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# What is Mathematics? An Exploration of Teachers' Philosophies of Mathematics during a Time of Curriculum Reform

Kimberly White-Fredette

ACCEPTANCE

This dissertation, WHAT IS MATHEMATICS? AN EXPLORATION OF TEACHERS' PHILOSOPHIES OF MATHEMATICS DURING A TIME OF CURRICULUM REFORM, by KIMBERLY WHITE-FREDETTE, was prepared under the direction of the candidate's Dissertation Advisory Committee. It is accepted by the committee members in partial fulfillment of the requirements for the degree Doctor of Philosophy in the College of Education, Georgia State University.

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ABSTRACT

WHAT IS MATHEMATICS?
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Current reform in mathematics teaching and learning is rooted in a changing vision of school mathematics, one that includes constructivist learning, student-centered pedagogy, and the use of worthwhile tasks (National Council of Teachers of Mathematics, 1989, 1991, 2000). This changing vision not only challenges teachers' beliefs about mathematics instruction but their philosophies of mathematics as well (Dossey, 1992). This study investigates the processes that four teachers' go through as they implement a new task-based mathematics curriculum while exploring their personal philosophies of mathematics. The participants were part of a graduate-level course that examined, through the writings of Davis and Hersh (1981), Lakatos (1976), Polya (1945/1973), and others, a humanist/fallibilist philosophy of mathematics. These participants shared, through reflective writings and interviews, their struggles to, first, define mathematics and its purpose in society and in schools, and second, to reconcile their views of mathematics with their instructional practices. The study took place as the participants, two classroom teachers and two instructional coaches, engaged in the initial implementation of a reform mathematics curriculum, a reform based in social constructivist learning theories.

Using narrative analysis, this study focuses on the unique mathematical stories of four experienced educators. Each of the participants initially expressed a traditional, a priori view of mathematics, seeing mathematics as right/wrong, black/white, a subject outside of human construction. The participants' expressed views of mathematics changed as they attempted to align their personal philosophies of mathematics with their (changing) classroom practices. They shared their personal struggles to redefine themselves as mathematics teachers through a process of experimentation, reflection, and adaptation. This process was echoed in their changing philosophies of mathematics. These participants came to see mathematics as fluid and a human construct; they also came to see their *philosophies* of mathematics as fluid and ever-changing, a process more than a product.

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I entered the doctoral program five years ago, unclear of my focus and unsure of my direction. What I brought to my doctoral studies was a passion for mathematics education, a passion I found echoed in my major advisor, Dr. David Stinson. Thank you, David, for showing me that passion and scholarly work go hand-in-hand, and challenging me and supporting me throughout these past five years.

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# CHAPTER 1

# INTRODUCTION

The teaching and learning of mathematics is going through tremendous changes. The National Council of Teachers of Mathematics (NCTM, 2000) *Principles and Standards for School Mathematics* calls for changes to curriculum and mathematics instruction. Constructivist learning, student-centered classrooms, worthwhile tasks, and reflective teaching are all a part of NCTM's vision of mathematics in the $21^{st}$ century. In Georgia, a new curriculum, the Georgia Performance Standards, is ushering in not just mathematical content changes, but calls for instructional changes as well:

> The implementation of this curriculum will require that mathematics classrooms at every grade be student-focused rather than teacher-focused. Working individually or collaboratively, students should be actively engaged in inquiry and discovery related to real phenomena. Knowledge and procedural skills should be developed in this context. Multiple representations of mathematics, alternative approaches to problem solving, and the appropriate use of technology are all fundamental to achieving the specified goals of the curriculum (Georgia Department of Education, n.d., p. 3).

Along with calls for changes in *how* mathematics is taught, there are numerous calls for changes in *who* engages in higher-level mathematics courses. The National Council of Teachers of Mathematics' *Principles and Standards for School Mathematics* places the equity principle first (NCTM, 2000). NCTM's Equity Principle calls for high expectations, challenging curriculum, and high-quality instructional practices for all students. Closing the achievement gaps that exist in mathematics education between white students and students of color, between upper-middle class students and working

class students, demands more than high expectations and teachers who are knowledgeable about mathematical content (although these are, of course, important). This changing vision of school mathematics—mathematics success for all, student-centered pedagogy, constructivist learning in our classrooms—cannot come about without a radical change in instructional practices and an equally radical change in teachers' views of mathematics teaching and learning, and the discipline of mathematics itself.

As state curriculums, assessment practices, and teacher qualifications are re-examined and revamped, a clear and discernable theoretical framework is essential to the reform process (Brown, 1998). This theoretical framework must include a re-examination of mathematics as a subject of learning. What are teachers' beliefs about mathematics as a field of knowledge? Do teachers believe mathematics is an accessible subject, one in which all student may learn successfully?

Recent studies have explored teacher change, examining teacher beliefs and mathematical reform (see, e.g., Becker, Pence, & Pors, 1995; Bibby, 1999; Chapman, 2002; Cooney, Shealy, & Arvold, 1998; Foss & Kleinsasser, 1996; Hart, 2002a; Mewborn, 2002; Preston & Lambdin, 1995; Steele, 2001; Sztajn, 2003). Research is needed that seeks to understand how teachers view mathematics and mathematics teaching and learning. Such research, unlike many previous studies, should examine teachers' philosophies, not simply their beliefs, regarding mathematics. Philosophy and beliefs, although similar, are not identical. Beliefs are defined in many ways. Beswick (2006) asserted that there is no agreed upon definition of the term beliefs, but that it can refer to "anything that an individual regards as true" (p. 96). Pajares (1992) affirmed the

importance of researching teacher beliefs, although he acknowledged that "defining beliefs is at best a game of player's choice" (p. 309). Not only is any definition of beliefs tenuous, but distinguishing beliefs from knowledge is also a difficult process (Pajares, 1992). I argue that a study of philosophy moves beyond the tenuousness of beliefs and/or knowledge, in that philosophy is a creative process, what Deleuze and Guattari (1991/1994) termed "knowledge through pure concept" (p. 7). "Philosophy is not a simple art of forming, inventing, or fabricating concepts, because concepts are not necessarily forms, discoveries, or products. More rigorously, philosophy is the discipline that involves *creating* concepts" (Deleuze & Guattari, 1991/1994, p. 5). A goal of this study is to affect the creation of philosophy by engaging teachers in an examination of philosophies of mathematics that might indeed be new and foreign to them.

## Why Philosophy?

*Webster's Dictionary* (2003) defines philosophy as "the critical study of the basic principles and concepts of a particular branch of knowledge, especially with a view to improving or reconstituting them" (p. 1455). A study examining philosophy, therefore, seeks to better understand those basic principles and concepts that a teacher holds regarding the field of mathematics. Current calls for reform in mathematics education are not without controversy (Schoenfeld, 2004). This controversy, and the reluctance towards change, may be rooted in philosophical considerations (Davis & Mitchell, 2008). But philosophy, not just philosophy of mathematics teaching and learning, but philosophy of mathematics, is rarely examined explicitly: "Is it possible that teachers' conceptions of mathematics need to undergo significant revisions before the teaching of mathematics can be revised?" (Davis & Mitchell, p. 146).

Teachers are not often asked to explore their *philosophy* of the mathematics they teach. But my study is in keeping with the writings of Davis and Hersch (1981), Restivo (1993), Hersh (1997), Tymoczko (1998), and others in the field of mathematics that have, in recent years, sought to problematize[1] the concept of mathematics. If we are to change the nature of mathematics teaching and learning, we have to look beyond the traditional view of mathematics as fixed and rigid, a subject of absolute truths, what Lerman (1990) terms an *absolutist* view of mathematics. Constructivist teaching and inquiry-based learning demands a new view of mathematics, the *fallibilist* view, which "focuses attention on the context and meaning of mathematics for the individual, and on problem-solving processes . . . a library of accumulated experience, to be drawn upon and used by those who have access to it" (Lerman, p. 56).

Ernest (2004) called upon educators to look more deeply at the subject of mathematics and to ask five essential questions about the mathematics that is taught. What is mathematics? How does mathematics relate to society? What is learning mathematics? What is teaching mathematics? What is the status of mathematics education as a field of knowledge? These questions challenge educators to not only reflect on their instructional practices, but also to question their own beliefs about mathematics and mathematical teaching. Is true reform in mathematics education possible if we do not ask ourselves these questions?

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[1] The term *problematize* is based on Freire's (1970/2000) model of *problem-posing education*: an education that emphasizes consciousness, intentionality, and the practice of freedom. Problem-posing education, wrote Freire, "strives for the emergence of consciousness and critical intervention of reality" (p. 81). Crotty (1998) termed Freire's problematization a "demystification" (p. 156), a transformative process in which people cast off a culture of silence and construct a new view of reality along with a hope for freedom. Thus to problematize is to let go of accepted meanings and values, and to seek a more critical, reflective view, a view tied explicitly to issues of power.

In the preceding paragraph, and throughout this paper, I use the term *reform*. While that term is not without controversy (see, e.g., Amit & Fried, 2002; Apple, 2000; Frykholm, 2004; Martin, 2003), it is a difficult term to avoid when describing the mathematics educational landscape of the early 21st century. Educational reform in mathematics is certainly not new. There is, in fact, a long and well documented history of mathematics reform in K–12 educational policy and research (see, e.g., Apple, 2000; Bracey, 2007; Kilpatrick, 1992; Schoenfeld, 2004). Bracey described the cyclical nature of reforms in mathematics education that have occurred since the Soviet launching of Sputnik in 1957. These reforms have been driven by varying factors: scientific competition during the Cold War of the 1950s and 60s, a collapsing industrial market that led to fears of dominance by the Japanese in the 1980s, and a continued worry of the (slipping) place of the United States in the world economy in the 1990s, continuing to present day. Reform of mathematics education, often rooted in economic concerns, has been more recently tied to issues of equity and achievement (see, e.g., Martin, 2003; Secada, 1995). Questions of who is engaged in the learning of higher-level mathematics and how is mathematics taught have become the focus of mathematics educational reform in the last 15 years. While those two questions are related to this study, it is not my intent to address specifically the use of the term, reform. I do not dispute that reform is a contested term, one which continues to require examination and dissection. But as I write of mathematics education reform in regards to this study, I am employing the term as Ellis and Berry (2005) envisioned it, a transformative process that goes beyond the (almost constant) periodic *revisions* common to educational process:

> Reform raises questions about the core beliefs of mathematics education, moving to restructure thinking about the nature of mathematics, how it is

> taught, how it is learned, and, ultimately, what constitutes success in learning it. (p. 8)

Specifically, in this study, I address the mathematics education reform occurring in Georgia through the introduction of a new curriculum (Georgia's curricular changes will be described further in the following section), and I address the instructional changes that teachers are being asked to implement as they engage in that new curriculum.

## A Personal Story

This study began as a personal journey, an exploration of the philosophy of mathematics. The journey began as I engaged in an extensive reading of the works of Paul Ernest (1988, 1991, 1994, 19998a, 1998b, 1999, 2004). Ernest (2004) challenged me to ask, "What is the purpose of teaching and learning mathematics?" (p 1). As a mathematics teacher, in elementary and secondary schools, I had tried to engage my students in understanding the way mathematics worked. Rather than just memorize formulas, I wanted students to make sense of the formulas; before learning prescribed algorithms, I had my students act out the mathematics with manipulatives; instead of completing 25 "naked" mathematics problems, I engaged my students in solving problems with mathematics. I had always been a good mathematics student, not because I could memorize rules and formulas, but because I could *reason* things out and come to understand *how* the mathematics worked. I sought to bring the same understanding to my students and to engage all students in the same love of mathematics I had always had.

In the fall of 2005, I left the classroom and became a "teacher of teachers." I took a job as a mathematics consultant, providing professional development to K–12 teachers of mathematics, as well as instructional coaches and administrators. What I found, as I left my own classroom and entered the larger educational sphere, was that what I had

valued and emphasized in the teaching and learning of mathematics, was by no means universal. I found elementary school teachers who engaged in mathematics instruction reluctantly, bringing their own fears and insecurities about mathematics to their classrooms. I found high school teachers who viewed mathematics almost as a contest, with winners (the mathematically able) and losers (the mathematically inept). I discovered that many teachers, although defining mathematics as a problem-solving activity, taught it as only rote procedures, rules and formulas to memorize and follow blindly. I began to wonder, is this disparity in our views of mathematics about pedagogy, or is it really about philosophy? Are teachers' views of mathematics grounded in a set of beliefs and values that truly keep us worlds apart as we engage in the teaching and learning of mathematics?

My personal journey, and this study, were spurred on, not only by my professional role and my studies of mathematics education, but by the changes taking place in Georgia's K–12 schools. In the 2005–2006 school year, as I began my work with teachers, Georgia introduced a new mathematics curriculum. Over the next several years, the Georgia Performance Standards Mathematics Curriculum was rolled out in K–8 schools. My work with teachers began just as the new curriculum was instituted, therefore the focus of my work was to support teachers in the implementation of the Georgia Performance Standards (GPS). The GPS rely on the use of "performance tasks" in the day-to-day instruction of mathematics:

> Performance tasks involve the application of knowledge and skills rather than recall and result in tangible products or observable performances. They involve meaning-making, encourage self-evaluation and revision, require judgment to score and are evaluated using predetermined criteria. (Georgia Department of Education, 2007, p. 5)

As I worked with teachers to support their adoption of the new curriculum, I concentrated on the pedagogical changes called for in the GPS. Mathematics was no longer about memorization and "recall," it was about making sense, doing tasks, and solving problems. But I came to realize, in my work with teachers, that this view of mathematics, while it aligned with my own personal views, did not always agree with how teachers viewed mathematics (or at least how they had traditionally taught mathematics). I began to wonder, have teachers ever examined their personal views of mathematics and questioned its purpose in our schools?

These questions led me on a personal exploration of mathematics. While I had always enjoyed the learning of mathematics and was comfortable teaching in ways that the new curriculum now emphasized, I had never really asked myself, what is mathematics? With the assistance of my major advisor, David Stinson, I began to investigate that question. The investigation began with Davis and Hersh's (1981) often-cited book, *The Mathematical Experience*, and included Russell's (1919/1993) *Introduction to Mathematical Philosophy*, Lakatos' (1976) *Proofs and Refutations: The Logic of Mathematical Discovery*, Polya's (1954/1973) *How to Solve It: A New Aspect of Mathematical Method*, and Hersh's (1997) *What is Mathematics, Really*? As Dr. Stinson and I explored the idea of a philosophy of mathematics and discussed our own philosophies of mathematics, we also began to develop what was to become a course on the philosophy of mathematics. This graduate-level course, described in further detail in chapter 4, became the basis of this study. For through my own reading about philosophy, I came to see that changing pedagogy, teaching mathematics through tasks with an emphasis on mathematical discourse and conceptual understanding, challenged teachers'

views of mathematics. What was needed, I believed, was an exploration of teachers' personal philosophies of mathematics, questioning just what teachers define as mathematics, and then guiding them to explore other, perhaps opposing, philosophies of mathematics. The question I had asked teachers in my work as a professional developer, why do we teach mathematics, became, in my role as a researcher, a new question: what is mathematics? This study links those two questions through the stories of four mathematics educators.

## Research Questions

My study examines the process of teachers' development of a personal philosophy of mathematics. What happens when teachers explore modern philosophies of mathematics, particularly fallibilist philosophies of mathematics, as they attempt to implement curricular changes in their classrooms? My study guides a group of elementary and secondary teachers through this philosophical exploration and examines, through their personal narratives, the processes they each go through as they struggle to define their own philosophies of mathematics and examine their teaching practices in light of that philosophy and the changing curriculum in Georgia. Guiding questions of this research are:

1. How do teachers define their personal philosophies of mathematics and mathematics teaching and learning?
2. As teachers explore humanist/fallibilist philosophies of mathematics, how do their perceptions of mathematics and mathematics teaching and learning change?

3. How do teachers perceive their own instructional practices in light of their personal philosophies of mathematics and the current mathematics curricular reform?

# CHAPTER 2

# LITERATURE REVIEW

The purpose of this literature review is to present an overview, as well as a critique, of recent studies that have explored teacher beliefs and conceptions about mathematics. This review will examine how research into teacher beliefs has developed over the past several decades, since Thompson's (1984) pivotal study, "The Relationship of Teachers' Conceptions of Mathematics and Mathematics Teaching to Instructional Practice." I will demonstrate that, although there have been a number of studies that explored teacher beliefs and advocated for changing teacher beliefs about mathematics in order to affect changes in classroom practices, few studies have investigated the implications of a focused exploration of teachers' philosophies of mathematics.

The structure of this literature review strengthens my assertion that research needs to move beyond teacher beliefs and into an investigation of teachers' philosophies. I have organized the review into five themes: (a) research that explored the relationship between teachers' beliefs and their instructional practices; (b) research that sought to identify the particular beliefs that align with current mathematics education reform; (c) research that investigated the process of changing teachers' beliefs; (d) research that examined the difficulties of defining beliefs; and (e) research that pushed beyond beliefs, that opened up a discussion of teachers' philosophies. Although this review is not meant to be a historical presentation of research, the studies do, in some ways, present an evolution of the past 25 years of research in the area of teachers' conceptions of mathematics. An

extensive search of the available research was conducted for this literature review and I believe the studies included to be representative of the significant studies in this area, with a focus towards more recent research. I conclude the literature review with a brief discussion of the terms *beliefs*, *conceptions*, *knowledge,* and *philosophy* and a clarification of the use of these terms in my study.

## Identifying Teacher Beliefs

### *Teacher Beliefs and Instructional Practices*

Thompson’s (1984) pivotal research on teacher conceptions was one of the first studies to explore the relationship between teachers’ views of mathematics and their classroom practices, and to make the mathematics the central focus of the research (NCTM, 2004). Using case studies, Thompson looked at teachers’ conceptions of mathematics—defining conceptions as their beliefs, views, and preferences—and examined how those conceptions shaped teachers’ instructional practices. The case studies focused on three junior high mathematics teachers. Data was collected over a period of 4 weeks—the initial 2 weeks involved daily classroom observations and the next 2 weeks included both classroom observations and daily interviews focused on what the researcher had observed in the classroom. Thompson gathered additional information about the participants’ views of doing mathematics by having each participant complete a series of six tasks and then discuss the results.

Thompson (1984) examined how the teachers’ differing views of mathematics affected their differing practices in the classroom. In particular, she analyzed the *integratedness* and *reflectiveness* of each of her participants. *Integratedness* is defined as the extent to which someone’s views and beliefs form a coherent system, as opposed to

each belief existing in isolation. *Reflectiveness* is the individual's tendency to think about her actions in relation to her beliefs. What Thompson found was that a teacher's beliefs and practices exist in a complex relationship, although she observed a consistency in teacher beliefs and classroom practices. Thompson called for further studies that explore a teacher's reflectiveness and its relationship to her beliefs and practices. Thompson's study is significant because, in highlighting the complexity of the intersection of beliefs and instructional practices, she cited the limitations of the traditional experimental or large-scale correlational studies that had, up to that point, been the norm in mathematics education research.

Other studies further examined the complex relationship between teachers' views about mathematics and their instructional practices. Taylor (1996) used an action research approach, combining autobiographical narrative with case study, to explore the cultural foundations of teachers' beliefs and values about mathematics. Taylor's study was influenced by his own work as a teacher educator who provided postgraduate professional development opportunities to secondary mathematics teachers. The case study focused on Ray, an experienced mathematics teacher working within the constraints of the British educational system that required the administration of state-produced end-of-year exams as well as periodic practice testing. Taylor defined his role as collaborative researcher as he worked with Ray to study Ray's instructional practices and his students' conceptual development. Taylor's work is framed by a critical constructivist epistemology, which he articulated through the work of Paul Ernest in social constructivism and mathematics, as well as Ole Skovsmose in critical mathematics education. Taylor's critical constructivism "serves as a powerful theoretical framework

for making visible and deconstructing repressive cultural myths that distort social roles and discursive practices" (p. 159).

What Taylor (1996) found were two influential cultural myths that served to limit the relationship between a teacher's professed constructivist views of mathematics teaching and learning and his classroom instructional practices. The myth of *cold reason* includes "a belief in the certainty of mathematical knowledge which leads to the perception that disembodied mathematical facts are knowable by means of an asocial cognitive activity of pure reason that transcends human lifeworlds" (p. 162), and the myth of *hard control* "renders as natural the teacher's classroom role of teacher as controller and that locks teachers and students into grossly asymmetrical power relationships designed to reproduce (rather than challenge) the established culture" (p. 165). Taylor concluded that these two repressive myths stand firmly in the way of mathematics educational reform and, unless directly challenged, continue to block any lasting change.

It is the myth of cold reason that is particularly relevant to my own study, in that this myth influences how teachers view the mathematics they teach. "In mathematics classes where the myth of cold reason prevails," wrote Taylor, " students work in splendid cognitive isolation, striving to (re)discover, by means of cold reason, the a priori universal Truths of mathematics" (p. 163). Taylor concluded that both these myths had to be, first, acknowledged and then, challenged in order to affect change in teachers' instructional practices. He advocated a historical exploration of mathematical development, such as non-Euclidean geometries, to assist in shaking teachers' assumptions about the infallibility of mathematics and the certainty of mathematical

knowledge. My own study, in many ways, picks up on Taylor's conclusions from over a decade ago by having teachers problematize the perception of mathematics as infallible and unchanging.

Bibby's (1999) study focused on what she termed the "mathematical histories" of elementary school teachers and the part those histories play of teachers' beliefs and practices. Through a series of interviews with four primary school teachers, Bibby explored each teacher's early experiences as a learner of mathematics and identified two major themes: time and confidence. Each participant spoke often of feeling rushed as a student by teachers who did not seem to care about students but about covering curriculum. The idea of building up or destroying confidence also occurred frequently in the interviews. Bibby's research took place in England and each of her study's participants was schooled in the British school system. Thus, the theme of confidence (or lack thereof) often was expressed through the participants' experiences with the exam system of British schools. Bibby concludes that the common view of professional development as "unproblematic" fails to take into consideration teachers' previous experience with subject matter, in this case mathematics, and that beliefs are often formed early in a teachers' career, in fact, long before they even begin teacher education programs. Although much of the teacher interviews focused on processes unique to the British education system, Bibby's study does support Thompson's earlier study in pointing out the complexities of researching teachers' beliefs and practices. What is most significant in Bibby's study is her use of teachers' stories to uncover their beliefs and the effects of their own mathematical histories on their current teaching practices.

Recent research by Skott (2001) also examined the complexities of studying teacher beliefs and practices. Skott's earlier research focused on teachers' *school mathematics images* (SMIs) that he defined as "expressions of unique personal interpretations of and priorities in relation to mathematics, mathematics as a school subject, and the teaching and learning of mathematics" (p. 6). The 2001 study was a follow-up to his earlier research, conducted in 1997, in which 115 Danish preservice teachers completed a questionnaire on school mathematics just prior to their graduation. From the original 115, Skott choose 11 individuals (who demonstrated a variety of SMIs) to interview. Four of these individuals were then part of a continued 18 month study as they entered the classroom.

In the later study, Skott (2001) uses the case of one of the four novice teachers, Christopher, to investigate how a teacher's SMIs relate to the ways in which he deals with the complexities of the mathematics classroom. In particular, Christopher was chosen because his school mathematics images seemed to align well with what Skott terms *reformist educational practice*—viewing mathematics as a process through which students learn by experimentation and investigation. Christopher taught sixth-grade mathematics and music in a grade 1 to 10 school in a suburb of Copenhagen. Skott's data included the earlier questionnaire Christopher had completed, a series of interviews, and videotaped classroom observations. In addition, Skott shared and discussed the videotapes with Christopher. Skott found that Christopher's teaching practices did not consistently align with his images of what school mathematics should be. At times, Christopher was more didactic and traditional in his instruction. Yet Christopher did not express concern when observing his changing instructional style. Skott concluded that a

teacher is often faced with "multiple and sometimes conflicting education priorities" (p. 18), and that one's beliefs about mathematics are never the sole determinant of one's instructional practices.

Although Skott's (2001) study took place in Denmark and there are certain cultural aspects to Christopher's classroom that are not applicable to a study conducted in the United States, his research does demonstrate the difficulties in drawing conclusions about a teacher's beliefs, and judging the consistency of beliefs and practices, through researcher observations. Thompson's earlier study highlighted the complications of bringing research out of the laboratory and into the real classroom setting, and Skott's study only reinforced those complications. Whether it be the socially situated myths of cold reason and hard control that Taylor (1996) identified, the personal mathematical histories that Bibby's (1999) participants related, or the myriad of instructional decisions that Christopher made in Skott's (2001) case study, researchers continue to bring to light the complexities of classroom instruction that seem to interfere with a teacher's beliefs about mathematics and mathematics teaching and learning.

*Identifying Beliefs That Align With Reform*

One underlying assumption in all of the studies in this literature review is the importance of changing instructional practices in mathematics classrooms to align with the intent of current reform. Higher value is therefore placed on inquiry learning, problem-based teaching and learning, and student-centered instruction. A number of studies have focused on changing what are thought to be "traditional" beliefs about mathematics and mathematics education to more reform minded beliefs. Before looking at studies specifically focused on changing teacher beliefs, I want to describe several

studies that sought to define those beliefs that are more aligned with current mathematics education reform. Although most of these studies took place outside of the United States, their discussions warrant review here, if for no other reason than to point out the dearth of studies in the United States that focus on defining teacher beliefs about mathematics. Too often, the research on teachers' beliefs and conceptions of mathematics conducted in the United States do not deeply explore just what beliefs teachers hold and how those beliefs relate to the curriculum teachers are called upon to teach.

Bibby (2002) developed her study as a search for a teacher's epistemology of primary school mathematics (this study, like the Bibby study addressed earlier, was conducted in England) as "constructed through an exploration of the teachers' experiences, values, and beliefs" (p. 165). Her goal, as stated, was to seek the underlying philosophy of school mathematics by asking, as Ernest (2004) did: What is mathematics? Bibby interviewed seven primary school teachers whose experience ranged from 2 to 20 years in the classroom. In her analysis of over 40 interviews, Bibby identified two dominant metaphors of mathematics: mathematics as a hierarchy that builds on the basics and mathematics as a tool. In her exploration of the metaphor of mathematics as a tool, Bibby highlighted instances where teachers often felt a lack of control over the tool and encouraged the use of "tricks and cheats" to aid their students in their learning. Bibby acknowledged in her study that, given the non-negotiable status of England's national curriculum, teachers seldom questioned the mathematics that they taught, even if they did question its utility. Although the United States is not currently faced with similar restrictions of a national curriculum, Bibby's 2002 study (along with her earlier cited 1999 study) have implications for current reform movements here in the United States.

Teachers bring a personal history as well as a sense of school mathematics to their classrooms that may, or may not, align with the official idea of mathematics embraced by their state or school system. How they then negotiate those differences certainly bears further research.

MacNab and Payne's (2003) large-scale study looked at preservice teachers' mathematics beliefs, attitudes, and practices. MacNab and Payne surveyed all primary school teacher education students in Scotland, both those pursuing the traditional 4 year education degree, as well as those pursuing a post-graduate certification in education. Over 1,000 students were surveyed. MacNab and Payne used a questionnaire designed to "elicit information about the student teachers' feelings, confidence and understanding of mathematics, together with their attitudes towards teaching and learning of the subject and the primary school mathematics curriculum" (p. 56). The researchers then conducted semi-structured interviews with a small, self-selected group of 25 students in order to provide additional insight into the views of the participants.

Overall, the results of the survey found the student teachers were confident about their ability to teach primary mathematics, but less positive about the mathematics itself. In addition, their survey responses indicated that the future teachers expected mathematics to be the least exciting and most taxing of the academic subjects they would be teaching. The questionnaires were sent to both Year-1 and Year-4 undergraduate education students, and the authors state that, by Year-4, the students had changed their views on teaching and learning mathematics to a more constructivist view. This conclusion does not seem substantiated as the questionnaire was given only once; the Year-1 and Year-4 groups were, therefore, different respondents. The authors state that

"there is no evidence of significant change in the character of the student group as it moves through…from Year 1 to Year 4" (2003, p. 65). A different research design would be needed to test the authors' conclusion. All in all, this study does provide interesting data regarding student teachers' beliefs and attitudes about the teaching and learning of primary school mathematics and establishes the need for further study into teachers' ideas about mathematics as a subject of learning. Why do teachers feel confident about themselves as teachers of primary school mathematics, yet carry negative feelings about both mathematics and the teaching and learning of mathematics? And what are the implications of those negative feelings on teachers' instructional practices?

One additional study from outside the United States that sought to identify beliefs of mathematics teachers is Beswick's (2007) recent study that took place in the Australian state of Tasmania. Beswick acknowledges the difficulty in defining beliefs and, even more, the difficulty in distinguishing beliefs from knowledge, particularly from a constructivist perspective. What her study examined were the beliefs, in action, that resulted in a classroom environment that was consistent with constructivist principles. These principles are outlined by Beswick as: (a) a focus on the students—their needs, backgrounds, interests, and existing mathematical understandings; and (b) facilitation of dialogue.

Although many of the studies I found that sought to tie particular teacher beliefs of mathematics with mathematics education reform took place in the United Kingdom and Australia, I did find one relevant study that took place in the United States. Simon, Tzur, Heinz, and Kinzel (2000) investigated the development of teachers' perspectives of mathematics through a 30-month study that involved both preservice and inservice

elementary school teachers. Nineteen participants were included in the study and data were collected through interviews as well as classroom observations (which also included follow-up discussions). Interviews were audio-taped and classroom observations were videotaped. Analysis and interpretation was done in a negotiated manner, making the participants an active member of the research team.

Simon, Tzur, Heinz, and Kinzel (2000) identified two major perspectives of mathematics: a *conception-based perspective*, whereas mathematics is seen as a human activity and mathematical learning is a "process of transformation of one's knowing and ways of acting" (p. 584); and the *perception-based perspective*, whereas mathematics is seen as an interconnected and understandable body of knowledge that exists independent of human activity, and mathematics learning depends upon firsthand experiences in perceiving or discovering mathematical ideas and is generally the same for each individual, dependent only upon their prior knowledge and understanding of mathematics. It is the researchers' assertion that the conception-based perspective is generally aligned with current reform and widely accepted in academic circles while the perception-based perspective was widely identifiable with their preservice and inservice teacher participants. The researchers therefore saw a major goal of teacher education and professional development as the "advancement" of teachers from one perspective—perception-based—to the other, "more correct" perspective—conception-based. (The use of the terms "advancement" and "more correct" is my own; the authors of this study use the term *paradigm shift* to describe the transformation they intend.) Yet this study is telling of the divide in perspectives that both researchers and teachers bring, not just to research, but to education in general. Researchers focusing on teacher beliefs and

conceptions rarely bring a value-free perspective to that research. Change is usually what is sought even if that change is not the initial focus of the research. The next section, though, will deal with research that explicitly investigated changing teacher beliefs.

## Changing Teacher Beliefs and Conceptions about Mathematics

A frequent assumption in the studies I have reviewed is that, if changing teacher practices is our goal in mathematics education, one must first change teachers' beliefs about mathematics and mathematics education to reflect current educational reforms. There are a number of studies that investigated the beliefs of both preservice and inservice teachers and the changes that occurred (or did not occur) in those beliefs when certain interventions were put in place. I will conclude this section with several studies that examined what caused teachers to change who were not involved in direct interventions (e.g.., educational coursework, staff development, research projects, etc.)

### *Preservice Teachers*

A common focus in belief studies has been to engage preservice teachers in coursework that exposes them to new pedagogical practices and, therefore, seeks to change their beliefs about what it means to teach and learn mathematics. If teachers implement the instructional practices with which they are most familiar (Wilson & Cooney, 2002), in other words, if teachers continue to teach the way they were taught, then what are the results of engaging them in different instructional practices? If inquiry-based instructional practices are the goal, should preservice teachers not themselves be taught in that manner? And is that learning experience sufficient to change teachers' beliefs about mathematics instruction? I will review several studies that investigated, with

mixed results, the effects of a constructivist-based instructional model on perspective teachers.

Foss and Kleinsasser (1996) used a mixed methodology to investigate the effects of an interactive methods course, where preservice elementary teachers were engaged in experiencing mathematics through active problem solving and collaborative learning. The course instructor (not one of the researchers) identified herself as constructivist and "through her organizational finesse, [she] integrates variable instructional strategies, mathematics content, and educational theory in almost every class session, confronting the preservice teachers with constructivism and developmentalism weekly" (p. 432). Foss and Kleinsasser collected quantitative data from surveys, demographic questionnaires, course evaluations, and teaching evaluations. Their qualitative data included interviews, preservice teachers' written coursework, observations of student teaching experiences as well as interactions within the mathematics methods course. At least three interviews were conducted with each of the participants of the study, which included 22 preservice teachers and the instructor of the methods course.

Overall, Foss and Kleinsasser (1999) found that the beliefs and conceptions of the preservice teachers changed very little, if at all, during the time they spent in the methods course. The preservice teachers, for the most part, continued to view mathematics as primarily a subject of rote memorization and computational skill, not a subject of creativity or reasoning, despite their involvement in the methods course. The preservice teachers seemed more influenced by their childhood education and experience when formulating their conception of mathematics than the philosophy and instructional practices of the methods course instructor. They expressed a continued belief that drill

and practice was essential to mathematics learning, and they discounted the effectiveness of the techniques advocated (and utilized) in the methods course. Foss and Kleinsasser's study emphasized the difficulty in changing teacher beliefs about the subject of mathematics, as well as the teaching and learning of mathematics. The study also underscored the limitations of a single exposure to constructivist teaching practices. In fact, the researchers recommended that the contrasting beliefs between instructor and preservice teachers be openly discussed and debated, and that a teacher education ethos being created where "attention to preservice teachers' beliefs is in the forefront" (p. 441).

Other studies, perhaps picking up on Foss and Kleinsasser's (1996) recommendations, found more success in changing preservice teacher beliefs about mathematics. Cooney, Shealy, and Arvold (1998) explored the beliefs and belief structures of four preservice secondary mathematics students and frame their study within the constructivist framework. The researchers collected data over a full year of teacher education coursework. Based on an initial survey, classroom observations, and written assignments, four student educators were chosen for the study. The four individuals (two men and two women) were selected due to their varying beliefs regarding mathematics teaching and learning. During the year-long study, the students participated in an integrated content and pedagogy curriculum course, a methods course that included middle and high school field experience, a post-student teaching seminar, as well as a problem-solving course and two technology courses. Four open-ended interviews and a follow-up survey were administered throughout the year. The researchers explored the catalyst for reflective practice—what spurred these preservice teachers to reflect on their instructional practices as well as their beliefs about mathematics and mathematics

teaching and learning. If our goal in teacher education is to produce “reflective connectionists” (Cooney et al., 1998, p. 330), then what are the implications when some education students are more reflective than others? One of the authors’ goals was to better understand teachers’ belief structures and the processes through which those belief structures change. They recognized the need for more long-term involvement in constructivist practices, as well as the importance of reflective practice for preservice teachers. Their study did not simplify the process of changing teacher beliefs, but instead placed that change in a context of prolonged involvement with a consistent pedagogy of constructivism, as well as reflective practice.

