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# Aspects of Tax Spillovers: Is There a "Worldwide" Tax Burden?

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# ASPECTS OF TAX SPILLOVERS: IS THERE A “WORLDWIDE” TAX BURDEN?

by

Sandeep Bhattacharya

A Dissertation Submitted in Partial Fulfillment
of the Requirements for the Degree
of
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in the
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# ACCEPTANCE

This dissertation was prepared under the direction of the candidate's Dissertation Committee. It has been approved and accepted by all members of that committee, and it has been accepted in partial fulfillment of the requirements for the degree of Doctor of Philosophy in Economics in the Andrew Young School of Policy Studies of Georgia State University.

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ABSTRACT

ASPECTS OF TAX SPILLOVERS: IS THERE A “WORLDWIDE” TAX BURDEN?

By
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The objective of this dissertation is to develop a model to examine the concept of a “worldwide” tax burden. The notion is that due to differential mobility of factors developed nations may be passing on a share of their tax burden to less developed countries while effectively indulging in a form of tax competition. This is important for many reasons especially since it may affect the distribution of income between countries, and influence the flow of capital. As globalization increases, “the race to the bottom” in taxation (which implies tax-cutting) suggests that these spillovers should be reduced over time. The traditional view of taxation implies that taxation imposes an excess burden and increasing most types of taxes will increase this burden. But for whom does this burden increase? Are developed countries passing on a burden to locations that are less able to shift the burden forward?

If this phenomenon of tax spillovers can be quantified, we can examine the extent and nature of shifting of the tax burden. Using a version of the famous general equilibrium model first developed by Prof Harberger in 1962, we analyze the extent of tax spillovers in the presence of a public input in an open economy setting. We model two different taxes, the Capital Income Tax and a Consumption Tax and two different types of expenditure patterns, a government input and a transfer payment.

The dissertation answers the following research questions:

- Can the extent of tax spillovers be quantified using a general equilibrium model that is not dependent on functional forms?
- Does the extent of spillovers depend on the type of tax used?
- Does the extent of spillovers depend on the use to which the taxes are put?
- What are the policy implications?

We find that the tax cutting economy can gain from cutting a distorting tax only when the expenditure pattern is neutral, while imposing a cost to the rest of the world in terms of sources and uses of GDP. When revenues are used to provide productive public goods; neither country gains from tax cuts that lower inputs.

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# CHAPTER I - INTRODUCTION

The objective of this dissertation is to develop a model to examine the concept of a “worldwide” tax burden. The notion is that due to differential mobility of factors developed nations may be passing on a share of their tax burden to less developed countries while effectively indulging in a form of tax competition. This is important for many reasons especially since it may affect the distribution of income between countries, and influence the flow of capital. As globalization increases, “the race to the bottom” in taxation (which implies tax-cutting) suggests that these spillovers should be reduced over time. The traditional view of taxation implies that taxation imposes an excess burden and increasing most types of taxes will increase this burden. But for whom does this burden increase? Are developed countries passing on a burden to locations that are less able to shift the burden forward?

There are two views of what might happen in situations of tax competition. The first is the prediction from theory that countries competing for mobile factors like capital will cut tax rates (this is only of the several different ways they might compete).

The other is the observation that the tax-cutting race to the bottom that the theory predicts has not actually happened in practice, and tax rates have fallen little over the last decade in many OECD countries. One explanation for the second feature is that jurisdictions may be able to pass on some of their tax burden to other jurisdictions, and so have little incentive to cut taxes.

In this dissertation, we concentrate solely on the effects of one tax-competing jurisdiction cutting its tax rates under four different scenarios to compete for mobile capital. We examine the effects of this tax cutting on the competing country as well as the rest of the world. In my opinion, this can answer questions raised by both views of tax competition theory and actual practice discussed above. If the tax cutting country benefits from cutting its taxes and the rest of the world loses, then we have the effect predicted by theory, and have to look for other explanations for the observation that tax rates have not fallen as predicted. If there are situations where the tax competing country can corner a disproportionate share of reduction in the excess burden and make the rest of the world worse-off while making itself better off, it has an even stronger incentive to compete by cutting certain taxes.

However, if we find plausible circumstances where the tax competing country loses income when cutting taxes, then this may explain the real-life observation discussed above; even without assuming that the competing country is able to resist competing because it is able to pass on some of its tax burden. It may be keeping its tax rates high because it may actually lose in some situations rather than gain, not only because it is explicitly passing on part of its current burden.

There is an extensive debate on tax competition and its effects in the literature and the intention of this dissertation is not to investigate the origins of such competition or whether it leads to a “race to the bottom” but to make explicit one of the effects of such competition in a general equilibrium framework.[1] By examining the effects of reductions

[1] This dissertation also does not seek to model or explain specific forms of tax competition.

of hypothetical taxes on all capital income and consumption in one country, we build models to investigate the phenomenon of tax "spillovers."

Traditional tax theory shows that taxation has a "burden" on the economy beyond payment of the tax revenue. As Harberger (1962) showed, even if we were to return all tax revenue to the economy, a tax still has a negative excess burden on the sources of GDP. So, we should expect that when we lower a tax, we should see a reduction in the excess burden. But is such a reduction of the burden inevitable, and does it mean an improvement in GDP for everyone?

One aspect of tax competition involves providing mobile capital with lower rates of taxation, especially in the early stages of a project. Whether tax relief is provided directly by lowering tax rates ("preferential" rates), or indirectly through exemptions, accelerated depreciation, rebates, or other tax expenditures or subsidies, we can think of many forms of tax competition (ignoring other forms of non-tax competition) as reducing the effective tax rate faced by a project. In this dissertation, we concentrate solely on this aspect: tax competition is modeled as a reduction in the tax rate in the OECD country, and we investigate its effects on the sources and uses of GDP worldwide. Obviously, tax competition involves more than just a tax rate reduction, but that is the only aspect considered in this dissertation as many other forms of tax competition have a similar end result as a reduction in the effective tax rate.

In the present economic environment, one of the major policy objectives of many countries is to increase public spending to stimulate aggregate demand. The constraint on such demand management policies is the need to keep the deficit and public debt under

control. Another issue being currently debated is the demand from several sections of the economy to reduce certain taxes that are viewed as detrimental to competition. What if it were possible to meet both policy objectives? If a country could increase its income while reducing taxes, would this invariably be at the cost of the income of the rest of the world? Would this depend on the type of tax? Does this mean that a large enough country could have its cake and eat it too? If this phenomenon of tax spillovers can be quantified, we can examine the extent and nature of shifting of the tax burden.

In this dissertation, we define "spillovers" in the following way: we develop a proxy measure of welfare for the countries involved. The measure is based on the sources and uses of income (GDP). "Spillovers" occurs when the "country" (ROW) not reducing its tax faces a negative change in the proxy measure of welfare as a result of the OECD country lowering its tax rate.

The Dissertation is divided into six chapters. In chapter one, we present an overview of the problem and review the existing literature. The next five chapters approach the problem in the following way:

First, we develop a three sector, two "country" open economy version of the Harberger model of general equilibrium taxation incidence. The two countries are meant to represent a very large open economy (such as the U.S. or the E.U. as a whole) and the rest of the world taken together. We incorporate a publicly provided input to production paid for out of current tax revenues. In this chapter, we examine the effects of a tax on all capital in the large country. We develop the analytical model staying as close as possible

to versions of the Harberger analysis (1962, 1995 and 2008). We derive analytical expressions for the extent of spillovers without assuming functional forms.

Second, we develop a second model with government input in chapter three, this time using a general consumption tax in the large country. We develop comparable analytical expressions for this case using the same model structure. As in the previous chapter, we measure spillovers by developing expressions to measure changes in GDP in the large country and the rest of the world from the sources and uses sides.

The next chapter develops the capital income tax and consumption tax models of the previous chapters using a different pattern of expenditure. Instead of a government input, the model assumes that all tax revenues are returned as a lump-sum transfer payment to the consumer in the taxing country.

The last two chapters examine existing literature and data with the purpose of collecting a set of reasonable parameters to calculate the magnitude of spillovers. The aim is to eliminate those parameters on which we have some sort of consensus in the existing literature from the analytical expressions, so that we can concentrate on answering the research questions in the final chapter. Using such “non-controversial” data, we reduce the analytical expressions to unknowns in the variables of interest. Closely following Prof Harberger’s work and subsequent contributions, we analyze changes in GDP that would result from variations in the remaining variables of interest and draw conclusions with respect to the research questions.

**POLICY ISSUES**

There are many aspects of this question that are important for policy analysis. The standard justification for lowering tax rates is summarized in Fullerton (1982). Tax competition introduces another element: of tax rates that are too "high" in relation to competing jurisdictions and the sharing of the fixed capital stock "pie." A more sophisticated version introduces the combined effect of taxation and expenditure on public goods. We can think of this as a Tiebout (1956) model with jurisdictions being countries and capital the mobile taxpayer.

A comprehensive recent treatment of issues involved in an optimal tax and spend package is available in Benassy-Quere, Gobalraja and Trannoy (2007). If taxation is linked to expenditure; can the ability to pass on part of the burden result in a redistribution of income between countries or between owners of factors of production worldwide? As Harberger (1995) has pointed out, high tax rates in developed countries are not necessarily harmful to the rest of the world. When capital flows out from a taxing developed country and into the non-corporate sector of a developing country, it might happen that the returns to labor in the developing countries increase due to an increased availability of capital. In that case workers in the non-corporate sector in a developing country may actually benefit from a high corporate tax rate in the developed world! In this situation, a reduction in excess tax rates in OECD countries will result in a loss in welfare (income) for the rest of the world. This hypothesis was not explicitly tested by Harberger (1995) but is examined in this dissertation.

Another general policy objective in many countries is to increase public spending to stimulate aggregate demand. The constraint on such demand management is the need to increase public expenditure without increasing taxation. This issue and other reforms pertinent to recessions have been reviewed and discussed by Slemrod (2009) and Viard (2009). The issue of reducing taxes that are viewed as detrimental to competition has been reviewed by Slemrod (2008).

Certain behavioral assumptions on policy issues are necessary to proceed. First, a country is presumed to be interested in maximizing domestic welfare (or a proxy measure based on sources and uses of GDP), and not just in attracting more capital or increasing revenue. Therefore it will not consider a reduction that lowers its own welfare (GDP).

Second, the competing country is not interested in what this does to countries in the rest of the world, or in the strictly long-run effects as long as domestic welfare (sources and uses of GDP) increases. While we may assume that the growth of social responsibility and altruism means that governments take into account global concerns, it is reasonable to assume that such considerations are secondary to domestic welfare when domestic tax policy issues are involved.

Third, a reduction in rates can take many forms. One way to think about this is a rationalization of the tax base that allows us to reduce the statutory rate while maintaining revenue; another could be eliminating differentials in tax rates.

Fourth, the models in this dissertation are static. We are not dealing with growth and the accumulation of capital. The stock of capital is fixed worldwide, as is the stock of labor, and the problem is one of allocation with perfect mobility of capital and perfectly

immobile labor. There is no consideration of savings and possible growth of capital stock. We ignore possible effects while recognizing that such effects do exist.

**RESEARCH QUESTIONS**

The main objective of this dissertation is to determine the worldwide tax "burden" and the impact of changes (reductions) in taxes on capital and consumption under different expenditure patterns. We analyze the changes to the worldwide tax burden by comparing consumption taxes with taxes on capital when the tax revenue is either refunded through a transfer payment or used to provide a public input to production.

The specific research questions that we examine are: (1) Can the extent of spillovers in the case of a capital income tax (CIT) and a consumption tax be estimated using reasonable assumptions and data in situations of tax competition? Using definitions developed above: tax competition is confined to a tax rate reduction, welfare is proxied by the sources and uses of income (GDP) and spillovers are defined as a situation where one country reduces its taxes and the result is a lowering in the other country's welfare as measured by sources and uses of GDP. (2) If "spillovers" are quantifiable, how large are these effects and how are they distributed between the taxing country and the rest of the world? (3) Does expenditure assumptions matter for measuring the extent and type of spillovers and do the effects vary by type of tax in addition to use of tax revenue? (4) Are results very sensitive to parameters used?

We model one "country" (like the U.S.A.) as a large open economy (calling it the OECD country), and the other "country" as the Rest of the World (ROW). Secondly, we focus on whether the reduction in taxes by the OECD country imposes a burden on the

ROW while benefiting the OECD since we assume that the OECD country will reject a policy change that reduces its own GDP. We abstract away from changes in the internal distribution between corporate and non-corporate sectors and the distribution between labor and capital.

The case of the home country being a small open economy and the ROW being a large open economy consisting of everyone else has been examined by Harberger (2008) and Gravelle and Smetters (2006). The unexplored part of this type of analysis is to examine what would happen in reverse-if the ROW (in my model, a large open economy called the OECD) were to impose or reduce taxes and we traced the effects on the small open economy (ROW in this dissertation are small open economies-a rough approximation for most developing countries). We will use the Harberger (1962) model explicitly; its assumptions and notation as far as possible to intentionally keep our model comparable to the original model.

Tax competition models (such as Zodrow and Mieszkowski, 1986) suggest that competition between jurisdictions to attract mobile factors of production, especially capital, lead to a lowering of tax rates across countries and regions within a country. This phenomenon has been studied extensively and a definitive survey of the literature on tax competition is available in Wilson (1999). The efficiency and welfare aspects of tax competition have also been studied – one approach is to look at the use of tax revenues-assuming that all revenues are raised to provide public goods(s). If tax competition leads to a lower than optimal tax rate on mobile factors, then this could lead to either a higher than desired tax rate on relatively immobile factors (land, and to some extent, labor) or an inefficient under-provision of public goods. If welfare/efficiency is measured based on

the change in national income/consumption, and the consumption of the public good is part of the income/welfare of the jurisdiction, this leads to a lower level of welfare than may have been possible otherwise.

However, Wilson and Wildasin (2004) have also discussed cases in which tax competition may be welfare enhancing. One of these is the case where tax competition can enhance welfare by allowing jurisdictions to attract capital and thereby “transfer” a part of the burden to non-residents.

## TYPES OF TAX CHANGES THAT RESULT IN SPILLOVERS

There are two classes of possibilities in trying to model tax changes that lead to spillovers: The first possibility is suggested by Noiset (2003). If competition drives tax rates to zero in the limit, and maximizes the base by attracting all foreign capital etc. possible; with the tax rate equal to zero, revenue equals zero. If rates are raised from zero to some positive rate, some capital leaves the jurisdiction-the positive tax rate is applied to a smaller base, but tax revenue is non-zero. If all capital leaves the jurisdiction for some rate that is high enough, then the base is zero, and revenues are again zero. For all values of the tax base and tax rate in between, we get positive tax revenues, and as long as non-residents own some of the capital or there is some change in the returns to labor or capital world-wide, we get tax spillovers (the quantity of such spillovers depends on many factors, however, including whether the changes cancel out). When there are changes in returns to labor and capital, we get effects beyond the tax revenue itself, depending on the model used. These effects can be measured either as changes in the sources of income or the uses of income.

The Harberger model (1962) analyzes this case–the distribution of burdens when a new tax is imposed on the sources side of national income. All taxes are zero to begin with, and a tax is imposed. The net incidence of this tax (such as the corporate income tax) on the rest of the world (ROW) constitutes tax spillovers in the international case. The important issue here is that when a non-lump sum tax is imposed, we have an increase in excess burden. When a tax is lowered, there should be a reduction in this burden, and/or an increase in output. Harberger concentrates on the sources side of income in the 1962 model since there is only one consumer in the closed economy version, so relative price changes on the uses side cancel out for the single consumer.

However, relative prices matter in the international case since there are two consumers. Even if income effects are absent, as long as one consumer consumes more of one good/some goods relative to the other, there are distributional effects based on relative price changes. The existence of non-traded goods in the open economy ensures this. Indeed, Harberger (2008) considers the impact on both the sources and uses sides.

The second case (following Wilson and Wildasin, 2004) is that the jurisdiction may have target revenue, and be imposing a tax rate that is too high to attract foreign capital. It may then lower tax rates in some form of tax competition, and thus broaden the base. This attracts foreign capital and for some values of the relevant elasticity, could increase revenue with a lower rate. An important underlying assumption is whether we may or may not care about short-run or long-run effects on revenues, or are willing to sacrifice short-term revenues for expansion in activity.

The case of a jurisdiction willing to accept a permanent decline in revenue without expecting that increased investment will lead to more revenue in the future is rather unlikely. The jurisdiction then also spills over part of its revenue burden by taxing capital owned by non-residents and can increase welfare by providing more public goods to its residents. Beyond the tax revenue itself we can even ignore the effects of ownership; if there is a change in world-wide returns to labor or capital, some excess burden is exported.

If we include the uses side of GDP, a change in relative consumption prices with no corresponding income compensation also constitutes part of the burden. Even if tax revenues are fully refunded, if relative prices are allowed to change, there are distributional effects depending on who consumes more of which commodity. To model this, we can start with an initial tax, and lower the rate.

## PREVIOUS STUDIES

Two major studies that have given form to the models in Harberger (1995, 2008) are Gravelle and Smetters (2006) using a CGE framework and Randolph (2006) using an analytical solution based on Jones (1965). These models use an open economy framework, but model a corporate tax without an intermediate government input. They do not explicitly focus on the consumption tax but Randolph (2006) does discuss the effects of several taxes besides the CIT.

Several earlier versions of government and taxes in the Harberger model are available; one early model within the Harberger framework is McLure (1969), where tax revenues are used to fund government expenditures that impact final demand since

government procures either final goods or factors of production. Models with externalities in the general equilibrium framework are also available, such as Fullerton and Metcalfe (2001) and are related in the sense that provision of a public good also involves externalities, albeit positive. Further explorations of the general equilibrium nature of environmental taxes and mandates in the context of un-priced inputs in production like pollution are provided in Fullerton and Heutel (2005, 2007). Zodrow and Mieszkowski (1986) have examined the issue of under-provision of public goods in the context of tax competition and their studies have also led to several important extensions such as by Matsumoto (1998, 2000), Noiset (1995, 2003) and Benassy-Quere, Gobalraja and Trannoy (2007).

We extend the existing literature in two ways. We explicitly introduce a non-neutral government within the Harberger model to examine tax spillovers and tax competition-the focus is not on optimal taxation or forms of competition but on modeling spillovers. We introduce the public good as in input into production and then compare models of two taxes with active public input with two models of taxes with no active government input. The introduction of an active or non-neutral government input was first suggested explicitly to me by Prof Glenday. The idea is also discussed in McLure (1975, sec 6.2, pg 150)…. "But it would be useful to investigate in a formal model just how the relationships between demands for public and private goods interact with the various other parameters in the model. In a similar vein, we could examine the effects of public expenditures that tend to substitute for or be complementary to, one or the other of the factors of production. But we cannot consider government capital formation without using a growth model."

In most cases, if we follow the original Harberger model, it is assumed that expenditure effects are absent, or neutral in terms of incidence. If, however, issues of expenditure incidence/tax price are brought in, there can be welfare effects beyond those caused by the tax. McLure (1969) has examined cases in which government expenditures can be incorporated into the standard Harberger analysis. He has considered the cases where the government either buys final goods, or factors of production. A comprehensive analysis of cases involving a tax on labor is available in Wallace (1993). To highlight how this dissertation is different from Randolph (2006) and Gravelle and Smetters (2006) a brief comparison of the modeling is presented below. Our model seeks to extend and improve on the existing literature since we have not assumed functional forms as the CGE model of Gravelle and Smetters does, and we have formalized the Harberger (1995, 2008) models while adding a non-neutral government using the same model structure as Harberger (1962) unlike in Randolph (2006) which is based on Jones (1965):

**Table 1 The Dissertation Compared to Important Existing Studies**

| **Dissertation** | **Randolph(2006)** | **Gravelle and Smetters(2006)** |
|---|---|---|
| Analytical | Analytical | CGE |
| 2 countries: domestic (OECD) and foreign(ROW) | 2 countries: domestic (OECD) and foreign(ROW) | 2 countries: domestic (OECD) and foreign(ROW) |
| 2 sectors OECD, 1 Sector ROW | 5 sectors per country | 4 sectors per country |
| 3 factors: labor, capital and government services vs. models with two factors | 3 factors: land, labor and capital; land is only used in non-corporate tradable (agriculture) sector in each country | 3 factors: land, labor and capital |

| | | |
|---|---|---|
| Constant Returns to Scale and product exhaustion for labor and capital, with IRS in government input models | Constant Returns to Scale and product exhaustion | CES production functions |
| Producer level perfect competition, combined with country level market power | Producer level perfect competition, combined with country level market power and perfect substitution | Producer level perfect competition, combined with country level market power/perfect substitution |
| Labor perfectly mobile within countries, not internationally-supply fixed in each country | Labor perfectly mobile within countries, not internationally-supply fixed in each country | Labor perfectly mobile within countries, not internationally-supply fixed in each country |
| Capital perfectly mobile-supply fixed worldwide | Capital perfectly mobile-supply fixed worldwide | Capital perfectly mobile and substitution elasticity based on rates of return-supply fixed worldwide ( but also consider variations) |
| No ownership, GDP studied and not GNP | Ownership of factors of production allowed-labor confined to own country, capital deployed in either country, but capital owners can't move | Ownership of factors of production allowed- labor confined to own country, capital deployed in either country, but capital owners can't move |
| Consumers identical, can consume all home goods and traded goods | Consumers identical, can consume 5 home goods and/or 3 foreign goods | Nested CES consumption functions. Consumers identical, can consume 4 home goods and/or 2 foreign goods |
| Marginal return to investment same (excluding producer level taxes and risk premia) everywhere in the world | Marginal return to investment same (excluding producer level taxes) everywhere in the world | Case of imperfect capital substitution across countries, domestic and foreign rates of return not same considered |
| No perfect substitutes, each country produces unique goods | Outputs of sectors One and Four (1st corporate tradable and the non-corporate tradable sectors) in each country are perfect substitutes; output of sector two (2nd corporate | Imperfect product substitution in sector one(traded corporate), perfect substitution in traded non-corporate good |

| | | |
|---|---|---|
| | tradable sector) in each country are unique | |
| Production and consumption shares based on literature review | Initial consumer expenditures on all 6 consumed goods in each country proportional to shares of worldwide production | Domestic consumption and exports of sector one equal to output for each country, worldwide production of sector two equal to worldwide consumption, and consumption equal to output in non-traded goods for each country |
| Government expenditures to be examined and two alternatives compared | Neutral government expenditure – no government expenditures effects | Neutral government expenditure – no government expenditures effects |
| No country capital investment risk | No risk | Portfolio substitution elasticity that is not infinite |
| Existing Capital Income Tax, Consumption tax | New Corporate Income Tax and several Replacement Taxes (income from capital, wages and consumption) discussed | Corporate Income Tax |
| Sources and Uses side impacts | Sources and Uses side impacts | Sources and Uses side impacts |
| Income effects included | No income effects | Real burdens calculated using different weighted price indices |
| ROW single product is numeraire | Sector One output in each country identical and its price is set as numeraire | Sector Two output in each country identical and used as numeraire |
| Data driven | Share of capital and labor explicitly the same for sector one in each country | Factor shares and output shares same for all 4 sectors in each country |
| Data driven | Input Substitution Elasticities the same across all sectors and both countries | Initial Input Substitution Elasticities the same across all sectors and both countries and equal to one, simulations done |

## GENERAL ASSUMPTIONS USED IN ALL MODELS

If only one representative consumer is assumed per economy and we do not consider issues of cross-border ownership of factors of production, then distribution issues can be considered only between countries. All income generated in a country will go to the same consumer, regardless of ownership of factors of production. Even if ownership of factors of production is introduced, the one consumer in a country gets all the labor income accruing to that country, since labor is internationally immobile. The difference is in the case of capital, which is mobile across countries. Here the distinction between GDP and GNP can have significant effects, depending on whether we assume substantial cross-border ownership of capital relative to total stocks or not and whether the jurisdiction is a net importer or exporter of capital.

If ownership or deployment of either resource is substantially more skewed across countries than relative income then redistribution between capital and labor will impact national incomes, and at the national level overall welfare still can change significantly due to taxes and expenditures. For our dissertation, we have used GDP. In the base case, if the OECD country is not a net exporter or importer of capital, GDP equals GNP in this type of model. Secondly, while acknowledging that GNP is the more appropriate measure of national income, with immobile labor and equal proportions of cross-border investment in both countries to start with, the difference is negligible. It also allows us to abstract away from issues such as home bias and risk premia and focus on the effects of spillovers. For the U.S., as well as the E.U. (the two jurisdictions large enough to change the worldwide rate of return to capital by their actions alone) the difference is not very

significant. A quick comparison for the U.S. based on WDI data is presented in table 2 below.

The only other entity as large as the U.S. in the sense that it could affect the world return to capital unilaterally would be the E.U. taken together, or at least the countries of the Euro zone. Perhaps one can see that other countries such as China may be large enough to be included in this category in the future, but in my opinion it is not so large at present.

**Table 2 Ratio of U.S. GDP to GNI Based on WDI Data**

| ***Series Name*** | ***YR2003*** | ***YR2004*** | ***YR2005*** | ***YR2006*** | ***YR2007*** |
|---|---|---|---|---|---|
| GNI (current U.S.$) MILLIONS | 10916100 | 11687900 | 12439300 | 13209000 | 13827247 |
| GDP (current U.S.$) MILLIONS | 10908000 | 11630900 | 12376100 | 13132900 | 13751395 |
| | | | | | |
| ***RATIO OF GDP TO GNI*** | ***0.99926*** | ***0.99512*** | ***0.99492*** | ***0.99424*** | ***0.99451*** |

SOURCE: World Development Indicators Online (Provided by the World Bank) available at: http://databank.worldbank.org/ddp/home.do?Step=12&id=4&CNO=2, accessed through GSU library on 5/1/2010

The same comparison is presented below for Euro-Zone countries from WDI data, and for the Euro-Zone countries the results are similar, there is almost no difference between the GDP and GNI.

**Table 3 Ratio of Combined Euro-Zone GDP to GNI Based on WDI**

| ***Series Name*** | ***YR2003*** | ***YR2004*** | ***YR2005*** | ***YR2006*** | ***YR2007*** |
|---|---|---|---|---|---|
| GNI (current U.S.$) MILLIONS | 8472745 | 9766332 | 10132559 | 10757603 | 12300256 |
| GDP (current U.S.$) MILLIONS | 8527649 | 9765862 | 10148194 | 10743647 | 12319397 |
| | | | | | |
| ***RATIO OF GDP TO GNI*** | ***1.006480*** | ***0.999952*** | ***1.001543*** | ***0.998703*** | ***1.001556*** |

SOURCE: World Development Indicators Online (Provided by the World Bank) available at: http://databank.worldbank.org/ddp/home.do?Step=12&id=4&CNO=2, accessed through GSU library on 6/1/2010

Harberger (1995 and 2006), Gravelle and Smetters (2001 and 2006) and Randolph (2006), have used up to three factors of production in their models:  land, labor and capital. Adding government introduces a fourth factor. In our models, we have omitted land, and introduced an active government in two models and a passive or neutral transfer program in the other two.

Harberger (1995 and 2006), Gravelle and Smetters (2001 and 2006) and Randolph (2006) use four or five sectors in their models. The five sectors per country assumption have specific effects on the derivation of results for the analytical model, and the introduction of land as a factor of production into the non-corporate tradable sector (agriculture) also has a specific reason. Dropping the distinction between corporate and non-corporate sectors in the OECD country and collapsing the rest of the world into one sector allows us to concentrate on the issue of spillovers across countries while allowing for the existence of non-traded goods. It also allows us to abstract away from the problem of mobility of labor within the consolidated ROW, since farmers in China could not actually work on farms in Armenia.

This dissertation explicitly follows the Harberger model as laid out in Harberger (1962, 1995 and 2008) as closely as possible to keep results comparable. The basic Harberger (1962) model has the following sets of equations:

(a) Demand     (b) Supply       (c) Price formation      (d) Substitution

(e) Adding-up or balance equations.

The steps envisaged in each of the chapters that follow are  (1) The assumptions of the model in general will be laid out (2) The assumptions for deriving each set of

equations will be discussed in detail and each set of equations derived (3) The model will be reduced by eliminating unknowns and equations to a set of 4 equations in 4 unknowns (4) The equations for estimating GDP and welfare effects will be derived (5) Solutions for the model will be discussed and the procedure to use these for analysis will be discussed. Subsequent chapters will compare the capital income tax model and the consumption tax model with the more traditional case where the government provides a transfer payment instead of an input into production; discuss data and finally analyze solutions and draw conclusions.

**GENERAL ASSUMPTIONS**

There are two "countries," an OECD country such as the U.S. that is large enough to be treated as a large open economy, and the Rest of the World (ROW). The subscript "O" will be used to denote variables particular to the OECD country and "R" for the other. When no country subscript is used, it can be assumed that the variable is common to both or encompasses both.

There are two productive sectors in the OECD country, X (the tradable goods sector) and Y (the non-tradable goods sector). The ROW has one consolidated sector Z, and no assumption is made about it except that the good is tradable. The only difference from the standard analysis is that since there is no separate corporate and non-corporate sector, the corporation tax is replaced by a capital income tax that applies to both the tradable and non-tradable sectors of the OECD country.

Capital is perfectly mobile all over the world. Following Harberger (1962) we assume that the net rate of return to capital is always equal around the world. This does

not imply that there are no risk differentials across countries, but that given those (known) differentials; we are able to isolate the pure, risk-free rate of return. This helps us to focus on the gross rate of return in the OECD country including the capital income tax, which is expressed as an ad valorem rate.

The tax rate on capital in OECD is $T_{ko}$ initially, and is assumed to be a known parameter, as is the proposed rate change $dT_{ko}$. As pointed out in Ballentine and Eris (1975), the expression $P_k*(1+T_{ko})$ reduces to $P_k$ only when $T_{ko}$ =0, and the change in gross price $d[P_k*(1+T_{ko})] = dP_k + T_{ko}$ only when the initial tax is zero. We are interested in modeling the case when the OECD country decides to lower its capital income tax to attract a greater share of the world's capital (simulating tax competition); and so the existing tax is not zero.

The gross rate of return to capital in the OECD country is defined as $P_k*(1+T_{ko})$, and is the same in both OECD sectors, X andY. The rate of return to capital in the ROW is simply $P_k$. Thus, we do not use any country or sector subscript when we talk about capital's net rate of return; it is the same across all sectors and countries. The same reasoning applies to the consumption tax or $T_{Co}$. While the price of capital remains $P_k$ or $P_k + dP_k$ , the consumer prices in the OECD country are $P_x*(1+T_{Co})$, $P_y*(1+T_{Co})$, $P_z*(1+T_{Co})$, and are $P_x$ and $P_z$ in ROW to begin with.

The total amount of capital is fixed worldwide; there is no saving, capital accumulation or growth in productive resources. $\bar{K} = K_x + K_y + K_z$ is fixed. While $dK_x \neq 0$, $dK_y \neq 0$ and $dK_z \neq 0$ in general, we get the relationship $dK_x + dK_y + dK_z =0$ for the world as a whole. Since $T_{ko} \neq 0$ and $dT_{ko} \neq 0$ and $(T_{ko} + dT_{ko}) \neq 0$ in general (though we

may assume it to be so if necessary), we always have a positive tax on capital as well as on consumption. Note that this does not need to be interpreted literally. This could simply be the excess or differential tax on capital in the “taxing” country, which it views as too high and seeks to reduce it to compete more effectively with other countries for mobile capital. In case countries feel their consumption tax rates are too high, they may consider reducing these to attract investment as well, or to stimulate aggregate demand.

Labor is constant and confined within each country. Since the ROW is treated as one country and one sector (Z), the total labor available to sector Z is equal to the total ROW labor and is fixed at $\overline{L_z}$ which is equal to $L_{row}$. The total labor available to the OECD country is $L_{oecd}$ and this is also fixed as the sum of the labor employed in sectors X and Y. Thus $L_x + L_y = L_{oecd}$ and $\overline{L_{oecd}}$ is fixed. This leads to two prices for labor: $P_{lo}$ and $P_{lr}$ for OECD and ROW respectively that are not in general the same. While we have assumed a fixed worldwide labor supply and abstracted away from issue of (a) mobility within the ROW (b) less than full employment and (c) taxes lost when there are taxes on labor: since the return to labor falls as a result of a tax on capital due to capital flight this implies lower yields from labor taxes as well if they are not reduced if the total income of labor falls.

Adding up of a diverse set of countries (developed, newly industrialized and developing) into a consolidated ROW with one sector where full employment is assumed is an abstraction from reality to keep our model simple and follow the existing literature. Doing so ignores issues of labor immobility in the unskilled labor market, extensive under-employment in the developing countries and effects of labor taxation in developed countries. Introducing such complications is left to further research, and the focus here is

on the main contribution – adding a non-neutral government. Further, the consumption tax does tax labor input as well, and presents some contrast to the CIT. This again does not imply that adding more realism to the labor side of the models is not desirable, but effects in the simpler models we deal with will be present in more advanced models as well, and it is standard to abstract away from more complicated reality in theory to focus on effects we are more interested in to begin with.

The labor assumptions also lead to two further balancing or adding up equations, namely that $dL_z = 0$ and that $dL_x + dL_y = 0$. Therefore each labor price can also change independently implying $dP_{lo} \neq 0$ and $dP_{lr} \neq 0$ in general. However, as we shall see later, due to the $dL_z = 0$ assumption, in our model $P_{lr}$ is effectively a residual. This is not at all necessary, it is purely for convenience. All models have assumed labor immobility across countries, as well as full employment in each country. To introduce market pricing for labor and substitution of inputs in ROW, we simply need to have two sectors in ROW as well, a tradable and non-tradable sector. This has been avoided only to simplify the analytical solutions of the reduced form models. Since the results are derived using the extended version of each model in MATLAB, it is relatively easy to introduce these elements if desired.

There is one consumer in each country. The consolidated demand function refers to the OECD single consumer and the ROW single consumer. The country superscript is interchangeable with the consumer subscript. We also ignore ownership issues by using GDP instead of GNP. This allows us to treat income generated in a country as income belonging to the consumer in that country. In a GDP based measure, income earned within the geographical area of a country counts as GDP, even if the income belongs to

non-residents. Similarly, income earned by nationals outside the geographical limits of the country is not included in home GDP.  A GNP measure (and national income usually relates to GNP and not GDP) would consider income earned by nationals abroad, and exclude the income of foreign nationals earned in the home country. However, the distinction in terms of our model is not quite so sharp. Since labor is immobile between countries by assumption, the only difference is in the return to capital.

It is possible to use a GNP measure as well in this model, and it is not used mainly for analytical convenience even though it is a better measure of welfare. One issue that arises is that for the U.S. and Euro-zone, GDP and GNP are almost the same. This implies that net factor income from abroad is either negligible, or that net labor and capital income balance each other. We have assumed that labor is immobile and lives, earns and consumes in their domestic area. If we continue to assume that capital owners earn capital income from abroad, we ignore the balancing factor seen in the data and we then have a model where we consider the effects on one source of factor income from abroad and ignore the equal and opposite other source.

If we had used GNP (as in Randolph 2006), we would make a data-based allocation of capital ownership between residents of the OECD and ROW that would have to be static in the sense that we would have no way to tell beforehand how capital owned by residents of OECD would behave differently from capital owned by ROW residents. Using GDP also allows us to abstract away from issues of “home bias” in capital investment that has been extensively documented which, in my opinion, does some damage to a model with perfectly mobile capital and ownership that does not include risk.

In the base case the two measures (GDP and GNP) are equal given no net capital importing and spillovers if labor is immobile. For the U.S. and E.U. the tables above show that that is also statistically the case. Given perfect mobility of capital and an equalized rate of return and no risk premium, using a GDP measure is a close approximation of GNP based welfare (income) and reduces the complexity of assigning capital income without accounting for other effects seen in the data.

When we use a GNP measure, foreigner’s capital attracted to the home country still counts as foreign capital and the income still counts as foreign capital income. In a GDP measure, capital attracted from one country to another counts as a diversion of capital and its earnings. If we think of spillovers as foreigners paying our tax, a GNP based measure would capture this. A GDP measure however, does not. The income from capital that moves to another country now becomes part of the other country’s GDP and the effect on the ROW depends on the change in earnings of labor and loss of capital income.

A perfect capital income tax that works on the GNP principle would tax residents’ capital income regardless of where that capital is located. However, as discussed by Randolph (2006) this condition of Capital Export Neutrality (CEN) is violated in practice by the U.S. and most foreign corporate income taxes. Other studies based on actual investment behavior by corporations such as Grubert (2004) and Altshuler and Grubert (2008) also support the view that CEN is not achieved in practice by the U.S. tax system as a result of international tax rules, transfer pricing, shifting of assets and corporate behavior in general. Therefore, it does not do great violence to the model to use GDP instead of GNP for this reason either.

The production functions in each sector X, Y and Z are homogenous of degree one (HOD 1) in capital and labor. When the Government input G is added in the OECD sectors, the production functions behave like they are homogeneous of degree greater than one (increasing returns to scale) in all three inputs. This can be modeled as a case where the government input is both labor and capital augmenting, or where the addition of G acts a shift factor to the supply curve which is drawn for capital and labor. A somewhat similar formulation is the technology-augmented version of the Solow growth model discussed in Romer (2006) and Mankiw, Romer and Weil (1992). This point is illustrated more fully in each relevant chapter.

Perfect competition is assumed and taxes are the only distortion. Adjustments to changes in discretionary parameters are instantaneous. Like the literature before us, we also extend the model from infinitesimally small changes to slightly larger, discrete ones, concentrating only on first round effects and ignoring higher order terms.

Following Harberger (1962), we assume that all prices are unity to begin with, and that all quantity units are defined accordingly to make their prices equal to one. This allows us to measure quantities as dollar amounts. Accordingly, $P_k$, $P_{lo}$ and $P_{lr}$ (factor prices) and $P_x$, $P_y$ and $P_z$ (output prices) are all equal to 1 and to each other to begin with. Since our existing tax is ≠ 0, this implies that $P_k*(1+T_{ko}) \neq 1$ and analogously, $P_x*(1+T_{Co}) \neq 1$.

No assumption is made about how the historical (excess) rate of tax on capital in the OECD country was set. Zodrow and Meiszkowski (1986), Noiset (1995), Matsumoto (1998, 2000) and Benassy-Quere, Gobalraja and Trannoy (2007) have examined the issue

of the optimal provision of public goods and the optimal tax rate in situations of tax competition in some detail, and this is not the objective here. We assume that a known historical rate exists, whether optimal or not, and that a policy decision is made to reduce it, with the intention of making the OECD country more competitive in attracting capital vis-à-vis the ROW.

The total amount of the public good provided, G is simply equal to the tax revenue or government expenditure. It is assumed that every dollar of expenditure on the government good leads to one unit of increase in the level of G (and the reverse is also true). There is no price for the government good since it is not a produced input, there are no compliance or administrative costs.

We also assume that the government good G is an input into production in the OECD sectors X and Y, it is a non-rival and non-excludable public good and its quantity is dependent solely upon available tax revenues in the current period. In the chapters with transfer payments, there is no active government input. We can assume therefore that it is government current services (perhaps maintenance) and not new investment that is being provided with the tax revenue. Alternatively, we can assume that it is infrastructure that does not last beyond one period, and that does not add to the stock of capital.

The CIT tax is collected like an excise tax on capital employed by the X and Y sectors only. The consumption tax is collected on sales of final goods in the OECD country only and includes imports. It is not applied to OECD exports. The ROW sector has no tax on its capital, consumption and no government input in its production function. The government good G, once provided, can be used by both X and Y in the same

quantity, and no separate payment is made for its use, other than the tax on capital employed by X and Y, or the consumption tax paid.

No explicit restrictions are imposed on demand curves other than that they can be separated into OECD and ROW demand. The equations are derived without assuming that they are HOD Zero in prices and incomes, and any set of elasticities that satisfy consistency conditions for Walrasian general equilibrium are possible. In practice, however, it is demonstrated that the HOD Zero retriction with matching elasticities may be crucial to the results, and a review of the literature does not support clearly any stand on this issue.

In chapter two, we develop the model of the capital income tax with government input. In chapter three, we model the consumption tax with government input. In chapter four, we present both models with a transfer payment only. Chapter five is used to conduct an extensive literature review to obtain usable parameters. The analytical solutions of chapters two, three and four and the data from chapter five are used to simulate results and draw conclusions with respect to our research questions in chapter six.

One last caveat is inserted here to ensure that we do not pre-judge what follows in this dissertation by using terms such as “burden” in conjunction with spillovers. This is simply with reference to the most famous result in public economics based mostly on several papers by Prof Harberger that show that imposing a tax (or raising it) in the initial state without distortions imposes an excess “burden.” We take this result as given for the models specified in his work, and base our work on modifications of the models that gave

rise to this result. In light of the result (Harberger, 1962) that when there are no other distortions, imposing a tax (or increasing it) should reduce GDP in a closed economy (our excess "burden"): we have gone on to investigate the opposite case. Reducing a tax or removing it should have the opposite relative effect to this base case; it should increase GDP and lower the excess burden. We state this only with reference to and relative to the existing literature, and not to anticipate results.

# CHAPTER II - THE CAPITAL INCOME TAX

The purpose of this chapter is to extend the Harberger model to the case of tax competition in an open economy setting where the OECD country lowers its capital income tax to compete with the rest of the world for mobile capital. The intention is to follow the Harberger model as laid out in Harberger (1962, 1995 and 2008) as closely as possible to keep results comparable. As noted earlier, this treatment is new in three respects. First, no distinction is made between corporate and non-corporate sectors and the distribution of income within a country. A tax on all capital deployed in the OECD is used. Second, a government-provided public input paid for out of tax revenues is used in the production of goods in both sectors in the tax-imposing country. Third, in later chapters, the model of this chapter is compared to models with a consumption tax and to two models without the government input. These comparisons allow us to isolate burdens on both the sources and uses sides of GDP in both jurisdictions.

This chapter focuses on setting up the capital income tax model with government input and deriving the necessary equations. We also reduce the model to 4 equations in 4 unknowns and discuss how an analytical solution may be obtained for reductions in the tax on capital. The complicated nature of these solutions for the unknown terms (as can be seen from symbolic solutions in the appendix) makes such an exercise hard to interpret. In later chapters we will arrive at possible values for parameters and discuss solutions based on estimates available in the literature. In chapters 2-4, the models are set up and derived; chapter 5 discusses data possibilities, and chapter 6 discusses results and solutions based on this data. We shall reiterate the assumptions made for this model, and

when general assumptions specified in chapter one are used they shall not be elaborated further.

**ASSUMPTIONS**

There are two "countries," an OECD country such as the U.S. that is large enough to be treated as a large open economy, and the Rest of the World (ROW). There are two productive sectors in the OECD country, X (the tradable goods sector) and Y (the non-tradable goods sector). The ROW has one consolidated sector Z, and no assumption is made about it except that the good is tradable.

Capital is perfectly mobile all over the world. Following Harberger (1962) we assume that the net rate of return to capital is always equal around the world. Since we assume ( as in Harberger 1962) that the pure, risk-free rate of return to capital can be isolated in the large OECD country while the rest of the world pays a risk premium for capital, the OECD country cannot tax the premium at all with its differential existing tax, only the pure rate of return is taxed.

If we were to include a risk premium for any sectors or the ROW or OECD country, it would have been added to the pure rate of return, and it need not be constant. We could have a risk premium/premia that is/are increasing in the proportion of capital stock attracted by a particular country or sector(s) as implied in Gravelle and Smetters (2006). To the extent that such risk premia are paid in some sector of the OECD country, they are taxed by the (differential) tax on capital, when they apply to the ROW exclusively; they are not taxed. This need not be taken literally to mean that the ROW never taxes its capital. We have discussed in earlier sections how the tax on capital in

OECD may be viewed as an excess or differential tax, over and above the worldwide rate.

The tax rate on capital in OECD is $T_{ko}$ initially, and is assumed to be a known parameter, as is the proposed rate change $dT_{ko}$. The gross rate of return to capital in the OECD country is defined as $P_k*(1+T_{ko})$, and is the same in both OECD sectors, X and Y. The rate of return to capital in the ROW is simply $P_k$. The total amount of capital is fixed worldwide and the model is static; there is no saving, capital accumulation or growth in productive resources. $\bar{K} = K_x + K_y + K_z$ is fixed. While $dK_x \neq 0$, $dK_y \neq 0$ and $dK_z \neq 0$ in general, we get the relationship $dK_x + dK_y + dK_z = 0$ for the world as a whole. Since $T_{ko} \neq 0$ and $dT_{ko} \neq 0$ and $(T_{ko} + dT_{ko}) \neq 0$ in general, we always have a positive tax on capital.

Labor is constant and confined within each country. Since the ROW is treated as one country and one sector (Z), the total labor available to sector Z is equal to the total ROW labor and is fixed at $\overline{L_z}$ which is equal to $L_{row}$. The total labor available to the OECD country is $L_{oecd}$ and this is also fixed as the sum of the labor employed in sectors X and Y.

There is one consumer in each country. The consolidated demand function refers to the OECD single consumer and the ROW single consumer. The country superscript is interchangeable with the consumer subscript. The issues of factor ownership and income calculation have been discussed in chapter one and are not repeated here.

We also assume that the government good G is an input into production in the OECD sectors X and Y, it is a non-rival and non-excludable public good and its quantity is dependent solely upon available tax revenues in the current period. We can assume

therefore that it is government current services and not investment that is being provided with the tax revenue. Alternative construction is possible as in McLure (1969). This would involve G being produced using K and L, and would have a price that would be a weighted average of the prices of K and L. The total amount of G produced, weighted by the price of G or $P_G$ would be equal to tax revenues. The demand for G would not be derived demand since producers don't pay directly for G.

However, implications of the simpler model would also exist in a more complicated one as well, there would of course be further elements added to the analysis by fully defining production of G. In the interests of simplicity and transparency, and keeping in mind that we can gain the essentials without repeating McLure's (1969) more developed analysis of G where G was a final consumption good and not an input, we have chosen not to model production of G fully and left this for future models.

The differential tax on capital in the OECD is collected like an excise tax on capital employed by the X and Y sectors only. The ROW sector has no tax on its capital and no government input in its production function. The government good G, once provided, can be used by both X and Y in the same quantity, and no separate payment is made for its use, other than the tax on capital employed by X and Y.

The production functions in each sector X, Y and Z are homogenous of degree one (HOD 1) in capital and labor. When the Government input G is added in the OECD sectors, the production functions behave like they are homogeneous of degree greater than one (increasing returns to scale) in all three inputs. This can be modeled as a case where the government input is both labor and capital augmenting, or where the addition

of G acts a shift factor to the supply curve which is drawn for capital and labor. If we define the production functions as Cobb-Douglas, the G would enter as a scale or shift factor, in a manner similar to technology or human capital in growth models. This point is illustrated more fully below.

Perfect competition is assumed and taxes are the only distortion. Changes are instantaneous, the model is an exercise in comparative statics, and there is no time dimension or dynamic factors such as savings, accumulation and growth. Following Harberger (1962), we concentrate only on first difference terms based on small changes. To illustrate, this means that for a pair of variables P*X that can both change, the total change is calculated thus: $d(P*X) = (P+ dP)*(X+dX) – P*X = dP*X + dX*P + dX*dP$. However, if both dP and dX are small, their product dX*dP is even smaller and is ignored.

Following Harberger (1962), we employ the convenient formulation that all prices (output and input) are set to unity to begin with and are allowed to change with respect to the numeraire ($P_z$), and that all quantity units are defined accordingly to make their prices equal to one. We can do this since we use quantities only as ratios in our equations, and so their units do not matter. This allows us to measure quantities as dollar amounts in the initial state. Accordingly, $P_k$, $P_{lo}$ and $P_{lr}$ (factor prices) and $P_x$, $P_y$ and $P_z$ (output prices) are all equal to 1 and to each other to begin with and are allowed to change with respect to the numeraire. There is no price for G, so we are in effect formulating G as invariant to inflation. Since our existing tax is $\neq 0$, this implies that $P_k*(1+T_{ko}) \neq 1$.

The total amount of the public good provided, G is simply equal to the tax revenue or government expenditure. It is assumed that every dollar of expenditure on the government good leads to one unit of increase in the level of G (and the reverse is also true). There is no price for the government good since it is not a produced input.

Lastly, we take the price of the ROW tradable good $P_z$ as the numeraire good, so $dP_z = 0$. We have effectively imposed the condition that imports equal exports while formulating the demand equations and choosing elasticities in chapter five. We could have instead had trade imbalance with BOP balance. To do this, we could have borrowed an idea from open economy macroeconomics where the capital account balances the trade account. The capital account in this model could have been represented by net capital inflows or outflows that balance the trade deficit or surplus. BOP balance conditions have been imposed in some of the other models cited but has not been attempted here, and the simpler formulation of trade balance has been used instead. This allows us to retain the real model with no currency considerations, and the use of the single foreign good Pz as the numeraire allows us to treat other goods prices $P_x$ and $P_y$ as some measure of terms of trade between OECD and ROW.

**PROFIT MAXIMIZATION BEHAVIOR BY FIRMS (SECTORS)**

The production functions for each sector can be written as a function of productive resources:

$X = f(K_x, L_x, G)$ $\qquad$ $Y = g(K_y, L_y, G)$ $\qquad$ $Z = h(K_z, L_z)$

{Where the amount of G is common to both X and Y, G is a public good}

The sector is synonymous with the firm. Since CRS industries imply that the size of the firm is indeterminate, we speak of the sector and firm interchangeably, like each sector were one giant firm. However, we have also assumed perfect competition, so each firm (sector) acts like an atomistic price-taker in all markets. Specifically, we assume that firms do not recognize the externality associated with paying taxes that support the public good G. When sector X employs more capital, this implies more tax revenue and thus more of the public good, provided sector Y does not employ less capital. However, we assume that each sector behaves as if it cannot predict what the other would do.

Thus, each firm behaves in the following way in the OECD country: they take as given the present or historical level of G, they believe that production function they face is CRS (HOD 1) in capital and labor employed by them for this fixed given level of G, they ignore the effect of their actions in employing capital on the level of G since they cannot tell if the other firm (sector) will free ride and reduce the amount of capital hired, they take all factor and output prices as given, and they take the price of capital in OECD as $P_k*(1+T_{ko})$. Firms then try to maximize profit by choosing the levels of capital and labor they can employ.

Following Matsumoto (1998), the firm's choices can be modeled in the following way. The firm perceives the production function faced by it as:

$X = f(K_x, L_x, \bar{G})$ $\qquad$ $Y = g(K_y, L_y, \bar{G})$ $\qquad$ $Z = h(K_z, L_z)$

Where $\bar{G}$ is taken as fixed and given at the historical level which is assumed unchanged. The profit maximization problem can then be set up in the following way.

FIRM (SECTOR) X

Maximize Π (profit) = { $P_x$* f ($K_x$ , $L_x$, $\bar{G}$) - $P_k$*(1+$T_{ko}$)*$K_x$ – $P_{lo}$*$L_x$ } with respect to $K_x$ and $L_x$

First order conditions:

1. $P_x * \frac{\partial f}{\partial K_x} \quad - \quad P_k*(1+T_{ko}) = 0$

And

2. $P_x * \frac{\partial f}{\partial L_x} \quad - \quad P_{lo} = 0$

Since $P_x = 1$, this implies that $\frac{\partial f}{\partial K_x} = P_k*(1+T_{ko})$ and $\frac{\partial f}{\partial L_x} = P_{lo}$

Each firm pays each factor its marginal product at the current level of provision of the public good. In the case of capital in OECD, this is the gross of tax rate of return. Firms do not have to pay for G directly.

FIRM (SECTOR) Y

Behaves in exactly the same way as X, and so we get $\frac{\partial g}{\partial K_y} = P_k*(1+T_{ko})$ and $\frac{\partial g}{\partial L_y} = P_{lo}$

FIRM (SECTOR) Z

Since there is no government good or tax in the ROW we have only the production function Z = h ($K_z$, $\overline{L_z}$ ) and the problem for firm Z:

Maximize Π (profit) = { $P_z$* h ($K_z$ , $\overline{L_z}$ ) - $P_k$*$K_z$ – $P_{lr}$*$\overline{L_z}$ } with respect to $K_z$ { since we assume full employment always in both countries, choosing $L_z$ is not a decision variable } and we get $\frac{\partial h}{\partial K_z}$ = $P_k$

Suppose alternatively that each firm had recognized that G is a function of K employed by it and that by choosing K, it was also choosing the level of G. For firm X, the first order condition would then have been:

$$P_x*\frac{\partial f}{\partial K_x} + P_x*\frac{\partial f}{\partial G}*\frac{\partial G}{\partial K_x} - P_k*(1+T_{ko}) = 0$$

Both firms in the OECD (X and Y) ignore the externality implied by the second term since they view G = $\bar{G}$ or $\frac{\partial G}{\partial K_x}$ = 0 and /or $\frac{\partial G}{\partial K_y}$ = 0. This is also possible since they do not have to make any payment for the use of G once it is provided; they only have to pay the tax on capital employed by them. Each firm is also not sure of how their use of capital is related to the others. If each firm assumes the other will be a free rider, then increasing use of K by one may lead to no change in total K in OECD if the other firm (sector) reduces its capital use.

**CHANGES IN THE AD VALOREM TAX RATE**

We represent the ad valorem tax rate as $P_k*(1+T_{ko})$, with $P_k$ =1 but $P_k*(1+T_{ko}) \neq 1$. When $P_k = 1$, $P_k*(1+T_{ko}) = 1 + T_{ko}$ and when we want to know $d(P_k*(1+T_{ko}))$ we use the expansion $d(P_k*(1+T_{ko})) = d(P_k + P_k*T_{ko}) = (1 + T_{ko})*dP_k + dT_{ko}$ . In the Harberger (1962) model, $T_{ko}$ is zero initially and $dT_{ko} = T_{ko}$ when the tax is set, leading to the expression $d(P_k*(1+T_{ko}))= dP_k + T_{ko}$ when $P_k$ =1 for this model. As discussed in

Ballentine and Eris (1975), this is not the case when the initial tax is not zero, and thus $dT_{ko} \neq T_{ko}$ in general. We use $d(P_k*(1+T_{ko})) = d(P_k + P_k*T_{ko}) = (1 + T_{ko})*dP_k + dT_{ko}$

**GOVERNMENT**

We have assumed that the government good G does not suffer from loss in value due to inflation, and that the dollar value of expenditure on G equals the amount of G available. Thus, in the initial case $G = P_k*T_{ko}*(K_x + K_y)$ or $G = T_{ko}*(K_x + K_y)$ since $P_k$ = 1 to start with. We do not have a derived demand for this factor due to two reasons; the factor does not have a direct price, and neither firm can control how much is provided since both pay for the input through tax revenue. Since G actually equals tax revenue collected, the only decision variable for the government is the tax rate. Once the rate is set (arbitrarily) the tax revenue and thus the amount of G is determined by the economy.

