A Tale of Two Telescopes: Taking a Closer Look at the Multiplicity Properties of Massive Stars in Cygnus

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ABSTRACT

Massive stars profoundly influence the evolution of the Universe, from Galactic dynamics and structure to star formation. They are often found with bound companions. However, our knowledge of O-type multiple systems with periods in the range from years to thousands of years is incomplete due their great distances. We present results from a high angular resolution survey to find angularly resolved companions using the Fine Guidance Sensor (FGS) on the Hubble Space Telescope and using ground-based adaptive optics at Gemini North. We observed 75 O- and early B-type stars in Cyg OB2 and determined that 42% of the sample have at least one companion that meets a statistical criterion for gravitationally bound status.

As a case study, we present an examination of high resolution, ultraviolet spectroscopy from Hubble Space Telescope of the photospheric spectrum of the O-supergiant in the massive X-ray binary HDE 226868 = Cyg X-1. We analyzed the ultraviolet and ground-based optical
spectra to determine the effective temperature and gravity of the O9.7 Iab supergiant. Using non-LTE, line blanketed, plane parallel models from the TLUSTY grid, we obtain $T_{\text{eff}} = 28.0 \pm 2.5$ kK and $\log g \gtrsim 3.00 \pm 0.25$, both lower than found in previous studies. The optical spectrum is best fit with models that have enriched He and N abundances. We fit the model spectral energy distribution for this temperature and gravity to the UV, optical, and IR fluxes to determine the angular size of and extinction towards the binary. By assuming that the supergiant rotates synchronously with the orbit, we can use the radius – distance relation to find mass estimates for both components as a function of the distance and the ratio of stellar to Roche radius. Our results indicate masses of $23^{+8}_{-6} M_\odot$ for the supergiant and $11^{+5}_{-3} M_\odot$ for the black hole. These results agree with subsequent mass estimates by Orosz et al. (2011) based on the trigonometric parallax distance measurements of Reid et al. (2011).

The results of this survey provide fundamental information on the impact of environment on massive binaries and also the role multiplicity has on massive star formation and evolution.

INDEX WORDS: Massive stars, Binary stars, Adaptive Optics, Cygnus OB2, Fine Guidance Sensor, Cygnus X-1, HDE 226868
A TALE OF TWO TELESCOPES: TAKING A CLOSER LOOK AT THE
MULTIPlicity PROPERTIES OF MASSIVE STARS IN CYGNUS

by

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DEDICATION

To La Tribu Caballero

For showing me to always “echar pa’lante, pa’tra ni pa’ impulso,” encouraging me to pursue my dreams and instilling in me the belief that anything is possible.
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<tr>
<th>Abbreviation</th>
<th>Description</th>
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<tbody>
<tr>
<td>2MASS</td>
<td>Two-micron All Sky Survey</td>
</tr>
<tr>
<td>AMA</td>
<td>Articulated Mirror Assembly</td>
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<tr>
<td>AO</td>
<td>Adaptive Optics</td>
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<tr>
<td>AU</td>
<td>Astronomical Unit (mean distance from Earth and Sun)</td>
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<tr>
<td>BH</td>
<td>black hole</td>
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<tr>
<td>CCD</td>
<td>Charged Coupled Device</td>
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<td>Cyg</td>
<td>Cygnus</td>
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<tr>
<td>Dec.</td>
<td>Declination (equivalent to latitude on the sky, in degrees)</td>
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<tr>
<td>FGS</td>
<td>Fine Guidance Sensor</td>
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<tr>
<td>FWHM</td>
<td>full-width at half-maximum</td>
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<tr>
<td>HST</td>
<td><em>Hubble Space Telescope</em></td>
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<tr>
<td>IDL</td>
<td>Interactive Data Language</td>
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<tr>
<td>IR</td>
<td>infrared</td>
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<td>IUE</td>
<td><em>International Ultraviolet Explorer</em></td>
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<tr>
<td>LTE</td>
<td>Local thermodynamic equilibrium</td>
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<td>MAST</td>
<td>Mikulski Archive for Space Telescopes</td>
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<tr>
<td>mag</td>
<td>Magnitude</td>
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<tr>
<td>mas</td>
<td>Milli-arcsecond</td>
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<tr>
<td>NASA</td>
<td>National Aeronautic and Space Administration</td>
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<tr>
<td>NIRI</td>
<td>Near InfraRed Imager and Spectrograph</td>
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<td>pc</td>
<td>Parsec</td>
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<td>Abbreviation</td>
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<tr>
<td>POS</td>
<td>Position observation mode of the FGS</td>
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<tr>
<td>R.A.</td>
<td>Right Ascension (equivalent to longitude on the sky, in hours)</td>
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<tr>
<td>SED</td>
<td>spectral energy distribution</td>
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<tr>
<td>STIS</td>
<td>Space Telescope Imaging Spectrograph</td>
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<td>TRANS</td>
<td>Transfer observation mode of the FGS</td>
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<td>UCAC3</td>
<td>Third release of the USNO CCD Astrograph Catalog</td>
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<tr>
<td>UKIDSS</td>
<td>UKIRT Infrared Deep Sky Survey</td>
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<td>UKIRT</td>
<td>United Kingdom Infrared Telescope</td>
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<td>USNO</td>
<td>United States Naval Observatory</td>
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<td>UV</td>
<td>ultraviolet</td>
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1

INTRODUCTION

Massive stars play a fundamental role in the evolution of the universe, from influencing Galactic dynamics and structure to triggering star formation through their spectacularly violent deaths. They are major sources of ultraviolet (UV) and ionizing radiation and drive the chemical evolution of the interstellar medium (ISM), especially via starburst regions.

In the spectral classification scheme, massive stars are of O- and early B-type. These stars have a mass greater than about 8 $M_\odot$ (Zinnecker & Yorke 2007). They are the hottest, brightest, and biggest (in terms of mass and size) stars. Due to their large luminosities, they are observed at much larger distances than lower mass stars. Throughout this dissertation, we use the terms hot stars, massive stars, and O-type stars synonymously.

Massive stars spend most of their formative years shrouded in their natal clouds, so that when they shed these clouds and a hot star is revealed, it is usually well into the Main Sequence stage of its life. We hope to understand the formation process of these influential stars through their binary fraction.

Despite their relative rarity with respect to lower mass stars, massive stars love company. These stars are mostly found in either dense, compact, young clusters or more expansive, loosely affiliated associations. Only about 20% of the massive stars constitute isolated, field stars (Gies 1987; Mason et al. 1998). For these cases, it is still uncertain whether they are formed independently or if they are runaway stars that were ejected from an association or cluster through a gravitational encounter or a supernova explosion. High mass stars are found mainly in the disk of the Galaxy where extinction is high and star formation is still active.
Due to large distances, high extinction, short lifetimes, and few numbers, our picture of massive star formation and particularly their early evolution is incomplete. Massive stars evolve relatively quickly, and therefore, by definition, they are young objects. The multiplicity of massive stars must play an important role in their formation because so many are members of binary systems. By studying the multiplicity properties of massive stars we can test the role of companions in formation theories of massive stars, and we can use orbital motion to measure fundamental stellar properties for these behemoths.

1.1 Observation Techniques

Binary stars are studied through a variety of techniques. These can be grouped into three distinct detection methods: periodic light variations, radial velocity variations and angularly resolved systems.

Eclipsing binaries found through periodic light variations are close systems with orbital planes that appear nearly edge-on to our line of sight (i.e., our line of sight is nearly parallel to the orbital plane of the system). The orbital periods range from hours to a few days, and their detection depends on the observing timeline. Shorter period systems are easier to detect than longer period systems that require an extensive observing program on a single object.

Spectroscopic binaries cover similar period and inclination regimes as eclipsing binaries. The systems are close in nature, and their large orbital motion causes observable shifts in the spectral lines of one or both components due to the Doppler effect. Velocity variability is easier to detect the closer the system is to edge-on. Similar to eclipsing variations, the detection and measurement of the long period systems are costly in terms of telescope time.
The advantage of these two methods is that they are distance independent. At a distance of 100 parsecs (pc) a spectroscopic binary with a period of 5 days will take the same amount of observing time to get complete phase coverage as a binary at 1 kiloparsec (kpc) (ignoring the fact that the system at 1 kpc will be fainter). On the other hand, the angular resolution of a binary or multiple system requires that the projected separation on the sky is larger than the limiting resolution of the telescope. At larger distances, the projected separation becomes smaller. Massive visual binaries, or astrometric systems, typically have periods on the order of years to millennia. One advantage of this method over the two mentioned above is that you are not restricted to systems that are edge-on, but can detect those over the entire range of inclination angle. Furthermore, it is easier to detect systems with more than two components. However, the timescales are prohibitive to obtain complete orbits for very widely separated cases, but a complete orbit is not necessary to establish duplicity. Observing the partial motion of the companion with respect to the primary star is enough to establish the system is gravitationally bound.

Observing orbital motion is also costly in terms of telescope time. As mentioned above, the period timescales for astrometrically detected systems are on the order of years or more. There exists a gap between systems that are detected spectroscopically and photometrically versus systems resolved astrometrically. In Figure 1.1, Mason et al. (1998) plot the number of known massive binary systems versus the logarithm of their orbital periods. It shows a bimodal distribution with shorter period, spectroscopic systems to the left and longer period, astrometric binaries to the right. The arrows on top indicate the regions that different observing techniques cover. The coverage gap can be bridged in two ways. The first is to
extend the timeline coverage for the spectroscopic and eclipsing systems. The second method is to improve the angular resolution limits of the imaging surveys done to date.

In this study we apply two high angular resolution techniques, adaptive optics and space interferometry, to look for astrometric companions to some of the most massive stars in our Galaxy. The Gemini North Near Infrared Imager and Spectrograph (NIRI) with the Altair adaptive optics (AO) system is able to detect astrometric companions in the range of $0\farcs08 - 15\arcsec$ with a magnitude difference less than $10\text{ mag}$. Adaptive optics reduces the effects of atmospheric distortion by using a deformable mirror to produce an image near the
diffraction limit of the optics of the telescope. We observed the 75 stars in Table 1.1 with the NIRI instrument.

Complementing this, the *Hubble Space Telescope (HST)* Fine Guidance Sensor (FGS) delves in slightly closer, resolving systems with separations of $0\,'01 - 2''$ and differential magnitudes less than 3 mag. The FGS is a shearing interferometer using the interference properties of the light of a star to improve the resolution limits of the single telescope. The FGS survey comprises those 58 stars in Table 1.1 marked with a ‘y’ in the final column.

### 1.2 Multiplicity Properties

It has been well established that massive stars have a higher binary frequency than lower mass stars (Mason et al. 2009; Raghavan et al. 2010). There is growing evidence that the binary fraction is as high as 100% (Mason et al. 1998, 2009; Kouwenhoven et al. 2007) for massive stars in clusters and associations. Stars in a cluster or association are at the same stage in their life cycles and have formed from the same collapsing cloud of gas, and therefore, have similar chemical abundances. As most massive stars are found in clusters and associations, the multiplicity properties of stars in an association are directly correlated to the formation scenarios of these stars (Zinnecker & Yorke 2007 references therein).

The true binary fraction remains incomplete due to the missing binaries with periods in the range of years to thousands of years (Mason et al. 1998). We need milliarcsecond (mas) resolution to start filling in this observational gap (Sana et al. 2008). High angular resolution techniques start to make inroads in this regime. Searching for binaries in clusters and associations is essential in establishing the relationship between high mass star formation and multiplicity.
1.3 Cygnus OB2

Cygnus OB2 (hereafter Cyg OB2) provides a relatively nearby environment \(d \approx 1.45 - 1.7\) kpc; Massey & Thompson 1991; Hanson 2003) that is rich in massive stars. The association has approximately 2600 ± 400 members (Knödlseder 2000) with about 100 O-stars within the central 1° (Comerón et al. 2002), making it one of largest concentration of OB stars in the Galaxy. It is home to some of the most massive \(M > 100\ M_\odot\); Herrero et al. 2001; Massey & Thompson 1991; Knödlseder 2000) and intrinsically brightest stars (MT 304; Massey & Thompson 1991) known in our Galaxy. It also harbors two of the rare stars classified with the early spectral type of O3 (MT 417 and MT 457; Walborn 1973a). Studying the collection of stars in this association will provide the necessary guidelines to interpret observations of more distant massive stars.

Cyg OB2 has been the subject of several multi-wavelength studies since its discovery by Münch & Morgan (1953). In one of the most comprehensive optical studies of the association, Massey & Thompson (1991) performed optical photometry on ~ 500 members of the association. They estimated the distance to the association to be \(d = 1.7\) kpc. Hanson (2003) obtained spectroscopic observations of a sub-sample of the Massey & Thompson (1991) stars. Age determinations of the association proved difficult due to evidence of some older stars at the same distance (Hanson 2003; Comerón et al. 2008; Negueruela et al. 2008). However, the majority of early stars suggest an age of about 2.5 Myr (Hanson 2003; Negueruela et al. 2008), making this a very young association where the low mass population has not yet reached the Main Sequence. Study of these fainter members is made difficult, not only by the circumstellar obscuration around young objects, but also the varying amount of
extinction throughout the association. MT 304 is one of the most reddened members, with an estimated extinction $A_V = 10.31$ mag. The optical extinction in the association varies from $A_V = 4 - 11$ mag (Massey & Thompson 1991), but the extinction is greatly reduced to $A_K = 0.4 - 0.95$ mag in the near infrared (Negueruela et al. 2008), making longer wavelength observations opportune for delving into the fainter population of the association.

The binary properties of Cyg OB2 have been studied extensively in the spectroscopic survey of Kiminki & Kobulnicky (2012) (and references therein). A few wider systems have been identified through the high angular resolution study of Maíz Apellániz (2010). In Table 1.1 we list 75 O- and early B-type stars that are the focus of our study. These stars were selected based on two criterion: (1) all of them have spectral classifications at the start of our survey and (2) they have $V < 14$ mag. The first column in Table 1.1 lists the designation we use in this paper, mainly taken from Massey & Thompson (1991) for the stars with MT # designations. Column (2) lists the identification from Schulte (1958). The numbering here corresponds to the Cyg OB2 designation used by Maíz Apellániz (2010). Column (3) lists the Washington Double Star (WDS)$^1$ identification appended with a letter corresponding to the component in the system. In column (4) we list the Comerón et al. (2002) (CPR) designation. Column (5) provides the numbering taken from the Bonner Durchmusterung catalog (Argelander 1903). The 75 stars in the sample range between 8.99 mag and 13.61 mag in $V$, between 2.7 and 14.2 in $K$, and between O3 and B3 in spectral type. As of July 2012, 22 stars in this sample are known multiple systems from radial velocity work (17) or high resolution observations listed in the Washington Double Star catalog (9; Mason et al. 2001).

$^1$http://ad.usno.navy.mil/wds/
Table 1.1: Cyg OB2 Naming Conventions

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<td></td>
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</tbody>
</table>
Table 1.1 – Continued

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<td>⋮</td>
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<td>⋮</td>
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<tr>
<td>S 5</td>
<td>S 5</td>
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<td>⋮</td>
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<td>y</td>
</tr>
<tr>
<td>S 73</td>
<td>S 73</td>
<td>⋮</td>
<td>⋮</td>
<td>⋮</td>
<td>y</td>
</tr>
<tr>
<td>WR 145</td>
<td>⋮</td>
<td>⋮</td>
<td>⋮</td>
<td>⋮</td>
<td>y</td>
</tr>
</tbody>
</table>

Note. — List of the 75 stars in our sample with other commonly used nomenclature in other references.

(1) Designation we use in this paper. MT # from Massey & Thompson (1991).
(2) Designation from Schulte (1958). The numbering here corresponds to the Cyg OB2 designation used by Maíz Apellániz (2010).
(3) Washington Double Star (WDS) identification appended with a letter corresponding to the component in the system.
(5) Numbering taken from the Bonner Durchmusterung catalog (Argelander 1903).
(6) ‘y’ indicates the stars that were observed as part of the FGS survey.

* Same star as MT 267 according to Kobulnicky et al. (2012), but different sources according the SIMBAD database. In this work we assume that they are two distinct objects.

In this study we explore the binary properties of this set of 75 stars. We resolve systems with physical separations greater than 15 Astronomical Units (AU) and orbital periods greater than 20 yrs. These two approaches start filling in the gaps in the picture of the multiplicity fraction of the most massive stars in Cyg OB2, as shown in Figure 1.1.

1.4 Cygnus X-1

A few degrees offset in the sky from Cyg OB2 is the high mass X-ray binary Cygnus X-1 (hereafter Cyg X-1). The system consists of an O9.7 Iab primary with an unseen compact companion. Cyg X-1 was the first black hole candidate, discovered in 1964 (Bowyer et al. 1965) as an X-ray source and later identified with the star HDE 226868 (Bolton 1972; Webster & Murdin 1972). With a period of $P \approx$ 5.6 days (Gies et al. 2003) giving an approximate projected separation of 0.1 mas, this system is well below the detection limits

of our high angular resolution survey. However, it provides an interesting case study of a tighter, more evolved high mass binary system that is thought to be a member of the nearby Cygnus OB3 association (Mirabel & Rodrigues 2003). At about the same distance to us as Cyg OB2 \(d \approx 1.5 \text{ kpc}\), the fundamental properties of this system have been the subject of many studies, but they continue to range greatly (Herrero et al. 1995; Ziolkowski 2005; Shaposhnikov & Titarchuk 2007; Karitskaya et al. 2008).

Our goal in studying Cyg X-1 is to determine if we can reconcile previous parameter estimates through an analysis of the supergiant’s spectrum to determine its stellar temperature and gravity. We present an analysis of the photospheric features for the supergiant based upon ground-based optical spectra and high-resolution UV spectra from the \textit{HST} Space Telescope Imaging Spectrograph (STIS). We compare the optical and UV line profiles of HDE 226868 with synthetic spectra from the TLUSTY/SYNSPEC codes (Lanz & Hubeny 2003, 2007), line-blanketed opacities, non-local thermodynamic equilibrium (LTE) models based on plane-parallel geometry, and solar abundances. From the comparisons we determine the stellar temperature and gravity that lead us to estimate the masses of both components (Caballero-Nieves et al. 2009).

1.5 Science Goals

In this study, we set out to accomplish the following science goals:

1. Estimate a lower limit to the binary frequency of Cyg OB2.

2. Provide initial binary parameters for future confirmation and orbital measurements.

3. Search for low mass companions.
4. Search for systems where the luminosity of the primary has been overestimated due to a companion.

5. Determine fundamental parameters of the high mass, X-ray binary, Cyg X-1.

We note here that in the search for companions we are really identifying candidate companions that may not be gravitationally bound to the target star. However, for simplicity we just refer to these as companions throughout the paper, and we specify true or statistically probable gravitationally bound companions found as a result of the survey. We also use the term binary interchangeably with multiple system for the cases where there are two or more components.

The results of both the infrared AO survey with the Gemini NIRI instrument (section 2) and the optical interferometric survey with the HST FGS instrument (section 3) provide preliminary estimates for goals 1 and 2. The large dynamic range of the NIRI instrument provides the opportunity to detect systems of very unequal mass, addressing goal 3. The FGS is efficient in undertaking goal 4 by resolving the closer systems where the brightness of the primary may have been overestimated by ignoring the flux of the companion. We address goal 5 with the results of our UV and optical study of Cyg X-1 in section 4. Finally, in section 5 we discuss the implications of the results and the direction of future work.
2
ADAPTIVE OPTICS SURVEY OF MASSIVE STARS IN CYGNUS OB2

2.1 Introduction to the Adaptive Optics Survey

Massive stars have a higher frequency of multiplicity than cooler, less massive stars, especially when they are found in clusters (Mason et al. 2009). Kouwenhoven et al. (2007) estimate about 100% binarity for the intermediate mass population in Scorpio OB2. Massive stars are known to have a higher binary fraction than lower mass stars (Duquennoy & Mayor 1991; Raghavan et al. 2010).

At a distance of 1.45-1.7 kpc (Massey & Thompson 1991; Torres-Dodgen et al. 1991; Hanson 2003), Cygnus OB2=Cyg OB2 is the second closest OB association that provides a nearby, young stellar environment, rich in high-mass stars. Due to extinction towards the region, the cluster begins to be unveiled in the infrared (IR). The association is close enough that with modern-day adaptive optics (AO) we are able to resolve wide companions with separations larger than 10 mas. The AO system at the Gemini North Observatory provides an effective tool to search for binaries in this association. With a resolving power of \( \sim 0\farcs06 \) and a sensitivity limit of about 10 mag for differential photometry, the Gemini AO infrared system can delve into the depths of the association and find faint companions with periods in the range from thousands to millions of years. Our results complement the radial velocity survey of Kiminki & Kobulnicky (2012) (and references therein) who searched for short period, spectroscopic systems in Cyg OB2. They determined there are 24 spectroscopic binaries in Cyg OB2 with periods less than 35 days.
In section 2.2 we describe the observations of the sample in Cygnus OB2. We present the results of the survey in section 2.3 along with notes about special cases. The binary properties are discussed in section 2.4.

2.2 Data and Observations

We were able to observe 75 of the brightest O- and B-type stars in Cygnus OB2 using the infrared AO system at the Gemini North Observatory. Our complete sample is listed in Table 1.1 along with other commonly used aliases from prior works. In Table 2.1 we list the sample along with position, spectral classification, and photometry information. The spectral classifications are taken from a variety of sources, indicated in the notes below the table. The $V$ magnitudes reported are from Massey & Thompson (1991) for the MT # stars and from Straizys & Laugalys (2008) for others. The infrared $JHK$ photometry are from the Two Micron All Sky Survey (2MASS; Skrutskie et al. 2006). In this paper, we provide initial measurements of $J$-, $H$-, and $K$-band relative photometry and position of candidate companions to our target stars. These initial results provide the first step in determining the true multiplicity fraction of widely separated systems.

In Figure 2.1 we plot the positions of our 75 Cyg OB2 targets in the sky. The majority of stars in this study were selected from the optical survey of Massey & Thompson (1991). This survey provides Johnson $B$ and $V$ magnitudes measurements for the brighter stars in the sample, as well as reddening towards each star. They find that the reddening in the association is not uniform. Another survey by Torres-Dodgen et al. (1991) in the optical and infrared also overlaps with some stars in our sample. Torres-Dodgen et al. (1991) estimate the age of the association to be least 3 Myr through analysis of their Strömgren and infrared
14
photometry. The young nature of the association is further established by the detection of
X-rays from young, low mass stars in the region (Albacete Colombo et al. 2007; Wright &
Drake 2009; Wright et al. 2012). A spectroscopic survey by Hanson (2003) establishes the
early types of these stars, as well as an estimate of the association distance at 1.45 kpc, a
little closer than that advocated by Massey & Thompson (1991) and Torres-Dodgen et al.
Table 2.1: Sample of Stars in Cygnus OB2
Star
Name
A 11
A 12
A 15
A 18
A 20
A 23
A 24
A 25
A 26
A 27
A 29
A 32
A 37
A 38
A 41
A 46
B 17
MT 5
MT 59
MT 70
MT 83
MT 138
MT 140
MT 145
MT 213
MT 217
MT 227
MT 250
MT 258
MT 259
MT 299
MT 304
MT 317
MT 339
MT 376
MT 390
MT 403
MT 417
MT 421
MT 429
MT 431
MT 448
MT 455
MT 457

R. A.
(HH:MM:SS)
20:32:31.539
20:33:38.219
20:31:36.909
20:30:07.879
20:33:02.920
20:30:39.710
20:34:44.110
20:32:38.441
20:30:57.730
20:34:44.719
20:34:56.061
20:32:30.330
20:36:04.520
20:32:34.870
20:31:08.378
20:31:00.200
20:30:27.299
20:30:39.820
20:31:10.549
20:31:18.330
20:31:22.038
20:31:45.400
20:31:46.011
20:31:49.659
20:32:13.130
20:32:13.830
20:32:16.560
20:32:26.079
20:32:27.660
20:32:27.739
20:32:38.579
20:32:40.958
20:32:45.458
20:32:50.019
20:32:59.190
20:33:02.920
20:33:05.269
20:33:08.801
20:33:09.600
20:33:10.508
20:33:10.751
20:33:13.258
20:33:13.690
20:33:14.110

Dec.
(DD:MM:SS
+41:14:08.22
+40:41:06.41
+40:59:09.25
+41:23:50.47
+40:47:25.45
+41:08:48.98
+40:51:58.51
+40:40:44.54
+41:09:57.57
+40:51:46.56
+40:38:18.06
+40:34:33.26
+40:56:12.98
+40:56:17.42
+42:02:42.28
+40:49:49.75
+41:13:25.31
+41:36:50.72
+41:31:53.55
+41:21:21.66
+41:31:28.41
+41:18:26.75
+41:17:27.07
+41:28:26.52
+41:27:24.63
+41:27:12.03
+41:25:35.71
+41:29:39.36
+41:26:22.08
+41:28:52.28
+41:25:13.75
+41:14:29.16
+41:25:37.43
+41:23:44.68
+41:24:25.50
+41:17:43.14
+41:43:36.80
+41:13:18.21
+41:13:00.54
+41:22:22.44
+41:15:08.20
+41:13:28.74
+41:13:05.79
+41:20:21.81

Spectral
Classification
O7.5 III-11
B0 Ia 3
O7 Ib(f)3
O8 V3
O8 II((f))5
B0.75
O6.5 III((f))3
O8 III3
O9.5 V3
B0 Ia3
O9.7 Iab5
O9.5 IV5
O5 V5
O8 V3
O9.7 II5
O7 V5
O7:1
O6 V2
O8 V1
O9 II1
B1 I2
O8 I2
O9.5 Iab:5
O9 III1
B0 V2
O7 IIIf2
O9 V2
B2 III2
O8 V1
B0 Ib2
O7 V2
B3 Iae2
O8 V2
O8 V2
O8 V2
O8 V2
B1 V2
O3 I6
O9 V1
B0 V1
O5:1
O6 V2
O8 V2
O3 If2

V
(mag)
···
···
···
···
···
···
···
···
···
···
···
···
···
···
···
···
···
12.93
11.06
12.99
10.61
12.26
9.38
11.62
11.95
10.07
11.47
12.88
10.90
11.50
11.12
11.40
10.65
11.71
11.91
12.95
12.94
11.68
12.86
12.98
10.78
13.61
12.92
10.50

Continued on next page. . .

J
(mag)
7.817
6.904
7.913
9.397
7.251
6.928
8.405
8.347
9.093
6.683
7.440
7.892
8.568
9.382
7.828
8.378
7.630
9.098
7.968
8.607
8.075
8.065
8.240
9.074
9.521
7.582
8.714
10.427
8.535
9.191
8.194
4.667
7.953
8.579
8.886
8.718
9.286
7.110
8.655
9.537
6.468
8.982
9.034
7.248

H
(mag)
7.094
6.170
7.208
8.739
6.632
6.328
7.796
7.705
8.514
6.062
6.859
7.365
7.968
8.858
7.292
8.016
6.850
8.574
7.556
8.046
7.750
7.552
8.061
8.768
9.248
7.248
8.389
10.150
8.193
8.895
7.918
3.512
7.617
8.188
8.524
8.165
8.854
6.540
8.135
9.113
5.897
8.346
8.559
6.818

K
(mag)
6.664
5.745
6.811
8.365
6.274
5.980
7.448
7.383
8.198
5.731
6.545
7.070
7.685
8.564
7.023
7.826
6.445
8.313
7.365
7.746
7.628
7.259
8.048
8.634
9.071
7.105
8.185
9.993
8.021
8.766
7.716
2.704
7.421
7.982
8.314
7.873
8.624
6.226
7.764
8.897
5.570
8.009
8.280
6.611


Seventeen of our targets were selected from the infrared survey by Comerón et al. (2002, 2008) (referenced as CPR in Table 2.1). These are redder sources that are not readily detected in the optical surveys, but V-band and spectral information are available for some of these from Straizys & Laugalys (2008).

Our observations were made in three queue observing programs at the 8.1-m Gemini North Observatory during the 2005B, 2008A and 2008B observing semesters. Using the Near InfraRed Imager and Spectrograph (NIRI) with the Altair adaptive optics (AO) system (Hodapp et al. 2003), we collected high resolution images (0.022 pixel\(^{-1}\) with the \(f/32\)
camera) of 75 O- and B-type stars in Cyg OB2 with a field of view (FOV) of approximately $22'' \times 22''$. The only exception is for our observations of MT 304 in $K$. Due to its brightness in the infrared ($K = 2.704$) MT 304 was observed with the shortest exposure time possible, and therefore, a smaller FOV ($11'' \times 11''$), so that the data could be read out without over-exposing the images. The CCD chip used the deep well setting for improved dynamic range and the 2008 data used the Altair field lens which improves the AO correction. The telescope sits on an Altitude-Azimuth mount, so that when NIRI is held fixed, the sky appears to rotate between frames. Exposure times for each frame range between 0.02 s to 800 s in $K$ and 0.1 s to 1869 s in $J$, depending of the brightness of the target star in each band.
Every target was observed in the $K$-band with the $K$ continuum filter, $K_{\text{con}}(209)$, to detect possible companions. We followed up 43 stars with $J$-band observations to get additional color information on those systems with obvious companions. The 2005 data were obtained using the $J$ continuum filter, $J_{\text{con}}(112)$. The wider $J_{\text{con}}(121)$ filter was used for the 2008 observations because the companions appear fainter in the $J$-band than in the $K$-band. The seven targets observed during the 2005B semester were also imaged in the $H$-band with the $H$ continuum filter, $H_{\text{con}}(157)$, with the exception of MT 304 which was only observed in $J$ at the time. Table 2.2 provides the central wavelength and the band path for each filter. These filters are narrow-band $JHK$ filters that were needed because the stars are so bright in the infrared.

In addition to the NIREI $K$-band observation, MT 421 was observed with the Palomar High Angular Resolution Optics (PHARO) camera and AO system on the 5-m Hale telescope during an engineering night in 2009 July. We were able to get observations in all three IR bands, $J$, $H$, and $K_S$, with a field of view comparable to that of NIREI ($\sim 25'' \times 25''$). The filter information for PHARO is also listed in Table 2.2. The PHARO images provide a comparable pixel scale of $0''025$ pixel$^{-1}$ (Hayward et al. 2001).

<table>
<thead>
<tr>
<th>Instrument</th>
<th>Filter Name</th>
<th>Central Wavelength ($\mu$m)</th>
<th>Bandpass ($\mu$m)</th>
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</thead>
<tbody>
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<td>NIREI</td>
<td>$J_{\text{con}}(112)$</td>
<td>1.122</td>
<td>0.009</td>
</tr>
<tr>
<td>NIREI</td>
<td>$J_{\text{con}}(121)$</td>
<td>1.207</td>
<td>0.018</td>
</tr>
<tr>
<td>NIREI</td>
<td>$H_{\text{con}}(157)$</td>
<td>1.570</td>
<td>0.024</td>
</tr>
<tr>
<td>NIREI</td>
<td>$K_{\text{con}}(209)$</td>
<td>2.0975</td>
<td>0.027</td>
</tr>
<tr>
<td>PHARO</td>
<td>$J$</td>
<td>1.246</td>
<td>0.162</td>
</tr>
<tr>
<td>PHARO</td>
<td>$H$</td>
<td>1.635</td>
<td>0.296</td>
</tr>
<tr>
<td>PHARO</td>
<td>$K_S$</td>
<td>2.145</td>
<td>0.310</td>
</tr>
</tbody>
</table>

Each observation consisted of approximately 90 frames. Table 2.3 lists the observation dates of the beginning of the first exposure and the number of frames combined to produce
Figure 2.2 Relative positions of the nine dither positions on the frame numbered according to the order in which they were observed.

The final co-added image for each filter. Each target was observed at nine dither positions, set up on a $3 \times 3$ grid, offset by about 50 pixels and with 10 exposures at each position.

Figure 2.2 shows the relative positions of the dither placements in a field, numbered according to the order they were observed. For the cases where the observations were taken over two nights, observations from each night were combined individually and also combined together.

For the detection of sources, the images from each night were analyzed separately due to differences in image quality, but only data from one night were used for photometric and astrometric measurements (denoted by *). For A 25 in $K$ and A 41 in $J$, we analyzed the combined image from both nights (denoted by $C$) because they were of comparable quality.

In the fourth column we list the average full-width at half-maximum (FWHM) of each point.
spread function detected in the frame. This is an indication of the image quality, and smaller
values of the FWHM are better.

Table 2.3: Observations of Stars in Cyg OB2

<table>
<thead>
<tr>
<th>Star</th>
<th>JD - 2,454,000</th>
<th>Filter</th>
<th>FWHM (pix)</th>
<th># Frames</th>
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<tbody>
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<td>A 11</td>
<td>4741.250</td>
<td>Keon(209)</td>
<td>4.12</td>
<td>91</td>
</tr>
<tr>
<td>A 12</td>
<td>4741.242</td>
<td>Keon(209)</td>
<td>3.76</td>
<td>90</td>
</tr>
<tr>
<td>A 15</td>
<td>4741.234</td>
<td>Keon(209)</td>
<td>4.07</td>
<td>90</td>
</tr>
<tr>
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<td>4741.220</td>
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<td>4.18</td>
<td>90</td>
</tr>
<tr>
<td>A 20</td>
<td>4741.210</td>
<td>Keon(209)</td>
<td>5.30</td>
<td>90</td>
</tr>
<tr>
<td>A 23</td>
<td>4590.621</td>
<td>Keon(209)</td>
<td>3.78</td>
<td>90</td>
</tr>
<tr>
<td>A 24</td>
<td>4741.201</td>
<td>Keon(209)</td>
<td>3.82</td>
<td>90</td>
</tr>
<tr>
<td>A 25</td>
<td>4740.329&lt;sup&gt;C&lt;/sup&gt;</td>
<td>Keon(209)</td>
<td>6.42</td>
<td>69</td>
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<td></td>
<td>4741.197&lt;sup&gt;C&lt;/sup&gt;</td>
<td>Keon(209)</td>
<td>6.42</td>
<td>22</td>
</tr>
<tr>
<td>A 26</td>
<td>4740.292</td>
<td>Keon(209)</td>
<td>5.04</td>
<td>90</td>
</tr>
<tr>
<td>A 27</td>
<td>4741.261</td>
<td>Jcon(121)</td>
<td>5.90</td>
<td>89</td>
</tr>
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<td>3.66</td>
<td>90</td>
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<td>3.82</td>
<td>90</td>
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<td>A 38</td>
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<td>Keon(209)</td>
<td>5.67</td>
<td>106</td>
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<td>Jcon(121)</td>
<td>4.34</td>
<td>60</td>
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<td></td>
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<td>Jcon(121)</td>
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<td>30</td>
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The data were reduced using the tools provided as part of the NIRI reduction package for both IRAF and Python. In Appendix A.1 we provide step-by-step instructions of the reduction process. Also included in Appendix A.1 are the shell scripts and IRAF tasks used for the reduction.

NIRI images have a characteristic pattern noise that is represented by a $16 \times 4$ fixed pixel pattern repeated over the image. The observing log for several nights noted the presence of the fixed pattern in some of the frames. We used the Python program, NIRNOISE.PY, written by instrument scientist Dr. Andrew Stephens, on all of the frames to remove the characteristic pattern as the first step in the reduction process. The noise and program are described in detail at Dr. Stephens’ website\footnote{http://staff.gemini.edu/~astephens/niri/patternnoise}.

