Three Essays in Public Finance

Shiyuan Chen
PERMISSION TO BORROW

In presenting this dissertation as a partial fulfillment of the requirements for an advanced degree from Georgia State University, I agree that the Library of the University shall make it available for inspection and circulation in accordance with its regulations governing materials of this type. I agree that permission to quote from, to copy from or to publish this dissertation may be granted by the author or, in his or her absence, the professor under whose direction it was written or, in his absence, by the Dean of the Andrew Young School of Policy Studies. Such quoting, copying, or publishing must be solely for scholarly purposes and must not involve potential financial gain. It is understood that any copying from or publication of this dissertation which involves potential gain will not be allowed without written permission of the author.

____________________
Signature of the Author
NOTICE TO BORROWS

All dissertations deposited in the Georgia State University Library must be used only in accordance with the stipulations prescribed by the author in the preceding statement.

The author of this dissertation is:
Shiyuan Chen
1322 Briarwood Road NE, APT I-12
Atlanta, GA, 30319

The director of this dissertation is:
Dr. Sally Wallace
Andrew Young School of Policy Studies
Georgia State University
P.O.Box 3992
Atlanta, GA, 30302-3992

Users of this dissertation not regularly enrolled as students at Georgia State University are required to attest acceptance of the preceding stipulations by signing below. Libraries borrowing this dissertation for the use of their patrons are required to see that each user records here the information requested.

Name of User Address Date Type of use (Examination only or copying)
THREE ESSAYS IN PUBLIC FINANCE
BY
SHIYUAN CHEN

A Dissertation Submitted in Partial Fulfillment
of the Requirements for the Degree
of
Doctor of Philosophy
in the
Andrew Young School of Policy Studies
of
Georgia State University

GEORGIA STATE UNIVERSITY
2008
ACCEPTANCE

This dissertation was prepared under the direction of the candidate’s Dissertation Committee. It has been approved and accepted by all members of that committee, and it has been accepted in partial fulfillment of the requirements for the degree of Doctor of Philosophy in Economics in the Andrew Young School of Policy Studies of Georgia State University.

Dissertation Chair: Dr. Sally Wallace
Committee:        Dr. Yongsheng Xu
                   Dr. David Sjoquist
                   Dr. Dillon Alleyne

Electronic Version Approved:
James R. Alm, Dean
Andrew Young School of Policy Studies
Georgia State University
August 2008
ACKNOWLEDGEMENTS

This dissertation is one of the most important achievements in my life so far. While I feel a little relaxed now and try to recall the past, what I most want to say is: it is never easy. To survive the doctorate program, complete a dissertation and in the end earn a Doctor of Philosophy in Economics is not something that I could do it by myself alone. I thank all of you who have supported me in financing, study, personal life, and many other places over these years. All of these make things possible for me, and thank you! Due to limited space, I am only allowed to give my special thanks to a selected group of people who have been involved closely with me in the dissertation writing process.

First I would like to gratefully and sincerely thank the chair of my committee, Dr. Sally Wallace, for your guidance and supports in numerous ways. As a graduate research assistant, I have worked with you for a couple of years. During these years I have the chance to access various research projects and accumulate academic research experience. As my dissertation committee chair, you are always very supportive in helping me to form the ideas, structure my work, write the essays, and finalize the dissertation. For everything you have done for me, Dr. Wallace, thank you! I have learned so much from you and I will work hard to grow myself as an economics like you!

I am also indebted to my committee members, Dr. Yongsheng Xu, Dr. David Sjoquist, and Dr. Dillon Alleyne. Dr. Xu has provided me enormous helps from the very beginning of the first essay. I really appreciate your help and advice both in academic research and my career development. You are always there when I need helps. Without the helps from you, the models in this dissertation would take much more time from me. You are a great teacher as well as a great friend! To Dr. Sjoquist, your extensive knowledge and wisdom have helped hugely. You have given me so many useful comments and suggestions for all three essays. The essays have avoided many weakness and mistakes with your help. To Dr. Dillon Alleyne, although you are far away in Jamaica, you are always kindly offering me various helps. Especially I would like to thank you for offering help in the Jamaica Survey of Living Conditions data. I believe I have to continue to ask for more helps from you in the future.

I would also like to thank Dr. Mary Beth Walker, Dr. Bruce Seaman, and Dr. Neven T. Valev. As readers of my dissertation, you have provided helps in econometric techniques, economic modeling, and research methodology for the proposal of this dissertation. All of your suggestions and comments are integrated into this dissertation. Thank you!

I would like to thank the Department of Economics at Georgia State University and its faculty. The doctorate funding makes it financially possible for me to complete the doctorate program. The classes here have taught me a wide range of economics knowledge and research methodology. The years at Georgia State University are sure to be most important time in my life.

I would like to thank Xiamen University and my friends, classmates, and teachers there. I had a best time there with all of you. Without your help, it’s not possible for me to come to the United States and pursue a degree of Doctor of Philosophy in Economics. I miss all of you so much! We should meet soon. My special thanks will go to Dr. Chongming Qiu, my
Here I would like to thank my friends and colleagues at Georgia State University. You have helped me in so many aspects of my study and daily life. Riatu M. Qibthiyyah, you are my classmate for these years at Georgia State University and we have overcome so many difficulties together. Thank you! Minghung Yao, Artidiatun D. Adji, Nandya Yuwono, Juan Sun, Meng Wang, Ki-Whan Choi, and Zhenghua Hu, thank you all for your helps too! To the best roommates I could ever have, Guangzhi Zheng and Wenyi Wu, you made my life easier and more interesting in these years, thank you! To Dr. Felix Rioja, Bess Blyler, Wanda Cooley, Mary Kenyatta, LaTonya Edwards, Christopher Peters, Bo Ko, and Gardner Neely, thank you for your excellent administration and technical assistance work, and you are also great friends!

Finally, I would like to thank my families. You support me and encourage me to go ahead all the time. I know you are always there for me: this dissertation is also yours. To my father, Yicha Chen, and my mother, Kuang Lin, thank you for always believing in me. I finally make it! To my brother, Shidian Chen, and my sister, Lian Chen, thank you for always encouraging me, supporting the family, and being with Mom and Dad in these years. I have been far away from you for too long time, and I shall be at home very soon.
CONTENTS

ACKNOWLEDGEMENTS.......................................................................................................... vi
LIST OF FIGURES ................................................................................................................... x
LIST OF TABLES .................................................................................................................... xii
ABSTRACT ........................................................................................................................... xiii

Essay 1: A Theory of Trading of Club Goods and Its Application to Jurisdiction Formation ..1
  Literature Review and Motivation ...................................................................................... 1
  A Model for Clubs with Trading Club Goods ................................................................. 5
    The Start Point: A World without Trading of Club Goods .......................................... 7
  A Model of Trading Club Goods .................................................................................... 12
  Conclusions ...................................................................................................................... 33

An Analysis Framework for the Formation of Jurisdictions .............................................. 38
  A Model for Jurisdiction Formation .............................................................................. 38
  Jurisdiction Formation Process .................................................................................... 41
  Conclusions ...................................................................................................................... 42

Essay 2: Food Consumption in Jamaica: A Household Behavior as well as a Social Behavior
  ............................................................................................................................................. 44
  Literature Review and Motivation ................................................................................... 44
  Spatial Models and Diagnostics Methods ...................................................................... 46
    Spatial Models ................................................................................................................ 46
    Diagnostics for Spatial Dependence .......................................................................... 48
  Data: Jamaica Survey of Living Conditions .................................................................. 50
    Data Introduction and Summary ................................................................................. 50
    Weight Matrix ............................................................................................................ 54
  Model Specification and Estimation .............................................................................. 56
    Model Specification .................................................................................................... 56
    Diagnostics for Spatial Lag and Spatial Error Dependence .................................... 57
    Model Estimation ....................................................................................................... 59
  Discussion and Summary ............................................................................................... 63
    Effect of Income ........................................................................................................ 63
    Effect of Household Structure .................................................................................. 63
    Effect of Social factors: Income Level in the Society and Neighborhood ............... 64
    Summary ...................................................................................................................... 66
  Application to Poverty Problem .................................................................................... 67

Essay 3: A Survival Analysis of Education Duration in Jamaica ....................................... 70
# LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (Essay 1) Figure 1 The Possibility of Trading Club Goods: Increasing MC Case</td>
<td>10</td>
</tr>
<tr>
<td>2 (Essay 1) Figure 2 The Possibility of Trading Club Goods: Decreasing MC Case</td>
<td>11</td>
</tr>
<tr>
<td>3 (Essay 1) Figure 3 The Possibility of Trading Club Goods: Constant MC Case</td>
<td>12</td>
</tr>
<tr>
<td>4 (Essay 1) Figure 4 A Market for Club Goods</td>
<td>17</td>
</tr>
<tr>
<td>5 (Essay 1) Figure 5 Formation of the Market for Club Goods</td>
<td>19</td>
</tr>
<tr>
<td>6 (Essay 1) Figure 6-1 No Adjusting When P&gt;Min(ATC) - Poor Clubs/Sellers</td>
<td>25</td>
</tr>
<tr>
<td>7 (Essay 1) Figure 6-2 Adjusting Process When P&lt;Min(ATC) - Rich Clubs</td>
<td>25</td>
</tr>
<tr>
<td>8 (Essay 1) Figure 7 Actual Cost vs Shadow Cost (I: Actual; II: Shadow)</td>
<td>29</td>
</tr>
<tr>
<td>9 (Essay 1) Figure 8 Club Formation- Decreasing MC</td>
<td>32</td>
</tr>
<tr>
<td>10 (Essay 1) Figure 9 Club Formation- Constant MC</td>
<td>33</td>
</tr>
<tr>
<td>11 (Essay 2) Figure 1 Household Expenditure Structure in Jamaica, 2001</td>
<td>52</td>
</tr>
<tr>
<td>12 (Essay 2) Figure 2-1 Plots for Totexp, Totfood, and Engel's Coefficient</td>
<td>53</td>
</tr>
<tr>
<td>13 (Essay 2) Figure 2-2 A Plot for Totexp, Totfood, and Engel's Coefficient by Popdec</td>
<td>53</td>
</tr>
<tr>
<td>14 (Essay 2) Figure 3 Jurisdiction Structure &amp; Weight Matrix</td>
<td>55</td>
</tr>
<tr>
<td>15 (Essay 2) Figure 4-1 Plots of SAC Estimation-1</td>
<td>61</td>
</tr>
<tr>
<td>16 (Essay 2) Figure 4-2 Plots of SAC Estimation-2</td>
<td>62</td>
</tr>
<tr>
<td>17 (Essay 2) Figure 5 Plot for Marginal Increase in Engel's Coefficient</td>
<td>64</td>
</tr>
<tr>
<td>18 (Essay 3) Figure 1 Student Distribution in Jamaica 2002 (Grade 1-13)</td>
<td>83</td>
</tr>
<tr>
<td>19 (Essay 3) Figure 2 Educational Attainment in Jamaica 2002 (by Age Group)</td>
<td>85</td>
</tr>
<tr>
<td>20 (Essay 3) Figure 3 Number of People by Marriage and Age</td>
<td>91</td>
</tr>
</tbody>
</table>
21 (ESSAY 3) FIGURE 4-1 ESTIMATED HAZARD CURVES FOR TYPICAL INDIVIDUALS ............ 101
22 (ESSAY 3) FIGURE 4-2 ESTIMATED SURVIVAL CURVES FOR TYPICAL INDIVIDUALS ........ 101
23 (ESSAY 3) FIGURE 5-1 LIFE TABLE ANALYSIS-SURVIVAL CURVE, BY AGE GROUP AND GENDER ........................................................................................................................................ 102
24 (ESSAY 3) FIGURE 5-2 LIFE TABLE ANALYSIS-DROP OUT HAZARD CURVE, BY AGE GROUP AND GENDER ........................................................................................................................................ 102
LIST OF TABLES

<table>
<thead>
<tr>
<th>Table</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (ESSAY 1) Table 1 A SUMMARY OF FORMATION OF CLUBS WITH TRADING</td>
<td>34</td>
</tr>
<tr>
<td>2 (ESSAY 1) Table 2 DEPENDENT AND SINGLE JURISDICTION STRUCTURE</td>
<td>41</td>
</tr>
<tr>
<td>3 (ESSAY 2) Table 1 A SUMMARY OF DATA (1)</td>
<td>51</td>
</tr>
<tr>
<td>4 (ESSAY 2) Table 2 A SUMMARY OF DATA (2)</td>
<td>51</td>
</tr>
<tr>
<td>5 (ESSAY 2) Table 3 AN ILLUSTRATION OF WEIGHT MATRIX DEFINED BY EDS</td>
<td>56</td>
</tr>
<tr>
<td>6 (ESSAY 2) Table 4 AN OLS ESTIMATE</td>
<td>58</td>
</tr>
<tr>
<td>7 (ESSAY 2) Table 5 AN OLS ESTIMATE FOR THE REDUCED MODEL</td>
<td>58</td>
</tr>
<tr>
<td>8 (ESSAY 2) Table 6 DIAGNOSTIC TESTS</td>
<td>59</td>
</tr>
<tr>
<td>9 (ESSAY 2) Table 7 MLE ESTIMATE FOR SAC, SAR, SEM MODELS</td>
<td>60</td>
</tr>
<tr>
<td>10 (ESSAY 2) Table 8 GMM ESTIMATE FOR SAC, SAR, SEM MODELS</td>
<td>60</td>
</tr>
<tr>
<td>11 (ESSAY 2) Table 9 CHANGE IN ENGEL’S COEFFICIENT WITH 1% INCREASE IN TOTEXP BY POPDEC</td>
<td>63</td>
</tr>
<tr>
<td>12 (ESSAY 3) Table 1 ENROLLMENT IN JAMAICA 2002 (BY SCHOOL TYPE)</td>
<td>80</td>
</tr>
<tr>
<td>13 (ESSAY 3) Table 2 EDUCATIONAL ATTAINMENT IN JAMAICA 2002 (BY SCHOOL TYPE)</td>
<td>81</td>
</tr>
<tr>
<td>14 (ESSAY 3) Table 3 HOUSEHOLD DEMOGRAPHIC CONSTITUENTS IN JAMAICA 2002</td>
<td>82</td>
</tr>
<tr>
<td>15 (ESSAY 3) Table 4 STUDENT DISTRIBUTION IN JAMAICA 2002 (GRADES 1-13)</td>
<td>83</td>
</tr>
<tr>
<td>16 (ESSAY 3) Table 5 EDUCATIONAL ATTAINMENT IN JAMAICA 2002: GRADE REPORTED (BY AGE GROUP)</td>
<td>85</td>
</tr>
<tr>
<td>17 (ESSAY 3) Table 6-1 MARITAL STATUS REPORTED (AGE &gt;=15)</td>
<td>86</td>
</tr>
<tr>
<td>18 (ESSAY 3) Table 6-2 UNION STATUS REPORTED (AGE &gt;=15)</td>
<td>86</td>
</tr>
<tr>
<td>19 (ESSAY 3) Table 7 ESTIMATION OF DISCRETE TIME COX MODEL</td>
<td>94</td>
</tr>
<tr>
<td>20 (ESSAY 3) Table 8 ESTIMATION OF DISCRETE TIME SURVIVAL MODEL-LOGIT MODEL</td>
<td>99</td>
</tr>
</tbody>
</table>
ABSTRACT
THREE ESSAYS IN PUBLIC FINANCE
BY SHIYUAN CHEN
August 2008

Committee Chair: Dr. Sally Wallace
Major Department: Economics

This dissertation comprises three essays in public finance. The first essay is a research of a theory of trading of club goods and its application to jurisdiction. The essay establishes a model of trading of club goods among clubs, and illustrates its effects on the process and outcome of club formation. Cost function as well as disutility of crowdedness is emphasized and integrated into the process of club formation, after allowing for exchanging club good among clubs. In the process, the essay develops a market for club goods. Then the model is revised and applied to the formation of jurisdictions.

The second essay comes out of an interest regarding household demand, poverty and public goods in developing countries. The essay explores household food consumption in Jamaica and estimates the effects of related variables. With Jamaica Survey of Living Conditions 2001 data, the essay estimates an Engel curve which reflects the relation between household food consumption and related variables. What’s more, to investigate the possible neighborhood effect on food consumption, the essay tests and estimates the spatial correlation among neighborhood food consumption. The estimated results can be applied to poverty reduction policy.

The third essay extends the theme of poverty, consumption, and government programs by analyzing one other public program—education. Education is closely linked to poverty alleviation. Determining the demand for education and the return to education will help government focus programs aimed at reducing drop-out rates and in the long run, poverty in
the country. The essay applies discrete time survival analysis techniques to analyze education duration in Jamaica. Based on Jamaica Survey of Living Conditions 2002, the essay estimates the effects of household, individual and other related covariates on dropout risks of students. The essay compares discrete time Cox model and discrete time Logit model and concludes that the two estimations are consistent. The estimation results could be used to predict the effects of changes in the covariates, or be used to predict the dropout risks of particular students in each grade, both of which could provide useful policy implications to improve education in Jamaica.
Essay 1: A Theory of Trading of Club Goods and Its Application to Jurisdiction Formation

Club theory originated from Buchanan (1965) and Tiebout (1956). Since then, it has attracted great interests of economists and been developed and applied to different aspects of group formation, varying from local groups (e.g., local jurisdictions) to international organization in terms of size, and also from public groups to private groups in terms of ownership, etc. In this paper, we mostly focus on the application of club theory to the formation of local jurisdictions, although the conclusions of our analysis should be able to apply to any other forms of clubs as long as our basic assumptions hold. We’ll establish a model of trading of club goods among clubs, and illustrate its effects on the process and outcome of club formation. The paper has three sections: in Section 1 we will introduce the motivation and have a brief literature review; in Section 2, we’ll build a model of trading of club goods among clubs; in Section 3, the model will be applied to analyze the formation of jurisdictions.

Literature Review and Motivation

Buchanan (1965) defined a club as an impure (congestible) public good for which exclusion is possible. Tiebout (1956) showed how the optimal size of local jurisdiction can be reached when consumer-voter are fully mobile. In his paper, the congestible and excludable characteristics are assumed implicitly.\(^1\) A club is defined by Sandler and Tschirhart (1980) as ‘a voluntary group deriving mutual benefits from sharing one or more of the following: production costs, the members’ characteristics, or a good characterized by excludable benefits’. In the past 50 years, club theory has been developed in many directions and we are

---

\(^1\) Scotchmer (2002): ‘local public goods’, which blends group formation with geography and sometimes with voting mechanisms. Thus local public goods can be treated as a special case of club goods.
most interested in these two directions: the first one is the development in private provision of club goods; the second one is the development in the interaction among clubs.

The first direction is a discussion about the difference of club production and private production: some authors even argue that they are equivalent and clubs can be replaced by private firms. (Berglas, 1976) Some authors see the possible separation of production and provision of club goods, which is the theoretical base of the popular practice of contracting out government services in the local jurisdictions in the United State after 1980s. (Musgrave, 1959; Oakerson, 1987; Warner & Hebdon, 2001) Thus clubs can outsource their production to outside producers, such as private firms, non-profit organization, or other clubs, etc. However, as we’ll show later, the literature also shows that many governmental services have to be produced and provided within the public sectors. There are reasons such as retaining public control, market failure for the goods already, the complexity of public service delivery, and the limits of market approaches, etc. (Oakerson, 1987; Morgan & Hirlinger, 1991; Parks & Oakerson, 1993; Lowery 1998; de Leon & Denhardt 2000; Warner & Hebdon, 2001; Hefetz & Warner, 2004). Although these are not our focus in this paper, they are very important for our model: they’ll provide the support from the real world for one of our model’s fundamental assumptions as we’ll show in the second part of the paper: the club goods in our model are assumed to be produced and provided only by clubs. Other alternatives are not our concern in this paper.

The second direction is our main interest. The development in the interaction among clubs has two interesting literatures: one is about the trading of private goods among clubs; the other one is about the sharing of club goods among clubs (specifically, sharing uncertainty and risk among clubs) (Sandler, Sterbenz, & Tschirhard, 1985; Sterbenz & Sandler, 1992). It seems natural that there should be a theory about trading of club goods among clubs following this direction. However, surprisingly we can find few research of it.
Are there any reasons that such a theory can’t be developed, or can a club good be traded among clubs at all? This question boggled us for quite some time. In the end, we overcame this problem with the help from another literature: New Public Management theory (NPM). This literature shows that there are many, very popular and common, of trading local public goods among local jurisdictions. We are excited at the ‘evidence’ in the real world that supports the idea of developing a theory of trading of club goods among clubs, and such a theory should be able to rationalize this kind of intergovernmental behavior theoretically in return and even provide help to correct those old views such as putting intergovernmental contracting into the category of privatization.

New Public Management (NPM) that rises from 1980s (Hood, 1991) is a global phenomenon that emphasized deregulation, downsizing, and outsourcing (Cooper, 2003). Outsourcing (Contracting out) is that government agencies provide services to the public by employing private firms, nonprofit organizations, or even other governments. Contracting out is the commonest form of privatization (Rehfuss, 1989). Most economists treat contracting out as part of privatization, however, a few authors point out “Another myth about government contracting is that it is something that only happens in conjunction with the private sector” (Lavery, 1999) and argued that “the failure of past studies to distinguish inter-municipal cooperation from private sector is serious flaw” (Kodrzychi, 1994, 1998; Lopez-de-Silanes, Shleifer, & Vishny, 1995; Warner & Hebdon, 2001). Actually, according to ICMA data the contracting among governments is very common and about 10% of privatization is intergovernmental contracting out. It is called as intergovernmental contracting, agreements, or cooperation, etc. One of the well-known cases is the Lakewood Plan in the LA county, CA.² Obviously, this kind of intergovernmental contracting has

² For details of the Lakewood Plan, see Lavery’s ‘Smart Contracting for Local Government Services’ (Lavery, 1999).
nothing to do with private sectors and should not be put into the category of privatization. Thus the theories of privatization might not be appropriate to explain it either.

Some economists do try to give intergovernmental contracting economic reasons: economies of scale, competition brought by establishing a quasi market, and higher government efficiency when managing outside organization, etc. (Ostrom, Tiebout, & Warren, 1961; Ferris & Graddy, 1986; Stein, 1990; Lavery, 1999; Savas, 2000; Jang & Feiock, 2003) According to ACIR (1985), the reason for entering an IG service contract: economies of scale (52%), the need for a larger area (38%), a lack of facilities (32%). (Morgan & Hirlinger, 1991) However, all of these seem to be based on intuition and seldom have been proven or illustrated by sound theories. A lack of a theory causes many misunderstandings of intergovernmental cooperation: such as not being able to see the real reasons, mistreating it as part of privatization, and ignoring its effects on jurisdiction formation, etc. Even for those literatures that separate it as an independent category from the other governmental restructuring practices, it’s still treated the same way in terms of analytical methodology. This might reflect that although the authors recognize the difference, they can’t find a better way to handle it. Apparently, a theory is needed here. On the other hand, as we recall our above brief review of club theory, there is also a lack of a theory of trading club goods in the literature of club theory. Now it seems very natural for us to link these two literatures together: is there a theory of clubs that can be applied to this kind of intergovernmental cooperation? The theory of trading of club goods in this paper, as a joint product of these two literatures, is a step toward this effort.3

---

3 Most of examples here are from the NPM literature and are about governments, however we think our theory should be able to be used to analyze general clubs such as golf courses, swimming pools, fishing, etc., as long as they meet our assumptions.
A Model for Clubs with Trading Club Goods

In this section, we will try to build a model for clubs with trading of club goods. However, before that, we think we must present an unsolved theoretical issue that is ignored by most economists till now. It can be called as the problem of non compatibility of collective consumption and production in a club without trading. We’ll illustrate the problem by comparing Tiebout (1956) to Buchanan (1965) as follows.

