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A Theory and Test of Credit Rationing: Comment

By Corry F. Azzi and James C. Cox*

One frequently encounters the casual empirical conclusion that some consumers and firms are not able to borrow as much as they would like at market rates of interest. The existence of these rejected offers to pay market rates of interest is then said to constitute "credit rationing." Marshall Freimer and Myron Gordon, in addition to Dwight Jaffee and Franco Modigliani, assume that rejected market interest rate offers exist and then attempt to explain why lenders might engage in such "credit rationing."

Our analysis begins by questioning the prevalent identification of credit rationing with rejected offers to pay market rates of interest. This concept of credit rationing is apparently derived by analogy with the theory of commodity markets under certainty. In that theory, any economic agent who makes an effective demand for a commodity, that is, who offers to pay its market price, is subject to nonprice commodity rationing if his demand is not supplied. The common extension of this conclusion to credit markets is that any economic agent who offers to pay the market rate of interest on some type of loan is subject to credit rationing if his "demand" for credit is not supplied. We argue that this concept of credit rationing is not useful because it is based on an inappropriate implicit assumption that an offer to pay the market rate of interest on a loan constitutes an effective demand for credit. In Section I we show that the distinction between a borrower's wants and demands for credit depends not only on the rate of interest offered, but also on the amount of collateral offered and on the borrower's equity. Therefore, if one is to have a concept of credit rationing that refers to nonsupplied effective demands for loans, rather than unsatisfied wants, it must involve analysis of lender response to offers of interest rate-collateral-equity combinations rather than only interest rate offers.

Freimer and Gordon consider the case of a risk-neutral lender who faces a certain cost of funds and observe that his supply of credit to a borrower may not be an increasing function of the rate of interest offered by the borrower. They attach significance to this observation, saying that it raises the possibility of unstable equilibria and protracted excess demand in credit markets; this has been called "disequilibrium credit rationing." In Sections II and III we show that, under various conditions, the supply of credit to a borrower is an increasing function of the amounts of collateral and equity offered by the borrower. Thus under the conditions assumed by Freimer-Gordon, and under more general conditions, a borrower will be supplied more credit if he offers more collateral or equity.

Jaffee and Modigliani's primary concern is with "equilibrium credit rationing." They assume that a lender can act as a discriminating monopolist and conclude that he will ration some borrowers if he is subject to an institutional constraint which requires him to charge the same interest rate to borrowers with different demand curves for credit. In Section IV we demonstrate that credit rationing is not optimal for any lender unless there are effective institutional constraints on the collateral and equity terms of loan contracts in addition to an effective constraint on interest rates. Therefore, given the Jaffee and Modigliani assumption of a single interest rate constraint, their conclusion that credit rationing is rational for a monopolistic lender is shown to be false.

I. Collateral, Equity, and Effective Demand for Loans

We proceed to an analysis of the role of equity and collateral in transforming a desire for credit into an effective demand for a loan. Assume that a lender has preferences defined over his random terminal wealth $\boldsymbol{x}$, and that

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he prefers more wealth to less. He begins with some initial wealth $w>0$ and lends an amount l, where $0 \leqq l \leqq w$, to a borrower who invests it in an opportunity which yields the constant stochastic rate of return θ, where $\theta \geqq -1$. The lender is assumed to invest the rest of his wealth $(w-l)$ at the constant stochastic rate of return ρ, where $\rho \geqq -1$. If the loan is repaid, then the lender's terminal wealth is the sum of the principal and interest on the loan $(1+r)l$, and the value of his other investment $(1+\rho)(w-l)$, and can be written as $(1+\rho)w+(r-\rho)l$.

The amount the borrower invests is the sum of the amount of the loan l, and the amount of the borrower's equity y, where $y \geqq 0$. The loan will be in default if θ is less than the ***default rate of return*** θ^*, which is the lowest rate of return on the borrower's investment sufficient to pay the principal and interest on the loan.