Once the noise pattern was removed, we ran the IRAF script REDUCE.SCRIPT, which is provided in Appendix A.1. The first step in the script is to create an initial pixel mask using GEMCOMBINE. This mask flags those pixels with extreme values relative to the median value of pixels across each frame. This step is especially critical in cleaning up the

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& 4607.554 & Kcon(209) & 5.04 & 90 \\
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& 4617.606 & Kcon(209) & 4.43 & 91 \\
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WR 145 & 4740.194 & Kcon(209) & 5.80 & 90 \\
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\textsuperscript{C} Denotes that the combined image from both nights was used for analysis.

* Denotes which night was used for analysis.

Note. — Listed are the start times of the observations for the first frames and the total number of frames used to compile the final image.
upper left portion of the frames, where the density of bad pixels is highest. The bad pixel mask is continuously updated throughout the reduction process.

All data must be run through NPREPARE because it adds certain essential header keywords for the subsequent reduction steps. In our case, NPREPARE also computes the variance and data quality planes when first used with flat-field, calibration frames. This updates the bad pixel mask that is subsequently applied when the science frames are run through NPREPARE.

The next step in the reduction is to create a flat field image using NIFLAT. This program uses a set of exposures with the shutter closed to combine and subtract from a set with the shutter open taken either the same night or near in date from the science observations to construct the flat as well as to further update the bad pixel mask.

The script then runs NRESIDUAL to flag saturated pixels that may leave a faint residual that can be confused with a real object in subsequent images. NRESIDUAL uses the variance information of each pixel from the images taken immediately prior and combines them, allowing the effects of saturation to be taken into account in subsequent exposures.

The next step, NISKY, derives a sky image by identifying and masking objects in the input science images. The use of a dither pattern shifts the position of the stars and allows sky observations from several frames to be recorded for every pixel. NISKY averages the sky level for each pixel over those frames where objects are not masked.

Once the flat field and sky images are created and the bad pixel mask is complete, the script runs NIREDUCE on the science images. This program sky subtracts and flat divides the science data.
Our observations were performed using the deep well setting of the CCD. This allows for fainter sensitivities in the exposures. The downside is that this setting creates a number of hot pixels. Dr. Stephens was instrumental in suggesting the necessary procedures for creating a robust bad pixel mask to remove a significant number of these bad pixels. He provided the IRAF task GEMTWEAKDQ to flag the low or negative valued pixels due to over subtraction from the reduction process.

Before combining, the final step is to run NIROTATE. This program rotates all the frames to the orientation of the first image in the list. This is necessary because the objects were observed with the Cassegrain rotator fixed in position. NIROTATE uses the orientation information from the World Coordinate System (WCS) parameters recorded in the header information. The WCS provides the position and scaling information of the frame to convert to sky coordinates. The data quality information that has been revised throughout the reduction process is used here to properly rotate each image without smearing pixels of poor quality.

With the images rotated, reduced, and the data quality robustly quantified through the various reduction steps, we used two different combining programs to co-add all of the frames. The images were initially co-added using the IRAF tool IMCOADD to derive an average image taking into account the bad pixel mask. To align, each frame is displayed and the user has the option of selecting the location of the brightest star. The program centers on the brightest pixel and carries out fractional pixel shifts to create a Point Spread Function (PSF). The alignment can fail for a variety of reasons. When the seeing was of poor quality or if the observations were done over multiple nights with different seeing quality, the combined image appears smeared. If the central PSF is a blend of two stars, then IMCOADD can
confuse which star should be centered. For these cases, GEMCOMBINE was used with the user giving the pixel position of the central star. GEMCOMBINE produces a slightly different median image than the mean co-added IMCOADD, but the capability of allowing the user to define the pixel shifts makes the final co-added image better aligned than is the case when IMCOADD fails.

The final images from GEMCOMBINE and IMCOADD produce a slightly larger field of view than the $22'' \times 22''$ FOV of a single frame, but depending on the observing conditions (e.g., exposure time, and whether the frames span multiple nights) some fields can be bigger than others. In Appendix A.2 we present the final reduced $K$-band images for the entire sample.

We took three different approaches to identifying possible point sources. The first approach was just by visually inspecting each frame using SAO Image display software. By adjusting the scaling and contrast we were able to identify at least one source in addition to the main target in each $K$-band frame. For a more automated approach, we used the astronomical source extractor SExtractor (Bertin & Arnouts 1996). This program builds a catalog of objects in an image either by giving it approximate pixel positions, if already known, or by setting a brightness threshold above the background for detection. This program is capable of distinguishing some blended sources, and for some close cases it was able to extract both components. We also made use of the IDL program called STARFINDER (Diolaiti et al. 2000). This program works by selecting several stars and measuring characteristic PSFs. The program then creates an average PSF to fit and identify all the other stars in the field. Then it obtains position and relative brightness information for each. Both STARFINDER and SExtractor would identify bad pixels as point sources, and they failed to identify faint
sources that fell near the bad part of the chip in the upper left. Through trial and error we determined that the best tool to identify sources in the frames is the human eye.

After visually identifying each point source and getting an approximate pixel position, this information is given to SExtractor, which is able to identify each source and measure the centroid position and relative brightness. We chose SExtractor over STARFINDER for two reasons. First, SExtractor can be automated, making processing each frame faster. Second, SExtractor has the option of accepting initial pixel positions, which reduces the amount of “weeding out” of wrongly identified bad pixels and avoids the problem of overlooking unidentified faint stars. The relative magnitude returned by SExtractor is measured using the MAG_APER parameter. MAG_APER estimates the flux above the background within a circular aperture. SExtractor returns a fractional pixel position of the centroid for each source extracted. We adopted an aperture diameter of 10 pixels because this includes the FWHM of all the average PSFs listed in Table 2.3. We note however, that the brighter stars, like the target stars, have extended halos that are not encompassed in the aperture. The NIRI halos resemble convection cells on the surface of the sun, while the Palomar halos appear like waffle patterns. In Figure 2.3 we show typical PSFs for the PHARO observations (left) and NIRI (center). The right panel shows how different a PSF can look from the same frame (center).

For close systems with blended PSFs ($\rho \lesssim 0\farcs1$), we used a fourth method, FITSTARS, to measure the differential magnitude and separation. This is further discussed below in section 2.3.3.1.
2.3 Results

We present the astrometric and photometric results for all the stars in Table A.1 in Appendix A.3. In every $K$-band frame of our 75 targets, we identified at least one candidate companion near the primary. The relative brightnesses and positions reported are determined with respect to the target stars. For cross-references purposes, companions are identified by their right ascension (R.A.) and declination (Dec.). We do not attempt any type of naming convention for the companions due to the uncertainty of their binary status (physically bound or line of sight). We also do not consider the possibility of a companion star being bound to another star in the field.

2.3.1 Astrometric Results

In Figure 2.1 the companions are plotted as small dots, but because of the small FOV, all the dots fall within the disc of the plotting symbol. In Table A.1 we provide separation and position angle measurements with respect to the primaries. The R.A. and Dec. reported are based on the 2MASS positions of the primaries. The position information listed for MT 421 in Table A.1 is taken mainly from the NIRI $K$-band frame. For the stars only observed with
the PHARO camera (denoted by P in the notes column) the position on the Palomar frame is used.

2.3.1.1 Astrometric Error

Precise astrometry has proved challenging with the NIRI data. The FITS files include World Coordinate System (WCS) position information in the header. However, after initial inspection of the images, the position information was found to be offset by a few arcseconds from that of the 2MASS values. For better agreement we aligned the center of the target star in the header to that reported in the 2MASS survey. This in itself introduces a source of uncertainty because of the poor angular resolution (∼2") of 2MASS. For close binaries 2MASS will give the position of the center of light, which is different than the position of the resolved target. 2MASS does have the advantage that it includes observations of our complete sample.

For stars that are observed in overlapping fields around multiple targets, we report the R.A. and Dec. values taken from the co-added frame, where the stars are closer to the center. These stars are noted in the Notes column in Table A.1 with a “see star.”

In addition to the issues with the WCS information on the absolute position, closer inspection shows that the pixel scaling information in the header is not quite accurate either. In 2008, our observing setup included the use of the ALTAIR field lens. According the instrument description at the observatory website\(^2\), the lens is designed to improve the image quality at large distances from the guide star. It also changes the average pixel scaling factor from 0\textquoteleft\textquoteright 0219 pixel\(^{-1}\) to 0\textquoteleft\textquoteright 0214 pixel\(^{-1}\). This is not reflected in the header information for

\(^2\)http://www.gemini.edu/
the 2008 data. Nevertheless, we adopted a scaling factor of 0\textquoteleft 0219 pixel\(^{-1}\) for the 2005 data and 0\textquoteleft 0214 pixel\(^{-1}\) for the 2008 data.

To complicate matters even further, the NIRI field has been shown to be subject to radial barrel distortion with our observing mode of f/32. According to the instrument website\(^3\) the distortion is quantified by the equation,

\[ r' = r + k \times r^2 \]  

(2.1)

where \(k = (1.32 \pm 0.02) \times 10^{-5}\) and \(r\) is the separation in pixels. Thus, a star separated by about 500 pixels will appear about 3 pixels closer to the center than actual. This corresponds to an angular offset of about 0\textquoteleft 07. However, when comparing our actual data on stars in adjoining fields the difference can be even larger. For example, the two stars MT 462 and MT 465 happen to fall on the edge of the frame of the other star. Their measured separation from the two frames differs by 0\textquoteleft 2. Ideally, accounting for radial distortion requires correcting for it before co-adding frames and creating the final image.

Due to the complexity of the uncertainty in our astrometric measurements, we use values determined from previous works to quantify the error in our measurements. We compare our values to those from the the third release of the USNO CCD Astrograph Catalog (UCAC3; Zacharias et al. 2010). This optical survey provides a better angular resolution (100\textquoteleft/ mm) and positional measurements (from 5500 Å to 7100 Å) than 2MASS. For comparison, UCAC3 astrometry RA and Dec values are converted to separation and position angle. Also, in the high angular resolution optical study by Maíz Apellániz (2010), five of our targets were observed using the AstraLux camera at the Calar Alto 2.2-m telescope. In Table 2.4 we

\(^3\)http://www.gemini.edu/sciops/instruments/niri/imaging/pixel-scales-and-fov
list the comparison of our values of separation and position angle with respect to the other studies and our Fine Guidance Sensor (FGS) results from section 3. The last two columns of the table list the standard deviation of the mean for separation and position angle from the various measurements. The uncertainty in the position is distance dependent and ranges in value from $0\farcs001$ to $0\farcs1$ for $\rho$ and from $0\degr3$ to $2\degr4$ in $\theta$. A more comprehensive analysis of the uncertainty in positions is forthcoming.

### 2.3.2 Photometric Results

In this study we present relative photometric measurements of the NIRI observations with respect to the target star. Figure 2.4 shows that observing these objects at longer wavelengths reduces the contrast between the hot, bright primary and the cooler, redder companions. The stars are more easily identified in the $K$-band than in $J$. There were cases where SExtractor failed to extract sources due to either being too close together or too faint. In Table A.1 such objects are listed only with an approximate position from the visual inspection.
Figure 2.4 NIRI images of MT 465. The panels correspond to the $J$-, $H$-, and $K$-band images from left to right, respectively. Arrows are $1''$ in length and indicate East as counter-clockwise to North.

Only eight stars were observed in the $H$-band and we present the results separately in Table 2.5. Note that MT 421 $H$-band observations were made using the PHARO instrument on Palomar (see section 2.3.3.2). Missing from Table 2.5 is MT 431, the only star observed in the $H$-band without another star visible in the $H$ frame.

Table 2.5: $H$-band Photometry

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<th>Dec.</th>
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<th>Notes</th>
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<td>(mag)</td>
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<td>+41:13:14.20</td>
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Note. — The magnitudes of the stars around MT 421 were measured using the PHARO observations.

The cases where a star fell on the edges of frames (i.e., the parts of the image where where all the individual frames do not overlap due to the dither pattern) are denoted in Table A.1 with an ‘E’, for edge, following the magnitude value. The differential magnitude
values listed were determined from the final co-added frame, but because these edge stars
did not fall on a region formed from the same number of frames as the central portion, the
resulting magnitude measurement has a significant error.

2.3.2.1 Photometry Error

The main error in the photometric measurements was estimated by comparing measure-
ments of overlapping stars from adjoining fields. For two cases, a target star appears on the
frame of another target and vice versa. By comparing the photometric measurements from
the different frames, we take the average difference as the observational error for our sample.
We used the pairs MT 213/MT 217 and MT 462/MT 465 to estimate the observational error
of 0.89 mag in $J$ and 0.83 mag in $K$. Because we lack $H$-band photometry on these pairs,
we cannot determine the error for $H$-band data and we adopt 0.86 mag, the average from
the $J$ and $K$ errors in Table 2.5. We suspect that the error in these cases is high due to
the fact that these stars are well separated, which means that the PSF near the edge of the
frame is different (see Figure 2.3). The error listed in most cases in Table A.1 is the sum
of the observational error calculated above added in quadrature to the error outputted by
SExtractor for the individual star.

There is a region around the edges of the final co-added frame that only a subset of the
individual frames cover. This edge region introduces another source of error in the magnitude
measurement for the stars that fall there. We took a representative sample of stars that fell
along the edges to estimate the error in magnitude. We compared the differential magnitudes
measured from the complete, co-added image, to that of a co-added image created from a
subset of frames that does have the stars in the field of view. In Figure 2.5 we plot the
Figure 2.5 Edge star comparisons between the complete co-added image and a co-added image from a subset of the frames ($\Delta \text{mag}_{\text{corr}}$). The measured differential magnitudes follow along the line of equality. However, there is a larger scatter among the fainter objects. The size of the scatter, 0.38 mag, is added to the error in magnitude for edge objects.

values measured from the complete image versus those of the subset image. The values are scattered evenly around the line of equality and do not deviate greatly, except for the faintest companions. We take the average of the deviation 0.38 mag, and add it in quadrature to the error estimate for these stars.

Due to the distortion in the field, the PSFs of stars near the edge of the frame show variations from the PSFs of stars near the center of the frame. The use of a constant aperture for all stars in a frame introduces another source of uncertainty for these cases. If the PSF changes with separation, then errors in photometry increase with separation.
a larger aperture (20 pixels rather than 10 pixels) on the “distorted” PSFs partially mitigates
this problem.

We compared the NIRI and the PHARO data sets of $K$-band photometry for the MT 421
observations. Figure 2.6 shows that the PHARO observations agree fairly well with that of
the NIRI. The average standard deviation in the $K$-band between the two sets of observ-
ations is $\sim 0.81$ mag, well within the uncertainty of the NIRI data. Figure 2.7 shows an
approximately flat distribution between the PHARO and NIRI observations with respect
to separation. However, Figure 2.6 and Figure 2.7 show a systematic difference between
the PHARO and NIRI $\Delta K$ values. NIRI measurements are consistently larger than those
from PHARO (i.e., companions appear fainter). Until we are able to constrain the NIRI
uncertainty further, we do not report uncertainties for the PHARO observations.

We compare our photometric results to those from the same studies used for the astro-
metric calibration (Skrutskie et al. 2006; Maíz Apellániz 2010; Zacharias et al. 2010) along
with our FGS measurements from section 3. In the optical and infrared, we are sampling
the Rayleigh-Jeans tail of the spectral energy distribution of the massive stars. Ignoring the
reddening effects, the difference in magnitude between the hot primary and its cooler com-
panion should be approximately equal over these wavelength ranges. In Figure 2.8 we plot
our results with those of previous studies and find that the values agree to within $\sim 0.5$ mag
on average, within the 0.8 mag error estimated above. Of note is the fact that the 2MASS
measurements consistently underestimate the differential magnitudes. This may be in part
due to the poor angular resolution of 2MASS ($\sim 2''$ pixel$^{-1}$). This causes the magnitude
measurements of the companions to be diluted with light from the primary because they
appear blended in the 2MASS observations while resolved in the NIRI frames. The compar-
Figure 2.6 Comparison plot of the PHARO and NIRI $\Delta K$ values.

Comparison with other studies indicates that the uncertainty in photometry increases with fainter companions. We hope in a future analysis to constrain the relationship between uncertainty and the brightness of the companion. In the meantime, for this paper we adopt a large value of uncertainty.

2.3.3 Special Cases

2.3.3.1 Close Systems

We define close systems as those with separation less than about $0'.5$, where the secondary falls within the halo of the primary’s PSF. In our sample, there were a total of nine systems that met this criterion: A 20, A 26, A 41, MT 5, MT 429, MT 605, MT 632, MT 642, and
Figure 2.7 Comparison of the difference between the NIRI and PHARO $\Delta K$ values and separation.

SCHULTE 73. The closest resolved binary, MT 429, is shown in Figure 2.9. With a projected separation of $\rho = 0.08$ these components are near equal in $K$ brightness ($\Delta K = 1.59$). We are less sensitive to fainter companions at such close separations.

The PSFs are blended for these close systems, making the simple aperture photometry performed by SExtractor inaccurate. Instead, photometric measurements reported here were made using the program FITSTARS, a PSF deconvolving program based on the method described in ten Brummelaar et al. (1996, 2000). FITSTARS is capable of fitting blended PSFs and measuring relative brightnesses and positions. In Figure 2.10 we show a comparison plot of the results of the close systems that were successfully measured with both SExtractor
Figure 2.8 Comparison plot between our photometric results to previous studies.

Figure 2.9 NIRI AO, $K$-band image of MT 429, the closest pair ($\rho = 0'.08$) found in our survey.

and FITSTARS ($J$ and $K$: A 41, MT 5, MT 632; $K$: A 26, MT 429, MT 605, MT 642, S 73).

On average, SExtractor underestimates the magnitude difference by about 0.55 magnitudes in both $J$ and $K$. However, for MT 605 and S 73 in $K$, FITSTARS underestimates the differential magnitude with respect to SExtractor. This is due to the fact that at larger
Figure 2.10 Differential magnitude comparison plot between SExtractor and FITSTARS magnitude measurements. FITSTARS was used to measure photometry and separation of close systems with \( \rho < 0.5'' \). The different symbols are for the different observing bands, \( \Delta J \) (cross marks) and \( \Delta K \) (diamonds). SExtractor tends to underestimate the differential magnitude by 0.5 mag.

separations, where the PSFs are not blended, FITSTARS does not center correctly on the companion star. At these larger separations SExtractor is a more accurate tool for differential photometry.

2.3.3.2 \textit{MT 421}

We detected 47 objects within \( \sim 25'' \) of the star MT 421, by far the most stars detected in our sample. We obtained additional \( JHK \) observations from an engineering night with the PHARO camera on the 200-in Palomar telescope because we only had \( K \)-band photometry
from the NIRI observations at the end of the survey. For consistency, the $K$-band values reported in Table A.1 are from the NIRI observations, unless otherwise noted by a P in the Notes column. The $J$- and $H$-band photometry in Tables A.1 & 2.5 are from the Palomar images. We adopted the $K$-band NIRI measurements here because the NIRI images have a larger dynamical range.

Figure 2.11 is a false-color image composed of the $J$, $H$, $K_S$ images taken with the PHARO AO. Many of the fainter objects are relatively much brighter in the $K_S$ band due to their red color, the varying and patchy extinction towards Cyg OB2, and/or contamination by foreground, cooler stars.

The white circle corresponds to a projected radius of 10,000 AU. This is approximately the radius where the orbital speed is on the order of the random motion in a cluster. Companions found at separations larger than this are easily disrupted in dense clusters. We found that 65 stars in our total sample had at least one star within that radius.
Figure 2.11 False color Palomar image ($J=$blue, $H=$green, $K_S=$red) of MT 421, the most populated field with 47 sources detected. The circle is at a projected radius of 10,000 AU (5.88 for an adopted distance of 1.7 kpc).

2.4 Discussion

We made one epoch imaging of 75 O- and B-type stars in Cyg OB2 with high angular resolution methods in the infrared $JHK$ bands. We presented initial photometric and position information for stars found in the field around our targets. We found at least one star in the field around each of our targets, for a total of 544 possible companions.
Figure 2.12 This figure shows the detectable dynamic range of our observations as a function of projected radius. Within 0.5′ we are able to detect objects with $\Delta K < 5$ mag.

Figure 2.12 shows the dynamical range of our detections as a function of separation. This figure demonstrates the sensitivity and completeness of our survey. The separation axis has been plotted in log space to show the sensitivity at close distances, such as the closest resolved binary, MT 429, shown in Figure 2.9. We detected stars with a magnitude differences as large as $\Delta K \approx 5$ mag even with small separations from the target of $\rho < 0.5′$.

In Figure 2.13 we show the number density of the entire sample over square area as a function of separation. The density levels off at larger separations. This very likely corresponds to the average number density of the association. Stars found at these separations are more likely to be chance alignments (see equation 2.2). The number density per area
Figure 2.13 The graph shows how the number density of stars falls with angular separation but levels off at larger distances. At a distance of 1.45 kpc, 1″ corresponds to a projected size of 1450 AU.

increases greatly with $\rho < 1''$ and stars found within this separation are more likely to be physical companions.

2.4.1 Determining Companions

As we are presenting initial detections of the field around our target sample, we need to differentiate between chance alignments and gravitationally bound systems. The best way to do so is to use multi-epoch observations, in conjunction with a proper motion study and spectroscopic information of the companions. However, because we only have access to single-epoch astrometric observations, we apply a statistical argument developed by Correia et al. (2006) to determine likely companions. The statistical probability for each object was
calculated by Correia et al. (2006) using

\[ P(\Sigma_K, \Omega) = 1 - e^{-\pi \Sigma K \Omega^2}. \] (2.2)

Here \( P \) is the probability of a star being a chance alignment at a separation of \( \Omega \), in arc-seconds, from the target. \( \Sigma_K \) is the surface density of stars in the surrounding field down to a magnitude of \( K \). We calculated \( K \) magnitudes for our stars based on the 2MASS \( K \) magnitude of the primary added to the \( \Delta K \) from the NIRI observations. The stellar field density was determined using a combination of data from 2MASS (Skrutskie et al. 2006) and UKIDSS (Lawrence et al. 2007) of the surrounding area around each of our targets. 2MASS provided photometry information of the stars with \( K < 14 \) mag and UKIDSS provided the information for \( 14 < K < 16 \) mag. The magnitudes of faint stars in UKIDSS were set by comparing the magnitudes of stars in the range of \( 8 - 14 \) mag where the two sets of observations overlapped. This was done by identifying stars in the UKIDSS frame that had 2MASS \( K \) magnitudes. Then the magnitudes of fainter stars were determined from the common stars in 2MASS and the differential magnitudes measured in the UKIDSS frame. Figure 2.14 shows how the average stellar density in our fields increases with \( K \). To determine the probability, we fit a spline function to the binned data in Figure 2.14 and used the stellar density of the spline function at our observed \( K \) value. This allows us to estimate the probability of our faintest companions with \( K > 16 \) mag by extrapolation of the spline fit.

The companions with \( P(\Sigma_K, \Omega) < 1\% \) we take to be gravitationally bound to their respective target star. In Table 2.6 we list the number of stars found with \( P(\Sigma_K, \Omega) < 0.01 \) in column 3. These stars have a very high probability of being physical companions. We also provide numbers on the detections from our FGS survey (section 3) in column 4. Previously
Figure 2.14 Magnitude dependence of the stellar density around our field using 2MASS and UKIDSS observations.

detected astrometric companions from the Washington Double Star (WDS) catalog are listed in column 5. A few of the stars listed as companions in the WDS are beyond the field of view of our AO data, which raises the question of whether they are physically bound or not. All of our statistically probable companions lie within a projected radius of 10,000 AU from their primary. However, not all companions that lie within this projected radius are probably companions. Of the 65 target stars with companions within 10,000 AU, 32 have at least one statistically probable companion. This suggests that most of the stars in the WDS that are found at larger separations are not physically bound companions. In column 6 we list the numbers of spectroscopic systems presented in Kobulnicky et al. (2012) and by Muntean et al. (2009) for the case of WR 145. In column 7 we provide the total number of unique companions from all these methods. In the final column we provide the number of
new detections around our target stars. We determined that 32 out of our 75 targets or 42% have at least one statistically probable companion. This does not include companions from other sources.

Table 2.6: Multiplicity properties of stars in Cyg OB2

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<td>...</td>
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</tbody>
</table>

(1) Star Name
(2) Total number of the stars found in the field around the target in the AO field.
(3) Number of stars that have a high probability of being real companions.
(4) Number of companions detected with FGS (Table 3.4).
(5) Number of companions listed in the Washington Double Star Catalog (Mason et al. 2001).
(6) Number of companions found through radial velocity measurements (Kobulnicky et al. 2012, or for the case of WR 145, Muntean et al. 2009, and references therein).
(7) Total number of unique companions from columns (2) through (6).
(8) New companions detected during this work.

2.4.2 Color-Magnitude Relationship

For color analysis, we use 2MASS apparent magnitudes for our observations without performing a filter correction between the 2MASS broad-band $JHK$ filters and the NIRI narrow-band $JHK$ filters. In Figure 2.15 we show the color-magnitude diagram of stars observed in both $J$ and $K$, corrected for reddening and converted to absolute magnitudes. Overlaid are a 2 Myr Pre-Main Sequence track (Siess et al. 2000) and a 3 Myr Zero-Age Main Sequence isochrone (Girardi et al. 2003). In determining the intrinsic color, we assumed that all stars in a given frame were at the same distance and had the same reddening as the target star. The extinction values were adopted from one of three sources, in order of preference and availability: Negueruela et al. (2008) for $E(J – K)$, Massey & Thompson (1991) for...
Figure 2.15 Magnitude-color diagrams in $J$ and $K$ of our target stars (black circles) and the other stars in the frames (blue Xs), dereddened according to estimates from Negueruela et al. (2008), Massey & Thompson (1991) and Torres-Dodgen et al. (1991), and converted to absolute magnitudes using the distance modulus ($DM = 10.8$) of Hanson (2003). Overlaid are a ZAMS isochrone at 3 Myr (dashed, black line) from Girardi et al. (2003) and a Pre-MS track at 2 Myr (solid, red line) from Siess et al. (2000).

$E(B-V)$ and Torres-Dodgen et al. (1991) for $E(b-y)$. Table 2.7 lists the reddening values available for our sample. There were two stars with no reddening estimates. B 17 is not plotted in Figure 2.15. MT 140 was only observed in $K$, and so is also not plotted. We applied the extinction correction transformations from Fitzpatrick (1999) to convert optical selective extinction ($E(B-V)$ and $E(b-y)$) to the total extinction terms in the infrared ($A_J$, $A_H$ and $A_K$). The transformations are provided in Table 2.8. The reddening corrected, absolute magnitude is calculated using $M_{corr} = M - 10.8 - A_M$, where $M$ is the infrared magnitude ($J$, $H$, or $K$) from 2MASS, and 10.8 is the distance modulus for a distance of $d = 1.45$ kpc (Hanson 2003). The large scatter seen is due to several factors. Some of the companions maybe foreground objects. The assumption that the companions have the same reddening as the primary may be incorrect. Also, some of the scatter maybe due to the
fact that several of our companions appear as blended sources in 2MASS and the primary appears over-luminous. This leads to a smaller $K$ value of both the target and its respective companions. Finally, some of the scatter may be due to young, red objects that have yet to reach the Main Sequence.

Table 2.7: Reddening Information

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Table 2.7 – Continued

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Table 2.8: Extinction Transformations

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<th>$A_J$</th>
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<td>$(0.34 \times E(J - K)) + A_K$</td>
<td>$E(J - K) + A_K$</td>
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<td>$E(B - V)$</td>
<td>$0.36 \times E(B - V)$</td>
<td>$0.53 \times E(B - V)$</td>
<td>$0.86 \times E(B - V)$</td>
</tr>
<tr>
<td>$E(b - y)$</td>
<td>$0.36 \times E(b - y)/0.74$</td>
<td>$0.53 \times E(b - y)/0.74$</td>
<td>$0.86 \times E(b - y)/0.74$</td>
</tr>
</tbody>
</table>

Cyg OB2’s high content of massive stars indicates that it is a relatively young association. The empirical data in Figure 2.15 follow the trend of the isochrones but the fact that we see a large scatter implies that our assumptions might not be true for all the stars in the field. This may be due to circumstellar reddening for young stars or they may be foreground objects or reddened early type stars. Further color and spectroscopic information is needed to determine the nature of these objects. However, the positions of the target stars with respect to the isochrones indicates that the target stars are at the same distance, young and coeval. Future observations would be useful to detect orbital motion and to further establish the true multiplicity of these systems.
3
FINE GUIDANCE SENSOR MULTIPLICITY SURVEY OF MASSIVE
STARS IN CYGNUS OB2

3.1 Introduction to the Fine Guidance Sensors

We present the results of an optical survey of close binaries around 58 stars that are members of the Cygnus OB2 association using the Fine Guidance Sensors (FGS) on the Hubble Space Telescope (HST). The FGSs are a shearing interferometers that sense the tilt of an incoming beam as projected along each of the instrument’s $x$ and $y$ axes. While the primary function of HST’s three FGSs is to provide guide star positions for accurate pointing of the observatory, one of them, FGS 1r, is available for use as a science instrument.

The FGS can be operated in two modes, POSITION and TRANSFER. In POSITION mode, which is used for guiding HST or for astrometric observations, the FGS locks onto the zero point crossing of the $S$-curve to provide precise measurement of the star’s position in the focal plane. In TRANSFER or TRANS mode, the FGS is used to scan across an object, and the resulting interference fringes can be reconstructed from the output signals (Nelan et al. 2011). The objects are scanned in the instantaneous field of view ($5'' \times 5''$) along two orthogonal axes with typical scan lengths of about $1''$ (Nelan et al. 2011). Figure 3.1 shows the typical S-like interference fringe pattern of a single point source, or $S$-curve (also transfer function), in the two orthogonal directions of the scan. We adopt interferometry vocabulary in describing the S-like portion of the curve as the fringe and the surrounding, flat portion of the curve as the wings. The slight imperfections in the curves are due to the spherical aberration of the primary mirror. These imperfections are mitigated by the Articulated Mirror Assembly (AMA) installed as part of FGS 1r during the 1997 servicing
mission (Nelan et al. 2011). With these improved optics the high angular resolution TRANS mode FGS is capable of detecting equal brightness binaries down to about 10 mas.

From low mass, faint red and white dwarfs to massive, O-stars, the FGSs have been applied to the study of binaries. In many cases the instrument was used to resolve known binaries to get system parameters, such as improved orbits (Franz et al. 1991) or magnitude differences to use in mass-luminosity relations (Henry et al. 1999). Horch et al. (2006) applied a Fourier analysis method to study TRANS mode data on a known spectroscopic binary. They explored the effects of color on the appearance of the transfer function and determined the system parameters, finding results consistent with those from the standard least-squares fitting method. Horch et al. (2006) also presented a detailed discussion of the detection limits of their method. The FGSs have also been applied to search for new binary systems. For example, Nelan (2007) resolved the double white dwarf system LB11146 into a pair with a separation of 15 mas. Nelan et al. (2004) searched for binaries in the Carina
Nebula region, and they made five new detections. Good single star calibrator observations are required for binary star detection and analysis, and Franz et al. (1992) described how such calibration curves can be selected from apparently single stars among a survey sample, an approach that we also adopt with our set of observations.

In section 3.2 we describe the sample selection, the observations and the FGS reduction pipeline and process. There. Section 3.3 describes how binary stars are detected and the fitting routines used to determine the system parameters (differential magnitude, angular separation, and position angle). The detection limits for our sample are also discussed there. The error analysis is discussed in section 3.4, and our results of the FGS survey are presented in section 3.5.

3.2 Sample and Observations

We present FGS 1r in TRANSFER mode observations of 58 of the most massive stars in Cyg OB2 that we use to search for companions within an arcsecond of the primary. The stars were selected from among the brightest, most massive stars in Cyg OB2 as cataloged in Schulte (1958), Massey & Thompson (1991), and Comerón et al. (2002). The observations were scheduled as part of a SNAP survey to fill in gaps between restricted observations of targets in other science programs. Table 3.1 lists the stars that were successfully observed. The stars are identified according to the numbering scheme used in the optical photometric survey of Massey & Thompson (1991) (e.g., MT 138) and the infrared spectroscopic study of Comerón et al. (2002) (e.g., A 23). Stars not included in those surveys are identified by the number assignation in Schulte (1958) (e.g., SCHULTE 5, S 5, or Cyg OB2-5) or by its Wolf-Rayet number, as in the case of WR 145. Table 3.1 also lists observational prop-
erties of the target stars (J2000 coordinates and spectral classification) along with remarks identifying known binary systems detected by our adaptive optics campaign or from spectroscopic observations of Kiminki et al. (2012). ‘NIRI’ in the remarks column denotes systems also resolved in the AO survey. ‘RV constant’ indicates objects observed for radial velocity variations by Kobulnicky et al. (2012) but that have not shown radial velocity variability during the course of their survey. Spectroscopic binaries are denoted by SB1 and SB2 for binaries detected by single spectral line radial velocity variations and double spectral line radial velocity variations, respectively. Eclipsing binaries are denoted by EW/KE, EA, and EB. EW/KE stands for hot eclipsing systems of the W UMa type, systems with ellipsoidal (tidal) variations and periods, $P < 1$ day. EA stands for Algol type (detached) systems with flat maxima. EB stands for $\beta$ Lyr type (semi-detached) with ellipsoidal light curves.

Table 3.1: Cygnus OB2 Target List

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## Table 3.1 – Continued

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<td>SB1 (^1)</td>
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\(^1\) from Muntean et al. (2009)

The sample was observed using FGS 1r with the F583W filter. This filter has an effective wavelength of 5830 Å with a 1500 Å bandpass (Nelan et al. 2011). Each observation consists of approximately 10-20 scans that are 1" long, except for those of MT 417 which are 2.5" long, and with a sampling rate of 1 mas per scan step. These scans are aligned, co-added, and smoothed to create the final transfer functions for two orthogonal baselines, \(x\) and \(y\) (section 3.2.1). The final smoothed, co-aligned \(S\)-curve is compared to an average point source \(S\)-curve to determine multiplicity (section 3.3). Table 3.2 is a complete list of our observed targets along with \(V\) magnitude, \(B - V\) color, date of observation, the name of the single star used as the calibrator for the binary fitting, and the number of components.
detected. The stars from Comerón et al. (2002) do not have known $B - V$ colors, but because they are bright infrared sources, they are assumed to be redder in nature than the target star.

Table 3.2: Observation and Calibrator Summary

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<th>Calibrator</th>
<th>No. Components</th>
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Table 3.2 – Continued

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<td>1.63</td>
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Note. — The Calibrator column lists the comparison, single star whose observations were used to make the binary model. The selection of the calibrator star was based on both color and date of observation.

3.2.1 Data Reduction

In the past, any data taken with the FGS were generally processed and analyzed by the instrument scientist, and the final results were sent to the principal investigator. Due to the large quantity of data from this survey and others at Georgia State University, with the help of the instrument scientist, Dr. Edmund Nelan, the processing pipeline for FGS was successfully imported to a Macintosh computer at Georgia State University.

Appendix B provides a tutorial for the reduction of FGS TRANS mode data on the system at Georgia State University using scripts developed at the Space Telescope Science Institute (STScI). The tutorial describes retrieving proprietary data from the STScI website\(^1\) and each step leading to determining binary parameters. Once the downloaded data are formatted using STRFITS from the STScI IRAF package, running the CALFGSA script splits the data into the individual scans. CALFGSA also produces an unaligned, co-added postscript image of both the $x$- and $y$-axis transfer functions.