Now we want an expression for $dG = d(P_k*T_{ko}*(K_x + K_y))$

$$dG = T_{ko}*(K_x + K_y)*\ dP_k \ + \ P_k*(K_x + K_y)*dT_{ko} \ + \ P_k*T_{ko}*(dK_x + dK_y)$$

Expressing this as a proportional change:

$$\frac{dG}{G} = dP_k + \frac{dT_{ko}}{T_{ko}} + \left(\frac{K_x}{K_x+K_y}\right)*\frac{dK_x}{K_x} + \left(\frac{K_y}{K_x+K_y}\right)*\frac{dK_y}{K_y}$$

**SUPPLY**

The production functions for the three sectors are:

$X = f\ (K_x\ , L_x, G)$ $\quad$ $Y = g\ (K_y, L_y, G)$ $\quad$ $Z = h\ (K_z, L_z)$

{Where the amount of G is common to both X and Y, G is a pure public good}

SECTOR X

$X = f(K_x, L_x, G)$

The total change in supply (output) or the total differential can be split into:

$$dX = \frac{\partial f}{\partial K_x} * dK_x + \frac{\partial f}{\partial L_x} * dL_x + \frac{\partial f}{\partial G} * dG$$

Remembering that firms in sector X (at the existing level of G) equate $\frac{\partial f}{\partial K_x} =$

$P_k*(1+T_{ko})$ and $\frac{\partial f}{\partial L_x} = P_{lo}$ and that $P_x = 1$

$$\frac{dX}{X} = \frac{P_k*(1+T_{ko})*K_x}{P_x*X} * \frac{dK_x}{k_x} + \frac{P_{lo}*L_x}{P_x*X} * \frac{dL_x}{L_x} + \frac{G}{P_x*X} * \frac{\partial f}{\partial G} * \frac{dG}{G}$$

Writing $\theta_{kx} = \frac{P_k*(1+T_{ko})*K_x}{P_x*X}$ the tax-inclusive share of capital's product in sector X

And $\theta_{lx} = \frac{P_{lo}*L_x}{P_x*X}$ the share of labor's product in sector X, we get

S1…………………..  $\frac{dX}{X} = \theta_{kx} * \frac{dK_x}{K_x} + \theta_{lx} * \frac{dL_x}{L_x} + \frac{G}{P_x*X} * \frac{\partial f}{\partial G} * \frac{dG}{G}$

The last term in this equation works a little like a "Solow residual" in the sense that like technology, the factor augmenting government good provides a third path for output to grow, beyond what would have been captured by growth in capital and labor alone (Solow 1957, Romer 2006). When production is CRS in K and L, and factor shares $\theta_{kx} + \theta_{lx} = 1$, this term shows the growth in output that cannot be accounted for by changes in K and L.

Similarly, for SECTOR Y we have:

S2……….............. $\frac{dY}{Y} = \theta_{ky} * \frac{dK_y}{K_y} + \theta_{ly} * \frac{dL_y}{L_y} + \frac{G}{P_y * Y} * \frac{\partial g}{\partial G} * \frac{dG}{G}$

Where $\theta_{ky} = \frac{P_k * (1+T_{ko}) * K_y}{P_y * Y}$ the tax-inclusive share of capital's product in sector Y

And $\theta_{ly} = \frac{P_{lo} * L_y}{P_y * Y}$ the share of labor's product in sector Y.

SECTOR Z

There is no tax and no government good in the ROW, so the relevant terms for this sector:

S3a……….............. $\frac{dZ}{Z} = \theta_{kz} * \frac{dK_z}{K_z} + \frac{1}{Z} * L_z * \frac{\partial h}{\partial L_z} * \frac{dL_z}{L_z}$

But $dL_z = 0$ by assumption ($L_{row} = \overline{L_z}$ is fixed), so we can write the above as:

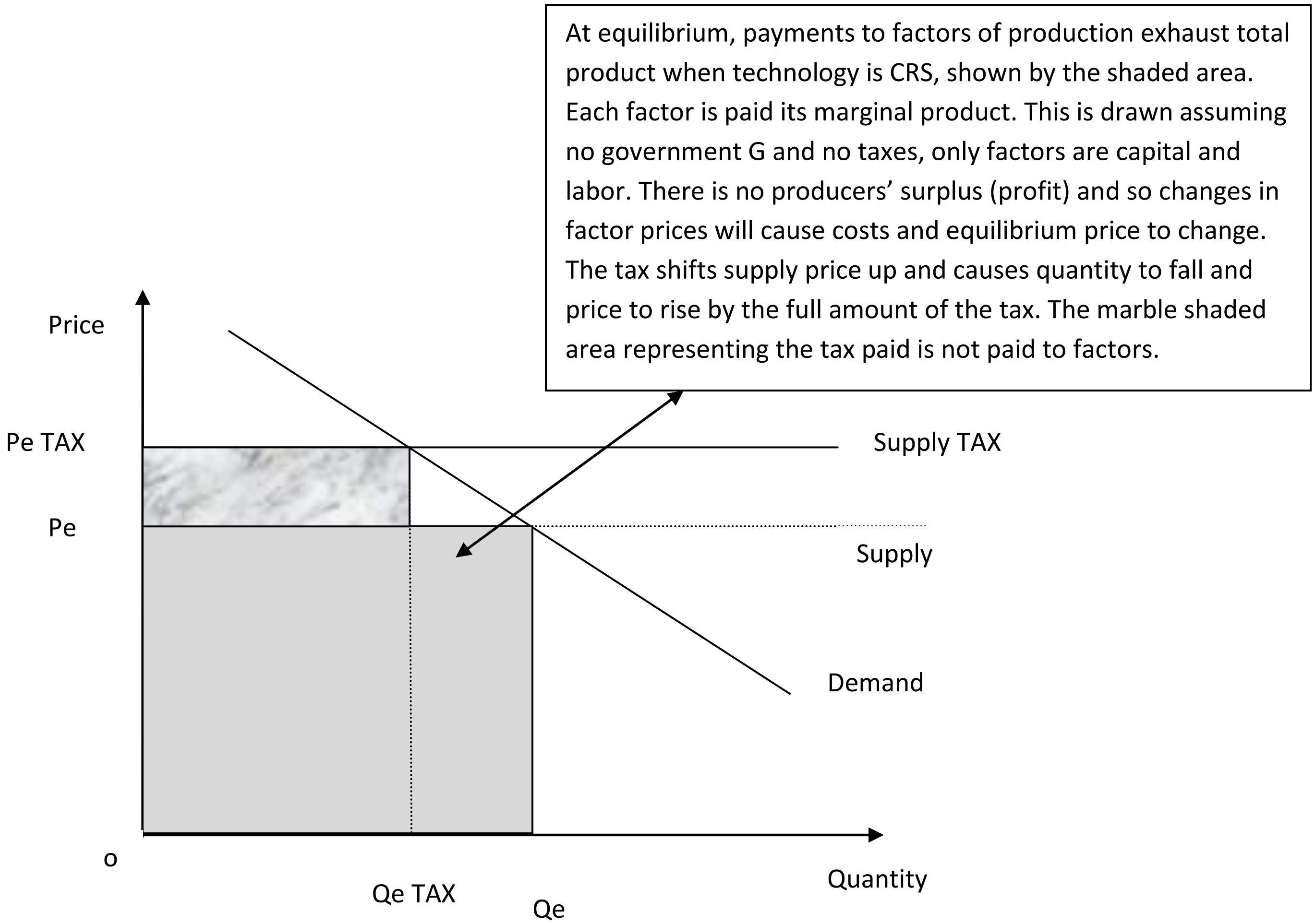
S3……….............. $\frac{dZ}{Z} = \theta_{kz} * \frac{dK_z}{K_z}$

This gives us our three supply equations.

**PRICE FORMATION**

When the firm (sector) has a production function that is HOD 1, and the supply curve exhibits CRS technology, the market equilibrium can be shown by the following graph that shows what would happen in the CRS situation when a fresh tax is imposed but no G is provided:

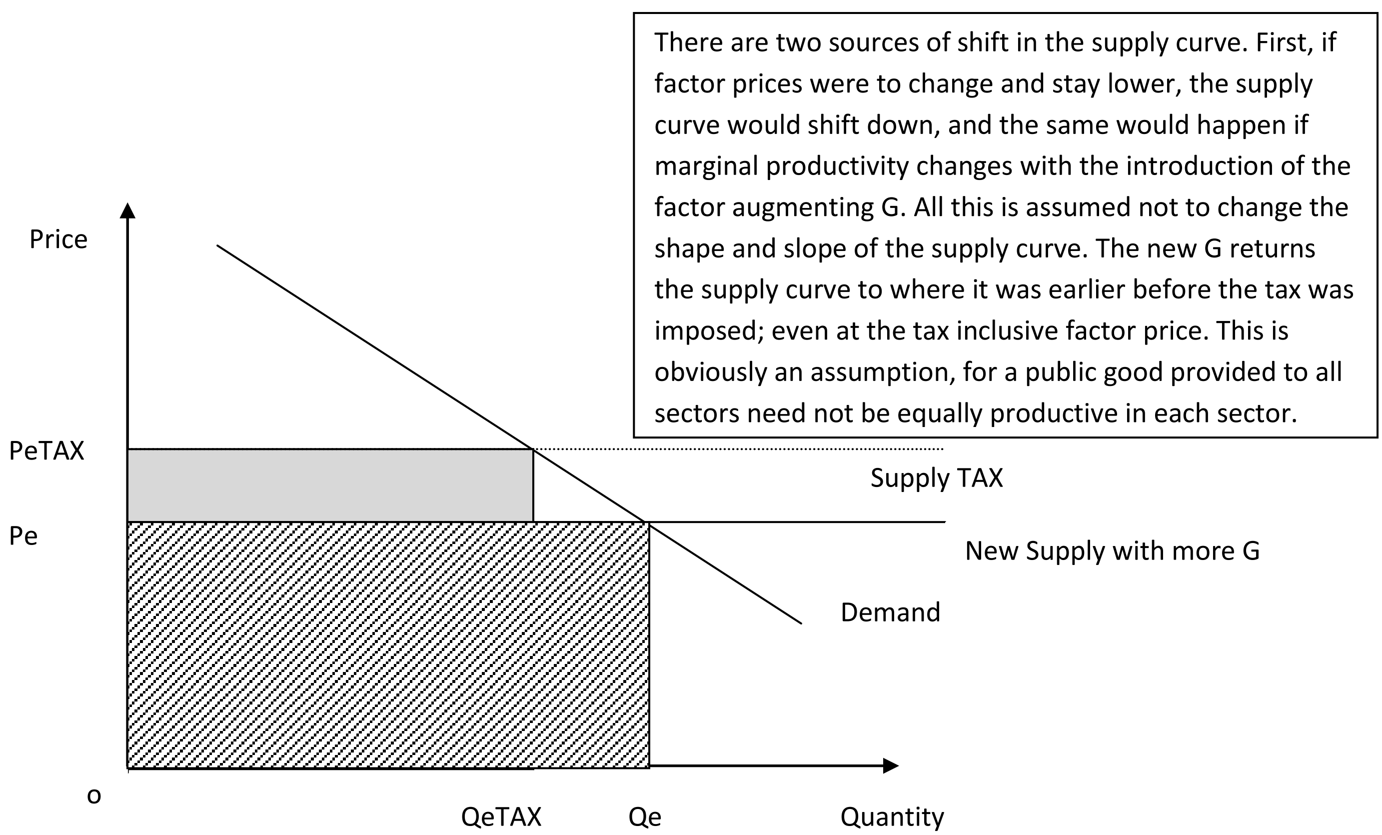
**FIGURE 1 Initial Tax in the CRS Case**




However, what happens when we have a certain level of G? Here we assume that G increases productivity of both factors such that it shifts the entire supply curve down without changing the shape. It is this supply curve with the fixed G = $\bar{G}$ that the firms (sectors) X and Y in OECD took as given and paid capital and labor their marginal product. Suppose G were to increase in such a way as to leave the CRS supply curve's shape unchanged, this would lead to a lowering of price and an expansion of equilibrium output. The reverse would happen if G were lowered.

Thus, there are two sources of change in price: one caused by changes in the price paid to capital (inclusive of the tax) and labor and the second when output expands or contracts due to change in the level of G. Suppose the level of G is increased so as not to change the slope of the supply curve, so that factors of production (capital and labor only since G is not paid for directly by the sector) are still paid the total product at the new level of G. If the total productivity increase due to G is exactly equal to total loss in income due to a tax, and total product is exactly equal to total income the result is no deficiency in aggregate demand. Since we have no savings and investment, all product must eventually return to capital and labor.

**FIGURE 2 Taxes and Provision of G in the CRS Case**



We do not know a priori if total payments to factors are equal to, less than or more than the previous case when G is changed. When we impose a tax, VMP would increase if G is provided. However, what prevents a corner solution is the fact that (a) imposing the tax also has an excess burden that reduces VMP and (b) the fact that the marginal efficiency of investment/capital is not constant. Therefore, a tax would only be imposed to the extent that the net increase in VMP was positive. If we have a declining MEI and an increasing excess burden of taxation that should be sufficient for a large enough economy to ensure that we do not have a corner solution.

Even the proposition that the change in tax and provision of G causes no change in the CRS shape of the supply curve but shifts it can be modified as shown below to accommodate IRS or DRS effects. However, the assumption that changes in factor prices are passed on to output prices in the same proportion as existing factor shares that is characteristic of Harberger (1962) and variants based both on his famous model and Jones (1965). Without CRS, we cannot have a one-to-one correspondence between the tax rate and supply price changes.

We continue to use this feature for price formation, with the caveat that there is now an extra term, that of change in output price caused by the variation in G in addition to the changes in factor prices caused by variation in the tax. The increase in G is assumed to add back to factor returns through the increased product exactly as much as the initial tax had taken away (without G). While this is only required to hold in the aggregate for total incomes to equal total product, whether this holds in each industry is open to debate. Keynesian economics is based on the contention that aggregate factor incomes are too little to purchase output, but when we eliminate money, savings,

investment and force trade balance, such an assumption (inequality in each industry) implies specific knowledge of the productivity of government input by sector, which we may not have.

SECTOR X

Suppose we did not know what the production function for X looked like when we included all factors K, L and G. If it is CRS for K and L alone, we can think of it as IRS with all three being varied.  From Euler's law we can write this as:

$\frac{\partial f}{\partial K_x} * K_x + \frac{\partial f}{\partial L_x} * L_x + \frac{\partial f}{\partial G} * G = A*X$   (where A > 1 and A is the degree of homogeneity)

Now we know that sector X does not have to pay for G, but has to pay the tax on K used. If K and L are paid their marginal products with no tax, and if G had to be paid a user fee equal to its marginal product, then payments to factors would equal A*X> X, total product would be more than exhausted. However, $\frac{\partial f}{\partial K_x}$ would have been equal to $P_k$ and not $P_k*(1+T_{ko})$, so more capital would have been employed. Since G is not paid for and we pay a tax on capital instead while assuming that for the economy as a whole the total tax revenue equals the size of the government input which in turn is at least as valuable as the tax revenue, we are interested in what the tax inclusive payment for capital and payments to labor add up to in each sector. Let us say that:

$\frac{\partial f}{\partial K_x} * K_x + \frac{\partial f}{\partial L_x} * L_x = B*X$ (where X is a function of K, L and G, and the marginal product paid to K includes the tax)

We don't know what B is. B could be <, > or = 1 and < or = A. If we were to proceed with B being unknown:

$\frac{\partial f}{\partial K_x}$ * $K_x$ + $\frac{\partial f}{\partial L_x}$ * $L_x$ = B*$P_x$*X since $P_x$ =1 and if B is further assumed to be a constant, then:

$$d(B*P_x*X) = B*d(P_x*X) = B*[\ P_x*dX\ +X*dP_x] = d[\ \frac{\partial f}{\partial K_x} * K_x + \frac{\partial f}{\partial L_x} * L_x]$$

If we then continue with our assumption that firms pay marginal products:

$$B*[\ P_x*dX\ +X*dP_x] = d[P_k*(1+T_{ko})* K_x + P_{lo}* L_x]$$

$$\Rightarrow \quad [\ P_x*dX\ +X*dP_x] = (\tfrac{1}{B})* d[(P_k+P_k*T_{ko})* K_x + P_{lo}* L_x]$$

$$\Rightarrow \quad P_x*dX + X*dP_x = (\tfrac{1}{B})* [d(P_k+P_k*T_{ko})* K_x + (P_k+P_k*T_{ko})* dK_x + dP_{lo}* L_x + P_{lo}* dL_x\ ]$$

From the supply equations and assumptions $P_x$ = 1 we know that $P_x$*dX = dX and

$$dX = \frac{\partial f}{\partial K_x}* dK_x + \frac{\partial f}{\partial L_x} * dL_x + \frac{\partial f}{\partial G} * dG$$

$$\Rightarrow \quad P_x*dX = P_k*(1+T_{ko})* dK_x + P_{lo} * dL_x + \frac{\partial f}{\partial G} * dG$$

Substituting in our original equation:

$$P_x*dX + X*dP_x = (\tfrac{1}{B})* [d(P_k+P_k*T_{ko})* K_x + (P_k+P_k*T_{ko})* dK_x + dP_{lo}* L_x + P_{lo}* dL_x\ ]$$

$$X*dP_x = (\tfrac{1}{B})* [d(P_k+P_k*T_{ko})* K_x + (P_k+P_k*T_{ko})* dK_x + dP_{lo}* L_x + P_{lo}* dL_x\ ] - P_x*dX$$

$$\Rightarrow \quad X*dP_x = (\tfrac{1}{B})*[d(P_k+P_k*T_{ko})*K_x + (P_k+P_k*T_{ko})*dK_x + dP_{lo}*L_x + P_{lo}*dL_x] - P_k*(1+T_{ko})*dK_x - P_{lo}*dL_x - \frac{\partial f}{\partial G}*dG$$

Now, if we make the “heroic” assumption that factor payments to K and L continue to just exhaust output, this implies that B =1. While we require that for the economy as a whole, total payments to factors have to equal the value of total output to prevent Keynesian deficiency of demand, we do not have to have this equality in each sector. The total amount of G has to equal tax revenue for the economy, and the amount of G is the same for each sector, but we can have that equality constraint for the economy as a whole while total payments in each sector do not equal product in that sector. This may be viewed as G having differential impacts on productivity across sectors, but it can be seen that this can be easily incorporated into the model by placing suitable restrictions on B in each sector. Continuing with B = 1:

$$X*dP_x = [d(P_k+P_k*T_{ko})*K_x + (P_k+P_k*T_{ko})*dK_x + dP_{lo}*L_x + P_{lo}*dL_x] - P_k*(1+T_{ko})*dK_x - P_{lo}*dL_x - \frac{\partial f}{\partial G}*dG$$

$$\Rightarrow \quad X*dP_x = d(P_k+P_k*T_{ko})*K_x + dP_{lo}*L_x - \frac{\partial f}{\partial G}*dG$$

$$\Rightarrow \quad X*dP_x = K_x*dP_k + K_x*(dP_k*T_{ko} + P_k*dT_{ko}) + dP_{lo}*L_x - \frac{\partial f}{\partial G}*dG$$

$$\Rightarrow \quad X*dP_x = K_x*\{(1+T_{ko})*dP_k + P_k*dT_{ko}\} + dP_{lo}*L_x - \frac{\partial f}{\partial G}*dG$$

Using $P_k = 1$, $P_{lo} = 1$ and $P_x = 1$:

$$dP_x = \frac{P_k*(1+T_{ko})*K_x}{P_x*X}*dP_k + \frac{P_k*(1+T_{ko})*K_x}{P_x*X}*\frac{dT_{ko}}{(1+T_{ko})} + \frac{P_{lo}*L_x}{P_x*X}*dP_{lo} - \frac{G}{P_x*X}*\frac{\partial f}{\partial G}*\frac{dG}{G}$$

$$\Rightarrow \quad dP_x = \theta_{kx}*dP_k + \theta_{kx}*\frac{dT_{ko}}{(1+T_{ko})} + \theta_{lx}*dP_{lo} - \frac{G}{P_x*X}*\frac{\partial f}{\partial G}*\frac{dG}{G}$$

…………………(P1)

Similarly for SECTOR Y

$$\Rightarrow \quad dP_y = \theta_{ky}*dP_k + \theta_{ky}*\frac{dT_{ko}}{(1+T_{ko})} + \theta_{ly}*dP_{lo} - \frac{G}{P_y*Y}*\frac{\partial g}{\partial G}*\frac{dG}{G}$$

…………………(P2)

SECTOR Z

There is no tax here. Suppose that total payments to capital in Z are a constant proportion of the product. The residual is paid to labor. If b represents the baseline payments proportion to capital in Z, and if b is assumed to be a constant, then:

$b*P_z*Z = P_k*K_z$ : $b<1$and b is constant

$[P_z*dZ + Z*dP_z] = (\frac{1}{b})*[dP_k*K_z + P_k*dK_z]$

Now from the supply equation for Z we know that: $P_z*dZ = dZ$ (since $P_z = 1$) $= P_k*dK_z$

$\Rightarrow Z*dP_z = (\frac{1}{b})*[dP_k*K_z + P_k*dK_z] - P_z*dZ$

$\Rightarrow Z*dP_z = (\frac{1}{b})*[dP_k*K_z + P_k*dK_z] - P_k*dK_z$

$\Rightarrow dP_z = (\frac{1}{b})*[\theta_{kz}*dP_k + \theta_{kz}*\frac{dK_z}{K_z}] - \theta_{kz}*\frac{dK_z}{K_z}$

(Since $P_k$ and $P_z = 1$, and $\theta_{kz}$ is tax-exclusive)

$\Rightarrow dP_z = (\frac{1}{b})* \theta_{kz} *dP_k + (\frac{1}{b} - 1)* \theta_{kz} * \frac{dK_z}{K_z}$

Now since we have chosen $P_z$ as our numeraire ($dP_z = 0$), we get a relationship between $dP_k$ and $\frac{dK_z}{K_z}$ and ultimately between $dP_k$ and $\frac{dK_z}{K_z} = -(\frac{K_x}{K_z})*\frac{dK_x}{K_x} - (\frac{K_y}{K_z})*\frac{dK_y}{K_y}$

$0 = (\frac{1}{b})* \theta_{kz} *dP_k + (\frac{1-b}{b})* \theta_{kz} * \frac{dK_z}{K_z}$

$dP_k = (b - 1) * \frac{dK_z}{K_z}$ …………………(P3)

Though we have assumed that the proportion of product that goes to capital in sector Z is a constant (b), this is not a necessary assumption. We could have proceeded as if b was a variable. However, to be able to derive an exact expression for the change in b, we need to know the functional form, and to maintain generality, we assume b is constant. The alternative is to introduce the nontradable sector in ROW, and have the market determine $P_{lr}$ ; in that case no further assumptions are necessary to fix b. This is not a departure from the various versions of the Harberger model or the Jones version, since b is replaceable by $\theta_{kz}$ , which is usually assumed fixed. From these expressions we can see that $P_{lr}$ and $dP_{lr}$ can be derived in terms of either total product $P_z$*Z, a combination of $P_z$*Z and $P_k$*$K_z$ or $P_k$*$K_z$ alone:

$\overline{L_z} *P_{lr} = (1 – b)* P_z*Z$ Or $dP_{lr} = (1 – b)* [P_z*dZ]* (\frac{1}{\overline{L_z}})$ since $dP_z = 0$

Or $\overline{L_z} *P_{lr} = P_z*Z - P_k*K_z$

Or $\overline{L_z} *P_{lr} = (\frac{1}{b})* [P_k* K_z ]- P_k*K_z$

$\Rightarrow P_{lr} = (\frac{1}{b} - 1) * [P_k * K_z] * (\frac{1}{L_z})$

$\Rightarrow dP_{lr} = (\frac{1}{b} - 1) * [dP_k * K_z + P_k * dK_z] * (\frac{1}{L_z})$

**SUBSTITUTION**

The direct elasticity of substitution is (with CRS and competition) defined as:

$$S_x = \frac{\frac{d(\frac{K_x}{L_x})}{\frac{K_x}{L_x}}}{\frac{d(\frac{P_k*(1+T_{ko})}{P_{lo}})}{\frac{P_k*(1+T_{ko})}{P_{lo}}}}$$

This can be rewritten as:

$$\frac{d(\frac{K_x}{L_x})}{\frac{K_x}{L_x}} = S_x * \frac{d(\frac{P_k*(1+T_{ko})}{P_{lo}})}{\frac{P_k*(1+T_{ko})}{P_{lo}}}$$

Since the amount of G is taken as fixed and given by the firms, and once provided the fixed amount of G is available to both firms, there is no substitution between G and other factors of production. This is also especially true since the amount of G cannot be varied by the firm since the firm ignores the externality from paying the tax on capital, and no payment has to be made for G. Therefore, we have a single elasticity of substitution per sector. Since the derivation follows exactly the same steps as in Tresch (2002) it is not derived here, the results are stated, noting that we have defined it to be negative:

SECTOR X

$$\frac{dK_x}{K_x} - \frac{dL_x}{L_x} = S_x * (dP_k + \frac{dT_{ko}}{(1+T_{ko})} - dP_{lo})$$

…………………………………(U1)

SECTOR Y

$$\frac{dK_y}{K_y} - \frac{dL_y}{L_y} = S_y * (dP_k + \frac{dT_{ko}}{(1+T_{ko})} - dP_{lo})$$

……………………………………(U2)

SECTOR Z

In the ROW $dL_z = 0$ by assumption, so any amount of capital is combined with the fixed amount of labor, which is not varied at all. Therefore, there is no substitution elasticity or equation for this sector.

**DEMAND**

There are two alternative formulations considered for the demand functions. Firstly, following Ballentine and Eris (1975) and Alm (1985), we cannot work with income compensated demand curves when we have an input financed by taxes. In the original Harberger (1962) formulation, the assumption is that the government either consumes out of tax revenues in exactly the same way as the consumer would have or that the government returns lump-sum to the consumer the proceeds of the tax. Either way, demand for output is assumed to not be affected at all by the change in income caused by the tax (though it is affected by any price changes). This allowed the demand

functions to be treated as if they were income compensated demand functions, and elasticities to be considered Hicksian (or "Harbergerarian").

However, when there are two countries, and the proceeds are spent on providing an input used only by the taxing country rather than purchasing final output or even returning money lump-sum to consumers, we cannot ignore income effects. There is a second reason for this. One way of thinking about "who pays the tax" is to identify where the tax revenue comes from.

The second way, followed by Harbereger (1962) is to identify who faces changes in factor income as a result of the tax. If the tax changes the worldwide income of capital or labor, the ROW also "pays" for the OECD tax in the Harberger sense. Secondly, as pointed out in Ballentine and Eris (1975), when the initial tax is not zero, a reduction in the tax has an income effect as well. The tax on capital is part of factor income, and disposable income changes due to the tax change. However, total factor returns are also affected by the government input, since we have assumed that total factor payments with input exhaust the product with input.

In our models, either tax revenue is not returned to the consumer due to provision of G, or a transfer payment is provided. In both cases we introduce income effects in the demand curves.

There is a third and fourth issue here. With two countries, we have two income effects, not one (unless we take total world income, which would defeat a lot of interest in this dissertation). The fourth issue is that when the tax is spent on providing a government input, what enters the demand curve is disposable income and not national or

total income- simply because the tax revenue is not spent on purchasing final output. However, due to the national income identity, total output has to equal total income, as well as disposable income since there is no transfer payment when G is provided. Otherwise, with disposable income being less than total product, we have a tendency towards recession built in. If total output were more than national income or total returns to factors, this would be similar to assuming that there is an effect of government input on total product but the increased product returned to factors of production does not compensate for the tax lost. Therefore we have to assume that even if disposable income falls due to a tax, it is returned to the consumer due to higher productivity.

We need not make this assumption at all if we did not want to have CRS price changes. As we have discussed earlier, modeling commodity price changes depends on what we assume about the productivity of government input and the assumptions we make about total factor payments. We therefore assume that in the absence of saving and abnormal profits, and the fact that perfect competition prevails and supply functions are HOD one in capital and labor, all domestic product has to be returned to either capital or labor as factor income.

Therefore, the modifications to the demand curve can be considered in two ways. We either have one consolidated demand curve each for the two tradable goods X and Z representing total world demand or we have two separable components representing each country's demand. If we work with a consolidated demand curve, we still have two separate income effects representing OECD and the ROW, and if we have additive and separate demand curves, we assume that they do not interact with or affect each other, other than through the joint determination of incomes. The demand curve for Y (the non-

tradable good) is completely dependent on the OECD country since the assumption is that it is not consumed at all by the ROW. We proceed by assuming separate and additive demand curves to begin with, but can later consider how these may be related to the single equation case as well.

SECTOR X

$X = X^o(P_x, P_y, P_z, M^o) + X^r(P_x, P_z, M^r)$

$X^o$ represents OECD demand for X. it is a function of all three prices since the OECD consumer consumes the products of all sectors. $M^o$ is OECD consumer's disposable income; we have formulated it such that it equals GDP for reasons discussed in chapter one, but other equations are possible. This is a Marshallian demand curve. $X^r$ is the ROW demand for X. it is a function of prices of only the two tradable goods since the ROW does not consume Y. $M^r$ is the disposable income of the ROW consumer. Since there are no taxes in the ROW, disposable income is obviously equal to GDP. This is also a Marshallian demand curve. Explicit formulations for consumer income follow.

We note that when a tax is imposed, the tax revenue is not returned to the consumer, but the input provided out of the tax revenue raises the total product to at least the previous level without the tax. If this were not so, then there was no justification for the tax or input in the first place. The only extra assumption made in the context of retaining CRS industries is that it holds for each industry with equality and not just for the economy as a whole.

$M^o = P_k*(1+T_{ko})* (K_x + K_y) + P_{lo}*(L_x + L_y)$

$dM^o = (1+T_{ko})* P_k*(dK_x + dK_y) + (1+T_{ko})* (K_x + K_y)*dP_k +(K_x + K_y) *dT_{ko} +$

$P_{lo}*(dL_x + dL_y) + (L_x + L_y)*dP_{lo}$

But $dL_x + dL_y = 0$ by definition, so

$dM^o = (1+T_{ko})*P_k*(dK_x + dK_y) + (1+T_{ko})*(K_x + K_y)*dP_k +(K_x + K_y)*dT_{ko} +$

$(L_x + L_y)*dP_{lo}$

$dM^o = (1+T_{ko})*K_x*\frac{dK_x}{K_x} + (1+T_{ko})*K_y*\frac{dK_y}{K_y} + (1+T_{ko})*(K_x + K_y)*dP_k + (K_x + K_y)*dT_{ko}$

$+(L_x + L_y)*dP_{lo}$

For $M^r$ there is no tax, so GDP is the same as disposable income:

$M^r = P_k * K_z + P_{lr} * L_z$

But $P_{lr} * L_z = \overline{L_z} *P_{lr} = P_z*Z - P_k*K_z = (\frac{1}{b})* [P_k* K_z ]- P_k*K_z$

⇨ $M^r = P_k * K_z + (\frac{1}{b})* [P_k* K_z ]- P_k*K_z$

⇨ $M^r = (\frac{1}{b})* [P_k* K_z ]$

$dM^r = (\frac{1}{b})* [K_z*dP_k + P_k*dK_z]$

But $dP_k = (b - 1 ) * \frac{dK_z}{K_z}$ …………………(P3)

⇨ $dM^r = (\frac{1}{b})* [K_z*(b - 1 ) * \frac{dK_z}{K_z} + P_k*dK_z]$

$\Rightarrow$ $dM^r = (\frac{1}{b})* [(b - 1)*dK_z + dK_z]$

$\Rightarrow$ $dM^r = dK_z$

$\Rightarrow$ $dM^r = - K_x*\frac{dK_x}{K_x} - K_y*\frac{dK_y}{K_y}$

Taking the total differential of the demand for X we get:

$$dX = \frac{\partial X^o}{\partial P_x} * dP_x + \frac{\partial X^o}{\partial P_y} * dP_y + \frac{\partial X^o}{\partial P_z} * dP_z + \frac{\partial X^o}{\partial M^o} * dM^o + \frac{\partial X^r}{\partial P_x} * dP_x + \frac{\partial X^r}{\partial P_z} * dP_z + \frac{\partial X^r}{\partial M^r} * dM^r$$

Dividing by X we get:

$$\Rightarrow \quad \frac{dX}{X} = \frac{X^o}{P_x * X} * E^o_{xx} * dP_x + \frac{X^o}{P_y * X} * E^o_{xy} * dP_y + \frac{X^o}{P_z * X} * E^o_{xz} * dP_z + \frac{X^r}{P_x * X} * E^r_{xx} * dP_x + \frac{X^r}{P_z * X} * E^r_{xz} * dP_z + (\frac{1}{X})*[\frac{\partial X^o}{\partial M^o} * dM^o + \frac{\partial X^r}{\partial M^r} * dM^r]$$

Remembering that all prices are = 1 to begin with, we can write the above as:

$$\Rightarrow \quad \frac{dX}{X} = [\frac{X^o}{X} * E^o_{xx} + \frac{X^r}{X} * E^r_{xx}] * dP_x + [\frac{X^o}{X} * E^o_{xy}]* dP_y + [\frac{X^o}{X} * E^o_{xz} + \frac{X^r}{X} * E^r_{xz}]* dP_z + (\frac{1}{X})*[\frac{\partial X^o}{\partial M^o} * dM^o + \frac{\partial X^r}{\partial M^r} * dM^r] \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..\text{(D1)}$$

Since Y is not consumed by ROW, $\frac{X^o}{X} * E^o_{xy} = E_{xy}$

We have not imposed the restriction that demand curves are HOD zero in all prices and income here. If we are willing to use that very reasonable condition, we can substitute out some elasticities using the Euler equation, the relation being that the sum of

all income and price elasticities for an HOD zero demand curve add up to zero. Further, the Marshallian elasticities can also be converted to Hicksian elasticities using the Slutsky equation in elasticity form as in Ballentine and Eris (1975). A third possibility is to write the weighted partial elasticities as total elacticies as we would have obtained from a unified demand curve.

SECTOR Y

Since Y is non-tradable, the demand curve consists of demand only from the OECD country.

$Y = Y^o(P_x, P_y, P_z, M^o)$

$$\frac{dY}{Y} = E_{yx} * dP_x + E_{yy} * dP_y + E_{yz} * dP_z + \left(\frac{1}{Y}\right) * \left[\frac{\partial Y^o}{\partial M^o} * dM^o\right] \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(D2)}$$

SECTOR Z

$$\frac{dZ}{Z} = \left[\frac{Z^o}{Z} * E^o_{zx} + \frac{Z^r}{Z} * E^r_{zx}\right] * dP_x + \left[\frac{Z^o}{Z} * E^o_{zy}\right] * dP_y + \left[\frac{Z^o}{Z} * E^o_{zz} + \frac{Z^r}{Z} * E^r_{zz}\right] * dP_z + \left(\frac{1}{Z}\right) * \left[\frac{\partial Z^o}{\partial M^o} * dM^o + \frac{\partial Z^r}{\partial M^r} * dM^r\right] \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(D3)}$$

**SUMMARY OF EQUATIONS FOR THE MODEL SO FAR**

***Supply***

$$\frac{dX}{X} = \theta_{kx} * \frac{dK_x}{K_x} + \theta_{lx} * \frac{dL_x}{L_x} + \frac{G}{P_x * X} * \frac{\partial f}{\partial G} * \frac{dG}{G} \quad \ldots. \text{(S1)}$$

$$\frac{dY}{Y} = \theta_{ky} * \frac{dK_y}{K_y} + \theta_{ly} * \frac{dL_y}{L_y} + \frac{G}{P_y * Y} * \frac{\partial g}{\partial G} * \frac{dG}{G} \quad \ldots (S2)$$

$$\frac{dZ}{Z} = \theta_{kz} * \frac{dK_z}{K_z} \quad \ldots (S3)$$

### *Price Formation*

$$dP_x = \theta_{kx} * dP_k + \theta_{kx} * \frac{dT_{ko}}{(1+T_{ko})} + \theta_{lx} * dP_{lo} - \frac{G}{P_x * X} * \frac{\partial f}{\partial G} * \frac{dG}{G} \quad \ldots . (P1)$$

(Where $\theta_{kx}$ is tax inclusive)

$$dP_y = \theta_{ky} * dP_k + \theta_{ky} * \frac{dT_{ko}}{(1+T_{ko})} + \theta_{ly} * dP_{lo} - \frac{G}{P_y * Y} * \frac{\partial g}{\partial G} * \frac{dG}{G} \quad \ldots (P2)$$

(Where $\theta_{ky}$is tax inclusive)

$$dP_z = \left(\frac{1}{b}\right) * \theta_{kz} * dP_k + \left(\frac{1}{b} - 1\right) * \theta_{kz} * \frac{dK_z}{K_z} \quad \ldots (P3)$$

### *Substitution*

$$\frac{dK_x}{K_x} - \frac{dL_x}{L_x} = S_x * \left(dP_k + \frac{dT_{ko}}{(1+T_{ko})} - dP_{lo}\right) \quad \ldots (U1)$$

$$\frac{dK_y}{K_y} - \frac{dL_y}{L_y} = S_y * \left(dP_k + \frac{dT_{ko}}{(1+T_{ko})} - dP_{lo}\right) \quad \ldots (U2)$$

### *Demand*

$$\frac{dX}{X} = \left[\frac{X^o}{X} * E^o_{xx} + \frac{X^r}{X} * E^r_{xx}\right] * dP_x + \left[\frac{X^o}{X} * E^o_{xy}\right] * dP_y +$$

$$\left[\frac{X^o}{X} * E^o_{xz} + \frac{X^r}{X} * E^r_{xz}\right] * dP_z + \left(\frac{1}{X}\right) * \left[\frac{\partial X^o}{\partial M^o} * dM^o + \frac{\partial X^r}{\partial M^r} * dM^r\right] \quad \ldots\ldots\ldots\ldots (D1)$$

$$\frac{dY}{Y} = E_{yx} * dP_x + E_{yy} * dP_y + E_{yz} * dP_z + \left(\frac{1}{Y}\right) * \left[\frac{\partial Y^o}{\partial M^o} * dM^o\right] \quad \ldots\ldots\ldots\ldots\ldots (D2)$$

$\frac{dZ}{Z}$ = [ $\frac{Z^o}{Z}$ * $E^o_{zx}$ + $\frac{Z^r}{Z}$ * $E^r_{zx}$] * $dP_x$ + [ $\frac{Z^o}{Z}$ * $E^o_{zy}$ ]* $dP_y$ +

[ $\frac{Z^o}{Z}$ * $E^o_{zz}$ + $\frac{Z^r}{Z}$ * $E^r_{zz}$ ]* $dP_z$ + ( $\frac{1}{Z}$)*[ $\frac{\partial Z^o}{\partial M^o}$ * $dM^o$ + $\frac{\partial Z^r}{\partial M^r}$ * $dM^r$ ] …………. (D3)

Where elasticities are Marshallian and income effects $M^o$ and $M^r$ is disposable income.

### *Adding up*

$dK_x + dK_y + dK_z = 0$ ………….(A1)

$dL_x + dL_y = 0$ ………….(A2)

$dL_z = 0$ ………….(A3)

$dP_z = 0$ [Numeraire]

$P_k$, $P_{lo}$ and $P_{lr}$ (factor prices) and $P_x$, $P_y$ and $P_z$ (output prices) are all equal to 1 and to each other to begin with

### *Other relationships*

$d(P_k*(1+T_{ko})) = dP_k + T_{ko}*dP_k + P_k*dT_{ko} = dP_k + T_{ko}*dP_k + dT_{ko}$ …..(O1)

$G = P_k*T_{ko}*(K_x + K_y)$

$dG = T_{ko}*(K_x + K_y)* dP_k + P_k*(K_x + K_y)*dT_{ko} + P_k*T_{ko}*(dK_x + dK_y)$

$\frac{dG}{G} = dP_k + \frac{dT_{ko}}{T_{ko}} + \left(\frac{K_x}{K_x+K_y}\right)*\frac{dK_x}{K_x} + \left(\frac{K_y}{K_x+K_y}\right)*\frac{dK_y}{K_y}$ …..(O2)

$dM^o = (1+T_{ko})*K_x*\frac{dK_x}{K_x} + (1+T_{ko})*K_y*\frac{dK_y}{K_y} + (1+T_{ko})*(K_x + K_y)*dP_k + (K_x + K_y)*dT_{ko} +$

$(L_x + L_y)*dP_{lo}$

$dM^r = - K_x*\frac{dK_x}{K_x} - K_y*\frac{dK_y}{K_y}$

$\overline{L_z} *P_{lr} = (\frac{1}{b})* [P_k* K_z ]- P_k*K_z$

$P_{lr} = (\frac{1}{b} - 1)* [P_k* K_z ]* (\frac{1}{\overline{L_z}})$

$dP_{lr} = (\frac{1}{b} - 1)* [dP_k* K_z + P_k* dK_z ]* (\frac{1}{\overline{L_z}})$

Or $dP_{lr} = (1 – b)* [P_z*dZ]* (\frac{1}{\overline{L_z}})$ since $dP_z = 0$

$(\frac{1}{b})* \theta_{kz} *dP_k + (\frac{1-b}{b})* \theta_{kz} * \frac{dK_z}{K_z} = 0$

$dP_k = (b - 1 ) * \frac{dK_z}{K_z}$

**SOLUTION PROCEDURES**

Using $dK_x + dK_y + dK_z =0$ …………(A1) we can write:

$\frac{dK_z}{K_z} = - (\frac{K_x}{K_z})*\frac{dK_x}{K_x} - (\frac{K_y}{K_z})*\frac{dK_y}{K_y}$

And using $dL_x + dL_y = 0$ …………(A2) we can write:

$\frac{dL_x}{L_x} = - (\frac{L_y}{L_x})*\frac{dL_y}{L_y}$

Then we equate the demand and supply equations for sector X and Y, remembering that due to Walras' Law, the market for Z is in equilibrium when the first

two are balanced. We substitute $dP_z = 0$ from the numeraire equation in the demand functions and $dL_z = 0$ wherever it appears. We also substitute $dP_k = (b - 1) * \frac{dK_z}{K_z}$ or $dP_k = (b - 1) * [- (\frac{K_x}{K_z})*\frac{dK_x}{K_x} - (\frac{K_y}{K_z})*\frac{dK_y}{K_y}]$ where we can. Now the substitution equations are rewritten as:

$$\frac{dK_x}{K_x} - \frac{dL_x}{L_x} = S_x * (dP_k + \frac{dT_{ko}}{(1+T_{ko})} - dP_{lo}) \qquad \dots (U1)$$

$$\Rightarrow \quad \frac{dK_x}{K_x} + (\frac{L_y}{L_x})*\frac{dL_y}{L_y} = S_x *(b - 1) * \frac{dK_z}{K_z} + S_x * \frac{dT_{ko}}{(1+T_{ko})} - S_x *dP_{lo}$$

$$\Rightarrow \quad \frac{dK_x}{K_x} + (\frac{L_y}{L_x})*\frac{dL_y}{L_y} = S_x *(b - 1)*\{ - (\frac{K_x}{K_z})*\frac{dK_x}{K_x} - (\frac{K_y}{K_z})*\frac{dK_y}{K_y} \}+ S_x * \frac{dT_{ko}}{(1+T_{ko})} - S_x *dP_{lo}$$

$$\frac{dK_y}{K_y} - \frac{dL_y}{L_y} = S_y * (dP_k + \frac{dT_{ko}}{(1+T_{ko})} - dP_{lo}) \qquad \dots (U2)$$

$$\Rightarrow \quad \frac{dK_y}{K_y} - \frac{dL_y}{L_y} = S_y *(b - 1)*\{ - (\frac{K_x}{K_z})*\frac{dK_x}{K_x} - (\frac{K_y}{K_z})*\frac{dK_y}{K_y} \} + S_y* \frac{dT_{ko}}{(1+T_{ko})} - S_y*dP_{lo}$$

Next we equate the supply and demand for X, and remembering that $dP_z = 0$ and $\frac{dL_x}{L_x} = - (\frac{L_y}{L_x})*\frac{dL_y}{L_y}$:

$$\theta_{kx} *\frac{dK_x}{K_x} - \theta_{lx} * (\frac{L_y}{L_x})*\frac{dL_y}{L_y} + \frac{G}{P_x*X} * \frac{\partial f}{\partial G} * \frac{dG}{G} = [\frac{X^o}{X} * E^o_{xx} + \frac{X^r}{X} * E^r_{xx}] * dP_x + [\frac{X^o}{X} * E^o_{xy}]* dP_y + (\frac{1}{X})*[\frac{\partial X^o}{\partial M^o} * dM^o + \frac{\partial X^r}{\partial M^r} * dM^r ]$$

Equating demand and supply for Y:

$$\theta_{ky} *\frac{dK_y}{K_y} + \theta_{ly} *\frac{dL_y}{L_y} + \frac{G}{P_y*Y} * \frac{\partial g}{\partial G} * \frac{dG}{G} = E_{yx} *dP_x + E_{yy} *dP_y + (\frac{1}{Y})*[\frac{\partial Y^o}{\partial M^o} * dM^o ]$$

We now have 4 equations in the following unknowns: $\frac{dK_y}{K_y}$ , $\frac{dL_y}{L_y}$, $\frac{dG}{G}$ ,$dP_x$ ,$dP_y$ , $dM^o$, $\frac{dK_x}{K_x}$ , $dM^r$, $dP_{lo}$ and $dP_k$ . We can substitute for some variables using:

$dP_x = \theta_{kx} * (b - 1) * [- (\frac{K_x}{K_z})*\frac{dK_x}{K_x} - (\frac{K_y}{K_z})*\frac{dK_y}{K_y}] + \theta_{kx} * \frac{dT_{ko}}{(1+T_{ko})} + \theta_{lx} * dP_{lo} - \frac{G}{P_x * X} * \frac{\partial f}{\partial G} * \frac{dG}{G}$ (Where $\theta_{kx}$ is tax inclusive)

$dP_y = \theta_{ky} * (b - 1) * [- (\frac{K_x}{K_z})*\frac{dK_x}{K_x} - (\frac{K_y}{K_z})*\frac{dK_y}{K_y}] + \theta_{ky} * \frac{dT_{ko}}{(1+T_{ko})} + \theta_{ly} * dP_{lo} - \frac{G}{P_y * Y} * \frac{\partial g}{\partial G} * \frac{dG}{G}$ (Where $\theta_{ky}$is tax inclusive)

$\frac{dG}{G} = (b - 1) * [- (\frac{K_x}{K_z})*\frac{dK_x}{K_x} - (\frac{K_y}{K_z})*\frac{dK_y}{K_y}] + \frac{dT_{ko}}{T_{ko}} + \left(\frac{K_x}{K_x+K_y}\right)*\frac{dK_x}{K_x} + \left(\frac{K_y}{K_x+K_y}\right)*\frac{dK_y}{K_y}$

$dM^o = (1+T_{ko})*K_x*\frac{dK_x}{K_x} + (1+T_{ko})*K_y*\frac{dK_y}{K_y} + (1+T_{ko})*(K_x + K_y)*(b - 1) * [- (\frac{K_x}{K_z})*\frac{dK_x}{K_x} - (\frac{K_y}{K_z})*\frac{dK_y}{K_y}] + (K_x + K_y)*dT_{ko} + (L_x + L_y)*dP_{lo}$

$dM^r = (\frac{1}{b}) * [K_z*(b - 1) * \frac{dK_z}{K_z} + P_k*dK_z]$

$\Rightarrow dM^r = (\frac{1}{b}) * [(b - 1)*dK_z + dK_z]$

$\Rightarrow dM^r = dK_z$

$\Rightarrow dM^r = - K_x*\frac{dK_x}{K_x} - K_y*\frac{dK_y}{K_y}$

And we are left with 4 equations in 4 variables, namely: $\frac{dK_y}{K_y}$ , $\frac{dL_y}{L_y}$ , $\frac{dK_x}{K_x}$ and $dP_{lo}$

Next we use symbolic notation for some of the parameters involved:

$$\epsilon xx = [\frac{X^o}{X} * E^o_{xx} + \frac{X^r}{X} * E^r_{xx}] \qquad \epsilon xy = [\frac{X^o}{X} * E^o_{xy}]$$

These weighted elasticities are Marshallian and are derived assuming the demand curves are separate for OECD and ROW. However, if the two are not separable, they can be replaced by a joint elasticity from a combined demand curve, which the Є represent. The elasticities would still be Marshallian, only the marginal income effects would be modified to reflect that they are income effects from a combined demand curve. There will still be two income effects for OECD and ROW disposable income. Further, there is no elasticity of demand for ROW for X with respect to price of Y since ROW does not consume Y. The elasticities here are not necessarily symmetrical either, since they are not Hicksian. Following Ballentine and Eris (1975) they can be decomposed using the Slutsky equation and re-written as Hicksian elasticities that are symmetric; this will involve a few extra income effect terms. At present, we only note that either representation is possible, and move on. We can write for symmetry since $Y^o = Y$:

$$\epsilon yx = E_{yx} \quad \text{and} \quad \epsilon yy = E_{yy}$$

**REDUCED FORM VERSION OF THE MODEL**

$$[1+ S_x*(b - 1)*(\frac{K_x}{K_z})]* \frac{dK_x}{K_x} + S_x*(b - 1)*(\frac{K_y}{K_z})*\frac{dK_y}{K_y} + (\frac{L_y}{L_x})*\frac{dL_y}{L_y} + S_x*dP_{lo} = S_x * \frac{dT_{ko}}{(1+T_{ko})}$$

…..(1)

$$S_y*(b - 1)*(\frac{K_x}{K_z})*\frac{dK_x}{K_x} + [1+ S_y*(b - 1)*(\frac{K_y}{K_z})]* \frac{dK_y}{K_y} - \frac{dL_y}{L_y} + S_y*dP_{lo} = S_y* \frac{dT_{ko}}{(1+T_{ko})}$$

…..(2)

$$\{\theta_{kx} + [\frac{G}{X} * \frac{\partial f}{\partial G}*(1+ \epsilon xx) + \frac{G}{Y} * \frac{\partial g}{\partial G}*\epsilon xy]*[\left(\frac{K_x}{K_x+K_y}\right) - (b-1)*\left(\frac{K_x}{K_z}\right)] +$$

$$[\theta_{kx}* \epsilon xx + \theta_{ky}* \epsilon xy]*[(b-1)*\left(\frac{K_x}{K_z}\right)] + \frac{K_x}{X} * \frac{\partial X^r}{\partial M^r} - \frac{(1+T_{ko})*K_x}{X} * \frac{\partial X^o}{\partial M^o} +$$

$$\frac{(1+T_{ko})*(Kx + Ky)}{X}*(b-1)*\left(\frac{K_x}{K_z}\right)*\frac{\partial X^o}{\partial M^o}\}* \frac{dK_x}{K_x} +$$

$$\{[\frac{G}{X} * \frac{\partial f}{\partial G}*(1+ \epsilon xx) + \frac{G}{Y} * \frac{\partial g}{\partial G}*\epsilon xy]*[\left(\frac{K_y}{K_x+K_y}\right) - (b-1)*\left(\frac{K_y}{K_z}\right)] +$$

$$[\theta_{kx}* \epsilon xx + \theta_{ky}* \epsilon xy]*[(b-1)*\left(\frac{K_y}{K_z}\right)] + \frac{K_y}{X} * \frac{\partial X^r}{\partial M^r} - \frac{(1+T_{ko})*K_y}{X} * \frac{\partial X^o}{\partial M^o} +$$

$$\frac{(1+T_{ko})*(Kx + Ky)}{X}*(b-1)*\left(\frac{K_y}{K_z}\right)*\frac{\partial X^o}{\partial M^o}\}* \frac{dK_y}{K_y} - \theta_{lx} * (\frac{L_y}{L_x})*\frac{dL_y}{L_y} - \{ \epsilon xx* \theta_{lx} + \epsilon xy* \theta_{ly} +$$

$$\frac{(Lx + Ly)}{X} *\frac{\partial X^o}{\partial M^o}\}* dP_{lo} = - \{ \frac{G}{X} * \frac{\partial f}{\partial G}*(1+ \epsilon xx) + \epsilon xy* \frac{G}{Y} * \frac{\partial g}{\partial G} \}* \frac{dT_{ko}}{T_{ko}} +$$

$$\{\epsilon xx* \theta_{kx} + \epsilon xy* \theta_{ky} + [\theta_{kx}+ \frac{K_y}{X} *(1 + T_{ko})]* \frac{\partial X^o}{\partial M^o} \}* \frac{dT_{ko}}{(1+T_{ko})} \qquad \text{.....(3)}$$

$$\{[\frac{G}{Y} * \frac{\partial g}{\partial G}*(1+ \epsilon yy) + \frac{G}{X} * \frac{\partial f}{\partial G}*\epsilon yx]*[\left(\frac{K_x}{K_x+K_y}\right) - (b-1)*\left(\frac{K_x}{K_z}\right)] +$$

$$[\theta_{kx}* \epsilon yx + \theta_{ky}* \epsilon yy]*[(b-1)*\left(\frac{K_x}{K_z}\right)] - \frac{(1+T_{ko})*K_x}{Y} * \frac{\partial Y^o}{\partial M^o} +$$

$$\frac{(1+T_{ko})*(Kx + Ky)}{Y}*(b-1)*\left(\frac{K_x}{K_z}\right)*\frac{\partial Y^o}{\partial M^o}\}* \frac{dK_x}{K_x} +$$

$$\{\theta_{ky} + [\frac{G}{Y} * \frac{\partial g}{\partial G}*(1+ \epsilon yy) + \frac{G}{X} * \frac{\partial f}{\partial G}*\epsilon yx]*[\left(\frac{K_y}{K_x+K_y}\right) - (b-1)*\left(\frac{K_y}{K_z}\right)] +$$

$$[\theta_{kx}* \epsilon yx + \theta_{ky}* \epsilon yy]*[(b-1)*\left(\frac{K_y}{K_z}\right)] - \frac{(1+T_{ko})*K_y}{Y} * \frac{\partial Y^o}{\partial M^o} +$$

$$\frac{(1+T_{ko})*(Kx+Ky)}{Y}*(b-1)*\left(\frac{K_y}{K_z}\right)*\frac{\partial Y^o}{\partial M^o}\}*\frac{dK_y}{K_y} \quad +$$

$$\theta_{ly}*\frac{dL_y}{L_y} \;-\{\, Єyx*\theta_{lx} + Єyy*\theta_{ly} + \frac{(Lx+Ly)}{Y}*\frac{\partial Y^o}{\partial M^o}\,\}*\, dP_{lo} \;=$$

$$-\{\; \frac{G}{Y}*\frac{\partial g}{\partial G}*(1+Єyy) + Єyx*\;\frac{G}{X}*\frac{\partial f}{\partial G}\quad \}*\frac{dT_{ko}}{T_{ko}} \qquad +$$

$$\{Єyx*\theta_{kx} + Єyy*\theta_{ky} + [\theta_{ky} + \frac{K_x}{Y}*(1+T_{ko})]*\frac{\partial Y^o}{\partial M^o}\}*\frac{dT_{ko}}{(1+T_{ko})} \qquad \ldots.(4)$$

These, as before are 4 equations in 4 variables. If variables are denoted in the following order by subscript j = 1, ..., 4; $\frac{dK_x}{K_x}$, $\frac{dK_y}{K_y}$, $\frac{dL_y}{L_y}$, $dP_{lo}$ and subscript i = 1, ..., 4 represents the equation, the 4 equations above can be written in symbolic form with $a_{ij}$ representing coefficients attached to the left hand side variables and $b_i$ the constants on the right hand side.

$$A_{11}*\frac{dK_x}{K_x} + A_{12}*\frac{dK_y}{K_y} + A_{13}*\frac{dL_y}{L_y} + A_{14}*dP_{lo} = B_1 \qquad \ldots\ldots\ldots\ldots \qquad (1)$$

$$A_{21}*\frac{dK_x}{K_x} + A_{22}*\frac{dK_y}{K_y} + A_{23}*\frac{dL_y}{L_y} + A_{24}*dP_{lo} = B_2 \qquad \ldots\ldots\ldots\ldots \qquad (2)$$

$$A_{31}*\frac{dK_x}{K_x} + A_{32}*\frac{dK_y}{K_y} + A_{33}*\frac{dL_y}{L_y} + A_{34}*dP_{lo} = B_3 \qquad \ldots\ldots\ldots\ldots \qquad (3)$$

$$A_{41}*\frac{dK_x}{K_x} + A_{42}*\frac{dK_y}{K_y} + A_{43}*\frac{dL_y}{L_y} + A_{44}*dP_{lo} = B_4 \qquad \ldots\ldots\ldots\ldots \qquad (4)$$

Where

$A_{11}= [1+ S_x*(b - 1)*(\frac{K_x}{K_z})]$ $\qquad A_{12} = S_x*(b - 1)*(\frac{K_y}{K_z})$ $\qquad A_{13} = (\frac{L_y}{L_x})$ $\qquad A_{14} = S_x$

$B_1 = S_x * \frac{dT_{ko}}{(1+T_{ko})}$

$A_{21} = S_y *(b - 1)*(\frac{K_x}{K_z})$ $A_{22} = [1+ S_y*(b - 1)*(\frac{K_y}{K_z})]$ $A_{23} = -1$ $A_{24} = S_y$

$B_2 = S_y* \frac{dT_{ko}}{(1+T_{ko})}$

$A_{31} = \{\theta_{kx} + [\frac{G}{X} * \frac{\partial f}{\partial G}*(1+ Єxx) + \frac{G}{Y} * \frac{\partial g}{\partial G}*Єxy]*[\left(\frac{K_x}{K_x+K_y}\right) - (b-1)*\left(\frac{K_x}{K_z}\right)] +$

$[\theta_{kx}* Єxx + \theta_{ky}* Єxy]*[(b-1)*\left(\frac{K_x}{K_z}\right)] + \frac{K_x}{X} * \frac{\partial X^r}{\partial M^r} - \frac{(1+T_{ko})*K_x}{X} * \frac{\partial X^o}{\partial M^o} +$

$\frac{(1+T_{ko})*(Kx + Ky)}{X}*(b-1)*\left(\frac{K_x}{K_z}\right)*\frac{\partial X^o}{\partial M^o}\}$

$A_{32} = \{[\frac{G}{X} * \frac{\partial f}{\partial G}*(1+ Єxx) + \frac{G}{Y} * \frac{\partial g}{\partial G}*Єxy]*[\left(\frac{K_y}{K_x+K_y}\right) - (b-1)*\left(\frac{K_y}{K_z}\right)] +$

$[\theta_{kx}* Єxx + \theta_{ky}* Єxy]*[(b-1)*\left(\frac{K_y}{K_z}\right)] +$

$\frac{K_y}{X} * \frac{\partial X^r}{\partial M^r} - \frac{(1+T_{ko})*K_y}{X} * \frac{\partial X^o}{\partial M^o} + \frac{(1+T_{ko})*(Kx + Ky)}{X}*(b-1)*\left(\frac{K_y}{K_z}\right)*\frac{\partial X^o}{\partial M^o}\}$

$A_{33} - \theta_{lx} * (\frac{L_y}{L_x})$ $A_{34} = - \{ Єxx* \theta_{lx} + Єxy* \theta_{ly} + \frac{(Lx + Ly)}{X} *\frac{\partial X^o}{\partial M^o} \}$

$B_3 = - \{ \frac{G}{X} * \frac{\partial f}{\partial G}*(1+ Єxx) + Єxy* \frac{G}{Y} * \frac{\partial g}{\partial G} \}* \frac{dT_{ko}}{T_{ko}} +$

$\{Єxx* \theta_{kx} + Єxy* \theta_{ky} + [\theta_{kx}+ \frac{K_y}{X} *(1 + T_{ko})]* \frac{\partial X^o}{\partial M^o} \}* \frac{dT_{ko}}{(1+T_{ko})}$

$A_{41} = \{[\frac{G}{Y} * \frac{\partial g}{\partial G}*(1+ Єyy) + \frac{G}{X} * \frac{\partial f}{\partial G}*Єyx]*[\left(\frac{K_x}{K_x+K_y}\right) - (b-1)*\left(\frac{K_x}{K_z}\right)] +$

$[\theta_{kx} * Єyx + \theta_{ky} * Єyy] * [(b-1) * \left(\frac{K_x}{K_z}\right)] - \frac{(1+T_{ko}) * K_x}{Y} * \frac{\partial Y^o}{\partial M^o} +$

$\frac{(1+T_{ko}) * (Kx + Ky)}{Y} * (b-1) * \left(\frac{K_x}{K_z}\right) * \frac{\partial Y^o}{\partial M^o}\}$

$A_{42} = \{\theta_{ky} + [\frac{G}{Y} * \frac{\partial g}{\partial G} * (1 + Єyy) + \frac{G}{X} * \frac{\partial f}{\partial G} * Єyx] * [\left(\frac{K_y}{K_x + K_y}\right) - (b-1) * \left(\frac{K_y}{K_z}\right)] +$

$[\theta_{kx} * Єyx + \theta_{ky} * Єyy] * [(b-1) * \left(\frac{K_y}{K_z}\right)] - \frac{(1+T_{ko}) * K_y}{Y} * \frac{\partial Y^o}{\partial M^o} +$

$\frac{(1+T_{ko}) * (Kx + Ky)}{Y} * (b-1) * \left(\frac{K_y}{K_z}\right) * \frac{\partial Y^o}{\partial M^o}\}$

$A_{43} = \theta_{ly}$ $A_{44} = - \{ Єyx * \theta_{lx} + Єyy * \theta_{ly} + \frac{(Lx + Ly)}{Y} * \frac{\partial Y^o}{\partial M^o} \}$

$B_4 = - \{ \frac{G}{Y} * \frac{\partial g}{\partial G} * (1 + Єyy) + Єyx * \frac{G}{X} * \frac{\partial f}{\partial G} \} * \frac{dT_{ko}}{T_{ko}} +$

$\{Єyx * \theta_{kx} + Єyy * \theta_{ky} + [\theta_{ky} + \frac{K_x}{Y} * (1 + T_{ko})] * \frac{\partial Y^o}{\partial M^o}\} * \frac{dT_{ko}}{(1+T_{ko})}$

Now this system can be solved in at least 3 ways, namely using Cramer's rule, or by directly inverting the coefficients matrix, or by solving the system of simultaneous equations using MATLAB. The third route is followed in this dissertation, and the symbolic solutions are provided in the appendix. The reason that this was done was to retain one of the greatest benefits of the Harberger model; simplicity and transparency achieved through the use of minimum necessary assumptions. CGE models assume functional forms and have underlying assumptions about utility maximization and specific forms of production functions etc., all of which are avoided in the Harberger models.

