing-to-learn, while another 20% stated they rarely used the strategy. With a growing expectation for students to explain their mathematical thinking and reasoning, the use of constructed written response items is becoming a standard practice on mathematics assessments because writing is a practical way to assess student explanations, as writing is a record of mathematical thought (Lee, 2006).

Writing in the mathematics classroom is an effective way to encourage students to examine their mathematical thinking (NCTM, 2000) as writing requires the individual to examine their thinking and plan how they will express their thinking in writing (Price & Jackson, 2015). Burns (1995) explains that the focus of elementary mathematics classrooms is often arithmetic. Children learn to compute without making sense of the procedures taught. Therefore, they have a difficult time justifying the reasonableness of solutions to story problems. The act of writing encourages the writer to reflect on what they have learned, which extends and deepens their mathematical understanding. The following studies were included in this literature review because of their relevance in the area of written discourse within the mathematics classroom. Due to a paucity of research in this area, research from all grade levels was included to gain a fuller picture of the impact writing in the mathematics classroom has students.
Banes (2016) worked with two fifth-grade teachers for one year to implement practices that would support students in writing mathematical explanations. Throughout this process it became evident that deciding what a good written explanation should include was important for teachers to consider in thinking about how to support student writing. At the beginning of the study, many of the written explanations provided by the students included only the procedures used to get to a solution. The two fifth-grade teachers agreed that a good written explanation should share more than the procedure used to solve the problem. A good explanation shows reasoning and provides evidence. Both teachers also valued using multiple solution strategies and the use of drawings as they supported children in communicating their reasoning. This study demonstrates the importance of developing an understanding of what it means to write a good written explanation.

Pugalee (2001) explored the written explanations of 20 ninth-grade algebra students. Students wrote about their solution process as they worked through the problems, finding that using writing to describe solution processes demonstrated the presence of metacognitive awareness in the problem-solving process, supporting the idea that “writing can provide a source of information for teachers to assess how their students learn and think about mathematics” (p. 242).

In a second study Pugalee (2004) examined the verbal and written descriptions of ninth-grade algebra students through oral problem-solving interviews, and written explanations of their problem-solving process. The study found that students who constructed plans were more likely to solve the problem accurately. The study also found that when students wrote about their problem-solving process, produced correct solutions at higher rates than when they solved the problem using a think-aloud process. Written explanations demonstrated more use of orientation and execution statements than verbal explanations, demonstrating the benefits of written
explanations of problem-solving strategies on student’s metacognition. This study identifies benefits of writing in the high school mathematics classroom. However, the benefit of writing in the mathematics classroom has not been determined with younger children has not been determined with younger children.

Bicer, Capraro, Capraro (2013) explored the impact of the writing process on 96 middle school students’ problem-solving skills. Participants were divided into two groups in a STEM after school program. The first group used the writing process to solve mathematical problems, while the other focused on problem solving through test preparation. They found that writing acted as a mediator in the problem-solving process. The students in the writing group showed more growth in tests for cognitive complexity and problem generation, further demonstrating the effects of using writing to develop problem-solving skills in middle school children.

Freeman, Higgins, Horney (2016) explores the impact of writing mathematics notes using a digital notepad and a social mathematics blog on 42 eight to thirteen-year-old students’ ability to communicate mathematical ideas. They found that the informality of using digital tools might help younger students with the challenges of formal mathematical writing, and the barrier it can sometimes create. While responses on the blog were not high in quality, they were accurate, demonstrating that writing can help students think logically. Freeman and colleagues suggest the use of caution and guidance when asking children to write about their mathematical thinking as guiding students in their writing and discussion facilitates achieving the mathematical goals. Further, Freeman and colleagues found that younger children posted three times more frequently than older children in the study, demonstrating the importance of introducing writing to learn mathematics to children as soon as possible.
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Kim and colleagues (2014) looked at the relationships between 531 first grade writing samples scored using an adapted version of 6+1 traits, a writing scoring system based on commonly referred to dimensions of writing, and oral language, reading, spelling, and letter writing automaticity. They found that children’s oral language skills are related to ideas, organization, word choice, and sentence quality 6+1 traits, demonstrating the importance of developing oral language skills, as well as the connection between oral and written communication.

In their study Williams and Casa (2012) facilitated oral discourse about symmetry through the use of the Talk Frame (Casa, 2013), a sequence of three steps where students think about the problem, gather for discussion of the problem, and record conclusions agreed upon by the class. After participating in the Talk Frame first-grade students then wrote about whether or not a picture of a leaf was symmetrical. Williams and Casa found that the students were able to explain how they knew if the leaf was symmetrical or not. They also found that the students were able to combine ideas from the talk frame with their own reasoning. This study demonstrates first-grade children’s ability to write about their thinking with the use of a tool such as the Talk Frame.
Cohen and colleagues (2015) explored the characteristics of second graders’ mathematical writing in the geometry mathematical domain. Students in this study participated in six-week Project M2 units, which encourage verbal discourse and writing about reasoning. Through qualitative analysis of written explanations, Cohen and colleagues developed six categories of written explanations, use of a linking word, reasons, informal vocabulary, formal vocabulary, attempt at math writing, and complete sentences. They found that students in the intervention group outperformed the comparison group in providing reasons, use of formal vocabulary, complete sentences and linking verbs. Cohen and colleagues concluded that specifically addressing vocabulary, use of formal labels, presenting terms after students developed conceptual understanding and use of talk frames contributed to the development of written mathematical explanations.

*First Graders as Communicators*

While this study focuses on first-grade children and their ability to communicate their mathematical thinking, understanding their development as communicators in general is important. Oral language develops before written language (Vygotsky, 1978), therefore, first grade children are often stronger oral communicators than written communicators (Berninger, 2000). This is especially true if transcription skills such as spelling and handwriting have not yet become automatized (Puranik et al., 2012). A common strategy for developing a foundation for writing as children begin to automatize is the use of a mediating tool, referred to as rehearsal by Graves (1983) and Calkins (1994). First-grade children often use pictures as a way to plan for writing. Graves (1983) also suggests that this process helps children become more aware of what they are doing, helping them communicate their thinking. Often teachers recognize that children have developed their oral communication, and see school as a time to develop written communication (Berninger & Abbot, 2010). However oral language continues to develop as children begin learning to write
A question of interest in the current study is whether or not oral mathematical explanations are significantly different than written mathematical explanations. However, to the researcher’s knowledge, no studies have been conducted to examine these relationships. While there is a paucity of research in this area within the literacy field (Shanahan, 2016) a few studies have been conducted. This next set of studies includes those literacy studies that support the development of young children’s oral and written expression, as well as how these two modes of communication are related.

Bourdin & Fayol (1994) compared adults and children’s (ages 7-8 and 9-10) ability to recall and produce a list of words in written and oral language modes. While controlling for the length of the list based on the age of the participant, they found that adults were better able to recall words when writing the list. However, the 7-8-year-old children were superior in their ability to recall the words using oral language.

Kim, Otaiba, Folsom, Greulich, and Purank (2014) investigated oral language and literacy skills and as related to the multiple dimensions of written composition among 527 first-grade children using structural equation modeling. The found four dimensions of writing including, substantive quality, syntactic complexity, productivity, and spelling and writing conventions. Further, they found language and literacy predictors were related to the four writing dimensions, and that children’s oral language “was uniquely related to the substantive quality dimension” (p. 208).

Berninger and Abbot (2010) assessed children’s language by “Ear, Moth, Eye, and Hand” (p. 635) in cohorts beginning in grades 1 or 3 and again in grades 5 or 7 using confirmatory factor analysis. They found that “language skills are related, yet unique” (p. 643). Specifically of interest is their finding that written expression contributed to oral expression in grades 3 and 7. They
also found that there was a relative weakness on the written expression subtest, but do not con-
sider this weakness to be because this subtest is more difficult than the other language subtests.
Rather this is the weakest subtest because written language is the last to fully mature.

Problem Based Learning

Many students find communicating their mathematical thinking challenging because they do not have conceptual understanding of the concepts they are trying to explain. Conceptual understanding is defined by the National Research Council (2001) as “an integrated and functional grasp of mathematical ideas. Students with conceptual understanding know more than isolated facts and methods. They understand why a mathematical idea is important and the kinds of contexts in which it is useful” (p. 118). The following studies were included because they speak to the importance of using problem-based instruction to increase children’s conceptual understanding of addition and subtraction.

Kamii and Dominick (1998) examined first through fourth grade children’s ability to solve problems mentally in individual interviews such as 7+52+186. Of the classrooms, none of the first-grade teachers taught algorithms, two of three second-grade teachers did not use algorithms, one of the three third-grade teachers did not teach algorithms and all four fourth-grade teachers taught algorithms. The no algorithms second and third-grade classes produced the most correct answers. Explicitly teaching algorithms is harmful to a child’s mathematical development because it does not require children to use their own thinking and algorithms ‘unteach’ place value, hindering the development of number sense (Kamii & Dominick, 1998).

Kamii and colleagues (2005) conducted a first-grade math course through the use of physical knowledge activities, such as playing pick-up sticks, and then continuing on to easy addition games such as piggy bang, a card game that requires students to make five. As students were successful, they moved on to more difficult games. Twenty-six low performing first graders were
compared with the performance of 20 low-performing first graders in a traditional math program. The pretest scores for both groups were approximately 79%. The posttest contained 17 mental math problems and an interview of four word problems. Students participating in physical knowledge activities did significantly better than the students in the traditional group on both sections of the posttest demonstrating the value and significance of using physical knowledge activities in young children’s classrooms.

Hiebert and Wearne (1992) explored the impact of conceptually based lessons on place value and two-digit addition and subtraction on 151 first grade students. They found that students in the four classrooms that received conceptually based instruction did statistically better on place value and two-digit addition and subtraction problems. They also found that the decrease in traditional procedural skill did not adversely impact the children’s proficiency in routine problems. This study demonstrates the importance of teaching two-digit addition and subtraction through problem-based instruction rather than traditional procedures and algorithms.

Peltenburg, van de Heuvel-Panuizen and Robitzch (2012) examined the use of indirect addition as a solution strategy for two-digit subtraction among 56 special education students ages eight to twelve. They found that special education children are likely to solve subtraction problems through the use of indirect addition. Further, children who had not been explicitly taught this strategy used it more frequently than students who had received explicit instruction in indirect addition to solve subtraction. The use of indirect addition to solve subtraction demonstrates conceptual understanding in addition and subtraction operations because children using these strategies understand what happens when numbers are added or subtracted. The results of this study demonstrate the importance of providing children with the opportunity to explore mathematics and develop their own solution strategies.
Sociomathematical Norms in the Elementary Mathematics Classroom

The use of oral and written discourse is strongly encouraged by the NCTM Principles and Standards for Mathematics (2001), and the Common Core State Standards for Mathematics (CCSSM, 2010). Traditionally, there is a deeply rooted belief about the way mathematics should be taught, and changing that belief requires teachers to turn the classroom into a place where students can experience mathematics as a creative activity, including the use of writing (Rose, 1989). Children must feel comfortable within their community to share their mathematical ideas and strategies (Lee, 2001; NCTM, 2000). Boaler (2016) explains that children are often unsure of the teacher’s expectations. Being clear about expectations and supporting expectations through action from the beginning of the school year can create a foundation for positive norms within the mathematics classroom.