First, let's take a look at the Tiebout’s definition of optimal local public good size: ‘This optimum is defined in terms of the number of residents for which this bundle of services can be produced at the lowest average cost. This, of course, is closely analogous to the low point of a firm’s average cost curve.’ In another word, Tiebout argued that an optimal size (of local public good) should meet with the production efficiency of the club good, i.e. a production level associated with the lowest average cost of the production function. Tiebout tried to use the cost to constrain the size of local public goods. However, for pure public goods, the size will go to infinite no matter what kind of cost functions or what the fixed cost is. The reason is that for pure public goods, there is no need to increase the production when the size is increasing. The only possible reason, which should be also an underlying assumption of Tiebout’s paper, is the crowdedness of local public goods, i.e., local public goods is subject to crowdedness. Thus we see a connection or fundamentally equivalence between Tiebout and Buchanan. Regarding determining the size of club (or local jurisdictions, etc), Buchanan’s club model integrated with the crowdedness is more persuasive. However, we also find that the attention paid to cost function and production efficiency by Tiebout is interesting: do we need to pursue the lowest average cost when we determine the optimal size? If not, is this optimal size also optimal in terms of production efficiency?

In Buchanan’s model, the objective function is the individual utility function with the income constraint. The forms of cost function are unimportant and not specified. However, we can’t find reasons why the outcome of maximizing individual utility will automatically
reach the lowest average cost. If the production and provision is separated and done by
competitive private firms and local governments or clubs respectively, the problem could be
solved. However, for those local public goods/club goods produced and provided both by
local governments/ clubs, in general the problem exists. As Deacon (1979) argued: ‘Under
traditional supply, with services produced in local government bureaus or departments, scale
(the size of the population served) may influence both production costs and costs of collective
consumption (i.e. group decision-making costs). Presumably, the population size of such
jurisdictions evolves as a product of both influences.\textit{With both activities (collective
consumption and production) operated at the same level in terms of population, there is no
reason to expect that production activities, taken separately, are efficiently scaled. … If
demand expression and production are separated across a market the two activities may reach
efficient scales separately.’ Thus it seems for us that there is a possible way to solve the
problem of non compatibility of collective consumption and production: a model
emphasizing on cost function and allowing for trading of club goods. These ideas are very
important for our model.\textit{Crowdedness of collective consumption, cost function of
production, and trading of club goods} are three basic constituents integrated in our model,
which maximizes individual utility of consumption and also seeks to get a lowest average
provision cost at the same time.\textsuperscript{4} In terms of general equilibrium, we believe such a model
will give us an optimum superior to traditional ones without trading of club goods.\textsuperscript{5}

We will start with the classic clubs without trading. Then the concept of trading of club
goods will be introduced and a model of trading of club goods will be built. In the process of

\textsuperscript{4} As we’ll see later, the average provision cost has two components: own production cost and purchasing cost.
Generally, the lowest average provision cost is not equal to the lowest average cost of the production function.
In some cases of our model, the former is associated with the latter.

\textsuperscript{5} Please note that our paper only focuses on club goods that have to be produced by clubs, i.e. that couldn’t be
produced by private firms. For those club goods produced by private firms, the provision and production are
separated and assumed by the clubs and the private firms respectively, which has been discussed for long time
by the traditional contracting out theories.
solving the objective function we’ll ‘establish’ a market for club goods. Several other issues will be discussed too. We’ll give our assumptions for our model in the beginning.

Assumptions:

1. The society is large enough such that the problem of integer number of clubs can be ignored. Let the size of the society be \( N \).

2. The society has two kinds of individuals, rich and poor, with the income endowments of \( I_r \) and \( I_p \) respectively. ( \( I_r > I_p \) ) They only differ in income. Furthermore, the size of the rich is \( N_r \) and that of the poor individuals is \( N_p \). So we will have \( N = N_r + N_p \). \( N_r \) and \( N_p \) are large enough too.

3. An individual will face the substitutive consumption between \( X \), a private good, and \( Y \), a club good. Both \( X \) and \( Y \) are normal goods. The individual has the utility function \( u = u(x, y, n) \), where \( x \) is the consumption of \( X \), \( y \) is the consumption of \( Y \), and \( n \) is the club size. We’ll have \( \partial u / \partial x > 0 \), \( \partial^2 u / \partial x^2 < 0 \), \( \partial u / \partial y > 0 \), \( \partial^2 u / \partial y^2 < 0 \), \( \partial u / \partial n \leq 0 \). ⁶

4. We assume the club good can only be produced by clubs. ⁷

The Start Point: A World without Trading of Club Goods
In this section, we’ll start from the world without trading of club goods, as most literature has done. The society will form homogeneous clubs. ⁸ In addition, we assume a cost function of production of club goods, \( C = C(y) \). For a typical individual, its objective function can be described as ⁹

\[
\max_{x,y,n} u(x, y, n) \\
\text{s.t. } x + C(y)/n = I
\]

---

⁶ The utility function simply fits the classic assumption of decreasing marginal utility for \( X \) and \( Y \). The negative marginal utility is used to reflect of the disutility of crowdedness of the clubs.

⁷ This assumption will simplify our analysis which focuses on the interaction behaviors among clubs. Please see Section 1 for the legitimacy of the assumption.

⁸ It can be proved that the homogeneous clubs are the outcomes of Nash equilibrium under our assumptions. Intuitively, in the homogeneous clubs, every member is the median voter and reaches his optimal utility level. Nobody will have incentives to migrate to the other clubs. Without the assumption of infinite population size, it may not be true.

⁹ In our model, we assume there is a unit price for private good \( X \). We also assume that the cost of club good is shared equally by members, regardless of their income levels. The cost sharing scheme is a crucial institutional arrangement, and other forms of cost sharing scheme, such as cost sharing according to income, etc., will change the budget condition of the objective function and thus change the outcomes. We suggest further study could be conducted based on various cost sharing schemes.
The solution need to meet with the following FOCs:

\[ \frac{u'_x}{u'_y} = \frac{n}{C'_y} \]  
\[ \frac{u'_x}{u'_n} = -n^* \frac{C'_y}{C} \]  
\[ x + \frac{C(y)}{n} = I \]

As usual, after solving the first order conditions, we can get \( x^* = x(I), y^* = y(I), \) and \( n^* = n(I). \) We denote those for the poor as follows: \( x_p^* = x(I_p), y_p^* = y(I_p), \) and \( n_p^* = n(I_p). \) We denote those for the rich as follows: \( x_r^* = x(I_r), y_r^* = y(I_r), \) and \( n_r^* = n(I_r). \) Furthermore, we can get the number of clubs in the society as \( N_r/n_r \) and \( N_p/n_p \) for the rich and the poor respectively.

If both \( X \) and \( Y \) are normal goods, we will have \( \partial x^*/\partial I > 0 \) and \( \partial y^*/\partial I > 0, \) i.e. as the income increases, the consumption of \( Y, y^*, \) and \( X, x^*, \) will increase. Under some assumptions, we can get that \( \partial n^*/\partial I > 0, \) i.e. the club size will increase along with increase in income. Thus we have \( y_r^* < y_p^*, \) and \( n_r^* < n_p^*. \)

A traditional club good has a constant-marginal-cost cost function without fixed cost. However, as we have discussed before, it’s more general to discuss the cost functions with constant marginal cost, increasing marginal cost, and decreasing marginal cost. What’s more, the fixed cost will also be very important as we will see in the later discussion of our paper.

In a word, the cost function, \( C = C(y), \) can have the characteristics: (1) \( \partial C / \partial y > 0; \) (2) \( \partial^2 C / \partial y^2 > 0, \partial^2 C / \partial y^2 < 0, \) or \( \partial^2 C / \partial y^2 = 0; \) (3) \( C(0) = c, \) \( c \) is a nonnegative constant parameter.

---

10 Please see the appendix for details.
CASE 1: cost function with increasing marginal cost

In the case of increasing marginal cost, we will have $\frac{\partial^2 C}{\partial y^2} > 0$. That’s to say, the more the club good the club produces, the higher the marginal cost will be.

As we can see from above, $y_p^* < y_r^*$, thus we will have $C'(y_p^*) < C'(y_r^*)$, i.e. the marginal production cost of the poor clubs is lower than that of the rich ones.\(^{11}\)

This can be demonstrated on Figure 1.\(^{12}\) The rich and the poor have the same utility function and thus they will face the same indifferent curves. On the other hand, they also face the same cost function. The marginal cost is equal to the slope of the tangent line on the production possibility curve at the optimal point.

Now, it can be known that the poor clubs can produce an additional unit of club goods with a lower incremental cost than the rich. Thus it’s possible for the poor clubs to produce more and sell some club goods to the rich clubs, while the rich clubs produce less and purchase some from the poor ones. This kind of trading between the rich and the poor will lead to a Pareto improvement in economic efficiency.

\(^{11}\) We assume both clubs have the same cost function $C(y)$, since it’s reasonable to assume that the technology is accessible for everyone. When the cost functions can vary according to geography, income, etc., we can get more general results which fit the real world better. However, to simplify our analysis, we will assume the same cost function for every club through our paper.

\(^{12}\) The individual indifferent curves are conditional on club size. To simplify our illustration, we’ll ignore the difference in club size. In this graph and the following graphs in this paper, we’ll ignore the process of adjusting optimal club size.
CASE 2: cost function with decreasing marginal cost

In the case of decreasing marginal cost function, we will have $\frac{\partial^2 C}{\partial y^2} < 0$. That’s to say, the more the club produces, the lower the marginal cost will be.

As we can see from above, $y^*_p < y^*_r$, thus we will have $C'(y^*_p) > C'(y^*_r)$, i.e. the marginal cost of the poor clubs is higher than that of the rich.

In this case, it is more efficient for the rich clubs to produce an additional unit of club good. The poor can purchase some from the rich with cheaper cost. The trading of club good in this case will also lead to a Pareto improvement in economic efficiency. This case has been discussed by much literature as the scale economy of production of club goods.
CASE 3: cost function with constant marginal cost

In the case of constant marginal cost, we’ll have \( C'(y) = 0 \). That’s to say, all the clubs have the same marginal cost no matter how much they produce. It seems that it doesn’t matter who produce and how much they produce. However, if we consider the fixed cost, there is still room for trading- the society will be better off if there are fewer producers and thus save some fixed cost. The results here are quite similar to the case with decreasing marginal cost.\(^\text{13}\)

\(^{13}\) We can think when the fixed cost is high enough, the small clubs will have no ability to produce it. The only way to consume the club good is to purchase club goods from big clubs or dissolve the small clubs and join the big clubs. We’ll discuss it in details later.
A Model of Trading Club Goods

From the analysis of the first part, we know that it is possible for trading club goods among clubs. In this part, we will continue to develop the trading model. In the previous classic models without trading among clubs, the incentive for grouping is to share the cost by consuming club goods together; the disincentive for grouping is the crowdedness of consumption caused by club members. We’ll discuss the trading behaviors following up the previous analysis. Now let’s suppose the clubs will trade certain amount of club goods. Let the amount of own production is $y_0$, then the trading amount is $y_1 = y - y_0$. When $y_1$ is positive, it represents the amount purchased in; when $y_1$ is negative, it represents the amount sold out; when $y_1$ is 0, there is no trade taking place.
**CASE 1: cost function with increasing marginal cost**

**individual's problem**

First let’s focus on the cost function with increasing marginal cost. Suppose the purchase price of \( y_1 \) is \( p \). Now we have the individual objective function:

\[
\max_{x,y,n} u(x, y, n)
\]

\[
\text{s.t. } x + C(y_0) + y_1 \cdot p / n = I
\]

\[
y = y_0 + y_1
\]

\[
p = C'(y_0)
\]

The purchase price \( p \) needs to be equal to the marginal cost of own production, \( C'(y_0) \). This is because additional own production will be cheaper than purchasing if the price is higher than the marginal cost of production. That’s to say, \( p = C'(y_0) \). We can let its inverse function as \( y_0 = h(p) \). If we assume that \( p \) is exogenous, we can derive the first order conditions:

\[
u'_y - u'_x \cdot p / n = 0
\]

\[
u'_n + u'_x \cdot [C(h(p)) / n^2 + y_1 \cdot p / n^2] = 0
\]

\[
x + C(h(p)) + y_1 \cdot p / n = I
\]

Now we have three equations (4)-(6). The first one can be rewritten as

\[
u'_y / u'_x = p / n = MC_y / MC_x
\]

i.e. \( MRS_{y,x} = MRTS_{y,x} \), which is the provision constraint of club goods. The second one can be rewritten as

\[
u'_n / u'_x = -[C(h(p)) / n^2 + y_1 \cdot p / n^2] = MC_n / MC_x
\]

i.e. \( MRS_{n,x} = MRTS_{n,x} \), which is the admission constraint for the club. The third one is the budget constraint for the club. Now we have four unknown variables, \( x, y_1, n \) with three equations. We can solve \( x, y_1, n \). Assume they are as follows respectively,
\[ x^* = x(p, I) \quad (7) \]
\[ y_1^* = y(p, I) \quad (8) \]
\[ n^* = n(p, I) \quad (9) \]

(and \( y^* = y_1^* + y_0 = y_1^* + h(p) \))

From (8), we can have \( y_1^* = y(p, I) \). Let its inverse function as \( p = f(y_1, I) \), which shows that when \( I \) is given, \( p \) and \( y_1 \) depend on each other.

Definition: when \( y_1 > 0 \), \( f \) is an individual demand function for the club good. When \( y_1 < 0 \), it is an individual supply function of the club good. When \( y_1 = 0 \), the club has no demand or supply of the club good in the market.

\textit{the society’s problem}

For a given price \( p \), we will have the \( y_1 = f^{-1}(p, I) \). As we summing up all of the demand/supply of club goods in the society given the price, we will get the social demand/supply of club goods.

\[ D = \sum y_{1i} = \sum f^{-1}(p, I_{ii}) \text{ when } y_{1i} > 0; \]
\[ S = -\sum y_{1i} = -\sum f^{-1}(p, I_{ii}) \text{ when } y_{1i} < 0. \text{ (i is the index of clubs.)} \]

Thus, for social demand function we can get

\[ \frac{\partial D}{\partial p} = \sum \frac{\partial y_{1i}}{\partial p} = \sum \left( \frac{\partial y_{0i}}{\partial p} - \frac{\partial y_{0i}}{\partial p} \right) \]

From \( p = C'(y_{0*}) \), we can get \( \frac{\partial y_{0}}{\partial p} > 0 \), i.e. as \( p \) increase, part of purchased club good will be replaced by own production. Since \( Y \) is a normal good, as the average cost (of own production and purchase) increases, both the income effect and substitution effect will give us \( \frac{\partial y}{\partial p} < 0 \). In a word, as \( p \) increases, \( y_0 \) will increase, and \( y \) and \( y_1 \) will decrease. Thus, we have

\[ \frac{\partial D}{\partial p} < 0, \text{ where } D > 0. \quad (10) \]

\[ ^{14}\text{ Measured by average production and price, respectively, and both are increased.} \]
Thus, for social supply function we can get

$$ \frac{\partial S}{\partial p} = -\sum \frac{\partial y_i}{\partial p} = -\sum (\frac{\partial y_i}{\partial p} - \frac{\partial y_{0_i}}{\partial p}) $$

We have $\frac{\partial y_{0_i}}{\partial p} > 0$. The direction of $\frac{\partial y}{\partial p}$ ambiguous because it will depend on the income effect and substitution effect: the former will have a positive effect on own consumption $y$ and the latter will have a negative effect. Only in few cases that most club good is produced to sell will the income effect overwhelm the substitution effect plus the effect of increase in own production (or $\frac{\partial y_{0_i}}{\partial p} > 0$). Thus, in general we will have:

$$ \frac{\partial S}{\partial p} > 0, \text{ where } S > 0. \quad (11) $$

Proposition 1: When the price equal to the marginal cost of club production before trading, the club has not formed supply or demand for club good; When the price is higher than that marginal cost, there is a supply of club good from the club; When the price is lower than that marginal cost, there is a demand for club good from the club.

If we let own production of club goods without trading be $y^0$, and the associated marginal cost is $C'(y^0)$, it will be easy to get:

$$ y_1 = 0, y = y_0 = y^0, \text{ as } p = C'(y^0); $$
$$ y_1 > 0, y_0 < y^0 < y, \text{ as } p < C'(y^0); $$
$$ y_1 < 0, y^0 < y_0, y < y_0, \text{ as } p > C'(y^0). $$

Now let’s go back to the assumption that allows for two income groups. In the previous part, we have shown that under some assumptions the pre-trading production of club goods for the poor will be less than that of the rich, i.e. $y_{0_p}^0 < y_{0_r}^0$ and furthermore, we can get $C'(y_{0_p}^0) < C'(y_{0_r}^0)$. If there exists an exogenous market price for club good, then we’ll have:

For the poor clubs, the aggregated demand/supply is:

$$ D_p = N_p\cdot p - I_p, \text{ when } p < C'(y_{0_p}^0). $$
\[ Sp = \frac{N_p}{n_p} f^{-1}(p, I_p), \text{ when } p > C'(y_{0p}^\cdot) \]

\[ D_p = Sp = 0, \text{ when } p = C'(y_{0p}^\cdot). \]

For the rich clubs, the aggregated demand/supply is:

\[ Dr = \frac{N_r}{n_r} f^{-1}(p, I_r), \text{ when } p < C'(y_{0r}^\cdot) \]

\[ Sr = \frac{N_r}{n_r} f^{-1}(p, I_r), \text{ when } p > C'(y_{0r}^\cdot) \]

\[ Dr = Sr = 0, \text{ when } p = C'(y_{0r}^\cdot). \]

*a market for club goods*

The demands and supplies derived in the above part can be illustrated in Figure 4. In Figure 4, when the price \( P \) for club good is higher than marginal cost of own production of club good, \( C'(y_{0r}^\cdot) \) and \( C'(y_{0p}^\cdot) \) for rich clubs and poor clubs respectively, there will form a supply for club good in the market: \( Sr \) and \( Sp \) for rich clubs and poor clubs respectively. On the other hand, when the price \( P \) is lower than marginal cost of own production of club good, \( C'(y_{0r}^\cdot) \) and \( C'(y_{0p}^\cdot) \) for rich clubs and poor clubs respectively, there will form a demand for club good in the market: \( Dr \) and \( Dp \) for rich clubs and poor clubs respectively. As we horizontally sum up \( Sr \) and \( Sp \), we’ll get the market supply curve \( S = Sr + Sp \); meanwhile, as we horizontally sum up \( Dr \) and \( Dp \), we’ll get the market supply curve \( D = Dr + Dp \). As the demand for club goods is equal to the supply, i.e. \( S = D \), there is the equilibrium of the market, \((P^*, Q^*)\). We can see obviously that at the equilibrium, the market supply is from the poor clubs, and the demand is from the rich clubs. This is consistent to our analysis in the previous section.
For mathematical solutions, we’ll need to solve Equations (4), (5) and (6) for every clubs and get the individual supply/demand functions of club good given the price of club good. After solving equation $S = D$ and getting the equilibrium price and quantity in the market, we can go back to get the club size, production, consumption and trading amounts of club good. There exists a unique solution.15

Definition: As we allow for the trading of club good, given a certain price, the society will form the demand force on club good as individual clubs want to purchase the club good for the given price, which can be called as the demand on club good. At the same time, the supply of club good is formed as some clubs want to sell part of their club good. Thus there will be an equilibrium point, on which the demand and the supply are equal and the market

---

15 Proof: let $F(p) = S(p) - D(p)$. Then we have $\frac{dF}{dp} = \frac{dS}{dp} - \frac{dD}{dp} > 0$, i.e. $F$ is a monotonic increasing function. Furthermore, we have $F(0) = S(0) - D(0) < 0$, and $F(\infty) = S(\infty) - D(\infty) > 0$, thus there exists a unique solution for $F(p) = 0$. 

---
price and amount of club good in the market are determined. This is a market for club goods.

When we pull the graphs above together, we can get a graph illustrating the formation of the market as shown in Figure 5. Now we start from area III, clubs are faced with a cost function with increasing marginal cost. Without trading the amount of club good produced by the poor and the rich clubs will be $y^p_0$ and $y^r_0$, which are associated with the marginal cost $C'(y^p_0)$ and $C'(y^r_0)$ in area II respectively. Every output level of club good is associated with a marginal cost. In area II, we have an increasing marginal cost function, this gives us $C'(y^p_0) < C'(y^r_0)$. As we have shown above, the equilibrium price has to be within this range. Through $P=MC$, we connect the area II to area I. Every price within $C'(y^p_0)$ and $C'(y^r_0)$ is associated to a demand and a supply. As the demand equal to the supply of club good in the market, there is the equilibrium $(P^*, Q^*)$. The equilibrium price $P^*$ will go to determine the marginal cost and the production of club goods in area II. The equilibrium price $P^*$ will change budget constraints, and so change the optimal choices of clubs. In the end, the sum of sale of the club good will be equal to the sum of purchase of the club good, and furthermore, equal to the equilibrium quantity $Q^*$. In this process of adjusting price and production, the club size will be adjusted simultaneously.

---

16 Scotchmer (2002): club theory is a theory of general equilibrium, which tries to maximize the individual utilities. The finding of a market of club good is an attempt toward this direction.

17 As we refer to it, we’ll ignore the adjusting process of club size. However, this will not affect our illustration. In the graph, we also assume that all gains from trading are used to consume private goods. In fact, club size and consumption bundle will be adjusted in the process toward the equilibrium.
We can prove the following lemmas:

(1) The higher the price is, the higher utility the seller will have and the lower utility the buyer will have. This is because the higher price will bring the seller more revenue and also profit from selling club good, while it will increase the cost of the buyers.

(2) The equilibrium price is limited to the area between the marginal production costs before trading, i.e. \( C'(y0^+, p) < p < C'(y0^+, p) \). It can be gotten directly from the above analysis.\(^\text{18}\)

(3) The population size is important. The larger population is, the more clubs can be formed. Thus the demand/supply will increase. Increase in poor population will increase the supply and reduce the equilibrium price. Increase in rich population will increase the demand and raise the equilibrium price.

---
\(^\text{18}\) We’ll expect a smooth demand and supply curve with no constraint on the equilibrium if we allow for varying in income and cost function.
(4) The income increase of the rich will increase the demand and raise the equilibrium price, and vice versa. The income increase of the poor will decrease the supply and raise the equilibrium price, and vice versa. This is because club good is normal good.

(5) After trading, the consumption of club good for rich clubs will increase because of price effect. The change in consumption of private good for rich clubs is ambiguous and determined by the income effect and substitution effect from the price change of club good. The change in size of rich clubs is ambiguous too. The change in consumption of club good for poor clubs will be ambiguous and determined by the income effect from selling and price effect of own consumption of club good. The consumption of private good for poor clubs will increase because the income effect and price effect. The change of club size of poor clubs is ambiguous too.