The scans are co-added and smoothed using PTRANS. PTRANS is an interactive program that allows the user to decide if any scans should be discarded from the final $S$-curve. Each scan is co-aligned so that the fringe crosses the vertical zero-point at the horizontal zero-point. Once aligned, the scans are co-added and smoothed according to a smoothing

\(^1\)http://archive.stsci.edu
parameter. The user increases the smoothing parameter to increase the signal-to-noise ratio of the co-added $S$-curve without reducing the fringe amplitude or changing the shape of the $S$-curve. PTRANS smooths the transfer function according to a cubic spline of the Reinsch type (Nelan, private communication; Franz et al. 1992). Figure 3.2 shows the un-smoothed (left panel) and smoothed (right panel) $x$-axis fringes of the calibrator stars MT 213 (solid line) and MT 145 (dashed line). The amplitude of the deviation curve is intrinsically the same in both cases and no information is lost by smoothing. Typical values for the smoothing parameter ranged from 2,000 to 10,000.

Figure 3.2 A comparison of un-smoothed (left panel) versus smoothed (right panel) $S$-curves. The curves for MT 213 are shown as solid lines and those for MT 145 as dashed lines.

The final co-added, smoothed $S$-curve is then compared to a calibrator $S$-curve to determine if a second source is present. The final $S$-curves for all of the stars in the sample are presented in Appendix B.2 in alphabetical order. The $S$-curves presented there have been rectified according the process described in section 3.3.2.2 and include panels plotting the results of the binary detection methods described in sections 3.3.1.2 and 3.3.1.3.
3.3 Binary and Multiple Star Fitting Methods

3.3.1 Determining Multiplicity

Binaries have an $S$-curve that differs from that of an unresolved point source (Figure 3.1) of the same color. A binary detection is determined by comparing its $S$-curve to that of a point-source transfer function of similar color. The comparison star, or calibrator, is taken from unresolved sources within our sample that meet 4 criteria:

1. Observed with the same FGS

2. Observed using the same filter

3. Close in $B - V$ color

4. Close in observation date

Criteria 1 and 2 are met by all the stars in our sample (section 3.2). Criterion 3 is necessary because the appearance of a point source $S$-curve is color dependent due to the change in the interference pattern with effective wavelength. The effect that color produces on the shape of an $S$-curve is to stretch out the fringe for redder targets, similar to the bellows of an accordion, without changing the fringe amplitude (Horch et al. 2006). Figure 3.3 shows the $x$- and $y$-axis $S$-curves of the bluest and reddest unresolved stars in the sample, and we see that the fringe of the redder star (dashed line) is wider than that of the bluer star (solid line). We chose calibrators whose colors are within 0.2 mag from the target $B - V$, except for the very red star MT 304 (section 3.3.2). Among the set of stars that meet criterion 3, we chose as the best calibrator the one closest in observation date to that of the target. The appearance of the $S$-curves can change with time due to the settling of the instruments.
and adjustments from servicing missions. Our observing program was not affected by any changes made due to a servicing mission, but our large range of observation dates (2005 December to 2008 June) may span a range where the long term changes are significant. The binary systems we are able to resolve are close systems with nearly equal brightness and hence color, so we apply the same calibrator to both components.

3.3.1.1 Visual Inspection

The transfer function of a binary consists of the superposition of the individual transfer functions of two point sources. For widely separated systems, the scan will show two shifted transfer functions, whose amplitudes are directly related to the magnitude difference (see eq. 3.3). For closer systems, the $S$-curve can look obviously different from that of a single star (such as the case of MT 429 shown in Figure B.25), because of the interference between...
the fringe patterns. The next step is to compare directly the smoothed, co-aligned target
$S$-curve with that of a single star in plots like those in Appendix B.2.

Another indicator of a resolved binary is the relative fringe amplitude, calculated by
CALFGSA as the value of $sppr$, or the $S$-curve peak-to-peak ratio. The ratio of the extrema
points of the fringe to that of a single star calibrator fringe can be expressed as

$$sppr = \frac{S_{obs,max} - S_{obs,min}}{S_{F583W,max} - S_{F583W,min}}$$ (3.1)

where $S_{F583W,max} = 0.61$ and $S_{F583W,min} = -0.55$ along the $x$-axis and $S_{F583W,max} = 0.30$ and
$S_{F583W,min} = -0.65$ along the $y$-axis for typical single star observations using the F583W
filter. The cases where $sppr \leq 0.9$ indicate that the target is a resolved binary. However,
when $0.9 \leq sppr \leq 1.0$, then it is more difficult to determine if FGS resolved the target as
a binary. In these cases, if the system is a binary, then it may be near the limits of the
resolving power of the instrument, and such cases are difficult to distinguish visibly from
point-source $S$-curves.

3.3.1.2 Wide Binary Detection and Limits

The next situation to consider is the case where the absolute projected separation of
the binary companion is greater than the width of the fringe ($\gtrsim 50$ mas) but the secondary
is very faint. The best approach in these cases is to calculate the cross-correlation function
(CCF) of a target $S$-curve with that of a calibrator star. This method has the advantage
of using more of the $S$-curve than just the extrema points, and it potentially helps unravel
those cases where the fringes overlap. The cost of this approach is a slight decrease in the
working angular resolution limit (but see below for a discussion of the close binary case).
The top panel of Figure 3.4 shows an example of a model S-curve of a binary star with a projected separation of $+0'.07$ and a flux ratio $r = \frac{F_2}{F_1} = 0.1$ (constructed using calibrator $x$-axis scans). The dotted line shows the calibrator S-curve while the solid line shows that for the target binary. The fringe patterns overlap significantly at this separation, and the main differences are a dilution of the main fringe pattern and a change in outer fringe structure near $x = 0'.08$. The lower panel shows the CCFs of the target with the calibrator (solid line) and of the calibrator with itself (dotted line). Unlike the S-curves, the CCFs show one main peak for each stellar component. In order to isolate the companion, the calibrator CCF is shifted to the peak of the target CCF and scaled to the peak of the target CCF. The shifted and scaled calibrator CCF is then subtracted from the target CCF to produce the residual CCF shown as a dashed line in an expanded scale in the lower panel (and offset by $-0.8$ for clarity). Now the peak from the companion is clearly visible at the offset position of $\Delta x = 0'.07$.

We need a working criterion to establish whether or not a peak in the residual CCF makes a significant detection of a companion. Because the dominant source of uncertainty in the shape of the S-curves is the inherent scatter observed between S-curve observations of the calibrators, the criterion was set by running the CCF procedure for any given target with an ensemble of calibrator S-curves for stars of similar color (usually a set of four or more calibrators). Then the detection criterion was set by requiring the peak in the mean of the residual CCFs to exceed $4\sigma(x)$, where $\sigma(x)$ is the standard deviation of the residual CCFs at the peak position $x$.

The limiting detection threshold for a binary of a given separation is set by running models using multiple examples of model binary and calibrator S-curves based upon the
Figure 3.4 Example of the cross-correlation function. The top panel shows an example of a model $S$-curve of a binary star (solid line) and that of a single star calibrator (dotted line). The lower panel shows the cross-correlation functions of the target with the calibrator (solid line) and of the calibrator with itself (dotted line). The difference between these two is shown as a dashed line in an expanded scale, offset for clarity.
calibrator observations. This was done using a set of 21 calibrator observations of similar
color stars for both the model and calibrator curves (for a total of 441 test cases), and the
faintest flux ratio was set that met the $4\sigma$ detection criterion. The positive (solid line) and
negative (dashed line) branches are folded onto one separation axis in Figure 3.5 in the left
and right panels for the $x$- and $y$-axis scans, respectively. Here the limiting flux ratio, $r$,
is shown as a magnitude difference $\Delta m = -2.5 \log r$ for trial separations, and any binary
brighter than the limit (i.e., below the line plots) would exceed the $4\sigma$ detection criterion.
The smallest separation detected in favorable, equal flux cases is slightly better for positive
separations (16 mas versus 17 mas), and the faintest detectable companions have $\Delta m = 4.5$
at large separations. The oscillation in these curves seen near $x = y = 0''06$ is due to the
changing and relatively larger uncertainties in the calibrator $S$-curves at such distances from
their zero-crossing (see the discussion of the calibrator observation uncertainties, section 3.4).
The results in these figures compare well with the advertised limits in Figure 3.3 of the Fine
Guidance Sensor Instrument Handbook (Nelan et al. 2011) except for the case of very close
binaries that is discussed below. In fact, the peaks in the residual CCFs in the close binary
models are rarely found at separations of less than $0''04$ even in favorable cases. This is due
to the fact that blending of the fringe patterns becomes so severe that the calibrator CCF
is positioned at the maximum that occurs between the actual positions of the components,
and consequently, the residual CCF shows two peaks: one for the companion and a mirror
one for the primary. At the smallest separations where the method can be applied ($0''016$
in $x$ and $0''017$ in $y$), the two peaks in the residual CCF approach equal intensity, and the
method can no longer distinguish the direction of primary to secondary.
Figure 3.5 Binary detection limits as a function of projected separation and magnitude difference. The solid (dashed) lines indicate the limits for positive (negative) offsets in \(x\) (left panel) and \(y\) (right panel), respectively. Binaries with separations and magnitude differences below these limits should be detected. The dotted lines indicate detection limits for close binaries using the second derivative method.

### 3.3.1.3 Close Binary Detection and Limits

Close systems \((\rho \lesssim 25\text{ mas})\) can slightly reduce the fringe amplitude and widen the fringe shape. If the fringe pattern of a single calibrator star is \(S(x)\), then the observed fringe pattern for more than one star will be

\[
S(x)_{\text{obs}} = \sum_{i=0}^{n} f_i S(x - x_i)
\]

where each of \(n\) stars has a flux fraction \(f_i = \frac{F_i}{\sum F_j}\) and a relative projected offset position \(x_i\). For a binary star with a companion flux ratio \(r = \frac{F_2}{F_1}\) and a projected separation \(\Delta x\), the observed pattern simplifies to

\[
S(x)_{\text{obs}} = \frac{S(x)}{1 + r} + \frac{r}{1 + r} S(x - \Delta x).
\]
An analytical representation of the difference between the binary and calibrator $S$-curves can be estimated by making a second-order expansion for small offset $\epsilon$,

$$S(x + \epsilon) = S(x) + \epsilon S'(x) + \frac{1}{2} \epsilon^2 S''(x)$$

(3.4)

where $S'$ and $S''$ are the first and second derivatives of the $S$-curve. In the frame of reference where $S(0) = 0$ for the binary, the primary and secondary $S$-curves will be respectively shifted by amounts $\epsilon_1 = -(\frac{r}{1+r})\Delta x$ and $\epsilon_2 = +\left(\frac{1}{1+r}\right)\Delta x$, where $\Delta x$ is the projected separation of secondary from primary. Then the difference between the binary and calibrator $S$-curves is

$$S(x)_{\text{bin}} - S(x)_{\text{cal}} = \frac{1}{1+r} S(\epsilon_1) + \frac{r}{1+r} S(\epsilon_2) - S(x)$$

$$= \frac{1}{2} \frac{r}{(1+r)^2} (\Delta x)^2 S''(x).$$

(3.5)

This second-order expression has several important features. First, the observed difference in the core of the $S$-curve will appear to have the same functional shape as the second derivative of the $S$-curve, so we can directly search for close companions by looking for a difference that has a second derivative shape. Second, the amplitude of the difference depends on a product involving both the separation $\Delta x$ and the flux ratio $r$, so in the absence of other information, neither parameter can be determined uniquely. Third, the amplitude of the difference depends on the separation squared, so no information can be reliably extracted on the direction of the companion from the primary.

Figure 3.6 shows examples of such $S$-curve differences for model binaries. The dashed line shows the difference for a model of equally bright stars with a separation of $0.015$ made from a mean $S$-curve from a collection of calibrator x-axis scans. According to the
analytical expression above, the coefficient leading the second derivative is $0.015^2/8 = 2.8 \times 10^{-5}$ arcsec$^{-2}$, and the solid line shows the product of this coefficient and a numerical solution of the second derivative of the calibrator $S$-curve (smoothed by convolution with a Gaussian of FWHM = 0.005'). The good match between the detailed model and analytical solution verifies the second derivative character of the difference curve. The same coefficient is found for $r = 0.5$ and $\delta x = 0.0159$, and the dotted-dashed line shows the difference of the binary and calibrator curves for these binary parameters. Again, the agreement between this model and the analytical curve shows that two models with the same product $a = \frac{1}{2} \frac{r}{(1+r)^2} (\Delta x)^2$ have very similar $S$-curves.

Therefore, in the case of close systems the difference curve between the suspected binary and a single star should look like that of the second derivative of the point-source transfer function scaled by the coefficient product term $a$. Unless the flux ratio is determined independently, there is not a unique solution for the system.

The method was applied by considering the difference between the target and calibrator $S$-curves over the range within $\pm 100$ mas of the center of the fringe. The coefficient was then estimated by a least-squares fit of equation 3.5 over the restricted range. The coefficient $a$ was determined in practice with an ensemble of like-color calibrators, and the criterion for detection was set by a mean coefficient $a$ with a positive value greater than $4\sigma$, where $\sigma$ is the standard deviation of the coefficient.

The detection limits using the second derivative method were estimated by running the scheme with multiple binary models from a large sample of calibrators. The working criteria from these models for $4\sigma$ detection are $a > 5.3 \times 10^{-6}$ and $6.3 \times 10^{-6}$ arcsec$^{-2}$ for the $x$- and $y$-axes, respectively. These limits are shown in the Figure 3.5 as dotted lines that
Figure 3.6 Example plots of the difference between a model binary and calibrator $S$-curves. The dashed line shows the difference for a binary with $r = 1$ and $\Delta x = 0''015$, and the dotted-dashed line shows the difference for a binary with $r = 0.5$ and $\Delta x = 0''0159$. Both resemble the second derivative of the calibrator $S$-curve (solid line) for the same amplitude $a$, but differ from that caused by the dilution of an off-scan companion (dotted line).

trace the upper envelope for detection by the second derivative approach. In the best cases ($r = 1$), the estimates suggest that binaries as close as 0''013 can be detected with FGS TRANS mode scans. The second derivative and CCF methods are probably both sensitive to binary detection in the 0''020 – 0''025 range. One further cautionary remark about distant binaries should be noted. In cases where a companion is widely separated (or even beyond the recorded scan) the fringe for the primary will appear diluted. Recall that a very close binary may also cause the amplitude to decline, but in the close case, the fringe will also be
widened. Consequently, one can differentiate between these cases by the appearance of the difference curve. The dotted line in Figure 3.6 shows that the difference curve for a very wide binary appears like a negatively scaled version of the $S$-curve itself, which looks very different than the second derivative curve. Also, the CCFs differences are good diagnostics of close companions versus far off companions. A characteristic double-peaked feature appears in the CCF once the primary is removed (see top-right panel of Figure B.50). Thus, it is important to inspect the shapes of the difference curves in order to decide if a positive detection indicates a very wide or very close companion. This method led to the detection of only one close binary in our sample. This detection (MT 304) was only along one axis, and the binary nature was confirmed by a wider component found along the orthogonal axis.

3.3.1.4 Off-scan Components

There is also the case of very widely separated binaries, where the light of the companion falls within the instrument FOV ($5'' \times 5''$), but the projected separation is greater than the length of the scan. This is the case for MT 531 where the system is resolved along the $y$-axis, but the secondary is off the scan along the $x$-axis (see Figure B.39). The CCF method will fail to detect the binary because a second peak is missing in the residual CCF. However, such binaries will still cause the $S$-curve of the primary to appear with an amplitude reduced by a factor $\frac{1}{(1+r)}$, and hence a comparison of target to calibrator $S$-curve amplitude provides an additional criterion to check for very wide binaries. In practice, the scatter between calibrator $S$-curves of similar color indicates that a $4\sigma$ detection may be claimed if the target $S$-curve amplitude is less than 92% of the mean calibrator $S$-curve amplitude. A second check can be made through inspection of results from our NIRI survey (section 2).
The NIRI observations have a larger dynamic range (i.e., can detect fainter companions) and have a larger field of view, so any system observed with FGS with $\rho \gtrsim 50$ mas will also be detected in the NIRI image. The infrared images allow us to identify those companions with projected separations longer the scan length, but still within the FOV of the FGS. With the help of the NIRI observations, we were able to conclude that distant companions influence the FGS results for MT 59 and MT 531.

3.3.2 Model Fitting

If a binary is detected, we assume sources we find are close binaries that are true, physical companions due to the short length of the scans ($\sim 1''$). The binary transfer function is modeled using the calibrator to recreate a synthetic binary. From there the model is compared to the target and a best fit is determined. From the best fit model we calculate the relative brightness and projected separation along an axis. Combining the information for both axes we can also calculate the total projected separation and position angle of the binary. We used two routines to calculate the best fit model, BINARY_FIT, part of the STScI reduction package, and the IDL routine TRIPFIT, which we used for the special cases where BINARY_FIT was not able to converge on a satisfactory answer (as described below).

With both programs each binary is compared to four calibrators. The calibrators are selected based primarily on $B - V$ color such that calibrators are within $\pm 0.2$ mag of the target color. The only exception is MT 304, with a $B - V = 3.35$, which is the reddest star in our sample by more than 1 mag. In this case, we selected calibrators from the reddest, single star in our sample, MT 448, and the stars from Comerón et al. (2002) (A 23, A 27,
A 41, A 46), which are red objects according to their brightness in the infrared. We adopt the fit made using the calibrator closest in color and observation date, and the spread in results from fits made using the other calibrators is used to determine the error. The same calibrator is used to model both the primary and secondary S-curves, assuming that any color difference between the two components is negligible.

3.3.2.1 BINARY_FIT

BINARY_FIT uses a least squares approach to determine the projected separation and magnitude difference of the system. The binary S-curve is compared with the model S-curve based on a calibrator star of similar color. BINARY_FIT fits data from one axis at a time, and the user may select the individual calibrators used to model the primary and secondary, if the components are of different colors. For our sample, we always applied the same calibrator for both components. Then the user enters initial guess values for the separation along the axis and differential magnitude. Once the program converges on a best fit for each axis and and the results from both axes agree within 0.2 mag in $\Delta m_{F583W}$, it calculates the projected separation and position angle. The $\Delta m_{F583W}$ values reported in Table 3.4 are taken from the axis with the larger separation using the calibrator listed in Table 3.2. However, the data for the axis with the shorter separation is not refit with the new $\Delta m_{F583W}$, and the original value is reported in Table 3.4.

While BINARY_FIT will always converge to an answer, it will not necessarily find the global best fit. BINARY_FIT allows the user to direct the program to a preferred answer by fixing the differential magnitude. This is done when initially supplying the differential magnitude or, in the case where the solutions from the two axes do not agree, by assigning
the value from the axis with the better fit to the solution for the other axis. The user selects which axis solution is a better fit. When prompted to do so, we would select the axis with the larger projected separation as the better fit. We would then rerun BINARY_FIT on the axis with the shorter separation, fixing the differential magnitude and converging to a new projected separation along that axis.

The fitting is limited by the scan length of the calibrator. If the separation of the binary is larger than the scan length of the calibrator, the program is not able to recreate a binary wide enough to properly model the target. BINARY_FIT also only considers solutions where both components are within the scan length. For example, if the component is in the scan along one axis but off on the other, then the program will converge to a solution of a very close system for the axis where the star is absent. Also, BINARY_FIT cannot be applied to systems with more than two components, such as the triple MT 417.

3.3.2.2 TRIPFIT

We coded the IDL program TRIPFIT to find the best fit model for triple systems along one axis using a Levenberg-Marquardt least-squares fit. The program is capable of making fits of triple systems, binaries, and off-scan components. The user selects the positions of the components and inputs an initial estimate for the differential magnitudes. The program fits the $S$-curves one axis at a time and returns the best fit for the differential magnitudes and separations. The code is provided in Appendix B.1 along with the function TTRIPLE that creates the model system scan.

Before fitting the systems, TRIPFIT subtracts the trend in the wings, a process similar to rectifying and normalizing a spectrum, using the program RECTTF. This is necessary for
the case of MT 417 (Figure 3.9) because it has a longer scan length with wing variations that are larger than those of the other, shorter curves. RECTTF fits second order polynomials to the wings, in both $x$ and $y$, of a simulated single star $S$-curve created by averaging our calibrators. To rectify a short scan $S$-curve, the polynomial is subtracted from each corresponding axis. For MT 417 and widely separated systems, the default polynomial does not follow the trend of the wings. For these cases, RECTTF uses a spline function to fit the trend in the wings based on representative points along the $S$-curve. This is done for both axes independently. By normalizing the curve, we are able to fit the wider separated binaries, like S 5, by adding zero points to the edges of the comparison $S$-curve. RECTTF was also used in the second derivative and cross-correlation analysis. The figures in Appendix B.2 have all been processed using RECTTF, whereas the curves in Figure 3.1 and Figure 3.9 were not.

Both BINARY_FIT and TRIPFIT model the binary to determine the best separation and flux ratio along each axis. The flux ratio is converted to differential magnitude using

$$
\Delta m_{F583W} = -2.5 \log \left( \frac{F_2}{F_1} \right)
$$

(3.6)

where $F_1$ and $F_2$ are the fluxes of the primary and secondary, respectively. BINARY_FIT is able to determine the projected separation and position angle of the component if the flux ratio results agree between the solutions for both axes. TRIPFIT does a single axis at a time, but the projected separation can be calculated using

$$
\rho = \sqrt{\Delta x^2 + \Delta y^2}
$$

(3.7)
Figure 3.7 Figure showing the values of $\phi$ in each quadrant. This is used to determine the position angles of binaries. Positive $x$ values are to the right and positive $y$ values are up.

where $\Delta x$ and $\Delta y$ are the separation along the $x$ and $y$ axes, respectively. The IDL program PANGLE (Appendix B.1) determines the position angle, $\theta$, using the aperture position angle of the telescope, a value found under PA_APER in the star’s file with the extension TAB outputted by CALFGSA. PA_APER is the position angle of the FGS $y$-axis measured from North to East. The binary position angle $\theta$ is the sum of the aperture position angle and an angle, $\phi$, determined from the $x$ and $y$ separations:

$$\theta = PA_{APER} + \phi.$$  \hspace{2cm} (3.8)

Figure 3.7 shows how the angle $\phi$ is determined based on the secondary’s relative position to the primary in the frame of the telescope, where positive $x$ values are to the right and positive $y$ values are up.

3.4 Error Analysis

Once a fit is determined, both BINARY_FIT and TRIPFIT return error estimates for the parameters. However, the error is underestimated and is not representative of the true error.
There are two sources of uncertainty in the FGS data. One comes from the internal error of the scan due to brightness of the source, the telescope jitter, and other sources of noise due to the instruments. The second source of error is due to systematic differences between the calibrators. As described below, we find that the main source of error is associated with the selection of the calibrator.

3.4.1 Systematic Errors

The model of a binary is created using the $S$-curve of a point source. Each binary $S$-curve was fitted using the $S$-curve of four different calibrator stars. BINARY.FIT produced a unique solution for each calibrator. The standard deviation of the solutions for the $x$ and $y$ axes are used to determine the error. The quality of the fits varies from calibrator to calibrator. Instead of averaging the solutions from the different calibrators, one calibrator was chosen for the binary model fits given in Table 3.4 along with an error determined from the solutions of the other calibrators. The selection of the “best fit” calibrator is described above. Figure 3.8 provides plots of the standard deviation of 40 of our calibrators to show the uncertainty at each point in the curve due to differences in color and other effects. All of the curves were aligned and normalized as described in section 3.3.2.2. These plots indicate that variations on the order of 10% of the fringe amplitude are not unusual among the set of calibrators we used.

3.4.2 Internal Error

The internal error of an $S$-curve is dependent on both the standard deviation between co-added each scans and the amount of smoothing applied. In PTRANS, the program prints
Figure 3.8 The standard deviation in the fringe amplitude among the $S$-curves of the calibrator stars (solid lines). The dotted lines show the mean fringe pattern reduced by a factor of 10 in amplitude. Note that the standard deviation is zero at the origin because all the scans were aligned to cross from positive to negative at the origin.

a table with the standard deviation of each scan in the wings and the core of the fringe. The standard deviation in the core of the fringe is on average higher than that of the wings, and the fringe core is the part of the interferogram of most interest. The smoothing introduces a complication in the error estimate because the signal-to-noise ratio is increased with greater smoothing. However, as discussed above, the shape of the fringe is not altered by smoothing.

To determine how the internal errors of a smoothed $S$-curve influence the binary parameters, we selected four binary systems (MT 5, MT 429, MT 605, and MT 632) and binned their scans into three or four independent subsets. Each subset includes approximately five scans that were shifted, co-added and smoothed using the same approach applied to the complete set of scans. The resultant subset of co-aligned scans were then fit with the best calibrator using BINARY_FIT. Table 3.3 lists the results for each binned curve and the standard deviation of the parameters from the fits. For the case of MT 5, the first subset of scans was
Table 3.3. Binary Solutions from Scan Subsets

<table>
<thead>
<tr>
<th>Name</th>
<th>Scans</th>
<th>$\Delta x$ (mas)</th>
<th>$\Delta y$ (mas)</th>
<th>$\rho$ (mas)</th>
<th>$\theta$ (°)</th>
<th>$\Delta m_{F583W}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>MT 5</td>
<td>4-8</td>
<td>-126.7</td>
<td>203.2</td>
<td>320.7</td>
<td>91.6</td>
<td>2.71</td>
</tr>
<tr>
<td></td>
<td>9-13</td>
<td>-120.0</td>
<td>203.2</td>
<td>320.7</td>
<td>91.6</td>
<td>2.71</td>
</tr>
<tr>
<td></td>
<td>15-17,19,20</td>
<td>-131.3</td>
<td>203.3</td>
<td>321.3</td>
<td>91.4</td>
<td>2.71</td>
</tr>
<tr>
<td>$\sigma_{rms}$</td>
<td>2.4</td>
<td>1.9</td>
<td>0.86</td>
<td>0.47</td>
<td>0.66</td>
<td></td>
</tr>
<tr>
<td>MT 429</td>
<td>1-7</td>
<td>-35.9</td>
<td>-96.1</td>
<td>102.6</td>
<td>23.3</td>
<td>1.13</td>
</tr>
<tr>
<td></td>
<td>8-14</td>
<td>-35.0</td>
<td>-94.6</td>
<td>100.9</td>
<td>23.1</td>
<td>1.01</td>
</tr>
<tr>
<td>$\sigma_{rms}$</td>
<td>0.5</td>
<td>0.7</td>
<td>0.85</td>
<td>0.2</td>
<td>0.064</td>
<td></td>
</tr>
<tr>
<td>MT 605</td>
<td>1-5</td>
<td>99.2</td>
<td>61.6</td>
<td>116.7</td>
<td>254.9</td>
<td>0.68</td>
</tr>
<tr>
<td></td>
<td>6-10</td>
<td>98.9</td>
<td>61.4</td>
<td>116.4</td>
<td>254.9</td>
<td>0.69</td>
</tr>
<tr>
<td></td>
<td>11-15</td>
<td>99.1</td>
<td>61.5</td>
<td>116.6</td>
<td>254.9</td>
<td>0.68</td>
</tr>
<tr>
<td>$\sigma_{rms}$</td>
<td>0.12</td>
<td>0.096</td>
<td>0.12</td>
<td>0.050</td>
<td>0.04</td>
<td></td>
</tr>
<tr>
<td>MT 632</td>
<td>1-5</td>
<td>165.1</td>
<td>-144.3</td>
<td>219.3</td>
<td>247.2</td>
<td>2.71</td>
</tr>
<tr>
<td></td>
<td>6-10</td>
<td>165.1</td>
<td>-144.3</td>
<td>219.2</td>
<td>247.2</td>
<td>2.68</td>
</tr>
<tr>
<td></td>
<td>11-15</td>
<td>165.0</td>
<td>-144.0</td>
<td>219.0</td>
<td>247.1</td>
<td>2.71</td>
</tr>
<tr>
<td>$\sigma_{rms}$</td>
<td>0.06</td>
<td>0.17</td>
<td>0.15</td>
<td>0.068</td>
<td>0.017</td>
<td></td>
</tr>
</tbody>
</table>

not compared because those problematic scans were not included in the complete S-curve.

The small spread in the values of separation and magnitude shows that the internal error is not a significant source of uncertainty for the binary parameters of MT 429, MT 605 and MT 632. However, this is not the case for MT 5, where the internal error is not negligible, and in section 3.5.1.1 we describe how we determined the uncertainties for this system. The other binaries are better representatives of the general internal quality of S-curves in our sample.

### 3.5 Multiplicity Results

In Appendix B.2 we provide figures of the transfer functions for all 58 stars in our sample. The figures show the $x$- (left) and $y$-axis (right) rectified S-curves in the central panel. Also shown as a dashed line is a preliminary model fit based upon the components derived from the CCF analysis and the mean S-curve of the calibrator set selected. The top panel plots the mean of the residual cross-correlation functions of the target with the calibrator (solid, black lines) after the peak of the primary has been subtracted off. The vertical dashed lines
Table 3.4. Multiplicity Parameters for Resolved Systems

<table>
<thead>
<tr>
<th>Star Name</th>
<th>$\Delta x$ (mas)</th>
<th>$\Delta y$ (mas)</th>
<th>$\rho$ (mas)</th>
<th>$\theta$ (°)</th>
<th>$\Delta m_{F583W}$ (mag)</th>
</tr>
</thead>
<tbody>
<tr>
<td>MT 5</td>
<td>-129.6 ± 0.7</td>
<td>294.0 ± 0.3</td>
<td>321.3 ± 0.6</td>
<td>91.7 ± 0.1</td>
<td>2.79 ± 0.09</td>
</tr>
<tr>
<td>MT 39</td>
<td>-751.3 ± 0.8</td>
<td>...</td>
<td>&gt; 751.3 ± 0.8</td>
<td>2.58 ± 0.04</td>
<td></td>
</tr>
<tr>
<td>MT 304</td>
<td>61.8 ± 3.1</td>
<td>9.7 or -14.9 ± 2.8</td>
<td>634.6 ± 3.5</td>
<td>279 or 189</td>
<td>2.31 ± 0.21</td>
</tr>
<tr>
<td>MT 417</td>
<td>-214.1 ± 5.1</td>
<td>1700.0 ± 2.5</td>
<td>1715.7 ± 2.9</td>
<td>147.9 ± 0.4</td>
<td>2.66 ± 0.32</td>
</tr>
<tr>
<td>MT 429</td>
<td>-36.2 ± 0.7</td>
<td>-95.8 ± 0.3</td>
<td>101.9 ± 0.3</td>
<td>23.5 ± 0.4</td>
<td>1.09 ± 0.02</td>
</tr>
<tr>
<td>MT 516</td>
<td>374.5 ± 0.04</td>
<td>-618.0 ± 0.1</td>
<td>722.6 ± 0.09</td>
<td>325.7 ± 0.1</td>
<td>0.28 ± 0.02</td>
</tr>
<tr>
<td>MT 531</td>
<td>99.0 ± 0.15</td>
<td>61.2 ± 0.5</td>
<td>116.4 ± 0.4</td>
<td>255.0 ± 0.2</td>
<td>0.69 ± 0.02</td>
</tr>
<tr>
<td>MT 605</td>
<td>163.0 ± 0.3</td>
<td>-144.1 ± 0.3</td>
<td>219.1 ± 0.2</td>
<td>247.2 ± 0.09</td>
<td>2.00 ± 0.16</td>
</tr>
<tr>
<td>MT 696</td>
<td>(±) 9.7 ± 1.5</td>
<td>20.6 ± 2.7</td>
<td>22.8 ± 3.0</td>
<td>175.1 - 225.6</td>
<td>0.94 ± 0.40</td>
</tr>
<tr>
<td>SCHULTE 5</td>
<td>-801.1 ± 2.2</td>
<td>439.6 ± 2.1</td>
<td>913.9 ± 2.3</td>
<td>56.4 ± 0.11</td>
<td>2.93 ± 0.29</td>
</tr>
</tbody>
</table>

indicates the position of each component resolved. The solid, gray lines show the standard deviation of the residual cross-correlation functions, and a peak is considered significant only if the mean residual CCF (black) exceeds the standard deviation (gray) by four times. The bottom panels show the difference curves between the star and calibrator $S$-curves (solid line). The shaded, gray region is the uncertainty envelope determined from the standard deviation at each point along the $S$-curves for the calibrators (Fig. 3.8). In the case where the second derivative test resolved a close pair (MT 304), the second derivative function of the calibrator is overplotted and scaled by the a coefficient (solid, gray line).

Tables 3.4 lists the 11 resolved systems within our Cyg OB2 sample and the model fitting results for separation and differential magnitude. In the cases where the fitted magnitude differences of the $x$- and $y$-axes agreed according to BINARY_FIT, the differential magnitude listed is from the fit for the axis with the larger separation. For the cases where the differential magnitudes did not agree, the axis with the shorter projected separation was refitted using the differential magnitude from the other axis. We describe below the cases where variations to the BINARY_FIT method or TRIPFIT were used.
3.5.1 Special Fitting Cases

3.5.1.1 MT 5

MT 5 was fit using BINARY_FIT in the standard method described above. After splitting the S-curve into subsets, the internal error was found to be a non-negligible source of uncertainty in this case. The errors listed for MT 5 in Table 3.4 are the errors due to both the spread among the calibrators and between the subsets of MT 5, added in quadrature.

3.5.1.2 MT 59

TRIPFIT was used to model MT 59 due to the binary being too widely separated in the x-axis for BINARY_FIT and the secondary lying off the scan in the y-axis. The values listed in Table 3.4 are from using the MT 601 data to model the x-axis scan and, the errors are from the standard deviation of fits from five calibrators.

The possibility of the y-axis scan having overlapping fringes is eliminated by the second derivative test (i.e., the difference does not match the second derivative curve) and by inspecting the NIRI image. If the y-axis fringe results from the blend of two close components, then the total projected separation, $\rho$, would be approximately $0.'75$. In the adaptive optics image, MT 59 has a companion at $\rho = 1.''20$ with $\Delta K = 2.75$, but there is no evidence of another component at $\rho \approx 0.'75$. If the companion detected along the x-axis is the same star detected in the NIRI image, then we can calculate expected projected separation along the y-axis by using $\rho^2 = \Delta x^2 + \Delta y^2$ and solving for $\Delta y$. This has led us to identify the secondary in the x-axis as the same star in the NIRI image with $\Delta y = 0.'95$, which is beyond the edge of the scan.
This B-supergiant is suspected to be one of the intrinsically brightest stars in our galaxy (Massey & Thompson 1991). MT 304 or Schulte 12 was particularly difficult to analyze. From the visual inspection of the $x$-axis scan and the detection of the $y$-axis as a close system using the second derivative test, we determined that this star is a binary. By far the reddest star in our sample ($B - V = 3.35$), this made finding a suitable calibrator from our sample a problem. We used MT 448 as the “best” calibrator because it is the second reddest star in our sample with known color, $B - V = 2.15$. Due to the infrared brightness of A 23, A 27, A 41 and A 46 (Comerón et al. 2002), we assumed they are among the redder stars in our sample. We used these stars to model MT 304 with BINARY_FIT and to determine the error in the fit. In this case, the differential magnitude from $x$ was used to calculate the separation in $y$. For close systems, where the projected separation is less than the size of the fringe, there exists a $180^\circ$ ambiguity in the position of the secondary. There are two possible solutions: $\Delta y = 9.7$ and $\theta = 279^\circ$ or $\Delta y = -14.9$ and $\theta = 189^\circ$. Both solutions are indicated in the values for the position angle in Table 3.4.