$$\theta^* = \frac{rl - y}{l + y} \tag{1}$$

If the borrower provides some collateral, the lender can obtain payment of principal and interest at some rates of return that are below the default rate of return on the borrower's investment. Let the borrower provide as collateral an asset that has value z, where $z \geqq 0$, at the time the loan contract is written. The subsequent value of the collateral is the random variable $(1+\pi)z$, where π is the constant stochastic rate of return on the collateral asset and $\pi \geqq -1$. The lender will obtain payment of principal and interest on a collateralized loan as long as the total returns on the investment $(1+\theta)(l+y)$, plus the value of the collateral $(1+\pi)z$, exceed the principal and interest due on the loan $(1+r)l$. Thus the lender will collect principal and interest if θ is not less than the ***repayment rate of return*** $\hat{\theta}$, where

$$\hat{\theta} = \frac{rl - y - (1 + \pi)z}{l + y} \tag{2}$$

If θ is less than $\hat{\theta}$ then the lender's terminal wealth is the sum of the values of his alternative investment and the borrower's investment and collateral. Thus the lender's terminal wealth x for all values of θ is given by the following function:

$$x = \begin{cases} (1 + \rho)w + (\theta - \rho)l + (1 + \theta)y \\ \quad + (1 + \pi)z, \quad \text{for } -1 \leqq \theta < \hat{\theta} \\ (1 + \rho)w + (r - \rho)l, \text{ for } \theta \geqq \hat{\theta} \end{cases} \tag{3}$$

If a loan transaction is to be made, the terms of the transaction must provide the lender with a distribution of terminal wealth that he prefers to all other attainable distributions. Unless the borrower has monopoly control of the probability distribution of θ, the lender has the option of investing some of his initial wealth in an opportunity which yields θ. If the lender can invest in such an opportunity, then one of the investment options in his feasible choice set is provided by investing the amount $(w-l)$ in an opportunity which yields ρ and an amount l in an opportunity which yields θ. This provides the terminal wealth function

$$x = (1+\rho)(w-l)+(1+\theta)l = (1+\rho)w + (\theta-\rho)l, \text{ for all } \theta \tag{4}$$

If y and z are both zero, then (4) dominates (3) and the potential lender will never prefer the loan to making the investment himself. Since a desire for credit by a nonmonopolistic potential borrower who does not supply collateral or equity will never be supplied, such a desire cannot be an effective demand for credit. We thus have:

PROPOSITION 1: *A nonmonopolistic potential borrower must provide a positive amount of collateral or equity to transform a desire for credit into demand for a loan.*

No lender will ever supply a loan to a potential borrower who does not provide a positive amount of collateral or equity as long as the borrower does not have monopoly control of a return distribution. Monopoly control of a return distribution is a stronger condition than monopoly control of an investment opportunity. The former requires that the lender be unable, through any combination of portfolio and direct investment, to duplicate the distribution of returns on the potential borrower's investment opportunity.

The preceding proposition is based on the hypothesis that the lender's feasible choice set includes an investment opportunity that yields the same probability distribution of returns as the potential borrower's prospective investment. We will next extend the analysis to include a case where the potential borrower can have monopoly control of a return distribution. The subsequent propositions will depend on the hypothesis that the lender's optimal loan satisfies first- and second-order conditions for maximization of a von Neumann-Morgenstern utility of wealth function. This will be called *hypothesis H.1*. Given this hypothesis, we need to examine the first- and second-order conditions for maximization of the von Neumann-Morgenstern utility function

$$(5)\quad \int_{-1}^{\infty}\int_{-1}^{\infty}\int_{-1}^{\infty} u(x)g(\theta,\rho,\pi)d\theta d\rho d\pi$$

where x is the terminal wealth variable defined in statement (3) and $g(\cdot)$ is a joint probability density function. The first- and second-order conditions for maximization of (5) with respect to l are

$$(6)\quad \int_{-1}^{\infty}\int_{-1}^{\infty}\left\{\int_{-1}^{\hat{\theta}} u'(x)[\theta-\rho]g(\theta,\rho,\pi)d\theta + \int_{\hat{\theta}}^{\infty} u'(x)[r-\rho]g(\theta,\rho,\pi)d\theta\right\} d\rho d\pi = 0$$

$$(7)\quad D = \int_{-1}^{\infty}\int_{-1}^{\infty}\left\{u'(\hat{x})[\hat{\theta}-r][(y+ry+z+\pi z)/(l+y)^2]g(\hat{\theta},\rho,\pi) + \int_{-1}^{\hat{\theta}} u''(x)[\theta-\rho]^2 g(\theta,\rho,\pi)d\theta + \int_{\hat{\theta}}^{\infty} u''(x)[r-\rho]^2 g(\theta,\rho,\pi)d\theta\right\} d\rho d\pi < 0$$

where $\hat{x}$ denotes the function that is derived from (3) by setting θ equal to $\hat{\theta}$.