(6) The market for club goods reaches its equilibrium status, and the clubs are stable. The equilibrium price and quantity of club good in the market, as well as club sizes, production, consumption and trading amounts of each club maximizes the individual utility function, so each individual also reach its best situation and nobody will have incentives to move. Thus our solution is a NE solution under our assumptions.

A discussion about fixed cost

In our cost function, we have $C(0) = c$, i.e. $c$ is the fixed cost. Little literature have taken serious consideration in the fixed cost. However, as we will show in the following, the fixed cost affects greatly the formation of clubs. For example, some small clubs might not be economic to exist anymore if the fixed cost is significant.

In our individual objective function in the analysis of individual’s problem, we circumvent the problem of fixed cost by assuming that the clubs will continue to produce until its marginal cost equal to the market price. Actually there should have a start point to produce for the clubs. The following proposition will be used to define it.
Proposition 2-A: A necessary condition for the clubs to produce is that the market price for the club good will not be lower than the lowest average cost of the cost function for the club good.

Proof: Hold the assumption in Case 1. In addition, let’s assume \( p < \text{Min}(\text{ATC}) \). Suppose now the clubs produce \( y \) amount of club good and its member’s consumption bundle is \((y',x')\). Then the budget constraint is \( x + y^*p \leq x' + y'^*p = I' \). Now we can find an available budget constraint \( x + y^*p = I \). In the following, we’ll prove that \( I' < I \).

\[
\text{TC}(y') = y'^*\text{ATC}(y') > y'^*p. \text{ Thus } x' + y'^*p < x' + \text{TC}(y') = I, \text{ i.e. } I' < I. \text{ So we prove that if } p < \text{Min}(\text{ATC}), \text{ there will be a better solution than the one get from the analysis of individual’s problem. In the new position, the club will purchase all the club good with the market price } p. \text{ We will find that this is true for both the rich and the poor clubs. Thus, all clubs want to purchase from the market with the price and nobody want to produce. The market will fail if the price is fixed as } p < \text{Min}(\text{ATC}).
\]

Intuitively, the proposition means if the price is too cheap, it will be more economic to purchase than to produce for all clubs. One important implication of this proposition is that when the fixed cost is very significant in the cost function, even if the MCs are significantly different between rich and poor clubs before trading, i.e. there exists the possibility of trading club goods, the market can’t be formed: to avoid the fixed cost of producing, everybody becomes purchaser and nobody wants to produce club good. In this case, only a non-market arrangement among clubs can solve the problem and realize the mutual benefit trading of club good. From this proposition, we can deduce another proposition directly:

Proposition 2-B: If the market price derived from the section of a market for club goods is higher than or equal to the lowest average total cost of the cost function, then the equilibrium is stable. If the market price is lower than it, the formation of clubs will follow without trading club goods with \( C(y) = \text{min}(\text{ATC})*y \). The difference between optimal \( y \) and ATC-
1(Min(\text{ATC})) will be the amount of club goods traded. (The positive difference means purchase and forms a demand, and the negative means sells and forms a supply.) There may exist some clubs who are pure purchasers to balance the aggregated supply and demand. In the equilibrium the ATC=MC, and might not be within \( C'(y^0_\lambda) \) and \( C'(y^0_\mu) \).

Proof: From the section of a market for club goods, we know that there exists a unique price \( p \) such that \( D(p) = S(p) \) if we don’t consider the fixed cost. If \( p \) is higher than or equal to the lowest average total cost of the cost function, nobody has incentive to quit production and become pure buyers. Thus the equilibrium is table. (See Proposition 2-A and its proof.)

When the fixed cost is included and the market price is lower than the lowest average cost, from Proposition 2-A, we know that the clubs will not produce anymore. Thus the price in the market needs to be raised, say to be \( p^\sim \geq \text{min}(\text{ATC}) \). However, from (10) and (11), we can get \( S(p^\sim) - D(p^\sim) > 0 \), i.e. the society has a net positive supply. There exists a dilemma between economic production and market efficiency: on one hand, the price has to be raised for economic production; on the other hand, the market can’t be liquated with the price. The only way is that some of the clubs who are suppliers before will quit production and become pure buyers. Since when the price is equal to \( \text{min}(\text{ATC}) \), it will not be different whether the clubs get the club good by purchase or production, some clubs will voluntarily quit production and become pure buyers, till \( p^\sim \) is raised to \( \text{min}(\text{ATC}) \). If the price continue to raise and \( p^\sim > \text{min}(\text{ATC}) \), it will better for the poor clubs to produce than to purchase because \( p^\sim = \text{MC} > \text{ATC} \) when \( p^\sim > \text{min}(\text{ATC}) \) and poor clubs can gain from producing to sell, which will increase the supply and pull back the price. In conclusion, when \( p^\sim = \text{min}(\text{ATC}) \), nobody wants to change their decision anymore. The outcome is Nash equilibrium.

Directly following proposition B, we can get the lemmas:
(1) If there is no fixed cost, the formation of clubs will follow the section of a market for club goods. The proof is quite straightforward. When there is no fixed cost, ATC<MC=p. so the budget x’+y’*p=I’ is better than x+y*p=I. The clubs have no incentive to (x’, y’).

(2) In equilibrium, every production unit has the same production, which equal to MC-1(p), where p is market price.

(3) There exists a production floor. Proof: since we know that p>=Min(ATC), the production y0=MC-1(p)>= MC-1(min(ATC)). The production floor of \( \frac{MC-1}{MC} \) can be called as the economic production constraint for the clubs.

(4) The increase of population size will not influence the equilibrium price directly, but will increase the total traded amount in the market. The population size affects the equilibrium by adjusting the demand and the supply.

(5) The existence of non-negative profit in the long run for club with selling.

Proof: The above two propositions not only illustrate the formation of clubs and the equilibrium, but also connect the theory of clubs to the theory of firm. If we compare clubs to firms, the two propositions ensure the clubs have non-negative profit from selling club good in the market (p-=MC=>ATC when p=>min(ATC)). New firms can be created in a market with a positive economic profit, which lead to the zero economic profit in the long run. However, new clubs can’t be created because everybody has been in some club already. On the other hand, nobody wants to sell more club good because all people have already maximized their utilities. Thus, the non-negative profit of clubs with selling can exist in the long run.

The graph in Figure 6 will be used to illustrate the above deduction. In Figure 6-1 we demonstrate a poor club with selling sell when p>Min(ATC). We can see that it has no

---

19 This is a special result when the club has the same cost function. The production of different type clubs can be variable if we allows for difference in cost function, which may be affected by income, geography, or even club size in the real world.
incentive to change to pure buyer because that’ll lower its budget line and the utility level. In Figure 6-2, we demonstrate a rich club that is a purchaser before considering the fixed cost. Then after considering the fixed cost, it will become pure buyer so that its budget line and utility level will go up because the market price is low than Min(ATC). However, as we have talked before, the price will increase till Min(ATC). As shown in Figure 6-2, before considering fix cost, the rich club is a buyer and producer (Status I); After considering fix cost, it will become pure buyer at first (Status II); In the end, after adjusting price in the market, it will go to Status III, where it’s no difference for it to buy or sell. We can see that in Figure 6-1 the seller can earn a profit from selling because p>ATC.
6 (Essay 1) Figure 6-1 No Adjusting when p>Min(ATC) -Poor Clubs/Sellers

7 (Essay 1) Figure 6-2 Adjusting process when p<Min(ATC) -Rich Clubs
the effect on welfare and individual consumption

Based on the above analysis, we can see that comparing to traditional club theory, all the clubs trading in a market for club goods will be better off (i.e. jump to higher indifferent curves).

After trading, the consumption of club good for rich clubs will increase and the consumption of private good for poor clubs will increase. The change in consumption of private good for rich clubs and the change in consumption of club good for poor clubs are ambiguous and determined by the income effect and price effect. The change in size of both rich and poor clubs is ambiguous too.

Furthermore, with simple proof, we can prove nobody wants to move to the clubs of different income levels, i.e. homogeneous clubs are NE.

Proof: In equilibrium in the market, the price p for club good is given by the market. Let’s assume some individuals be forming groups to consume the club good together and the group decision is determined by majority rule. For heterogeneous clubs, let’s assume the decision maker (or median voter) be with income Im. Then the amount of club good to produce, consume, sell or purchase will be determined by equations (4), (5) and (6), which can give us solutions of $x_{m}^{i}, y_{m}^{i}, n_{m}^{i}$ (from equations (7), (8), and (9)), all of which depend on Im. The decision maker’s utility level will be $U_{m}^{i} = u(x_{m}^{i}, y_{m}^{i}, n_{m}^{i})$. However, for a club member with income $I_{i} \neq I_{m}$, his utility level is $U_{i} = u(x_{i}, y_{i}, n_{i})$, where

$x_{i} = I_{i} - C(y_{0}^{m})/n_{m}^{i} + y_{1}^{m}/n_{m}^{i}$.

Now suppose that the individual with income $I_{i}$ is currently in a homogeneous club, then its utility level will determined by itself, $U_{i}^{i} = u(x_{i}, y_{i}, n_{i}^{i})$, which maximizes the utility under budget constraint. We’ll have $U_{i}^{i} \geq U_{i}$, because both have the same utility function and budget constraint and the former maximizes the utility function. Even when $I_{i}=I_{m}$, we still have $U_{i}^{i} = U_{m}^{i}$. Thus, we prove that in equilibrium all club members in homogeneous clubs
have no incentive to move to other clubs no matter homogeneous or heterogeneous clubs. i.e. Homogeneous clubs are NE.

the difference between the market for club good and the market for private good

In the section of a discussion about fixed cost, we connect club theory to firm theory by the economic production constraint. In this section, we will illustrate the difference between the markets for club goods and those for private goods.

(1) The agents in the market for club good are individual clubs, or a group of people, while the agents in normal market are firms and individuals. A club can be the producer, the purchaser/seller, or both. A club is always a consumption unit. Clubs as sellers keep part of production for their own consumption, and clubs as purchasers may produce some club goods for themselves. For example, in the Lakewood Plan, each city of Los Angeles County can contract with the county or other appropriate agencies for municipal services for the city as a whole; it also has the option of producing municipal services for itself. In the private market, the firms produce and sell out all the products, and the individuals purchase and consume the products.

(2) In a private good market, firms form the supply of the private good, and individuals form the demand. In a club good market, demand and supply are both formed by clubs. Whether a club forms a demand or supply depends on its own characteristics and the other clubs’.

(3) Club goods are consumed together, while the consumption of private goods is rival and exclusive to each other.

(4) In a market for private good, the firms produce and sell its product for the purpose of maximizing profit, and the consumers are trying to maximize individual utilities through adjusting their consumption bundles. In a market for club good, all clubs try to maximize the utility of their members. The purpose of trading club goods is to lower the consumption cost
of the club goods. The equilibrium price in a private market depends on the cost function, and there is no economic profit for the firms in the long run. However in a market for club good, the equilibrium price depends on the demand and supply of club good, and the clubs have nonnegative profit after selling.

**membership problem**

(1) actual price and shadow price:

In the equilibrium, the actual total cost of the consumption of club good will equal to the difference between the expenditure of private goods and the income endowment. We call the average consumption cost of club good as the actual price for the club good. In contrast to the actual cost, we can call the market price of club good as the shadow price for the club good.

Proposition 3: The shadow price is not lower than the actual prices for the sellers and buyers. When the shadow price is equal to Min(ATC), the actual price and the shadow price will be the same for all clubs.

Proof: These results can be deduced directly from our analysis in the section of discussion about fixed cost. From Proposition 2-A, we know that the shadow price (or market price) can’t be lower than Min(ATC). When the shadow price is equal to Min(ATC), then the actual price (Min(ATC)) and the shadow price (or market price) are the same; when it’s higher than Min(ATC), the shadow price (or market price) will be higher than the actual price (ATC) because when marginal cost (equal to shadow cost) is higher than Min(ATC), it will higher than ATC (or the actual price).
(2) membership vs. fees for nonmember

The difference between actual price and shadow price is important as we analyze the membership problem in club theory. If we assume that a club provide some amount of club good for members and sell the extra production to the nonmembers, the actual price and shadow price can be used to discern members and nonmembers.\(^\text{20}\) The actual price along with the consumption amount and club size is used to calculate membership, and the shadow price is used to calculate the fee for the nonmembers.

We need to point out one limitation here: the fee is paid by another club and generally its use will depend on its discretion.\(^\text{21}\) The actual cost of the individuals is amount paid as a group divided by the group size. Thus although the shadow price is not lower than the actual price, we can’t know in which club the individual pay less or more per unit for the club good. The fees changed from nonmember will affect the consumption of both club and private

\(^\text{20}\) Note: for a club who is purchaser, there is no problem of members and nonmembers.

\(^\text{21}\) To extend our model to explain the fee paid by individuals, we might assume the seller sets the fee per person such that the individuals who want to purchase with the fee actually form an optimal group to consume together. We need to prove that the fee is optimal for the seller too.
CASE 2: cost function with decreasing marginal cost

If the cost function has a decreasing marginal cost, the analysis will be relatively simple. As we have discussed in the first section, the economy of scale will request that all the production is conducted by single production unit. Similar to the monopoly case, the difficulty here is how to share the cost or define a price. Marginal cost is not suitable to be the price anymore.

Proposition 4: if the club sizes are insignificant to the population size, when there is economy of scale in production, all production will be conducted by one club. The average cost is a Nash equilibrium price.

Proof: Suppose the total production of club good is \( Q = \sum y_i \), where \( y_i \) is the consumption of club good in club \( i \). Now the clubs’ objective function is as that in the first section with the price \( p = ATC(Q) \) and \( C(y) = ATC(Q) \cdot y \). Then we can solve \( y_i \) as a function of \( Q \), say \( y_i = g(Q) \). Then with \( Q = \sum y_i \), we can solve \( Q \) and all \( y_i \). Since the price \( p = ATC(Q) \) is the lowest price the clubs can find either by producing themselves or buying form the others, the buyers will be in the market with the price. Now we will prove that for the one seller, it has not incentive to increase the price. Suppose the seller increases the price to \( p' > p \), then we can find a \( Q' < Q \) such that \( p' = ATC(Q') \). That’s to say, the buyers can form a new production unit and produce \( Q'' \), \( Q > Q'' > Q' \), thus the average cost for the buyers group is \( p'' = ATC(Q'') \), \( p' > p'' > p \). In fact, we can find a \( p'' \) such that \( Q'' = Q - y_s \), where \( y_s \) is the original producer and seller. So this buyers group will be better off after form a new production unit with production of \( Q'' \). Now he average production cost of the old production
unit will become ATC(yₙ)>ATC(Q’’)=p’. So it’s not economic for the seller to increase the price. Thus it’s a Nash equilibrium.²²

In the literature about club theory without trading, the production of club good is constrained by consumption (they are equivalent.). The disutility of crowdedness also prevents the forming of one club for the whole society.²³ Thus the economy of scale could not be made use of. In our model, we don’t have this problem. In Figure 8, we illustrate the unique production unit for all the clubs. We can see that clubs budget line change from the production line to a new one with slope equal to ATC(Q) as well as the price of club good. The utility level is increased, and the consumption of club good is increased. The consumption of private good is ambiguous since the price effect and income effect have different directions. The change in club size will is ambiguous.

²² If the seller’s size is significant, then it has some room to increase the price till it equal the ATC when all the other clubs form a production unit.
²³ Kennedy (1990) argued that when economies of scale are so significant relative to population size in the efficient allocation the entire population is included in a single club. However, such a single club as combination of production and consumption will incur enormous disutility of crowdedness. After separation of consumption and production, the production can still be burdened by a single club and the other clubs purchase from this single club. The trading of club good avoids such kind of crowdedness disutility from forming a single club for the entire population.
CASE 3: cost function with constant marginal cost

When the cost function has a constant marginal cost, the problem reduced to that in the first section with \( C(y) = c + p \cdot y \), where \( p \) is the constant marginal cost. If there is no fixed cost, i.e. \( c = 0 \), then there is not necessary to trade. The model will be the same as that one in Buchanan (1965)’s paper.

If there exist a fixed cost, \( c > 0 \), then the problem will be the same as that in Case 2. (Although the marginal cost is the same, the average cost is decreasing along the production increases. It’s similar to the effect of economy of scale, especially when the fixed cost is high.) In Figure 9, we show when there is a fixed cost and the marginal cost is constant. We can see that the budget line shifts up to pass through \((0, x)\) with the same slope. The utility level is increase, and the consumption of club good and private good is increase due to income effect. Proposition 4 in Case 2 still hold here and the average cost is a Nash equilibrium price.
**Conclusions**

(1) A summary of formation of clubs with trading

Comparing to clubs without trading, with the available cheaper way to get the club goods, the welfare, the consumption of club good and the size of the clubs with trading will change. From the previous analysis, we can know the trading of club goods depends not only on income and utility function, but also on cost function and population distribution. As a summary of our previous analysis, we can make a table of the formation of club with trading.

(We will include capacity constraint here to better fit the real world.)

From Table 1, we can see in most cases (except 9 and 11), trading affect the formation and behavior of clubs. Only in the case of 9 and 11, (fixed cost is insignificant and marginal cost is constant) the trading is not important. Thus, we can’t ignore the trading behavior as most literature did.
# Table 1 A Summary of Formation of Clubs with Trading

<table>
<thead>
<tr>
<th>Market</th>
<th>Without capacity limit</th>
<th>With capacity limit²⁴</th>
</tr>
</thead>
<tbody>
<tr>
<td>Marginal cost</td>
<td>Fixed cost insignificant²⁵</td>
<td>Fixed cost significant</td>
</tr>
<tr>
<td>Increasing</td>
<td>Equilibrium price is between the pre-trade marginal costs of rich and poor clubs</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Sellers: poor clubs</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Buyers: rich clubs</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Both are producers and produce the same amount of club good.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Equilibrium price is equal to Min(ATC). The producer produce the amount of club good.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>It doesn’t matter whether the clubs produce, sell, or purchase. Some clubs will quit production and become pure buyers.</td>
<td></td>
</tr>
<tr>
<td>Decreasing</td>
<td>It will be optimal for one club to produce for all clubs and the average cost is the NE price.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Seller: one club</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Purchaser: all the other clubs</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Same as 5</td>
<td></td>
</tr>
<tr>
<td></td>
<td>The society will have several clubs producing for all the others. Each producer reaches the capacity limit. The average cost is the NE price.</td>
<td></td>
</tr>
<tr>
<td>constant</td>
<td>The average cost is equal to the marginal cost. It doesn’t matter whether the clubs produce, sell, or purchase.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Same as 5 and 6.</td>
<td></td>
</tr>
</tbody>
</table>

²⁴ Capacity limit prevent the production of each club from being ‘too big’.
²⁵ Insignificant fixed cost represents there is no fixed cost or the fixed cost doesn’t large enough to affect the formation of clubs, while significant one means it has effects.
(2) Club production and a market for club good v.s. those for private good

In the section of A Model of Trading Club Goods, we connect them together and also distinguish them. In a word, we find the economic production constraint for the club production and then we find the club good markets under different situations.

(3) A market for club good v.s. private provision of club good.

In our model we illustrate a model for the trading of club goods, or the market for club good. Many literatures have been done about the private provision of club good. It will be interesting to compare our analysis to these literatures. The most significant connection of them is that both treat the production and the consumption separately, which should not be according to traditional club theories.

The second connection is that in our model, some clubs purchase club good from the others. The part purchased can be treated as the private provision of club good (from the other clubs in the market), i.e. our model comprehend the idea of private provision of club good.

Although these significant connections, there are much more difference between them. First, the frameworks are different. The private provision club good theory mostly is based on the idea that the private sector is more efficient and then separates the producers (firms) from the consumers (clubs), while in our model both producers and consumers are clubs. What’s more, we allows for the clubs to be both producers and consumers at the same time. This difference derived the second difference-- the models have different setups. Third, our model derives the market of club good and gets the equilibrium price and consumption bundle, which can’t be gotten from the private provision models. Fourth, many literatures of the private provision theories are still struggling in the problem whether it is better or feasible for the market to provide the club good. We have no this problem since the market is endogenous in our model. Especially for those club goods without appropriate private markets, our model
shows that there exists another market for them which can improve efficiency. Many case 
studies about contracting out government services among governments had already shown us 
how this will happen in the real world.

(4) Club as a consumption group v.s. clubs as a production group

Some literature argued clubs are formed by individuals for the purpose of sharing the cost 
and many of them assumed that the cost function is constant or decreasing. However, as we 
discuss before, this assumption is unnecessary: whether the marginal cost is increasing, 
decreasing, or constant, the optimal size can be reached anyway. What’ more, after allowing 
for trading among clubs, the cost functions will become complicated and even be dependent 
on the market for club good. Thus in general we shouldn’t have this assumption.

Thus the traditional idea that clubs are formed because of economy of scale in production 
is in doubt. As we can see from above analysis, the trading of club goods also separates the 
production from the clubs when they are pure purchasers. So the only reason for club’s 
formation is the sharing of consumption. Thus we can give a definition for club: a group 
which is formed for the purpose of sharing the consumption of the club goods. The clubs 
happen to produce if it’s more efficient. As Ostrom, Tiebout and Warren (1961) said: “…the 
production of (public) goods and services needs to be distinguished from their provision at 
public expense…So, a public agency by contractual arrangements with private firms-or with 
other public agencies-can provide the local community with public services without going 
into the business of producing them itself…The separation of the provision of public goods 
and services from their production opens up the greatest possibility of redefining economic 
functions in a public service economy…The separation of production from provision may 
also have the consequence of turning local governments into the equivalents of associations 
of consumers.”
(5) A smooth demand/supply curve

In our model, we simplify the problem by assuming there be only two income levels, the poor and the rich, in the society. However, our main results in this paper will still hold without this assumption. The key point in our analysis is the marginal cost of the production of club goods. It should be always hold that the marginal costs are equal through all the clubs who are producing. With the assumption that the cost functions are the same, all of the producers will have the same output. The difference between their output and their consumption is the amount produced to trade. When the income distribution is continuous in the society, we can get smooth demand/supply curves which are similar to those of the competitive private market.

(6) Multiple levels of clubs

As we can see, some clubs will connect tightly to each other due to the trading among them. If we treat them as a special group, it can be called as a big club of small clubs. In the real world, when the transaction cost is high, it is highly possible that they establish some institutions (member clubs, state-local jurisdiction, international organization, etc) to be responsible for the trading. For example, in the state and local economics, this can be some kind of state and local arrangements, such as revenue sharing scheme, or inter-governmental transfer scheme, etc. Obviously these institutional arrangements are important and will affect the formation of clubs.

We have talked several aspects of trading of club goods in the paper, however there are still many interesting research topics in the future: 1. What happens in a world where the club goods are non exclusive to nonmembers? 2. What happens if the cost sharing scheme is based on individual income while the cost is not equally shared by all the members? 3. In the real world, many clubs are bounded not only by size but also by geographic conditions, thus we
might want to develop a model to reflect the spatial pattern of clubs with trading and the associated market for club good.

**An Analysis Framework for the Formation of Jurisdictions**

As we develop a model of clubs with trading, it can be applied in the jurisdiction formation directly. As we’ll show later, the trading of club good will affect the formation process of jurisdictions and the interaction among them.