We were not able to detect the infrared counterpart in the NIRI image. The projected separation of this system is $\rho = 63.6$ mas and the limiting resolution for NIRI is about 80 mas. MT 304 is an interesting target for follow-up observations.

3.5.1.4 MT 417

MT 417 is the only triple system resolved in our sample and so we developed TRIPFIT to model it. Visual inspection of the $y$-axis $S$-curve (right panel of Fig. 3.9 and Fig. B.24)
shows that there is a very faint third component. Confirming this with the NRI images led us to conclude that the third component fringe is blended with that of the secondary in the $x$-axis. Figure 3.10 shows the NRI image with the projected FGS axes indicated by the arrows. Of note, in Figure B.24 the $x$-axis CCF analysis detected the blended third component, as indicated by the vertical dashed line in the top panel, but placed it closer to the primary than it should. We provide this in the figure to show the initial estimates provided by the CCF analysis.

![Figure 3.9 Unrectified S-curves for MT 417. Arrows indicate the positions of the three components.](image)

The solution for differential magnitudes for the secondary from the two axes did not agree with each other ($\Delta m_x = 0.85$ and $\Delta m_y = 0.45$). This is due to the fact the second and third component are blended in the $x$-axis, making the amplitude of the secondary’s fringe appear smaller. This is not the case for the $y$-axis, where the two components are well separated. The value listed in Table 3.4 is from the $y$-axis, which has the larger projected
Figure 3.10 NIRI image of MT417 with arrows indicating the projection of the FGS axes with respect to the system.

separation, and the error is estimated from the standard deviation between the fits of the different calibrators

3.5.1.5 MT 531

MT 531 is an obvious binary in $y$ and the second component is off the scan in $x$, similar to MT 59. We confirm this by looking at the NIRI image for this system using similar arguments as MT 59, and we determined that the infrared counterpart is the source with a total projected separation of $\rho = 1''45$ and $\Delta K = 0.65$. This corresponds to an off-scan position of $\Delta x = 1''38$.

3.5.1.6 MT 696

MT 696 was not detected using the second derivative test, but along the $y$-axis through the cross-correlation method. We interpret the $S$-curve to be that of a close system, where the
fringes from the two components overlap. We used BINARY_FIT to converge to consistent solutions with multiple calibrators. We list the estimated system parameters in Table 3.4. To arrive at this solution we used the differential magnitude from the $y$-axis because it is more widely separated. There exists a $180^\circ$ ambiguity in the position of the secondary along the $x$-axis ($\Delta x = \pm 9.7$). The ambiguity is reflected in the two values of the position angle in Table 3.4. The second derivative test returned a ratio of $a/\sigma(a) = 2.12$ along the $y$-axis, well below our detection threshold for close binaries.

3.5.1.7 Schulte 5

Schulte 5 is an obvious binary in both axes. Along the $x$-axis, it is too wide for BINARY_FIT to model, so we applied TRIPFIT to this system. The program arrived at two, though close, solutions for the differential magnitude and we list the average in Table 3.4. The errors were determined from the standard deviation between the differential magnitude from both axes.

3.5.1.8 Marginally Resolved

There were at least eight additional cases where the CCF analysis indicates a marginally resolved and/or faint companion (near the limits of the instrument). We list these stars in Table 3.5 as objects of interest for follow-up analysis along with initial separation estimates from the CCF analysis. We note that the second derivative analysis for these stars did not produce an $a$ coefficient larger than $3\sigma$ from the mean, whereas our cutoff for detection is $4\sigma$. We suspect that the two marginally resolved stars around A 41 and MT 138 are the nearby companions observed in NRI with $\rho = 0'35$ and $\rho = 1'34$, respectively. Some of
Table 3.5. Marginally Resolved Systems

<table>
<thead>
<tr>
<th>Star Name</th>
<th>( \Delta x ) (mas)</th>
<th>( \Delta y ) (mas)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A 27</td>
<td>( \cdots ) 53</td>
<td>31</td>
</tr>
<tr>
<td>A 41</td>
<td>( \cdots )</td>
<td>32</td>
</tr>
<tr>
<td>A 46</td>
<td>( \cdots )</td>
<td>68</td>
</tr>
<tr>
<td>MT 138</td>
<td>( \cdots )</td>
<td>68</td>
</tr>
<tr>
<td>MT 259</td>
<td>( \cdots )</td>
<td>62</td>
</tr>
<tr>
<td>MT 317</td>
<td>31 ( \cdots )</td>
<td>( \cdots )</td>
</tr>
<tr>
<td>MT 448</td>
<td>( \cdots )</td>
<td>76</td>
</tr>
<tr>
<td>MT 692</td>
<td>( \cdots )</td>
<td>501</td>
</tr>
</tbody>
</table>

these marginally resolved cases have been used as calibrators for the above analysis. The only impact this has in our results is in overestimating our errors.

3.6 Conclusions

In the sample of 58 Cyg OB2 stars, we resolved 11 systems with the FGS 1r TRANSFER mode observations. A 19\% binary detection is consistent with the results from Nelan et al. (2004), who resolved 5 out of 23 OB stars (20\%) in the Carina Nebula. However, our binary fraction may be as high as 33\% if further analysis determines that our eight marginally resolved pairs are real. We were able to find the infrared counterpart for nine of the resolved systems in the NIRI observations. We were unable to verify with NIRI is the close pairs of MT 304 and MT 696. In chapter 5 we discuss the direction of future work and the broader application of our results.
THE ULTRAVIOLET SPECTRUM AND PHYSICAL PROPERTIES OF THE
MASS DONOR STAR IN HD 226868 = CYGNUS X-1

4.1 Cygnus X-1 Introduction

A few degrees away from Cygnus OB2 in the sky is an interesting massive binary, Cygnus X-1. The massive X-ray binary Cygnus X-1 = HD 226868 consists of an O9.7 Iab primary (Walborn 1973b) with a black hole (BH) companion. The fundamental properties of this system have been the subject of many studies, but they continue to be controversial. For example, Shaposhnikov & Titarchuk (2007) determined a relationship between black hole mass and observed X-ray properties in the low frequency, quasi-periodic oscillation – spectral index plane to derive a BH mass of \(8.7 \pm 0.8 M_\odot\) for Cyg X-1, a value at the low end of previous estimates (Gies & Bolton 1986a; Abubekerov et al. 2005). On the other hand, Ziolkowski (2005) used temperature – luminosity relations in conjunction with evolutionary models to calculate the mass of the bright, mass donor star. Then, with the orbital mass function from Gies et al. (2003) and the method outlined by Paczyński (1974), he estimated the mass of the BH as \(13.5 - 29 M_\odot\), at the high end of prior estimates.

Our goal in this chapter is to determine if mass estimates from these two methods can be reconciled through a re-examination of the supergiant’s spectrum to determine its stellar temperature, mass, and radius. Shortly after the X-ray source Cyg X-1 was identified with the star HD 226868 (Bolton 1972; Webster & Murdin 1972), Walborn (1973b) classified it as a normal O9.7 Iab star. The stellar temperature of a star of this type depends critically upon the model atmosphere assumptions adopted to match the line spectrum (Martins et al. 2005). From a classical curve of growth analysis of the optical spectrum of HD 226868, Canalizo et al.
(1995) estimated an effective temperature of $T_{\text{eff}} = 32\pm 2$ kK, and found an overabundance of He in the photosphere. Herrero et al. (1995) also estimated the temperature of the star $T_{\text{eff}} \approx 32$ kK based upon fits of the optical line spectrum with calculated profiles from unified model atmospheres that included a non-LTE treatment of H and He, but neglected line-blanketing from transitions of heavier elements. In addition, they determined values for gravity from fits of the Balmer lines that ranged from $\log g = 3.03$ for plane-parallel models to $\log g = 3.21$ for spherical models that included wind effects. Their results led to mass estimates of 17.8 and 10.1$M_\odot$ for the supergiant and BH, respectively. More recently, Karitskaya et al. (2008) classified HD 226868 as an ON star with a temperature of $T_{\text{eff}} = 30.4 \pm 0.5$ kK and gravity of $\log g = 3.31 \pm 0.07$ using a semi-gray model atmosphere that accounts for non-LTE effects in some lines and for X-ray illumination.

Here we present an analysis of the photospheric parameters for the supergiant based upon ground-based optical spectra and high-resolution, UV spectra from the Hubble Space Telescope Imaging Spectrograph (STIS). These STIS spectra were first presented by Gies et al. (2008) and Vrtilek et al. (2008) in discussions of the orbital variations observed in the stellar wind lines. We compare the optical and UV line profiles of HD 226868 with synthetic spectra based on line blanketed, non-LTE photospheric models in order to determine the stellar temperature and gravity (section 4.2). Since the continuum flux and spectral lines of the supergiant could be influenced by X-ray heating, we search for heating effects in the orbital UV flux variations using the low/hard state International Ultraviolet Explorer (IUE) archival spectra and the high/soft state HST spectra (section 4.3). A stellar radius – distance relation can be determined from fits of the stellar energy distribution. We use the observed flux distribution and spectra of field stars in the same region of the sky
to estimate the reddening and extinction in the direction of Cyg X-1 and to determine the angular size of the star (section 4.3). Finally, we use this radius – distance relation with the method developed by Paczyński (1974) to set mass limits as a function of distance and to estimate the probable masses using constraints from the rotational line broadening and ellipsoidal light curve (section 4.4).

4.2 Ultraviolet and Optical Line Spectrum

We need to rely on the line spectral features to estimate temperature since the UV and optical continuum falls in the long wavelength, Rayleigh-Jeans part of the flux distribution where the shape of the continuum is insensitive to temperature. Some of the best line diagnostics for late O-supergiants are found in the optical spectrum where several ionization state line ratios and the Balmer line profiles change dramatically with temperature and gravity (Walborn & Fitzpatrick 1990; Searle et al. 2008). In this section, we use $\chi^2$ fits of the optical and UV spectra with model spectra to estimate $T_{\text{eff}}$ and $\log g$. Our data consist of high resolution UV spectra taken with the HST/STIS (G140M grating, resolving power $R = 14500$) and two sets of optical spectra from the Kitt Peak National Observatory Coudé Feed telescope (CF; 3759 – 5086 Å, $R = 2990$) and 4 m Mayall Telescope and RC spectrograph (RC; 4182 – 4942 Å, $R = 5700$). Details of these observations are given in Table 4.1 of Gies et al. (2008). All of these flux-rectified spectra were shifted to the rest frame (using the orbital solution given by Gies et al. 2008) and co-added to increase the signal-to-noise ratio.

We compared these spectra with model spectra from the TLUSTY/SYNSPEC codes given in the grids OSTAR2002 (Lanz & Hubeny 2003) and BSTAR2006 (Lanz & Hubeny
The model atmospheres are based upon a plane parallel geometry, solar abundances, line blanketed opacities, and non-LTE calculations of atomic populations for H, He, and representative atoms up to Fe. The model spectra are presented as a function of four parameters: the microturbulent velocity of gas in the line forming region, $\xi$, the stellar effective temperature, $T_{\text{eff}}$, logarithm of the gravitational acceleration in the photosphere, $\log g$, and the chemical abundance of the gas. The model spectra were transformed to the observed wavelength grids by wavelength integration and convolution with rotational and instrumental broadening functions. We adopted a projected rotational velocity of $V \sin i = 98 \text{ km s}^{-1}$ (Gies & Bolton 1986a), linear limb darkening coefficients from Wade & Rucinski (1985), and Gaussian representations of instrumental broadening using the projected slit FWHM (Gies et al. 2008).

It was clear from inspection that a large microturbulence is required to match the observed and model spectra. The OSTAR2002 grid uses $\xi = 10 \text{ km s}^{-1}$ throughout while the BSTAR2006 grid adopts $\xi = 2 \text{ km s}^{-1}$ for the full grid and $\xi = 10 \text{ km s}^{-1}$ for a selection of low gravity (supergiant) models. We found that the best fits were obtained in all three spectral bands with the $\xi = 10 \text{ km s}^{-1}$ models, and this was especially true in the FUV where the observed deep spectral lines were not matched with the lower microturbulent velocity models. An atmospheric microturbulence of $\xi = 10 \text{ km s}^{-1}$ is typical for late-O supergiants (Ryans et al. 2002), and Canalizo et al. (1995) derived an estimate of $\xi = 10.7 \text{ km s}^{-1}$ from a curve of growth analysis of the optical N$\text{III}$ lines in the spectrum of HD 226868.
We tested the goodness-of-fit for each of the models of interest by calculating the reduced
χ²ν statistic,
\[ \chi^2_\nu = \sum_{i=0}^{N} \frac{[F_{\text{obs}}(\lambda_i) - F_{\text{model}}(\lambda_i)]^2}{\sigma_{\text{err}}^2(\lambda_i)(N-1)}. \] (4.1)

Here \( N \) is the total number of wavelength points used in the fit and \( \sigma_{\text{err}}(\lambda_i) \) is the standard
deviation of the mean rectified flux (determined from the scatter at wavelength \( \lambda_i \) among
the individual spectra in the co-added mean). We selectively omitted from the summation
spectral regions that contained stellar wind features or interstellar lines that are not present
in the model spectra. Our results are listed in Table 4.1 for a wide range of model spectra
with a microturbulence of \( \xi = 10 \) km s⁻¹. Column (1) indicates the spectral region fit by
“HST” for the FUV spectrum, “CF” for the blue, KPNO Coudé Feed spectrum, and “RC”
for the green, KPNO 4 m RC spectrum. Column (2) gives the grid value of gravity \( \log g \),
and column (3) gives a code for the spectral model (“O” for spectra from the OSTAR2002
grid and “B” and “BCN” for spectra from the BSTAR2006 grid). Then follow 10 columns
that list the measured \( \chi^2_\nu \) for grid values of \( T_{\text{eff}} \) (at increments of 2.5 kK and 1 kK for the
OSTAR2002 and BSTAR2006 grids, respectively).

The trends in Table 4.1 are represented in a combined contour diagram in Figure 4.1.
Here the gray-scale contours represent the goodness-of-fit for the FUV spectrum, the solid
lines for the blue spectrum, and the dashed lines for the green spectrum. Since there is not
exact agreement between the predictions of the OSTAR2002 and BSTAR2006 grids at their
boundary, Figure 4.1 shows contours based only on the OSTAR2002 grid for high gravity
models \( \log g \geq 3.0 \), while the contours in the lower gravity region \( \log g \leq 3.0 \) are based upon
the BSTAR2006 grid. Note that there are no models available for high temperature, low
**Table 4.1.** $\chi^2$ for Spectral Fits with Models

<table>
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gravity region in the lower right part of the diagram because such atmospheres approach or exceed the Eddington luminosity limit. The higher gravity, lower temperature region is empty because the BSTAR2006 grid contains only models for a lower microturbulent velocity $\xi = 2 \text{ km s}^{-1}$ for this parameter range. Note that the $\chi^2_\nu$ minima in Table 4.1 have values much larger than the expected value of unity. This is due to the inclusion of spectral regions where there is evident mismatch because of incomplete removal interstellar features, marginal differences in the continuum placement, and real differences between the observed and models even in the best fit cases. For the purposes of intercomparison in Figure 4.1, we have subtracted from each $\chi^2_\nu$ set the minimum minus one value, so that the figure represents variance increases that result only from changes in the assumed temperature and gravity.

The temperature and gravity properties of the $\chi^2_\nu$ fits shown in Figure 4.1 result primarily from the dependence of the spectral lines on the ionization levels in the gas. The Saha ionization equilibrium equation shows that the number ratio of atoms of one ion to the next higher ionization state is equal to the electron density times a function that decreases with increasing temperature. Thus, in order to match the ionization ratios represented in the spectral line strengths, the best fits are found along a diagonal zone of increasing log $g$ with increasing temperature, i.e., increasing the electron density with log $g$ compensates for the functional drop related to temperature increase. However, there are other spectral dependences that help us determine the minimum $\chi^2_\nu$ position along the valley in the $(T_{\text{eff}}, \log g)$ diagram. In particular, the H Balmer line wings in the optical spectrum are sensitive to pressure broadening (linear Stark effect) and hence model gravity. The lowest contours of $\chi^2_\nu$ in Figure 4.1 indicate that the best fits are found for $T_{\text{eff}} = 26.5 - 28.5$ kK and log $g = 2.9 - 3.1$. Both of these estimates are lower than found in earlier studies (Canalizo et al. 1995; Herrero et al.}
Figure 4.1 The variation within the \((T_{\text{eff}}, \log g)\) plane of the net reduced \(\chi^2_{\nu}\) statistic that measures the goodness of fit between the solar abundance models and the observed spectrum of HD 226868. The contours show the value of \(\chi^2_{\nu}\) above the best fit minimum (arbitrarily set to one), and they nominally represent the intervals of \(2\sigma, 4\sigma, \ldots 10\sigma\) where \(\sigma\) is the error in the parameter estimate. The contours for the FUV, KPNO CF, and KPNO 4 m spectral fits are shown as different gray-shaded regions, solid lines, and dashed lines, respectively. The contours for \(\log g \leq 3.0\) are for fits with the BSTAR2006 models, while those for \(\log g \geq 3.0\) are for fits with the OSTAR2002 models.

1995; Karitskaya et al. 2008), but they are consistent with recent studies that demonstrate that the inclusion of line blanketing in stellar atmosphere models tends to lower the derived effective temperature (Repolust et al. 2004; Martins et al. 2005; Lefever 2007; Searle et al. 2008).

The run of \(\chi^2_{\nu}\) fits shown in Figure 4.1 are for solar abundance models, but given the reports that the spectrum of HD 226868 has strong N lines, we also explored fits with
CN abundance models included in the BSTAR2006 grid for gravities of log $g = 2.75$ and 3.00. These models assume a He to H number ratio of 0.2 (compared to 0.085 for the solar abundance models), a C abundance equal to one half the solar value, and a N abundance five times the solar abundance. These adjustments demonstrate the kind of changes that are expected when the atmosphere becomes enriched in CNO-processed gas. While the CN models do not improve the fits of the FUV spectrum, they are significantly better fits of the optical spectrum (where a number of strong He I and N III lines are present; see Figure 4.2 below). When we compare the $\chi^2$ fit of a solar model to a CN model at the same gravity in Table 4.1, the CN models fit better at higher temperature, especially for fits of the optical spectra. Following this trend, we estimate that the supergiant’s spectrum is fit best by the CN models with $T_{\text{eff}} = 28 \pm 2.5$ kK and log $g = 3.00 \pm 0.25$ dex. Thus, we will focus on these CN models for the rest of this section.

We compare the two mean optical spectra (CF and RC) with our best fit model spectrum in Figure 4.2 ($T_{\text{eff}} = 28$ kK, log $g = 3.00$) and with a marginally acceptable fit in Figure 4.3 ($T_{\text{eff}} = 26$ kK, log $g = 2.75$). Also shown for comparison is the spectrum of a similar O9.7 Iab star, $\mu$ Nor (HD 149038; from Walborn & Fitzpatrick 1990). The spectra appear in three plots: the short wavelength range from the lower resolution CF data is shown in the top panel while the bottom two panels illustrate the longer wavelength region from the higher resolution RC spectra (compare with Figure 4.1 in Karitskaya et al. 2008). In general the agreement between the observed and model spectrum is satisfactory. The largest discrepancies are seen in the He II $\lambda 4686$ and H$\beta$ lines where incipient emission from the stellar wind of the supergiant alters the profiles (Gies & Bolton 1986b; Ninkov et al. 1987a; Gies et al. 2003, 2008). The H Balmer line emission is strongest in H$\alpha$, and for simple estimates of the
Balmer decrement for Case B recombination (Osterbrock & Ferland 2006), we expect some measurable degree of wind emission for all the Balmer lines shown in Figures 4.2, 4.3. We find that the H line cores do appear shallower, while the Balmer line wings agree well with the models. Searle et al. (2008) observed this effect in other supergiants and they suggest that stellar wind emission from the outer atmosphere tends to fill in the line core. On the other hand, the Balmer line wings are formed in higher density gas, deeper in the photosphere where we expect the TLUSTY/SYNSPEC results to be quite reliable. The H line wings become narrower with lower gravity, and the predicted H profiles for the lower gravity model illustrated in Figure 4.3 appear to be significantly narrower than the observed ones.

The He I and He II line strengths are well matched in the He enriched CN model spectra. In particular the temperature sensitive ratio of He II λ4541 to Si III λ4552 (equal for the O9.7 classification) is better reproduced by the $T_{\text{eff}} = 28$ kK and $\log g = 3.00$ model (Figure 4.2) than the $T_{\text{eff}} = 26$ kK and $\log g = 2.75$ model (Figure 4.3). The N III λλ4097, 4379, 4510, 4514, 4630, 4634, 4640 lines are also reasonably well fit in the five times overabundant CN models. On the other hand, O lines like O II λλ4069, 4072, 4075, 4590, 4596 are too strong in the model spectra, which suggests that the O abundance should be revised downwards from solar values as expected for CNO-processed gas. The other differences between the observed and model spectra are related to the presence of interstellar features (Ca II λλ3933, 3968 in the CF spectrum; most of the deep ISM features were removed from the RC spectra).

In Figure 4.4 we present the averaged UV spectrum made with HST/STIS with the best TLUSTY/SYNSPEC model superimposed as a lighter line. Figure 4.4 also includes an average UV spectrum of μ Nor, based upon 34 high resolution, archival spectra from IUE.
Figure 4.2 Rectified optical spectra (dark line) together with a $T_{\text{eff}} = 28$ kK and $\log g = 3.0$ TLUSTY CN model (light line). The top panel shows the mean spectrum obtained with the KPNO Coudé Feed telescope while the bottom two panels show the mean spectrum obtained with the KPNO 4-m telescope. The spectrum of the O9.7 Iab star $\mu$ Nor is offset by 0.15 from the model and HD 226868 spectra for comparison. The horizontal lines below the spectra indicate the wavelength regions included in the $\chi^2$ calculation. The He II $\lambda_{4686}$ emission line originates in the focused wind from the star.

Horizontal line segments indicate those regions where the lines primarily originate in the photosphere, i.e., free from P Cygni stellar wind lines and from regions where interstellar lines were removed by interpolation (Gies et al. 2008). Overall, the line features in the observed UV spectrum agree well with the model UV spectrum based upon the optimal $T_{\text{eff}}$ and $\log g$ parameters derived from the optical and FUV spectral fits. Note that the He II $\lambda_{1640}$ feature appears in absorption as predicted, so there is no evidence of the Raman scattering emission that was observed by Kaper et al. (1990) in the massive X-ray binary 4U1700–37. There
are, however, a few specific regions where the match is less satisfactory. For example, the blends surrounding Fe $\text{V}$ $\lambda$ 1422 and Fe $\text{IV}$ $\lambda\lambda$1596, 1615 appear stronger in both the spectra of HD 226868 and $\mu$ Nor, which suggests that the models are underestimating the Fe line opacity in these wavelength regions. The S $\text{V}$ $\lambda$1502 line (Howarth 1987) has a strength in the spectrum of HD 226868 that falls between that of the model and of $\mu$ Nor. The deep feature near 1690 Å is an instrumental flaw near the edge of the detector at one grating tilt.

There are huge variations in the stellar wind lines between the orbital conjunctions that are due to X-ray ionization of the wind (Gies et al. 2008; Vrtilek et al. 2008), and it is possible that X-ray heating might also affect some of the photospheric lines. Figure 4.5 compares
Figure 4.4 Observed UV spectrum (dark line) plus a TLUSTY CN model spectrum for \( T_{\text{eff}} = 28 \text{ kK} \) and \( \log g = 3.0 \) (light line). The spectrum of the O9.7 Iab star \( \mu \) Nor is offset by 0.6 from the model and HD 226868 spectra for comparison. The horizontal lines below the spectra indicate regions included in the \( \chi^2 \) calculation.

The average UV spectra at the two conjunction phases \( \phi = 0.0 \) and 0.5 (inferior and superior conjunction of the supergiant, respectively). With the exception of the known wind line changes, we find that the spectra are almost identical between conjunctions. Some slight differences are seen in very strong features, such as the Si \( \text{iii} \) \( \lambda 1300 \) complex and the Fe \( 5 \) line blends in the 1600 – 1650 Å region. The deep lines appear somewhat deeper at \( \phi = 0.0 \) and have slightly extended blue wings compared to those observed at \( \phi = 0.5 \) (when the black hole is in the foreground). We speculate that the deeper cores and blue extensions result from line opacity that forms in the upper atmosphere where the outward wind acceleration
begins. This outer part of the atmosphere in the hemisphere facing the black hole may also experience X-ray ionization (like the lower density wind) that promotes Si and Fe to higher ionization levels and reduces the line opacity of the observed transitions.

Our spectral fits are all based upon the existing OSTAR2002 and BSTAR2006 grids, and it would certainly be worthwhile to explore more specific models, for example, to derive reliable estimates of the He and N overabundances. A determination of the He abundance in particular will be important for a definitive temperature estimate. It is also important in such an analysis to consider the full effects of the stellar wind in HD 226868. Herrero et al. (1995) compared analyses of the spectrum of HD 226868 from static, plane-parallel
models with unified, spherical models (that treat the photosphere and wind together), and they found their log $g$ estimate increased by about 0.2 dex (with no change in temperature) in the unified models. Thus, we suspect that our gravity estimate derived from the plane-parallel TLUSTY code is probably a lower limit (approximately consistent with the results of Herrero et al. 1995 and Karitskaya et al. 2005).

4.3 UV – IR Spectral Energy Distribution

We can use the derived model flux spectrum to fit the observed spectral energy distribution (SED) and reassess the interstellar extinction and the radius – distance relation. We collected the archival low dispersion $IUE$ spectra and combined these fluxes with the $HST$ spectra in wavelength bins spanning the FUV and NUV regions. We transformed the $UBV$ magnitudes from Massey et al. (1995) into fluxes using the calibration of Colina et al. (1996), and the near-IR fluxes were determined from a calibration of the 2MASS $JHK_s$ magnitudes (Cohen et al. 2003; Skrutskie et al. 2006). Then we fit the observed fluxes with the optimal BSTAR2006 flux model (CN model, $\xi = 10$ km s$^{-1}$, $T_{\text{eff}} = 28$ kK, log $g = 3.0$) to find the best reddening curve using the extinction law from Fitzpatrick (1999). We placed additional weight on the six optical and IR points to compensate for the larger number of UV points. Figure 4.6 shows the observed and best fit model fluxes for HD 226868 that we obtained with a reddening $E(B - V) = 1.11 \pm 0.03$ mag and a ratio of total to selective extinction $R_V = 3.02 \pm 0.03$. These values agree well with the previous reddening estimates that are collected in Table 4.2.

For comparison we examined the colors and reddening of six field stars within 10 arcminutes of HD 226868 in the sky. These stars were observed with the KPNO 4 m telescope
Figure 4.6 The spectral energy distribution of HD 226868 (plus signs) together and the TLUSTY best fit (solid line) for $T_{\text{eff}} = 28$ kK and $\log g = 3.0$. The UV points were binned from the average HST and IUE spectra. The three optical points are the $UBV$ measurements from Massey et al. (1995) and the three IR points are taken from 2MASS. Also shown is the extrapolation of the accretion disk flux model of Miller et al. (2002) (dotted line).

Table 4.2. Interstellar Reddening Estimates

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Table 4.3. Interstellar Reddening for HD 226868 and Nearby Stars

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<td>0.84</td>
</tr>
<tr>
<td>19 58 44.45</td>
<td>+35 08 09.8</td>
<td>F5 V</td>
<td>15.76</td>
<td>16.87</td>
<td>0.68</td>
<td>1.07</td>
</tr>
</tbody>
</table>

and RC spectrograph using the same blue region arrangement selected for our observations of HD 226868 (Gies et al. 2008). We made spectral classification of the stars, and then we used the observed $UBV$ colors from Massey et al. (1995) and the intrinsic color and absolute magnitude for the classification (Gray 1992) to estimate reddening and distances to these stars. Our results are collected in Table 4.3 with the reddening estimate from above listed for HD 226868. Bregman et al. (1973) estimated the distance of HD 226868 as $d \approx 2.5$ kpc, and set a lower limit of 1 kpc based upon the colors of other nearby field stars. We find that there are two stars at distances just under 1 kpc that have a similar reddening to that of HD 226868, which is consistent with a distance $d \gtrsim 1.0$ kpc.

The normalization of the fit to the SED yields the star’s limb-darkened angular diameter, $\theta = 96 \pm 6 \mu$as. Then we can calculate the luminosity and radius of the star as a function of distance $d$ (in kpc) to HD 226868,

$$\frac{L_1}{L_\odot} = (5.9 \pm 2.1) \times 10^4 d^2, \quad (4.2)$$

$$\frac{R_1}{R_\odot} = (10.3 \pm 0.7)d. \quad (4.3)$$

It is important to check that these SED results are not affected by long term or orbital flux variability, so we examined the archival $IUE$ low dispersion spectra (Gies et al. 2008) and
the HST spectra to determine the amplitude of any flux variations in the UV. We calculated the average continuum flux over three wavelength spans (1252 – 1380 Å, 1410 – 1350 Å, and 1565 – 1685 Å) that excluded the main wind features. We then converted the UV fluxes to differential magnitudes $\Delta m$. We found no significant differences between fluxes from times corresponding to the X-ray low/hard state (IUE) and high/soft state (IUE and HST; see Gies et al. 2008 for X-ray state information), nor were there any long term variations over the 25 year time span between the IUE and HST observations. On the other hand, we do find marginal evidence of the orbital flux variations related to the tidal distortion of the supergiant. We plot in Figure 4.7 the mean orbital flux variations of the three wavelength intervals for both IUE and HST spectra that are averaged into eight bins of orbital phase. For comparison we also include the $V$-band ellipsoidal light curve from Khaliullin & Khaliullina (1981). The UV and $V$-band light curves appear to have similar amplitudes, consistent with past estimates (Treves et al. 1980; van Loon et al. 2001). Note that the minima have approximately equal depths (consistent with the optical results; Balog et al. 1981), which suggests that there is little if any deep heating by X-rays of the hemisphere of the supergiant facing the black hole. Since the amplitude of the light curve is small and the average UV fluxes plotted in the SED in Figure 4.6 cover the full orbit, the ellipsoidal variations have a minimal impact on the quantities derived from the SED.

Finally, we need to consider if the SED has a non-stellar flux contribution from the accretion disk around the black hole or from other circumstellar gas. Bruevich et al. (1978) estimated that the disk contributes about 2% of the optical flux, and there are reports of small optical variations with superorbital periods that may correspond to the precession of the accretion disk (Kemp et al. 1987; Brocksopp et al. 1999; Szostek & Zdziarski 2007;
Figure 4.7 UV light curve from both the HST and IUE spectra (diamonds) compared with the V-band light curve (circles) from Khaliullin & Khaliullina (1981). The UV data were divided into eight orbital phase bins, and the error bars indicate the standard deviation within each bin.

Poutanen et al. 2008). Furthermore, Dolan (2001) observed rapid UV variations that he argued originate in dying pulse trains of infalling material passing the event horizon of the BH. Miller et al. (2002) developed a multi-color disk SED to model the X-ray continuum of Cyg X-1, and Dr. Miller kindly sent us the model fluxes extrapolated into the UV and optical. These are also plotted in Figure 4.6 after accounting for interstellar extinction. Both the photospheric and disk SEDs correspond to the Rayleigh-Jeans tail of a hot continuum, and the model predicts that the disk contributes approximately 0.01% of the total flux in the UV to IR range. This small fraction is consistent with our successful fitting of the UV and
optical line features that would otherwise appear shallower by flux dilution if the disk was a significant flux contributor. Thus, our SED fitting is probably unaffected by any non-stellar flux source.

4.4 Mass of the Supergiant

In this section we will explore the mass consequences of our relations for radius and luminosity as a function of distance. Paczyński (1974) derived model-independent, minimum mass estimates for both components as a function of distance based on the lack of X-ray eclipses (setting a maximum orbital inclination) and the assumption that HD 226868 is not larger than its Roche lobe (setting a lower limit on the ratio of the supergiant to black hole mass, $M_1/M_2$). We repeated his analysis using our revised radius – distance relationship (eq. [3]), stellar effective temperature $T_{\text{eff}} = 28$ kK, and current values for the mass function $f(m) = 0.251 \pm 0.007 \, \text{M}_\odot$ and period $P = 5.599829$ days (Gies et al. 2003). The resulting minimum masses are presented in Table 4.4 as a function of distance $d$.

We can make further progress by assuming the supergiant has attained synchronous rotation with the orbit since the stellar radius is probably comparable in size to the Roche radius (Gies & Bolton 1986b). We take the ratio of the star’s spin angular velocity and orbital angular velocity, $\Omega$, to be 1. Then the projected rotational velocity $V \sin i$ is related to the inclination $i$ by

$$V \sin i = \frac{2\pi}{P} R_1 \sin i$$

(4.4)

where $P$ is the orbital period. The projected rotational velocity, after correction for macro-turbulent broadening, is estimated to be $V \sin i = 95 \pm 6$ km s$^{-1}$ (Gies & Bolton 1986a; Ninkov et al. 1987b; Herrero et al. 1995). Inserting equation (4.3) for $R_1$ we obtain an
Table 4.4. Mass and Luminosity versus Distance for HD 226868

<table>
<thead>
<tr>
<th>$d$ (kpc)</th>
<th>$i$ (°)</th>
<th>$R_1$ ($R_\odot$)</th>
<th>log $L_1$</th>
<th>log $L_1^{1}(0.9)$</th>
<th>log $L_1^{1}(1.0)$</th>
<th>$M_1^{\text{min}}$ ($M_\odot$)</th>
<th>$M_1^{\text{sync}}(0.9)$ ($M_\odot$)</th>
<th>$M_1^{\text{sync}}(1.0)$ ($M_\odot$)</th>
<th>$M_2^{\text{min}}$ ($M_\odot$)</th>
<th>$M_2^{\text{sync}}(0.9)$ ($M_\odot$)</th>
<th>$M_2^{\text{sync}}(1.0)$ ($M_\odot$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>67.5</td>
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<td>4.85</td>
<td>3.06</td>
<td>2.77</td>
<td>5.0</td>
<td>6.6</td>
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<td>4.5</td>
<td>7.7</td>
<td>6.6</td>
</tr>
<tr>
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<td>39.4</td>
<td>16.5</td>
<td>5.18</td>
<td>4.99</td>
<td>4.57</td>
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<td>6.05</td>
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<td>60.6</td>
<td>45.6</td>
<td>9.2</td>
<td>27.9</td>
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<tr>
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<td>51.9</td>
<td>9.9</td>
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<tr>
<td>2.5</td>
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<td>25.9</td>
<td>5.57</td>
<td>6.27</td>
<td>6.02</td>
<td>45.9</td>
<td>77.8</td>
<td>58.6</td>
<td>10.6</td>
<td>35.9</td>
<td>30.5</td>
</tr>
</tbody>
</table>
inclination estimate in terms of distance $d$ (kpc) of

$$i = \arcsin((1.02 \pm 0.09)/d).$$

(4.5)

These inclination estimates are given in column 2 of Table 4.4. Note that this argument suggests a lower limit to the distance of $\approx 1.0$ kpc, similar to that found by reddening considerations.