Facilitating meaningful discourse in the mathematics classroom relies in part on the establishment of sociomathematical norms. Yackel and Cobb (1996) state, “these norms are distinct from general classroom social norms in that they are specific to the mathematical aspects of students’ activity” (p. 458). In a classroom focused on sociomathematical norms, learning opportunities occur when teachers ask children to attempt to make sense of their peers’ mathematical explanations, compare their solutions to the solution strategies of others, and consider and make judgments about the similarities and differences in those solution strategies (Yackel & Cobb, 1996). In a classroom with a solid sociomathematical foundation, the student’s role during mathematical discussions is to explain their thinking to their teacher and classmates, as well as to challenge and question the thinking of their peers (Yackel, 1995). The teacher’s role is to facilitate the discussion, and help children develop a sense of what a meaningful mathematical explanation constitutes (Yackel & Cobb, 1996). The following studies were
included because they inform thinking regarding developing sociomathematical norms within a classroom.

A major aspect of sociomathematical norms is to determine what counts as a mathematical explanation within the classroom. McClain and Cobb (2001) examines sociomathematical norms in Ms. Smith’s first-grade classroom through analysis of daily field notes of three observers over a four-month period. At the beginning of the study, they found that Ms. Smith felt it was important for all of her students to share in mathematical discourse, not wanting to challenge the thinking of her students. However, this produced an environment where students shared their strategies, but they were not engaged in listening to one another. The established norm was that all strategies that were shared were counted as different. McClain and Cobb worked with Ms. Smith to help her understand the mathematical value of sharing varying strategies creating discussions that build on student contributions to achieve the goal of the lesson. After receiving encouragement from the researchers, Ms. Smith began to explicitly describe what it meant to share a different strategy with the course. For example, in a lesson involving subitizing, Ms. Smith gave students a few seconds to look at five chips placed three in the top row and two in the bottom. She then asked her to tell her how many chips were there without counting each chip. One student explained that she knew there were five chips because “I saw three on top and two on bottom.” Another student explained “I saw three at the top and two at the bottom, and um, and um, I could just see three up here, and I knew when you turned it off I could just count 4,5.” Ms. Smith’s response to the second student was that her response was the same as the first. After a third student says, “I saw three plus two, Ms. Smith interrupts and explains what she means by different strategies. She says, “I don’t mean just another way to count, but if you grouped them in a different way, or you saw them in a different way, that’s what will help us.” In lesson, Ms.
Smith wanted students to begin to use grouping as a solution. Ms. Smith developed the sociomathematical norm of understanding what different strategies look like with her students.

Sociomathematical norms are different from classroom to classroom. Lopez and Allal (2007) compared the norms in two third-grade classrooms in Switzerland. The teachers are referred to as Paula and Luke. They found different norms were involved and the role of the students and teachers regarding assessment of the students problem-solving procedure differed. For example, both classrooms developed a norm to explain problem-solving procedures and propose different procedures. Paula’s class furthered their discussion by re-explaining procedures or attempting to explain the procedure of another student. Luke’s class expressed opinions about other student’s procedures, specifically commenting on the effectiveness of the procedures. Paula’s class tried and used different procedures, while Luke’s class tried different procedures, but were encouraged to adopt the most effective procedure. Lopez and Allal attribute this to the varying micro cultures of classrooms, and the way in which transactions between the teacher and students occur.

Levenson, Tirosh, and Tsamir (2009) consider the complexity of sociomathematical norms by investigating the teachers’ endorsed norms, the students’ enacted norms, and the students’ perceived norms. They found that while it was clear that students’ participation in class entailed explaining solutions to problems, students’ enacted norms did not always match the teachers’ endorsed norms. This study demonstrates a need to explicitly establish sociomathematical norms within a community of learners through discourse. Explaining mathematical thinking is difficult for children. This study demonstrates that knowing what to include in a mathematical explanation must be established within a community of learners.
Conclusion

The literature demonstrates the value of integrating oral and written discourse in mathematics instruction. Sharon, the young girl in Levenson et al (2015) asked, “What does it mean to explain?” Many children have a difficult time communicating their mathematical thinking. Developing sociomathematical norms within the classrooms helps children understand what is expected of them when they are asked to explain their reasoning both orally and in writing.

The Common Core State Standards for Mathematics and achievement assessments such as the PARCC are asking children explain their mathematical reasoning. Summative assessments such as the PARCC typically assess children through the use of multiple choice questions. However, this question format does not lend itself to assessing children’s thinking. The most practical way to assess a child’s ability to explain their reasoning on statewide assessments is through writing, leaving many teachers wondering how to support their students in developing the ability to communicate their reasoning.

The literature demonstrates that writing can be used to develop children’s problem-solving ability (Pugalee, 2004). However, many of these studies have been conducted with older children, leaving the question, can writing be used to help young children develop their ability to solve problems? Further, how do teachers begin to build a foundation for written mathematical explanations? Casa (2015) suggests supporting young children in writing about their mathematical thinking by connecting oral explanations with written explanations. Williams and Casa (2012) draw a link between oral and written discourse, however, it does so at a point in time and does not look for change or correlations between oral and written discourse.
Though there have been studies conducted to consider children’s ability to talk and write about their mathematical thinking, few studies have been conducted that consider the relationship between talking and writing about mathematical reasoning and the impact each has on the other. Further, many of the studies conducted look at a point in time rather than changes that occur when increased discourse is introduced through a problem-based instructional format.

This study seeks to explore how oral and written discourse are related prior to the problem-based lessons and after the problem-based lessons and if the use of written mathematical explanations improves children’s conceptual understanding of base ten operations. The children participating in this study engaged in lessons designed to develop base ten understanding within a problem-solving environment. The classes participating in the study also developed sociomathematical norms necessary to determine what qualifies as oral and written mathematical explanations.
3 METHODOLOGY

Overview of the Study

This study involved two groups of first-grade students at a title I elementary school in a large metropolitan area in the southeastern United States. Both groups participated in six problem-based lessons focused on developing base ten understandings within a problem-based learning environment. The intervention group engaged in oral and written discourse during each lesson. The comparison group engaged in oral discourse, but not written discourse. Pre-and Post-assessments of each group included oral and written explanations of the strategies they used to solve a simple story problem. The explanations were scored using a rubric adapted by the researcher from Gavin (2006-2008). The research questions are as follows:

(1) Is there a significant difference between children’s oral and written mathematical explanations before and after the problem-based lessons?

(2) Is there a significant difference in the mean score of the oral and written mathematical explanations of the intervention and comparison groups?

Pilot Study

This study was informed by a pilot study conducted during Spring 2016. The participants in the pilot were 20 first grade students in a suburban southeastern U.S. elementary school. The researcher of this study was a classroom teacher in the building. At the invitation of the first grader’s classroom teacher the researcher provided the students with Problem-Based Lessons (PBL), similar to the structure of the lessons in the current study, once a week over the fall 2015 semester as part of the students’ regular mathematics instruction. After that instruction and in preparation for the dissertation study, the researcher conducted a pilot study with these students in the spring 2016. Specifically, a series of six 45-minute, grade-level appropriate PBL lessons
were presented once a week. The lessons emphasized oral and written discourse. Assessment of student written discourse was conducted before and after the intervention instruction.

With parent permission, data collection occurred at the beginning and at the end of this pilot intervention unit. Student written responses were collected and scored using a 3-point holistic rubric from the New York State Department of Education (2014, p. 11) (see Appendix C) to explore development of written discourse. A paired sample t-test was conducted to compare the pretest scores and posttest scores of all the students. There was a significant difference in the scores on the pretest ($M=1.1$, $SD = .72$) and posttest ($M=2.1$, $SD = .97$); $t(19) = 4.156$, $p = .001$. The standard effect size index, $d$, was 1.17, a large effect size. However, the use of this rubric to score written mathematical explanations proved difficult in analyzing the pilot study data (no oral mathematical explanations were collected during the pilot study). For example, a child receiving three points according to this rubric had the “correct solution to the question and demonstrated a thorough understanding of the mathematical concepts and or procedures in the task. A two-point response “demonstrates a partial understanding of the mathematical concepts and/or procedures in the task,” and a one-point response “demonstrates only a limited understanding of the mathematical concepts and/or procedures in the task.” Determining the difference between assigning a piece of writing a point value of zero, one, two or three based on criteria became problematic as many children’s writing samples did not fit into one category or another, nor did the rubrics reflect the complexities of written mathematical explanation.

The pre-and post-writing samples from the Spring 2016 pilot study were re-analyzed using the adapted rubric outlined below in the Fall of 2016. Another graduate student scored the writing samples using the rubric to establish interrater reliability. Percent absolute agreement was established for each indicator on the rubric (mathematical concepts .90, mathematical
communication of .80 and vocabulary 1). Cohen’s Kappa was also found for each indicator (mathematical concepts .84, mathematical communication .75 and vocabulary 1. The three-item rubric was analyzed using data collected in the pilot study and found reliable. Cronbach’s alpha for the 3 items was .84. There was a significant difference in the mean scores on the pretest ($M=3.1, SD = 2.5$) and posttest ($M = 5.35, SD = 2.43$); $t(19) = 4.265, p = .001$. The standard effect size index, $d$, was .91, demonstrating large practical significance.

Though the results of the pilot study were significant, reflection on the lessons and student writing revealed areas in which the intervention and assessment could be improved. As a result of this work, the following methods were developed for use in the current dissertation study reported here.

**Participants and Context**

The participants in the current study were 50 first grade students in four classrooms in a suburban southeastern U.S. Title I elementary school. The most recent demographic information from Governor’s Office of Student Achievement (2015) indicates that many of the children in this school are from low-socioeconomic families as 87% of the children are eligible for Free/Reduced meals. The racial make-up of the school is, 63% African-American, 19% Hispanic, 13% Caucasian, 4% multiracial, and 1% Asian. Fifteen percent of the student body is classified as Limited English Proficient. The teachers of the students included four female teachers, three African-American and one Caucasian. The two comparison teachers are African American. One Intervention teacher is African American while the other is Caucasian. All students from each of the four classrooms with consent to participate were included in the study. Of those students, 20 are male, 30 are female, 9 are English Language Learners, 6 have an IEP, and 8 are in the Early Intervention Program. There were 23 students in the comparison group, 7
males, 16 females, 4 have an IEP, 3 are in the Early Intervention Program, 2 are English Language Learners, and 2 are in the gifted program. There were 27 students in the intervention group, 13 males, 14 females, 2 have an IEP, 5 are in the Early Intervention Program, 7 are English Language Learners, and 2 are in the gifted program.

The school’s current mathematics curriculum is Eureka Math. Eureka Math is a widely utilized K-12 mathematics curriculum aligned with the Common Core State Standards that is free for use in schools (Great Minds, 2016). The first-grade teachers proceed through the Eureka Math Module lessons four days a week. Students participated in the problem-based lessons with the research one day per week. Classroom teachers on the team were asked to proceed through their current Eureka module lessons as usual throughout the study. Each lesson contains the same lesson structure including fluency practice, application problem, concept development, and 10 minutes for student debrief. While participating in this study, the first-grade students were working within the Geometry modules, however, they had just completed their unit on base-ten addition and subtraction.

