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Let’s Talk About Math: Exploring How Elementary Teachers Plan And Implement Mathematical Discourse In The Classroom

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ACCEPTANCE

This dissertation, LET'S TALK ABOUT MATH: EXPLORING HOW ELEMENTARY TEACHERS PLAN AND IMPLEMENT MATHEMATICAL DISCOURSE IN THE CLASSROOM, by ANNE ROBERTSON GREEN was prepared under the direction of the candidate's Dissertation Advisory Committee. It is accepted by the committee members in partial fulfillment of the requirements for the degree, Doctor of Philosophy, in the College of Education & Human Development, Georgia State University.

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LET'S TALK ABOUT MATH: EXPLORING HOW ELEMENTARY TEACHERS PLAN AND IMPLEMENT MATHEMATICAL DISCOURSE IN THE CLASSROOM

by

ANNE ROBERTSON GREEN

Under the Direction of Susan Swars Auslander

ABSTRACT

Mathematical discourse is a critical component of effective mathematics instruction, but it remains one of the least implemented strategies in mathematics classrooms. This exploratory case study examined how elementary teachers understand, plan, and implement mathematical discourse practices, with a particular focus on connections to the curriculum. Participants included three kindergarten teachers in an urban-situated elementary school. Data were collected through: two sets of documents, including the provided curriculum and teacher-created lesson plans; two individual, semi-structured interviews with each participant; and two classroom observations of each participant's mathematics instructional practices. The data were analyzed through the constant comparative method. Interview data were coded through a three-stage coding cycle, resulting in emergent themes. Data collected through documents and observations were categorized and compared to the interview codes created through the coding cycle to

determine themes. The findings show teachers had a desire to engage in mathematical discourse but there were barriers to implementation such as time, academic language, and COVID-19 protocols. Additionally, there was no appreciable influence of the written curriculum on the enacted curriculum. However, the utilization of a curriculum with supports for classroom discourse may give teachers tools to engage students in more high press lines of questioning. In the context of a stressed work environment, teachers did alter the curriculum to simplify and lessen the cognitive load of the discourse for students. The findings illuminate how teachers would benefit from practice with mathematical discourse in the classroom within the contexts that already exist for them, such as collaborative planning sessions where teachers can become more comfortable anticipating lines of questioning. Additionally, teachers may benefit from attending to the types of questions with which they engage students during the lesson planning process.

INDEX WORDS: Mathematics, Mathematical discourse, Mathematics education, Discourse, Teacher role in discourse, Curriculum

LET'S TALK ABOUT MATH: EXPLORING HOW ELEMENTARY TEACHERS PLAN AND IMPLEMENT MATHEMATICAL DISCOURSE IN THE CLASSROOM

by

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1 INTRODUCTION

Over time, discourse and mathematical language have been emphasized by many as an important learning medium in the classroom (Pimm, 1987). The National Council of Teachers of Mathematics' (NCTM, 2014) publication, *Principles to Actions: Ensuring Mathematical Success for All,* describes a vision of reformed mathematics education involving teachers' use of eight high leverage instructional practices. One of these practices, facilitation of meaningful mathematical discourse, is considered central to building students' conceptual understanding and critical thinking skills. This study focuses on mathematical discourse in elementary classrooms in an urban school context. The overarching research question guiding this qualitative exploratory case study is: *How do teachers understand, plan for, and implement mathematical discourse in elementary classrooms?* A secondary research question also guides the study, specifically: *What role does written curriculum play in the planning and implementation of mathematical discourse in elementary classrooms?*

Problem

While mathematical discourse or *math talk,* to which it is commonly referred, is an educational buzzword that has recently gained attention, the view that mathematics could be taught through the careful sequencing of teacher questions leading to student learning and understanding dates back to Warren Colburn's approach in 1821 (Jones & Coxford, 1970). This sparked the great mathematics debate that persists in present day: Should classroom instruction occur through teachers presenting facts, procedures, and information to students or through teachers facilitating students' understanding through questioning and interaction?

In the U.S., the "New Math" movement of the 1950s and 1960s brought with it a return to Colburn's ideas of classroom instruction emphasizing conceptual understanding of mathematics

rather than the isolated routine of mathematical memorization with practice (Graumann, 2019). However, it was again met with backlash because the introduction of new ways of conceptualizing mathematics were confusing and irrational to those who had experienced a more traditional mathematics schooling. Once again, the pendulum swung back in the other direction with the “Back to Basics” movement of the 1970s and 1980s, and with it came the return in focus on mindless computations and procedures, skills and practice, and extensive use of standardized testing (Fey & Graeber, 2003). In the late 1980s and early 1990s, the NCTM created standards for school mathematics focused on teaching, learning, curriculum, and assessments. By the mid-1990s, 41 states had adopted or created standards consistent with the NCTM’s recommendations (McLeod, 2003). The NCTM’s *Professional Standards for Teaching Mathematics* (1991) stressed the importance of conceptual understanding, calling for student participation in discourse-based classrooms as central to their learning. The role of the teacher should be that of a guide who is active rather than passive, giving students space to meaningfully explore mathematical concepts. However, with the federal introduction of the No Child Left Behind (NCLB) Act in 2001, the priority shifted to standardized testing across content areas, which for mathematics meant an emphasis on skills and low-level understanding. More recently, mathematics reform initiatives have included the Common Core State Standards-Mathematics (CCSS-M), which has been adopted by many states in the U.S. and has the aim of building students’ conceptual understanding through problem solving and reasoning, justification, and use of representations (National Governors Association Center for Best Practices & Council of Chief State School Officers (NGACBP & CCSSO), 2010).

When the CCSS-M (NGACBP & CCSSO, 2010) was published, these standards were intended to be a roadmap of the content students should learn at each grade level and the

associated mathematical practices through which this content should be learned. Changing mathematics education in the U.S., including states' standards so they align with the CCSS-M, is a monumental endeavor often fraught with discord. The extent of needed changes was illuminated by Porter et al. (2011), who determined that the overlap between existing state standards and the CCSS-M was on average only 20% to 30% similar across the elementary grade levels. They also found a significant difference in the required levels of cognitive demand of student learning between the two sets of standards. Existing state standards had a greater emphasis on memorization and performing procedures, while the CCSS-M included more focus on demonstrating understanding and solving non-routine problems. After the first year of adoption, the Center on Education Policy conducted a survey and determined two-thirds of the school districts that had adopted the CCSS viewed the guidance on implementation of the standards as inadequate (Center on Education Policy, 2011).

Overall, teachers shifting their mathematics instruction from delivering information to facilitating learning and understanding is difficult. Several obstacles to changing their instructional practices, including the use of classroom discourse, are evident. For example, in an attempt to cover large quantities of information within the school year, Karp et al. (2014) suggest that teachers teach students *math tricks,* or shortcuts, that do not hold up when they begin to apply them to more complex situations. In addition, Ball and Forzani (2011) name two reasons for the lack of instructional improvement, including "no agreed-upon curriculum and no system for developing skilled teaching practice" (p. 18). NCTM's *Principles to Actions* (2014) attempted to develop what Ball and Forzani (2011) call a "common core for teaching practices" (p. 19) for mathematics, which gave teachers insight into *how* to teach students so that they would learn the new CCSS-M. It named "facilitation of meaningful mathematical discourse"

(NCTM, 2014, p. 29) as one of the eight essential components of mathematics instruction. Even still, discourse is a component of teaching mathematics that remains one of the least employed strategies (Herbel-Eisenmann, 2007). Additionally, as the standards and expectations of teachers and students change, curriculum guidelines change as well to support the goals of the current standards (Gewertz, 2011). With discourse being an important component of the current CCSS-M (NGACBP & CCSSO, 2010), it should be considered whether curricula include mathematical discourse resources in their materials and if teachers are able to understand and use them effectively.

An additional layer to the difficulties associated with changing teachers' mathematics instruction is the larger issue of the failings of mathematics education in the U.S., especially in regard to historically marginalized groups. In 2015, the Trends in Mathematics and Science Study (TIMSS) ranked the U.S. 10th out of 49 developing and developed countries in mathematical performance of fourth grade students, which is at least an improvement from 1997 when the U.S. ranked 16th (International Association for the Evaluation of Educational Achievement, 1997). Notably, the findings of assessments such as TIMMS have contributed to global competition in which countries attempt to gain advantage by replicating another country's programs without regard to culture and context (Tsai & Li, 2017). Further, these assessments have consistently revealed mathematics achievement gaps in the U.S. between students of color and their white peers. These gaps have contributed to a culture of *teaching to the test* becoming rampant in K-12 schools, especially in those with high minority populations and in urban settings, in an attempt to appear successful (Kitchen et al., 2016). This occurrence has caused the mathematical practices in the CCSS-M (NGACBP & CCSSO, 2010) and effective mathematics

teaching practices in *Principles to Actions* (NCTM, 2014) to become much lower priorities (Hargreaves & Shirley, 2009; Kitchen et al., 2016).

Research Questions

The primary question guiding this qualitative exploratory case study is:
How do teachers understand, plan for, and implement mathematical discourse in elementary classrooms? The secondary research question is: What role does the written curriculum play in the planning and implementation of mathematical discourse in elementary classrooms? For the purposes of this study, discourse is defined as "the purposeful exchange of ideas through classroom discussion, as well as through other forms of verbal, visual, and written communication" (NCTM, 2014, p. 29). Further, curriculum is defined as the printed material that is provided to teachers through adoption of a text or set of resources by the school or district, which is also referred to as the *written curriculum.*

Theoretical Framework

This study uses dialogism (Bakhtin, 1981) and socioconstructivism (Cobb, 1994; Cobb & Bauersfeld, 1995; Cobb, Wood, & Yackel, 1990; Cobb et al., 1993; Hufferd-Ackles, Fuson, & Sherin, 2004) to explore the ways teachers understand, plan, and implement mathematical discourse in elementary classrooms, including the use of curriculum. By examining the processes and resources teachers use to help students make meaning of mathematical concepts, findings can help build an understanding of the affordances and challenges surrounding mathematical discourse in the elementary classroom. Socioconstructivism provided a lens, not only to the social nature of the construction of knowledge *within* the classroom, but also the social nature of the construction of knowledge as teachers plan their lessons (Cobb, 1994; Cobb & Bauersfeld, 1995; Cobb, Wood, & Yackel, 1990; Cobb, Yackel, & Wood, 1993; Hufferd-Ackles et al.,

2004). While teaching as a profession often feels isolating and segmented, teachers often seek out peers for collaboration and the sharing of ideas.

Dialogism

Bakhtin's (1981) theory of dialogism asserts that words carry the past in them, with nothing being said in isolation. Every time we speak, we are speaking in response to the past and in anticipation of the future. Dialogic word exists to be in relationship with others' words, in order to alter or inform it. Bakhtin's work critiques the viewpoint that in a disagreement, one person must be wrong. Instead, he asserts that because there are many different viewpoints, the truth requires multiple perspectives in order to establish truth in a particular context. Bakhtin (1981) developed a relational approach to language in which all that is spoken or thought is connected to the things and people and language that exist around us in conjunction with those that have existed in the past. Discourse carries "multiple voices, perspectives or intentions" (Barwell, 2016, p. 336). In this way, discourse is not merely a *tool* for mathematical thinking; it *is* the mathematical thinking itself. Sfard and Avigail (2007) additionally support this idea by adding that discourse is not created as new, instead it is changed based on the fluent discourses that already exist within a student. The process of thinking, including informing ourselves, arguing, asking questions, and waiting for a self-response, is a dialogic exercise (Bakhtin, 1981; Barwell, 2016; Sfard & Avigail, 2007). In a non-dialogic (or authoritative) exchange, the teacher pushes a scientific view, directs the discourse in a prescribed manner, micromanages student points of view, corrects or redirects student conjectures, places constraints on the direction of discourse, and rejects ideas inconsistent with their own (Bakhtin, 1981; Mortimer & Scott, 2003; Saglam et al., 2015).

This framework is relevant to this study in that dialogic discourse relies on others' perspectives in order to come to a mathematical truth that can be agreed upon within the context presented. It also grounds several assumptions of the study: (a) discourse is more than a *tool* for learning, it is the learning *itself*; (b) construction of knowledge is a social process; and (c) context of dialogue and discourse matters.

Socioconstructivism

Socioconstructivism is a framework in which knowledge is constructed based on culture, experience, and the outside world (Cobb, 1994; Cobb & Bauersfeld, 1995; Cobb et al., 1990; Cobb et al., 1993; Hufferd-Ackles et al., 2004). Meaning in mathematics is constructed within community through reflection and discussion (Cobb & Bauersfeld, 1995; Hufferd-Ackles et al., 2004). A critical element of this framework is that students must be active participants in their learning in order to construct knowledge and accordingly, learners should be contributing significantly within discourse.

Truxaw and DeFranco (2008), however, state that the "mere presence of talk does not ensure that understanding follows" (p. 489). It is the *quality* and *type* of discourse that contribute to the mathematical thinking of students (Kazemi & Stipek, 2001; Truxaw & DeFranco, 2008). Many researchers argue that a "give-and-take communication that uses dialogue as a process for thinking" (Truxaw & DeFranco, 2008, p. 489) or dialogic discourse, leads to deeper conceptual understanding (Bakhtin, 1981; Bruner, 1996; Hobson, 2004; Knuth & Peressini, 2001; Tomasello, 2001; Truxaw & DeFranco, 2008; Wood, et al., 2006) rather than simply producing a "maximally accurate transmission of a message" or univocal discourse, (Lotman, 1988, p. 68) in which the teacher or student is concisely delivering the steps taken toward a correct answer. Classrooms that incorporate effective mathematical discourse are more than simply "strategy

reporting," (Wood et al., 2006, p. 224) or having students recalling the step-by-step processes that they performed in order to arrive at a solution. They are participating in a cycle of inquiry and critical thinking, alongside argument and defense both student-to-teacher and student-to-student (Wood et al., 2006).

Mathematical discourse also includes the "purposeful exchange of ideas" (NCTM, 2014, p. 29) during dialogue and discussion as well as written formats (NCTM, 2014). Additionally, mathematical discourse is centered on reasoning and problem solving in order to build strong foundations of mathematical concepts and promote meaningful learning (Michaels et al., 2008; NCTM, 2014). Discussions should involve dialogic discourse in which students hold equal or primary talking space within the classroom. Mathematical discourse should encourage growth in conceptual understandings, supporting all students (NCTM, 2014).

The interplay of dialogism and socioconstructivism undergirds the framework on which this study is designed. There is a strong connection between mathematical discourse, dialogism, and socioconstructivism as they all operate under the assumption that knowledge is constructed through collaborative community discussion (Bakhtin, 1981; Bruner, 1996; Cobb & Bauersfeld, 1995; Hufferd-Ackles et al., 2004; Truxaw & DeFranco, 2008; Hobson, 2004; Knuth & Peressini, 2001; Tomasello, 2001; Wood, et al., 2006). Where the frameworks diverge is in connection to the mathematics curriculum. Herbel-Eisemann (2007) characterizes the disconnect between mathematics curriculum and the implementation of mathematical discourse not as an intentional disregard of the vision and goals of the standards, but a result of the susceptibility of curriculum developers to "conventional mathematics and mathematics education discourses" (p. 361).

Need for the Study

This study is significant because it examines teachers understanding, planning, and implementing mathematical discourse practices, with a focus on how these relate to the curriculum provided to teachers. This research aims not to focus on the bigger systems at play such as corporations and politicians that dictate curriculum choices and implementation policies for mathematical discourse, instead, focusing on teacher moves and choices when planning for and implementing mathematical discourse that contribute to student voices being validated (or invalidated) within the classroom. It also seeks to examine the role that mathematics curricula play as teachers interpret, modify, and implement them.