Gies & Bolton (1986a) showed that the mass ratio can be estimated from the ratio of the projected rotational velocity to the orbital semiamplitude of the supergiant,

$$\frac{V \sin i}{K} = \rho (Q + 1) \Phi(Q)$$

(4.6)

where $\rho$ is the fill-out factor, i.e., the ratio of volume equivalent radii of the star and Roche lobe, $Q = M_1/M_2$ is the mass ratio, $\Phi$ is the ratio of the Roche lobe radius to the semimajor axis (Eggleton 1983), and synchronous rotation is assumed. Thus, given the observed values of $V \sin i$ and $K$ and an assumed value of $\rho$, we can find the mass ratio and, with the inclination, the masses of each star. These masses are listed in Table 4.4 under columns where the fill-out factor is given in parentheses. The run of masses is also shown in Figure 4.8 that illustrates the mass solutions as a function of distance and fill-out factor. Loci of constant $\rho$ are denoted by dotted lines (increasing right to left from $\rho = 0.85$ to 1.0) while loci of constant distance (and inclination angle) are shown by dashed lines. The derived gravity values from these masses of $\log g \approx 3.3$ reinforces the idea that our spectral estimate of $\log g = 3.0$ is a lower limit (see section 4.2).

We assumed synchronous rotation in the relations above because both observations and theory indicate that the orbital synchronization time scale in close binaries is shorter than
the circularization time scale (Claret et al. 1995), and since the orbit is circular, it follows that the star must rotate at close to the synchronous rate. However, it is straightforward to see how the solutions will change if the synchronism parameter $\Omega$ differs from unity. In equation (5) the distance $d$ can be replaced by the product $\Omega d$, while in equation (6) the fill-out parameter $\rho$ can be replaced by $\Omega \rho$. If, for example, $\Omega = 0.95$, then the mass solutions can be obtained from Table 4.4 and Figure 4.8 by selecting a distance of $0.95d$ and a fill-out ratio of $0.95\rho$.

The other important constraint comes from the ellipsoidal light curve. The tidal distortion of the star results in a double-wave variation (Figure 4.7) whose amplitude depends on the inclination (maximal at $i = 90^\circ$) and degree of tidal distortion (maximal for fill-out $\rho = 1.0$). In order to determine which parts of mass plane are consistent with the observed variation, we constructed model $V$-band light curves using the GENSYN code (Mochnacki & Doughty 1972; Gies & Bolton 1986a) for the four values of fill-out factor illustrated in Figure 4.8. There is a unique solution for the best fit of the light curve along each line of constant fill-out factor, since the light curve amplitude monotonically decreases with decreasing inclination (increasing distance). The solid line in Figure 4.8 connects these best fit solutions (indicated by plus sign symbols). These light curve solutions differ slightly from those presented by Gies & Bolton (1986a) because we chose to fit the light curve from Khaliliullin & Khaliullina (1981) instead of that from Kemp et al. (1983), and the differences in the solutions reflect the uncertainties in the observed light curve.

There are several other constraints from hints about the mass transfer process, luminosity, and distance that can provide additional limits on the acceptable mass ranges. Both Gies & Bolton (1986b) and Ninkov et al. (1987a) presented arguments that the unusual $\text{He II} \lambda 4686$
emission in the spectrum of HD 226868 originates in a tidal stream or focused wind from the supergiant towards the black hole. Furthermore, Gies & Bolton (1986b) made radiative transfer calculations of the focused wind emission profiles for models of the asymmetric wind from Friend & Castor (1982), and they determined that the fill-out factor must exceed $\rho = 0.90$ in order to increase sufficiently the wind density between the stars to account for
the observed strength of the He II λ4686 emission. Thus, the presence of a focused wind implies that the fill-out factor falls in the range \( \rho = 0.9 - 1.0 \).

Paczyński (1974) and Ziolkowski (2005) argue that massive stars evolve at near constant luminosity, and, therefore, the best solutions will obey the observed mass – luminosity relation. Table 4.4 lists the derived luminosity as a function of distance (eq. [2]), \( \log L_1 \), plus the predicted luminosities for the mass solutions determined for the \( \rho = 0.9 \) and 1.0 cases, \( \log L_1^*(0.9) \) and \( \log L_1^*(1.0) \), respectively. These predictions are based upon the mass – luminosity relations for \( T_{\text{eff}} = 28 \text{ kK} \) stars from the model evolutionary sequences made by Schaller et al. (1992). We find that the observed and predicted luminosities match over the distance range of \( d = 1.7 \) (\( \rho = 0.9 \)) to 2.0 kpc (\( \rho = 1.0 \)), closer than the range advocated by Ziolkowski (2005) who adopted a higher temperature and hence higher luminosity. Note that some stars in mass transfer binaries appear overluminous for their mass, so these distances should probably be considered as upper limits.

Several authors have suggested that the position and proper motion of HD 226868 indicates that it is a member of the Cyg OB3 association (Mirabel & Rodriguez 2003) that has a distance of 1.6-2.5 kpc (Uyaniker et al. 2001). However, a radio parallax study by Lestrade et al. (1999) indicates a smaller (but possibly consistent) distance of \( 1.4^{+0.9}_{-0.4} \) kpc for Cyg X-1. Our fits of the ellipsoidal light curve suggest that the maximum allowable distance is \( d \approx 2.0 \) kpc (for \( \rho = 1.0 \)). The interstellar reddening indicates a distance of at least 1.0 kpc (section 4.3), which is probably consistent with the strength of interstellar Ca II lines. Megier et al. (2005) present a method for determining the distance to O supergiants using the equivalent width \( W_\lambda \) of the Ca II λ3933 feature. Using their calibration with the value of \( W_\lambda = 400 \pm 10 \) mA from Gies & Bolton (1986a) yields a distance \( d = 1.2 \) kpc. Since
Table 4.5. Mass Estimates

<table>
<thead>
<tr>
<th>Source</th>
<th>$M_1$ (M$_\odot$)</th>
<th>$M_2$ (M$_\odot$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Balog et al. (1981)</td>
<td>20 – 27</td>
<td>7 – 12</td>
</tr>
<tr>
<td>Gies &amp; Bolton (1986a)</td>
<td>23 – 38</td>
<td>10 – 20</td>
</tr>
<tr>
<td>Ninkov et al. (1987b)</td>
<td>20</td>
<td>10</td>
</tr>
<tr>
<td>Herrero et al. (1995)</td>
<td>17.8</td>
<td>10.1</td>
</tr>
<tr>
<td>Abubekerov et al. (2005)</td>
<td>22</td>
<td>8.2 – 12.8</td>
</tr>
<tr>
<td>Ziółkowski (2005)</td>
<td>30 – 50</td>
<td>13.5 – 29</td>
</tr>
<tr>
<td>Shaposhnikov &amp; Titarchuk (2007)</td>
<td>· · ·</td>
<td>7.9 – 9.5</td>
</tr>
<tr>
<td>This paper</td>
<td>17 – 31</td>
<td>8 – 16</td>
</tr>
</tbody>
</table>

the reddening of HD 226868 is approximately the same as that for the much more distant Cepheid, V547 Cyg (Bregman et al. 1973), the ISM must have a relatively low density beyond $\approx 1$ kpc along this line of sight through the Galaxy, so we suspect that the distance derived from the interstellar Ca II line is probably a lower limit.

All of these constraints are consistent with the mass solutions for a fill-out factor range of $\rho = 0.9 – 1.0$, and the corresponding mass ranges are listed in Table 4.5. We also list mass estimates from earlier investigations. Our downward revision of the effective temperature results in lower luminosity estimates than adopted by Ziółkowski (2005), and consequently, our mass estimates (based upon the light curve) are significantly lower than his mass estimates (based upon the mass – luminosity relation from models). In fact, the lower limit for the black hole mass now overlaps comfortably with the mass determined by Shaposhnikov & Titarchuk (2007) using the correlation between the X-ray quasi-periodic oscillation frequency and spectral index, so the apparent discrepancy in black hole mass estimates from X-ray and optical data is now resolved. If the X-ray derived mass is accurate, then the mass solution for fill-out factor $\rho = 0.91$ is preferred ($M_1 = 19M_\odot$ and the distance is $d = 1.6$ kpc).

Our analysis of the first high resolution UV spectra of HD 226868 and of the complementary optical spectra shows that the photospheric line spectrum can be matched
by adopting an atmosphere mixed with CNO-processed gas with an effective temperature $T_{\text{eff}} = 28.0 \pm 2.5 \text{kK}$ and $\log g \gtrsim 3.0 \pm 0.25$. Assuming synchronous rotation ($\Omega = 1$) and using the fill-out factor range from above, the mass of the supergiant ranges from $M_1 = 17 - 31 M_\odot$ and the black hole mass ranges from $M_2 = 8 - 16 M_\odot$. This corresponds to an inclination of $i = 31^\circ - 43^\circ$ and a distance of $d = 1.5 - 2.0 \text{kpc}$. Better estimates of the masses may be possible in the future. For example, both the GAIA (Jordan 2008) and SIM Lite (Unwin et al. 2008) space astrometry missions will provide an accurate parallax and distance. Furthermore, pointed observations with SIM Lite will measure the astrometric motion of the supergiant around the system center of mass, yielding independent estimates of both the orbital inclination and distance (by equating the astrometric and radial velocity semi-major axes; Tomsick et al. 2009). Finally, future high dispersion X-ray spectroscopy with the International X-ray Observatory\(^1\) will measure the orbital motion of the black hole through the orbital Doppler shifts of accretion disk flux in the Fe K\(\alpha\) line (Miller 2007). By comparing the optical and X-ray orbital velocity curves, we will have a secure mass ratio that, together with the distance estimate, will lead to unique and accurate mass determinations of the supergiant and black hole.

\(^1\)http://ixo.gsfc.nasa.gov/index.html
DISCUSSION

The course of this study has truly been the tale of two telescopes. Through the best of times and the worst of times, the development of new techniques for the analysis of our Hubble Space Telescope and the Gemini North Observatory observations has provided an initial snapshot of some of the massive binaries in Cygnus. The results form a basis from which future work can be done to confirm companions and determine system parameters. Here we discuss the implications of some of our results and the direction of future work.

5.1 NIRI Results

The Gemini infrared AO observations revealed a host of 26 new astrometric binaries in Cyg OB2, for future follow-up observations. We found stars near the resolution limit of the telescope (MT 429, \( \rho = 0'08 \)) and pushed the dynamic range to the limits (A 11, \( \Delta K = 10.2 \)). Immediate work entails a more robust analysis of the calibration and uncertainty of both the astrometric and photometric results.

The astrometric data ideally would be corrected using position information from other sources to create a map used for the co-addition of frames. In a more manageable approach, we envision a scheme in which we first transform the native \((x, y)\) positions from SExtractor to a uniform spatial grid that accounts for the known barrel distortion of the fields. Then, for stars observed in other studies, we will compare separations and position angles derived from an assumed pixel scale with published values to revise the pixel scale and rotational zero-point for both the AO lens in and out observations. We will then use the calibrations to derive new astrometric positions for the companions to those stars. Initial tests indicate
that this approach will significantly reduce the positional errors for those companions near the edge of the field of view. The calibration of observed stars will then be applied to the calibration of the position of stars without previously published measurements.

The uncertainties in the photometry involve further investigation of effects of the PSF variations in a frame. The apparent discrepancies become smaller when a larger flux inclusion aperture is used in SExtractor. This suggests that there are significant variations in the shape of the PSF with position in the field of view, and these cause a change in the ratio of flux in the core to flux in the halo of the PSF. Consequently, our photometric measurements that use the integrated flux in the core for a fixed aperture size are probably significantly affected by PSF variations. Comparison of our measurements with those from other studies will help to determine if there exists a dependence of the differential magnitude on separation and pixel position. In addition, comparison of all overlapping stars to determine the dependence on differential magnitude will help further constrain the uncertainties in the photometry. Finally, we need to make subset frames for all edge stars for more accurate differential photometry.

Once the uncertainty in the photometry is better understood, we can use the $J$, $H$ and $K$ observations to create a color-color plot to inspect color differences with respect to the primary stars. Recall that we found a significant number of very red objects that appear brighter in the $K$-band frames. Are these foreground red objects? Are they young, pre-main sequence stars, such as those found around the massive star $\sigma$ Orionis by Caballero (2005, 2007) (no relation to this author)? If they are young objects in Cyg OB2, this can help answer the question, which came first, the chicken or the egg? In this case we are referring to massive stars or low mass stars. Do massive stars trigger star formation of low mass stars,
or do low mass stars form first, before all the star forming material is shed by the high-winds of the forming, massive stars? Through a collaboration with scientists at Chandra, we hope to use X-ray detections of some of these objects to provide evidence that they are young, low mass objects. Spectroscopic observations could provide the decisive observations to eliminate their explanation as foreground objects.

This also brings up the question of the masses of the companions to massive stars. Are massive stars usually found in twin systems, like the stars of the planet Tatooine, or are massive stars equal-opportunists with companions of all sizes? A future study of the mass distribution combining the spectroscopic results of Kobulnicky et al. (2012) and our results will start to answer this question, keeping in mind the bias of both studies towards equal mass systems.

The arrangement of the stars around MT 421 is particularly interesting. They appear to form a straight line. It would be interesting to investigate if the filamentary structure seen in the massive star formation models of Bonnell & Bate (2005), Seifried et al. (2012), and Naranjo-Romero et al. (2012) would produce stars in a line like those of MT 421.

Second epoch observations and proper motion studies will be able to determine if some of these companions are bound or not. Also looking for binaries among the fainter stars will be of broader interest for overall multiplicity properties, not just those of massive stars. There is still a lifetime (and more!) of work to be done in the study of multiplicity properties of Cyg OB2.
5.2 FGS Results

The complementary nature of the FGS and NIRI surveys allows us to start filling in the observational gap in the period distribution of massive binaries found by Mason et al. (1998). With the FGS, we found systems with periods in the range of $20 < P < 20,000$ yrs. The lower limit may be increased through the further investigation of the marginally resolved binaries in Table 3.5. For the closer systems, especially MT 304, these new detections are important for the interpretation of the spectra and luminosity measurements of these stars.

The availability of the NIRI data to help interpret some of the FGS observations has led us to develop a robust process for detecting binaries with the FGS and fitting systems with more than two components. We will apply the methods outlined in this paper to a larger sample of Galactic O-stars spanning different environments.

5.3 Cyg X-1

The greatest uncertainty in determining the parameters of Cyg X-1 is the distance. Over the distance range we consider in Table 4.4 the masses of the star and BH vary by factors of 2.3 and 11.7, respectively. In subsequent work, Orosz et al. (2011) were able to put additional constraints on the parameters of the system thanks to a radio, trigonometric parallax measurement for the distance (Reid et al. 2011). Based on our temperature and gravity analysis, they estimate log $g = 3.30$-3.45 and $30,000 \, \text{K} \leq T_{\text{eff}} \leq 32,000 \, \text{K}$ for the supergiant. They were then able to further constrain the masses for the system to $M_{\text{opt}} = 19.16 \pm 1.90 \, M_\odot$ and $M = 14.81 \pm 0.98 \, M_\odot$ for the O-star and black hole, respectively. With the improved distance measurement to Cyg X-1, other system parameters, such as black hole
spin, will be better constrained, and such studies will enrich our knowledge of this complex system.

5.4 Everything Before Us

As Charles Dickens once wrote in his epic novel *A Tale of Two Cities*, “we ha[ve] everything before us.” The work presented here provides the foundation for future work on massive star multiplicity. One day we will be able to measure the system parameters in a similar analysis to that done with Cygnus X-1. Understanding the fundamental properties of massive binaries will lead to understanding both the births and deaths of some of the most influential stars in our universe.
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A.1 NIRI IRAF Reduction Tools

Here we provide the necessary reduction documentation used on the NIRI observations. REDUCTION.NOTES provides step by step instructions on how to reduce the NIRI data.

NIRI Reduction Process

SCRIPTS NEEDED:
-reduce.script
-d1.cl
-gemtweakdq.cl
-nirinoise.py
-nirinoise.script

SETUP

This assumes that the data is sorted according to star and then the filter used for the observation. If the star was observed on multiple nights, then that will be a subdirectory in the stars_filter folder.

1.- Start up an IRAF session
	xgterm &

2.- Image data (in IRAF)

Use hselect to find out information about the images. Use the command:

```bash
hselect data/*.fits[0] $I,title,filter1,filter2,filter3,obsclass,gcalshut yes > info.txt
```

to find out the information needed to create the lists for the following steps.

3.- Make subdirectories for each star, filter combination (e.g., mt456_j, a23_k, etc.)

4.- Copy target and calibration files into subdirectory.

All fits files can be found in the folder "data".

In some cases, a calibration from another star or night may be available.

```bash
files data/[].fits > list
ed list (select files)
copy @list MT417_K/ (destination directory)
cd MT417_K
dir
```

OR

On a command line (not in IRAF), type (not including [comments]):

```bash
for i in 1 2 3 4 5 [whatever numbers/variables are needed] [enter]
do [enter]
cp filename_$i.fits ../mt456_k [or whatever directory] [enter]
done [enter]
```

And wait. It can take a while, as the files are pretty big.
5.- Out of IRAF, you will need to make 3 lists:

- star.list #This contains the filenames of the frames of the object
  # (in info.txt, OBSCLASS = SCIENCE)
- flat.on.list #This is a list of the flats with the lamp on
  # (GCALSHUT = OPEN)
- flat.off.list #This is a list of the flats with the shutter closed
  # (GCALSHUT = CLOSED)

******************************
NIRNOISE.PY
******************************

We are going to use Python to run the python script nirinoise.py that
is used to remove the NIRI Noise Pattern. First, you need to make sure
that your machine has the NUMPY and PYFITS modules. You can check this
by typing the following in a terminal.

%python
>>>import numpy #or try import _numpy, if that works you will need to
>>>import pyfits #change that in nirinoise.py
>>>CTRL-d #to exit

If you type these and get no error message than go on to the steps on
running nirinoise.py. If you got an error message for one or both of
these, then you will need to go on the astronet support page
(www.chara.gsu.edu/support) and create a task. In the description, let
them know you need the NUMPY and PYFITS python modules and the name of the
machine you need them on. Once that has been taken care you can run the
nirinoise.py script.

1.- You are going to want to run nirinoise.py in C-shell. Make sure it is
   executable.

%csh
>cd path/star_filter
>source path/nirinoise.script

******************************
NIRI Reduction
******************************

1.- In your IRAF session load the gemini/niri packages and define the two tasks
called by the script. Also change to the folder where you have your data.

cl> gemini
gi>niri
ni>task dl = path/dl.cl
ni>task gemtweakdq = path/gemtweakdq.cl
ni>c path/mt259_k

2.- Make sure you have a ds9 window open with your IRAF session.

   cl>!ds9 &

3.- Edit reduce.script the outimage for imcoadd (at the bottom of the
    file, line 58) and set it to the name of your star_filter (i.e. mt259_k):

    ********
imcoadd rotrbn//@star.list outimage=mt259_k geofitgeom=shift fwhm=3.25 \ 
    ********

    Then just run the script and it will go through all the NIRI reduction
discussed in Emma Hogan’s website
(http://www.star.le.ac.uk/~eh54/data/niri.html) as well as:
- Creating another bad-pixel mask using gemcombine
- Using nresidual to flag saturated pixels from individual frames
- Using gemtweakdq to actually change the default badpixel mask of the
  individual frames from the first two.

   cl>c cl < path/reduce.script

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
The shell script NIRINOISE.SCRIPT runs the PYTHON script, NIRINOISE.PY, used to remove a characteristic noise pattern each individual frame. The noise pattern is further described at http://staff.gemini.edu/~astephens/niri/patternnoise/.

```bash
#!/bin/csh

This should link to path of your nirinoise.py
alias nirinoise.py /nfs/morgan/users/scaballero/ostars/niri/python/nirinoise.py

set list =(*.fits)
foreach file ($list)
    nirinoise.py -o c$file -f -b -c 0.5 $file;
end

exit 0
```

REDUCE.SCRIPT is the script used to run the IRAF reduction procedures for NIRI images.

```bash
#GN-2008A-Q-85 NIRI + Altair CR=FIXED Imaging
#Created lists:
# star.list - file with a list off all the data fits files
# flat.on - and flat.off.list

cat star.list flat.on.list flat.off.list > all.list

# Create a rough bad-pixel mask:
unlearn gemcombine
gemcombine c//@star.list test combine=median sci=1
display test[1] 1 zs- zr- z1=0 z2=500
imhist test[1] z1=-100 z2=500
imcopy test[1] temp
imreplace temp value=-999 lower=300 upper=INDEF
imreplace temp value=-999 lower=INDEF upper=0
imreplace temp value=0 lower=0 upper=300
imreplace temp value=1 lower=-999 upper=-999
imarith temp / 1 bpm.pl pixtype=integer
hedit bpm.pl title "Bad pixel mask (bad=1 good=0)" verify-
display bpm.pl 2 zs- zr- z1=0 z2=1

unlearn nprepare
nprepare in=c//@flat.on.list
nprepare in=c//@flat.off.list
nprepare in=c//@star.list bpm=bpm.pl
dl nc//@all.list ext=1

unlearn niflat
niflat lampson=nc//@flat.on.list lampsoff=nc//@flat.off.list flatfile=flat
dl flat ext=1

unlearn nresidual
```
## Procedure Description

DL.CL is an IRAF display task that uses SAO image display a list of files. This task is run after each reduction step to inspect each individual frame after each change is made.

### Procedure DL (images)

#### # 2001.02.13 - original version - Andrew W. Stephens - OSU Astronomy
#### # 2005.02.23 - extension
#### # 2007.03.21 - overlay DQ plane
#### # 2009.06.13 - add rawpath parameter

```plaintext
string images {"", prompt="Image list"}
int extension{1, prompt="Image extension"}
int buf {1, prompt="Buffer number"}
real delay {1, prompt="Delay between displaying images"}
bool imexam {no, prompt="Imexamine each image?"}
bool dq {yes, prompt="Overlay DQ plane as red mask?"}
real z1 {INDEF,prompt="Lowest pixel value to display"}
real z2 {INDEF,prompt="Highest pixel value to display"}
char rawpath {"", prompt="Path for input image if not included in name"}
```
bool silent {no, prompt="Silent mode"}
struct *imagelist
char sversion {"2009 Jul 18", prompt="Software version"}

begin

int n
string image
string bpmask = ""
string bpdisplay = "none"
string lastchar = ""
bool zscales = yes

string tmplist = "dl.list"
if (access(tmplist)) delete(tmplist)

if ( (z1 != INDEF) && (z2 != INDEF) ) { zscales = no }
if ( rawpath != "" ) {
    lastchar = substr(rawpath, strlen(rawpath), strlen(rawpath))
    if ((lastchar != "/") && (lastchar != "$")) rawpath = rawpath//"/
}

sections ( images, option="fullname", > tmplist )
imagelist = tmplist
n=0
while ( fscan(imagelist,image) != EOF ) {
    n = n+1
    image = rawpath // image
    print ("Image ",n," : ",image)

    if (dq) {
        bpmask = image//"[DQ]"
        bpdisplay = "overlay"
    }

    # nprepare sets the DQ flag to 4 if over the saturation limit and 2 if over the linearity limit
    display (image//"[/extension/]", buf,
            zscales=zscales, zrange-, z1=z1, z2=z2,
            bpmask=bpmask, bpdisplay=bpdisplay, bpcolors="red,2=209")

    if (imexam) imexamine (keeplog=no)

    sleep (delay)
}
if (access(tmplist)) delete(tmplist)

end

GEMTWEAKDQ.CL is a task that flags low pixel values or negative valued pixels as poor. This is used for the combining process to exclude pixels of poor quality when combining each frame.

GEMTWEAKDQ.CL is a task that flags low pixel values or negative valued pixels as poor. This is used for the combining process to exclude pixels of poor quality when combining each frame.
# Tweak values in the data quality extension
#
# 2009.08.21 - Andrew W. Stephens

string images {"", prompt="Image list"}
real z1 {INDEF, prompt="Lowest pixel value to mark as bad"}
real z2 {INDEF, prompt="Highest pixel value to mark as bad"}
int value {1, prompt="Value to insert into DQ extension"}
bool display {yes, prompt="Display image?"}
real delay {1, prompt="Delay between displaying images"}
char raopath {"", prompt="Path for input image if not included in name"}
bool verbose {no, prompt="Verbose (debugging) output"}
struct *scanfile {"", prompt="Internal use only"}
char sversion {"2009 Aug 21", prompt="Software version"}

begin

int n
char image, expr
char oldbpm, newbpm, tmplist

oldbpm = mktemp ("gentweakdq.oldbpm.")
newbpm = mktemp ("gentweakdq.newbpm.")
tmplist = mktemp ("gentweakdq.list.")

if (z1==INDEF && z2==INDEF) {
  print ("ERROR: at least one cut level (z1 or z2) must be specified")
  goto clean
}
if (z1 == INDEF) z1 = -9e9
if (z2 == INDEF) z2 = 9e9
if (verbose) print ("...z1 = ", z1, " z2 = ", z2)

sections (images, option="fullname", > tmplist) ###ORIGINAL
scanfile = tmplist
n=0
while ( fscan(scanfile, image) != EOF ) {  
  n = n+1
  print ("Image ",n, ": ", image)
  gimverify (image)
  if (gimverify.status != 0) gimverify (rawpath//image)
  if (gimverify.status == 1) {
    print ("ERROR: Cannot access image " // image)
    goto clean
  } else if (gimverify.status != 0) {
    print ("ERROR: Image " // image // " is not a MEF file")
    goto clean
  }
  image = gimverify.outname // ".fits"
imcopy (image//"[DQ]", oldbpm, verbose=verbose)
fxdelete (image//"[3]", verbose=verbose)
expr = "(a>"//z1//" && a<"//z2//") ? "/value //" : b"
imexpr (expr, newbpm, image//"[SCI]", oldbpm, verbose=verbose)
fxinsert (newbpm, image//"[3]", group="", verbose=verbose)
nhedit (image//"[3]", "EXTNAME", "DQ", comment="Extension name",
update+, add+, verify-, show=verbose)
nhedit (image//"[3]", "EXTVER", 1, comment="Extension version",
update+, add+, verify-, show=verbose)
nhedit (image//"[3]", "INHERIT", "F", comment="Inherits global header",
update+, add+, verify-, show=verbose)

if (display) {
  if (verbose) print ("displaying image...")
  display (image//"[SCI]", 1, bpmask=image//"[DQ]",
  bpdisplay="overlay", bpcolors="red,2=209")
sleep (delay)
}
imdelete (oldbpm, verify-)
imdelete (newbpm, verify-)
}

print ("Done.")

clean:
  if (access(oldbpm)) imdelete (oldbpm, verify-)
  if (access(newbpm)) imdelete (newbpm, verify-)
  if (access(tmplist)) delete (tmplist, verify-)
end

#%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

A.2 NIRI K-band Images

Here we present the NIRI K-band images for all 75 stars in our sample. Each figure consists of two panels. The top, smaller panel is a low contrast image of the central star to show any bright, close companions. The larger image is the complete, high contrast co-added image to bring out the fainter stars. The length of the direction arrows corresponds to 1 ″, with East being 90° counter-clockwise from North.
Figure A.1 A 11 K-band image
Figure A.2 A 12 $K$-band image
Figure A.3 A 15 K-band image
Figure A.4 A 18 $K$-band image
Figure A.5 A 20 K-band image
Figure A.6 A 23 $K$-band image
Figure A.7 A 24 K-band image
Figure A.8 A 25 $K$-band combined from 2 different nights
Figure A.9 A 26 $K$-band image
Figure A.10 A 27 K-band image
Figure A.11 A 29 $K$-band image
Figure A.12 A 32 $K$-band image
Figure A.13 A 37 $K$-band image
Figure A.14 A 38 K-band image
Figure A.15 A 41 $K$-band image
Figure A.16 A 46 K-band image
Figure A.17 B 17 K-band image
Figure A.18 MT 5 $K$-band image combined using GEMCOMBINE.
Figure A.20 MT 70 $K$-band image
Figure A.21 MT 83 $K$-band image
Figure A.22 MT 138 K-band image
Figure A.23 MT 140 $K$-band image
Figure A.24 MT 145 $K$-band image
Figure A.25 MT 213 K-band image
Figure A.26 MT 217 K-band image
Figure A.27 MT 227 $K$-band image
Figure A.28 MT 250 $K$-band image
Figure A.29 MT 258 K-band image
Figure A.30 MT 259 K-band image
Figure A.31 MT 299 K-band image
Figure A.32 MT 304 $K$-band image combined using GEMCOMBINE.
Figure A.33 MT 317 K-band image
Figure A.34 MT 339 K-band image
Figure A.35 MT 376 $K$-band image
Figure A.36 MT 390 K-band image
Figure A.37 MT 403 $K$-band image
Figure A.38 MT 417 K-band image
Figure A.39 MT 421 K-band image
Figure A.40 MT 429 $K$-band image combined using GEMCOMBINE.
Figure A.41 MT 448 K-band image
Figure A.42 MT 455 $K$-band image
Figure A.43 MT 457 K-band image
Figure A.44 MT 462 K-band image
Figure A.45 MT 465 $K$-band image
Figure A.46 MT 470 $K$-band image
Figure A.47 MT 473 $K$-band image
Figure A.48 MT 480 $K$-band image
Figure A.49 MT 483 $K$-band image
Figure A.50 MT 485 $K$-band image
Figure A.51 MT 507 K-band image
Figure A.52 MT 516 $K$-band image combined using GEMCOMBINE.
Figure A.53 MT 531 $K$-band image
Figure A.54 MT 534 $K$-band image
Figure A.55 MT 555 $K$-band image
Figure A.56 MT 556 $K$-band image
Figure A.57 MT 588 $K$-band image
Figure A.58 MT 601 $K$-band image
Figure A.59 MT 605 $K$-band image
Figure A.60 MT 611 $K$-band image
Figure A.61 MT 632 K-band image
Figure A.62 MT 642 $K$-band image
Figure A.63 MT 692 $K$-band image
Figure A.64 MT 696 K-band image
Figure A.65 MT 716 $K$-band image
Figure A.66 MT 734 $K$-band image
Figure A.67 MT 736 $K$-band image
Figure A.68 MT 745 K-band image
Figure A.69 MT 771 K-band image
Figure A.70 MT 793 K-band image
Figure A.71 Schulte 3 K-band image
Figure A.72 Schulte 5 $K$-band image
Figure A.73 Schulte 73 $K$-band image
Figure A.74 WR 145 $K$-band image
A.3 Photometric and Astrometric Results

Here we present the position and photometric results of our survey of 75 O- and B-type stars in Cygnus OB2 observed at Gemini North. We list the target star followed by the other stars found in the field of view ordered according to distance from the target star. The separations and position angles, in arcseconds and degrees, respectively, were determined with respect to the target stars. We list the Right Ascension and Declination (J2000) determined from the World Coordinate System information included in the image header. The relative magnitudes in $J$- and $K$-bands were also determined with respect to the target stars. The values followed by a superscript E denote the stars whose magnitude errors were determined as edge frame stars. This is described further in section 2.3.2. Also reported here is the probability of chance alignment determined from equation 2.2. In the last column we make note of any stars that have previously been identified in other studies with the corresponding component designations. Also noted are the stars observed in multiple frames, referring the reader to the other frame. Finally, the close companions whose magnitudes were determined using FITSTARS are denoted with an FS.
Table A.1: List of Stars Detected in Sample