The modules focusing on the content standards for this study are found in module 4. Topics in module 4 include tens and ones, comparison of two-digit numbers, addition and subtraction of tens, addition of tens or ones to a two-digit number, varied problems within 20, addition of tens and ones to a two-digit number. Within this module, students were presented with the strategy of direct modeling counting on, single-digit sums, and add ones and ones or tens and tens. The focus of the research study was to add and subtract tens and ones to a two-digit number, or adding and subtracting multiples of ten because this set of standards explicitly states that students explain reasoning used.
Design

A pretest-posttest nonequivalent groups quasi-experimental design was used to determine if the writing intervention had an impact on children’s ability to explain their mathematical thinking in writing. The four first-grade classes were randomly assigned either as part of the comparison group or as part of the intervention group with students in two classes in the intervention group and students in two classes in the comparison group. A pre/post design was enacted with individual written and oral solutions to simple word problems collected by the researcher from each student before and after the problem-based lessons. The problem-based lessons consisted of six 45-minute, grade-level appropriate lessons that focused on developing base ten understandings. Lessons in all four classes included an opportunity to participate in oral discourse to explain and share solution strategies. Students in the two intervention classes also had an opportunity to write about their strategies during each lesson.

Prior to the problem-based lessons, the researcher observed a mathematics lesson in each classroom to develop a clearer understanding of the context within which she would be teaching and to observe a typical mathematics lesson, as well as give the students time to become familiar with the researcher. The researcher also interviewed each teacher prior to the unit to determine how their mathematics lessons are structured and the ways in which the students typically participate and to gain an understanding of the sociomathematical norms that were already be in place (McClain & Cobb, 2001).

Instrument Development: Rubric for Mathematical Explanations

Teachers are encouraged to engage children in writing in mathematics (NCTM, 2000). However, there are very few holistic rubrics available to assess young children’s written mathematical explanations. A 3-Point Holistic Rubric from the New York State Department of
Education Grade 3 Common Core Mathematics Test Guide (2015) was used to score the pilot study writing samples, however, it did not account for the complexities of written mathematical explanations. Therefore, it was determined that a different rubric would need to be used to score the writing samples.

After reaching out to other researchers and conducting further searches, a rubric was identified from the Project M³ Series (Gavin, et al., 2008). Project M³, a curriculum developed for talented elementary students, includes two “Think Deeply” questions for each lesson. These questions require students to engage in writing about mathematical concepts. The Rubric for Student Mathematicians Journal (Gavin et al., 2006-2008) is a part of the Project M³ curriculum for teachers to assess the written responses of talented elementary students in grades three through five. This rubric includes three indicators, mathematical concepts, mathematical communication, and mathematical vocabulary. While the indicators of this rubric were appropriate for the context of this study, for interrater reliability reasons, the descriptors for each indicator needed to be more specific. Therefore, the descriptors for each indicator in this rubric were adapted using the data collected through the pilot study, existing research conducted on children’s written mathematical explanations, and sociomathematical norms. See Appendix C for the original Rubric for Student Mathematicians Journal and adapted Rubric for Mathematical Explanations.

The first category identified by Gavin and colleagues (2006-2008) is Mathematical Concepts. In this category, the assessor is identifying a child’s conceptual understanding of the mathematical concepts about which they are writing. In this study, this category focuses on conceptual understanding of the use of base-ten to add and subtract. Specifically, demonstration of base ten understanding through expressing that tens are added to tens and ones are added to
ones, or counting by tens from one of the numbers in the story problem indicate understanding of this concept as indicated in the standard (Carpenter et. al., 2015; CCSSM, 2010). Therefore, students who did not provide an explanation, or did not have access to the problem will receive zero points for the indicator. Oral and written explanations that demonstrate the student had access to the problem, but has a lack of procedural and conceptual understanding of base-ten concepts will receive one point. Oral and written explanations that demonstrate partial or inconsistent base-ten understanding will receive two points. Oral and written explanations that demonstrate a consistent conceptual understanding of base-ten concepts will receive three points for the indicator. For example, a child in the pilot study wrote, “He was at his home with his collection he went to the store and got 40 more he counted his baseball cards and he got 76,” to explain her solution for 36+40. While this response indicates access to the problem, and the correct solution, it would be a 1 on this indicator because it does not indicate that the child used base ten understanding to find a solution to the problem. Another child wrote “I put 3 tens and six ones then I put seven tens and I made the answer,” as the explanation for 36 + 40. This child would receive two points because they referred to the use of tens and ones, but they have not demonstrated an understanding of the use of base ten in their solution strategy as they referred only to one quantity. As an explanation for solving 36 + 40 the same child wrote, “So first I put down 3 10s and 6 1s and then I put 40 10s down and then I added.” This response would be a 2 because child is demonstrating conceptual understanding of base ten, while the explanation begins with accurate understanding (3 tens and 6 ones) the child made a minor error in the explanation, writing 40 tens rather than 4 tens. An example of a three-point explanation from the pilot study is, “I used 3 tens 6 ones and made 36. I used 4 tens and 0 ones and made 40. I counted them and made 76.”
The second indicator in Gavin and colleagues (2006-2008) rubric is *Mathematical Communication*. The Common Core State Standards for Mathematics (2010) this study focuses on specifically requires that children explain the reasoning used. Within the context of this study, a student who does not provide an oral or written response, or the response is unintelligible, will receive zero points. An oral or written explanation that only states the tools used to find a solution, but does not refer to how the tools were used will receive one point. An oral or written explanation that states a partially developed solution, or reasoning is incomplete will receive two points. An oral or written explanation that adequately states a developed solution or clearly describes the steps taken to find a solution will receive three points. For example, a child in the pilot study wrote, “I used ten blocks and one blocks and I got my answer 58.” This response would receive one point because it attempts mathematical communication, but only states the tools used with no mention of how the tools were used. Another child wrote, “I put 36 lines. I put 40 lines. I put my answer 76.” This child’s response would receive a 2 because they have explained that they have used thirty-six and forty, however, the reasoning is incomplete and does not clearly state how they got to 76. A third child wrote, “I put 2 10s and put 28 3 10s and made 50 and then I put 28+30=50.” This response would receive 3 points because it clearly communicates what the child did to solve the problem.

The use of mathematical vocabulary is an important indicator in written mathematical explanations. Gavin and colleagues (2006-2008) approach vocabulary in terms of use and appropriateness, while Cohen et. al (2015) counts the number of terms used within a response. In order to quantitatively compare pre-and post-vocabulary usage, a word count will be used in this indicator. Words that are used incorrectly or do not refer specifically to the mathematics involved in finding a solution will not be counted. For example, children in the pilot study (2016)
said they “drew circles.” While circle is a mathematics term, it does not refer to the mathematics necessary to solve this problem. If the child said they drew ones, ones would count as a vocabulary term because it is a formal place-value term. Due to a child’s difficulty transcribing (Berninger et al., 1992) words they may not know how to spell or frequently see written as numerals or words will be accepted. For example, 10s will be excepted as tens. Cohen et. al (2015) differentiates the use of formal and informal vocabulary, identifying the total number of terms used for each. Both formal and informal vocabulary will be included as one indicator in this study because the NCTM (2000) recommends that children not be pushed to use formal terms, allowing children to grapple with their ideas can help them develop ownership of formal mathematical vocabulary. Therefore, some informal mathematical language may be acceptable in the writing samples of young children. Children will receive 0 points for including 0 vocabulary words, 1 point 1 word, 2 points for 2 words or 3 points for 3 or more vocabulary words. If a child uses the same word more than once, it will be counted as one word. A list of mathematical terms was compiled from the vocabulary used by children in the pilot study and the Corresponding Georgia Mathematics Unit (tens, ones, add, subtract, counted, equals, together, apart, take away, more, solved).

Children in first grade are beginning writers who have not achieved automaticity of the transcription process (Puranik et al., 2012). Young children typically employ invented spelling prior (Steffler et al., 1998), therefore, children’s use of invented spelling, numerals, and mathematical symbols were acceptable in their written explanations, and counted as vocabulary where applicable.

The complete Mathematical Explanation Rubric and examples of scoring can be found in Appendix C.
Data Collection

Data for the study were collected by the researcher at two time points: before the 6-lesson unit and after the 6-lesson unit. The Common Core State Standards for Mathematics Glossary (2010) Table 1 indicates 12 problem types. Asking first grade children to write mathematical explanations for all 12 problem types would be overwhelming and time-consuming. Therefore, the pre-and post-assessments was comprised of one story problem type. The Add To: Result Unknown problem type was selected because the children in the pilot study demonstrated that this problem type is accessible to most children. For continuity, the problem types for the pre-assessment and post-assessment remained the same.

Written responses to problems were collected pre/post in a whole-class lesson format where students are asked to solve a problem and write their solution process. A separate story problem, using the same Add To: Result Unknown problem type, was used to conduct pre/post individual interview assessments of each child. Interviews were conducted by the researcher with children individually. The children were read the assessment question, given time to find the solution, and then asked to explain how they found the solution. Their explanations were audio recorded and transcribed. If they chose to, students were able to use manipulatives as support for solving the problems on the oral and written assessments (Carpenter et al., 1999). The pre-and post-assessments can be found in Appendix A. A research journal was kept at all points of data collection so that the researcher would have reflective notes to use throughout analyses. Teacher interviews were conducted prior to the intervention lessons and at the completion of the intervention lessons. The interviews were audio recorded and transcribed.

The Problem-Based Lessons

Once pre-assessments were completed, each class participated in the six problem-based lessons designed to develop the children’s conceptual understanding of adding or subtracting
two-digit numbers and multiples of ten; each lesson included the components described below. The researcher served as the teacher in all the problem-based lessons.

At the beginning of each lesson, the teacher engaged students by telling them a story problem, purposefully engaging the students in a real-world situation. For example, in the second lesson of the unit, the teacher read the students the story, The Penny Pot by Stuart Murray. The students were then asked to solve the problem, “Jessie had 39 cents. She needed 50 cents to get her face painted. How many pennies does she need to collect in her penny pot to get her face painted? Oral discourse was used as a medium to unpack the problem and determine if the students have understood the situation, without giving away too much about how the problem should be solved (Carpenter et al., 2015). Therefore, the teacher may pose a question to the students such as “Will she need to collect more than 50 pennies or less than 50 pennies? “How do you know?” This type of questioning helps children begin to think about what the solution may look like and that in this case, the solution will be greater than 50. Once the teacher was certain most students understood the problem, children began independently working on their solution strategies.

During this time, students used their strategies to solve the story problem. Students typically began solving problems using a direct modeling approach (Baroody, 1993; Carpenter et al., 2015), representing both quantities using manipulatives or pictures to find a solution. As children continued to gain a deeper understanding of mathematical concepts, and as they listened to the solution strategies of their peers, they began to transition to more abstract strategies such as counting techniques and then invented algorithms (Baroody, 1993; Carpenter et al, 2015).

While children worked on their solution strategies, the teacher monitored students and facilitated oral discourse with individual students to understand students’ mathematical reasoning
(Smith & Stein, 2011; Yackel, 1995). Discourse was facilitated through questioning. For example, the teacher asked questions such as, “How did you solve the problem?” “What is your next step going to be?”