It is important to assess mathematics discourse practices in elementary classrooms. When it comes to the body of research on classroom discourse in mathematics, the elementary classroom is a particularly heavily researched environment because these spaces lay the foundation for how students come to build conceptual understanding later on. It is widely accepted that classroom discourse is an effective practice for building conceptual understanding in the elementary classroom (Hiebert & Carpenter, 1992; Kazemi & Stipek, 2001; Michaels et al., 2008; NCTM, 2014). There is extensive research that shows when elementary students are provided with appropriate instructional support, their mathematical arguments and deductive reasoning skills improve (Ball & Bass, 2000; Jacobs, et al., 2007; Maher & Martino, 1996; Morris, 2007; Morris & Sloutsky, 1998). Interestingly, Ben-Yehuda et al. (2005) compare mathematics to traveling because there are a multitude of ways to reach a destination and any student can be successful, but teachers must provide the space and flexibility so that students can use their individual strengths and find ways to push through their weaknesses.

Although classroom discourse, specifically dialogic discourse, are widely accepted best practices, there are many reasons why they are not widely implemented effectively in elementary classrooms. A few of the reasons identified herein are curricular restrictions, equity issues, and minimal ability to standardize the practice and assessment of discourse.

Curriculum may be a mitigating factor in implementation, along with other obstacles. Curricular materials in public schools are often mandated or part of a limited pool from which teachers may select. These materials are often easily standardized and set up in a way that limits user error. Consequently, they tend to have a more scripted or rigid flow to them. Standards typically have a breadth, rather than depth, coverage pattern resulting in minimal time to spend on each mathematical concept (Knuth & Peressini, 2001). In turn, curricular materials that follow state and national standards tend to cover one standard after another with sometimes-scripted responses to student questioning in order to stay on track. Many curriculum options incorporate mathematical discourse in a narrow manner where students may report strategies and explain their process, but do not further extend thinking. Consequently, the primary voice and authority may come primarily from the textbook or the teacher, limiting the students' opportunity to construct their own knowledge and understanding. Even when opportunity for discourse is presented, the results are "strategy-reporting" (Wood et al., 2006, p. 224) with little room for argument or challenge. Many teachers find it easier to follow a problem/solution format than to push student thinking beyond what is provided to them (Carpenter et. al., 1996; Chazan & Ball, 1995; Fennema et al., 1996; Heaton, 1992; Kazemi & Stipek, 2001).

Disconcertingly, high quality mathematics curricula and the rigorous opportunity to learn important mathematics in the U.S. are generally reserved for the most affluent students (Diversity in Mathematics Education (DiME), 2007; Schmidt et. al., 2015; Tate, 1995). The

outcome of this disparity is that students in low-income communities have less access to mathematics instruction that pushes discourse as well as other high leverage practices.

Beyond the constraints of curriculum and mandates, the learning experience of classroom discourse can result in some challenges for students' academic self-image. This way of learning can cause frustration and despair rather than confidence if the learning environment is not carefully constructed (Ball, 1993). Particularly for historically marginalized groups, dynamics at play within classroom discourse can emphasize issues of power and privilege. While the national conversation centers mainly around representation of people of color in the field of mathematics, there are many studies now examining equity issues within day-to-day classroom discourse (Esmonde & Langer-Osuna, 2013; Herbel-Eisenmann, 2007, Choppin et. al., 2011; Reinholz & Shah, 2018). Research has shown over and over that mathematical discourse is vital for learning and, yet, there still exist inequities with respect to opportunities for certain marginalized groups such as students of color, females, and English Language Learners (ELLs) to participate in discourse (McAfee, 2014; Planas & Gorgorió, 2004; Reinholz & Shah, 2018; Sadker & Zittleman, 2009). This does not necessarily mean that teachers are intentionally excluding students from these marginalized groups from participating in classroom discourse, however, there are barriers that exist within the education system itself that can preclude marginalized students from equitable participation. Bids for authority within the classroom from historically marginalized groups can create tension and conflict (Esmonde & Langer-Osuna, 2013; Langer-Osuna, 2011, 2017; Wood & Kalinec, 2012). This could potentially result in less participation or risk-taking, specifically by historically marginalized groups within the context of dialogic discourse. There is a need for further examination here as researchers are still in the beginning

stages of exploring the impact of race, gender, and primary language on participation in dialogic discourse.

Additionally, conflict in understanding may arise during collaborative work, whether in community with the whole class or in small groups. In small groups, if teachers are not diligent with checking in and pushing or challenging student thinking, the conflict may go unresolved resulting in an incomplete or inaccurate understanding of the concept. The teacher may even be completely unaware that a problem exists in the first place (Sánchez & García, 2014). Developing an environment where discourse can be effective is a huge challenge (Hufferd-Ackles et al., 2004). This could cause teachers to shy away from this type of teaching because it requires more planning and monitoring of students than a typical lesson, and the teacher is not guaranteeing that they will catch every misconception or misunderstanding presented by the students.

The inability to standardize mathematical discourse to ensure teachers are effectively implementing discourse practices is a piece of the puzzle that may be a barrier to widespread implementation. Without the ability to monitor the implementation in a standardized way (other than existing formats), it poses a large risk to states, districts, and schools. However, researchers argue that the standardization of mathematics leads to the dehumanizing of the subject and can result in an erosion of ethical considerations (Kelman, 1973; Sriraman & Ernest, 2016). As an example, Morris (2007) found in her study of preservice teachers' ability to assess student mathematical arguments that there was little to no consistency in their assessments of the arguments. Many of the assessments were flawed in regard to what the preservice teachers considered to be valid mathematical arguments because they were seeking responses that followed the format of an argument rather than assessing understanding. Preservice teachers

tended to accept the argument of a single case as sufficient to generalize (Morris, 2007). Discourse relies heavily on teacher discretion and decision-making in many ways, so without meaningful and impactful professional development and instruction for teachers, it may result in more harm than good. States and districts are already severely underfunded, so for many places, offering the kind of professional development needed to ensure teachers are implementing these practices effectively is out of the question.

Significance of the Study

Findings from recent studies in the field of mathematics education suggest that effective pedagogical practices implemented by a perceptive teacher have great potential for contributing to a high-quality learning environment and deeper mathematical understandings (Reznitskaya et. al., 2012). As Walshaw (2013) states, "effective mathematics pedagogy, so the saying goes, is a gatekeeper to lifetime opportunities, signifying upward mobility, and meritocracy for the successful individual student." One of those effective pedagogical practices, which is the focus of this study, is mathematical discourse. NCTM (2014) names the facilitation of meaningful mathematical discourse as one of eight mathematics teaching practices that are "necessary to promote deep learning of mathematics" (p. 9).

Mathematics learning, and learning itself, is a social activity (Stamps, 1997). Thus, mathematical discourse provides the means to learn from a variety of experts in the room including peers, teachers, and written material. Discourse also allows students to interact with misconceptions and misunderstandings in a way that challenges their own mathematical conceptions and understandings on a regular basis.

Gaps in the current research exist around the role that written curriculum plays as teachers plan for and implement mathematical discourse in the classroom. This is a significant

piece of the puzzle that cannot be ignored. Curriculum often restricts teachers in the ways that they are able to adapt and use the mathematics material. Additionally, curriculum can guide teachers toward the direction that the designers intend for their goals and purposes (Dietiker & Riling, 2018). If the curriculum designers and/or publishers do not recognize the value of mathematical discourse, it may not be an integral part of the design of the curriculum, thus limiting teachers' exposure to this effective practice. This study is significant in that it explores teachers' use of curriculum in relation to mathematics discourse and their understanding, planning, and implementation of this high leverage instructional practice, particularly in a setting of elementary school classrooms with students who are from historically marginalized groups. It provides insights into the challenges and affordances these teachers encounter when it comes to mathematical discourse.

2 REVIEW OF LITERATURE

In this chapter, I examine the current research around (a) mathematical discourse, (b) mathematics curriculum, and (c) discourse in action.

Mathematical Discourse

According to NCTM's *Principles to Actions* (2014), there are five main components necessary for teachers to implement effective mathematical discourse within the classroom. Those components are (a) teacher role, (b) teacher questioning, (c) explaining mathematical thinking, (d) mathematical representations, and (e) building student responsibility within the classroom community. These components will be addressed in the following sections to paint a comprehensive picture of the spectrum of mathematical discourse within the classroom.

Teacher Role

Teachers hold the primary responsibility of supporting students in learning specific mathematical ideas and concepts, generally as prescribed by state or national standards. However, teachers should not merely deliver the material and send the students off to practice (Ball, 1993). Teachers are the driving force that assists in moving the practice from memorization to conceptual understanding (Hiebert & Carpenter, 1992; Kazemi & Stipek, 2001; NCTM, 2014). In order for productive dialogic discourse to exist in the elementary mathematics classroom, teachers must create a mathematical environment in which students are comfortable and willing to take risks in discussion and the problems/tasks presented in the discourse are complex and interesting to talk and argue about (Michaels et al., 2008). Stein et al. (2008) refer to such tasks as "cognitively demanding tasks", which are designed to promote critical thinking and problem-solving skills.

Additionally, the teacher is not only present during the verbal discourse portion of the learning period. Jacobs et al. (2015) found that teacher support during the students' active problem-solving time fell into five major categories: "(a) ensuring the child is making sense of the problem, (b) exploring details of a child's existing strategy, (c) encouraging the child to consider other strategies, (d) inviting the child to generate symbolic notation, and (e) adjusting the problem to match the child's understandings." Therefore, the teacher should be supporting the students' idea development throughout the thinking process rather than just the verbalization portion of the lesson. This can help develop students' thinking process in the moment rather than in hindsight.

Creating a Safe Mathematical Environment

In order for students to feel comfortable and willing to take risks in discussion, there must be accountability and community (Michaels et al., 2008). If there is accountability, but no community, students may not risk presenting poorly formulated ideas that could potentially lead to productive discourse. On the other hand, if there is community without accountability the resulting discourse is simply "polite but empty of content" (Michaels et al., 2008, p. 292). Teachers can build accountability through intentional sequencing of student strategies discovered through the monitoring phase of the work period (Smith & Stein, 2018). Teachers can develop a sense of community through the building of norms for discourse that are mutually created and agreed upon by most, if not all, students (Kazemi & Stipek, 2001; Sánchez & García, 2014).

Teachers must have the ability to simultaneously compensate for misrepresentations or misunderstandings which students present, without disrespecting the students' thinking (Ball, 1993). For example, in Ball's (1993) examination of dilemmas of teaching mathematics, she recounts a moment in a lesson on odd and even numbers in which a student, Sean, argued that a

number could be *both* odd and even at the same time. While it might have been easy for the teacher to dismiss or correct Sean, she simply listened to what Sean had to say. In turn, the teacher gave the numbers he was referring to a new name, “Sean numbers.” This opened up discussion and dialogue around the rules for Sean numbers. Although the teacher was concerned about confusing the other students or disrupting the “conventional” understandings of even and odd, they found that it did not create any confusion when students were assessed on their understanding, but it provided them an opportunity to define rules and defend their understanding in a way that respected the students’ voices.

Presenting Complex and Interesting Ideas

Some researchers argue that knowledge has to precede reasoning or discourse in order for discourse to be effective in promoting conceptual growth in the mathematical classroom, but studies show they are best developed together (Michaels et al., 2008). Ideas and concepts should, therefore, be presented in a way that allows students to grapple with the unknown. Many students view mathematics as a fixed set of facts and algorithms that are firm truths and could not possibly be argued to the contrary (Hamm & Perry, 2002; Schoenfeld, 1992; Stodolsky, 1988). If teachers leave students to believe this misconception, then opportunities for expansion of conceptual understanding and critical thinking are dismissed.

One way that researchers are exploring this component of dialogic discourse is by examining the practice of co-development of mathematical concepts and definitions (Anderson et al, 2004; Kobiela & Lehrer, 2015). Kobiela and Lehrer (2015) found that these practices were underemphasized in mathematics classrooms and that definitions and concepts were delivered from an authority rather than by utilizing components of dialogic discourse. They found that when teachers led the discourse in this way, the role of the teacher became a force to “destabilize

consensual definitions by proposing monsters" (Kobiela & Lehrer, 2015, p. 449). Meaning that alternative theories or exceptions were presented in order to disrupt the thought process of the students and therefore requiring deeper investigation in order to combat these *monsters.* This process led to an increase in contributions by students, which indicated that they were invested in the work (Kobiela & Lehrer, 2015). When used in this way, discourse is the process of thought that results in deeper comprehension of concepts and meaning.

Stein et al. (2008) developed a framework for presenting and orchestrating meaningful discourse in which the teacher selected a cognitively demanding task aimed toward a particular mathematical concept and then anticipated as many potential strategies and responses that they might encounter with their students. Then, students are presented with the task and given space to explore the task. During this time, the teacher is monitoring and pushing student thinking through questioning. Students are then selected to present their strategy and/or thinking based on purposeful sequencing by the teacher based on observations during the monitoring phase. This opens the door for discourse to occur around connections between strategies, challenges from classmates that may have arrived at a different solution, and key concepts related to the mathematical topic selected by the teacher. This circles the discussion intentionally around a mathematical topic but leaves space for students to engage and explore the multiplicity of "correct."

Mathematical Tasks. Alongside the notion that teachers should present ideas that are complex and interesting, are the types of opportunities given to students to explore those ideas. Mathematical tasks are the problems given to students to elicit strategies, skills, or understanding of specific concepts identified by the teacher or the curriculum. The recent reforms of mathematical standards call for movement away from *skill and drill* worksheets that reduce

learning to a practice in performing algorithms correctly toward more meaningful problem-solving opportunities (Hiebert & Wearne, 1993). High quality mathematical tasks are open-ended enough to provide space for discourse and debate within the classroom and require teachers to have a deep understanding of the mathematical concept in order to facilitate conversations around a particular mathematical task. According to Stein et al. (2008), a cognitively demanding task should be intentionally selected to address a specific mathematical concept but can also be solved in multiple ways. The task must be open-ended enough to allow for diverse ways of thinking so that when students share their thinking, they are able to see a variety of successful strategies to arrive at a conclusion.

Effective Teacher Questioning

The NCTM's *Principles to Actions* (2014) positions the teacher as a facilitator of discourse that pushes students to reason and allows students to be the authors of ideas. In addition, the teacher makes explicit connections between strategies and reasoning. There are four main categories of questioning that NCTM (2014) found are utilized in classrooms: (1) Gathering Information, or asking students to "recall facts, definitions, or procedures"; (2) Probing Thinking, or having students "explain, elaborate, or clarify their thinking, including articulating the steps in solution methods or the completion of a task"; (3) Making the Mathematics Visible, or having students "discuss mathematical structures and make connections among mathematical ideas and relationships"; and (4) Encouraging Reflection and Justification, where students are asked to "reveal deeper understanding of their reasoning and actions, including making an argument for the validity of their work". This is a realistic and manageable practice for teachers to implement in the classroom on a regular basis. However, studies suggest looking one step further, creating an environment in which students are the ones seeking out

connections and defending or challenging lines of reasoning (Ball, 1993; Kazemi & Stipek, 2001; Michaels et al., 2008; Truxaw & DeFranco, 2008). The teacher must use effective teacher questioning through exchanges that further student thinking and understanding (Kazemi & Stipek, 2001; NCTM, 2014). Herbel-Eisenmann (2007) found that in her examination of the *Thinking with Mathematical Models (TMM)* textbooks, when the authors would say they were going to ask a question, they would use imperatives, or instructions to direct actions, rather than asking a question.