**A Model for Jurisdiction Formation**

Suppose there are two groups of people, the rich and the poor, in which the individuals have the endowment income of $I^r$ and $I^p$ respectively. The population size of the rich group is $n^r$, and the poor population size is $n^p$, i.e. the society’s total population is $n = n^r + n^p$.

Furthermore, we assume that all people of the same income level will stay in one group. Thus they’ll form two jurisdictions by income level. With these assumptions, we can start to analyze the interaction between these two jurisdictions.

Now we can have three kinds of jurisdiction structures for the two groups of people:

1. The rich and the poor form their jurisdictions independently, and there is no relation between the jurisdictions. We call it the *parallel structure*.

2. The rich and the poor groups form a single jurisdiction. We call it the *single structure*.

3. The rich and the poor form their own jurisdictions. However, there is some kind of cooperation between them, such as sharing or trading local public goods. In many cases, for the purpose of better cooperation, the two jurisdictions will form a higher level jurisdiction in charge of the cooperation. We call it the *dependent structure*.

---

26 The literature in migration had shown that the same type of residents tend to stay together in the real world. With this assumption, we try to analyze the interaction among different groups.
If we assume the individual’s consumption of the private good X is x, and that of consumption of the local public good Y is y.\textsuperscript{27} The objective functions of the residents in the jurisdictions with the jurisdiction structures above are as follows respectively:\textsuperscript{28}

1. parallel structure:
   \[
   \begin{align*}
   \text{Max} & \quad u_p(x, y) \\
   \text{s.t.} & \quad x + C(y)/n = I
   \end{align*}
   \]

   Thus we can solve the objective function and get the optimal choices are \(x'_s, y'_s, \text{and} \ u'_s\) for the rich jurisdiction and \(x'_p, y'_p, \text{and} \ u'_p\) for the poor jurisdiction.

2. single structure:
   If this case, we assume the majority rule is used to determine the level of local public goods. When the rich population size is bigger, then the objective function will be:
   \[
   \begin{align*}
   \text{Max} & \quad u'_r(x, y) \\
   \text{s.t.} & \quad x + C(y)/n = I' \\
   & \quad n' > n^p
   \end{align*}
   \]

   We can solve it and get \(x'_r, y'_r, \text{and} \ u'_r\) for the rich residents, where \(y'_r\) is the consumption of Y for the jurisdiction. For the poor residents, they have no choice, \(y'_p = I^p - C(y'_p)/n, \text{and} \ u'_p\).

   When the poor population size is bigger, then the solutions will be
   \[
   x'_p = I' - C(y'_p)/n, y'_p, \text{and} \ u'_p \text{ for the rich and } x'_p, y'_p, \text{and} \ u'_p \text{ for the poor.}
   \]

\textsuperscript{27} We assume that the local public good Y is exclusive to the residents outside the jurisdiction.

\textsuperscript{28} Compared to the objective functions in section 2, we will find that there is not the adjusting process of jurisdiction size here.
3. dependent structure:

In this case, for the rich jurisdiction, we have

$$\begin{align*}
\text{Max}_{x,y} & \quad u_d^r(x,y) \\
\text{s.t.} & \quad x + C(y_0)/n^r + y_1 \cdot p/n^r = I^r, \\
& \quad y = y_0 + y_1.
\end{align*}$$

For the poor jurisdiction, we have

$$\begin{align*}
\text{Max}_{x,y} & \quad u_d^p(x,y) \\
\text{s.t.} & \quad x + C(y_0)/n^p + y_1 \cdot p/n^p = I^p, \\
& \quad y = y_0 + y_1.
\end{align*}$$

Where $y_1^r + y_1^p = 0$. When the marginal cost function is increasing, $p = C'(y_0^r) = C'(y_0^p)$ if $y_0^r, y_0^p$ exist; when it is constant or decreasing,

$$p = ATC(y^r + y^p).$$

After solving the objective function together, we can get $x_d^r, y_d^r, and u_d^r$ for the rich and $x_d^p, y_d^p, and u_d^p$ for the poor.
**Jurisdiction Formation Process**

From our analysis in the previous section, we know that the parallel structure is dominated by the dependent structure. So we only need to compare the latter two structures.

2 (Essay 1) Table 2 Dependent and Single Jurisdiction Structure

<table>
<thead>
<tr>
<th></th>
<th>Dependent structure</th>
<th>Single structure$^{29}$</th>
<th>Difference$^{30}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Poor group</td>
<td>$u_d^p$</td>
<td>$u_{si}^p$</td>
<td>$D^p = u_d^p - u_{si}^p$</td>
</tr>
<tr>
<td>Rich group</td>
<td>$u_d^r$</td>
<td>$u_{si}^r$</td>
<td>$D^r = u_d^r - u_{si}^r$</td>
</tr>
</tbody>
</table>

We will have the following statements:

(1) If both $D^p$ and $D^r$ are positive, then dependent structure will be dominant.

(2) If both $D^p$ and $D^r$ are negative, then single structure will be dominant.

(3) If one is positive, and the other one is negative, the dependent structure is dominant if there is no income transfer between the groups. However it’s possible for the group with negative difference to pay an amount of money, $T$, to remedy the other group so that the two groups can merge to a big single group.$^{31}$

Statements (1) and (2) are quite straight forward. The two groups have the same tendency towards either the dependent structure or the single structure. Now let’s see statement (3).

---

$^{29}$ When the rich population size is larger, $i=1$; otherwise, $i=2$.

$^{30}$ Whether the difference is positive or negative depends on two effects: (1) collective consumption efficiency (or crowdedness of collective consumption); (2) production efficiency. For example, when $MC$ is increasing and there is no fixed cost, if we don’t consider collective consumption efficiency, then the dependent structure is preferred because it has lower production cost. However, when we take collective consumption efficiency into consideration, if both group sizes are far smaller than optimal sizes, then it might be better for them to form a single group. Thus on one hand they have higher production efficiency to produce separately; on the other hand, they have higher consumption efficiency to consume the club good together. The final outcome will depend on the tradeoff of these two effects.

$^{31}$ In our model we assume that either group has the right to choose its status. If this is not the case, i.e. either group’s status depends on the agreement of the other group, then we’ll be faced with an opposite case: the group with positive difference will pay money to the other group and keep it out.
Suppose the rich club transfer \( T \) amount of money to the poor. (When \( T \) is negative, it means the rich receives \( T \) amount of money from the poor.) \( T \) belong to \((-I_p, I_r)\).

Max \( U_r(x, y, nr+np) \) if \( nr>np \)

\[
s.t. \ x+c(y)/(nr+np)=Ir-T
\]

We can solve and get \( x_m, y_m, \) and \( U^r_m(x_m, y_m^r, nr^r + np^r), U^p_m(x_m, y_m^p, nr^p + np^p) \), where

\[
y_m^p = y_m^r, \text{ and } x_m^p = I^p + nr^p/np^p * T - c(y_m^p)/(nr^p + np^p).
\]

Max \( U_p(x, y, nr+np) \) if \( nr<np \)

\[
s.t. \ x+c(y)/(nr+np)=Ip+nr^p/np^p * T
\]

We can solve and get \( x_m^p, y_m^p, \) and \( U^r_m(x_m, y_m^r, nr^r + np^r), U^p_m(x_m, y_m^p, nr^p + np^p) \), where

\[
y_m^r = y_m^p, \text{ and } x_m^r = I^r - T - c(y_m^p)/(nr^r + np^r).
\]

Suppose there exists a set of solutions for \( T \), say \( T \), within \(-I_p \) and \( I_r \), such that \( U^r_m > U^r_a \) and \( U^p_m > U^p_a \), then we will have:

(1) If \( T \) includes 0, then both groups prefer to the single structure. No intergroup income transfer is needed. The final outcome is single structure.

(2) If \( T \) is a positive set, then the rich pays the money to the poor; on the other hand, if \( T \) is a negative set, the poor pays the money to the rich. The final outcome is single structure with intergroup income transfer.

(3) If \( T \) is empty, then the final outcome is dependent structure.

Conclusions
From the analysis above, we know there are four outcomes of the interaction between the two groups of people: (A) a dependent structure without income transfer (both groups prefer to it); (B) a dependent structure without income transfer (one group prefers to it, but the other one prefers to the single structure); (C) a single structure without income transfer (both
groups prefer to it); (D) a single structure with income transfer (different preferences to structures before transfer, but with income transfer, both groups prefer to a single structure).

If a higher level jurisdiction is formed to lower transaction cost of trading or income transfer, we can have three types of hierarchy government structures:

(1) From outcome (A) and (B), we’ll have a government structure as follows: a higher level government in charge of trading among local governments which provide local public good within their jurisdictions;

(2) From outcome (C), we’ll have a government structure as follows: a higher level government in charge of local public good provision for all people within its jurisdiction.

(3) From outcome (D), we’ll have a government structure as follows: a higher level government in charge of income transfer among local governments and also in charge of local public good provision for all people within its jurisdiction;
Essay 2: Food Consumption in Jamaica: A Household Behavior as well as a Social Behavior

Literature Review and Motivation

Engel’s curve is used to show the relationship between consumption and income (ceteris paribus). The literature on Engel’s curve traces back to Engel (1857). Based on surveys of families’ budgets and spending patterns, Engel found that the income elasticity of the demand for food was relatively low. This is the well-known Engel’s law: As incomes increase, the share of expenditures for food declines.

Engel’s curve is important because of its wide application. It can be used to estimate income elasticity of various household consumptions, including food. It can be used to measure the economic development level of a region or a country. Poor countries usually have higher Engel’s coefficients in such studies, and rich countries have lower ones. In research on poverty, Engel’s curve helps draw a poverty line in a region or a country (Rowntree 1901, Fisher 1992, Ravallion & Bidani 1994, Boltvinik 1998).

A huge literature has grown up about Engel’s curve. Most of it shares a common assumption about the independence of consumption behavior. This assumption says, that people (or a family or a household, etc.) will be unaffected by the behavior of others. However, the recent literature takes more and more interest in interactive behavior. These studies in interdependent preferences show that preferences and choices of behavior are influenced not only by an individual’s own tastes but also by the tastes of others. Such interactions are called, among others terms, “social norms,” “bandwagons,” “neighborhood effects,” “peer influences,” “conformity,” and “herd behavior” (Hyman, 1942; Merton, 1957; Granovetter, 1979; Manski, 2000).

Efforts to estimate the interdependence of preference have absorbed many economists lately. Alessie and Kapteyn (1991) modeled this interdependence by making current budget
shares of a household dependent upon mean budget shares in the reference group of this household. Kapteyn, Geer, Stadt and Wansbeek (1997) modeled it by making parameters in the Linear Expenditure System (LES) dependent upon current quantities in the reference group of a household. In our paper, we find that the spatial econometric technique can be very useful as a way to estimate interdependence. The development of spatial econometrics enables economists to estimate the magnitude of the interactions among neighbors and thus has become one of the most popular econometric techniques. It’s been used to study interdependent preferences in consumer expenditure (Darough, Pollak and Wales, 1983; Alessie and Kapteyn, 1991; Case, 1991; Kaptyen et. al., 1997), labor supply (Aronsson, Blomquist and Sacklen, 1999), political science (Smith and LeSage, 2000; Darmofal 2006), poverty policy (Daimon, 2001), marketing (Yang and Allenby, 2001; Bronnenberg, 2004), and public finance/taxation (Franzse and Hays, 2005).

This paper uses JSLC 2001 data to create spatial econometric models to estimate an Engel’s curve for food consumption in Jamaica. Effects to be considered are household factors (household income, household size, household structure, etc.) as well as the effects of social factors (income level in the society, neighborhood’s food consumption, etc.). Various forms of Engel’s curve have been estimated, and this paper uses the Working-Leser share expenditure system (Working, 1943; Leser, 1963), which has been developed into the widely used Almost Ideal Demand System (AIDS) by incorporating price variables (Deaton & Muellbauer, 1980). The application of the Working-Leser system enables us to compare our estimation to the recent literature. Meanwhile, a preliminary exploration shows that the interdependence of household food consumption might exist in terms of Engel’s coefficient, and the interdependence is ambiguous and inconsistent in terms of food consumption amount (or its log form). Because we’ll estimate an Engel’s curve for household food consumption and the other expenditures are not our interest, the Working-Leser system can be simplified
as (without price variables):

Engel's coefficient = constant + b*\log(\text{total income}) + c*\text{other variables} + \text{residual}.

(where b and c are coefficients, the residual is i.i.d.)

The paper is organized as follows: the second section introduces the spatial models and the model specification; the third section introduces the data and defines the weight matrix; the fourth section reports the estimations of the spatial models and compares them; the fifth section discusses and summarizes the results of the estimations; and the last section assesses the policy implications of our estimations.

**Spatial Models and Diagnostics Methods**

**Spatial Models**

Spatial models can be used to estimate the interdependence or correlation among neighborhoods. The widely used general form of spatial models can be expressed as (SAC):

\[
y = x\beta + \rho wy + u, \\
u = \lambda wu + \epsilon, \\
\epsilon \sim i.i.d.
\]

where \(y\) is a dependent variable, \(x\) is a covariate, and \(u\) and \(\epsilon\) are error terms. \(w\) is a predefined weight matrix, \(wy\) and \(wu\) are the spatial lag and spatial error terms. \(\rho\) and \(\lambda\) will measure the spatial correlation (or interdependence) among neighborhoods. A positive (negative) sign of \(\rho\) or \(\lambda\) reflects the positive (negative) spatial correlation among the neighborhood’s \(y\)s or errors. In the spatial models, the weight matrix is crucial, and neighbors affect each other through the spatial lag and/or spatial error terms.

The interaction among neighborhood behaviors has three possibilities (Manski, 2000). These are (1) endogenous interactions: neighbors affect each other’s behavior directly; (2) contextual interactions: the behavior of people is affected by the exogenous variables of their neighbors; and (3) correlated effects: neighborhoods tend to behave similarly because of common characteristics or similar environments.
In empirical work, contextual interactions are typically assumed not to exist. In other words, researchers assume that people will not be affected by the exogenous variables of their neighbors. Thus, the interactions of the behaviors among neighbors will be either endogenous interactions or correlated effects. In the general spatial models described above, the endogenous interactions can be estimated by the coefficient of the spatial lag term ($\rho$), and the correlated effects can be estimated by the coefficient of the spatial error term ($\lambda$).

If $\lambda = 0$, the spatial model can be reduced to a spatial autoregressive model or to spatial lag model (SAR):

\[
y = x\beta + \rho wy + u, \\
\epsilon \sim i.i.d.
\]

If $\rho = 0$, the spatial model can be reduced to a spatial autoregressive error model or spatial error model (SEM):

\[
y = x\beta + u, \\
u = \lambda wu + \epsilon, \\
\epsilon \sim i.i.d.
\]

If $\rho = 0$ and $\lambda = 0$, it’s a normal linear model and the ordinary least square model (OLS) model is appropriate. However, for spatial models, OLS will be either biased or inefficient. If the spatial lag term is ignored but $\rho$ isn’t equal to zero, then the OLS estimate will be biased and inconsistent. Conversely, if the spatial error term is ignored but $\lambda$ isn’t equal to zero, then the OLS will be inefficient because of the heterogeneity in errors. The spatial models can be estimated by the maximum likelihood estimation (MLE), the generalized method of moments (GMM) or other econometric methods. However, before estimating the spatial models, we need to identify the real forms of the spatial models. Several statistical tests for this are available.
Diagnostics for Spatial Dependence
Lagrange Multiplier tests

There are two basic LM tests: The first one is a test for spatial lag dependence, and the other is for spatial error dependence. According to Anselin and Rey (1991), the two LM tests take the following forms:

LM test for spatial lag dependence:

$$LM_{\text{lag}} = [N(e'e)'e'N]^{-1} [N(W_i X \hat{\beta})' M(W_i X \hat{\beta}) + e'e' + \text{tr}(W_i W_i + W_i^2)]$$

LM test for spatial error dependence:

$$LM_{\text{error}} = [N(e'e)'e'N]^{-1} [\text{tr}(W_2 W_2 + W_2^2)]$$

where $N$ is the number of observations, $e$ is the OLS residuals, $M = I - X'(X'X)^{-1}X'$, $\hat{\beta}$ is the OLS estimate of $\beta$, and $W_1$ and $W_2$, respectively, are the weight matrixes for the spatial lag and spatial error terms.

Both tests share the hypothesis (H0) of no correlation ($\rho = 0$ and $\lambda = 0$ respectively). When we reject H0 by either the LM lag test or the LM error test, we can expect either spatial lag dependence or spatial error dependence. We can use MLE, GMM or other methods to estimate the specified spatial models.

Robust Lagrange Multiplier tests

One main limitation of the LM tests is that they are nonnested. That is, they will reject H0 even if $\rho = 0$ (or $\lambda = 0$), if spatial error (or spatial lag) dependence exists $\lambda = 0$ (or $\rho = 0$). The robust Lagrange Multiplier tests overcome this limitation by accounting for any spatial error (or spatial lag) when testing for spatial lag (or spatial error) dependence. This more robust LM application also accounts for the noncentrality problem of LM tests. (Anselin, Bera, Florax, & Yoon, 1996) However, robust LM tests have less reduced power than the LM lag
(or error) test when no spatial error (or spatial lag) dependence exists. Robust LM tests take these forms:

Robust LM test for spatial lag dependence:

\[ \text{LM}_{\text{lag}} = \frac{(e'W_y e/s^2 - e'W_e e/s^3)^2}{(N\tilde{J}_{1p,\beta})^{-1} - t_1} \]

Robust LM test for spatial error dependence:

\[ \text{LM}_{\text{error}} = \frac{[(e'W_e e/s^2 - t_2) - (N\tilde{J}_{1p,\beta})^{-1}(e'W_y e/s^3)]^2/[t_2 - t_2^2(N\tilde{J}_{1p,\beta})^{-1}]} \]

where \( s^2 = e'e/N \), \( (N\tilde{J}_{1p,\beta})^{-1} = [t_1 + (W_1 X\beta)' M (W_1 X\beta)/s^2]^{-1} \), \( M = I - X'X)^{-1} X' \),

\( t_1 = \text{tr}(W_1'W_1 + W_1^2) \), \( (N\tilde{J}_{2p,\beta})^{-1} = [t_2 + (W_2 X\beta)' M (W_2 X\beta)/s^2]^{-1} \), \( t_2 = \text{tr}(W_2'W_2 + W_2^2) \).

Moran’s I test

\[ I = \frac{N e'W_e e}{S e'e} \]

Moran’s I test also is used to test for spatial error dependence. However, it is powerful against both error and lag dependence and consequently should be used conservatively.
Data: Jamaica Survey of Living Conditions

Data Introduction and Summary
Our data comes from the Jamaica Survey of Living Conditions 2001. It's a survey of randomly selected households and contains 1,668 observations. The survey covered 14 parishes, 58 constituencies, and 159 districts. The following is a list of variables we’ll use in our model:

**Engel:** Engel’s coefficient of household and equal to total household food consumption divided by total household expenditure. This is the independent variable.

**totfood:** Total household food consumption.

**totexp:** Total household expenditure. In this paper, total household expenditure is assumed to be equivalent to total household income.

**logexp:** Equal to the log of total household expenditure.

**popdec:** Deciles of household’s total per capita consumption in the society. This paper uses it to reflect the household’s social status.

**popexp:** Interaction of popdec and logexp.

**sex:** Gender of the head of household.

**logsize:** Log of household size (label as size).

**marr1-5:** Dummy variables for the five indicators of marriage status: “Married,” “Never married,” “Divorced,” “Separated,” and “Widowed.”

**union1-5:** They are dummy variables for five marriage statuses: “Married,” “Common Law,” “Visiting” “Single,” and “None.”

**adsex1-4:** Number, respectively, of male adults, female adults, male children, and female children.

A summary of the data is in Table 1 and Table 2. There are also several graphs. Figure 1 shows the household expenditure structure in Jamaica. Figure 2 displays a pattern of negative correlation between Engel’s coefficient and household total expenditure, which is exactly what we expect according to Engel’s law. After the log transform of household total expenditure, a linear correlation between Engel’s coefficient and household total expenditure comes up. Meanwhile, we also see that the Engel’s coefficient of a group decreases as population deciles measured by household total expenditure (or income) goes up.
### 3 (Essay 2) Table 1 A Summary of Data (1)

<table>
<thead>
<tr>
<th></th>
<th>N</th>
<th>MIN</th>
<th>MAX</th>
<th>MEAN</th>
</tr>
</thead>
<tbody>
<tr>
<td>Engel</td>
<td>1665</td>
<td>3.09%</td>
<td>93.88%</td>
<td>48.77%</td>
</tr>
<tr>
<td>totfood</td>
<td>1665</td>
<td>888</td>
<td>706,866</td>
<td>118,980</td>
</tr>
<tr>
<td>totexp</td>
<td>1668</td>
<td>7,872</td>
<td>4,137,979</td>
<td>284,186</td>
</tr>
<tr>
<td>size</td>
<td>1668</td>
<td>1</td>
<td>28</td>
<td>3.42</td>
</tr>
<tr>
<td>adsex1</td>
<td>1668</td>
<td>0</td>
<td>8</td>
<td>1.10</td>
</tr>
<tr>
<td>adsex2</td>
<td>1668</td>
<td>0</td>
<td>6</td>
<td>1.18</td>
</tr>
<tr>
<td>adsex3</td>
<td>1668</td>
<td>0</td>
<td>7</td>
<td>0.61</td>
</tr>
<tr>
<td>adsex4</td>
<td>1668</td>
<td>0</td>
<td>7</td>
<td>0.53</td>
</tr>
</tbody>
</table>

### 4 (Essay 2) Table 2 A Summary of Data (2)

<table>
<thead>
<tr>
<th>sex</th>
<th>N</th>
<th>totfood</th>
<th>totexp</th>
<th>Engel</th>
</tr>
</thead>
<tbody>
<tr>
<td>Male</td>
<td>944</td>
<td>117,090</td>
<td>285,428</td>
<td>49%</td>
</tr>
<tr>
<td>Female</td>
<td>719</td>
<td>121,223</td>
<td>282,838</td>
<td>49%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>union</th>
<th>N</th>
<th>totfood</th>
<th>totexp</th>
<th>Engel</th>
</tr>
</thead>
<tbody>
<tr>
<td>others</td>
<td>19</td>
<td>159,076</td>
<td>365,500</td>
<td>48%</td>
</tr>
<tr>
<td>1</td>
<td>465</td>
<td>139,473</td>
<td>369,193</td>
<td>45%</td>
</tr>
<tr>
<td>2</td>
<td>239</td>
<td>140,946</td>
<td>310,404</td>
<td>51%</td>
</tr>
<tr>
<td>3</td>
<td>288</td>
<td>113,823</td>
<td>259,585</td>
<td>48%</td>
</tr>
<tr>
<td>4</td>
<td>362</td>
<td>95,908</td>
<td>226,259</td>
<td>51%</td>
</tr>
<tr>
<td>5</td>
<td>290</td>
<td>98,568</td>
<td>218,388</td>
<td>51%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>marr</th>
<th>N</th>
<th>totfood</th>
<th>totexp</th>
<th>Engel</th>
</tr>
</thead>
<tbody>
<tr>
<td>Others</td>
<td>6</td>
<td>119,088</td>
<td>230,969</td>
<td>53%</td>
</tr>
<tr>
<td>1</td>
<td>507</td>
<td>138,575</td>
<td>366,855</td>
<td>45%</td>
</tr>
<tr>
<td>2</td>
<td>940</td>
<td>110,354</td>
<td>244,539</td>
<td>51%</td>
</tr>
<tr>
<td>3</td>
<td>18</td>
<td>121,147</td>
<td>366,324</td>
<td>40%</td>
</tr>
<tr>
<td>4</td>
<td>29</td>
<td>114,373</td>
<td>312,076</td>
<td>42%</td>
</tr>
<tr>
<td>5</td>
<td>163</td>
<td>107,115</td>
<td>244,862</td>
<td>49%</td>
</tr>
</tbody>
</table>
Figure 1: Household Expenditure Structure (Jamaica, 2001)
12 (Essay 2) Figure 2-1 Plots for Totexp, Totfood, and Engel's Coefficient

13 (Essay 2) Figure 2-2 A Plot for Totexp, Totfood, and Engel’s Coefficient by Popdec
**Weight Matrix**

To account for the neighborhood effect, we used spatial econometric techniques. With its help, we can determine whether there is an interaction of consumption behavior among neighbors and the degree of such interaction. To apply spatial econometric techniques, however, we must first define a suitable weight matrix, which will be used to identify the neighbors or measure the magnitude of influence among subjects. Most literature defines a weight matrix either by geographical proximity or by similarity in characteristics (such as income). For example, in using a weight matrix defined by ge proximity, subjects are considered connected to each other geographically, and thus the weight matrix is defined by either sharing the same border or the inverse of the distance among subjects. In our paper, we want to define a weight matrix that could reflect social connections among households that group to form a neighborhood/jurisdiction. These households in the same neighborhood might affect each other’s behaviors for various reasons that we have referred to previously. Thus, in our paper, we define a neighborhood as consisting of people within a certain jurisdiction.