Calkins (1994) suggests that young children begin to engage in writing by drawing a picture first. Therefore, the children were asked to show their work by drawing representations. These representations also served as a formative assessment piece throughout the unit to monitor the solution strategies of the students and any misconceptions they may have had about addition, subtraction, and the use of base-ten.

Mathematical thinking should be shared among students (Ernest, 1998; NCTM, 2000). Therefore, students in both the intervention group and comparison group shared their solution strategy with a partner. Sociomathematical norms for sharing solution strategies with partners and establishing what counts as explaining a mathematical solution (Yackel & Cobb, 1996) were established in each classroom.

Children in the intervention group then had time to write about how they found the solution, while the children in the comparison group did not write about their solution strategies and moved to the whole group discourse portion of the lesson. Establishing the sociomathematical norm of what counts as a written mathematical explanation (Yackel & Cobb, 1996) was essential in the intervention group.

During worktime, the teacher selected students to share their solution strategy with the class. The teacher selected a variety of solution strategies (direct modeling, counting, invented algorithm) and sequenced the students sharing based on the sophistication of the strategy (Smith & Stein, 2011). Students with the least sophisticated strategy shared first so that students who did not have access to the problem, had a chance to hear a solution strategy they were able to
understand (Smith & Stein, 2011). When students engage with mathematics using their thinking, it becomes more natural for them to explain their solution process, while also allowing the teacher to facilitate more meaningful discourse (NCTM, 2015). Sociomathematical norms for the class discourse were established, including what counts as a mathematical solution, what should be included when sharing solution strategies with the class and how to ask each other questions about solution strategies (Yackel & Cobb, 1996).

After the students shared their solution strategies, Carpenter and colleagues (2015) suggest facilitating discourse about the similarities and differences of the solution strategies. This was also when mathematical goals were pushed into the lesson (McClain & Cobb, 2001). Throughout this unit, the mathematical goals focused on the CCSSM (2010) standards for adding and subtracting a two-digit number and multiple of ten. To develop conceptual understanding, questions were asked based on student thinking, moving children from a direct modeling solution strategy to a counting strategy, which demonstrates a child’s ability to conserve one number and count up or down (Carpenter et al., 2015). See Appendix B for a sample lesson plan and lesson reflection.

**Data Analysis**

Oral explanations were transcribed and scored using the adapted *Rubric for Mathematical Explanations* described above and provided in Appendix C. Written explanations were also scored using the same rubric. A sample of 20 assessments oral and 20 written assessments were scored by two different scorers using the Rubric for Mathematical Explanations Rubric to establish interrater reliability prior to scoring the samples from this study. If the scorers disagreed on a score, they engaged in discussion to agree on a score for the assessment. Percent absolute agreement was established to determine the percent agreement for each level of the
rubric, while Cohen’s Kappa was calculated to account for the agreement occurring by chance. Percent absolute agreement will be good if the interrater reliability outcome is .8 or greater and Cohen’s Kappa is between .61 and .81 (Graham, Milanowski, & Miller, 2012). The three-item rubric was analyzed for internal consistency using Cronbach’s alpha. Cronbach’s alpha is used to determine internal consistency of assessments with no correct answer. A reliable assessment will have a Cronbach alpha greater than .70 (Salkind, 2013; Cortina, 1993).

**Research Questions**

(1) Is there a significant difference between children’s *oral and written* mathematical explanations before and after the intervention?

Once a score for each child’s written and oral pre-and post-assessment was determined, dependent *t*-test analyses were conducted to determine if a significant difference existed within the children’s ability to talk and write about their mathematical thinking before and after the problem-based lessons. Young children often have a difficult time communicating their mathematical thinking (Moyer, 2000). The problem-based lessons in this study were designed to increase oral discourse and children’s conceptual understanding of the mathematics. A significant difference in the comparison group’s ability to talk and/or write about their solution strategies would mean the problem-based lessons with increased oral discourse was enough to increase the children’s ability to talk and/or write about their solution strategies. A significant difference in the intervention group’s ability to talk and/or write about their solution strategies would mean that problem-based lessons with increased oral and written discourse improved children’s ability to talk and write about their solution strategies.

*T*-tests were also conducted to determine if a significant difference exits between children’s oral and written responses of the intervention group and the comparison group before and after the problem-based lessons. Young children are expected to explain their reasoning, but their
mathematical reasoning is not yet assessed through writing (CCSSM, 2010). If there is not a significant difference between oral and written mathematical explanations, this would mean that using written explanations in young children’s classroom is warranted as a means for assessing a child’s ability to explain his or her reasoning. It also further justifies the use of oral discourse as a foundation for developing written mathematical discourse (Casa, 2015). Further, Pugalee (2004) found that older students included more detail and reasoning about their strategies in written explanations compared to their oral explanations. However, Bourdin and Fayol (1994) found that oral production is easier than written production for young children. A dependent $t$-test analysis will contribute to understanding if the process of translation when writing significantly interferes with young children’s written mathematical explanations.

(2) Is there a significant difference in the mean score of the oral and written mathematical explanations of the comparison and intervention groups?

While the $t$-test analyses were conducted to look at differences within each group, an analysis of covariance (ANCOVA) was conducted to determine if a significant difference exists between the mean written rubric scores of the intervention group and the comparison group, as well as to determine the effect of the added writing component in each lesson on the intervention group’s written mathematical explanations. ANCOVA was used to account for differences in written mathematical explanations before the problem-based lessons, increasing the power of the findings. The covariate was the pre-assessment rubric scores. The dependent variable was the post assessment rubric scores, where $k=2$, $n \approx 50$. The minimal detectable effect size where $k = 2$ is $.41$ for a sample size of $50$. This effect size is considered moderate significance (Cohen, 1988).
4 RESULTS

Data Analysis

This study used a pre/post comparison design where data were collected from two groups of first graders before and after a six-week instructional intervention in which writing about mathematics was the independent variable. Results from the pre-and post-assessments follow.

Reliability

Data from both groups were collected in two forms, children’s oral explanations of Add To: Result Unknown problems and children’s written explanations of Add To: Result Unknown problems before and after the six-week instruction unit. Each of the pre-and post-assessments were scored independently by two scorers using the *Rubric for Mathematical Explanations adapted from Gavin and colleagues (2006-2008)* found in Appendix C. Percent absolute agreement was calculated for each of the three indicators, Concepts ($\kappa = .89$), Communication ($\kappa = .86$) and Vocabulary ($\kappa = .93$). Cohen’s Kappa was also determined for each of the three indicators, Concepts ($r = .81$), Communication ($r = .80$), and Vocabulary ($r = .89$). Internal consistency of the rubric was determined using Cronbach’s alpha. The Cronbach alpha value for the Rubric for Mathematical Explanations was .85. Percent absolute agreement is considered reliable if greater than .6 and Cohen’s Kappa is considered reliable if above .6 (Graham, Milanowski, & Miller, 2012), therefore the raters were reliable in their scoring using the *Rubric for Mathematical Explanations*. Cronbach alpha, used to determine internal consistency for assessments with no correct response, is considered reliable if greater than .7 (Salkind, 2013; Cortina, 1993), therefore the rubric can be considered internally reliable.

Results

*Research Question 1*
Is there a significant difference between children’s oral and written mathematical explanations before and after the problem-based lessons?

In order to answer question one, a paired-sample t-tests were conducted to determine differences in the children’s oral and written mathematical explanations before and after the problem-based lessons. Because more than one t-test was conducted using the same set of data, the Bonferroni Method was used to determine the alpha level to avoid a type I error, falsely finding significant results (Armstrong, 2014). Since eight t-tests were conducted (see Table 1 and 2), an alpha level of .006 was used to determine significance for each t-test. Effect size was also calculated and reported for each t-test using G*Power 3.1 (Faul, Erdfelder, Buchner, & Lang, 2009). Effect size is reported to communicate the size of the effect of results (Wright, 2003). Cohen (1988) interprets an effect size around .2 as small, .5 as moderate, and .8 as large. The greater the effect size, the more practical significance of the results. Normality checks were carried out and the assumptions met.

Comparing Means Between Discourse Modes

A paired-sample t-test was conducted to compare the mean oral and mean written scores of the comparison group and the intervention group prior to the problem-based lessons. There was a significant difference in the mean scores for oral explanation scores ($M = 5.43, SD = 2.5$) and written explanation scores ($M = 3.48, SD = 2.52$), $t(22) = 4.8914, p = .0001$ of the comparison group. The standardized effect size index, $d$, was .60. There was also a significant difference in the mean oral explanation scores ($M = 4.85, SD = 2.33$) and mean written explanation scores ($M = 3.37, SD = 2.31$), $t(27) = 3.9502, p = .0005$ of the intervention group. The standardized effect size index, $d$, was .46, a moderate effect size. These results suggest that prior to the problem-based lessons, there was a significant difference within both group’s ability to explain their
mathematical thinking using oral language versus their ability to explain their thinking using written language.

Paired-sample *t*-tests were conducted to compare the mean oral and written scores of the students in the comparison and intervention groups at the completion of the problem-based lessons. There was *not* a significant difference in the mean scores for oral explanations (*M* = 5.62, *SD* = 2.50) and written explanation mean scores (*M* = 4.67, *SD* = 2.61), *t*(21) = 2.3171, *p* = .0312 for the comparison group. A small standardized effect size index, *d* = .27 was found. There was also *not* a significant difference in the mean scores for oral explanations (*M* = 6.36, *SD* = 1.87), and mean scores for written explanations (*M* = 6.04, *SD* = 2.14), *t*(25) = .4102, *p* = .4102 for the intervention group. A small standardized effect size index, *d* = .11 was found. The post results showed no significant difference in the mean scores of students from both groups on the oral and written explanations assessment. However, there was less difference between the intervention groups oral and written scores than the comparison groups oral and written scores.

**Table 1. Results of t-test and Descriptive Statistics for Comparing Between Discourse Modes**

<table>
<thead>
<tr>
<th>Group</th>
<th>Discourse Mode</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Oral</td>
<td>Written</td>
</tr>
<tr>
<td></td>
<td><em>M</em></td>
<td><em>SD</em></td>
</tr>
</tbody>
</table>
| Pre-Comparison | 5.43 | 2.50 | 23 |  3.48 | 2.25 | 23 | 22 | 4.8914* | .60  
| Pre-Intervention | 4.85 | 2.33 | 27 |  3.37 | 2.31 | 27 | 27 | 3.9507* | .46  
| Post Comparison | 5.62 | 2.50 | 21 |  4.67 | 2.61 | 21 | 20 | 2.3271 | .27  
| Post Intervention | 6.36 | 1.87 | 25 |  6.04 | 2.14 | 26 | 24 | 0.8382 | .11  

*p* < .006
Comparing means within Discourse Modes

Paired-sample *t*-tests were conducted to compare the mean oral explanation scores each group received before and after the problem-based lessons (See Table 2). For the intervention group, there was a significant difference between the oral explanation mean score before the problem-based lessons ($M = 4.85$, $SD = 2.33$), and the mean oral explanation scores ($M = 6.36$, $SD = 1.87$), $t(24) = 4.1167$, $p = .0004$, at the completion of the problem-based lessons. A moderate effect size, $d = .50$, was found. However, for the comparison group, there was not a significant difference in the mean oral explanation scores prior to the lessons ($M = 5.43$, $SD = 2.50$) and the mean oral explanation scores at the completion of the lessons ($M = 5.62$, $SD = 2.50$), $t(20) = -.17$, $p = .8667$. The standardized effect size index, $d = .06$ was small. These results suggest that adding writing to problem-based lessons may increase students’ ability to explain their mathematical thinking orally.