Kazemi and Skipek's (2001) study of teacher questioning found that teachers in reform-oriented classrooms typically appeared to be concerned with the deeper understanding of mathematics; however, the teachers' exchanges typically fell into two categories: low press and high press. In low press exchanges, teachers asked questions that might ask for a show of hands in agreement, or yes/no questions that resulted in general or global responses that showed little about the students' understanding of the concepts (Kazemi & Stipek, 2001). Whereas students may still collaborate and share strategies, the focus remained on the *how* over the *why*. Additionally, when students were working in small groups low press exchanges typically limited teacher-student interactions to managerial instructions and led to very little, if any, push to deeper thinking or different thinking by the teacher. Contrastingly, high press questioning was found to exhibit four outcomes generated through sociomathematical norms (Kazemi & Stipek, 2001; Yackel & Cobb, 1996): (1) explanation produced mathematical argument rather than procedural summary, (2) multiple strategies were connected through deep understanding of their relationship to each other, (3) misconceptions and errors offered opportunities to rethink the problem and explore alternative solutions, and (4) students were held accountable individually and collectively through collaborative work (Kazemi & Stipek, 2001). These and other studies

confirm that teachers have a primary role in developing the dialogic discourse in the classroom by simultaneously stepping up to challenge thinking and stepping back to let the students' voices be the guide of the discourse.

Shared Space

By letting student-talk and teacher-listening dominate the classroom space during discourse, it can benefit both parties by allowing teachers to assess student understanding and utilize that information to make decisions about follow up questions and/or tasks (Franke et al., 2009). Shared space may appear to imply that students and teachers should both be contributing equally to the discussion; however, the role of the teacher in effective mathematical discourse is a facilitator. Although the importance of more student-talk beyond simply answering low-level, short answer questions has been thoroughly demonstrated, teacher-centered instruction continues to be pervasive in classrooms around the country (Cazden, 2001; Cuban, 1993; Graesser & Person, 1994; Serin, 2018; Stephan, 2020).

Explaining Mathematical Thinking

Listening to student explanations of mathematical thinking during mathematical discourse can offer teachers: (1) information about student understanding, (2) insight into misconceptions, and (3) opportunities to plan for follow up questioning and future lessons (Franke et al., 2009; Vanderhye & Zmijewski Demers, 2007). Often the explanation of strategy and mathematical thinking allows students to process their process and be challenged on their work. Teachers, however, may still be on the lookout for the "right" answer or are holding onto some authority they have to bestow knowledge onto students.

The "Right" Answer

The education system has changed very little in the last 200 years regarding the ways in which mathematics classrooms are set up and operated. While teachers are attempting to encourage discovery-based approaches to teaching mathematics, the "right answer" still lies with the teacher as the final authority. McCarthy et al. (2016) found that teachers make efforts to direct students by giving them hints or asking leading questions during discourse. They also noted that teachers gave "verbal checkmarks" (p. 8) such as *okay*, *yep*, *yeah*, *right*, or *good*, indicating to students that they were on the right track or were responding in the way that the teacher expected or hoped.

Mathematics in particular is a discipline that is perceived as concrete and absolute by design. Small (2010) suggests one belief about the teaching of mathematics that is commonly held by classroom teachers is that "each math question should have a single answer" (p. 29). By teaching mathematics in this way, teachers perpetuate the false idea that the student's job is to answer in a singular way dictated by the authority in the classroom, the teacher.

Authority in the Classroom

One component of mathematical discourse that can cause discomfort with teachers is the release of the idea that they are the one mathematical authority in the classroom. Authority is a resource that teachers deploy to maintain control within the classroom. One distinction that Wagner and Herbel-Eisenmann (2014) draw is the difference between being *an* authority and being *in* authority. They note that authority is not a limited resource in which the teacher holds a bank of authority and giving away half leaves them with only half of the authority. Authority and power are still being utilized but often in more concealed ways (Wagner & Herbel-Eisenmann,

2014; Oyler, 1996) such as choosing how and when students can share their thinking or creating rules around how students must present ideas.

While engaging in mathematical discourse where student voice is valued, not only can students develop their mathematical knowledge, but teachers can develop their own mathematical knowledge as well (Khuzwayo & Bansilal, 2012). Depaepe et al. (2012) identified three aspects of authority that are observed and perceived within the mathematics classroom: (1) who could provide help, (2) who could answer student questions, and (3) who could assess student responses. In many mathematics classrooms, the only person that could hold authority in these areas is the teacher. Offering students the opportunity to be an authority in one or more of these aspects may feel like a loss of control for some teachers.

Mathematical Representations

Mathematical representations may not seem like an important aspect of mathematical discourse; however, it helps to connect the shared conversation around mathematical concepts to a concrete visual depiction of strategy or thinking. Mathematical representations can come from several authorities in the classroom. They can be existing representations provided as a resource with the written curriculum (i.e., strategy posters, base 10 charts, etc.), teacher designed representations based on their expectations for students (i.e., anchor charts, powerpoint slides, etc.), teacher representations of student explanations, or authentic, original student work. Wood (2009) found that making connections between students' different visual mathematical representations plays an important role in building autonomy. While some teachers might shy away from authentic student representations, especially when they are in conflict with each other or the "right answer," this encourages student exploration and autonomous activity (Wood, 2009). Schukajlow and Krug (2014) found, similarly, that encouraging students to think about

and explore multiple solutions to a task or problem strengthened their understanding of mathematical concepts as well as increased students' interest in mathematics.

Bal (2014) found that mathematics teachers typically represent mathematical concepts in four ways: verbal, graphical, algebraic (symbolic), and numeric (table/matrix). However, they most often used verbal and algebraic representations, which limited the students' exposure to other types of mathematical representations. While teacher representations can perpetuate the notion that the teacher is the primary authority on mathematics in the classroom, it is important that when teacher representations are necessary, a variety of types of representations are utilized.

Building Student Responsibility within the Community

As discussed previously, it is the teacher's responsibility to create a classroom community in which students feel safe to take risks as well as collaboratively build norms around discourse. In order for discourse to be productive and effective, students must understand their role and responsibility within that community. One key component for establishing norms in the mathematical classroom is the intention of autonomy. The goal of norms is for them to be created by and for the participants in the classroom. This must be explicitly stated and agreed upon by all parties involved. Specific ways that students are expected to participate might include justifying answers, showing their work, and coming to a consensus with others (Weber et al., 2010; Mullins, 2018).

As students are building their mathematical identity, it is especially important for elementary educators to posit students as "experts" of mathematics, thus accepting for themselves the idea that all students are capable of complex mathematical thinking and reasoning (Howard, 2010; Kent, 2017). Building students' self-efficacy in this way can result in higher participation rates and perseverance with challenging tasks (Ozdemir & Pape, 2013).

Collaboration

Placing students into groups for group work is not enough to draw out collaborative practices. As the facilitator of discourse, teachers must find ways to guide without taking away students' autonomy and ownership over a task (Francisco & Maher, 2011). Collaboration can exist in a variety of settings (i.e., whole group discussion, small group tasks, partner work, etc.). It is important that teachers emphasize the importance of working with and learning from peers, however, there must be a clear distinction drawn between autonomous help-seeking and dependent help-seeking. Students must be able to consider the "need for help" (Yamaji, 2016, p. 256) in order to be considered autonomous in their help-seeking (Seo, 2007; Yamaji, 2016).

Cobb et al. (1997) also explored the impact of "reflective" or "mathematizing" discourse (p. 258) on first grade students. They found that when students were working in collaboration with each other, students were able to make conceptual shifts towards understanding much more easily than working alone. Collaboration also resulted in a general orientation toward mathematical understanding where mathematical culture was being built, laying the foundation for deeper understanding (Bauersfeld, 1995; Cobb et al., 1997).

Mathematical Argumentation

The NCTM *Principles to Actions* (2014) calls for students to "listen carefully to and critique the reasoning of peers, using examples to support or counterexamples to refute arguments" (p. 35). Therefore, it is the responsibility of students to challenge and/or verify claims made by their peers or the teacher. Rumsey and Langrall (2016) define mathematical argumentation as "a process of dynamic social discourse for discovering new mathematical ideas and convincing others that a claim is true" (p. 414). They argue that this component of

mathematical discourse allows students the opportunity to deepen their understanding of concepts rather than procedural understanding (Rumsey & Langrall, 2016; Rumsey 2012).

Curriculum

For some purposes, the terms *curriculum* and *standards* have been used interchangeably to refer to the activities and concepts students learn in schools. However, this is a conflation of the two terms. *Standards* refers to the content of which students are expected to demonstrate mastery in a particular grade or course. This may also include instructional resources that aid in that work. *Curriculum* refers to the program utilized by the teacher or school to help students meet those standards (NCTM, 2014; Polly, 2017). This may include "instructional materials, activities, tasks, units, lessons, and assessments" (NCTM, 2014, p. 70).

Mathematics curriculum is not simply one instrument that tells a complete story from words on paper to teacher delivery. It is, as Stein et al. (2007) describe, the relationship between the written, intended, and enacted curriculum. Each of these pieces of curriculum plays a role in the way in which the student eventually interacts with the information. In this section, described are these three aspects of curriculum.

Written Curriculum

For the purpose of this study, the term *written curriculum* is used to refer specifically to the printed material that is provided to teachers through adoption of a text or set of resources by the school or district. The "voice" and language choice of the written curriculum is intentionally designed by the authors to achieve the goals that they have set. Typically, the goals relate in some way to ensure standards are met. Herbel-Eisenmann (2007) found in her study of mathematics textbooks that the authors of these resources tended to take on the "sole authoritative voice" (p. 363). This often undermined their own goals of student-centered or

experiential learning. Rosenblatt (1988, 1994) argues that a text on its own does not hold meaning, instead, meaning is made through the reading and interpretation of the text, which is read through a particular lens. This means that, while the designers of written curriculum may try to establish the intention of their lessons clearly, there is no true way to guarantee that teachers will read and interpret that message as intended.

The final product of a written curriculum distributed by a publisher does not necessarily reflect the authentic work of the designers. It is often the result of compromises between the designers and the publishers to put forth a marketable product for distribution. These compromises could potentially disrupt the intentions of the authors and, consequently, the teachers' interpretations of the material. Curriculum designers are generally moving away from *textbooks* in the traditional sense as the national standards push for more hands-on, exploratory ways of learning. Instead, they may opt for a teacher-educative curriculum that offers teachers tools to enhance their understanding of the purpose and methods used to teach the lessons (Stein et al., 2007).

Intended Curriculum

The movement from the written curriculum to the intended, or planned, curriculum is where teachers utilize their subject-matter knowledge, interpretation of the material, and beliefs about instruction in order to evaluate and adapt the curriculum to fit the teacher's style of instruction and student needs. Teacher's adaptations of the written curriculum as they plan their mathematics lessons can range from minor changes in delivery or materials used to major deviations from the curriculum designers' goals and visions of the lesson (Sherin & Drake, 2009).

Remillard (2000) and Sherin and Drake (2009) found that the way in which teachers read the mathematics curriculum can impact the way they plan their instruction. Some teachers read for a general overview of the learning outcomes and activities, while others read through every detail of the lesson for how each part of the lesson will go. After reading, teachers then make decisions about replacing, creating, or omitting parts of the lesson (Sherin & Drake, 2009). For this reason, many curriculum designers intentionally design their text and supporting materials to support some variation (Taylor, 2013). Stein et al. (2007) refer to this as "adopting and adapting" the written curriculum. Teachers adopt certain parts or tasks from the provided material and then adapt the rest based on their prior knowledge and experience. The ways teachers adapt curricula varies widely and can become problematic if the purpose behind the adaptation is not clear or considered. For example, teachers have access to sites such as Teachers Pay Teachers to purchase and download teacher created materials to supplement or adapt their curriculum. Sometimes, the adaptation is simply a visually appealing craft rather than an extension of valuable learning.

Nicol and Crespo (2006) considered that preservice teachers, when attempting to adapt or supplement the provided curriculum, often based their decisions on "what might be considered as merely more fun for students" (p. 352). Polly (2017) found, in his study of teacher use of primary mathematics curriculum, that 23.43% of teachers used supplemental materials outside of the provided curriculum and 48.89% of those supplemental materials were not professionally developed resources (e.g., internet-based resources, Teachers Pay Teachers, teacher-created resources, Pinterest, etc.). While Polly (2017) and Nicol and Crespo (2006) did not directly address the reason for the utilization of supplementary resources, Remillard et al. (2019) may have found a potential link in their recent examination of teacher enactment of the goals set forth

by the curriculum designers. Specifically, teachers enacted the curriculum more closely to the intended design when (1) the learning goals of the lesson are clearly identified and extensively detailed, and (2) the connection between the lesson activities and the learning outcomes are explicitly stated.

Enacted Curriculum

The enacted curriculum is the way in which teachers implement the curriculum dispensed in their textbooks, frameworks, or national, state, and local standards (Usiskin & Thompson, 2014). There is often a conflict between the intentions of the designers of the written curriculum and the teachers that enact it. Curriculum designers have goals and visions for how the lessons will be enacted in the classroom; however, teachers and students have their own goals and visions related to the teaching and learning of mathematics that can end up at odds with each other. In a time of standards-based curricular reform, curriculum designers are walking a fine line between leaving enough ambiguity and flexibility for teachers to adapt the curriculum to their student, school, and district needs, while maintaining fidelity of implementation (Dietiker & Riling, 2018).

Additionally, teachers typically have developed a *way of doing* in their classroom that follows a particular structure or foundational belief held by the teacher. If the curriculum adopted by the school or district reflects the beliefs and/or follows an existing structure utilized by the teacher, the teacher is more willing to integrate or include the new curriculum into their teaching practice (Pepin et al., 2013). On the other hand, curricula that do not fit with the teacher's existing beliefs may likely be supplemented or replaced with lessons and activities that the teacher sees as a better fit for their understanding of the learning goals. This leads to a disruption in the fidelity of implementation of the provided curriculum.

Fidelity of curriculum implementation (FOI) is an area of enacted curriculum that developers research heavily to assure that the programs they design are effective. FOI can be described as an examination of the extent to which the enacted curriculum reflects the intent of the written curriculum. This can be measured in a variety of ways, including adherence to the program, exposure, quality of instruction, student performance, and differentiation. While these components ensure that the curriculum is followed as prescribed, it leaves very little room for teacher interpretation and modification (Castro Superfine et al., 2015). In addition to the curriculum designers pushing FOI onto teachers, many districts and principals push their own interpretations of curriculum goals and visions onto teachers as well. As a result, teachers may be left feeling like they have little autonomy over the content or delivery of mathematics instruction.

Discourse in Practice

Since the introduction of the CCSS-M, many teacher preparation programs have used the NCTM's *Principles to Actions* (2014) as a guide for helping preservice teachers develop proficiency with the eight mathematics teaching practices. Lui and Bonner (2016) found in their study that teachers had little formal background in mathematics and struggled to analyze student work. Although many teachers in the study endorsed constructivist beliefs, when planning instruction, they reverted back to traditional methods for teaching mathematics (Lui & Bonner, 2016). This seems to be common as new teachers are overwhelmed with new curriculum, school-specific expectations, and managing a classroom of students. While many *want* to implement the practices they may have learned in their preparation programs, they return to the ways they were taught because these methods are familiar. Purnomo et al. (2017) found, similarly, that the teacher in their study often held more traditional beliefs in the content area of mathematics than

other areas. They noted several reasons for the teacher's restriction of more constructivist practices in the mathematics classroom including "previous school experience, social norms, mathematical knowledge for teaching, the attitude that dares not to act out of habit, time constraints, high-stakes testing, curriculum, student behavior and the learning environment" (Purnomo et al., 2017, p. 638). These seem to be common threads through much of the current research on teachers' practices in the mathematics classroom.