Hart’s (2002a) study also emphasized the importance of reflection in the process of change. Hart’s study focused on 14 early childhood education students at a large urban research university. The students were involved in an alternative certification program with two phases. First, students (all of whom already had a 4-year non-teaching undergraduate degree) participated in needed coursework for teacher certification, as well as student teaching experiences. The next phase followed teachers into the classroom during their first year of employment, and supported teachers as they completed necessary coursework for a masters-level degree in education. Hart’s study followed the teachers during their initial phase of coursework and student teaching. In order to challenge student teachers’ understanding of mathematics and mathematics education, the program integrated the methods and mathematics courses. Because the goal was to intimately familiarize the future teachers with a social constructivist view of mathematics, the integrated coursework sought to teach mathematics to the student teachers in a manner consistent with constructivism.

The education students, wrote Hart (2002a), “need to construct new thinking models through hands-on, problem-solving experiences that required analysis and reflection” (p. 5). Hart collected both survey data and written participant data (weekly teaching logs) throughout the three semesters of the coursework. One goal of the program was to encourage preservice teachers to reflect on their coursework as well as their student teaching experiences. The weekly teaching logs were used to respond to open-ended questions (such as “what is mathematics?”), as well as to comment on their emerging instructional practices. Overall, Hart found that preservice teachers became more comfortable with the constructivist view of both learning and teaching mathematics, and felt prepared to teach elementary students utilizing a constructivist approach. Hart cautioned, however, that these changing beliefs had not yet been tested in the classroom. What happened when a teacher is on her own, behind closed doors, confronted with the realities of public schooling, and no longer engaged in the reflective practices of teacher preparation?

Szydlik, Szydlik, and Benson (2003), conducted a large scale, quantitative study with 93 preservice elementary teachers enrolled in a public university undergraduate program. The participants, who were involved in a series of three required mathematics courses, completed pre- and post-coursework questionnaires designed to measure their perceptions of mathematics and the learning of mathematics. The coursework was designed to engage students in student-centered mathematical inquiry that emphasized problem solving, reasoning and proof, and communication. A collaborative effort was emphasized, with the instructor serving as guide, not expert. Szydlik, et al. noted significant changes in preservice teacher beliefs and cite the reflective nature of their

study: “All participants, but particularly those who participated in the interviews, had the opportunity to reflect on their mathematical beliefs through their work on the questionnaire items, as well as through course assignments” (pp. 276–277). I question the reflective nature of completing a questionnaire but acknowledge that this study emphasizes the importance of prolonged experience with constructivist learning, rather than the single course described in the Foss and Kleinsasser (1996) study. And I agree with Szydlik, et al. that their study would have been improved by making the reflective processes more “transparent” to the participants.

A number of quantitative studies replicate the findings of Szydlik, et al. (2003). Ambrose (2004), Wilkins and Brand (2004), and Barlow and Cates (2006) each used statistical analysis of pre- and post-coursework survey data to conclude that involvement in an inquiry-based, constructivist learning experience changed the beliefs of preservice elementary teachers to align more with current reform. Several concerns are raised in each of these studies. In each study, one of the researchers was an instructor of the coursework, thus opening up questions of bias and influence (and none of the studies explored the implications of these dual roles). The question raised by previous studies must also be raised here: Does this perceived change in preservice teacher beliefs continue as they move into the realities of day-to-day teaching? In addition, as Thompson’s (1984) earlier study demonstrated, survey data is limited in what it reveals when researching the complexities of teacher beliefs and practices.

*Novice Teachers*

Studies of preservice teachers present researchers with obvious limitations. Change may seem significant within a teacher preparation program but what happens

when the teacher moves into the realities of the classroom? Is the change observed in beliefs and instructional practices carried into the classroom environment? In order to address the issue of sustained change, Mewborn (2002) conducted a longitudinal case study of an elementary school teacher. Using Green's work on the structure of belief systems and how those beliefs are held, as well as Dewey's call for reflective thinking development in educators, Mewborn collected data on "Carrie" during her teacher education experience as well as her first 2 years as an educator. Using Carrie's autobiographical writings, interviews, journals, classroom observations, and audiotapes of small group instruction, Mewborn explored how Carrie's beliefs in mathematics and mathematics instruction changed as a result of her educational experiences. Carrie's initial inconsistencies between her general beliefs about teaching and learning, and her specific beliefs about mathematics teaching and learning lessened as she became more comfortable about the mathematics she was teaching and her role as a mathematics teacher. Mewborn concluded that "as teacher educators, we need to take a broader view of our students and their beliefs and try to understand not just their beliefs about mathematics but also their wider beliefs about education, human relationships, and a person's role in society" (p. 27). Her study emphasized the need to help elementary school teachers explore their beliefs about mathematics and mathematics instruction, as well as their general beliefs and philosophy of education, if they are to teach students in an accessible and constructivist manner.

Steele (2001) also conducted a longitudinal study of four elementary school teachers, beginning with their participation in her university methods course. Steele collected quantitative (pre- and post-surveys) data during the preservice teachers'

coursework. Four years later, after the teachers had 2 years of classroom experience, Steele collected qualitative data including formal and informal interviews with the participants, their principals and co-workers (other teachers in the schools), as well as participant observations during classroom instruction, lunch-time interactions, and school-wide meetings. In addition, Steele collected written data including teacher plans, worksheets, curriculum guidelines, and tests. Steele found, initially, that preservice teachers' beliefs about mathematics and mathematics instruction changed during their university training, becoming more aligned with constructivist, reform models. But as Steele followed her four participants into the classroom, that change did not hold consistently. Two of the teachers felt pressured by their working environments to teach mathematics in a more traditional manner. Textbook driven instruction in mathematics, and pressure from other teachers and administrators to conform, forced these teachers to let go of many of their previously expressed beliefs about mathematics.

Steele's (2001) study offers compelling evidence of the limitations of changing preservice teacher beliefs and ignoring their journey into the classroom, suggesting that "school culture is a major problem when implementing reform-based mathematics instruction" (p. 169). Mathematics educators cannot focus solely on changing preservice teacher beliefs and conceptions about mathematics. Change needs to occur at the school level as well: "When preparing reform-oriented mathematics teachers, mathematics educators should help prepare the context of teaching by working with schools in order to change the culture of teaching mathematics in schools" (p. 169). Therefore, studies are needed that address reform and teacher beliefs at the school level, studies that examine

how school culture affects a teacher's ability to change beliefs as well as the ability to embrace reform curricular ideas.

*Inservice Teachers*

Changing the school culture means a focus on classroom teachers (i.e., inservice teachers). As previously cited studies have demonstrated, changing instructional practices is often tied to changing teachers' beliefs and conceptions about mathematics. If the goal of mathematics reform is to change teachers' beliefs and practices about mathematics, what has research taught us about such change among inservice mathematics teachers? I will next review two general areas in the research of practicing teachers: attempting teacher change through intervention and investigating teachers who have changed on their own, that is, teachers who were not involved in coursework or professional development that focused on changing instructional practices.

Research may, itself, affect the participants and result indirectly, or directly, in change. Wood, Cobb, and Yackel's (1991) study was initially designed to examine student learning in an elementary classroom. Their investigation soon focused on the teacher in the classroom and her changing instructional practices. Through her involvement in the research project, the veteran teacher underwent what the researchers termed "pedagogical conflict" as the research team guided her towards implementing problem-centered activities and collaboratively learning in her second-grade classroom. This pedagogical conflict resulted in changes in the teacher's beliefs about mathematics and mathematics teaching and learning: "The teacher found that accepting her students' incorrect answers unconditionally created a contradiction with her beliefs about the nature of mathematics and her role as a teacher and member of the wider society" (p.

602). This study reinforces the notion that changing beliefs about *mathematics,* and not just exposure to new (constructivist) instructional practices, is needed to change how mathematics is taught in our schools. Wood, et al. contended:

> If reform in learning mathematics, which has been advocated by mathematicians and mathematics educators, is to be successful, attention must be given to the way in which math is currently taught and the existing practices of elementary school mathematics as well as the development of preservice teaching. As the view of learning mathematics changes, so must the practice of teaching mathematics. . . . Research that focuses on beginning teachers only without considering their later acculturation into the traditional practice of teaching ignores a critical aspect of teachers' learning. (pp. 588–589)

Thus, changing teacher beliefs must be carried into the *practicing* teacher's classroom where the realities of teaching often draw teachers back into a traditional idea of both mathematics and mathematics instruction.

Simon and Schifter (1991) recognized the need to work towards changing perspectives of inservice teachers. They designed a four stage intervention program (the Educational Leaders in Mathematics Project, or ELM) that sought to stimulate the development of a constructivist view of learning among elementary and secondary teachers. The program included a summer institute, during which teachers engaged in mathematical tasks and discussions; classroom follow-up during the following school year that included weekly visits and interviews; continued professional development during the school year; and a leader apprenticeship, through which the program participants co-led workshops and other professional learning opportunities. One of the guiding principles of the ELM project was: "Teachers must be encouraged to examine the nature of mathematics and the process of learning mathematics as a basis for deciding how to teach mathematics" (p. 312).

The study by Simon and Schifter (1991) focused on the 14 participants of the program's pilot year (1985), as well as the 30 participants in each following year. Data collected included teachers' writings and interviews with the participating teachers. Generally, the teachers indicated that their participation in the program had a substantial impact on their beliefs about mathematics and on their teaching practices. Simon and Schifter concluded that the ELM Project provided evidence that an inservice program in mathematics education could be successful in guiding teachers to a vision of mathematics consistent with recent reform movements, as well as helping them to develop new instructional strategies. Yet, they asserted, the ELM Project was "labor, cost, and time intensive" (p. 328). But the single most important feature that the Simon and Schifter pointed out, one quite relevant to my own study, was:

> That teachers were encouraged to develop their own theories of learning as the basis for their curriculum and instructional decisions. How teachers think about mathematics learning is a key determinant of how they teach. . . . Whereas previously teachers may have looked to be told what to teach and how to teach it, the development of their own epistemological view enables them to base decisions on their own, informed, professional judgment. . . . Empowerment of this type may contribute to teachers' development as educational leaders. (p.329)

While Simon and Schifter speculated that such change is possible without the same level of time, labor, and cost, they left it to future professional developers and researchers to pursue.

It is as important to investigate teachers who navigate change successfully as it is to explore why others fail to change. Preston and Lambdin (1995) examined why two teachers involved in a middle school mathematics reform project chose to drop out of the program. The teachers had been involved in the Connected Mathematics Project (CMP), a National Science Foundation (NSF) funded reform curriculum for middle school

mathematics. Due to their identified strength in mathematical content knowledge, supportive work environments, and perceived reform-aligned instructional philosophies, it had been expected that these two teachers would successfully implement the CMP curriculum. The researchers used teacher and student questionnaires, as well as classroom observation in their data collection. Their analysis of the data found that, although teachers may verbalize agreement with the intentions of educational reform (e.g., student-centered instruction, open-ended problem solving), their teaching practices can actually reflect a much different approach to instruction, thus making it difficult to implement change. Tied to their lack of change seemed to be the teachers' limited *depth* of understanding of mathematical content (their identified strength in content knowledge being limited to a more rote, procedural understanding). This study again reinforces the need for support, both in terms of teacher beliefs as well as teacher knowledge, when implementing change in mathematics instruction.

Becker, Pence, and Pors (1995) studied high school mathematics teachers, examining the effects of a staff development project on the teachers' pedagogical practices and beliefs about mathematics teaching and learning. The focus of the staff development was the implementation of a new reform curriculum as well as issues of equity in the classroom, specifically the concept of "algebra for all." How well did teachers implement change in their instructional practices? Did the staff development project result in lasting philosophical changes in the participating teachers? And did the goal of "algebra for all" actually translate into equitable pedagogical practices in their classrooms? Through in-depth interviews with five participating teachers (selected purposefully based on demographic diversity, including gender, race, ethnicity, and years

of experience), the researchers gathered data about the teachers' perceptions of the project and its impact on their teaching. Triangulation was achieved through classroom observation and quantitative analysis of student achievement data. The results of the study showed that changes in teacher beliefs and practices come slowly and over time, and only with on-going support and mentoring. Becker, et al. reinforced the notion that change and, hence, educational reform, is a journey that must be supported on an ongoing basis if it is to be achieved. Along with Preston and Lambdin (1995), the Becker et al. study raised issues of empowerment also addressed by Simon and Schifter (1991)—how much do teachers really change when change is thrust upon them? How important is it that teachers feel invested in the change process? And how is this change related to teachers' basic philosophical beliefs about teaching and learning as well as mathematics?

Hart (2002b) addressed some of those issues in a study with classroom teachers (see earlier cited study with preservice teachers, Hart, 2002a). Working with experienced teachers involved in the Atlanta Math Project (AMP), Hart's study asked, "Why are some teachers reluctant to change and hold fast to their traditional methods while others are embracing reform practices and changing the environment of their mathematics classroom? … Why are some projects, courses, experiences, etc., able to impact teachers' beliefs about teaching and learning mathematics and others are not? " (2002b, p. 162). These are vital questions for researchers, like me, whose goal is to positively impact reform practices in mathematics education. Hart's study examined teachers' beliefs about educational change as they attempted to reform their teaching. Through the use of a 16-item survey, Hart identified factors that affected teacher change. Interviews with a select group of surveyed teachers were then used to confirm and expand the survey data. Survey

results were analyzed using descriptive statistics and qualitative methods were used to analyze the interview results. The overall results indicated that teachers saw changes in their teaching practices in the areas of classroom discourse, the use of multiple representations, and increased problem-solving activities. They attributed those changes to increases collaboration with colleagues, the use of modeling techniques, and their own use of reflective practice. They also cited learning from their students and becoming a student themselves as factors in their professional growth. Hart's study is important in that she allowed the teachers to speak for themselves; it is their voices we hear in the research through the use of extended quotations. In this study, Hart again pointed out the importance of reflection in the change process. But questions regarding the staying power of the changes remain, just as they did in earlier cited studies with preservice teachers. Are there lasting effects to these professional development opportunities or do teachers "revert" soon after a project ends?

*Inservice Teachers—Changing on Their Own*

Perhaps one way to answer the previous questions is to look at teachers who change on their own and not due to any one particular professional development project. If, as Simon and Schifter (1991) asserted, change is linked to empowerment, then the teacher who changes her practices independently is perhaps a good case to study. Chapman (2002) studied just that in an interpretative study of the professional growth of four high school mathematics teachers who changed their instructional practices without participating in any specific staff development program. The four teachers, whose experience ranged from 16 to 33 years in the classroom, were all involved, at the start of Chapman's study, in reviewing and/or writing mathematics textbooks, conducting

workshops, and presenting at professional conferences. Thus they were purposefully chosen by the researcher for their accomplishments in the field of mathematics education. As Chapman noted, “they were very articulate and open about their thinking and experiences in teaching mathematics” (p. 180).

Chapman’s (2002) study focused on mathematics educators’ thinking in teaching mathematical word problems. Data were collected and analyzed following what Chapman described as a humanistic approach framed in phenomenology. Data included interviews, role-plays, and classroom observations. The teachers were asked to give detailed narrative accounts of their lived experiences. Further interviews explored why particular stories were chosen for re-telling. The focus of Chapman’s analysis was the mathematical beliefs of the participants, in the assumption that beliefs strongly affect behavior. Thus, reasoned Chapman, if teachers are to change their instructional practices, they must first change instructional beliefs. Chapman did not utilize a framework of belief structures, but instead looked at each individual participant’s expressed beliefs. What she found were strong themes of *pedagogical conflict* in two of her participants. *Pedagogical conflict* refers to conflicts in the teaching act, the teacher’s intentions and expectations, and the outcome of the teaching act as reflected in students’ performance, behavior, and attitudes, and was cited in the Wood, et al. (1991) study as well. As teachers realized that their beliefs about teaching were misaligned with their actual teaching behavior, change occurred. Continued conflict resulted in continued change. This detailed study reinforces the role of beliefs (and time) in educational change. What I find particularly relevant to my own study is the author’s contention that both the *deconstruction* of an existing set of beliefs and attitudes incompatible with a reform

perspective as well as the *construction* of a new set of beliefs compatible with the intended reform are essential if change is to occur.

Sztajn (2003) also used case study to investigate the beliefs and practices of two elementary school teachers. The goal of Sztajn's research was to unveil "factors that shape how teachers adapt reform rhetoric when trying to adopt it" (p. 54). Sztajn chose to study a third-grade teacher and a fourth-grade teacher, neither of whom were involved in any formal reform project. Both teachers had been in the classroom for 9 years and had met the researcher through their participation in graduate-level coursework (one teacher was actually a doctoral student). They taught in public school classrooms in two small Midwestern towns. Data were collected through classroom observation, five semi-structured interviews, and artifacts such as classroom handouts and teachers' planning notes. In addition, Sztajn interviewed the principals, other teachers, and a few parents at each school. Sztajn developed, through her analysis of the data, *portraits* of each teacher that included a "justified description of her practice, that is, a summary of her teaching combined with her explanations for specific actions" (p. 60). Sztajn used the 1989 NCTM *Curriculum and Evaluation Standards for School Mathematics* as a lens through which to view the teachers' practices and beliefs.

Sztajn's (2003) was a descriptive study and she was able to provide rich descriptions of each teacher's personal beliefs (about mathematics and mathematics education) and instructional practices. Although Sztajn found that both teachers thought their practices were aligned with recommendations for change in school mathematics (based on the 1989 NCTM *Standards*), their instructional practices and conceptions of mathematics were very different. Teresa was a very traditional teacher who emphasized

drill and practice, rote memorization, and the importance of structure and order, which she believed mathematics exemplified. Julie, on the other hand, believed students needed to be interested and happy in order to learn, and she emphasized experiential problem solving and higher-order thinking in her mathematics instruction.

How do two such different teachers each believe that they are adopting a reform curriculum? Sztajn (2003) found that teachers' concepts of what their students *need* influenced their adaptation to and adoption of reform rhetoric. Teresa worked in a low-income area with a majority of single-parent families and believed her students needed to be taught the value of hard work, structure, and discipline. Julie worked in a much more affluent area with traditional two-parent families who wanted, she believed, to know their children where happy at school. Each teacher was, therefore, responding to what they identified as the students' primary educational needs. Sztajn concludes that reform rhetoric does not challenge teachers' conceptions of mathematics or their perceptions of what their students need when learning mathematics.

Gates (2006) sought to understand why attempts to change teacher beliefs often fail. His study examined teacher beliefs, not as a cognitive construct, but as a social construct, asserting that the "hegemonic nature of these beliefs may be responsible for the widespread failure of the history of reform in mathematics education" (p. 349). Gates drew on three key components to substantiate his social perspective of teacher beliefs: *habitus*, *ideology*, and *discourse*. Gates wrote that habitus is "history turned into second nature" (p. 352). A teacher's habitus influences her structures of thought, style of dress, figures of speech, and day-to-day practices. Ideology is what makes us the same as or different from others. Ideology, wrote Gates, are "particular sets of dispositions which

become organized through social engagement" (p. 353). Finally, discourses are the "interactional means whereby we live out and act out ideological framework and dispositions" (p. 354).

Gates' (2006) study investigated the beliefs of two secondary school teachers of mathematics. Data collected included interviews, observations, and written documents. The site of the study was a state comprehensive high school in Great Britain. In particular, Gates analyzed the differing beliefs and practices of Fran, the former Head of Mathematics, and Alan, the current (at the start of the study) Head of Mathematics. Gates found that Alan was a very traditional teacher whose primary goal was student success on a key examination. Alan supported ability grouping of students, was well organized, dressed formally, and, as department head, felt empowered to impose unilateral decisions. Fran, on the other hand, was committed to seeing strengths in all students. She dismissed the importance of formal assessments, and saw education as bigger than preparation for examinations. As head of department, she had exercised shared responsibility and collaborative work. Gates' goal in this study was to emphasize how entrenched certain instructional beliefs and practices are, and to explore the implications for educational change: "Attention needs to shift from surface aspects of the discursive positions which teachers adopt, to the deeper ideological frameworks which constitute teachers and play a significant role in the establishment of their work with children" (p. 365). Gates' study is relevant here because he stresses the social aspects of teacher beliefs and advocates a deeper look at what he terms *ideological frameworks*, deeply embedded belief structures that may indeed be in conflict with the principles of educational and mathematical reform.

## Difficulties in Defining "Beliefs"

One dilemma common to the previously reviewed research is the difficulty in defining *beliefs*. As cited earlier, Beswick (2006) described beliefs as "anything a person regards as true" (p. 96). Thompson's (1984) study focused on conceptions of mathematics that she characterized as "teachers' professed beliefs, views, and preferences about mathematics and mathematics teaching" (p. 107). In a review of study on beliefs, Thompson (1992) acknowledged the difficulty of distinguishing between beliefs and knowledge, and added that "researchers interested in studying teachers' beliefs should give careful consideration to the concept, both from a philosophical as well as a psychological perspective" (p. 129). Wilson and Cooney (2002) defined knowing as a stronger claim than believing, yet recognize that "when the emphasis of research shifts towards a sense-making perspective, boundary lines between knowing and believing become blurred as we seek to understand the phenomena of teacher change and what drives that change" (p. 131).

Skott (2001) moved beyond the elusive notion of beliefs and adopted the term *school mathematics images* (SMI) to describe teachers' "idiosyncratic priorities in relation to mathematics, mathematics as a school subject and the teaching and learning and learning of mathematics in schools" (p. 6). Skott went on to describe these images as "expressions of unique personal interpretations of and priorities in relations to mathematics, mathematics as a school subject, and the teaching and learning of mathematics in schools" (p. 6). Hart (2002a, 2002b) surveyed her participants regarding their beliefs yet did not offer her definition of beliefs. Steele (2001) used the term *conceptions* in her longitudinal case study but did not define conceptions. Studies by

Cooney, Shealy, and Arvold (1998) and Mewborn (2002) gave detailed explanations of beliefs and belief structures, using Green's metaphorical analysis that focused on three different aspects of belief structures: the quasi-logical relation between beliefs—central beliefs held strongly and peripheral beliefs that are more susceptible to change; the psychological strength of beliefs; and the clustering of beliefs, which prevents cross-fertilization or confrontation among conflicting beliefs. Gates (2006) adopted a more social perspective to belief structure when he identified the three essential components of habitus, ideology, and discourse.

Leatham (2006) warned against identifying teachers' beliefs as inconsistent: "Beliefs become viable for an individual when they make sense with respect to that individual's other beliefs. This viability via sense making implies an internally consistent organization of beliefs, referred to herein as a sensible system" (p. 93). Leatham also argued:

> The challenge for teacher education is not merely to influence what teachers believe—it is to influence how they believe it…The sensible system framework offers teacher educators a constructive approach for viewing teachers' belief systems as well as changes in those systems. Through this framework, teachers are seen as complex, sensible people who have reasons for the many decisions that make. When teachers' belief systems are viewed in this way, we have a basis for constructing a different type of teacher education. Teacher educators should provide teachers with opportunities to explore their beliefs about mathematics, teaching and learning. (p. 100)

I agree that researchers should refrain from critiquing the consistency of teachers' beliefs and should instead focus on the social structures that influence both beliefs and instructional practices. In fact, Leatham seemed to be advocating for a teacher education program that begins with an exploration of teacher beliefs, which then makes that

exploration essential to anything else. What knowledge of mathematics is sufficient if teachers are never expected to explore their beliefs about mathematics?

Some researchers have advocated a view beyond beliefs. As the definition of beliefs grows to include “conceptions, personal ideologies, worldviews, and values that shape practice and orient knowledge” (Speer, 2005, p. 365), are we not looking at a teacher’s philosophy? The next section will examine a number of studies that expanded the research of teacher beliefs about mathematics to examine philosophy. As Ernest (1988) stated, “Teachers’ conceptions of the nature of mathematics by no means have to be consciously held views; rather they may be implicitly held philosophies” (¶ 4).

## Exploring Philosophy

Lerman’s (1990) study is one of the first that focused on teachers’ philosophies of mathematics, which he termed teachers’ attitudes towards the nature of mathematics. Identifying two distinct philosophies of mathematics—*absolutism* (mathematics as certain, absolute, value-free and abstract) and *fallibilism* (mathematics as a social construction, focusing on the context and meaning of the mathematics for each individual)—Lerman developed a questionnaire that he used to gather data on student teachers’ views of mathematics. He surveyed 42 graduate-level education students, all working towards initial certification. The students were then shown a videotaped teaching session of a mathematics lesson. The videotape presented a 5-minute didactic, teacher-centered (yet engaging) lesson on solving an algebraic equation. Lerman later interviewed, at depth, four of the student teachers—two identified as extremely *absolutist* and two at the extreme *fallibilist* paradigm based on the questionnaire results. Not surprisingly, the two absolutist student teachers criticized the videotaped lesson as not

directive enough and not providing the students enough support. The two student teachers Lerman identified as fallibilist felt the teacher was too directive and not open enough in her instruction. How do four preservice teachers view an identical lesson and come away with such different interpretations? Lerman's study has many implications for the postmodern researcher. We each view experiences through our own individual lenses and researchers must be careful to let participants' interpretations be recognized and understood (as well as they can be). Teachers bring to the classroom their own, often embedded, philosophies, not just of mathematics teaching and learning but about the very nature of mathematics.

Wiersma and Weinstein (2001) investigated the mathematical sophistication and educational philosophies of first-year secondary mathematics teachers. The researchers used Weinstein's Ways of Knowing Mathematics, a hierarchical scale based on intellectual development theory, as well as Ernest's (1991) five mathematical philosophies of education—Industrial Trainer, Technological Pragmatist, Old Humanist, Progressive Educator, and Public Educator. Weinstein's hierarchy examines the impact of beliefs on the behavior of the teacher as a learner, whereas Ernest's classifications explore the impact of philosophy on the teacher's instructional practices (Wiersma and Weinstein use the terms beliefs and philosophies interchangeably in their description of the study).

Ernest's (1991) five classifications move from a conservative, rigid, fixed view of mathematics (the Industrial Trainer) to a social constructivist, cultural, reform-based view of mathematics (the Public Educator). Each philosophical view of the nature of mathematics in Ernest's classifications influences the teaching and learning of the

subject—the Industrial Trainer, viewing mathematics as a set of absolute truths, emphasizes hard work, memorization, drill and practice, and the absence of technology. The Technological Pragmatist, who also views mathematics as an unquestioned body of useful knowledge, recognizes the usefulness of mathematical learning. The Old Humanist sees the beauty in mathematics yet transmits this idea in education by promoting mathematics as a remote ideal. The Progressive Educator embraces a child-centered idea philosophy of education, and values the process view of mathematics. The Progressive Educator embraces the *ideal* of mathematics for all without questioning the mathematics to be learned. And finally, in Ernest's classifications, the Public Educator sees mathematics as a social construct, accepts the fallibility of mathematics, and believes that mathematics must be recreated in each student's mind to be made relevant and worthwhile.

Wiersma and Weinstein (2001) gathered data on five first-year math teachers through 45-minute structured interviews. The transcripts were then coded using key words extrapolated from the Ernest and Weinstein frameworks. The teachers, four females and one male, were all White, middle-class teachers with diverse educational backgrounds. They were in their early-to-late 20s. The four teachers with undergraduate degrees in mathematics demonstrated a higher level of mathematical sophistication (using the Weinstein framework) than the fifth teacher, whose degree was in business. The teachers' philosophies of mathematics education fit primarily into the Technological Pragmatist or Industrial Trainer models, meaning a view of mathematics as fairly rigid, no-nonsense and absolutist, espousing set rules and procedures, and dependent on teacher-centered instruction. Wiersma and Weinstein recommended using such

frameworks in mentoring novice teachers, particularly when the goal is to change instruction and teacher behavior to a more student-centered, constructivist approach. In addition, the researchers noted the application of the frameworks in building teacher reflection, especially in times of educational reform.

Lloyd's (2005) research is a more recent case study of a preservice high school teacher, one in which the researcher closely examined the teacher's beliefs about his role as a teacher in the mathematics classroom, particularly his role in shaping mathematical discourse within his classroom. Lloyd defined mathematical classroom discourse as the "ways that teachers and students interact during classroom activities and the ways that mathematics is represented and develop through those interactions" (p. 442). Rather than look for contradictions between a teacher's beliefs and practices, Lloyd sought to understand how a preservice teacher makes sense of his beliefs while learning to negotiate the realities of classroom teaching. Although she does not define her use of the term beliefs, Lloyd looked at her participant's images of mathematics through the use of fictional accounts as well as other data.

Lloyd's (2005) participant, Todd, was enrolled in a graduate-level secondary mathematics certification program and was, at the time of the study, completing an 8-week field experience. Data collected included Todd's written assignments (for his methods course), including both fiction and non-fiction writings; interviews; and observations of Todd's student teaching. Among Todd's written assignments was an analogy assignment, in which the student teacher identified an analogy that fit his idea of what it meant to be a mathematics teacher. Lloyd used narrative analysis to analyze that and other written assignments. The research findings demonstrated that, although Todd

embraced *non-traditional* notions of teaching—eschewing lecture, embracing teacher as facilitator and questioner—his views of mathematics were fairly static: "Todd's views did not appear to include the notion that increased student exploration and communication may actually develop and change the mathematical content of classroom tasks and activities, thus allowing classroom discourse to encompass a more dynamic treatment of mathematics" (p. 454). In other words, Todd was ready to challenge the traditional view of mathematics instruction but not the traditional view of mathematics.

Lloyd's (2005) study has tremendous implications for my research, one goal of which is to begin to challenge, to deconstruct and reconstruct teachers' conceptions of mathematics. Lloyd's findings strengthen the argument for a detailed investigation into teachers' philosophies of mathematics. A limitation of Lloyd's study, which she touches on only briefly, is that the researcher was also the course instructor, as well as Todd's field-experience supervisor. The issue of overlapping roles, and the effect of those roles on both the researcher and the participant, is one that I share with Lloyd, and is addressed in the methodology, analysis, and discussion sections of this study.

None of the studies described here examined specifically the results of an exploration of philosophy. What happens when teachers are presented with non-traditional views of mathematics, when teachers explore philosophical writings about mathematics? At a university in Greece, Toumasis (1993) developed a course for preservice secondary school mathematics teachers that focused on readings on the history and philosophy of Western mathematics, as well as "discussion and an exchange of views" (p. 248). The purpose of the course was to develop a reflective mathematics teacher:

> To be a mathematics teacher requires that one know what mathematics is. This means knowing what its history, its social context and its philosophical problems and issues are. . . . The goal is to humanize mathematics, to teach tolerance and understanding of the ideas and opinions of others, and thus to learn something of our own heritage of ideas, how we came to think the way we do. (p. 255)

According to Toumasis, mathematics teachers continued to be shortchanged when their teacher preparation programs focused only on participation in higher level mathematics—Linear Algebra, Discrete Mathematics, Analysis. Knowledge of mathematics, especially if one is to teach mathematics, must include a reflexive study of mathematics.

Toumasis (1997), in a later article, argued that the philosophical and epistemological beliefs about the nature of mathematics are intrinsically bound with the pedagogy of mathematics. In his examination of the philosophical underpinnings of NCTM's *Curriculum and Evaluation Standards for School Mathematics* (1989), Toumasis identified a clear fallibilist point of view—a view that mathematics truth is uncertain, fallible, and tentative; that mathematics is "a dialogue between people tackling mathematical problems" (p. 320). Yet in our current attempts to reform mathematics, a reform certainly based on both the 1989 Standards and the later *Principles and Standards for School Mathematics* (NCTM, 2000), an investigation of philosophy is rarely undertaken. Although teachers are often identified as having an absolutist view of mathematics (i.e., mathematics as a subject of objective knowledge and absolute, certain truths), as evidenced in the studies included in this literature review, I was unable to find any research that investigated teachers' personal exploration of philosophy of mathematics. Toumasis, although recommending a similar course for other mathematics teacher preparation programs, did not investigate the results of his course.

## Summary

The studies on which I have focused inspire many new research questions. Studies exploring the beliefs and attitudes of elementary school teachers towards mathematics demonstrate the need for more work in this area, particularly with a focus on teachers at the secondary level. Do teachers with deeper mathematics backgrounds, having participated in higher-level mathematics courses, and many of whom experienced success in traditional mathematics instructional models, face greater challenges when implementing reform-based models of instruction? Several studies that I cited emphasized the importance of the participants' voices being heard in the research. Lerman's (1990) study is a reminder that individuals each interpret events through a unique lens, a lens influenced by beliefs and attitudes, as well as by lived experiences. Although frameworks are useful in examining teacher beliefs and attitudes, Chapman (2002) let her participants speak for themselves and, thus, provided a clearer, more complete picture of who a teacher is and why she makes the decisions she does in the classroom. Hart (2002b) also used the voices of her participants to strengthen her study. Thompson's (1984) landmark study highlighted the intricacies of teacher beliefs and practices, reinforcing the need to look beyond surveys and questionnaires, and to deeply investigate the instructional practices and beliefs of classroom teachers. More recent studies, including Sztajn (2003), and Becker, Pence, and Pors (1995) examined teacher change and its relationship to teachers' beliefs and values. A few studies have even investigated the lack of teacher change and its tie to beliefs, including Preston and Lambdin (1995) and Gates' (2006) more recent study.

Each of these studies has in common a research focus on *how* mathematics is taught in schools. But additional research is needed that focuses on *what* we teach as mathematics and, even more so, how teachers view the mathematics that they teach. Is mathematics transcendental and pure, something that exists outside of humanity, or is it a social activity, a social construction whose rules and procedures are defined by humanity (Restivo & Bauchspies, 2006)? An extensive review of the literature found no studies that lead teachers to explore their philosophies of mathematics and investigated the results. Yet Restivo and Bauchspies recognized the need to push teachers' understanding of mathematics beyond the debate of mathematics as a social construction. To understand mathematics (and thus to teach mathematics) is to understand the social, cultural, and historical worlds of mathematics (Restivo & Bauchspies). Should we not then explore mathematics, explore it in a philosophical sense, its "basic principles and concepts…with a view to improving or reconstituting them" (*Webster's Dictionary*, 2003, p. 1455)?

Change in classroom practices may not be possible without first "improving or reconstituting" teachers' philosophies of mathematics. But, as Taylor (1996) pointed out, school mathematics continues to be heavily influenced by certain myths—including the myth of cold reason. It is this myth—the myth that mathematics is a dispassionate and impersonal subject, a subject of pure reason—that my study leads teachers of mathematics to confront through an exploration of readings from modern philosophers of mathematics and mathematicians.

I end this literature review by revisiting a definition of philosophy, this time specifically, philosophy of mathematics: "The philosophy of mathematics is basically concerned with systematic reflection about the nature of mathematics, its methodological

problems, its relations to reality, and its applicability" (Rav, 1993, p. 81). If our goal in mathematics education reform is to make mathematics more accessible and more applicable to real-world learning, should we not then help guide today's teachers of mathematics, those who develop and nurture tomorrow's mathematicians, to delve into this realm of systematic reflection and to ask themselves, what is mathematics?

# CHAPTER 3

# THEORETICAL FRAMEWORK

Mewborn (2005) compared a theoretical framework to a picture frame, a bed frame, or the frame of a house. A researcher's theoretical framework, wrote Mewborn, "can help 'set off' ideas from other data to draw attention to them, giving them names and robust definitions. It can support the building up and deepening of an idea, or it can provide a structure on which to hang new ideas" (p. 2). For Kilbourn (2006), the "theoretical framework represents a point of view that legitimizes the manner in which the interpretations are justified or warranted" (p. 533). The theoretical framework for my study stems from two sources—modern explorations in the philosophy of mathematics that troubles the traditional absolutist view of mathematics (Davis & Hersh, 1980; Hersh, 1997; Lakatos, 1976; Tymoczko, 1998), which will be explored further in a later section, and Ernest's (1988, 1991, 1998b, 2004) work on social constructivism as a *philosophy* of mathematics. If radical instructional change is our goal—and it is my contention that the current reform movement in mathematics teaching and learning calls for just that—then research is needed that more deeply explores teachers' philosophies of mathematics (and not just their philosophies of education or mathematics education). Thus, challenges to traditional ideas of mathematics must be brought into the mathematics classroom.