<table>
<thead>
<tr>
<th>Name</th>
<th>( \rho ) (( ^{\prime} ))</th>
<th>( \theta ) (( ^{\circ} ))</th>
<th>R.A. (HH:MM:SS)</th>
<th>Dec. (DD:MM:SS)</th>
<th>( \Delta J )</th>
<th>( \Delta K )</th>
<th>( P_{unrelated} )</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>A 11</td>
<td>( \cdots )</td>
<td>( \cdots )</td>
<td>20:32:31.539</td>
<td>+41:14:08.22</td>
<td>( \cdots )</td>
<td>( \cdots )</td>
<td>( \cdots )</td>
<td>( \cdots )</td>
</tr>
<tr>
<td></td>
<td>1.02</td>
<td>286.0</td>
<td>20:32:31.451</td>
<td>+41:14:08.50</td>
<td>( \cdots )</td>
<td>7.02 ± 0.83</td>
<td>4.0E-4</td>
<td>( \cdots )</td>
</tr>
<tr>
<td></td>
<td>1.31</td>
<td>176.0</td>
<td>20:32:31.547</td>
<td>+41:14:06.90</td>
<td>( \cdots )</td>
<td>8.42 ± 0.83</td>
<td>5.2E-3</td>
<td>( \cdots )</td>
</tr>
<tr>
<td></td>
<td>2.25</td>
<td>277.2</td>
<td>20:32:31.341</td>
<td>+41:14:08.50</td>
<td>( \cdots )</td>
<td>4.19 ± 0.83</td>
<td>2.0E-3</td>
<td>( \cdots )</td>
</tr>
<tr>
<td></td>
<td>3.74</td>
<td>104.2</td>
<td>20:32:31.861</td>
<td>+41:14:07.29</td>
<td>( \cdots )</td>
<td>10.17 ± 0.83</td>
<td>9.9E-2</td>
<td>( \cdots )</td>
</tr>
<tr>
<td></td>
<td>5.38</td>
<td>195.4</td>
<td>20:32:31.411</td>
<td>+41:14:03.03</td>
<td>( \cdots )</td>
<td>6.04 ± 0.87</td>
<td>4.3E-2</td>
<td>( \cdots )</td>
</tr>
<tr>
<td></td>
<td>6.02</td>
<td>179.7</td>
<td>20:32:31.541</td>
<td>+41:14:02.20</td>
<td>( \cdots )</td>
<td>8.70 ± 0.83</td>
<td>1.8E-1</td>
<td>( \cdots )</td>
</tr>
<tr>
<td>A 12</td>
<td>( \cdots )</td>
<td>( \cdots )</td>
<td>20:33:38.219</td>
<td>+40:41:06.41</td>
<td>( \cdots )</td>
<td>( \cdots )</td>
<td>( \cdots )</td>
<td>( \cdots )</td>
</tr>
<tr>
<td></td>
<td>5.96</td>
<td>239.2</td>
<td>20:33:37.768</td>
<td>+40:41:03.35</td>
<td>( \cdots )</td>
<td>7.72 ± 0.83</td>
<td>8.1E-2</td>
<td>( \cdots )</td>
</tr>
<tr>
<td></td>
<td>9.67</td>
<td>255.3</td>
<td>20:33:37.396</td>
<td>+40:41:03.96</td>
<td>( \cdots )</td>
<td>8.67 ± 0.83</td>
<td>2.8E-1</td>
<td>( \cdots )</td>
</tr>
<tr>
<td>A 15</td>
<td>( \cdots )</td>
<td>( \cdots )</td>
<td>20:31:36.909</td>
<td>+40:59:09.25</td>
<td>( \cdots )</td>
<td>( \cdots )</td>
<td>( \cdots )</td>
<td>( \cdots )</td>
</tr>
<tr>
<td></td>
<td>5.32</td>
<td>79.9</td>
<td>20:31:37.372</td>
<td>+40:59:10.18</td>
<td>( \cdots )</td>
<td>8.89 ± 0.83</td>
<td>1.6E-1</td>
<td>( \cdots )</td>
</tr>
<tr>
<td></td>
<td>12.87</td>
<td>261.2</td>
<td>20:31:35.785</td>
<td>+40:59:07.29</td>
<td>( \cdots )</td>
<td>7.75E± ± 0.91</td>
<td>4.5E-1</td>
<td>( \cdots )</td>
</tr>
<tr>
<td></td>
<td>14.10</td>
<td>350.9</td>
<td>20:31:36.713</td>
<td>+40:59:23.17</td>
<td>( \cdots )</td>
<td>6.39E± ± 0.91</td>
<td>3.3E-1</td>
<td>( \cdots )</td>
</tr>
<tr>
<td>A 18</td>
<td>( \cdots )</td>
<td>( \cdots )</td>
<td>20:30:07.879</td>
<td>+41:23:50.47</td>
<td>( \cdots )</td>
<td>( \cdots )</td>
<td>( \cdots )</td>
<td>( \cdots )</td>
</tr>
<tr>
<td></td>
<td>4.26</td>
<td>190.4</td>
<td>20:30:07.810</td>
<td>+41:23:46.28</td>
<td>( \cdots )</td>
<td>6.94 ± 0.83</td>
<td>9.3E-2</td>
<td>( \cdots )</td>
</tr>
<tr>
<td></td>
<td>4.54</td>
<td>99.2</td>
<td>20:30:08.278</td>
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<td></td>
</tr>
<tr>
<td>11.70</td>
<td>269.2</td>
<td>20:31:57.463</td>
<td>+41:13:20.89</td>
<td>6.68 ± 0.89</td>
<td>6.92 ± 0.83</td>
<td>1.8E-1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>S 5</td>
<td>...</td>
<td>...</td>
<td>20:32:22.431</td>
<td>+41:18:19.10</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>S 5A</td>
</tr>
<tr>
<td>0.95</td>
<td>55.5</td>
<td>20:32:22.500</td>
<td>+41:18:19.64</td>
<td>...</td>
<td>3.37 ± 0.83</td>
<td>1.9E-5</td>
<td>S 5B</td>
<td></td>
</tr>
<tr>
<td>5.68</td>
<td>226.1</td>
<td>20:32:22.067</td>
<td>+41:18:15.15</td>
<td>...</td>
<td>7.13 ± 0.83</td>
<td>1.1E-2</td>
<td>S 5D</td>
<td></td>
</tr>
<tr>
<td>S 73</td>
<td>...</td>
<td>...</td>
<td>20:34:21.929</td>
<td>+41:17:01.60</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td></td>
</tr>
<tr>
<td>0.57</td>
<td>118.1</td>
<td>20:34:21.974</td>
<td>+41:17:01.33</td>
<td>...</td>
<td>4.63 ± 0.83</td>
<td>5.7E-4</td>
<td>FS</td>
<td></td>
</tr>
<tr>
<td>2.93</td>
<td>65.5</td>
<td>20:34:22.166</td>
<td>+41:17:02.82</td>
<td>...</td>
<td>8.29 ± 0.83</td>
<td>6.0E-2</td>
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<td></td>
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<tr>
<td>3.33</td>
<td>107.3</td>
<td>20:34:22.212</td>
<td>+41:17:00.61</td>
<td>...</td>
<td>7.30 ± 0.83</td>
<td>5.0E-2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4.54</td>
<td>21.2</td>
<td>20:34:22.075</td>
<td>+41:17:05.84</td>
<td>...</td>
<td>8.65 ± 0.83</td>
<td>1.5E-1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4.80</td>
<td>50.1</td>
<td>20:34:22.257</td>
<td>+41:17:04.68</td>
<td>...</td>
<td>8.10 ± 0.83</td>
<td>1.3E-1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5.62</td>
<td>28.6</td>
<td>20:34:22.169</td>
<td>+41:17:06.54</td>
<td>...</td>
<td>8.24 ± 0.83</td>
<td>1.9E-1</td>
<td></td>
<td></td>
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<tr>
<td>6.76</td>
<td>42.3</td>
<td>20:34:22.333</td>
<td>+41:17:06.60</td>
<td>...</td>
<td>7.31 ± 0.83</td>
<td>1.6E-1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8.62</td>
<td>185.4</td>
<td>20:34:21.857</td>
<td>+41:16:53.02</td>
<td>...</td>
<td>4.92 ± 0.83</td>
<td>7.3E-2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8.62</td>
<td>285.2</td>
<td>20:34:21.191</td>
<td>+41:17:03.87</td>
<td>...</td>
<td>7.58 ± 0.83</td>
<td>3.1E-1</td>
<td></td>
<td></td>
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<tr>
<td>9.17</td>
<td>268.6</td>
<td>20:34:21.116</td>
<td>+41:17:01.38</td>
<td>...</td>
<td>6.88 ± 0.83</td>
<td>2.4E-1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>11.73</td>
<td>9.4</td>
<td>20:34:22.101</td>
<td>+41:17:13.18</td>
<td>...</td>
<td>6.94 ± 0.83</td>
<td>3.9E-1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>12.73</td>
<td>166.7</td>
<td>20:34:22.189</td>
<td>+41:16:49.21</td>
<td>...</td>
<td>6.23 ± 0.83</td>
<td>2.8E-1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>WR 145</td>
<td>...</td>
<td>...</td>
<td>20:32:06.289</td>
<td>+40:48:29.54</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td></td>
</tr>
<tr>
<td>12.74</td>
<td>80.8</td>
<td>20:32:07.397</td>
<td>+40:48:31.57</td>
<td>...</td>
<td>8.52 ± 0.83</td>
<td>4.5E-1</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
B.1 FGS Reduction and Analysis Tools

Here we present the tools developed and used for the reduction and analysis of the FGS observations. The first section provides a copy of the documentation about the reduction pipeline on the Georgia State University Mac workstation named NELAN. This is a step-by-step set of instructions that describes the procedures from acquiring the data to fitting binaries using BINARY_FIT. The process is summarized in section 3.2.1.
FGS Data Reduction Tutorial

(Last updated 12-Apr-12)

§1 DATA RETRIEVAL:

1. The FGS data can be found at the MAST website: http://archive.stsci.edu

2. For non-proprietary data you may retrieve it anonymously
   Username = anonymous, Password = your e-mail
   or create an account
   http://archive.stsci.edu/registration/registration_form.html
   You must register to retrieve proprietary data and the PI needs to notify the
   MAST archive help desk (archive@stsci.edu) giving the account name for
   authorization.

3. From the drop down menu of Mission_Search choose Hubble.

4. To retrieve all your data enter your Proposal ID (e.g. 11943). Otherwise, you
   can use any of the other search criteria.
   a. Each row in the output corresponds to a separate observation of the
      target. The two main columns to look at are the Dataset and the Target
      Name.
   b. Taking the first Dataset as an example: FB9C2W01M
      F   - Stands for the instrument used, in this case FGS.
      B9C - Is unique to the proposal ID, in this case 11943.
      2W  - Is a base 36 number corresponding to the visit number.
      01  - Is the exposure number. Multiple exposures are due to long
           and short scans.
      M   - Means that the data is merged, which is the case for all FGS
           data.

5. Once you mark the appropriate rows that you would like retrieve or click on
   Mark All, click Submit marked data for retrieval from STDADS.

6. For username and password see Step 2 above. Use the Delivery Option
   STAGE: Put the data onto the Archive staging Disk. After you click on Send
   Retrieval request to ST-DADS you will receive an e-mail with your request
   number. You will receive another e-mail notifying you when your data is
   available to download via FTP.

7. In a terminal window, go to the directory where you want to store the raw data
   files (/Users/fgs/fgsdata/<Proposal ID>). FTP into the archive
   %ftp archive.stsci.edu
   and use your account information. (see Step 2)

8. Change to the directory where your data is stored.
   ftp>cd /stage/<user name>/<request number>

9. Transfer the files from the archive onto your computer.
   ftp>mget *fits
   mget <filename>.fits [anpqy]? a

10. Logout ftp>bye.
§2 PRE-REDUCTION SETUP:

- **Calibrator List**
  1. Change to the directory with the master list for targets
    ```
    cd /Users/fgs/fgsdata/Calibrators
    ```
  2. Open the file `cal_list.dat` in your preferred editor. This list keeps track of relevant information with respect to targets in the proposal 11943 and 11944. This is meant as a master list to look for possible calibrators when fitting binaries.
  3. Update the **Owner** column for your target with a 3-letter ID
     Example: Wei-Chun Jao = WCJ
  4. Update the color information where applicable (columns V, B-V, V-I, V-K)
  5. After reducing the data (section §3) you will come back and update the **binary**, **Reducer**, **Redc-Date** and **Note**.
     - Currently the **binary** column is based on preliminary results from Ed Nelan, where “0” means single and “1” means double or more.
     - **Reducer** is a three letter ID used to keep track of who reduced the data.
     - **Redc-Date** is the date when the data is fully reduced. Format is YYYY-MM-DD.
     - **Note** use this column to make a note of any problems with the data or during the reduction process.
  6. Make sure to save any changes you may make to the file and close it when done.

- **Sorting the data**
  1. After updating the Calibrator directory run the `sort.data` script.
    ```
    sort.data
    ```
    This script creates a new directory and copies the data you identified in the **Owner** column of `cal_list.dat` to it.
  2. Change to the newly created directly and reduce the data from here.
    ```
    cd ~/fgsdata/<yourname>.data.dir
    ```
  3. To be able to use the clean up script at the end of the reduction, each target should have its own directory and each visit should be either in its own directory or subdirectory.
§3 TRANSFER MODE REDUCTION:

- IRAF and STSDAS
  1. Run IRAF in the terminal. (NOTE: login.cl is in /Users/fgs/iraf/ directory.)
     ```bash
     %> cl
     ```
  2. Load the STSDAS package.
     ```bash
     ecl> stdas
     stdas> set imtype=hhh
     ```
  3. Change to the correct directory.
     ```bash
     stdas> cd ../fgsdata/<yourname>.data.dir
     ```
  4. Use the fits reader to make GEIS files out of the fits files.
     ```bash
     stdas> strfits *fits
     IRAF filename: <Hit Enter>
     ```
  5. Exit IRAF.
     ```bash
     stdas> logout
     ```

This routine creates 2 sets of files for each FGS:

- `<filename>.a1d` & `<filename>.a1h`
  - `l` - corresponds to FGS1r. This is the nominal science FGS, or the astrometer.
  - `d` - stands for data file.
  - `h` - stands for header file.
- `<filename>.trl`
- `<filename>_cvt.dmh`

See the FGS Data Handbook Section 2.1.1 for a more detailed description.

The figure below shows an example of the output from STRFITS.
IRAF and STSDAS

Welcome to IRAF. To list the available commands, type ? or ?? . To get detailed information about a command, type help <command> . To run a command or load a package, type its name . Type 'bye' to exit a package, or 'logout' to get out of the CL . Type 'news' to find out what is new in the version of the system you are using.

Visit http://iraf.net if you have questions or to report problems.

The following commands or packages are currently defined:

apropos  images,  noac,  proto,  system,  
dataio,  language,  obsdate,  software,  tables,  
dbase,  lists,  plot,  stsdas,  utilities,  
ecl>

--- Space Telescope Science Data Analysis System ---
  STSDAS Version 2.9
---
  Space Telescope Science Institute, Baltimore, Maryland
  Copyright (C) 2003 Association of Universities for
  Research in Astronomy, Inc. (AURA)
  See stsdas/COPYRIGHT, stsdas for terms of use.
  For help, send e-mail to help@stsci.edu
---

stsdas> set imtype = hhh
stsdas> cd ../fgsdata/test
stsdas> strfits *fits
stsdas> "fits_file  IRAFNAME Dimensions   BP DATE   OBJECT
        f89r0101m_t1.f1 f89r0101m_t1   733987  8  2006-12-22 L311145
        f89r0101m_a2.f1 f89r0101m_a2   733987  32  2006-12-22 L311145
        f89r0101m_a3.f1 f89r0101m_a3   733987  32  2006-12-22 L311145
        f89r0101m_d.mf f89r0101m_7.mw   8  2006-12-22 L311145
        f89r0101m_tr1.f1 f89r0101m_tr1  8  2006-12-22

        f89r0101m_tr1  132  5  Nmode= 1
fits Error reading record 1
AFTER RTF_READ_FITS
ERROR: RTF_READ_HEADER: Not a FITS file
stsdas> lsn
[nelan:"/iraf] fgs% cd ../fgsdata/test/
[nelan:"/fgsdata/test] fgs%  

FGS PLOT

1. For a preliminary look at the data, use the FGS PLOT tool.

   \texttt{%>fgsplot}

   FGS PLOT provides many options to view the data. The following steps will only describe how to view a coadded image. For information on the acquisition path, see the FGS Data Handbook Section 1.3.

2. For the following steps I will use as an example the data in \texttt{/Users/fgs/fgsdata/test}. This is data for the star LB11146 from the Proposal ID 11944. FGS1r was used as the astrometer. When you are prompted, enter the header filename for the astrometer data file.

   \texttt{enter header file name: fb9r0101m.a1h}

3. To see the actual scans use the \texttt{(zx)} and/or \texttt{(zy)} options. These images are coadded, but are not shifted.

4. Hit enter for defaults for scan numbers used in averaging, First sample and Last Sample.

5. Use a binning number of 2. or 3.

6. Hit enter for the sub scan length, or enter a value smaller than the measured scan length.

7. For the graph title enter the star name or leave it blank. Then enter your preference for line type.

8. Hit enter to continue and use the default for the vertical scale.

9. You do not have to output the plot to a file.

10. Enter y if you wish to plot something else. This will return to the initial menu options. n will exit FGS PLOT.
FGSPLIT
[n李某"/fgdata/test] fgs fgsplot

enter header file name: fbgro16d.msh

naxis: 73398
cst_id: 1
mode: TRANSFER
obs_date: 2006 3 25:32:46

Data loaded. #bytes: 2055144
Flags located
data quality assessed

Table of graphics options:
Select the desired graphics option

(1) X vs. Y
(2) S-curve vs. x-axis (a) X (b) [PMTYA]
(3) S-curve vs. y-axis (c) Y (d) [PMTYB]
(4) R vs. X (e) point vs. time (f) [PMTKA + PMTKB]
(5) R vs. Y (g) S-curve (j) [PMTKA + PMTKB]
(6) x-position vs. time (e) point vs. time (i) [PMTKA + PMTKB]
(7) y-position vs. time (f) PMTKA (n) Theta A
(8) S vs. time (g) PMTKB (n) Theta B
(9) S vs. time
(10) [PMTKA + PMTKA] vs. time
(11) [PMTKA + PMTKB] vs. time
(12) PMTSUM vs. time

(a) co-added X s-curves
(b) co-added Y s-curves
(c) co-added X point counts
(d) co-added Y point counts
(e) co-added X,B point counts
(f) co-added X,Y,B point counts
(g) co-added X,A point counts
(h) co-added X,A,B point counts
(i) co-added X,A,B point counts

Your choice (? for help) -> zx

OBS mode -> TRANSFER

Flags are:
Search......................... 1008
Coarse Track.................... 1032
Coarse Track Data Valid........ 1512
Fine Lock....................... 3670
Fine Lock Data Valid........... -1
SSH......................... 3642

###################################
Scan 1: Start..... 4182 End..... 7821
Scan 2: Start..... 7382 End..... 11221
Scan 3: Start..... 10707 End..... 14521
Scan 4: Start..... 14030 End..... 13022
Scan 5: Start..... 17507 End..... 21429
Scan 6: Start..... 20906 End..... 24924
Scan 7: Start..... 24307 End..... 28320
Scan 8: Start..... 27702 End..... 31528
Scan 9: Start..... 31117 End..... 35026
Scan 10: Start..... 34514 End..... 38422
Scan 11: Start..... 57921 End..... 42922
Scan 12: Start..... 42304 End..... 46220
Scan 13: Start..... 45709 End..... 49613
Scan 14: Start..... 49106 End..... 53025
Scan 15: Start..... 52506 End..... 56421
Scan 16: Start..... 55907 End..... 59821
Scan 17: Start..... 59316 End..... 62711
Scan 18: Start..... 62705 End..... 65652
Scan 19: Start..... 66117 End..... 70026
Scan 20: Start..... 69516 End..... 73598

Maximum Number of Samples..... 73598

enter scan numbers to be included in averaging (RET=all) -> []
FGS PLOT

CBS node -> TRANSFER

Flags are:
Search.......................... 1008
Coarse Track.................... 1032
Coarse Track Data Valid........ 1512
Fine Lock Data Valid............ 2630
LOS................................ -1
Stop................................ -1
SSP............................... 3542
Star Presence False.............. -1
Maximum Number of samples ..... 73398

First sample (default = 1, -1 for TURBO) -> 
Last sample (default = 73398) ->
enter binning interval (in milli arc seconds) .... -> 2.

........................... measured scan length -> 0.96
........................... enter sub scan length ->

graph title ->
enter line type ([solid] [labels] or [bo]th) -> so

x_min -> -2.709
g_min -> -0.479
x_max -> -1.745
g_max -> 0.571

continue? ->
enter the vertical scale (0.8-default) ->
warnings this program uses gets(), which is unsafe,
output the plot to a file? ................. -> n

plot another? ([y], [n], or [zoom]) ->
• FGSPLOT

plot another? ([y], [n], or [z]oom) -> y

Table of graphics options:
Select the desired graphics option

(1) X vs. Y
(2) S-curve vs. x-axis (a) X (d) PMTYA
(3) S-curve vs. y-axis (e) Y (i) PMYB
(4) Xs vs. X (c) X-spline (j) PMTYA + PMYB
(5) Ys vs. Y (g) S-spline (k) PMYB + PMYB
(6) x-position vs. time (e) Time (l) PMYB
(7) y-position vs. time (f) PMTYA (n) Theta A
(8) Xs vs. time (g) PMYB (n) Theta B
(9) Ys vs. time
(10) [PMTYA + PMYB] vs. time
(11) [PMTYA + PMYB] vs. time (z) 1/0 Parameters

Flags are:
Search........................................ 1008
Coarse Track............................... 1092
Coarse Track Data Valid................. 3522
Fine Lock.................................. 3630
Fine Lock Data Valid..................... 3642

Maximum Number of Samples ......... 73398

Enter scan numbers to be included in averaging (RET=all) ->

035 node - > TRANSFER

Flags are:
Search........................................ 1008
Coarse Track............................... 1092
Coarse Track Data Valid................. 3522
Fine Lock.................................. 3630
Fine Lock Data Valid..................... 3642

Maximum Number of samples ......... 73398

First sample (default = 1, -1 for END) ->
Last sample (default = 73398) ->
Enter binning interval (in milli arc seconds) ....... 2.

First sample (default = 1, -1 for END) ->
Last sample (default = 73398) ->
Enter binning interval (in milli arc seconds) ....... 2.
FGSPLT

........................ measured scan length -> 0.94
........................ enter sub scan length ->

graph title ->
enter line type ([col]id or [sy]mbols or [bo]th) -> so

x_min -> 726.219  y_min -> -0.554
x_max -> 729.164  y_max -> 0.102

continue? ->
enter the vertical scale (0.0=default) ->
output the plot to a file? ................. -> n

plot another? (ful, lin. or [l]oom) -> n
• **CALFGSA**
  1. CALFGSA is used on both TRANS and POS Mode Data to separate the GEIS files into individual scans. To run, just type CALFGSA in the terminal window:

     ```
     %>calfgsa
     ```

  2. You will be prompted for “observation rootname”. This can be in the form of an individual file

     ```
     enter observation rootname: fb9r0101m
     ```

    or a list (obs.lis) with the rootnames of the header files

     ```
     enter observation rootname: @obs.lis
     ```

    CALFGSA will create individual files for each scan and for each FGS:

    `<filename>.1s5`

    - 1 – corresponds to FGS1r. This is the nominal science FGS, or the astrometer.
    - s – stands for scan.
    - 5 – is the scan number.

    The program also outputs:

    `<filename>.coadded_scan`
    `<filename>.scan_summary`
    `<filename>.tab`
    `<filename>.coadd.x.ps` (preliminary co-added files for both x- and y- axes)

    On the top right corner of each co-added plot are the target name, observation date, V-magnitude and an Sppr number. Sppr is the observed ratio of the observed peak-to-peak of the central S-curve to that of a calibration point source of the same magnitude. Any value less than Sppr ≤ 0.9 is a fairly reliable indicator that the star is not an isolated point source. This is used as the preliminary determination for a star’s binarity noted in the file `cal_list.dat` and is especially useful for identifying possible, tight binaries.

    **NOTE:** If the code gets hung up at towards the end, without returning to the prompt, then there is an issue with the supermongo plotting routine. To get around this, use `calfgsa2` instead. The bounding box around the plots will not be outputted in the PS files (see plots below).

    **NOTE:** Check the `<filename>.tab` file for any error messages. This file is like a header file with all the important observation information in it. If there is a message such as “Guide Star Acquisition Failed” then the star was not observed and CANNOT be reduced.
- CALFGSA

```
[relom:~/fsgdata/test] fsg2 calfgsa
  enter observation rootname: FS0-0101m

processing obst .......... -> FS00101m
observing mode .......... -> TRANSFER
number of samples .......... -> 73336
filter .................. -> F548m
  target magnitude .......... -> 14.22
  target_id .................. -> 11346.462
  target name .................. -> EB1146
  obs date .................. -> 2008-355 23:32:46
  telemetry format .......... -> FN
  FCS Y2 OFFSET (align pt) .. -> 0.00
  FCS Y3 OFFSET (align pt) .. -> 0.00

sl. vmax/min, vmean/min: 1.35 -1.25 -2.12 730.01 729.16
writing data files for scan# : 1
writing data files for scan# : 2
writing data files for scan# : 3
writing data files for scan# : 4
writing data files for scan# : 5
writing data files for scan# : 6
writing data files for scan# : 7
writing data files for scan# : 8
writing data files for scan# : 9
writing data files for scan# : 10
writing data files for scan# : 11
writing data files for scan# : 12
writing data files for scan# : 13
writing data files for scan# : 14
writing data files for scan# : 15
writing data files for scan# : 16
writing data files for scan# : 17
writing data files for scan# : 18
writing data files for scan# : 19
writing data files for scan# : 20
[relom:~/fsgdata/test] fsg3
```

Figure 1 – X-axis output to PS file from CALFGSA

Figure 2 – X-axis output to PS file from CALFGSA2
• PTRANS
1. To process the TRANS data and make the final S-curve run PTRANS.
   
   %>ptrans

2. Like CALFGSA, when prompted for the name of the file(s) you can enter an
   individual file rootname
   
   enter: name of the obs or file with list of scans -> fb9r0101m
   or a file list preceded by an @ sign
   
   enter: name of the obs or file with list of scans -> @obs.lis

3. Next you will be prompted to perform a cross correlation. For anything fainter
   than V > 14 mag, you run the risk of cross correlating the noise, so you should
   perform two runs: once with cross correlating and once without cross
   correlating. Otherwise, always cross correlate. The default is yes.
   
   perform cross correlation (y/n) ................. -> y

4. For the following prompts use the default by hitting enter <CR>, but I have
   included here what each default is:
   
   use 100 mas range for cross correlation (y/n) .... -> <CR = y>
   process which axis (<CR> = both X and Y) ........ - > <CR>
   shall the individual scans be output? ....... - > <CR = n>
   set x, y limits of data to be analyzed ?
   --CAUTION, you must know the range of the raw data
   set x,y limits ----> <CR = n>
   de-jitter scans (y=default, n=no-dejitter) ? … - > <CR>
   add constants to PMT data? .................. -> <CR>

5. After PTRANS processes the data, you will work with the X-axis data first.
   When prompted to specify the reference scan number, choose the middle scan
   (i.e. if there are 20 scans, enter 10)
   
   specify the X-axis reference scan number: 10
   PTRANS will shift and bin all the other scans to this reference scans. If you run
   into problems (i.e. the sd_fr, sd_wg, etc. list –Nan), rerun PTRANS with a
   different scan number as the reference scan.

6. The program will print out a table with information about each scan. The
   columns to take special note of are:
   
   scan – the scan number.
   shift – how much the scan is was shifted (in mas) to line up with the
   reference scan. Throw out scans with significantly large shifts.
   rel_diff – is the ratio between sd_fr and ave_sd_fr. This should be about
   1 but a good range in values in from 0.7 to 1.4.
   The other 4 columns are the standard deviations and average standard deviations
   from the mean S-curve in the fringe (fr) and the wings (wg).
After taking a look at the shifts and rel_diff, you are prompted with several options. After performing each, you will return to this menu:

- **enter scan numbers to omit (r#)** – type the letter ‘r’ followed by the individual scan number of a scan with a large shift and/or a bad rel_diff that you do not want in the final co-added S-curve.
  
  \[ \ldots \rightarrow r20 \]
  
  If you accidentally omit a scan, you can restore it by entering a negative sign in front.

  \[ \ldots \rightarrow r-1 \]

  If you want to omit more than one scan, just enter the numbers, separated by commas. You can also omit and restore in a single line.

  \[ \ldots \rightarrow r-1, 20 \]

  The X-axis co-added S-curve and the Y-axis co-added S-curve do NOT need to have the same scans included. For example, the X-axis you may have omitted scan #1 but in the Y-axis scan #1 is perfectly acceptable.

- **set maximum allowable threshold (fx.xx)** – this is the maximum threshold for rel_diff. If you want to exclude a rel_diff larger than 1.2 just enter:

  \[ \ldots \rightarrow f1.20 \]

- **choose to plot a scan (s#)** – if you want to plot an individual scan, enter the letter ‘s’ followed by the scan number (no space).

  \[ \ldots \rightarrow s20 \]

  This will plot the individual scan in blue and the unsmoothed, co-added scan in red. You will be prompted for the range to plot the data. Use a (all) for the first plot. If you decide to enter a range, make sure you always have a decimal, regardless of whether it is a whole number. For the default range in the Y-axis type 0, not just `<cr>`. Then you will be asked to output plot to a file, re-plot data with different limits and mark scan for deletion. The default for all of these is no (n). You are then given the option to plot another scan (enter just the scan number) or if you hit enter, you will return to the main menu.

- **compare smooth/un-smoothed data or change the smoothing parameter (cp)** – Typing cp will plot the smoothed, co-added data in red and the unsmoothed, co-added data in blue. Use this to make sure you are not compromising the quality of the final smoothed S-curve and whether you need to change the smoothing parameter.

  You will be prompted for the smoothing parameter. You can hit enter to use the default or current value or enter an integer. (see below how to choose smoothing parameter)

  \[ \ldots \rightarrow 4000 \]

  Next you can choose the plot range by either entering a for the whole plot range, s to plot the ranges used previously in PTRANS or you can choose the X and Y min and max values.

  \[ \text{--------------------------} \rightarrow \text{xmin: s} \]

  Then you will be prompted if you want to output the file and if you would like to re-plot with different limits. Choosing a smaller X-range (e.g. -0.2
to 0.2) will let you see more detail in the smoothing, as well as remove the edges of the scan that are bad due to instrumentation effects. I would set the limits of the plot here. You will get a better fit when using BINARY_FIT below with smoother data. The default value is 1000 and the maximum is around 11000. For a faint star V~14 mag, the smoothing parameter will be roughly around 4000. It will be higher for brighter objects. You may want to start by changing the value in steps of 1000 and refine it to steps of 50. This is a case where bigger is better. The smoothing parameter will be approximately the same for both axes. In the case where you have multiple fringes due to more than one star in the FOV, first smooth the curve to the primary and double check the secondary.

- **Smooth the signal from the PMTs (pa/pb)**
  Same concept as for the smoothing above, but for the signal from the photomultiplier tubes (PMTs). This is important for fitting widely separated S-curves because the PMT signal is not flat. Follow the same procedure for cp above.

- At the main menu, after hitting enter you will be prompted to set the smoothing parameter and set the scans used for the final S-curve.
  
  **ready to finish this scan --> y**
  For label #1, enter the target name.
  For label #2, enter the date.
  (NOTE: The name and data are outputted when you run STRFITS or you can find that information in the <filename>.tab)
  Hit enter when prompted to output to a file and re-do with new limits or title.
  If this is the X-axis and you are doing both, the program will repeat steps 5-6 for the Y-axis.
  Otherwise the program ends here.

PTRANS outputs a total of 6 files:
4 files for both the X-axis and Y-axis.
  - fb9r0101m.cx – co-added, cross correlated and de-jittered X-axis S-curve
  - fb9r0101m.cx0 – same as above, but shifted to the reference scan
  - fb9r0101m.cx0s – same as .cx0 but also smoothed

Optional output files:
  - <filename>.ilx.ps, where il is the iteration number after removing scans or smoothing and x is the axis
  - <filename>.coadded_scan
  - <filename>.scan_summary
  - <filename>.zero_pt

When finished, run the cleanup script to organize all the files in the directory. Assumes that the data has long and short scans.

%> cleanup
PTRANS

```
[relan:~/Test/nt5] fgs% pttrans

enter name of the obs or file with list of scans -> f9ea0101m
perform cross correlation (y/n) .............. ->
use 100 mas range for cross correlation (y/n) .... ->
using range = 100,0 mas
process which axis (CR = both x and y) ........ ->

shall the individual scan files be output? ...... ->

set x,y limits of data to be analyzed ?
--- CAUTION, you must know the range of the raw data
set x,y limits -------- ->
de-jitter scans (y=defaul, n=no de-jitter) ? ... ->

add constants to PMT data ? ................. ->
donormant guide star in FGS #2

processing raw data files:
scan: 1 -1.833 0.661 -0.640 1.391
727.324 0.360 -0.672 1.031
scan: 2 -1.833 0.647 -0.619 1.256
727.327 0.413 -0.631 1.044
scan: 3 -1.832 0.639 -0.609 1.248
727.325 0.396 -0.661 1.056
scan: 4 -1.837 0.661 -0.610 1.271
727.319 0.406 -0.655 1.059
scan: 5 -1.829 0.716 -0.594 1.310
727.332 0.410 -0.655 1.065
scan: 6 -1.833 0.673 -0.653 1.325
727.327 0.401 -0.681 1.092
scan: 7 -1.832 0.656 -0.606 1.255
727.325 0.418 -0.650 1.089
scan: 8 -1.827 0.720 -0.650 1.371
727.325 0.346 -0.654 1.000
scan: 9 -1.833 0.654 -0.602 1.256
727.323 0.555 -0.659 1.015
scan: 10 -1.829 0.713 -0.629 1.340
727.335 0.378 -0.676 1.053
scan: 11 -1.829 0.672 -0.573 1.250
727.327 0.365 -0.643 1.014
scan: 12 -1.822 0.647 -0.614 1.261
727.325 0.397 -0.685 1.082
scan: 13 -1.831 0.662 -0.598 1.259
727.337 0.355 -0.666 1.040
scan: 14 -1.827 0.703 -0.597 1.298
727.324 0.417 -0.643 1.060
scan: 15 -1.823 0.704 -0.635 1.357
727.327 0.405 -0.661 1.066
scan: 16 -1.824 0.684 -0.643 1.327
727.333 0.396 -0.679 1.074
```
### PTRANS

scan: 17  -1.823  0.675  -0.591  1.257  
   727.339  0.389  -0.642  1.031  
scan: 18  -1.824  0.672  -0.606  1.279  
   727.332  0.380  -0.646  1.027  
scan: 19  -1.835  0.669  -0.587  1.248  
   727.335  0.425  -0.685  1.076  
scan: 20  -1.086  0.692  -0.594  1.276  
   727.313  0.390  -0.635  1.025  

specify the X-axis reference scan number: 10  
reference scan ID: 10

processing x axis, iteration # 1

xzero: -1.807942903679479

--- x axis ---

<table>
<thead>
<tr>
<th>scan</th>
<th>shift</th>
<th>rel_diff</th>
<th>sd_frb</th>
<th>sd_w</th>
<th>ave_sd_frb</th>
<th>sd_wings</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.16</td>
<td>0.0581</td>
<td>0.0463</td>
<td>0.0499</td>
<td>0.0463</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>-0.643</td>
<td>1.20</td>
<td>0.0539</td>
<td>0.0443</td>
<td>0.0499</td>
<td>0.0463</td>
</tr>
<tr>
<td>3</td>
<td>-3.05</td>
<td>0.84</td>
<td>0.0413</td>
<td>0.0456</td>
<td>0.0499</td>
<td>0.0463</td>
</tr>
<tr>
<td>4</td>
<td>-0.321</td>
<td>0.98</td>
<td>0.0478</td>
<td>0.0466</td>
<td>0.0499</td>
<td>0.0465</td>
</tr>
<tr>
<td>5</td>
<td>-3.36</td>
<td>0.89</td>
<td>0.0433</td>
<td>0.0451</td>
<td>0.0499</td>
<td>0.0465</td>
</tr>
<tr>
<td>6</td>
<td>-3.32</td>
<td>1.06</td>
<td>0.0526</td>
<td>0.0457</td>
<td>0.0499</td>
<td>0.0463</td>
</tr>
<tr>
<td>7</td>
<td>-3.07</td>
<td>0.89</td>
<td>0.0442</td>
<td>0.0462</td>
<td>0.0499</td>
<td>0.046</td>
</tr>
<tr>
<td>8</td>
<td>-1.57</td>
<td>1.09</td>
<td>0.0544</td>
<td>0.0484</td>
<td>0.0499</td>
<td>0.046</td>
</tr>
<tr>
<td>9</td>
<td>-1.23</td>
<td>1.00</td>
<td>0.0498</td>
<td>0.0464</td>
<td>0.0499</td>
<td>0.046</td>
</tr>
<tr>
<td>10</td>
<td>0.00</td>
<td>1.17</td>
<td>0.0586</td>
<td>0.0484</td>
<td>0.0499</td>
<td>0.046</td>
</tr>
<tr>
<td>11</td>
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<td>0.85</td>
<td>0.0425</td>
<td>0.0479</td>
<td>0.0499</td>
<td>0.0463</td>
</tr>
<tr>
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<td>1.09</td>
<td>0.0544</td>
<td>0.0478</td>
<td>0.0499</td>
<td>0.0463</td>
</tr>
<tr>
<td>13</td>
<td>3.05</td>
<td>1.06</td>
<td>0.0530</td>
<td>0.0457</td>
<td>0.0499</td>
<td>0.0463</td>
</tr>
<tr>
<td>14</td>
<td>4.32</td>
<td>0.82</td>
<td>0.0410</td>
<td>0.0445</td>
<td>0.0499</td>
<td>0.0463</td>
</tr>
<tr>
<td>15</td>
<td>3.20</td>
<td>0.96</td>
<td>0.0481</td>
<td>0.0443</td>
<td>0.0499</td>
<td>0.0463</td>
</tr>
<tr>
<td>16</td>
<td>3.62</td>
<td>0.89</td>
<td>0.0445</td>
<td>0.0474</td>
<td>0.0499</td>
<td>0.0463</td>
</tr>
<tr>
<td>17</td>
<td>3.55</td>
<td>0.94</td>
<td>0.0467</td>
<td>0.0472</td>
<td>0.0499</td>
<td>0.0463</td>
</tr>
<tr>
<td>18</td>
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<td>1.01</td>
<td>0.0502</td>
<td>0.0451</td>
<td>0.0499</td>
<td>0.0463</td>
</tr>
<tr>
<td>19</td>
<td>3.52</td>
<td>1.00</td>
<td>0.0497</td>
<td>0.0461</td>
<td>0.0499</td>
<td>0.0463</td>
</tr>
<tr>
<td>20</td>
<td>2.91</td>
<td>1.03</td>
<td>0.0512</td>
<td>0.0459</td>
<td>0.0499</td>
<td>0.0463</td>
</tr>
</tbody>
</table>

enter scan numbers to omit, (e.g., r1.2,5)  
set maximum allowable threshold (fx.xx)  
choose to plot a scan (#)  
compare smoothed/unsmoothed data  
change the S-curve smoothing parameter (cp)  
change the PHTA smoothing parameter (pa)  
change the PHTB smoothing parameter (pb).... -> s16

npts, npts coadded: 3651 3651  
x_plot(1), x_plot(npts): -0.734324963453045 0.72489467270725740