We now proceed to proof of a second proposition on effective demand. Assume hypothesis *H.1* and that the lender is risk neutral. As a consequence of *H.1* we know that the lender's optimum loan satisfies the second-order condition (7). The risk neutrality assumption on preferences implies that the second and third integral expressions in (7) are everywhere equal to zero. Thus statement (7) requires that the first integral expression be negative. Since y and z are nonnegative, this expression can be negative only if y or z is positive and $[\hat{\theta}-r]$ is negative. But statement (2) implies that $[\hat{\theta}-r]$ is negative only if y or z is positive. Therefore, if a loan is to be supplied given the above hypothesis, the borrower must provide a positive amount of collateral or equity. Therefore, we have:

PROPOSITION 2: *Given hypothesis H.1, any potential borrower must provide a positive amount of collateral or equity to transform a desire for credit into demand for a loan from risk-neutral lenders.*

Propositions 1 and 2 inform us that analysis of credit supply responses must involve study of lender response to changes in borrower equity and collateral as well as lender response to interest rate changes. One cannot explain "credit rationing," meaning unsupplied effective demands for credit, without introducing the collateral and equity components of loan contracts that make the credit demand effective. We now proceed to examine the comparative statics of the supply of credit.

II. Collateral, Equity, and the Supply of Loans

Derivation of the comparative statics of loan supply with respect to the interest rate leads to indeterminate results in the present model, as it did in the special case examined by previous authors. These results will not be reproduced here; instead, we examine lender responses to changes in collateral and equity.

Considering the effect of changes in the amount of collateral, we differentiate (6) with respect to z and find that

$$(8)\quad \frac{\partial l}{\partial z} = \frac{A}{D}$$

where

$$(9) \quad A = \int_{-1}^{\infty}\int_{-1}^{\infty}\left\{u'(\hat{x})[\hat{\theta} - r] [(1+\pi)/(l+y)]g(\hat{\theta},\rho,\pi) + \int_{-1}^{\hat{\theta}} u''(x)[\rho - \theta] [1 + \pi]g(\theta,\rho,\pi)d\theta\right\}d\rho d\pi$$

and D is defined in statement (7).

Displacing the equilibrium with respect to y, we get

$$(10) \qquad \frac{\partial l}{\partial y} = \frac{M}{D}$$

where

$$(11) \quad M = \int_{-1}^{\infty}\int_{-1}^{\infty}\left\{u'(x)[\hat{\theta} - r] [(1 + \hat{\theta})/(l + y)]g(\hat{\theta},\rho,\pi) + \int_{-1}^{\hat{\theta}} u''(x)[\rho - \theta] [1 + \theta]g(\theta,\rho,\pi)d\theta\right\}d\rho d\pi$$

Since D is negative by the second-order condition (7), the signs of the relationships between lender's optimal loan size and amounts of collateral and equity depend, respectively, on the signs of A and M, and will be positive if A and M are negative.

We will proceed to examine various special cases of the model developed above. We will begin with the case of a risk-neutral lender. For such a lender, the expressions in (9) and (11) which contain $u''(x)$ are everywhere equal to zero. The second-order condition (7) and statement (2) tell us that $[\hat{\theta}-r]$ is negative. We have proved the following proposition.

PROPOSITION 3: *Given hypothesis H.1, a borrower can increase the size of a loan from a risk-neutral lender by offering more collateral or equity.*

We will next extend the analysis to comprehend supply responses of risk-averse lenders. The resulting propositions will vary with the assumptions made about the random returns on the lender's alternative investment and on the collateral asset. We will begin with the assumption that the lender's alternative investment yields the same constant stochastic rate of return as the borrower's investment, θ. Substituting $\rho=\theta$ in (9) and (11), the second integral expression in each equation vanishes. Since the lender's alternative investment yields the random rate of return θ, the borrower does not have a monopoly of this return. Therefore, by Proposition 1, either y or z must be positive. Then from statement (2) we know that $[\hat{\theta}-r]$ is negative. We have proved the following proposition.

PROPOSITION 4: *Given hypothesis H.1 and the hypothesis that the lender's alternative investment yields the same random rate of return as the borrower's investment, a borrower can increase the size of a loan from a risk-averse lender by offering more collateral or equity.*