Jamaica is divided into 14 administrative regions called parishes. Each of these parishes is then subdivided into several constituencies (typically four constituencies in a parish: northwest, north east, south west, and southeast). An enumeration district (ED) is a group of dwellings established for the national census. Figure 3 describes the Jamaican jurisdictional structure. In this paper, people living in the same neighborhood are expected to tend to interact with each other, and EDs are considered a suitable neighborhood for such interaction. Thus, our paper defines a neighborhood as households within the same enumeration district (ED).
Within the same district, defined as \( w \)

Neighborhood is defined as households within the same district.
Measuring the neighborhood effect requires a weight matrix: A unit weight is assigned to all households within the same district; otherwise, the assigned weight is 0. Then the weight matrix is standardized so that for each household the sum of the weights is equal to one. Table 3 is an example of the weight matrix.

**5 (Essay 2) Table 3 An Illustration of Weight Matrix defined by Eds**

<table>
<thead>
<tr>
<th>Weight Matrix</th>
<th>ED1:Dwelling 1</th>
<th>ED1:Dwelling 2</th>
<th>ED1:Dwelling 3</th>
<th>ED2:Dwelling 1</th>
<th>ED2:Dwelling 2</th>
<th>ED2:Dwelling 3</th>
<th>...</th>
</tr>
</thead>
<tbody>
<tr>
<td>ED1: Dwelling 1</td>
<td>0</td>
<td>1/2</td>
<td>1/2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>ED1: Dwelling 2</td>
<td>1/2</td>
<td>0</td>
<td>1/2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>ED1: Dwelling 3</td>
<td>1/2</td>
<td>1/2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>ED2: Dwelling 1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1/2</td>
<td>1/2</td>
<td>0</td>
</tr>
<tr>
<td>ED2: Dwelling 2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1/2</td>
<td>0</td>
<td>1/2</td>
<td>0</td>
</tr>
<tr>
<td>ED2: Dwelling 3</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1/2</td>
<td>1/2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>...</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>...</td>
</tr>
</tbody>
</table>

**Model Specification and Estimation**

**Model Specification**

We undertook to estimate whether a household’s food consumption as a ratio to total household expenditure will be affected by the household’s characteristics such as household income, head of household structure, and other factors and also whether this ratio will be affected by its neighborhood. Thus, our dependent variable will be the ratio of household food consumption to household total expenditure, or in other words, the household Engel coefficient. Our explanatory variables will include variables such as household total expenditure (household income), household income deciles in the society, household size, marriage status, union status, and household structure (numbers of adults and children by gender). The models are also integrated with the spatial (spatial lag/error) terms to estimate the neighborhood effect. A summary of the factors affecting the household Engel’s coefficient is as follows:

a. Household factors: log of household income, marriage status, union status, and household structure. A huge of literature has discussed the effect of household factors on household consumption behavior.
b. Social factors: household income deciles in the society; spatial effects from neighbors.

Fan and Abdel-Ghany (2004) tested the importance of integrated permanent and relative income model in explaining consumer expenditure behavior and argued that both are important determinants of household expenditure behavior even in the presence of the other. Thus in our model we will integrate the variable of household income deciles and estimate its effect on household consumption behavior. The spatial effects have been discussed previously in this paper.

**Diagnostics for Spatial Lag and Spatial Error Dependence**

Before we can choose a spatial model for our analysis, we first need to test statistically for spatial dependence. These tests for spatial dependence use Moran’s I test for spatial dependence, the LM tests and Robust LM tests for spatial lag and spatial error dependence.

**A Preliminary OLS Regression**

All of these tests assumed that without spatial dependence, the model would be linear and OLS would be appropriate. First estimating the model by OLS yields the result shown in Table 4 in which variables for sex, marr1, marr3, marr4, union1, union2, union3, union4, adsex2, adsex4 are highly insignificant. Thus, these variables can be removed and a regression run for a reduced model, as in Table 5, with the result that F test, Root MSE, and Adjust R-Squared are all improved. Because of this improved performance, the reduced model will be used in the further steps of diagnosis and estimation of spatial models.
6 (Essay 2) Table 4 An OLS Estimate

| Engel Coefficient | Std. Err | t   | P>|t| |
|-------------------|----------|-----|------|
| _cons             | 1.671782 | 0.175809 | 9.51 | 0.001 |
| logexp            | -0.112315 | 0.016671 | -6.74 | 0.001 |
| popdec            | 0.123040 | 0.016512 | 7.45 | 0.001 |
| popexp            | -0.008947 | 0.001401 | -6.39 | 0.001 |
| sex               | -0.003050 | 0.007874 | -0.39 | 0.699 |
| logsize           | 0.096417 | 0.016575 | 5.82 | 0.001 |
| marr1             | 0.001892 | 0.019200 | 0.10 | 0.922 |
| marr2             | 0.026955 | 0.011360 | 2.37 | 0.018 |
| marr3             | -0.013359 | 0.031042 | -0.43 | 0.667 |
| marr4             | -0.028455 | 0.025116 | -1.13 | 0.257 |
| union1            | -0.020555 | 0.018552 | -1.11 | 0.268 |
| union2            | 0.000230 | 0.012346 | 0.02 | 0.985 |
| union3            | -0.007818 | 0.011181 | -0.70 | 0.485 |
| union4            | -0.005959 | 0.009977 | -0.60 | 0.550 |
| adsex1            | 0.015117 | 0.004977 | 3.04 | 0.002 |
| adsex2            | -0.000571 | 0.005547 | -0.10 | 0.918 |
| adsex3            | 0.006069 | 0.004790 | 1.27 | 0.205 |
| adsex4            | -0.000062 | 0.004970 | -0.01 | 0.990 |

7 (Essay 2) Table 5 An OLS Estimate for the Reduced Model

| Engel Coefficient | Std. Err | t   | P>|t| |
|-------------------|----------|-----|------|
| _cons             | 1.677970 | 0.170320 | 9.85 | 0.001 |
| logexp            | -0.114510 | 0.016189 | -7.07 | 0.001 |
| popdec            | 0.122163 | 0.016127 | 7.57 | 0.001 |
| popexp            | -0.008858 | 0.001361 | -6.51 | 0.001 |
| logsize           | 0.095705 | 0.013343 | 7.17 | 0.001 |
| marr2             | 0.036624 | 0.006386 | 5.73 | 0.001 |
| adsex1            | 0.015723 | 0.003917 | 4.01 | 0.001 |
| adsex3            | 0.006690 | 0.004325 | 1.55 | 0.122 |
Diagnostics for Spatial Lag and Spatial Error Dependence

Diagnostic tests using Moran’s I, LMs and Robust LM tests for spatial error and spatial lag dependence were run on the model.\textsuperscript{32} Moran’s I test yielded a p value lower than 0.001, which reflects the existence of spatial dependence. Moreover, not only the LM tests for spatial lag/error dependence, but also their robust counterparts showed a high degree of spatial lag and error dependence in the model. Accordingly, based on our discussion in the previous section, we conclude that both spatial lag and spatial error dependence are present to a significant degree. Thus, a general spatial model (or SAC) as described before is suggested.

\textbf{8 (Essay 2) Table 6 Diagnostic Tests}

<table>
<thead>
<tr>
<th>Diagnostics Tests</th>
<th>Statistic</th>
<th>df</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Moran’s I</td>
<td>15.145</td>
<td>1</td>
<td>&lt;0.001</td>
</tr>
<tr>
<td>Spatial error:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lagrange multiplier</td>
<td>224.298</td>
<td>1</td>
<td>&lt;0.001</td>
</tr>
<tr>
<td>Robust Lagrange multiplier</td>
<td>54.997</td>
<td>1</td>
<td>&lt;0.001</td>
</tr>
<tr>
<td>Spatial lag:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lagrange multiplier</td>
<td>178.938</td>
<td>1</td>
<td>&lt;0.001</td>
</tr>
<tr>
<td>Robust Lagrange multiplier</td>
<td>9.636</td>
<td>1</td>
<td>0.002</td>
</tr>
</tbody>
</table>

\textbf{Model Estimation}

We used both MLE and GMM to estimate the SAC spatial models.\textsuperscript{33} At the same time, for comparison, we also estimate SAR and SEM models. As we can see, the estimated coefficients of different spatial models are quite similar for both MLE and GMM. However, when using MLE, we can see that the SAC model has a higher log likelihood than either the SAR or SEM model. At the same time, when we compare the results with the OLS estimate, we also find similar estimated coefficients, but the spatial model gives us a much lower standard error. Thus, we are able to conclude that the SAC model is suitable.

\textsuperscript{32} The statistical software we used for OLS estimation and diagnosis for spatial dependence is STATA. The STATA package is sg162: Tools For Spatial Data Analysis, downloaded from http://www.stata.com/stb/stb60.

\textsuperscript{33} We use MATLAB for our estimation. The MATLAB toolbox is available from “Econometrics Toolbox” created by James P. LeSage. (http://www.spatial-econometrics.com)
9 (Essay 2) Table 7 MLE Estimate for SAC, SAR, SEM models

<table>
<thead>
<tr>
<th>Variable</th>
<th>SAC</th>
<th>SAR</th>
<th>SEM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Const</td>
<td>1.454668***</td>
<td>1.321773***</td>
<td>1.555184***</td>
</tr>
<tr>
<td>Logexp</td>
<td>-0.102501***</td>
<td>-0.097878***</td>
<td>-0.103335***</td>
</tr>
<tr>
<td>Popdec</td>
<td>0.119934***</td>
<td>0.116896***</td>
<td>0.120293***</td>
</tr>
<tr>
<td>Popexp</td>
<td>-0.008733***</td>
<td>-0.008436***</td>
<td>-0.008811***</td>
</tr>
<tr>
<td>marr2</td>
<td>0.035220***</td>
<td>0.036578***</td>
<td>0.034160***</td>
</tr>
<tr>
<td>Logsize</td>
<td>0.091366***</td>
<td>0.088860***</td>
<td>0.091153***</td>
</tr>
<tr>
<td>adsex1</td>
<td>0.015170***</td>
<td>0.014369***</td>
<td>0.015376***</td>
</tr>
<tr>
<td>adsex3</td>
<td>0.006377*</td>
<td>0.006645*</td>
<td>0.006173*</td>
</tr>
<tr>
<td>Rho</td>
<td>0.176000***</td>
<td>0.328985***</td>
<td></td>
</tr>
<tr>
<td>Lambda</td>
<td>0.263999***</td>
<td></td>
<td>0.405982***</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.404</td>
<td>0.348</td>
<td>0.408</td>
</tr>
<tr>
<td>Rbar-squared</td>
<td>0.401</td>
<td>0.346</td>
<td>0.406</td>
</tr>
<tr>
<td>sigma^2</td>
<td>0.014</td>
<td>0.014</td>
<td>0.014</td>
</tr>
<tr>
<td>log-likelihood</td>
<td>2696.424</td>
<td>1736.226</td>
<td>1739.173</td>
</tr>
</tbody>
</table>

(*: 15% significant level; **: 10% significant level; ***: 1% significant level.)

10 (Essay 2) Table 8 GMM estimate for SAC, SAR, SEM models

<table>
<thead>
<tr>
<th>Variable</th>
<th>SAC</th>
<th>SAR</th>
<th>SEM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Const</td>
<td>1.104769***</td>
<td>1.378375***</td>
<td>1.554316***</td>
</tr>
<tr>
<td>Logexp</td>
<td>-0.10208***</td>
<td>-0.09758***</td>
<td>-0.10326***</td>
</tr>
<tr>
<td>Popdec</td>
<td>0.119744***</td>
<td>0.124798***</td>
<td>0.120548***</td>
</tr>
<tr>
<td>Popexp</td>
<td>-0.008709***</td>
<td>-0.00913***</td>
<td>-0.00883***</td>
</tr>
<tr>
<td>marr2</td>
<td>0.035427***</td>
<td>0.037047***</td>
<td>0.034205***</td>
</tr>
<tr>
<td>Logsize</td>
<td>0.091216***</td>
<td>0.089462***</td>
<td>0.091153***</td>
</tr>
<tr>
<td>adsex1</td>
<td>0.015093***</td>
<td>0.014645***</td>
<td>0.015374***</td>
</tr>
<tr>
<td>adsex3</td>
<td>0.006413*</td>
<td>0.006438*</td>
<td>0.00617*</td>
</tr>
<tr>
<td>Rho</td>
<td>0.201735***</td>
<td>0.21183***</td>
<td></td>
</tr>
<tr>
<td>Lambda</td>
<td>0.230879***</td>
<td></td>
<td>0.399549***</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.403</td>
<td>0.389</td>
<td>0.407</td>
</tr>
<tr>
<td>Rbar-squared</td>
<td>0.400</td>
<td>0.387</td>
<td>0.404</td>
</tr>
<tr>
<td>sigma^2</td>
<td>0.014</td>
<td>0.015</td>
<td>0.014</td>
</tr>
</tbody>
</table>

(*: 15% significant level; **: 10% significant level; ***: 1% significant level.)

Then, we can write our estimated model as follows (using the GMM estimate for the SAC model):

\[ Engel = 1.104769 - 0.10208 \cdot \log \exp + 0.119744 \cdot \text{popdec} - 0.008709 \cdot \text{popexp} + 0.035427 \cdot \text{marr2} + 0.091216 \cdot \text{logsize} + 0.015093 \cdot \text{adsex1} + 0.006413 \cdot \text{adsex3} + 0.201735 \cdot \text{w} \cdot Engel + u \]
\[ u = 0.230879 \cdot u + \varepsilon \]
When the predicted Engel’s coefficient is plotted against the logexp in Figure 4-1&2, it captures the trend quite well. The normality plot of the residual also suggests a normal distribution of the residual.

15 (Essay 2) Figure 4-1 Plots of SAC Estimation-1
Normal Probability Plot for SAC's Residual
Discussion and Summary

Effect of Income

As demonstrated by the estimation results (for the SAC model by the GMM estimate), the coefficient of logexp is -0.10208. At the same time, the interaction term of logexp and popdec is -0.008709. So the effect of total expenditure (or income) on Engel’s coefficient will depend on popdec: As popdec increases, the effect of total expenditure becomes larger. The following table shows how much Engel’s coefficient decreases in each popdec in response to each 1% increase in total expenditure.

11 (Essay 2) Table 9 Change in Engel’s Coefficient With 1% Increase in Totexp by Popdec

<table>
<thead>
<tr>
<th>Popdec</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Change</td>
<td>-0.0011</td>
<td>-0.0012</td>
<td>-0.0013</td>
<td>-0.0014</td>
<td>-0.0015</td>
<td>-0.0015</td>
<td>-0.0016</td>
<td>-0.0017</td>
<td>-0.0018</td>
<td>-0.0019</td>
</tr>
</tbody>
</table>

Effect of Household Structure

(1) The gender of the head of household and the union status have no significant effects. This negative finding was strongly expressed in the preliminary OLS estimation.

(2) “Never Married” status for a head of household differs significantly from the other marital statuses (“Married,” “Divorced,” “Separated,” and “Widowed”). These households will have an Engel Coefficient that is 0.035427 higher than the others, ceteris paribus. This is a large difference and means that these households spend 3.5% more of their total income on food.

(3) Household size and structure effect food consumption significantly. The coefficient of logsize is 0.091216. The effect of an additional household member on Engel’s coefficient is illustrated in the following graph. As documented by the graph, the effect of additional members decreases as household size increases. The graph in Figure 5 also shows the economy of household food consumption: Food consumption doesn’t increase proportionately to an increase in household size, and thus per capita food consumption will decrease when household size increases.
We can also see that male members of a household have a positive effect on Engel’s coefficient. A male adult will increase the Engel’s coefficient by 0.015093 (or increase food share as a proportion to total household expenditure by 1.51%); a male child will increase the Engel’s coefficient by 0.006413 (or increase food share as a proportion to total household expenditure by 0.64%). Neither female adults nor female children have a significant effect.

Effect of Social factors: Income Level in the Society and Neighborhood

The household deciles variable has a very significant effect. It shows that when a household moves upward from one decile to the next, the Engel’s coefficient will increase by 0.119744, which translates into an 11.97% increase in food share. (Interestingly, the same decile upward mobility will increase the negative effect of logexp.) And although the popdec variable increases the Engel’s coefficient, it also accelerates the decrease of Engel’s coefficient with the increase in log income. This can be described as a typical tendency by people to decrease the percentage of income they spend on food as income increases, but an atypical jump in income that moves the same people into an upper decile may be accompanied by an increase in consumption of food as they indulge because of their higher status. In the higher deciles, the percentage of income spent on food declines at a faster rate.
than in the lower deciles when the same increase of income in log form is involved, however it declines at a slower rate when the same increase of income is involved.

We have talked about the improvement in performance gained by adopting the SAC model for our analysis. The SAC model not only increases the efficiency of estimation, but also corrects the problem of bias in the OLS estimate. (The OLS estimate is biased if spatial term dependence is encountered.) In addition, the SAC model also estimates the effects of the neighborhood on a household. The significant positive coefficients of spatial lag term and spatial error term suggest a strong positive correlation among neighbors. The estimated coefficients for spatial lag and error terms differ slightly. The GMM estimates are

\[
\rho = 0.201735 \quad \text{and} \quad \lambda = 0.230879.
\]

The positive spatial lag dependence thus can be expressed as when the neighborhood’s Engel coefficients increase 1% on average, this increase will be followed by a 0.20% increase in the Engel coefficients for households. The positive spatial error dependence reflects the high correlated unspecified factors of neighbors. It may be caused by geographic backgrounds.

Spatial lag dependence can lead us to a very interesting deduction: A household will have a higher (or lower) food share as a proportion to total household expenditure if it is located in a poor (or rich) neighborhood. Stated another way, a poor (or rich) household located in a rich (or poor) neighborhood will consume less (or more) food compared to their counterparts in the poor (or rich) neighborhood. It’s interesting to see that the interaction is through food share (or expenditure) but not through the amount of food consumed.\(^{34}\)

\(^{34}\) If interaction were through the amount of consumption, then on the effect on Engel’s coefficient is the opposite: A household will consume more (or less) food and have a higher (or lower) food share if it is in a rich (or poor) neighborhood with higher (or lower) food consumption amount.
Summary
We have examined the effect of household factors and social factors on Engel’s coefficient (or food share). Joint consideration of household factors and social factors not only improves our estimation, but also shows us that household food consumption is both a household behavior and a social behavior. In summary, we find that:

(1) Household income has a significant negative effect, which enlarges as the income deciles in the society increase;

(2) The gender and the union status of the head of household have no significant effects. The head of household’s marital status has a significant effect in the sense that heads of household who report “never married” tend to have a higher Engel’s coefficient than others. This may be a topic for further research.

(3) Household size has significant effects. The increase in household size will lead to increase in total food consumption; however, marginal food consumption decreases, and so the food share decreases, too. It might reflect the economy of household food consumption. The presence of male members (adults and children) has a significant effect not found with female adults or children in a household This might reflect discrimination against females in household food consumption.

(4) The income deciles have significant effects. These deciles increase Engel’s coefficient on one hand, and enlarge the negative effect of log income on Engel’s coefficient on the other.

(5) Neighborhood effects are significantly positive. An increase in a neighborhood’s average Engel’s coefficient will lead to an increase in its household members’ Engel’s coefficient. The unknown variables of neighbors that affect Engel’s coefficient are positively correlated with each other.
Application to Poverty Problem

Poverty Line

Ravallion and Bidani (1994) proposed the use of household level data to estimate an Engel’s curve for food consumption and then used the estimated curve to calculate the poverty line. Suppose that we estimate an Engel’s curve as \( \text{Engel} = f(\text{Income} \mid X) \), where income is the total household income (or expenditure) and \( X \) represents the conditional variables, such as the previously discussed household factors and social factors. If we think the household food poverty line is \( F \), then we can find the household income poverty line \( I \) by solving \( \frac{F}{\text{Income}} = f(\text{Income} \mid X) \); or if we think household poverty is measured by a certain Engel coefficient \( E \), we can find household income poverty line \( I \) by solving \( E = f(\text{Income} \mid X) \). The reason to determine a poverty line measured by household income is that such a line is easy to define by comparing a household’s Engel’s coefficient and household food consumption.

The poverty incidence in Jamaica was 16.9% in 2004. The poverty line in Jamaica is defined as the minimum food basket divided by the average food share for the lowest income quintile. The minimum food basket is based on the nutritional requirements established by the World Health Organization, Pan American Health Organization, and the Jamaica Ministry of Health. The poverty line is computed for a reference family of five, which includes one adult male, one adult female, an infant, a teenager and a pre-teen child. The poverty line was J$221,130.78 for a “Reference Family of 5” in 2004 (or J$167,083.1 in 2001). The poverty line for individuals was J$58,508.5 in 2004 (or J$44,208.2 in 2001).35

The average food share for the lowest income quintile might be inappropriate for all households and might cause a biased estimate. The Engel’s curve estimated in this paper

---

should be able to be used to set up a household income poverty line for different prespecified groups of households (grouped by the independent variables in our model, such as household size, household structure, neighborhood, etc.).

**An Estimation of Income Elasticity of Food Consumption**

As mentioned before, Engel’s curve by itself is a demand curve and useful for estimating income elasticity, a determination that is not only important for assessing food consumption by poor people but also serves to estimate food demand in a society. From the estimated curve, we are able to estimate the income elasticity of food consumption in Jamaica. It can be described as: \(^{36}\)

\[
\tau = 1 - 1.01 \cdot \frac{0.10208 + 0.008709 \cdot \text{popdec}}{\text{Engel}}
\]

For example, for three households with (\text{popdec}=2, \text{Engel}=.7), (\text{popdec}=5, \text{Engel}=.5) and (\text{popdec}=9, \text{Engel}=.4), respectively, we can calculate their income elasticity as 0.827581, 0.705838, and 0.544336. We can easily find there is a negative relationship between income elasticity and income, and there is also a negative relation between it and income deciles, i.e. the income elasticity is not only less than 1, but also decreases as income increases. Equation (6.1) should be useful when policy makers want to measure the effect on poor people of subsidies such as food stamps.