"x" data ranges from (min,max) = (-0.734, 0.725)
enter range to be plotted ("a" plots all, "s" plots same as before)

--------------------------> xmin: a

warning, this program uses gets(), which is unsafe.  
output the plot to a file? ................. -> n  
re-plot data with different limits? ........... -> n  
mark scan for deletion? ....................... -> n  
Enter next scan to plot ........................ -> 17
PTRANS

\[ npts, npts\_coadded: \quad 3651 \quad 3651 \]
\[ x\_plot(1), x\_plot(npts): \quad -0.7341924963453045 \quad 0.7248946727073740 \]

Enter scan numbers to omit, (e.g., r1,2,5)
or set maximum allowable threshold (fx,xx)
or choose to plot a scan (s#)
or compare smoothed/un-smoothed data
or change the S-curve smoothing parameter (cp)
or change the PMTA smoothing parameter (pa)
or change the PMTB smoothing parameter (pb),... \rightarrow cp

Smoothing parameter currently set as \( \text{sn} = 1000 \)
Enter new value (CR retains current value) \( \rightarrow > 4000 \)

"x" data ranges from (min,max) = (-0.734, 0.725)
Enter range to be plotted ("a" plots all, "s" plots same as before)

\[ x_{\text{min}}, x_{\text{max}}: \quad -0.200000000 \quad 0.200000000 \]

"y" data ranges from (min,max) = (-0.554, 0.618)
Enter range to be plotted (CR plots +/-0.8) ymin:

\[ y_{\text{min}}, y_{\text{max}}: \quad -0.8 \quad 0.8 \]

Warning: this program uses get() which is unsafe.
Output the plot to a file? \( ................. \rightarrow \)
Re-plot data with different limits? \( ........ \rightarrow \)

Enter scan numbers to omit, (e.g., r1,2,5)
or set maximum allowable threshold (fx,xx)
or choose to plot a scan (s#)
or compare smoothed/un-smoothed data
or change the S-curve smoothing parameter (cp)
or change the PMTA smoothing parameter (pa)
or change the PMTB smoothing parameter (pb),... \rightarrow cp

Smoothing parameter currently set as \( \text{sn} = 4000 \)
Enter new value (CR retains current value) \( \rightarrow > 3100 \)

"x" data ranges from (min,max) = (-0.734, 0.725)
Enter range to be plotted ("a" plots all, "s" plots same as before)

\[ x_{\text{min}}: \quad \rightarrow \]
Output the plot to a file? \( ................. \rightarrow \)
Re-plot data with different limits? \( ........ \rightarrow y \)

"x" data ranges from (min,max) = (-0.734, 0.725)
Enter range to be plotted ("a" plots all, "s" plots same as before)

\[ x_{\text{min}}, x_{\text{max}}: \quad -0.200000000 \quad 0.200000000 \]

"y" data ranges from (min,max) = (-0.554, 0.618)
Enter range to be plotted (CR plots +/-0.8) ymin:

\[ y_{\text{min}}, y_{\text{max}}: \quad -0.8 \quad 0.8 \]
Output the plot to a file? \( ................. \rightarrow \)
Re-plot data with different limits? \( ........ \rightarrow \)
• PTRANS

- enter scan numbers to omit. (e.g., r1,2,5)
- or set maximum allowable threshold [fx,xx]
- or choose to plot a scan (s#)
- or compare smoothed/un-smoothed data
- or change the S-curve smoothing parameter (cp)
- or change the PMTA smoothing parameter (pa)
- or change the PMTB smoothing parameter (pb), ..., \( \rightarrow \) cp

Smoothing parameter currently set as sm = 3100
enter new value (CR retains current value) \( \rightarrow \) 3050

"x" data ranges from \((\text{min}, \text{max}) = [-0.734, 0.725]\)
enter range to be plotted ("a" plots all, "s" plots same as before)

\[ \text{-----------} \rightarrow \text{xmin: } s \]
output the plot to a file? ................ \( \rightarrow \)
re-plot data with different limits? ........ \( \rightarrow \)

- enter scan numbers to omit. (e.g., r1,2,5)
- or set maximum allowable threshold [fx,xx]
- or choose to plot a scan (s#)
- or compare smoothed/un-smoothed data
- or change the S-curve smoothing parameter (cp)
- or change the PMTA smoothing parameter (pa)
- or change the PMTB smoothing parameter (pb), ..., \( \rightarrow \) pa

PMTA smoothing parameter currently set as ipa = 1000
enter new value (CR retains current value) \( \rightarrow \)

"x" data ranges from \((\text{min}, \text{max}) = [-0.734, 0.724]\)
enter range to be plotted ("a" plots all, "s" plots same as before)

\[ \text{-----------} \rightarrow \text{xmin: } s \]
output the plot to a file? ................ \( \rightarrow \)
re-plot data with different limits? ........ \( \rightarrow \)

- enter scan numbers to omit. (e.g., r1,2,5)
- or set maximum allowable threshold [fx,xx]
- or choose to plot a scan (s#)
- or compare smoothed/un-smoothed data
- or change the S-curve smoothing parameter (cp)
- or change the PMTA smoothing parameter (pa)
- or change the PMTB smoothing parameter (pb), ..., \( \rightarrow \) pa

PMTA smoothing parameter currently set as ipa = 0
enter new value (CR retains current value) \( \rightarrow \) 4000

"x" data ranges from \((\text{min}, \text{max}) = [-0.734, 0.724]\)
enter range to be plotted ("a" plots all, "s" plots same as before)

\[ \text{-----------} \rightarrow \text{xmin: } s \]
output the plot to a file? ................ \( \rightarrow \)
re-plot data with different limits? ........ \( \rightarrow \)
• PTRANS

- enter scan numbers to omit (e.g., r1,2,5)
- or set maximum allowable threshold \( f_{x,xx} \)
- or choose to plot a scan (s#)
- or compare smoothed/un-smoothed data
- or change the S-curve smoothing parameter (cp)
- or change the PNTA smoothing parameter (pa)
- or change the PNTB smoothing parameter (pb),..., \( \rightarrow \) pb

PNTB smoothing parameter currently set as \( pb = 4000 \)
enter new value \( (\geq 4000) \) retains current value \( \rightarrow \) \( > \) 4000

\( x \) data ranges from \( \text{min} \rightarrow \text{max} = [ -0.734, 0.724] \)
enter range to be plotted \( (a \) plots all, \( s \) plots same as before)

\( \rightarrow \) \( \text{min} \rightarrow \text{a} \)

output the plot to a file? \( \ldots \ldots \ldots \rightarrow \Pi \)

- enter scan numbers to omit (e.g., r1,2,5)
- or set maximum allowable threshold \( f_{x,xx} \)
- or choose to plot a scan (s#)
- or compare smoothed/un-smoothed data
- or change the S-curve smoothing parameter (cp)
- or change the PNTA smoothing parameter (pa)
- or change the PNTB smoothing parameter (pb),..., \( \rightarrow \)

ready to finish this axis? \( \rightarrow \) \( y \)

Enter label #1 for final plot:

MT5

Enter label #2 for final plot:

SK3x50
\( x \) data ranges from \( \text{min} \rightarrow \text{max} = [ -0.734, 0.724] \)
enter range to be plotted \( (a \) plots all, \( s \) plots same as before)

\( \rightarrow \) \( \text{min} \rightarrow \text{a} \)

output final plot to a file? \( \ldots \ldots \ldots \rightarrow \) \( y \)
renA final plot with new limits or tit,A? \( \rightarrow \) \( n \)
specify the Y-axis reference scan number: 10
reference scan ID: 10

processing y axis, iteration # 1
xzero: 727.35158618902300

\[ \text{----------- y axis ----------} \]
\[ \begin{array}{cccccccc}
\text{scan} & \text{shift} & \text{rel_diff} & \text{sd_fr} & \text{sd_wg} & \text{ave_sd_fr} & \text{sd_wings} \\
1 & -7.42 & 1.08 & 0.0643 & 0.0475 & 0.0598 & 0.0451 \\
2 & -6.95 & 0.86 & 0.0613 & 0.0477 & 0.0598 & 0.0451 \\
3 & -4.93 & 0.83 & 0.0624 & 0.0472 & 0.0598 & 0.0451 \\
4 & -4.75 & 1.08 & 0.0647 & 0.0462 & 0.0598 & 0.0451 \\
5 & -2.25 & 0.94 & 0.0661 & 0.0451 & 0.0598 & 0.0451 \\
6 & -3.33 & 1.01 & 0.0603 & 0.0454 & 0.0598 & 0.0451 \\
7 & -2.17 & 0.94 & 0.0563 & 0.0454 & 0.0598 & 0.0451 \\
8 & 0.50 & 0.93 & 0.0585 & 0.0459 & 0.0598 & 0.0451 \\
9 & -0.26 & 0.94 & 0.0563 & 0.0457 & 0.0598 & 0.0451 \\
10 & 0.00 & 1.17 & 0.0699 & 0.0457 & 0.0598 & 0.0451 \\
11 & 0.20 & 1.09 & 0.0693 & 0.0471 & 0.0598 & 0.0451 \\
\end{array} \]
- **PTRANS**

  enter scan numbers to omit, (e.g., r1.2.5)
  or set maximum allowable threshold (fx.xx)
  or choose to plot a scan (s#)
  or compare smoothed/un-smoothed data
  or change the S-curve smoothing parameter (cp)
  or change the PHTA smoothing parameter (pa)
  or change the PHTI smoothing parameter (pb),... → cp

  smoothing parameter currently set as sm = 4000
  enter new value (<CR> retains current value) → 3500

  "x" data ranges from [min,max] = (-0.680, 0.774)
  enter range to be plotted ("a" plots all, "s" plots same as before)

  ----------------> xmin
  output the plot to a file? ................. → n
  re-plot data with different limits? ........ → y

  "x" data ranges from [min,max] = (-0.680, 0.774)
  enter range to be plotted ("a" plots all, "s" plots same as before)

  ----------------> xmin: -0.2
  ----------------> xmax: 0.2

  xmin, xmax: -0.20000000 0.20000000

  "y" data ranges from [min,max] = (-0.601, 0.302)
  enter range to be plotted (<CR> plots +/-0.8) ymin: -0.8
  umax: 0.9

  output the plot to a file? ................. → y
  enter scan numbers to omit, (e.g., r1.2.5)
  or set maximum allowable threshold (fx.xx)
  or choose to plot a scan (s#)
  or compare smoothed/un-smoothed data
  or change the S-curve smoothing parameter (cp)
  or change the PHTA smoothing parameter (pa)
  or change the PHTI smoothing parameter (pb),... → pb

  PHTI smoothing parameter currently set as pb = 1000
  enter new value (<CR> retains current value) → 4000

  "x" data ranges from [min,max] = (-0.680, 0.774)
  enter range to be plotted ("a" plots all, "s" plots same as before)

  ----------------> xmin
  output the plot to a file? ................. → y
  re-plot data with different limits? ....... → y
  enter scan numbers to omit, (e.g., r1.2.5)
  or set maximum allowable threshold (fx.xx)
  or choose to plot a scan (s#)
  or compare smoothed/un-smoothed data
  or change the S-curve smoothing parameter (cp)
  or change the PHTA smoothing parameter (pa)
  or change the PHTI smoothing parameter (pb),... → pb

  ready to finish this axis? →
• PTRANS
Enter label#1 for final plot:

MT5

Enter label#2 for final plot:

SM3100

"x" data range from (min, max) = (-0.690, 0.774)
Enter range to be plotted ("a" plots all, "s" plots same as before)

--------------> xmin: a

output final plot to a file? .................. -->
re-do final plot with new limits or title? -->
CP_SCURVES
1. You can use the CP_SCURVES routine to look at the output S-curves from PTRANS and also to check if the star is a binary. The program also creates postscript files of the plots. Run CP_SCURVES:
   
   > cp_scurves

2. You can compare any two sets of S-curves, as long as they are from the same axis (X or Y). When prompted to specify the files, enter the filename with the extension:
   
   specify s-curve 1 file: fb9r0101m.cx0
   specify s-curve 2 file: fb9r0101m.cx0

   NOTE: This procedure is best for comparing the reduced s-curves of two different stars.

3. Then you will be asked which axis (x or y):
   
   enter fgs axis (x or y) …… -> x

4. Enter the name of the output file:
   
   enter output file name …… -> LB11146.x.ps

5. You must specify the graph boundary for the x-axis
   
   specify graph boundaries …. -> y <CR=n>
   
   n – Default will plot x from -0.2 to 0.2
   
   y – Specify the graph boundaries
   
   enter xmin ---> -0.5
   enter xmax ---> 0.5
   enter ymin ---> -0.8
   enter ymax ---> 0.8

6. The program will plot the first file in red, the second in blue and the difference between the two curves in white. The white curve should be centered on S(x or y) = 0. You will be prompted if the white curve needs to be adjusted, meaning if you want it to be shifted up to be in the plot, or shifted down so that it is not plotted in the window.
   
   Does diff curve need to be adjusted? …… -> y <CR=n>
   
   If you want to shift it, then you will be prompted how much you would like to shift the white curve in units of the ordinate (vertical) axis. A positive number will shift it down; a negative number will shift it up.
   
   Enter increment for curve to be shifted (from zero) -> -0.5
   
   Does diff curve need to be adjusted? …… -> <CR=n>

7. Finally you will be prompted if you would like to output plot to a file. If you choose to do so, it will create a postscript file of the final plot with the name you specified above.
   
   Output the plot to a file? …………………. -> y
CP_SCURVES

```
[rmln:"/f/gdata/temp] fgs2 cp_scuvres
specify s-curve 1 file : fbg0001a.cyg
specify s-curve 2 file : fbg0002a.cyg
s2_file: fbg0002a.cyg

r1, n2, np: 3992 968 1561

from least_squares: s10, s11, s12: 0.098378 0.62843 0.019433
s20, s21, s22: 0.08993 0.698986 0.021246

shift (use) ...... : -0.0003

in fringe: number of points : 793
normalized difference : -0.004513
total fringe area : 0.09682
total fringe diff : 0.00679

in wings: number of points : 1561
normalized difference : 0.355433
total wings area : 0.054023
total wings diff : 0.02128

enter figs axis (x or y) ...... → y
enter output file name ...... → fbg1114_gy
specify graph boundaries .... → y

enter xmin → -0.5
enter xmax → 0.5
enter ymin → -0.8
enter ymax → 0.8

warning: this program uses gets(), which is unsafe.
Does diff curve need to be adjusted? ......... → y
Enter increment for curve to be shifted (from zero) → .5
Does diff curve need to be adjusted? ......... →
Output the plot to a file? .................. → y
```

```
[rmln:"/f/gdata/temp] fgs2 cp_scuvres
specify s-curve 1 file : fbg0001a.cyg
specify s-curve 2 file : fbg0002a.cyg
s2_file: fbg0002a.cyg

r1, n2, np: 4089 968 1566

from least_squares: s10, s11, s12: -0.113469 -0.011306 0.020646
s20, s21, s22: -0.052158 -0.074723 0.008268

shift (use) ...... : -0.0002

in fringe: number of points : 827
normalized difference : 0.228036
total fringe area : 0.082151
total fringe diff : 0.005291

in wings: number of points : 1566
normalized difference : 0.111025
total wings area : 0.022533
total wings diff : 0.002504

enter figs axis (x or y) ...... → y
enter output file name ...... → fbg1114_gy
specify graph boundaries .... → y

enter xmin → -0.5
enter xmax → 0.5
enter ymin → -0.8
enter ymax → 0.8

warning: this program uses gets(), which is unsafe.
Does diff curve need to be adjusted? ......... → y
Enter increment for curve to be shifted (from zero) → .5
Does diff curve need to be adjusted? ......... →
Output the plot to a file? .................. → y
```
§4 FINDING CALIBRATORS

There are six main criteria for choosing a calibrator:
1. The star must be single. Compare the S-curve with other single stars to make sure that they look the same.

2. Make sure that the calibrator and the target star were observed on the same side of January 22, 2009. There was an adjustment of the y-axis of FGS1r from 9:34.07 to 11:51:45 and the S-curves prior to this date are different than those after this date. You can find the data of observation in the <filename>.tab file. The data will be in the following format:
yyyy/ddd – the year, followed by the day of the year (January 1 = 001). You can use the program UTDATE (see §5 below) to convert to the more conventional date format.

3. You want the calibrator and the target to have been observed near in date. See step 2 above on how to determine the date of observation.

4. The calibrator and target should have similar B-V colors. You can look these up in SIMBAD.

<table>
<thead>
<tr>
<th>(B-V)$_1$</th>
<th>(B-V)$_2$</th>
<th>Acceptable?</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0.5</td>
<td>Yes</td>
</tr>
<tr>
<td>-0.3</td>
<td>2.1</td>
<td>No</td>
</tr>
</tbody>
</table>

NOTES:
- Targets fainter than V = 14 mag the dark current is incoherent with the star. You do NOT want an equally faint calibrator. Use a brighter calibrator and the BINARY_FIT routine will take it into account.
- For targets brighter than V = 8 mag you need to account for dead time (10% of photons not accounted for)

5. Both the calibrator and target need to be observed with the same FGS (1r,2,3).

6. The observations need to be done in the same filter (e.g. Neutral Density (ND) or F583W).
§5 BINARY FITTING

* BINARY_FIT

1. When you have a star that is a binary and have found a suitable, single star calibrator, run BINARY_FIT in the terminal.

   %>binary_fit

   NOTE: Make sure to include the `<starname>.tab` for both the calibrator and science star when running this. The file must follow the same naming convention that was used for the scans (e.g. `mt145.cx0s & mt145.tab, mt696.cx0s & mt696.tab`)

2. Enter the corresponding number to the axis you would like to fit.

   choose axis to fit: 1=x, 2=y, 3=x&y --> 3

3. Then you will be prompted to enter the name of the output file. This can overwrite pre-existing files.

   enter name of the output file to be created ........ -> mt145_mt696.fit

4. Then enter the shifted and smoothed filename for the corresponding axis.

   enter X-axis data filename ..................... -> mt145.cx0s

5. When you are prompted for a bright model star file name, you enter the filename of your calibrator.

   enter X-axis bright model star file name .......... -> mt696.cx0s

6. You will be prompted for a faint model. You can use this option if the companion to the primary is a different color. Otherwise, it will use the bright model S-curve to fit both components.

   enter X-axis faint model star file name .......... ->

7. When asked if you want to adjust the target magnitude, you especially want to do this for faint objects. You will be asked this twice.

   adjusting point source S-curve to target magnitude
   proceed with adjustment? -------------------------- > y

8. From looking at the S-curves you should get an idea of the separation and magnitude difference of the components. First the program, asks for the separation (in arcseconds). Make sure you estimate is within 50mas, because that is the search window when the program is looking for a companion to fit.

   enter N if the separation is narrow <[0.025]:
   enter W if the separation is wide <[0.100]:
   enter V if the separation is very wide <[0.300]:
   enter G for specific response initial estimate [N/W/V/G]: G

9. You are then asked for the magnitude difference. Usually an estimate of 1.0 for close companions is a good starting point.

   enter the estimated magnitude difference -> 1.
10. The magnitude difference should not be held fixed unless the value is from another source and known very well.

   magnitude difference to be held fixed? \textit{\textasciitilde-CR=n}\textit{

11. If you did NOT enter ‘G’ in step 8, the program will repeat steps 4-10 for the y-axis, if fitting both axes. Otherwise, it will continue to step 12.
   If you chose ‘G’ in step 8, you will be prompted to enter the separation in mas. A positive number will mean the secondary is to the right of the primary, a negative number will mean that the companion is to the left of the primary.

   Enter separation (mas) \textit{\textasciitilde-CR=9.4}
   Enter zero point \textit{\textasciitilde-CR=0}.

12. If your x and y fits are consistent or if you are only doing one axis, then continue to step 13.
   More often than not, they will not be consistent and you will be asked choose which axis you would like to refit. Usually, the X-axis is the better and more consistent fit, but choose whichever axis has the largest \textit{Sum of Squares}. The closer the value is to zero, the better the result.

   \textbf{Choose which axis to refit (X or Y): X}
   It will repeat step 11 if applicable.

13. The program will then output the final results of the fits and plot the axes that were fit.

   enter desired range to plot: (\textit{\textasciitilde-CR} plots -0.2,0.2)
   (“a” plots all)

   xmin: \textit{\textasciitilde-CR}

   The plot will show the S-curve of your binary in red and the model fit in blue. The difference between the two is the white line.

14. You will then be prompted if you want to shift the position of the “\textbf{diff curve}” or the white line in the plot. (see CP_SCURVES step 6).

   Does diff curve need to be adjusted? \textit{\textasciitilde-CR=n}

15. Then you are asked if you want to output the plot. Default is no.

   Output the plot to a file? \textit{\textasciitilde-CR=n}

16. You will be given the option to replot with different ranges, which will repeat from step 13. You must enter something here to continue.

   re-plot the date with different (x,y) limits \textit{\textasciitilde-CR=n}
   If this is the last axis, then the program will end here. Other wise it will repeat from step 13.
The output we are interested in is the difference in magnitude ($\Delta m$), separation ($\rho$) and position angle ($\theta$). This is printed to the screen and to the output file. (e.g. mt696.dat)

The errors calculated by BINARY_FIT are too small. Good estimates for the errors are:

- $\Delta m$: 0.1 mag
- $\theta$: 0.02°
- $\rho$: 1-2 mas

You can also calculate errors by creating binaries with TBINARY, changing the parameters slightly and comparing to the observed star using CP_SCRUVES.

Other output files:
- `<filename>.shift` – copy of the science star
- `<filename>diff` – the difference between the fit and the binary
- `<calibrator_starname>.xbin` – the best binary fit created from the calibrator
- `<calibrator_starname>.ybin` – see xbin above
• **BINARY_FIT**

```plaintext
[nlens="/F3e0data/test"] f3e0 binary_fit
choose axis to fit: 1=x, 2=y, 3=axy --> 3

enter name of the output file to be created .......... -> n3e006pce
enter X-axis data file name ................................ -> f3e0a0600m.coxs
enter <k-axis data file name ............................. -> f3e0a0601m.coxs
enter X-axis faint model star file name .............. -> f3e0a0701m.coxs
data tab file -> f3e0a0600m.tab
modfile1: f3e0a0601m.coxs
modfile2: f3e0a0701m.coxs
model_tab: f3e0a0701m.tab
da filter: F5830
mod file: f3830

adjusting point source S-curve to target magnitude 
proceed with adjustment? -------------------------- > y

model S-curve being adjusted to magnitude of science target, axis: X
background counts (xy,xy,yb,yb): 3.6 1.5 3.7 5.3

adjusting point source S-curve to target magnitude 
proceed with adjustment? -------------------------- > y

model S-curve being adjusted to magnitude of science target, axis: Y
background counts (xy,yb,yb,yb): 5.6 1.5 3.7 5.3

enter N if the separation is narrow (10,025):
enter W if the separation is wide (10,100):
enter V if the separation is very wide (10,200):
enter S for specific initial estimate [(M-W)/V]: 6

enter estimated magnitude difference --> 1.
delta mag, b1, b2 --> 1,000 0.715 0.285

magnitude difference to be held fixed? --> n
nfit: 4
bias held at 0.0
Enter separation in arcsec .......... -> -9.4
Enter zero point ............... -> 0.
X-axis Solution:
Star brightnesses: 0.674 0.305 +/- 0.075
Separation: 5.0 +/- 0.6 mas
Zero-point offset: -5.4 +/- 1.1 mas
Bias: 0.004 +/- 0.000
Sum of squares: 0.0786

enter Y-axis data file name ................................ -> f3e0a0600m.coxs
enter Y-axis bright model star file name ............. -> f3e0a0601m.coxs
enter Y-axis faint model star file name .............. -> f3e0a0701m.coxs
modfile1: f3e0a0601m.coxs
modfile2: f3e0a0701m.coxs

adjusting point source S-curve to target magnitude 
proceed with adjustment? -------------------------- > y

model S-curve being adjusted to magnitude of science target, axis: Y
background counts (xy,xy,yb,yb): 3.6 1.5 3.7 5.3

adjusting point source S-curve to target magnitude 
proceed with adjustment? -------------------------- > y

model S-curve being adjusted to magnitude of science target, axis: Y
background counts (xy,xy,yb,yb): 5.6 1.5 3.7 5.3
```
**BINARY_FIT**

enter N if the separation is narrow \(<10,025\):
enter W if the separation is wide \(<10,100\):
enter V if the separation is very wide \(<10,200\):
enter G for specific initial estimate [W/V/W/V/G]: G

enter the estimated magnitude difference --> 1.
delta mag. id1, id2 --> 1.000 0.715 0.395

magnitude difference to be held fixed? --> n
nfix: 4
Bias held at 0.0

Enter separation (mas) ------ \(\rightarrow 22.4\)
Enter zero point ............ \(\rightarrow 0.0\)
Y-axis Solution:
Star brightnesses: 0.839 0.052 \(+/- 0.005\)
Separation: \(22.5 \pm 0.4\) mas
Zero-point offset:
Bias: \(0.005 \pm 0.000\)
Sum of squares: 0.0406

Fit converged in X and Y but magnitudes not consistent.
Choose which axis to refit (X or Y): x
Bias held at 0.0
Brighter star intensity held at: 0.039
Enter separation (mas):
\(-9.4\)
Enter zero point (asec): 0.0

X-axis Solution:
Star brightnesses: 0.039 0.052 \(+/- 0.005\)
Separation: \(-11.0 \pm 0.2\) mas
Zero-point offset:
Bias: 
\(0.004 \pm 0.000\)
Sum of squares: 0.0680

phi: \(-28.1149284185767\)
PHI, PER: 0.0000000000000000

\(Y:\) \(-11.0\) mas 
\(Y:\) \(22.5\) mas

Total separation: \(\rightarrow 25.0\) mas
Magnitude diff for larger separation: \(\rightarrow 1.75\)

Binary position angle \(\rightarrow 335.3881\)

star name: NGC6

\(X:\) \(1.7835349641994767\)
\(Y:\) \(1.7835349641994767\)

\(X:\) \(-11.040171059704\)
\(Y:\) \(22.767635789471395\)

\(i = 1\)

separation (mas) \(\rightarrow -11.0\)
delta, magnitude \(\rightarrow 1.0\)
e^\_curve max \(\rightarrow 0.675\)
e^\_curve min \(\rightarrow -0.544\)
e^\_curve pk-pk \(\rightarrow 1.159\)

shift (mas) \(\rightarrow -0.239\)
in fringes: number of points: 300
normalized difference: 0.048548
total fringe area: 0.041295

in wings: number of points: 400
normalized difference: 0.116278
total wings area: 0.006539

in wings: number of points: 300
normalized difference: 0.028614

in wings: number of points: 400
normalized difference: 0.007532

in wings: number of points: 300
normalized difference: 0.004635
**BINARY_FIT**

\[(x_{\text{min}}, x_{\text{max}}) = (-0.7, 0.7)\]

Enter desired range to plot: \(<\text{cr}>\) plots -0.2,0,2)

\(<\text{a}>\) plots all

Calling sm_plot

Warning: this program uses gets(), which is unsafe.

Does diff curve need to be adjusted? ....... ->

Output the plot to a file? ................. ->

Re-plot the data with different \((x,y)\) limits -->

\[n = 2\]

Separation (mas) ... -> 22.5

Delta magnitude ....... -> 1.8

s_curve max .......... -> 0.257

s_curve min .......... -> -0.546

s_curve pk-pk ........ -> 0.845

Shift (mas) ...... : 2.454

In fringe: number of points : 200

Normalized difference : 0.033757

Total fringe area : 0.037902

Total fringe diff : 0.001808

In wings: number of points : 400

Normalized difference : 0.027395

Total wings area : 0.029920

Total wings diff : 0.000521

\[(x_{\text{min}}, x_{\text{max}}) = (-0.7, 0.8)\]

Enter desired range to plot: \(<\text{cr}>\) plots -0.2,0,2)

\(<\text{a}>\) plots all

Calling sm_plot

Does diff curve need to be adjusted? ....... ->

Output the plot to a file? ................. ->

Re-plot the data with different \((x,y)\) limits -->
§6 OTHER ROUTINES

• TBINARY

1. This routine creates binary S-curves from point source S-curve. It allows the user to enter the parameters, (e.g. differential magnitude, separation, etc.) of the binary.

   \%>tbinary

2. When prompted for the FGS ID, enter the FGS number as a numeral with no letters.

   enter the FGS ID .......... -> 1

3. Enter which axis you are working with.

   enter FGS axis (x or y) ...... -> x

4. The filter information can be found in the <filename>.tab file. It will usually be F583W.

   enter the FGS FILTER ....... -> F583W

5. Enter the S-curve file name for a single star.

   enter file with input s-curve -> mt213.cx0s

6. Enter the magnitude of the single star, found as the actual target magnitude in the <filename>.tab file.

   enter magnitude of cal target -> 11.69

7. Name the output file. You can use a naming convention that includes the parameters you will enter for the binary. Example: mt213xm0.67s99.4 will be a binary created from the x-axis S-curve of MT213 with a magnitude difference of 0.67 and a separation of 99.4mas.

   enter name of the output file -> mt213xm0.67s99.4

8. Enter the separation in milliarcseconds (mas). The recreated binary curve will have the primary component centered on zero and the secondary offset by the separation along that axis. Positive is to the right and negative to the left.

   enter the separation (mas, real) ...... -> 99.4

9. Enter the magnitude difference between the primary and the recreated secondary.

   enter magnitude difference ........ -> 0.67

10. Enter the magnitude of the target system you are trying to model. For this case, that would be MT605.

    enter system magnitude ............ -> 11.74

11. You must enter ‘y’ or ‘n’ when prompted to correct for the background. You cannot leave this question blank.

    adjust the S-curves for background? ->

    TBINARY will output the binary file that you created with the specified parameters.
This program converts the UT date, in the form day of the year, to month and the day. It also takes into account leap years:

```
> utdate
enter date (mm/dd) or day of the year (ddd) -> 247
is this a leap year? -------------------------------> n
the calendar date is -> sep 4.
```

UTDATE also converts from calendar date to day of the year:

```
> utdate
enter date (mm/dd) or day of the year (ddd) -> 09/04
is this a leap year? -------------------------------> n
the ut date is -> 247
```

§7 USEFUL MAC TIPS & TRICKS

- **aqua** – text editor, equivalent to emacs
- **nedit** – text editor, equivalent to gedit

**Copy and paste string in X11:**

1. Highlight the string
2. Press Command key (apple key) and C to copy
3. Press Alt key and click scroll wheel on mouse to paste
The figures in Appendix B.2 were produced with the program BINXY.PRO (below).

This program makes use of both the wide and close binary detection IDL codes provided below.

```
pro binxy, starname, x, rsx, y, rsy
    ; Run detection procedures binfarcal.pro and binclosecal.pro
    ; for both x and y scans, and plot results.
    ; Input:
    ;   starname = star name for plot (string)
    ;   x = spatial grid for x-scan
    ;   rsx = rectified x-scan
    ;   y = spatial grid for y-scan
    ;   rsy = rectified y-scan
    ; Last four can be obtained with getxy.pro where
    ;   filenames = path and root name for scans
    ; Output:
    ;   - plot with name 'binxy_-'+starname+'.eps'
    ;   - summary data printed to screen and to file 'binxy_-'+starname+'.log'
    ;
    ; select calibrators
    getcals, x, y, xcal, ycal, xcss, ycss
    ; calculate sigma vectors for x and y from scatter in calibrators
    xsig = fltarr(n_elements(x))
    for i=0, n_elements(x)-1 do xsig(i) = stddev(xcss(i, *))
    ysig = fltarr(n_elements(y))
    for i=0, n_elements(y)-1 do ysig(i) = stddev(ycss(i, *))
    ; fix ends where cal=0
    gend = where((xsig eq 0.) and (x lt 0.), cnt)
    if (cnt gt 0) then xsig(gend) = mean(xsig(max(gend)+1:max(gend)+9))
    gend = where((xsig eq 0.) and (x gt 0.), cnt)
    if (cnt gt 0) then xsig(gend) = mean(xsig(min(gend)-9:min(gend)-1))
    gend = where((ysig eq 0.) and (y lt 0.), cnt)
    if (cnt gt 0) then ysig(gend) = mean(ysig(max(gend)+1:max(gend)+9))
    gend = where((ysig eq 0.) and (y gt 0.), cnt)
    if (cnt gt 0) then ysig(gend) = mean(ysig(min(gend)-9:min(gend)-1))
    ; run binfarcal.pro and form differences
    binfar, x, rsx, xcal, xpo, xccfr
    binfarcal, x, rsx, xcss, xo, xccfmr, xccfam, xfratio, xpeaksig, xmodel, xscaleratio
    xdiff = rsx - xmodel
    binfar, y, rsy, ycal, ypo, yccfr
    binfarcal, y, rsy, ycss, yo, yccfmr, yccfam, yfratio, ypeaksig, ymodel, yscaleratio
    ydiff = rsy - ymodel
    ;
    ; run binclosecal.pro
    ; binclose, x, rsx, xcal, xsig, xmean, xsig, xdiff, xddavs, xdiffex
    binclosecal, x, rsx, xcss, xfratio, xpeaksig, xam, xas
    ; binclose, y, rsy, ycal, ysig, ymean, ysig, ydiff, yddavs, ydiffex
    binclosecal, y, rsy, ycss, yfratio, ypeaksig, yam, yas
    print
    ;
    ; plot results
    !x.thick=2
    !y.thick=2
    !p.thick=4
    !p.charsize=1.5
    !p.multi=[0, 2, 3, 0, 1]
    !x.minor=2
    ; set up positional vectors for plots
```
\text{x0}=[0.16, 0.16, 0.16, 0.16, 0.62, 0.62, 0.62, 0.62, 0.16, 0.16]
\text{y0}=[0.75, 0.30, 0.06, 0.75, 0.30, 0.06]*0.0
\text{dx}=0.38

!p.font=0
set_plot,'ps'
device,filename='binxy_+starnx+_.eps'/landscape/encapsulated

; \text{ x ccf plot}
xcat=[xccfrm, xccfrs]
plot,xpo,xccfrm,xstyle=1,xrange=[min(x), max(x)],xtitle='y',ytitle='RCCF!8(x)!3',$
    \text{ xrange}=[\text{min}(\text{xcat})*1.1, \text{max}(\text{xcat})*1.1],ymminor=2,$
    \text{ xticklen=0.02+2, xtitlename=REPLICATE( ' ', 9), position=[x0(0),y0(0),x0(0)+dx,y0(0)+0.24],/nndata}
oplot,xpo,xccfrs,color=130
oplot,xpo,xccfrm
\text{nsigx}={\text{nex_elements(xfratio)}}
for \text{i}=0,\text{nsigx}-1 do oplot,\text{[0.0, .]}*xpseparation(\text{i}),[-1..1.],linestyle=2

; \text{ x S-curve plot}
plot,x,rsx,xstyle=1,xtitle='',yrange=[-0.75, 0.75],ystyle=1,ytitle='!8S(x)!3',$
    \text{xticklen}=0.02+2, \text{position}=[x0(1),y0(1),x0(1)+dx,y0(1)+0.45]$
oplot,x,xmodel,linestyle=2
xpos=0.7*(max(x)-min(x))+min(x)
ypos=0.58
xyouts,xpos,ypos,starname,size=1.
sppr=(max(rsx)-min(rsx))/(max(xcal)-min(xcal))
xyouts,xpos,ypos-0.1,'sppr = '+\text{string(sppr,format='(f4.2)')},size=1.
; print,' Excess flux from x sppr estimate = ',1./sppr -1.