## Social Constructivism

Social constructivism, a learning theory that grew out of the early 20$^{th}$ century work of Russian Lev Vygotsky, is in many ways the driving force behind mathematical

reform in the United States and other nations (Forman, 2003). Forman and others (e.g., Restivo & Bauchspies, 2006; Toumasis, 1997) have argued that NCTM's *Professional Standards for Teaching Mathematics* (1991) and the later *Principles and Standards for School Mathematics* (2000) clearly build upon a social constructivist model of learning. But Ernest (1991, 1994, 1998b, 1999) argues that social constructivism is more than just a learning theory applicable to the teaching and learning of mathematics. Social constructivism, according to Ernest, is a *philosophy* of mathematics, one that views mathematics as a *social construction*. In this section, I will define social constructivism as it applies to the teaching and learning of mathematics and will then describe Ernest's view of social constructivism as a philosophy of mathematics. This section will close with an explanation of how Ernest's work can be used as a theoretical framework, a lens through which to view a research study that explores and challenges teachers' philosophies of mathematics. The next section will further develop the use of Ernest's work as a theoretical frame.

Constructivism as a theory of learning may be defined in a number of ways. Using Confrey's (1990) definition:

> Constructivism can be described as essentially a theory about the limits of human knowledge, a belief that all knowledge is necessarily a product of our own cognitive act…We construct our understanding through our experiences, and the character of our experience is influenced profoundly by our cognitive lenses…When one applies constructivism to the issue of teaching, one must reject the assumption that one can simply pass on information to a set of learners and expect that understanding will result. (p. 108–109)

Thus constructivism requires an active educational process, not merely the memorization of facts and the practicing of procedures, but the engagement of the learner in *doing* mathematics.

Constructivism is often divided into two camps: radical constructivism and social constructivism (Davis, Maher, & Noddings, 1990; Lerman, 1994). Radical constructivism focuses primarily on the individual in the learning process and is greatly influenced by the work of Piaget and von Glasersfeld. Social constructivism focuses on the community of the mathematics classroom and the communication that takes place there (Noddings, 1990). As cited earlier, social constructivism grew out of the work of Vygotsky (1978) in social learning theory and has been further developed in mathematics teaching and learning through the work of Confrey (1990); Lerman (1990, 1998, 1999); Damarin (1999); and others. This theory is in keeping with NCTM's (2000) views of mathematics instruction that emphasizes the social interplay necessary to mathematics instruction:

> Students' understanding of mathematical ideas can be built throughout their school years if they actively engage in tasks and experiences designed to deepen and connect their knowledge. Learning with understanding can be further enhanced by classroom interactions, as students propose mathematical ideas and conjectures, learn to evaluate their own thinking and that of others, and develop mathematical reasoning skills. Classroom discourse and social interaction can be used to promote the recognition of connections among ideas and the reorganization of knowledge. (p. 21)

Overall, social constructivism moves educators from a view of mathematical learning as something people *gain* to a broader view of mathematical learning as something people *do* (Forman, 2003).

It is upon this foundation, mathematics as something people do, that Ernest (1998b) built his theory of social constructivism as a philosophy of mathematics. He argued that the teaching and learning of mathematics is indelibly linked to philosophy of mathematics:

> Thus the role of the philosophy of mathematics is to reflect on, and give an account of, the nature of mathematics. From a philosophical

> perspective, the nature of mathematical knowledge is perhaps the central feature which the philosophy of mathematics needs to account for and reflect on. (p. 50)

Without that link, Ernest argued, we cannot truly understand the aims of mathematics education. Ernest (2004) emphasized the need for researchers, educators, and curriculum planners to ask “what is the purpose of teaching and learning mathematics?” (p. 1). But, in order to answer that, both mathematics and its role and purpose in society must be explored. Dossey (1992) echoed Ernest’s emphasis on philosophy of education: “Perceptions of the nature and role of mathematics held by our society have a major influence on the development of school mathematics curriculum, instruction, and research” (p. 39). Yet there is in the educational sphere a lack of conversation about and exploration of philosophy: “The lack of a common philosophy of mathematics has serious ramifications for both the practice and teaching of mathematics. This lack of consensus, some argue, is the reason that differing philosophies are not even discussed” (Dossey, p. 39).

Ernest (1991, 1998b) described two dichotomist philosophical views of mathematics—the absolutist and the fallibilist. The Platonists and formalists schools of philosophy, which will be further described in a later section, both stem from an absolutist view of mathematics—mathematics as a divine gift or mathematics as a consistent, formalized language without error or contradiction. Both of these schools of thought hold mathematics up to be infallible, without error owing either to its existence beyond humanity, waiting to be discovered (the Platonist school) or to its creation as a logical, closed set of rules and procedures (the formalist school). The fallibilist philosophy, what Hersh (1997) termed a philosophy of humanism, views mathematics as

a human construction and, therefore, fallible and corrigible. One important implication of the fallibilist philosophy of mathematics is that if mathematics is a human construct then so must the learning of mathematics be a human construct. Mathematics is no longer knowledge that is simply memorized in a rote fashion, but it is knowledge with a societal purpose that must be learned in a manner that brings meaning to the subject. The constructivist approach to learning, therefore, aligns well with the fallibilist philosophy of mathematics.

Ernest (1991, 1998b) characterized a cycle of subjective knowledge to objective knowledge back to subjective knowledge to further support his view of the social constructivist foundations of mathematical knowledge. In this cycle, new knowledge begins as subjective knowledge, the mathematical thoughts of an individual. This new thought becomes objective knowledge through a social vetting process; it enters the public domain and is tested, reformulated and refined, a method akin to Lakatos' (1976) system of proofs and refutations (described further in a later section). This objective knowledge then becomes internalized and understood by the individual, thus becoming once again subjective knowledge. Through this cycle, Ernest explained how objective knowledge, knowledge that may appear to exist independent of humanity, is actually knowledge made legitimate and real through social interaction and acceptance. The social process of learning and then knowing mathematics is, therefore, intricately linked to society's ideas of what is and is not mathematics. Using this cyclical nature of knowledge building, Ernest was able to connect a learning theory, social constructivism, with a philosophy of mathematics.

## A Postmodern View of Mathematics

During the past 50 years, there has been a growing questioning of the historical and philosophical foundations of mathematics. What was once seen as certain and unquestionable, existing beyond humankind and waiting to be discovered, is now viewed by some as a historical and social construction, changing and malleable, as subjective as any social creation. Aligned with these changing views of mathematics are new ideas about mathematics instruction. The absolutist view of mathematics is associated with a behaviorist approach, utilizing drill and practice of discrete skills, individual activity, and an emphasis on procedures. The fallibilist view of mathematics aligns itself with the constructivist pedagogy, utilizing problem-based learning, real-world application, collaborative learning, and an emphasis on process (Threlfall, 1996). But, while there have been numerous calls to change and adapt our teaching of mathematics, embracing a constructivist epistemology, little has been done to challenge teachers' conceptions of mathematics. The push towards constructivist instructional practices in the mathematics classroom and the modern challenges to our views of mathematics have been brought together through Ernest's work over the past 20 years: "Teaching reforms cannot take place unless teachers' deeply held beliefs about mathematics and its teaching and learning change" (Ernest, 1988).

Ernest's (2004) more recent work is embedded in the postmodern[2]—challenging absolutes about human knowledge and understanding while questioning the rationality

[2] Lyotard (1979/1984) defines the postmodern, quite simply as an "incredulity toward metanarratives" (p. xxiv). Leistyna, Woodrum, and Sherblom (1996) define the postmodern as a rejection of universal truths and values: "Postmodernists don't believe that the mind has an innate, universal structure; rather, they see consciousness, identities, and meaning as socially and historically produced" (p. 341). Although acknowledged as a contested terrain, Usher and Edwards (1994) emphasize that the postmodern rejects the modernist view of science and scientific research as value-free and objective. And, in fact, "the significant

and objectivity of mathematics. He seeks to break down the influence of what he terms the "narratives of certainty" that have resulted in "popular understandings of mathematics as an unquestionable certain body of knowledge" (p. 16). Certainly this understanding still predominates in mathematics classrooms today (see, e.g., Bishop, 2002; Brown, Jones, & Bibby, 2004; Davison & Mitchell, 2008; Golafshani, 2004; Handel & Herrington, 2003). But Ernest draws upon postmodern philosophers such as Lyotard, Wittgenstein, Foucault, Lacan, and Derrida, to challenge traditional views of mathematics and mathematics education. He embraces the postmodern view because it rejects the certainty of Cartesian[3] thought and places mathematics in the social realm, a human activity influenced by time and place. Ernest is joined by others who have explored mathematics and mathematics instruction through the postmodern (see, e.g. Brown, 1994; Walkerdine, 1994; Walshaw, 2004). Neyland (2004) calls for a postmodern perspective in mathematics education to "address mathematics as something that is enchanting, worthy of our esteem, and evocative of wonder" (p. 69). In so doing, Neyland hopes for a movement away from mathematics instruction emphasizing procedural compliance and onto a more ethical relationship between teacher and student, one that stresses not just enchantment in mathematics education but complexity as well.

Walshaw (2004) ties sociocultural theories of learning to postmodern ideas of knowledge and power, drawing, as Ernest does, on the writings of Foucault and Lacan: "Knowledge, in postmodern thinking, is not neutral or politically innocent" (p. 4). Thus,

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thing is that in postmodernity uncertainty, the lack of a centre and the floating of meaning are understood as phenomena to be celebrated rather than regretted" (Usher & Edwards, p. 10).

[3] Cartesian thought stems from the work of 17th century philosopher and mathematician, Rene Descartes. Descartes believed in the certainty of mathematics and viewed it as the primary tool for revealing truths about nature. Through his 1637 *Discourse in the Methods, Optics, Geometry, and Meteorology*, Descartes "proclaims the importance of individual autonomy in the search for truth" (Hersh, 1997, p. 111).

for example, issues of equity in mathematics can be seen in ways other than who can and cannot *do* mathematics. Indeed, societal issues of power and reproduction must be considered. A postmodern analysis forces a questioning of mathematics as value-free, objective, and apolitical (Walshaw, 2002). Why are the privileged mathematical experiences of the few held up as the needed (but never attained) mathematical experiences of all? Furthering a postmodern view of mathematics, Fleener (2004) draws on Deleuze and Guattari's idea of the rhizome[4] in order to question the role of mathematics as lending order to our world: "By pursuing the bumps and irregularities, rather than ignoring them or 'smoothing them out,' introducing complexity, challenging status quo, and questioning assumptions, the smoothness of mathematics is disrupted" (p. 209). The traditional view of mathematics has ignored the bumps and holes, and forced a vision of mathematics as smooth, neat, and orderly.

Another postmodern view is that our representations of mathematics cannot be divorced from the language we use to describe those representations:

> Any act of mathematics can be seen as an act of construction where I simultaneously construct in language mathematics notions and the world around me. Meaning is produced as I get to know my relationships to these things. This process is the source of the post-structuralist notion of the human subject being constructed in language. (Brown, 1994, p. 156)

Brown uses Derrida's ideas on deconstructing[5] language to examine how the social necessity of mathematical learning means that mathematics is always, in some way,

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[4] Building from the botanical definition of rhizome, Deleuze and Guattari (1980/1987) used the analogy of the rhizome to represent the chaotic, non-linear, postmodern world. Like the tubers of a canna or the burrows of a mole, rhizomes lead us in many directions simultaneously. Deleuze and Guattari described the rhizome as having no beginning or end; it is always in the middle.

[5] Although an elusive term to define, *deconstruction* involves the continuous critique of text and language, a way to look beyond accepted meanings and explore the "multiplicity of truth" (Usher & Edwards, 1994, p. 120). "The goal is to keep things in process," wrote Lather (1991), "to disrupt, to keep the system in play, to set up procedures to continuously demystify the realities we create, to fight the tendencies for our

constructed. And, in examining new mathematical ideas, the learners cannot help but bring their entire mathematical and personal history to the process (Brown, 1994). This view strengthens Ernest's (1998b) own contention of the philosophical basis of social constructivism. Mathematical ideas begin as social constructions but "become so embedded within the fabric of our culture that it is hard for us to see them as anything other than givens" (Brown, p. 154). Thus the establishment of mathematical meta-narratives[6] camouflages the social/culture roots of mathematical knowledge. As a result, mathematics continues to be viewed as something discovered, not constructed, especially, I believe, by K–12 teachers and students.

Siegel and Borasi (1994) describe the pervasive cultural myths that continue to represent mathematics as the discipline of certainty. In order to confront this idealized certainty, they contend, what is needed is an inquiry epistemology that "challenges popular myths about the truth of mathematical results and the way in which they are achieved, and suggests, instead, that: mathematical knowledge is fallible…[and] mathematical knowledge is a social process that occurs within a community of practice" (p. 205). This demystifying process is necessary, argue Siegel and Borasi, if teachers are to engage students in *doing* mathematics, not simply memorizing rote procedures and discrete skills. The inquiry classroom described by Siegel and Borasi mirrors the

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categories to congeal" (p. 13). "The word 'deconstruction', like all other words," wrote Derrida (1985), "acquires its value only from its inscription in a chain of possible substitutions, in what is too blithely called a 'context'…What deconstruction is not? Everything of course! What is deconstruction? Nothing of course!" (p. 3–4)

[6] A meta-narrative, wrote Kincheloe and Steinberg (1996), "analyzes the body of ideas and insights of social theories that attempt to understand a complex diversity of phenomena and their interrelations" (p. 171). It is, in other words, a story about a story; a meta-narrative seeks to provide a unified certainty of knowledge and experience, removed from its historic or personalized significance.

mathematics teaching and learning outlined in NCTM’s 2000 *Principles and Standards for School Mathematics*.

If, as Lerman (2001) asserts, the object of educational research is a particular moment in the zoom of a lens, then using Ernest’s work as the lens opens up the possibilities of a postmodern view of both mathematics and mathematics education. Framing my research around Ernest, one goal of my study is to have teachers problematize the idea of mathematics as objective and value-free, thus opening new and, perhaps, unexplored views of mathematics. My research, therefore, utilizes Ernest’s theories to merge the social constructivist approach of current educational reform in mathematics with the philosophical questioning of mathematics, described in the next section.

## Mavericks in the Philosophy of Mathematics

> A world of ideas exists, created by human beings, existing in their shared consciousness. These ideas have objective properties, in the same sense that material objects have objective properties. The construction of proof and counterexample is the method of discovering the properties of these ideas. This branch of knowledge is called mathematics. (Hersh, 1997, p. 19)

A reawakening of the philosophy of mathematics occurred during the last part of the 20th century (Hersh, 1997). Davis and Hersh (1981) explored ideas of mathematics as a human invention, a fallibilist construct, in their landmark book, *The Mathematical Experience*. In their book, Davis and Hersh described several schools of philosophical thought regarding mathematics—including Platonism and formalism. The Platonist views mathematics as being outside of human beings, bigger than humans: “Mathematics, in this view, has evolved precisely as a symbolic counterpart of the universe. It is no wonder, then, that mathematics works; that is exactly its reason for existence. The universe has

imposed mathematics upon humanity" (p. 68). The Platonist not only accepts, but embraces god's (God?) place in mathematics. For what is mathematics but god's gift to us mortals? (Plato, trans. 1956) The Platonist, forever linking god and mathematics, sees the perfection of mathematics. If there are errors made in our mathematical discoveries (and, of course, they are discoveries not inventions because they come from a higher power), then the errors are ours as flawed humanity, not inherent to the god-given mathematics. And because mathematics is this higher knowledge, a gift from god, it follows that some will succeed at mathematics while others fail. Mathematics, in the Platonic view, becomes a proving ground, a place where those who are specially blessed can understand mathematics' truths (and perhaps even discover further truths) while the vast numbers are left behind. Euclid's Elements was (and still is) the bible of belief for mathematical Platonists. So naturally, as Davis and Hersh pointed out, "the appearance a century and a half ago of non-Euclidean geometries was accompanied by considerable shock and disbelief" (p. 217). The creation of non-Euclidean geometries—systems in which Euclid's fifth postulate (commonly known as the parallel postulate) no longer held true—momentarily shook the very foundations of mathematical knowledge:

> The loss of certainty in geometry was philosophically intolerable, because it implied the loss of all certainty in human knowledge. Geometry had served, from the time of Plato, as the supreme exemplar of the possibility of certainty in human knowledge. (Davis & Hersh, p. 331)

A result of the uncertainty brought on by the formation of non-Euclidean geometries was the development of formalism. As Davis and Hersh (1981) wrote, "the formalist makes a distinction between geometry as a deductive structure and geometry as a descriptive science" (p. 341). While the first, the deductive logic of geometry, is acknowledged as mathematics by the formalist, the second, the descriptive aspect of

geometry, utilizing pictures and diagrams, are non-mathematical according to the formalist, as they merely describes physical world around us. In formalism, mathematics is the science of rigorous proofs, a language for other sciences (Davis & Hersh). “The formalist says mathematics isn’t about anything, it just *is*” (Hersh, 1997, p. 212). In the early part of the 20th century, Frege, Russell, and Hilbert, each attempted to formalize all of mathematics through the use of the symbols of logic and set theory. Russell and Whitehead’s “unreadable masterpiece” (Davis & Hersh, p. 138), *Principia Mathematica*, attempted the complete logical formalization of mathematics. But the attempts to complete the logical formalization of mathematics were doomed to failure as demonstrated later by Gödel’s Incompleteness Theorem that proved that any formal system of mathematics would remain incomplete, not provable within its own system (Goldstein, 2005).

*Proofs and Refutations: The Logic of Mathematical Discovery*, a beautifully written exploration of the philosophy of mathematics penned by Imre Lakatos and published posthumously in 1976, offers a third philosophy of mathematics—one that has been termed the humanist philosophy by Davis, Hersh, Ernest, and other modern philosophers of mathematics. In *Proofs and Refutations*, Lakatos used the history of mathematics as well as the structure of an inquiry-based mathematics classroom to explore ideas about proof. Through a lively Socratic discussion between fictional teacher and students, Euler’s formula (V – E + F = 2) is dissected, investigated, built upon, improved, and finally made nearly unrecognizable. Lakatos used the classroom dialogue to challenge accepted ideas about proof—forcing the reader to question whether proofs are ever really complete or do mathematicians just agree to ignore that which contradicts

the proof, what Lakatos' students termed *monsters*. Through this analogy, Lakatos demonstrated that in mathematics there are many monsters, most of which are ignored, as though the mathematical community has made a tacit agreement to turn away from that which makes it uncomfortable.

Ernest built much of his philosophy of mathematics and mathematics education on the writings of Lakatos. Like Lakatos, Ernest (1998b) saw mathematics as indubitably tied to its creator—humankind: "Both the creation and justification of mathematical knowledge, including the scrutiny of mathematical warrants and proofs, are bound to their human and historical context" (p. 44). Hersh (1997), in his book, *What is Mathematics, Really?*, included both Lakatos and Ernest on his list of "mavericks"—thinkers who see mathematics as a human activity and, in so doing, having impacted the philosophy of mathematics. Others are included as well—philosophers Charles Sanders Peirce (cited by Siegel & Borasi, 1994) and Ludwig Wittgenstein (cited by Ernest, 1991, 1998b); psychologists Jean Piaget and Lev Vygotsky (cited by Confrey, 1990, and Lerman, 1994); and mathematicians George Polya and Philip Kitcher.

Polya's (1945/1973) classic, *How to Solve It: A New Aspect of Mathematical Method*, revived the study of the methods and rules of problem solving—called heuristics—in mathematics. Although he eschewed philosophy, Polya certainly influenced the philosophers of mathematics who succeeded him by viewing mathematics as a human endeavor. He described the messiness of the mathematician's work:

> Mathematics in the making resembles any other human knowledge in the making. You have to guess a mathematical theorem before you prove it; you have to guess the idea of the proof before you carry through the details. You have to combine observations and follow analogies; you have to try and try again. The result of the mathematician's creative work is demonstrative reasoning, a proof; but the proof is discovered by plausible

> reasoning, by guessing…Demonstrative reasoning is safe, beyond controversy, and final. Plausible reasoning is hazardous, controversial, and provisional. (Polya, 1954/1998, pp. 99–100)

Both Polya and Lakatos led mathematicians into new areas that questioned the very basis of mathematical knowledge. Their combined impact on the philosophy of mathematics was as important as the development of non-Euclidean geometries and Gödel’s Incompleteness Theorem (Davis & Hersh, 1981). By defining mathematics as a social construct, they opened up the field to new interpretations. No longer was mathematics a subject for the elite. Polya’s heuristic emphasized the accessibility of problem solving—and mathematics for Polya was about solving problems. Lakatos, by using dialogue to trace the evolving knowledge of mathematics—the proofs and refutations—stressed the social aspects of mathematical learning as well as the fallibility of mathematical knowledge and defined mathematics as quasi-empirical. Ernest (1998b) credited Lakatos with a synthesis of epistemology, history, and methodology in his philosophy of mathematics—a synthesis that impacted the sociological, psychological, and educational practices of mathematics.

Ernest (1998b), like Hersh (1997), also referred to Kitcher as a maverick, in that Kitcher stressed the importance of both the history of mathematics and the philosophy of mathematics, a point Ernest makes as well. Kitcher (1983/1998), in his work, underscored the concept of change in mathematics: “Why do mathematicians propound different statements at different times? Why do certain questions wax and wane in importance? Why are standards and styles of proof modified?” (p. 217). His conclusion: that mathematics changes in practice, not just in theory. Kitcher identified five components of mathematical practice—language, metamathematical views, accepted

questions, accepted statements, and accepted reasonings—that are compatible (Hersh, 1997). As one component changes, others must change as well. Kitcher's five components emphasized the social aspect of mathematics, mathematics as a community activity with agreed upon norms and practices. Kitcher's view of mathematics mirrors Ernest's cycle of *subjective knowledge → objective knowledge → subjective knowledge* and Lakatos' idea of proofs and refutations in that each generation simultaneously critiques, internalizes, and builds upon the mathematics of the previous generation (Hersh, 1997).

I will add to the list of mavericks one more name (although there are many more): Sal Restivo. Restivo is a sociologist and social scientist at Rensselaer Polytechnic Institute who wrote that "math worlds are social worlds" (1993, p. 269). Hersh (1997) terms Restivo a constructivist sociologist. Ernest (1998b) credited Restivo with challenging the Cartesian dualism of mind and body by viewing human thinking and doing as one. But Restivo views mathematics as a political entity as well (for if it's social, is it not also political?). Restivo (1993) highlighted issues of equality and power when he describes mathematics as a tool of ruling elites:

> As a social institution, modern mathematics is itself a social problem in modern society…It tends to serve ruling-class interest; it can be a resource that allows a professional and elite group of mathematicians to pursue material rewards independently of concerns for social, personal, and environmental growth, development and well-being; aesthetic goals in mathematics can be a sign of alienation or of false consciousness regarding the social role of mathematicians; and mathematical training and "education" may stress "puzzle solving" rather than ingenuity, creativity, and insight…If we adopt the constructivist perspective that *social* talk about mathematics is the key to understanding mathematics (including mathematical knowledge), then our approach to solving the social problems of mathematics *and* the problems of "mathematics as a social problem" will necessarily focus on social roles and institutions. (pp. 275–276)

Thus, Restivo asserted, mathematics educational reforms cannot occur without a view towards the broader issues of power, social structure, and values. How these issues of power, social structure, and values intersect with Ernest's philosophy of mathematics and my own research is the focus of the next section.

## A Personal Theoretical Lens

Although I have long been a feminist, I first became familiar with the idea of postmodern feminism through the writings of Elizabeth St. Pierre and Patti Lather.[7] My feminism is inextricably linked with my role as a mathematics educator and researcher. As a student, my experience of mathematics was distinctly female, constantly feeling awkward and out of place for succeeding where society thought I should not. As a teacher, I was shocked to find my mathematically-talented female students affecting ignorance and misunderstanding in my advanced algebra courses. Had the world not changed for these young women in the past 30 years? As I move into a research sphere, although the experiences of girls and women are not the focus of my study, I cannot help but be influenced by issues of equity and power in the mathematics classroom. Poststructural feminist use of critiques such as deconstruction lead researchers to investigate how women and other disenfranchised groups are left out of the mainstream of educational success, particularly in the areas of mathematics and science. And, as St. Pierre (2000) noted, "deconstruction is not about tearing down but about rebuilding; it is not about pointing out an error but about looking at how a structure has been constructed, what holds it together, and what it produces" (p. 482). Thus a feminist view of deconstruction

[7] The terms postmodern and poststructural are often used interchangeably, although some scholars insist on distinct meanings (Crotty, 1998). St. Pierre generally uses the term poststructural, perhaps due to her emphasis on language and text. I will follow Lather's (1991) lead, however, and use the terms interchangeably, primarily using postmodern unless the cited author has used a different term.

is not a destructive one but an affirming one. The postmodern feminist is more than willing to let go of the meta-narratives of the past and to look forward to newly constructed understandings, understandings that may vary from person to person, influenced by social relations, as well as varied representations of race, class, and gender (Lather, 1991). But the postmodern feminist does not forgo responsibility:

> Poststructuralism does not allow us to place blame elsewhere, outside our own daily activities, but demands that we examine our own complicity in the maintenance of social injustice. Feminism's slogan that everything is political must be joined with the poststructural idea that "everything is dangerous" (Foucault, 1984/1983, p. 343). (St. Pierre, 2000, p. 484)

Postmodern feminism, therefore, holds us each accountable to examine the meta-narratives and to challenge their effects on teachers and students. Pedagogy is not neutral, Lather (1991) told us. The teacher is not a neutral transmitter of knowledge, the student is not a passive receiver, and the knowledge itself is never value-free and immutable. Thus Lather echoed Ernest's pedagogical belief in social constructivism. Yet how do we as mathematics educators and education researchers embrace the postmodern while still acknowledging our understanding of the mathematical and scientific? "What does it mean to recognize the limits of exactitude and certainty, but still to have respect for the empirical world and its relation to how we access and formulate theory?" (Lather, 1991, pp. 124–125).

Walkerdine (1994), Burton (1995), and Walshaw, (2001) help us to bridge the gap between postmodern feminism and philosophies of mathematics and mathematics education. Walkerdine used a postmodern analysis in her research to "understand how I, and other oppressed and exploited peoples come to see themselves as unable to think" (p. 61). Walkerdine sought to question not only the meta-narratives of science and scientific

knowledge, but the very foundations of developmental theory. Do our understandings of development privilege the male way of knowing and reasoning while diminishing that of our girls?

> My argument therefore is that it is not simply the case that girls are poor at mathematics, reasoning and so forth, but that the "truth" of child development pathologizes and defines their performance in such a way as to read it as bad. (Walkerdine, p. 68)

But Walkerdine (1994) was not just looking at the differences between girls and boys in our educational system; others such as non-Europeans, Black and White working-class children are also devalued in their mathematical thinking, argued Walkerdine.

Burton (1995), though describing a "feminist epistemology" of mathematics, also moved the focus beyond the education of females as she describes a "colonization of mathematics" that has diminished the contributions of non-Western cultures. And, although Burton championed social constructivism as a philosophical position with profound implications for pedagogy, she wanted educators to go further than challenging and changing their classroom practices: "Scratch a pedagogical or philosophical constructivist and underneath you are likely to expose an absolutist" (p. 214). Why are our views of mathematics are non-negotiable, Burton asked, even as we attempt to bring about pedagogical reform.

Walshaw (2002) echoed this "epistemological double standard," whereas we adduce mathematics as objective and value-free while, at the same time, privilege the mathematical experiences and knowledge of some at the expense of others. But Walshaw (2001) moved away from Burton's standpoint theory of knowledge, that in some ways essentialized the learning of girls, and looked instead at the complexity of learning and learners in her postmodern "gaze." The desire to stay clear of essentializing the learning

of mathematics has led me to blend the ideology and epistemology of postmodern feminism with recent concepts of critical pedagogy as well as the theories of progressive educators such as Dewey and Freire.

Kincheloe and Steinberg (1996) drew on postmodern feminist theory as well as critical pedagogy to frame their "critical constructivism (a constructivism grounded on an understanding of critical theory and postmodernism)" (p. 171). Kincheloe and Steinberg called for this critical constructivism to transcend cognitive essentialism and acknowledge the complexities of what they term "post-formal thinking." Issues of power and knowledge are ineffaceably linked and must be uncovered and affirmed: "Critical postmodern theory has taught us that little is as it appears on the surface" (p. 181). Like other postmodern researchers and educators, Kincheloe and Steinberg emphasized the process of deconstruction, noting that what is absent in a text might often be as important as what is present. These authors recall Dewey's concerns, nearly a century before, of decontextualized knowledge, and his contention that knowledge should never be viewed outside the context of its relationship to other information.

Both Ernest (1991, 1998b) and Noddings (1990) recognized Dewey's contributions to social constructivism. By stressing the essential role that social experiences play in constructing knowledge, Ernest (1998b) credited Dewey as one of the first philosophically-minded theorists to give attention to social interaction. Ernest (1991) even described Dewey's philosophy of mathematics as "more empiricist, even fallibilist, than absolutist" (p. 184). Unfortunately, Ernest noted, Dewey did not further develop his philosophy of mathematics and others failed to take up the standard. "It is," wrote Dewey (1938/1963), "easier to walk in the path that has been beaten" (p. 30).

Dewey's philosophy of education rested on a philosophy of experience. Students bring to their learning their own prior experiences and (mis)understandings that cannot be discounted. In addition, Dewey believed, students learn *through* experience, as well as through social interaction with others. Dewey (1938/1963) also anticipated the rapidly changing future that education is faced with today:

> The principle of continuity in its educational application means, nevertheless, that the future has to be taken into account at every stage of the educational process. This idea is easily misunderstood and is badly distorted in traditional education. Its assumption is, that by acquiring certain skills and by learning certain subjects which would be needed later…pupils are as a matter of course made ready for the needs and circumstances of the future…It is a mistake to suppose that acquisition of skills in reading and figuring will automatically constitute preparation for their right and effective use under conditions very unlike those in which they were acquired. (p. 47)

Dewey (1937/1987) also warned educators against indoctrination: "the systematic use of every possible means to impress upon the minds of pupils a particular set of political and economic views to the exclusion of every other" (p. 415). Skovsmose (2005) carried this idea even further when he warned us about the hidden agenda of mathematics: "Could it be that mathematics education operates as an efficient social apparatus for selection, precisely by leaving a large group of students as not being 'suitable' for any further and expensive technological education?" (p. 11). For a long time, mathematics has served as the gatekeeper (Stinson, 2004)—the course of study that separates those who will reap the financial and intellectual rewards of our educational system from those who will remain economically disenfranchised. Seventy years ago, Dewey (1937/1987) wrote "that society is in process of change, and that the schools tend to lag behind" (p. 408). Others now also warn of the dangers of elitism in our mathematics educational policies: "In effect, math instruction weeds out people and you wind up with what

amounts to a priesthood, masters of the arcane secrets of math through what appears to be some god-given talent or magic" (Moses & Cobb, 2001, pp. 9–10). Nearly a century ago, Dewey challenged society to change its view of education, just as Moses and Cobb (as well as Ernest) challenge us today to change our views of mathematics and mathematics education. Paulo Freire offered the same challenge:

> To teach is not to transfer knowledge, to transfer contents. To teach is to struggle, together with the students; it is to create conditions for the construction of knowledge, for the reconstruction of knowledge. For me, this is to teach. (Freire, D'Ambrosio, & Mendonca, 1997, p. 9)

In *Pedagogy of the Oppressed*, Freire (1970/2000) warned educators that sciences and technology, including mathematics, were powerful instruments of oppression. To fight this oppression, as well as the banking concept of education in which "knowledge is a gift bestowed by those who consider themselves knowledgeable upon those whom they consider to know nothing" (p. 72), Freire advocated an empowering ideal of education, one in which teachers are partners with their students. Similar to Dewey's vision of experiential education, Freire described a problem-posing education that embodied communication—a liberating education. Freire emphasized the need for dialogue in this educational model, a model in which both teacher and student are learners. Ernest (1991) embraced this view of education and aligned his philosophy of mathematics education with Freire's pedagogy of the oppressed: "Freire argues, as we have done, that objective knowledge is continually created and re-created as people reflect and act on the world" (p. 84). Ernest, Freire, Dewey—all educators who underscore the need for learners to make sense of their learning and to be active participants in the learning process, as well as the importance of the social process of education.

Like Stinson (in press), I have found it necessary to adopt an eclectic theoretical framework. As a researcher, I am still many selves: woman, mother, teacher, scholar, learner, agent of change. My framework, that which supports and legitimizes my study, must encompass all that I am. And so I have blended the social constructivist philosophy of Ernest, the humanist philosophies of mathematics, a postmodern feminism, and the progressive educational theories of Dewey and Freire. I strive to be ethical in my research as in my life and I adopt what I see as an ethical framework—one that recognizes the diverse needs of the students and teachers in our classrooms. I share a goal with the many others I have cited, to improve mathematics teaching and learning in our schools, but in so doing I want to embrace all that is unique and extraordinary in our learners. “Purposes are constructed as well as knowledge,” wrote Noddings (1993), and my eclectic theoretical framework constructs and frames the purposes of my research. Recognizing that mathematics often serves as a filter, as a gatekeeper, Hart (2003) advocated new avenues in research on equity and justice in mathematics education. An exploration of teachers’ philosophies of mathematics, challenging traditional views of mathematics and mathematics teaching and learning, is one such avenue.

# CHAPTER 4

# METHODOLOGY

In this chapter, I present a detailed explanation of my study, including a general description of the study, the conceptual framework that guides the design of the research and the analysis of the data, and a detailed account of narrative analysis as a research methodology. In addition, I explain the type of data that was collected and the method of analysis of that data. I conclude the chapter by addressing my role as a researcher, as well as issues of validity, confidentiality, and researcher ethics.

## Description of Study

The purpose of this study is to examine the process that educators go through as they formulate a personal philosophy of mathematics at a moment in time when they are also implementing a new, task-based mathematics curriculum. Four mathematics educators who had participated in a graduate-level course in the philosophy of mathematics were interviewed over a period of 18 months. Discussions of philosophy, mathematical background, and instructional practices were the foci of those interviews with the intent of creating a “mathematical story” for each of the four participants.