**Neighborhood Matters**

From the previous section, we know that when the neighborhood’s Engel’s coefficients increase 1% on average, this increase will cause a 0.20% increase in Engel’s coefficient for households. The effect is significant. We know the household’s Engel’s coefficient varies from 3% to 98%. If there are two identical households and they live in different neighborhoods with an average Engel’s coefficient, respectively, equal to 40% (a rich neighborhood).

---

\(^{36}\) A change in one household’s Engel’s coefficient has a very small effect on the neighborhood, and the feedback effect is minimal. Thus we can ignore the neighborhood effect here.
neighborhood) and 70% (a poor neighborhood), then the first household will have a 6% lower Engel’s coefficient compared with the second one. This might cause distortion in the household’s expenditure. The unknown factors that cause neighbors to “copy” each other’s expenditure structure have important implications for the issue of poverty. Will the neighborhood improve or worsen the poverty problem? And will it cause the migration of people? With a poverty policy that helps poor people through subsidies, the first problem can be solved by establishing appropriate income poverty lines in different neighborhoods as described in 6.1. The second question raises an issue for further research on the relationship between migration and preference interdependence among neighborhoods.

The positive correlation in the error term can be caused by correlated factors not captured in our model, such as geographic factors, the local economic environment, common risks, or common expectations. This correlation may also suggest that poverty policy, which now mostly targets households, should be able to target the common background of neighborhoods.

**Possible Discrimination against Females**

Our estimation shows that male members of a household have a significant positive effect. The increase in Engel’s coefficient caused by an additional male adult (or child) is 0.015093 (or 0.006413), more than that caused by an additional female member (either adult or child). We can see that even male children could affect food consumption more than female adults do. It’s very possible that in Jamaica, as in many other developing countries, discrimination exists against females in food consumption.
Essay 3: A Survival Analysis of Education Duration in Jamaica

Jamaica is a small island country with a population of 2.7 million and a GDP per capita in 2005 of 4,482 US dollars (PPP). In spite of its low rank in GDP per capita (111th among world countries in 2005), Jamaica stands out in the developing world as a country with a strong commitment to education: Public spending on education grew to 7.6% of GDP in 1997-1998. This was complemented by an estimated household spending of about 6% of GDP. With significant public and private investment in education as well as successful governmental education policies, the country has made impressive progress in providing primary and secondary education.

In the 1990s, the Jamaica Ministry of Education and Culture (MOE&E) embarked on a 15-year “Reform of Secondary Education Program” (ROSE). The first phase focusing on lower secondary schools has been completed. The second phase focuses on improving upper secondary schools and on improving education quality at all levels.\(^{37}\) Although the education system in Jamaica is complex and includes different types of schools that are constantly evolving\(^{38}\), the system can be summarized and described as follows: Primary education is from Grade 1 to Grade 6, lower secondary education is from Grade 7 to 9, and upper secondary education is from Grade 10 to 11 (or 13\(^{39}\)). In 1997-1998, the gross enrollment rates were more than 100 percent in primary education, 97 percent in lower secondary education, and 66 percent in upper secondary education. The net enrollment rates were 93


\(^{38}\) Note: “Time” is one of the most important key words in our study. However, in our paper “time” in different contexts represents different things: It represents grade when we are talking about a time period in the survival analysis, such as in the “discrete time survival analysis.”; It represents real time, such as year or age when we are talking about the time/age effect on the dropout risks of students. Readers should be aware and refer to the right meaning based on context.

\(^{39}\) Grades 12-13 are not required for attending tertiary education institutions in Jamaica.
percent in primary education, 82 percent in lower secondary education, and 49 percent in upper secondary education.40

According to the World Bank's World Development Indicators (2005), in 2002-03 the gross enrollment rates in primary and secondary education in Jamaica were 101% and 84%, respectively, and the net enrollment rates were 95% and 75%. In comparison, the gross enrollment rates in 2002-03 for primary and secondary education in middle income countries averaged 112% and 74%, respectively. The gross enrollment rate in secondary education in Jamaica in 2002-03 was 10% higher than the average for middle income countries.

Although these are significant and impressive achievements, we also find that the net enrollment rates decreased more than 10% from primary education to lower secondary education, and furthermore, decreased more than 30% from lower secondary education to upper secondary education. We might be eager to know “Where, What and How”: Where (or in which grade) the students tend to drop out of school, what factors affect dropout behavior, and how to increase education duration in Jamaica. We believe that research on these questions will be informative and helpful to continued improvement of education in Jamaica.

In this paper, we will apply discrete time survival analysis to analyze education duration in Jamaica: First we will analyze the data from JSLC 2002, investigate the pattern of educational attainment in Jamaica, and summarize some related variables; then the discrete time Cox model and the Logit model will be applied to estimate the effects of the related covariates; in the end, we want to discuss the policy implications of our research to the continued improvement of education in Jamaica.

The paper is structured as follows: In the first section, we will briefly review the literature for education studies and the application of survival analysis. In the second section, we will introduce the data and the survival techniques (the discrete time Cox model and the discrete

time Logit model). In the third section, we will apply these survival techniques to exploration of education duration in Jamaica. In fourth section, we will discuss the policy implications of our findings. And in the last section, we will discuss the conclusions drawn from our research and its limitations.

Literature Review

Education Literature

Since 1940s, the concept of “dropout” has been used to represent a category of people who do not complete secondary education. For more than 50 years, this subject has attracted enormous interest from economists, and substantial research has been done on the determinants of dropping out. The dropout problem attracts much attention from economists for various reasons. These reasons fall into three categories that can be summarized as follows: (1) Dropping out undermines the individual’s future welfare: Dropouts tend to have lower income and higher unemployment; Dropouts are also more likely to have health problems, and as an economy upgrades, dropouts will have an even harder time surviving economically. (2) Dropouts generate large social costs. They tend to receive more public assistance and also tend to engage in criminal activities (Catterall, 1987; Rumberger, 1987 and 2001; Murnane & Levy, 1996). (3) Dropouts decrease human capital accumulation in a country and thus in the long run damage the country’s economic growth. For developing countries, education is the principal way to escape poverty and ignite sustainable economic growth. However, dropouts may lead to a vicious cycle in developing countries: poor education>poverty>pofor education. The dropout rate is used to measure the probability of

41 Fulci (1999) said, ‘Education is the key to development. Quality basic education, as well as secondary and higher education, vocational training, and skill acquisition throughout life are indispensable tools to eradicate poverty.’ (Reported by Singh, 1999)
dropping out of school. Reducing the dropout rate can break the cycle of poverty and is a crucial part of poverty reduction in developing countries. For all of these reasons and more, this issue has drawn the attention of economists and spurred their research into why students drop out and how they can be retained in school. The importance of this issue has attracted much attention from economists who have explored the reasons for dropping out and the ways to deal with this problem.

One of the most popular economic theories for education/dropout issues is the theory of human capital (Becker, 1967, 1981; Becker & Tomes, 1979, 1986). In this framework, children’s human capital derives from two sources: First, the inheritance of genetic and cultural endowments from parents, and second, the investments in children by parents. Inheritance depends on the parents’ abilities, education, and cultural background. The investments by the parents depend on parental preferences, income, and other factors. The human capital theory emphasizes the effect of family factors on children’s educational attainment. Other social disciplines such as sociology and psychology contribute many other theories to the literature. These theories include peer/role models, a life span development approach, a stress theory, the “working mother perspective,” and the “economic deprivation perspective,” (Haveman & Wolfe, 1995).

Drawing on these various theories, Rumberger (2001) presents two conceptual frameworks: One is from an individual perspective, and the other one is from an institutional perspective. The individual framework focuses on the attributes of students, such as their values, attitudes, and behaviors, and how these attributes contribute to their decisions about their education. The institutional framework focuses on the settings and environment in which children live, such as families, schools, and communities.

Haveman and Wolfe (1995) summarized three primary categories that will affect the educational attainment of children: the choices made by the society/government that
determine the opportunities available to both children and their parents (i.e. the social investment in children); the choices made by parents regarding the quantity and quality of family resources devoted to children (the parental investment in children); and the choices that children make given the investments in and opportunities available to them. The children’s educational attainment is the outcome of these choices. These three categories are consistent with Remberger’s two frameworks. Haveman and Wolfe (1995) also provided a useful review of the literature on dropout research, and we will summarize their main finding as follows.

In one of earliest studies, Blau and Duncan (1967) used a system of recursive regression equations to estimate the relationships among time-ordered, life cycle family background characteristics and children’s educational attainment. In 1970s, the researchers of the Wisconsin Longitudinal Study estimated the determinants of educational attainment based on a life cycle framework that included the number of family members, school, and aspiration variables. (Haveman & Wolfe, 1995)

The factors affecting children’s educational attainment in recent research can be summarized as including social, family, choice, and background characteristics. Social factors include background and quality of students in the school, race structure in the school, school location (such as urban/rural and South/North), neighborhood, state and local education expenditures, unemployment rates, etc. Family factors include characteristics of the head of family (sex, education, and so on), family structure (parents, siblings, and so on), parents’ occupation, family income, family birth plan, distance to schools, etc. Factors related to students’ own choices include scores at school, expectations/self-esteem, pregnancy, religious activities, etc. The background characteristics include race, gender, time, age, opportunity wages, etc. The measurements of children’s educational attainment include a high school graduation dummy variable, years of schooling, dropout risk, etc. Although
Ordinary Least Square (OLS) and Logit/Probit models are widely used, survival analysis techniques have become more and more popular in the most recent literature (Haveman & Wolfe, 1995).

**Survival Analysis**

(1) Survival Analysis and Its Application to Education

Survival analysis is a statistical technique for studying the occurrence and timing of events. It is also known as event history analysis, reliability analysis, failure time analysis, duration analysis, and transition analysis (Allison, 1995). Willet and Singer (1991, 1995) and Singer and Willet (1992, 1993) introduced survival analysis (especially discrete time survival analysis) to education issues (such as students’ education paths and teachers’ career paths). Since then, it has become more and more popular in the analysis of education issues.

Some of the key concepts of survival analysis include time, event, survival function, hazard, and the hazard function. Time is recorded when an event happens and can be continuous or discrete. An event can be death, dropping out of school/some programs, or any event that is of research interest. Survival and the hazard function will be introduced in detail later.

(2) Advantages of Survival Analysis Compared with Traditional Methods

Enrollment rates are widely used to measure education status in a country. Enrollment rates are typically available for primary, secondary, and college levels. Gross enrollment rates are calculated as the number of enrolled students divided by the total population within the specific age range eligible for enrollment in school. Net enrollment rates adjust the gross enrollment numerator for students within the designated age range.  

42 Here is an example of the calculation of an enrollment rate: $n_1$ is the number of students in the primary schools (Grades 1-6); $n_2$ is the number of people within a certain age range who are eligible for enrollment in the primary schools; $n_3$ is the number of students in the primary schools who are also within a certain age range who are eligible for enrollment in the primary schools. We have $n_3 \leq n_1$. The gross enrollment rate will equal
limitations of enrollment rates: (1) it makes little sense to calculate an enrollment rate for each grade, because students of the same age are not necessary to enroll in the same grade. Enrollment rates by school level cannot accurately reflect education in each grade; (2) enrollment rates are inappropriate as a reflection of the educational attainment of people beyond the age for school; (3) it is difficult to measure the enrollment rates at different school levels for the same cohort of people; thus, we do not have a way to “follow” a group of people and see when they drop out of school; (4) and enrollment rates can be manipulated easily. Researchers can get a higher or lower enrollment rate simply by changing the research group. For example, including (excluding) a group of students (mostly in the lower grades) with higher enrollment will increase (decrease) the enrollment rates.

Some empirical studies have applied Logit models to estimate graduation rates in primary schools, secondary schools and colleges. In this line of research, the authors usually ask if a student has graduated from school. These studies are helpful as a way to explore if the students have made it through school. Their two limitations are that (1) they cannot handle those students who are still in schools, and (2) they have little information on when (in which grade) students tend to drop out. (Willett & Singer, 1991)

Some other studies have applied OLS model to estimate the duration of education and to determine at what point students tend to dropout. (Duration at school is most often the dependent variable.) However, OLS model still cannot overcome these problems: (1) Some students are out of school because they are transferring to other schools or because of illness, death, etc., but they are not dropouts and cannot be counted as such; consequently their observed duration at school will be shorter than the actual duration; (2) for those students still in school, the observed duration of education will be also shorter than the actual duration; (3) n1/n2 and the net enrollment rate will equal to n3/n2. We can see that the gross enrollment rate is not less than the net one, and it can be more than 100%.
and the estimation duration from OLS could be longer than the possibly longest duration. Because of these limitations, the OLS estimate will tend to be biased.

Survival analysis can overcome all of these shortcomings. By estimating the dropout risk for each grade, it provides detailed information on the dropout risks for students in each grade. On the other hand, by treating as “censored observations” those students who do not drop out during the period of research, it can account for those students who are still in school but have transferred to other schools, or temporarily left school because of illness or other reasons but are not dropouts. Moreover, survival analysis has one more advantage: the ability to incorporate time varying data. While it is difficult for traditional models like OLS and Logit models to incorporate time varying data, some survival models can do so easily and naturally.

Because of the above advantages (ability to identify the timing of the problem, ability to handle censored data, and ability to incorporate time varying variables) compared with traditional methods, survival analysis has become more and more popular in social science and natural science. The development of survival statistical techniques, its availability in standard statistical software (such as SAS and STATA) and the improvement in data quality has enabled researchers to apply survival analysis to education duration research.

Data and Empirical Methods

Data: Introduction and Summary

The data is from the Jamaica Survey of Living Conditions 2002, which was undertaken to establish baseline measurements of household welfare and subsequently to monitor the impact of Jamaica’s Human Resources Development Program on health, education and nutrition (The World Bank, 2002). The survey covers 6,976 households randomly selected
from the Survey of Labor Force\textsuperscript{43}. All household members older than 3 in 2002 were included in the survey. After matching education data, household consumption data, and household demographic data, etc., we were able to develop a data set that includes the variables that might affect individual educational attainment. Understanding education and related demographic variables can provide basic information for our survival modeling; thus we summarize these variables as follows.

(1) School Enrollment

Table 1 shows Jamaican school enrollment in 2002, when 65.03\% of the population was not enrolled in any type of school (including institutes, colleges and universities). However, 7.11\% of the people were enrolled in the Basic/Infant/Nursery/Kindergarten schools, and 15.79\% of the people were enrolled in Grades1-6, which included primary schools, all-age schools, and primary/junior high schools. The primary schools have better reputation than the all-age and primary/junior high schools.)

About 10.69\% of people were enrolled in the secondary schools of all types. About 8.67\% were in secondary high schools (7.61\%), technical schools (0.76\%), and vocational/agricultural schools (0.30\%) (Grades 7-11 or 13); 2.02\% were in all-age schools, primary/junior high schools, and junior high schools (Grades 7-9), which have inferior educational quality.

Universities or other tertiary schools accounted for 0.93\% of the population, and 0.3\% of the people were enrolled in adult and special schools.

Table 1 also shows the difference in school enrollment between males and females. Females were a relatively larger proportion of the enrollment in secondary high schools, technical schools, vocational/agricultural schools, universities/post-secondary schools, and

\textsuperscript{43} LFS used a two-stage stratified sampling strategy, and the sample is self-weighted. That is, each household in Jamaica is equally likely to be included in the survey sample. For details, please see Jamaica Survey of Living Conditions 1988-2000: Basic Information, the World Bank (2002).
adul education/night schools. The gap between female-male enrollments in favor of females grew larger in upper level education.
### 12 (Essay 3) Table 1 Enrollment in Jamaica 2002 (by school type)

<table>
<thead>
<tr>
<th>School Type</th>
<th>MALE</th>
<th>FEMALE</th>
<th>Grand Total</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>BASIC/INFANT/NURSERY /KINDERGARTEN</td>
<td>838</td>
<td>792</td>
<td>1,630</td>
<td>7.11%</td>
</tr>
<tr>
<td>PRIMARY</td>
<td>1,080</td>
<td>1,037</td>
<td>2,118</td>
<td>9.24%</td>
</tr>
<tr>
<td>ALL AGE (GRADES 1-6)</td>
<td>579</td>
<td>515</td>
<td>1,094</td>
<td>4.77%</td>
</tr>
<tr>
<td>ALL AGE (GRADES 7-9)</td>
<td>138</td>
<td>105</td>
<td>243</td>
<td>1.06%</td>
</tr>
<tr>
<td>Primary/JUNIOR HIGH (GRADES 1-6)</td>
<td>206</td>
<td>203</td>
<td>409</td>
<td>1.78%</td>
</tr>
<tr>
<td>Primary/JUNIOR HIGH (GRADES 7-9)</td>
<td>91</td>
<td>60</td>
<td>151</td>
<td>0.66%</td>
</tr>
<tr>
<td>JUNIOR HIGH (GRADES 7-9)</td>
<td>36</td>
<td>33</td>
<td>69</td>
<td>0.30%</td>
</tr>
<tr>
<td>SECONDARY HIGH</td>
<td>812</td>
<td>932</td>
<td>1,744</td>
<td>7.61%</td>
</tr>
<tr>
<td>TECHNICAL</td>
<td>77</td>
<td>97</td>
<td>174</td>
<td>0.76%</td>
</tr>
<tr>
<td>VOCAT/AGRIC</td>
<td>27</td>
<td>41</td>
<td>68</td>
<td>0.30%</td>
</tr>
<tr>
<td>UNIVERSITY</td>
<td>29</td>
<td>44</td>
<td>73</td>
<td>0.32%</td>
</tr>
<tr>
<td>Other Tertiary (PUBLIC)</td>
<td>28</td>
<td>63</td>
<td>91</td>
<td>0.40%</td>
</tr>
<tr>
<td>Other Tertiary (PRIVATE)</td>
<td>16</td>
<td>32</td>
<td>48</td>
<td>0.21%</td>
</tr>
<tr>
<td>Adult Literacy Classes</td>
<td>4</td>
<td>3</td>
<td>7</td>
<td>0.03%</td>
</tr>
<tr>
<td>Adult Education/Night</td>
<td>14</td>
<td>32</td>
<td>46</td>
<td>0.20%</td>
</tr>
<tr>
<td>Special School</td>
<td>10</td>
<td>7</td>
<td>17</td>
<td>0.07%</td>
</tr>
<tr>
<td>None</td>
<td>7,313</td>
<td>7,591</td>
<td>14,904</td>
<td>65.03%</td>
</tr>
<tr>
<td>Total</td>
<td>11,315</td>
<td>11,603</td>
<td>22,920</td>
<td>100.00%</td>
</tr>
</tbody>
</table>

(2) Educational Attainment

The JSCLC 2002 also provided information on the educational attainment of people not in schools in 2002. Table 2 resembles Table 1, but the two tables have totally different meanings: Table 1 reflects the current education enrollment in schools in 2002, and Table 2 reflects the educational attainment by school type for persons not in school in 2002. Thus, the two tables have very different distributions.

Table 2 shows that 19.98% of the people not enrolled in school in 2002 had a primary school education (including primary, all-age, and primary/junior high schools). 71.52% had a secondary school education. 27.58% of these people had been in all-age schools (Grades 7-9). 42.72% had been in new secondary (12.83%), comprehensive (5.27%), secondary high (20.04%), technical (2.63%), and vocational/agricultural schools (1.95%).
Here again we also see a gender difference in educational attainment. After secondary school, more females than males are present at each level, and this gender gap grows larger as the level of education increases. For secondary schools, males had a relatively larger presence in the lower and lesser quality secondary schools, and females had a relatively larger proportion of enrollment in the upper and better quality secondary schools. (Note: The percentages of males and females not in schools in 2002 were 49% and 51%, respectively. The numbers are shown in the last row in Table 2.)

13 (Essay 3) Table 2 Educational Attainment in Jamaica 2002  (by school type)

<table>
<thead>
<tr>
<th>School Type</th>
<th>MALE</th>
<th>FEMALE</th>
<th>Grand Total</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>BASIC/INFANT/NURSERY /KINDERGARTEN</td>
<td>32</td>
<td>41</td>
<td>73</td>
<td>0.50%</td>
</tr>
<tr>
<td>Primary</td>
<td>1,025</td>
<td>1,066</td>
<td>2,091</td>
<td>14.36%</td>
</tr>
<tr>
<td>ALL AGE (GRADES 1-6)</td>
<td>390</td>
<td>389</td>
<td>779</td>
<td>5.35%</td>
</tr>
<tr>
<td>ALL AGE (GRADES 7-9)</td>
<td>2,176</td>
<td>1,841</td>
<td>4,017</td>
<td>27.58%</td>
</tr>
<tr>
<td>Primary/JUNIOR HIGH (GRADES 1-6)</td>
<td>18</td>
<td>21</td>
<td>39</td>
<td>0.27%</td>
</tr>
<tr>
<td>Primary/JUNIOR HIGH (GRADES 7-9)</td>
<td>50</td>
<td>37</td>
<td>87</td>
<td>0.60%</td>
</tr>
<tr>
<td>JUNIOR HIGH (GRADES 7-9)</td>
<td>58</td>
<td>33</td>
<td>91</td>
<td>0.62%</td>
</tr>
<tr>
<td>NEW SECONDARY 1</td>
<td>920</td>
<td>949</td>
<td>1,869</td>
<td>12.83%</td>
</tr>
<tr>
<td>COMPREHENSIVE 2</td>
<td>362</td>
<td>406</td>
<td>768</td>
<td>5.27%</td>
</tr>
<tr>
<td>SECONDARY HIGH</td>
<td>1,353</td>
<td>1,565</td>
<td>2,918</td>
<td>20.04%</td>
</tr>
<tr>
<td>TECHNICAL</td>
<td>199</td>
<td>184</td>
<td>383</td>
<td>2.63%</td>
</tr>
<tr>
<td>VOCAT/AGRIC</td>
<td>91</td>
<td>193</td>
<td>284</td>
<td>1.95%</td>
</tr>
<tr>
<td>UNIVERSITY</td>
<td>117</td>
<td>133</td>
<td>250</td>
<td>1.72%</td>
</tr>
<tr>
<td>OTHER TERTIARY (PUBLIC)</td>
<td>148</td>
<td>290</td>
<td>438</td>
<td>3.01%</td>
</tr>
<tr>
<td>OTHER TERTIARY (PRIVATE)</td>
<td>60</td>
<td>112</td>
<td>172</td>
<td>1.18%</td>
</tr>
<tr>
<td>ADULT LITERACY CLASSES</td>
<td>10</td>
<td>15</td>
<td>25</td>
<td>0.17%</td>
</tr>
<tr>
<td>ADULT EDUCATION/NIGHT SPECIAL SCHOOL</td>
<td>14</td>
<td>43</td>
<td>57</td>
<td>0.39%</td>
</tr>
<tr>
<td>NONE</td>
<td>33</td>
<td>25</td>
<td>58</td>
<td>0.40%</td>
</tr>
<tr>
<td>TOTAL</td>
<td>7,144</td>
<td>7,419</td>
<td>14,563</td>
<td>100.00%</td>
</tr>
</tbody>
</table>

Note: 1. New secondary schools converted to comprehensive high in the 1998/1999 academic year.
2. Comprehensive high schools converted to secondary high in 2000/2001 academic year.
(3) Household Demographics

The JSLC 2002 recorded every household member’s relation to the head of household, which provides useful information on household demographics. Table 3 shows the survey comprised 6,795 households. However, only 2,911 heads of household had spouses or partners. Furthermore, only 121 of 8,154 children of the heads had spouses. However, there were 2,616 grandchildren of the heads in the households.) The exploration of household demographic structure will help our modeling because family characteristics are expected to greatly affect educational attainment.