; \text{ x difference plot}
xcat=[-abs(xsig), abs(xsig), xdiff]
plot,x,xdiff,xstyle=1,xtitle='!8x!3 (arcsec)',ytitle='!8S(x)-C(x)!3',$
    \text{ yrange}=[\text{min}(\text{xcat})*1.1, \text{max}(\text{xcat})*1.1],$\text{ yminor=2,}$
    \text{ xticklen=0.02+2, position=[x0(2),y0(2),x0(2)+dx,y0(2)+0.24],/nndata}$xx=[x, reverse(x)]
\text{yy}=[xsig, reverse(-xsig)]
polyfill,xx,yy,color=210
oplot,x,xdiff
\text{xdav}=deriv(x,xmodel) ; first derivative
\text{xddav}=deriv(x,xdav) ; second derivative
\text{gsmooth,xdav,xddavs,5.} ; smoothed version of second derivative
\text{xflag=strarr(nsigx)+' '}
for \text{i}=0,\text{nsigx}-1 do begin
if (xam(\text{i})/xas(\text{i}) ge 0.) then begin
  \text{answer}=' ,
  print, i,' x scan close coefficient / sigma = ',xam(i)/xas(i)
  read,' Plot second derivative for this component (y or n)? ',answer
  if (answer eq 'y') then begin
    gclose=where(abs(x-xpseparation(i)) lt 0.17,cnt)
    ;offset=0.1*(max(xcat)-min(xcat))\text{+max(xdiff(gclose))}
 xfit=xam(i)*xddavs ;+offset ; second derivative fit
 oplot,x(gclose),xfit(gclose),color=130,thick=4. ;color=30,thick=0.5
 xflag(i)='*'
  endif
endif
endfor

; \text{ y ccf plot}
cyat=[ycffrm, yccfrs]
plot,ypo,ycffrm,xstyle=1,xrange=[min(y), max(y)],xtitle='',ytitle='RCCF!8(y)!3',$
    \text{ yrange}=[\text{min}(\text{ycat})*1.1, \text{max}(\text{ycat})*1.1],ymminor=2,$
    \text{ xticklen=0.02+2, xtitlename=REPLICATE( ' ', 9), position=[x0(3),y0(3),x0(3)+dx,y0(3)+0.24],/nndata}$
oplot,ypo,ycfrs,color=130
oplot,ypo,ycffrm
\text{nsigy}={\text{nex_elements(yfratio)}}
for \text{i}=0,\text{nsigy}-1 do oplot,\text{[0.0, .]}*ypseparation(\text{i}),[-1..1.],linestyle=2

; \text{ y S-curve plot}
plot,y,rsy,xstyle=1,xtitle='',yrange=[-0.75,0.75],ystyle=1,ytitle='!8S(y)!3',$
xtickname=REPLICATE(' ',9),position=[x0(4),y0(4),x0(4)+dx,y0(4)+y0(4)+0.45]

plot,y,ymodel,linestyle=2
xpos=0.7*(max(y)-min(y))+min(y)
ypos=0.58
xyouts,xpos,ypos,starname,size=1.
sppr=(max(rsy)-min(rsy))/(max(ycal)-min(ycal))
xyouts,xpos,ypos-0.1,'sppr = '+string(sppr,format='(f4.2)'),size=1.
;print,' Excess flux from y sppr estimate = ',1./sppr -1.

; y difference plot
ycat=[-abs(ysig),abs(ysig),ydiff]
plot,y,ydiff,xstyle=1,xtitle='!8y!3 (arcsec)',ytitle='!8S(y)-C(y)!3',$
yrange=[min(ycat)*1.1,max(ycat)*1.1], $ $ xticklen=0.02*2,position=[x0(5),y0(5),x0(5)+dx,y0(5)+0.24],/nodata

xx=[y,reverse(y)]
yy=[ysig,reverse(-ysig)]
polyfill,xx,yy,color=210

oplot,y,ydiff
ydav=deriv(y,ymodel); first derivative
yddav=deriv(y,ydav); second derivative
gsmooth,yddav,yddavs,5.; smoothed version of second derivative
yflag=strarr(nsigy)+'
for i=0,nsigy-1 do begin
  if (yam(i)/yas(i) ge 0.) then begin
    answer=','
    print, i,' y scan close coefficient = ',yam(i)/yas(i)
    read,' Plot second derivative for this component (y or n)? ',answer
    if (answer eq 'y') then begin
      gclose=where(abs(y-ypseparation(i)) lt 0.17,cnt)
      offset=0.1*(max(ycat)-min(ycat))+max(ydiff(gclose))
yfit=yam(i)*yddavs; +offset; second derivative fit
      oplot,y(gclose),yfit(gclose),color=130,thick=4.
yflag(i)='*'
    endif
    endif
endfor
device,/close

set_plot,'x'
!p.font=-1
!p.multi=0
; print results
print,''
print,' x primary fringe scale ratio = ',xscaleratio,format='(a,f5.2)'
print,' y primary fringe scale ratio = ',yscaleratio,format='(a,f5.2)'
print,' x/y = ',' fr ',' sep. ',' S/N = ',' a10-6++ ' a10-6++ ' a/s '
for i=0,nsigx-1 do print,' x: ',xfratio(i),xpseparation(i),xpeaksig(i), 1.e6*xam(i),1.e6*xas(i),xam(i)/xas(i),xflag(i), format='(a,f6.3,f8.4,f7.2,3f9.2,a)'
for i=0,nsigy-1 do print,' y: ',yfratio(i),ypseparation(i),ypeaksig(i), 1.e6*yam(i),1.e6*yas(i),yam(i)/yas(i),yflag(i), format='(a,f6.3,f8.4,f7.2,3f9.2,a)'
openw,1,'binxy_'+starname+'.log'
printf,1,starname
printf,1,' x primary fringe scale ratio = ',xscaleratio,format='(a,f5.2)'
printf,1,' y primary fringe scale ratio = ',yscaleratio,format='(a,f5.2)'
printf,1,' x/y = ',' fr ',' sep. ',' S/N = ',' a10-6++ ' a10-6++ ' a/s '
for i=0,nsigx-1 do printf,1,' x: ',xfratio(i),xpseparation(i),xpeaksig(i), 1.e6*xam(i),1.e6*xas(i),xam(i)/xas(i),xflag(i), format='(a,f6.3,f8.4,f7.2,3f9.2,a)'
for i=0,nsigy-1 do printf,1,' y: ',yfratio(i),ypseparation(i),ypeaksig(i), 1.e6*yam(i),1.e6*yas(i),yam(i)/yas(i),yflag(i), format='(a,f6.3,f8.4,f7.2,3f9.2,a)'
close,1
pro rtf, file, x, s
openr, 1, file
x=fltarr(10000)
s=fltarr(10000)
n=0
while (not EOF(1)) do begin
  readf, 1, a, b
  x(n)=a
  s(n)=b
  n=n+1
endwhile
x=x(0:n-1)
s=s(0:n-1)
close, 1
return
end

pro getxy, filein, x, rsx, y, rsy
; read in and rectify scans
rtf, filein+'.cx0s', x, sx
recttf, 0, x, sx, rsx
rtf, filein+'.cy0s', y, sy
recttf, 1, y, sy, rsy
return
end

pro getcals, x, y, xcal, ycal, xcss, ycss
; get calibrator list
caltab='cygob2-cal.txt'
openr, 1, caltable
a=' '
readf, 1, a
n=0
name=strarr(1000)
bmv=fltarr(1000)
date=strarr(1000)
while (not EOF(1)) do begin
  readf, 1, a
  ;012345678901234567890123456789
  ;mt259 1.00 5/17/06
  name(n)=strtrim(strmid(a,0,7),2)
bmv(n)=float(strmid(a,7,5))
date(n)=strtrim(strmid(a,16,8),2)
n=n+1
endwhile
close, 1
name=name(0:n-1)
bmv=bmv(0:n-1)
date=date(0:n-1)

; print calibrator list
print, ' List of calibrators: '
for i=0,n-1 do print, i, name(i), bmv(i), date(i), format='(i4,a7,f6.2,a12)'

; select calibrators for use
read, ' Index number of primary calibrator = ', ipcal
read, ' Index number of lowest B-V calibrator = ', ical
read, ' Index number of highest B-V calibrator = ', i2cal

; Transform calibrators to observed grid
xgridv=readfits('xgridv.fits')
xcalss=readfits('xcalss.fits')
ygridv=readfits('ygridv.fits')
ycalss=readfits('ycalss.fits')
xcal=interpol(xcalss(*,i1cal),xgridv,x)
ycal=interpol(ycalss(*,i1cal),ygridv,y)
ncss=i2cal-i1cal+1
xcss=fltarr(n_elements(x),ncss)
ycss=fltarr(n_elements(y),ncss)
for i=0,ncss-1 do begin
   xcss(0,i)=interpol(xcalss(*,i1cal+i),xgridv,x)
   ycss(0,i)=interpol(ycalss(*,i1cal+i),ygridv,y)
endfor
return
end

;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;
@recttf.pro
@bincfarcal.pro
@binclosecal.pro
;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;}

Systems with separations > 20 mas and $\Delta m_{F583W} \gtrsim 4.0$ mag can be detected using BIN-FAR.PRO. This program applies the cross-correlation method described in section 3.3.1.2.
; pad TF with extra zeros
dxg=mean(deriv(xg))
ns=n_elements(xg)
nsd2=fix(max(abs(xg))/dxg+0.5)
nsd2=fix((ns-1)/2)
n=2*nsd2+1 ; odd number for ccf
pad=fltarr(ns)
sp=[pad,stf,pad]
sa=[pad,av,pad]
xstep=mean(deriv(xg))
po=(findgen(n)-nsd2)*xstep

; run the cross correlation
crosscore,sa,sa,n,5,ccfa,zero,ezero
crosscore,sp,sa,n,5,ccf,zero,ezero

; remove primary contribution
ccfmax=max(ccf)
ccfmaxa=max(ccfa)
f1=ccfmax/ccfmaxa
fshift,ccfa,zero,ccfas
ccfas=ccfas*f1
ccfr=ccf-ccfas
f2=max(ccfr)/ccfmaxa
psep=(!c - nsd2 - zero)*xstep
fre=f2/f1

return
end

; PRO CROSSCORE,a,z,n,nfit,c,zero,ezero
; cross-correlation with error estimates
; assumes some padding on ends so that edge effects are negligible
; Input:
; a = spectrum to measure
; z = template spectrum
; n = number of points in ccf (odd)
; nfit = number of points for parabolic fit around minimum (odd)
; Output:
; c = ccf
; zero = shift of spectrum from template in pixels
; ezero = error in zero

; get offsets
n=fix(n)
nd2=fix((n-1.)/2.)
s=indgen(n)-nd2

; form continuum subtracted spectra
ntot=n_elements(a)
fn=a-mean(a)
zn=mean(z)
sf2=total(fn^2)/ntot
sg2=total(gn^2)/ntot
sf=sqrt(sf2)
sg=sqrt(sg2)

; form ccf
c=fltarr(n)
for i=0,n-1 do begin
  gns=shift(gn,a(i))
  c(i)=total(fn*gns)/ntot/sf/sg
endfor
g=[indgen(20),n-1-reverse(indgen(20))]
back=mean(c(g))
c=(c-back)/(1.-back)

; find peak
top=max(c)
toppix=C
nlim=3
if (!c lt 2) or (!c gt nlim) then begin
    print, ' zero too close to edge of cc function !'
    print, ' returning 0 ...
    zero=0.
ezero=0.
endif else begin
    nfitd2=fix((nfit-1)/2.)
    cf=poly_fit(-nfitd2:findgen(nfit),c(!c-nfitd2:!c+nfitd2),2)
    ptop=-0.5*cf(1)/cf(2)
    zero=toppix+ptop*nd2
    ctop=cf(0)+cf(1)*ptop+cf(2)*ptop^2
    good=where(z ne 1.0,neff)
    ezero=-neff*2.*cf(2)*ctop/(1.-ctop^2)
    ezero=1./sqrt(ezero)
    ;print,' minimum position of cc = ', zero,' +/- ',ezero
    endelse
return
end

;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;
@~/IDL/gsmooth.pro
;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;;

;%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
BINFARCAL.PRO uses BINFAR.PRO to test the detection limits of the Cross-Correlation method based on calibrator S-curve comparisons.
;%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
pro binfarcal,xg,stf,ss,po,ccfrm0,ccfrs,ccfam,fratio,pseparation,peaksig,model,scaleratio
; Test detection limits of wide binary signal using binfar.
; Run with multiple calibrator TFs to get uncertainties in signal.
; Check if detected peak exceeds 3 sigma.
; Input:
; xg = grid of scan position (arcsec)
; stf = Transfer Function of target on xg grid
; ss = Transfer Functions of calibrators in matrix (xpos,cal#)
; Output:
; po = grid for ccf in arcsec
; ccfrm0 = mean of ccfs with primary removed
; ccfrs = standard deviation of residual ccfs
; ccfam = mean of calibrator autocorrelations
; fratio = vector of flux ratio estimates
; pseparation = vector of estimates of separation in arcsec
; peaksig = vector of ratios of ccfr amplitude to sigma at peak
; model = model S-curve based on identified components and mean calibrator
; scaleratio = ratio of observed to predicted fringe amplitude for primary
; run do loop of binfar for each calibrator in ss
ncal=n_elements(ss(0,*))
for i=0,ncal-1 do begin
    binfar,xg,stf,sspo,ccfrm0,ccfrs,ccfam,fratio,pseparation,peaksig,model,scaleratio
    ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; ; }
ccfrall=fltarr(nccf,ncal)
ccfaall=fltarr(nccf,ncal)
;fratio=fltarr(ncal)
pseparation=fltarr(ncal)
ampratio=fltarr(ncal) ; for primary fringe
endif
ccfrall(*,i)=ccf ; ccf
ccfaall(*,i)=ccfa
;fratio(i)=fre
;pseparation(i)=psep
ampratio(i)=ar
endfor

; Determine mean ccf and uncertainty vector
ccfrm=fltarr(nccf)
ccfrs=fltarr(nccf)
ccfam=fltarr(nccf)
for i=0,nccf-1 do begin
  ccfrm(i)=mean(ccfrall(i,*)) ; mean ccf
  ccfrs(i)=stddev(ccfrall(i,*)) ; standard deviation
  ccfam(i)=mean(ccfaall(i,*)) ; mean autocorrelation
endfor

; adjust sigma at boundaries for artificial decline
; due to zero padding
g=where(po lt (min(po)+0.1))
sigav=mean(ccfrs((max(g)+1):(max(g)+10)))
ccfrs(g)=sigav

; amplitude parameters for average calibrator
calav=rebin(ss,n_elements(xg),1) ; average calibrator
calmin=min(calav)
xmin=xg(!c)
calmax=max(calav)
xmax=xg(!c)
ampcalibrator=calmax-calmin

; Find most significant peaks and check significance
fratio=fltarr(100)
pseparation=fltarr(100)
peaksig=fltarr(100)
; sampratio=fltarr(100)
nsig=0

; set parameters for primary
;fratio(nsig)=1.
pseparation(nsig)=0.
;peaksig(nsig)=100.
sig=nsig+1

; iteration for other signals
ccfres=ccfrm
peakinclude='y'
xstep=mean(deriv(xg))
answer=''
ccfinclude=0.*ccfres+1.
minxg=min(xg)
maxxg=max(xg)
while(peakinclude eq 'y') do begin
  maxtestpeak=max(ccfres*ccfinclude)
  maxtestpeak=max(ccfres)
  if (!c eq 0) then begin
    print,' Warning: maximum found at edge.'
!c=1
endif
if (!c eq n_elements(xg)-1) then begin
  print,' Warning: maximum found at edge.'
  !c=n_elements(xg)-2
endif
peakcentroid=total(ccfres(!c -1:!c +1)*po(!c -1:!c+1))/total(ccfres(!c -1:!c +1))
fsfshift,ccfam,peakcentroid/xstep,ccfams
scale=mean(ccfres(!c -1:!c +1)/ccfams(!c -1:!c +1))
ccfams=scale*ccfams
plot,po,ccfres,xstyle=1
oplot,po,ccfams,linestyle=1
oplot,po,ccfrs,linestyle=2
oplot,[peakcentroid,peakcentroid],[-1.,1.],linestyle=1
read,' Press <CR> to continue ... ',answer
plot,po,ccfres,xrange=[-0.1,0.1]*po(!c)
oplot,po,ccfams,linestyle=1
oplot,po,ccfrs,linestyle=2
oplot,[peakcentroid,peakcentroid],[-1.,1.],linestyle=1
; over plot limits of scan
oplot,[minxg,minxg],[-1.,1.],linestyle=9
oplot,[maxxg,maxxg],[-1.,1.],linestyle=9
print,' Candidate peak location = ',peakcentroid
print,' (max ccf) / sigma = ',ccfres(!c)/ccfrs(!c)
; query about inclusion and if included save parameters
peakok='y'
read,' Include this peak (y or n)? ',peakok
if (peakok eq 'y') then begin
  pseparation(nsig)=peakcentroid
  peaksig(nsig)=ccfres(!c)/ccfrs(!c)
  fratio(nsig)=scale
  ccfres=ccfres-ccfams
  ; keep residual ccf with primary only removed
  if (nsig eq 0) then ccfrm0=ccfres ; for plotting purposes
  nsig=nsig+1
endif
; omit this region from next search
gomit=where(ccfams gt 0.2*max(ccfams),cnt)
if (cnt gt 0) then ccfinclude(gomit)=0.
; query for another iteration
read,' Try another iteration (y or n)? ',peakinclude
endwhile
fratio=fratio(0:nsig-1)
pseparation=pseparation(0:nsig-1)
peaksig=peaksig(0:nsig-1)
; sampratio=sampratio(0:nsig-1)

; model of S-curve
model=0.*stf
for i=0,nsig-1 do begin
  fshift,calav,pseparation(i)/xstep,calavs
  calavs=fratio(i)*calavs
  model=model+calavs
endfor
;plot,xg,stf
;oplot,xg,model,linestyle=1

; check peak-to-peak amplitude scaling
scaleobs=mean(ampratio)
scalemod=(max(model)-min(model))/ampcalibrator
model=model*scaleobs/scalemod

; revise fratio for peak-to-peak amplitude scaling
Binary systems with a separation lower than 25 mas can be detected using BINCLOSE.PRO.
The detection limits of the second derivative method were estimated using BINCLOSECAL.PRO applied to several calibrator S-curves.
Along with the BINARY_FIT procedure, we used other approaches to analyze multiple systems where BINARY_FIT is unable to do so, like the case for the triple MT417. The codes, TRIPFIT.PRO and TTRIPLE.PRO, developed for treating triples and widely separated systems are included below.

```idl
PRO TRIPFIT,xy
;This program finds the best fit for a triple or binary system using
;the IDL the least-squares approach of MPFIT along one axis (X or Y)
; Input:
; xy = 0 for x-axis, 1 for y-axis

;Common variables of Calibrator S-curves
COMMON TFIT,x1,s1,x2,s2,x3,s3

;Structure used to determine if parameters are to be fixed or floated
TSTRUC = {FIXED:0,$ ;Tells MPFIT to fix or float parameter (0-float, 1-fix)
   LIMITED:[0,0],$ ;Tells MPFIT whether to limit the lower/upper side
   LIMITS:[0.D,0.D]$;If limited, then the lower/upper limits
}

;Structure array, where each element of the array corresponds to the following:
;TSTRUC[0] = Position of Primary
;TSTRUC[1] = Position of Secondary
;TSTRUC[2] = Position of Tertiary
;TSTRUC[3] = Flux of secondary to primary
;TSTRUC[4] = Flux of tertiary to primary
TSTRUC = REPLICATE(tstruc,5)

;Read in binary or triple data
READ, file1, PROMPT='What is the filename of the system? (e.g. mt59.cx0s) '
RDCRV, file1, xs, ss
recttf, xy, xs, ss, rs
ss=DOUBLE(rs)

;Read in model data for primary
READ, file2, PROMPT='What is the filename of the primary calibrator? (e.g. mt601.cx0s) '
RDCRV, file2, x1, sx
recttf, xy, x1, sx, s1 ;flattens and normalizes the s-curve to zero
```
; Read in model data for secondary

csm = ''
file=''
READ,'Use primary calibrator for secondary? (y or [n])',csm
IF (CSM EQ 'y') OR (CSM EQ 'Y') THEN BEGIN
  x2 = x1
  s2 = s1
ENDIF ELSE BEGIN
  READ,file,PROMPT='What is the filename of the secondary calibrator? '
  RDCRV,file,x2,sx
  recttf,xy,x2,sx,s2 ; flattens and normalizes the s-curve to zero
ENDELSE

; Read in model data for tertiary

csm = ''
READ,'Use primary calibrator for tertiary? (y or [n])',csm
IF (CSM EQ 'y') OR (CSM EQ 'Y') THEN BEGIN
  x3 = x1
  s3 = s1
ENDIF ELSE BEGIN
  READ,file,PROMPT='What is the filename of the tertiary calibrator? '
  RDCRV,file,x3,sx
  recttf,xy,x3,sx,s3 ; flattens and normalizes the s-curve to zero
ENDELSE

; Pad S-curve with zeroes for extrapolation when shifting and scaling
; to create artificial triple s-curves
x1 = [min(x1)-0.002,min(x1)-0.001,x1,max(x1)+0.001,max(x1)+0.002]
s1 = [0.,0.,s1,0.,0.]
x2 = [min(x2)-0.002,min(x2)-0.001,x2,max(x2)+0.001,max(x2)+0.002]
s2 = [0.,0.,s2,0.,0.]
x3 = [min(x3)-0.002,min(x3)-0.001,x3,max(x3)+0.001,max(x3)+0.002]
s3 = [0.,0.,s3,0.,0.]

;;;;;;;;;;;;;;;;;;;;;; Component Relative Brightness;;;;;;;;;;;;;;;;;;;;;;
fix = ''
READ,dm12,PROMPT='Enter diff mag between the primary and secondary'
if12 = 10.^(-dm12/2.5)
READ,fix,PROMPT='Limit the delta mag? (y or [n])'
IF (FIX EQ 'y') OR (FIX EQ 'Y') THEN BEGIN
  TSTRUC[3].LIMITED = [1,1]
  TSTRUC[3].LIMITS = [if12-0.05,if12+0.05] ; Limits the delta mag between +/- 1 dex
ENDIF

PRINT,'Enter diff mag between the primary and tertiary'
READ,dm13,PROMPT='(Negative number indicates there is no 3rd component): '
if (dm13 lt 0.) then BEGIN
  if13 = 0.
  TSTRUC[4].FIXED = 1 ; Fix flux ratio of tertiary to primary
  TSTRUC[2].FIXED = 1 ; Fix position of the tertiary
ENDIF else BEGIN
  if13 = 10.^(-dm13/2.5)
  READ,fix,PROMPT='Limit the delta mag? (y or [n])'
  IF (FIX EQ 'y') OR (FIX EQ 'Y') THEN BEGIN
    TSTRUC[4].LIMITED = [1,1]
    TSTRUC[4].LIMITS = [if13-0.05,if13+0.05] ; Limits the delta mag between +/- 1 dex
  ENDIF
ENDIF

;;;;;;;;;;;;;;;;;;;;;; Select Component Positions;;;;;;;;;;;;;;;;;;;;;;;
SET_PLOT,'X'
PLOT,xs,ss,XSTYLE=1
OPlot,[min(xs),max(xs)],[0,0],LINESTYLE=1
PRINT, 'Click zero crossing point of primary'
CURSOR,x,y
WAIT,1
ipos1 = x ;position of the primary
PRINT, 'Click zero crossing point of secondary'
PRINT, 'Click "off-screen" if not in scan'
CURSOR,x,y
WAIT,1
ipos2 = x ;position of the secondary
;If user clicks off screen
IF (ipos2 GT max(ss)) OR (ipos2 LT min(ss)) THEN TSTRUC[1].FIXED = 1
PRINT, 'Click zero crossing point of tertiary'
PRINT, 'Click "off-screen" if not in scan or there is no third component'
CURSOR,x,y
WAIT,1
ipos3 = x ;position of the tertiary
;If user clicks off screen
IF (ipos3 GT max(ss)) OR (ipos3 LT min(ss)) THEN TSTRUC[2].FIXED = 1

;;;;;;;;;;;;;;;;;;;;Calculate best fit;;;;;;;;;;;;;;;;;;;;;;;
iparm = [ipos1,ipos2,ipos3,if12,if13] ;starting parameter fed into MPFITFUN
for i=0,1 do begin ; iterate noise model with better parameters
    noisecalc,xy,xs,ss,iparm,noise
    parm = MPFITFUN('ttriple', xs, ss, noise, iparm, perror=perror, $  
                        bestnorm=bestnorm, dof=dof, /quiet,PARINFO=TSTRUC)
    IF N_ELEMENTS(DOF) EQ 0 THEN DOF = N_ELEMENTS(X) - N_ELEMENTS(PARM)
    rchisqr=bestnorm/dof
    iparm=parm
endfor

;Calculate parameters in term of separation and delta mag, convert to 
;STRING for output
;Secondary to Primary
sep12 = STRTRIM(STRING((parm[1]-parm[0])*1000.),2)
perror12 = STRTRIM(STRING(1000.*SQRT((perror[0]^2.)+(perror[1]^2.))),2)
dm12 = STRTRIM(STRING(-2.5*ALOG10(parm[3])),2)
dmerr12 = STRTRIM(STRING(ABS(perror[3]*(-2.5)/(parm[3]*ALOG(10.)))),2)

;Tertiary to Primary
sep13 = STRTRIM(STRING((parm[2]-parm[0])*1000.),2)
perror13 = STRTRIM(STRING(1000.*SQRT((perror[0]^2.)+(perror[2]^2.))),2)
dm13 = STRTRIM(STRING(-2.5*ALOG10(parm[4])),2)
dmerr13 = STRTRIM(STRING(ABS(perror[4]*(-2.5)/(parm[4]*ALOG(10.)))),2)

;Print to screen
print, ' Reduced chi-squared of fit = ',rchisqr  
print, ' Separation of 1 to 2 (mas) = ', sep12, ' +/- ', perror12  
print, ' Separation of 1 to 3 (mas) = ', sep13, ' +/- ', perror13  
print, ' Delta mag 1 to 2 = ', dm12, ' +/- ', dmerr12  
print, ' Delta mag 1 to 3 = ', dm13, ' +/- ', dmerr13  
smodel=ttriple(xs,parm) ;calculate model with best fit parameters
plot,xs,ss,xstyle=1
oplot,xs,smodel,linestyle=1
file1 = STRSPLIT(file1,'.',/EXTRACT)
file2 = STRSPLIT(file2,'.',/EXTRACT)

;Output results to file
if xy EQ 0 then ax = 'x' else ax='y'
TRIPFIT.PRO creates a triple model with given separation and differential magnitudes.

FUNCTION TTRIPLE,x,param
; Creates artificial triple or binary S-curves from a single S-curve
; along one axis based on the input parameters
; Called by TRIPFIT
; x = input angular grid points
; param(0) = pos1 = position primary S-curve zero point crossing (DOUBLE)
; param(1) = pos2 = position secondary S-curve zero point crossing (DOUBLE)
; param(2) = pos3 = position tertiary S-curve zero point crossing (DOUBLE)
; param(3) = f12 = flux ratio of primary to secondary (DOUBLE)
; param(4) = f13 = flux ratio of primary to tertiary (DOUBLE)

COMMON TFIT,x1,s1,x2,s2,x3,s3
; TFIT COMMON block of the transfer function for the three components.
; The curves are assumed to be "rectified" and "normalized" to zero. The common block is initialized by TRIPFIT.PRO

;**************************************************************
; RELATIVE BRIGHTNESS
;**************************************************************

f12=param(3)
f13=param(4)
br1 = 1. / (1. + f12 + f13) ; brightness factor
br2 = f12 / (1. + f12 + f13)
br3 = f13 / (1. + f12 + f13)
Due the large variation in the appearance of the wings of MT 417, the S-curves were rectified and normalized with RECTTF.PRO for more accurate comparison when creating the models in TRIPFIT.PRO.
; rs = rectified transfer function (to zero in wings)

; rectify TF

if (xy eq 0) then xlim=0.384
if (xy eq 1) then xlim=0.391

g=where(abs(x) ge xlim)

;if (xy eq 0) then c=[0.0312817d, 0.0563176d,-0.0220172d] ;original
if (xy eq 0) then c=[0.0298723d, 0.0556486d,-0.0159361d]
if (xy eq 1) then c=[-0.0922049d,0.0210046d,-0.0181979d]
fit=poly(x,c)

rs=s-fit ; subtract off parabolic part

offset=mean(rs(g))

rs=rs-offset

plot,x,s
oplot,x,fit+offset,linestyle=1

answer=' 
read,' Correct fit (y or [n])? ',answer

if ((answer eq 'y') or (answer eq 'Y')) then begin ; better fit

fit: print,' Select rectification points; select above range to end.'

n=0
xf=fltarr (1000)
xcursor=0.
while (xcursor lt max(x)) do begin
cursor,xcursor,y & wait,1
xf(n)=xcursor
print,xcursor
n=n+1
endwhile

xf=xf(0:n-2)

dx=0.05 ; +/- dx arcsec around each sample point
nxf=n_elements(xf)
xfit=fltarr(nxf)
yfit=fltarr(nxf)
for i=0,nxf-1 do begin

g=where(abs(x-xf(i)) le dx)

xfit(i)=mean(x(g))
yfit(i)=mean(s(g))
endfor

sfit=spline(xfit,yfit,x)

plot,x,s
oplot,x,sfit,linestyle=1

redo=' 
read,' Try another fit (y or [n])? ',redo

if ((redo eq 'y') or (redo eq 'Y')) then goto, fit

rs=s-sfit

; cut off edges

g=where((x ge min(xfit)) and (x le max(xfit)))

x=x(g)
s=s(g)
rs=rs(g)

endif

; align transfer function at zero crossing
glim=where(abs(x) lt 0.018)
xcen=interpol(x(glim),rs(glim),[0.])
PANGLE.pro calculates the position angle of system given the separation in $x$ and $y$ and the position angle of the aperture. The position angle of the aperture is given in the header TAB file outputted by CALFGSA.

```idl
xstep=mean(deriv(x))
pshift=xcen(0)/xstep
fshift,rs,pshift,rss
rs=rss
return
end

@~/IDL/fshift.pro

;%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

PRO PANGLE,x,y,pa_aper,pa

;x and y are the separation in arcseconds along the corresponding axis
;pa_aper is the aperture position angle read in from the '.tab' file

ifgs = 1 ;assume using FGS 1r

;Determine the quadrants where the quadrants correspond to:
;4|1
;---
;3|2
IF (x GE 0 AND y GE 0) THEN quad = 1
IF (x LT 0 AND y GE 0) THEN quad = 2
IF (x LT 0 AND y LT 0) THEN quad = 3
IF (x GE 0 AND y LT 0) THEN quad = 4

;Determine the angle with respect to the y-axis going clockwise
IF (x EQ 0 AND y GE 0) THEN BEGIN ;Along positive y-axis
phi = 0.D
ENDIF ELSE BEGIN
IF (x EQ 0 AND y LT 0) THEN BEGIN ;Along negative y-axis
phi = 180.D
ENDIF ELSE BEGIN
IF (x GT 0 AND y EQ 0) THEN BEGIN ;Along positive x-axis
phi = 90.D
ENDIF ELSE BEGIN
IF (x LT 0 AND y EQ 0) THEN BEGIN ;Along negative x-axis
phi = 270.D
ENDIF ELSE BEGIN
ratio = ABS(x/y)
phi = ((180.D)/!pi)*ATAN(ratio) ;Any point not on an axis
IF Quad EQ 2 THEN phi = -1.*phi
IF Quad EQ 3 THEN phi = phi-180.D
IF Quad EQ 4 THEN phi = 180.-phi
ENDIF ELSE BEGIN
pa = pa_aper+phi
IF (pa GE 360.) THEN pa = pa - 360.
IF (pa LT 0.) THEN pa = pa +360.
END
ENDIF ELSE BEGIN
END
END

;%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```
B.2 FGS S-Curves

All transfer functions included in this sample are provided here ordered by the primary star name. The figures show the $x$- (left) and $y$-axis (right) rectified $S$-curves in the central panel. Also shown as a dashed line is a preliminary model fit based upon the components derived from the CCF analysis and the mean $S$-curve of the calibrator set selected. The top panel plots the mean of the residual cross-correlation functions of the target with the calibrator (solid, black lines) after the peak of the primary has been subtracted off. The vertical dashed line indicates the position of each component resolved. The solid, gray lines show the standard deviation of the residual cross-correlation functions, and a peak is considered significant only if the mean residual CCF (black) exceeds the standard deviation (gray) by four times. The bottom panels show the difference curves between the star $S$-curves and that of the calibrator (solid line). The shaded, gray region is the uncertainty envelope determined from the standard deviation at each point along the $S$-curves for the calibrators (Fig. 3.8). In the case where the second derivative test resolved a close pair (MT 304), the second derivative function of the calibrator is overplotted and scaled by the $a$ coefficient (solid, gray line).
Figure B.1 A 23 $S$-curves
Figure B.2 A 27 S-curves
Figure B.3 A 41 $S$-curves
Figure B.4 A 46 $S$-curves
Figure B.5 MT 5 S-curves
Figure B.6 MT 59 S-curves
Figure B.7 MT 70 S-curves
Figure B.8 MT 83 $S$-curves
Figure B.9 MT 138 S-curves
Figure B.10 MT 145 S-curves
Figure B.11 MT 213 S-curves
Figure B.12 MT 217 S-curves
Figure B.13 MT 227 $S$-curves
Figure B.14 MT 250 S-curves
Figure B.15 MT 258 S-curves
Figure B.16 MT 259 S-curves
Figure B.17 MT 299 S-curves
Figure B.18 MT 304 S-curves
Figure B.19 MT 317 S-curves
Figure B.20 MT 339 S-curves
Figure B.21 MT 376 S-curves
Figure B.22 MT 390 S-curves
Figure B.23 MT 403 S-curves
Figure B.24 MT 417 S-curves
Figure B.25 MT 429 S-curves
Figure B.26 MT 431 S-curves
Figure B.27 MT 448 S-curves
Figure B.28 MT 455 S-curves
Figure B.29 MT 457 S-curves
Figure B.30 MT 462 S-curves
Figure B.31 MT 465 S-curves
Figure B.32 MT 470 S-curves
Figure B.33 MT 473 S-curves
Figure B.34 MT 480 S-curves
Figure B.35 MT 483 S-curves
Figure B.36 MT 485 S-curves
Figure B.37 MT 507 S-curves
Figure B.38 MT 516 S-curves
Figure B.39 MT 531 S-curves
Figure B.40 MT 534 S-curves
Figure B.41 MT 555 $S$-curves
Figure B.42 MT 556 S-curves
Figure B.43 MT 588 S-curves
Figure B.44 MT 601 S-curves
Figure B.45 MT 605 S-curves
Figure B.46 MT 611 S-curves
Figure B.47 MT 632 S-curves
Figure B.48 MT 642 S-curves
Figure B.49 MT 692 S-curves
Figure B.50 MT 696 $S$-curves
Figure B.51 MT 734 S-curves
Figure B.52 MT 736 S-curves
Figure B.53 MT 745 S-curves
Figure B.54 MT 771 S-curves
Figure B.55 MT 793 S-curves
Figure B.56 Schulte 5 $S$-curves
Figure B.57 Schulte 73 S-curves
Figure B.58 WR 145 S-curves