As can be seen we need to replace coefficients for which we have some agreed numerical values to be able to analyze the results in terms of the unknowns we are interested in. Fortunately, we can substitute numbers for elasticities, factor shares, capital and labor amounts and ratios, and we can choose to represent the tax rates symbolically or use prevailing values from the U.S. (or any OECD country that is large). Data issues are discussed in detail in chapter V, and at this stage it can be noted that the reduced form equations can also be solved symbolically without using parameters. For interpretation of effects, however, we need to reduce the number of unknowns.

**CHANGES IN GDP AND USES SIDE**

$GDP^o$ (OECD) $= P_k*(1 + T_{ko})*(K_x + K_y) + P_{lo}*(L_x + L_y)$

$$dGDP^o = P_k* dK_x + dP_k* K_x + P_k*T_{ko}*dK_x + P_k*dT_{ko}*K_x + dP_k*T_{ko}*K_x + P_k*dK_y + dP_k*K_y + P_k*T_{ko}*dK_y + P_k*dT_{ko}*K_y + dP_k*T_{ko}*K_y + dP_{lo}*(L_x + L_y) + P_{lo}*d(L_x + L_y)$$

But $d(L_x + L_y)= 0$, so we ignore the last term:

$$dGDP^o = dP_k*(1 + T_{ko})*(K_x + K_y) + (1 + T_{ko})*(dK_x + dK_y) + dT_{ko}*(K_x + K_y) + dP_{lo}*(L_x + L_y)$$

$GDP^r = M^r = P_k * K_z + P_{lr} * L_z$

$dGDP^r = dM^r = d(P_k * K_z + P_{lr} * L_z)$

But $P_{lr} * L_z = \overline{L_z} * P_{lr} = P_z*Z - P_k*K_z = (\frac{1}{b})* [P_k* K_z ]- P_k*K_z$

$\Rightarrow M^r = P_k * K_z + (\frac{1}{b})* [P_k* K_z ]- P_k*K_z$

$\Rightarrow M^r = (\frac{1}{b})* [P_k* K_z ]$

$dM^r = (\frac{1}{b})* [K_z*dP_k + P_k*dK_z]$

But $dP_k = (b - 1 ) * \frac{dK_z}{K_z}$ …………………(P3)

$\Rightarrow dM^r = (\frac{1}{b})* [K_z*(b - 1 ) * \frac{dK_z}{K_z} + P_k*dK_z]$

$\Rightarrow dM^r = (\frac{1}{b})* [(b - 1 )*dK_z + dK_z]$

$\Rightarrow dM^r = dK_z$

$\Rightarrow dGDP^r = dM^r = - K_x*\frac{dK_x}{K_x} - K_y*\frac{dK_y}{K_y}$

Next we formulate a measure of change in the sources and uses of income (welfare) caused by the fact that relative user prices of final goods also change. Though this was not a part of closed economy models as in Harberger (1962), since open economy models mean that non-tradable commodities exist, changes in output prices also impact welfare, since each country consumer consumes a different bundle of goods. Following Randolph (2006) and Gravelle and Smetters (2001, 2006), we have to find a way to compute this effect as well. We can use a Laspeyres' index defined as the change in the cost of purchasing the base year's consumption bundle. Since G is an input into production, it does not enter into the consumption bundle at all. For the OECD country, the index is defined as:

$$LAS^o = \frac{(P_x + dP_x)^* X^o + (P_y + dP_y)^* Y^o + (P_z + dP_z)^* Z^o}{P_x * X^o + P_y * Y^o + P_z * Z^o}$$

Similarly, for ROW we can estimate:

$$LAS^r = \frac{(P_x + dP_x)^* X^r + (P_z + dP_z)^* Z^r}{P_x * X^r + P_z * Z^r}$$

Next we know that $dP_z = 0$ and initial prices are equal to 1, this gives:

$$LAS^o = \frac{dP_x * X^o + dP_y * Y^o + Z^o + X^o + Y^o}{X^o + Y^o + Z^o} = 1 + \frac{dP_x * X^o + dP_y * Y^o}{X^o + Y^o + Z^o}$$

Similarly, we have:

$$LAS^r = \frac{X^r + dP_x * X^r + Z^r}{X^r + Z^r} = 1 + \frac{dP_x * X^r}{X^r + Z^r}$$

We also have to find a way to combine the total welfare effect of the change in income (GDP) and the change in the uses side or cost-of-living index. This can be done in a number of ways; one very logical way is available in Randolph (2006). The method proposed here is more basic. It starts with the assumption that the social welfare function is a weighted function of proportionate change in income and the proportional change in

the cost of living, with equal weights. This would imply that we could write OECD welfare as:

$W^o = 1 + dGDP^o/GDP^o - LAS^o$ and ROW welfare $W^r = 1 + dGDP^r/GDP^r - LAS^r$

This is a pure assumption, and we could have chosen any form for the welfare function, such as $W^o = \frac{1+\frac{dGDP^o}{GDP^o}}{LAS^o}$ instead or more complicated forms.

# APPENDIX A

```
% Dissertation model chapter#2- lowering tax on capital- AMENDED ( dt 7/22/09) %
% 4 equations in 4 variables:- dKx_Kx, dLy_Ly, dPlo, dKy_Ky %
clear;
clear all;

eq1 = 'A11*dKx_Kx   + A12*dKy_Ky  + A13*dLy_Ly    +  A14*dPlo = B1';

eq2 = 'A21*dKx_Kx   +A22*dKy_Ky     + A23*dLy_Ly    + A24*dPlo = B2' ;

eq3 = 'A31*dKx_Kx   + A32*dKy_Ky    + A33*dLy_Ly    + A34*dPlo   =  B3' ;

eq4 = 'A41*dKx_Kx   +A42*dKy_Ky     +  A43*dLy_Ly    + A44*dPlo   =   B4' ;

s = solve (eq1, eq2, eq3, eq4, 'dKx_Kx', 'dKy_Ky', 'dLy_Ly', 'dPlo');

dKx_Kx= s.dKx_Kx

dKy_Ky = s.dKy_Ky

dLy_Ly = s.dLy_Ly

dPlo = s.dPlo


%  RESULTS %

dKx_Kx =

-(A12*A23*A34*B4 - A12*A23*A44*B3 - A12*A24*A33*B4 + A12*A24*A43*B3 +
A12*A33*A44*B2 - A12*A34*A43*B2 - A13*A22*A34*B4 + A13*A22*A44*B3 +
A13*A24*A32*B4 - A13*A24*A42*B3 - A13*A32*A44*B2 + A13*A34*A42*B2 +
A14*A22*A33*B4 - A14*A22*A43*B3 - A14*A23*A32*B4 + A14*A23*A42*B3 +
A14*A32*A43*B2 - A14*A33*A42*B2 - A22*A33*A44*B1 + A22*A34*A43*B1 +
A23*A32*A44*B1 - A23*A34*A42*B1 - A24*A32*A43*B1 + A24*A33*A42*B1)
-----------------------------------------------------------------------------------

(A11*A22*A33*A44 - A11*A22*A34*A43 - A11*A23*A32*A44 +
A11*A23*A34*A42 + A11*A24*A32*A43 - A11*A24*A33*A42 -
```

A12*A21*A33*A44 + A12*A21*A34*A43 + A12*A23*A31*A44 -
A12*A23*A34*A41 - A12*A24*A31*A43 + A12*A24*A33*A41 +
A13*A21*A32*A44 - A13*A21*A34*A42 - A13*A22*A31*A44 +
A13*A22*A34*A41 + A13*A24*A31*A42 - A13*A24*A32*A41 -
A14*A21*A32*A43 + A14*A21*A33*A42 + A14*A22*A31*A43 -
A14*A22*A33*A41 - A14*A23*A31*A42 + A14*A23*A32*A41)

dKy_Ky =

(A11*A23*A34*B4 - A11*A23*A44*B3 - A11*A24*A33*B4 + A11*A24*A43*B3 +
A11*A33*A44*B2 - A11*A34*A43*B2 - A13*A21*A34*B4 + A13*A21*A44*B3 +
A13*A24*A31*B4 - A13*A24*A41*B3 - A13*A31*A44*B2 + A13*A34*A41*B2 +
A14*A21*A33*B4 - A14*A21*A43*B3 - A14*A23*A31*B4 + A14*A23*A41*B3 +
A14*A31*A43*B2 - A14*A33*A41*B2 - A21*A33*A44*B1 + A21*A34*A43*B1 +
A23*A31*A44*B1 - A23*A34*A41*B1 - A24*A31*A43*B1 + A24*A33*A41*B1)
*DIVIDED BY*

-----------------------------------------------------------------------------------------

(A11*A22*A33*A44 - A11*A22*A34*A43 - A11*A23*A32*A44 +
A11*A23*A34*A42 + A11*A24*A32*A43 - A11*A24*A33*A42 -
A12*A21*A33*A44 + A12*A21*A34*A43 + A12*A23*A31*A44 -
A12*A23*A34*A41 - A12*A24*A31*A43 + A12*A24*A33*A41 +
A13*A21*A32*A44 - A13*A21*A34*A42 - A13*A22*A31*A44 +
A13*A22*A34*A41 + A13*A24*A31*A42 - A13*A24*A32*A41 -
A14*A21*A32*A43 + A14*A21*A33*A42 + A14*A22*A31*A43 -
A14*A22*A33*A41 - A14*A23*A31*A42 + A14*A23*A32*A41)

dLy_Ly =

-(A11*A22*A34*B4 - A11*A22*A44*B3 - A11*A24*A32*B4 + A11*A24*A42*B3 +
A11*A32*A44*B2 - A11*A34*A42*B2 - A12*A21*A34*B4 + A12*A21*A44*B3 +
A12*A24*A31*B4 - A12*A24*A41*B3 - A12*A31*A44*B2 + A12*A34*A41*B2 +
A14*A21*A32*B4 - A14*A21*A42*B3 - A14*A22*A31*B4 + A14*A22*A41*B3 +
A14*A31*A42*B2 - A14*A32*A41*B2 - A21*A32*A44*B1 + A21*A34*A42*B1 +

A22*A31*A44*B1 - A22*A34*A41*B1 - A24*A31*A42*B1 +
A24*A32*A41*B1)*DIVIDED BY*

------------------------------------------------------------

(A11*A22*A33*A44 - A11*A22*A34*A43 - A11*A23*A32*A44 +
A11*A23*A34*A42 + A11*A24*A32*A43 - A11*A24*A33*A42 -
A12*A21*A33*A44 + A12*A21*A34*A43 + A12*A23*A31*A44 -
A12*A23*A34*A41 - A12*A24*A31*A43 + A12*A24*A33*A41 +
A13*A21*A32*A44 - A13*A21*A34*A42 - A13*A22*A31*A44 +
A13*A22*A34*A41 + A13*A24*A31*A42 - A13*A24*A32*A41 -
A14*A21*A32*A43 + A14*A21*A33*A42 + A14*A22*A31*A43 -
A14*A22*A33*A41 - A14*A23*A31*A42 + A14*A23*A32*A41)

dPlo =

(A11*A22*A33*B4 - A11*A22*A43*B3 - A11*A23*A32*B4 + A11*A23*A42*B3 +
A11*A32*A43*B2 - A11*A33*A42*B2 - A12*A21*A33*B4 + A12*A21*A43*B3 +
A12*A23*A31*B4 - A12*A23*A41*B3 - A12*A31*A43*B2 + A12*A33*A41*B2 +
A13*A21*A32*B4 - A13*A21*A42*B3 - A13*A22*A31*B4 + A13*A22*A41*B3 +
A13*A31*A42*B2 - A13*A32*A41*B2 - A21*A32*A43*B1 + A21*A33*A42*B1 +
A22*A31*A43*B1 - A22*A33*A41*B1 - A23*A31*A42*B1 + A23*A32*A41*B1)
*DVIDED BY*

---------------------------------------------------------------------------

(A11*A22*A33*A44 - A11*A22*A34*A43 - A11*A23*A32*A44 +
A11*A23*A34*A42 + A11*A24*A32*A43 - A11*A24*A33*A42 -
A12*A21*A33*A44 + A12*A21*A34*A43 + A12*A23*A31*A44 -
A12*A23*A34*A41 - A12*A24*A31*A43 + A12*A24*A33*A41 +
A13*A21*A32*A44 - A13*A21*A34*A42 - A13*A22*A31*A44 +
A13*A22*A34*A41 + A13*A24*A31*A42 - A13*A24*A32*A41 -
A14*A21*A32*A43 + A14*A21*A33*A42 + A14*A22*A31*A43 -
A14*A22*A33*A41 - A14*A23*A31*A42 + A14*A23*A32*A41)

# CHAPTER III – THE CONSUMPTION TAX

The purpose of this chapter is to develop a model of the consumption tax and simulate tax competition in the OECD country, using a similar model set-up as in chapter two. The major variation is that instead of a tax on all capital in the OECD country, there is now a tax on consumption (not production) in the OECD. The tax initially drives a wedge between the price paid by the OECD consumer and producers on the one hand and between the prices paid by the ROW and OECD consumers on the other.

If total demand can be split as in chapter two in the following manner:

$X = X^o + X^r$ , $Y = Y^o$ , and $Z = Z^o + Z^r$ , then the tax applies to $X^o$ , $Y^o$ and $Z^o$ only, not to $X^r$ and $Z^r$. Further, the price paid by the consumer in OECD is $P_x*(1+T_{Co})$, $P_y*(1+T_{Co})$ and $P_z*(1+T_{Co})$, the prices paid by the consumer in the ROW are $P_x$ and $P_z$ and the prices received by producers in both countries are also $P_x$ , $P_y$ and $P_z$ . As before, $P_x$ , $P_y$ and $P_z$ are still initially equal to 1, and $P_x*(1+T_{Co})$, $P_y*(1+T_{Co})$ and $P_z*(1+T_{Co})$ are $\neq 1$. We will continue with all the previous assumptions, except where specified.

## PROFIT MAXIMIZATION BEHAVIOR BY FIRMS (SECTORS)

The production functions for each sector can be written as a function of productive resources:

$X = f(K_x , L_x, G)$ $\qquad$ $Y = g(K_y, L_y, G)$ $\qquad$ $Z = h(K_z, L_z)$

{Where the amount of G is common to both X and Y, G is a public good}

The sector is synonymous with the firm. Since CRS industries imply that the size of the firm is indeterminate, we speak of the sector and firm interchangeably, like each sector were one giant firm. However, we have also assumed perfect competition, so each firm (sector) acts like an atomistic price-taker in all markets. Specifically, we assume that firms do not recognize the externality associated with paying taxes that support the public good G. The main difference is that the tax is not imposed on capital. Since producers receive only the supply price, and they take the tax rate as a given constant, they seek to maximize the objective function based on the production functions:

$X = f(K_x , L_x, \bar{G})$ $\qquad$ $Y = g(K_y, L_y, \bar{G})$ $\qquad$ $Z = h(K_z, L_z)$

Where $\bar{G}$ is taken as fixed and given at the historical level which is assumed unchanged.

The profit maximization problem can then be set up as:

FIRM (SECTOR) X

Maximize Π (profit) = $\{ P_x * f(K_x , L_x, \bar{G}) - P_k * K_x - P_{lo} * L_x \}$ with respect to $K_x$ and $L_x$

First order conditions:

1. $P_x * \frac{\partial f}{\partial K_x} - P_k = 0$

   And

2. $P_x * \frac{\partial f}{\partial L_x} - P_{lo} = 0$

Since $P_x = 1$, this implies that $\frac{\partial f}{\partial K_x} = P_k$ and $\frac{\partial f}{\partial L_x} = P_{lo}$

Each firm pays each factor its marginal product at the current level of provision of the public good. The demand price includes the tax, but since the tax is assumed to go to the government and the tax rate is independent of the firm's actions, it does not enter the objective function. Revenue for a firm is independent of the tax, so even if we included the demand price in our objective function we would have to subtract tax payable. As in the previous chapter, the firm ignores the externality arising from changes in G.

FIRM (SECTOR) Y

Behaves in exactly the same way as X, and so we get $\frac{\partial g}{\partial K_y} = P_k$ and $\frac{\partial g}{\partial L_y} = P_{lo}$

FIRM (SECTOR) Z

Since there is no government good or tax in the ROW we have only the production function

$Z = h\ (K_z, \overline{L_z}\ )$ and the problem for firm Z:

Maximize Π (profit) = { $P_z$* h ($K_z$ , $\overline{L_z}$ ) - $P_k$*$K_z$ - $P_{lr}$*$\overline{L_z}$ } with respect to $K_z$ {since we assume full employment always in both countries, choosing $L_z$ is not a decision variable} and we get $\frac{\partial h}{\partial K_z} = P_k$

We have chosen to keep the total amount of labor fixed in ROW, and to maintain a full employment restriction to keep the model simple and transparent. To equate labor price in ROW to marginal products, and to allow labor to move between sectors in ROW, all we have to do is introduce two sectors in ROW instead of one. We will then have a 4 sector open economy model. Alternatively, we could have modeled labor supply in the

ROW on the lines of existing substantial under-employment in the informal and agricultural sector, a la Arthur Lewis' famous model to give us an independent equation for labor in ROW. It will need to account for the fact that in competitive conditions in this model with full price and wage flexibility, unemployment should imply that $P_{lr}$ has to be driven down to zero.

we leave this issue to a future extension of this model, and for the present, assume that full employment exists for ROW, there is a positive labor price and that $dL_z = 0$. We do this for two reasons. The first is to keep the model as transparent and simple as possible and incorporate only essential complications. This is in line with most of the significant previous literature that also assumes full employment in the OECD and ROW, albeit with more sectors. The second is that if we were to contemplate a fixed compensation for labor in ROW, or allow $P_{lr}$ to fall to zero, we cannot capture the effect noted in Harberger (1995): when the tax in the OECD is changed, capital flows into or out of the ROW, changing the marginal productivity of labor, especially for constant employment.

This has to mean that even if total labor supply in ROW is fixed, $P_{lr}$ is only fixed for a given amount of capital. Thus, instead of assuming both in this chapter and in chapter two that movement of capital into and out of ROW changes the unemployment level with $dP_{lr} = 0$ we prefer $L_z = \overline{L_z}$ , we treat $P_{lr}$ as a residual determined by the level of $K_z$ and instead take $P_z$ as the numeraire. However, we could have proceeded in both chapters by using $P_{lr}$ as the numeraire instead of $P_z$ , or by allowing $L_z$ to vary in terms of some independent labor supply equation.

**SUBSTITUTION**

The elasticity of substitution is (with CRS and competition) defined as: $S_x = \frac{\frac{d(\frac{K_x}{L_x})}{\frac{K_x}{L_x}}}{\frac{d(\frac{P_k}{P_{lo}})}{\frac{P_k}{P_{lo}}}}$

This can be rewritten as:

$$\frac{d(\frac{K_x}{L_x})}{\frac{K_x}{L_x}} = S_x * \frac{d(\frac{P_k}{P_{lo}})}{\frac{P_k}{P_{lo}}}$$

Since the amount of G is taken as fixed and given by the firms, and once provided the fixed amount of G is available to both firms, there is no substitution between G and other factors of production. This is also especially true since the amount of G cannot be varied by the firm, and no payment has to be made for G. Since the derivation follows exactly the same steps as in Tresch (2002) it is not derived here, the results are stated:

SECTOR X

$$\frac{dK_x}{K_x} - \frac{dL_x}{L_x} = S_x * (dP_k - dP_{lo})$$

…………………………………….(U1)

SECTOR Y

$$\frac{dK_y}{K_y} - \frac{dL_y}{L_y} = S_y * (dP_k - dP_{lo})$$

……………………………………(U2)

SECTOR Z

There is no substitution equation in sector Z, since the total supply of labor in ROW is fixed and full employment is assumed.

**BALANCING**

$dK_x + dK_y + dK_z = 0$

$dL_x + dL_y = 0$

$dL_z = 0$

$dP_z = 0$ [Numeraire]

$P_k$, $P_{lo}$ and $P_{lr}$ (factor prices) and $P_x$, $P_y$ and $P_z$ (output supply prices) are all equal to 1 and to each other to begin with

**SUPPLY**

The production functions for the three sectors are:

$X = f(K_x, L_x, G)$ $Y = g(K_y, L_y, G)$ $Z = h(K_z, L_z)$

{Where the amount of G is common to both X and Y, G is a public good}

SECTOR X

$X = f(K_x, L_x, G)$

The total change in supply (output) or the total differential can be split into:

$$dX = \frac{\partial f}{\partial K_x} * dK_x + \frac{\partial f}{\partial L_x} * dL_x + \frac{\partial f}{\partial G} * dG$$

$$\frac{dX}{X} = \frac{P_k * K_x}{P_x * X} * \frac{dK_x}{k_x} \quad + \frac{P_{lo} * L_x}{P_x * X} * \frac{dL_x}{L_x} \quad + \frac{1}{X} * \frac{\partial f}{\partial G} * dG$$

Writing $\theta_{kx} = \frac{P_k * K_x}{P_x * X}$ the tax-exclusive share of capital's product in sector X

And $\theta_{lx} = \frac{P_{lo} * L_x}{P_x * X}$ the tax-exclusive share of labor's product in sector X, we get

S1…………………..  $$\frac{dX}{X} = \theta_{kx} * \frac{dK_x}{K_x} \quad + \quad \theta_{lx} * \frac{dL_x}{L_x} \quad + \frac{1}{X} * \frac{\partial f}{\partial G} * dG$$

The last term in this equation works a little like a "Solow residual" in the sense that like technology, the factor augmenting government good provides a third path for output to grow, beyond what would have been captured by growth in capital and labor (Solow 1957, Romer 2006). When production is CRS in K and L, and factor shares $\theta_{kx}$ + $\theta_{lx}$ = 1, this term shows the growth in output that cannot be accounted for changes in K and L.

Similarly, for SECTOR Y we have:

S2……….............  $$\frac{dY}{Y} = \theta_{ky} * \frac{dK_y}{K_y} \quad + \quad \theta_{ly} * \frac{dL_y}{L_y} \quad + \frac{1}{Y} * \frac{\partial g}{\partial G} * dG$$

Where $\theta_{ky} = \frac{P_k * K_y}{P_y * Y}$ the tax-exclusive share of capital's product in sector Y

And $\theta_{ly} = \frac{P_{lo} * L_y}{P_y * Y}$ the share of labor's product in sector Y.

SECTOR Z

There is no tax and no government good in the ROW, so the relevant terms for this sector:

S3a……….............. $\frac{dZ}{Z} = \theta_{kz} * \frac{dK_z}{K_z} + \frac{1}{Z} * L_z * \frac{\partial h}{\partial L_z} * \frac{dL_z}{L_z}$

But $dL_z = 0$ by assumption ($L_{row} = \overline{L_z}$ is fixed), so we can write the above as:

S3……….............. $\frac{dZ}{Z} = \theta_{kz} * \frac{dK_z}{K_z}$

This gives us our three supply equations. The only difference between this case and the previous chapter is in the shares of capital in the supply equations. Since the producer does not receive or pay the tax, the shares are all tax-exclusive. We now turn to price formation. The differences from chapter two are that (1) there is no tax on any factor, and that supply price here means the prices received by producers. (2)The demand price in each case is the producers' price inflated by the ad valorem consumption tax.

**PRICE FORMATION**

SECTOR X

Suppose we did not know what the production function for X looked like when we included all factors K, L and G. If it is CRS for K and L alone, we can think of it as IRS with all three being varied. From Euler's law we can write this as:

$\frac{\partial f}{\partial K_x} * K_x + \frac{\partial f}{\partial L_x} * L_x + \frac{\partial f}{\partial G} * G = A*X$ (where A > 1 and A is the degree of homogeneity)

Now we know that sector X does not have to pay a user fee for G, the consumer pays the tax on consumption in the OECD. If K and L are paid their marginal products in the presence of G, and if G had to be paid a user fee equal to its marginal product, then payments to factors would equal $A*X > X$, total product would be more than exhausted.

Since G is not paid for directly, we are interested in what the payment for capital and payments to labor add up to. Let us say that:

$$\frac{\partial f}{\partial K_x} * K_x + \frac{\partial f}{\partial L_x} * L_x = B*X$$

$$\frac{\partial f}{\partial K_x} * K_x + \frac{\partial f}{\partial L_x} * L_x = B*P_x*X \qquad \text{since } P_x = 1$$

If we then continue with our assumption that firms pay marginal products:

$B*P_x*X = P_k* K_x + P_{lo}* L_x$

The assumption we make here, to maintain consistency between total factor incomes and expenditure is that:

$B = (1+T_{Co})$, so that implies: $(1+T_{Co})*P_x*X = P_k* K_x + P_{lo}* L_x$ where $X = f(K, L)$, or

$P_x*X = P_k* K_x + P_{lo}* L_x$ when $X = f(K, L, G)$.

What we are saying here is that without T, G, the total product would have been exactly exhausted by factor incomes (CRS). When the tax was imposed in the first instance, demand went down, quantity went down and there was a wedge between payments by consumers and payments to factors. This wedge however, was used to provide G which increased output, reduced costs and thus resulted in additional product. We assume simply that this additional product, in the absence of savings will end up with the single consumer in the economy and is exactly equal to tax revenue. It looks at first instance that this extra product should go to the government as tax revenue but not to the consumer.

This is true if we were to assume that our model is dynamic in some sense, or if we were to consider the next period. To maintain parity with the transfer case, we have to assume that at some stage, the extra product from the input is also returned to the economy. In the standard Harberger (1962) model and others, the transfer payment also could not go back to the consumer as a transfer payment and flow back to the government as tax revenue. If this happens, then we would not have been able to claim income compensation in the demand curve, or to be able to isolate the excess burden from the revenue itself.

In the transfer payments model, to have income compensated demand curves, authors assume that the tax is collected in the first round and then returned to the consumers, and focus only on the distortion caused by the change in relative prices. Similarly in the input models we assume the exact analogue of these transfers: the extra output due to the input also has to go to the consumers. The additional assumption is that it holds in each sector separately, though this is not strictly necessary and is assumed only to allow us to continue with CRS price formation equations as in the Harberger (1962) model. The increase in taxes that resulted in extra G added back as much product in each market as was lost due to the tax wedge.

$$d(B*P_x*X) = d((1+T_{Co})*P_x*X) = (1+T_{Co})*P_x*dX + (1+T_{Co})*X*dP_x + X*dT_{Co}=$$

$$d[\frac{\partial f}{\partial K_x} * K_x + \frac{\partial f}{\partial L_x} * L_x]$$

This is possible when we remember that: $P_x*X (f (K_x , L_x, \bar{G})) = (1+T_{Co})*P_x*X (f (K_x , L_x))$ or that the government input paid for out of taxes adds back as much value to

product as taken out by taxes. From the supply equations and assumptions $P_x = 1$ we know that $P_x*dX = dX$ and

$$dX = \frac{\partial f}{\partial K_x} * dK_x + \frac{\partial f}{\partial L_x} * dL_x + \frac{\partial f}{\partial G} * dG$$

$$\Rightarrow \quad P_x*dX = P_k * dK_x + P_{lo} * dL_x + \frac{\partial f}{\partial G} * dG$$

Substituting in our original equation:

$$(1+T_{Co})*P_x*dX + (1+T_{Co})*X*dP_x + X*dT_{Co} =$$

$$[dP_k* K_x + P_k* dK_x + dP_{lo}* L_x + P_{lo}* dL_x ]$$

$$\Rightarrow \quad (1+T_{Co})*X(K,L)*dP_x = [dP_k* K_x + P_k* dK_x + dP_{lo}* L_x + P_{lo}* dL_x ] - (1+T_{Co})*P_x*dX(K, L) - X(K,L)*dT_{Co}$$

$$\Rightarrow \quad (1+T_{Co})*X*dP_x = [dP_k* K_x + dK_x + dP_{lo}* L_x + dL_x ] - dK_x - dL_x - T_{Co} *dK_x - T_{Co}*dL_x - \frac{\partial f(K,L,G)}{\partial G} * dG - X*dT_{Co}$$

$$dP_x = \theta_{kx} *\frac{1}{1+T_{Co}} *dP_k + \theta_{lx} * \frac{1}{1+T_{Co}} *dP_{lo} - \frac{1}{X(K,L,G)} *\frac{\partial f}{\partial G} *dG - \frac{T_{Co}}{1+T_{Co}} * \theta_{kx} *\frac{dK_x}{K_x} - \frac{T_{Co}}{1+T_{Co}}*\theta_{lx} *\frac{dL_x}{L_x} - \frac{dT_{Co}}{1+T_{Co}} \qquad \text{.....................(P1)}$$

Similarly for SECTOR Y

$$dP_y = \theta_{ky} *\frac{1}{1+T_{Co}} *dP_k + \theta_{ly} * \frac{1}{1+T_{Co}} *dP_{lo} - \frac{1}{Y(K,L,G)} *\frac{\partial g}{\partial G} *dG - \frac{T_{Co}}{1+T_{Co}} * \theta_{ky} *\frac{dK_y}{K_y} -$$

$$\frac{T_{Co}}{1+T_{Co}} * \theta_{ly} * \frac{dL_y}{L_y} \quad - \quad \frac{dT_{Co}}{1+T_{Co}} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(P2)}$$

SECTOR Z

There is no tax and no government good here. Suppose that total payments to capital in Z are a constant proportion of the product (output). This is a small restriction we have to place for analytical convenience since we have only one sector in Z, and full employment is assumed. Even if we had assumed two ROW sectors, the standard assumption in the literature is that full employment of labor exists to ROW as a whole. Only the allocation of labor between ROW sectors would be governed by labor's marginal product and we would have substitution equations between labor and capital in ROW as well. In both the Harberger (1962) and Jones (1965) versions fixed factor shares are used for labor and capital in the price formation and supply equations. What we have given up for the convenience of one ROW sector is the payment of marginal product to labor. Capital is still paid marginal product since it is mobile. We have no substitution equation in Z since the amount of labor is fixed.

To introduce marginal product payments to labor in ROW, all we have to do is introduce two ROW sectors. We would still use fixed proportions between labor and capital for supply and price formation, and we would still use full employment of labor in ROW as a whole. Thus, this is a trade-off between the convenience of a single ROW sector, and the restricted behavior of labor in ROW. Since we are interested primarily in movement of capital and income between the countries, and not between sectors in ROW, or the relative distribution between capital and labor, we believe that we do not lose much in terms of insights with this modification. The residual output in Z after

capital is paid marginal product is paid to labor. If b represents the baseline payments proportion to capital in Z, and if b is assumed to be a constant, then:

$b* P_z* Z = P_k * K_z$ : b<1and b is constant

$[ P_z*dZ + Z*dP_z] = (\frac{1}{b})* [dP_k* K_z + P_k* dK_z ]$

Now from the supply equation for Z we know that: $P_z*dZ = dZ$ ( since $P_z = 1$) $= P_k * dK_z$

$\Rightarrow Z*dP_z = (\frac{1}{b})* [dP_k* K_z + P_k* dK_z ] - P_z*dZ$

$\Rightarrow Z*dP_z = (\frac{1}{b})* [dP_k* K_z + P_k* dK_z ] - P_k * dK_z$

$\Rightarrow dP_z = (\frac{1}{b})* [\theta_{kz} *dP_k + \theta_{kz} * \frac{dK_z}{K_z} ] - \theta_{kz} * \frac{dK_z}{K_z}$

(Since $P_k$ and $P_z = 1$, and $\theta_{kz}$ is tax-exclusive)

$\Rightarrow dP_z = (\frac{1}{b})* \theta_{kz} *dP_k + (\frac{1}{b} - 1)* \theta_{kz} * \frac{dK_z}{K_z}$

Now since we have chosen $P_z$ as our numeraire ($dP_z = 0$), we get a relationship between $dP_k$ and $\frac{dK_z}{K_z}$ and ultimately between $dP_k$ and $\frac{dK_z}{K_z} = - (\frac{K_x}{K_z})*\frac{dK_x}{K_x} - (\frac{K_y}{K_z})*\frac{dK_y}{K_y}$

$0 = (\frac{1}{b})* \theta_{kz} *dP_k + (\frac{1-b}{b})* \theta_{kz} * \frac{dK_z}{K_z}$

$dP_k = (b - 1 ) * \frac{dK_z}{K_z}$ …………………(P3)

From these expressions we can see that $P_{lr}$ and $dP_{lr}$ can be derived in terms of either total product $P_z*Z$, a combination of $P_z*Z$ and $P_k*K_z$ or $P_k*K_z$ alone:

$\overline{L_z} * P_{lr} = (1 - b) * P_z * Z$

Or $\overline{L_z} * P_{lr} = P_z * Z - P_k * K_z$

Or $\overline{L_z} * P_{lr} = (\frac{1}{b}) * [P_k * K_z] - P_k * K_z$

$\Rightarrow P_{lr} = (\frac{1}{b} - 1) * [P_k * K_z] * (\frac{1}{\overline{L_z}})$

$\Rightarrow dP_{lr} = (\frac{1}{b} - 1) * [dP_k * K_z + P_k * dK_z] * (\frac{1}{\overline{L_z}})$

Or $dP_{lr} = (1 - b) * [P_z * dZ] * (\frac{1}{\overline{L_z}})$ since $dP_z = 0$

**DEMAND**

The major difference with the previous chapter is in this area.

SECTOR X

$X = X^o(P_x * (1+T_{Co}), P_y * (1+T_{Co}), P_z * (1+T_{Co}), M^o) + X^r(P_x, P_z, M^r)$

$X^o$ represents OECD demand for X. it is a function of all three prices since the OECD consumer consumes the products of all sectors. $M^o$ is OECD consumer's disposable income. This is a Marshallian demand curve. Here, the total factor income is also equal to disposable income, since there are no direct taxes. $X^r$ is the ROW demand for X. it is a function of prices of only the two tradable goods since the ROW does not consume Y. $M^r$ is the disposable income of the ROW consumer. Since there are no taxes in the ROW this is equal to GDP. This is also a Marshallian demand curve. Explicit formulations for disposable income follow:

For both $M^o$ and $M^r$ there is no tax on incomes:

$M^o = P_k*(K_x + K_y) + P_{lo}*(L_x + L_y)$

$dM^o = P_k*(dK_x + dK_y) + (K_x + K_y)*dP_k + P_{lo}*(dL_x + dL_y) + (L_x + L_y)*dP_{lo}$

But $dL_x + dL_y = 0$ by definition, so

$dM^o = P_k*(dK_x + dK_y) + (K_x + K_y)*dP_k + (L_x + L_y)*dP_{lo}$

$\Rightarrow dM^o = P_k*K_x*\frac{dK_x}{K_x} + P_k*K_y*\frac{dK_y}{K_y} + (K_x + K_y)*dP_k + (L_x + L_y)*dP_{lo}$

$M^r = P_k * K_z + P_{lr} * L_z$

But $P_{lr} * L_z = \overline{L_z} * P_{lr} = P_z*Z - P_k*K_z = (\frac{1}{b})* [P_k* K_z ]- P_k*K_z$

$\Rightarrow M^r = P_k * K_z + (\frac{1}{b})* [P_k* K_z ]- P_k*K_z$

$\Rightarrow M^r = (\frac{1}{b})* [P_k* K_z ]$

$dM^r = (\frac{1}{b})* [K_z*dP_k + P_k*dK_z]$

But $dP_k = (b - 1 ) * \frac{dK_z}{K_z}$ …………………(P3)

$\Rightarrow dM^r = (\frac{1}{b})* [K_z*(b - 1 ) * \frac{dK_z}{K_z} + P_k*dK_z]$

$\Rightarrow dM^r = (\frac{1}{b})* [(b - 1 )*dK_z + dK_z]$

$\Rightarrow dM^r = dK_z$

$\Rightarrow dM^r = - K_x*\frac{dK_x}{K_x} - K_y*\frac{dK_y}{K_y}$

Taking the total differential of the demand for X we get:

$$dX = \frac{\partial X^o}{\partial[P_x*(1+T_{Co})]} * d[P_x *(1+T_{Co})] + \frac{\partial X^o}{\partial[P_y*(1+T_{Co})]} * d[P_y *(1+T_{Co})] +$$

$$\frac{\partial X^o}{\partial[P_z*(1+T_{Co})]} * d[P_z *(1+T_{Co})] + \frac{\partial X^o}{\partial M^o} * dM^o + \frac{\partial X^r}{\partial P_x} * dP_x + \frac{\partial X^r}{\partial P_z} * dP_z + \frac{\partial X^r}{\partial M^r} * dM^r$$

Dividing by X we get:

$$\Rightarrow \frac{dX}{X} = \frac{X^o}{P_x*X*(1+T_{Co})} * E^o_{xx} * d[P_x *(1+T_{Co})] + \frac{X^o}{P_y*X*(1+T_{Co})} * E^o_{xy} * d[P_y *(1+T_{Co})] +$$

$$\frac{X^o}{P_z*X*(1+T_{Co})} * E^o_{xz} * d[P_z *(1+T_{Co})] + \frac{X^r}{P_x*X} * E^r_{xx} * dP_x + \frac{X^r}{P_z*X} * E^r_{xz} * dP_z$$

$$+ (\frac{1}{X})*[\frac{\partial X^o}{\partial M^o} * dM^o + \frac{\partial X^r}{\partial M^r} * dM^r ]$$

Remembering that all prices are = 1 to begin with and that $d[P_x *(1+T_{Co})]= [(1+T_{Co})* dP_x + P_x *dT_{Co} ]$ and that $dP_z =0$ due to the numeraire assumption we can write the above as:

$$\Rightarrow \frac{dX}{X} = [\frac{X^o}{X} * E^o_{xx} + \frac{X^r}{X} * E^r_{xx}] * dP_x + [\frac{X^o}{X} * E^o_{xy} ]* dP_y + \frac{X^o}{X} * E^o_{xx} * \frac{dT_{Co}}{(1+T_{Co})}$$

$$+ \frac{X^o}{X} * E^o_{xy} * \frac{dT_{Co}}{(1+T_{Co})} + \frac{X^o}{X} * E^o_{xz} * \frac{dT_{Co}}{(1+T_{Co})} + (\frac{1}{X})*[\frac{\partial X^o}{\partial M^o} * dM^o + \frac{\partial X^r}{\partial M^r} * dM^r ]$$

…………………………..(D1)

All elasticities for OECD are defined with respect to demand prices here, inclusive of the consumption tax. In the ROW there is no tax, and producer and consumer

prices are the same, so there is no difference. Also, Y is not consumed in the ROW. We have not assumed that demand curves are HOD zero in all prices and income.

SECTOR Y

Since Y is non-tradable, the demand curve consists of demand only from the OECD country.

$Y = Y^o(P_x *(1+T_{Co}), P_y*(1+T_{Co}), P_z *(1+T_{Co}), M^o)$, and $Y^o = Y$

$$\frac{dY}{Y} = E_{yx} *dP_x + E_{yy} *dP_y + E_{yz} *dP_z + (\frac{1}{Y})*[\frac{\partial Y^o}{\partial M^o} * dM^o] + E_{yx} *\frac{dT_{Co}}{(1+T_{Co})} + E_{yy}*\frac{dT_{Co}}{(1+T_{Co})} + E_{yz} *\frac{dT_{Co}}{(1+T_{Co})} \quad \text{.......................................(D2)}$$

SECTOR Z

$$\frac{dZ}{Z} = [\frac{Z^o}{Z} * E^o_{zx} + \frac{Z^r}{Z} * E^r_{zx}] * dP_x + [\frac{Z^o}{Z} * E^o_{zy}]* dP_y + \frac{Z^o}{Z} * E^o_{zx} * \frac{dT_{Co}}{(1+T_{Co})} + \frac{Z^o}{Z} * E^o_{zy} * \frac{dT_{Co}}{(1+T_{Co})} + \frac{Z^o}{Z} * E^o_{zz} * \frac{dT_{Co}}{(1+T_{Co})} + (\frac{1}{Z})*[\frac{\partial Z^o}{\partial M^o} * dM^o + \frac{\partial Z^r}{\partial M^r} * dM^r] \quad \text{...............................(D3)}$$

**GOVERNMENT**

We have assumed that the government good G does not suffer from loss in value due to inflation, and that the dollar value of expenditure on G equals the amount of G available. We could have had a price for G that would have been a weighted sum of labor and capital prices in OECD. However, since we have simplified away from this more complete model of the government good production to keep the model simple, we simply assume that tax revenue gets transformed into the input without complications raised by

the changing prices of labor and capital. As explained in the previous chapter, this is relatively simple to do following McLure (1969) and can be left to future models. Thus, in the initial case G = tax revenue received. Since the tax is on consumption in OECD and there is no tax on exports from OECD while imports are taxed,

$$G = T_{Co} * P_x * X^o + T_{Co} * P_y * Y^o + T_{Co} * P_z * Z^o$$

Since there are no savings and capital accumulation in this model, Total consumption = Total GDP in OECD and ROW separately. Total OECD expenditure =

$$P_x *(1+T_{Co}) * X^o + P_y *(1+T_{Co}) * Y^o + P_z *(1+T_{Co}) * Z^o$$

$$= P_x * X^o + P_y * Y^o + P_z * Z^o + G$$

From the income side, we have: OECD GDP = $P_k*(K_x + K_y) + P_{lo}*(L_x + L_y) = P_x * X^o + P_y * Y^o + P_z * Z^o + T_{Co} * P_x * X^o + T_{Co} * P_y * Y^o + T_{Co} * P_z * Z^o$

$$\Rightarrow G = P_k*(K_x + K_y) + P_{lo}*(L_x + L_y) - P_x * X^o - P_y * Y^o - P_z * Z^o$$

Now we want to convert this into a function of GDP alone, so we choose the rate $T_c$ such that the following is true:

$$(1 - T_c)*[P_k*(K_x + K_y) + P_{lo}*(L_x + L_y)] = P_x * X^o + P_y * Y^o + P_z * Z^o$$

In the price formation equations we had assumed that $(1+T_{Co})*P_x*X = P_k* K_x + P_{lo}* L_x$ and $(1+T_{Co})*P_y*Y = P_k* K_y + P_{lo}* L_y$, so it is easy to check that

$P_k*(K_x + K_y) + P_{lo}*(L_x + L_y) = (1+T_{Co})*P_x*X + (1+T_{Co})*P_y*Y$ ; and since we have budget balance internationally, $X^r = Z^o$ with all prices equal to one, so the relation

$P_k*(K_x + K_y) + P_{lo}*(L_x + L_y) = P_x *(1+T_{Co})* X^o + P_y*(1+T_{Co}) * Y^o + P_z *(1+T_{Co}) * Z^o$ holds.

Following McLure (1975) and Musgrave (1959), it could very well be proved with certain restrictions that $T_{Co}$ and $T_c$ are directly related in a constant manner. This is conveniently illustrated by the tax-inclusive and tax-exclusive rate analogy. If $T_{Co}$ is the "tax-exclusive" rate that inflates consumption ($P_x * X^o + P_y * Y^o + P_z * Z^o$ ) to income ($P_k*(K_x + K_y) + P_{lo}*(L_x + L_y)$ ), then $T_c$ is the "tax-inclusive" rate that deflates income to consumption. The relationship between changes in the two rates is also similarly derived.

We note that $(1 - T_c)$ is the fraction of OECD GDP that goes to the producer in the first instance before the tax revenue is used to provide the input, the rest being spent on the government good. If such a $T_c$ exists, is positive and < 1, we discuss in the chapter on data issues how it may be calculated.

Now:

$$G = P_k*(K_x + K_y) + P_{lo}*(L_x + L_y) - P_x * X^o - P_y * Y^o - P_z * Z^o$$

And:

$$(1 - T_c )*[P_k*(K_x + K_y) + P_{lo}*(L_x + L_y)] = P_x * X^o + P_y * Y^o + P_z * Z^o$$

Therefore:

$$G = \{ 1- (1 - T_c )\}*[P_k*(K_x + K_y) + P_{lo}*(L_x + L_y)]$$

$$= T_c *[P_k*(K_x + K_y) + P_{lo}*(L_x + L_y)]$$

Thus:

$dG = [dP_k*(K_x + K_y) + P_k*d(K_x + K_y) + dP_{lo}*(L_x + L_y)]* T_c +$

$[P_k*(K_x + K_y) + P_{lo}*(L_x + L_y)]* dT_c$ since $d(L_x + L_y) = 0$

We have assumed again that such an appropriate $dT_c$ exists, and has all the required attributes. $T_c$ and $dT_c$ are chosen such that the following is true:

$(1 - T_c)*d[P_k*(K_x + K_y) + P_{lo}*(L_x + L_y)] - [P_k*(K_x + K_y) + P_{lo}*(L_x + L_y)]* dT_c$

$= d[P_x * X^o + P_y * Y^o + P_z * Z^o]$ since

$(1 - T_c)*[P_k*(K_x + K_y) + P_{lo}*(L_x + L_y)] = P_x * X^o + P_y * Y^o + P_z * Z^o$

**SUMMARY OF EQUATIONS FOR THE MODEL SO FAR**

***Supply***

$$\frac{dX}{X} = \theta_{kx} *\frac{dK_x}{K_x} + \theta_{lx} *\frac{dL_x}{L_x} + \frac{1}{P_x*X} * \frac{\partial f}{\partial G} * dG \qquad \text{.... (S1)}$$

$$\frac{dY}{Y} = \theta_{ky} *\frac{dK_y}{K_y} + \theta_{ly} *\frac{dL_y}{L_y} + \frac{1}{P_y*Y} * \frac{\partial g}{\partial G} *dG \qquad \text{... (S2)}$$

$$\frac{dZ}{Z} = \theta_{kz} *\frac{dK_z}{K_z} \qquad \text{... (S3)}$$

***Substitution***

$$\frac{dK_x}{K_x} - \frac{dL_x}{L_x} = S_x * (dP_k - dP_{lo}) \qquad \text{... (U1)}$$

$$\frac{dK_y}{K_y} - \frac{dL_y}{L_y} = S_y * (dP_k - dP_{lo}) \qquad \text{... (U2)}$$

### *Price Formation*

$$dP_x = \theta_{kx} * \frac{1}{1+T_{Co}} * dP_k + \theta_{lx} * \frac{1}{1+T_{Co}} * dP_{lo} - \frac{1}{X(K,L,G)} * \frac{\partial f}{\partial G} * dG -$$

$$\frac{T_{Co}}{1+T_{Co}} * \theta_{kx} * \frac{dK_x}{K_x} - \frac{T_{Co}}{1+T_{Co}} * \theta_{lx} * \frac{dL_x}{L_x} - \frac{dT_{Co}}{1+T_{Co}} \qquad \text{...............(P1)}$$

(Where $\theta_{kx}$ and $\theta_{lx}$ are tax exclusive)

$$dP_y = \theta_{ky} * \frac{1}{1+T_{Co}} * dP_k + \theta_{ly} * \frac{1}{1+T_{Co}} * dP_{lo} - \frac{1}{Y(K,L,G)} * \frac{\partial g}{\partial G} * dG -$$

$$\frac{T_{Co}}{1+T_{Co}} * \theta_{ky} * \frac{dK_y}{K_y} - \frac{T_{Co}}{1+T_{Co}} * \theta_{ly} * \frac{dL_y}{L_y} - \frac{dT_{Co}}{1+T_{Co}} \qquad \text{..................... (P2)}$$

(Where $\theta_{ky}$ and $\theta_{ly}$ are tax exclusive)

$$dP_z = \left(\frac{1}{b}\right) * \theta_{kz} * dP_k + \left(\frac{1}{b} - 1\right) * \theta_{kz} * \frac{dK_z}{K_z} \qquad \text{.................. (P3)}$$

### *Demand*

$$\frac{dX}{X} = \left[ \frac{X^o}{X} * E^o_{xx} + \frac{X^r}{X} * E^r_{xx} \right] * dP_x + \left[ \frac{X^o}{X} * E^o_{xy} \right] * dP_y +$$

$$\frac{X^o}{X} * E^o_{xx} * \frac{dT_{Co}}{(1+T_{Co})} + \frac{X^o}{X} * E^o_{xy} * \frac{dT_{Co}}{(1+T_{Co})} + \frac{X^o}{X} * E^o_{xz} * \frac{dT_{Co}}{(1+T_{Co})} +$$

$$\left(\frac{1}{X}\right) * \left[ \frac{\partial X^o}{\partial M^o} * dM^o + \frac{\partial X^r}{\partial M^r} * dM^r \right] \qquad \text{...........................(D1)}$$

$$\frac{dY}{Y} = E_{yx} * dP_x + E_{yy} * dP_y + \left(\frac{1}{Y}\right) * \left[ \frac{\partial Y^o}{\partial M^o} * dM^o \right] +$$

$$E_{yx} * \frac{dT_{Co}}{(1+T_{Co})} + E_{yy} * \frac{dT_{Co}}{(1+T_{Co})} + E_{yz} * \frac{dT_{Co}}{(1+T_{Co})} \qquad \text{................................(D2)}$$

$$\frac{dZ}{Z} = \left[ \frac{Z^o}{Z} * E^o_{zx} + \frac{Z^r}{Z} * E^r_{zx} \right] * dP_x + \left[ \frac{Z^o}{Z} * E^o_{zy} \right] * dP_y +$$

$$\frac{z^o}{z} * E^o_{zx} * \frac{dT_{Co}}{(1+T_{Co})} + \frac{z^o}{z} * E^o_{zy} * \frac{dT_{Co}}{(1+T_{Co})} + \frac{z^o}{z} * E^o_{zz} * \frac{dT_{Co}}{(1+T_{Co})} +$$

$$(\frac{1}{z})*[\ \frac{\partial z^o}{\partial M^o} * dM^o + \frac{\partial z^r}{\partial M^r} * dM^r\ ] \qquad \text{……………….(D3)}$$

Where elasticities are Marshallian and income effects $M^o$ and $M^r$ is disposable income.