Additionally, Hiebert and Wearne (1993) examined the differences between discourse-based instruction and non-discourse-based instruction in 5 second grade classes over the course of a 12-week unit. The questioning used by teachers in the non-discourse-based classrooms was naturally more mechanical and procedural in nature, whereas the teachers in the discourse-based classrooms used questioning designed to push student thinking, calling for more complex analysis and comparison of strategies. While in the discourse-based classrooms they spent twice as long on each problem, the outcomes for students were a deeper understanding of the mathematical concepts. The teacher questioning also influenced how students viewed the tasks they were presented with and set expectations for how to approach the task. In the non-discourse-based classrooms, students approached tasks with a procedural or mechanical framework, while students in the discourse-based classrooms tended to approach tasks with a more analytical framework. Franke et al. (2009) specifically examined the follow-up questioning teachers use in the context of pushing student strategy explanation. They assert in their findings that while teachers used a variety of types of follow up questions, the pattern and quantity of questioning had more impact than the specific type of question. Single questions were simply not sufficient in helping teachers uncover the details of students' strategies in order to make thinking known.

Instead, series of specific probing questions helped students correct and complete their explanation.

As discussed previously, teachers struggle when asked to bestow authority to students who are solving tasks and participating in discourse as it introduces confusion and unpredictability into the classroom (Depaepe et al., 2012). In their study of two Flemish elementary classrooms, Depaepe et al. (2012) found that there were inconsistencies in what teachers claimed to do and what they actually did when considering student authority. They discovered that teachers claimed that authority had been given to students, but their opportunities to exercise this authority were limited or non-existent for a variety of reasons, including time, maintaining school expectations, rise in complexity and unpredictability, and hesitancy of students to accept authority.

In their 2017 study, Martin et al. found that after professional learning opportunities around both content and practices within the context of a provided curriculum (i.e., *Investigations*), teachers shifted their practices from largely teacher-centered to largely student-centered. Piccolo et al. (2008) asserted that the curriculum that teachers are required to use matters. Their study determined that classroom teachers find it difficult to implement high quality mathematical discourse while also ensuring that the required curriculum is presented (Piccolo et al., 2008). In a similar vein, Francisco and Maher's (2011) study on teacher observations of students' mathematics activity determined that teachers were often shocked by the capability of their students' mathematical reasoning. In the classroom, these instances of rich mathematical reasoning by students often go unnoticed by teachers, as the priority is to fulfill the requirements of the curriculum and/or standards.

In a different context, Anderson et al. (2004) examined early mathematical discourse practices in the context of shared reading with Pre-Kindergarten students. Parents read a specific, non-“mathematical” storybook to their children and the discourse between parent and child was examined. They found that parents often built in mathematical discourse into the shared reading, even when there were no *obvious* mathematical concepts presented. The researchers found that this practice helps to lay the foundation for students’ mathematical discourse and that this practice should be utilized more in the elementary classroom because it assists in children attending to mathematics in the context of meaning making. Many times, elementary teachers use shared reading as a way to *springboard* into an activity based around the reading, but the researchers argue that the reading *itself* can build the framework for students to talk about mathematics.

COVID-19 Health Pandemic

The COVID-19 health pandemic shut down schools and most other businesses in the United States around the second week of March 2020 as the country went into a lockdown to attempt to slow the spread of the virus. In order to avoid major gaps in instructional time, many schools pivoted to e-learning, or virtual learning using platforms such as Zoom, Microsoft Teams, Google Meet, and other various video conferencing platforms. This shift to e-learning affected, and continues to affect teaching and learning in a variety of ways. Murgatrotd (2020) identified “accessibility, affordability, flexibility, learning pedagogy, life-long learning and educational policy” as a few major challenges related to e-learning specifically. Reliable internet, access to devices, lack of teacher training, and over-exposure to screens are some of the issues that contribute to these challenges. Additionally, pedagogy that teachers were most familiar with for face-to-face instruction was not always feasible for e-learning. Many teachers needed

additional training to facilitate e-learning effectively and because the shift was so rapid, that training was not always available to teachers (Doucet et al., 2020). Concerns around mental and physical health also arose during this time with cases of domestic violence and child abuse on the rise (Ravichandran & Shah, 2020). This finding facilitated a large outcry from the public to get children back into schools face-to-face. Once schools began returning to face-to-face instruction, schools had to get creative in order to enforce safety standards such as keeping 6 feet apart and masking. It must be noted that this study occurred during the COVID-19 health pandemic, and the researcher was mindful of this contextual element throughout the inquiry.

3 METHODOLOGY

The purpose of this qualitative case study is to explore the classroom mathematical discourse of three elementary teachers in an urban school setting. The study addresses the primary research question: How do teachers understand, plan for, and implement mathematical discourse in elementary classrooms? As well as the secondary research question: What role does written curriculum play in the planning and implementation of mathematical discourse in elementary classrooms?

Methodology and Rationale

Exploratory case study is typically utilized to contribute to the understanding of real-life events when the behaviors or events cannot be modified or controlled (Yin, 2017). The primary sources of evidence in case study are direct observations and interviews. These components allow for a well-rounded view of the event being studied. One of the chief concerns with case study is the perceived lack of rigor in the research. Accordingly, in this study, the researcher aims to maintain a systematic set of procedures and clearly outline the processes in detail to ensure the trustworthiness of the study (Yin, 2017).

This study is designed as an exploratory case study for several reasons: (1) to provide a focus on particularization overgeneralization (Stake, 1978; Yin, 2017); (2) to explore the issue of the role of curriculum in mathematical discourse research (Kohn, 1997); and (3) to emphasize understanding rather than proof (Stake, 1978). While statistical data may provide a snapshot of a bigger picture, examination of specific experiences in classrooms provides a smaller but much more detailed picture, allowing for particularization. Case studies do not represent a population through *statistical generalization* but can provide expansion of theories through *analytic generalization* (Yin, 2017). A case study helps to begin the discussion of what might be worth

studying further by focusing on particular cases that could draw out themes and findings (Kohn, 1997; Stake, 1978). Additionally, it brings out real-life examples that give insight to actual challenges, nuances, and inner workings of an event that may be lost in a more generalizable research design. This exploratory case study aims to provide insights into the research questions in fine-grained ways. This study also lends itself to focus on a new component of mathematical discourse, because the role of curriculum within mathematical discourse in elementary classrooms has not been considerably studied. Further, this study focuses on understanding rather than proof through a statistically generalizable realization. The focus is to draw out themes and particulars of specific experiences of teachers and students in the elementary classroom, offering a foundation for other researchers to build more generalizable studies based on the findings and experiences of individual cases. The aim of case studies is to focus on the explanation of the "how" and/or "why" components of a particular issue (Yin, 2017). Because this study aims to understand the "how" and "why" components of mathematical discourse in elementary classrooms, case study is an appropriate research design for this inquiry.

Components of Research Design

Yin (2017) emphasizes five important components of case study research design: research questions, propositions, unit(s) of analysis, logic linking the data to the propositions of the study, and criteria for interpreting findings. The research questions and the lack of propositions of the study due to the exploratory nature of the study have been addressed in previous sections, so in this section, I will focus on the components that have not yet been described.

Unit of Analysis

The unit of analysis for this exploratory case study is a group of three elementary teachers in an urban charter school as they understand, plan for, and implement mathematical discourse practices in the elementary classroom. While the curriculum is an element that was considered, the exploration of curriculum is directly linked to the teachers' use, interpretation, and modification of the curriculum; therefore, the unit of study remains as the group of teachers.

Linking Data to Purpose

The purpose of the study, as previously discussed, is to explore a group of primary elementary teachers' understanding, planning, and implementation of mathematical discourse, including their use of curriculum, in an urban public school. Due to the exploratory nature of the case study, I used constant comparative analysis to guide my coding process. I began by using in vivo coding to preserve the words of the participants as codes themselves. Next, I used axial coding to begin to group and create conceptual categories of codes. Finally, I used selective coding to draw out the central or core categories that support and align the codes (Saldana, 2014; Charmaz, 2014). Throughout the coding process, I utilized a manual color-coding system to keep track of codes and patterns emerging from the data.

Study Setting and Participants

The setting for this study is an urban elementary school in a large city in the southeastern U.S. The race/ethnicity of the students at the school include approximately 71% white, 14% Black or African American, 10% two or more races, 4% Hispanic/Latinx, and 1% Asian. Nine percent of students attending the school are from low-income households, as evidenced by eligibility for the federally-funded free and reduced lunch program. Eighteen percent of students at the school qualify as students with disabilities (SWD). One percent of students attending the

school are identified as English Language Learners (ELLs). The school mobility rate, the rate at which students enroll and withdraw within the same school year, is about 2.5%. The school spends approximately $19,788 per pupil and has an average financial efficiency rating score of 2.5, which compares spending to CCRPI (College and Career Readiness Performance Index). A score of 2.5 represents above-average spending and average CCRPI scores. Approximately 65% of the students are performing at the “proficient” level or higher on the mathematics portion of the state’s standardized assessment, as measured in third and fifth grades by the Georgia Milestones (Public School Review, 2021).

This elementary school is a public charter school, which means they receive public funding per pupil, but retain the flexibility to dictate how they utilize their funding. This particular charter school is founded on principles of constructivism and prioritizes having two teachers in every classroom. The study site also employs two academic coaches that provide professional development opportunities to the staff as well as coaching cycles with a hands-on approach for developing strong teaching strategies within classrooms. Additionally, the study site provides opportunities for teachers to attend professional development sessions outside of the school building each year. In the 2 years leading up to this study, the school sent two teachers to be trained as curriculum trainers for the Everyday Math curriculum. Those teachers provided professional development to the rest of the staff regularly, as well as offered coaching cycles specifically around the implementation of the Everyday Math curriculum within the classroom. Teachers are afforded 45 minutes of partnership planning time each day and 90 minutes of dedicated collaborative planning time with their grade level team each week. This collaborative planning time is designed for teachers to collaborate on “big-picture” elements of the curriculum such as pacing, assessment, and sharing ideas with one another, rather than planning individual

lessons together. The partnership planning time is designed as a space for teachers to do more of the day-to-day planning of lessons and activities.

The participants for this study are three teachers in kindergarten classrooms. Data were collected from the participants during the 2020-2021 school year. During this school year, students were primarily learning virtually due to the COVID-19 health pandemic. The study site had been closed to students from March 2020 until February 2021. During this time, all students and teachers were fully virtual with no option for students to be in the school building. Beginning in February 2021, the study site allowed students and teachers to opt in to a hybrid model of instruction. During this time, some students were physically present in the class, while the remaining students were participating virtually through a large-screen projection of the Zoom platform that teachers utilized to engage virtual students in the classroom activities. All teachers at this grade level opted to teach in person and were physically present in the school building for the duration of the study. Participants were chosen using convenience sampling (Dörnyei, 2007), with the aim of identifying participants teaching students who were physically in the building for in-person instruction. Four participants were asked to participate in the study, and three agreed after reviewing the consent forms and discussing the goals of the study with the researcher.

Descriptions of the participants are provided here, with pseudonyms. Ms. Standen has been teaching for 9 years. She is a white female in her mid-30s. Ms. Standen holds a Bachelor’s Degree in Early Childhood Education with an ESOL endorsement. She has primarily taught Kindergarten and has used the Everyday Math curriculum for over 5 years. Ms. Standen did not enjoy mathematics as a child and did not prefer teaching mathematics in comparison to other subject areas. She primarily utilizes the Everyday Math curriculum’s resources and rarely ventures outside of the curriculum for supplementary tools or lessons. Ms. Standen works with

an associate teacher in her classroom and collaborates with that teacher to serve the needs of all the students.

Ms. Kyle has been teaching for 4 years. Ms. Kyle is a Black female in her late 20s. She currently holds a Bachelor's Degree and came into teaching non-traditionally. Ms. Kyle is currently working toward her Master's Degree in Curriculum and Instruction. She also does not prefer to teach mathematics in comparison to other subject areas. Ms. Kyle primarily utilizes the Everyday Math curriculum and rarely utilizes outside resources to supplement her lessons. She has only taught Kindergarten and has used the Everyday Math curriculum for all 4 years. Ms. Kyle works with an associate teacher in her classroom and collaborates with that teacher on lesson planning, behavior, and instruction.

Ms. McDonald has been teaching for 3 years. She is a white female in her early 20s. She currently holds a Bachelor's Degree in Early Childhood Education. Ms. McDonald was the most comfortable teaching mathematics. She enjoyed mathematics as a student and gets excited about mathematics instruction. Ms. McDonald primarily utilizes the Everyday Math Curriculum resources and rarely supplements her instruction with outside resources. She has only taught Kindergarten and has used the Everyday Math curriculum for all 3 years. Ms. McDonald works with an associate teacher in her classroom and collaborates with that teacher consistently.

Instrumentation and Data Collection

For this study, data were collected in three ways: (a) documents, (b) teacher interviews, and (c) classroom observations. Two types of documents were collected, and there were two observations of classroom mathematics instruction for each teacher. Each teacher also participated in one individual interview before the classroom observations and one after both classroom observations were completed. See Table 1 below for more details on data collection.

Table 1

Data Collection Purpose and Timeline

Research Questions	Data Collection	Purpose	Timeline (data collected across 2 months)
How do teachers understand, plan for, and implement mathematical discourse in elementary classrooms?	Interviews	- Examining how teachers understand, plan for, and implement mathematical discourse	- Pre-interview prior to initial observation and document submissions -Post-interview after each observation
	Observations	- Examining how teachers implement mathematical discourse	- Following pre interview and review of submitted documents
What role does curriculum play in the planning and implementation of mathematical discourse in elementary classrooms?	Documents	- Examining discourse in the written and intended curriculum	- Prior to each observation
	Interviews	- Examining discourse in the intended and enacted curriculum	- Pre-interview prior to initial observation and document submissions -Post-interview after each observation
	Observations	- Examining discourse in the enacted curriculum	- Following pre-interview and review of submitted documents

Documents

Two types of documents were collected. First, the researcher collected and reviewed the mathematics lesson as written in the provided curriculum, as well as any supplemental materials provided to the teachers by the school and/or district for the observed lessons. Additionally, the researcher collected and reviewed the teacher-created lesson plans provided by the teachers for the observed lessons. However, the study site does not require teachers to submit lesson plans or follow a particular lesson plan format, so there was very little data obtained from these documents.

The provided mathematics curriculum was reviewed and analyzed prior to observing the classroom lessons using researcher-created document analysis tools adapted from NCTM’s *Principles to Actions* (2014) sections on facilitating meaningful mathematical discourse and posing purposeful questions (Appendices D and E). The *Teacher and Student Actions* tool

(Appendix D) was utilized to examine the curriculum for evidence and examples of the following teacher moves: (a) the teacher engages students in purposeful sharing of mathematical ideas, reasoning, and approaches, using varied representations; (b) the teacher selects and sequences student approaches and solution strategies for whole-class analysis and discussion; (c) the teacher facilitates discourse among students by positioning them as authors of ideas, who explain and defend their approaches; and (d) the teacher ensures progress toward mathematical goals by making explicit connections to student approaches and reasoning. The categories of not evident, partially evident, and evident were utilized to summarize the extent to which each specific move was evident within the provided lessons of the curriculum. The rating of "not evident" was given when the document (curriculum and/or teacher-created lesson plans) included no examples for the given category. The rating of "partially evident" was given when the document included 2 or fewer examples for the given category or if all examples were recorded during one exchange (e.g., several sample questions were provided for one interaction). The rating of "evident" was given when the document included 3 or more examples for a given category across the entirety of the lesson (e.g., students were asked to compare their solutions to their peers 4 times across several exchanges within the lesson).