Of course if the borrower has monopoly control of a return distribution, the lender would have to make his alternative investment in an investment opportunity that yields a rate of return that is distinct from θ. We will examine two cases where ρ and θ are distinct and the lender is risk averse. The first case will employ the assumption used by previous authors that the lender's alternative investment is made at a certain rate of interest i. In addition, we assume that the rate of return on the collateral asset is this same certain rate of interest. In this case, equations (9) and (11) can be rewritten as:

$$(9') \quad A = u'(\hat{x})[\hat{\theta} - r][(1 + i)/(l + y)]f(\hat{\theta}) + \int_{-1}^{\hat{\theta}} u''(x)[i - \theta][1 + i]f(\theta)d\theta$$

$$(11') \quad M = u'(\hat{x})[\hat{\theta} - r][(1 + \hat{\theta})/(l + y)]f(\hat{\theta}) + \int_{-1}^{\hat{\theta}} u''(x)[i - \theta][1 + \theta]f(\theta)d\theta$$

where $f(\cdot)$ is the probability density function for θ. A sufficient condition for both (9′) and (11′) to be negative is that $\hat{\theta}$ is less than both i and r. Statement (2) implies that $\hat{\theta}$ will be less than i if the collateral plus equity to loan ratio satisfies the condition

$$\frac{z+y}{l} > \frac{r-i}{1+i} \tag{12}$$

Clearly, a lender who prefers more wealth to less wealth will not make a loan if the rate of interest on the loan is less than the rate of interest on his alternative investment opportunity. Therefore, condition (12) is sufficient to ensure that $\hat{\theta}$ is less than both i and r. Thus we have:

PROPOSITION 5: *Given hypothesis H.1 and the hypothesis that the collateral asset and the lender's alternative investment yield the same certain rate of interest, a borrower can increase the size of a loan from a risk-averse lender by offering more collateral or equity if* $[(z+y)/l] > [(r-i)/(1+i)]$.

The maximum of $[(r-i)/(1+i)]$ on the set $\{(i,r)$: 4 percent$\leqq i \leqq r$; 4 percent$\leqq r \leqq 18$ percent$\}$ is 13.5 percent at $(i,r)=$(4 percent, 18 percent). Thus condition (12) is satisfied by the values typically observed in credit markets.

Finally, we consider the case where ρ, θ, and π are distinct random variables and the lender is risk averse. This case requires that we evaluate A and M as given in (9) and (11). Since $y \geqq 0$ and $z \geqq 0$, statement (2) implies that $\hat{\theta} \leqq r$. Therefore the first terms on the right-hand sides of (9) and (11) are nonpositive. Statement (2) also tells us that $\hat{\theta} < r$ if $y>0$ or $z>0$; in this case, the first terms on the right-hand side of (9) and (11) are negative. The second terms on the right-hand sides of (9) and (11) are nonpositive if $prob\{\rho \geqq \theta$ for all $\theta \leqq \hat{\theta}\}=1$, and are negative if $prob\{\rho \geqq \theta$ for all $\theta \leqq \hat{\theta}\}=1$ and $\rho > \theta$ for some $\theta < \hat{\theta}$. Thus we have:

PROPOSITION 6: *Given hypothesis H.1, a borrower can increase the size of a loan from a risk-averse lender by offering more collateral or equity if:*

$$prob\{\rho \geqq \theta \textit{ for all } \theta \leqq \hat{\theta}\} = 1 \textit{ and } (z+y) > 0;$$

or

$$prob\{\rho \geqq \theta \textit{ for all } \theta \leqq \hat{\theta}\} = 1 \textit{ and } \rho > \theta \textit{ for some } \theta \leqq \hat{\theta}$$

We have proved various propositions on effective demand for loans and on the relation of the amount of credit supplied to amounts of collateral and equity. All of the propositions follow from a model in which the proceeds of a loan are used to acquire a capital asset. This formulation applies to "consumer loans" such as mortgages and loans on consumer durables but does not apply to loans to consumers for expenditures on services and nondurable commodities. The next section is concerned with the supply of pure consumption loans, where a *pure consumption loan* is any loan the proceeds of which are not used to acquire a capital asset.

III. Collateral and the Supply of Pure Consumption Loans

The supply model for pure consumption loans can be developed easily by analogy with the model developed above. Given the rate of interest r on the loan and the random rate of return ρ on the lender's alternative investment, the lender's terminal wealth if the loan is repaid is $(1+\rho)w+(r-\rho)l$. Define q as the random total amount of payment that the consumer makes on the loan. If q is less than the sum of principal and interest due on the loan then the loan is in default. Thus the *default amount of payment* q^* is $(1+r)l$. Let the consumer provide a nonnegative amount of collateral z in the form of an asset with random rate of return π. The lender will obtain payment of principal and interest on the loan, even though the loan may be in default, as long as the sum of the borrower's payment q and the value of the collateral $(1+\pi)z$ is not less than the principal and interest on the loan. Thus the *repayment amount of payment* $\hat{q}$ is $(1+r)l-(1+\pi)z$. If the amount of payment on the loan is less than $\hat{q}$ then the lender's terminal wealth is $q+(1+\rho)(w-l)+(1+\pi)z$. Given hypothesis *H.1*, we can use the first- and second-order conditions for maximization of a von Neumann-Morgenstern utility function with joint probability density function for q, ρ, and π. Finding $\partial l/\partial z$ by straightforward differentiation of the first-order condition, and using the negativity of the second-order condition and the nonpositivity of $u''(x)$, one can easily prove the following proposition.