### *Adding up*

$dK_x + dK_y + dK_z = 0$ …………(A1)

$dL_x + dL_y = 0$ …………(A2)

$dL_z = 0$ …………(A3)

$dP_z = 0$ [Numeraire]

$P_k$, $P_{lo}$ and $P_{lr}$ (factor prices) and $P_x$, $P_y$ and $P_z$ (output prices) are all equal to 1 and to each other to begin with

### *Other relationships*

$G = T_{Co}* P_x * X^o + T_{Co}* P_y * Y^o + T_{Co}* P_z * Z^o$

$G = P_k*(K_x + K_y) + P_{lo}*(L_x + L_y) - P_x * X^o - P_y * Y^o - P_z * Z^o$

$G = T_c *[P_k*(K_x + K_y) + P_{lo}*(L_x + L_y)]$

$$dG = [dP_k*(K_x + K_y) + P_k*d(K_x + K_y)+ dP_{lo}*(L_x + L_y) ]* T_c +$$
$$[P_k*(K_x + K_y) + P_{lo}*(L_x + L_y)]* dT_c$$

$$dM^o = K_x*\frac{dK_x}{K_x} + K_y*\frac{dK_y}{K_y} + (K_x + K_y)*dP_k + (L_x + L_y)*dP_{lo}$$

$dM^r = - K_x*\frac{dK_x}{K_x} - K_y*\frac{dK_y}{K_y}$

**SOLUTION PROCEDURES**

Using $dK_x + dK_y + dK_z = 0$ …………(A1) we can write:

$$\frac{dK_z}{K_z} = -(\frac{K_x}{K_z})*\frac{dK_x}{K_x} - (\frac{K_y}{K_z})*\frac{dK_y}{K_y}$$

And using $dL_x + dL_y = 0$ …………(A2) we can write:

$$\frac{dL_x}{L_x} = -(\frac{L_y}{L_x})*\frac{dL_y}{L_y}$$

We equate the demand and supply equations for sector X and Y, remembering that due to Walras' Law, the market for Z is in equilibrium when the first two are balanced. We substitute $dP_z = 0$ from the numeraire equation in the demand functions and $dL_z = 0$ wherever it appears. We use $dP_k = (b - 1) * \frac{dK_z}{K_z}$ or

$dP_k = (b - 1)*[-(\frac{K_x}{K_z})*\frac{dK_x}{K_x} - (\frac{K_y}{K_z})*\frac{dK_y}{K_y}]$when required.

Now the substitution equations are rewritten as:

$$\frac{dK_x}{K_x} - \frac{dL_x}{L_x} = S_x * (dP_k - dP_{lo}) \qquad \text{… (U1)}$$

$$\Rightarrow \quad \frac{dK_x}{K_x} + (\frac{L_y}{L_x})*\frac{dL_y}{L_y} = S_x *(b - 1)*[-(\frac{K_x}{K_z})*\frac{dK_x}{K_x} - (\frac{K_y}{K_z})*\frac{dK_y}{K_y}] - S_x *dP_{lo}$$

$$\frac{dK_y}{K_y} - \frac{dL_y}{L_y} = S_y * (dP_k - dP_{lo}) \qquad \text{… (U2)}$$

$$\Rightarrow \quad \frac{dK_y}{K_y} - \frac{dL_y}{L_y} = S_y*(b-1)*[-(\frac{K_x}{K_z})*\frac{dK_x}{K_x} - (\frac{K_y}{K_z})*\frac{dK_y}{K_y}] - S_y*dP_{lo}$$

Next we equate the supply and demand for X, and remembering that $dP_z = 0$ and

$\frac{dL_x}{L_x} = -(\frac{L_y}{L_x})*\frac{dL_y}{L_y}$ :

$$\theta_{kx}*\frac{dK_x}{K_x} - \theta_{lx}*(\frac{L_y}{L_x})*\frac{dL_y}{L_y} + \frac{1}{X}*\frac{\partial f}{\partial G}*dG = [\frac{X^o}{X}*E^o_{xx} + \frac{X^r}{X}*E^r_{xx}]*dP_x + [\frac{X^o}{X}*E^o_{xy}]*dP_y + \frac{X^o}{X}*E^o_{xx}*\frac{dT_{Co}}{(1+T_{Co})} + \frac{X^o}{X}*E^o_{xy}*\frac{dT_{Co}}{(1+T_{Co})} + \frac{X^o}{X}*E^o_{xz}*\frac{dT_{Co}}{(1+T_{Co})} + (\frac{1}{X})*[\frac{\partial X^o}{\partial M^o}*dM^o + \frac{\partial X^r}{\partial M^r}*dM^r]$$

Equating demand and supply for Y:

$$\theta_{ky}*\frac{dK_y}{K_y} + \theta_{ly}*\frac{dL_y}{L_y} + \frac{1}{Y}*\frac{\partial g}{\partial G}*dG = E_{yx}*dP_x + E_{yy}*dP_y + (\frac{1}{Y})*[\frac{\partial Y^o}{\partial M^o}*dM^o] + E_{yx}*\frac{dT_{Co}}{(1+T_{Co})} + E_{yy}*\frac{dT_{Co}}{(1+T_{Co})} + E_{yz}*\frac{dT_{Co}}{(1+T_{Co})}$$

Then we have 4 equations in the following unknowns: $\frac{dK_y}{K_y}$, $\frac{dL_y}{L_y}$, $dG$, $dP_x$, $dP_y$, $dM^o$, $\frac{dK_x}{K_x}$, $dM^r$, $dP_{lo}$ and $dP_k$. We can substitute for some variables using:

$$dP_x = \theta_{kx}*(b-1)*\frac{1}{1+T_{Co}}*[-(\frac{K_x}{K_z})*\frac{dK_x}{K_x} - (\frac{K_y}{K_z})*\frac{dK_y}{K_y}] + \theta_{lx}*\frac{1}{1+T_{Co}}*dP_{lo} - \frac{1}{X}*\frac{\partial f}{\partial G}*dG - \frac{T_{Co}}{1+T_{Co}}*\theta_{kx}*\frac{dK_x}{K_x} - \frac{T_{Co}}{1+T_{Co}}*\theta_{lx}*\frac{dL_x}{L_x} - \frac{dT_{Co}}{1+T_{Co}}$$

(Where $\theta_{kx}$ is tax exclusive)

$$dP_y = \theta_{ky}*(b-1)*\frac{1}{1+T_{Co}}*[-(\frac{K_x}{K_z})*\frac{dK_x}{K_x} - (\frac{K_y}{K_z})*\frac{dK_y}{K_y}] + \theta_{ly}*\frac{1}{1+T_{Co}}*dP_{lo} - \frac{1}{Y}*\frac{\partial g}{\partial G}*dG - \frac{T_{Co}}{1+T_{Co}}*\theta_{ky}*\frac{dK_y}{K_y} - \frac{T_{Co}}{1+T_{Co}}*\theta_{ly}*\frac{dL_y}{L_y} - \frac{dT_{Co}}{1+T_{Co}}$$

(Where $\theta_{ky}$ is tax exclusive)

$$dM^o = K_x*\frac{dK_x}{K_x} + K_y*\frac{dK_y}{K_y} + (K_x + K_y)* (b - 1 )*[- (\frac{K_x}{K_z})*\frac{dK_x}{K_x} - (\frac{K_y}{K_z})*\frac{dK_y}{K_y}] + (L_x + L_y)*dP_{lo}$$

$$dM^r = - K_x*(\frac{dK_x}{K_x}) - K_y*(\frac{dK_y}{K_y})$$

$$dG = \{(b - 1 )*[- (\frac{K_x}{K_z})*\frac{dK_x}{K_x} - (\frac{K_y}{K_z})*\frac{dK_y}{K_y}]*(K_x + K_y) + K_x*(\frac{dK_x}{K_x}) + K_y*(\frac{dK_y}{K_y})+$$

$$dP_{lo}*(L_x + L_y) \}* T_c + [(K_x + K_y) + (L_x + L_y)]* dT_c$$

And we are left with 4 equations in 4 variables, namely: $\frac{dK_y}{K_y}$, $\frac{dL_y}{L_y}$, $\frac{dK_x}{K_x}$ and $dP_{lo}$

Next we use symbolic notation for some of the parameters involved:

$$\text{Є}xx = [\frac{X^o}{X} * E^o_{xx} + \frac{X^r}{X} * E^r_{xx}] \qquad\qquad \text{Є}xy = [\frac{X^o}{X} * E^o_{xy}]$$

These weighted elasticities are Marshallian and are derived assuming the demand curves are separate for OECD and ROW.

We can write for symmetry since $Y^o = Y$: $\text{Є}yx = E_{yx}$ and $\text{Є}yy = E_{yy}$

**REDUCED FORM VERSION OF THE MODEL**

$$[1+S_x *(b - 1 )*(\frac{K_x}{K_z})]*\frac{dK_x}{K_x} + (\frac{L_y}{L_x})*\frac{dL_y}{L_y} + S_x *(b - 1 )*(\frac{K_y}{K_z})*\frac{dK_y}{K_y} + S_x *dP_{lo} = 0$$

………… (1)

$$S_y *(b - 1 )*(\frac{K_x}{K_z})*\frac{dK_x}{K_x} - \frac{dL_y}{L_y} + [1+S_y *(b - 1 )*(\frac{K_y}{K_z})]*\frac{dK_y}{K_y} + S_y *dP_{lo} = 0$$

………… (2)

$\{\theta_{kx} + [\ T_c{*}\ \theta_{kx}{*}(1{+}\beta){*}\ \frac{\partial f}{\partial G}]{*}(1{+}\ \epsilon xx) + \frac{(b-1)}{(1+T_{Co})}{*}\ \frac{K_x}{K_z}{*}(\ \epsilon xx\ {*}\theta_{kx} + \epsilon xy\ {*}\theta_{ky}) +$

$\theta_{kx}{*}(\ \frac{\partial X^r}{\partial M^r} - (1\text{-}\ K_z\ {*}\beta){*}\frac{\partial X^o}{\partial M^o}\ ) + \epsilon xy\ {*}(1{+}\beta){*}\frac{T_c * K_x}{Y} * \frac{\partial g}{\partial G}\ + \epsilon xx\ {*}\theta_{kx}\ {*}(\frac{T_{Co}}{(1+T_{Co})})\ \}{*}\ \frac{dK_x}{K_x}\ +$

$\{\ [\ T_c{*}\ \frac{K_y}{X}{*}(1{+}\beta){*}\ \frac{\partial f}{\partial G}]{*}(1{+}\ \epsilon xx) + \frac{(b-1)}{(1+T_{Co})}{*}\ \frac{K_y}{K_z}{*}(\ \epsilon xx\ {*}\theta_{kx} + \epsilon xy\ {*}\theta_{ky}) +$

$\frac{K_y}{X}{*}(\ \frac{\partial X^r}{\partial M^r} - (1\text{-}\ K_z\ {*}\beta){*}\frac{\partial X^o}{\partial M^o}\ ) + \epsilon xy\ {*}\theta_{ky}{*}[(1{+}\beta){*}T_c * \frac{\partial g}{\partial G}\ + (\frac{T_{Co}}{(1+T_{Co})})]\ \}{*}\ \frac{dK_y}{K_y}\quad -$

$\{\ [\theta_{lx} * (\frac{L_y}{L_x}){*}(1 \mp \epsilon xx\ * (\frac{T_{Co}}{(1+T_{Co})})] - \epsilon xy\ * (\frac{T_{Co}}{(1+T_{Co})}) *\ \theta_{ly}\ \}{*}\ \frac{dL_y}{L_y}\ +$

$\{\text{-}\ \frac{1}{(1+T_{Co})}{*}(\ \epsilon xx{*}\ \theta_{lx}\ + \epsilon xy{*}\ \theta_{ly}\ ){+}\ (\ 1 + \epsilon xx){*}\ T_c{*}[\ \theta_{lx} + \frac{L_y}{X}\ ]{*}\frac{\partial f}{\partial G}\ + \epsilon xy\ {*}\ T_c{*}[\ \theta_{ly} +$

$\frac{L_x}{Y}\ ]{*}\frac{\partial g}{\partial G} -$

$[\ \theta_{lx} + \frac{L_y}{X}\ ]\ {*}\frac{\partial X^o}{\partial M^o}\}{*}\ dP_{lo}\ =\ \{\ \frac{X^o}{X} * E^o_{xy} + \frac{X^o}{X} {*}E^o_{xx} + \frac{X^o}{X} * E^o_{xz}\ \text{-}\ \ [\epsilon xx + \epsilon xy\ ]\}{*}\ \frac{dT_{Co}}{(1+T_{Co})}$

$\{\ \frac{\partial f}{\partial G}{*}\ (1{+}\ \epsilon xx){*}[\ \ \theta_{kx}\ + \frac{K_y}{X} + \theta_{lx} + \frac{L_y}{X}\ ]\ \ + \frac{\partial g}{\partial G}{*}\ \epsilon xy{*}[\ \ \theta_{ky}\ + \frac{K_x}{Y} + \theta_{ly} + \frac{L_x}{Y}\ ]\}{*}\ dT_c$

………… (3)

$\{[\ T_c{*}\frac{K_x}{Y}\ {*}(1{+}\beta){*}\ \frac{\partial g}{\partial G}]{*}(1{+}\ \epsilon yy) + \frac{(b-1)*K_x}{(1+T_{Co})*K_z}{*}(\ \epsilon yx\ {*}\ \theta_{kx} + \epsilon yy\ {*}\theta_{ky})\ -\ \frac{K_x}{Y}{*}(1\text{-}\beta){*}\frac{\partial Y^o}{\partial M^o}\ +$

$\epsilon yx\ {*}\theta_{kx}\ {*}[(\frac{T_{Co}}{(1+T_{Co})}) + (1{+}\beta){*}T_c{*}\ \frac{\partial f}{\partial G}]\}{*}\ \frac{dK_x}{K_x}\ +$

$\{\theta_{ky}\ + [\ T_c{*}\theta_{ky}{*}(1{+}\beta){*}\ \frac{\partial g}{\partial G}]{*}(1{+}\ \epsilon yy) + \frac{(b-1)*K_y}{(1+T_{Co})*K_z}{*}(\ \epsilon yx\ {*}\ \theta_{kx} + \epsilon yy\ {*}\theta_{ky}) - \theta_{ky}{*}(1\text{-}$

$\beta){*}\frac{\partial Y^o}{\partial M^o}\ + \epsilon yy\ {*}\theta_{ky}\ {*}[(\frac{T_{Co}}{(1+T_{Co})}) + \epsilon yx\ {*}(1{+}\beta){*}T_c * \frac{K_y}{X} * \frac{\partial f}{\partial G}\}{*}\ \frac{dK_y}{K_y}\quad +$

$$\{[\theta_{ly}*(1 \mp \epsilon yy * (\frac{T_{Co}}{(1+T_{Co})})] - \epsilon yx * \frac{L_y}{L_x} * (\frac{T_{Co}}{(1+T_{Co})}) * \theta_{lx}\}*\frac{dL_y}{L_y} +$$

$$\{ ( 1 + \epsilon yy)* T_c*[ \theta_{ly} + \frac{L_x}{Y} ]*\frac{\partial g}{\partial G} + \epsilon yx * T_c*[ \theta_{lx} + \frac{L_y}{X} ]*\frac{\partial f}{\partial G} - [ \theta_{ly} + \frac{L_x}{Y} ] *\frac{\partial Y^o}{\partial M^o}$$

$$- \frac{1}{(1+T_{Co})} *( \epsilon yx* \theta_{lx} + \epsilon yy* \theta_{ly} ) \}* dP_{lo} = \{ E^o_{yx} + E^o_{yy} + E^o_{yz} - [\epsilon yx + \epsilon yy ] \}*$$

$$\frac{dT_{Co}}{(1+T_{Co})} - \{ \frac{\partial g}{\partial G}* (1+ \epsilon yy)*[ \theta_{ky} + \frac{K_x}{Y} + \theta_{ly} + \frac{L_x}{Y} ] + \frac{\partial f}{\partial G}* \epsilon yx*[ \theta_{kx} + \frac{K_y}{X} + \theta_{lx} + \frac{L_y}{X} ]\}*$$

$$dT_c$$

………… (4)

Where $\beta = [- \frac{(b-1)*(K_x+K_y)}{K_z} ]$

These, as before are 4 equations in 4 variables. If variables are denoted in the following order by subscript j = 1, ..., 4; $\frac{dK_x}{K_x}$, $\frac{dK_y}{K_y}$ , $\frac{dL_y}{L_y}$, $dP_{lo}$ and subscript i = 1, ..., 4 represents the equation, the 4 equations above can be written in symbolic form with $a_{ij}$ representing coefficients attached to the left hand side variables and $b_i$ the constants on the right hand side.

$$A_{11}*\frac{dK_x}{K_x} + A_{12}*\frac{dK_y}{K_y} + A_{13} *\frac{dL_y}{L_y} + A_{14}*dP_{lo} = B_1 \quad \ldots\ldots\ldots\ldots \quad (1)$$

$$A_{21}*\frac{dK_x}{K_x} + A_{22}* \frac{dK_y}{K_y} + A_{23} *\frac{dL_y}{L_y} + A_{24}* dP_{lo} = B_2 \quad \ldots\ldots\ldots\ldots \quad (2)$$

$$A_{31}*\frac{dK_x}{K_x} + A_{32}*\frac{dK_y}{K_y} + A_{33} *\frac{dL_y}{L_y} + A_{34}* dP_{lo} = B_3 \quad \ldots\ldots\ldots\ldots \quad (3)$$

$$A_{41}* \frac{dK_x}{K_x} + A_{42}*\frac{dK_y}{K_y} + A_{43} *\frac{dL_y}{L_y} + A_{44}*dP_{lo} = B_4 \quad \ldots\ldots\ldots\ldots \quad (4)$$

Where

$A_{11} = 1+S_x *(b-1)*(\frac{K_x}{K_z})$ $A_{12} = S_x *(b-1)*(\frac{K_y}{K_z})$ $A_{13} = (\frac{L_y}{L_x})$ $A_{14} = S_x$ $B_1 = 0$

$A_{21} = S_y *(b-1)*(\frac{K_x}{K_z})$ $A_{22} = 1+S_y *(b-1)*(\frac{K_y}{K_z})$ $A_{23} = -1$ $A_{24} = S_y$ $B_2 = 0$

$A_{31} = \{\theta_{kx} + [T_c * \theta_{kx}*(1+\beta)* \frac{\partial f}{\partial G}]*(1+ \epsilon xx) + \frac{(b-1)}{(1+T_{Co})} * \frac{K_x}{K_z}*(\epsilon xx *\theta_{kx} + \epsilon xy *\theta_{ky}) +$

$\theta_{kx}*( \frac{\partial X^r}{\partial M^r} - (1- K_z *\beta)*\frac{\partial X^o}{\partial M^o}) + \epsilon xy *(1+\beta)*\frac{T_c * K_x}{Y} * \frac{\partial g}{\partial G} + \epsilon xx *\theta_{kx} *(\frac{T_{Co}}{(1+T_{Co})}) \}$

$A_{32} = \{[T_c* \frac{K_y}{X}*(1+\beta)* \frac{\partial f}{\partial G}]*(1+ \epsilon xx) + \frac{(b-1)}{(1+T_{Co})} * \frac{K_y}{K_z}*(\epsilon xx *\theta_{kx} + \epsilon xy *\theta_{ky}) +$

$\frac{K_y}{X}*( \frac{\partial X^r}{\partial M^r} - (1- K_z *\beta)*\frac{\partial X^o}{\partial M^o}) + \epsilon xy *\theta_{ky}*[(1+\beta)*T_c * \frac{\partial g}{\partial G} + (\frac{T_{Co}}{(1+T_{Co})})] \}$

$A_{33} = -\{[\theta_{lx} * (\frac{L_y}{L_x})*(1 \mp \epsilon xx * (\frac{T_{Co}}{(1+T_{Co})})] - \epsilon xy * (\frac{T_{Co}}{(1+T_{Co})}) * \theta_{ly} \}$

$A_{34} = \{- \frac{1}{(1+T_{Co})} *(\epsilon xx* \theta_{lx} + \epsilon xy* \theta_{ly}) + (1 + \epsilon xx)* T_c*[\theta_{lx} + \frac{L_y}{X}]*\frac{\partial f}{\partial G} +$

$\epsilon xy * T_c*[\theta_{ly} + \frac{L_x}{Y}]*\frac{\partial g}{\partial G} - [\theta_{lx} + \frac{L_y}{X}] *\frac{\partial X^o}{\partial M^o}\}$

$B_3 = \{ \frac{X^o}{X} * E^o_{xy} + \frac{X^o}{X} *E^o_{xx} + \frac{X^o}{X} * E^o_{xz} - [\epsilon xx + \epsilon xy]\} * \frac{dT_{Co}}{(1+T_{Co})}$ -

$\{ \frac{\partial f}{\partial G}* (1+ \epsilon xx)*[\theta_{kx} + \frac{K_y}{X} + \theta_{lx} + \frac{L_y}{X}] + \frac{\partial g}{\partial G}* \epsilon xy*[\theta_{ky} + \frac{K_x}{Y} + \theta_{ly} + \frac{L_x}{Y}]\}* dT_c$

$A_{41} = \{[T_c*\frac{K_x}{Y} *(1+\beta)* \frac{\partial g}{\partial G}]*(1+ \epsilon yy) + \frac{(b-1)*K_x}{(1+T_{Co})*K_z}*(\epsilon yx * \theta_{kx} + \epsilon yy *\theta_{ky}) - \frac{K_x}{Y}*(1-$

$\beta)*\frac{\partial Y^o}{\partial M^o} +$

$Єyx * \theta_{kx} *[(\frac{T_{Co}}{(1+T_{Co})}) + (1+\beta)*T_c* \frac{\partial f}{\partial G}]\}$

$A_{42} = \{\theta_{ky} + [ T_c*\theta_{ky}*(1+\beta)* \frac{\partial g}{\partial G}]*(1+ Єyy) +\frac{(b-1)*K_y}{(1+T_{Co})*K_z}*( Єyx * \theta_{kx} + Єyy *\theta_{ky})$

$- \theta_{ky}*(1-\beta)*\frac{\partial Y^o}{\partial M^o} + Єyy *\theta_{ky} *[(\frac{T_{Co}}{(1+T_{Co})}) + Єyx *(1+\beta)*T_c * \frac{K_y}{X} * \frac{\partial f}{\partial G}\}$

$A_{43} = \{[\theta_{ly}*(1 \mp Єyy * (\frac{T_{Co}}{(1+T_{Co})})] - Єyx * \frac{L_y}{L_x} * (\frac{T_{Co}}{(1+T_{Co})}) * \theta_{lx} \}$

$A_{44} = \{( 1 + Єyy)* T_c*[ \theta_{ly} + \frac{L_x}{Y} ]*\frac{\partial g}{\partial G} + Єyx * T_c*[ \theta_{lx} + \frac{L_y}{X} ]*\frac{\partial f}{\partial G} - [ \theta_{ly} + \frac{L_x}{Y} ] *\frac{\partial Y^o}{\partial M^o}$

$- \frac{1}{(1+T_{Co})} *( Єyx* \theta_{lx} + Єyy* \theta_{ly} ) \}$

$B_4 = \{ E^o_{yx} +E^o_{yy}+ E^o_{yz} - [Єyx + Єyy ] \}* \frac{dT_{Co}}{(1+T_{Co})} -$

$\{ \frac{\partial g}{\partial G}* (1+ Єyy)*[ \theta_{ky} + \frac{K_x}{Y} + \theta_{ly} + \frac{L_x}{Y} ] + \frac{\partial f}{\partial G}* Єyx*[ \theta_{kx} + \frac{K_y}{X} + \theta_{lx} + \frac{L_y}{X} ]\}* dT_c$

This can be solved using MATLAB or any other procedure as discussed in chapter two.

**CHANGES IN GDP AND USES SIDE**

Here GDP from the income side equals disposable income since there are no direct taxes.

$dM^o = dGDP^o = P_k*K_x*\frac{dK_x}{K_x} + P_k*K_y*\frac{dK_y}{K_y} + (K_x + K_y)*dP_k + (L_x + L_y)*dP_{lo}$

$dM^r = dGDP^r = - K_x*\frac{dK_x}{K_x} - K_y*\frac{dK_y}{K_y}$

Now, as in chapter two we construct Laspeyres index for both countries to get an estimate of the change in the cost of living. Since the consumption tax does not apply to the ROW, the index for ROW is the same as in chapter two.

$$LAS^r = \frac{(P_x + dP_x)* X^r + (P_z + dP_z)*Z^r}{P_x * X^r + P_z *Z^r}$$

Next we know that $dP_z = 0$ and initial prices are equal to 1, this gives:

$$LAS^r = \frac{X^r + dP_x* X^r + Z^r}{X^r + Z^r} = 1 + \frac{dP_x* X^r}{X^r + Z^r}$$

The major difference is in the index in the OECD country since the tax is imposed on consumption here:

$$LAS^o = \frac{d[P_x*(1+T_{Co})]* X^o + d[P_y*(1+T_{Co})]* Y^o + d[P_z*(1+T_{Co})]* Z^o + [P_x*(1+T_{Co})]* X^o + [P_y*(1+T_{Co})]* Y^o + [P_z*(1+T_{Co})]* Z^o}{[P_x*(1+T_{Co})]* X^o + [P_y*(1+T_{Co})]* Y^o + [P_z*(1+T_{Co})]* Z^o}$$

$$= 1 + \frac{d[P_x*(1+T_{Co})]* X^o + d[P_y*(1+T_{Co})]* Y^o + d[P_z*(1+T_{Co})]* Z^o}{[P_x*(1+T_{Co})]* X^o + [P_y*(1+T_{Co})]* Y^o + [P_z*(1+T_{Co})]* Z^o}$$

$$= \quad 1 + \frac{dT_{Co}*[X^o + Y^o + Z^o] + (1+T_{Co})[dP_x*X^o + dP_y*Y^o]}{(1+T_{Co})*[X^o + Y^o + Z^o]}$$

$$LAS^o = 1 + \frac{dT_{Co}}{(1+T_{Co})} + \frac{[dP_x*X^o + dP_y*Y^o]}{[X^o + Y^o + Z^o]}$$

As in chapter two, to measure total welfare, we can either take a linear combination of the changes in the sources and uses side, or deflate the change in GDP by the change in prices.

OECD welfare:

$W^o = 1 + dGDP^o/GDP^o - LAS^o$ and

ROW welfare $W^r = 1 + dGDP^r/GDP^r - LAS^r$

This is a pure assumption, and we could have chosen any form for the welfare function, such as $W^o = \frac{1+\frac{dGDP^o}{GDP^o}}{LAS^o}$ instead or more complicated forms.

In the next chapter, we examine the models of the CIT and the consumption tax with different expenditure assumptions. We assume in chapter 4 that government does not provide an active input, but instead returns the tax revenue to the consumer in the OECD through a transfer payment. We compare these models of the transfer payment with the ones developed in chapters two and three to examine the impact of the expenditure assumption made.

# CHAPTER IV – THE TRANSFER MODELS

The purpose of this chapter is to develop models of the tax competition case with largely the same set-up as in chapters two and three; the main difference will be that there is no government provided input. In line with Randolph (2006), Harberger (1962, 1995 and 2008) and others, tax revenue will be returned lump-sum to the single consumer in the OECD. This may be thought of as a tax and transfer program, but since there is only one consumer in the OECD, the net effect is that what is taken in tax revenue is returned without administrative costs. However, taxation creates an excess burden when relative prices change, and so a change in the tax implies a change in this burden.

In the Harberger (1962) model, taxes are returned to the single consumer. The model seeks to quantify only the change in the sources side of income caused by imposing the tax even after the tax revenue itself is returned. Although Harberger (1962) concentrates on small tax rate changes, the effects of such a tax in terms of excess burden are not negligible.

In the case where the tax is imposed on one factor of production, the tax changes the relative price of this factor. If the producer equates this distorted factor price to marginal product, the higher tax inclusive price means that relatively less capital is employed relative to the no-tax situation. This inefficiency or "excess burden" implies that total production and therefore GDP is lower in the situation with taxes than in the no-tax case.

Even if the tax revenue raised is returned lump-sum to the consumer, the wedge between factor prices remains, and the welfare cost does too. This burden arises purely on the tax side since there is no government good here, and all tax revenues are raised and returned without administrative cost. Since the GDP equals the single consumers' income, a change in GDP implies an income effect. Thus, a reduction in this distorting tax should imply a reduction in the excess burden or an increase in GDP.

In the case of a consumption tax, one country is able to tax only the goods consumed within that country's borders. This imposes a wedge between the prices of the import and export goods prevailing in the ROW and OECD countries. Reducing taxes removes this wedge and affects demand only in the OECD country since the demand curves include income effects.

Once two models for the transfer case are developed (one for the capital income tax and the other for the consumption tax), we can answer one of the research questions set out in chapter one: Does the use to which taxes are put have an effect on the quantum and nature of burdens on the sources and uses of income even if no extra assumptions are made about the non-neutrality of expenditure? In case the differential effects on income as measured are different in the two cases (for each tax) we can say that the use to which the tax is put implies that there is an expenditure effect in addition to the tax effect, and it could very well happen that the government ends up imposing a higher or lower excess burden on the ROW due to the expenditure choice, even with the same tax revenue.

The models in this chapter differ from the Harberger (1962, 1995 and 2008) models in the following ways: (1) We still have taxes on all capital income and on

consumption, not on corporate income (2) There is no separate corporate sector-we focus on international burdens only (3) There is an existing tax that is lowered (4) The OECD country is a large open economy and burdens are measured on the sources and uses sides. Though this version is not solved explicitly in Harberger (1995 and 2008), it is closer in spirit to the models discussed in Gravelle and Smetters(2006) and Randolph (2006) with the introduction of the capital income tax and consumption tax being new features.

## THE CAPITAL INCOME TAX CASE WITH TRANSFERS

### Profit maximization behavior by firms (sectors)

We remind ourselves that $P_k$, $P_{lo}$ and $P_{lr}$ (factor prices) and $P_x$, $P_y$ and $P_z$ (output prices) are all equal to 1 and to each other to begin with. Since our existing tax is ≠ 0, this implies that $P_k*(1+T_{ko}) \neq 1$.

The production functions for each sector can be written as a function of productive resources:

$X = f(K_x, L_x)$ $\qquad$ $Y = g(K_y, L_y)$ $\qquad$ $Z = h(K_z, L_z)$

Since there is no government provided input, the production functions in sectors X and Y are functions of capital and labor alone. For sector Z, since we have assumed full employment, the only variable is $K_z$ since $L_z$ is a constant. We can write the production function for Z as: $Z = h(K_z, \overline{L_z})$. The profit maximization problem can then be set up as:

FIRM (SECTOR) X

Maximize Π (profit) = { $P_x$* f ($K_x$ , $L_x$) - $P_k$*(1+$T_{ko}$)*$K_x$ – $P_{lo}$*$L_x$ } with respect to $K_x$ and $L_x$

First order conditions:

3. $P_x * \frac{\partial f}{\partial K_x} \; - \; P_k*(1+T_{ko}) = 0$

And

4. $P_x * \frac{\partial f}{\partial L_x} \; - \; P_{lo} = 0$

Since $P_x = 1$, this implies that $\frac{\partial f}{\partial K_x} = P_k*(1+T_{ko})$ and $\frac{\partial f}{\partial L_x} = P_{lo}$

Each firm pays each factor its marginal product at the current level of provision of the public good. In the case of capital in OECD, this is the gross of tax rate of return.

FIRM (SECTOR) Y

Behaves in exactly the same way as X, and so we get $\frac{\partial g}{\partial K_y} = P_k*(1+T_{ko})$ and $\frac{\partial g}{\partial L_y} = P_{lo}$

FIRM (SECTOR) Z

In relative terms, we can behave as if there is no government good or tax in the ROW. We can assume that the amount of taxation is fixed, and does not vary in the sense that we are only modeling the excess tax in the OECD country (over and above the level of taxes and public services provided in the ROW). Furthermore, the tax and level of G in the ROW does not change at all, so it is built into prices and output. We can thus proceed

as if there is no tax or G in ROW, although this may be true only in relative terms. Since no changes are being made to T and G in ROW, we assume that there are no marginal effects. However, this is an approximation, as in previous chapters. If there is an ad valorem tax in the ROW either on capital or on consumption, changing factor and commodity prices will affect revenues even without rate changes. As in previous chapters, we have ignored such effects. We therefore have only the production function:

$Z = h(K_z, \overline{L_z})$ and the problem for firm Z:

Maximize Π (profit) = { $P_z * h(K_z, \overline{L_z}) - P_k*K_z - P_{lr}*\overline{L_z}$ } with respect to $K_z$ { since we assume full employment always in both countries, choosing $L_z$ is not a decision variable } and we get $\frac{\partial h}{\partial K_z} = P_k$

**CHANGES IN THE AD VALOREM TAX RATE**

We use $d(P_k*(1+T_{ko})) = d(P_k + P_k*T_{ko}) = dP_k + T_{ko}*dP_k + dT_{ko} = dT_{ko} + (1 + T_{ko})*dP_k$

**SUPPLY**

The production functions for the three sectors are:

$X = f(K_x, L_x)$ $\quad$ $Y = g(K_y, L_y)$ $\quad$ $Z = h(K_z, L_z)$

Since there is no government input here, G does not enter any supply function.

SECTOR X

$X = f(K_x, L_x)$

The total change in supply (output) or the total differential can be split into:

$$dX = \frac{\partial f}{\partial K_x} * dK_x + \frac{\partial f}{\partial L_x} * dL_x$$

Remembering that firms in sector X equate $\frac{\partial f}{\partial K_x} = P_k*(1+T_{ko})$

and $\frac{\partial f}{\partial L_x} = P_{lo}$ and that $P_x = 1$:

$$\frac{dX}{X} = \frac{P_k*(1+T_{ko})*K_x}{P_x*X} * \frac{dK_x}{k_x} + \frac{P_{lo}*L_x}{P_x*X} * \frac{dL_x}{L_x}$$

Writing $\theta_{kx} = \frac{P_k*(1+T_{ko})*K_x}{P_x*X}$ the tax-inclusive share of capital's product in sector X

And $\theta_{lx} = \frac{P_{lo}*L_x}{P_x*X}$ the share of labor's product in sector X, we get

S1…………………..  $\frac{dX}{X} = \theta_{kx} * \frac{dK_x}{K_x} + \theta_{lx} * \frac{dL_x}{L_x}$

Similarly, for SECTOR Y we have:

S2……….............. $\frac{dY}{Y} = \theta_{ky} * \frac{dK_y}{K_y} + \theta_{ly} * \frac{dL_y}{L_y}$

Where $\theta_{ky} = \frac{P_k*(1+T_{ko})*K_y}{P_y*Y}$ the tax-inclusive share of capital's product in sector Y

And $\theta_{ly} = \frac{P_{lo}*L_y}{P_y*Y}$ the share of labor's product in sector Y.

SECTOR Z

There is no tax in the ROW, so the relevant terms for this sector:

S3a…………............ $\frac{dZ}{Z} = \theta_{kz} * \frac{dK_z}{K_z} + \frac{1}{Z} * L_z * \frac{\partial h}{\partial L_z} * \frac{dL_z}{L_z}$

But $dL_z = 0$ by assumption ($L_{row} = \overline{L_z}$ is fixed), and $\theta_{kz}$ is tax-exclusive, so we can write the above as:

S3…………............ $\frac{dZ}{Z} = \theta_{kz} * \frac{dK_z}{K_z}$

This gives us our three supply equations.

**PRICE FORMATION**

Sectors X and Y, as in the traditional case are assumed to be CRS. Since there is no government input, they continue to be so. For sector Z, however, since we assume full employment, the production function has only one argument, that is $K_z$. We can no longer say that the production function is CRS in K alone since $L_{row} = \overline{L_z}$ and any amount of $K_z$ is combined with the fixed $L_z$ which is not variable. The production function has to show diminishing returns to the single variable factor $K_z$. Payments to labor in Z are a residual. Since the amount of $K_z$ depends on $P_k$ which is determined jointly with other sectors, whatever is left over of the product is paid to labor. Given the fixed amount of labor, we get $P_{lr}$ and $dP_{lr}$ as a residual.

SECTOR X

We know that production is HOD One, and that there are only two arguments in the production function, capital and labor.

From Euler's law:

$$\frac{\partial f}{\partial K_x} * K_x + \frac{\partial f}{\partial L_x} * L_x = X$$

$$\frac{\partial f}{\partial K_x} * K_x + \frac{\partial f}{\partial L_x} * L_x = P_x * X \qquad \text{since } P_x = 1$$

$$d(P_x * X) = [\, P_x * dX + X * dP_x] = d[\frac{\partial f}{\partial K_x} * K_x + \frac{\partial f}{\partial L_x} * L_x]$$

If we then continue with our assumption that firms pay marginal products:

$$[\, P_x * dX + X * dP_x] = d[P_k * (1 + T_{ko}) * K_x + P_{lo} * L_x]$$

$$\Rightarrow \qquad [\, P_x * dX + X * dP_x] = d[(P_k + P_k * T_{ko}) * K_x + P_{lo} * L_x]$$

$$\Rightarrow \qquad P_x * dX + X * dP_x = [d(P_k + P_k * T_{ko}) * K_x + (P_k + P_k * T_{ko}) * dK_x + dP_{lo} * L_x + P_{lo} * dL_x]$$

From the supply equations and assumptions $P_x = 1$ we know that $P_x * dX = dX$ and

$$dX = \frac{\partial f}{\partial K_x} * dK_x + \frac{\partial f}{\partial L_x} * dL_x$$

$$\Rightarrow \qquad P_x * dX = P_k * (1 + T_{ko}) * dK_x + P_{lo} * dL_x$$

Substituting in our original equation:

$$P_x * dX + X * dP_x = [d(P_k + P_k * T_{ko}) * K_x + (P_k + P_k * T_{ko}) * dK_x + dP_{lo} * L_x + P_{lo} * dL_x]$$

$$X * dP_x = [d(P_k + P_k * T_{ko}) * K_x + (P_k + P_k * T_{ko}) * dK_x + dP_{lo} * L_x + P_{lo} * dL_x] - P_x * dX$$

$$\Rightarrow \qquad X * dP_x = [d(P_k + P_k * T_{ko}) * K_x + (P_k + P_k * T_{ko}) * dK_x + dP_{lo} * L_x + P_{lo} * dL_x] -$$

$$P_k * (1 + T_{ko}) * dK_x - P_{lo} * dL_x$$

$$\Rightarrow \qquad X * dP_x = d(P_k + P_k * T_{ko}) * K_x + dP_{lo} * L_x$$

⇨ $X*dP_x = K_x*dP_k + K_x*( dP_k*T_{ko} + P_k*dT_{ko}) + dP_{lo}* L_x$

⇨ $X*dP_x = K_x*\{(1 + T_{ko})* dP_k + P_k*dT_{ko}\} + dP_{lo}* L_x$

Using $P_k = 1$, $P_{lo} = 1$ and $P_x =1$:

$$dP_x = \frac{P_k*(1+T_{ko})*K_x}{P_x*X} *dP_k + \frac{P_k*(1+T_{ko})*K_x}{P_x*X} * \frac{dT_{ko}}{(1+T_{ko})} + \frac{P_{lo}*L_x}{P_x*X} *dP_{lo}$$

⇨ $dP_x = \theta_{kx} * dP_k + \theta_{kx} * \frac{dT_{ko}}{(1+T_{ko})} + \theta_{lx} * dP_{lo}$

…………………(P1)

Similarly for SECTOR Y

⇨ $dP_y = \theta_{ky} * dP_k + \theta_{ky} * \frac{dT_{ko}}{(1+T_{ko})} + \theta_{ly} * dP_{lo}$

…………………(P2)

SECTOR Z

There is no tax here. Suppose that total payments to capital in Z are a constant proportion of the product. The residual is paid to labor. As discussed in previous chapters, we impose this restriction for the analytical convenience of collapsing ROW to one sector. This is not so damaging, however, when we consider that constant factor proportions are an implication of several well-known functional forms such as Cobb-Douglas. Harberger (1962) assumes CRS production functions, as do many other models based on his paper and Jones (1965). Further, constant proportions are assumed in the price formation and supply equations in Harberger (1962) even without assuming

functional forms. If b represents the baseline payments proportion to capital in Z, and if b is assumed to be a constant, then:

$b^* P_z^* Z = P_k * K_z$ : b<1and b is constant

$[ P_z^*dZ + Z^*dP_z] = (\frac{1}{b})^* [dP_k^* K_z + P_k^* dK_z ]$

Now from the supply equation for Z we know that: $P_z^*dZ = dZ$ ( since $P_z = 1$) $= P_k * dK_z$

$\Rightarrow Z^*dP_z = (\frac{1}{b})^* [dP_k^* K_z + P_k^* dK_z ] - P_z^*dZ$

$\Rightarrow Z^*dP_z = (\frac{1}{b})^* [dP_k^* K_z + P_k^* dK_z ] - P_k * dK_z$

$\Rightarrow dP_z = (\frac{1}{b})^* [\theta_{kz} *dP_k + \theta_{kz} * \frac{dK_z}{K_z} ] - \theta_{kz} * \frac{dK_z}{K_z}$

(Since $P_k$ and $P_z = 1$, and $\theta_{kz}$ is tax-exclusive)

$\Rightarrow dP_z = (\frac{1}{b})^* \theta_{kz} *dP_k + (\frac{1}{b} - 1)^* \theta_{kz} * \frac{dK_z}{K_z}$

Now since we have chosen $P_z$ as our numeraire ($dP_z = 0$), we get a relationship between $dP_k$ and $\frac{dK_z}{K_z}$ and ultimately between $dP_k$ and $\frac{dK_z}{K_z} = - (\frac{K_x}{K_z})^*\frac{dK_x}{K_x} - (\frac{K_y}{K_z})^*\frac{dK_y}{K_y}$

$0 = (\frac{1}{b})^* \theta_{kz} *dP_k + (\frac{1-b}{b})^* \theta_{kz} * \frac{dK_z}{K_z}$

$dP_k = (b - 1 ) * \frac{dK_z}{K_z}$ …………………(P3)

Though we have assumed that the proportion of product that goes to capital in sector Z is a constant (b), this is not a necessary assumption. We could have proceeded as

if b was a variable. However, to be able to derive an exact expression for the change in b, we need to know the functional form, and to maintain generality, we assume b is constant. This is not a departure from the various versions of the Harberger model or the Jones version, since b is replaceable by $\theta_{kz}$ , which is usually assumed fixed. From these expressions, we can see that $P_{lr}$ and $dP_{lr}$ can be derived in terms of either total product $P_z*Z$, a combination of $P_z*Z$ and $P_k*K_z$ or $P_k*K_z$ alone:

$\overline{L_z}*P_{lr} = ( 1 - b )* P_z*Z$

Or $\overline{L_z}*P_{lr} = P_z*Z - P_k*K_z$

Or $\overline{L_z}*P_{lr} = (\frac{1}{b})* [P_k* K_z ]- P_k*K_z$

$\Rightarrow P_{lr} = (\frac{1}{b} - 1)* [P_k* K_z ]* (\frac{1}{\overline{L_z}})$

$\Rightarrow dP_{lr} = (\frac{1}{b} - 1)* [dP_k* K_z + P_k* dK_z ]* (\frac{1}{\overline{L_z}})$

Or $dP_{lr} = (1 - b)* [P_z*dZ]* (\frac{1}{\overline{L_z}})$ since $dP_z = 0$

**SUBSTITUTION**

The elasticity of substitution is (with competition) defined as: $S_x = \dfrac{\dfrac{d(\frac{K_x}{L_x})}{\frac{K_x}{L_x}}}{\dfrac{d(\frac{P_k*(1+T_{ko})}{P_{lo}})}{\frac{P_k*(1+T_{ko})}{P_{lo}}}}$

This can be rewritten as:

$$\frac{d\left(\frac{K_x}{L_x}\right)}{\frac{K_x}{L_x}} = S_x * \frac{d\left(\frac{P_k*(1+T_{ko})}{P_{lo}}\right)}{\frac{P_k*(1+T_{ko})}{P_{lo}}}$$

Since the derivation follows exactly the same steps as in Tresch (2002) it is not derived here, the results are stated:

SECTOR X

$$\frac{dK_x}{K_x} - \frac{dL_x}{L_x} = S_x * (dP_k + \frac{dT_{ko}}{(1+T_{ko})} - dP_{lo}) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(U1)$$

SECTOR Y

$$\frac{dK_y}{K_y} - \frac{dL_y}{L_y} = S_y * (dP_k + \frac{dT_{ko}}{(1+T_{ko})} - dP_{lo}) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(U2)$$

SECTOR Z

In the ROW $dL_z = 0$ by assumption, so any amount of capital is combined with the fixed amount of labor, which is not varied at all. Therefore, there is no substitution elasticity or equation for this sector.

**DEMAND**

SECTOR X

$$X = X^o(P_x, P_y, P_z, M^o) + X^r(P_x, P_z, M^r)$$

$X^o$ represents OECD demand for X. it is a function of all three prices since the OECD consumer consumes the products of all sectors. $M^o$ is OECD consumer's disposable income. This is a Marshallian demand curve. Even though, following

Harberger (1962), all tax revenue is returned as a lump-sum transfer to the OECD consumer; the change in the tax changes the excess burden, and creates an income effect. If income or utility is not held constant, the demand curve is not Hicksian, but Marshallian.

There is another difference with chapter two. Since all tax revenue is returned to the consumer, disposable income is equal to GDP in the OECD in this model by virtue of the transfer and not the increase in product caused by the government input. This formulation follows Ballentine and Eris (1975), and the Marshallian elasticities could have been represented as compensated elasticities as well, using the Slutsky substitution.

$X^r$ is the ROW demand for X. It is a function of prices of only the two tradable goods since the ROW does not consume Y. $M^r$ is the disposable income of the ROW consumer. Since there are no taxes in the ROW this is equal to GDP. This is also a Marshallian demand curve. Explicit formulations for disposable income follow:

$$M^o = P_k*(1 + T_{ko})*(K_x + K_y) + P_{lo}*(L_x + L_y)$$

$$dM^o = P_k*(1 + T_{ko})*(dK_x + dK_y) + (K_x + K_y)*d[P_k*(1 + T_{ko})] + P_{lo}*(dL_x + dL_y) +$$

$$(L_x + L_y)*dP_{lo}$$

But $dL_x + dL_y = 0$ by definition, so

$$dM^o = P_k*(1 + T_{ko})*(dK_x + dK_y) + (K_x + K_y)* dP_k*(1 + T_{ko}) +(K_x + K_y)* dT_{ko} +$$

$$(L_x + L_y)*dP_{lo}$$

$$\Rightarrow dM^o = P_k*(1 + T_{ko})*K_x*\frac{dK_x}{K_x} + P_k*(1 + T_{ko})*K_y*\frac{dK_y}{K_y} + (K_x + K_y)*dP_k *(1 + T_{ko}) +$$

$(K_x + K_y)* dT_{ko} + (L_x + L_y)*dP_{lo}$

For $M^r$ there is no tax, so GDP is the same as disposable income:

$M^r = P_k * K_z + P_{lr} * L_z$

But $P_{lr} * L_z = \overline{L_z} * P_{lr} = P_z*Z - P_k*K_z = (\frac{1}{b})* [P_k* K_z ]- P_k*K_z$

$\Rightarrow M^r = P_k * K_z + (\frac{1}{b})* [P_k* K_z ]- P_k*K_z$

$\Rightarrow M^r = (\frac{1}{b})* [P_k* K_z ]$

$dM^r = (\frac{1}{b})* [K_z*dP_k + P_k*dK_z]$

But $dP_k = (b - 1 ) * \frac{dK_z}{K_z}$ …………………(P3)

$\Rightarrow dM^r = (\frac{1}{b})* [K_z*(b - 1 ) * \frac{dK_z}{K_z} + P_k*dK_z]$

$\Rightarrow dM^r = (\frac{1}{b})* [(b - 1 )*dK_z + dK_z]$

$\Rightarrow dM^r = dK_z$

$\Rightarrow dM^r = - K_x*\frac{dK_x}{K_x} - K_y*\frac{dK_y}{K_y}$

Taking the total differential of the demand for X we get:

$$dX = \frac{\partial X^o}{\partial P_x} * dP_x + \frac{\partial X^o}{\partial P_y} * dP_y + \frac{\partial X^o}{\partial P_z} * dP_z + \frac{\partial X^o}{\partial M^o} * dM^o + \frac{\partial X^r}{\partial P_x} * dP_x + \frac{\partial X^r}{\partial P_z} * dP_z + \frac{\partial X^r}{\partial M^r} * dM^r$$

Dividing by X we get:

$$\Rightarrow \quad \frac{dX}{X} = \frac{X^o}{P_x * X} * E^o_{xx} * dP_x \; + \frac{X^o}{P_y * X} * E^o_{xy} * dP_y \; + \frac{X^o}{P_z * X} * E^o_{xz} * dP_z \; + \frac{X^r}{P_x * X} * E^r_{xx} * dP_x$$

$$+ \quad \frac{X^r}{P_z * X} * E^r_{xz} * dP_z \; + \; (\frac{1}{X})*[\frac{\partial X^o}{\partial M^o} * dM^o \; + \frac{\partial X^r}{\partial M^r} * dM^r \; ]$$

Remembering that all prices are = 1 to begin with, and that $dP_z = 0$ we can write the above as:

$$\Rightarrow \quad \frac{dX}{X} = [\frac{X^o}{X} * E^o_{xx} \; + \; \frac{X^r}{X} * E^r_{xx}] \; * dP_x \; + [\frac{X^o}{X} * E^o_{xy} \; ]* dP_y \; +$$

$$(\frac{1}{X})*[\frac{\partial X^o}{\partial M^o} * dM^o \; + \frac{\partial X^r}{\partial M^r} * dM^r \,] \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(D1)$$

SECTOR Y

Since Y is non-tradable, the demand curve consists of demand only from the OECD country.

$Y = Y^o(P_x, P_y, P_z, M^o)$

$$\frac{dY}{Y} \; = \; E_{yx} *dP_x \; + E_{yy} *dP_y \; + (\frac{1}{Y})*[\frac{\partial Y^o}{\partial M^o} * dM^o] \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(D2)$$

SECTOR Z

$$\frac{dZ}{Z} = [\frac{Z^o}{Z} * E^o_{zx} \; + \; \frac{Z^r}{Z} * E^r_{zx}] \; * dP_x \; + [\frac{Z^o}{Z} * E^o_{zy} \; ]* dP_y \; +$$

$$(\frac{1}{Z})*[\frac{\partial Z^o}{\partial M^o} * dM^o \; + \frac{\partial Z^r}{\partial M^r} * dM^r \,] \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(D3)$$

**SUMMARY OF EQUATIONS FOR THE MODEL SO FAR**

### *Supply*

$$\frac{dX}{X} = \theta_{kx} * \frac{dK_x}{K_x} + \theta_{lx} * \frac{dL_x}{L_x} \qquad \text{.... (S1)}$$

(Where $\theta_{kx}$ is tax inclusive)

$$\frac{dY}{Y} = \theta_{ky} * \frac{dK_y}{K_y} + \theta_{ly} * \frac{dL_y}{L_y} \qquad \text{... (S2)}$$

(Where $\theta_{ky}$is tax inclusive)

$$\frac{dZ}{Z} = \theta_{kz} * \frac{dK_z}{K_z} \qquad \text{... (S3)}$$

### *Price Formation*

$$dP_x = \theta_{kx} * dP_k + \theta_{kx} * \frac{dT_{ko}}{(1+T_{ko})} + \theta_{lx} * dP_{lo} \qquad \text{.... (P1)}$$

(Where $\theta_{kx}$ is tax inclusive)

$$dP_y = \theta_{ky} * dP_k + \theta_{ky} * \frac{dT_{ko}}{(1+T_{ko})} + \theta_{ly} * dP_{lo} \qquad \text{... (P2)}$$

(Where $\theta_{ky}$is tax inclusive)

$$dP_z = \left(\frac{1}{b}\right) * \theta_{kz} * dP_k + \left(\frac{1}{b} - 1\right) * \theta_{kz} * \frac{dK_z}{K_z} \qquad \text{... (P3)}$$

### *Substitution*

$$\frac{dK_x}{K_x} - \frac{dL_x}{L_x} = S_x * \left(dP_k + \frac{dT_{ko}}{(1+T_{ko})} - dP_{lo}\right) \qquad \text{... (U1)}$$

$$\frac{dK_y}{K_y} - \frac{dL_y}{L_y} = S_y * \left(dP_k + \frac{dT_{ko}}{(1+T_{ko})} - dP_{lo}\right) \qquad \text{... (U2)}$$

### *Demand*

$$\frac{dX}{X} = [\frac{X^o}{X} * E^o_{xx} + \frac{X^r}{X} * E^r_{xx}] * dP_x + [\frac{X^o}{X} * E^o_{xy}]* dP_y$$

$$+ (\frac{1}{X})*[\frac{\partial X^o}{\partial M^o} * dM^o + \frac{\partial X^r}{\partial M^r} * dM^r] \quad \text{………… (D1)}$$

$$\frac{dY}{Y} = E_{yx} *dP_x + E_{yy} *dP_y + (\frac{1}{Y})*[\frac{\partial Y^o}{\partial M^o} * dM^o] \quad \text{…………(D2)}$$

$$\frac{dZ}{Z} = [\frac{Z^o}{Z} * E^o_{zx} + \frac{Z^r}{Z} * E^r_{zx}] * dP_x + [\frac{Z^o}{Z} * E^o_{zy}]* dP_y$$

$$+ (\frac{1}{Z})*[\frac{\partial Z^o}{\partial M^o} * dM^o + \frac{\partial Z^r}{\partial M^r} * dM^r] \quad \text{………… (D3)}$$

Where elasticities are Marshallian and income effects $M^o$ and $M^r$ is disposable income.