14 (Essay 3) Table 3 Household Demographic Constituents in Jamaica 2002

<table>
<thead>
<tr>
<th>Relation to head of household</th>
<th>Number</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Head</td>
<td>6,975</td>
<td>30.43%</td>
</tr>
<tr>
<td>Spouse/Partner</td>
<td>2,911</td>
<td>12.70%</td>
</tr>
<tr>
<td>Child of Head/Spouse</td>
<td>8,154</td>
<td>35.58%</td>
</tr>
<tr>
<td>Spouse of Child</td>
<td>121</td>
<td>0.53%</td>
</tr>
<tr>
<td>Grandchild</td>
<td>2,616</td>
<td>11.41%</td>
</tr>
<tr>
<td>Parent of Head/Spouse</td>
<td>237</td>
<td>1.03%</td>
</tr>
<tr>
<td>Other Relative</td>
<td>1,537</td>
<td>6.71%</td>
</tr>
<tr>
<td>Helper/Domestic</td>
<td>31</td>
<td>0.14%</td>
</tr>
<tr>
<td>Other Not Relative</td>
<td>335</td>
<td>1.46%</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>22,917</td>
<td>1.00%</td>
</tr>
</tbody>
</table>

(4) Student Distribution by Grade

In Table 1, we saw the school enrollment of Jamaica in 2002. Now we will continue to explore student distribution by grade. Table 4 lists the number of students and percentages for Grades 1-13 in 2002. The progression in grade levels is marked by a visible trend of diminishing enrollment. This enrollment decline accelerates after Grade 9: from Grade 9 to Grade 10, the decrease is about 1.82%; from Grade 10 to 11, the decrease is about 1.99%; and from Grade 11 to 12, the decrease jumps to about 4.75%. The table shows that after
lower secondary school, the number of students declined sharply. The huge decrease from Grade 11 to Grade 12 is not especially surprising because only a few upper secondary schools have Grades 12 and 13.

15 (Essay 3) Table 4 Student Distribution in Jamaica 2002 (Grades 1-13)

<table>
<thead>
<tr>
<th>Grade</th>
<th>Number</th>
<th>Percentage</th>
<th>Grade</th>
<th>Number</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>661</td>
<td>11.09%</td>
<td>8</td>
<td>515</td>
<td>8.64%</td>
</tr>
<tr>
<td>2</td>
<td>623</td>
<td>10.45%</td>
<td>9</td>
<td>532</td>
<td>8.93%</td>
</tr>
<tr>
<td>3</td>
<td>621</td>
<td>10.42%</td>
<td>10</td>
<td>424</td>
<td>7.11%</td>
</tr>
<tr>
<td>4</td>
<td>589</td>
<td>9.88%</td>
<td>11</td>
<td>305</td>
<td>5.12%</td>
</tr>
<tr>
<td>5</td>
<td>568</td>
<td>9.53%</td>
<td>12</td>
<td>22</td>
<td>0.37%</td>
</tr>
<tr>
<td>6</td>
<td>559</td>
<td>9.38%</td>
<td>13</td>
<td>9</td>
<td>0.15%</td>
</tr>
<tr>
<td>7</td>
<td>532</td>
<td>8.93%</td>
<td>Total</td>
<td>5,960</td>
<td>100.00%</td>
</tr>
</tbody>
</table>

18 (Essay 3) Figure 1 Student Distribution in Jamaica 2002 (Grade 1-13)
(5) Educational Attainment-by Grade Reported

Now we turn to analyzing the educational attainment (by grade and age group) of those people not currently in school in 2002. Table 5 and Figure 2 show that the younger generations generally have higher educational attainment than their elders. For people older than 60, the educational peaks are at Grade 6 (around 45%) and Grade 9 (around 25%); for people between ages 41 and 60, the peaks are at Grade 6 (around 21%), Grade 9 (around 40%), and Grade 11 (around 18%); for people between 21 and 40, the peaks are at Grade 9 (28%) and Grade 11 (53%); and for people between 3 and 20, the peaks are at Grade 9 (22%) and Grade 11 (58%)\(^{44}\). The increase in educational attainment over time is obvious.

\(^{44}\) A large proportion of people between ages 3 and 20 are still in school, which will be treated as censored observations in the following survival analysis.
16 (Essay 3) Table 5 Educational Attainment in Jamaica 2002: Grade Reported (by age group)

<table>
<thead>
<tr>
<th>Grade</th>
<th>Age 3-20</th>
<th>Age 21-40</th>
<th>Age 41-60</th>
<th>Age&gt; 60</th>
<th>Total</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6</td>
<td>13</td>
<td>18</td>
<td>20</td>
<td>57</td>
<td>0.44%</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
<td>20</td>
<td>32</td>
<td>60</td>
<td>118</td>
<td>0.91%</td>
</tr>
<tr>
<td>3</td>
<td>11</td>
<td>55</td>
<td>68</td>
<td>115</td>
<td>249</td>
<td>1.93%</td>
</tr>
<tr>
<td>4</td>
<td>13</td>
<td>46</td>
<td>86</td>
<td>168</td>
<td>313</td>
<td>2.43%</td>
</tr>
<tr>
<td>5</td>
<td>39</td>
<td>155</td>
<td>137</td>
<td>133</td>
<td>464</td>
<td>3.60%</td>
</tr>
<tr>
<td>6</td>
<td>34</td>
<td>147</td>
<td>741</td>
<td>1,149</td>
<td>2,072</td>
<td>16.06%</td>
</tr>
<tr>
<td>7</td>
<td>28</td>
<td>90</td>
<td>82</td>
<td>65</td>
<td>265</td>
<td>2.05%</td>
</tr>
<tr>
<td>8</td>
<td>47</td>
<td>166</td>
<td>134</td>
<td>101</td>
<td>448</td>
<td>3.47%</td>
</tr>
<tr>
<td>9</td>
<td>308</td>
<td>1,627</td>
<td>1,335</td>
<td>602</td>
<td>3,872</td>
<td>30.01%</td>
</tr>
<tr>
<td>10</td>
<td>92</td>
<td>374</td>
<td>72</td>
<td>7</td>
<td>545</td>
<td>4.22%</td>
</tr>
<tr>
<td>11</td>
<td>813</td>
<td>3,018</td>
<td>607</td>
<td>39</td>
<td>4,477</td>
<td>34.69%</td>
</tr>
<tr>
<td>12</td>
<td>5</td>
<td>11</td>
<td>7</td>
<td>1</td>
<td>24</td>
<td>0.19%</td>
</tr>
<tr>
<td>Total</td>
<td>1,402</td>
<td>5,722</td>
<td>3,319</td>
<td>2,460</td>
<td>12,904</td>
<td>100.00%</td>
</tr>
</tbody>
</table>

19 (Essay 3) Figure 2 Educational attainment in Jamaica 2002 (by age group)
(6) Other Demographics

The marital and union statuses were also reported in the survey. They are summarized in Table 6-1 and 6-2.

**17 (Essay 3) Table 6-1 Marital Status Reported (age >=15)**

<table>
<thead>
<tr>
<th>Marital Status</th>
<th>Number</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>MARRIED</td>
<td>4,046</td>
<td>24.96%</td>
</tr>
<tr>
<td>NEVER MARRIED</td>
<td>10,795</td>
<td>66.60%</td>
</tr>
<tr>
<td>DIVORCED</td>
<td>123</td>
<td>0.76%</td>
</tr>
<tr>
<td>SEPARATED</td>
<td>185</td>
<td>1.14%</td>
</tr>
<tr>
<td>WIDOWED</td>
<td>905</td>
<td>5.58%</td>
</tr>
<tr>
<td>UNKNOWN</td>
<td>154</td>
<td>0.95%</td>
</tr>
</tbody>
</table>

**18 (Essay 3) Table 6-2 Union Status Reported (age >=15)**

<table>
<thead>
<tr>
<th>Union Status</th>
<th>Number</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>MARRIED</td>
<td>3,903</td>
<td>24.09%</td>
</tr>
<tr>
<td>COMMON LAW</td>
<td>2,606</td>
<td>16.08%</td>
</tr>
<tr>
<td>VISITING</td>
<td>2,028</td>
<td>12.52%</td>
</tr>
<tr>
<td>SINGLE</td>
<td>5,633</td>
<td>34.77%</td>
</tr>
<tr>
<td>NONE</td>
<td>1,754</td>
<td>10.83%</td>
</tr>
</tbody>
</table>

(7) Summary

In this section, we have briefly described the educational attainment and demographic variables in Jamaica. We also have explored the basics of gender and time/age effects on educational attainment. From the tables, especially Tables 4 and 5, we can get useful information on educational attainment in 2002 by grade for both students and nonstudents. As we discussed in the previous section, integration of the information in these two tables is
difficult with traditional methods. However, survival analysis easily handles this integration issue through its “censored data” analytical strategy.

As already observed, survival analysis has other superior attributes for this type of analysis. The next section will introduce the survival techniques used in this paper and then apply them to explore dropout risk by grade and estimate the effects of covariates such as household income (household total expenditure), gender, head of household, household demographic variables, time, and government policy, etc.

Empirical Methods: Introduction to Survival Analysis

(1) Basic Concepts

Survival Function

Survival function (or survivor function, survivorship function) gives the probability of surviving after a specific time. In survival analysis, surviving not only refers to the status of being alive but also to a status of not having experienced or engaged in or performed a specific event or action, such as divorce or dropping out of a program or, to use a business example, a company’s bankruptcy, and so on. In this paper, surviving refers to the specific status of not having dropped out of school. The survival function can be written as \( S(t) = \Pr(T>t) \), where \( t \) is time, and \( T \) is the time of the occurrence of an event.

Hazard Function

The hazard function is crucial in survival analysis. The hazard at a specific time corresponds directly to the risk of the occurrence of an event. In this paper, hazard is the dropout risk/rate of students. The hazard function can be defined as:

\[
 h(t) = \lim_{dt \to 0} \frac{S(t) - S(t + dt)}{S(t)} / dt
 = f(t) / S(t)
\]

where \( dt \) is a time period, \( f(t) \) is the density distribution function of an event occurrence.
The relationship between the survival function and the hazard function can be written as

\[ S(t) = \exp\left(-\int_0^t h(t) \, dt\right) \]

For discrete time, the hazard is defined as the conditional probability of the occurrence of an event, given that no previous event has occurred (Singer and Willet, 1993). It can be referred to as discrete time hazard function:

\[ h(t) = \Pr[T = t | T \geq t] \]

\[ = \frac{S(t-1) - S(t)}{S(t-1)} \]

where \( T \) is the time of event occurrence.

The relationship between the discrete time hazard function and the survival function can be written as:

\[ S(t) = \prod_{i=1}^t (1 - h(t)) \]

In this paper, school grade level is used as the measurement of educational attainment, and the discrete time survival analysis is applied.

(2) Semi-parametric Analysis: Cox Regression

Cox (1972) proposes a semi-parametric regression, which has become one of the most popular methods in survival analysis. Compared with other regression methods, Cox regression does not need to choose some particular probability distribution to represent survival times. Such probability distributions, when they have been used, have been typically arbitrary assumptions. Without them, the results of Cox regression will be more robust. The Cox method can handle continuous and discrete data. However, we will only introduce the discrete time method, which is the method applied in our paper.

The basic idea of the Cox regression for discrete time data is to estimate a binary model to predict whether an event does or does not occur in each time period. Cox regression then
relates the conditional probability of event occurrence \((p(t))^{45}\) to the covariates by a Logit equation:

\[
\log\left(\frac{p(t)}{1-p(t)}\right) = \alpha(t) + \beta x
\]

In the equation, \(\alpha(t)\) is a set of constants for each time period and controls the time variance of the regression model. \(x\) is the covariate array that can include time unvarying variables as well as time varying variables. In this model, the odds ratio between any two individuals does not depend on time, thus the model can be described as a proportional odds model. The semi-parametric method will make use of this characteristic and will not estimate \(\alpha(t)\).\(^{46}\) That is to say, the estimation ignores the baseline hazard function and focuses only on the estimation of the effects of the covariates. However, after estimating the coefficients of the covariates, the nonparametric maximum likelihood method can still be used to estimate the survival function, \(S(t)\).

(3) Parametric Analysis: Logit Model

In the Cox regression, the semi-parametric estimation discards the information of \(\alpha(t)\). However, the traditional Logit model can be used to estimate the model. With the Logit model, we can directly estimate the effect of time, \(\alpha(t)\). Meanwhile, the maximum likelihood method to estimate the Logit model is more computationally efficient than the semi-parametric method used in the Cox regression.

Allison (1995) argues that uncontrolled heterogeneity confounds the hazard function and, thus research on the shape of the hazard function might yield incorrect information. He suggests that researchers should be cautious when using alternatives to the Cox regression to study the time dependence of the hazard function.

\(^{45}\) The conditional probability of event occurrence is also known as a discrete time hazard in the Cox regression, which is the same as in Singer and Willet (1993).

\(^{46}\) For details of the estimation algorithm, please see Cox (1972).
Empirical Analyses

In this section, survival analysis is applied as follows: First, we will apply semi-parametric survival analysis using Cox regression’s discrete time survival model. Cox regression makes no assumption of the baseline hazard function of time, and therefore the estimation of covariates’ coefficients will be robust. Second, we will apply parametric survival analysis, i.e., the Logit model for the survival analysis, to estimate the effects of covariates as well as the baseline hazard function of time. The comparison of the estimation result to the result obtained by the Cox regression could test the accuracy of the model.

However, the use of current variables to explain the occurrence of a past event is a problem common to other studies and also occurs here. This problem is resolved by narrowing the research field from the entire population to the youngest group, i.e., people ages 3 to 20. The variables in the survey should be applicable for modeling of this group. For example, Figure 3, shows that almost all people in this group (ages 3-20) are unmarried. Because marriage is a crucial event that will affect household formation and thus variables at the household level, this fact thus supports our assumption to some extent.

On the other hand, although information on older generations will be lost by narrowing the research focus, we believe that such loss is justifiable because the refined study will provide more up-to-date and useful information for policy makers. Thus, people aged 3-20 will be the focus of our survival modeling.
Semi-parametric Survival Analysis: Cox Regression

(1) Covariates in the Survey

Based on previous analysis and on the literature, the model could include income, gender, age, educational attainment, and gender of heads of households, number of siblings, geography, and distance to schools, etc. We will expect the following effects on educational attainment:

**Income:** A positive effect; educational attainment should increase when household income increases. The survey does not report total household income, so household total expenditure is used as a proxy for household income.

**Gender:** Females will have positive effect because females have better educational attainment in general. In the model, females have a value of 1, and males have a value of 0.

**Age (or Time):** A negative effect; younger generations should have higher educational attainment because of the improvement of education quality by government policy, etc.

**Educational attainment of the head of household:** A positive effect because a household head with more education should lead to higher educational attainment of children.

**Gender of head of household:** Some literature suggests that a female head of household might have a negative effect. In the model, females have a value of 1, and males have a value of 0.

**Number of siblings:** A negative effect; more siblings will lead to lower educational attainment of children.
**Geographic area:** Jamaica can be divided into three geographic areas: Kingston Metropolitan Area (KMA), Other Towns, and Rural Areas. People living in KMA and Other Towns areas should have higher educational attainment.

**Distance to schools:** A negative effect; the farther the distance to schools, the higher the education cost will be, which will lead to lower educational attainment. Because not all people report the distance to schools, we used the average distance (in miles) to schools for people within the same district. Thus, we obtained the distance to the nearest primary school and the distance to the nearest secondary school.

Meanwhile, we also suspect that the effect of age (time) may vary in primary, lower, and upper secondary education. Consequently, we created interaction variables between age and these school types. As a result of these considerations, we have individual variables (gender, age), family variables (household income, gender, and education of heads of households, number of siblings), and school variables (distance to schools). An age variable can also be used to control the time effect.

(2) **Estimation**

After getting the variables available from JSLC 2002, we can estimate a discrete time Cox model. The result is shown in Table 7. We can see that 6,982 individuals are included in the model and 92.17% of them are censored. This should not be a surprise, because the research subjects are ages 3-20 and most of them were at school at that time. As noted earlier, the capability to handle censored data is one of the biggest advantages of survival analysis, and that advantage is also one of the main reasons we chose to use survival analysis in the study.
The table shows that all three global null hypothesis tests (likelihood ratio test, score test and Wald test) are significant (p<.0001). Household income, gender, age, grade level attained by the head of household, number of siblings, and distance to the nearest secondary school are all significant, but the gender of the head of household, distance to nearest primary school, and the geographic dummy variables (KMA, Other Towns?, and Rural Areas) are not statistically significant. The interaction between age and the dummy variable for primary school is significant, but the interaction between age and the lower secondary school dummy variable is not significant. The full expression of the model can be written as:

\[
\log\left(\frac{P(d)}{1-P(d)}\right) = \alpha(d) \\
= -1.0212 \times \text{logexp} - 0.5891 \times \text{female} \\
+ 0.0729 \times \text{Age} + 0.3669 \times (\text{Age} = \text{Primary}) - 0.0451 \times (\text{Age} = \text{Low Secondary}) \\
- 0.1060 \times \text{Grade of Head} - 0.0662 \times \text{Gender of Head} \\
+ 0.0788 \times \text{Number} \\
- 0.0113 \times \text{Distance to Primary} + 0.0058 \times \text{Distance to Secondary} \\
+ 0.0788 \times \text{KMA} + 0.0610 \times \text{Other Town} \\
(0.0786) \quad (0.1264)
\]

47 Standard errors for the coefficients are reported inside the parenthesis below the coefficients. Please note that Cox regression doesn’t estimate time effect, i.e. \( \alpha(d) \) in the model. 47

***: .0001 significant level; **: .001 significant level; *: .01 significant level
As noted earlier, we also can estimate a Logit model for the discrete time survival model if we want to know the effects of time. For convenience, we will continue to estimate a Logit model and then compare it with the result from this model. After that, we can discuss these comparative findings.

19 (Essay 3) Table 7 Estimation of Discrete Time Cox Model

<table>
<thead>
<tr>
<th>Covariates</th>
<th>Coefficient</th>
<th>Chi-Square</th>
<th>Pr&gt;ChiSq</th>
</tr>
</thead>
<tbody>
<tr>
<td>Household Income (log form)</td>
<td>-1.0213</td>
<td>170.2995</td>
<td>&lt;.0001</td>
</tr>
<tr>
<td>Gender</td>
<td>-0.5001</td>
<td>27.6982</td>
<td>&lt;.0001</td>
</tr>
<tr>
<td>Age</td>
<td>0.1224</td>
<td>11.693</td>
<td>0.0006</td>
</tr>
<tr>
<td>Age*Primary</td>
<td>0.1689</td>
<td>7.8053</td>
<td>0.0052</td>
</tr>
<tr>
<td>Age*Lower Secondary</td>
<td>-0.0721</td>
<td>1.5199</td>
<td>0.2176</td>
</tr>
<tr>
<td>Grade of Head</td>
<td>-0.1060</td>
<td>27.2024</td>
<td>&lt;.0001</td>
</tr>
<tr>
<td>Gender of Head</td>
<td>-0.0362</td>
<td>0.1504</td>
<td>0.6981</td>
</tr>
<tr>
<td>Sibling Number</td>
<td>0.0763</td>
<td>18.0029</td>
<td>&lt;.0001</td>
</tr>
<tr>
<td>Distance-Primary</td>
<td>-0.0113</td>
<td>0.2661</td>
<td>0.6059</td>
</tr>
<tr>
<td>Distance-Secondary</td>
<td>0.0698</td>
<td>25.6078</td>
<td>&lt;.0001</td>
</tr>
<tr>
<td>KMA Area</td>
<td>0.0788</td>
<td>0.2563</td>
<td>0.6127</td>
</tr>
<tr>
<td>Other Towns</td>
<td>0.0519</td>
<td>0.1444</td>
<td>0.704</td>
</tr>
</tbody>
</table>

Testing Global Null Hypothesis

<table>
<thead>
<tr>
<th>Test</th>
<th>Chi-Square</th>
<th>DF</th>
<th>Chi-Square</th>
</tr>
</thead>
<tbody>
<tr>
<td>Likelihood Ratio</td>
<td>396.5562</td>
<td>12</td>
<td>&lt;.0001</td>
</tr>
<tr>
<td>Score</td>
<td>394.1473</td>
<td>12</td>
<td>&lt;.0001</td>
</tr>
<tr>
<td>Wald</td>
<td>370.7448</td>
<td>12</td>
<td>&lt;.0001</td>
</tr>
</tbody>
</table>

Parametric Survival Analysis: Logit Model

In this section we will apply a traditional Logit model to survival analysis and estimate the time effect. The Logit model has become popular for discrete time survival analysis because it is easier to understand and use. Before the Logit model can be used, however, the cross-sectional data must be transformed into a new data set categorized by time and individual. That is to say, each individual will have several observations that correspond to his or her “current” grade level. Moreover, each observation must include new time/grade dummy variables. Singer and Willett (1993) demonstrate in detail how to transform the
relevant data. The estimation of the Logit model is shown in Table 8. In comparing Table 8 to Table 7, the close similarity of the estimation of the coefficients of the covariates is obvious because the Cox model applied a semi-parametric method to the estimation of a Logit model that does not include the time effect. Table 8 also includes the time/grade effect. Because the estimation results are very similar, we can focus on the estimation result from the Logit model and discuss its findings.

(1) Income Effect

Household income has a positive and significant effect on educational attainment. The coefficient of the log form of household total expenditure is -1.0222, and the odds ratio is 0.360. The result shows that the increase in household income will decrease the risk of dropping out of school and thus increase individual educational attainment. An increase in log income by 1 will reduce the odds ratio by 64%. This finding is consistent with the previous study.

(2) Gender Effect

The gender effect is very significant. The coefficient is -0.5005, and the odds ratio 0.606. The result reflects that females are at lower risk of dropping out and achieve higher educational attainment. Females have an odds ratio that is 60.6% of males’.

(3) Time-Age Effect

We estimate age and its interaction variables with primary and lower secondary school types. The upper secondary school dummy variable is left out of the model. The coefficient of age variable, 0.1125, is significant. The coefficient of the interaction between age and the primary school dummy variable is 0.169, which is also significant. However, the interaction between age and the lower secondary school dummy variable is -0.0721 and not significant with a 20% confidence level. The result shows that in general the dropout risk increases as age increases, i.e., younger people are associated with lower dropout risks. This finding tells
that educational attainment improves over time. The improvement is much greater for those in primary school education than for those in secondary schools. Lower secondary school education does not perform better than upper secondary school education. In fact, the performance for lower secondary schools is slightly worse but not significant. Nevertheless, this result for the lower secondary schools surprises us because our research subjects (aged 3-20) were in the secondary schools during the ROSE I project (implemented after 1994), which tried to reform lower secondary school education.