Paired-sample *t*-tests were also conducted to compare the mean written explanation scores each group received before and after the problem-based lessons. For the intervention group, there was a significant difference between the written explanation mean score before the problem-based lessons ($M = 3.37$, $SD = 2.31$) and at the completion of the problem-based lessons ($M = 6.04$, $SD = 2.14$), $t(25) = 5.2021$, $p = .0001$. The standardized effect size index, $d$, was .87, indicating a large effect size and practical significance. For the comparison group, there was not a significant difference between the written explanation mean scores before the problem-based lessons ($M = 3.48$, $SD = 2.52$) and at the completion of the problem-based lessons ($M = 4.67$, $SD = 2.61$), $t(20) = 1.8199$, $p = .0838$. The standardized effect size index, $d$, was .34, indicating a small to moderate effect size. These results suggest that adding writing to an instructional
problem-based unit may increase student’s ability to communicate their thinking in writing in a short 6-week period of time. Further, these results suggest that simply engaging in problem-based lessons where oral discourse is used as a medium to explain mathematical thinking was not enough to produce more detailed written explanations among the children in the comparison group.

Table 2. Results of t-test and Descriptive Statistics for Oral and Written Explanation Scores

<table>
<thead>
<tr>
<th>Group</th>
<th>Time</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Pre</td>
<td></td>
</tr>
<tr>
<td></td>
<td>M</td>
<td>SD</td>
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<tr>
<td>Oral Comparison</td>
<td>5.43</td>
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<tr>
<td>Oral Intervention</td>
<td>4.85</td>
<td>2.33</td>
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<tr>
<td>Written Comparison</td>
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<td>2.52</td>
</tr>
<tr>
<td>Written Intervention</td>
<td>3.37</td>
<td>2.31</td>
</tr>
</tbody>
</table>

* p < .006.

Research Question 2

Is there a significant difference between the mean score of the oral and written mathematical explanations of the intervention and comparison groups?

Intervention effects on oral mathematical explanations were evaluated using a one-way Analysis of Covariance using groups (comparison and intervention) as the independent variable, post oral explanation scores as the dependent variable, and the pre-oral explanation scores as a covariate (See Table 3). Levene’s test and normality checks were carried out and the assumptions met.
ANCOVA analysis revealed significant differences between the post oral explanation scores of the intervention and comparison groups, $F(1,43) = 8.992, p = .004$. The comparison group’s adjusted mean score ($M = 5.299, SD = 2.4995$) was significantly lower than the intervention group’s adjusted mean ($M = 6.629, SD = 1.8682$) on the post oral explanations rubric. The standardized effect size index, $d$, was .46, indicating moderate practical significance. These results suggest that adding written discourse to problem-based mathematics lessons that encourage oral discourse increased the children in intervention group’s ability to orally explain their mathematical thinking.

Table 3. Descriptive Statistics for Oral Mathematical Explanations Scores

<table>
<thead>
<tr>
<th>Group</th>
<th>Observed Mean</th>
<th>Adjusted Mean</th>
<th>SD</th>
<th>n</th>
</tr>
</thead>
<tbody>
<tr>
<td>Comparison Group</td>
<td>5.619</td>
<td>5.299</td>
<td>2.4995</td>
<td>21</td>
</tr>
<tr>
<td>Intervention Group</td>
<td>6.360</td>
<td>6.629</td>
<td>1.8682</td>
<td>25</td>
</tr>
</tbody>
</table>

Table 4. ANCOVA Results for Oral Mathematical Explanations Scores

<table>
<thead>
<tr>
<th>Source</th>
<th>$df$</th>
<th>$SS$</th>
<th>$MS$</th>
<th>$F$</th>
<th>$\eta^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-Oral</td>
<td>1</td>
<td>115.473</td>
<td>115.473</td>
<td>53.254*</td>
<td>.553</td>
</tr>
<tr>
<td>Group</td>
<td>1</td>
<td>19.497</td>
<td>19.47</td>
<td>8.992*</td>
<td>.173</td>
</tr>
<tr>
<td>Error</td>
<td>43</td>
<td>93.239</td>
<td>2.168</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*p<.05

Intervention effects on written mathematical explanations were evaluated using a one-way Analysis of Covariance using groups (comparison and intervention) as the independent variable,
post written explanation scores as the dependent variable, and pre-written explanation scores as
the covariate (See Table 4). An ANCOVA revealed statistically significant differences between
the post written explanation scores of the intervention group and the comparison groups $F(1,44) = 6.173, p = .017$. The comparison group’s adjusted mean score ($M = 4.446, SD = .2.61$) was
significantly lower than the intervention group’s adjusted mean ($M = 6.127, SD = 2.14$) on the
post written explanation rubric. The standardized effect size index, $d$, was .37, indicating
moderate practical significance. These results suggest that adding written discourse to problem-
based mathematics lessons that encourage oral discourse increased the children in intervention
group’s ability to write about their mathematical thinking.

| Table 5. Descriptive Statistics for Written Mathematical Explanations Scores |
|---------------------------|---------------------------|----------------|--------|
| Group                     | Written Mathematical Explanations | | |
|                           | Observed Mean | Adjusted Mean | SD | n |
| Comparison Group          | 4.667         | 4.556         | 2.61 | 21 |
| Intervention Group        | 6.038         | 6.127         | 2.14 | 26 |

| Table 6. ANCOVA Results for Written Mathematical Explanations Scores |
|---------------------------|----------------|----------------|--------|
| Source                    | $df$ | $SS$ | $MS$ | $F$ | $\eta^2$ |
| Pre-Writing               | 1    | 49.148 | 49.148 | 10.680* | .195 |
| Error                     | 44   | 202.48 |        |        |        |

*p<.05
Summary

In this study, the mean scores of the mathematical explanations were compared in three different sets of analyses. See Table 7 and 8 for a brief explanation of each set of analyses.

<table>
<thead>
<tr>
<th>Analyses</th>
<th>Group</th>
<th>Comparison</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between Discourse Modes</td>
<td>Intervention</td>
<td>Pre-Oral and Pre-Written Explanations</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Post-Oral and Post-Written Explanations</td>
</tr>
<tr>
<td></td>
<td>Comparison</td>
<td>Pre-Oral and Pre-Written Explanations</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Post-Oral and Post-Written Explanations</td>
</tr>
<tr>
<td>Within Discourse Modes</td>
<td>Intervention</td>
<td>Pre-Oral and Post-Oral Explanation</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Pre-Written and Post-Written Explanation</td>
</tr>
<tr>
<td></td>
<td>Comparison</td>
<td>Pre-Oral and Post-Oral Explanation</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Pre-Written and Post-Written Explanation</td>
</tr>
</tbody>
</table>

Table 8. Explanation of ANCOVA Analyses

<table>
<thead>
<tr>
<th>Analyses</th>
<th>Language Mode</th>
<th>Comparison</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between Groups</td>
<td>Oral</td>
<td>Intervention Group and Comparison Group</td>
</tr>
<tr>
<td></td>
<td>Written</td>
<td>Intervention Group and Comparison Group</td>
</tr>
</tbody>
</table>

In summary, when comparing mean scores of the oral and written explanations within between discourse modes, the results indicate that prior to the problem-based lessons, there was a significant difference between both groups ability to talk and write about their mathematical thinking. However, at the completion of the problem-based lessons, there was not a significant difference between both groups ability to talk and write about their mathematical thinking. These
results indicate that the problem-based lessons decreased the difference in the children’s ability to talk and write about their thinking. In this case, the null hypothesis for the comparison group would normally have been rejected, however, due to the multiple comparisons, a stricter than the typical alpha of .05 ($p = .006$) was used to avoid a Type I, more serious error, therefore leading to a possible Type II error of not rejecting the null hypothesis when there is actually a statistical difference (Armstrong, 2014). Therefore, it is difficult to generalize that there was no significant difference in the children’s ability to talk and write about their mathematical thinking within these findings beyond the students in this study.

When comparing the pre- and post-assessments *within* discourse modes the results indicate that adding writing to the problem-based lessons increased the children in the intervention group’s ability to write about their mathematical thinking as there was a significant difference, with a large effect size ($d = .87$) between their ability to write about their thinking before and after the problem-based lessons where they had the opportunity to write about their thinking during each lesson. Further, the intervention group’s ability to talk about their mathematical thinking was also significantly different, with a moderate effect size, ($d = .5$) between their ability to talk about their thinking before and after the problem-based lessons where they had the opportunity to write about their thinking. However, there were no significant differences in the comparison groups ability to talk or write about their thinking before and after the intervention. These results suggest that the problem-based lessons, focusing only on oral discourse was not enough to improve children’s ability to talk about their mathematical thinking for this group of children.

When comparing mean scores of the intervention group and comparison group, the results indicate that there was a significant difference between the groups ability to talk and write
about their mathematical thinking. Since both groups participated in the similar problem-based lessons, the results suggest that adding written discourse to problem-based lessons that encourage oral discourse increased the students in the intervention group’s ability to both talk and write about their mathematical thinking more than the comparison group, who did not have the opportunity to write about their mathematical thinking.
5 DISCUSSION

Learners who have the opportunity to reflect on their thinking and justify their solution strategies increase their conceptual understanding of mathematics (NCTM, 2015; NRC, 2001). When students also have the chance to make sense of the mathematical explanations of their peers and compare others’ solutions with their own, even more mathematical growth occurs (Carpenter et al.; NCTM, 2015). However, communicating mathematical thinking is difficult for young children (Moyer, 2000). This is often because of a traditional procedurally-based curriculum (NRC, 2001), low cognitive demand tasks (Stein et al., 2009), and lack of sociomathematical norms (Cobb, Wood, & Yackel, 1993).

A pilot study, conducted in the Spring of 2016, found that adding writing to problem-based lessons increased children’s ability to explain their mathematical thinking (Venuto & Hart, 2017). However, several questions resulted from this study. For example, the researcher/teacher wondered if the problem-based lessons also had an impact on the children’s ability to use oral language to explain their mathematical thinking. Further, the pilot study left questions about whether or not children’s use of oral language to discuss solution strategies was enough to result in more detailed written explanations. Therefore, two research questions were posed: (1) Is there a significant difference between children’s oral and written mathematical explanations before and after the problem-based lessons, and (2) Is there a significant difference between the mean score of the oral and written mathematical explanations of the intervention and comparison groups?

Overview of the Study

A quasi-experimental pre/post design was used in this study where pre- and post-oral and written mathematical explanations were collected at the beginning and completion of a six-lesson
problem-based unit in the numbers and operations domain with first graders. The purpose of this study was to explore first-grade children’s ability to talk and write about their mathematical thinking within the context of problem-based lessons. Often, children have a difficult time explaining their thinking because they have not developed conceptual understanding, and are simply trying to explain a procedure (NRC, 2001). Therefore, problem-based lessons were utilized to help children develop conceptual understanding and in turn deepen their ability to communicate their thinking through talking and writing. Both groups of students (comparison group and intervention group) participated in problem-based lessons rich in oral discourse that focused on adding and subtracting two-digit numbers and multiples of ten. The intervention group also wrote about how they solved problems during each lesson.