### *Adding up*

$$dK_x + dK_y + dK_z = 0 \quad \text{………….(A1)}$$

$$dL_x + dL_y = 0 \quad \text{………….(A2)}$$

$$dL_z = 0 \quad \text{………….(A3)}$$

$dP_z = 0$ [Numeraire]

$P_k$, $P_{lo}$ and $P_{lr}$ (factor prices) and $P_x$, $P_y$ and $P_z$ (output prices) are all equal to 1 and to each other to begin with

### *Other relationships*

$$d(P_k*(1+T_{ko})) = dP_k + T_{ko}*dP_k + P_k*dT_{ko} = dP_k + T_{ko}*dP_k + dT_{ko} \quad \text{…..(O1)}$$

$$dM^o = P_k*(1 + T_{ko})*K_x*\frac{dK_x}{K_x} + P_k*(1 + T_{ko})*K_y*\frac{dK_y}{K_y} +$$

$(K_x + K_y)*dP_k *(1 + T_{ko}) + (K_x + K_y)* dT_{ko} + (L_x + L_y)*dP_{lo}$

$dM^r = - K_x*\frac{dK_x}{K_x} - K_y*\frac{dK_y}{K_y}$

$\overline{L_z} *P_{lr} = (\frac{1}{b})* [P_k* K_z ]- P_k*K_z$

$P_{lr} = (\frac{1}{b} - 1)* [P_k* K_z ]* (\frac{1}{\overline{L_z}})$

$dP_{lr} = (\frac{1}{b} - 1)* [dP_k* K_z + P_k* dK_z ]* (\frac{1}{\overline{L_z}})$

Or $dP_{lr} = (1 - b)* [P_z*dZ]* (\frac{1}{\overline{L_z}})$ since $dP_z = 0$

$(\frac{1}{b})* \theta_{kz} *dP_k + (\frac{1-b}{b})* \theta_{kz} * \frac{dK_z}{K_z} = 0$

$dP_k = (b - 1 ) * \frac{dK_z}{K_z}$

**SOLUTION PROCEDURES**

Using $dK_x + dK_y + dK_z =0$ …………(A1) we can write:

$\frac{dK_z}{K_z} = - (\frac{K_x}{K_z})*\frac{dK_x}{K_x} - (\frac{K_y}{K_z})*\frac{dK_y}{K_y}$

And using $dL_x + dL_y = 0$ …………(A2) we can write:

$\frac{dL_x}{L_x} = - (\frac{L_y}{L_x})*\frac{dL_y}{L_y}$

Then we equate the demand and supply equations for sector X and Y, remembering that due to Walras' Law, the market for Z is in equilibrium when the first two are balanced. Now the substitution equations are rewritten as:

$$\frac{dK_x}{K_x} - \frac{dL_x}{L_x} = S_x * (dP_k + \frac{dT_{ko}}{(1+T_{ko})} - dP_{lo}) \qquad \dots \text{(U1)}$$

$$\Rightarrow \quad \frac{dK_x}{K_x} + (\frac{L_y}{L_x})*\frac{dL_y}{L_y} = S_x *(b - 1) * \frac{dK_z}{K_z} + S_x * \frac{dT_{ko}}{(1+T_{ko})} - S_x *dP_{lo}$$

$$\Rightarrow \quad \frac{dK_x}{K_x} + (\frac{L_y}{L_x})*\frac{dL_y}{L_y} = S_x *(b - 1)*\{ - (\frac{K_x}{K_z})*\frac{dK_x}{K_x} - (\frac{K_y}{K_z})*\frac{dK_y}{K_y} \}+ S_x * \frac{dT_{ko}}{(1+T_{ko})} - S_x *dP_{lo}$$

$$\frac{dK_y}{K_y} - \frac{dL_y}{L_y} = S_y * (dP_k + \frac{dT_{ko}}{(1+T_{ko})} - dP_{lo}) \qquad \dots \text{(U2)}$$

$$\Rightarrow \quad \frac{dK_y}{K_y} - \frac{dL_y}{L_y} = S_y *(b - 1)*\{ - (\frac{K_x}{K_z})*\frac{dK_x}{K_x} - (\frac{K_y}{K_z})*\frac{dK_y}{K_y} \} + S_y* \frac{dT_{ko}}{(1+T_{ko})} - S_y*dP_{lo}$$

Next we equate the supply and demand for X, and remembering that $dP_z = 0$ and

$$\frac{dL_x}{L_x} = - (\frac{L_y}{L_x})*\frac{dL_y}{L_y} :$$

$$\theta_{kx} *\frac{dK_x}{K_x} - \theta_{lx} * (\frac{L_y}{L_x})*\frac{dL_y}{L_y} = [ \frac{X^o}{X} * E^o_{xx} + \frac{X^r}{X} * E^r_{xx}] * dP_x + [ \frac{X^o}{X} * E^o_{xy} ]* dP_y +$$

$$(\frac{1}{X})*[ \frac{\partial X^o}{\partial M^o} * dM^o + \frac{\partial X^r}{\partial M^r} * dM^r ]$$

Equating demand and supply for Y:

$$\theta_{ky} *\frac{dK_y}{K_y} + \theta_{ly} *\frac{dL_y}{L_y} = E_{yx} *dP_x + E_{yy} *dP_y + (\frac{1}{Y})*[ \frac{\partial Y^o}{\partial M^o} * dM^o ]$$

We now have 4 equations in the following unknowns: $\frac{dK_y}{K_y}$ , $\frac{dL_y}{L_y}$ ,$dP_x$ , $dP_y$ , $dM^o$, $\frac{dK_x}{K_x}$ , $dM^r$, and $dP_{lo}$ . We can substitute for some variables using:

$$dP_x = \theta_{kx} *(b - 1 )*\{ - (\frac{K_x}{K_z})*\frac{dK_x}{K_x} - (\frac{K_y}{K_z})*\frac{dK_y}{K_y} \}+ \theta_{kx} * \frac{dT_{ko}}{(1+T_{ko})} + \theta_{lx} * dP_{lo}$$

(Where $\theta_{kx}$ is tax inclusive)

$$dP_y = \theta_{ky} *(b - 1 )*\{ - (\frac{K_x}{K_z})*\frac{dK_x}{K_x} - (\frac{K_y}{K_z})*\frac{dK_y}{K_y} \} + \theta_{ky} * \frac{dT_{ko}}{(1+T_{ko})} + \theta_{ly} * dP_{lo}$$

(Where $\theta_{ky}$is tax inclusive)

$$dM^o = P_k*(1 + T_{ko})*K_x*\frac{dK_x}{K_x} + P_k*(1 + T_{ko})*K_y*\frac{dK_y}{K_y} +$$

$$(K_x + K_y) *(b - 1 )*\{ - (\frac{K_x}{K_z})*\frac{dK_x}{K_x} - (\frac{K_y}{K_z})*\frac{dK_y}{K_y} \}*(1 + T_{ko}) + (K_x + K_y)* dT_{ko} +$$

$$(L_x + L_y)*dP_{lo}$$

$$dM^r = - K_x*\frac{dK_x}{K_x} - K_y*\frac{dK_y}{K_y}$$

And we are left with 4 equations in 4 variables, namely: $\frac{dK_y}{K_y}$ , $\frac{dL_y}{L_y}$ , $\frac{dK_x}{K_x}$ and $dP_{lo}$

Next we use symbolic notation for some of the parameters involved:

$$Єxx = [ \frac{X^o}{X} * E^o_{xx} + \frac{X^r}{X} * E^r_{xx}] \qquad Єxy = [ \frac{X^o}{X} * E^o_{xy}]$$

We can write for symmetry since $Y^o = Y$:

$$Єyx = E_{yx} \qquad \text{and} \qquad Єyy = E_{yy}$$

**REDUCED FORM VERSION OF THE MODEL**

$$[1+ S_x*(b - 1)*(\frac{K_x}{K_z})]* \frac{dK_x}{K_x} + S_x*(b - 1)*(\frac{K_y}{K_z})*\frac{dK_y}{K_y} + (\frac{L_y}{L_x})*\frac{dL_y}{L_y} + S_x*dP_{lo} = S_x * \frac{dT_{ko}}{(1+T_{ko})}$$

…..(1)

$$S_y*(b - 1)*(\frac{K_x}{K_z})*\frac{dK_x}{K_x} + [1+ S_y*(b - 1)*(\frac{K_y}{K_z})]* \frac{dK_y}{K_y} - \frac{dL_y}{L_y} + S_y*dP_{lo} = S_y* \frac{dT_{ko}}{(1+T_{ko})}$$

…..(2)

$$\{\theta_{kx} + [\theta_{kx}* Єxx + \theta_{ky}* Єxy]*(b-1)*\left(\frac{K_x}{K_z}\right) - \theta_{kx}*\frac{\partial X^o}{\partial M^o} + \frac{\theta_{kx}}{(1+T_{ko})}*\frac{\partial X^r}{\partial M^r} +$$

$$[\theta_{kx} + \frac{K_y}{X}*(1 + T_{ko})]*(b-1)*(\frac{K_x}{K_z})*\frac{\partial X^o}{\partial M^o}\}*\frac{dK_x}{K_x} +$$

$$\{[\theta_{kx}*Єxx + \theta_{ky}*Єxy]*(b-1)*\left(\frac{K_y}{K_z}\right) - \frac{K_y}{X}*(1 + T_{ko})*\frac{\partial X^o}{\partial M^o} + \frac{K_y}{X}*\frac{\partial X^r}{\partial M^r} +$$

$$[\theta_{kx} + \frac{K_y}{X}*(1 + T_{ko})]*(b-1)*(\frac{K_y}{K_z})*\frac{\partial X^o}{\partial M^o}\}*\frac{dK_y}{K_y}$$

$$- \theta_{lx}*(\frac{L_y}{L_x})*\frac{dL_y}{L_y} - \{Єxx*\theta_{lx} + Єxy*\theta_{ly} + \frac{(Lx + Ly)}{X}*\frac{\partial X^o}{\partial M^o}\}*dP_{lo} =$$

$$\{\theta_{kx}*Єxx + \theta_{ky}*Єxy + [\theta_{kx}+ \frac{K_y}{X}*(1 + T_{ko})]*\frac{\partial X^o}{\partial M^o}\}*\frac{dT_{ko}}{(1+T_{ko})} \qquad ………… \qquad (3)$$

$$\{[\theta_{kx}*Єyx + \theta_{ky}*Єyy]*(b-1)*\left(\frac{K_x}{K_z}\right) - \frac{K_x}{Y}*(1 + T_{ko})*\frac{\partial Y^o}{\partial M^o} +$$

$$[\theta_{ky} + \frac{K_x}{Y}*(1 + T_{ko})]*(b-1)*(\frac{K_x}{K_z})*\frac{\partial Y^o}{\partial M^o}\}*\frac{dK_x}{K_x} +$$

$$\{\theta_{ky} + [\theta_{kx}*Єyx + \theta_{ky}*Єyy]*(b-1)*\left(\frac{K_y}{K_z}\right) - \theta_{ky}*\frac{\partial Y^o}{\partial M^o} +$$

$[\theta_{ky} + \frac{K_x}{Y} *(1 + T_{ko})]*(b-1)*(\frac{K_y}{K_z})* \frac{\partial Y^o}{\partial M^o} \}* \frac{dK_y}{K_y}$

$+ \ \theta_{ly}*\frac{dL_y}{L_y} - \{ Єyx* \theta_{lx} + Єyy* \theta_{ly} + \frac{(Lx + Ly)}{Y} *\frac{\partial Y^o}{\partial M^o} \}* dP_{lo} =$

$\{ \theta_{kx}*Єyx + \theta_{ky}*Єyy + [\theta_{ky} + \frac{K_x}{Y} *(1 + T_{ko})]* \frac{\partial Y^o}{\partial M^o}\}*\frac{dT_{ko}}{(1+T_{ko})}$ ………… (4)

These, as before are 4 equations in 4 variables. If variables are denoted in the following order by subscript j = 1, ..., 4; $\frac{dK_x}{K_x}$, $\frac{dK_y}{K_y}$ , $\frac{dL_y}{L_y}$, $dP_{lo}$ and subscript i = 1, ..., 4 represents the equation, the 4 equations above can be written in symbolic form with $a_{ij}$ representing coefficients attached to the left hand side variables and $b_i$ the constants on the right hand side.

$A_{11}*\frac{dK_x}{K_x} + A_{12}*\frac{dK_y}{K_y} + A_{13} *\frac{dL_y}{L_y} + A_{14}*dP_{lo} = B_1$ ………… (1)

$A_{21}*\frac{dK_x}{K_x} + A_{22}* \frac{dK_y}{K_y} + A_{23} * \frac{dL_y}{L_y} + A_{24}* dP_{lo} = B_2$ ………… (2)

$A_{31}*\frac{dK_x}{K_x} +A_{32}*\frac{dK_y}{K_y} + A_{33}* \frac{dL_y}{L_y} + A_{34}* dP_{lo} = B_3$ …………. (3)

$A_{41}* \frac{dK_x}{K_x} + A_{42}*\frac{dK_y}{K_y} + A_{43}*\frac{dL_y}{L_y} + A_{44}*dP_{lo} = B_4$ ………… (4)

Where

$A_{11}= [1+ S_x*(b - 1 )*(\frac{K_x}{K_z})]$ $A_{12} = S_x *(b - 1 )*(\frac{K_y}{K_z})$ $A_{13} = (\frac{L_y}{L_x})$ $A_{14} = S_x$

$B_1 = S_x * \frac{dT_{ko}}{(1+T_{ko})}$

$A_{21} = S_y *(b - 1)*(\frac{K_x}{K_z})$ $A_{22} = [1+ S_y*(b - 1)*(\frac{K_y}{K_z})]$ $A_{23} = -1$ $A_{24} = S_y$

$B_2 = S_y* \frac{dT_{ko}}{(1+T_{ko})}$

$A_{31} = \{\theta_{kx} + [\theta_{kx}* Єxx + \theta_{ky}* Єxy]*(b-1)*\left(\frac{K_x}{K_z}\right) - \theta_{kx}*\frac{\partial X^o}{\partial M^o} + \frac{\theta_{kx}}{(1+T_{ko})}*\frac{\partial X^r}{\partial M^r} +$

$[\theta_{kx} + \frac{K_y}{X}*(1 + T_{ko})]*(b-1)*(\frac{K_x}{K_z})*\frac{\partial X^o}{\partial M^o}\}$

$A_{32} = \{[\theta_{kx}*Єxx + \theta_{ky}*Єxy]*(b-1)*\left(\frac{K_y}{K_z}\right) - \frac{K_y}{X}*(1 + T_{ko})*\frac{\partial X^o}{\partial M^o} + \frac{K_y}{X}*\frac{\partial X^r}{\partial M^r} +$

$[\theta_{kx} + \frac{K_y}{X}*(1 + T_{ko})]*(b-1)*(\frac{K_y}{K_z})*\frac{\partial X^o}{\partial M^o}\}$

$A_{33} = - \theta_{lx} * (\frac{L_y}{L_x})$ $A_{34} = - \{ Єxx* \theta_{lx} + Єxy* \theta_{ly} + \frac{(Lx + Ly)}{X}*\frac{\partial X^o}{\partial M^o}\}$

$B_3 = \{ \theta_{kx}*Єxx + \theta_{ky}*Єxy + [\theta_{kx}+ \frac{K_y}{X}*(1 + T_{ko})]*\frac{\partial X^o}{\partial M^o}\}*\frac{dT_{ko}}{(1+T_{ko})}$

$A_{41} = \{ [\theta_{kx}*Єyx + \theta_{ky}*Єyy]*(b-1)*\left(\frac{K_x}{K_z}\right) - \frac{K_x}{Y}*(1 + T_{ko})*\frac{\partial Y^o}{\partial M^o} +$

$[\theta_{ky} + \frac{K_x}{Y}*(1 + T_{ko})]*(b-1)*(\frac{K_x}{K_z})*\frac{\partial Y^o}{\partial M^o}\}$

$A_{42} = \{ \theta_{ky} + [\theta_{kx}*Єyx + \theta_{ky}*Єyy]*(b-1)*\left(\frac{K_y}{K_z}\right) - \theta_{ky}*\frac{\partial Y^o}{\partial M^o} +$

$[\theta_{ky} + \frac{K_x}{Y}*(1 + T_{ko})]*(b-1)*(\frac{K_y}{K_z})*\frac{\partial Y^o}{\partial M^o}\}$

$A_{43} = \theta_{ly}$ $A_{44} = - \{ Єyx* \theta_{lx} + Єyy* \theta_{ly} + \frac{(Lx + Ly)}{Y}*\frac{\partial Y^o}{\partial M^o}\}$

$B_4 = \{ \theta_{kx}*Єyx + \theta_{ky}*Єyy + [\theta_{ky} + \frac{K_x}{Y} *(1 + T_{ko})]* \frac{\partial Y^o}{\partial M^o}\}*\frac{dT_{ko}}{(1+T_{ko})}$

**CHANGES IN GDP: SOURCES AND USES SIDES**

**Sources side:**

$GDP^o$ (OECD) $= P_k*(1 + T_{ko})*(K_x + K_y) + P_{lo}*(L_x + L_y)$

⇨ $GDP^o = P_k* K_x + P_k*T_{ko}*K_x + P_k*K_y + P_k*T_{ko}*K_y + P_{lo}*(L_x + L_y)$

$dGDP^o = P_k* dK_x + dP_k* K_x + P_k*T_{ko}*dK_x + P_k*dT_{ko}*K_x + dP_k*T_{ko}*K_x + P_k*dK_y +$

$dP_k*K_y + P_k*T_{ko}*dK_y + P_k*dT_{ko}*K_y + dP_k*T_{ko}*K_y + dP_{lo}*(L_x + L_y) + P_{lo}*d(L_x + L_y)$

But $d(L_x + L_y)= 0$, so we ignore the last term:

$dGDP^o = P_k* dK_x + dP_k* K_x + P_k*T_{ko}*dK_x + P_k*dT_{ko}*K_x + dP_k*T_{ko}*K_x + P_k*dK_y +$

$dP_k*K_y + P_k*T_{ko}*dK_y + P_k*dT_{ko}*K_y + dP_k*T_{ko}*K_y + dP_{lo}*(L_x + L_y)$

$dGDP^o = P_k* K_x*\frac{dK_x}{K_x} + dP_k* K_x + P_k*T_{ko}*K_x*\frac{dK_x}{K_x} + P_k*dT_{ko}*K_x + dP_k*T_{ko}*K_x +$

$P_k*K_y*\frac{dK_y}{K_y}+ dP_k*K_y + P_k*T_{ko}*K_y* \frac{dK_y}{K_y} + P_k*dT_{ko}*K_y + dP_k*T_{ko}*K_y + dP_{lo}*(L_x + L_y)$

$dGDP^o = dM^o = P_k*(1 + T_{ko})*K_x*\frac{dK_x}{K_x} + P_k*(1 + T_{ko})*K_y*\frac{dK_y}{K_y} +$

$(K_x + K_y) *(b - 1 )*\{ - (\frac{K_x}{K_z})*\frac{dK_x}{K_x} - (\frac{K_y}{K_z})*\frac{dK_y}{K_y} \}*(1 + T_{ko}) + (K_x + K_y)* dT_{ko} +$

$(L_x + L_y)*dP_{lo}$

$GDP^r = M^r = P_k * K_z + P_{lr} * L_z$

$GDP^r = M^r = (\frac{1}{b}) * [P_k * K_z]$

$dM^r = (\frac{1}{b}) * [K_z * dP_k + P_k * dK_z]$

But $dP_k = (b - 1) * \frac{dK_z}{K_z}$

$\Rightarrow dM^r = (\frac{1}{b}) * [K_z * (b - 1) * \frac{dK_z}{K_z} + P_k * dK_z]$

$\Rightarrow dM^r = (\frac{1}{b}) * [(b - 1) * dK_z + dK_z]$

$\Rightarrow dM^r = dK_z$

$\Rightarrow dGDP^r = dM^r = - K_x * \frac{dK_x}{K_x} - K_y * \frac{dK_y}{K_y}$

**USES SIDE:**

We can use a Laspeyres' index defined as the change in the cost of purchasing the base year's consumption bundle. For the OECD country, the index is defined as:

$$LAS^o = \frac{(P_x + dP_x) * X^o + (P_y + dP_y) * Y^o + (P_z + dP_z) * Z^o}{P_x * X^o + P_y * Y^o + P_z * Z^o}$$

Similarly, for ROW we can estimate:

$$LAS^r = \frac{(P_x + dP_x) * X^r + (P_z + dP_z) * Z^r}{P_x * X^r + P_z * Z^r}$$

Next we know that $dP_z = 0$ and initial prices are equal to 1, this gives:

$$LAS^o = \frac{dP_x{*}\,X^o + dP_y{*}\,Y^o + Z^o + X^o + Y^o}{X^o + Y^o + Z^o} = 1 + \frac{dP_x{*}\,X^o + dP_y{*}\,Y^o}{X^o + Y^o + Z^o}$$

Similarly, we have:

$$LAS^r = \frac{X^r + dP_x{*}\,X^r + Z^r}{X^r + Z^r} = 1 + \frac{dP_x{*}\,X^r}{X^r + Z^r}$$

We also have to find a way to combine the total welfare effect of the change in income (GDP) and the change in the uses side or cost-of-living index. This can be done in a number of ways; one very logical way is available in Randolph (2006). The method proposed here is more basic. It starts with the assumption that the social welfare function is a weighted function of proportionate change in income and the proportional change in the cost of living, with equal weights. This would imply that we could write OECD welfare as:

$W^o = 1 + dGDP^o/GDP^o - LAS^o$ and ROW welfare $W^r = 1 + dGDP^r/GDP^r - LAS^r$

This is a pure assumption, and we could have chosen any form for the welfare function, such as $W^o = \frac{1+\frac{dGDP^o}{GDP^o}}{L^o}$ instead or more complicated forms.

## THE CONSUMPTION TAX CASE WITH TRANSFERS

## GOVERNMENT

Since there are no savings and capital accumulation in this model, Total consumption = Total GDP in OECD and ROW separately. Total OECD expenditure /consumption =

$$P_x *(1+T_{Co})* X^o + P_y*(1+T_{Co}) * Y^o + P_z *(1+T_{Co}) * Z^o$$

From the income side, we have: OECD GDP = $(1+T_{Co})* [P_k*(K_x + K_y) + P_{lo}*(L_x + L_y)]$ and this = $P_x *(1+T_{Co})* X^o + P_y*(1+T_{Co}) * Y^o + P_z *(1+T_{Co}) * Z^o$

In this case we have assumed that all revenue collected is returned lump-sum to the single OECD consumer. Thus, whatever is paid as tax is returned, and no government good exists, so the production and price formation equations are functions of capital, tax and labor alone. However, since the tax is returned to the consumer and not spent on G, the effect has to be as if the income of the consumer in OECD went up by the amount of the tax. This is not clearly reflected when we consider GDP in terms of $P_k*(K_x + K_y) + P_{lo}*(L_x + L_y)$, since GDP has to equal factor payments in either case to maintain parity between aggregate production and demand.

To keep this case comparable with chapter three, we now can assume that even if total product was returned as factor payments to the extent of $P_x * X^o + P_y* Y^o + P_z * Z^o$ since $X^r = Z^o$ due to balanced trade; all the tax revenue collected is returned as well and is available for spending. Since we have a proportionate ad valorem tax on all consumption in the OECD, prices of all goods changed proportionately and there was no change in relative prices. There are no savings and the tax was returned, so the only effect

could be the change in GDP through the change in excess burden. The only price differences were the wedge between consumer and producer prices, and the differences in prices faced by the OECD and ROW consumer.

When all tax revenue collected from the OECD consumer is returned to him, it is as if the OECD consumer spends $P_x *(1+T_{Co})* X^o + P_y*(1+T_{Co}) * Y^o + P_z *(1+T_{Co}) * Z^o$ and then gets back $G = T_{Co}* P_x * X^o + T_{Co}* P_y * Y^o + T_{Co}* P_z * Z^o$ which would have gone to finance the government good. Thus, his total expenditure on the same quantities purchased as in chapter three is $P_x * X^o + P_y * Y^o + P_z * Z^o$ but he has unspent income equal to tax revenue returned. There are no savings and no leisure, and no untaxed consumption. He has to spend the returned income on the same goods. When demand curves are HOD Zero in prices and income, the effect on quantities purchased in the limit is the same as if he were facing the lower prices $P_x$, $P_y$ and $P_z$ instead of $P_x *(1+T_{Co})$, $P_y*(1+T_{Co})$ and $P_z *(1+T_{Co})$ .

When the amount of the tax is returned to the same consumer, he spends it as is he had received income equal to $T_{Co}* P_x * X^o + T_{Co}* P_y * Y^o + T_{Co}* P_z * Z^o$ . Tax revenue out of this income is $T_{Co} * [T_{Co}* P_x * X^o + T_{Co}* P_y * Y^o + T_{Co}* P_z * Z^o]$. Since $T_{Co} < 1$, the amount of tax revenue in successive rounds keeps decreasing until it converges to zero. Then the entire spending is on goods, and the effect is the same as the consumer facing prices of $P_x$, $P_y$ and $P_z$ . Since there is no change in relative prices in either case and there are no savings, (i.e. $\frac{P_i *(1+T_{Co})}{P_j *(1+T_{Co})} = \frac{P_i}{P_j}$ ), the effect of returning the tax lump-sum is the same as reducing the prices faced by the same proportion.

However, as before we have note that we have avoided placing the perfectly reasonable restrictions on utility functions that guarantee that demands are HOD Zero in prices and income. This simply means that we do not rule out beforehand that demand curves do not incorporate “money illusion,” or other effects (Veblen, “Bandwagon” or “Snob” effects). Since the demand curves are not demand curves for single goods but consolidated ones, we minimize the set of restrictions in our model in the Harberger spirit by not placing this restriction in the first instance although the curves do have to satisfy the Walrasian consistency conditions for a system of general equilibrium. This also allows for the empirical possibility that the consolidated demand curves may have a set of elasticities that are not consistent with HOD Zero.

**PROFIT MAXIMIZATION BEHAVIOR BY FIRMS (SECTORS)**

The production functions for each sector can be written as a function of productive resources:

$X = f(K_x, L_x)$ $\qquad$ $Y = g(K_y, L_y)$ $\qquad$ $Z = h(K_z, \overline{L_z})$

The profit maximization problem can then be set up as:

FIRM (SECTOR) X

Maximize Π (profit) = $\{ P_x * f(K_x, L_x) - P_k * K_x - P_{lo} * L_x \}$ with respect to $K_x$ and $L_x$

First order conditions:

1. $P_x * \frac{\partial f}{\partial K_x} - P_k = 0$

And

2. $P_x * \frac{\partial f}{\partial L_x}$ - $P_{lo}$ = 0

Since $P_x = 1$, this implies that $\frac{\partial f}{\partial K_x} = P_k$ and $\frac{\partial f}{\partial L_x} = P_{lo}$

Each firm pays each factor its marginal product. The demand price includes the tax, but since the tax is assumed to go to the government and is then refunded to the single consumer, it does not go to the firm. The tax rate is independent of the firm's actions, so it does not enter the objective function as it is not a part of the firm's revenue.

FIRM (SECTOR) Y

Behaves in exactly the same way as X, and so we get $\frac{\partial g}{\partial K_y} = P_k$ and $\frac{\partial g}{\partial L_y} = P_{lo}$

FIRM (SECTOR) Z

Since there is no tax in the ROW we have only the production function

Z = h ($K_z$, $\overline{L_z}$ ) and the problem for firm Z:

Maximize Π (profit) = { $P_z$* h ($K_z$ , $\overline{L_z}$ ) - $P_k$*$K_z$ – $P_{lr}$*$\overline{L_z}$ } with respect to $K_z$ { since we assume full employment always in both countries, choosing $L_z$ is not a decision variable } and we get $\frac{\partial h}{\partial K_z} = P_k$

**SUBSTITUTION**

The elasticity of substitution is (with CRS and competition) defined as: $S_x = \frac{\frac{d(\frac{K_x}{L_x})}{\frac{K_x}{L_x}}}{\frac{d(\frac{P_k}{P_{lo}})}{\frac{P_k}{P_{lo}}}}$

This can be rewritten as:

$$\frac{d(\frac{K_x}{L_x})}{\frac{K_x}{L_x}} = S_x * \frac{d(\frac{P_k}{P_{lo}})}{\frac{P_k}{P_{lo}}}$$

SECTOR X

$$\frac{dK_x}{K_x} - \frac{dL_x}{L_x} = S_x * (dP_k - dP_{lo}) \quad \text{.......................................(U1)}$$

SECTOR Y

$$\frac{dK_y}{K_y} - \frac{dL_y}{L_y} = S_y * (dP_k - dP_{lo}) \quad \text{......................................(U2)}$$

SECTOR Z

There is no substitution equation in sector Z.

**BALANCING**

$dK_x + dK_y + dK_z = 0$

$dL_x + dL_y = 0$

$dL_z = 0$

$dP_z = 0$ [Numeraire]

$P_k$, $P_{lo}$ and $P_{lr}$ (factor prices) and $P_x$, $P_y$ and $P_z$ (output supply prices) are all equal to 1 and to each other to begin with

**SUPPLY**

The production functions for the three sectors are:

$X = f\,(K_x\,,\,L_x)$ $\quad\quad$ $Y = g\,(K_y, L_y)$ $\quad\quad$ $Z = h\,(K_z, L_z)$

SECTOR X

$X = f\,(K_x\,,\,L_x)$

The total change in supply (output) or the total differential can be split into:

$$dX = \frac{\partial f}{\partial K_x} * dK_x + \frac{\partial f}{\partial L_x} * dL_x$$

$$\frac{dX}{X} = \frac{P_k * K_x}{P_x * X} * \frac{dK_x}{k_x} + \frac{P_{lo} * L_x}{P_x * X} * \frac{dL_x}{L_x}$$

Writing $\theta_{kx} = \frac{P_k * K_x}{P_x * X}$ the tax-exclusive share of capital's product in sector X

And $\theta_{lx} = \frac{P_{lo} * L_x}{P_x * X}$ the tax-exclusive share of labor's product in sector X, we get

S1…………………..  $\frac{dX}{X} = \theta_{kx} * \frac{dK_x}{K_x} + \theta_{lx} * \frac{dL_x}{L_x}$

Similarly, for SECTOR Y we have:

S2……….............  $\frac{dY}{Y} = \theta_{ky} * \frac{dK_y}{K_y} + \theta_{ly} * \frac{dL_y}{L_y}$

Where $\theta_{ky} = \frac{P_k * K_y}{P_y * Y}$ the tax-exclusive share of capital's product in sector Y

And $\theta_{ly} = \frac{P_{lo} * L_y}{P_y * Y}$ the share of labor's product in sector Y.

SECTOR Z

There is no tax in the ROW, so the relevant terms for this sector:

S3a……….............. $\frac{dZ}{Z} = \theta_{kz} * \frac{dK_z}{K_z} + \frac{1}{Z}*L_z * \frac{\partial h}{\partial L_z} * \frac{dL_z}{L_z}$

But $dL_z = 0$ by assumption ($L_{row} = \overline{L_z}$ is fixed), so we can write the above as:

S3……….............. $\frac{dZ}{Z} = \theta_{kz} * \frac{dK_z}{K_z}$

This gives us our three supply equations. The only difference between this case and the capital income tax is in the shares of capital and labor in the supply equations. Since the producer does not receive or pay the tax, the shares are all tax-exclusive. Since there are no compliance and administrative (collection) costs, we have the full payment to owners of capital and labor being retained by them, and the impact of the tax being confined, if at all to the difference between the price paid by the consumer, and that received by the producer for sales in the OECD alone.

We now turn to price formation. The differences from chapter two are that (1) there is no tax on any factor, and that supply price here means the prices received by producers. (2)The demand price in each case is the producers' price inflated by the ad valorem consumption tax.

**PRICE FORMATION**

SECTOR X

If the production function is CRS and we use Euler's law:

$$\frac{\partial f}{\partial K_x} * K_x + \frac{\partial f}{\partial L_x} * L_x = X$$

$$(1+T_{Co}) * [\frac{\partial f}{\partial K_x} * K_x + \frac{\partial f}{\partial L_x} * L_x] = P_x * (1+T_{Co}) * X \qquad \text{since } P_x = 1$$

$d(P_x*(1+T_{Co})*X) = [(1+T_{Co})*dX + dT_{Co}*X + (1+T_{Co})*X*dP_x] =$

$d\{(1+T_{Co})*[\frac{\partial f}{\partial K_x}*K_x + \frac{\partial f}{\partial L_x}*L_x]\}$

If we then continue with our assumption that firms pay marginal products:

$[(1+T_{Co})*dX + dT_{Co}*X + (1+T_{Co})*X*dP_x] = d\{(1+T_{Co})*[P_k*K_x + P_{lo}*L_x]\}$

$\Rightarrow \quad [(1+T_{Co})*dX + dT_{Co}*X + (1+T_{Co})*X*dP_x] = [dP_k*K_x*(1+T_{Co})* + P_k*dK_x$

$*(1+T_{Co}) + dP_{lo}*L_x*(1+T_{Co}) + P_{lo}*dL_x*(1+T_{Co}) + dT_{Co}*[P_k*K_x + P_{lo}*L_x]]$

From the supply equations and assumptions $P_x = 1$ we know that $P_x*dX = dX$ and

$dX = \frac{\partial f}{\partial K_x}*dK_x + \frac{\partial f}{\partial L_x}*dL_x$

$\Rightarrow \quad P_x*dX = P_k*dK_x + P_{lo}*dL_x$

Substituting in our original equation:

$[(1+T_{Co})*dX + dT_{Co}*X + (1+T_{Co})*X*dP_x] = [dP_k*K_x*(1+T_{Co})* + P_k*dK_x*(1+T_{Co}) +$

$dP_{lo}*L_x*(1+T_{Co}) + P_{lo}*dL_x*(1+T_{Co}) + dT_{Co}*[P_k*K_x + P_{lo}*L_x]]$

$(1+T_{Co})*X*dP_x = [dP_k*K_x*(1+T_{Co})* + P_k*dK_x*(1+T_{Co}) + dP_{lo}*L_x*(1+T_{Co}) +$

$P_{lo}*dL_x*(1+T_{Co}) + dT_{Co}*[P_k*K_x + P_{lo}*L_x]] - (1+T_{Co})*dX - dT_{Co}*X$

$\Rightarrow \quad (1+T_{Co})*X*dP_x = dP_k*K_x*(1+T_{Co})* + P_k*dK_x*(1+T_{Co}) + dP_{lo}*L_x*(1+T_{Co}) +$

$P_{lo}*dL_x*(1+T_{Co}) + dT_{Co}*[P_k*K_x + P_{lo}*L_x] - (1+T_{Co})*[P_k*dK_x + P_{lo}*dL_x] -$

$dT_{Co}*X$

$\Rightarrow \quad X*dP_x = dP_k*K_x + dP_{lo}*L_x$ as long as $[P_k*K_x + P_{lo}*L_x] = X$

Using $P_k = 1$, $P_{lo} = 1$ and $P_x = 1$:

$$dP_x = \frac{P_k * K_x}{P_x * X} * dP_k + \frac{P_{lo} * L_x}{P_x * X} * dP_{lo}$$

$\Rightarrow$ $dP_x = \theta_{kx} * dP_k + \theta_{lx} * dP_{lo}$ …………………(P1)

Similarly for SECTOR Y

$\Rightarrow$ $dP_y = \theta_{ky} * dP_k + \theta_{ly} * dP_{lo}$ …………………(P2)

SECTOR Z

There is no tax here. Suppose that total payments to capital in Z are a constant proportion of the product. The residual is paid to labor. If b represents the baseline payments proportion to capital in Z, and if b is assumed to be a constant, then:

$b * P_z * Z = P_k * K_z$ : b<1and b is constant

$[P_z * dZ + Z * dP_z] = (\frac{1}{b}) * [dP_k * K_z + P_k * dK_z]$

Now from the supply equation for Z we know that: $P_z * dZ = dZ$ ( since $P_z = 1$) $= P_k * dK_z$

$\Rightarrow Z * dP_z = (\frac{1}{b}) * [dP_k * K_z + P_k * dK_z] - P_z * dZ$

$\Rightarrow Z * dP_z = (\frac{1}{b}) * [dP_k * K_z + P_k * dK_z] - P_k * dK_z$

$\Rightarrow dP_z = (\frac{1}{b}) * [\theta_{kz} * dP_k + \theta_{kz} * \frac{dK_z}{K_z}] - \theta_{kz} * \frac{dK_z}{K_z}$

(Since $P_k$ and $P_z = 1$, and $\theta_{kz}$ is tax-exclusive)

$\Rightarrow dP_z = (\frac{1}{b}) * \theta_{kz} * dP_k + (\frac{1}{b} - 1) * \theta_{kz} * \frac{dK_z}{K_z}$

Now since we have chosen $P_z$ as our numeraire ( $dP_z = 0$), we get a relationship between $dP_k$ and $\frac{dK_z}{K_z}$ and ultimately between $dP_k$ and $\frac{dK_z}{K_z} = -(\frac{K_x}{K_z})*\frac{dK_x}{K_x} - (\frac{K_y}{K_z})*\frac{dK_y}{K_y}$

$$0 = (\frac{1}{b})* \theta_{kz} *dP_k + (\frac{1-b}{b})* \theta_{kz} * \frac{dK_z}{K_z}$$

$$dP_k = (b - 1) * \frac{dK_z}{K_z} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(P3)$$

From these expressions we can see that $P_{lr}$ and $dP_{lr}$ can be derived in terms of either total product $P_z*Z$, a combination of $P_z*Z$ and $P_k*K_z$ or $P_k*K_z$ alone:

$$\overline{L_z} *P_{lr} = ( 1 – b )* P_z*Z$$

Or $\overline{L_z} *P_{lr} = P_z*Z - P_k*K_z$

Or $\overline{L_z} *P_{lr} = (\frac{1}{b})* [P_k* K_z ]- P_k*K_z$

$\Rightarrow P_{lr} = (\frac{1}{b} - 1)* [P_k* K_z ]* (\frac{1}{\overline{L_z}})$

$\Rightarrow dP_{lr} = (\frac{1}{b} - 1)* [dP_k* K_z + P_k* dK_z ]* (\frac{1}{\overline{L_z}})$

Or $dP_{lr} = (1 – b)* [P_z*dZ]* (\frac{1}{\overline{L_z}})$ since $dP_z = 0$

**DEMAND**

SECTOR X

$$X = X^o((1+T_{Co})* P_x , (1+T_{Co})* P_y, (1+T_{Co})* P_z , M^o) + X^r(P_x, P_z, M^r)$$

$X^o$ represents OECD demand for X. it is a function of all three prices since the OECD consumer consumes the products of all sectors. $M^o$ is OECD consumer's disposable income. This is a Marshallian demand curve. Here, the total factor income is also equal to disposable income, since there are no direct taxes and the transfer equals the tax revenue taken out of total expenditure and not paid to the factors of production. $X^r$ is the ROW demand for X. It is a function of prices of only the two tradable goods since the ROW does not consume Y. $M^r$ is the disposable income of the ROW consumer. Since there are no taxes in the ROW this is equal to GDP. This is also a Marshallian demand curve. Explicit formulations for disposable income follow:

For both $M^o$ and $M^r$ there is no tax on incomes:

$$M^o = (1+T_{Co})* [P_k*(K_x + K_y) + P_{lo}*(L_x + L_y)]$$

$$dM^o = (1+T_{Co})* P_k*(dK_x + dK_y) + (1+T_{Co})* (K_x + K_y)*dP_k + (1+T_{Co})* P_{lo}*(dL_x + dL_y) + (1+T_{Co})* (L_x + L_y)*dP_{lo} + dT_{Co}* [P_k*(K_x + K_y) + P_{lo}*(L_x + L_y)]$$

But $dL_x + dL_y = 0$ by definition, so

$$dM^o = (1+T_{Co})* P_k*(dK_x + dK_y) + (1+T_{Co})* (K_x + K_y)*dP_k + (1+T_{Co})* (L_x + L_y)*dP_{lo} + dT_{Co}* [P_k*(K_x + K_y) + P_{lo}*(L_x + L_y)]$$

$$\Rightarrow dM^o = (1+T_{Co})* P_k*K_x*\frac{dK_x}{K_x} + (1+T_{Co})* P_k*K_y*\frac{dK_y}{K_y} + (1+T_{Co})* (K_x + K_y)*dP_k + (1+T_{Co})* (L_x + L_y)*dP_{lo} + dT_{Co}* [P_k*(K_x + K_y) + P_{lo}*(L_x + L_y)]$$

For $M^r$ there is no tax, so GDP is the same as disposable income:

$$M^r = P_k * K_z + P_{lr} * L_z$$

But $P_{lr} * L_z = \overline{L_z} * P_{lr} = P_z*Z - P_k*K_z = (\frac{1}{b})*[P_k*K_z] - P_k*K_z$

$\Rightarrow M^r = P_k * K_z + (\frac{1}{b})*[P_k*K_z] - P_k*K_z$

$\Rightarrow M^r = (\frac{1}{b})*[P_k*K_z]$

$dM^r = (\frac{1}{b})*[K_z*dP_k + P_k*dK_z]$

But $dP_k = (b-1)*\frac{dK_z}{K_z}$ …………………(P3)

$\Rightarrow dM^r = (\frac{1}{b})*[K_z*(b-1)*\frac{dK_z}{K_z} + P_k*dK_z]$

$\Rightarrow dM^r = (\frac{1}{b})*[(b-1)*dK_z + dK_z]$

$\Rightarrow dM^r = dK_z$

$\Rightarrow dM^r = -K_x*\frac{dK_x}{K_x} - K_y*\frac{dK_y}{K_y}$

Taking the total differential of the demand for X we get:

$$dX = \frac{\partial X^o}{\partial[P_x*(1+T_{Co})]} * d[P_x*(1+T_{Co})] + \frac{\partial X^o}{\partial[P_y*(1+T_{Co})]} * d[P_y*(1+T_{Co})] +$$

$$\frac{\partial X^o}{\partial[P_z*(1+T_{Co})]} * d[P_z*(1+T_{Co})] + \frac{\partial X^o}{\partial M^o} * dM^o + \frac{\partial X^r}{\partial P_x} * dP_x + \frac{\partial X^r}{\partial P_z} * dP_z + \frac{\partial X^r}{\partial M^r} * dM^r$$

Dividing by X we get:

$$\Rightarrow \quad \frac{dX}{X} = \frac{X^o}{P_x*X*(1+T_{Co})} * E^o_{xx} * d[P_x *(1+T_{Co})] + \frac{X^o}{P_y*X*(1+T_{Co})} * E^o_{xy} * d[P_y *(1+T_{Co})] +$$

$$\frac{X^o}{P_z*X*(1+T_{Co})} * E^o_{xz} * d[P_z *(1+T_{Co})] + \frac{X^r}{P_x*X} * E^r_{xx} * dP_x + \frac{X^r}{P_z*X} * E^r_{xz} * dP_z$$

$$+ \left(\frac{1}{X}\right)*\left[\frac{\partial X^o}{\partial M^o} * dM^o + \frac{\partial X^r}{\partial M^r} * dM^r\right]$$

Remembering that all prices are = 1 to begin with and that $d[P_x *(1+T_{Co})]=$

$[(1+T_{Co})* dP_x + P_x *dT_{Co}]$ and that $dP_z =0$ due to the numeraire assumption we can write the above as:

$$\Rightarrow \quad \frac{dX}{X} = \left[\frac{X^o}{X} * E^o_{xx} + \frac{X^r}{X} * E^r_{xx}\right] * dP_x + \left[\frac{X^o}{X} * E^o_{xy}\right]* dP_y + \frac{X^o}{X} * E^o_{xx} * \frac{dT_{Co}}{(1+T_{Co})}$$

$$+ \frac{X^o}{X} * E^o_{xy} * \frac{dT_{Co}}{(1+T_{Co})} + \frac{X^o}{X} * E^o_{xz} * \frac{dT_{Co}}{(1+T_{Co})} + \left(\frac{1}{X}\right)*\left[\frac{\partial X^o}{\partial M^o} * dM^o + \frac{\partial X^r}{\partial M^r} * dM^r\right]$$

…………………………..(D1)

All elasticities for OECD are defined with respect to demand prices as in chapter three, inclusive of the consumption tax. In this case, the elasticities can be the same or equivalent to tax free ones since the rate is very low compared to the price, and the effect of returning tax revenue to the single consumer is the same as him facing tax-free prices. In the ROW there is no tax, and producer and consumer prices are the same, so there is no difference. Also, Y is not consumed in the ROW.

SECTOR Y

Since Y is non-tradable, the demand curve consists of demand only from the OECD country.

$Y = Y^o(P_x*(1+T_{Co}), P_y*(1+T_{Co}), P_z*(1+T_{Co}), M^o)$, and $Y^o = Y$

$$\frac{dY}{Y} = E_{yx}*dP_x + E_{yy}*dP_y + E_{yz}*dP_z + (\frac{1}{Y})*[\frac{\partial Y^o}{\partial M^o}*dM^o] + E_{yx}*\frac{dT_{Co}}{(1+T_{Co})} + E_{yy}*\frac{dT_{Co}}{(1+T_{Co})} + E_{yz}*\frac{dT_{Co}}{(1+T_{Co})} \quad \text{.........................................(D2)}$$

SECTOR Z

$$\frac{dZ}{Z} = [\frac{Z^o}{Z}*E^o_{zx} + \frac{Z^r}{Z}*E^r_{zx}]*dP_x + [\frac{Z^o}{Z}*E^o_{zy}]*dP_y + \frac{Z^o}{Z}*E^o_{zx}*\frac{dT_{Co}}{(1+T_{Co})}$$

$$+\frac{Z^o}{Z}*E^o_{zy}*\frac{dT_{Co}}{(1+T_{Co})} + \frac{Z^o}{Z}*E^o_{zz}*\frac{dT_{Co}}{(1+T_{Co})} + (\frac{1}{Z})*[\frac{\partial Z^o}{\partial M^o}*dM^o + \frac{\partial Z^r}{\partial M^r}*dM^r]$$

$$\text{................................(D3)}$$

**SUMMARY OF EQUATIONS FOR THE MODEL SO FAR**

***Supply***

$$\frac{dX}{X} = \theta_{kx}*\frac{dK_x}{K_x} + \theta_{lx}*\frac{dL_x}{L_x} \quad \text{.... (S1)}$$

$$\frac{dY}{Y} = \theta_{ky}*\frac{dK_y}{K_y} + \theta_{ly}*\frac{dL_y}{L_y} \quad \text{... (S2)}$$

$$\frac{dZ}{Z} = \theta_{kz}*\frac{dK_z}{K_z} \quad \text{... (S3)}$$

***Substitution***

$$\frac{dK_x}{K_x} - \frac{dL_x}{L_x} = S_x*(dP_k - dP_{lo}) \quad \text{... (U1)}$$

$$\frac{dK_y}{K_y} - \frac{dL_y}{L_y} = S_y*(dP_k - dP_{lo}) \quad \text{... (U2)}$$

### *Price Formation*

$$dP_x = \theta_{kx} * dP_k + \theta_{lx} * dP_{lo} \qquad \text{.... (P1)}$$

(Where $\theta_{kx}$ and $\theta_{lx}$ are tax exclusive)

$$dP_y = \theta_{ky} * dP_k + \theta_{ly} * dP_{lo} \qquad \text{... (P2)}$$

(Where $\theta_{ky}$ and $\theta_{ly}$ are tax exclusive)

$$dP_z = (\frac{1}{b}) * \theta_{kz} * dP_k + (\frac{1}{b} - 1) * \theta_{kz} * \frac{dK_z}{K_z} \qquad \text{... (P3)}$$

### *Demand*

$$\frac{dX}{X} = [\frac{X^o}{X} * E^o_{xx} + \frac{X^r}{X} * E^r_{xx}] * dP_x + [\frac{X^o}{X} * E^o_{xy}] * dP_y +$$

$$\frac{X^o}{X} * E^o_{xx} * \frac{dT_{Co}}{(1+T_{Co})} + \frac{X^o}{X} * E^o_{xy} * \frac{dT_{Co}}{(1+T_{Co})} + \frac{X^o}{X} * E^o_{xz} * \frac{dT_{Co}}{(1+T_{Co})} +$$

$$(\frac{1}{X}) * [\frac{\partial X^o}{\partial M^o} * dM^o + \frac{\partial X^r}{\partial M^r} * dM^r] \qquad \text{..............................(D1)}$$

$$\frac{dY}{Y} = E_{yx} * dP_x + E_{yy} * dP_y + (\frac{1}{Y}) * [\frac{\partial Y^o}{\partial M^o} * dM^o] + E_{yx} * \frac{dT_{Co}}{(1+T_{Co})} + E_{yy} * \frac{dT_{Co}}{(1+T_{Co})} +$$

$$E_{yz} * \frac{dT_{Co}}{(1+T_{Co})} \qquad \text{.........................................(D2)}$$

$$\frac{dZ}{Z} = [\frac{Z^o}{Z} * E^o_{zx} + \frac{Z^r}{Z} * E^r_{zx}] * dP_x + [\frac{Z^o}{Z} * E^o_{zy}] * dP_y +$$

$$\frac{Z^o}{Z} * E^o_{zx} * \frac{dT_{Co}}{(1+T_{Co})} + \frac{Z^o}{Z} * E^o_{zy} * \frac{dT_{Co}}{(1+T_{Co})} + \frac{Z^o}{Z} * E^o_{zz} * \frac{dT_{Co}}{(1+T_{Co})} +$$

$$\left(\frac{1}{Z}\right)*\left[\ \frac{\partial Z^o}{\partial M^o} * dM^o \ + \ \frac{\partial Z^r}{\partial M^r} * dM^r\ \right] \qquad \text{...............................(D3)}$$

Where elasticities are Marshallian and income effects $M^o$ and $M^r$ is disposable income.

### *Adding up*

$dK_x + dK_y + dK_z = 0$ ............(A1)

$dL_x + dL_y = 0$ ............(A2)

$dL_z = 0$ ............(A3)

$dP_z = 0$ [Numeraire]

$P_k$, $P_{lo}$ and $P_{lr}$ (factor prices) and $P_x$, $P_y$ and $P_z$ (output prices) are all equal to 1 and to each other to begin with

### *Other relationships*

$$dM^o = (1+T_{Co})*\ P_k*K_x*\frac{dK_x}{K_x} + (1+T_{Co})*\ P_k*K_y*\frac{dK_y}{K_y} +$$

$$(1+T_{Co})*\ (K_x + K_y)*dP_k \ + (1+T_{Co})*\ (L_x + L_y)*dP_{lo} +$$

$$dT_{Co}*\ [P_k*(K_x + K_y) + P_{lo}*(L_x + L_y)]$$

$$dM^r = -\ K_x*\frac{dK_x}{K_x} - K_y*\frac{dK_y}{K_y}$$

**SOLUTION PROCEDURES**

As can be seen, only the demand functions remain a function of the tax rate. When a tax was imposed on all consumption at a uniform rate, with no savings or leisure or untaxed goods, no relative prices changed. When the tax was returned to the single consumer, his income is also inflated by the same amount. The only effects of the tax

wedge are the difference between the prices faced by the OECD and ROW consumer with respect to the prices of X and Z. This is similar to the tax wedge on the price of capital faced by the corporate and non-corporate sector in the Harbereger (1962) model. One the income side, the impact of this tax and transfer program in the OECD should be neutral. However, once we take the uses side price indices into account, we may not have uniform effects. Also, if we had been willing to assume that demand was HOD Zero in income and prices, inflating all prices and income by the same factor of $(1+T_{Co})$would have left all demand unchanged. Since we are not making such an assumption, our demand curves include the tax rate.

Using $dK_x + dK_y + dK_z = 0$ …………(A1) we can write:

$$\frac{dK_z}{K_z} = -\left(\frac{K_x}{K_z}\right)*\frac{dK_x}{K_x} - \left(\frac{K_y}{K_z}\right)*\frac{dK_y}{K_y}$$

And using $dL_x + dL_y = 0$ …………(A2) we can write:

$$\frac{dL_x}{L_x} = -\left(\frac{L_y}{L_x}\right)*\frac{dL_y}{L_y}$$

Then we equate the demand and supply equations for sector X and Y, remembering that due to Walras' Law, the market for Z is in equilibrium when the first two are balanced. We substitute $dP_z = 0$ from the numeraire equation in the demand functions and $dL_z = 0$ wherever it appears. We use $dP_k = (b - 1) * \frac{dK_z}{K_z}$ or

$dP_k = (b - 1)*[-\left(\frac{K_x}{K_z}\right)*\frac{dK_x}{K_x} - \left(\frac{K_y}{K_z}\right)*\frac{dK_y}{K_y}]$when required.

Now the substitution equations are rewritten as:

$$\frac{dK_x}{K_x} - \frac{dL_x}{L_x} = S_x * (dP_k - dP_{lo}) \qquad \text{… (U1)}$$

$$\Rightarrow \quad \frac{dK_x}{K_x} + (\frac{L_y}{L_x})*\frac{dL_y}{L_y} = S_x *(b - 1)*[- (\frac{K_x}{K_z})*\frac{dK_x}{K_x} - (\frac{K_y}{K_z})*\frac{dK_y}{K_y}] - S_x *dP_{lo}$$

$$\frac{dK_y}{K_y} - \frac{dL_y}{L_y} = S_y * (dP_k - dP_{lo}) \qquad \text{… (U2)}$$

$$\Rightarrow \quad \frac{dK_y}{K_y} - \frac{dL_y}{L_y} = S_y *(b - 1)*[- (\frac{K_x}{K_z})*\frac{dK_x}{K_x} - (\frac{K_y}{K_z})*\frac{dK_y}{K_y}] - S_y*dP_{lo}$$

Next we equate the supply and demand for X, and remembering that $dP_z = 0$ and

$\frac{dL_x}{L_x} = - (\frac{L_y}{L_x})*\frac{dL_y}{L_y}$ :

$$\theta_{kx} *\frac{dK_x}{K_x} - \theta_{lx} * (\frac{L_y}{L_x})*\frac{dL_y}{L_y} = [\frac{X^o}{X} * E^o_{xx} + \frac{X^r}{X} * E^r_{xx}] * dP_x + [\frac{X^o}{X} * E^o_{xy}]* dP_y$$

$$+ \frac{X^o}{X} *E^o_{xx} * \frac{dT_{Co}}{(1+T_{Co})} + \frac{X^o}{X} * E^o_{xy} * \frac{dT_{Co}}{(1+T_{Co})} + \frac{X^o}{X} * E^o_{xz} * \frac{dT_{Co}}{(1+T_{Co})} +$$

$$(\frac{1}{X})*[\frac{\partial X^o}{\partial M^o} * dM^o + \frac{\partial X^r}{\partial M^r} * dM^r]$$

Equating demand and supply for Y:

$$\theta_{ky} *\frac{dK_y}{K_y} + \theta_{ly} *\frac{dL_y}{L_y} = E_{yx} *dP_x + E_{yy} *dP_y + (\frac{1}{Y})*[\frac{\partial Y^o}{\partial M^o} * dM^o] + E_{yx} *\frac{dT_{Co}}{(1+T_{Co})} +$$

$$E_{yy}*\frac{dT_{Co}}{(1+T_{Co})} + E_{yz} *\frac{dT_{Co}}{(1+T_{Co})}$$

We now have 4 equations in the following unknowns: $\frac{dK_y}{K_y}$, $\frac{dL_y}{L_y}$, $dG$, $dP_x$, $dP_y$, $dM^o$, $\frac{dK_x}{K_x}$, $dM^r$, $dP_{lo}$ and $dP_k$. We can substitute for some variables using:

$$dP_x = \theta_{kx} * (b - 1)*[- (\frac{K_x}{K_z})*\frac{dK_x}{K_x} - (\frac{K_y}{K_z})*\frac{dK_y}{K_y}] + \theta_{lx} * dP_{lo} \quad \text{.... (P1)}$$

(Where $\theta_{kx}$ and $\theta_{lx}$ are tax exclusive)

$$dP_y = \theta_{ky} * (b - 1)*[- (\frac{K_x}{K_z})*\frac{dK_x}{K_x} - (\frac{K_y}{K_z})*\frac{dK_y}{K_y}] + \theta_{ly} * dP_{lo} \quad \text{... (P2)}$$

(Where $\theta_{ky}$ and $\theta_{ly}$ are tax exclusive)

$$dM^o = (1+T_{Co})* P_k*K_x*\frac{dK_x}{K_x} + (1+T_{Co})* P_k*K_y*\frac{dK_y}{K_y} + (1+T_{Co})* (K_x + K_y)*dP_k + (1+T_{Co})*$$

$$(L_x + L_y)*dP_{lo} + dT_{Co}* [P_k*(K_x + K_y) + P_{lo}*(L_x + L_y)]$$

$$dM^r = - K_x*(\frac{dK_x}{K_x}) - K_y*(\frac{dK_y}{K_y})$$

And we are left with 4 equations in 4 variables, namely: $\frac{dK_y}{K_y}$, $\frac{dL_y}{L_y}$, $\frac{dK_x}{K_x}$ and $dP_{lo}$

Next we use symbolic notation for some of the parameters involved:

$$Єxx = [\frac{X^o}{X} * E^o_{xx} + \frac{X^r}{X} * E^r_{xx}] \qquad Єxy = [\frac{X^o}{X} * E^o_{xy}]$$

These weighted elasticities are Marshallian and are derived assuming the demand curves are separate for OECD and ROW.