The participants of this study were all, at the onset of the study, graduate students in a major urban university in a large southeastern city. The graduate students chosen for the study were public school educators, enrolled part-time in a Specialist or Doctoral level degree program. The study participants included elementary and secondary school mathematics educators working as classroom teachers or instructional coaches. Each of

the participants was part of a graduate-level mathematics education course that was offered during the summer of 2007. The focus of that course (referred to in this paper as the summer course) was to challenge teachers' conceptions of mathematics through an exploration of (perhaps) new and different philosophies of mathematics and mathematics education. Modern philosophical writings in mathematics used in the summer course included Russell's (1919/1993) *Introduction to Mathematical Philosophy*, Lakatos' (1976) *Proof and Refutations: The Logic of Mathematical Discovery*, Davis and Hersh's (1981) *The Mathematical Experience*, Tymoczko's (1998) *New Directions in the Philosophy of Mathematics*, and Hersh's (1997) *What is Mathematics, Really?* (see Appendix A for the course syllabus). Course participants submitted a reading journal, a personal reflection on each of the readings, at the conclusion on the course. In addition to the reading journal, the course participants wrote two reflective essays during the course to help them explore their own philosophies of mathematics and to investigate the influence of their personal philosophies on their instructional practices. Course discussions were conducted in a Socratic method, with participants being encouraged to question themselves and each other during their philosophical explorations. Each week a course participant would be responsible for leading the class discussion, while another would scribe, or keep minutes of the discussion. Course participants were given graduate-level credit for the class. I was a co-creator as well as a co-instructor of the course, working with Dr. David Stinson, my major advisor. The summer course grew out of an exploration of philosophy of mathematics that Dr. Stinson and I had undertaken earlier the previous year, and the readings focused on the "mavericks" in the philosophy of mathematics. As my advisor, David Stinson, and I organized the readings that became the focus of our summer course,

our intent was not to present a “balanced” view of mathematical philosophies. Our intent was to trouble teachers’ philosophies of mathematics, to deconstruct the meta-narratives of mathematics as value-free, objective, and privileged. We chose readings that challenged the traditional view of mathematics.[8] Our goal was to engage the course participants in a problematization of the nature of mathematics, and to begin a process that led them to question previously held perceptions about mathematics and mathematics education. The intent was not “innocent” as this study was not merely descriptive, but actively sought change in teachers’ perceptions and even instructional practices. I will address our intent further in the following section.

## Conceptual Framework

It is important to be explicit about the conceptual framework of a research project, particularly a project that seeks change. As Lather (1991) posited, no research is neutral. Each researcher and each research project represents a particular lens, a “point of view that legitimizes the manner in which the interpretations are justified or warranted” (Kilbourn, 2006, p. 533). The conceptual framework should be interwoven throughout a research project: from design of the project and choice of data collected, to method of data analysis, interpretation of the findings, and even the manner in which the data is presented (Mewborn, 2005). In addition, Brown (1998) argued quite convincingly that pragmatic reasons alone do not justify research in mathematics education, that a fully articulated theoretical framework is essential if a research project is to impact educational

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[8] We also, unfortunately, chose readings that privileged the Western, Greek-based mathematics that is favored in our educational system. We did this because, like Hersh (1997), we found that the literature on non-Western mathematics was limited in its philosophical focus. I believe that a comprehensive examination of non-Western mathematics and its influences on the mathematics taught in this country, is a much needed topic and hope that future research will move in that direction.

practices. I will, therefore, spend some time addressing the conceptual framework of this study, in particular how it relates to this study's methodology.

There has been a call, in recent years, to challenge the traditions of mathematics education research and to pursue alternative research directions, including both constructivist and postmodern research paradigms (see, e.g., Ernest, 1998a, 1998b, 2004; Lerman, 1998, 2000, 2001; Lester & Lambdin, 1998; Valero, 2004). Other researchers have explored issues of equity and access in mathematics education through a new lens as well, advocating new ways to view not just how we teach mathematics, but how we view the mathematics that we teach (see, e.g., Boaler, 2002; Hart, 2003; Martin, 2003; NCTM Research Committee, 2005). The conceptual framework of this project follows suit by embracing a postmodern lens through which to view the research. At the same time, I acknowledge the importance of social constructivist epistemologies of teaching and learning, both in the classroom and in research as well. A researcher embracing a constructivist paradigm believes that knowledge is socially constructed and that one of the goals of research is to understand the complexity of the social world, the world in which we all live and learn, from the point of view of those who live in it (Mertens, 2005). Hart (2003) offers a criticism of mathematics education research that is too narrowly focused when she writes that "to understand what is happening in an individual mathematics classroom it is important to examine the experiences of teachers both inside and outside the classroom" (p. 36). Both the constructivist and the postmodern views of education and research are needed in this study, which examines and challenges teachers' philosophical views of mathematics, views that influence their instructional practices as well as their responses to the current reform initiatives in mathematics education.

A postmodern approach to mathematics education research seeks to critique the predominance of scientific rationalism in mathematics educational research (and other research domains as well). As Valero (2004) wrote, this critique is necessary for two reasons: “The first is to support a search for plausible, alternative understandings of mathematical education in schools; the second is to break with the deeply entrenched modern systems of reason in which our discipline (mathematics education) has built” (p. 51). Teppo (1998) also acknowledged this changing view of mathematics education research:

> The multiple perspectives from which the nature of mathematics is now being considered and the variations in processes and contexts that are increasingly being used to characterize school mathematics reflect a paradigm shift from a modernist to a postmodernist worldview. This shift represents a reconceptualization of the nature of knowledge from a single and external reality to a set of multiple and subjective realities. (pp. 9–10)

By recognizing the complexities of knowing and learning, some educators and researchers are changing their views on what it means to “do math” as well as what it means to “do research.” Research in mathematics education must be more reflexive (Ernest, 1998a); researchers and educators must look not only at the teaching and learning of mathematics, but at the mathematics itself. In fact, this study is framed around Ernest’s ongoing work in the field of philosophy of mathematics. Ernest (2004) identified five key questions that can focus our exploration of mathematics using a philosophical approach. What is mathematics? How does mathematics relate to society? What is learning mathematics? What is teaching mathematics? What is the status of mathematics education as a knowledge field? Ernest (1998a, 1998b) called for research that challenges not only our views of mathematics teaching and learning, but our views of mathematics itself. Ernest (1998b) described:

> A move to reconceptualize accounts of mathematics to accommodate greater plurality and diversity, including external, social dimensions of mathematics—its history, applications, and uses…The outcome is a demystification of mathematics to the benefit of the discipline and mathematicians and also to students, teachers, and other users of mathematics in society. (pp. 25–26)

This study, which examines teachers' philosophies of mathematics and challenges them to explore what it means to "do mathematics" in their classroom and in society, also examines what it means to do research. Just as Hersh and Davis (1981) and other "humanist" philosophers of mathematics question societal assumptions of mathematics as value-free, this study does not consider research to ever be value-free. All research takes on a certain point of view (Lather, 1991); it assumes an often unspoken view of the world. This study, by acknowledging and embracing the postmodern ideas of complexity and non-certainty, also embraces an empowering agenda. Lather (1991) advocated for the "development of research approaches which empower those involved to change as well as to understand the world" (p. 3). The understandings sought in this study were not predetermined; the participants and the researcher were challenged to question their ideas of mathematics, mathematics education, and mathematics education research. The results of that challenge are not fixed and the data presented in the next chapter, as well as my analysis of that data, are only a snapshot of a particular time in the lives of the participants and the researcher.

## Narrative Analysis

Several authors have sought to differentiate between the terms *methods* and *methodology* (see, e.g., Bogdan & Biklen, 2003; Grbich, 2007; Mertens, 2005), sometimes with differing conclusions. While the previous section described the methodological or conceptual framework of this study, this section will describe how

narrative analysis framed the specific methods of data collection and analysis that were used in the study.

Pirie (1998a) called on the mathematics education researcher to ask herself numerous questions about her research focus and methodology, terming the choice of research methods a “very personal decision” (p. 21). Methodology must not only be determined by our research questions but must be consistent with both our theoretical perspective and our research framework. As a feminist, postmodernist, and mathematics educator, I am not seeking answers but instead am seeking insight into teachers’ perceptions of mathematics. Narrative analysis, sometimes referred to as *narrative inquiry* (Clandinin & Connelly, 2000), *narrative construction* (Barone, 2007), or simply *narrative research* (Casey, 1996), focuses on the stories of research participants (Grbich, 2007). Researchers use narrative analysis based on the belief that “we find and construct meaning in our lives by telling our stories” (Johnson-Bailey, 2004). Participants share their stories which are then accepted as their beliefs and understandings at a particular time within a particular context. Thus, narrative analysis does not seek an objective truth; it does not question the “reality” of a participant’s story:

> Narrative analysis takes as its object of investigation the story itself. . . . The purpose is to see how respondents in interviews impose order on the flow of experience to make sense of events and actions in their lives. The methodological approach examines the informant’s story and analyzes how it is put together, the linguistic and cultural resources it draws on, and how it persuades a listener of authenticity. Analysis in narrative studies opens up the forms of telling about experience, not simply the content to which language refers. We ask, why was the story told that way? (Riessman, 1993, pp. 1–2)

Narrative analysis does not seek truth in the traditional sense. Researchers engaged in narrative analysis do not employ triangulation of multiple data sources to test

the validity of data. Instead narrative researchers seek prolonged engagement in the field. In this study, multiple interviews, reflective essays, and reading journals were all used to elicit multiple stories centering on a common theme. By exploring stories on multiple occasions, both participant and researcher made sense of the experience. Cortazzi (2001) cites the usefulness of narrative analysis as "an especially powerful research tool if the narratives are accounts of epiphanic moments, crises, or significant incidents in people's lives, relationships or careers" (p. 384). Narrative analysis therefore is an informative methodology in a study that pushes teachers to explore their own philosophies of mathematics at a historical moment when they are being challenged to shift their instructional practices in their classrooms due to educational reform. Narrative analysis celebrates the lives of ordinary people and recognizes the political nature of their personal and professional struggles (Casey, 1996). Narrative analysis is often viewed as an empowering methodology—it invites participants to make meaning and to interpret the events of their own lives, it gives them a voice (Johnson-Bailey, 1999). But the voices of the participants are not alone in the research. As the researcher, I directed the study. I chose the design of the research, I framed the questions. Although I attempted to follow the lead of the participants as they told their stories, and to allow their stories to point the way, I came to each interview with an outline, an idea of the questions I wanted to ask and a direction in which I planned to guide the study. Riessman (2008) cautioned against viewing narrative analysis (or any research methodology) as empowering, especially when working with marginalized groups, of which teachers must be included (Apple, 1986). Although I view the participants in my study as fellow students and fellow educators, I must be aware that my role as teacher (in the summer course) and as

researcher changed the dynamics of our relationship and created unequal position of power. As I conducted the study and analyzed my data, I had to be aware not to overstate issues of empowerment and voice.

Cortazzi (2001) cited narrative analysis as an effective tool of professional development. Smeyers and Verhesschen (2001) described narrative analysis as a philosophical endeavor, one that seeks to change how both researcher and participant see, if not the world, then at least themselves within the world. In education, narrative analysis has been used in studies that explored the social and personal transformations of mature college students (Britton & Baxter, 1999); examined the effects of a co-principal shared leadership initiative (Court, 2004); explored the experiences of preservice to inservice elementary school teachers as they learned and taught science (Mulholland & Wallace, 2003); examined the field of teacher knowledge research (Rosiek & Atkinson, 2007) and other teacher practices (Clandinin, Pushor, & Orr, 2007); and investigated the effects of race, class, and gender on educational settings (see, e.g., Johnson-Bailey, 1999, 2004).

Given that narrative analysis allows for the systemic study of personal experience and meaning (Riessman, 2002), it seemed an appropriate methodology for this research study. If, as Hatch and Shiu (1998) claimed, teachers are a critical part of mathematics education research and should have access to the outcomes of that research, a methodology that intimately involves teachers as participants is essential: "Teachers of mathematics are uniquely placed to investigate—and record—aspects of their teaching, their classroom and their students that are hidden from others" (Hatch & Shiu, p. 297). Narrative analysis is an overtly political act (Barone, 2007), one whose aim "is not to

seek certainty about correct perspectives on educational phenomena but to raise significant questions about prevailing policy and practice that enrich an ongoing conversation" (p. 466). Narrative analysis is a divergent methodology (Rosiek & Atkinson, 2007); it does not seek resolution and conclusion but rather experiential understanding, a story as metaphor for life. Narrative analysis aligns with a social constructivist paradigm of learning: "By acknowledging the social construction of knowledge, narrative has provided a methodology that has taken into account the situated, partial, contextual, and contradictory nature of telling stories" (Hendry, 2007, p. 489).

## Data Collection

The data collected for this study included written course assignments, class discussions, and interviews with the participants. The teachers enrolled in the summer course wrote an initial reflective essay on their personal philosophy of mathematics. Each course member was responsible for a summary of one week's readings and led that week's discussion. Weekly class discussions were scribed by another member of the course. A final, more scholarly paper (including citations of class readings) was turned in at the conclusion of the course. This written data, along with the interview data, were utilized during data analysis.

There were four interview phases to this study. The first phase, conducted as the participants were enrolled in the graduate course in the summer of 2007, included initial interviews with three participants—one middle school mathematics instructional coach and two high school teachers. The initial interview focused on each teacher's mathematics background and professional experiences, the intent being to build a relationship as well as collect background information. The information discussed in the

interview included personal school experiences with mathematics, educational background, and information about each participant's current teaching assignment (see Appendix C for a copy of the Initial Interview Protocol). The protocol was not strictly followed as I responded to each participant's conversation in order to establish a positive and comfortable relationship.

The second phase of the study included the addition of two new participants—a high school mathematics instructional coach and an elementary school mathematics instructional coach. (The middle school instructional coach was no longer available for interviews and her data was not used in this dissertation study.) I conducted initial interviews with these two teachers using the same interview protocol in order to gather background information. These interviews took place in the spring of 2008. The third phase, which took place in the summer of 2008, included a second interview with each of the four participants that further explored each participant's philosophy of mathematics. Prior to these second interviews, I provided the participants with copies of their own reflective essays written during the summer course, as well as transcripts of their initial interviews. This form of member checking allowed participants to reflect back on their earlier statements, to remember the context of our initial interview, and to consider the previous readings and writings from the summer course on their current philosophies of mathematics.

The final interview phase took place in November 2008. The purpose of the third and final interview was to investigate the relationship between the participants' philosophies of mathematics and their perceived instructional practices, especially in light of the curricular and pedagogical changes being implemented in Georgia through

the Georgia Performance Standards (GPS). The participants were asked to explore the effects (if any) their exploration of the philosophy of mathematics had had on their perceptions of mathematics as a field of knowledge and to share any changes they might have observed in their professional practices. This final interview took place well into the school year (the school year having begun in early to mid-August) so that the participants could reflect on the curriculum changes that took place in high school that year (as three of the four participants were high school mathematics educators).

Although the initial interview included a list of possible questions, like Riessman (1993), I preferred a less structured interview style: "Interviews are conversations in which both participants—teller and listener/questioner—develop meaning together, a stance requiring interview practices that give considerable freedom to both" (p. 55). Unlike a typical social conversation, though, an interview is guided by the researcher through the use of focused questions, questions that encourage the interviewee to speak in depth and at length (Rubin & Rubin, 2005). It was important, in the interview process, to put the interviewee at ease, to create a relaxed, safe environment, one in which the interviewee felt comfortable sharing what may well be personal and painful or awkward memories and events (Weiss, 1994). As a researcher, it was important that I be interested in my participants' stories, that I demonstrate a willingness to learn from their stories (Rubin & Rubin, 2005). The goal in data collection was to gather a substantial amount of rich, thick descriptive data (Geertz, 1973), over a period of time, through the words of the participants. Then through the analysis of this data, teacher stories emerged and meaning was constructed. But, as I analyzed my data, I remained aware of my own "conscious and unconscious baggage" (Scheurich, 1997, p. 73), the biases and background that I brought

to the research process. I did strive to be transparent, as a postmodern researcher, and recognize the impossibility of removing my own voice from the research.

## Data Analysis

Riessman (1993) outlined five levels of representation in the research process of narrative analysis: *attending, telling, transcribing, analyzing*, and *reading*. I will address the first four of these five levels in this section (reading, the presenting of the study, will be addressed in a later section). Attending to an experience, research participants note certain features of that experience, that is, certain images, thoughts, memories, and visions. Their telling of those experiences then depends on their memories—what made an impression, what did not. We acknowledge that there is always a gap between the lived experience and the telling of it (Riessman, 1993). But meaning also shifts within the actual experience of telling; it is no longer just an experienced event, it has now been shared with another. Meaning is constructed anew with each additional telling.

The transcribing of experience is in many ways unique to the research process. It is through transcription that the researcher begins the interpretation of the lived experiences. Within this study, interviews were audio-recorded and transcribed. The transcriptions included pauses, facial expressions, and body language. Thus the transcriptions noted *how* a story was told not just the words that were used because, as Riessman (1993) asserted, “forms of transcription that neglect features of speech miss important information” (p. 20). My analysis of the data began as I transcribed each interview because “analysis cannot be easily distinguished from transcription” (Riessman, p. 60). Following an initial transcription, I listened again to each recording, adding my initial observer comments to the transcription, which began my reflective work as a

researcher. At that second listening, I noted my reactions to the participants' words, the emotions I sensed, and the expressions I observed.

Analysis of the narrative continued with readings and re-readings of the transcripts. Narratives are not read simply for content; it is often the structure of a narrative that tells a story. How is it organized, why does a participant choose to tell a particular story, or to tell it in a particular manner? I have utilized two methods of narrative analysis—thematic analysis and dialogic/performance analysis (Riessman, 2008). Thematic analysis has been used for the interview data as well as the written data collected during the course. Thematic analysis focuses on *what* is said in a narrative. Thematic narrative analysis differs from more traditional approaches such as grounded theory and interpretive phenomenological analysis in that narrative analysis attempts to "keep the story intact for interpretive purposes and…attends to time and place of narration and, by historicizing a narrative account, rejects the idea of a generic explanation" (Riessman, 2008, p. 74). Through thematic analysis, I have identified themes *within* a participant's narrative rather than attempted to identify themes common to all the study's participants, what Riessman terms a "case-centered commitment" (p. 74). Thus, in my analysis chapter, each participant's story stands alone. As part of the thematic analysis of each participants' story, I created a concept map to organize the data. The use of the concept map helped me to organize the themes I observed in the data and served as a tool to share my initial analysis with the participants (Raymond, 1997). I forwarded the applicable concept maps to each participant and invited comments on my interpretations (only one participant responded with a clarifying point). Although I

received little feedback from my participants on the concept maps, they served as a useful tool in my analysis process.

My thematic analysis is integrated with a dialogic/performance analysis. Just as thematic analysis focuses on what is said, a dialogic/performance analysis focuses on who a participant is speaking to, as well as when and why that participant speaks. Thus, through dialogic/performance analysis, the researcher is made transparent, for a participant is responding to a particular set of questions, asked at a particular time, and for a particular purpose. It is this contextual component of dialogic/performance analysis that appeals to me. As Riessman (2008) asserted, "investigators carry their identities with them like tortoise shells into the research setting, reflexively interrogating their influences on the production and interpretation of narrative data" (p. 139). Thus it is through the dialogue between speaker and listener, between participant and researcher, that meaning is made. It has been important for me to recognize my active role in this study, to recognize that it is my choice of readings, my questions, and my own beliefs about mathematics that are guiding this study. In my analysis, as I tie each participants' story to both a social constructivist frame of mathematics education and a postmodern view of mathematics, I have sought to focus on the *telling* of the stories, in other words, which stories were chosen, the manner in which they were told, and the words selected in the telling. Yet this form of analysis places me dangerously in the center of the research, a construct I will explore in the following section.

## Troubling the Role of the Researcher

An important issue to consider in narrative analysis, or any qualitative research project, is the role of the researcher. As a qualitative researcher, I have a tremendous

impact on the research but I must not be myopic in my analysis and see and hear only what I expect to see and hear. It is necessary, in my role as a researcher, to address issues of *transparency* and *integrity* (Rogan & de Kock, 2005): transparency by sharing who I am and what history I bring to this research, and integrity by acknowledging the political nature of research, as well as educational and social relationships. Yet I must beware of foregrounding my own perspective: “How do we explore our own reasons for doing research without putting ourselves back at the center?” (Lather, 1991, p. 91).

My research project must include, therefore, some autobiography—who am I and what has brought me to this research. I am not neutral observer. I am, as Amit and Fried (2002) describe, a reform researcher. My work includes professional development with K–12 teachers with a focus on changing instructional practices toward a social constructivist paradigm. My research is influenced by my professional experiences. All mathematics education research, argued Amit and Fried, should be connected to reform, as their ultimate end must be the improvement of mathematics education. Being a reform researcher, whose acknowledged goal is to change what goes on in the mathematics classroom, it is essential that I adopt a methodology that is not only active but reflective. Narrative analysis is that active methodology, but the onus is on me, the researcher, to utilize a reflective stance as I collect and analyze data.

To maintain transparency and integrity, I kept a researcher journal during data collection and analysis. My journal included the following: comments on data collection—difficulties or road blocks that I encountered and concerns that I had; reflections on analysis—what I was observing and what struggles I had in making sense of my data; reflections on methods—comments on procedures and strategies as well as

decisions I made regarding the design of the study; points of clarification—what I needed to pursue further to understand better; and reflections of ethical dilemmas and conflicts as well as my own state of mind—to be self-reflective and self-critical (Bogdan & Biklen, 2003). These journal entries served to uncover my own biases and prejudices, to ensure not objectivity but transparency and integrity. One particular issue I focused on in my journal was the problematic nature of my overlapping roles in this study—I am a teacher and a researcher; I am researcher and professional developer. I was the co-creator and co-instructor (along with my major advisor, David Stinson) of the graduate-level course, an active participant (and, at times, leader) of class discussions, the assessor (and grade-giver) of the participants' work, as well as the researcher. Dr. Stinson and I purposefully chose the readings with an eye, not towards a balanced presentation of 20$^{th}$ century philosophies of mathematics, but towards challenge and disequilibrium. The readings and discussion of mathematics philosophy in which the educators in this study participated affected the stories they then told. My professional work—supporting and guiding teachers as they implement statewide curriculum mathematics reform—is proactive, not a passive role. Professionally, I have a goal in mind—changing instructional practices. Does this interfere with my role as a researcher? My study is an active one; reform is my goal as a researcher as well—to change teachers' conceptions of mathematics. I am not the proverbial fly-on-the-wall. Yet I did not enter this research claiming objectivity. It is my voice that may be the clearest in this study and I must be self-critical in how I use that voice (Tierney & Lincoln, 1997).

An additional dilemma I had as a researcher was how to make sense of my data. In many ways, the qualitative researcher is in the midst: "in the midst of a three-

dimensional narrative inquiry space, and…in the midst of a temporal storied flow" (Clandinin & Connelly, 2000, p. 65). As teachers share their stories, the researcher is placed into a real time and space (or at least the participant's perception of a real time and space). Narratives move back and forth in time; stories change from telling to telling. How do I make sense of this complex process that is narrative analysis (Rogan & de Kock, 2005)? In many ways, this process required what Hendry (2007) termed faith: "Research is not ultimately about interpretation but about faith. Trusting in the stories and the storyteller" (p. 494). This faith, Hendry argued, is a political act, one that acknowledges research participants as whole, as meaning makers of their own lives and as central to the researcher's meaning making. My task as a narrative researcher, then, was to *listen*; to let my participants' stories guide the direction that the research took; and to be humble in my role as researcher. Clandinin and Connelly (2000) wrote of the "mutuality of the interaction" (p. 91) in narrative research. Thus interviews are not formalized but are, instead, conversations, unstructured and informal, led often by the participants.

## Addressing Validity

Ultimately, the researcher's analysis of the narrative are tied to the evolving research questions, and are also linked to the researcher's own history, her own values and biases (Riessman, 1993). Thus, issues of validity are raised within narrative analysis and must be addressed. Validity within the research process concerns the believability of a statement or research claim (Polkinghorne, 2007). Within narrative analysis, validity means not truthfulness but trustworthiness. Narratives are not static; they are dependent on the teller, on the time, and even on the listener's point of view. How then does the

researcher address issues of trustworthiness? And what indeed is meant by trustworthiness? Riessman (1993) presented four criteria to address issues of validity or trustworthiness: *persuasiveness*, *correspondence*, *coherence*, and *pragmatic use*. Persuasiveness asks "is the interpretation reasonable and convincing" (Riessman, 1993, p. 65)? To increase persuasiveness, the researcher should support claims with ample evidence from the participants' narratives. Hence my analysis in the following chapter includes detailed words from the participants. Correspondence refers to the researcher sharing results with the participants of the study, what other researchers often call *member checking*. What do our participants think of our interpretations? As I developed my analysis of the data, I shared my ideas both through the interview process and in the concept maps I had created, inviting the reactions of the participants.

Riessman's coherence criterion refers to the manner of the storytelling—what are the overall goals of the narrator, what linguistic devices are employed, what themes are repeated and reiterated throughout the narration? The coherence of my interpretation is, again, presented through the use of the participants' words, and illustrated through the use of similar stories from multiple sources (e.g., several interviews, reflective essays, reading journals). And lastly, pragmatics use—to what extent is a study useful to others; what will be its future uses? The pragmatic uses of my study will be addressed in the closing chapter. Riessman stressed the fluidity of her criteria; they are not meant as a checklist for validity but as guides for the researcher in the development and evolution of a study.

Mulholland and Wallace (2003) offered their own set of criteria for assessing the value of knowledge claims within narrative research. The first, the *strength* criterion,

focuses on issues of trustworthiness of data, and is addressed through sufficient time in the field, use of multiple data sources, member checks, presence of the participants' voices, documentation of researcher bias, and discussions of the limitations of a study. Polkinghorne (2007) recommended at least three interviews spread out over a period of time. My own study included extended time with participants, multiple data sources (interviews, written reflections, and class discussions) and three interviews with each participant. Mulholland and Wallace's second criterion, the *sharing* criterion, refers to the "validation or legitimation that is won from the reader" (p. 8). This criterion, argued the authors, comes about through thick description, allowing the reader to make contact with the participants of the study, to connect with them at a personal level. The authors recommended telling the story of the research itself along with the participants' stories to meet this criterion. Throughout this paper, I have sought to share my own story as researcher with the reader, in order to share the journey I experienced as well as the journeys of my participants. A final criterion is the *service* criterion, similar to Riessman's criterion of pragmatic use. Will this study provide service to the field of mathematics education; will it be useful to future researchers, educators, and students? The service criterion is a future oriented criterion—the goal of all research is to serve a purpose but, although my intent will be shared in the final chapter, the service criterion is the most difficult to document in the present.

Some points need to be raised here as to the differences between qualitative and quantitative research as regards to issues of validity, such as ideas of *generalizability*, and *reliability*. These concepts are used frequently to assess the credibility of a research project but must carry different meanings when applied to qualitative research studies

such as narrative analysis. Bogdan and Biklen (2003) offered substantial clarification on the relevance of these criteria to qualitative research, which I will touch on briefly here. Regarding generalizability, first, not all researchers are concerned with issues of generalizability, or whether the findings of a particular study may hold true in other settings, with other participants, and in other times. Postmodern researchers, for one, question the very idea of generalizability (Lather, 1991). Some researchers focused the question not on whether findings can generalize, but to what other particular settings and subjects the findings may generalize (Bogdan & Biklen, 2003). Other researchers looked towards a plethora of related studies to address the issue of generalizability (Mertens, 2005).

This is a study of four educators, four mathematics teachers engaged in the struggle to engage students and other teachers in the learning of mathematics. Did I attempt to choose a demographic cross section of teachers? While this study included three White teachers, and one Black teacher, three female teachers and one male teacher, those demographics were not key to their participation. Was it my intent to focus on educators employed in a particular type of school or working within a particular grade band? While my participants included both elementary and secondary educators, teachers in upper-middle class schools as well as working-class schools, the participants were chosen more purposefully based on their initial reflections in our graduate course, and their varying ideas about mathematics and mathematics education. My goal in this study was not to generalize about teachers' philosophies of mathematics or instructional practices. Pirie (1998) encouraged us to ask: "Do we seek insight or do we want

generalizability?" (p. 21) as we engage in educational research. Insight was clearly my intent in this study.

As to the issue of reliability, in other words, will a second researcher be able to reproduce the findings of a particular study, qualitative researchers again define the issue in a different manner. Reliability in a research study is tied to issues of subjectivity. Qualitative researchers acknowledge that each researcher brings to a study her own philosophical, theoretical, and experiential background that directly influence analysis and interpretation of data. The qualitative researcher argues that all research is subjective, that objectivity is a falsehood that is often hidden behind sterile numbers and statistics. Issues of reliability in qualitative research are actually addressed through laying bare a researcher's biases and subjectivity, to guide the reader towards an understanding of how a particular interpretation was obtained (Bogdan & Biklen, 2003). By providing sufficient data examples and a transparency of researcher biases, it is my hope that this study sufficiently addresses the issues of validity and reliability.

## The Research Report

Earlier I presented Riessman's (1993) five levels of representations involved in narrative analysis: attending, telling, transcribing, analyzing, and reading. *Reading* is the (final) stage in which a researcher shares her work. Although the final product of narrative analysis can often itself be a narrative, a story with a beginning, middle, and end, it has been my intent to present the findings of this research project in a traditional research format. The participants' narratives, however, cannot be lost through a conglomeration of analysis; hence this written report includes an accumulation of lengthy quotes and excerpts from my collected data. Like Lather (1991), my use of quotes and

excerpts is “an effort to be ‘multi-voiced,’ to weave varied speaking voices together as opposed to putting forth a singular ‘authoritative’ voice” (p. 9). The goal is to be true to my participants and yet create a reading that is accessible and interesting. It is at the reading level of representation that issues of validity come to the forefront:

> It is the readers who make the judgment about the plausibility of a knowledge claim based on the evidence and argument for the claim reported by the researcher. The confidence a reader grants to a narrative knowledge claim is a function of the cogency and soundness of the evidence-based arguments presented by the narrative researcher. (Polkinghorne, 2007, pp. 484-485)

Yet even the reading of a study is a changing experience, one based in a given time and a particular context. “All texts stand on moving ground,” wrote Riessman (1993, p. 15). As a researcher, my goal in this project is to affect reform in mathematics instruction, to add to the research on mathematics teaching and learning, and to share the stories of teachers as they examined their own personal philosophies of mathematics and explore the relationship between philosophy and practice. I know the interpretations each reader brings to this research may vary, may not be my own, and may not be that of the participants. Again, there is no one truth that I am seeking through this research:

> Writing ethnography as a practice of narration is not about capturing the real already out there. It is about constructing particular versions of truth, questioning how regimes of truth become neutralized as knowledge, and thus pushing the sensibilities of readers in new directions. (Britzman, 2000, p. 38)

Will the reader experience the research text as a transformative process, one that compels that reader to examine her own mathematical and educational philosophies, and to begin to explore the relationship between her philosophies and her teaching practices? As a reform researcher, my goal is just that—to produce research that, in some way, transforms the reader, the participants, and me, the researcher.

## Concluding Remarks on Methodology

How one learns, how one teaches, how one conducts research—these ideas should align philosophically, epistemologically, ontologically, and methodologically (Conle, 2001; Moen, 2006). If one adopts a constructivist paradigm, with the understanding that knowledge is socially constructed, then that paradigm that influences how one views the learning process should also affect how one views the research process (Mertens, 2005). Qualitative research is not, however, a monolithic concept (Preissle, 2006). Qualitative researchers differ greatly in methods and design, as well as in theoretical and philosophical frameworks. What is common to qualitative research, Preissle argued, are three ideas. First, qualitative research emphasizes the description of direct experience and meaning. Second, qualitative research "specifies conceptual framing while leaving open the 'what and why' of experience and meaning that vary by the philosophical, theoretical and disciplinary orientations that researchers bring to their studies" (p. 687). And, third, qualitative research is a "theory-practice nexus" that brings together theory and practice, that necessitates theory and practice as interactive and interdependent entities. Lather (1991) reiterates this third point in her description of research as praxis as "philosophy becoming practical" (p. 11).

The writer of qualitative research invites the reader to join her in the interpretation of the data. Through researcher transparency, the use of thick descriptions, and the inclusion of her participants' voices, the researcher traces the path of her own interpretations. Whether the reader comes to agree with those interpretations is not a necessity; what qualitative research expects is that the reader at least comes to understand how the researcher reached her conclusions. In keeping with a constructivism paradigm,

the qualitative researcher acknowledges that interpretation, like knowledge, is constructed. Qualitative research, in many ways, emphasizes *process* more than *product*. Interestingly, this parallels the focus of the current reform movement in mathematics education that stresses the significance of process in mathematics teaching and learning, and the use of a social constructivist epistemology (Ernest, 1999).

# CHAPTER 5

# TEACHER STORIES

I present in this chapter my analysis of the mathematical stories of my participants. Over a period of 18 months, each participant and I explored his or her personal philosophy of mathematics, troubled what it means to teach and learn mathematics, and discussed the struggles of changing one's instructional practices as a mathematics educator. My analysis of the data—including interviews and reflective writings—seeks to keep each participant's story distinct. Thus this chapter is organized by participant; my analysis of each participant's relationship with mathematics is presented in a separate section. I describe each participants' background as a mathematics student and, later, teacher, recognizing the importance of what Bibby (1999) called "mathematical histories." I also describe the processes these educators go through as they implement a new curriculum, recognizing the need for reflection (see, e.g., Hart, 2002a, 2002b; Thompson, 1984) to guide the change process. Each participant's story is presented as a unique experience; there are, however, commonalities to the stories and these will serve as the organizing structure to each section and are addressed in more detail in the closing section.

My four participants, as indicated in the previous chapter, were all graduate students involved in a course during the summer of 2007 (see Table 1 for a summary of the participants' teaching experience and graduate enrollment status). Two of the participants, Katie and Michael, were part of a pilot study I had conducted in 2007. Both

Katie and Michael were working on an Education Specialist degree in mathematics education during the initial pilot study. They were also both public high school mathematics teachers. The other two participants in this study, Julia and Diane, were both Ph.D. students—Julia in the area of mathematics education and Diane in early childhood. Both Julia and Diane, at the time of the summer course, served as mathematics instructional coaches in public schools. Julia worked as a mathematics coach at a metropolitan area high school. Diane worked as an elementary school mathematics coach in a small, rural school system. These four participants wrote and spoke of their personal relationships with mathematics, their views of mathematics as a subject, their ideas about the teaching and learning of mathematics, and their struggles as mathematics educators in a time of tremendous curriculum change in Georgia. My analysis of those stories follows.

Table 1

*Summary of Study Participants*

| Name | Years of professional experience in education | Professional role | Graduate student status |
|---|---|---|---|
| Katie | 8 years | High school mathematics teacher | Completing Ed.S. in mathematics education |
| Michael | 8 years | High school mathematics teacher | Completing Ed.S. in mathematics education |
| Julia | 15 years | Mathematics Instructional Coach (high school) | Ph.D. student in mathematics education |
| Diane | 26 years | Mathematics Instructional Coach (elementary school) | Ph.D. student in early childhood education |

# Katie's Story

*Background*

Katie is a White woman who has taught for 8 years at a predominately White, middle class high school in a suburban school system. She was just beginning coursework for an Education Specialist degree at the time of our summer graduate course in Philosophy of Mathematics. After completing undergraduate and Master's degrees in mathematics education at two local state universities, Katie was thrilled to be offered a teaching position at Lake High School—the school from which she had graduated just 6 years earlier. Katie enjoyed the comfort and familiarity of being back again at her old high school, describing it as "like a dream come true." "I mean, it hadn't been that many years so most of the teachers that taught me were still there. It was just comfortable" (Interview 1). Throughout our three interviews, Katie often referred to herself as a "new" teacher, probably a result of working beside teachers who had taught her just a few years earlier.