The document analysis tools also examined the following student moves: (a) during the lesson students present and explain ideas, reasoning, and representations to one another in pair, small-group, and whole-class discourse; (b) during the lesson students listen carefully to and critiquing the reasoning of peers, using examples to support or counterexamples to refute arguments; (c) during the lesson students seek to understand the approaches used by peers by asking clarifying questions, trying our others' strategies, and describing the approaches used by others; and (d) during the lesson students identify how different approaches to solving a task are

the same and how they are different. Again, the categories of not evident, partially evident, and evident were utilized to summarize the extent to which the specific move was evident in the provided lessons.

The *Question Types* tool (Appendix F) was used to determine the numerical amount of each of the four question types present in the curriculum along, with examples in each category. The first question type is gathering information, which prompts students to recall facts, definitions, or procedures. The second question type is probing thinking, which prompts students to explain, elaborate, or clarify their thinking, including articulating the steps in solution methods or the completion of a task. The third question type is making the mathematics visible, which prompts students to discuss mathematical structures and make connections among mathematical ideas and relationships. The fourth, and final, question type is encouraging reflection and justification, which prompts students to reveal deeper understanding of their reasoning and actions, including making an argument for the validity of their work (NCTM, 2014).

Teacher Interviews

Each teacher participated in an individual, semi-structured interview prior to the submission of documents and the first observation. Teachers participated in an additional individual, semi-structured interview following both observations and submission of documents. All interviews were completed via Zoom and audio recorded directly on that platform. They were then transcribed. The initial interview focused on general teaching philosophy, curriculum utilization, planning habits, and understanding of mathematical discourse (see Appendix B for interview protocol). The second interview focused on clarifying specific pedagogical and content choices within lessons and lesson plans based on observations and document reviews (Appendix C for interview protocol). Within all interviews, teachers were asked to reflect on their

understanding of, planning for, and implementation of mathematical discourse, including the use of curriculum. The interviewer strived to maintain a conversational approach to the interviews as the participants were familiar with the researcher prior to the study. While an interview protocol was outlined for each round of interviews, follow-up questions and/or exploratory topics were included to allow for a more complete picture (Vogt et al., 2012).

Classroom Observation

Classroom observations, approximately 30-45 minutes in duration, occurred two times over the course of 2 months. The lessons selected by the teacher included at least 15 minutes of whole-class instruction. The participants audio recorded their lessons via Zoom and sent them to the researcher due to visitor restrictions in place to prevent the spread of COVID-19. The recording was reviewed and transcribed following receipt of the recording. The transcription was used to detail the types and levels of questioning used by the teacher during specific activities. Researcher-created observation tools adapted from NCTM's *Principles to Actions* (2014) sections on facilitating meaningful mathematical discourse and posing purposeful questions were used to categorize question types directly related to discourse (Appendix F). These tools both collect the same data as the document analysis tools, however, the context in which the data were collected is researcher observation of enacted teacher and student moves (Appendix E) and teacher question types (Appendix F).

Rationale for Researcher Created Tools

After a thorough search of existing observational tools to analyze mathematical discourse, I found few options. The existing tools tended to have one or more of the following three issues. Some tools had too broad of a focus, limiting the scope of data collected directly related to mathematical discourse. Some tools were quantitative in nature, boiling down mathematical

discourse to a checklist or a numerical quantity only, which does not fit the purpose of this study. Finally, some tools did not align with the NCTM's *Principles to Actions* (2014) definition and description of quality mathematical discourse around which this study is designed. For these reasons, I felt that researcher-created tools that were explicitly focused on discourse, qualitative in nature, and aligned with the NCTM's *Principles to Actions* (2014) definition and description of quality mathematical discourse would provide the data necessary to fit the purpose of this study.

Data Analysis

Constant Comparative Method

Glaser and Strauss (1967) developed the constant comparative method in the 1960s as a systematic analysis approach to qualitative inquiry. The process typically relies on applying specific coding processes to data through a series of coding cycles that lead to the development of conceptual links between and among categories and properties (Merriam, 1998). Therefore, the data collection and analysis processes are concurrent, and each affects and influences the other toward the goal of finding connections within and across the data. In order to understand the participants' experiences, the researcher worked to refrain from assumptions of meaning in the participants' words (Charmaz, 1996).

Coding Cycles

For the teacher interview transcripts, analysis involved the following coding protocol that was documented in a coding manual using Microsoft Excel to track codes over the course of the coding process. Once the two coding cycles were complete, a manual color-coding method was employed to draw out patterns and similarities between the coded data and data organized through the researcher-created tools. This process resulted in emergent themes.

In Vivo Coding. During the initial cycle of coding, the researcher used in vivo coding (Charmaz, 2014; Saldaña, 2016). In vivo coding uses the exact words of the participants to code in order to avoid interpretation too early leading to possible misinterpretations (Seale, 2011). The researcher imported the transcribed text into an Excel spreadsheet where each sentence received its own line. Each line was then boiled down to a 1-3 word or phrase that summarized the sentence. For the duration of the initial cycle of coding, all codes assigned to the data remained temporary and flexible to remain open to the multiplicity of meaning within the data.

Axial Coding. During the second phase of coding, the researcher used axial coding (Saldaña, 2016). The goal of axial coding is to strategically reassemble the data that was split during the initial coding cycle through in vivo coding. This process included finding patterns and categories that the initial codes could be grouped into as well as defining conceptual categories that were found within the data. Each new cycle of coding was input in a new column in the Excel spreadsheet. This helped to keep the essence of the participants' story intact as much as possible. Major categories and subcategories were established and defined during this cycle of coding (Charmaz, 2014; Glaser & Strauss, 1967; Saldaña, 2016) such as "Strategies for Teaching," "Math as a Child," and "Student Confidence".

Manual Color Coding. In this final coding phase, the researcher used manual color-coding to visualize the crossover connections between coded interviews and organized data within the tools. The aim of this coding process was to find the essence of what the major themes were across all of the data. During this coding cycle, the researcher integrated and synthesized the categories created during the first and second coding cycles as well as finding matching evidence across the other data. The purpose of this coding cycle was to draw out consistent themes of the explored phenomenon (Saldana, 2016).

Document and Observation Analysis

Document analysis is a "systematic procedure for reviewing or evaluating documents" (Bowen, 2009, p. 27). Document analysis, like other qualitative analysis methods, requires that data be examined and interpreted in order to "elicit meaning, gain understanding, and develop empirical knowledge" (Bowen, 2009, p. 27). The purpose of the document analysis was to gain insight into the matches and mismatches between written curriculum and enacted curriculum, along with providing supporting examples for the categories of not evident, partially evident, and evident in the document analysis tools (Appendices D and E). The data collected and categorized through the researcher-created document analysis tools were constantly compared to the codes created through the interview transcript coding process. As needed, new codes and categories were created to include data not represented in the interviews. Specific examples were also utilized to contribute to the rich, narrative description of the case.

A third layer of data collected through observation was also utilized. The purpose of the observational data was to provide insight into the discourse practices of the teacher and the matches and mismatches between the written and enacted curriculum. The data collected and categorized through the researcher-created observation tools (Appendices D and E) include the categories of not evident, partially evident, and evident with examples and were constantly compared to the codes created through the interview transcript coding process. As needed, new codes and categories were created to include data not represented in the interviews.

Table 2

Coding Cycles

Coding Cycle 1 (In Vivo)	Coding Cycle 2 (Axial)	Coding Cycle 3 (Color)
Each line of transcribed text was coded line by line. Codes were established using the participants' own words.	Initial codes were synthesized and broader categories and patterns emerged. 23 codes generated during this phase.	All data were compiled and compared to the codes generated during cycle 2. 8 code categories were finalized during this phase. Final themes emerged.

Trustworthiness

Lincoln and Guba (1986) presented a series of techniques to ensure the trustworthiness of a qualitative study. Three criteria must be met through a series of tests, each increasing the probability that the criteria have been met. The criteria are credibility, transferability, and dependability and confirmability.

This study meets the criteria of credibility through crystallization (Ellingson, 2009) and peer debriefing. Crystallization helps to confirm themes and connections within and between the data (Campbell & Fiske, 1959; Denzin, 2009; Ellingson, 1999; Thurmond, 2001). Crystallization is achieved in the study through the combination of the documents, observations, and interviews, which report three unique perspectives on the same data. The constant comparison of the three data sources confirms the themes that emerge in each context while also highlighting the mismatches between the sources. Peer debriefing with a professional peer having no direct interest in the study was utilized to assist in checking themes and categories from the data to "keep the enquirer honest" (Lincoln & Guba, 1986, p. 77).

This study meets the criteria of transferability through the dynamic and rich description of the case, alongside the use of the participants' direct words, which ensures that the contexts to

which the findings are applicable are abundantly clear (Lincoln & Guba, 1986). Specific examples and vignettes from observations, interviews, and documents were utilized to paint a vivid picture of mathematical discourse and the role of curriculum as it relates.

Finally, this study meets the criteria of dependability and confirmability by ensuring that the methods chosen for data collection are appropriate to the problem being studied. The techniques of analysis utilized are consistent with the way in which data are collected and assembled. The researcher additionally ensures that reports of the data are "coherent, credible, and exhibit structural corroboration" (Lincoln & Guba, 1982, p. 6) and that all assertions made by the researcher may be traced to "authentic data units or categories" (Lincoln & Guba, 1982, p. 6).

Considerations

A major consideration of this study is the state of the world during the time period in which these data were collected. There were major hurdles in collecting data and physically being able to observe teachers due to restrictions in place to prevent the spread of COVID-19. Teachers had also been teaching virtually for the majority of the year leading up to the collection of this data. Additionally, some students remained virtual during the time these data were collected, so the participants were coordinating students both virtually and physically in the classroom. This most definitely affected the ways that teachers planned for and implemented the curriculum. Technology also added another layer of coordination for the participants to engage with their students. Teachers often struggled to hear their students who were participating virtually, due to internet connection issues, microphone malfunctions, technology glitches, and a myriad of other challenges. Background noise and distractions were an additional barrier for

students participating virtually. These factors should be heavily considered when reviewing the findings of this study.

4 FINDINGS

This chapter contains the findings of the exploratory case study that sought to answer the primary question: *How do teachers understand, plan for, and implement mathematical discourse in elementary classrooms?* Additionally, the study aimed to answer a secondary research question: *What role does written curriculum play in the planning and implementation of mathematical discourse in elementary classrooms?* The goal of this exploratory case study was to examine teachers' use of mathematical discourse in the classroom as it relates to their own understanding and the curricula they are provided. The purpose of this chapter is to communicate the major themes and other findings that became evident from the data. First are shared the data and themes related to the teachers' understanding, planning, and implementation of mathematical discourse. Then data relating to the understanding, adaptation, and utilization of the written curriculum are presented and compared to answer the secondary question. Finally, the findings are summarized.

Teachers and Mathematical Discourse

To answer the primary research question, *How do teachers understand, plan for, and implement mathematical discourse in elementary classrooms?,* The data across interviews, documents, and observations were constantly compared to understand the process that teachers undergo from receiving the curriculum materials to modifying (or not) for the specific purposes of their classroom and finally enacting those plans with their students. By constantly comparing each data source to another, it revealed the ways in which the teachers' intentions and knowledge intersected with the curriculum and students. As a result, three major findings emerged: (a) the participants understood and desired to engage in meaningful discourse with and between their students, (b) factors such as time, language, and COVID-19 protocols impacted their ability to

plan for and implement meaningful discourse in their classrooms, and (c) there was limited influence from the written curriculum on the enacted curriculum.

Desire to Engage in Meaningful Discourse With and Between Students

Through interviews with the participants, both before the observations and after, it was clear that their understanding of meaningful mathematical discourse aligned with the NCTM's indicators. All three participants spoke extensively about discourse as an effective strategy for building student understanding. They mentioned benefits for students such as learning from each other, understanding that they can solve problems in more than one way, and allowing a space to process their thinking aloud. The participants also named a variety of ways that they teach and encourage discussion in their classrooms such as utilizing discussion protocols, modeling appropriate discourse behaviors, thinking aloud, asking open-ended questions, and connecting the students' own stories and experiences to mathematics. They all felt that discourse was an important part of learning for their students and wanted to offer those opportunities to their students. Within this theme, major findings clustered around peer-to-peer interlocution, student-to-teacher interlocution, and assessing student responses.

Peer-to-Peer Interlocution. Peer-to-peer interlocution is the verbal interaction between peers within the classroom. Each participant spoke about the benefits of students engaging with others in their classroom during mathematics instruction. When asked about how she encourages her students to talk about mathematics, Ms. McDonald said,

> Putting them in partnerships to explain their thinking, or, um, to play like a math game together. Like when they're playing a math game. So, for example, Top It, where each kid has cards, they laid out a card, and they figure out which one, you know, if we're playing highest number or lowest number, and then, um, I'll tell them, and then talk

> about why that person won instead of just going through the game, like, alright, I get this, I get this, I get this, I get this. Um, talk about how you know that's the highest card.

Ms. McDonald also expressed that she felt that "students best learn through each other" as well as "listening to each other's answers, a lot of different strategies, being able to access it in different ways." Ms. Standen had a similar sentiment when she stated,

> So in partnerships, there's a lot of discussion, we do discussion models at the beginning of the school year, where we say this is how you have a productive conversation. You might say something like, 'Well I think this because, or I think differently because' and, you know, we give them those language stems so that they can work in partnerships in that way.

Ms. Standen spoke at length about the benefits of peer-to-peer interlocution and the ways that they support students in that work throughout the year. Ms. Kyle did not have as much to say about peer-to-peer interlocution, but she did share about opportunities she offers for her students to participate in discourse or discussion such as asking questions and working in partnerships to utilize those discussion skills.

In contrast, none of the lessons provided from the written curriculum explicitly encouraged peer-to-peer interlocution such as partner work or peer critique (though other lessons throughout the Everyday Math curriculum do encourage these types of interactions). In addition, peer-to-peer interlocution was only partially evident in three of the six observed lessons, meaning that peers were sharing out to each other and expected to listen, but did not engage in discourse as a result. Peer-to-peer interlocution was not evident in the remaining three of the six observed lessons. When asked about this, Ms. Standen and Ms. McDonald attributed this discrepancy to the difficulties of virtual learning. Ms. Standen stated,

> In a typical year, I would say that we would have a lot more discourse and discussion and we would actually teach them how to do that, like I agree with you, because my evidence is this. But this year with COVID, it's so hard for them to hear each other behind the Plexiglass and do partnership work. So I would say that there was not as much discussion as usual and not as much as--as we would like.

Ms. McDonald shared those views, saying,

> When we do give them opportunities to play math games together or do things together, it's so difficult for them to hear each other, that often they spend most of the time like, 'What? Can you repeat that? What are you saying?' So I would definitely like more opportunities. (...) It's hard to work in a group at like a square table with Plexiglass dividers. So it's definitely something that I would like to have more talk like between the kids.

Overall, teachers expressed their desire for students to engage with one another through discourse, activities, games, etc. However, the observations within the classroom showed far fewer opportunities for students to engage with other members of the classroom community aside from the teacher. With the restrictions in place due to COVID-19, it is understandable that the peer-to-peer interlocution may have had to decrease, even though the teachers feel the importance of it being utilized in the classroom and desire for it to occur in the future.