At the completion of the study, the assessments were scored by two independent scorers using the Rubric for Mathematical Explanations. The rubric was adapted from the Student Mathematicians Journal Rubric (Gavin et al., 2006-2008) found in Appendix C. Once scores were determined for each of the work samples, reliability was established for both inter-rater reliability and internal consistency of the rubric. The assessment scores were then analyzed using t-test and ANCOVA analysis. T-tests were conducted to determine if there was a significant difference in the intervention and comparison groups ability to write and talk about their mathematical thinking before and after the problem-based lessons. T-tests were also conducted to compare the mean oral explanation scores within the comparison and intervention groups, as well as to compare the mean written explanation scores within the comparison and intervention groups. ANCOVA analysis were conducted to determine if there was a difference in the mean oral and written explanation scores between the intervention group and comparison group.
Summary of Findings

Results of the $t$-test analysis indicated that prior to the problem-based lessons, both groups were better able to talk about their mathematical thinking than to write about their mathematical thinking. These findings are supported by the understanding that written language develops later than oral language (Vygotsky, 1978) as well as current literacy research findings. For example, Kim et al. (2014) found that first-grade children demonstrate stronger oral language skills than written language skills.

While there was a difference in the first-grade children’s ability to talk and write about their mathematical thinking at the beginning of this study, no significant difference was found with either group at the completion of the problem-based unit. These results suggest that participating in problem-based lessons that focused on oral discourse decreased the gap between the children’s ability to talk and write about their mathematical thinking. Further, when comparing the pre-and post-oral explanations or the pre-and post-written explanation scores, there was a difference for the intervention group, while there was not for the comparison group. This suggests that adding writing to the problem-based instruction not only improved children’s ability to write about their mathematical thinking, it also increased children’s ability to talk about their mathematical thinking.

The results are supported by the understanding that oral and written language develop concurrently in early and middle childhood (Harrell, 1957; Shanahan, 2006; Beers & Nagy, 2008). While there is a paucity of research on the relationship between oral and written language, Berninger and Abbot (2010) found that developing children’s oral expression is often neglected, however, it continues to develop as children learn written language. All of the children in this study participated in problem-based lessons that focused on increasing oral discourse by giving
students the opportunity to talk about their solution strategies with peers. However, only the children in the intervention group used both talking and writing as a mode for communicating their mathematical thinking. The close connection between oral and written language supports the change in the students in the intervention group’s ability to both talk and write about their thinking as the two discourse modes develop concurrently.

**Comparing Groups**

When accounting for children’s ability prior to the problem-based lessons, there was a significant difference in the mean score of the oral and the mean score of the written mathematical explanations between the intervention and comparison groups. These findings indicate that adding written discourse to problem-based lessons had a greater impact on the children in the intervention group’s ability to both talk and write about their mathematical thinking, and are consistent with other research. For example, Cohen et al. (2015) found that explicit attention to mathematical writing with second-grade students had the greatest impact on children’s ability to explain their mathematical thinking followed by verbal discourse and the use of graphic organizers, as they helped students make connections between spoken and written language. Williams and Casa (2012) found that first-grade children were able to use written discourse to explain whether or not they knew a leaf was symmetrical or not after participating in classroom discourse and the use of a graphic organizer. Bicer, Capraro, and Capraro (2013) found that adding written discourse to the problem-solving process among middle school students acted as a mediator in the problem-solving process. Freeman, Higgins, and Horney (2016) found that using digital notepads with third through fifth graders to write social mathematics blogs increased students’ ability to communicate their thinking through writing. Further, they found that the younger children posted three times more frequently than older children, demonstrating the importance of introducing writing about mathematical thinking as a
mode of communication at an early age. Pugalee (2004) found that high school students who wrote about their problem-solving process and received feedback were significantly more successful than students who only verbally explained their processes.

**Discussion**

The results of this study indicate that adding written discourse to problem-based instruction had several benefits for the children in this study. Increasing oral discourse within the context of problem-based instruction helped improve children in both groups ability to write about their thinking. Adding written discourse to the lessons increased the children in the intervention groups ability to both write and talk about their mathematical thinking. However, consistent with research, these findings would not have been possible without problem-based framework instruction and the establishment of sociomathematical norms.

*Developing Sociomathematical Norms within the Context of Problem-Based Instruction*

Many children have a difficult time communicating their mathematical thinking because of the traditional, procedure based curriculum in which they participate (NRC, 2001). An increase in ability to talk and write about mathematical thinking in this study can be attributed at least partially to the development of sociomathematical norms during problem-based lessons that facilitated productive mathematical discourse.

In their regular classroom the children in this study typically participated in a procedures-based curriculum, that sometimes allows for exploration. However, the strategies used during application problems are often prescribed (Great Minds, 2016). The Eureka curriculum is not unique, it is typical. Young children are often taught procedures and algorithms for solving addition and subtraction problems without first developing conceptual understanding of the mathematics when two quantities are added or subtracted (Kamii & Dominick, 1998). The use of prob-
lem-based lessons in this study allowed children to utilize their own solution strategies and facilitated the development of important mathematical concepts such as understanding that tens are added to tens and ones are added to ones (Carpenter et al., 2015) that were the content focus in this study. This is consistent with research that children who utilize their own strategies rather than the procedures prescribed by their teacher are more likely to apply those strategies when in problem-solving situations of their own (Peltenburg, van den Heuvel-Panhuizen, Robitzch, 2012).

An important component of problem-based instruction is the sharing of mathematical ideas (Carpenter et al., 2015). Allowing children to share their thinking with their peers gives them ownership and makes them the author of their mathematical thinking. However, facilitating this type of mathematical discourse is difficult for many teachers, often turning into a “show and tell” where teachers have all students share their work without drawing connections between student work and mathematical goals of the lesson (Ball, 2001). While giving students the opportunity to share is important, without making connections between strategies or mathematical goals, conceptual understanding is not always developed (Baxter & Williams, 2009). To encourage teachers to engage in more meaningful mathematical discourse, Stein and colleagues (2008) developed the 5 Practices for Orchestrating Productive Mathematical Discourse, anticipating, monitoring, selecting, sequencing, and connecting. These practices were used as a framework for engaging in discourse throughout the six problem-based lessons in this study. However, these practices can be difficult for novice teachers as they have not yet developed the pedagogical or content knowledge necessary to be able to anticipate strategies children may use (Borko & Livingston, 1989).
Understanding what it means to explain is difficult for many children, as seen in Levinson (2013) when Sharon asks, “What does it mean to explain” when she is asked to explain how she solved a problem. Sociomathematical norms are established within a classroom to help children understand what constitutes a mathematical explanation (Yackel & Cobb, 1996). Many teachers believe they have developed norms for their students, however their endorsed norms, the norms the teacher believes are established, differ from the enacted norms, the norms the students understand, of the classroom (Levenson, Tirosh, Tsamir, 2009). Many of the established norms have to do with procedural fluency rather than developing conceptual understanding (Brooks, 2016), likely because most teachers recognize the importance of establishing norms in their classroom, they lack sufficient understanding of these norms (Zembat & Yasa, 2015).

Research in the area of sociomathematical norms usually refers to oral discourse, however, the same norms could be applied to written discourse. For example, in literacy research, discourse knowledge, refers to understanding what one knows about writing, including characteristics of specific genres (McCutchen, 1996). Olinghouse & Graham (2009) found that discourse knowledge of various forms of writing is important in the development of second and fourth-grade children’s writing. In this study, writing to communicate solution strategies or mathematical explanations was a new genre for students. Therefore, it was necessary to establish what it means to communicate mathematical thinking both orally and in writing.

Also in this study, children in both groups had the opportunity to develop their mathematical explanation discourse knowledge through increased use of talk during mathematics lessons. The intervention group also developed discourse knowledge through conversations about what to include when you write about how you solved a problem. Further, sociomathematical norms
were developed so that children understood what was meant by explaining solution strategies, developing the genre of talking and writing about mathematical thinking.

*Oral and Written Discourse as Formative Assessment*

Another important outcome of this study is the development of a rubric to assess children’s mathematical explanations. The classrooms in this study had an average of 18 students in each classroom. Providing children with the opportunity to share their thinking, hear the thinking of others, and engage in discussion is an important part of problem-based instruction. However, it is not always possible for the teacher to have conversations with every student during each lesson.

There are several things that were used in lessons in this unit that a teacher could do to encourage all students to have the opportunity to talk during the course of a lesson. For example, the researcher/teacher had children share their thinking with a partner. While the teacher still was not able to hear from every student, every student had the opportunity to share with at least one other student during every lesson. Another strategy used in the lessons was the *monitoring* step of the Five Practices for Orchestrating Productive Mathematical Discourse. During this step, teachers monitor student thinking as they work. While it still may not be possible to hear from every child, the teacher can ask several children about their mathematical thinking and keep a record of that thinking. An additional strategy used in the study that teachers could employ was to assess student learning by looking at the pictures, models, or invented algorithms they produce while problem-solving. However, this must be done cautiously since they are often vague and misleading regarding a child’s thinking. For example, in this unit many children had the correct solution, but their pictures did not accurately represent the strategies used to obtain that solution.

While writing is not traditionally used as a formative assessment in young children’s mathematics classrooms, the results from this study suggest that when sociomathematical norms
and discourse knowledge are established, young children can begin to clearly communicate their thinking in writing in a similar way that they communicate their thinking through talking. Writing creates a record of mathematical thought that can be evaluated over time to assess a student’s growth.

While teachers can qualitatively reflect on the growth of a child, there are few quantitative measures available for teachers to assess growth over time, and most are written for use with older children. Further, many rubrics are holistic in nature, assigning a writing sample a score for the whole piece of writing. This score often takes into account whether or not the student had a mathematically appropriate procedure. For example, the New York 3-Point Holistic Rubric (2015) assigns a score of 0-3, a three representing a solution that “includes the correct solution to the question and demonstrates a thorough understanding of the mathematical concepts and/or procedures in the task” (p. 1). While this is an important aspect in the development of children’s mathematics, it does not encompass all of the components of children’s mathematical explanations. Further, what a “thorough understanding of mathematical concepts” means may vary based on the teacher’s mathematical expertise (Borko & Livingston, 1989).

A rubric with various important indicators, including content, communication and vocabulary is a more detailed way to track children’s growth. An example of this type of rubric is the Rubric for Student Mathematician’s Journal found in Appendix C. The Rubric for Student Mathematician’s Journal from Project M3 is a dependable tool for teachers, however, it was designed for older children, mathematically talented children. Further, it gives broad, general descriptions of each indicator and can be applied across the mathematics curriculum. While this may be useful for well trained teachers with deep content knowledge themselves, it leaves many teachers wondering what “demonstrates strong understanding of concepts, and if applicable, uses efficient
strategy to solve the problem correctly” (Gavin, 2006-2008, p. 1) means and looks like in young children’s writing. Therefore, there is a need for content specific rubrics, such as the one utilized in this study, across mathematical domains.