Katie told me she had known she wanted to be a teacher for a long time and, in fact, had planned on teaching mathematics since the 8$^{th}$ grade. Her mother had been an elementary school teacher and so Katie felt it was natural to be drawn to teaching. She chose to teach mathematics because it was a subject she liked and was good at although "I was no genius" (Interview 1). In fact, Katie spoke often of her struggles to do well in mathematics and how hard work and determination resulted in success, not any natural ability:

> I was good at it but I always took just the normal courses that are, I mean it was never like I was in any honors anything. I just, I really did, I liked it. And I still, you know, there's a huge satisfaction of taking a lot of time, working through a problem and then getting the right answer. (Interview 1)

This belief in overcoming her own "natural" limitations through hard work and perseverance affected both how Katie viewed the purpose of mathematics in school and her approach to teaching mathematics as evidenced in the next section.

*Views of Mathematics and Mathematics Teaching*

Katie's views of mathematics did not change greatly throughout our 18 months together. At the beginning of our course, Katie described her views of mathematics as well as her views of teaching mathematics as traditional. She saw mathematics as being everywhere through the numbers in our daily lives, that we cannot live without mathematics. To Katie, mathematics made sense; it was right or wrong, black or white. That is what drew Katie to mathematics as a child:

> I have always liked mathematics because the material seemed so straightforward. You follow certain steps when solving problems and then you arrive at your answer. Example problems were given in class and then you had more practice on these same types of problems for homework. By the time you were tested over the material, it seemed pretty clear that you either knew it or you didn't. (Initial reflection paper)

Katie never much thought about the why's of mathematics until she got to college courses. Prior to this, she had memorized steps and repeated them back. Higher mathematics courses in college forced her to think about why rules and procedures worked as they did. This led Katie, as a teacher, to want to help her students understand how we derive formulas not just get them to memorize the formulas. But Katie continued to see mathematics as absolute and unquestionable. During our summer course, discussions about religious views and views of mathematics were not uncommon. Katie drew parallels between her unquestioning religious faith and her ideas about mathematics:

> Prior to this class, I had not thought about religion and mathematics having any connections. However, through our reading I have seen

> religion compared to mathematics quite often…This idea was quite interesting to me. My favorite point made relating these two was when Davis and Hersh (1997) said that “God is a Mathematician” (p. 51). I just thought that of course He is, God is anything and everything. I do see that my philosophy of mathematics and my philosophy of religion are very similar in that I do not need to have proof to believe. Just as I believe in God without proof of His existence, I believe the mathematics I have been taught without proof that it exists. (Final reflection paper)

Thus Katie was more comfortable accepting mathematics as is than questioning either its (supposed) absolute nature or its purpose in school. Mathematics, for Katie, just *was*.

More than anything, Katie viewed mathematics as a subject that rewarded hard work and perseverance. It was her goal to get her students to believe in themselves and in their own abilities through success in mathematics. Throughout her teaching career Katie had asked to work with students that many experienced teachers chose to avoid—the struggling students just entering high school. Katie’s goal was to instill in these students a sense of purpose and accomplishment as they struggled and succeeded in mathematics. Katie hoped that her students would remember her for having taught them to work hard to achieve what they wanted and that mathematics was just the tool to help her do that. Just as she felt proud that she had struggled through mathematics and achieved knowledge through that struggle, she wanted her students to feel that same sense of accomplishment. When asked what she would want her students to most remember about her, it was this idea of hard work, teaching students to persevere that she described:

> I think I would want my kids to come back and say that, you know, you taught me never to give up or you taught me that I can do anything that I put my mind to as long as I work hard at it…But I think more than anything it would be that they felt motivated when they left and encouraged and they didn’t want to give up in anything. (Interview 1)

As Katie and I developed a relationship over the 18 months of this study, she continued to emphasize the importance of hard work in mathematics. In this, she was

echoing a common theme in school mathematics: that effort is a significant factor in the success, or lack thereof, in mathematics (see, e.g., National Mathematics Advisory Panel, 2008). Although Katie felt that some people had a gift, a natural ability to do mathematics (and she did not identify herself as one of those individuals), she also believed strongly that all students could succeed in mathematics:

> I think there are definitely people who are better at math than others but I don't, I really don't think that people can't do it. I think, and my attitude has always been, if you apply yourself and you want to do well at whatever, anything, you can do it. (Interview 1)

For Katie, the idea that her success in mathematics did not come naturally but was, instead, the result of hard work, made that success even more worthwhile.

Katie's personal experiences with mathematics as a subject that was difficult but rewarded her efforts with correct answers and a sense of satisfaction were paramount to her teaching style. She strove to make the learning of mathematics accessible to her young students. She hoped they would feel the same sense of accomplishment that she had known when solving a difficult problem. But for Katie, a large part of that satisfaction came from her own view of the closed nature of mathematics—it was right or it was wrong. To view mathematics as an open-ended subject, one with many solutions and no clear answers, was not easy for Katie. Both her readings in our summer course and her professional development experiences a year later in preparation for the teaching of a new high school curriculum began pushing her to look at mathematics in a very different light. Those changes will be explored in the next section.

*Struggles Amidst Change*

Over the years, Katie had adapted her teaching methods to help her students succeed in her classes; she encouraged student questions, had them worked in small

groups to share and teach each other, demonstrated multiple methods to solve problems, and displayed a caring demeanor with her students. But Katie never questioned what it meant to learn and teach mathematics. She believed in traditional, teacher-directed instruction. She was in the front of the classroom, sharing examples, guiding her students through various problems, and working them towards independence in completing the examples. That, for Katie, was what it meant to teach mathematics as evidenced in our initial interviews as she struggled to view mathematics instruction differently:

> When you do a task it's supposedly going to teach them the same thing as if you'd done the real teaching! (Interview 1)

> It's going to be hard for me not to be the one, not to teach. Like I just feel like everything I've done has been so traditional and I'm not one to say, you know, here's a formula, memorize it. I mean I do, I want them to, I want it to make sense. I do try to go over, well this is where the distance formula comes from or whatever the case is. But I just think, for me the hardest part is thinking that they're going to get everything that I normally would teach to them just from doing this task. (Interview 1)

> But as far as my role as a teacher, I mean I really feel like my role is to try, you know, to have every student in my class master what they should master and pass the class. (Interview 1)

> I think it's going to be a big challenge. And I think it's going to be hard for me because I've been so used to, in the past, helping kids. You know, answering questions more straightforward. And teaching, I mean really using direct instruction. (Interview 2)

Katie believed in teacher-directed instruction, with the teacher's role being to guide students through a process, show them how it was done, and then watch as they duplicated her efforts. She had never questioned this idea of teaching even as she sought to help her students find success in mathematics, she continued to envision a teacher-centered instructional model. She was aware that the new curriculum was going to demand something different from her but she struggled with just what that would look

like. In our second interview, which took place the summer prior to her implementation of the new high school curriculum, Katie remained optimistic:

> But in a way I think this year's going to be good because you don't have any ideas of what students are going to say or how they're going to work things out which in a way makes it probably more interesting and better in the respect that I'm not going to lead them in a direction…because I don't know where they're going to be going. (Interview 2)

The implementation of the new Georgia high school curriculum for 9th graders, termed Mathematics I, began during the 2008–2009 school year. I met with Katie for our final interview 3 months into that school year. Katie began our interview expressing frustration about teaching through tasks and her students' ability to engage in the new curriculum:

> It really has been a disappointment, I'll be honest. When I first started this year, I wanted to do exactly what the state, you know, had told us, and here is a task, hand it out to the kids, and let them work together in groups, which groups I have very strategically picked, and let them try the task! And I was getting nothing! Nothing. Absolutely nothing. (Interview 3)

In many ways, Katie no longer felt like a mathematics teacher, as she had always defined the role. Watching students do tasks (or, as she described, not do the tasks) left her feeling removed from the mathematics. Her previously identified strengths as a teacher—hard-won knowledge of mathematics, ability to present the "why's" of rules and formulas, guiding students through the difficult procedures—did not seem to be needed in a classroom where students were left to work through tasks independently.

Katie continued to discuss the frustration she was feeling, not just with her students, but with her changing role as teacher. For her, teaching mathematics meant helping students to see the right answers and, when I asked her what her biggest challenge was as a teacher with the new curriculum, she discussed the difficulties:

> I think trying to not be the teacher who is just giving the direct instruction. Trying to step away and let the kids struggle more than they have in the past. And it's been hard. . . . Well it is hard because I just feel like too often they just give up. . . . It's hard to just walk around and knowing that they're not getting anywhere. It's like, where do you draw the line, you know, and I try to let them struggle and I try to let them talk about it. But I think for me, that's been the hardest. I mean obviously it's a lot easier to say, here's what we're doing and here's how to do it. (Interview 3)

Katie was tied to what Taylor (1996) had described as the *myth of hard control*, where the mathematics was hers to give and the students to receive. This, to Katie, was what mathematics teaching entailed. Otherwise, what was the purpose of all her mathematical knowledge?

Katie's struggles to implement a constructivist teaching style, to guide her students through tasks without providing answers and step-by-step instruction, was stymied by her own experiences as a (successful) learner of mathematics. Her personal knowledge of social constructivism as an instructional theory were just that—theoretical. When asked, Katie could not recall ever learning mathematics in a social constructivist setting:

> I know in high school, I mean it was definitely traditional all the way around. And really in college too. So no, if anything I've probably seen it more possibly in like my education classes. I can't think of specific examples but that would probably be more of where I've seen it. Not in a real math class. (Interview 1)

Katie did not really grasp her role in this new mathematics classroom as it was outside her own personal experience. Her personal history as a mathematics student and as a mathematics teacher had never prepared her to view the instructional method differently.

In fact, Katie admitted to giving very little thought to her philosophy of mathematics or of mathematics education prior to her enrollment in the summer course:

> And I've never, to be honest, sat down and thought about what is my philosophy of math? Honestly. And for me it's always, it always goes back to teaching of course. I mean that's my only, that's my passion. But it's, it's, yeah, I really haven't thought about why I teach the way I do and it's just kind of, I feel like I've just been kind of molded into that type of teacher. (Interview 1)

Katie's words reinforce Davison and Mitchell's (2008) contention that "students trained in an absolutist philosophy who become teachers will likely teach from the absolutist standpoint" (p. 147).

Although Katie had expressed an understanding that change was needed in her teaching style, and a willingness to change, her ability to embrace a social constructivist instructional model was limited by her lack of experience as a learner of mathematics through social constructivism. Katie taught the way she had learned (Cooney, Shealy, & Arvold, 1998). Her teaching style was reinforced by her own view of mathematics as a rigorous activity that provided satisfaction and a sense of pride through the conquering of its rules and procedures. In addition, Katie had been drawn to mathematics by its structure, its absolute nature, in her view, of right/wrong, black/white. Katie acknowledged that in a constructivist classroom, she would lose the absolute certainty of mathematics that is such a comfort to her:

> And my question is how will I know? You know, how will I know, did they get it? And then again, how do I know just what I normally would teach them is really what they should get out of that. (Interview 1)

Here Katie expressed concern about not just a changed instructional style but the very nature of the mathematics she had traditionally taught. Her views of mathematics—a subject that rewards discipline and hard work, a subject of unchanging, absolute rules and procedures—are troubled by a humanist view of mathematics that stresses context, problem solving, and personal sense making.

In many ways, Katie had been caught *in the midst* (Clandinin & Connelly, 2000) in this study—caught between the teacher she was and the teacher the new curriculum expected her to be. Her views of mathematics, although not essentially changed, were being questioned. She knew her traditional teaching style must also change. But she is unsure of where to go and how to get there. Katie struggled to redefine what it means to *teach*. A constructivist view of mathematics education envisions students engrossed in dialogue, immersed in mathematical tasks that call for multiple pathways toward multiple solutions, a reflective process whereas students question, explore, and justify the mathematics in which they are engaged (Confrey, 1990). This view was neither familiar nor comfortable for Katie. During our final interview, Katie's frustrations with this new idea of teaching were evident:

> I think that the kids are not doing the critical thinking that is expected of them. And so I'm hoping again after, maybe, two or three, four more years, it's better. But overall, I think that part has been a big disappointment, to be honest. (Interview 3)

But Katie's struggles were not just in redefining the *teaching* of mathematics, she also struggled to redefine the *mathematics* itself. In many ways, Katie no longer recognized the mathematics with which she had felt so secure and knowledgeable:

> I really, you know, I think of math as being numbers. Regardless of if you're in the grocery store or out to dinner or in a math textbook, math is just everywhere…I never thought of math as being so wordy. You know, because you, I mean I really do think more numbers. And the new curriculum, especially going through these tasks, it is word, after word, after word, after word. (Interview 3)

The persistent contextual nature of the mathematical tasks that the new curriculum entailed troubled Katie. She felt that, somehow, the mathematics was being lost in this new curriculum. In this, Katie was caught between the traditional view of mathematics as

an abstract, organized structure, and mathematics as a social construct, one that best makes sense through the solving of real-world problems (Sfard, 2003).

As the year progressed, Katie knew she was slipping back into instructional practices not aligned with the new constructivist curriculum: “So, to answer your question, I really think that I am, I don’t want to say spoon feeding, but I feel like I’m more back into that routine of doing a lot for them” (Interview 3). And Katie found herself turning to the traditional textbook for assistance: “It doesn’t hurt to give them some real practice out of the book” (Interview 3).

Katie’s experiences demonstrate the difficulties of implementing new pedagogical practices in our mathematics classrooms. Katie understood that change was required and, to some extent, understood what that change should look like (e.g. mathematical discussion, use of tasks, multiple solutions). For Katie, the challenge was how to create that environment in her classroom. With no concrete example in her own learning experiences to work from, Katie struggled to change the environment for her high school mathematics students. Her hope was for students in “2, 3, or 4 more years,” to come to her changed, ready to engage in the mathematical discourse the new curriculum demanded. In many ways, Katie was asking the same question asked by Sfard (2003) in her critique of building mathematical discourse in the classroom: “How can the children play a game whose rules they do not know?” (p. 375). Katie was not sure how to teach her students these new rules; that role was outside of her experiences as a mathematics teacher. It was her hope that they would come to her, sometime in the future, already knowing the rules of the game.

Katie knew how to be a mathematics teacher in a traditional classroom. But to facilitate tasks, to teach students to communicate about mathematics, to guide students to mathematical understandings through their engagement in contextual problems, none of those activities seemed to fit Katie's perception of the role of mathematics teacher. Katie did not see the need for her strengths—knowledge of mathematics and the ability to help students see why procedures and rules worked in mathematics—in the new instructional model.

As we ended our final interview, Katie remained optimistic. Her own reliance on hard work and effort was evidenced in her not giving up on the new curriculum, her willingness to strive to make things work even as she was not sure how:

> And I really, I mean I really have enjoyed this year, it really overall has been a great year. I can't, I can't complain. I mean, it has been so much work but I have enjoyed it, really and truly, it has been fun for me. Because the curriculum is so different. I mean, I guess my point is that teaching Algebra I for probably like  years, something needed to change. You know, we were pretty much doing the same thing. And there needed to be a change. So I think this change is good. And it's been good for me to be challenged and to do things differently. And it's kind of cool to have a year where there's no fear in trying something different because if it fails, it fails, you know, just learn from it and, and do it differently the year after. (Interview 3)

Katie was beginning to see the freedom that the new curriculum was granting her—the freedom to try new instructional methods, the freedom to see mathematics teaching and learning in new ways, and perhaps even the freedom to explore new ideas of just what is mathematics. This freedom, along with the "pedagogical conflict" (Wood, et al., 1991; Chapman, 2002) she was experiencing, hinted at the tremendous changes that Katie was in the midst of undertaking.

## Michael's Story

### *Background*

Michael is a White man who, like Katie, had worked at one high school during his 8 years of teaching. The school where Michael taught is predominately White and one of several high schools located in an upper-middle class suburb. Throughout his teaching career, Michael has taught primarily Algebra 1 and 2, Advanced Algebra/Trigonometry, and the first of a 2-year sequence of algebra known as the "applied" classes. Also like Katie, Michael enjoys and requests to work with the students that many other teachers avoid—the struggling freshmen.

Michael comes from an extended family of educators—both of his parents were teachers, as were his grandparents, uncles, and aunts. By high school, Michael knew he wanted to be a teacher as well, primarily due to practical reasons. Teaching was a familiar profession and would allow him the extended summer vacations he had enjoyed with his family. Choosing mathematics was also a practical decision; Michael found equal interest in all subjects but realized, with the aid of his mother, a school guidance counselor at the time, that the need for teachers of the maths and sciences would be greatest. In high school, Michael had attended a small, private Christian school, and he chose a similar setting for college, attending a small Christian college less than 2 hours from his home. Michael majored in mathematics education but, due to the small size of the college, he attended many classes alongside mathematics and science majors. Michael found benefits to studying with non-education majors:

> I had a group of folks I studied with, and three of us were math ed majors and four of them were pure math or science majors. And I learned a lot from them because, because they thought totally differently than I did. . . . It was really interesting because I would learn a lot from them because

> they just thought completely differently than I did. Because I really had, I struggled with the usefulness of things. (Interview 2)

These two ideas, that mathematics needed to be useful plus an identified dichotomy between what Michael terms pure mathematics and applied mathematics were common themes in Michael's conversations and will be explored further in the following section.

After completing his undergraduate degree, Michael taught for a few years, then worked on a Masters degree in mathematics education at a local state university while continuing to work full-time. A few years later, he once again went back to graduate school at the same state university while still working full-time. Michael was close to completing his Education Specialist degree in mathematics education during the summer graduate course (he finished his degree the following semester). In addition to teaching full-time, Michael also began work that summer as a contracted trainer for the state Department of Education, facilitating professional development for the high school mathematics teachers who were preparing to implement the new statewide mathematics curriculum. In that position, Michael and I worked together on several occasions. Thus we had a professional relationship I did not share with the other participants.

Michael described himself as a mediocre student of mathematics. Although he had done well in average track mathematics classes in his small high school, he had found himself struggling in college:

> It was good [in high school] in the sense that, not necessarily all A's but I felt good about what I did. Got into college, struggled a lot. Questioned whether I'd made the right decision about a major, you know, and was a very B, C student in calc, in college, and struggled. Failed a class, went back and took it again, you know, Calc 2 and me did not get along. But no, it was very mediocre and really a struggle for me to make sense of a lot of things because I didn't really speak, I didn't feel like I spoke the language well. Which I think has been one of my biggest attributes in teaching. (Interview 1)

In many ways, then, Michael felt shut out of the mathematical community. His experiences as a struggling mathematics student who didn't quite "speak the language" helped him reach out to the students he would later teach:

> I do think I have a better connection with students who don't do as well or not, it doesn't come natural because it came natural to me all the way through high school and then, all of a sudden, I found out in a hurry what didn't come natural so…I feel like I struggle instead of getting there by intuition, I feel like I get there through sweating, by sweating a little bit and that, I think that makes a difference. But it's also been a good point, it's been a good avenue for me to talking to kids I work with to keep them motivated, you know. Some of them go, what, you failed a class in college? You're going to be our math teacher? And it's like, well hang on. I think that having to go through that process made a big difference for me. It really kind of pushed me on to do what I wanted to do. I mean to follow through with math. (Interview 1)

Michael explained how he continued to struggle through mathematics courses in his graduate studies but, interestingly, never avoided the difficult coursework. One lesson he believed his struggles had taught him was not to value those for whom mathematics came naturally over other students. This lesson influenced his views not just on mathematics but on the structures of mathematics education, particularly in the school system in which he taught. Those issues will be addressed in the following sections.

*Views of Mathematics and Mathematics Teaching*

Michael described his views of mathematics and of mathematics education as "joined at the hip" (Initial Reflection paper). He reiterated throughout our conversations that he could not separate his philosophy of mathematics from his philosophy of mathematics education. Thus his philosophy of mathematics was tied to the usefulness of mathematics:

> I see mathematics as a tool that grows and cultivates logical understanding. It is the nature of our growth and maturity that can be demonstrated in the

> understanding that we possess through the mathematics that we can do. The use of mathematics in the education field is to promote thinking skills. I do not feel that mathematics is some set of rules and operations to be learned for the sake of knowing them. The knowing of these things that we have determined to be mathematics is not the end goal. It is often the aspect that we evaluate, but it is not the end goal for me. I want to see that my students have grown their understanding of the world around them. (Initial Reflection paper)

For Michael, mathematics not only teaches logic and critical thinking skills, but also is used to explain the "world around them."

In his reflection papers and throughout our three interviews, Michael described himself firmly as a Platonist in his views of mathematics. And he described his firm Platonic views of mathematics as stemming from his strong religious beliefs:

> I hold to the idea that our world has origins that are far beyond myself, and that origin is the work of God. I am not sure how to interpret all the references to God that I see within the text that has been read so far, but I do believe in God as creator of the world we live in. I also see our world as more than a chaotic place of coincidence. Our world seems to be created with great order and purpose. It is an incredible experience to explore and study it. With this study comes the realization of the great detail it contains. Mathematics gives us the ability to record, measure, and make predictions about the patterns that are occurring within our world. (Initial Reflection paper)

> Mathematical objects exist. I can describe them and I can see them. Mathematicians are discovers. I am not here to create mathematics, only to discover what is here already. I find that this holds closely to my religious beliefs. A created world is created with order, not out of chaos. The patterns that are used by the One that creates are here for us to discover and apply. (Final Reflection paper)

> I live in a space where I have the utmost faith in God, and I live my life with perspective of his creation. With this belief system, I can only see that our world is a created space with attention to the smallest detail. I see mathematics as the study of the discovery of patterns within our world….Standing on the premise of a created world, there must be a respect for the Creator. I know this is going to isolate me from many philosophers, but this is the foundation on which I have chosen to stand. From here, everything else in mathematics begins to build. (Final Reflection paper)

Michael realized, in his reflection papers, that his views were different than others in the class, were certainly different than many of the readings we had done. But he did not embrace a postmodern view of mathematics (and the world) that left uncertainty and unanswered questions. Michael was quite clear on his need for a foundation on which to stand, and he used a mathematical analogy to explain this perspective:

> Strangely enough, the foundation is the key part of our conversation this semester that seems to be missing. The fact that everything I have read is built from an axiomatic system. Axioms give us a starting point, but to call it a foundation is an overstatement. The starting point is fine, but I cannot stand on a point. Please pardon the mathematics humor, but a point is not proven to exist. A foundation is a place to stand and build upon. When considering the fact that I do not sit in an ivory tower and contemplate mathematics, I need a foundation to stand on. I base my livelihood, my career, on this mathematics. To stand in front of a crowd, or classroom, substantial support is needed. (Final Reflection paper)

This need for a foundation was, for Michael, answered by his belief in God. In his reading journal, he compared himself to Lambda in *Proofs and Refutations* (1945/1976), searching for certainty: "I still trust that the light of absolute certainty will flash up when refutations peter out!" (p. 53).

Later, Michael dismissed a discussion by Hersh (1997) on the mathematical properties of the 4-cube: "A 4-cube does not exist, as far as I know, so I feel like I am wasting my time discussing it" (Reading Journal). Michael needed a mathematics routed in the practical. Although he acknowledged the existence of this other mathematics, the pure mathematics, it served no purpose in his world.

Michael consistently described himself as a Platonist in his views of mathematics, yet he also embraced a constructivist view of the learning of mathematics. His own experiences as a struggling student of mathematics taught him the importance of building

interest in the subject. What you are interested in, you are motivated to learn (Sfard, 2003). And Michael believed that engaging students in doing mathematical tasks, working collaboratively, discussing multiple solutions, and asking them open-ended questions would succeed in piquing their interest in mathematics. Even before the implementation of the new high school curriculum, Michael described the changes he had begun in his classroom instruction:

> A lot of it is, you know, an activity or a word problem or something that you can discuss and get some information about and be open-ended with or at least open method about. You know the fact that they can, they can present different ways, they can turn in multiple representations of things but there still may be one answer. But just in that sense of getting beyond me at the board, talking, calling on them, putting an overhead up, and then going on to the next thing. (Interview 1)

Michael saw no contradiction between his Platonic beliefs of mathematics as a given, something out there waiting to be discovered, and his constructivist beliefs in facilitating student learning of mathematics. Mathematics, for Michael, need not be a human construct in order to discover its meaning and purpose. His goal was to move students away from a rote recitation of the rules and procedures of mathematics, to a discovery of its wonders:

> I want them to find that discovery, not just me tell them what it is. . . . I think giving them a chance to really, and not necessarily be just so discovery-based but even if I had showed them some things, that they can do the discover part of actually seeing how it works, and where it comes from. And they get a little application with that. That allows them to see the value of it. And I think we'll find things to study more deeply. Find more in it than just wanting to do, to get to this test or just to finish this assignment. Though that's the teacher's dream, you know the pipe dream. (Interview 2)

Michael felt comfortable experimented with task-based learning because the contextual nature of tasks was in keeping with his own appreciation of the applicability of mathematics and his rejection of the more abstract realm of pure mathematics.

Michael believed that all kids should be given the opportunity to succeed in mathematics because he understood the power that mathematics held for his students. His own experiences in college as a mathematics learner had demonstrated for him how the knowledge of mathematics could open or close doors. For Michael, teaching both the “lower-tracked” and the “on-tracked” mathematics students at a predominately upper-middle class high school crystallized the different opportunities provided to the students at his school. And he realized those opportunities were tied to something beyond mathematical ability, but were tied to class, race, and other factors. Michael spoke frequently and passionately about issues of socioeconomic class and its ties to mathematics during our first two interviews:

> Um, kids are, it’s not necessarily that the kids were different. They came with different backgrounds. A lot of kids came in with family, right off the bat, families volunteered information that was very different from. . . personal information. . . . Just even in race, there’s a big difference between the two classes [his Applied Problem Solving, a lower-tracked course, and Algebra 1, the typical freshmen level mathematics course]. But in the sense, in some things I pursued more just out of curiosity’s sake, but in our free-and-reduced lunch numbers. Much greater in one class that the other. I was mailing home letters at one time and I realized over half of the letters that I mailed home to my Applied Problem Solving class were all in one apartment complex. . . . You’re dealing with kids who ride the bus to school together, take classes all day together, and then go home together. (Interview 1)

What Michael observed was a group of students, primarily Black, living in the same neighborhood, who were isolated from the rest of their schoolmates by their daily schedule:

> I think you can track kids sometimes but yet they don't appear tracked because they may have different classes. But when there's so few, that tracking ends up, they just, they follow right through the schedule together and they go from Applied English, you know, they put you into Applied Math to Science, you know, they're all in the same science class. . . . We are very tiered, even in the sense of college-prep and the AP kids, or the gifted-honor kids. I mean, you really fall into one of three molds. You are taking the applied track, you are taking the basic college-prep track, which is broader, or you're in the honors-gifted AP route. And there's a lot of value put on those kids [the honors-gifted]. (Interview 1)

As Michael described the rigid tracking within his high school, I envisioned an apartheid-like system, where students' futures are prescribed, not by what they can or cannot do, but by the neighborhood in which they live. Michael realized that mathematics, particularly school mathematics, was neither neutral nor innocent (Walshaw, 2002), that it held great power.

Michael looked forward to the changes the new curriculum would bring in its elimination of the applied track of study. He had learned, in his own teaching experiences, that students deemed "not capable" often embraced his constructivist style of instruction, a style emphasized in the new high school mathematics curriculum. He told me several stories of students who had traditionally struggled in mathematics but were excited to be working through tasks and learning concepts instead of memorizing rules:

> But just given time she could, you know, see concepts, understand, and she was dealing with some learning disabilities as well but there were just a lot of things when it came to the task, she had much more opportunity to show what she could learn as opposed to, you know, as opposed to just popping out with materials. But I think I saw that with a lot of kids, and it took me looking for that because it hit me in the face, it hit me in the face once when I realized, wait, hold on, my kid with a 55 average is doing well on this task but he's not doing anything else. (Interview 1)

> Because she's accomplishing, she is working on, she's working to answer the question, doing the best way she knows how, using stuff we've done in class. You know, she's shown me more in the last week, with stuff that we've done in the geometry unit, than she has all year long. And she's an

> autistic kid at that. . . . She answered questions today. We have, we have come somewhere. This girl is now raising her hand, and verbally from across the room, asking me a question. We have accomplished something with her. . . . And it's showing up, it's in the format of math. Like I said, it's been an avenue for her to, it's allowed that to happen. (Interview 3)

So Michael saw that mathematics could become an arena to empower students who had traditionally been shut out. And he believed that teaching mathematics differently, in a constructivist manner, was making that happen.

Michael also related his own views of mathematics to issues of class. In describing the difference between what he termed pure versus applied mathematics, he said, "It's almost a white collar, blue collar difference in looking at math" (Interview 2). And Michael definitely aligned himself with the blue collar mathematics through his need for the usefulness of mathematics and his rejection of pure mathematics that seemed, at least for him, to have lost its purpose (e.g., the 4-cube). But even as he rejected the importance, in his own life, of the pure mathematics, he did not discard the idea of himself as mathematician:

> In my current curriculum, there's no, there's no room for me to be a mathematician. I am delivering material and they're coming back with it. There's nothing new…And some open-endedness I think almost gives me a chance, forces me a little bit, but gives me a chance to be a mathematician. (Interview 2)

For Michael, being a mathematician was not about working in the abstract world of pure mathematics; it was about doing mathematics in a useful and practical way.

Michael was very comfortable with his views of mathematics. His preference for the applied mathematics worked well with his students and aligned with the task-driven instruction of the new curriculum. His desire to open doors, through mathematics, also agreed with the elimination of the applied track that the new curriculum promised.

Michael knew mathematical concepts existed that did not fit into his own personal views of mathematics, but he did not need those troubling concepts to be a part of his world as a mathematics teacher. For Michael, mathematics was a tool to understand the world and to empower his students, to make them feel valued: “I had someone say, you know, I felt important in your class, and that’s huge! I think that’s more my philosophy of how teaching is, you know, I want kids to feel valued” (Interview 1). Michael viewed mathematics, in his life, as having a higher purpose:

> You know, it’s almost more, it’s almost more of a platform. I mean there’s, I mean math is the platform that I stand on to be able to have an impact on kids. I don’t think you could just have a group of kids in your room and say, okay now I’m going to teach you. Okay, I’m going to teach you what? What I want! You know, you’ve got to have a platform to stand on and then run from there. . . . I mean, that’s legitimate, that’s the thing. And in the process, if I can have a positive impact on them along the way while we have a meaningful thing, an important thing to discuss, it’s going to be beneficial to them in the learning process. But just to, to tie all that together. (Interview 1)

Michael saw a consistency in his views of mathematics and teaching mathematics with his world view: “I can’t really, they’re not two distinct things. But I think it’s too, it’s just because mathematics to me is a part of life” (Interview 1). In our summer course, we had talked often of fragmented philosophies, putting together philosophies from various scholars that make up our own personal philosophy. But Michael did not see his philosophy as fragmented: “You know, our philosophy’s probably not that fragmented but the language that we use to describe it is fragmented because it comes from different places” (Interview 1). And for Michael, it all came together in a way that made sense:

> Where I walk is where I walk altogether and everything comes together and I carry all of that with me, and that’s important because I don’t want one part of my life to conflict with another part of my life. You know, whether I teach mathematics or if I coach baseball or if I, you know,

> whatever it may be, I mean, I want those things to go together. (Interview 1)

Michael felt there was a consistency in his teaching, in his religious faith, and in his perceptions of mathematics. In our discussions, he seemed to find strength and accord in each of these areas even while recognizing that others might see contradictions.

*Struggles Amidst Change*

I met with Michael for our final interview 4 months into the school year. It was the first year of the implementation of the new high school curriculum. Although Michael had experimented with new instructional practices (e.g., use of open-ended tasks, collaborative learning, sharing multiple solution paths) and was intimately involved with the changes in curriculum through his role as a state trainer for other high school teachers, he found he was struggling with the reality of implementation. During our previous interviews, Michael had expressed eagerness about the "de-tracking" of the mathematics curriculum:

> I'm interested to see what happens when we bring these kids together. I don't know if it's going to work every time. And I know that our community, particularly our school, is not going to embrace that. But if we start to do that, start just trying to cross some things up adjust see what these kids can bring to each other. (Interview 1)

> I was just thinking about, one of the biggest things I'm excited about with this curriculum, I think it a lot of people's biggest fear, from what I hear from folks, but I am really excited about having a variety of kids in the same classroom. I really think that's going to bring a lot. (Interview 2)

Michael had believed the new curriculum would eliminate the pervasive "tracking by neighborhood" that he had witnessed at his school and, thus he hoped, usher in a more egalitarian instructional policy.

During our final interview, Michael talked about complex struggles he was experiencing as he implemented the task-based, open-ended learning of the new curriculum. The foremost issue he noted was issues of maturity that aligned with his view of mathematics as critical thought:

> You know, I do want them to, I know we're going to push them to do things earlier and do more with them. But they're still 13 and 14 year old students, and we're asking a lot of them. I don't know that their brains sometimes are, I don't know that they're capable of reasoning in a way that I'm asking them to reason. (Interview 3)

> And I wonder if it does take a kid, and I guess I would say to that, and that would get tied back to the maturity, even if it is mathematic maturity and I don't know if that's even appropriate to say, but something that has, they have a perspective anyway to see why it does make sense. Where a kid hasn't gotten to that point yet, this is still foreign to them. (Interview 3)

Michael had begun to connect this idea of mathematical maturity with the need for the tracking that had previously occurred in his school. And so Michael was unsure if the de-tracking he had looked so forward to was helping all students:

> There is another student that is working hard, and she's capped out at her abilities, and there's some emotional stuff that is also a struggle. We're looking at giving her credit for the course even though she is not, because she's not at the same spectrum as the other, it's kind of operating at a different level. So I haven't seen the benefit that I was expecting from that. Um, almost quickly swinging the other way. I love the thought behind this, but it's not happening yet. (Interview 3)

But as we talked further, it was clear that what Michael was questioning was the true constructivist nature of the new mathematics classroom. Although the tasks were routinely described as open-ended, Michael realized that it was not truly discovery-based learning, that the intent of the curriculum was for students to "discover" a certain mathematical concept and to move through the task in a given direction:

> We're almost forcing, it's horrible, are we forcing them to all think the same way when we're saying we want to approach, we want to approach

> them as individual thinkers, when we're trying to, we're shuffling them all in this direction. (Interview 3)

Michael was realizing the limits of a task-based curriculum, at least as adopted in his school. His struggle was in keeping his students engaged in the mathematical tasks while still ensuring they learned the defined mathematics concepts identified in the curriculum.

Michael shared a story about a particular student, during an activity exploring centers of triangles (orthocenter, circumcenter, and incenter), who refused to follow the mathematical line of reasoning as identified in the curriculum:

> And we say, where are you going to place the airport? And she said, I want to put it here. And she just put it in a random spot. And I said, well why? And she just said, well it's in the middle of everything. Well okay, what about it (that random spot) is special? And so we had to walk through that process and it's kind of those things where she got to the point where she read a question and she had an answer for it. She just didn't have any support for her answer. And did not desire any support for her answer. This works, I would put it here. It looks good there…You don't see the need for my incenter or my circumcenter to be helpful for you in this process, to answer this question. You don't see the need for it. (Interview 3)

This experience led Michael to question the intent of the task-based learning he had so heartily endorsed: "When we start to really apply, answer a question and give me a why, am I setting myself up as a math teacher to have to be okay with your why?" (Interview 3). In this, Michael is echoing what Ernest (1998) termed the dangers of a relativist epistemology such as social constructivism. Do we give up, in social constructivism, the (previously inherent) qualities of mathematical knowledge such as necessity, stability, and autonomy? Do we even allow our students to question what mathematics is needed and when? In many ways, the interaction Michael described questioned the very nature of mathematical knowledge and mathematics teaching and learning. If we are to ask students why, must we then accept their responses (if the

response do indeed work)? If Michael's student can find, using mathematical precision, the correct incenter, orthocenter, and circumcenter of the three given vertices, yet chooses to disregard those in her response, has she somehow come up short in our estimation of her mathematical ability and understanding? By searching through these questions, Michael has begun questioning just what it means to teach mathematics, particularly teaching mathematics as a social construction.