Student to Teacher Interlocution. Student-to-teacher interlocution refers to the discourse interactions between students and teachers. Participants spoke very differently about student-to-teacher interlocution when compared to their reflections on peer-to-peer interlocution. Ms. McDonald, when reflecting on the opportunities for discourse that she created during one of her lessons, said,

> Having them tell me what they notice about the hundreds chart was an opportunity for them to talk. I didn't give them an opportunity to talk in partners, though. Typically, I'll have them discuss in partners trying to do some things and then tell me, so that's an area where they could have had more discussion between themselves. (...) But in these particular lessons, the opportunities for them to respond was just to me in answering the questions I had.

It seemed apparent throughout the interviews that the participants placed more theoretical value in peer-to-peer interlocution and less in student-to-teacher interlocution, however all six of the lessons contained far more student-to-teacher interlocution than peer-to-peer interlocution. When reflecting on her role in group discourse, Ms. Standen stated,

> I've always said that I feel my role should be more of a facilitator. So, like I said, like teaching them the structure for discussions, and then giving them the time and opportunity to do that. I don't feel like these lessons were really good examples of what typically happens in the classroom.

Ms. Kyle echoed Ms. Standen's sentiment, saying,

> I feel like my role is just to start the conversation, I would love to be able to start the conversation and just to take a step back and let them speak. So my role is more so of a facilitator or coach if they--if they lose sight of where we're supposed to be going.

In five out of the six lessons from the written curriculum, it was evident that the teacher facilitates discourse among students by positioning them as authors of ideas, who explain and defend their approaches. These included suggestions such as "How do you know?" or "Which one and why?" or "Which would describe your estimate and why?" These types of questions are designed to push children's thinking and encourage them to defend their rationale. The transfer

from the curriculum suggestions to the enacted lesson occurred in only one observed lesson for this particular indicator. In Ms. Standen's lesson on rules and patterns, she asked questions during the teaching portion of the lesson such as "Do you have an idea how we can put them into groups based on what is the same?" and then followed up with questions such as "Why can't we put these together?". Additionally, during the work portion of the lesson, Ms. Standen was asking students questions such as "How are you sorting?", "What do you think his rule is?", "How did he sort?", and "Can anyone sort this another way?" This lesson was marked as evident for this indicator because Ms. Standen positioned the students as the authors of the ideas and pushed them to explain their reasoning throughout the lesson. Even when she was demonstrating during the lesson, she ensured that the students were presenting ideas instead of her. This particular lesson in the curriculum also was marked as evident for this indicator. The lesson repeatedly places the students as the idea generators and follows every question with a request for defense or explanation such as, "How did you decide which string is the longest?" and "Which group has a greater number of children? How do you know?"

In contrast, the teacher facilitating discourse among students by positioning them as authors of ideas, who explain and defend their approaches was evident in only one out of the six observed lessons and partially evident in five out of the six observed lessons. Teachers wanted this type of engagement to occur in their classroom and it was clear that there were attempts to facilitate these interactions, but they were not always successful or consistent across the lesson. In the lesson titled "Estimation Jar," the curriculum suggested partner discussion around strategies and solutions for generating estimates, in addition to suggesting questions such as, "How did you use the reference jar to help you make your estimate?" and "When would an estimate, instead of an exact count, be fine?" However, in Ms. Standen's implementation of that

lesson, it was only partially evident because while she did solicit estimates from students, the push to have students explain how they got to their estimate was less apparent. Another example is from the lesson titled, “Growing Train.” In the curriculum, children are expected to play a game in pairs and the teacher circulates and asks children to describe their turns. Then, when the students are ready to move to recording their work, the teacher is expected to ask questions such as, “What does the 8 show?” “Why did I write ‘+3’? What does it mean?” and “Why did I write ‘=11’? What does it mean?”. In Ms. McDonald’s implementation of the lesson, she asked the students to verbalize their moves while playing the game with their partner, but it was difficult for her to interact with all of the students while managing students attempting to play the same game virtually. The time and space for the additional push for students to defend their thinking was limited.

Assessing Student Responses. All three participants utilized targeted lines of questioning to push students toward the mathematical goals that they were working to accomplish in the observed lessons. The participants were making split-second assessments of the student responses in order to help guide them toward an understanding of the concept. Ensuring progress toward mathematical goals by making explicit connections to student approaches and reasoning was partially evident in six out of six lessons in the written curriculum. One major reason for this is that the curriculum cannot anticipate student responses. In the lessons where it was partially evident, the curriculum gave suggested responses and connections to make if none of the students are able to, but those were limited to one or two questions, and they provided limited follow up questions, whereas, teachers are able to react to the student input they receive on the spot. However, in the observed lessons this indicator was evident in five out of the six lessons and partially evident in one out of the six lessons. The participants were consistently restating

student responses and connecting them back to the mathematical goal of the lesson, using lines of questioning to push student thinking toward the mathematical goal, and reframing their work with additional vocabulary and language.

Using intentional questioning appeared to be an expectation from both the curriculum and school leadership, but the specific lines of questioning were not necessarily planned for as they had to adapt based on their student interactions. Upon reflection, the participants recognized that the types of questions they were using were intended for a specific purpose toward achieving the mathematical goal for the lesson. For example, Ms. McDonald, when asked if she felt that her question, "What do you notice?" achieved her intended responses from students, she said, "Probably, you know, at the end, after asking more specific questions. I think if I just stuck with like, 'Alright, what do you notice?' the whole lesson, it wouldn't have got at noticing the patterns in the hundreds chart." In Ms. Kyle's lesson on sorting, she asked, "What do all the fish have in common?" and received no responses from the students. She stated that her intention with that question was "to see where they were, and then I realized, okay we have to start from ground zero because none of them could answer the question."

It was evident that the participants spoke much more about engaging with and between students in a variety of ways far more than they were enacting those methods in their classrooms. During the initial interviews, each of the three participants spoke about the importance of students engaging in a variety of ways with the teacher, with peers, in small groups, in a whole group, and in partnerships. When compared to the observational data, the results were much more singular in practice, where the teacher was the primary communicator in the room. This may very well have been a result of other factors that inhibited discourse implementation.

Factors That Impact Discourse Implementation

A multitude of factors were discussed as the participants qualified the minimal presence of mathematical discourse in their classrooms during the observed lessons. Across all three participants, the major impediments were time, measures of student success, and COVID-19 protocols.

Time. Time seemed to impact mathematical discourse because the teachers saw discussion time as a lengthy process and that their students may miss out on content and/or practice opportunities. Additionally, time for planning and following a pacing structure added another impediment to discourse implementation. Particularly in the context of the study, the participants were forced to make tough decisions about what content to keep and what to eliminate in order to cover enough content that the students would be prepared for the following school year. Unfortunately, a climate has been created in schools to push students through content faster with mastery expected at lightning speed. The top-down approach where schools are handed pacing guides and standards for mastery by individuals or groups outside of the classrooms, schools, or any educational setting has forced teachers to compromise what they may feel is best for students, so that they can give them the best chance to succeed on a standardized test that may stand between them and a paycheck.

Each of the participants described the differences they have experienced between their normal pacing structure and planning time, and the alterations they were having to make due to COVID-19. Ms. Standen explained,

> Some of my colleagues have taken on math as a subject. And they basically go in, and they have picked out content that is most important because Everyday Math is a cyclical program. So instead of maybe teaching seven lessons on one topic, they were like, you

know, oh actually, maybe let's see, how's everyone's kids doing with this? Do you have assessments to back it up? Yes. Why don't we just teach three, and then we can focus on, you know, more of the content that they don't know as well. And then we come together, and we discuss whether or not those plans look good, and then we just follow that.

Ms. McDonald expounded on this saying,

We're moving through the curriculum a lot more slowly. I still have students who can't count consecutively in the, like, the teen numbers. So we've not been able to see what they really know very well. (...) I would say I don't spend as much time planning for math because we have a shorter period, a smaller amount of time. (...) So the planning hasn't--and then also like cutting out a lot of things because of time.

Ms. Kyle confirmed this as well when she said,

This year, we're all planning together. So it's a collaborative effort when deciding what materials and what lessons we're gonna teach. Since we're--we have a shorter, we have a limited amount of days in the school year. So it's really a collaborative effort, and we decide as a kindergarten team.

However, all of the participants agreed that mathematics was a priority in their school and that, despite the time restrictions, when planning out their schedules, the team made sure to prioritize mathematics with a dedicated instructional block every day at the expense of other subject areas like social studies and science.

Math is pretty important. We--we really prioritize math. Like, for instance, when in our virtual schedule, we have a limited amount of time, as I said, and a limited amount of days, and we li--we literally only have two blocks for--two huge blocks for all of our

subjects. And we made the intentional decision to let math have it's own block, because we feel like it's a priority with our students. (Ms. Kyle)

Ms. McDonald added more detail to the picture when she described the decisions that had to be made when planning the schedule.

It's [math] been made a priority and moving to this hybrid schedule I know over, you know, maybe some like social studies and science. So it's taken priority. So it seems pretty important to the school.

All three participants noted their concern about the amount of time that discussion takes up, especially when engaging students both virtually and in-person simultaneously. Ms. Kyle shared her take on how discussions are going in her classroom.

I think I don't know if as many kids are paying attention, you know, or listening because they can simply look like they're listening. But virtually, you really can't tell, especially if you're sharing your screen and you can't see all the kids. I really hate that. Because I don't know what they're--like, are they listening? Are they at their computer? And a lot of times, I will stop sharing my screen just to look at them. And that takes a lot of time. But--and then if I call on them, I have to repeat the question.

Ms. McDonald also cited issues with technology as a time hindrance.

So we don't have as much time for kids to answer. It's difficult to get a lot of input. And, you know, in the class, we can have a kid like quickly answer, but with the whole unmute thing, that definitely lags. And so we're not able to hear as much student input.

Ms. Standen doesn't see this as solely a COVID-19 issue, saying, "But--but time because there's just never enough time to get it all in and to, you know, give them enough time to really get it." She gave an example from one of the observed lessons,

> There were some kids who were doing crazy numbers like 1500. And we were going to show them what those things actually looked like, we were going to pull up like a 1000s chart and 100s chart. But we didn't have time to do that.

Time may be a factor that is a significant barrier to teacher implementation of discourse practices. When the participants felt that time was not on their side, their lessons focused primarily on delivering instruction to students and practice. The discourse that they desired and planned for was watered down or cut out altogether to get through the lesson.

Measures of Student Success. Discourse was a common theme when the participants were talking about ways that they measure student success as they all felt that it was a helpful way to gauge how well their students were understanding a concept. However, it was also clear that all three participants struggled to trust the discourse interactions as evidence of understanding without the backup of more traditional paper and pencil assessments.

Ms. Standen started off by saying that she felt that if kids were having fun and building confidence, that helped her know that her kids were doing well. However, she then focused heavily on formal assessments such as "beginning of the year check-in, the mid-year check-in, the end of the year check-in. Those I use from the Everyday Math program. (...) I'm able to keep track of, you know, how they're progressing." When Ms. Standen was asked how she knew that her students understood the content of the observed lessons, she said,

> By observation, and by them applying it in their own way, like coming up with their own rules based off of like, what everyone looks like, or things we have in the classroom. (...) So we just ask them questions, we always come back around to the lessons that we've taught.

She also mentioned that they later gave a "pencil paper" assessment and had them graph and sort which she used as evidence of student success as well in addition to the observations and anecdotal notes.

Ms. McDonald, when talking about how she knew that her students "got it," said, "If they're able to answer the questions, I know I'm effective. (...) So just kind of based on their performance, like what they're producing, lets me know, like, hey, that was a good lesson, I got it." In the context of the observed lessons, Ms. McDonald stated that she hoped at the end of the lesson they would be able to come up with their own work and notice patterns. When pressed on how she would know that, she spoke primarily about physical work from the students such as name posters, number posters, showing a number in different ways, and writing number sentences.

While all three participants mentioned the use of anecdotal or observational notes in determining student success with a concept, they mainly spoke about these notes in regard to student-produced products rather than in the context of discourse. Ms. Kyle was much more straightforward in her methods used to determine student success. She stated that she knew her kids understood the concept of sorting "because when we had the assessment, all the kids knew their shapes and all the kids knew how to sort in two ways." However, she also stated that she planned on taking anecdotal notes but ended up not doing that during these lessons.

COVID-19 Protocols. Lastly, COVID-19 protocols and restrictions were put in place at the study site to slow the spread of COVID-19. These included mandatory masks for all teachers and students, Plexiglass dividers on all tables, reducing the number of children at one table to allow for social distancing, as well as the option to remain learning virtually through Zoom in a hybrid teaching model. Teachers felt this impacted their ability to engage students in discourse

simply because of the logistics of communicating with one another. The physical barriers between students meant that they often could not hear what their peers were saying. If they were learning virtually, the delay in student responses due to muting and unmuting sometimes stalled discussions.

For example, Ms. Standen felt that her ability to offer opportunities for discourse was limited by the protocols placed on her due to COVID-19,

> On a typical year, I would say that we would have a lot more discourse and discussion and we would actually teach them how to do that, like I agree with you because my evidence is this. But this year, with COVID, it's so hard for them to hear each other behind the Plexiglass and do partnership work. So I would say that there was not as much discussion as usual and not as much as–as we would like.

Ms. Kyle spoke about how the restrictions have forced teachers to have to be more creative, but also come up with solutions that are less than ideal for a classroom community and discourse, especially for students that need more support or enrichment opportunities.

> There's a lot more planning because we have to make everything virtual. It just means that I have to put in a lot more work into differentiating. I have a few high fliers that have to go in breakout rooms a lot and they don't get to be with the class a lot, so I have to plan for them and plan for the rest of the class.

Influence of Written Curriculum on Enacted Curriculum

While the personal philosophies of the three participants aligned fairly strongly with the curriculum and many indicators aligned between the written and enacted curriculum, there is limited connection that can be made between the two due to the factors discussed previously around time and COVID-19. The written curriculum can only offer so much to teachers because

the final product handed out in schools is the result of a compromise between curriculum writers, educators, and publishers in order to sell a product. There are certain aspects of the classroom that are impossible to standardize such as cultural responsiveness, prior knowledge that students bring, and the individual students' needs.

However, there was a surprising emphasis on language by all three participants as a barrier due to the heavy emphasis by the curriculum on vocabulary and language development. Some participants felt that it was difficult and confusing for students to understand the language of mathematics. It, again, took time to develop discussion protocols, practice language stems with students, and teach them vocabulary so that they could use it effectively in their explanations. Time was a factor that they felt like they did not have. This particular finding was interesting due to the framework around which this study was designed. Bakhtin (1981) describes discourse as the learning itself and that vocabulary understanding can be developed simultaneously with content understanding, however, the participants felt very differently about the development of language and vocabulary in their mathematics lessons.

Language and Vocabulary. Two of the participants described the language suggested in the curriculum as difficult for the students in their classes to grasp. Ms. Kyle stated,

> I think sometimes the language suggested in the curriculum is not effective for all of our students. And when I say that, I just mean, some kids are just not familiar with--they're not complicated words, you know, they just don't use them at home. Like, we had a really hard time last year with positional words like, near, beside, below, in front of, and kids are like, what? I, you know, that was very tough for them. And I, I don't think, I just think it's a background thing. You know, they're not --they haven't heard those type of words, we use words like angles and vertices. And they're like what are you talking

> about? You know, we can explain it to them because they start like, a lot of them don't know what that means. So it's, it's fine to go from there, like from, 'Okay, we're gonna tell you what that word means.' But a lot of times, some of the language in the, you know, curriculum that they suggest is just tough for them.