We can write for symmetry since $Y^o = Y$: $Єyx = E_{yx}$ and $Єyy = E_{yy}$

**REDUCED FORM VERSION OF THE MODEL**

$$[1+S_x *(b - 1)*(\frac{K_x}{K_z})]*\frac{dK_x}{K_x} + (\frac{L_y}{L_x})*\frac{dL_y}{L_y} + S_x *(b - 1)*(\frac{K_y}{K_z})*\frac{dK_y}{K_y} + S_x *dP_{lo} = 0$$

........... (1)

$$S_y *(b-1)*(\frac{K_x}{K_z})*\frac{dK_x}{K_x} - \frac{dL_y}{L_y} + [1+S_y *(b-1)*(\frac{K_y}{K_z})]*\frac{dK_y}{K_y} + S_y *dP_{lo} = 0 \quad \ldots\ldots\ldots\ldots(2)$$

$$\{\theta_{kx} + \frac{(b-1)*K_x}{K_z}*(\,\epsilon xx *\theta_{kx} + \epsilon xy *\theta_{ky}) + \theta_{kx}*(\frac{\partial X^r}{\partial M^r} - (1+T_{Co})*\frac{\partial X^o}{\partial M^o}) +$$

$$(1+T_{Co})*\frac{\partial X^o}{\partial M^o} * \frac{(b-1)*K_x}{K_z} *\frac{(K_x+K_y)}{X}\}* \frac{dK_x}{K_x} +$$

$$\{\frac{(b-1)*K_y}{K_z}*(\epsilon xx *\theta_{kx} + \epsilon xy *\theta_{ky}) + \frac{K_y}{X}*(\frac{\partial X^r}{\partial M^r} - (1+T_{Co})*\frac{\partial X^o}{\partial M^o}) +$$

$$(1+T_{Co})*\frac{\partial X^o}{\partial M^o} * \frac{(b-1)*K_y}{K_z} *\frac{(K_x+K_y)}{X}\}* \frac{dK_y}{K_y} -$$

$$\{[\theta_{lx} * (\frac{L_y}{L_x})\}* \frac{dL_y}{L_y} - \{\epsilon xx* \theta_{lx} + \epsilon xy* \theta_{ly} +(1+T_{Co})*[\theta_{lx} + \frac{L_y}{X}] *\frac{\partial X^o}{\partial M^o}\}* dP_{lo}$$

$$= \{\frac{X^o}{X} * E^o_{xy} + \frac{X^o}{X} *E^o_{xx} + \frac{X^o}{X} * E^o_{xz} + \frac{X^o}{X} * \epsilon_{MX}*(1+T_{Co})\}* \frac{dT_{Co}}{(1+T_{Co})}$$

$$\ldots\ldots\ldots\ldots \quad (3)$$

$$\{\frac{(b-1)*K_x}{K_z}*(\,\epsilon yx * \theta_{kx} + \epsilon yy *\theta_{ky}) - \frac{K_x}{Y}*(1+T_{Co})*\frac{\partial Y^o}{\partial M^o} +$$

$$(1+T_{Co})*\frac{\partial Y^o}{\partial M^o} * \frac{(b-1)*K_x}{K_z} *\frac{(K_x+K_y)}{Y}]\}* \frac{dK_x}{K_x} +$$

$$\{\theta_{ky} + \frac{(b-1)*K_y}{K_z}*(\,\epsilon yx * \theta_{kx} + \epsilon yy *\theta_{ky}) - \theta_{ky} *(1+T_{Co})*\frac{\partial Y^o}{\partial M^o} +$$

$$(1+T_{Co})*\frac{\partial Y^o}{\partial M^o} * \frac{(b-1)*K_y}{K_z} *\frac{(K_x+K_y)}{Y}\}* \frac{dK_y}{K_y} + \theta_{lx} *\frac{dL_y}{L_y} -$$

$$\{[\theta_{ly} + \frac{L_x}{Y}] *\frac{\partial Y^o}{\partial M^o} + \epsilon yx* \theta_{lx} + \epsilon yy* \theta_{ly}\}* dP_{lo} =$$

$\{\ E^{o}_{yx} + E^{o}_{yy} + \ E^{o}_{yz} + \text{Є}_{MY}*(1+T_{Co})\}* \frac{dT_{Co}}{(1+T_{Co})}$ ………… (4)

Considering the two terms in equations 3 and 4 after the = that are constants:

$\{\ \frac{X^{o}}{X} * E^{o}_{xy} + \frac{X^{o}}{X} * E^{o}_{xx} + \frac{X^{o}}{X} * E^{o}_{xz} + \frac{X^{o}}{X} * \text{Є}_{MX}*(1+T_{Co})\}* \frac{dT_{Co}}{(1+T_{Co})}$ And

$\{\ E^{o}_{yx} + E^{o}_{yy} + \ E^{o}_{yz} + \text{Є}_{MY}*(1+T_{Co})\}* \frac{dT_{Co}}{(1+T_{Co})}$. Had we been willing to assume that the demand curves were HOD Zero in tax inclusive prices and tax inclusive income, they would sum to zero in each case due to the homogeneity identity and Euler's law: The sum of all own price, cross price and income elasticities in a HOD Zero demand curve are equal to zero (Mas-Colell et al 1999). Since we have not made the assumption we have these terms in both equations. Had the terms dropped out instead, each equation would have been free of the $dT_{Co}$ term, implying that the change in the tax rate had no impact on the solution.

These, as before are 4 equations in 4 variables. If variables are denoted in the following order by subscript j = 1, ..., 4; $\frac{dK_x}{K_x}$, $\frac{dK_y}{K_y}$ , $\frac{dL_y}{L_y}$, $dP_{lo}$ and subscript i = 1, ..., 4 represents the equation, the 4 equations above can be written in symbolic form with $a_{ij}$ representing coefficients attached to the left hand side variables and $b_i$ the constants on the right hand side.

$A_{11}*\frac{dK_x}{K_x} + A_{12}*\frac{dK_y}{K_y} + A_{13}*\frac{dL_y}{L_y} + A_{14}*dP_{lo} = B_1$ ………… (1)

$A_{21}*\frac{dK_x}{K_x} + A_{22}*\frac{dK_y}{K_y} + A_{23}*\frac{dL_y}{L_y} + A_{24}*dP_{lo} = B_2$ ………… (2)

$$A_{31}*\frac{dK_x}{K_x} + A_{32}*\frac{dK_y}{K_y} + A_{33}*\frac{dL_y}{L_y} + A_{34}*dP_{lo} = B_3 \quad \ldots\ldots\ldots\ldots \quad (3)$$

$$A_{41}*\frac{dK_x}{K_x} + A_{42}*\frac{dK_y}{K_y} + A_{43}*\frac{dL_y}{L_y} + A_{44}*dP_{lo} = B_4 \quad \ldots\ldots\ldots\ldots \quad (4)$$

Where

$A_{11} = 1+S_x *(b - 1)*(\frac{K_x}{K_z})$ $\quad A_{12} = S_x *(b - 1)*(\frac{K_y}{K_z})$ $\quad A_{13} = (\frac{L_y}{L_x})$ $\quad A_{14} = S_x$

$B_1 = 0$

$A_{21} = S_y *(b - 1)*(\frac{K_x}{K_z})$ $\quad A_{22} = 1+S_y *(b - 1)*(\frac{K_y}{K_z})$ $\quad A_{23} = -1$ $\quad A_{24} = S_y$

$B_2 = 0$

$$A_{31} = \{\theta_{kx} + \frac{(b-1)*K_x}{K_z}*(\,\text{Є}xx *\theta_{kx} + \text{Є}xy *\theta_{ky}) + \theta_{kx}*(\,\frac{\partial X^r}{\partial M^r} - (1+T_{Co})*\frac{\partial X^o}{\partial M^o}) +$$

$$(1+T_{Co})*\frac{\partial X^o}{\partial M^o} * \frac{(b-1)*K_x}{K_z} * \frac{(K_x+K_y)}{X}\}$$

$$A_{32} = \{\,\frac{(b-1)*K_y}{K_z}*(\text{Є}xx *\theta_{kx} + \text{Є}xy *\theta_{ky}) + \frac{K_y}{X}*(\,\frac{\partial X^r}{\partial M^r} - (1+T_{Co})*\frac{\partial X^o}{\partial M^o}) +$$

$$(1+T_{Co})*\frac{\partial X^o}{\partial M^o} * \frac{(b-1)*K_y}{K_z} * \frac{(K_x+K_y)}{X}\}$$

$$A_{33} = -\{\theta_{lx} * (\frac{L_y}{L_x})\} \qquad A_{34} = -\ \{\,\text{Є}xx*\,\theta_{lx} + \text{Є}xy*\,\theta_{ly} + (1 + T_{Co})*[\,\theta_{lx} + \frac{L_y}{X}\,] *\frac{\partial X^o}{\partial M^o}\}$$

$$B_3 = \{\,\frac{X^o}{X} * E^o_{xy} + \frac{X^o}{X} *E^o_{xx} + \frac{X^o}{X} * E^o_{xz} + \frac{X^o}{X} * \text{Є}_{MX}*(1+T_{Co})\}* \frac{dT_{Co}}{(1+T_{Co})}$$

$$A_{41} = \{\frac{(b-1)*K_x}{K_z}*(\,\text{Є}yx * \theta_{kx} + \text{Є}yy *\theta_{ky}) - \frac{K_x}{Y}*(1+T_{Co})*\frac{\partial Y^o}{\partial M^o} +$$

$(1+ T_{Co})*\frac{\partial Y^o}{\partial M^o} * \frac{(b-1)*K_x}{K_z} *\frac{(K_x+K_y)}{Y}]\}$

$A_{42} = \{\theta_{ky} + \frac{(b-1)*K_y}{K_z}*( Єyx * \theta_{kx} + Єyy *\theta_{ky}) – \theta_{ky} *(1+ T_{Co})*\frac{\partial Y^o}{\partial M^o} +$

$(1+ T_{Co})*\frac{\partial Y^o}{\partial M^o} * \frac{(b-1)*K_y}{K_z} *\frac{(K_x+K_y)}{Y}\}$

$A_{43} = \theta_{lx}$ $A_{44} = - \{[ \theta_{ly} + \frac{L_x}{Y} ] *\frac{\partial Y^o}{\partial M^o} + Єyx* \theta_{lx} + Єyy* \theta_{ly} \}$

$B_4 = \{ E^o_{yx} +E^o_{yy}+ E^o_{yz} + Є_{MY}*(1+ T_{Co})\}* \frac{dT_{Co}}{(1+T_{Co})}$

This can be solved using MATLAB or any other procedure as discussed in chapter two.

**CHANGES IN GDP AND USES SIDE**

Here GDP from the income side equals disposable income since there are no direct taxes.

$dM^o = dGDP^o = (1+T_{Co})* P_k*K_x*\frac{dK_x}{K_x} + (1+T_{Co})* P_k*K_y*\frac{dK_y}{K_y} + (1+T_{Co})* (K_x + K_y)*dP_k +$

$(1+T_{Co})* (L_x + L_y)*dP_{lo} + dT_{Co}* [P_k*(K_x + K_y) + P_{lo}*(L_x + L_y)]$

We get $dGDP^r = P_k*K_z*\frac{dK_z}{K_z} = - K_x*\frac{dK_x}{K_x} - K_y*\frac{dK_y}{K_y}$

Now, as in chapter two we construct Laspeyres index for both countries to get an estimate of the change in the cost of living. Since the consumption tax does not apply to the ROW, the index for ROW is the same as in chapter two.

$$LAS^r = \frac{(P_x + dP_x)* X^r + (P_z + dP_z)*Z^r}{P_x * X^r + P_z *Z^r}$$

Next we know that $dP_z = 0$ and initial prices are equal to 1, this gives:

$$LAS^r = \frac{X^r + dP_x * X^r + Z^r}{X^r + Z^r} = 1 + \frac{dP_x * X^r}{X^r + Z^r}$$

The major difference is in the index the OECD country since the tax is imposed on consumption here. Therefore $LAS^o$ =

$$\frac{d[P_x*(1+T_{Co})]* X^o + d[P_y*(1+T_{Co})]* Y^o + d[P_z*(1+T_{Co})]* Z^o + [P_x*(1+T_{Co})]* X^o + [P_y*(1+T_{Co})]* Y^o + [P_z*(1+T_{Co})]* Z^o}{[P_x*(1+T_{Co})]* X^o + [P_y*(1+T_{Co})]* Y^o + [P_z*(1+T_{Co})]* Z^o}$$

$$= 1 + \frac{d[P_x*(1+T_{Co})]* X^o + d[P_y*(1+T_{Co})]* Y^o + d[P_z*(1+T_{Co})]* Z^o}{[P_x*(1+T_{Co})]* X^o + [P_y*(1+T_{Co})]* Y^o + [P_z*(1+T_{Co})]* Z^o}$$

$$= 1 + \frac{dT_{Co}*[X^o + Y^o + Z^o] + (1+T_{Co})[dP_x*X^o + dP_y* Y^o]}{(1+T_{Co})* [X^o + Y^o + Z^o]}$$

$$LAS^o = 1 + \frac{dT_{Co}}{(1+T_{Co})} + \frac{[dP_x*X^o + dP_y*Y^o]}{[X^o + Y^o + Z^o]}$$

As in chapter two, to measure total welfare, we can either take a linear combination of the changes in the sources and uses side, or deflate the change in GDP by the change in prices.

OECD welfare: $W^o = 1 + dGDP^o/GDP^o - LAS^o$ and

ROW welfare $W^r = 1 + dGDP^r/GDP^r - LAS^r$

This is a pure assumption, and we could have chosen any form for the welfare function, such as $W^o = \frac{1+\frac{dGDP^o}{GDP^o}}{LAS^o}$ instead or more complicated forms.

To compare the models of chapter two and three with those in chapter four, we simply note that for each tax, the tax rate change enters the expressions for GDP and uses through the solutions for individual terms. Therefore the differential impact for each input model is the difference between welfare index terms in chapter two and chapter three and the corresponding transfer model in chapter four. The results for the sources (GDP) and uses (LAS) are also presented separately for each model. A sample table from chapter six is presented below for ease of comparison.

**Table 4 Results: Sources, Uses & Total Welfare Changes with Baseline Data**

| | **OECD** | | | **ROW** | | |
|---|---|---|---|---|---|---|
| | *SOURCES* | *USES* | *WELFARE* | *SOURCES* | *USES* | *WELFARE* |
| **CIT and INPUT** | 1.3793 | 1.3535 | 0.02583 | 0.96805 | 1.0207 | -0.052605 |
| **CIT and TRANSFER** | 0.97619 | 0.97905 | -0.00286 | 1.0 | 0.998776 | 0.001225 |
| **CONSUMPTION and INPUT** | 1.3939 | 1.3193 | 0.07457 | 0.96792 | 1.01899 | -0.05107 |
| **CONSUMPTION and TRANSFER** | 0.99067 | 0.99100 | -0.0003 | 1.0001 | 0.99995 | 0.00013 |

We will derive values for all parameters used in the four models (elasticities, shares, tax rates etc.) from the existing literature. We will then substitute these in the model equations. For the tax change, we take the base case to be a 50% reduction in the differential tax. MATLAB models are built for all 4 cases in chapters two, three and four. There models are solved to yield numerical values for the variables that enter the sources and uses side calculations. As an illustration, the baseline MATLAB model and results from it used to calculate burdens for the CIT with transfer case is appended below. A set of four such distinct models in MATLAB (for CIT with input, consumption with input, CIT with transfer and consumption with transfer) with appropriate parameter values yields each results table in chapter six.

```
% Dissertation model chapter 4 dec- lowering tax on capital-  %

% Reduced for is 4 equations in 4 variables:- dKx_Kx, dLy_Ly, dPlo, dKy_Ky %

clear;
clear all;
clc;
eq5='(1/3)*dKx_Kx + (2/3)*dLx_Lx  = dX_X';
eq6='(1/3)*dKy_Ky + (2/3)*dLy_Ly  = dY_Y';
eq7='(1/3)*dPk - (1/3)*(0.025/1.05) + (2/3)*dPlo  = dPx';
eq4='(1/3)*dPk - (1/3)*(0.025/1.05) + (2/3)*dPlo  = dPy';
eq1='dPk = - (2/3)*dKz_Kz';
eq2='dKz_Kz = - (3/36.75)*dKx_Kx - (12/36.75)*dKy_Ky';
eq3='dLx_Lx = -4*dLy_Ly';
eq8='dKx_Kx - dLx_Lx = -0.6*dPk + (0.6)*(0.025/1.05) + 0.6*dPlo';
eq9='dKy_Ky - dLy_Ly = -0.6*dPk + (0.6)*(0.025/1.05) + 0.6*dPlo';
```

```
eq10='dX_X = -.5*dPx - .3*dPy + 0.08*(1/3)*dKx_Kx  + 0.08*(4/3)*dKy_Ky +
0.08*(5/3)*dPk + 0.08*(10/3)*dPlo - 0.036*(1/3.15)*dKx_Kx - 0.036*(4/3.15)*dKy_Ky
- 0.025*0.08*(5/3.15)';
eq11='dY_Y = -.1*dPx - .8*dPy + 0.8*(1/12)*dKx_Kx  + 0.8*(1/3)*dKy_Ky +
0.8*(5/12)*dPk + 0.8*(10/12)*dPlo - 0.025*0.8*(5/12.6)';
s=solve(eq1,eq2,eq3,eq4,eq5,eq6,eq7,eq8,eq9,eq10,eq11,'dKx_Kx','dKy_Ky','dLx_Lx','d
Ly_Ly','dPk','dPlo','dPx','dPy','dX_X','dY_Y','dKz_Kz');
dKx_Kx= s.dKx_Kx
dKy_Ky = s.dKy_Ky
dLy_Ly = s.dLy_Ly
dPlo = s.dPlo
dLx_Lx = s.dLx_Lx
dPk = s.dPk
dPx = s.dPx
dPy = s.dPy
dX_X= s.dX_X
dY_Y = s.dY_Y
dKz_Kz = s.dKz_Kz
Kx = (1/3.15)*20
Ky = (1/3.15)*80
Ly = (2/3)*80
Kz = (1/3)*(700/3)
Lx = (2/3)*20
dKx = dKx_Kx*Kx
dKy = dKy_Ky*Ky
dLy = dLy_Ly*Ly
dKz = dKz_Kz*Kz
dLx = dLx_Lx*Lx
GDPO = 100
dGDPO = (1.05)*(dKx + dKy) + (1.05)*(Kx + Ky)*dPk - (0.025)*(Kx + Ky) + (Lx +
Ly)*dPlo
```

Xo = 8

Yo = 80

Zr = 221.3333333

Zo = 12

Xr = 12

LASO = 1 + (dPx*0.08) + (dPy*0.8)

WELFO = 1+ (dGDPO/GDPO) - LASO

GDPR = 700/3

dGDPR = -dKx -dKy

LASR = 1 + dPx*(36/700)

WELFR = 1+ (dGDPR/GDPR) - LASR

SOURCESR = 1+ (dGDPR/GDPR)

SOURCESO = 1+ (dGDPO/GDPO)

**%RESULTS%**

dKx_Kx =0.0095238095238095238095238095238095

dKy_Ky =-0.0023809523809523809523809523809526

dLy_Ly =-0.0023809523809523809523809523809524

dPlo =-0.023809523809523809523809523809524

dLx_Lx =0.0095238095238095238095238095238096

dPk =(-3.7844965146552448139749727051315)*10^(-35)

dPx =-0.023809523809523809523809523809524

dPy =-0.023809523809523809523809523809524

dX_X =0.0095238095238095238095238095238096

dY_Y =-0.0023809523809523809523809523809525

dKz_Kz =5.6767447719828672209624590576973*10^(-35)

Kx = 6.3492

Ky = 25.3968

Ly = 53.3333

Kz = 77.7778

Lx = 13.3333

dKx =0.060468631897203325774754346182917

dKy =-0.060468631897203325774754346182922

dLy =-0.12698412698412698412698412698413

dKz =4.4152459337644522829708014893201*10^(-33)

dLx =0.12698412698412698412698412698413

GDPO = 100

dGDPO =-2.3809523809523809523809523809524

Xo = 8

Yo = 80

Zr = 221.3333

Zo = 12

Xr = 12

LASO =0.97904761904761904761904761904762

WELFO =-0.0028571428571428571428571428571429

GDPR = 233.3333

dGDPR =4.4522829799670587500411415562129*10^(-33)

LASR =0.99877551020408163265306122448980

WELFR =0.0012244897959183673469387755102041

SOURCESR =1.0

SOURCESO =0.97619047619047619047619047619048

# CHAPTER V – DATA

## BASIC ECONOMY DATA

In this section we report the data to be used as parameters in the four models, obtaining data from existing studies and standard data sources. At the outset, it is clarified that following the standard literature on which this dissertation is based (Harbereger 1962, 1995 and 2008; Gravelle and Smetters 2001, 2006 and Randolph 2006) we use data for the U.S. to represent the OECD country, and the ROW includes all other countries in the world. The only country of comparable size in the world in our opinion that can have an impact similar to the U.S. would be the E.U. or Euro-Zone countries taken together and perhaps China in the not-too-distant future.

While it seems that lumping all other countries - developed, NICs and LDCs together as ROW may be a bit of a stretch, we can always isolate the major trading (investment) partners of the OECD country and have them represent ROW instead. This group may be more homogenous. Both characterizations of the OECD country and ROW are standard in most important models of this type that we have come across in the literature, and so we follow the same route in the dissertation.

The objective is to substitute out those parameters for which estimates are available in the literature so that we can concentrate on analyzing the solutions dependent on parameters which fall in the “unknowable” category. To begin with, the sources of information for basic economy data from Randolph (2006) are:

**Table 5 Initial Assumptions Randolph (2006)**

| | Share of Value Added in Sector | | | Share of |
|---|---|---|---|---|
| | **Labor** | **Capital** | **Land** | **Output** |
| **Corporate Sectors** | | | | |
| Sectors 1 and 2: Tradable | 82% | 18% | … | 28% |
| Sector 3: Nontradable | 76% | 24% | … | 45% |
| **Non-Corporate Sectors** | | | | |
| Sector 4: Tradable, agriculture | 49% | 17% | 34% | 3% |
| Sector 5: Nontradable | 47% | 53% | … | 24% |
| **Total** | **70%** | **29%** | **1%** | **100%** |

| | |
|---|---|
| Domestic (OECD) economy's share of world output | 30% |
| Domestic (OECD) ownership share of world capital | 30% |
| Partial elasticity of substitution, capital and labor | 0.6 |

**Source: Randolph (2006), Based on Gravelle and Smetters (2006).**

Randolph (2006) also reports (implied) calculated statistics form Harberger (1995) for the same variables.

**Table 6 Shares Consistent With Harberger (1995)**

| | Share of Value Added in Sector | | | Share of |
|---|---|---|---|---|
| | **Labor** | **Capital** | **Land** | **Output** |
| **Corporate sectors** | | | | |
| Sectors 1 and 2: Tradable | 71% | 29% | … | 25% |
| Sector 3: Nontradable | 82% | 18% | … | 40% |
| **Non-corporate sectors** | | | | |
| Sector 4: Tradable, agriculture | 49% | 17% | 34% | 3% |
| Sector 5: Nontradable | 57% | 43% | … | 32% |
| **Total** | **70%** | **29%** | **1%** | **100%** |

| | |
|---|---|
| Domestic economy's share of world output | 37.5% |
| Domestic ownership share of world capital | 37.5% |
| Partial elasticity of substitution, capital and labor | 0.6 |

**Source: Randolph (2006) and Based on Harberger (1995)**

The data above have been derived to be consistent with results in Harberger (1995), although all the variables have not been specified in the paper itself. This is especially apparent in the case with the labor share for nontradable corporate sector which yields a figure of 82% consistent with the results. While this figure is consistent, it should not be interpreted as intentional since this sector (consisting of items like housing) is generally taken to be more capital intensive than the labor share of 82% suggests.

Further, we need to clarify what constitutes the corporate tradable sector since we know that the non-corporate tradable sector refers to agriculture. Harberger (2008) classifies the corporate tradable sector to include manufacturing, the corporate non-tradable sector to include utilities and transport, and the non-corporate non-tradable sector to include services. However, without a clear specification of exactly which sectors go where and how the output of sub-sectors is allocated we cannot hope to replicate the results. For example, we may argue that with the advent of the internet, some services such as higher education are now tradable, and could constitute a major export (invisibles).

The distinction is between goods actually traded and those potentially tradable. While goods actually traded might be a specific fraction of GDP in the U.S. (exports and imports), tradables refer to a different class of goods. To illustrate, let us follow a strict interpretation of the Harberger (2008) typology using 2007 BEA data and see what we get. Here, the tradable sector includes manufacturing, agriculture and mining. The non-tradable sector includes everything else, services, utilities and transport, government et al.

There does not appear to be a good source to estimate the world's total stock of capital, even though there is an exact estimate of U.S. capital stock from BEA. The elasticity of substitution of 0.6 is based on Hamermesh and Grant (1979) and common to both sectors, but as reported by Randolph (2006), results are not very sensitive to change within the range 0.6 to 1.0. We will use this elasticity as negative (- 0.6) since it was defined as such in our model.

**Table 7 Initial Assumptions Following Harberger (2008)**

| | Share of Value Added in Sector | | Share of |
|---|---|---|---|
| | **Labor** | **Capital (= 1- labor)** | **Value Added** |
| **Tradable Sector** | **52%** | **48%** | **15%** |
| Manufacturing | 60% | 40% | 11.7% |
| Mining | 23% | 77% | 2% |
| Agriculture | 25% | 75% | 1.2% |
| **Non-tradable Sector** (Everything else) | **57%** | **43%** | **85%** |
| **Total** | **57%** | **43%** | **100%** |

Domestic economy's share of world output 29% (WDI 2007)

Therefore, there does not appear to be any reason based on WDI estimates for 2007 to dispute the Randolph/GandS estimates of:

Domestic economy's share of world output 30%

However, there is no straightforward source for the next statistic:

Domestic ownership share of world capital 30%

**Issues with basic economy data: tradables and nontradables**

There are several well-documented issues in the literature on each one of these figures. First we will briefly review the discussion on the issue of what constitutes tradables. DeGregorio, Giovannini and Wolf (1994) summarize one dominant view thus:

"The theoretical literature on real exchange rates relies upon a neat division of commodities into "tradables" and "nontradables". Unfortunately few real world commodities fall easily into the nontradable category. Indeed, as Roy Harrod pointed out, virtually all commodities are tradable within some area, with the extent of the area determined by transportation cost. Notwithstanding, most economists would argue that certain commodities are in some sense inherently "less tradable" than others."(pg 1231)

They proceed to define a sector as tradable if more than 10% of its output is exported, and find that for a sample of 14 OECD countries for the period 1970-1985, manufacturing, mining and agriculture are tradables and services are nontradables. However, within nontradables they find that transportation has higher export content, and thus include transport in tradables (this could be due to the higher tradable component of international airlines and shipping). Everything else, summing to about 50-60% of GDP is treated as nontradable. The authors report that this coincides with the classification of Stockman and Tesar (1991) as well, and find that the export threshold of 10% chosen does not affect the results qualitatively when varied in the range 5%-20%.

However, a more recent strand followed by Benetrix and Lane (2009), Canzoneri, Cumby and Diba (1999), Galystan and Lane (2008) and Obstfeld (2009) presents a slightly different classification. As described in Benetrix and Lane (2009):

"real output in the nontradable sector is the sum of the real added value in 'Construction', 'Wholesale and Retail Trade', 'Hotels and Restaurants', 'Transport and Storage and Communication', 'Finance, Insurance, Real Estate and Business Services', 'Public Administration and Defense; Compulsory Social Security', Education', 'Health and Social Work' and 'Other Community, Social and Personal Services'. Real output in the

tradable sector is the aggregate of the real added value in 'Agriculture, Hunting, Forestry and Fishing', 'Mining and Quarrying', 'Total Manufacturing' and 'Electricity, Gas and Water Supply'." (Page 5)

The share of "utilities" in BEA 2007 is 2% and for "transport" is 2.9%. Either way, the total tradables do not add up to more than 20% of total value added if we do not exclude other items such as taxes and government value added from the total or add other services to tradables. The relevant figure for Harberger (1995) is 28%. The share used in GandS/Randolph is 31% for tradables. Based on BEA 2007 and either definition above, there does not seem to be a strong case for a tradables ratio above 20%.

A recent study by Obstfeld and Rogoff (2007) also supports a tradables ratio of 25% for the U.S. Even if we add traded goods to the total from sectors otherwise classified as tradable, we have to remember that the classification into tradables and nontradables is different from traded and nontraded. For the U.S., exports and imports constitute around 13% to 17% of GDP respectively, including invisibles. Exports constitute around 13%- 14% in the last two years or so, but have averaged around 12% over the recent decade, going by WDI data.

Exports in this model correspond to $X^r$ and imports to $Z^o$. If we want to avoid issues of trade deficits, we have to assume that they are equal. Therefore, taking 12% as actually traded ratio, tradables constitute the sum of value added of the sectors from which traded goods are taken from. If exports are well-diversified, potentially, the entire economy could be included in tradables. However, if there were 20 sectors and each contributed 5% of own output to exports, this would lead to an overall export ratio of 5% but a tradables ratio of 100%. This would imply that each sector produces an output

consumed potentially by the ROW, and that the price for this product is determined in world markets as well as at home. Therefore, the essential distinction is between those sectors whose demand price is affected by the home country and those whose demand price is determined by the ROW as well as the home country. What those sectors are, in my opinion, is a matter of judgment, and several ratios are possible.

**LABOR SHARES**

The next issue concerns the share of labor in each sector of the U.S. economy. Here the debate really pertains to the relative shares of capital and labor in each sector, as well as the relative shares of each factor across tradables and nontradables. Unfortunately, estimates from different sources vary substantially. Can we go ahead and use the BEA 2007 figures in the following manner?

**Table 8 U.S. Shares Based on BEA (2007)**

| | Share of Value Added in Sector | | Share of |
|---|---|---|---|
| | **Labor** | **Capital (= 1- labor)** | **Output** |
| **Tradable Sector** | **52%** | **48%** | **15%** |
| **Non-tradable Sector** | **57%** | **43%** | **85%** |
| **Total** | **57%** | **43%** | **100%** |

Source: Bureau of Economic Analysis Website: GDPbyInd_VA_NAICS_1998-2008.XLS; available at: http://www.bea.gov/industry/gdpbyind_data.htm; accessed on 5/1/2010.

These figures have been obtained by dividing total compensation of employees by value added in each category. The problem with this approach has been well documented by several authors. In a recent ILO report, Lubker (2007) explains the issue thus: "…this commonly used calculation of the labour share is bound to be lower estimates since national accounts do not include incomes generated from self-employment under

total compensation, but record them as "mixed income." Their attribution to either labour or capital is unclear due to the fact that they reflect both the returns on labour inputs and on capital investment."(pg 1)

Another recent ILO study has suggested an alternative method of calculating the labor share. According to ILOs World of Work Report (2008)

"To adjust for the fact that "compensation of employees" only captures the income of salaried workers (not of self-employed persons), for a number of countries, "compensation of employees" was divided by the ratio of employees to total employment. As such, the assumption is that self-employed persons earn, on average, the same as employees…… Total labour cost divided by nominal output, where: Total labour cost = (compensation per employees * number of employees * hours worked employment)/ (hours worked employees), and nominal output refers to annual current price value added compiled according to the System of National Accounts 93."(pg 32)

According to the World Economic Outlook 2007, the 2005 share of income of employees to GDP for U.S.A. was around 57% while the share of total labor income to GDP was around 60%. The comparable figures for G-7 countries as a whole was around 55% and 62% respectively (pg 168, chap 5).

Gomme and Rupert (2004) conduct a detailed analysis of what this means for the U.S. economy. In addition to the issues raised in the ILO and WEO above, they identify several other differences between labor's share and BEA's simple ratio:

"Unfortunately, the deeper one digs into the national income and product account, the more one discovers that this simple calculation is woefully inadequate. Some important

considerations that should be kept in mind when interpreting measures of the income shares include:

● How should proprietors' income be divided between labor and capital...

● How should the government sector's lack of capital income be handled...

● How should the housing sector's lack of labor income be handled...

● How should indirect taxes less subsidies be handled...

● Should output (income) be measured on a *gross* or *net* basis?" (pg 5)

The authors then provide their own calculations for the shares of labor, net taxes and capital income (profits plus interest) for 1950-2004 for the corporate nonfinancial sector in the U.S., and the shares for the final quarter of 2004 are approx 74% labor, 11% for net taxes and 15% for capital. Using methodologies discussed elsewhere the authors derive a similar series for the economy and come to the conclusion that the relevant labor share for the economy as a whole is around 72%.

Unfortunately, this is not the end of the story either. Buchele and Christiansen (2007) use the data developed by Piketty and Saez (2003) to calculate the labor share series based on the work of Gomme and Rupert (2004), Krueger (1999) and Poterba (1997). They find that the relevant share for 2005 is 69%. However, they point out that this is inclusive of the compensation of top executives, whose pay includes stocks and options which are more akin to capital income than labor income. By making an adjustment for the top 0.5% of earners from the Piketty and Saez (2003) data, they find that the relevant figure for 2005 is 61%. This is similar to the WEO (2007) estimate. Similar figures are reported in Guscina (2006) and Harrison (2002) for different time periods.

However, none of these studies provide estimates for labor share broken down by tradables and nontradables, so the relative positions have to be estimated elsewhere. The most detailed recent estimates available are presented in Valentinyi and Herrendorf (2008) who have provided several different classifications. At producer prices they estimate the share of capital income to be 37% for tradables, 32% for nontradables and 33% for the economy as a whole. This implies that the ratio of tradables to GDP is taken as 20%. This also yields labor ratios (if capital plus labor shares add to 1) of 63% for tradables, 68% for nontradables and 67% for the economy as a whole. Applying the methodology to BLS data, the authors find that the shares for capital are for tradables is 35%, 34% for nontradables, and 34% for the entire economy, again at producer prices. Using purchaser prices for traded goods (imports plus exports) does not substantially change the shares. Using purchaser prices, capital shares are 33% for tradables and nontradables 34% and 33% overall.

We can summarize the main conclusions as:

1. Capital gets around 1/3 in tradables and nontradables when we take purchaser prices, labor gets 2/3
2. The share of capital is almost the same in both sectors and the economy as a whole when different data sources and prices are accounted for
3. The implied ratio of tradables to nontradables is estimated at 1:4.

There are some issues with using these data. Firstly, the ratios and coefficients are based on data covering a wide period in some cases extending from 1990-2000. Second, the sectoral breakdown of tradables and notradables is not completely specified in terms of the BEA 2007 data, the authors use a slightly different classification.

Based on the above discussion, it appears that a labor share of around 67% for nontradables is justified, as is the 1:4 ratio between tradables and nontradables. The problem arises with the share of tradables. The preferred estimates in Randolph (2006) based on Gravelle and Smetters (2006) imply a 78% share for labor in tradables. The Harberger (1995) estimate is 68% going to labor in tradables. Both studies also do not use the same ratio for labor in the tradable sector vis-a-vis nontradables, where the Gravelle and Smetters (2006) estimate is 67% for labor and the Harberger (1995) estimate is 72% for labor. This also means that the two studies have different capital intensities in the two sectors. Their ratios of tradables to nontradables in value added are also closer to 30% than 20%.

If we use the lower of the labor ratios in each case, we get results as noted above. Labor gets around two-thirds. More importantly, while there are studies that estimate higher labor shares in the corporate sectors, there is no evidence that shows labor shares substantially different from the average of two-thirds to 70% for nontradables. To correct one and not the other implies changing capital-labor intensities between the two sectors, for which there does not appear to be clear empirical support, at least in the magnitudes the authors have suggested. There also does not appear to be any justification provided to take the ratio of tradables at above 20%.

**WORLD AND U.S. CAPITAL STOCKS**

We now turn to the ratio of U.S. capital to world capital. The relevant measure for assets is the SNA 93 balance sheet entry – National wealth. This is available as a series for the U.S., but not for ROW. While several flow measures are available, such as gross fixed capital formation and FDI (as well as FDI stocks), we need a consistent

methodology to convert the GFCF flows into a stock measure. The preferred methodology employed by the OECD is detailed in Mienem, Verbiest and de Wolf (1998) to estimate the stock of capital for countries for which such data is available. The method of aggregation is known as the "Perpetual Inventory Method": it involves taking the stock of capital at a point of time ( opening balance), and adding to it the GFCF and major new additions such as subsoil assets, subtracting depreciation and losses due to disasters etc. and lastly accounting for revaluations. The important decision variables are service lives, discard patterns and depreciation methods.

Since such details for the world as a whole are not available the WDI GFCF series for 31 years (1965-2006) was used instead. The total of 31 years of GFCF was used as a proxy for the stock of capital in each case. For U.S. and the World the ratio of total GFCF comes to 23%. As a further proxy check, the 2007 WDI ratio of U.S. market capitalization of listed companies is around 30% of world market capitalization, but the ratio in the two preceding years is around 38%. This means that the Harberger (1995) estimates of 37.5% are probably at the highest end of the range and the WDI rough estimate of 23% at the bottom. The Randolph (2006) preferred figure of 30% is somewhere at the midpoint, so we will proceed with the 30% assumption for the ratio of U.S. capital to world capital.

The last item in this table is the elasticity of substitution of U.S. capital for labor in tradables and nontradables. A recent paper by Klump, McAdam and Willman (2007) has found estimates for 1953-1998 for U.S. of the same magnitude for the elasticity of substitution, with preferred estimates around the 0.6 mark. Several other studies have also used elasticities of substitution of less than one, such as Hamermesh (1993), Guscina

(2006), Decreuse and Maarek (2008) and Checchi and Penalosa (2007). Summarizing the discussion above gives:

**Table 9 Shares Based On Literature Review**

| | Share of Value Added in Sector | | Share of |
|---|---|---|---|
| | **Labor** | **Capital (= 1- labor)** | **Output** |
| **Tradable Sector** | **67%** | **33%** | **20%** |
| **Non-tradable Sector** | **67%** | **33%** | **80%** |
| **Total** | **67%** | **33%** | **100%** |
| Domestic economy's share of world output | | 30% | |
| Domestic ownership share of world capital | | 30% | |
| Partial elasticity of substitution, capital and labor | | - 0.6 | |

## INTERPRETATION OF TAX RATES

For the capital income tax, we can proceed by noting that a strict interpretation of the tax rate $T_{ko}$ as the total tax rate on capital imposed only by the OECD country is not necessary. Every country imposes some taxes on capital (with perhaps a few resource rich countries being exceptions), and very few countries tax all capital at the same constant average rate. In practice, average and marginal rates for many taxes vary substantially due to exemptions, allowances etc. It is more intuitive to think of $T_{ko}$ as a differential tax rate. This represents, for a country that taxes capital at a higher rate than other countries, its excess tax rate on capital income. Tax competition implies that the OECD country might seek to reduce or eliminate this differential. This differential tax could simply be the difference between the average tax rate on the income from capital in all sectors in the OECD country and the average tax rate on capital income in the rest of the world.

There are several different ways of looking at this differential tax rate on capital income. First, we can note that Corporate Income Tax rates (combined federal and local) vary widely between even the OECD countries. For 2009, the combined rate for the U.S. at 39.1% was the second-highest within the OECD, exceeded only marginally by Japan in 2008. The OECD average for the same year was 26.3% (Source: The Tax Foundation website; available at: http://www.taxfoundation.org/research/show/23473.html ). Further, the combined U.S. average rate for tax on dividends that combines the effects of the Corporate Tax and Income Tax is 49.6%, and only two countries, Denmark and France had a rate above 50%. (Source: OECD website; available at: http://www.oecd.org/document/60/0,3343,en_2649_34533_1942460_1_1_1_37427,00.html#cc)

Secondly, there is an even more interesting way of looking at this issue. Auerbach (2008) has pointed out that taxation of capital is not uniform. In addition to the well-known issue of double taxation of dividend income and the existence of the non-corporate sector, exemptions, thresholds, tax shelters and differential treatment of different forms of capital income ensure that the effective tax rate after including the excess burden of distortions due to non-uniform tax treatments is higher than the average tax rate assuming uniform taxation. Auerbach (1989) calculates that this differential could be of the order of 9% of capital income in the U.S.

Therefore, a reduction in the effective tax rate could take the form of a rationalization of the tax structure to remove distortions, and this could mean a reduction in the effective tax rate on capital income. Perhaps this rate is of the order of 5%, even if the complete distortion is not removable, tax reform could seek to rationalize capital

income taxation in a way to make it uniform and reduce distortions, and have the same effect in practice as reducing the tax rate on an imaginary uniform rate applied to all capital. Note that it does not apply to a specific tax on one form of capital, such as the corporate income tax, but rather to taxes on income from capital. This includes income taxes on capital income (dividends, profits and interest), property taxes on that part of assessable property value not accounted for by land and tax on capital gains. A government may seek to eliminate all the differential tax.

Since we have abstracted away from risk differentials and other distortions including transport costs, the only difference between prices in OECD and ROW is the tax wedge. This need not be interpreted literally as saying that one country has a tax and the other has none, it is perfectly reasonable to say that it represents only the difference in the average tax rate between the two countries. Since there are no exemptions, the average rate also equals the marginal rate.

In the capital income tax model, we have made the assumption that the difference between OECD and ROW is the differential tax on capital income. There is no difference in the rates of taxation of labor income. Let us proceed with the assumption that this difference is $T_{ko}$ = 5%. We can treat this not as the actual difference, but as one that is hypothesized. If we are willing to assume that the policymaker is interested in removing half this differential, we can further assume that $dT_{ko}$ = - 2.5%. This policy combination could also accommodate any other ratio of the total differential.

In practice, the differential may be greater or less than 5% (as discussed above), and the change aimed at may also be greater or less than the differential assumed. The

equations have been derived in general form and any combination of numbers can be used. Any combination is possible, although changes in the average tax rate of an order greater than 5% will be extremely rare for a large country. This also implies that in the case where the country tries to get rid of the entire estimated differential in the average tax rate at one go, $T_{ko} = - dT_{ko}$.

This set of assumptions and data are enough to derive several key variables in our analysis, given our assumption that all prices were equal to 1 to begin with (without taxes).

**RATIOS AND SHARES USED IN OECD AND ROW FOR CIT**

In the previous chapters on the capital income tax we had defined $\theta_{kx} = \frac{P_k*(1+T_{ko})*K_x}{P_x*X}$, $\theta_{lx} = \frac{P_{lo}*L_x}{P_x*X}$, $\theta_{ky} = \frac{P_k*(1+T_{ko})*K_y}{P_y*Y}$, $\theta_{ly} = \frac{P_{lo}*L_y}{P_y*Y}$, $b = \theta_{kz} = \frac{P_k*K_z}{P_z*Z}$ and $(1\text{-}b) = \theta_{lz} = \frac{P_{lr}*L_z}{P_z*Z}$

where $L_z = \overline{L_z}$

Now we can use the ratios in our table to get:

$$\theta_{kx} = \frac{P_k*(1+T_{ko})*K_x}{P_x*X} = \theta_{ky} = \frac{P_k*(1+T_{ko})*K_y}{P_y*Y} = 1/3$$

$$\theta_{lx} = \frac{P_{lo}*L_x}{P_x*X} = \theta_{ly} = \frac{P_{lo}*L_y}{P_y*Y} = 2/3$$

$$\frac{X}{X+Y} = 1/5 \quad \text{and} \quad \frac{X}{Y} = 1/4 \text{ and } \frac{X+Y}{X+Y+Z} = 3/10 \text{ and } \frac{(1+T_{ko})*(K_x+K_y)}{(1+T_{ko})*(K_x+K_y)+K_z} = 3/10$$

This implies that:

$\frac{X+Y}{Z}$ = 3/7 and $\frac{K_x+K_y}{K_z} = \frac{3}{7*(1+T_{ko})}$

We also know from our tables above that $\frac{P_k*(1+T_{ko})*(K_x+K_y)}{P_x*X+P_y*Y}$ = 1/3

And we had assumed $P_k$, $P_{lo}$ and $P_{lr}$ (factor prices) and $P_x$, $P_y$ and $P_z$ (output prices) are all equal to 1 and to each other to begin with.

Substituting X + Y = 3*Z/7 and $K_x + K_y = 3*K_z /7*(1+T_{ko})$ above, we get

$\frac{K_z}{Z} = b = \theta_{kz} = \frac{1}{3}$

This calculation is very sensitive to the assumption of the ratio of U.S. capital stock to world capital stock as well. If we had taken the WDI ratio of 0.22, we would have b = 0.50. That these shares are not so far off the mark for the ROW has some support from several sources such as ILO (2008) and Harrison (2002). They have generally found that the share of capital income in non-OECD countries is much higher on average than in developed countries, and has grown with globalization and increasing FDI. However, we have to remember that the observed share for the ROW will include the risk premium. If the risk premium is constant, we can isolate the return to capital from the risk free rate of return as b, and club the rest with labor income. This will reconcile the higher observed ratio in Harrison (2002) for capital share in the ROW with b = 1/3.

Next noting that we get (1- 1/3) = 2/3 = (1-b) = $\theta_{lz} = \frac{P_{lr}*L_z}{P_z*Z}$ where $L_z = \overline{L_z}$

And that $\frac{P_{Lo}*(L_x+L_y)}{P_x*X+P_y*Y}$ = 2/3 and that $\frac{X+Y}{Z}$ = 3/7, we get $\frac{L_x+L_y}{L_z}$ = 3/7

With some further simple manipulations we get:

$(\frac{K_x}{K_z})$ = 3/35*(1+$T_{ko}$) ; $(\frac{K_y}{K_z})$ = 12/35*(1+$T_{ko}$) and $(\frac{L_y}{L_x})$ = 4

$(\frac{K_x+K_y}{X})$ = 5/3*(1+$T_{ko}$) ; $\left(\frac{K_x}{K_x+K_y}\right)$ = 1/5; $\left(\frac{K_y}{K_x+K_y}\right)$ = 4/5 and $\frac{(Lx + Ly)}{X}$ = 10/3

$(\frac{K_x+K_y}{Y})$ = 5/12*(1+$T_{ko}$) ; $(\frac{K_x}{Y})$ =1/12*(1+$T_{ko}$) and $\frac{(Lx + Ly)}{Y}$ = 10/12 and $(\frac{K_y}{X})$ =

4/3*(1+$T_{ko}$)

By assumption, $\frac{dT_{ko}}{T_{ko}}$ = -1/2, and $\frac{dT_{ko}}{(1+T_{ko})}$ = - 0.02381 (any other combinations are

possible)

**ELASTICITIES OF INCOME USED IN THE MODELS**

The next sets of parameters we require are:

$Є xx$ , $Є xy$, $\frac{\partial X^r}{\partial M^r}$, $\frac{\partial X^o}{\partial M^o}$, $Є yx$ , $Є yy$ and $\frac{\partial Y^o}{\partial M^o}$

We have two further parameters other than these as well: the elasticity of output in the OECD country with respect to the public good. However, since this is ex-ante an "unknowable," we shall put this aside for the time being and review what we know about the other parameters. We can begin by reminding ourselves what they stand for:

$Є xx = [\frac{X^o}{X} * E^o_{xx} + \frac{X^r}{X} * E^r_{xx}]$ $\quad$ $Є xy = [\frac{X^o}{X} * E^o_{xy}]$ $\quad$ $Є yx = E_{yx}$ $\quad$ $Є yy = E_{yy}$

Since $X^r$ is the amount of X consumed by ROW, it is equivalent to OECD or U.S. exports. $X^o$ is the consumption of OECD tradables at home. As a proportion of GDP,

U.S. exports have averaged around no more than 12% in the 2000-06 periods, although according to Department of Commerce (2009), for 2008 the figure is 13.1%. In our model, there is no scope for a trade deficit, so even though we have U.S. imports averaging around 16-17% of GDP in the relevant periods, we have to assume that imports (or $Z^o$ in our model) equal exports or $X^r$. It should be pointed out that the model has been derived with sufficient flexibility to incorporate deficits, non-constant returns and risk premia, etc. The marginal propensities to consume can be derived from elasticities of income and some manipulation of identities. The elasticity of U.S. exports with respect to ROW income can be written as:

$\square^r_{x,m} = \frac{\partial X^r}{\partial M^r} * \frac{M^r}{X^r}$, so $\frac{\partial X^r}{\partial M^r} = \square^r_{x,m} * \frac{X^r}{M^r}$

Since the OECD country can consume only $X^o$, $Y^o$ (or Y) and $Z^o$, and there is no saving and no trade deficit, and prices $P_x$, $P_y$ and $P_z$ are = 1 to begin with, then we get:

$\frac{\partial X^o}{\partial M^o} + \frac{\partial Y^o}{\partial M^o} + \frac{\partial Z^o}{\partial M^o} = 1$, and that the U.S./OECD import elasticity with respect to income is

$\square^o_{z,m} = \frac{\partial Z^o}{\partial M^o} * \frac{M^o}{Z^o}$, so $\frac{\partial Z^o}{\partial M^o} = \square^o_{z,m} * \frac{Z^o}{M^o}$, we get $\frac{\partial X^o}{\partial M^o} + \frac{\partial Y^o}{\partial M^o} = 1 - \frac{\partial Z^o}{\partial M^o}$

The empirical trade literature has been interested in estimating price and income elasticities of trade for a long time, and most researchers point to the seminal early contributions for the U.S. starting with Orcutt (1950), Adler (1945), Chang (1945), Ball and Marwah (1962), Houthakker and Magee (1969), Leamer and Stern (1970) and Stern et al. (1976). A recent comprehensive study, including discussion of theoretical issues and empirical estimation is available in Marquez (2002). Mann and Pluck (2007) provide a literature review of the development of these models, and note that the single log linear

equation often used in the past also assumes that domestic and foreign tradable goods are also imperfect substitutes, as we have. This is in line with the vast literature documenting the "home bias" in consumption, Lewis (1999) and Karolyi and Stulz (2003) provide comprehensive surveys.

As pointed out by Mann and Pluck (2007), there has been some consistency in estimates for the U.S. for import and export income elasticities:

"Houthakker and Magee estimated the U.S. income elasticity for total imports of 1.7 (auto-correlation corrected estimate in the appendix) and the foreign income elasticity for U.S. exports at around 1. In their survey of import and export demand elasticities for the United States, Sawyer and Sprinkle (1996) find income elasticities for total merchandise imports ranging from 0.1322 (Welsch 1987) to 4.028 (Wilson and Takacs 1979). Estimates for foreign income elasticities for U.S. exports do not vary quite as much; still they range from 0.374 (Stern, Baum, and Greene 1979) to 2.151 (Wilson and Takacs 1979). The median (mean) estimate of the 24 studies on total U.S. imports referenced in Sawyer and Sprinkle is 2.02 (2.14). The median (mean) estimate of the 17 studies on total U.S. merchandise exports referenced in Sawyer and Sprinkle is 1.12 (1.02). In one of the more recent studies, Hooper, Johnson, and Marquez (2000) find that the long-run income elasticities for U.S. exports and imports are 0.8 and 1.8, respectively, and are stable over time" ( Footnote 1 on pg 4).

The regularity observed by them is that:

"The sizes of the coefficients on income and relative price vary greatly by study, time period, countries analyzed, coverage of commodity groups, and as to whether different or additional explanatory variables are in the model. Most studies estimate that the income

elasticity for U.S. exports is smaller than the income elasticity for U.S. imports and in this regard replicate the earliest and most well-known finding by H.S Houthakker and Stephen Magee. Subsequent studies often estimate higher export and import elasticities than the original findings but surprisingly find that the *ratio* of the import to export elasticity varies relatively little from the 1.7 found by Houthakker and Magee in 1969." (Pg 4)

The main results found are that the ratio of 1:1.7 for export to import elasticity persists, and that the average of long-term and short-terms elasticities for different studies and methodologies yields a figure somewhere near 1.2 for exports, and 2.0 for imports. Both Marquez (2002) and Mann and Pluck (2007) found that correction for the use of GDP and GDP related price indices as well as introduction of other variables and methods of calculations substantially reduce the estimation of short and long run elasticities. These estimates are on the higher side for short-run elasticities, much closer to those we would see in the long-run. They are also very close to an average of U.S. elasticities for the short and long run reported in Marquez (1990), Marquez, Hooper and Johnson (2000) and Senhadji (1998) and Senhadji and Montenegro (1999).

We then come to the issue of $\frac{\partial X^o}{\partial M^o}$ and $\frac{\partial Y^o}{\partial M^o}$. These are the U.S./OECD propensities to consume tradables and nontradables respectively. When multiplied by the inverse of income shares, we get income elasticities. A comprehensive survey of the relative sizes for the U.S. is available in Schettkat and Yocarini (2006). Several authors have classified tradables (main component being agriculture and manufactured goods) as "necessities." The fact that the function relating tradables consumption to income has a

positive intercept at zero income for subsistence implies that the income elasticity of consumption of tradables has to be less than one.

Similarly, since nontradables include mainly services, the income elasticity for this category is thought to be above one. This view (implying non-homothetic preferences) is supported by Bergstrand (1991), Kravis et al. (1982), Hunter and Markusen (1988) and deGregorio, Giovaninni and Wolf (1994) as well. Schettkat and Yocarini (2006) provide several detailed estimates for the U.S. over time by different authors for the two elasticities. The most recent estimates quoted therein, by Kalwij et al. (2006) have a tradable elasticity of 0.8 and a nontradable elasticity of budget expenditures for the U.S. of 1.38. Some care has to be taken while using these values since services has been used for nontradables and budget expenditure for income.

However, we must be careful in using these data. If the income elasticity of tradables is taken as 1.3, and the income elasticity of imports taken as 2.0, we can see that given the low residual weight of domestic tradables, it will take negative income elasticity for the weighted income elasticities to sum to one. It might be that this is precisely due to the growth of the U.S. deficit and imports over time that such data estimates have arisen, but in our model we cannot have a trade deficit without a change in relative prices that will force alignment; a deficit cannot persist.