And for Michael, teaching mathematics had been a way to hold on to certainties, to the concrete he needed but often found lacking in his previous studies of higher mathematics:

> You know I think we wanted, I wanted truth. And I think that's my perspective, I think that's me a little bit. I need truth, I need a concrete to stand on. I need a rock to stand on. And then let's go from there. . . . I think that's a little bit of my comfort in the teaching realm, it's because we have parameters. You know, we don't necessarily go back to an ideological thing. But we have the parameters of requirements and curriculum and things. And there are, yeah, we got a curriculum. There may be nothing underneath that curriculum but I've got a curriculum to stand on, as a teacher. (Interview 2)

But even as Michael was defining his comfort in teaching and the parameters it afforded him, he realized that those parameters were shifting:

> I think that, yeah I do, I find it interesting that part of the goal of our class was not that we would leave with answers but we would leave with more questions. Maybe better questions…I don't know if better is a good word there. But you know, we're leaving with questions that are different. And I think, see my personality was drawn to math maybe a little more so that this is concrete, this is really, this is solid. This is what we do, we've done this for the last 50 years, we'll do this for the next 50 years. So I'm safe in my little 30 year zone in that. But I found out there are questions still in that, in how we do it and the fact that you have the variable of people, you know, kids that are always changing to make that fit. (Interview 2)

For as much as Michael looked to mathematics, and the teaching of mathematics, for certainty, he accepted and embraced the uncertainties it often brought him:

> I feel like, from a lot of people I know, I'm much more comfortable being uncertain in front of my students. Because I don't have a problem looking at them and going, hey, we're going to come back to this tomorrow. I've got to do some more with this. Or, you know what, you're right, that doesn't make sense. (Interview 2)

This attitude, that it is okay to not know, was reflected in Michael's final judgment of our exploration of philosophy of mathematics: "I am starting to see that the questions are what is important, not necessarily the answers" (Final Reflection paper). As Michael struggled to implement a changing mathematics classroom, he seemed to be comfortable having more questions than answers. A new curriculum not only made him look at the teaching and learning of mathematics in new ways, but even made him think about mathematics in a changing light.

## Julia's Story

### *Background*

Julia is a Black woman who works as an instructional coach for mathematics in a suburban high school. She works for a large and diverse school system. Her high school is predominately Black and many of the teachers there are new to teaching and following an alternative route to certification (i.e., did not attend a traditional college or university education program). Julia began her teaching career as a high school mathematics teacher. Like Katie and Michael, she found herself drawn to the struggling students and the entering freshmen. After 3 or 4 years, she pursued certification in educational leadership and awaited a school administrative position. Instead Julia got a call to serve as a mathematics instructional coach at a local middle school:

> Then I aspired to be an administrator. I should have had my head checked. And I got a call, and I thought that was what it was for, but they, the principal said, no, I want you to be a math coach. I was told you were the best math teacher in this area. So, what is a math coach? (Interview 1)

Julia worked a few years at the middle school then moved on to her current position as a high school mathematics coach. Her responsibilities were varied:

> I am responsible for, I like to sum it up as helping math teachers transform their world and their work. . . . To help them implement best practices that will in turn help promote student achievement. I am on-site professional development…I am the resource for anything they need. I'm supposed to be the expert on strategies, classroom management, even be a parent resource, a facilitator, in some regards. I model lessons, especially if there's a different strategy that they have not, that they're not familiar with, I would model that lesson. . . . I observe their pedagogy and give feedback to them. Help with all different types of school-based initiatives, prepare them for high-stakes testing, pull-outs, you name it. Whatever needs to be done, I will do it. (Interview 1)

Julia received her undergraduate degree in mathematics, a subject she had "always loved" (Interview 1). Although she initially wanted to be a teacher, her father encouraged her to pursue a different career. So she came to teaching following 10 years in the military. Her move to teaching, though, was like "coming full circle" (Interview 1) as she came from a family of educators. Her brother was a special education teacher who "really inspired me" (Interview 1), and her father was a teacher of mathematics. Her father's influence was tremendous, as Julia was, at an early age, drawn to the subject he taught. In addition, Julia credited her success in mathematics to the encouragement she got at home:

> I would just pick up some of his books; try to figure out what was going on with them. I would finish my homework, grab some of his books, and just try to go through the problems. I guess the challenge of it, in that regard. It's the only place where I like a challenge is in math. (Interview 1)
>
> When I was in school, I just remember I could go through the problems, and that's when I was saying, I would pick up my daddy's books and do his math. Now, my daddy says that we were very nerdy when we were in school because when I finished my math book, I'd always look ahead. I was always like a week ahead. But that doesn't say I was smarter, I just, I just liked math and no one ever said to me that I couldn't do something or

> that my mama wasn't that good at math or she never said that. . . . My daddy did not because, again, he was a math teacher. I didn't come up in that environment where, you know, well you're not going to be able to do it because I didn't know. (Interview 1)

Julia did not identify herself as a "natural" in mathematics; she simply recognized the support in her home for mathematical learning and the access she had, through her father's textbooks, to push herself further. Throughout our conversations, Julia never described her talent in mathematics as a gift; she instead recognized the culture of learning her family had afforded her.

During our summer course and throughout the period of our interviews, Julia was a doctoral student in mathematics education. Prior to her participation in the summer course, Julia and I had taken several classes together and had a passing acquaintance. During the study, Julia joined the group of state curriculum trainers to which both Michael and I belonged. Julia was a very confident and thoughtful individual to interview. At times, though, I realized that our ways of conversing collided. Julia had been brought up in the rural south, and I had grown up in the urban northeast. Julia spoke slowly and carefully while I spoke rapidly, often talking over others. As time went on, I learned to listen better to Julia's deliberate speech, allowing her, not me, to fill in the pauses. Unlike my relationship with Katie and, somewhat, with Michael, Julia and I had many shared experiences as professional developers, working with adults, and not students, to facilitate change. Much of our focus, then, was on that work and its effects on Julia's views of mathematics and mathematics education.

*Views of Mathematics and Mathematics Teaching*

Julia's views of mathematics followed two basic themes: mathematics as beauty and mathematics as power. Throughout her writings and our interviews, Julia described the beauty she saw in mathematics:

> The beauty in math is making your own sense of the process of problem-solving and understanding. . . . The beauty of mathematics is doing mathematics. I see beauty in mathematical solutions which make connections between two areas of mathematics that at first seemed like they had nothing in common. The most intense experience of mathematical beauty for me comes from actively engaging in the mathematics. (Final Reflection paper)

> I guess I'm kind of relating to art and how he [the artist] would sit and capture this object he's going to paint and you begin to get, what colors and all that kind of stuff that you want to put in your picture and, as a math problem, I think I related it to the trig [solving trigonometric identities] because that's my favorite. I would pick the hardest one and okay, how am I going to take this apart? What do I know? And sometimes those trig identities would take 2 or 3 pages of a whole board. And then I would just sit back and look at it, like I had created this great masterpiece. (Interview 1)

For Julia, the beauty of mathematics was never a cold, dispassionate thing, disconnected from the humanity of the subject. Her view of mathematics' beauty aligns with Neyland's (2004) postmodern view of the wonder and enchantment of mathematics as an ethical choice. For Julia associated the beauty of mathematics with the doing of mathematics and, hence, with the potential power of mathematics: "I align my beliefs with those of Polya's that to know mathematics is to be able to do mathematics. When I do math, I am discovering patterns and order, trying to figure out how something works and making connections" (Final Reflection paper). It is this importance in the doing of mathematics that gives it power, not just power to those who come to know mathematics, but power to the teachers of mathematics, the dispensers of the knowledge. Thus

mathematics becomes a priesthood (Moses & Cobb, 2001), one in which some are accepted but many are shut out: "The math teachers use this math power to determine who will be successful in life and who will not" (Initial Reflection paper). This is a lesson Julia learned early in life as teachers dictated to this mathematically creative child their *right* way to do a math problem:

> Yeah, definitely. You've got to do it this way. And I would do it that, I guess, that was my thing then, back then, was multiple representations. I'll do it your way but here's my way on the other side. . . . But I, trust me, I learned early on, which is still happening now, the power that they possess! The power that they possess so I gave her what she wanted but I feel like my brain, my way of thinking to come out, I don't want it to be stifled otherwise I wouldn't progress. (Interview 1)

Thus Julia subverted the teachers' power, by utilizing her own representations even as she copied their "correct" procedures.

Julia believed this mathematical power could be transferred to students through their own success in problem solving. It was her goal to eliminate the priesthood of mathematics and make it, instead, accessible to all by making her students successful *doers* of mathematics:

> Problem solving places the focus of the student's attention on ideas and sense making. Problem solving develops the belief in students that they are capable of doing mathematics and that mathematics makes sense. Once they make the connections, the students have confidence and will tackle any problem you put before them. Problem solving develops *mathematical power* (Julia's emphasis). Wow, there's that power I was referring to earlier but look who has the power, the student. Understanding gives confidence and engagement; not understanding leads to frustrations and disengagement. (Interview 1)

So Julia's goal, as a teacher of mathematics, had been to ensure her students felt that power. To do that, she used what had often been taught as a rote process in high

school mathematics: the proof. And she encouraged what had been lacking in her own mathematics instruction—creativity:

> Just seeing it come, I guess seeing it come alive as something that I did, I created, because you know, I used some of the theorems or proofs, putting them all together in my own way. Because that's what I did, because you can prove those in so many different ways. You know, just arriving to the different answer. There were different ways to do it. And that's what I liked about those because I guess they gave students a realm of free expression. (Interview 1)

Julia was redefining proof, rejecting the traditional mathematically confining definition of proof for a more empowering, sense-making definition of proof. There was not one correct proof. Proof was, instead, understanding and justifying and finding one's own way.

The power of mathematics, for Julia, was tied as well to the hard work involved in working it through. Determination, what Julia termed "dig-in-ness" (Interview 1), to solve a mathematics problem, to see it through to the end, was something she saw missing in the traditional school mathematics curriculum:

> I think that's part of the problem, the way we teach math, we want a quick answer. Of course, you know, x + 7 = 11, that's quick. But we have to teach them that endurance, they need to understand, when they get out in the world, when they, when they are confronted with problems, they first, you know, try to figure out how to solve the problem, hang in there to see it through. (Interview 1)

This dig-in-ness, for Julia, connected to the bigger purpose of mathematics in our schools—to teach that endurance she believed her students, Black students in a racialized society, needed to succeed in life:

> Well, it's important I think to empower them here because, one of my philosophies is, not so much being concerned with them, of course I want them to be successful in school, but I'm more concerned about how you are outside of school. I take personal accountability for my students. So you want to see you, beyond these walls, be successful, be productive

> citizens. Teach them competence, being able to deal with change, stamina. That's another thing, to dig in. (Interview 1)

Like other educators in this study, Julia saw mathematics as a life lesson, a platform to teach the bigger ideas that she wanted students to learn.

Julia also wanted her students to know that it was not a predisposition, a natural ability that had led to her own success with mathematics, but perseverance, a desire to succeed:

> Our students haven't been as successful in math because they just don't stick with it. And I always tell my students, it's not hard. You say that cause you're good at math. No, because I sat down with math, hours after hours. . . . But that's because I wanted to know. (Interview 1)

Julia recounted a story of how her class had been struggling through trigonometric identities and how that struggle led them to understand the power that success in mathematics could bring:

> And that's why I remember my kids with those trigonometric identities, I mean they're such a challenge. They wanted to cry. Most of them dug in with, I mean we worked that problem one day, we didn't have it done. I mean, both boards were full. I said, okay, you work on it at home and when you get it, call me. Most of them called me at 11:30. Hey Ms. G, we got the answer! I said, I just found it myself! That's what I'm talking about, that's what I want to see. That they're just digging in, see it through. I used to have this word, dig-in-ness. (Interview 1)

So Julia wanted students to reject the idea of there being "math people" and "non-math people." Instead, she urged her students to take on that persona for themselves—I am a math person because I do math!

Julia realized that the power in mathematics was in doing the mathematics, not in watching the mathematics get done, a mistake she herself had learned and wanted to pass on to the teachers with whom she worked:

> Memories of a proof taking about an hour to prove and taking up the entire chalkboard were a real piece of artwork to me. Notice that I did say to me. As I reflect back to those days now, I wonder how much was learned by my students. I wonder, do we measure how much we have accomplished in class by how much we have proven in our lectures? (Final Reflection paper)

> The one who does the most talking does the most learning. The lecturer goes on and on to her him/herself speak or for self gratification of proving a proof. Let the students explore and take charge and follow their way of thinking. Let the math be their math. (Reading Journal)

When we first met, prior to the implementation of the new high school curriculum, Julia looked forward to the changes she hoped would occur. She believed the new Georgia Performance Standards (GPS) aligned much more with her own personal views of mathematics and mathematics education:

> One reason why I like the GPS, because a lot of those problems are problematic. What I mean by that is that they make sense to them (the students), they can apply them to the real world. They want to know the answer, so they're going to dig in with them. (Interview 1)

Julia also looked forward to the pedagogical changes called for in the new curriculum. She believed her own love of creativity in mathematics, of solving a problem in your own fashion, would be realized through the use of multiple pathways and multiple solutions in the tasks of the new curriculum:

> The beauty and elegance of math is also rooted in the realization that fundamental mathematical concepts can inevitably be explored from multiple directions. The strategy of solving a single problem with tools from a variety of approaches is imperative to student success. This strategy is also, however, central to my approach to teaching. (Final Reflection paper)

Again, the wonderment of mathematics was tied to the ethics of how mathematics is taught. For Julia, the beauty and power of mathematics were connected through its problem-solving nature. What became a struggle for Julia, as an instructional coach, was

bringing her own beliefs about just what mathematics is and what it means to learn mathematics, to the teachers with whom she worked.

*Struggles Amidst Change*

When I met with Julia for our second (final) interview[9] in November of 2008, she had been working with her teachers for 3 months on the implementation of the new high school curriculum. Her frustration was evident. Although Julia's job was to support teachers in their implementation of the Georgia Performance Standards, it was clear that her focus remained on the learning of the students. She began our interview by reiterating her goals for her students:

> I want my students to see math as a space that they are welcome in. That they can do it. I think too if they look at it as a means to, of empowerment, then they would probably embrace it more. Look at it, you know, they say they hate it, but look at it more as a challenge, a puzzle that I want to solve. . . . They see it as a wall. And I said, I think I said before was the perception that a teacher gives to them, that it is a privileged space. You know, if you don't have a certain skill, if you don't understand the way I do, then you're not welcome into this space. Whereas to validate how they see it, how they reason through it, you know, I think that would help them, helps them a whole lot more to embrace it. Like I was telling them, you don't have to take it to the dance, just have confidence that you can do it, you know. And you may not solve it the way I do, but if it makes sense to you and it works and you get the same answer every time, then that's good, that's good enough for me. (Interview 2)

Julia was disappointed in her teachers' unwillingness to embrace what she saw as the pedagogical intent of the new curriculum—learning mathematics by doing mathematics (i.e., engaging in tasks and mathematical talk, encouraging students' multiple representations and solution paths). Julia believed that it was teachers' own fear, fear of the unknown that stood in the way of change:

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[9] Julia and I had our initial interview in the summer of 2008 and we were only able to schedule one additional interview. Both of the interviews, therefore, lasted an extended amount of time. Our first interview was used to discuss Julia's mathematical background, teaching experience, and initial thoughts on philosophy of mathematics.

> It might be a fear that they cannot explain it. Once that student ventures to another way, and they can't explain it, or understand it, then that's where their fear or lack of confidence may be. And a lot of them [the teachers] are afraid to let that be seen. You know, I would easily say, I would never have thought of that when they solve a problem that I, that I wouldn't have done it that way! I quickly tell them that. And that builds their confidence even more. But I think that's part of it, them [the teachers] being afraid that they don't know and don't want the kids to know. (Interview 2)

Julia knew the teachers' fear was stopping them from seeing mathematics differently and that seeing mathematics differently was needed in order to teach it differently.

Julia understood the difficulty the teachers were having in changing their approach to the teaching of mathematics. Too many teachers had relied on textbook driven instruction. That is what they had experienced as students and that is what they were used to as teachers. Julia also realized that many of her teachers were challenged by the mathematical content which they were now teaching:

> It's not a stand and deliver approach. However they're trying to make it that way. It's more of discovery which is not a lot of the way they have taught. (Interview 2)

> It is because math teachers, a lot of them are linear. Freaking out because it's not page to page…I always say, just teach the standard. So therefore you may find resources here, something here, whatever. I'm not tied to a textbook. It is just a resource. It needs to go page to page [is the teachers' response]. They got bogged down on Pascal's Triangle [which is not in the curriculum] because it was the next section. I said, why are you teaching it? You know, it was the next section. (Interview 2)

> And because this is an integrated math, high math, they have no familiarity with it because, of course, as a first year teacher, well they've been teaching Algebra 1, okay? So now you're asking them to go into a world they really don't know anything about. So constantly it's like, what is this? So that adds on to the fear. Okay, and the uncertainty. They're starting to question their own abilities. (Interview 2)

Again, as Taylor (1996) described, Julia saw the hesitancy of letting go of the myth of hard control that mathematics had given teachers for so long. As long as the teacher

controlled the dispensing of knowledge, the control remained the teachers. Yet Julia knew the teachers had to let go of the control, to let the students explore the mathematics, wherever it may lead them.

Julia believed the teachers' response to the new curriculum, their clinging to their traditional ways of teaching was more than just habit, more than just what they had always known. Julia related this reluctance to change their teaching practices to the teachers' (perhaps unconscious) views of mathematics as power, as well as a clinging to what Freire (1970/2000) termed the banking concept of education—depositing knowledge into students as though they were what Julia termed "empty cups" (Initial Reflection paper):

> What I see more now is when a kid does not know math, like let's say the basic skills or whatever, how they're stripped, demoralized. And it bothers me that it continues instead of empowering. I feel like when they're given confidence that they can do it, at their level, then you are empowering that student, you're giving them the confidence to be able to do any and everything. . . so when teachers start to, I won't say dumb them down, but you know, keep them at that particular level, they are stripping them of any type of power that they will ever experience. And that's unfortunate because, and maybe that's how it's designed, I don't know, because teachers have to feel they're in this power mode, and so the kid must be at a certain, and if you don't come with it, you have to be at a certain level. . . a certain status. (Interview 2)

> They still believe that I am the giver of knowledge. And it's only by me, going back to that space, that the only way you're going to get it is through me giving it to you. (Interview 2)

Julia saw that on some level she was asking that her teachers change their philosophy of mathematics education, as well as their perceptions of mathematics. She wanted her teachers to embrace a constructivist model of teaching and learning, to understand and accept that students could learn mathematics through doing mathematics:

> The main thing that I try to do when working with them is changing their philosophy of their approach, and of their students, believing that they can. This is different. But we can do it! (Interview 2)

> But as I was teaching, I don't know, because I was in the military before I became a teacher, but I never gave a formula, they always derived it. Because I felt that they would remember them, they would remember them. So that I never gave a formula. And this is how I see the standards. That the learning that's supposed to come from the standards is you constructing your own understanding. Just like finding the area of a rectangle, a triangle. You give it to them, you say it over and over. Do they remember? No! But once you have them do the model, do the blocks, so they can see that. But then when you go half it from the rectangle, you draw the triangle, then you say, what did you do? Cut it in half. There's a triangle there. There it is. And I never see that done. I just see, here's the formula, on the sheet. Never see that done. I think that's where we've gong wrong. So we're trying to correct that now. And it's going to take awhile, it's going to take awhile. (Interview 2)

Julia was expressing here the same concept Ernest (2004) noted as the Topaze effect: "the more explicitly the teacher states what the learner is supposed to learn, the less possible the learning becomes" (p. 29). Julia rejected that way of learning and wanted her teachers to do the same. Julia remained hopeful that her teachers would begin to change. And her approach to teacher change was the same as her approach to instructing students—let them *experience* the learning:

> That's the big thing, that's it right there. Because sometimes they do get stuck in the old ways, so I try to give them confidence. Let's try it, we'll try it together. Always, first thing I say is, it may work, it may not. But we're going to try it. Cause we'll never know. Okay, then if it doesn't work then we'll go back and be reflective. Why doesn't, why didn't this work? What could we have done differently? Then we go back and try it again. And the, most time it works. And that's how I get them to believe, feel empowered, this stuff does work, what she's saying. And they can try it again. (Interview 2)

In many ways, Julia was working to get her teachers to see mathematics instruction as similar to the mathematical idea of proofs and refutations (Lakatos, 1976)—a continuous cycle of trying, adapting, rejecting, and reflecting. She encouraged

her teachers to view the new curriculum as an experiment, one in which they should believe, but one that will only succeed through reflection and adaptation. In this way, Julia empowered her teachers to take a risk, to feel confident, and to see it through.

As Julia worked with her teachers, strengthening their resolve as they struggled to implement the new curriculum, she realized that many of her old ideas about mathematics had been left behind, thanks, in part, to the readings from our summer course:

> Because I, you know, when I came in, when I started teaching math, one of the reasons I gave back then was math is objective, it's either, you know, it's a right answer or you're wrong. To me it was easy, versus another subject like English or whatever, and it's nice and clean. But then after that class [our summer course], I don't know. They [the readings] kind of start making me think a little more about how, you know, I used to regard it. I don't know how objective it really is. . . . And see now, to embrace open-ended, the different representations, the different methods of doing it. And now we're saying, it might be more, it's not one correct answer. I am now, but like I said, back then I used to go, yeah, this is clean, cut and dry. But now, now it's, but that's a good thing because I remember when I was in school, as a child, and if you did not do it the way the teacher did the problem, it was wrong. You know, you may have the same answer, and they would say, no you got to do it this way. And that bothered me. So when I began to teach, I would pray that I would see kids do it a different way. Because what it does is validate everyone's thinking. And that goes back to my empowerment, that empowers that kid that, oh, I can do this, I just did it differently. (Interview 2)

Yet in many ways, Julia's philosophy of mathematics had formed in her early years, when, as a child, she did the mathematics the teacher's way but then secretly did it her own way as well. Later, as a teacher, she had wanted her own students to feel that same sense of accomplishment, that same feeling of artistry that she had felt when "doing math." Now working with teachers, her goal had become to change their views of mathematics, to see learning mathematics as doing mathematics, to see that the power was in the doing, and to transfer that power from themselves, the teachers, to their students.

# Diane's Story

*Background*

Diane has been a teacher for 26 years. She began her career right out of college, teaching first in a middle school and then in a high school in New England. Diane, a White woman, has taught in diverse settings throughout the world. Her experience includes teaching at international schools in England and Japan. In London, Diane taught both high school and elementary school students; her students came primarily from the Middle East and Europe. In Japan, Diane taught 2nd grade at the Yokahama International School. In Georgia, Diane has taught elementary and middle school students. After teaching for nearly 10 years in a large, demographically diverse suburban school system, Diane had taken a position as a mathematics instructional coach for a small rural school system. Although her position required her to work with district-wide mathematics teachers in grades 3 through 12, her primary responsibility was to assist and support teachers in the upper elementary grades as they implemented the new statewide curriculum, the Georgia Performance Standards for mathematics.

Diane was first inspired to be an educator by her 1st grade teacher, Mrs. O'Connell. What Diane most remembered about Mrs. O'Connell was how good she made her students feel in her classroom so many years ago: "I guess that's more it, it was how she made me feel and how I saw her impacting the kids in the classroom. It was just that overall feeling" (Interview 1). Diane attended a small women's college in Massachusetts, earning a degree in elementary education, with a minor in mathematics. It was in college that she first became interested in both the teaching and learning of mathematics.

> I didn't like math until I got to college. I think people looked at me and thought, teachers would look at me and think I was a good math student but I guess I'm really good at memorizing things and following procedures and um, I had no clue what I was doing. I mean, I did what they told me to do and, no understanding. And when I went to college, I took classes, the college I went to [was primarily a teaching college] and so every class that you took, they knew that you were going to be a teacher. So there was no Math 101 things in a big lecture hall, there was, you know, all the classes that I took, our Math 101 was, had maybe 20 students in it and it was, it was all, we got to learn about manipulatives and we were taught higher level, they taught higher level math concepts but they did it, you know, in a constructivist way. And so then, I just remember, everything just started making sense to me. . . . When I got to college, it clicked and that's when I decided to minor in math because I really liked the math classes that I was taking. (Interview 1)

Diane's preparation through a teacher education program where "everything we did, you thought about how did this translate into you being a teacher in the classroom" (Interview 1), had a tremendous impact on her own instructional practices. More than the other participants of this study, Diane had long-term personal experience with constructivist teaching and learning. As a practicing teacher, she had independently sought out training that strengthened her understanding of what it meant to teach and learn mathematics. She attended several Marilyn Burns workshops that focused on the use of manipulatives and teaching conceptually; she became involved with the prepublication use of Investigation, a task-based curriculum being field tested in the 1990's; and she continues to the present day to seek out professional development opportunities that challenge her to question and explore what it means to teach mathematics. For Diane, teaching mathematics never meant opening a textbook and working your way through it with your students. Diane shared a story of her first years teaching that exemplified this belief:

> When I interviewed for the job, I asked a math coordinator, this just cracked me up, I asked the math coordinator, so where's the curriculum?

> He opened up the textbook and said, chapters 1 through 6. That's the curriculum. I'm like, okay, I don't think it's supposed to be this way. (Interview 1)

At the time of our summer course, Diane was a doctoral student in early childhood education, with a focus in mathematics education. Although Diane had spent years developing a strong philosophy of mathematics education by constantly challenging herself to explore what it meant to learn and teach mathematics, she admitted in her course writings that she had never much thought about her philosophy of mathematics:

> Earlier this summer, I didn't know that one could have a philosophy of mathematics. I now know better. My philosophy is still in its infancy. Through discussions, readings, and in unexpected places, the philosophy of mathematics has presented itself to me. (Final Reflection paper)

Through her writings and in our interviews, Diane was very open about her changing views of mathematics, her personal struggles to define what it meant to teach mathematics, and her developing role as an educator moving beyond her own personal classroom into the larger mathematics educational community.

*Views of Mathematics and Mathematics Teaching*

Diane first described mathematics in a Platonic manner: "mathematics is all around us waiting to be discovered or found" (Reading Journal). She expounded on this belief in her initial reflection:

> I believe that mathematics exists all around us. How can one not believe that mathematics is all around us when mathematical patterns are found in nature and in all types of human interactions? Everything fits together too neatly for mathematics not to exist. (Initial Reflection paper)

This emphasis on mathematical patterns is a recurring theme for Diane. And initially, for Diane, these patterns were proof of mathematics' a priori existence, the idea that mathematics exists independent of human experience:

> I feel the power of mathematics in the chill I get when I make yet another connection between different representations of numbers. Today I was talking about patterns with square numbers. Between square numbers, students identified a pattern of the number of non-square numbers. Between 1 and 4, there are two numbers; between 4 and 9 there are four numbers; between 9 and 16 there are six numbers, and so on. So from 1, 4, 9, 16, 25, 36, …students found the pattern 2, 4, 6, 8, 10, …When finding the change between each of these numbers you find 2, 2, 2, 2, 2, …When you find a constant rate of change at the second level, you know it is a quadratic function. In this case, $x^2$, which is how any square number can be represented. The interrelatedness of things and how everything fits together so neatly, as in this example, is what convinces me that mathematics not only exists but is so powerful. I don't mean powerful as in the power of God. I don't believe that mathematics was created. I believe it just is. Religiously, I guess I would describe myself as an agnostic…It may seem obvious that because my beliefs are agnostic, I wouldn't believe that mathematics was created. (Initial Reflection paper)

But Diane soon began to question her Platonic views of mathematics, especially in relationship to her views about god and religion:

> Am I a Platonist? I am agnostic which means I fit right in with the mathematicians and scientists. I can see how they need to keep things separate, but how does a Platonist see mathematics as a gift from God and yet be agnostic? (Reading Journal)

Despite seeing a disconnect between her religious beliefs and her views of mathematics, in her final course writings, Diane still believed in the independent existence of mathematics:

> Mathematics is about simplicity, beauty, and predictability. It is about comfort, fluidity, and life. It is familiar, structured, and artful. Mathematics exists all around us; it is a part of us. Everything is connected and fits together too neatly for mathematics not to exist. (Final Reflection paper)

For Diane, as for the other participants of this study, her philosophy of mathematics could not be separated from her teaching of mathematics. And in her teaching of mathematics, the importance of patterns and predictability was once again evident:

> In the classroom, my students are mathematicians. They discover, describe, prove, argue, and use their senses just as other mathematicians do. I go on their journey with them. I get excited when we, as a class, make discoveries. I may not always know how to get there, or what the answer is, but I do know that there is an answer, and we will (eventually) find it. (Final Reflection paper)

And like Julia, Diane associated the power of mathematics with the effort involved in working through and solving mathematical problems:

> Part of my job is to help students realize and appreciate the importance of perseverance, especially with mathematics. . . . In my classroom, students often perceive me as knowing the answer, while letting them find their own way. Many teachers like to know the answer and how to get there, propagating the idea that mathematics has one right way to find the right answer. This should not be the case. Sometimes I don't want to know the path to the answer because it takes the fun out of mathematics. The struggle is part of the process and without it, working on a problem is not nearly as interesting. I allow students to find the connections, beauty, and structure of mathematics just as mathematicians have done for thousands of years. (Final Reflection paper)

> I want my students to struggle with ideas. I believe they create knowledge for themselves and being aware of other people's strategies gives them power (choices) that they wouldn't have otherwise. But I struggle not to make math rote and predictable. I have seen some students struggle (even third grade) with ideas in mathematics, and they just want me to tell them so that it is easier. It is difficult as a nurturing teacher not to want to do that for them. (Reading Journal)

This idea of learning through effort, of gaining power through that struggle, was a recurring theme for Diane. But for Diane, as for Julia, it was not simply the ideal of working hard and finding success through that hard work. For both Diane and Julia, the power issues in mathematics instruction were more about teachers letting go of their position of power as the holder of mathematical knowledge, using an empowering notion of education (Freire, 1970/2000) that credits the mathematical knowledge of the learners. For Diane saw that mathematics was not just about finding an answer, but about the processes involved in finding that answer. With her young students, she emphasized

discourse: thinking aloud, talking about the problem-solving processes, exploring ideas through mathematical discussions. Diane termed this “bringing the math out of the kids instead of dumping the math into their heads” (Interview 1). And as Diane described the mathematical conversations her students had, the idea of mathematical beauty once again emerged:

> I just think that there’s a lot of beauty in elementary school. It’s like when kids see, just when kids take apart numbers in their heads and they’ll be like, oh well I multiplied four 20’s and then four 5’s. And the other kid said, well you know, I thought about money and did four quarters. I don’t know, those connections, to me, just is exciting. And I don’t know if you’d really call it beautiful but in some situations, it just seems really pretty to me. You know, it just looks really pretty up there when you’re sharing all different ways that people thought about something. (Interview 1)

Diane created, in her own classroom, a mathematical discourse much like that in Lakatos’ (1976) *Proofs and Refutations*, in which a teacher facilitates her students’ exploration, questioning, and redefining of mathematical ideas.

Diane believed firmly in a constructivist model of mathematics teaching and learning. This belief had first been formed in college when, through her own constructivist learning, mathematics “just started making sense” (Interview 1). Her constructivist instructional practices were reinforced by both her participation in a series of Marilyn Burns mathematics workshops and her introduction to the *Investigations* textbook series: “Using those materials was really transforming for me because it really changed the way, having those materials really changed the way that I taught” (Interview 1).

Diane realized that teaching through tasks, having students explore mathematical problems and talk extensively about the mathematics they were doing, was a radical

change for most students and their parents. So she reached out to her students' families to help them to view mathematics in a different light:

> I was very proactive when I used Investigations. Like I had math mornings and I would invite the parents in and I would model how I taught the kids. And I would model some whole group work. And then I'd let the kids teach their parents games that we had. I wanted the parents to hear how I was talking to their kids. You know, like what kinds of conversations we were having. And we had some great conversations when the parents were there. (Interview 1)

But even as Diane embraced a constructivist view of mathematics teaching and learning, she acknowledged the difficulties in its implementation. What scared her most was the inability to follow the thoughts and ideas of all the classroom students as they explored mathematical ideas:

> It makes me wonder, you know, when some kid makes a comment that seems so far out in left field, you're like, well that could be and you move on. Um, maybe that kid has a thought that if you could just rein it in enough so you could just understand what they're saying. And sometimes I try to, but not all the time. It always makes me wonder. But the whole idea of addressing the needs and thought of all the kids is so intimidating to me. And it, it always has been. It's just, it's overwhelming. (Interview 2)

Despite the immense effort, Diane never wavered from her commitment to having her students construct their own knowledge about mathematical problems and processes. Although Diane had taught mathematics at all grade levels, she felt most committed to working with upper elementary students. And she realized how much of the traditional mathematics curriculum in grades 3 to 5 was dedicated to the teaching and learning of computational algorithms. But Diane felt strongly that the teaching of algorithms was to be avoided or at least postponed: "Once you teach them the algorithms, thinking stops. And I absolutely believe that" (Interview 2). It was Diane's own personal experience as a

student of mathematics that reinforced this belief: “I was really good at remembering how to do things and doing it. Not real good at what I was doing” (Interview 2). Diane understood that working through mathematical problems, rather than simply applying an algorithm, was much more difficult:

> I think it is more work in that you have to process things and make sense of it yourself. And to me, that’s more work than saying, do it this way, do this, do this. You don’t have any understanding but you can get to an answer without much mental work. (Interview 2)

> When I taught 3rd grade, you know, we would talk about how you multiply two 2-digit numbers. And there would always be a kid, at least one who would say, but Mrs. D, it’s just so easy, you know, you just do this and carry the 2 and why are you making me do all this work, why are you making me think about this more? And there were always those kids who were questioning me. Saying I was making it too hard for them, I was making their life harder. And they would complain because I would, we would share all the different ways you could think about it and they would just be like, no just tell me one way to do it. (Interview 2)

Diane understood her role, as the teacher, was to not give in to the search for an “easy” way to do mathematics, but was to push her students toward mathematical understanding by facilitating their exploration of mathematical problems and patterns.