Ms. Standen had a similar perspective regarding vocabulary, saying,

> But I do think there are some children who come into school with less vocabulary. And so it is difficult for them to engage in conversations and--and learn the concepts well, and quickly, because in math, there are a lot of concepts, right? So like, even, more and less, you know, like that seems like a simple thing to some, but those words maybe haven't been used, you know, in their homes and so, but it's kind of like, I mean, I'm lucky that I have my ESOL--ESOL background, because I think naturally, I--I explain every single detail of the words that I'm using, or I try to break it down into kid friendly--friendly language or show what I mean, but the kids who have had less, you know, language in their upbringing, I think do struggle with discussions.

As a result of this belief around both Ms. Kyle and Ms. Standen's students' struggles with accessing the curriculum, they felt the need to simplify some of the concepts or lines of questioning to accommodate what they perceived as overly complicated language for her students. The language accommodation was an unplanned adaptation that the participants made to the written curriculum during the enacting of the curriculum based on what they perceived as their student's difficulty understanding or utilizing the expected language of the curriculum.

Conversely, Ms. McDonald had a slightly different perspective on the curriculum and did not perceive the curriculum as the barrier to her students' understanding, rather she felt her own over complicating of language and vocabulary resulted in confusion for the students.

> I think, you know, talking about the equals and the plus sign, there's so many different words when talking about like equal, like the same as, adding, adding on, plus, that it's hard to get kids to understand that it's all the same. I think just kind of like thinking back, starting with one term and using only that term for the lesson, I think I tried to over explain, like, oh, it could be this, this and this, and it confuses them.

Explanation. Having students present and explain ideas, reasoning, and representations to one another is one of the indicators of successful mathematical discourse according to the NCTM's *Principles to Actions* (2014). This indicator was evident in six of the six lessons in the written curriculum with directives such as "ask children to describe their turns" during a game or sharing strategies for making an estimate or "promote this by asking children to explain why they guessed a certain number." However, this did not fully transfer into the observed lessons. The indicator was evident in one observed lesson, partially evident in three observed lessons, and not evident in two observed lessons. Again, it was clear that the participants wanted these interactions to happen, with Ms. McDonald saying,

> Another kind of expectation is being able to explain your thinking in--in a lot of situations. Like, well, why do you think that, you know, it's--it might be the right answer, but, you know, we tell the kids like, it's more than just the right answer.

Ms. Standen also felt that it was important,

> Because language development in kindergarten is a thing. And sometimes they'll just be like, 'I just knew it.' (...) I want them to be able to transfer. So yeah, I feel like my job is to help them connect with their learning, as well as facilitate discussions and use more oral language.

However, she also felt that it was possibly detrimental for some students.

> I think that the discussion part, and the writing part of Everyday Math is what holds a lot of our students back. Kids who don't have, you know, like, we've been saying, don't have those language skills. They're not able to excel, because a lot of the Everyday Math is talking about what you know, talking about how you did it. And it's not necessarily just getting the right answer.

Written vs. Enacted Curriculum

To answer the secondary research question, *What role does written curriculum play in the planning and implementation of mathematical discourse in elementary classrooms?* The observed lessons (enacted curriculum) were compared to the teacher's edition of the curriculum (written curriculum) to observe patterns between the two. The indicator ratings for the lesson observations and the document analyses were evaluated independently and then compared to each other. When comparing the observation notes tool to the document analysis tool, it is evident that while many of the indicator ratings match up, or closely align with each other, the majority do not (see Tables 3 and 4). Both tools were used in comparison with each other. The document analysis of the written curriculum was compared to the observation analysis of the observed lesson and vice versa. For example, in Ms. McDonald's Lesson #1, four of the eight indicators were rated the same for both the document and the observation. This could be the result of a number of factors and therefore any conclusions drawn about the role of the curriculum are the result of the participants' expressed experiences. While looking at the teacher and student moves within the documents compared to the moves that occurred within the classroom, most of the modifications made to the written curriculum were in an attempt to simplify or reduce the cognitive load of the task. This resulted in some moves that were evident

in the written curriculum becoming not evident or only partially evident in the enacted curriculum.

Teacher and Student Moves

The teacher moves that were observed and compared in both the written curriculum and the enacted lessons are as follows: (1) The teacher engages students in purposeful sharing of mathematical ideas, reasoning, and approaches, using varied representations, (2) The teacher selects and sequences student approaches and solution strategies for whole-class analysis and discussion, (3) The teacher facilitates discourse among students by positioning them as authors of ideas, who explain and defend their approaches, and (4) The teacher ensures progress toward mathematical goals by making explicit connections to student approaches and reasoning.

The student moves that were observed and compared in both the written curriculum and the enacted lessons are as follows: (1) During the lesson students present and explain ideas, reasoning, and representations to one another in pair, small-group, and whole-class discourse, (2) During the lesson students listen carefully to and critiquing the reasoning of peers, using examples to support or counterexamples to refute arguments, (3) During the lesson students seek to understand the approaches used by peers by asking clarifying questions, trying our others' strategies, and describing the approaches used by others, and (4) During the lesson students identify how different approaches to solving a task are the same and how they are different.

Table 3

Teacher and Student Moves - Written Curriculum vs. Observed Lessons

	Ms. McDonald				Ms. Kyle				Ms. Standen			
	Lesson 1		Lesson 2		Lesson 1		Lesson 2		Lesson 1		Lesson 2	
	WC[c]	OL[d]	WC	OL	WC	OL	WC	OL	WC	OL	WC	OL
TM[a] 1	E	PE	E	E	PE	PE	E	NE	E	PE	E	E
TM 2	PE	PE	NE	PE	NE	NE	NE	PE	NE	PE	NE	PE
TM 3	E	PE	E	PE	PE	PE	E	PE	E	PE	E	E
TM 4	PE	PE	PE	E	PE	E	PE	E	PE	E	PE	E
SM[b] 1	E	PE	E	E	E	NE	E	PE	E	NE	E	PE
SM 2	NE	NE	NE	PE	NE	NE	NE	PE	NE	NE	NE	PE
SM 3	NE	NE	NE	NE	NE	NE	NE	NE	NE	PE	NE	NE
SM 4	PE	NE	NE	NE	PE	NE	E	NE	PE	NE	E	PE

Note. [a] TM indicates Teacher Move. [b] SM indicates Student Move. [c] WC indicates Written Curriculum. [d] OL indicates Observed Lesson.

Table 4

Summary of Indicator Matches Between Written Curriculum and Observations.

	Ms. McDonald	**Ms. Kyle**	**Ms. Standen**
Lesson #1	4/8 matches	5/8 matches	1/8 match
Lesson #2	4/8 matches	1/8 match	3/8 matches

Utilizing the NCTM's *Principles to Actions* (2014), the various question types that were recorded in both the written curriculum and the observed lessons were: (1) Gathering Information, where students are asked to recall facts, definitions, or procedures, such as "Which shape is 3D: circle or cylinder?" (2) Probing Thinking, where students are asked to explain, elaborate, or clarify their thinking, including articulating the steps in solution methods or the

completion of a task, such as “What did you do on your turn?” (3) Making the Mathematics Visible, where students are asked to discuss mathematical structures and make connections among mathematical ideas and relationships, such as “If I continue to follow my rule, what else can I catch?” and (4) Encouraging Reflection and Justification, where students are asked to reveal deeper understanding of their reasoning and actions, including making an argument for the validity of their work, such as “How do you know?” and “Why?”. These question types were investigated during the document analysis as well as the observed lessons. While the moves were then further analyzed on the researcher-created tools, Facilitating Meaningful Mathematical Discourse Teacher and Student Actions (Appendices D and E), the sheer volume of each question type was also recorded. In Table 5 below, you can see that the written curriculum aimed to offer questions across all four types, while the teachers tended to stick to one or two types primarily. Often, the question types that teachers tended toward were Gathering Information and Probing Thinking, which resulted in more yes/no or targeted answers. The participants sometimes got stuck in a “funneling” line of questioning where they were seeking a specific response from students by asking them more and more targeted questions to get to the answer they wanted, rather than “focusing” their questioning to get students to share more and different ways of doing.

Table 5

Question Types Present in Written Curriculum vs. Observed Lessons

		Gathering Information	Probing Thinking	Making the Mathematics Visible	Encouraging Reflection and Justification
M1*	**Written** **Observed**	14% 100%	57% 0%	14% 0%	14% 0%
M2	**Written** **Observed**	62% 62%	0% 13%	13% 25%	25% 0%
K1	**Written** **Observed**	53% 76%	0% 24%	29% 0%	18% 0%
K2	**Written** **Observed**	12% 75%	47% 25%	23% 0%	18% 0%
S1	**Written** **Observed**	33% 90%	11% 5%	44% 5%	11% 0%
S2	**Written** **Observed**	12% 71%	47% 29%	24% 0%	17% 0%

Note. M1/M2 indicates Ms. McDonald's Lessons 1 & 2. K1/K2 indicates Ms. Kyle's Lessons 1 & 2. S1/S2 indicates Ms. Standen's Lessons 1 & 2.

Conclusions

This chapter contains the results of the data analysis as well as connections back to the research questions with a constant comparative method. Three participants were interviewed and observed, and documents related to the curriculum were obtained and analyzed using researcher-created tools. All participants had between three and nine years of experience in education and held bachelor's degrees.

There were three levels of analysis, in vivo coding, axial coding, and selective coding. Twenty-three codes emerged during axial coding and those were refined down to 8 codes during selective coding. Constant comparison was used to look across interviews, document analyses, and observational data, resulting in three emergent themes that reflect how teachers understand,

plan for, and implement mathematical discourse in the classroom, as well as the influence curriculum has on the planning and implementation of mathematical discourse: (a) the participants understood and desired to engage in meaningful discourse with and between their students, (b) factors such as time, language, and COVID-19 protocols impacted their ability to plan for and implement meaningful discourse in their classrooms, and (c) there was no apparent influence from the written curriculum on the enacted curriculum. Chapter 5 includes further discussion of the themes.

5 DISCUSSION

The purpose of this study is to explore a group of primary elementary teachers' understanding, planning, and implementation of mathematical discourse, including their use of curriculum, in an urban public school. This chapter includes a discussion of major findings as related to the literature on mathematical discourse, curriculum, and discourse in practice. This chapter concludes with a discussion of the limitations of this study, areas for future research, and a brief summary.

This chapter contains discussion to help answer the research questions:

(a) How do teachers understand, plan for, and implement mathematical discourse in elementary classrooms?

(b) What role does the written curriculum play in the planning and implementation of mathematical discourse in elementary classrooms?

The participants in this study showed a solid understanding of mathematical discourse and had a strong desire to implement mathematical discourse in their lessons, however, there were many barriers that stood in the way of their ability to fully follow through with that desire including time, language, and COVID-19 protocols. As for the written curriculum, while the participants followed the outline of the lesson quite closely, discourse suggestions and/or protocols were not always aligned. Therefore, there was limited observed influential connection between the written curriculum and the enacted curriculum as it relates to classroom discourse.

The theoretical underpinnings of this study are dialogism and socioconstructivism. Throughout the analysis of the data, the ideas of meaning-making and social construction of knowledge were constantly revisited. As the themes began to emerge, particularly in the context of the COVID-19 pandemic, it was clear that teachers craved the opportunity to work in

community with one another and provide the opportunity for students to learn in community with their peers. The participants expressed that the barriers to mathematical discourse also created barriers to understanding for their students, confirming the ideas of dialogism (Bakhtin, 1981) and socioconstructivism (Cobb, 1994; Cobb & Bauersfeld, 1995; Cobb et al., 1990; Cobb et al., 1993; Hufferd-Ackles et al., 2004).

Interpretation of Findings

While the backgrounds and individual experiences of the participants varied, the three major themes were evident across all of the participants and their experiences with mathematical discourse in the classroom. The participants had a desire to implement mathematical discourse, encountered barriers related to discourse such as language, time, and COVID-19, and the curriculum had little effect on the question types they used in their instruction.

Desire to Implement Mathematical Discourse

This study concluded that while the teachers communicated a philosophical agreement with socioconstructivist theories and beliefs around mathematical discourse, they were more likely to use more traditional lecture or call and response engagement during their lessons. This aligns with the research from Lui and Bonner (2016), who determined that a reversion to more traditional styles of teaching is more likely in times of high stress and feelings of overwhelm. The COVID-19 pandemic has certainly added additional layers of stress to teachers, so it is no surprise that teachers reverted back to the ways in which they were taught as students. This finding is similarly supported by Purnomo, Suryadi, and Darwis' (2017) study, which determined that even teachers who held primarily constructivist beliefs implemented more traditional practices in the area of mathematics. This is evidenced by the amount of questions that were aimed at gathering information during the observed lessons. Percentages of the

gathering information question type ranged from 62% to 100% in the observed lessons, whereas, in the written lessons, they ranged from 12% to 62%. Conversely, the higher-level question types such as *Making the Mathematics Visible* and *Encouraging Reflection and Justification* were rarely used in the observed lessons. The usage of these question types in the written lesson ranged from 11% to 44%, while the usage in all but two observed lessons was 0%. Upon reflection on their observed lessons, the teachers lamented that they wished they were able to get students talking more. They recognized that the ways in which they delivered the questions were not necessarily getting at the understandings that they had hoped for.

In this study, the participants emphasized their desire to implement meaningful mathematical discourse in their classrooms. Two of the participants specifically noted that their professional development offered at their school focused heavily on constructivist practices and that they considered their beliefs to align with that philosophy. The school site itself promotes a constructivist philosophy and encourages discourse practices through their curriculum choices and professional development opportunities. Teachers at the site are expected to engage students in discourse. The curriculum supported this desire by presenting “cognitively demanding tasks” for teachers to engage with alongside students, which promoted a problem-solving environment (Stein et al., 2008).

All three of the participants’ classrooms were designed in an attempt to support discourse among students. The participants also presented complex and interesting ideas (Stein et.al., 2008) through math tasks and games, but they also expected specific “correct” answers from their students while engaging in mathematical discourse, aligning with the findings of Hamm and Perry (2002), Schoenfeld (1992), and Stodolsky (1988). Notably, these participants are following the recommendations of those in the field of mathematics education calling for the movement

away from *skill and drill* worksheets to more meaningful problem-solving opportunities (Hiebery & Wearne, 1993), however, given the context of the study, these teachers were given the nearly impossible task of continuing to support students with high-level learning opportunities while navigating a global health crisis. Many of the "missing pieces" can be attributed to the difficulty of the environment and compromises made in order to keep moving forward.

Barriers to Implementation

The second conclusion of this study is that time, language, and COVID-19 protocols inhibited teachers' ability to implement mathematical discourse in the way they desired. This conclusion related to time as a barrier is similarly evident in a study by Hiebert and Wearne (1993), who found that discourse-based classrooms spent twice as long on each problem. Time is a limited resource in schools. It seems there is never enough of it to go around. Effective discourse does take time, but it also the means in which learning takes place, as Bakhtin (1981) asserts. When considering the findings of Kazemi and Stipek's (2001), this study similarly determined that while teachers were concerned with depth of understanding, the exchanges between teacher and student were *low press*, or simple questions of agreement answered with a yes/no that may include some collaboration and strategy sharing, but with a focus on the *how* over the *why*. Instead of allowing the discourse to have natural tension and letting students come to their own conclusions about a strategy, the participants were pushing students toward the "right" answer with the so-called "verbal checkmarks" (McCarthy et.al., 2016, p. 8) indicating that students were on the right track or that their response was the one that the teacher desired. The participants in this study felt that they needed to cover the content as a first priority and, secondarily, go deeper if there was time. But, there was hardly ever time. As a result, their desire

for deeper understanding was pushed to the side to accommodate the coverage of material, so even during what the participants described as discourse, they were mostly low press questioning sessions. Although the participants' schedule was designed specifically to give mathematics its own time block during the day (at the expense of other subject areas), lessons were still having to be shortened or condensed in order to get through all the work. This resulted in more teacher-centered instruction to accommodate the time constraints the participants were given, which confirms several studies' findings that teacher-centered instruction tends to be the default in times of stress or pressure (Cazden, 2001; Cuban, 1993; Graesser & Person, 1994; Serin, 2018; Stephan, 2020).