Therefore, if we believe that the import elasticity estimate is higher than unity, and that the share of nontradables does not decline over time, we have to assume that the marginal propensities to import stays at no more than 0.14 to avoid a fall in the share of domestic nontradables, a sharp rise in the real exchange rate and a fall in the domestic terms of trade against nontradables prices.

Even if the elasticity for nontradables is taken at unity, it has to be higher than the elasticity of domestic tradables (which is less than one) whenever the elasticity of imports is greater than one and the share of total tradables cannot rise. The neutral course gives us MPCs of 0.12, 0.8 and 0.08 for imports, nontradables and domestic tradables respectively, corresponding to elasticities of 1.0 for all leaving relative shares unchanged, and does not affect the trade balance. To avoid choosing one strand of the literature over another, and to also avoid choosing between long and short run elasticities, it is best to assume neutrality: elasticities of income are such that trade will continue to be balanced and relative shares unchanged. If all income elasticities are assumed to be = 1 (we can think of this as a base, or neutral case), this gives us $\frac{\partial X^o}{\partial M^o} + \frac{\partial Y^o}{\partial M^o} + \frac{\partial Z^o}{\partial M^o}$ = 1 = 0.08 + 0.8 + 0.12.

The non-constant share of U.S. services over time assumption is not without critics as well as supporters. Summers (1985), Baumol (2001) and Heston et al. (2002) have all maintained that the share of services for the U.S. has been constant, and indirectly challenged the luxury- necessity classification.

The choices we made are: $\frac{\partial X^o}{\partial M^o}$ = 0.08, $\frac{\partial Y^o}{\partial M^o}$ = 0.8 and $\frac{\partial Z^o}{\partial M^o}$ = 0.12

Thus, the choices above have allowed us to retain 3 features: (1) Share of services (nontradables) have not declined (2) share of imports have not increased and the trade balance is not disturbed – it should be noted that we could have built in this feature as well, one extra equation would be required for trade balance, and the income elasticity of imports would have been greater than one, and the income elasticity of domestic tradables consumption less than one (3) income elasticity of nontradables is the same as for domestic tradables. In fact, we have taken all elasticities as 1.

This further allows us to automatically satisfy a proposition required by Walras's Law (Mas-Collel et. al. 1999). The weighted sum of income elasticities in a Walrasian system of demand equations must add to one. As long as the total weights add to one, this requirement is met when all income elasticities are equal to one. Given that exports are 12% of OECD/U.S. output and that U.S./World output is 30%, we get $\frac{X^r}{M^r}$ = 0.036. This implies in turn that $\frac{\partial X^r}{\partial M^r} = \square^r_{x,m} * \frac{X^r}{M^r}$ = 1.0*0.036 = 0.036.

**OWN-PRICE ELASTICITIES USED IN THE MODELS**

Now we can turn to the price elasticities we require. First, to relate these to Harberger (1962) we need to note that the elasticities we have in previous chapters are Marshallian and not the Hicksian ("Harbergerarian") elasticities used in Harberger (1962). Second, in Harberger (1962), since there were two sectors, the prices of the two commodities were expressed as a relative price: $\frac{P_x}{P_y}$ . Thus there is a single elasticity in relative prices and this obviates the need for a separate own-price and cross-price elasticity. Technically, the same could be done in this model.

With $P_z$ as the numeraire, both $P_x$ and $P_y$ are expressed relative to $P_z$ with the third relationship: relative price between X and Y, assuming no change in the relative prices with respect to Z. International economists have usually termed the relative price of imports and exports ($Z^o$ and $X^r$) the "real exchange rate," and the relative price of tradables and nontradables (X and Y) the domestic or internal "terms of trade," although the distinction is not always explicit in the literature since many models do not have all three sectors. However, we have chosen to have Marshallian elasticities in own and cross prices, all relative to the numeraire.

Second, these elasticities are elasticities of demand and not substitution or “Armington” elasticies. A short survey of the literature will help us decide on possible values of these parameters. We note that given certain assumptions, there is a relationship between the elasticity of substitution in demand and the price elasticities as discussed by Gravelle (2008). Neglecting outside substitutes, and considering Hicksian elasticities, she shows that the elasticity of substitution in consumption between two goods is equal to either the sum of the two own price elasticities, or the difference between an own price and cross price elasticity. This also implies a relation between the cross and own price elasticties. Taking the sum of the two own price elasticities (Hicksian), as the substitution elasticity, and noting that they both have to have the same sign, this has to imply that a high value of this elasticity implies that the two goods considered are almost perfect substitutes in consumption. While we have maintained that capital is perfectly mobile, we have not made any such assumption yet in consumption between any of the three sectors.

Krugman (1989), using data for the 1970s and 1980s, finds the U.S. export elasticities with respect to relative prices to be low, with absolute magnitudes less than 0.5. Mann and Pluck (2007) comparing two methods in their own study and two previous studies report elasticities of relative price for both exports and imports with the correct sign, but extremely low magnitudes, most cases the elasticities are well below 0.5. They also do not find, unlike previous authors that export price elasticities facing the U.S. are higher than U.S. import elasticities. Senhadji and Montenegro (1999) report average short-run export demand price elasticities of -0.21, and average long-run elsticities of -1.0. The sample includes 53 countries for the period 1960-93. Senhadji (1998) finds for a sample of 77 countries that the short-run price elasticity for imports is also low, at around

- 0.26. Though long-run price elasticities have been found to be much higher by most researchers for both incomes and prices,

Senhadji (1998) points out that the time it takes for a short run elasticity to reach 90% of its long-run value is on average around 5 years. Hooper and Marquez (1995) find an average price elasticity of total imports for the U.S. of - 0.5, and which goes down to - 0.3 when services are included. Hooper, Johnson and Marquez (2000) find short-run elasticities for the U.S. for both imports and exports close to - 0.5.

Goldstein, Khan and Officer (1980) find that most previous calculations generally aggregate tradable and nontradable goods while calculating import price elasticities. While the existence of the three separate categories of purchases in the bundle implies that there is a three way relationship, that imports and domestic tradables are substitutes, imports and nontradables are substitutes, this does not say automatically that domestic tradables and nontradables are also substitutes. If foreign tradables and domestic tradables are substitutes, and domestic tradables and nontradables are weak complements, then foreign tradables and domestic nontradables could still be Marshallian substitutes. As expected, they find that imports have the expected negative price elasticity of around - 0.7 for the U.S. with respect to the prices of domestic tradables, but the relationship between domestic tradables and nontradables is not so clear.

Bems (2008); notes that several researchers using a CES framework have used the elasticity of substitution in consumption between tradables and nontradables as a parameter in their research. Following the earlier literature estimating directly he uses a less than unitary elasticity, since a rising relative price level as well as a rising expenditure share on nontradables implies such a less than unitary elasticity. Given the

relation in Gravelle (2008) this implies that the own price elasticities of the right sign (Hicksian) must significantly be less than unitary elasticity as well. However, as we have noted elsewhere, the finding that the relative share of nontraded goods has increased is not without its critics either.

Matsumoto (2007), in a recent survey of findings defines the elasticity of substitution between traded and nontraded goods in consumption as $\theta > 0$ and the elasticity of substitution between foreign and domestic traded goods ( the Armington elasticity) as $\omega > 0$. He finds that

"The elasticity of substitution between nontraded goods and traded goods, $\theta$, is set to be 0.7. Ostry and Reinhart (1992) estimate $\theta$ in the range 1.22-1.28 for all regions and 0.66 - 1.44 for each individual region. Stockman and Tesar (1995) find that $\theta = 0.44$ and claim that $\theta$ tends to be low among industrialized countries. Mendoza (1995) estimates $\theta = 0.74$ for industrialized countries. While $\theta$ can potentially alter the moments in general, given other parameter values in this section, important correlations hardly responds to changes in $\theta$." (pp 15-16)

His position on the Armington is not readily apparent from the paper, however. He has cited the range of 0.6 to 0.8 as reasonable in the literature, but cited estimates of 1.5 as well as 2.0 in various contexts in the same paper. Hunt (2009) using the IMF GEM model for Australia and New Zealand uses an elasticity of substitution for manufactures and nontradables of 0.5. Bergstrand (1991), as discussed above, in addition to preferring non-homothetic preferences and income elasticities for tradables (necessities) less than one, and for nontradables (luxuries) greater than one, finds that the elasticity of substitution in consumption between tradables and nontradables is around 0.9.

**CROSS-PRICE ELASTICITIES USED IN THE MODELS**

The final issue in the price elasticities section concerns cross price elasticities. Carlton and Perloff (2005) and Hubbard and O'Brien (2008) lay out some basic characteristics of these. First, Hicksian compensated cross-price elasticities are symmetric, Marshallian cross price elasticities need not be equal and symmetric. This is clear when we note that the Marshallian elasticity includes the income effect. The Slutsky decomposition (Mas-Collel et al. 1999) reveals that the weighted income effect of a change in the price of Y (Nontradables) on consumption of X (Tradables) is greater simply because of the greater weight of nontradables in consumption, even if we assume the marginal propensities to consume are equal.

The Hicksian substitution effect implies that when the prices of nontradables rise (fall), more (less) tradables are consumed since their relative prices are lower (higher) and less (more) nontradables are consumed. However, higher prices of nontradables imply a greater effect on the budget since 80% of consumption is nontradables. Thus less of both goods will be consumed, reversing the substitution effect of nontradables on tradables. The dampening effect of a price change in tradables should be lower simply because the relative share is lower. All this is further complicated by the fact that marginal propensities to consume out of income may not be the same for both categories.

Also, noting that Єxy = [ $\frac{X^o}{X} * E^o_{xy}$ ] but Єyx = $E_{yx}$, we further find that one of the elasticities is share weighted. Further, it is not quite clear that tradables and nontradables are Marshallian substitutes. Given that domestic and foreign tradables are more likely to be better substitutes for each other than nontradables and tradables of any kind it is far more likely that domestic tradables and nontradables are weak complements and foreign

goods and domestic tradables are substitutes. This is further complicated by the fact that in a Walrasian system, the weighted sum of cross price elasticities and expenditure share for any commodity bears a specified relation (Mas-Collel et. al. 1999): $B_x{*}\ E_{xx} + B_y{*}E_{yx} + B_z{*}E_{zx} + B_x = 0$ where B is the share in total (world) income, and this holds separately for any good. Also, $B_x{*}E_{xy} + B_y{*}E_{yy} + B_z{*}E_{zy} + B_y = 0$ and $B_x{*}E_{xz} + B_y{*}E_{yz} + B_z{*}E_{zz} + B_z= 0$ have to hold as well for the chosen values where $B_k$ refers to the share of the good K in OECD consumption (since consumption = income and the budget constraint has to hold separately for each country).

It is a well-known argument in anti-trust theory that the narrower the definition of the market, the higher the demand elasticity is likely to be (Carlton and Perloff, 2005). A broad definition such as tradables and nontradables will yield lower absolute price elasticity than a narrow definition within industries. It is not even clear that once income effects are taken into account, the goods remain substitutes.

Heim (2007) finds that:

"Rising exchange rates can lower prices on imported consumer goods. The lower prices have two effects. A substitution effect shifts in demand from domestically produced goods to imports. An income effect also allows more import purchases. It also allows some income previously spent on imports to be shifted to domestic spending. This shift may or may not increase total demand for U.S. consumer goods. This paper finds it does, and that increases in demand for domestically produced consumer goods and services are about five times as large as the increase in demand for imported consumer goods and services. The paper also finds that the increase in demand for domestic goods is about

three times as large as the increase in the trade deficit resulting from the higher exchange rate" (pg 1).

Therefore, given the Gravelle (2008) relation, and the evidence on demand price elasticities discussed above (for tradables, imports and nontradables), the two own price elasticities are taken to be - 0.5 for tradables and - 0.8 for nontradables. This serves as an average of short and long run elasticities found in the literature. Textbooks such as Hubbard and O'Brien (2008) recommend that luxuries should have higher absolute price elasticity than necessities, given the evidence above such a guess is made while noting that these elasticities are somewhere at the mid-point of short and long run elasticities, but the extent to which nontradables are more elastic is debatable. The model itself, it should be noted is sensitive to elasticity assumptions that incorporate this assumption. The cross price elasticities are likely to be low.

If domestic tradables and nontradables are Marshallian complements, then the elasticities are negative and income effects complement the substitution effect. There is still no reason to believe that X and Y are good substitutes or good complements. If X and Y are complements, and the Hicksian cross-effects are equal, (say they are equal to -0.1 given very weak complementarity) we have a higher income effect of Y on X balanced by the weighting of X, but the smaller income effect of X on Y is unweighted. We have assumed that the income effects are more important, given that the literature allows for traded goods to be both substitutes and complements.

Following the literature also does not help us much in choosing the size of cross effects since there is only one study in my knowledge where (Goldstein, Khan and Officer, 1980) where the three sector model has been explicitly used to derive elasticities.

Therefore we proceed with $Єxx = - 0.5$ and $Єyy = - 0.8$, and we say that X and Y are gross complements, with $Єxy = - 0.3$ (Implying $E^{o}_{xy} = - 0.75$) due to the stronger income effect and weighting, and $Єyx = - 0.1$ with no weighting and a much smaller income effect. This allows us to retain foreign and domestic tradables as weak substitutes, and keep our Walrasian regularity condition equations intact. We have also used $E_{xz} = 0.5$ in the OECD (Marshallian substitutes) and $E_{yz} = - 0.1$.

**PRODUCTIVITY OF GOVERNMENT EXPENDITURE**

The very last set of issues is the values of the elasticities of output with respect to government input. How productive is government expenditure? The issues arise with respect to expenditure at the margin, and the type of expenditure. Without knowing something about the programs likely to be cut first when tax revenues fall, we cannot conclude anything about the productivity of government expenditure for the U.S. keeping other factors constant at the margin, or about how the elasticity would vary across tradables and nontradables. We could proceed therefore assuming that the elasticity is less than one (0.5) to begin with and is the same for both tradables and nontradables, noting that different values could be used depending on what we think the efficacy of expenditure is at the margin.

We have to remember that when increasing returns to scale prevail, the partial elasticities of output with respect to all factors add up to more than one, but how much more than one at the margin is an unknown. If we agree with the tax reduction lobby, then surely it is the case that they think of the elasticity of government input at the margin to be substantially less than one and that total revenues will increase with a rate reduction.

**RATIOS FOR CONSUMPTION TAX MODELS**

We now have to recalculate some of the ratios for our consumption tax models. Where we use different figures, we will specify them, but where we do not, we assume that they are the same as in the capital income tax case.

First, the revenue neutral rate $T_{Co}$ is 1/3 of the capital income tax rate $T_{ko}$ since it is applied to all consumption and not just capital. We had taken $T_{ko}$ to be 0.05, so this implies that $T_{Co}$ = 0.016667. We assume that $\theta_{kx} = \frac{P_k * K_x}{P_x * X}$, $\theta_{lx} = \frac{P_{lo} * L_x}{P_x * X}$, $\theta_{ky} = \frac{P_k * K_y}{P_y * Y}$, $\theta_{ly} = \frac{P_{lo} * L_y}{P_y * Y}$ $b = \theta_{kz} = \frac{P_k * K_z}{P_z * Z}$ and $(1\text{-}b) = \theta_{lz} = \frac{P_{lr} * L_z}{P_z * Z}$ where $L_z = \overline{L_z}$

Now we can use the ratios in our table to get with assumptions modified where necessary:

$\theta_{kx} = \theta_{ky} = 1/3$

$\theta_{lx} = \theta_{ly} = 2/3$

$\frac{X}{X+Y} = 1/5$ and $\frac{X}{Y} = 1/4$ and $\frac{(1+T_{Co})*(X+Y)}{(1+T_{Co})*(X+Y)+Z} = 3/10$ and $\frac{K_x+K_y}{K_x+K_y+K_z} = 3/10$

This implies that:

$\frac{X+Y}{Z} = \frac{3}{7*(1+T_{Co})}$ and $\frac{K_x+K_y}{K_z} = \frac{3}{7}$

We also know from our tables above that $\frac{P_k*(K_x+K_y)}{(P_x*X+P_y*Y)} = 1/3$

And we had assumed $P_k$, $P_{lo}$ and $P_{lr}$ (factor prices) and $P_x$, $P_y$ and $P_z$ (output prices) are all equal to 1 and to each other to begin with.

Substituting X + Y = $\frac{3*Z}{7*(1+T_{Co})}$ and Kx + $K_y$ = $\frac{3}{7}$*$K_z$ above, we get

$\frac{K_z}{Z}$ = b = $\theta_{kz}$ = $\frac{1}{3*(1+T_{Co})}$

Next noting that we get (1- 1/3*$(1+T_{Co})$) = 2/3*$(1+T_{Co})$ = (1-b) = $\theta_{lz}$ = $\frac{P_{lr}*L_z}{P_z*Z}$ where

$L_z$ = $\overline{L_z}$

And that $\frac{P_{Lo}*(L_x+L_y)}{(P_x*X+P_y*Y)}$ = 2/3 and that $\frac{X+Y}{Z}$ =$\frac{3}{7*(1+T_{Co})}$, we get $\frac{L_x+L_y}{L_z}$ = 3/7

With some further simple manipulations we get:

$(\frac{K_x}{K_z})$ = 3/35 ; $(\frac{K_y}{K_z})$ = 12/35 and $(\frac{L_y}{L_x})$ = 4

$(\frac{K_x+K_y}{X})$ = $\frac{5}{3}$ ; $\left(\frac{K_x}{K_x+K_y}\right)$ = 1/5 ; $\left(\frac{K_y}{K_x+K_y}\right)$ = 4/5 and $\frac{(Lx+Ly)}{X}$ = $\frac{10}{3}$

$(\frac{K_x+K_y}{Y})$= $\frac{5}{12}$; $(\frac{K_x}{Y})$= $\frac{1}{12}$ and $\frac{(Lx+Ly)}{Y}$ = 10/12 and $(\frac{K_y}{X})$= $\frac{4}{3}$

By assumption, $\frac{dT_{Co}}{T_{Co}}$ = -1/2, and $\frac{dT_{Co}}{(1+T_{Co})}$ = - 0.081967 (any other combinations are possible). To get $T_c$ we note that 100*(1+$T_{Co}$)*(1- $T_c$ ) = 100, so $T_c$ in our case = 0.016393.

The marginal propensities to consume are all the same as in the previous section, as are the figures for elasticities of output and demand. We need two extra elasticities, which are $\frac{X^o}{X}$ $E^o_{xz}$ and $E_{yz}$. We take $E^o_{xz}$ as = + 0.5 and $E_{yz}$ = - 0.1. This does not violate any

of the restrictions from the Walrasian conditions, and we note that the literature can support reversing the signs of the some of the cross price elasticities as well.

## EFFECTS OF TAX CHANGES ON TERMS OF TRADE

The final issue that we need to highlight before we proceed to discuss results is to remind ourselves of the link between the uses side of income and trade theory, which is also the source of this type of general equilibrium theory. As discussed in Randolph (2006), in a world with no international investment and no international mobility of capital, the relative prices of goods between countries can also serve as a vehicle to transmit burdens internationally. We also remind ourselves that not all authors support the uses approach to measuring income as a proxy for welfare. An extensive critique of this approach is available in Whalley (1984) who feels that using the uses and sources of income with an appropriate choice of parameters allows a researcher to generate almost any result desired. However, when we use the price of Z ($P_z$) as our numeraire, we express OECD prices relative to the world price. For at least the price of OECD tradables, this allows us to account for effects of taxation that are conveyed through the terms of trade, and not through input prices alone.

Randolph (2006) discusses the results in Melvin (1982); a model where there is no international investment but there is international trade, and both capital and labor are immobile internationally. The economy is a small open economy and cannot affect international prices. The analysis shows that when the small open economy imposes a corporate income tax the burden of this tax can be shifted to domestic labor. He also finds that in the open economy case the terms-of-trade effect could partially offset the tax effect.

Whalley (1980) considers a general equilibrium model of the U.S., the 9 member EEC and Japan to analyze the effects of removing distorting domestic factor taxes. In his model, factor supplies are fixed by country, and he does not assume an international capital market where the rate of return to capital is equalized across trading blocks. He finds that "For the United States existing factor taxes yield a significant terms-of-trade gain. Results indicate that the abolition of existing discriminatory features of distorting taxes in the United States, while leading to a domestic gain in production efficiency, would lead to welfare losses because of the movement in the terms of trade against the United States. A similar feature is also present for Japan, although it is not as pronounced. In the EEC, this terms-of-trade effect, while present, is milder; small gains occur from the removal of distortions in factor taxes with losses occurring when capital tax distortions are removed." (pg 1200)

In a more recent work Whalley (2002)[2] has further discussed the role and mechanisms by which different types of taxes (trade and non-trade) can have international effects. While trade taxes obviously are thought of as causing international burdens, he points out that non-trade taxes can also have international burdens; "Non trade taxes impact trade indirectly through forward shifting of production taxes into costs, including taxes on inputs such as fuels. The more that such taxes are backwards shifted, the smaller their impacts on trade" (pg 29)

Therefore, it should be clear that using the sources and uses of income to analyze non-trade taxes not only has a history, it helps capture effects of non-trade domestic taxes including factor taxes that work on the international terms of trade (in our models the

[2] Whalley, J. (2002) "*Taxes and Trade*" (revised draft). Available online at: http://siteresources.worldbank.org/INTTPA/Resources/WhalleyPaper.pdf, accessed on 6/1/2010.

uses of income relative price indices) as well as on production and input prices (the sources of income). The results presented in the next chapter should be viewed with this aspect in mind.

## PARAMETERS USED TO DERIVE SOLUTIONS

**Table 10 Summaries of Parameters Used in CIT Models, Sources and Simulations**

| PARAMETER | SOURCE | SYMBOL | INITIAL VALUES | ALTERNATIVE VALUES |
|---|---|---|---|---|
| Tax inclusive share of K in X | Valentinyi and Herrendorf (2008) | $\theta_{kx}$ | 1/3 | None |
| Share of L in X | Valentinyi and Herrendorf (2008) | $\theta_{lx}$ | 2/3 | None |
| Tax inclusive share of K in Y | Valentinyi and Herrendorf (2008) | $\theta_{ky}$ | 1/3 | None |
| Share of L in Y | Valentinyi and Herrendorf (2008) | $\theta_{ly}$ | 2/3 | None |
| Elasticity of X with respect to G | Assumed | $\frac{G}{P_x * X} * \frac{\partial f}{\partial G}$ | 0.5 | 0.3, 0.1 |
| Elasticity of X with respect to G | Assumed | $\frac{G}{P_y * Y} * \frac{\partial g}{\partial G}$ | 0.5 | 0.3, 0.1 |
| Differential CIT in OECD | Assumed | $T_{ko}$ | 0.05 | None used |
| Change in CIT | Assumed | $dT_{ko}$ | -0.025 | -0.0125, -0.0375 |
| Share of K in ROW | Derived in chapter 5 based on BEA, WDI and Gravelle and Smetters | $\theta_{kz}$ | 1/3 | None |
| Elasticity of substitution of K and L in X | Klump, McAdam and Willman (2007) | $S_x$ | -0.6 | -0.8, -1.0 |
| Elasticity of | Klump, | $S_y$ | -0.6 | -0.8,-1.0 |

| | | | | |
|---|---|---|---|---|
| substitution of K and L in Y | McAdam and Willman (2007) | | | |
| Elasticity of OECD demand for X in terms of price of X | Based on literature survey above (average value) | $E^{o}_{xx}$ | -0.5 | -0.8 |
| Elasticity of ROW demand for X in terms of price of X | Based on literature survey above (average value) | $E^{r}_{xx}$ | -0.5 | -0.8 |
| Elasticity of OECD demand for X in terms of price of Y | Based on literature survey above (average value) | $E^{o}_{xy}$ | -0.75 | -0.7 |
| Marginal change in OECD demand for X with respect to change in OECD income | Based on literature survey above (average value) | $\frac{\partial X^{o}}{\partial M^{o}}$ | 0.08 | None |
| Marginal change in ROW demand for X with respect to change in ROW income | Based on literature survey above (average value) | $\frac{\partial X^{r}}{\partial M^{r}}$ | 0.036 | None |
| Elasticity of demand for Y with respect to price of X | Based on literature survey above (average value) | $E_{yx}$ | -0.1 | None |
| Elasticity of demand for Y with respect to price of Y | Based on literature survey above (average value) | $E_{yy}$ | -0.8 | None |
| Marginal change in OECD demand for X with respect to change in OECD income | Based on literature survey above (average value) | $\frac{\partial Y^{o}}{\partial M^{o}}$ | 0.8 | None |
| Weighted total elasticity of | Based on literature | $Єxx = [\ \frac{X^{o}}{X} * E^{o}_{xx}$ | -0.5 | -0.8 |

| | | | | |
|---|---|---|---|---|
| demand for X with respect to price of X | survey above (average value) | $+ \frac{x^r}{x} * E^r_{xx}]$ | | |
| Share weighted total elasticity of demand for X with respect to price of Y | Based on literature survey above (average value) | $\epsilon xy = [\frac{x^o}{x} * E^o_{xy}]$ | -0.3 | -0.28 |
| Elasticity of demand for Y with respect to price of X | Based on literature survey above (average value) | $\epsilon yx = E_{yx}$ | -0.1 | None |
| Elasticity of demand for Y with respect to price of Y | Based on literature survey above (average value) | $\epsilon yy = E_{yy}$ | -0.8 | None |
| OECD capital as proportion of X | Derived in chapter 5 based on BEA, WDI and Gravelle and Smetters | $(\frac{K_x + K_y}{X})$ | $5/3*(1+T_{ko})$ | None |
| OECD capital as proportion of Y | Derived in chapter 5 based on BEA, WDI and Gravelle and Smetters | $(\frac{K_x + K_y}{Y})$ | $5/12*(1+T_{ko})$ | None |
| OECD labor as proportion of X | Derived in chapter 5 based on BEA, WDI and Gravelle and Smetters | $\frac{(Lx + Ly)}{X}$ | 10/3 | None |
| OECD labor as proportion of Y | Derived in chapter 5 based on BEA, WDI and Gravelle and Smetters | $\frac{(Lx + Ly)}{Y}$ | 10/12 | None |
| Capital in X as proportion of Y | Derived in chapter 5 based on BEA, WDI and | $\frac{K_x}{Y}$ | $1/12*(1+T_{ko})$ | None |

| | | | | |
|---|---|---|---|---|
| | Gravelle and Smetters | | | |
| Capital in Y as proportion of X | Derived in chapter 5 based on BEA, WDI and Gravelle and Smetters | $\frac{K_y}{X}$ | $4/3*(1+T_{ko})$ | None |

**Table 11 Summaries of Parameters Used In Consumption Tax Models, Sources and Simulations**

| PARAMETER | SOURCE | SYMBOL | INITIAL VALUES | ALTERNATIVE VALUES |
|---|---|---|---|---|
| Tax exclusive share of K in X | Valentinyi and Herrendorf (2008) | $\theta_{kx}$ | 1/3 | None |
| Tax exclusive share of L in X | Valentinyi and Herrendorf (2008) | $\theta_{lx}$ | 2/3 | None |
| Tax exclusive share of K in Y | Valentinyi and Herrendorf (2008) | $\theta_{ky}$ | 1/3 | None |
| Tax exclusive share of L in Y | Valentinyi and Herrendorf (2008) | $\theta_{ly}$ | 2/3 | None |
| Elasticity of X with respect to G | Assumed | $\frac{G}{P_x * X} * \frac{\partial f}{\partial G}$ | 0.5 | 0.3, 0.1 |
| Elasticity of X with respect to G | Assumed | $\frac{G}{P_y * Y} * \frac{\partial g}{\partial G}$ | 0.5 | 0.3, 0.1 |
| Differential Consumption tax in OECD-tax exclusive rate | Assumed | $T_{Co}$ | 0.016667 | None used |
| Differential Consumption tax in OECD-tax inclusive rate | Assumed | $T_C$ | 0.016393 | None used |
| Change in | Assumed | $dT_{Co}$ | -0.00833 | -0.004167, |

| | | | | |
|---|---|---|---|---|
| Consumption tax – tax exclusive rate | | | | -0.0125 |
| Change in Consumption tax – tax exclusive rate as percentage of original rate | Assumed | $dT_c$ | -1/2 | -1/4, -3/4 |
| Share of K in ROW | Derived in chapter 5 based on BEA, WDI and Gravelle and Smetters | $\theta_{kz}$ | $\frac{1}{3*(1+T_{Co})}$ | None |
| Elasticity of substitution of K and L in X | Klump, McAdam and Willman (2007) | $S_x$ | -0.6 | -0.8, -1.0 |
| Elasticity of substitution of K and L in Y | Klump, McAdam and Willman (2007) | $S_y$ | -0.6 | -0.8,-1.0 |
| Elasticity of OECD demand for X in terms of price of X | Based on literature survey above (average value) | $E^o_{xx}$ | -0.5 | -0.8 |
| Elasticity of ROW demand for X in terms of price of X | Based on literature survey above (average value) | $E^r_{xx}$ | -0.5 | -0.8 |
| Elasticity of OECD demand for X in terms of price of Y | Based on literature survey above (average value) | $E^o_{xy}$ | -0.75 | -0.7 |
| Elasticity of OECD demand for X in terms of price of Z | Based on literature survey above (average value) | $E^o_{xz}$ | +0.5 | None |
| Elasticity of OECD demand for Y in terms of price of Z | Based on literature survey above (average value) | $E^o_{yz}$ | -0.1 | None |
| Marginal change in OECD demand for X with respect to change in | Based on literature survey above (average value) | $\frac{\partial X^o}{\partial M^o}$ | 0.08 | None |

| | | | | |
|---|---|---|---|---|
| OECD income | | | | |
| Marginal change in ROW demand for X with respect to change in ROW income | Based on literature survey above (average value) | $\frac{\partial X^r}{\partial M^r}$ | 0.036 | None |
| Elasticity of demand for Y with respect to price of X | Based on literature survey above (average value) | $E_{yx}$ | -0.1 | None |
| Elasticity of demand for Y with respect to price of Y | Based on literature survey above (average value) | $E_{yy}$ | -0.8 | None |
| Marginal change in OECD demand for X with respect to change in OECD income | Based on literature survey above (average value) | $\frac{\partial Y^o}{\partial M^o}$ | 0.8 | None |
| Weighted total elasticity of demand for X with respect to price of X | Based on literature survey above (average value) | $\epsilon xx = [\frac{X^o}{X} * E^o_{xx} + \frac{X^r}{X} * E^r_{xx}]$ | -0.5 | -0.8 |
| Share weighted total elasticity of demand for X with respect to price of Y | Based on literature survey above (average value) | $\epsilon xy = [\frac{X^o}{X} * E^o_{xy}]$ | -0.3 | -0.28 |
| Elasticity of demand for Y with respect to price of X | Based on literature survey above (average value) | $\epsilon yx = E_{yx}$ | -0.1 | None |
| Elasticity of demand for Y with respect to price of Y | Based on literature survey above (average value) | $\epsilon yy = E_{yy}$ | -0.8 | None |
| OECD capital as proportion of X | Derived in chapter 5 based on BEA, WDI and Gravelle and Smetters | $(\frac{K_x + K_y}{X})$ | 5/3 | None |

| | | | | |
|---|---|---|---|---|
| OECD capital as proportion of Y | Derived in chapter 5 based on BEA, WDI and Gravelle and Smetters | $(\frac{K_x + K_y}{Y})$ | 5/12 | None |
| OECD labor as proportion of X | Derived in chapter 5 based on BEA, WDI and Gravelle and Smetters | $\frac{(Lx + Ly)}{X}$ | 10/3 | None |
| OECD labor as proportion of Y | Derived in chapter 5 based on BEA, WDI and Gravelle and Smetters | $\frac{(Lx + Ly)}{Y}$ | 10/12 | None |
| Capital in X as proportion of Y | Derived in chapter 5 based on BEA, WDI and Gravelle and Smetters | $\frac{K_x}{Y}$ | 1/12 | None |
| Capital in Y as proportion of X | Derived in chapter 5 based on BEA, WDI and Gravelle and Smetters | $\frac{K_y}{X}$ | 4/3 | None |

# CHAPTER VI – RESULTS AND CONCLUSION

## The "Thought Experiments" From Chapters 1-5

To begin with, we shall briefly review what we did in the previous chapters and summarize the main points.

- We started by asking ourselves the question: if a large, open developed economy were to contemplate reducing its taxes to attract mobile capital from the rest of the world, would it benefit both the OECD country as well as the ROW? Since imposing a tax results in an excess burden, reducing the tax should lower the burden. Is the burden reduced for the ROW as well as the OECD, or does the large country corner a disproportionate share of the benefit at the cost of the ROW?
- We decided that to answer this question we first need to define the terms "spillovers" of the burden of the tax in the OECD country on the ROW ; and how we define and measure this "burden" (benefit) and see if they can be quantified in some way. We define "spillovers" of the burden of the OECD tax as a situation where; as a result of the OECD country lowering its tax rate to compete for mobile capital, the sources and uses measure of GDP for the ROW declines, thereby effectively spillovers a burden. We constructed a measure of the sources and uses of GDP for both the OECD and ROW to answer this question, and chose the simplest possible method of combining the sources and uses sides to arrive at the measure of "spillovers" of the OECD tax burden.

- We asked the following question: is the effect of lowering a tax on the relative sources and uses of GDP measure for OECD and ROW different for different taxes?
- We also asked: does the use to which the tax revenue (expenditure) is put to qualify the answer to the previous question?
- We set up four different models to answer the preceding questions. Two of the models used a tax on all capital in the OECD country and two used a tax on all consumption in the OECD country. For each tax, we used two different expenditure patterns; a passive government that returned all revenues to the OECD economy, and an active government that spent the revenues on a public input that increased productivity within the OECD country.
- After specifying the models, we searched for data based on the existing literature that could be used to represent a large, open OECD economy (large enough to influence the worldwide return to capital on its own). We considered the U.S. as the best example of this economy and chose data for the U.S. that we considered both reasonable and representative to provide numerical solutions to our models.
- We compare the results on the sources and uses of GDP for the OECD country and ROW from reducing the excess tax rate in the OECD by half. When the net impact is positive for any country, we surmise that it benefits, and vice-versa.
- We check whether the results are sensitive to changes in the some of the key data used including the size of the tax change. We find that imposing the very reasonable and standard restriction of neo-classical utility maximization and the HOD Zero demand curve that follows from it on the tradable goods sector in

OECD yield the most believable results. The results are not very sensitive to most other data assumptions tested.

- Under the HOD zero assumption, we find that the OECD country may be gaining at the expense of the ROW when it lowers its tax rate if the taxes are returned through transfer. When we consider the input models for both taxes, both the OECD and ROW lose when the tax is lowered if the government input is productive although in all cases the loss or gain to the OECD is proportionately much larger for the OECD country.
- We find that this "gain" to the OECD in the tax and transfer cases is less pronounced in the case of the consumption tax than the capital income tax, as should be expected since the factor tax is more distorting. The plausible reason for the losses to the ROW in all 4 cases and the gains to the OECD only in the transfer cases is that a productive government input lowers the cost of production, and when withdrawn increases it. Therefore, reducing taxes that reduce a productive input will not benefit either country. This effect on relative prices, similar to a terms-of-trade effect, counteracts many of the benefits of attracting more capital to the OECD country. The loss of capital in the ROW reduces its GDP and the productivity of its labor, and thus impacts it negatively in all four cases. Where no input is provided, the OECD country gains through effects on lower prices of output from lower taxes, while the ROW still loses overall since the loss of capital outweighs the benefits of lower import prices.
- It appears from the results in our models that correct specification of expenditure and estimation of its productivity should be important to answering this type of

question. While we are relatively exact on the tax side, since we model different taxes differently and relatively precisely, we tend to be less specific about the expenditure to be cut, its nature, and its impact on the production function. The models show that when government is active, the total impact depends on both aspects-benefits or burdens depend on correctly modeling both expenditure and taxes, and the taxes side alone should not be expected to yield the answer on its own.

- These results may explain another phenomenon of concern. Tax competition theory predicts a tax-cutting “race to the bottom” between jurisdictions competing for mobile capital. This has not happened recently since most OECD countries have retained relatively high tax rates. In our models, the OECD country did not benefit from cutting taxes when the revenue was used to provide an active government input. The benefits were marginal in the transfer cases. The ROW did not benefit in any case studied. If cutting tax rates implies a fall in the level of overall revenue and therefore of productive public goods, there is no incentive to cut taxes.
- Most models that have considered the case of the government that we am aware of are not in the Harberger format, or have not considered the case of a productive government input being cut when taxes are lowered. The general approach seems to have an implicit underlying assumption that the program to be cut is wasteful or unproductive expenditure. There are obviously merits in doing so, even without cutting taxes. The tendency has been to mostly model the effects of one action (either cutting taxes or wasteful expenditure).The fact is that in practice, there is

no guarantee that "pork" will be eliminated first, and that productive government inputs will not be when taxes are reduced. Once we allow for the fact that cutting taxes can be accompanied by reduction in expenditure of a productive public input, we get the result that the benefits from reducing taxation may be outweighed by the negative effect of losing a productive public good, and that this effect need not be confined to the country cutting its taxes. In addition to modeling the effects of taxation isolated from effects of expenditure, we need to study the combined effect of both.

## THE RESEARCH QUESTIONS

The specific research questions we asked in chapter one was:

(1) Can the extent of "spillovers" in the case of a capital income tax (CIT) and a consumption tax be estimated using reasonable assumptions and believable data in situations of tax competition? This involved two steps. First, we decided that we do not explicitly model various forms of tax competition, but instead choose a tax rate reduction in the OECD as a proxy for several major forms of tax competition. Second, we define what we mean by "spillovers". We do not model it as foreigners' paying the OECD tax, but as a net reduction in ROW GDP from the sources and uses sides as a consequence of the OECD country lowering its tax.

(2) How large are these effects we have defined above and how are they distributed between the taxing country and the rest of the world?

(3) Does the type of tax (consumption or tax on capital) or the use to which tax revenues are put to (expenditure assumptions) matter in measuring the extent and type of spillovers?

(4) Are results very sensitive to parameters used, or to reasonable restrictions on behavior?

(5) What policy implications may be drawn from the results?

The answer to our first question is yes. We note that we have analytical expressions for sources and uses side changes in welfare in both the ROW and OECD. Even without using actual data, it is clear that we have computed burdens and benefits for both.

The models derived analytically can accommodate several variations such as price inflation for government; non-constant returns to scale in capital and labor, a risk premium for capital, different factor shares and elasticities and even different functional forms. They are thus robust in this sense to different assumptions and analytical specifications. While parameters have been chosen keeping the existing literature in mind, this does not restrict the theoretical models since minimal assumptions have been made while deriving the equations and several variations are possible. Each reduced form model is a system of four equations in four unknowns and can be solved analytically.

The answer to the second question is also yes. To answer question two and make a firm prediction about what we can expect, however, we need to use data given the extremely complicated results. Given the choices made and the data available, we have simulated answers to the questions. We have used several simulations to check the

robustness of our results, and indicated where more are possible. The reasons for the answers to one and two are explained below.

The burden on the sources side of GDP is expressed for the ROW as: $dM^r = - K_x*\frac{dK_x}{K_x} - K_y*\frac{dK_y}{K_y}$ which is the same as $dM^r = - dK_x - dK_y$ regardless of the tax and expenditure type. This is an answer to the first question only at this stage. The ROW burden on the sources side of GDP is unambiguously negative as long as capital is attracted away by the importing country (OECD in this case); the burden is simply the amount of capital attracted away by the country lowering its taxes. We cannot say whether this is large or small. We also cannot be sure that capital is attracted away from ROW to OECD without knowing the relevant parameters since it is possible to generate the opposite movement by choosing extreme parameters in this model. Further, we also need to compare relative sizes of burdens and not just the signs. Both burdens could be negative, but one may be larger or smaller. This is not possible to gauge without parameters.

This specific expression for the sources side burden for the ROW is the consequence of how we have chosen to measure burdens, the choice of numeraire and other model choices. Other formulations would yield expressions that use other variables. Once we take the uses side into account, the relationship between capital outflow and ROW GDP is no longer so stark. If the price of the imported good falls to a very large extent, this could potentially make up for the loss of capital. Given the low overall weight of the imports in ROW total consumption, this is also very unlikely. We have not made the distribution of burdens and benefits between labor and capital the main focus of our

work and chosen to concentrate on aggregate country effects instead. The addition of the uses side could also have a significant impact on the measurement of the distribution of burdens within countries and between factors.

The simple additive version of the measure of sources and uses of GDP (our proxy for welfare) function that we use is:

OECD net sources and uses of GDP impact measure $W^o = 1 + dGDP^o/GDP^o - LAS^o$

and ROW net sources and uses of GDP impact measure $W^r = 1 + dGDP^r/GDP^r - LAS^r$

LAS is the Laspeyre's index of consumer price changes for each country and represents the impact on the uses of GDP. The Laspeyre's index does not measure Consumer's Surplus, only the weighted price change. Consumer's surplus changes combined with producer's surplus measures could have been used to measure welfare changes. Since we have production functions that are CRS in L and K and there is no producer's surplus in CRS, it is difficult to consider producer's surplus for the models with G alone. We have instead measured the sources and uses of GDP and added the two ratios instead of explicitly measuring changes in consumer's surplus. While we have tried to avoid any confusion between the two, wherever we use the term welfare to save space, it means the sources and uses of GDP and not consumer's surplus or any other version of "welfare." Our measure is closer to a measure of real income.

To answer questions three and four, we used the results from all four models and the data discussed in chapter 5. We present first the results for all 4 models using the baseline data discussed in chapter five. We will then elaborate the answers to the other research questions and proceed to test sensitivity to data assumptions.

**DISCUSSION OF RESULTS FROM THE BASELINE CASE**

We consider first the exercise: the OECD country has an excess tax rate of 5% for the CIT and an excess rate of 1.6667% for the consumption tax. In the baseline case, it seeks to cut in half this excess rate, and lowers the CIT by 2.5% points or the consumption tax by 0.83333% points. We compare results across taxes, i.e. the two transfer models with the two input models. Then, we compare what happens in the CIT model with input to the CIT model with transfer, and the consumption tax model with input with what happens in the consumption tax model with transfer.

The next step will be to impose a key restriction on the demand for X. When we drop the necessity-luxury classification and use an HOD zero restriction on the demand for X in OECD (the Y sector was already restricted to be HOD zero), the general direction and order of results changes and make intuitive sense. Since demand curves are usually constructed with this restriction in mind, we proceed to discuss the results using the HOD zero restriction as our preferred results, since they are more plausible than results obtained using the baseline data.

For the very first table (12), using baseline data we present the results in the following way: the impact of two taxes with transfer models are presented first. The results for the two taxes with input models are presented next (13). This is followed by repeating the main results with our preferred parameters, when X is HOD zero (14 and 15). We discuss what these results are and some implications.

Following this, we present the results of simulations using baseline data, and some simulations using the preferred data. The last section is used to discuss implications and possible future models.

We remind ourselves that "sources" in the header of each table stands for changes in the sources side of GDP and are measured by 1 + dGDP/GDP. When the figure in this column is less than one, it means the GDP has fallen on the sources side of income. "Uses" stands for the uses of GDP and stands for the change in the Laspeyre's price index for each country of consumer goods. For the OECD country, all 3 goods are consumed and enter the index, using base year quantities as weights. However, since we chose the price of Z ($P_z$) as our numeraire, there is a change only in the prices of X and Y relative to this price. For the ROW, only X and Z enter the index as Y is not consumed by ROW. The only price change is therefore the price of X.

When the number in any table under the uses side is less than one, it means that the prices of goods consumed have fallen, weighted by initial consumption. "Welfare" is defined as the net change in the sources and uses of GDP, and is equal to sources minus uses. Consider the case when the sources of GDP fall by 5%. This will be reflected as a 95% sources side measure, or 0.95. Suppose the uses side price index fell by 6%. The uses side final figure or (LAS) would be 0.94. The net result is 0.95 - 0.94 = 0.01. This basically indicates that relative consumption prices fell by more than real factor income, so the country should be better off in some sense.

Let us begin by considering the case of lowering a CIT with public input (first row of table 13). There is a 2.6% real improvement in the net sources and uses of GDP

for the OECD country and a 5.3% loss on the net sources and uses of GDP for the ROW. The hypothetical excess tax rate (the difference between the total tax rate in OECD and ROW assumed, to be 5%) was reduced by half from 5% to 2.5%. This implies a tax rate reduction of 2.5 percentage points for the CIT in the OECD country. The sources side burden was calculated as the proportional change in GDP for each country, or 1 + $dGDP^o/GDP^o$ for the OECD country and 1+ $dGDP^r/GDP^r$ for the ROW.

When the change in GDP is negative for any country, the sources side measure falls below one. On the uses side, if prices fall, then the LAS measure also falls below one. The difference between the sources side measure and LAS gives us the net change, i.e. have sources fallen more than uses. If instead prices rise, then the LAS measure becomes greater than one. If GDP has fallen, then this is subtracted from sources (less than one), making it an even larger negative number. If both increase to a number greater than one, the net result is negative whenever the price increase is relatively greater than the income increase.

**Table 12 Impact of Lowering the (Excess) OECD Tax Rate by Half (With Transfer) on Sources and Uses of GDP & Net Sources and Uses of GDP (Welfare) Using Baseline Data**

| **TAX and EXPENDITURE** | **OECD** | | | **ROW** | | |
|---|---|---|---|---|---|---|
| | *SOURCES* | *USES* | *WELFARE* | *SOURCES* | *USES* | *WELFARE* |
| **CIT and TRANSFER** | 0.976 | 0.979 | -0.0029 | 1.000 | 0.999 | 0.001225 |
| **CONSUMPTION and TRANSFER** | 0.991 | 0.991 | -0.0003 | 1.000 | 0.999 | 0.00013 |

**Table 13 Impact of Lowering the (Excess) OECD Tax Rate by Half (With Input) on Sources and Uses of GDP & Net Sources and Uses of GDP (Welfare) Using Baseline Data**

| TAX and EXPENDITURE | OECD | | | ROW | | |
|---|---|---|---|---|---|---|
| | *SOURCES* | *USES* | *WELFARE* | *SOURCES* | *USES* | *WELFARE* |
| **CIT and INPUT** | 1.3793 | 1.3535 | 0.02583 | 0.96805 | 1.0207 | -0.05261 |
| **CONSUMPTION and INPUT** | 1.3939 | 1.3193 | 0.07457 | 0.96792 | 1.01899 | -0.05107 |

The figure from the first row of the table 13 above in the sources column for OECD is 1.3793. This implies a 38% gain on the sources side for the OECD country. The figure 0.96805 from the first row of table 13 for the sources side of the ROW implies a 3% loss for the ROW. Next we consider changes in the uses of income, which we had defined to be the change in the relative cost-of-living index. Thus a figure of 1.3535 in the first row of table 13 for OECD implies that consumption prices increased 35% for OECD, and the corresponding figure of 1.0207 implies that consumption prices increased 2% for ROW.

We have used the simplest version of net change in the sources and uses of income-the difference of GDP and price changes ($W^o = 1 + dGDP^o/GDP^o - LAS^o$ for OECD and ROW welfare $W^r = 1 + dGDP^r/GDP^r - LAS^r$ where LAS refers to the Laspeyre’s index of consumer price changes). The fourth column in the first row of table 13, or net change for OECD is simply the difference between the sources and uses side figures. It means that although GDP in OECD rose by almost 38%, a price rise of 35% wiped out part of that net gain resulting in an improvement in net income of 2.6%.

Note that this last number is purely the result of the manner in which we have chosen to combine price and GDP changes. Therefore, care has to be taken while interpreting the net change figure. The changes in the uses and sources are all proportionate changes, and the way we have combined them is also artificial in some sense and should not be taken literally. We can however discuss these changes in relative terms and avoid comparing absolute magnitudes.

Similarly, the effect of a GDP decline for ROW of 3% in the first row of table 13 is further exacerbated by the inflation of 2%, and that implies that ROW net sources and uses of GDP fell by 5%. For the ROW, the change in GDP expression is $dM^r = - dK_x - dK_y$ in all 4 models. This specific expression arises once we have chosen our numeraire as $P_z$, and we have opted to fix factor shares and maintain full employment in ROW. The loss to ROW on the sources side of GDP is the amount of capital lost. This capital loss and the fixed full employment labor assumption directly translate to lost output. When we add to it the increase in the relative price of X consumed by ROW, the welfare loss of lost income is exacerbated.

We also note that since ROW does not consume Y, and consumes X in a much smaller proportion than OECD, the loss on the uses side is smaller for ROW in the first row of table 13 than it is for the OECD. Even this need not be taken literally. We can say that the proportion of nontradables (Y) consumed by ROW is so small as to not matter for price formation of Y, while most of Y is consumed by OECD. Further, since the consumption of ROW is much larger than OECD overall, the small amount of Y consumed by ROW is negligible in terms of ROW overall consumption and thus does not affect its uses of GDP. However, since the sources side of ROW GDP falls due to loss of

capital, only a major fall in the price of X could have compensated for this in the welfare equation. (We have assumed that Z is the numeraire, so $P_z$ is always equal to 1). Given the small weight of X in the consumption of ROW, it is highly unlikely that the price of X will fall enough to make up for the loss in welfare on the sources side.

Intuitively, therefore, in the case of the CIT with input (first row of table 13), ROW's loss is dominated by the sources side effect- the loss of capital to OECD. OECD therefore gains through the extra capital it attracts away, but loses most of the benefit to inflation in the prices of X and Y.

The burdens are reversed in the transfer model (table 12) with the same parameters, although welfare effects are well below 1% in both cases. We remind ourselves that the excess tax rate is halved in all four models. The excess CIT is halved from 5% to 2.5%, and the excess consumption tax is halved from 1.667% to 0.8333%. No assumption has been made about the size of the tax revenues in these cases, only the tax rate changed is the same. Although we have not combined the models, the result of considering the four models is similar. First, we can look at the effects of the tax change on the two input models. Then, we consider the same tax change on the two transfer models. Since we have one input model with each tax and one transfer model with each tax, it is equivalent to considering two policy changes; a tax change and a switch from input to transfer.

In the CIT and Transfer case (first row of table 12), changes are much smaller in magnitude than the CIT and Input case since the tax collected was returned to the consumer in OECD- the only real change is in the excess burden caused by distorting

factor prices. Since the sources side burden is so small (almost zero) in the CIT and transfer case for the ROW (first row of table 12), this implies that given these parameters the ROW is likely to gain from the reduction in distortion caused by lowering the tax in the OECD country.

The sources side impact for the OECD country is negative (first row of table 12), which is counterintuitive. The addition of capital, and the rise in net return to capital should have lowered sources on income in ROW and increased it in OECD. This may be the result of the non-HOD zero demand curve in X in the baseline case. This assumes that while consuming X, consumers experience some form of "money illusion," and weight income changes more heavily than total price changes. When we consider the fact that the transfer payment is reduced as the tax is reduced, and the income effect depends on the transfer, this result might have been induced by the parameters. When we have not imposed a HOD Zero restriction on X; we allowed demand elasticities of X with respect to prices of X and Y to sum to less than income elasticity plus the price elasticity of X with respect to price of Z. We have weighted changes in a certain direction that may have let the income effect dominate. This can be confirmed by repeating the same exercise with HOD zero demand for X.

However, since the input improved productivity and lowered the cost of production in X and Y, we also see a price rise. The fact that we see expected changes directly in table 13 and not directly in table 12 also leads us to consider the effect of the luxury-necessity restriction, and whether we should replace it with the more standard and reasonable HOD Zero restriction instead.

When we consider the consumption tax (second row of table 13), we find that the results with a public input are similar to the CIT case with input, with one major exception. The improvement in sources and uses of GDP for the OECD country is around 7.5%, nearly 3 times the CIT case and the effect on ROW is virtually unchanged. Taking the transfer results (second row of table 12) we find that effects again get reversed as in the CIT case, but they are even more negligible, this time less than 0.1% on either side. This too is largely the effect of not imposing a HOD Zero restriction on the demand curve for X. Though we have not explicitly imposed the restriction on Y either, our choice of elasticities for Y does not violate the restriction $E_{yx} + E_{yy} + E_{yz} + E_{M,Y} = 0$ that is required for such curves. However, our choice of elasticities for X violates the condition for X.

Therefore, we get three central results for the baseline data chosen in chapter five:

- Lowering the tax rate allows the OECD country to gain (positive net change in sources and uses of GDP) while imposing a burden ( loss in net sources and uses of GDP) on the ROW when the tax was used to provide a public input
- The OECD country gains a lot more when the tax being lowered is a consumption tax rather than a CIT, if revenue was used to provide a public input
- When revenues are used for transfer payments, it is not clear any longer that the OECD country benefits at all. When lowering the CIT, it is the ROW that may benefit more.

The exact figures are obviously dependent on the parameters and the form of the sources and uses of GDP function and how we choose to weight changes in earnings and changes in user prices. At the moment, it suffices to note that (in table 12 and 13) the

increase in earnings from attracting capital into the OECD that the OECD country achieved was choked off considerably by inflation in the case of the CIT with input. In the both the transfer cases (12), the sources of GDP loss was zero for the ROW but the net sources and uses of GDP improved to the extent that some price deflation took place. When compared to the input case, roles were reversed in the two transfer cases and the OECD country actually lost in terms of net sources and uses of GDP.

**DISCUSSION OF THE PREFERRED RESULTS WITH HOD ZERO**

Now we turn to the results after imposing the restriction that X is also HOD Zero and there is no luxury-necessity dichotomy between Y and X. All other assumptions of the baseline case are unchanged other than this restriction. Specifically, we consider the situation where we impose an HOD Zero restriction on demand curves in X, as we have done for Y. In doing so, we also eliminate the difference in the own price elasticity of demand in X and Y for the OECD country. We assume that Єxx = - 0.8 and Єyy = - 0.8 unlike Єxx = - 0.5 and Єyy = - 0.8 assumed in the base case. We also revise $E^{o}_{xy}$ = - 0.75 to $E^{o}_{xy}$ = - 0.7, while leaving $E^{O}_{XZ}$ = 0.5 as in the baseline case. We can quickly check that the demand curve for X in OECD is HOD Zero since $E^{o}_{xx}$ +$E^{o}_{xy}$ + $E^{o}_{xz}$ + $E^{o}_{MX}$ = - 0.8 - 0.7 + 0.5 + 1.0 = 0, as is the demand curve for Y = $E^{o}_{yx}$ +$E^{o}_{yy}$ + $E^{o}_{yz}$ + $E^{o}_{MY}$ = - 0.1 - 0.8 - 0.1 + 1.0 = 0 in this case as well as the baseline case.

This also removes the necessity vs. luxury classification between tradables and nontradables that we had assumed in the baseline case. We note only at this stage that given the wide range of values for both short and long run elasticities available in the literature and the very wide categories of goods included in different sectors,

conservative or neutral elasticity estimates seem far more plausible than those based on theory that have no support in the empirical literature. No other assumptions have changed here, such as the fact that X and Z are substitutes while X and Z are both weak complements for Y. The only difference between tables 12 and 13 and tables 14 and 15 is that in tables 14 and 15 both X and Y have been restricted to be HOD Zero, while in 12 and 13, X is not HOD Zero.