Diane, like others in this study, saw that the struggle to gain mathematical understanding as more than just a school lesson. For her, the struggle to solve problems, to understand mathematical concepts and not just memorize rules and procedures, was a lesson about life:

> I want to say that, you have, everything’s not always going to come easy to you. And maybe, maybe that shouldn’t be a math lesson, maybe math shouldn’t be a life lesson or vice versa. But it just seems like when you struggle through something and you come out the other end and it’s all making sense, it was worth the effort. But you’re not going to feel that feeling of success or that feeling of satisfaction unless you persevere and work through the problem or work whatever it is. And maybe that shouldn’t be a math thing, but I feel like, I feel like math is maybe a good allegory for, you know, maybe it’s a good place to do that. (Interview 2)

And again, for Diane, this ability to see things through, to persevere and succeed, is what gave mathematics its power:

> I think you need to study math so you have that mathematical power. . . . Just feeling powerful, self-empowerment that, that you are capable and that you can deal with whatever situation, you know, you have enough background knowledge to be able to make decisions, depending on what situations you're in. (Interview 2)

One area of mathematics that Diane continued to question was the nature of higher mathematical learning, what she termed pure mathematics. Diane often wondered why mathematicians pursued concepts that seemed too complex for the vast majority of people to understand: "I get muddled when I think about the 'pure' mathematics that only a handful of people will ever understand. Is that math really 'there'?" (Reading Journal).

> While there is so much mathematics that is prevalent and so many connections that make mathematics seem simple and perfect, there is another side to mathematics that is not easy to understand and apply. This is where I wonder about its importance. If very few people understand it, and no application is found, is pure mathematics contrived? Was it there all along? Does it really exist? It seems like it is not being recognized but is forced into being. In my mind, it's the simplicity of mathematics that makes it real. (Initial Reflection paper)

> Mathematics is not so comfortable, fluid, and simple when I think about the pure mathematics that only a handful of people will ever understand. Is it mathematics that has not yet been realized? Or is it really mathematics at all? (Final Reflection paper)

In many ways, this struggle to define the mathematics that *really* existed as opposed to the higher mathematics that seemed somehow contrived crystallized for Diane a contradiction in her own views about mathematics, and began a self-questioning process regarding the a priori nature of mathematics.

*Struggles Amidst Change*

More than the other participants, Diane strove to align her views of mathematics with her philosophy of mathematics education. She saw a disconnect between the constructivist beliefs she had developed as a teacher of mathematics with the Platonic views she initially expressed regarding the nature of mathematics. Interestingly, this contradiction became clearer as she continued her reading of mathematics philosophy in a later class:

> My paper from the summer, I was still on board with Platonist views. But something we read, and I think it was Ernest, um, a Social Constructivist view of Mathematics [Paul Ernest's 1998 *Social Constructivism as a Philosophy of Mathematics*], when I read that and they started talking about, he started talking about the whole idea that math understanding is constructed and I thought, well, and my philosophy is constructivism, then how can I be a Platonist if, you know, and I had that, but that conflict didn't come until later. (Interview 1)

Diane was quite comfortable with a changing view of mathematics and had, in fact, expressed this idea of continuous change in one of her course writings:

> If nothing else, our understanding of mathematics is always changing. "At any given moment, mathematicians have only an incomplete and fragmentary vision of this world of ideas" (Thom, 1998, p. 72). A teacher's job is to help students realize that this is true for mathematicians and philosophers, so why shouldn't it be the same for them? If the world is always changing, wouldn't it make sense that mathematics is always changing as well? Often students are not let in on this secret about mathematics. They need to be told so that their experiences with mathematics is finding answers rather then getting the right answers. (Final Reflection paper)

This idea, that mathematics holds secrets that teachers do not share with students, secrets which perpetuate the myth of cold reason (Taylor, 1996)—mathematics as a priori and unchanging, transcending human experience—led Diane to question her previously stated Platonic philosophy of mathematics.

What began to unfold in our interviews was Diane's realization that she wanted a philosophy of mathematics that seemed more in line with what she believed about the teaching and learning of mathematics:

> Now my thinking has changed and, um, do you want me to go into that? I think, I think what we've done is, the things that are out there, that are mathematics, we've assigned a, a way of processing or dealing with it, using numbers. So it's more we've made sense of things or we've connected numbers to phenomena that happened. You know, I mean, whatever it is. Um, so I, the reason I changed camps is because I believe that people have to construct knowledge. Well if you construct knowledge then why do I believe that everything's just given to me. And so, but it took me awhile to even see that there was a conflict there when I wrote it [her final reflection paper]. All of a sudden I'm like, what was I thinking. Like this is so conflicting, these are two conflicting ideas that I have to make sense of so, that's how I make sense of it. (Interview 2)

For Diane, her emerging view of mathematics as a human construct not only aligned with her views of mathematics education, but also explained her discomfort with the study of higher mathematics:

> I think one thing that really struck me in our reading was, they had some story about how when you study mathematics, like you're on a river of mathematics and you get off, and you're in a stream of mathematics, and then it kind of dissipates, you know, just kind of gets so spread out…the whole idea that, it just seems like some mathematics have gotten off on this, this path or stream that's made sense to them. And they've continued on this path, but they're so far away from what we can recognize. But that doesn't mean that it doesn't make sense to them and they, it's explaining something for them. So, in that realm, it's the same as, you know, the mathematics that we use. It's explaining or it's a way to express something that we know. (Interview 2)

Another struggle that Diane discussed during our interviews was her growing realization that mathematics was not a culture-free discipline. During her own teaching experiences at a predominately African American and Hispanic middle school, Diane saw that her way of teaching mathematics, of building mathematical discourse, was dependent upon building a sense of community. Yet this sense of community was not coming

together in Diane’s middle school mathematics classrooms. Diane was not able to identify whether her struggles at the middle school where due to the age of her students or the lack of a shared culture with her students:

> I really want kids to be talking. And I wasn’t as successful, the last place I taught. It was a middle school and I had a much harder time getting kids to talk about math in middle school. And I think it was, I mean I think there’s a lot of cultural reasons. And it all came to play and I didn’t understand the culture that they were coming from and the whole idea that you, I don’t know, they just didn’t want to be verbal about. . . . You know, I wanted to get into mathematical discourse. But they, they either didn’t understand where I was going or there’s something about the way they perceived it, it just didn’t, it just didn’t agree with them. (Interview 1)

Diane realized that, in many ways, she was just not connecting on a personal level with her middle school students. This failure to connect emphasized, for her, that the mathematical community that she had previously created in her classrooms was dependent upon more than her own personal passion for mathematics teaching and learning:

> I taught at diverse schools, I taught in schools where it was, you know, a meshing of cultures and I just thought, I just thought, I have experience with this and it just frustrates me so much that it just didn’t go as well as I wanted it to go. That really, I mean I thought I had, you know, a background that would allow me to do well in that kind of environment but it didn’t. . . . I was really excited about teaching there but now I wish I could go back and teach there after having some of the conversations that I’ve had here [in her graduate program]. Because I, I’m much more aware of where I, my thinking was wrong and it maybe made my life more difficult. My thinking made my life more difficult. (Interview 1)

Diane was beginning to see that mathematics was not as simple as she had perceived it and that simply looking at patterns and connections in mathematics may not be enough to get all students engaged in the learning. She had begun to see the connection between mathematics learning and culture:

> You know, it kind of seems in a lot of cases the math that we teach, we teach it regardless of what they bring with them. And it seems like we

> could do a lot more to empower kids if we took into account the equity that they bring to the classroom. You know, the strength and skills and knowledge that they have. And a lot of times, it's ignored or it's ignored because they don't make those connections. You know, the teacher doesn't, doesn't connect. You know, this is what you're coming with, this is the math, let's see how they come together. It's just, this is the math, let's talk about it. So I think that there's a real disconnect with what kids bring in. (Interview 2)

Diane shared with me the story of one lesson she felt had been truly effective with her middle school students:

> One of the best things I did, that the kids were really engaged in, was, and I had no clue what I was doing, because I just put it out to them. You know, like if you turn on a radio station, that's where I started, what radio station would you turn on? Because I don't even know what station they listen to, and so they told me. And so, okay, what would be a really great song that you would hear? And it was some singer, I don't remember. And then I said, okay, so then I set up this whole scenario where, we were doing multiples, least common multiples, common multiples, whatever. So I said, okay, you're in line to get tickets and every second person is going to get a ticket on the floor. But every $10^{th}$ person, or every $7^{th}$ person, or whatever is going to also get a ticket to go backstage. And so the question would be, how many students, how many people in line are going to get both, a ticket for the floor and a ticket for backstage? And they really got into it, they did because they had given me the scenario. But see, it took them to give me the scenario and to work from there. (Interview 3)

Although Diane was coming to appreciate the importance of recognizing cultural difference in the learning of mathematics, she also realized that it was still the teacher's responsibility to get her students to connect to the mathematics, culturally as well as intellectually. And she knew that would best be done by knowing, and listening to, the students:

> I think the kids need to bring it in. I think it needs to be informed by those students. I don't think, I don't think if you're trying to, if you're trying to connect with the kids because of their backgrounds and their understandings, then you need to connect with the kids to get, you know, informed by the kids. And that means every single class is different. And that's really easily said, very difficultly done, very difficult to do. (Interview 3)

Diane’s changing role, from classroom teacher to coach and mentor of teachers, caused her to reflect further, not just on the nature of mathematics and mathematics teaching and learning, but also on how to change teachers’ views of both mathematics and mathematics education. At the time of our final interview, Diane was finding consistency in her views of mathematics and her views of mathematics education: “I’m starting to see math as a construct, something that we’ve constructed to represent what happens in the world” (Interview 3). She saw her changing views of mathematics as more representative of how she had, for years, taught mathematics:

> This doesn’t change it [my view of teaching mathematics] but it probably reaffirms the whole idea that students need to construct their understanding of mathematics, which I’ve always felt strongly about. So I guess it was kind of a dichotomy to think otherwise [about the mathematics]. But I hadn’t really, before the class, I hadn’t really thought about what my thinking was so I guess there wasn’t any distress because I hadn’t really given it any thought. . . . I think it’s been a process. Because during the class, if you had asked me, I would have said, math is discovered. And it’s everywhere. But I think it makes more sense for me now. And I think it was more, I’m very comfortable with my position now because that’s how I feel children should learn. So if that part made sense, but I guess I had to come full circle. You know like, didn’t know, figured it out, though something different, then tried to balance it with my teaching beliefs or student learning beliefs. (Interview 3)

That circle of understanding and defining one’s beliefs which Diane described above, of not knowing, exploring, thinking differently, and searching for balance, was one she now was helping her own teachers work through. She knew that many teachers were struggling to implement the new curriculum as well as struggling with their own identity as teachers of mathematics:

> I think there’s a real disconnect with how to do it [change instructional practices]. But the problem is, how do you address those needs without insulting the people who have been doing that for 20 years or 10 years or

> for as ever long as they've been teaching. . . . Because they don't know what they don't know. (Interview 3)

Diane, like Julia, knew that a great deal of teachers' reluctance to change their practices came from teachers' own fears—fears of the unknown mathematics in which their students might engage:

> And that's something that I've discovered over the last few years that the amount of math, I guess it came natural to me because I had different math courses. I didn't have that nervousness. You know, like, I was comfortable enough to listen to a kid and let them explain and not worry much about whether I'm going to understand them or not. I know I can understand them. Whereas I think some elementary teachers might be worried that they're not going to understand or they're not, they'll know it's wrong but they don't know why it's wrong. (Interview 3)

Diane was echoing previous research that emphasized the importance of teachers' mathematical knowledge (see, e.g., Fennema & Franke, 1992; Mewborn, 2003). She realized the discomfort teachers faced when the mathematical discourse in their classrooms ventured beyond their own mathematical knowledge.

Being the strong believer in the power of constructivist learning, Diane turned to a constructivist view of professional development to help her teachers begin to change their own instructional practices and to address their mathematical knowledge and beliefs:

> Well, I mean, you can lead a horse to water. But I just feel like if, and maybe this is personal because of me but I just feel like if you give them experiences where they're going to see the same connections that you see and that the kids could make, then that excitement is going to translate into them wanting their kids to have the same sense of excitement. So for me, it's just providing those experiences and thinking about it and can you do or can you explain, and having them process it and construct an understanding. And I just feel, for me, that gets me excited and I want them to have that. (Interview 3)

Diane's goal, in her new work with teachers, was to get them to understand the why's of mathematics. As a dedicated elementary school teacher and as a mathematics

educator, Diane had come to realize that the mathematics being taught in the early grades was much more complex than she, or most others, had recognized:

> I really want teachers to have an understanding of the math that they teach. I know I said this already, but it goes back to, I think that society as a whole that you're not a good math teacher unless you've had these really hard, high level math classes. And I disagree with that. I think that the math that even the kindergarten through fifth grade teachers teach, it's very intricate, and how you teach it, and the experiences you provide your kids, and knowing why you're providing those experiences for the kids, is much more important than having Differential Equations or whatever. (Interview 3)

So, for Diane, it was not about how much mathematics a teacher had learned, but how deeply the teacher had explored mathematical concepts.

Diane's goal was to give her teachers those in-depth experiences with elementary-level mathematics. And then, through exploration, discovery, and discussion, she hoped they would begin to understand the mathematics in ways that would then benefit their own students' learning. She had come to see mathematics differently, more in keeping with her perceptions of mathematics education, and she felt comfortable pushing the teachers with whom she worked to view mathematics differently as well. Just as she came to see mathematics as a human construction by engaging in constructivist learning, Diane worked with other teachers to offer them the same experiences.

## Summary

It has been my intent to present these individual teacher stories as just that—individual stories of teachers' struggles to define mathematics and to teach mathematics in a manner that aligns with their own personal beliefs about the nature of mathematics. Yet there are still questions that must be asked. What does this study tell the reader about teachers' personal philosophies of mathematics? How do those teachers' philosophies

change over time as they engage in a scholarly study of mathematics philosophy and as they attempt to implement a new task-based curriculum? What issues do teachers raise as they define and refine their views of mathematics education, and try to answer the question, what is mathematics?

The philosophy of each of the mathematics educators in this study was influenced by their personal experiences with mathematics, first as students and later as teachers. It was evident in the interviews that each participant formed a relationship with mathematics early in life. Julia's father, a teacher of mathematics, played a large role in her own love of mathematics. Through him, she had access to higher level mathematical learning. She was able to pursue more advanced mathematics by exploring his textbooks; this opened up possibilities not found in her own classroom experiences. Perhaps it was this influence that gave Julia the courage to do mathematics in her own way—to follow the prescribed procedures of her teacher and then to use her method "on the side" (Interview 1). Curiously, Julia was the only participant who did not immediately pursue a career in education. But after a career in the military, she came back "full circle" (Interview 1) to her love—mathematics and mathematics education.

Unlike Julia, the other three participants described an inconsistent relationship with mathematics. Katie, Michael, and Diane all knew early in life that they wanted to be teachers. They were then drawn to the teaching of mathematics, though, for varying reasons. Katie did not see herself as a "brilliant" student of mathematics (Interview 1), but she had always enjoyed mathematics and never imagined teaching any other subject. Michael also identified himself as an average mathematics student prior to college, but saw the practicalities of being a mathematics teacher because the demand was high. And

both Katie and Michael admitted to struggling in college-level mathematics courses. Diane began her college career in elementary education but enjoyed the constructivist instructional methods of her mathematics instructors so much that she then pursued a minor in mathematics. For all four of these educators, the joy of learning mathematics came from the struggle to understand, the determination to see it through, and the satisfaction of solving a mathematical problem. For each of them, mathematics represented an effort achieved through hard work; none of them saw mathematics as something that came easy. And that idea—mathematics as achievement, rewarded for hard work—was something that Katie, Michael, Julia, and Diane brought to their classrooms as mathematics teachers. It was a goal for each of them to help their students feel the same sense of pride they had felt through their successes in mathematics. For these educators, mathematics was satisfying not because it came easily or naturally, but because they had worked hard to understand it. But for each of them, it was never just the understanding of mathematical rules and procedures that gave them satisfaction and pride, it was the actual "doing" of mathematics—Julia's trigonometric identities, Diane's use of primitive algebra tiles to understand how to solve equations, Katie's and Michael's struggles through higher-level mathematics courses to understand differential equations or linear algebra.

Power and mathematics were inseparable for these four educators. They all talked about power in our interviews, although they each characterized the relationship between power and mathematics differently. For Katie, the issue of power was primarily focused on the sense of confidence and assurance she found in her own personal success in mathematics. Katie emphasized the importance of effort—effort to learn mathematics

despite its difficulties, effort to complete a problem and see it through to the end. When she tried hard and succeeded, she felt empowered because she felt successful. It was this feeling of empowerment through success that she wanted to pass onto her students. She enjoyed working with freshmen because she believed she could teach them the importance of putting forth effort to achieve a particular goal. In her class that goal was to learn mathematics but, for Katie, the primary focus was the importance of hard work, to teach kids that “I can do anything that I put my mind to as long as I work hard at it” (Interview 1). In her view of effort, Katie held similar views to those put forth by the National Mathematics Advisory Panel (2008) when they emphasized the importance of effort in student achievement. Katie did not deconstruct this notion of effort; she did not question the many factors that may contribute to a student’s own beliefs about mathematics or mathematics instruction. She was, in many ways, reacting from her own personal mathematics history (see Bibby, 1999) when she stressed the role of effort in the learning of mathematics. Her experience had shown her that effort made a difference and it was that experience that she sought to pass on to her students.

Michael viewed mathematics and power as inseparable as well. Like Katie, he saw mathematics as the “platform that I stand on” (Interview 1) in order to have an impact on his students. He too had struggled through the learning of mathematics and felt empowered by his eventual success. And he wanted his students to feel that same sense of accomplishment. He did not hesitate to share with his students his own struggles with mathematics. Both Michael and Katie felt comfortable making mistakes in front of their students and then working together to correct those mistakes. Thus they wanted their

students to see mathematics as a subject one struggles through, learns from errors, and gains strength through success.

But for Michael, issues of mathematics and power went beyond personal empowerment through the learning of mathematics. Michael was confronted with the realities in his school system of mathematics as an instrument of socio-economic power. His realization that certain students, students of a particular race and class, were being isolated from the school population at large through their participation in lower tracked courses, was troubling for him. Michael was proud to be working with those students, proud to be engaging them in interesting and challenging mathematics. But he also acknowledged the injustice of an educational system that kept those students apart from others and he hoped to see changes to that system with a new, less tracked mathematics curriculum. Michael had come to see mathematics not as empowering for many students, but as a way to perpetuate class and racial inequities.

Julia clearly viewed mathematics as a subject that both granted power and withheld power. Her own experiences as a student who did not want to follow the prescribed methods and procedures had taught her early on that teachers too often held all the mathematical power in a classroom. But she subverted that power by following their rules while secretly charting her own course as well. It had been her aim as a teacher to share the power with her students by guiding them to become mathematicians in their own right—by giving her students the tools they needed to become logical problem solvers and by encouraging them to explore their own solution paths. She did not want her students, primarily students of color, to be kept out of the privileged world of mathematical knower. Later, in her work with teachers, Julia advocated that they, the

teachers, must let go of the power in their classrooms and relinquish that power to their students. Julia knew it was often fear, fear of the unknown and fear of not being in control, that kept teachers from changing their instructional practices. Holding onto the knowledge, being the one who knew the mathematics, was how teachers maintained control. It was Julia's goal to get her teachers to recognize the relationship between the mathematics and power and to, at the very least, share that mathematical power with their students.

Diane also viewed the learning of mathematics as a powerful endeavor. For Diane, it was more than just the struggle to grasp a difficult topic that gave mathematics its power. She believed that young children who could talk mathematically, who could solve complex problems, who could understand the predictable nature of numbers, were mathematically powerful. She also knew that students brought many different mathematical views to the classrooms, views often influenced by culture, that may be discounted in a traditional classroom. Diane tried hard to listen to all her students but she knew how difficult that could be when a student's culture is not shared by the teacher. But listening to students, mathematically and culturally, was of the utmost importance to Diane. For it was by listening to students that she learned best how to teach them.

For these four teachers, learning mathematics was much more than simply learning rules and procedures. None of them believed that teaching was telling—standing in front of the classroom lecturing to the students about mathematics. In the 18 months that I interacted with these teachers, it was clear that they had come to see their relationship with mathematics and mathematics teaching and learning as a continuing journey, a process of change that would not end. And they each were in very different

places in that journey. Prior to the implementation of the new curriculum, Katie had prided herself on helping her students see the "behind-the-scenes" (Final Reflection) mathematics, what Hersh (1997, p. 36) termed the back of mathematics:

> Front of mathematics is formal, precise, ordered, and abstract. It's broken into definition, theorems, and remarks. At the beginning of each chapter, a goal is stated. At the end of the chapter, it's attained. Mathematics in back is fragmentary, informal, intuitive, tentative. We try this or that. We say maybe or it looks like.

Katie wanted her students to derive formulas not just memorize them. She used a great deal of guided practice in her class to help her students understand why things worked as they did. Yet Katie felt overwhelmed by the new curriculum. She was uncomfortable watching her students struggle; she wanted to help them find the right answers. What Katie had always valued in herself as a teacher—the ability to help students understand how mathematical procedures worked—no longer seemed relevant in the new mathematics classroom. Katie knew that she still had a ways to go in her journey and she was not sure how to get there. She remained optimistic that the future would be better, would be easier for her as a teacher. But Katie had seen mathematics as a subject where hard work paid off. Effort resulted in right answers. With the new curriculum, she saw students struggling with no apparent results. Their efforts did not seem to pay off and so they gave up. She felt her ability to help them was stymied by the need for them to be "discovering" the mathematics. She was searching for a more efficient way for her students to be learning mathematics than the lengthy tasks that had been provided by the state curriculum: "And I'm almost wondering too if, instead of doing the tasks per se, coming up with some sort of discovery learning that's quicker" (Interview 3). In some ways, Katie needed to redefine her ideas of mathematics to reconcile her difficulties with

the new curriculum. She struggled with answers that were not simply right or wrong; she struggled to be efficient in her grading when she had to carefully review each student's efforts at problem solving:

> And the grading is way, you know, in the past it's been right or wrong. Algebra 1 for the most part, there's very, very little partial credit and now everything is partial credit. And so grading is taking a very, very long time. (Interview 3)

As our interview time ended, Katie was still attempting to redefine both her role in the new classroom and just how the mathematics was now being defined.

Michael also struggled during the initial year of the new high school curriculum. Like Katie, he had always wanted his students to understand the why's of mathematics. He had begun, prior to the implementation of the new curriculum, to integrate the use of tasks in his mathematics classroom. But Michael was surprised to find that his biggest struggle was around a change to which he had looked forward—the integration of various tracks of students into one classroom. Whereas in the past, he had worked hard to engage his lower-tracked students in mathematical problem solving, he was now faced with the difficulty of teaching students with varying levels of mathematical knowledge in the same classroom. Michael was concerned that he was unable to assist students as he had in the past. He questioned if the new curriculum was pushing students to learn concepts before they were mathematically mature enough. Despite individual successes, he wondered whether a curriculum that was supposed to address issues of equity was not actually ignoring the individual needs of his students.

The struggles for Julia and Diane were different from those of Katie and Michael as their work centered on teachers and not students. Both Julia and Diane had explored constructivist teaching in their own classrooms and felt comfortable engaging students in

task-based learning, getting them to talk about mathematics, and encouraging multiple solution paths. But helping teachers change their instructional practices presented new challenges. Julia's goal was to help teachers understand the mathematics behind the tasks so that fear would not stand in their way. In many ways, she sought to empower her teachers by exposing them to constructivist learning so that they would experience those ah-ha moments that had so excited her students. For Julia, this idea of engaging teachers in constructivist learning experiences was also tied to changing teachers' philosophies:

> So we have to get past that, they always want to complain, they don't know the basics. If I had five dollars for every time I heard that, I'd buy a new car. So we have to go past that as well. The main thing that I try to do when working with them is changing their philosophy of their approach, and of the students, believing that they can. This is different. But we can do it! (Interview 2)

Although Julia identified the philosophical issues as tied to instructional practices and equality, by emphasizing a changed view of mathematics (not just basic computational skills but problem solving), Julia was also highlighting a changing philosophy of mathematics. Her biggest frustration was working with teachers who continued to teach basic skills prior to engaging students in any tasks: "Because [the teachers say] they don't know this, they don't know that, so I need to teach them basic skills. So I had one teach 'add and subtract fractions' for 3 weeks!" (Interview 2). And Julia knew that she was, in many ways, asking teachers to reject what had attracted them to teaching high school mathematics in the first place—the idea of mathematics as right or wrong, black or white:

> And I've always said that maybe why people came to teach math, because it's easy, cut and dry. It's not now. Because I've had teachers say no, you should be able just to check, you know, go down [and check answers]. But now I have to analyze. Okay? Not just quick, quick, quick, checklist. I have to analyze, and that's energy, endurance, and it's something that

> some of them don't want to do. So it's easier I guess to say, they [the students] can't do it. (Interview 2)

The challenge that Julia then faced was how to get her teachers to begin to view mathematics in a different light.

Diane's challenge, in working with elementary school teachers, was to get them to understand the complexity of mathematics, to see mathematics as more than just memorizing the algorithms of addition, subtraction, multiplication, and division. Diane also knew that she was pushing teachers outside of their comfort levels and that overcoming their fear of the unknown was a factor in getting them to change their instructional practices. Like Julia, her goal then was to get teachers to feel the excitement of constructing their own mathematical knowledge and understanding, perhaps for the first time, how numbers worked. In her work with teachers, Diane emphasized the power of numerical patterns and number sense, and de-emphasized the use of algorithms, rules, and procedures. But Diane understood that experience alone might not be enough to change a teacher's beliefs about teaching and learning: "I don't have any magic bullet. I mean a lot of teachers have beliefs about how kids learn and they're different than mine and short of showing them experiences, I don't know that there's a lot you can do" (Interview 3).

In some ways, each of these educators saw a connection between changing instructional practices, whether it be their own or the teachers' with whom they worked, and personal philosophies of mathematics. They each had identified the need to see mathematics as more than computation, more than algorithms, and more than memorizing formulas. And in their reflective essays and our conversations, as we explored the summer course readings and specifically talked about their philosophies of

mathematics, the participants of this study examined what it meant to have a “philosophy of mathematics.”

Katie admitted that the summer course and our study had made her think about ideas she would not normally have thought about: “You know, cause you’re not usually asked these types of questions” (Interview 3). But, for her, to be engaged in discussions about philosophy while in the midst of trying to implement changes in how she taught mathematics made sense. Because, ultimately, Katie saw the new curriculum demanding a changed view of mathematics not just for her, but for her students as well:

> But I think the students, overall, with the new curriculum, they are seeing math in a way that they would not ever really have though of math. It’s just constantly showing them how math is related to the real world. And I think there are times where, you know, they just don’t, they just think, well math is just something you do in school, out of a textbook and so I think it’s been good for them too. Because I think they are definitely getting more opportunities to see how math relates to the real world. (Interview 3)

Both Katie and her students were struggling to redefine mathematics and to understand its implications in the classroom.

Julia found a way to connect the ideas of Ernest (1998) with her own beliefs about teaching mathematics:

> Paul Ernest (1998) believes that mathematical knowledge is influenced by human activity and contends that mathematics knowledge is situated within and grows out of a community of individual mathematicians. The implication is that since mathematical knowledge is a product of the social nature of the mathematical community, then the development of mathematical knowledge cannot occur without human activity. The formation of mathematical knowledge relies upon the discourse and dialogue that students have in and out of school. Their conversations include agreements or disagreements on the definitions, processes, assumptions, and rules of math. The function of mathematical symbols as tools must be socially acquired and mastered if students are to experience academic success. (Final Reflection paper)

Just as Julia's goal as a classroom teacher had been to empower her students by providing them the tools needed to build mathematical knowledge, and facilitating their use of mathematical discourse and dialogue, her goals as an instructional coach were to pass on the same ideals to her teachers. Julia wanted both teachers and students to see mathematics as something growing and changing, not a static field of study, but one with many unanswered questions. "As teachers, shouldn't we let our students know that the question 'what is mathematics?' has yet to be answered?" (Final Reflection paper).

Diane seemed most comfortable with a constantly changing, evolving philosophy of mathematics. While she struggled initially to align her constructivist view of mathematics instructions with her stated Platonic view of mathematics, she realized later, even as she came to define her mathematical philosophy as humanist, that philosophy, like mathematics, was not fixed. Throughout our interviews, Diane was always questioning, revising, and adapting both her views of mathematics and her ideas about mathematics education. Relating her philosophical journey to that of the character in the children's book, *Math Curse* (Scieszka & Smith, 1995), Diane began to see questions about mathematics everywhere:

> In my case, everything I looked at or thought about became philosophical. Does math exist? If it doesn't then why is Fibonacci's sequence found in nature, the Golden Ratio pleasing to the eye, patterns prevalent in the calendar, a parabola formed by the arc of a basketball shot? (Final Reflection paper)

Diane was not always certain about the answers to her questions, but she seemed comfortable discussing the dilemma of a self-identified constructivist teacher who loves the beauty and the predictability of mathematics.

The participants in this study had not thought about their personal philosophy of mathematics prior to the summer course but our readings and our conversations helped them to bring words to ideas with which they had previously toyed. None of the educators in this study ended with a single, easily defined philosophy of mathematics. But the idea of questioning—questioning what it means to teach mathematics, questioning just what is mathematics—was one with which they all became more comfortable. As Michael told me, “I find it interesting that part of the goal of our class was not that we would leave with answers, but we would leave with more questions” (Interview 2).

# CHAPTER 6

# DISCUSSION AND CONCLUDING REMARKS

The purpose of this study was to explore four teachers' personal philosophies of mathematics and mathematics education. The study took place as each of the four teachers was involved in statewide mathematics curriculum reform that emphasized constructivist instructional practices, thus engaging students in learning mathematics through doing mathematical tasks. The teachers all participated in a graduate-level mathematics education course that focused on particular readings in the philosophy of mathematics. The intent of the course was to engage the teachers in an investigation of a humanist/fallibilist philosophy of mathematics. This study extended that investigation, through the use of personal reflections and interviews, as the participants and I explored their views of mathematics in light of the instructional changes they were implementing as part of the curriculum reform in mathematics. In this chapter, I will present a final discussion of the findings of this study and examine the implications of those findings. In addition, I will share the limitations of the study and offer suggestions for future research.

## Discussion of the Study

The four participants in this study shared, through writings and interviews, their struggles to, first, define mathematics and its purpose in society as well as in school, and second, to reconcile[10] their views of mathematics with their instructional practices. One

[10] I use the term reconcile here to mean "to bring into agreement or harmony; make compatible or consistent" (*Webster's*, 2001, p. 1612); thus emphasizing a balancing or harmonizing of what may have been opposing or conflicting views.

question served as a continued focus throughout this study: What is mathematics? Connecting to that question were issues of teaching and learning mathematics, for the participants of this study acknowledged the inseparable relationship between their views of mathematics and the teaching and learning of mathematics. That is to say, these four educators did not come to the mathematics classroom as newcomers (Britzman, 2007). They brought with them their personal mathematical and educational histories—their relationship with mathematics as student and as child, their beliefs about schooling and the place of mathematics in schools, and their ideas of what it means to teach and learn mathematics.

The teachers participating in this study acknowledged that, prior to our summer graduate course, they had not given much thought to their philosophy of mathematics. But this study presumes the necessity of a philosophy of mathematics for, as Dossey (1992) wrote:

> The conception of mathematics held by the teacher may have a great deal to do with the way in which mathematics is characterized in classroom teaching. The subtle messages communicated to children about mathematics and its nature may, in turn, affect the way they grow to view mathematics and its role in their world. (Dossey, 1992, p. 42)

It was, therefore, the goal of this study to engage teachers in a discussion of philosophy and an exploration of a new view of mathematics, a humanist/fallibilist view that might have differed from their more traditional views of mathematics:

> The development and acceptance of a philosophy of mathematics carries with it challenges for mathematics and mathematics education. A philosophy should call for experiences that help mathematician, teacher, and student to experience the invention of mathematics. It should call for experiences that allow for the mathematization, or modeling, of ideas and events. Developing a new philosophy of mathematics requires discussion and communication of alternative views of mathematics to determine a valid and workable characterization of the discipline. (Dossey, 1992, p. 42)

The participants in this study did begin, for the most part, with a stated view of mathematics that was quite traditional: a right/wrong, black/white, a priori view of mathematics. And they each reinforced, time and again, that their views of mathematics could not be separated from their views of mathematics education. Yet, as they continued to read and discuss philosophy, they came to see that their traditional philosophies of mathematics did not align with their views of mathematics in the classroom or their personal goals for mathematics education. For they each had embraced (to varying degrees) a constructivist pedagogy within their classroom instruction, that is, constructing one's own understanding, making sense of the mathematics in one's own way, empowering students through the struggles and successes of problem solving. Each participant saw the "doing" of mathematics as the purpose of mathematics teaching and learning. In other words, they valued mathematics for its problem-solving nature, echoing Polya's (1945/1973) emphasis on the heuristics of mathematics. And so they began to redefine their philosophies of mathematics, to trouble the previously unquestioned meta-narratives of mathematics, to demystify the mathematics. Mathematics began to loose its abstract perfection, its certain and ethereal nature.

But even as they began to change their philosophies of mathematics, to view it as a fluid subject, a human construct, they also came to see those philosophies as fluid, ever-changing, a process more than a product. Michael and Diane, especially, struggled to find the words that defined their elusive philosophies, recognizing, as Michael shared with me, that the philosophy was not fragmented or limited, only the words being used to describe it. In this way, they agreed with Deleuze and Guattari (1991/1994) on the dynamic nature of philosophy:

> Philosophy is becoming, not history; it is the coexistence of planes, not the succession of systems. . . . Philosophy thus lives in a permanent crisis. The plane takes effect through shocks, concepts proceed in bursts, and personae by spasms. . . . Philosophy does not exist in knowing and is not inspired by truth. (p. 82)

What became clear in this study was that each participant's views of mathematics were a result of her or his own personal story of mathematics, her or his relationship with mathematics as a student, as a teacher, as a child of an educator, as a learner, and as a mentor. Although the participants could not identify an early philosophy of mathematics, they could easily remember and relay their early experiences and feelings about mathematics. Katie spoke of her struggles to succeed in mathematics classes and her feelings of pride as she overcame those struggles. Julia spoke of her love of mathematics early on, pursuing problems beyond her own assignments because of the sense of power that mathematics afforded her. Diane saw mathematics open up to her as she learned to understand the concepts behind the procedures she had always just memorized. And each of these teachers wanted to bring to their students the positive aspects that mathematics had brought to them. Michael viewed mathematics as his platform to reach out to students and show them they each had value. Julia wanted her students, and her teachers, to experience the power of solving complex mathematical problems in their own way. Diane helped her students to talk about mathematics, exploring different ideas and sharing those ideas with each other. Mathematics was not just a school subject for any of these teachers. It was, in many ways, a microcosm for life lessons they wanted their students to learn: hard work will reward you with success (Katie); you are important and don't let anyone convince you otherwise (Michael); don't let anyone stop you from doing

things your own way (Julia); construct your own knowledge through exploration, discovery, and communicating (Diane).