Language as a barrier to implementing mathematical discourse was a surprising finding in this study. All three participants spoke about the lack of academic language with which their students came to school. They felt that they had a hard time engaging students in discourse because the participants were having to explain what words meant and spending time teaching the vocabulary of the concept. This became a frustration point for these teachers. Erath et al. (2018) drew a line between "explaining to learn" and "learning to explain", concluding that teachers were not placing enough emphasis on the "learning to explain" component and instead expected their students to simply *know* how to utilize the language of mathematical discourse in their explanations of their learning. This seems to connect with this study's finding quite well, because the participants expected their students to come to school with known academic language and discourse skills but were not willing or able to commit the time to teach their students the vocabulary and correct context in order to effectively participate. Additionally, turning back to the frameworks that guided this study, both dialogism and socioconstructivism are rooted in the ideas that discourse is the learning itself and that knowledge is created in community and context

(Bakhtin, 1981; Cobb, 1994; Cobb & Bauersfeld, 1995; Cobb et.al., 1993; Hufferd-Ackles et.al., 2004). While the participants wanted students to engage in discourse and learn in community, the teachers viewed lack of academic language as a barrier to entry for mathematical discourse. This view is somewhat paradoxical because academic language does not necessarily have to be present for mathematical understanding to take place.

The final barrier to implementation were the protocols put in place to reduce the spread of COVID-19 within the school. Some of the measures put in place by the school were mandatory mask-wearing for both teachers and students, Plexiglass dividers at each table separating students from each other, and an opt-in virtual module where students could engage in their classroom via remote learning. Studies by both Murgatrotd (2020) and Doucet et.al. (2020) illuminated similar concerns about the effects of virtual or hybrid instruction on students and the classroom environment.

All three participants referenced these specific measures as hindrances to their implementation of mathematical discourse. It was difficult for students to hear each other, technical difficulties made it hard to maintain a comfortable conversation flow, and engaging students both in-person and virtually simultaneously was challenging. While there was evidence of the participants' attempts to engage students in conversation, the dynamic was primarily between the teacher and one student before moving on or restating the same question to another student. These specific impacts on mathematical discourse have not been heavily researched due to the novelty of this particular barrier. However, there is evidence that masks impact the speech intelligibility in realistic classroom settings (Bottalico et al., 2020).

Connection to Written Curriculum

The final conclusion of this study is that there was little evidence connecting the curriculum with the implementation of mathematical discourse. All of the participants in the study followed the written curriculum fairly closely and as discussed in Chapter 4, the participants' indicator ratings for mathematical discourse sometimes aligned with the curriculum, but other times did not. Additionally, because of the particular limitations of the study that will be discussed further in this chapter, it is difficult to attribute the participants' mathematical discourse practices to the curriculum.

The participants all agreed that their beliefs about mathematical discourse aligned with the social constructivist perspective of both the school and the curriculum. Contrary to Herbel-Eisenmann's (2007) findings, the curriculum did not, in fact, undermine the goals of student-centered or experiential learning. The Everyday Math curriculum actually included much more high-level questioning and student-centered tasks than the participants implemented in their enacted lessons. The participants discussed the ways that they utilized the curriculum to help push their students' thinking and appreciated the incorporation of many discourse practices within the curriculum. In practice, however, the alignment was not as present in their observed lessons as they communicated. Because their beliefs were already aligned with the curriculum, these findings connect to Pepin et al.'s (2013) study, which concluded that teachers were more willing to incorporate the curriculum when it aligned with their already held beliefs and preferred practices. It also connects to Sherin and Drake's (2009) findings that teachers make decisions about what to keep and what to change based on the particular goals and priorities of the teacher. In this case, this study's participants' apparent priority was to cover as much material as possible in the short amount of time given and within the context of physical and mental barriers.

Implications

This study found that there are several perceived barriers in place preventing teachers from implementing meaningful mathematical discourse practices in their classrooms. The participants desired to engage their students in mathematical discourse and utilized the curriculum when planning these opportunities for engagement, however, in practice, the participants fell back to their comfort zones of univocal discourse when faced with stress such as time constraints, language confusion, and communication hurdles related to COVID-19 protocols.

The barriers related to COVID-19 protocols will eventually be removed as the pandemic wanes, which may allow teachers to engage in a more natural conversation flow and encourage more peer-to-peer discourse in the classroom. Language issues seem to be a self-imposed frustration point rather than a barrier to mathematical discourse implementation, as Bakhtin (1981) points out that mathematical discourse is the learning itself, not simply a tool for learning. By engaging in conversation and exploration with students, an understanding of language and terms comes to be collectively. So perhaps by spending more time engaging in discussion with and between students, the language will be discovered within the context of a mathematical setting. Additionally, this frustration could be assuaged by spending more time co-constructing knowledge through the lens of socioconstructivism, which builds through engagement with community. Time perhaps is a barrier that may simply always exist in the context of schedules and schooling systems that are designed to achieve specific learning goals and targets with strict timelines attached. However, the hesitancy to spend time engaging in discourse appears to be linked to the fear of "wasting time." Teachers may fear that spending so much time engaging in discourse around one problem or one concept with little demonstrable work is wasted or undesirable.

Based on the findings of this study, the participants were well prepared in their understanding of mathematical discourse through their college courses as well as on-site trainings specifically related to the topic. They all recognized the efficacy of the practice and desired to implement it in their classrooms. More support in the area of intentional planning and implementation of mathematical discourse may be helpful for teachers, such as collaborative time to walk through lessons with other teachers and play out some potential discussions that may come up or potential lines of questioning that might be beneficial in helping students reach a better understanding. This seemed to be the area where the participants struggled the most in making their desires a reality. Additionally, these participants benefitted from a dedicated time and space to engage in their own meaningful discourse with their peers around the goals and shared experiences of their students through collaborative planning blocks. This could be an additional way to support teachers in their planning and implementation practices. When teachers participate in shared discourse, they are able to build their understanding in community that deepens their experience with not only the mathematical goals, but also discourse as well.

The participants in this study had access to a curriculum that prioritized mathematical discourse as a core value of its product, so while there was no real influence found on the types of questions utilized from the written curriculum to the enacted curriculum, the participants expressed that they recognized the discourse supports that were included in their resources. They also all utilized discourse practices consistently, engaging students in questioning cycles and expecting them to engage in conversation around mathematical concepts. This curriculum provided a starting point and a strong focus on discourse itself rather than memorization or procedural steps. The more that curricular resources and materials focus on and provide supports

for mathematical discourse, the more likely teachers are to pull from and utilize those suggestions.

The finding that time was a barrier to implementation means that teachers need opportunities and tools to develop the skills of mathematical discourse and plan for them in the contexts that already exist for them. Teachers may benefit from observation-based coaching cycles during which a coach or mentor teacher could provide feedback for specific mathematical interactions and general classroom discourse on lessons they are already planning to teach. Additionally, sustained, intensive professional development opportunities during which teachers are able to observe coaches or master teachers modeling appropriate discourse practices may be beneficial for teachers struggling to implement discourse practices during their mathematics lessons. Lesson studies or professional learning communities (PLCs) devoted to developing mathematical discourse practices, coupled with book studies that provide research-based strategies for implementing discourse in the classroom, may also be helpful. Choosing a curricular program that prioritizes discourse could lessen the burden on teachers to create these experiences from scratch. These participants were not new to the idea of mathematical discourse, however, teachers that are beginning their journey toward understanding and implementing discourse practices in their classrooms would benefit from explicit professional development examining the types of questioning they currently use and revising to expand their understanding of engaging students in mathematical discourse.

Limitations and Recommendations for Future Research

The results of this study were majorly limited by the time period during which the data was collected. Because these data were collected during the height of the COVID-19 pandemic, the results are only for this particular context given the number of barriers and results that were

found to be directly linked to issues related to the COVID-19 pandemic. While some of the findings may not be connected solely to the COVID-19 pandemic, it is impossible to separate the two. More research is needed around mathematical discourse during a fully in-person school year experience in order to provide more information around which findings could be attributed to broader contexts.

This study was also limited by the non-traditional context of the school site from which the data were collected. This school in particular makes it exceptionally well known that they fully align with the social constructivist philosophy and encourage their teachers to implement these types of practices in their classroom. The school also purchases curricula that follow constructivist principles which, again, include extensive suggestions for discursive practices. In more traditional contexts, the findings may be very different. More research is needed in a variety of contexts to provide a broader picture of the experiences of teacher engagement in mathematical discourse.

The curriculum and resources examined in this study reflected only one perspective on the impact of curriculum on mathematical discourse. More research could be done to examine the emphasis on and recommendations for discourse across a wider spectrum of materials that teachers utilize in classrooms. By examining a range of published curricular materials in this context, there could be patterns in the planning and implementation process that were not able to be explored in this study. This may also help distinguish the teachers' own knowledge and ability to implement discourse from the curricular influence.

Final Thoughts

There is still work to do in our elementary classrooms to increase and encourage the use of meaningful mathematical discourse. Even with many resources in place, aligning philosophies,

and curricula that encourage mathematical discourse, there are still issues with implementation. More research is needed to expand the findings of this study outside of the context of the COVID-19 pandemic; however, this study suggests that teachers may have a desire to implement mathematical discourse, but barriers such as time, language, and COVID-19 protocols prevent them from acting on their philosophical beliefs regarding how students best learn. Additionally, the curriculum showed no appreciable impact on the teachers' implementation of mathematical discourse.

With more research around mathematical discourse, and more insight into the barriers for implementation in other contexts and settings, perhaps we can determine more ways to support teachers in their desire to implement meaningful mathematical discourse in the classroom. In turn, students would be able to develop more skills around language, expression, debate, and critical thinking by engaging with teachers and peers in productive discursive experiences. Hopefully, future studies can shed some light onto the supports and practices that are successful in encouraging mathematical discourse in the classroom. Until then, classroom discourse remains an effective learning opportunity for all students and any current perceived barriers should be actively addressed to increase accessibility to resources and support for teachers.

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APPENDICES

Appendix A

Initial Email Protocol

Teachers,

My name is Anne Green and I am an Ed. D. student studying at Georgia State University. I am conducting a research study about teacher practices in mathematics in elementary classrooms. I am emailing to ask if you would like to participate in this research project. Participation is completely voluntary and your information will be anonymous. For this study, you will be asked to complete 2 thirty minute interviews, 2 one-hour classroom observations, and submit provided curriculum and teacher-created lesson plans over the course of three months. If you are interested, please click on the link below for additional information and/or to sign up for participation: [www.linktosignup.com.] If you have any questions, please do not hesitate to contact me. Thank you for your time.

Anne Green

Georgia State University

Appendix B

Initial interview protocol

1. What do you believe should be the teacher's role during mathematics instruction? How should students learn mathematics?
2. The NCTM defines mathematical discourse as "the purposeful exchange of ideas through classroom discussion, as well as through other forms of verbal, visual, and written communication." Is mathematical discourse something that you implement in your classroom? If yes, in what ways is it implemented? If no, why not?
3. How do you plan for mathematical discourse? (*Probe for examples, etc.)*
4. Do the curricular resources provided help you plan for mathematical discourse? If so how? If not, why?
5. Do you have norms/rules about how students should participate in conversations or group discourse in your classroom? (*Probe for what the norms/rules are*) Why or why not?
6. When you are planning your math lessons, what types of questions do you typically plan to ask your students? (*Probe for examples)*
7. What is challenging about implementing mathematical discourse? (*Probe for examples, etc.*)
8. What do you think are the benefits, if any, of mathematical discourse for students?

Appendix C

Post Interview Protocol

1. How do you feel about the lesson I observed?
2. Was this a "typical" math lesson?
3. Was there anything that you planned to do that you decided to change or skip while you were teaching? Why?
4. How did you create opportunities for students to participate in discourse during this lesson? *[Probe if the teacher or the curriculum created these opportunities.]*
5. Do you feel that the level of student discourse in this lesson was adequate, or would you prefer to see changes? Explain. *[Probe how teacher might facilitate that change.]*
6. Analysis of specific discourse incidents … *[to be used as appropriate to the situation]*
    a. Teacher discourse –
        i. You said/asked " . . . " What were you hoping to elicit from students?
        i. Did you achieve your intended goal?
        ii. Would you say/ask it differently if you did it again? Why or why not?
    b. Group discourse –
        i. Explain your role during the group discussion.
        i. What were you hoping students were learning during {*specific discourse interaction}?*

Appendix D

Facilitating meaningful mathematical discourse Teacher and student actions (Document Analysis)			
TEACHER MOVES	EVIDENCE	STUDENT MOVES	EVIDENCE
The teacher engages students in purposeful sharing of mathematical ideas, reasoning, and approaches, using varied representations.	NE PE E Examples:	During the lesson students present and explain ideas, reasoning, and representations to one another in pair, small-group, and whole-class discourse.	NE PE E Examples:
The teacher selects and sequences student approaches and solution strategies for whole-class analysis and discussion.	NE PE E Examples:	During the lesson students listen carefully to and critiquing the reasoning of peers, using examples to support or counterexamples to refute arguments.	NE PE E Examples:
The teacher facilitates discourse among students by positioning them as authors of ideas, who explain and defend their approaches.	NE PE E Examples:	During the lesson students seek to understand the approaches used by peers by asking clarifying questions, trying our others' strategies, and describing the approaches used by others.	NE PE E Examples:
The teacher ensures progress toward mathematical goals by making explicit connections to student approaches and reasoning	NE PE E Examples:	During the lesson students identify how different approaches to solving a task are the same and how they are different.	NE PE E Examples:
Rating Scale: **NE** – Not Evident **PE** – Partially Evident **E** – Evident			

Appendix E

Facilitating meaningful mathematical discourse Teacher and student actions (Observation Notes)			
TEACHER MOVES	EVIDENCE	STUDENT MOVES	EVIDENCE
The teacher engages students in purposeful sharing of mathematical ideas, reasoning, and approaches, using varied representations.	NE PE E Examples:	During the lesson students present and explain ideas, reasoning, and representations to one another in pair, small-group, and whole-class discourse.	NE PE E Examples:
The teacher selects and sequences student approaches and solution strategies for whole-class analysis and discussion.	NE PE E Examples:	During the lesson students listen carefully to and critiquing the reasoning of peers, using examples to support or counterexamples to refute arguments.	NE PE E Examples:
The teacher facilitates discourse among students by positioning them as authors of ideas, who explain and defend their approaches.	NE PE E Examples:	During the lesson students seek to understand the approaches used by peers by asking clarifying questions, trying our others' strategies, and describing the approaches used by others.	NE PE E Examples:
The teacher ensures progress toward mathematical goals by making explicit connections to student approaches and reasoning	NE PE E Examples:	During the lesson students identify how different approaches to solving a task are the same and how they are different.	NE PE E Examples:
Rating Scale: **NE** – Not Evident **PE** – Partially Evident **E** – Evident			

Appendix F

Question Types (Document Analysis)			
Question Type	Description	Occurrences within the Lesson	Examples
Gathering information	Students recall facts, definitions, or procedures.		
Probing thinking	Students explain, elaborate, or clarify their thinking, including articulating the steps in solution methods or the completion of a task.		
Making the mathematics visible	Students discuss mathematical structures and make connections among mathematical ideas and relationships.		
Encouraging reflection and justification	Students reveal deeper understanding of their reasoning and actions, including making an argument for the validity of their work.		