It is clear that this has a major effect on the nature of the results from the model, in some ways making it more intuitive. First, in every case in both tables 14 and 15, the tax reduction has a negative impact on the sources side for ROW, although in the case of the consumption tax and transfer case (second row of table 14) the effects are very small for both OECD and ROW. However, in the case of the input (table 15), which is very productive when we assume the elasticity of output is 0.5 as in the baseline case: reduction of the tax reduces welfare for both OECD and ROW regardless of the type of tax but more so for the OECD. This is more intuitive since the reduction of the tax results in reduction of the quantity of the public good which causes inflation in X and Y, while attracting away capital from the ROW and lowering the marginal product of labor which declines in ROW.

**Table 14 Impact of Lowering the (Excess) OECD Tax Rate by Half (With Transfer) on Sources and Uses of GDP & Net Sources and Uses of GDP (Welfare) Using HOD Zero & Other Baseline Data**

| **TAX and EXPENDITURE** | **OECD** | | | **ROW** | | |
|---|---|---|---|---|---|---|
| | *SOURCES* | *USES* | *WELFARE* | *SOURCES* | *USES* | *WELFARE* |
| **CIT and TRANSFER** | 0.994 | 0.992 | 0.002308 | 0.999 | 0.999 | -0.00092 |
| **CONSUMPTION and TRANSFER** | 0.994 | 0.994 | 0.000699 | 0.999 | 1.000 | -0.00029 |

**Table 15 Impact of Lowering the (Excess) OECD Tax Rate by Half (With Input) on Sources and Uses of GDP & Net Sources and Uses of GDP (Welfare) Using HOD Zero & Other Baseline Data**

| TAX and EXPENDITURE | OECD | | | ROW | | |
|---|---|---|---|---|---|---|
| | *SOURCES* | *USES* | *WELFARE* | *SOURCES* | *USES* | *WELFARE* |
| **CIT and INPUT** | 1.152 | 1.266 | -0.11462 | 0.986 | 1.0156 | -0.02947 |
| **CONSUMPTION and INPUT** | 1.167 | 1.262 | -0.09414 | 0.968 | 1.0156 | -0.02926 |

In table 15, the sources side of GDP improves for the OECD country for both taxes, while there is considerable price inflation. The greater the reduction in the public good, the more inflation there is, and this chokes off the improvement in the sources of GDP in OECD. The ROW experiences loss of capital as well as a fall in the marginal product of labor. When combined with price inflation in X, this results in a net loss to ROW as well. Thus, with a very productive government input, neither ROW nor OECD benefit when the tax rate is cut by half and tax revenue overall shrinks. It is a matter for speculation, however, that we could find such highly productive inputs that would be in line for cutting. That is why we have performed simulations (tables 16-17) with lower values of this elasticity to see if the results hold for more moderately productive inputs. We have not considered the case of unproductive inputs.

At the same time, the reduction of the tax in the transfer cases (table 14) removes some excess burden without resulting in a price increase and so benefits the OECD country. There is a negligible change in the net sources and uses of GDP in the ROW, so we may say this is close to zero. This is an intuitive result as well; if no public good was being provided the only result of a CIT would have been distortion of factor prices in

OECD. This would have led to the outflow of some capital and reduced the worldwide rate of return. This is corrected when the tax is reduced, but some capital also then flows back to the OECD country and the worldwide return also rises slightly. The loss of capital in ROW leads to a decline in marginal product of labor and a shrinkage of output.

In the case of the consumption tax with transfer (second row of table 14), effects are so small that we can almost ignore them. Since no factor prices were distorted with a consumption tax, the assumption of HOD Zero will result in a negligible change as a response to the change in the tax rate; the only distortions are the wedge between prices of X and Z in OECD and ROW and rounding errors.

**SIMULATIONS WITH THE PREFERRED CASE DATA**

Taking the case of preferred demand elasticities noted above, we investigate only one final set of issues: what are the effects of lowering 50% of the differential tax burden on the OECD and ROW when government input is moderately productive at the margin ($\frac{G}{X} * \frac{\partial f}{\partial G} = \frac{G}{Y} * \frac{\partial g}{\partial G}$ =0.1) and quite productive ($\frac{G}{X} * \frac{\partial f}{\partial G} = \frac{G}{Y} * \frac{\partial g}{\partial G}$ =0.3), treating the $\frac{G}{X} * \frac{\partial f}{\partial G} = \frac{G}{Y} * \frac{\partial g}{\partial G}$ =0.5 ( the baseline case) reported above as too high for the marginal government spending programs.

The results from this set of simulations are more intuitive in several ways. We note first that the results from the transfer models are not affected by changes in the elasticity of output with respect to the government input and are the same in all three tables 16, 17 and 18.

**Table 16 Impact of Lowering the (Excess) OECD Tax Rate by Half on Sources and Uses of GDP & Net Sources and Uses of GDP (Welfare) Using HOD Zero &**

$\frac{G}{X} * \frac{\partial F}{\partial G} = \frac{G}{Y} * \frac{\partial G}{\partial G}$ **= 0.5 [This is table 14 &15 combined]**

| TAX and EXPENDITURE | OECD | | | ROW | | |
|---|---|---|---|---|---|---|
| | *SOURCES* | *USES* | *WELFARE* | *SOURCES* | *USES* | *WELFARE* |
| **CIT and TRANSFER** | 0.993948 | 0.991639 | 0.002308 | 0.99859 | 0.999511 | -0.000919 |
| **CONSUMPTION and TRANSFER** | 0.994219 | 0.993520 | 0.0006998 | 0.999803 | 1.000098 | -0.000295 |
| | | | | | | |
| **CIT and INPUT** | 1.15163 | 1.26625 | -0.11462 | 0.98609 | 1.01556 | -0.029465 |
| **CONSUMPTION and INPUT** | 1.167395 | 1.261533 | -0.094138 | 0.986368 | 1.01563 | -0.029263 |

**Table 17 Impact of Lowering the (Excess) OECD Tax Rate by Half on Sources and Uses of GDP & Net Sources and Uses of GDP (Welfare) Using HOD Zero &**

$\frac{G}{X} * \frac{\partial F}{\partial G} = \frac{G}{Y} * \frac{\partial G}{\partial G}$ **= 0.3**

| TAX and EXPENDITURE | OECD | | | ROW | | |
|---|---|---|---|---|---|---|
| | *SOURCES* | *USES* | *WELFARE* | *SOURCES* | *USES* | *WELFARE* |
| **CIT and TRANSFER** | 0.993948 | 0.991639 | 0.002308 | 0.99859 | 0.999511 | -0.000919 |
| **CONSUMPTION and TRANSFER** | 0.994219 | 0.993520 | 0.0006998 | 0.999803 | 1.000098 | -0.000295 |
| | | | | | | |
| **CIT and INPUT** | 1.0986 | 1.1739 | -0.07529 | 0.99029 | 1.01016 | -0.019865 |
| **CONSUMPTION and INPUT** | 1.114179 | 1.1801 | -0.06592 | 0.990701 | 1.01091 | -0.02021 |

**Table 18 Impact of Lowering the (Excess) OECD Tax Rate by Half on Sources and Uses of GDP & Net Sources and Uses of GDP (Welfare) Using HOD Zero &**

$\frac{G}{X} * \frac{\partial F}{\partial G} = \frac{G}{Y} * \frac{\partial G}{\partial G} = 0.1$

| TAX and EXPENDITURE | OECD | | | ROW | | |
|---|---|---|---|---|---|---|
| | *SOURCES* | *USES* | *WELFARE* | *SOURCES* | *USES* | *WELFARE* |
| **CIT and TRANSFER** | 0.993948 | 0.991639 | 0.002308 | 0.99859 | 0.999511 | -0.000919 |
| **CONSUMPTION and TRANSFER** | 0.994219 | 0.993520 | 0.0006998 | 0.999803 | 1.000098 | -0.000295 |
| | | | | | | |
| **CIT and INPUT** | 1.03297 | 1.05961 | -0.026634 | 0.99549 | 1.00348 | -0.007984 |
| **CONSUMPTION and INPUT** | 1.04069 | 1.06766 | -0.026974 | 0.996686 | 1.00439 | -0.007709 |

There is a loss in net sources and uses of GDP to both the OECD country and ROW when the tax rate is reduced in the presence of a government input regardless of the type of tax and productivity of input (third and fourth filled row of tables 16, 17 and 18). This loss in net sources and uses of GDP has different reasons for each country. In the OECD, the gain comes from the sources side in the input models since capital is attracted away from the ROW and this leads to an increase in the marginal product of labor as well. This is offset in each case by the rise in prices; since taxes were used to provide an input in X and Y that increased productivity and reduced output prices. The net result of lowering taxes in the OECD in the case of an input was a decline in net sources and uses of GDP in the presence of an input.

In the transfer cases (first and second rows of tables 16, 17 and 18), since we have assumed that there are no extra beneficial effects of transfers, taxation simply imposed an

excess burden in and a reduction in taxes thus has a small positive effect due to the reduction in consumption prices caused by a decline in the tax rates.

The loss in net sources and uses of GDP to the OECD increases with the elasticity of output of the marginal government input (third and fourth filled row of tables 16, 17 and 18), and the effect is negligible for the ROW. This is to be expected, since the more productive inputs are in X and Y in terms of lowering supply prices, the greater the loss to OECD when they are cut. The ROW loses on the sources side in every single case due to the loss of capital when OECD cuts taxes and the resulting decline in the product of labor. The slight overall gains or losses to ROW are determined by the changes in the uses side price index. Since the weight of X in ROW consumption is so small, the effects on the uses of GDP are very small for ROW and increasing with elasticity of G, noting however that the ROW loses net sources and uses of GDP overall in all cases.

The best results occur when the HOD zero restriction applies and the rest of the data are in conformity with the baseline case (while allowing elasticity of output in OECD with respect to G to vary between 0.1 and 0.5 as in tables 16-18, third and fourth filled rows). OECD loses by cutting the tax rate when tax revenues are used to provide productive inputs. The loss is smaller when productivity of government input is smaller since this leads to a smaller price increase. The loss to OECD originates mainly on the uses side in the inputs cases. There is a sources side gain to the OECD when taxes are cut in the presence of an input. This sources side gain also falls with the elasticity of output with respect to government input. This is the result of the fall in tax revenues and G being greater for lower values of the elasticity.

Losses to the ROW also increase in the input models with productivity of input. Since there are uses side losses to ROW in all cases, in the input models the greater price rise in X due to productivity loss increases the ROW burden as well.

The OECD gains by cutting taxes (tables 16-18, first and second rows) when there are transfers due to reduction in the excess burden. There is a sources side loss to ROW always. In the transfer models, there is a small sources side loss to OECD, but a much larger reduction in prices resulting in OECD gains overall, while ROW still loses overall.

The orders of magnitude also vary by elasticity of government input. In the transfer cases, the welfare benefits depend on the excess burden reduced when taxes are reduced, and the effects are small in this model. The benefit or loss to the OECD country is always larger regardless of expenditure pattern than the effects on the ROW. This should not surprise us since the OECD economy and ownership of capital are both 30% of that of the ROW. In the low productivity of input case (table 18), the loss to the OECD overall is around 3-4% for a reduction in the tax rate of 50% of the excess tax. This goes up to 6.5-7.5% in the medium productivity (table 17) case. Since good planning implies that the most distorting taxes should be cut first, and if tax revenues fall with the cut in the tax rate, then the least productive inputs should be cut first. We should then expect the lower elasticity case to be more realistic. The total tax revenue falls in each case with input, so this implies that when the tax rate is cut, it leads to less public goods, not more.

In the CIT case we should also expect an improvement in the net return to capital and that capital flows from the ROW to OECD. When both sources and uses are taken

into account, this implies a combined loss to the OECD in the input case that can vary from 3%- 4%.

The transfer case improvement cannot be taken literally either, since we know that taxes impose an excess burden unless they correct an externality including distortions caused by existing taxes. If redistribution had no welfare effects, there can never be a justification for taxation, since we will always improve GDP by lowering taxes. Therefore, we have to consider the following facts.

In both input models, there is a sources side improvement for OECD. The uses side effect is used to compensate for the fact that in the international case, the goods consumed by the two countries are different, so relative price changes that effect one country and not the other lead to a loss for one country.

The sources side shows losses to both countries of very small amounts when taxes are lowered in the transfer models. The reason for this is that the tax revenue was part of an income effect we have included in the demand curves. Since we have relatively price-inelastic demand curves, the reduction in transfers in OECD will initially reduce demand, while the reduction in tax rates will lower prices. In these cases as well, the results are reversed overall for the OECD due to the larger uses side effects.

If we were to account for the loss in relative welfare due to the loss in redistribution benefits, we could very well have an overall loss to the OECD in the transfer case as well. This effect is hidden in the transfer case since we have assumed one consumer per country.

The main point that is brought out by the comparisons above is that when tax revenues are used to provide inputs, using the same parameters in the models for inputs and transfers and regardless of the type of tax, a reduction in rates that leads to a fall in revenues can make the competing country worse off. A reduction in rates regardless of the use of revenues makes the ROW worse off since it loses capital resulting in a decline in the product of labor. The loss to the OECD country depends on how productive its expenditure was, and the same parameter affects the ROW in the same way. The loss and gain of benefits in relative terms are three times larger for the OECD country since it is 30% of the size of the ROW.

We also clarify that this should not be interpreted as an exercise in comparing tax revenue neutral changes. The tax revenue change in the two input cases is dependent on the elasticity of output with respect to government input assumed. If we assume that the tax base for each tax is the same to begin with, with the same tax rate the revenue is also the same initially. The consumption tax rate is taken to be 1/3 the CIT rate since capital in OECD is 1/3 the total product (the consumption tax base). The rate is halved in all cases. However, the change in tax revenue, and therefore G is not the same as a proportion of G across different assumptions of the elasticity of output with respect to G (table 19). Unfortunately, when we reduce the tax rate by the same proportion (1/2 of the initial excess tax rate), even with the assumption of similar initial bases and therefore revenues, we do not get revenue-neutral changes. Therefore, our exercise compares similar tax rate changes across models. Since we assume similar shares and bases to start with, similar rates should also result in similar starting revenues across models.

**Table 19 Changes in Tax Revenue (and the Government Input G) as Percentage of G When the Excess Tax Rate is Reduced by Half in the OECD Country and Demand is HOD Zero**

| ELASTICITY OF OUTPUT IN SECTORS X and Y WITH RESPECT TO G | CHANGE IN G AS PROPORTION OF G IN THE CASE OF CIT | CHANGE IN G AS PROPORTION OF G IN THE CASE OF CONSUMPTION TAX |
|---|---|---|
| 0.5 | -0.369986 | -0.3326 |
| 0.3 | -0.409281 | -0.38582 |
| 0.1 | -0.457915 | -0.4593 |

An examination of the assumptions about $dP_k$ and the price of the government good may provide a clue to this result. The proportionate reduction in G consequent to a reduction in the tax rate is smaller as the elasticity of output with respect to G increases. With the baseline elasticity at 0.5, the reduction in G is the lowest as a proportion of G when the tax rate is halved. This may be due to the following reasons.

First, we have modeled G as an input that is not produced. This has allowed us to abstract away from inflation in G. The price of G is assumed not to vary. Thus, greater G is available when we have more K in the case of CIT, a higher tax rate or a higher $P_k$ since the rate is ad valorem. However, if G required K and L to produce, and had a price formation equation, higher $P_k$ and $P_{lo}$ would also imply that the price of the government good goes up, and the amount of real G is reduced. The real increase in G would have come about due to increases in the tax rate and capital movements only. Ignoring the price of government goods has however allowed us to capture one feature that we would like to highlight: a reduction in the excess tax rate by 50% led to a 33-37% decline in the provision of the public good in the highest productivity case, and a 45% decline in the

lowest productivity case. The decline in the excess tax rate is accompanied by an increase in the base; especially if we ignore the inflation effect on government- the increase in the base is highest when government input is most productive. This is line with results in Bénassy-Quéré, Gobalraja and Trannoy (2007) and the growing "investment climate" literature.

It is relatively easy to introduce inflation in government by turning the government input into a produced input, with its own production and price formation equation, and with its supply determined by the tax revenue collected. For reasons discussed earlier this is left to future versions of the model.

The second research question about the size and distribution of effects is answered to the extent that we agree with the data. What becomes clear is that the country reducing taxes is likely to benefit to the extent it can attract capital, and to the extent the rise in prices caused by reducing government services does not remove these gains. The central question of whether tax rate reduction implies that "spillovers" occurs has been answered as well. It is also clear from the answers that the same set of data can produce different effects for different assumptions and that some parameters chosen clearly affect results. The exact numerical measure of the burdens is both dependent on model assumptions and data.

The third research question about expenditure patterns qualifying tax change effects is also answered in the affirmative. This becomes clear from the difference in results using baseline parameters in the two different models. Not only is the sign of the welfare changes reversed, it also becomes clear that the type of use to which taxes are put

to does matter. This holds true for both cases of tax change individually. The extent to which these results are a function of particular assumptions made about the productivity of government expenditure is another issue.

We also get some answer to the question of the impacts with respect to the type of tax considered. Taking the case of neutral expenditure as in the transfer models (first and second columns of tables 16-18), we see that the overall effect of reducing the excess tax by half in the OECD is much greater in the CIT case than the consumption tax. We should expect this to happen since the factor tax is expected to be more distorting than the consumption tax, so lowering the CIT should benefit the OECD country more.

Not only can we model changes in the values of many parameters, it is quite easy to model other expenditure patterns as well. For instance, a tax and spend model can be assumed with the same parameters with different types of expenditure, and the differential results of this model would constitute the difference due to expenditure patterns alone. Possible variations in the types of expenditure are a produced government input; a government provided final good that may or may not be produced, a public good that is an input as well as a consumption good (perhaps education) or even infrastructure investment that lasts beyond one period.

The final question is the impact of the data used. For the moment we ignore the issues of the aggregation of the impacts on the sources and uses sides. While our analytical expressions were enough to show that the burdens on the sources side would favor the importing country and harm the taxing country as long as capital is attracted away, it is also clear that the burdens on either side can be reduced or enhanced

considerably when changes in consumption prices are taken into account. How we combine the sources and uses sides impacts can change the numerical calculations of the burden. [3]

Finally, there are at least two important gaps in the data. First, there are no recent and reliable studies on the issues of cross-price elasticities. Second, studies of the elasticity of output with respect to government spending by sector are also limited, in my opinion, for our purpose. Without at least some sort of agreement in this area, the numerical calculations can only be specified within a range. The analytical solution are enough to show that tax competition (proxied by a tax rate reduction by the OECD) can and does result in “spillovers” that is dependent on the type of expenditure, and provide another important justification to limit this kind of inter-jurisdictional competition by large countries. At the same time, it also shows that under very general assumptions, the country that reduces taxes can gain considerably even if it is at the cost of the rest of the world, but it loses when the tax results in variations of expenditures on public goods and the reverse in the case of transfer payments.

## SIMULATIONS USING THE BASELINE DATA TO CHECK SENSITIVITY

Do these results hold when some key parameters which we have assumed due to lack of data are varied? Does imposing the HOD zero result give us the preferred results, or can we get the same result by varying other parameters? To answer this question, we

[3] Changing the capital intensities in OECD to reflect the dominant neo-classical vies expressed in Randolph (2006) that the tradables in OECD is more labor intensive has no material effect on the direction of overall results, and has effects only in the relative distribution between OECD and ROW. In all the tables, OECD and ROW still lose overall. The difference is that the relative loss to OECD is greater in all cases and the relative loss to ROW is smaller in all cases. The full analysis is provided in the appendix B.

perform some simulations, varying the elasticity of output with respect to government input, the elasticities of substitution in production, the quantum of tax change, imposing HOD Zero restrictions on demand and some key demand elasticities. However, we do this using the baseline data from chapter 5, repeated in tables 12 and 13, without imposing the HOD Zero restriction on X.

**Table 20 Sources, Uses & Total Net Changes Using Baseline Data from Chapter 5 with No HOD Zero Restriction on X and $\frac{G}{X} * \frac{\partial F}{\partial G} = \frac{G}{Y} * \frac{\partial G}{\partial G} = 0.1$**

| TAX and EXPENDITURE | OECD | | | ROW | | |
|---|---|---|---|---|---|---|
| | *SOURCES* | *USES* | *WELFARE* | *SOURCES* | *USES* | *WELFARE* |
| **CIT and INPUT** | 1.1306 | 1.12248 | 0.008134 | 0.98776 | 1.007158 | -0.019397 |
| **CIT and TRANSFER** | 0.97619 | 0.97905 | -0.00286 | 1.0 | 0.998776 | 0.001225 |
| **CONSUMPTION and INPUT** | 1.21035 | 1.17063 | 0.03972 | 0.98287 | 1.01037 | -0.0275 |
| **CONSUMPTION and TRANSFER** | 0.99067 | 0.99100 | -0.0003 | 1.0001 | 0.99995 | 0.00013 |

**Table 21 Sources, Uses & Total Net Changes Using Baseline Data from Chapter 5 with No HOD Zero Restriction on X and $\frac{G}{X} * \frac{\partial F}{\partial G} = \frac{G}{Y} * \frac{\partial G}{\partial G} = 0.3$**

| TAX and EXPENDITURE | OECD | | | ROW | | |
|---|---|---|---|---|---|---|
| | *SOURCES* | *USES* | *WELFARE* | *SOURCES* | *USES* | *WELFARE* |
| **CIT and INPUT** | 1.29399 | 1.27423 | 0.01976 | 0.97481 | 1.016026 | -0.04121 |
| **CIT and TRANSFER** | 0.97619 | 0.97905 | -0.00286 | 1.0 | 0.998776 | 0.001225 |
| **CONSUMPTION and INPUT** | 1.344697 | 1.27947 | 0.065223 | 0.971929 | 1.01668 | -0.04475 |
| **CONSUMPTION and TRANSFER** | 0.99067 | 0.99100 | -0.0003 | 1.0001 | 0.99995 | 0.00013 |

**Table 22 Sources, Uses & Total Net Changes Using Baseline Data from Chapter 5 with No HOD Zero Restriction on X and $\frac{G}{X} * \frac{\partial F}{\partial G} = \frac{G}{Y} * \frac{\partial G}{\partial G}$ = 0.5 [This is the Same Table as 12&13 Combined]**

| TAX and EXPENDITURE | OECD | | | ROW | | |
|---|---|---|---|---|---|---|
| | *SOURCES* | *USES* | *WELFARE* | *SOURCES* | *USES* | *WELFARE* |
| **CIT and INPUT** | 1.3793 | 1.3535 | 0.02583 | 0.96805 | 1.0207 | -0.052605 |
| **CIT and TRANSFER** | 0.97619 | 0.97905 | -0.00286 | 1.0 | 0.998776 | 0.001225 |
| **CONSUMPTION and INPUT** | 1.3939 | 1.3193 | 0.07457 | 0.96792 | 1.01899 | -0.05107 |
| **CONSUMPTION and TRANSFER** | 0.99067 | 0.99100 | -0.0003 | 1.0001 | 0.99995 | 0.00013 |

The results are counter intuitive. The higher the output produced by the input (table 22 compared to 21 and 20), the greater the welfare gain to the OECD country by reducing taxes and input, and the greater the losses inflicted on the ROW. The losses on the sources side in the case of the ROW arise due to the amount of capital attracted away by lowering the tax rate and the resulting fall in the product of labor. The sources side loss to the ROW is unchanged in the transfer models since the productivity of G does not matter. The sources side loss increases for the ROW with productivity of G in both tax models. This implies that the more productive the G was in OECD, the greater the loss to ROW due to capital leaving ROW for OECD when the tax was reduced.

It does not appear rational that for the same tax rate reduction, the gains to the OECD country on the sources side increase as the government input is more productive. Higher tax revenues imply more G and more G implies more output when G is productive. When G is less productive, lower tax revenue and therefore less G implies a

smaller loss in output than when G is more productive. However, the general orders of changes in welfare are not affected in any way in tables 20 to 22, making it clear that it is not the productivity of input that drives the direction of the results. For the transfer models, the elasticity of output with respect to input does not enter these functions at all since no input is being provided.

Next we simulate results for different values of the elasticity of substitution in production between capital and labor in OECD (X and Y sectors only). We still use the baseline case of chapter 5 where X is not HOD Zero. Since there is only one ROW sector (Z) and we have assumed full employment, there is no elasticity of substitution in production in Z. We use values of -0.8 and -1.0 for comparison with the base case of -0.6, and note that these elasticities enter both the input and transfer models.

**Table 23 Sources, Uses & Total Net Changes Using Baseline Data from Chapter 5 with No HOD Zero Restriction on X ($S_x$= $S_y$ = - 0.6) [This is the Same Table as 12&13 Combined]**

| TAX and EXPENDITURE | OECD | | | ROW | | |
|---|---|---|---|---|---|---|
| | *SOURCES* | *USES* | *WELFARE* | *SOURCES* | *USES* | *WELFARE* |
| **CIT and INPUT** | 1.3793 | 1.3535 | 0.02583 | 0.96805 | 1.0207 | -0.052605 |
| **CIT and TRANSFER** | 0.97619 | 0.97905 | -0.00286 | 1.0 | 0.998776 | 0.001225 |
| **CONSUMPTION and INPUT** | 1.3939 | 1.3193 | 0.07457 | 0.96792 | 1.01899 | -0.05107 |
| **CONSUMPTION and TRANSFER** | 0.99067 | 0.99100 | -0.0003 | 1.0001 | 0.99995 | 0.00013 |

**Table 24 Sources, Uses & Total Net Changes Using Baseline Data from Chapter 5 with No HOD Zero Restriction on X ($S_x$= $S_y$ = - 0.8)**

| TAX and EXPENDITURE | OECD | | | ROW | | |
|---|---|---|---|---|---|---|
| | *SOURCES* | *USES* | *WELFARE* | *SOURCES* | *USES* | *WELFARE* |
| **CIT and INPUT** | 1.31405 | 1.29604 | 0.018005 | 0.96805 | 1.01730 | -0.04925 |
| **CIT and TRANSFER** | 0.97619 | 0.97905 | -0.002857 | 1.0 | 0.998776 | 0.001225 |
| **CONSUMPTION and INPUT** | 1.36597 | 1.30028 | 0.06568 | 0.964479 | 1.01789 | -0.05341 |
| **CONSUMPTION and TRANSFER** | 0.990983 | 0.99125 | -0.00027 | 1.00008 | 0.999968 | 0.000112 |

**Table 25 Sources, Uses & Total Net Changes Using Baseline Data from Chapter 5 with No HOD Zero Restriction on X ($S_x$= $S_y$ = - 1.0)**

| TAX and EXPENDITURE | OECD | | | ROW | | |
|---|---|---|---|---|---|---|
| | *SOURCES* | *USES* | *WELFARE* | *SOURCES* | *USES* | *WELFARE* |
| **CIT and INPUT** | 1.27491 | 1.26160 | 0.013309 | 0.96805 | 1.01529 | -0.04724 |
| **CIT and TRANSFER** | 0.97619 | 0.97905 | -0.002857 | 1.0 | 0.998776 | 0.001225 |
| **CONSUMPTION and INPUT** | 1.346124 | 1.28675 | 0.05937 | 0.962034 | 1.01710 | -0.05507 |
| **CONSUMPTION and TRANSFER** | 0.991131 | 0.991367 | -0.00024 | 1.00007 | 0.999975 | 0.000099 |

Again we find that the elasticity of substitution does not change the order of effects at all in tables 23-25. The elasticity has no effect at all in the CIT and transfer case with the tax unchanged. In the consumption tax case, increasing the elasticity reduces the welfare gain for the OECD country but has a varying impact for the ROW. Thus within a reasonable range, we can hypothesize that this parameter has no large effect on the direction of the results.

We now turn to the question of the range of the proposed tax change. We will examine two cases, where the tax change is 25% of the existing tax and the case of 75% change. Our base case of 50% change lies in between. All other baseline assumptions are unchanged.

**Table 26 Sources, Uses & Total Net Changes Using Baseline Data from Chapter 5 with No HOD Zero Restriction on X and Tax Change is 25% Points**

| TAX and EXPENDITURE | OECD | | | ROW | | |
|---|---|---|---|---|---|---|
| | *SOURCES* | *USES* | *WELFARE* | *SOURCES* | *USES* | *WELFARE* |
| **CIT and INPUT** | 1.189639 | 1.176722 | 0.012916 | 0.984025 | 1.010327 | -0.026302 |
| **CIT and TRANSFER** | 0.988095 | 0.989523 | -0.001428 | 1.0 | 0.999387 | 0.000612 |
| **CONSUMPTION and INPUT** | 1.196952 | 1.159670 | 0.037282 | 0.983961 | 1.009496 | -0.025535 |
| **CONSUMPTION and TRANSFER** | 0.995335 | 0.995499 | -0.000164 | 1.000046 | 0.999976 | 0.000069 |

**Table 27 Sources, Uses & Total Net Changes Using Baseline Data from Chapter 5 with No HOD Zero Restriction on X and Tax Change is 50% Points [This is the Same Table as 12&13 Combined]**

| TAX and EXPENDITURE | OECD | | | ROW | | |
|---|---|---|---|---|---|---|
| | *SOURCES* | *USES* | *WELFARE* | *SOURCES* | *USES* | *WELFARE* |
| **CIT and INPUT** | 1.3793 | 1.3535 | 0.02583 | 0.96805 | 1.0207 | -0.052605 |
| **CIT and TRANSFER** | 0.97619 | 0.97905 | -0.00286 | 1.0 | 0.998776 | 0.001225 |
| **CONSUMPTION and INPUT** | 1.3939 | 1.3193 | 0.07457 | 0.96792 | 1.01899 | -0.05107 |
| **CONSUMPTION and TRANSFER** | 0.99067 | 0.99100 | -0.0003 | 1.0001 | 0.99995 | 0.00013 |

**Table 28 Sources, Uses & Total Net Changes Using Baseline Data from Chapter 5 with No HOD Zero Restriction on X and Tax Change is 75% Points**

| TAX and EXPENDITURE | OECD | | | ROW | | |
|---|---|---|---|---|---|---|
| | *SOURCES* | *USES* | *WELFARE* | *SOURCES* | *USES* | *WELFARE* |
| **CIT and INPUT** | 1.568918 | 1.530168 | 0.038749 | 0.952076 | 1.03098 | -0.078907 |
| **CIT and TRANSFER** | 0.964285 | 0.968571 | -0.004285 | 1.0 | 0.998163 | 0.001836 |
| **CONSUMPTION and INPUT** | 1.590858 | 1.47901 | 0.111847 | 0.951883 | 1.02849 | -0.076607 |
| **CONSUMPTION and TRANSFER** | 0.98600 | 0.986499 | -0.00049 | 1.00013 | 0.999930 | 0.000207 |

The results of this simulation are fairly simple. The quantum of tax change amplifies results proportionately without changing any orders or directions. Changes are almost 50% less with the tax change being 50% less than the baseline case, and changes are around 50% more when the tax change contemplated is 50% more. Thus, the choice of how much tax change to consider, like the choice of elasticities of output and substitution, does not materially affect results, except in terms of size.

**CONCLUSIONS**

The discussion above reveals that most of the parameters chosen do not materially affect the results of our models, with the exception of the demand elasticities. Despite the wide range of results reported in the literature, the choice of neutral elasticities close to one and to each other (all income elasticities are one, own price elasticites equal to each other and close to one, at - 0.8, and demand in OECD is HOD Zero ) produce the most intuitive results. The choice of HOD Zero is also considered standard and is the direct consequence of neo-classical utility maximization. These values chosen are the average

of the ranges reported in the literature as well as the average of the long and short run elasticities.

Conflicting estimates do not allow us to take the position that one set of estimates dominates another, and we do not have evidence to support either the luxury-necessity theory or strong complementarities or substitution. Thus, given the evidence, the best estimates of elasticities in my opinion are unitary elasticities of income (although a case may be made for the U.S. elasticity of income for imports to be higher than for exports, this is not pursued in this model due to the theoretical set-up); own-price elasticities close to one (we have chosen - 0.8 for both OECD sectors); fairly low cross-price elasticities for tradables and nontradables and slightly higher cross-price elasticities for OECD tradables and ROW consolidated output.

Assuming that the elasticity of demand for tradables is substantially less than for nontradables produces results that are almost opposite to this neutral case. Due to the lack of recent studies that provide evidence that this is the case or good estimates of cross-price elasticities, there is not much basis for such assumptions.

Further refinements to the model are possible and not very difficult. The most obvious ones are the addition of a second ROW sector, making G a produced input in OECD, using GNP instead of GDP, introducing balance of trade imbalances with a BOP condition and estimating cross-price elasticities using recent data. These are not attempted at this stage and are left for future studies.

In my opinion, the results are important for policy since they illustrate that:

- In line with existing literature on the subject, there can be a justification for lowering taxes through tax competition potentially leading to a “race to the bottom” based on improvements expected through increases in tax revenue, reduction of excess burdens, or a fixation on the sources side (attracting mobile capital) exclusively.
- These arguments are shown to have a weak basis when we consider that reducing taxes that result in a reduction in revenues actually lead to less public goods, and that efficiency arguments have small effects that may easily be outweighed by redistribution effects, and may rely on considering uses side effects in addition to sources of income effects. Similar observations made by Wilson and Wildasin (2004).
- Taking a holistic view of burdens on the sources and uses of GDP by including effects on the rest of the world, considering the uses as well as the sources sides of GDP equally important and considering possible redistribution losses can lead to a reversal of effects expected by the “reduce taxes” lobby. On the other hand, there are existing critiques of including the uses side burdens (Whalley, 1984) as an instrument that allows one to generate results in any direction that one wants by choosing appropriate measures. Further, since all sources side burdens are already measured in real terms, this is not likely to deter the advocates of tax reduction, who may argue that such adjustments are unnecessary.
- Reductions in tax rates that are “too high” relative to the ROW can be beneficial only when we consider that the reduction may not actually raise but lower ad valorem tax revenues depending on the elasticities of demand for goods and

factors, and that this reduction in revenues may lead to a reduction in the provision of public goods. As pointed out in Bénassy-Quéré, Gobalraja and Trannoy (2007), we have to take into account both the reaction of mobile capital with respect to the tax rate, and the total provision of public goods with respect to the tax rate. Once we have parameters that show that tax revenues (and therefore public goods) fall with the reduction in that tax rate, we have to consider issues of how productive those public goods were. Analogously, we cannot ignore how productive the redistribution resulting from taxation was in the transfer models.

- In terms of orders of magnitude, the results depend on the choice of elasticities, and methods used to calculate burdens. To be a strict guide to policy, one has to have a reasonable consensus on the range of these elasticities. At present, the plausible range is too vast to allow definitive predictions, since the choice of reasonable figures within the range can generate results of any type we choose. We need better and more recent, reliable estimates of these specific elasticities for policy analysis.
- Introducing country specific variations in the choice of elasticities and features such as balance of trade deficits, produced inputs, nontradables in ROW, risk premia etc. will certainly give more confidence in estimating the size of effects. When we consider the alternative in the form of CGE models, these involve much stronger assumptions such as specific functional forms for demand and supply curves. Models based on the Harberger analysis at least have the benefit of not having to make as many assumptions.

- There are at least three areas where improvements are possible. First, it will be useful to have a good estimate of the size of the stock of U.S. capital relative to the ROW. Second, the estimates of cross-price elasticities of demand are almost pure guesses. Third, we have assumed that production is CRS in OECD for both sectors. It would be useful to have better price formation equations based on recent estimates of the returns to scale in the relevant range for tradables and nontradables in the OECD and ROW, especially ones that take into account the role of government. Without these, we can have confidence in the general direction of results: to extend that to the magnitudes would be wishful thinking.

The results are not that far from existing opinions on tax competition effects. For example, the OECD report for 2004 on “Harmful Tax Practices” (available at: http://www.oecd.org/dataoecd/60/33/30901115.pdf; accessed on 5/1/2010) clearly highlights that the criteria enumerated in 1998 for determining whether a preferential tax regime was harmful had at the very top of the list “the regime imposes low or no taxes on the relevant income (from geographically mobile financial and other service activities).” However, the conclusion that a regime is harmful also depends on the breadth of the welfare measure used. The fact that taxation imposes an excess burden and that a reduction in that burden will increase GDP is not in dispute; what is debated is whether it will do so at the cost of the rest of the world, and whether it will do so after expenditure effects are considered.

We also have to consider that for many countries such as the U.S.A, and partially India, taxation by sub-national authorities is an important component of the total tax rate faced by a firm. The analysis above is from the view of a purely federal tax regime.

When we have large and strong sub-national jurisdictions waiting to occupy the fiscal space vacated by the federal government, or ones that compete amongst themselves, then the analysis has to be modified accordingly. Further, models of this kind can equally well be used to analyze competition between sub-national jurisdictions themselves for mobile resources (Wallace, 1993).

This also brings us to the next question: if the effects of tax competition (rate reductions) are global (spillovers of burdens), should tax policy be considered a strictly domestic issue? The question is not as far-fetched as one may think. "Beggar thy neighbor" effects in trade involve raising trade tariffs on imports to improve domestic agents' welfare. Trade tariffs, like all taxes, may be thought to be strictly in the domestic policy domain of any country. Yet, the WTO and the GATT before it, has had considerable success in reducing the general level of tariffs across the globe, and may take some of the credit for the increase in world trade and consumption levels that this has brought.

With the exception of a world body like the OECD (and only partially the IMF), there does not appear to be a "World Tax Organization" that is in a position to perform the World Trade Organization's role for other tax issues. The OECD tax unit already deals with such issues as tax competition, transfer pricing, evasion etc. The extent to which it has the ability to influence non-OECD countries is questionable. The Fiscal Affairs Department of the IMF is another body that may have considerable influence on debtor countries. Given the global effects of tax competition and the spillovers of burdens that can result, perhaps it is time to think about international cooperation in this field; setting up of a body that deals with the issues that the OECD's tax unit does with the

structure of a truly world body such as the WTO. The WTO already deals with several other policy issues other than tariffs that affect relative prices, such as wage supports, agricultural price supports, as they apply to international trade. It also deals for example with intellectual property rights and customs valuation. However, there is still a role for a World Intellectual Property Rights Organization and World Customs Organization. Both the international tax (public finance) organization and WTO and other bodies could peacefully co-exist.

Such a body would have the knowledge, capacity and global membership of a world body to deal with tax issues (competition, coordination, treaties, transfer pricing, spillovers of burdens, harmful practices, uniform tax codes and disclosure norms, etc.). Such a body will have to deal with the following rejoinder from advocates of tax cuts: higher distorting taxes in the OECD forced mobile capital out of the OECD into the ROW in the past. This raised ROW labor productivity and lowered OECD output. Reducing those taxes brought back capital that had been driven out previously. Therefore the loss in ROW welfare is simply a correction of past distortions, not a freshly spilt over burden.

This is precisely where a multilateral agency, on the lines of WTOs dispute resolution mechanism can be useful. That such an idea (of an international tax coordination and policy advisory body) is not completely far-fetched is evidenced by a recent article by J. Attali in the New York Times global edition (available at: http://www.nytimes.com/2010/02/27/opinion/27iht-edattali.html?hpw) that calls for a "European Ministry of Finance" to coordinate tax and fiscal policy as an inevitable consequence of a common Euro-zone.

The choice of the four models in this dissertation has been motivated in part by the desire to separate the effects of taxes on "exporting" and "burdens" (or benefits) and the effects of expenditures on the same. The models can be thought of as four parts of the same model of an economy where there are two possible types of distortion. The question of examining the effects of a tax involve isolating the effects due to the tax itself by assuming that the expenditure pattern before and after the tax remains the same. In our presentation, we can make at least two additional types of comparisons. First, the consumption tax with transfer may be thought of as a non-distorting tax and can serve as a base case (except for rounding errors). This can be compared with the capital income tax (which is a factor tax and therefore distorting, though probably less distorting than a partial factor tax like the corporate income tax). Further, the consumption tax with transfer may be compared to the consumption tax with input to isolate the effects of the input itself, since the tax is non-distorting. Presenting this in a simple schematic framework:

| **TAX and INPUT** | **NO TAX DISTORTION** | **TAX DISTORTION** |
|---|---|---|
| **NO EXPENDITURE DISTORTION** | CONSUMPTION TAX and TRANSFER | CIT and TRANSFER |
| **EXPENDITURE DISTORTION** | CONSUMPTION TAX and INPUT | CIT and INPUT |

The advantage of presenting it in this manner (at the suggestion of Dr Randolph) is that we can further isolate sub-categories while thinking of the work itself as four aspects of one model instead of four separate models.

The second aspect is related to some results from the investment climate literature that shows how the model can shed light on some more policy implications. As an example, Bennasy-quere et al. (2007) split their sample of European countries into higher and lower income countries and find that investment is less responsive to tax cuts in richer countries that cut government expenditure, while low tax rates are important for the less rich countries for attracting investment. This says something about the productivity of government expenditure paid for out of tax revenues. If investors view the investment climate as relatively poor (and government expenditure as relatively ineffective or wasteful), they are likely to go for the location with a lower tax rate ceteris paribus; and when they view government expenditure as effective and productive, they are more willing to support a high tax rate and a higher level of provision of public goods. While we have not modeled expectations or the perceived productivity of expenditure in our model, it can provide some theoretical basis for these results. Cutting tax rates that result in cutting marginally productive public inputs does not benefit anyone in our models, while cutting tax rates in neutral expenditure situations does.

Finally, we can examine the question of whether this is really a situation that can exist in the real world. From a theoretical perspective, it is correct to analyze the effects of a tax separately from expenditure. In the real world, taxes can be an end in themselves only if the objective is redistribution. If we adopt the more modern view that taxes are a weaker tool for redistribution than expenditures and that the major objective of taxation is to provide public goods; we should then focus on the combined effect of both on welfare. In a developing country like India, it is not necessary that the government will cut wasteful expenditures first if it needs to reduce the level of taxation. The problem is more

likely to be a low tax-GDP ratio to begin with, compounded by high marginal rates on easy-to-tax activities.

The marginal productivity of basic input expenditure can also be very high in these countries. If tax rates are cut and overall revenues fall, most developing countries do not have extensive social security systems or transfer systems to mitigate the conditions of those affected. Salaries and other current expenditures are not pruned much, what suffers is "development expenditure." Delayed projects, lack of maintenance, lack of spending on capacity-building, cutting programs aimed at improving the overall investment climate etc., are likely to suffer. If the planned inputs were productive in the first place, the overall benefit to the country would be lower.

We can then think of some further policy implications that flow from this.

- Tax cut proposals that are based on analysis of taxation assuming neutral transfers should be supplemented by an analysis of the "excess benefit" of expenditures. We normally deal with the tax side considering the tax revenues separately from the "excess burden," it is only symmetric to deal with policy analysis using excess burdens and benefits of both.
- Comparisons of taxes that rely on "revenue neutral changes" are correct in theory. In the real world, as our models have shown, revenues depend on the productivity of expenditures. Therefore, if tax cuts cause changes in spending, then the effects on revenue depend on the productivity of spending. The more appropriate design is to consider actual proposed cuts, their revenue implications and the full analysis of what is likely to happen on the expenditure side.

- Many countries already require an accounting for "tax expenditures" as part of budgeting. If this implicitly implies the tax revenue effects of a tax change, it follows that it should include an analysis of the expenditure impact with the same degree of precision. This is not just confined to the level of expenditure change. Its source, method of financing and excess burden need to be accounted for with the same degree of precision as the tax side.
- One can also suggest that doing the above will help with clarity of thought and better policy analysis by separating the muddled thinking that conflicting objectives create. As an illustration, if we think that "tax exporting" means that non-residents are paying our taxes when they deploy their capital or labor in our jurisdiction; and that this is good-we should follow through with only this analysis and not consider Harbergarian issues of changes in the sources and uses of income other than the tax revenue itself. In the real world, this is what is typically done. Citizens do not like to pay taxes, and the idea that a non-citizen pays part of our taxes while "we" (residents) get to benefit from the public goods is always an attractive proposition. Therefore tax cuts that allow this to happen are attractive.
- Further, the expectation from the above analysis, if we are focused exclusively on tax revenues has to be that we will eventually not lose revenue, since this would imply a cut in the level of public goods. In the best case, if we assume that the change contemplated is revenue-neutral, then it is no longer clear why the tax change is necessary except to redistribute the payment of the revenue to non-residents. If this is the case, a clear analysis should concentrate on only these effects and the tax revenue itself, not "welfare."

- The primary contribution of the Harbergerarian analysis is to show that even if we eliminate tax revenues from consideration, there are changes in overall income due to tax effects. The same is obviously true of expenditure effects.

Consider the case of a small jurisdiction that cannot change the overall rate of return to capital trying to account for the effects of a tax reduction to attract capital from non-residents. Under a GNP system, the only effect could be an increase in the net return to capital owners who deploy capital in their jurisdiction when taxes are cut. This is ruled out for a small economy since it cannot affect the net rate of return to capital. Residents may have deployed a part of their capital elsewhere, but their overall returns cannot fall. Labor in the domestic area benefits whether they are residents or non-residents; due to raised marginal productivity. If we further assume that tax revenues do not fall as a result and there is no change in public good provision, this means that in the Harbergarian sense there must have been no decline in the overall income of owners of capital of that jurisdiction on a GNP basis, and an increase in the productivity of labor in the jurisdiction, but on a GDP basis.

This further implies that a revenue-neutral change in these circumstances is very attractive whether we look at the tax revenue exclusively. For policy analysis, we have to consider other issues. First, will tax revenues change or not? To assume that they will not change is to say that the elasticity of tax revenues with respect to the tax rate is exactly zero. For most budgets around the world, tax cuts proposals are accompanied by revenue implications and these have expenditure effects. The analysis of these expenditure effects proposals should at the very minimum be as

sophisticated as the tax analysis. If taxes fall, exactly what expenditures are to be cut? What is the total impact of both?

Second, we should in advance choose the basis of analysis. Are we focused on the tax revenue itself (and GNP) or overall changes in the sources and uses of income? If on the former, it is natural to ignore the expenditure side, since we are worried only about who paid the tax and whether the change is tax revenue neutral. If the latter methodology is adopted, it becomes clear that GNP is important for looking at incomes of owners of capital, but not necessarily for labor, since labor has often to live where it works, or consume where it lives. It is also clear that who paid the tax is largely irrelevant since the person paying the tax may simultaneously benefit from an increase in the net return to capital. Expenditure effects are important, and considering transfers as neutral is also not valid where the purpose of a transfer is to redistribute income.

# APPENDIX B

The traditional and dominant view in the literature, as expressed in Randolph (2006) and others is to use different capital intensities in the tradables and nontradable sectors. As discussed in chapter 5, the figures from Gravelle and Smetters (2006) preferred by Randolph (2006), representing the dominant neo-classical literature have used the following data (with approximation):

**Table B1 U.S. Shares Based on Randolph (2006)**

| | Share of Value Added in Sector | | Share of |
|---|---|---|---|
| | **Labor** | **Capital (= 1- labor)** | **Output** |
| **Tradable Sector** | **78.8%** | **21.2%** | **31%** |
| **Non-tradable Sector** | **65.9%** | **34.1%** | **69%** |
| **Total** | **69.9%** | **30.1%** | **100%** |
| Domestic (OECD) economy's share of world output | | 30% | |
| Domestic (OECD) ownership share of world capital | | 30% | |
| Partial elasticity of substitution, capital and labor | | - 0.6 | |

**Source: Randolph (2006), Based on Gravelle and Smetters (2006).**

It should be noted that the nontradables figure is almost exactly the same as in the table we chose to use. However, tradables are more labor intensive and have a larger weight. For convenience, we can round these figures off to:

**Table B2 Adjusted Shares Based on Randolph (2006)**

| | Share of Value Added in Sector | | Share of |
|---|---|---|---|
| | **Labor** | **Capital (= 1- labor)** | **Output** |
| **Tradable Sector** | **80%** | **20%** | **30%** |
| **Non-tradable Sector** | **66%** | **34%** | **70%** |
| **Total** | **70%** | **30%** | **100%** |
| Domestic (OECD) economy's share of world output | | 30% | |
| Domestic (OECD) ownership share of world capital | | 30% | |
| Partial elasticity of substitution, capital and labor | | - 0.6 | |

**Source: Randolph (2006), Based on Gravelle and Smetters (2006).**

Now we can use the ratios in our table to get:

$\theta_{kx} = \frac{P_k*(1+T_{ko})*K_x}{P_x*X} = 1/5$ $\theta_{ky} = \frac{P_k*(1+T_{ko})*K_y}{P_y*Y} = 1/3$

$\theta_{lx} = \frac{P_{lo}*L_x}{P_x*X} = 4/5$ $\theta_{ly} = \frac{P_{lo}*L_y}{P_y*Y} = 2/3$

$\frac{X}{X+Y} = 3/10$ and $\frac{X}{Y} = 3/7$ and $\frac{X+Y}{X+Y+Z} = 3/10$ and $\frac{(1+T_{ko})*(K_x+K_y)}{(1+T_{ko})*(K_x+K_y)+K_z} = 3/10$

This implies that:

$\frac{X+Y}{Z} = 3/7$ and $\frac{K_x+K_y}{K_z} = \frac{3}{7*(1+T_{ko})}$

We also know from our tables above that $\frac{P_k*(1+T_{ko})*(K_x+K_y)}{P_x*X+P_y*Y} = 3/10$

And we had assumed $P_k$, $P_{lo}$ and $P_{lr}$ (factor prices) and $P_x$, $P_y$ and $P_z$ (output prices) are all equal to 1 and to each other to begin with.

Substituting $X + Y = 3*Z/7$ and $K_x + K_y = 3*K_z/7*(1+T_{ko})$ above, we get

$\frac{K_z}{Z} = b = \theta_{kz} = \frac{3}{10}$

Next noting that we get $(1- 3/10) = 7/10 = (1-b) = \theta_{lz} = \frac{P_{lr}*L_z}{P_z*Z}$ where $L_z = \overline{L_z}$

And that $\frac{P_{Lo}*(L_x+L_y)}{P_x*X+P_y*Y} = 7/10$ and that $\frac{X+Y}{Z} = 3/7$, we get $\frac{L_x+L_y}{L_z} = 3/7$

With some further simple manipulations we get:

$(\frac{K_x}{K_z})$ = 27/308*$(1+T_{ko})$ ; $(\frac{K_y}{K_z})$ = 105/308*$(1+T_{ko})$ and $(\frac{L_y}{L_x})$ = 35/18

$(\frac{K_x+K_y}{X})$ = 44/45*$(1+T_{ko})$ ; $\left(\frac{K_x}{K_x+K_y}\right)$ = 9/44; $\left(\frac{K_y}{K_x+K_y}\right)$ = 35/44 and $\frac{(Lx + Ly)}{X}$ = 106/45

$(\frac{K_x+K_y}{Y})$ = 44/105*$(1+T_{ko})$ ; $(\frac{K_x}{Y})$ =3/35*$(1+T_{ko})$ and $\frac{(Lx + Ly)}{Y}$ = 106/105 and $(\frac{K_y}{X})$ = 7/9*$(1+T_{ko})$

By assumption, $\frac{dT_{ko}}{T_{ko}}$ = -1/2, and $\frac{dT_{ko}}{(1+T_{ko})}$ = - 0.02381 (any other combinations are possible)

Using these, we have also to recalculate the trade ratios and marginal income effects used in chapter 5. MPC of imports = 0.12, MPC of domestic tradables is now 0.18 and MPC of domestic nontradables is now 0.7. $\frac{X^r}{M^r}$ = 0.036 still holds since exports are still 12% of domestic GDP and OECD is still 30% of world output and that $\frac{\partial X^r}{\partial M^r}$ = $\square^r_{x,m}$* $\frac{X^r}{M^r}$ = 1.0*0.036 = 0.036. Єxx = - 0.8 and Єyy = - 0.8, and we say that X and Y are gross complements, with Єxy = - 0.42 (Implying $E^o_{xy}$ = - 0.7) due to the stronger income effect and weighting, and Єyx = - 0.1 with no weighting and a much smaller income effect.

Now let us see what effect this has on the results. We have taken the preferred HOD zero case of the CIT with input since this is the very first model we derived in chapter two. To ensure that the results are not driven by the elasticity of output with respect to government, we present results of all three versions of the government input elasticity. The only difference is that the OECD tradables sector is more labor intensive using the data in Randolph (2006) discussed above.

**Table B3 Sources, Uses & Total Net Changes Using Baseline Data From Chapter 5 and Adjusted Factor Shares in OECD With HOD Zero Restriction on X and** $\frac{G}{X} * \frac{\partial F}{\partial G} = \frac{G}{Y} * \frac{\partial G}{\partial G} = 0.5$

| TAX and EXPENDITURE | OECD | | | ROW | | |
|---|---|---|---|---|---|---|
| | *SOURCES* | *USES* | *WELFARE* | *SOURCES* | *USES* | *WELFARE* |
| **CIT and INPUT WITH DISSERTATION CAPITAL INTENSITY** | 1.15163 | 1.26625 | -0.11462 | 0.98609 | 1.01556 | -0.029465 |
| | | | | | | |
| **CIT and INPUT WITH OECD TRADABLES LABOR INTENSIVE** | 1.0093 | 1.208269 | -0.198975 | 1.0000 | 1.012071 | -0.012028 |

**Table B4 Sources, Uses & Total Net Changes Using Baseline Data From Chapter 5 and Adjusted Factor Shares in OECD With HOD Zero Restriction on X and**

$\frac{G}{X} * \frac{\partial F}{\partial G} = \frac{G}{Y} * \frac{\partial G}{\partial G} = 0.3$

| TAX and EXPENDITURE | OECD | | | ROW | | |
|---|---|---|---|---|---|---|
| | *SOURCES* | *USES* | *WELFARE* | *SOURCES* | *USES* | *WELFARE* |
| **CIT and INPUT WITH DISSERTATION CAPITAL INTENSITY** | 1.0986 | 1.1739 | -0.07529 | 0.99029 | 1.01016 | -0.019865 |
| | | | | | | |
| **CIT and INPUT WITH OECD TRADABLES LABOR INTENSIVE** | 1.00546 | 1.12690 | -0.121438 | 0.99979 | 1.00737 | -0.00758 |

**Table B5 Sources, Uses & Total Net Changes Using Baseline Data From Chapter 5 and Adjusted Factor Shares in OECD With HOD Zero Restriction on X and**

$$\frac{G}{X} * \frac{\partial F}{\partial G} = \frac{G}{Y} * \frac{\partial G}{\partial G} = 0.1$$

| **TAX and EXPENDITURE** | **OECD** | | | **ROW** | | |
|---|---|---|---|---|---|---|
| | *SOURCES* | *USES* | *WELFARE* | *SOURCES* | *USES* | *WELFARE* |
| **CIT and INPUT WITH DISSERTATION CAPITAL INTENSITY** | 1.03297 | 1.05961 | -0.026634 | 0.99549 | 1.00348 | -0.007984 |
| | | | | | | |
| **CIT and INPUT WITH OECD TRADABLES LABOR INTENSIVE** | 1.001466 | 1.04196 | -0.040495 | 0.99953 | 1.002468 | -0.002936 |

As we can see, the change in capital intensities in OECD to reflect the dominant neo-classical vies expressed in Randolph (2006) that the tradables in OECD is more labor intensive has no material effect on the direction of overall results, and has effects only in the relative distribution between OECD and ROW. In all the tables, OECD and ROW still lose overall. The difference is that the relative loss to OECD is greater in all cases and the relative loss to ROW is smaller in all cases.

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