The stories that each of these teachers shared with me emphasized their own unique views of mathematics, mathematics education, and education in general. The teachers often struggled to transform their views of mathematics into their day-to-day instructional practices. But they each defined those struggles as part of a process, a continuous journey toward change. In this, they agreed with researchers who have emphasized the importance of reflexivity in bringing about teacher change (see, e.g., Cooney, et al., 1998; Hart, 2002a, 2002b). The four educators in this study recognized the continuous cycle of experimentation, reflection, and adaptation required to bring about change in instructional practices:

> If we characterize reform-oriented teaching as that teaching which attends to context, including basing instruction on what students know, then teaching becomes a matter of being adaptive (Cooney, 1994) rather than a matter of using a particular sequence of instructional strategies. The development of a reform-oriented teacher so characterized, is rooted in the ability of the individual to doubt, to reflect, and to reconstruct. (Wilson & Cooney, 2002, p. 132)

In addition, these teachers no longer viewed mathematics as culture-free. Whether viewed as a subject that perpetuated class inequities, as Michael did, or viewed as a subject that needed a cultural context if one's students were to become interested, as Julia and Diane did, these teachers knew that students did not enter a mathematics classroom as blank slates. They would agree with Brown (1994) that we bring our entire histories to our doing of mathematics: "If I am presented with a new piece of mathematics I bring to it a whole history of myself. Any construction I make in respect of this new task cannot be independent of this history" (p. 156). What these teachers then struggled with was the

enormity of bringing mathematics to a heterogeneous classroom, a room of varied cultures, varied understandings, varied beliefs, and varied feelings about the mathematics. The commonality they did bring to their students was their own personal love of mathematics, their belief in its beauty as well as its power. In many ways, they encouraged their own students, and the teachers they mentored, to see mathematics differently, to demystify the mathematics that they taught by seeing it as a human construct:

> If mathematics is conceived as inseparable from human contexts and practices, then social implications for mathematics education follow, enabling notions of accessibility, equity, and social accountability to be applied to the discipline of mathematics. The outcome is a demystification of mathematics, to the benefit of the discipline and mathematicians and also to students, teachers, and other users of mathematics in society. (Ernest, 1998c, p. 26)

## Implications of the Study

The teaching and learning of mathematics is a politically charged arena. Strong feelings exist in the debate on how “best” to teach mathematics in K–12 schools, feelings that are linked to varying perceptions about the nature of mathematics (Dossey, 1992). Is mathematics “an ideal, well-defined body of knowledge, faithfully mirroring certain mind-independent reality of abstract ideas” (Sfard, 1998, p. 491) or is it a human-construct, fallible and ever-changing? These perceptions of mathematics then drive beliefs about the appropriate instructional practices in mathematics. Is mathematics a body of knowledge that must be memorized and unquestionably mastered, or do we engage the learners of mathematics in personal sense making, in constructing their own mathematical knowledge? That we are in the midst of “math wars” is indisputable

(Schoenfeld, 2004). What it means to teach mathematics and the very nature of mathematics is at the center of these wars:

> Traditionalists or back-to-basics proponents argue that the aim of mathematics education should be mastery of a set body of mathematical knowledge and skills. The philosophical complement to this version of the teaching and learning of mathematics is mathematical absolutism. Reform-oriented mathematics educators, on the other had, tend to see understanding as a primary aim of school mathematics. Constructivism is often the philosophical foundation for those espousing this version of mathematics education. (Stemhagen, 2008, p. 63)

I agree with Schoenfeld (2004), Greer and Mukhopadhyay (2003), and others (e.g., Davison & Mitchell, 2008) that the math wars are based on philosophical differences. It has therefore been my intent to inject philosophy into the discussion of mathematics educational reform and research. This study then has implication in several areas: teacher preparation and teacher change; educational policy, particularly relating to mathematics curriculum reform; and research in mathematics education. I will address each of these areas briefly.

The literature review chapter of this study addressed, at length, the ongoing research on teachers' beliefs and teacher change. Much has been written regarding the need to engage teachers, both preservice and inservice, in constructivist learning in order to change their instructional practices (see, e.g., Hart, 2002a, 2002b; Mewborn, 2003; Thompson, 1992). Yet little has been done to engage teachers in a philosophical discussion of mathematics: "Teachers, as well, should be encouraged to develop professionally through philosophical discourse with their peers" (Davison & Mitchell, 2008, p. 151). Philosophy and mathematics have a long-standing connection, going back to the ancient Greeks and others (Davis & Hersh, 1981). Yet seldom are mathematics teachers asked to explore philosophy beyond an introductory Philosophy of Education

course. If one is going to teach mathematics day in and day out, should one not at some point ask, why? What is the purpose of teaching mathematics in our public schools? And what is the purpose of mathematics in society at large? And should not mathematics' *purpose* be tied to *how* we then teach it? These questions come back to teachers' perception of mathematics, and more specifically, their philosophies of mathematics.[11]

That then has been the focus of this study—to accompany and even guide teachers on an exploration of the philosophy of mathematics. Through readings and discussions, the participants of this study were encouraged to examine philosophical questions of the nature of mathematics and gained the language through which to define their own philosophies of mathematics. And these four teachers came to see that exploration as a journey, one similar to their journeys as educators. I believe they would agree with Edwards and Usher's (2001) account of learning: "In a postmodern condition, we (en)counter the issue that the growth of knowledge expands the field of ignorance, and with each step toward the horizon new unknown landscapes appear. Lifelong learning as travel and, no doubt, travail" (p. 285). This journey took place as each participant was engaged in the implementation of a new mathematics curriculum, a task-based curriculum that emphasized mathematical discourse, problem solving, and multiple pathways and solutions. Hence, the philosophical question, what is mathematics, became a guide for these educators as they struggled to support that implementation. The participants began to reject their previously held philosophies of the certainty of mathematics, the other-worldliness of the subject, and began to see mathematics as a human construction, fallible and amenable. Britzman (2007) called on teacher education

[11] I draw the reader's attention back to the definition of *philosophy* employed in chapter 1: "the critical study of the basic principles and concepts of a particular branch of knowledge, especially with a view to improving or reconstituting them" (Webster's, 2003, p. 1455).

to recognize and embrace the place of *uncertainty* in the human condition. This study calls for mathematics educators to embrace uncertainty not only in education but in mathematics as well.

Preservice and inservice teachers should engage in an exploration of the philosophy of mathematics and examine how those philosophies intersect with their philosophy of teaching and learning. By exploring their own personal mathematical histories and putting words to their personal philosophies of mathematics, preservice teachers may well find that their perceptions of mathematics and mathematics education do not align with the expectations of teaching mathematics in the constructivist-based, reform classroom. The summer course described in this study can serve as a guide for mathematics teacher educators who want to engage both their preservice and inservice teachers in philosophical explorations of mathematics.

Philosophy may well be at the root of the math wars (Schoenfeld, 2004), yet too often philosophy, particularly philosophy of mathematics, has been left out of the discussion. Education policy makers need to acknowledge the importance of philosophy in the current mathematics curriculum reform. The National Council of Teachers of Mathematics has avoided a direct conversation about philosophies of mathematics, yet this study demonstrates that teachers often cling to philosophies of mathematics that are at odds with the philosophical implications of the curriculum they are asked to implement. Philosophy, like religion and politics, is a discussion ignored. But the teachers in this study were not philosophy-free prior to their participation in this research. What they possessed, instead, were unexamined philosophies of mathematics. And when those (unexamined) philosophies clash with intended curriculum changes, is curriculum reform

possible? I do not argue that exploring philosophy in the field of mathematics and mathematics education offers a panacea. There is no magic bullet for mathematics curriculum reform. But ignoring philosophy as we seek to change how we view the teaching and learning of mathematics does not serve the community of teachers and learners of mathematics well.

I contend that discussions of philosophy, particularly philosophy of mathematics, should be brought to the forefront of mathematics education reform. Teachers will continue to resist change, to teach the way they were taught, if they are never asked to explore the philosophical basis of their perceptions of mathematics. This study demonstrates that teachers can engage in a philosophical investigation of mathematics through readings and discussions of historical and current writings in the philosophy of mathematics. Although not all teachers may wish to participate in the reading-intensive, graduate-level study that was the basis of this research, teachers involved in curriculum reform must be made aware that mathematics is not a static subject, that its transcendental nature has been questioned, and that seeing mathematics as a human construct may allow us to integrate constructivist instruction more easily into the mathematics classroom. Too often, in my work with mathematics teachers, I see their (unexplored) philosophies of mathematics impeding their ability to change their instructional practices. Therefore the examination of philosophy of mathematics needs to become a part of the reform process, and this study supports the capacity of teachers to engage in such an examination.

The growing philosophical investigations of mathematics (see, e.g., Davis & Hersh, 1981; Hersh, 1997; Restivo, Van Bendegen, & Fischer, 1993; Tymoczko, 1998) in

the past 30 years have not often been addressed in mathematics education research. This study sought to merge humanist/fallibilist philosophical writings in mathematics with research on mathematics education. Further research is needed in this area, continuing the bridge between the philosophical and the practical. We seem afraid to raise issues of philosophy as we implement curriculum reform and study teacher change. But philosophy too often lies hidden, an unspoken obstacle in the attempt to change mathematics education (Ernest, 2004). Researchers can continue to bring the hidden obstacle to light, to engage both policymakers and educators in a conversation about philosophy, not with the intent of enforcing the “right” philosophy but with the acknowledgement that, without a continued dialogue about philosophy, the curriculum reform they research may continue to fall short.

## Issues of Power

I began this study as a co-instructor in a graduate course. The course readings were selected purposefully based on my own personal exploration of philosophies of mathematics. Both my advisor (and co-instructor) and I sought out authors who examined mathematics from a non-traditional stance. Whether it be Whitehead and Russell’s (1910/1962) attempt at a formalist structuring of mathematics in *Principia Mathematica*, Lakatos’ (1976) troubling of the mathematical idea of proof in *Proofs and Refutations: The Logic of Mathematical Discovery*, or Davis and Hersh’s (1981) investigation of a humanistic philosophy of mathematics in *The Mathematical Experience*, the intent of the course readings was to engage teachers in a new view of mathematics. Did this role, as instructor and researcher, limit the views of my participants? Did they seek, in our

interviews, to mimic the readings, to echo what they had perhaps come to see as my views, the “preferred” views, the “right” views?

Issues of power cannot be removed from any study. By engaging the participants in conversations over a sustained period of time, 18 months, I hoped to lessen the impact of my own views on the study. It was my intent to form a reciprocal relationship with each of the participants in this study, to encourage them to express their own views, not what they thought I wanted to hear. I worried at times that a participant was searching for the right words, the words that would express their views in a “correct tone.” I tried to deflect that in our conversations, to keep them focused on what they experienced, and how they interpreted those experiences. But even in that, what the reader comes to know in this study is my own interpretation of the stories that each participant interpreted to me.

Issues of power arise, as well, in the nature of the relationship I have built up with each of the participants. In order to focus on their voices, on their views, I have spent months reading essays, discussing, interviewing, emailing, and sharing coffee and family stories with these four exceptional mathematics educators. I have come to like and respect each of them. At times, as we talked, I might have disagreed with ideas that they shared; I might have wanted to debate an instructional technique they described. My professional role, my day-to-day work, is that of a professional developer, one who guides teachers to change their instructional practices, who works within schools and school systems to affect changes in mathematics curriculum. Sometimes, as I talked to my participants, I found myself slipping from my role as researcher into my role as consultant, as professional developer. But I tried to keep a wall between those two selves; I did not want to debate practices with my participants; it was not my role to change what

went on in their classrooms. Did I influence my participants stories through the blurring of my professional and researcher roles? My own journaling, reflecting on the interviews, troubling where I felt I had "crossed the line," helped to keep me honest, to keep my focus on the researcher role, to keep me focused on the words on the participants, not the thoughts and judgments in my head. But I acknowledge that a blurring of roles might have occurred, that I am never separate people, researcher one hour and professional developer the next.

At the same time, I recognize the lack of power that teachers face in today's educational environment (Apple, 1986, 2000). The voices of teachers are often silenced in the debate to "better" America's public schools. Discussions of "good" teaching versus "bad" teaching dominate the media and political focus on education. I do not wish to add to that discourse. The teachers who participated in this study shared a commitment to their students and to mathematics education. My recognition of teachers' lack of a voice in today's educational landscape might have, at times, kept my own thoughts silenced. I strove to avoid criticism in my analysis of these teachers' stories due to my personal respect and admiration for my participants. Did that avoidance damage my study? I hope it did not but, from an ethical standpoint, I cannot add to the political climate surrounding education that seeks to portray some teachers as "low quality" and others as "high quality." I believe that research on teachers' philosophies, beliefs, and instructional practices must recognize the great effort most teachers put into their work with students and build from there.

I share with the reader the above concerns, not to diminish the impact of this study, but to be transparent regarding what I have brought to the research. I am not the

fly-on-the-wall, my role has been an active one. I continue the discussion of my own role in this study in the next section.

## Revisiting the Methodology

The use of narrative analysis in this study served to highlight the stories of these four mathematics educators. Teachers and others make sense of their personal experiences through the use of stories or narratives (Clandinin & Connelly, 1998). Hence, the narratives that teachers share open a window of understanding into their educational practices: “Narrative research in which teachers’ voices are heard in their stories of experience offers an opportunity to present the complexity of teaching to the public” (Moen, 2006, p. 10). My use of narrative analysis sought to keep those voices intact while, at the same time, provided my ideas, my interpretations of the reflective journeys of these educators. Yet my own role in this study is not one of objective observer. I have brought my own perceptions and beliefs, my own “baggage” (Scheurich, 1997) to the analysis of these teachers’ stories:

> Although we may come to know the knowledge of others by interpreting their language and actions through our own conceptual constructs, we must acknowledge that the others have realities that are independent of ours. Indeed, these realities of others along with our own realities are what we strive to understand in qualitative research, but we may never take these realities as fixed. (Ernest, 1998c, p. 30)

I share, therefore, through this study, my interpretations of teachers’ search for a personal philosophy of mathematics, one that is in keeping with their beliefs about mathematics as well as the teaching and learning of mathematics. As Scheurich (1997) asserted, there is an openness, an ambiguity that lies in the relationship between interviewer and interviewee, researcher and participant, an openness I have sought to fill with my analysis of the participants’ stories. But that analysis is not meant as a Truth. I

have shared my baggage, my biases and beliefs with the reader. Perhaps what the reader comes to know, after a review of this study, is my own interpretations, my own perceptions about what is important in mathematics and mathematics education:

> Like life, qualitative inquiry is fiction, in the sense that it is made or constructed, but not in the sense that it is pure invention, lies, or imaginings. In other words, qualitative inquiry has a grounding in "real" events and "real" lives, but learning about and representing events and lives is a process of constructing others' constructions of the constructions of the world. (Talburt, 2004, p. 81)

I have offered the reader, then, my construction of others' constructions of the constructions of mathematics and mathematics education. But it is a journey I feel I have taken alongside my participants. I have struggled through my own defining of philosophy just as they have struggled through theirs. And I have sought to understand the processes that teachers go through as they explore the philosophy of mathematics and as they implement a changed view of mathematics education in their classrooms, with the hope that sharing that process with others will open new avenues for research and practice in mathematics education. It is my hope that future researchers continue to explore teachers' philosophies of mathematics. I see a need, in particular, for research that engages preservice teachers in a similar course of readings in the humanist/fallibilist philosophies of mathematics as this study, and investigates the process those "becoming" teachers then go through as they transition to the mathematics classroom. In addition, a follow-up study is needed that continues to explore teachers' philosophical journeys. The teachers in my study were, for the most part, in the initial year of implementing a new and challenging curriculum; they were very much in the midst. To return to teachers, several years into a reform process, is necessary to better understand the processes these teachers go through.

## Closing Remarks

What do we truly come to know as we conclude a study? What answers can I now offer the reader? Can I state without question that an exploration of philosophy made these participants better teachers? Can I assert the importance of philosophical discussions for teachers attempting to change their instructional practices? Perhaps I can just offer the reader more ideas, more thoughts about the complexity of education, about the struggles good teachers go through to reach students, to connect to them day after day, and to get those students to connect to mathematics in a meaningful way. Perhaps as researchers and educators, we must come to feel comfortable with knowing what we do not know, that there is no magic potion in education, no silver bullet, just hard work and struggle, reflectivity and caring. Each study may move us closer to an understanding of just what it means to teach and to learn, but it also leaves us with a sense of what we still do not know:

> We cannot look into people's heads, but let us think about the conditions that education researchers find themselves in and ask whether knowing that one does not know is the sort of knowledge that is valued and encouraged. (Hostetler, 2005, p. 21)

Like my participants, I may end this study knowing only that I have more questions or, as Michael pointed out, "better questions." Curiously, what I struggled with most, throughout this study, was formulating my research questions, maybe because questions imply answers. And in this, I agree with Hostetler's (2005) concerns about good research:

> Of course, in some sense, all research starts with a question, awareness that one does not know something. The problem is that research tends to end with an answer. Hello? Of course, I am not saying researchers should not try to answer questions. The problem is ending with answers—being unaware of or uninterested in the ethical questions generated or avoided. The "answers" to research questions

> do not end things but offer new circumstances for exploring the persistent question of what is good for people. (p. 21)

And so what my research offers may not be answers but a continued dialogue about education, a dialogue focused on two very important yet often unasked questions: What is mathematics? What does it mean to teach and to learn mathematics? “What if higher education understood more of its inquiry as part of a conversation that cannot conclude with certainty?” (Talburt, 2004, p. 84). And so I conclude not with certainty but with the hope that I have carried on the conversation about the mathematics education in a meaningful way, that I have caused the reader to think about her own philosophy of mathematics, the educator to ponder the implications of the philosophy of mathematics in the classroom, and the researcher to plan a further investigation of the processes that occur when one asks: What is mathematics?

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APPENDIXES

# APPENDIX A

## COURSE SYLLABUS
The Study of Learning and Instruction in Mathematics:
What Is Mathematics, Really?[12]

EDMT 8290 – Summer 2007
Georgia State University
Department of Middle-Secondary Education and Instructional Technology

**Course Description:** This **READING INTENSIVE** seminar will explore various philosophical traditions of mathematics. Are you a Platonist, formalist, intuitionist, constructivist, humanist, or some eclectic combination of these traditions? Students, through reading some of the "classic" texts of Western mathematics, will explore their own philosophical foundations about mathematics. The seminar promises *no answers*, just a space for critical, intellectual, and open Freirian dialogue around the question: What is mathematics, really?

**Pre-requisite:** None

**Credit:** 3-hour graduate seminar

**Instructors:**

David W. Stinson, Ph.D.
dstinson@gsu.edu
617 College of Education (COE)
Office: (404) 413-8409

Kimberly White-Fredette, M.A., Ed.S.
(Ph.D. student)
kwhitefredettte1@student.gsu.edu

**Class Time/Location:**
Monday/Wednesday 4:00 PM–7:10 PM
COE 657
**Office Hours:**
Wednesday 2:30 p.m.–4:30 p.m.; by appointment
**College of Education Mission:**
To provide leadership and scholarship for the betterment of education and human development (Strategic Plan 2002–2007).

[12] Hersh, R. (1997). *What is mathematics, really?* New York: Oxford University Press.

**College of Education Purpose:**
The Professional Education Faculty is committed to planning, implementing, and assessing programs that prepare educational professionals focused on pupil learning and development.
**Programs:**
EDMT 8290 is an elective in the Doctor of Philosophy (Ph.D.) and the Education Specialist (Ed.S.) in Teaching and Learning with a concentration in mathematics education (repeatable).
**Assumptions Guiding These Programs:**
1. Learning and teaching must continually adapt to changes in society and the expanding knowledge base.
2. Learning is an active process.
3. Quality teaching takes into account individual differences, learning styles, and backgrounds.
4. Learning environments are based on the mutual respect of all participants.
5. A variety of teaching strategies and assessments are used to meet the needs of individual learners.
6. An integrated knowledge base consisting of content, skills, attitudes, technologies, and theories is developed and demonstrated in field-based applications.

**Knowledge Base** (Required text)**:**
Davis, P. J., & Hersh, R. (1995). *The mathematical experience*. Boston: Houghton Mifflin Company. (Original work published 1981)

Hersh, R. (1997). *What is mathematics, really?* New York: Oxford University Press.

Lakatos, I. (1999). *Proofs and Refutations: The logic of mathematical discovery*. Cambridge, UK: Cambridge University Press. (Original work published 1976)

Russell, B. (1993). *Introduction to mathematical philosophy*. New York: Dover. (Original work published 1919)

Tymoczko, T. (Ed.). (1998). *New directions in the philosophy of mathematics: An anthology* (Rev. and expanded ed.). Princeton, NJ: Princeton University Press.

Georgia Department of Education. www.doe.k12.ga.us

National Council of Teachers of Mathematics www.nctm.org

(Recommended text):
Ernest, P. (1998). *Social constructivism as a philosophy of mathematics*. Albany: State University of New York Press.

**Teaching Strategies:**
The instructor will utilizes lecture, small group discussion, whole group discussion, cooperative learning, jig sawing, WebCT, various technologies, and guest speakers to

facilitate the learning experience. Students will gain greater insight of these techniques for use in their own classrooms.

*According to Freire (1970/2000) in a problem-posing pedagogy…*

> the teacher is no longer merely the-one-who-teaches, but one who is himself taught in dialogue with the students, who in turn while being taught also teach. They become jointly responsible for the process in which all grow. …Here, no one teaches another, nor is anyone self-taught. People teach each other. (p. 80)
>
> Freire (1969/2000) explicitly defined dialogue, insisting that it must be a "horizontal relationship between persons…[a] relation of 'empathy' between two 'poles' who are engaged in a joint search" (p. 45). Later, Freire (1970/2000) elaborated on the concept of dialogue as he provided the elements that must be present for dialogue to exist, without expanding on the details, they are: love, humility, faith, trust, hope, and critical thinking. It is this Freirian definition of dialogue that I anticipate will be present during our class sessions.

**Academic Freedom:**

*According to Foucault (1984/1996)…*

> The role of an intellectual is not to tell others what they must do. By what right would he do so?... The work of an intellectual is not to mold the political will of others; it is, through the analysis that he does in his own field, to re-examine evidence and assumptions, to shake up habitual ways of working and thinking, to dissipate conventional familiarities, to re-evaluate rules and institutions and starting from this re-problematization (where he occupies his specific profession as an intellectual) to participate in the formation of a political will (where he has his role as a citizen to play). (pp. 462–463)

**Learning Objectives and Outcomes:**

*Learning objectives:*

The students will

- explore the philosophical (i.e., ontological, epistemological, and ethical) underpinnings of mathematics;
- explore the main tenets of mathematics education research (i.e., problem statement, research question, methodology, and representation) within their philosophy of mathematics;
- explore their pedagogical practices within their philosophy of mathematics; and
- explore the NCTM *Principles and Standards for School Mathematics* (2000) and Georgia Performance Standards (GPS) within their philosophy of mathematics.

*Learning outcomes:*

Future mathematics teacher-educators and education researchers will be increasingly

- aware of diversity of philosophical perspectives found in mathematics and mathematics education, developing an appreciation for various fields of study, such as anthropology, sociology, and philosophy, and understand what these fields offer to the analysis of mathematics, mathematics teaching and learning, and research in mathematics education;
- aware of cultural, economic, political, and social factors that influence mathematics, mathematics teaching and learning, and research in mathematics education;
- aware of aspects of mathematics teaching and learning and research in mathematics education that reach beyond mathematics content and the psychologized study of mathematics teaching and learning;
- aware of philosophical foundations and diversity of theoretical frameworks for research and methodological procedures available to mathematics education researchers;
- aware of merits and weaknesses of various theoretical frameworks and methodological procedures; and
- aware of impact that varying philosophical positions have on research design, implementation, and (re)production of knowledge.

**Expectations and Requirements:**
This is an advanced graduate level seminar and you will be expected to **READ INTENSIVELY**. There are no substitutes for, nor shortcuts to, in-depth reading. This preparation is essential for your own learning, for the quality of your own research, and for the benefit of others in class.

**Some Comments on Reading:**
*According to Stinson (2004)…*

> Each time I read and reread a book, essay, or interview by Dewey, Freire, or Foucault (and others) I am impelled into critical reflection—rethinking my rethinking. And ever since Foucault entered into the picture, I attempt to think the unthought. St. Pierre suggested that we get smarter as we read, and as we reread we will always find something different because we have changed since the last reading (E. A. St. Pierre, personal communication, fall 2002). But then again, as I read and reread text, I no longer have "dreams of deciphering a truth or an origin"; but rather, I think about how the text offers a different way of seeing, trying "to pass beyond man and humanism" (Derrida, 1978, p. 292). (p. xx)

*According to St. Pierre (2003)…*

> Reading in unfamiliar discourses is required, and students are encouraged to heed French poststructural philosopher Jacques Lacan's advice, "to read does not obligate one to understand. First it is necessary to read…avoid understanding too quickly" (as cited in Ulmer Gregory, 1985, p. 196). One might also heed Roland Barthes's (1974) advice on rereading:
>
> > Rereading, an operation contrary to the commercial and ideological habits of our society, which would have us "throw away" the story once it has been consumed (or "devoured"), so that we can then move on to another

> story, buy another book, and which is tolerated only in certain marginal categories of readers (children, old people and professors), rereading is here suggested at the outset, for it alone saves the text from repetition (those who fail to reread are obliged to read the same story everywhere)…Reading is no longer consumption, but play. (pp. 15–16)

**Course Requirements:**
*Class participation (40 pts):*
10 pts for overall class participation in the form of Freirian defined dialogue
20 pts for summary of readings (see example)
10 pts for scribing (see example)

*Reading journal (30 pts):*
Begin an EndNote reading journal (see example). This journal should use *Publication Manual of the American Psychological Association*-5$^{th}$ edition (APA) citation and writing format criteria; include a summary and/or abstract of the article, significant quotations, and comments regarding your struggles with the readings and/or how the readings will assist you in your teaching/research. (A reduction of one letter grade will result in not following APA format. In other words, you must become **very** familiar with APA format; the format preferred by most education journals.)

*Final paper (30 pts: 5 pts for initial reflections and 25 pts for final paper):*

**Initial Reflections:** Write a reflective statement (3–5 pages) regarding *What is Mathematics, Really*? In other words, as you begin the course how do you philosophical position mathematics, and how does this positioning impact your pedagogical philosophies and practices. (This paper is a statement of you beliefs, not so much a scholarly argument of those beliefs; that scholarly argument will be in your final paper, see below). **Due date is June 18, 2007**.

**Final Paper:** Write a focused, reflective, **academic** paper that outlines your philosophy of mathematics and positions your pedagogical practices and research within that philosophy. Include a discussion of where your philosophy of mathematics was/is, and the changes (if any) that the course readings and discussions have motivated, and how your pedagogical practices and research agendas have been transformed (or not). The paper, 8–10 <u>text</u> pages in length, should follow APA format style (see note above regarding APA). The paper is your opportunity to illustrate your learning from the class; therefore, it should use citational authority, citing not only the essays read in class, but also other scholarly works. **Due date is July 25, 2007**.

| **Assignments** | **Points Possible** | **Points Earned** |
|---|---|---|
| Class participation | 40 | |
| Reading journal | 30 | |
| Final paper | 30 | |
| **Total** | 100 | |

**Schedule** (planned, deviations may be necessary)**:**

| Dates | Agenda | Assignment |
|---|---|---|
| **June 11**<br>**Monday** | • Instructor and student introductions<br>• Review of syllabus | READ:<br>1. Davis & Hersh (1981): Introduction; Overture; chapter 1<br>2. Davis & Hersh (1981): chapter 2<br>3. Davis & Hersh (1981): chapter 3 |
| **June 13**<br>**Wednesday** | • Student summary of readings<br>• Freirian dialog of readings | READ:<br>1. Davis & Hersh (1981): chapter 4<br>2. Davis & Hersh (1981): chapter 5<br>3. Davis & Hersh (1981): chapter 6 |
| **June 18**<br>**Monday** | • Student summary of readings<br>• Freirian dialog of readings | READ:<br>1. Davis & Hersh (1981): chapter 7<br>2. Davis & Hersh (1981): chapter 8 |
| **June 20**<br>**Wednesday** | • Student summary of readings<br>• Freirian dialog of readings | READ:<br>1. Russell (1919): v–28<br>2. Russell (1919): 29–116 |
| **June 25**<br>**Monday** | • Student summary of readings<br>• Freirian dialog of readings | READ:<br>1. Russell (1919): 117–166<br>2. Russell (1919): 167–206 |
| **June 27**<br>**Wednesday** | • Student summary of readings<br>• Freirian dialog of readings | READ:<br>1. Lakatos (1976): ix– 42<br>2. Lakatos (1976): 42–106 |
| **July 2**<br>**Monday** | • Student summary of readings<br>• Freirian dialog of readings | READ:<br>1. Lakatos (1976): 106–126<br>2. Lakatos (1976): 127–154 |
| **July 4**<br>**Wednesday** | **NO CLASS** | |
| **July 9**<br>**Monday** | • Student summary of readings | READ<br>1. Tymoczko: Polya 99–124 |

| | | |
|---|---|---|
| | Freirian dialog of readings | 2. Tymoczko: Grabiner: 201–214<br>3. Tymoczko: Kitcher: 215–242 |
| **July 11 Wednesday** | • Student summary of readings<br>• Freirian dialog of readings | READ:<br>1. Tymoczko: Tymoczo: 243–266<br>2. Tymoczko: Chaitin: 287–311<br>3. Tymoczko: Thom: 67–78 |
| **July 16 Monday** | • Student summary of readings<br>• Freirian dialog of readings | READ:<br>1. Hersh (1997): xi–xxiv; chapters 1, 2<br>2. Hersh (1997): chapters 3, 4, 5<br>3. Hersh (1997): chapters 6, 7, 8 |
| **July 18 Wednesday** | • Student summary of readings<br>• Freirian dialog of readings | READ:<br>1. Hersh (1997): chapters 9, 10, 11<br>2. Hersh (1997): chapters 12, 13<br><br>**Reading Journal due!** |
| **July 23 Monday** | • Student summary of readings<br>• Freirian dialog of readings | |
| **July 25 Wednesday Final Exam** | **NO CLASS**<br>Social at Stinson's home? | **Philosophy of Mathematics Due! Hardcopy due in my mailbox by 5:00 PM!** |

**Policies:**

1. Students are expected to read, reflect, and participate in each class. If a student must miss a class or portion of one, she or he is expected to check with classmates first before instructor to determine what was missed.
2. Students are expected to read and observe the GSU policy on academic honesty, cheating, and plagiarism; attendance; and conduct (see current *Graduate Catalog*).
3. The syllabus provides a general plan for the course; deviations may be necessary.
4. Tentative rubrics for evaluating assignments are given. Students are invited to comment and discuss rubrics to assure equitable and fair grading.
5. **Assignments will not be accepted late. Assignments are due regardless of attendance to class.**
6. E-Mail Protocol:
    a. Give informative subject headings.
    b. Change subject heading as discussion changes in a series of communications.
    c. If attaching assignments, include your name, assignment title, and page numbers on each attachment.

d. When responding to a message, include the message in your response.
e. Emails will be responded to with 72 hours.

**Reference:**

Foucault, M. (1996). The concern for truth. In S. Lotringer (Ed.), *Foucault live: Interviews, 1961–1984* (pp. 455–464). New York: Semiotext(e). (Original work published 1984)

Freire, P. (2000). *Education for critical consciousness* (Continuum ed.). New York: Continuum. (Original work published 1969)

Freire, P. (2000). *Pedagogy of the oppressed* (M. B. Ramos, Trans., 30th anniversary ed.). New York: Continuum. (Original work published 1970)

St. Pierre, E. A. (2003). *ELAN 8045 The postmodern turn: Theories and methods*. Course syllabus, The University of Georgia, Athens.

Stinson, D. W. (2004). African American male students and achievement in school mathematics: A critical postmodern analysis of agency. *Dissertations Abstracts International*, *66* (12). (UMI No. 3194548)

**Grading Scale (will be exactingly followed):**

A (100–94) A- (93–90) B+ (89–88) B (87–84) B- (83–80) C+ (79–78) C (77–74)
C- (73–70) D (69–60) F (60 and below)

APPENDIX B

INFORMED CONSENT

Georgia State University
College of Education
Department of Middle and Secondary Education, and Informational Technology

**Informed Consent**

Title: Examining Philosophy in a Time of Reform: A Study of Teachers' Philosophies of Mathematics and Mathematics Education

Investigators: David Stinson, Principal Investigator
Kimberly White-Fredette, Co-Principal Investigator

I. Purpose:

You are invited to participate in a research study. The purpose of the study is to investigate teachers' philosophies of mathematics and mathematics education. You are invited to participate because you are a secondary school mathematics teacher and a graduate student in mathematics education. A total of three participants will be recruited for this study. Participation will require approximately three hours of your time over seven months (June through December, 2007).

II. Procedures:

Although all students enrolled in the summer course EDMT 8290 are being asked to sign this Informed Consent, only three will be chosen to participate in the study, based on current teaching position and the initial course assignment. The Principle Investigator and Co-Principle Investigator will invite three participants to take part in the study. If you decide to participate, you will take part in three one-to-one interviews with the research investigator. The interviews will be audio-taped and transcribed. Each interview will last about 60 minutes. Interviews will take place on the campus of Georgia State University. In addition, your written coursework, including your reading journals and final paper, will be used as data for this study.

III. Risks:

In this study, you will not have any more risks than you would in a normal day of

life.

IV. Benefits:

Participation in this study may or may not benefit you personally. You will be given the opportunity to talk freely about your educational experiences and philosophies. Overall, we expect to gain information about teachers' philosophies of mathematics and mathematics education to benefit the mathematics educational community as curricular reforms are implemented.

V. Voluntary Participation and Withdrawal:

Participation in research is voluntary. You have the right not to be in this study. If you decide to be in the study and change your mind, you have the right to drop out at any time. You may skip questions or stop participating at any time. Whatever you decide, you will not lose any benefits to which you are otherwise entitled.

VI. Confidentiality:

We will keep your records private to the extent allowed by law. We will use pseudonyms rather than your name on study records. Only the Principal Investigator and Co-Investigator will have access to the information you provide. Interviews will be digitally recorded and transcribed. Copies of written coursework will be stored in a locked file cabinet; digital recordings and transcriptions will be located on a password and firewall protected computer. Your name and other facts that might point to you will not appear when we present this study or publish its results. The findings will be summarized and reported in group form. You will not be identified personally.

VII. Contact Persons:

Contact David Stinson (mstdws@langate.gsu.edu) and/or Kimberly White-Fredette (kimwf@bellsouth.net / 770-507-4537) if you have questions about this study. If you have questions or concerns about your rights as a participant in this research study, you may contact Susan Vogtner in the Office of Research Integrity at 404-463-0674 or svogtner1@gsu.edu.

VIII. Copy of Consent Form to Subject:

We will give you a copy of this consent form to keep.
If you agree to be in this research study and be audio-taped, please sign below.

Participant Date

Principal Investigator or Researcher Obtaining Consent Date

## APPENDIX C

## INITIAL INTERVIEW PROTOCOL

Interview Questions for Initial Interview (follow-up interview questions will be developed based on responses during initial interview):

Why did you become a teacher?

Why did you become a mathematics teachers?

Tell me about your experiences as a mathematics student.

How do those experiences influence your role as a teacher of mathematics?

Tell me about your views of the new Georgia Performance Standards in mathematics.

How do you think the new curriculum will change the way you teach mathematics?

Describe a typical class period.

Describe your ideal teaching environment.

How do you think the way we teach mathematics affects our students' ability to learn mathematics?

How is mathematics changing as new methods and information and technologies emerge?

What are the aims of mathematics instruction?

How does mathematics instruction contribute to the overall goals of society and education?

How is mathematics viewed and perceived in society?

Does learning mathematics impact on the whole person in a positive or negative manner?

What means are adopted to achieve the aims of mathematics education? Are the ends and the means consistent?