If we look at the odds ratio, for every year that students age, the odds ratio will increase by 13% in general and by 31.4% in primary school. In other words, the odds ratio decreases by 13% annually in general and by 31.4% annually in primary school. This improvement can be attributed to the Jamaican government’s educational policies and to the nation’s determination and efforts to improve its education.

(4) Effects of Head of Household

The literature shows that head of household might have two effects on educational attainment: The educational attainments of the household’s children might be positively correlated to the attainments of the head of household or the gender of head of the household might affect the educational attainments of the children. Our estimation shows that the educational attainment of the head of the household has a significant effect, -0.1061, and the associated odds ratio is 0.964. That is to say that for every additional grade of educational attainment by the head of a household, the odds ratio related to the risk of a child in the household dropping out will be reduced by 3.6%. As for the impact of the gender of the head of household, however, the coefficient is negative but not statistically significant.

(5) Effect of the Number of Siblings

For example, Buchmann and Hannum (2001) performed a review of the literature that shows that in some African contexts female headship appears to be associated with greater educational opportunities for children in South Africa (Fuller and Liang, 1999) and in sub-African countries (Lloyd and Blanc, 1996).
The coefficient for the variable concerning the number of siblings is significant, 0.0764, and the odds ratio is 1.079. In other words, for every additional sibling in a family, the odds ratio for a child’s risk of dropping increases by 7.9%. This is also consistent with the literature. More siblings mean fewer family resources to put into education for each child, which might cause lower educational attainment. In explaining the effect of siblings, Buchmann and Hannum (2001) emphasize the importance of understanding the social and economic contexts in which families make educational decisions for their children.

(6) School Effect

The literature shows that the distance from home to school will affect educational attainment. In our estimation, the distance to primary school has no significant effect. However, the distance to secondary school has a significant effect, 0.0764, and the odds ratio is 1.072. That is to say, an increase in distance by 1 mile will increase the odds ratio by 7.2%.

(7) Geographic Effect

Jamaica is divided into three geographic areas: KMA, Other Towns, and Rural Areas. The literature highlights the difference in education quality between rural and urban areas. However, after we control other variables, we cannot find a significant difference between them. This suggests that geography does not directly influence educational attainment; instead it is the characteristics of geographic areas that have direct effects.

(8) Time-Grade Effect

The Logit model includes an estimate of a time effect. In this study, our research time span is from Grade 1 to 11: i.e., students who have completed Grade 11 are treated as survivors. Each time/grade is associated with a baseline hazard, which is derived by assuming that all other variables are 0.

Our estimation has 10 dummy variables, one each for Grades 1 to 10. These produce a clear pattern of coefficients in which the coefficients increase as the grade levels increase and
all of these coefficients are negative except for Grade 10. The coefficients for Grades 7-9 are negative but not significant. These results tell us two things. First, in general the baseline hazards increase as grade levels increase but Grade 10 is an exception and has a higher baseline hazard than Grade 11. Second, the coefficients of Grades 7-9 are not significant, which shows that grades in the secondary schools differ little in baseline hazard except for Grade 10, which has a much higher baseline hazard. This result for Grade 10 might reflect a space shortage in the upper secondary schools that prevents students from achieving higher educational attainment.
## 20 (Essay 3) Table 8 Estimation of Discrete Time Survival Model-Logit Model

<table>
<thead>
<tr>
<th>Covariates</th>
<th>Coefficient</th>
<th>Chi-Square</th>
<th>Pr&gt;ChiSq</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>8.4172</td>
<td>51.3959</td>
<td>&lt;.0001</td>
</tr>
<tr>
<td>grade1</td>
<td>-19.3456</td>
<td>0.0085</td>
<td>0.9267</td>
</tr>
<tr>
<td>grade2</td>
<td>-7.0268</td>
<td>36.915</td>
<td>&lt;.0001</td>
</tr>
<tr>
<td>grade3</td>
<td>-6.8301</td>
<td>35.5478</td>
<td>&lt;.0001</td>
</tr>
<tr>
<td>grade4</td>
<td>-6.4051</td>
<td>32.4019</td>
<td>&lt;.0001</td>
</tr>
<tr>
<td>grade5</td>
<td>-6.179</td>
<td>30.4034</td>
<td>&lt;.0001</td>
</tr>
<tr>
<td>grade6</td>
<td>-4.9914</td>
<td>20.7265</td>
<td>&lt;.0001</td>
</tr>
<tr>
<td>grade7</td>
<td>-0.995</td>
<td>0.9419</td>
<td>0.3318</td>
</tr>
<tr>
<td>grade8</td>
<td>-0.9229</td>
<td>0.7838</td>
<td>0.376</td>
</tr>
<tr>
<td>grade9</td>
<td>-0.3469</td>
<td>0.1088</td>
<td>0.7415</td>
</tr>
<tr>
<td>grade10</td>
<td>0.859</td>
<td>42.7507</td>
<td>&lt;.0001</td>
</tr>
<tr>
<td>Household Income (log form)</td>
<td>-1.0222</td>
<td>170.3635</td>
<td>&lt;.0001</td>
</tr>
<tr>
<td>Gender</td>
<td>-0.5005</td>
<td>27.7172</td>
<td>&lt;.0001</td>
</tr>
<tr>
<td>Age</td>
<td>0.1225</td>
<td>11.701</td>
<td>0.0006</td>
</tr>
<tr>
<td>Age*Primary</td>
<td>0.169</td>
<td>7.8084</td>
<td>0.0052</td>
</tr>
<tr>
<td>Age*Lower Secondary</td>
<td>-0.0721</td>
<td>1.5211</td>
<td>0.2175</td>
</tr>
<tr>
<td>Grade of Head</td>
<td>-0.1061</td>
<td>27.223</td>
<td>&lt;.0001</td>
</tr>
<tr>
<td>Gender of Head</td>
<td>-0.0362</td>
<td>0.1507</td>
<td>0.6979</td>
</tr>
<tr>
<td>Sibling Number</td>
<td>0.0764</td>
<td>18.0159</td>
<td>&lt;.0001</td>
</tr>
<tr>
<td>Distance-Primary</td>
<td>-0.0113</td>
<td>0.2662</td>
<td>0.6059</td>
</tr>
<tr>
<td>Distance-Secondary</td>
<td>0.0699</td>
<td>25.6257</td>
<td>&lt;.0001</td>
</tr>
<tr>
<td>KMA Area</td>
<td>0.079</td>
<td>0.2571</td>
<td>0.6121</td>
</tr>
<tr>
<td>Other Towns</td>
<td>0.0519</td>
<td>0.1445</td>
<td>0.7039</td>
</tr>
</tbody>
</table>

### Testing Global Null Hypothesis

<table>
<thead>
<tr>
<th>Test</th>
<th>Chi-Square</th>
<th>DF</th>
<th>Chi-Square</th>
</tr>
</thead>
<tbody>
<tr>
<td>Likelihood Ratio</td>
<td>2060.0773</td>
<td>22</td>
<td>&lt;.0001</td>
</tr>
<tr>
<td>Score</td>
<td>4180.3474</td>
<td>22</td>
<td>&lt;.0001</td>
</tr>
<tr>
<td>Wald</td>
<td>1210.4211</td>
<td>22</td>
<td>&lt;.0001</td>
</tr>
</tbody>
</table>
(9) Summary

After getting the estimates, we can write the estimated Logit model as follows: 49

\[
\begin{align*}
\log\left(\frac{P(Y=1)}{1-P(Y=1)}\right) &= 0.4172 \\
&= -19.8456 \times \text{Grade} - 7.0266 \times \text{Grade} - 6.5891 \times \text{Grade} - 4.6981 \times \text{Grade} - 6.4179 \times \text{Grade} \\
&\quad (2102) \quad (1.1964) \quad (1.1964) \quad (1.1964) \quad (1.1964) \\
&\quad -45.956 \times \text{Grades} - 6.995 \times \text{Grades} - 0.229 \times \text{Grades} - 0.3469 \times \text{Grades} + 0.859 \times \text{Grades}  \\
&\quad (10.964) \quad (1.0292) \quad (0.9151) \quad (0.1816) \\
&\quad -13.222 \times \log\text{exp} \quad -0.8065 \times \text{female}  \\
&\quad (0.0788) \quad (0.0801) \\
&\quad +0.1225 \times \text{Age} \quad +0.169 \times \text{Age} - \text{Primary} \quad -0.0731 \times \text{Age} - \text{Lower Secondary}  \\
&\quad (0.0825) \quad (0.0788) \quad (0.0883) \quad (0.0883) \\
&\quad -0.0861 \times \text{Grade of Head} \quad -0.0863 \times \text{Gender of Head}  \\
&\quad (0.0008) \quad (0.0008) \\
&\quad +0.0754 \times \text{Sibling Number}  \\
&\quad (0.0180) \\
&\quad -0.01113 \times \text{Distance to home} + 0.00995 \times \text{Distance to family}  \\
&\quad (0.0219) \quad (0.0138) \\
&\quad +0.079 \times \text{HIV} + 0.0019 \times \text{Other Towns}  \\
&\quad (0.1487) \quad (0.0265)
\end{align*}
\]

As a demonstration, we can use our estimation to draw hazard and survival curves for a typical individual with median values of those variables as follows: household income is J$283,322 50; age is 18; the head of household has completed Grade 10; there are five siblings; and the distance to a secondary school is 3.03 miles. We also will have hazard and survival curves by gender as in Figure 4. As we can see, the hazards at Grade 10 are almost two times as many as those at Grade 11. This is mostly caused by the shortage of space for students in the upper level secondary schools.

To demonstrate the application of our estimation, we will use as an example our analysis of the effect of the ROSE II project in its effort provide enough space in the upper secondary schools. Based on our estimation and on an assumption that after completion of the ROSE II

---

49 ****: .0001 significant level; ***: .001 significant level

project the peak at Grade 10 will decline to the same level as that at Grade 11, we can estimate that the probability that these typical individuals will complete Grade 11 will increase by about 5%, to 86% from 81%.\textsuperscript{51}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure4-1.png}
\caption{Estimated Hazard Curves for Typical Individuals}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure4-2.png}
\caption{Estimated Survival Curves for Typical Individuals}
\end{figure}

\textsuperscript{51} Specifically, the increases are 4.7% (from 85% to 89.7%) for females and 6.7% (from 77% to 83.7%) for males. The calculation process is omitted. Readers should be able to calculate the numbers based on Table 8.
23 (Essay 3) Figure 5-1 Life Table Analysis-Survival Curve, by age group and gender

24 (Essay 3) Figure 5-2 Life Table Analysis-Dropout Hazard Curve, by age group and gender
Policy Implications
In the previous sections, we examined the JSLC data and applied survival analysis techniques to explore education duration in Jamaica. To some extent, the findings should be able to answer the questions of “Where, What and How”: where (or in which grade) the students tend to drop out of school, what factors affect dropout behavior, and how to increase education duration.

Where Is the Problem?
From Table 8, we can see the coefficients of the dummy variables of Grades 1-10. This table tells us the distribution of dropout risk is as follows:

1. Primary Schools

In general, dropout risks were quite low in primary schools. Nevertheless, there was a trend of increasing risks from Grade 1 to Grade 6. Grade 6 especially had a huge increase in dropout risk compared with Grade 5. In Table 8, the coefficient of Grade 6 is 1.1876 higher than that of Grade 5, which means an odds ratio increase of about 2.28 times. Figure 4 shows that the estimated hazard for typical individuals in Grade 6 is significantly higher than for those in Grades 1-5, and Grade 6 has the same hazard level as in the lower secondary schools. (It is even slightly higher than the estimated hazards in Grade 7 and Grade 8.) This finding is very interesting and also confusing: Why did the dropout risk “suddenly” increase in the last year of primary schools when students were so near a graduation certification? Another thing worthy of note is that the curves for females and males diverge after Grade 6 as the effect of gender becomes increasingly obvious with the increase in the risk of dropping out.

2. Lower Secondary Schools

The dropout risks in lower secondary schools were similar in Grade 7 and in Grade 8. However, Figure 4 shows that the dropout risk almost doubled in Grade 9. Table 8 shows that the coefficient increased from -0.9229 in Grade 8 to -0.3469 in Grade 9, which means an odds ratio increase of about 0.779 times. Again we are confronted by the same question of
why the dropout risk “suddenly” increased in the last year of primary schools when a graduation certification was within reach.

The second problem in the lower secondary schools is that the improvement over time is slower than in primary schools (and even slower than in secondary schools, although not significant.). The lag in improvement in the lower secondary schools deserves attention because the ROSE I project that targeted lower secondary schools was completed in this period.

(3) Upper Secondary Schools

The upper secondary schools carried the highest dropout risks. This is obvious in Figure 4. Table 8 shows (1) that Grade 10 has a positive coefficient compared with Grade 11, and (2) Grades 1-9 have negative coefficients compared with Grade 11. As discussed earlier, one of main reasons for the high dropout risk in Grade 10 is because of space shortages in the upper secondary schools. However, even after we eliminate this effect by assuming that Grade 10 has the same dropout risk as Grade 11, thus controlling for the effect of a space shortage, the dropout risks in upper secondary high schools remain quite high. In Figure 4, the numbers are 6% and 4%, respectively, for males and females. Using the same assumption, we can estimate that 8% and 5%, respectively, of males and females could not go to upper secondary schools because of the space shortage. The unanswered question is why the dropout risks were so high despite the students’ successfully entry into the upper secondary schools?
What Factors Matter?
The analysis thus far has revealed the factors that could affect dropout risk. As a summary, we know that (1) household income, age, and the grade level achievement of the head of household have positive effects on educational attainment, and females typically surpass males in educational attainment; (2) the number of siblings and the distance to secondary schools have negative effects; and (3) the gender of the head of household head and the geographic area have no effect.

How: Some Policy Suggestions
Based on the foregoing analysis, some policy suggestions for continued improvement of education are:

(1) Target the Right Place

In general, secondary schools (especially upper secondary schools) have higher dropout risks. While this generalization is noteworthy, the most effective way to lessen dropout risks would be to focus on several “problematic” grades: Grade 6, Grade 9, and Grade 10. There is a need especially to determine why dropout risks increased so dramatically in the last year of primary school and lower secondary school. Were the students intimidated by the prospect of a difficult last year? The high dropout risk observed in Grade 10 was caused by two factors: admission limitation (a space shortage in upper secondary school) and voluntary dropouts. As we have estimated, providing enough space in upper secondary school might increase the percentage of graduation from secondary schools by 5%, to 86% from 81%.

(2) Gender Difference

In our estimation in Figure 4, we can see an 8% difference in survival rates between females and males. This is a significant percentage. If survivorship for males could be raised to the same level of females, it would mean about a 4% increase in the percentage of graduation from secondary schools. However, we do not know the reasons for the gender gap in education. Although much literature has been developed, little of it relates directly to
Jamaica. (And in many developing countries, the gender gap favors males because females receive less education because of social discrimination.) A study of this issue should be able to provide useful policy guidance for solving the problem of inferior educational performance by males in Jamaica.

(3) How to Break the Cycle of Poverty?

Our estimation shows that household income and educational achievement, as measured by the grade level achievement of the head of household, had significant effects on educational attainment. Household income directly affects the resources available for children’s education. The effect of the grade level achievement of the head of household might come from two sources: inheritance, and awareness of education’s importance. Human capital theories show that higher educational attainment is associated with higher income. And the absence of such attainment is associated with a cycle of poverty represented by: poverty (Generation I) > low educational attainment (Generation II) > low income (Generation II) + low educational attainment (Generation II) > low educational attainment (Generation III) …

This depiction of the poverty cycle shows that it can be broken by any one of three ways: (1) increased family income; (2) increased parental education; (3) increased educational attainment of children. Several projects have been undertaken in Jamaica to help the poor, such as the Jamaica National Poverty Eradication Programme, School Feeding Programme, Rural Electrification Programme (REP), and so on. Such programs have increased family income and thus have the potential to increase the educational attainment of poor people. Increasing parental education is more difficult, although there are adult schools. The more feasible way might be for the government to adopt a policy to increase the awareness among parents of the importance of education. To increase the educational attainment of children, we should focus on school quality and increased effort by students. Unfortunately, our
research does not yield much information directly related to the quality of schools. The only factor related to schools is the variable of distance to secondary schools, which had a significant negative effect on educational attainment. Thus, we suggest that increased space in the upper secondary schools should be achieved by building new schools instead of enlarging the current ones. (If new schools can only be created by much higher expenditures, this goal should still be pursued by emphasizing cost effective policies.) Another study focusing on school quality and the effect on students should be able to provide useful policy guidance.

(4) Uncontrollable Effects

The effects of age (time), gender of the head of household, number of siblings and geographic factors are uncontrollable. As we have seen in Figure 5, education in Jamaica is improving continuously and significantly over time. However, the time effect chronicled here relates mainly to the past efforts in education of the Jamaican people and government and may not have the same effect in the future. Gender of the head of household is uncontrollable, and in any case, the effect was insignificant. The number of siblings had a negative effect on educational attainment. It is common sense in terms of economics that more children mean fewer resources for each child. However, as a social phenomenon, the number of children in a family is hard to control by governmental policy. Geographic factors are also uncontrollable. However, after controlling for other covariates in our model, geographic effects became insignificant. A governmental policy that directly targets a geographic area might not be necessary.
Conclusions

In our study we have applied survival techniques to analyze education duration in Jamaica based on JSLC 2002 data. Because of the limitation of the data, we do not have time varying covariates. Thus we focused on the youngest cohort, who had an age range of 3-20, and we assumed that the values of the covariates would change little for them. Although this focus loses information on the older generations, such a study still could provide useful information for policy makers who deal with current issues in education.

The estimation results of the Cox regression and the Logit model are very similar, and the Logit model gives us an extra estimation of the time (grade) effect. The estimated model can be used to estimate an individual’s hazard function as we have shown in Figure 4. The analysis in this section provides information about where the problem was and what caused the problem. The results could be used by policy makers to target the right persons and right time to reduce dropout rates. They also could estimate the effects of education policies, their economic background, and their demographic characteristics. In the previous section, we provide some policy implications based on our research.

However, our study did have several obvious limitations. First, we did not have time varying variables from the survey data, a lack that might cause inaccurate estimation of the effects of the covariates. Longitudinal data based on several continuous surveys for the same cohort would improve our estimation. Second, we did not have enough variables. The Jamaica Survey of Living Conditions 2002 has a section on education, but the variables we can use from it are very limited. Our covariates can be grouped into approximately four categories: individual characteristics (gender, age), family characteristics (household income, gender and educational attainment of the head of household, and number of siblings), school characteristics (distances to primary and secondary schools), geographic characteristics (KMA, Other Towns and Rural Areas), and a time/grade variable. The variables are very
limited, and we cannot estimate the effects of many other important factors, such as school quality, exam scores, etc. Third, we measured educational attainment based on education duration of individuals (i.e. their grades completed or attending). However, education duration does not distinguish the difference in quality between types of schools. For example, a graduate from the all-age schools generally has lower educational attainment than a graduate from the primary schools because all-age schools generally are inferior to the primary schools. Further studies in these fields will allow better estimations.
APPENDIX

Solutions for Equations from Totally Differentiating FOCs (1), (2) and (3)

After totally differentiating FOCs, we can solve the equations as followings:

\[
\frac{\partial x}{\partial I} = \frac{|A^T|}{|A|}
\]
\[
\frac{\partial y}{\partial I} = \frac{|A^T|}{|A|}
\]
\[
\frac{\partial n}{\partial I} = \frac{|A^n|}{|A|}
\]

The sign of the derivatives are undetermined. However, if we know that X and Y are normal goods, we’ll have \(\frac{\partial x}{\partial I} > 0\) and \(\frac{\partial y}{\partial I} > 0\). Under some conditions, we can make \(\frac{\partial n}{\partial I} > 0\). i.e. Under these assumptions, we’ll have the conclusion that the club size will increase as income increases. In our paper, we’ll base on this assumption which can simplify our analysis and also not affect the analysis framework.

\[
A = \begin{pmatrix}
A_1 & A_2 & A_3 & A_4 & A_5 \\
A_6 & A_7 & A_8 & A_9 & A_{10} \\
A_{11} & A_{12} & A_{13} & A_{14} & A_{15} \\
A_{16} & A_{17} & A_{18} & A_{19} & A_{20} \\
A_{21} & A_{22} & A_{23} & A_{24} & A_{25}
\end{pmatrix}
\]

\[
A^2 = \begin{pmatrix}
A_{11}^2 & A_{12}^2 & A_{13}^2 & A_{14}^2 & A_{15}^2 \\
A_{16}^2 & A_{17}^2 & A_{18}^2 & A_{19}^2 & A_{20}^2 \\
A_{21}^2 & A_{22}^2 & A_{23}^2 & A_{24}^2 & A_{25}^2 \\
A_{26}^2 & A_{27}^2 & A_{28}^2 & A_{29}^2 & A_{30}^2 \\
A_{31}^2 & A_{32}^2 & A_{33}^2 & A_{34}^2 & A_{35}^2
\end{pmatrix}
\]

\[
A^3 = \begin{pmatrix}
A_{11}^3 & A_{12}^3 & A_{13}^3 & A_{14}^3 & A_{15}^3 \\
A_{16}^3 & A_{17}^3 & A_{18}^3 & A_{19}^3 & A_{20}^3 \\
A_{21}^3 & A_{22}^3 & A_{23}^3 & A_{24}^3 & A_{25}^3 \\
A_{26}^3 & A_{27}^3 & A_{28}^3 & A_{29}^3 & A_{30}^3 \\
A_{31}^3 & A_{32}^3 & A_{33}^3 & A_{34}^3 & A_{35}^3
\end{pmatrix}
\]
REFERENCES


Boltvinik. Poverty measurement methods-an overview.


Fagiolo, G. (2001). Engel curves specification in an artificial model of consumption dynamics with socially evolving preferences. Unpublished LEM Papers Series. Laboratory of Economics and Management (LEM), Sant'Anna School of Advanced Studies, Pisa, Italy.


Gibson, J. *Why does the Engel method work? Food demand, economies of size and household survey methods*: SSRN.

Goh, A.-T., & Wan, H. *Fragmentation, Engel's law, and learning*: SSRN.


VITA

Shiyuan Chen was born in the Fujian province in Southeast China. He graduated from high school and entered Xiamen University in 1994. He graduated with a Bachelor in Taxation degree and entered the masters program in Finance in 1998. In 2001, he received a Masters degree in Finance from Xiamen University. At the same time, he received an offer from the masters program in the Department of Economics at Georgia State University and came to Atlanta on January 1, 2002. He transferred to the doctorate program in August 2003. Then he received a Master of Arts degree in Economics in December 2005 and passed the final defense for the dissertation required by the doctorate program in March 2008.

Shiyuan’s research interests focus on public finance and taxation, development economics, finance, and education. He has published several academic papers in the banking and finance fields in China. While he attended Georgia State University, his research mostly focused on public finance, development economics and education issues.

Shiyuan taught finance classes at Xiamen University in 2000. He worked as a graduate research/teaching assistant at Georgia State University from 2002 to 2008. He worked as part time Statistician at Assurant from 2006 to 2007. Currently, he is a Senior Analyst at Microfinance Risk Management, LLC.