PROPOSITION 7: *Given hypothesis H.1, a borrower can increase the size of a pure consumption loan from a risk-averse or risk-neutral lender by offering more collateral.*

Propositions 2–7 depend on the assumption of price taking behavior by lenders and thus do not directly apply to the attempt by Jaffee and Modigliani to show that credit rationing is profitable for monopolistic lenders. The next section fills in this gap.

IV. Market Organization, Equilibrium, and Credit Rationing

Jaffee and Modigliani attempt to demonstrate that if lenders are not price takers and exogenous constraints exist on interest rates, then rationing can be optimal for lenders and can exist in equilibrium. We argue that whether lenders are or are not price takers, credit rationing cannot be optimal for them at a market equilibrium unless institutional constraints are placed on the equity and collateral terms of loans in addition to the interest rate.[1]

In Jaffee and Modigliani's discussion, lenders are assumed to be able to act like discriminating monopolists who face price-taking borrowers who differ in their demand functions for credit. Without exogenous constraints on interest rates, borrowers who differ in their demands for credit would in general be charged different interest rates. By analogy with commodity markets under certainty, Jaffee and Modigliani conclude that if all borrowers must be charged the same interest rate, then lenders who could otherwise act as discriminating monopolists would ration some borrowers. They arrive at this conclusion by implicitly assuming that a borrower's offer to pay the interest rate represents an effective demand. When collateral and equity are introduced into the model, one does not need the assumption that lenders are discriminating monopolists to explain why borrowers with different demand functions for credit may be charged different interest rates. In general the market equilibrating process would result in the demands of various borrowers being satisfied at different collateral-equity-interest rate combinations.

We can easily demonstrate that with or without exogenous constraints on interest rates, credit rationing cannot exist in equilibrium. The amount of credit that a borrower demands will depend on the interest rate he must agree to pay and on the amount of collateral and equity he must provide. If a borrower is rationed, then the amount of credit supplied to the borrower is less than the amount he demands. Since the amount of credit demanded is a function of the interest rate, collateral, and equity terms of the loan contract, any one of the three possible two-dimensional representations of the demand function must show that the amount of credit supplied is less than the amount demanded if credit rationing is to occur. Consider Figure 1 which contains the schedule which relates the amount of credit demanded to the amount of collateral for given

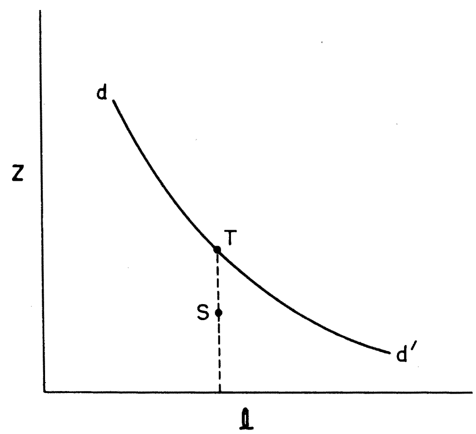

FIGURE 1

[1] We assume atomistic borrowers; in other words, we exclude bilateral monopoly.

values of the rate of interest and the amount of equity. If credit rationing is to be optimal for a lender then there must exist a point S in Figure 1 that is below the demand schedule and represents an optimal transaction for a lender. However, point S cannot be optimal for a lender because point T, a point on the demand schedule, is in the lender's feasible set of credit transactions. Points T and S involve the same amount of credit, the same interest rate, and the same equity financing but at T the lender gets more collateral. If a lender is rationing a borrower, that lender is foregoing collateral that he could obtain without altering the other terms of the credit transaction or the terms of other transactions including other loans. Since the partial derivative of the lender's expected utility function with respect to collateral is positive for all $z \geqq 0$, credit rationing cannot be optimal for any lender so long as there are no constraints on collateral. An analogous argument can be made for the equity component of credit transactions.[2]

[2] However, the demand for credit may not be a decreasing function of the amount of equity.

REFERENCES

M. Freimer and M. Gordon, "Why Bankers Ration Credit," *Quart. J. Econ.*, Aug. 1965, *79*, 397–416.

D. Jaffee and F. Modigliani, "A Theory and Test of Credit Rationing," *Amer. Econ. Rev.*, Dec. 1969, *59*, 850–72.