**Limitations and Future Research**

There were limitations in this study. The first limitations stem from the quasi-experimental design. While the classrooms participating in the comparison group and intervention group were selected randomly, all of the participants in the study attend the same school and are a part of a pre-selected class. Even though each of the classes participated in the same mathematics curriculum, they have had different experience with that curriculum as the teachers do not always strictly adhere to the scripted lessons. Teacher difference in implementation of the curriculum may certainly have impacted results.

Next, while much effort was taken to ensure students in all four classes received the same mathematical content during the study, it is not possible for this to always happen when using problem-based instruction. This is especially true when the discourse is driven by the students. Their ideas often lead in various directions. While a journal was kept to monitor this limitation, there were instances where children in some classes engaged differently in solving the problems than the children in other classes, leading to different discourse. There were also pre-established sociomathematical norms and discourse knowledge that varied between the classrooms. A third limitation of the study is that it only looked at first-grade children’s oral and written explanations in one mathematical domain.

Though there are limitations in this study, and the results are not generalizable beyond the first-grade numbers and operations domain, it does warrant future research. First, similar
studies to the one reported here should be conducted across elementary grade levels and mathematical domains to explore the effects of adding written discourse to problem-based instruction.

Another area of future research includes professional development in the use of problem-based instruction and the development of sociomathematical norms. The teacher/researcher in this study conducted all of the lessons, as she has both experience in this type of instruction and a deeper content knowledge base than most elementary teachers. The results of this study, and improving children’s ability to communicate their thinking are hinged upon the effective use of this type of instruction. Therefore, it is warranted to provide teachers with professional development in these areas as they begin to utilize these strategies to elicit oral and written discourse.

Conclusion

While this was a small-scale intervention study, the findings demonstrate that adding written discourse to mathematics lessons with first-grade children has several benefits. For both groups, participating in problem-based, discourse rich mathematics lessons decreased the gap between their ability to talk and write about their mathematical thinking by developing sociomathematical norms and discourse knowledge for explaining solution strategies. Providing children with the opportunity to solve problems in their own way and then share their ideas helps develop both mathematical concepts and communication. It was expected that adding writing to lessons would result in increased ability to write about solution strategies from the pre-and post-assessment. Interestingly, adding writing to the problem-based lessons also resulted in differences in oral language from pre-to post that were not found in the comparison group. The results of this study also indicate that, when sociomathematical norms and discourse knowledge are developed among learners, written discourse can be used as a formative assessment and serve as a record for mathematical thought.
REFERENCES


APPENDICES

Appendix A: Assessments

Interview Pre-Assessment

Story Problems

Skylar had 32 balloons for her birthday. She blew up 30 more balloons. How many balloons does Skylar have now?

Use numbers or pictures to find the solution to the story

Solution: ________________
Interview Post Assessment

Story Problems

Joe had 26 rocks in his collection. He went on a walk and found 40 more. How many rocks does Joe have now?

Use numbers or pictures to find the solution to the story

Solution: ________________
Written Pre-Assessment

Story Problems

Mrs. Smith had 28 pencils for the class. She found 30 more pencils. How many pencils does Mrs. Smith have now?

Use numbers or pictures to find the solution to the story

Solution: ________________

Use words to explain how you found your solution:

______________________________________________________________________________
______________________________________________________________________________
______________________________________________________________________________
______________________________________________________________________________
______________________________________________________________________________
______________________________________________________________________________

Written Post Assessment

Story Problems

Eric had 36 cookies for a class party. He baked 40 more cookies. How many cookies does Eric have now?
Use words to explain how you found your solution:

Use numbers or pictures to find the solution to the story

Solution: ________________
Appendix B: Sample Lesson Plan and Reflection

Lesson Two: The Penny Pot

1. Goals for student learning:

   **Standard:**

   **CCSS.MATH.CONTENT.1.NBT.C.4**
   Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used (emphasis added). Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten.

   **CCSS.MATH.CONTENT.1.NBT.C.5**
   Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used (emphasis added)

   **CCSS.MATH.CONTENT.1.NBT.C.6**
   Subtract multiples of 10 in the range 10-90 from multiples of 10 in the range 10-90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. (emphasis added)

   **Goals:**

   - Understand add to action with an unknown change
   - Understand a relationship between addition and subtraction
   - Conjecture: Sometimes you can either add or subtract and still get the same answer (Not sure if someone will add to solve this, if they do, I will have them share so we can talk about this)
   - Use a counting up or counting down by 10s strategy

   **Formative Assessment:** The teacher will collect student work at the end of each lesson to inform problem types and mathematical goals for future lessons. The teacher will also keep a journal with notes about discourse that occurred during the lessons.

2. Tools and other materials:

   - Base Ten Blocks
   - Chart paper
   - Student representation sheets
3. **Review: Look at the work the students completed the previous week. Ask students:**

   a. Who remembers what we did last week?
   b. How did we figure out how many Skittles were in the light bulb?
   c. How did we know we had to add 70 and 15?
   d. When you tell someone how you solved a problem, what do you need to include?

4. **Launch: Posing the Story Problem –**

   Begin reading the Penny Pot to the class. Stopping to ask questions on sticky notes. Then stop to figure out how much money Penny will need.

   a. **Statement of the problem to be solved:**

      Fran has 39 cents. She needs 50 cents to get her face painted. How many pennies does she need to collect in her penny pot to get her face painted?

   b. **Questions to be used to support student comprehension of the story and access to the problem:**
      - What can you tell me about this story?
      - What is the story asking you to figure out?
      - Do you think Fran will need more than 50 cents or less than 50 cents?

5. **Work Time: Questions to be used to support students’ thinking and problem solving:**

   - **Anticipated Strategies:**
     - Direct modeling –
       - Count on - draw 39 and then draw pennies until reaching 50.
       - Count back – draw fifty, take away pennies until getting to 39. Then count crossed out pennies.
     - Counting on – 49, 50
     - Counting back - counting backward 50, 40, 39.
     - Invented algorithm – 50 – 10 = 40, 40-1 = 39. 10 +1 = 11.
     - 
   - **Questions to ask students:**
     - Can you tell me about how you solved the problem?
     - Tell me about your answer. What does it mean?
     - Will you tell me about your thinking?
     - How did you count?
• Once students have found their solution, they will explain their solution strategy to their partner. This week partners will be selected based on a student who used a similar strategy near the student.
• Once students in the control group have shared with a partner, they will revise representations if necessary and meet for classroom discourse. Students in the intervention group will write about their solution strategy after sharing with a partner and then meet for classroom discourse.

6. Orchestrating Discourse:

(Choose 3 students to share, beginning with the least sophisticated.)
Try to find a student who counted up and a student who counted back.

a. Questions to be used to explore students’ thinking, strategies, and solutions:
   i. Ask the children sharing, “How much money does Penny need?”
   ii. Can you give me a number sentence that matches your work?

b. Questions to be used to contribute to the discourse and help students make connections to important mathematical ideas:
   i. How are the strategies we shared alike? How are they different?
   ii. What do you notice about the numbers in the number sentences?
   iii. Can you count by tens to find a solution?
   iv. How do you count by tens if your one’s place is not a zero?
   v. What do you notice about the number sentences?
   vi. Can you give me a number sentence that matches the story? Where will the box go in our number sentence? Ask the students to explain their thinking.
   vii. How could you use addition to solve this problem?

7. Assessing and analyzing students’ learning, addressing sociomathematical norms, and teaching

• Collect student representations and identify solution strategies used by students. Consider the next problem type that would best meet the needs of the class.
• Identify and address students who:
  o did not have access to the problem
  o had misconceptions about the comprehension question
  o were unable to write a number sentence that matched their work
  o need help with representations.
  o Consider what students included in their oral and written explanations and encourage children to establish norms about what is expected to be shared.
An example lesson: Lesson #2

Lesson Reflection: The Intervention Group

Most students had seemed comfortable with the Add to: Result Unknown type in the first lesson, so for the second lesson I increased cognitive demand by introducing an Add To: Change Unknown type with the goal of creating more opportunities for discourse during the lesson (McClain & Cobb, 2001) due to the need to explain alternative thinking used to solve the problem. For example, in this lesson (and most of the six lessons), the goal was to count up (adding) or count down (separating) by tens as a strategy to finding the solution, addressing the first-grade Common Core State Standards (2010) related to this study. Another goal of this lesson was to understand the relationship between addition and subtraction, and to establish a conjecture that you can either add or subtract to solve a problem and still get the same answer.

At the beginning of this lesson, I engaged students by reading the story, The Penny Pot by Stuart Murphy (1998). In this story, the main character Jessie, wants to get her face painted, but she only has $.39 and a face-paint costs $.50. The art teacher shows her that she has a penny pot for other students to put their extra pennies into. She tells Jessie that she can collect the pennies and use what she needs to get her face painted.

As I began reading The Penny Pot, the students were excited to discover how much money was going into Jessie’s pot. Without prompting, the students began figuring out how many pennies were left over and going to go into the penny pot after each student got their face painted. When it came time to figure out how many pennies Jessie needed in the penny pot to get 50 cents, someone said they thought it would be 20, but could not explain why. I asked if she would need more than 50 pennies or less than 50 pennies. At first a few students said she would need
more. Another student said no, it’s less because she already has 39 and she needs to get to 50. The students went back to their seats to begin solving the problem.

During this time, students used their strategies to solve the story problem. During this lesson, most of the children used a direct modeling strategy, using manipulatives to represent 39 and adding to them until they reached 50. Children with deeper conceptual understanding chose counting strategies. For example, some children in the class did not need to represent the 39. They simply wrote the numeral 39 and counted up to 50 by either tens or ones. A few students used an invented algorithm to solve the problem, by adding ten to 39 and then one more.

While children worked on their solution strategies for this problem, the I monitored students and facilitated oral discourse with individual students to understand students’ mathematical reasoning. For example, while monitoring the students, I noticed several children having a difficult time beginning. I asked these tables what the problem was asking them to figure out. Once they explained the problem, I asked them what they could do to get started. A few students suggested representing 39 and adding blocks until they reached 50.

Many of the children used the base-ten blocks or invented algorithms to solve the problem, while others began by drawing a representation. The children were then asked to show their work by drawing representations, even if they chose to find the solution using base-ten blocks. Students who used an invented algorithm used number sentences as their method for representing. The class was then asked to write about how they solved the problem.

Once students had the opportunity to solve the problem, talk with a peer, and write about their solution strategy, they met as a group to discuss the problem. In this particular lesson, three students were selected to share their strategies with the class. Since most of the children in the class began by using tens and ones to model thirty-nine and then added base-ten blocks until they
reached fifty, a student using this strategy was selected to share first. A few students used a
counting strategy and counted on from 39, therefore a student who used this strategy shared next.
Two students used an invented algorithm. One said, “39 plus 10 is 49 and one more is 50.” The
other child used a compensating strategy saying, “40 plus 10 is 40, so 39 plus 11 is 50. Since
these strategies were the most sophisticated (Smith & Stein, 2011), they shared last.

After the students shared their solution strategies, Carpenter et al (2015) suggests facilitating
discourse about the similarities and differences of the solution strategies. This was also when
mathematical goals were pushed into the lesson (McClain & Cobb, 2001). To develop
conceptual understanding, questions were asked based on student thinking, moving children
from a direct modeling solution strategy to a counting strategy, which demonstrates a child’s
ability to conserve one number and count up or down (Carpenter et al., 2015). For example, since
no children in the class used a subtraction strategy to solve the problem, I asked, “Could you
subtract to figure out how many pennies Jessie needed?” At first many of the children said you
could not subtract. However, one child said you could. I asked the children how we could use
subtraction. They said, we should draw 50 and take away 39. However, no children in the class
could explain why this equaled 11. Therefore, I decided to continue to use this problem type the
next week to continue to develop the student’s relationship between addition and subtraction.
Appendix C: Rubrics and Scoring

3-Point Holistic Rubric:

New York State Department of Education Grade 3 Common Core Mathematics Test Guide

<table>
<thead>
<tr>
<th>Points</th>
<th>Description</th>
</tr>
</thead>
</table>
| **3 Points** | A three-point response includes the correct solution(s) to the question and demonstrates a thorough understanding of the mathematical concepts and/or procedures in the task. This response includes:  
  - indicates that the student has completed the task correctly, using mathematically sound procedures  
  - contains sufficient work to demonstrate a thorough understanding of the mathematical concepts and/or procedures  
  - may contain inconsequential errors that do not detract from the correct solution(s) and the demonstration of a thorough understanding |
| **2 Points** | A two-point response demonstrates a partial understanding of the mathematical concepts and/or procedures in the task. This response includes:  
  - appropriately addresses most but not all aspects of the task using mathematically sound procedures  
  - may contain an incorrect solution but provides sound procedures, reasoning, and/or explanations  
  - may reflect some minor misunderstanding of the underlying mathematical concepts and/or procedures |
| **1 Point** | A one-point response demonstrates only a limited understanding of the mathematical concepts and/or procedures in the task. This response includes:  
  - may address some elements of the task correctly but reaches an inadequate solution and/or provides reasoning that is faulty or incomplete  
  - exhibits multiple flaws related to misunderstanding of important aspects of the task, misuse of mathematical procedures, or faulty mathematical reasoning  
  - reflects a lack of essential understanding of the underlying mathematical concepts  
  - may contain the correct solution(s) but required work is limited |
| **0 Points** | A zero-point response is incorrect, irrelevant, incoherent, or contains a correct solution obtained using an obviously incorrect procedure. Although some elements may contain correct mathematical procedures, holistically they are not sufficient to demonstrate even a limited understanding of the mathematical concepts embodied in the task. |

*Condition Code A is applied whenever a student who is present for a test session leaves an entire constructed-response question in that session completely blank (no response attempted).
**Student Mathematician’s Journal: Guides for Teaching and Assessing**

**Rubric for Student Mathematician’s Journal**

### Mathematical Concepts

<table>
<thead>
<tr>
<th>Score</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>Overall, student demonstrates a strong understanding of concepts and, if applicable, uses appropriate and efficient strategy to solve problems correctly. The student answers all parts of the question/prompt.</td>
</tr>
<tr>
<td>2</td>
<td>Overall, student demonstrates a good understanding of concepts and, if applicable, uses appropriate and efficient strategy but with minor errors or incomplete understanding. The student answers all parts of the question.</td>
</tr>
<tr>
<td>1</td>
<td>Overall, student demonstrates a partial understanding of concepts and, if applicable, uses appropriate strategy but may have major errors. The student may have not answered all questions.</td>
</tr>
<tr>
<td>0</td>
<td>Overall, student demonstrates a lack of understanding of concepts and, if applicable, does not use appropriate strategy. The student may not have answered all questions.</td>
</tr>
</tbody>
</table>

### Mathematical Communication

<table>
<thead>
<tr>
<th>Score</th>
<th>Description</th>
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</thead>
<tbody>
<tr>
<td>3</td>
<td>Student states ideas/generalizations that are well developed and reasoning is supported with clear details, perhaps using a variety of representations such as examples, charts, graphs, models and words.</td>
</tr>
<tr>
<td>2</td>
<td>Student states adequately developed ideas/generalizations and reasoning is supported with some details. When appropriate, representations may be limited.</td>
</tr>
<tr>
<td>1</td>
<td>Student states partially developed ideas/generalizations and reasoning is incomplete.</td>
</tr>
<tr>
<td>0</td>
<td>Student does not state ideas/generalizations correctly and reasoning is unclear with little or no support.</td>
</tr>
</tbody>
</table>

### Mathematical Vocabulary

<table>
<thead>
<tr>
<th>Score</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>Student uses all mathematical vocabulary appropriately, including mathematical vocabulary related to the major math concept(s) from the unit.</td>
</tr>
<tr>
<td>2</td>
<td>Student uses most mathematical vocabulary appropriately or may have minor misunderstanding. Student may have misused or omitted an appropriate vocabulary term.</td>
</tr>
<tr>
<td>1</td>
<td>Student uses some mathematical vocabulary or may have a major misunderstanding. Student may have misused or omitted several appropriate vocabulary terms or a key vocabulary term related to the major math concept(s) from the unit.</td>
</tr>
<tr>
<td>0</td>
<td>Student does not use any mathematical vocabulary.</td>
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</table>
# Rubric for Mathematical Explanations

<table>
<thead>
<tr>
<th>Indicator</th>
<th>0</th>
<th>1</th>
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<th>3</th>
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<tbody>
<tr>
<td><strong>Mathematical Concepts</strong></td>
<td>Student does not provide a written or oral explanation. Child did not have access to the problem.</td>
<td>Oral or written explanation demonstrates that the student has access to the problem, but has a lack of procedural and conceptual understanding of base-ten concepts. - <em>does not refer to base-ten counts all by ones</em> - <em>refers to tens and ones but does not elaborate</em></td>
<td>Oral or written explanation demonstrates partial or inconsistent understanding of base-ten addition and subtraction. - <em>only refers to one quantity or the result in tens and ones</em> - <em>exhibits counting by tens error, e.g., 46, 56, 76</em> - <em>adding digits</em></td>
<td>Oral or written explanation demonstrates a consistent conceptual understanding of base-ten concepts - <em>Counts by tens</em> - * Decomposes numbers into tens and ones to solve the problem* - <em>Uses an invented algorithm</em></td>
</tr>
</tbody>
</table>
| **Mathematical Communication**   | Student does not provide an oral or written response or the response is unintelligible. | Oral or written explanation only states the tools used to find a solution, but does not refer to how the tools were used. - *Refers to using tens and ones without telling how they were used* - *Does not indicate awareness of operation* | Oral or written explanation states partially developed solutions, reasoning is incomplete. - *Refers to the total quantity without explaining how it was obtained* - *Refers to the two quantities in the problem but does not tell how an* | Oral or written explanation states adequately developed solution. The explanation clearly describes the steps taken to find the solution. - *may not be mathematically accurate, but the explanation is clear, developed* - *explains how quantities were added (counted,
### Mathematical Vocabulary
Terms: (in written, symbol, or numeral form) tens, ones, add, subtract, counted, equals, together, apart, take away, more, solved, number sentence, made

<table>
<thead>
<tr>
<th>Student does not use a mathematical vocabulary term.</th>
<th>Student uses one mathematical vocabulary term.</th>
<th>Student uses 2 mathematical vocabulary terms.</th>
<th>Student uses three or more mathematical vocabulary terms.</th>
</tr>
</thead>
</table>

### Story Problems from Sample Items

1. Gabby had 28 seashells in her collection. Her family went to the beach and she found 30 more seashells. How many seashells does Gabby have now?

2. Eric had 36 baseball cards in his collection. He went to the store and got 40 more. How many baseball cards does Eric have now?
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<th>1</th>
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</table>
| Student does not provide a written or oral explanation. Child did not have access to the problem. | Oral or written explanation demonstrates that the student has access to the problem, but has a lack of procedural and conceptual understanding of base-ten concepts.  
- *I add because how are you supposed to get 76. I used tens and ones and numbers.*  
- *I put 36 lines. I put 40 lines. I put my answer 76.* |

| No response was given | No response was given |
Oral or written explanation demonstrates partial or inconsistent understanding of base-ten addition and subtraction.
- So first I put down 3 10s and 6 1s and then I put 40 10s down and then I added, \(3 + 4 = 7\) and then I added \(30 + 6 + 40 = 76\).
- I put 2 10s and 28 3 10s and made 50 and then I put \(28 + 30 = 50\).

Oral or written explanation demonstrates a consistent conceptual understanding of base-ten concepts
- I used 3 tens 6 ones and made 36. I used 4 tens 0 ones and made 40. I counted them and made 76.
- I added because Eric had 36 and bought 40 more. I put 3 ten lines and six circles and then I put 4 ten lines and my answer was \(36+40=76\).
Write a number sentence that matches your work: \( 8 + 7 = 36 \times 10 = 76 \)

Write a number sentence that matches the story: \( 26 + 40 = 66 \)

Use words to explain how you found the answer:

I added because \( 26 + 40 = 66 \)

I found \( 40 \) more. I put \( 3 \) ten lines and six circles. I then put \( 4 \) more ten lines. My answer was \( 66 \).
# Mathematical Communication

**Response to:**
Can you tell me how you got your answer? - OR - Can you write me a note about how you got your answer?

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<td><strong>Student does not provide an oral or written response or the response is unintelligible.</strong></td>
<td><strong>Oral or written explanation only states the tools used to find a solution, but does not refer to how the tools were used</strong></td>
</tr>
<tr>
<td>- I used ten blocks and one blocks and I got my answer 58.</td>
<td>- I used numbers to get the answer and number sentences.</td>
</tr>
</tbody>
</table>

**No Response on Paper**
Oral or written explanation states partially developed solutions, reasoning is incomplete.
- I put 36 lines. I put 40 lines. I put my answer 76.
- I did 5 base ten blocks and 8 ones and it equaled 58 for my mystery.

Oral or written explanation states adequately developed solution. The explanation clearly describes the steps taken to find the solution.
- So first I put down 310s and 6 1s and then I put 40 10s down and then I added. \(3 + 4 = 7\) and then I added 30 + 6 + 40 = 76.
- I put 2 10s and put 28 3 10s and made 50 and then I put 28 + 30 = 50
## Mathematical Vocabulary

Terms: in written, symbol, or numeral form
- tens, ten blocks, ones, one blocks, add, subtract, counted, equals, together, apart, take away,
  - more, solved, number sentence, made

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<td>Student uses one mathematical vocabulary term.</td>
</tr>
<tr>
<td>No response on paper</td>
<td>- I used numbers to get the answer and number sentences.</td>
</tr>
<tr>
<td></td>
<td>- I did 2 ten lines and squares to the end.</td>
</tr>
</tbody>
</table>

**Student uses 2 mathematical vocabulary terms.**

- **tens**

**Student uses three or more mathematical vocabulary terms.**

- **number**

- **sentence**

- **tens**
- I used ten blocks and one blocks and I got my answer 58.
- I use 10 blox and I counted and that’s how I got my answer.

- I did 5 base ten blocks and 8 ones and it equaled 58 for my mystery.
- So first I put down 3 10s and 6 1s and then I put 40 10s down and then I added. 3 + 4 = 7 and then I added 30+6+40=76

• ten blocks
• one blocks

• Base ten blocks
• Ones
• equaled

• 10 blocks

• 10s
• 1s
• Added
• Plus

• Counted

• Equals
• +
• =