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Experimental Development of Sealed-Bid Auction Theory; Calibrating Controls for Risk Aversion

By JAMES C. COX, VERNON L. SMITH, AND JAMES M. WALKER*

We offer a brief survey of bidding theory in high price auctions, of experimental studies of behavior in such auctions, and of the interplay between the design and results of the experiments and efforts to further develop the theory. Two new series of experiments are reported. The first applies a convex transformation of payoffs in an attempt to induce a lowering of subject bids “as if” the bidders had become less risk averse. The second applies a method for inducing any prespecified utility function (for risky choices) on an individual. We use it to induce “as if” risk-neutral behavior. Both series use baseline control to “calibrate” the hypothesized effect of the procedures on “risk-averse” behavior.

I. Bidding Theory and its Development under Testing

Early experimental papers testing William Vickrey’s (1961) noncooperative equilibrium model of bidding behavior for risk-neutral agents in single unit auctions report the robust result that subjects tend to bid significantly higher than the predictions of the model when the number of bidders is $N \geq 4$, but not when $N = 3$. (For citations to our experimental-theoretical work, see the references in our 1984 article.) The results for $N \geq 4$ are consistent with extensions of the Vickrey model which postulate that agents all have the same concave utility for monetary surplus (for example, Charles Holt, 1980). However, these extensions also imply

that all bidders use the same equilibrium bid function: $b_i(v_i) = b(v_i) \geq b_n(v_i)$, for all i , where v_i is the value of the auctioned item (known only) to i and $b_n(v_i) = (N - 1)v_i/N$ is the Vickrey risk-neutral bid function when each v_i is drawn independently from the constant density on $[0, \bar{v}]$. We have tested the null hypothesis that the bids submitted by the N bidders in each experimental group can be regarded as N samples from the same population. It is rejected in 13 of 23 experimental groups. A straightforward conclusion is that an appropriate extension of the model should be based on the assumption of heterogeneous risk-averse bidders. We have articulated such a model for single unit auctions and extended it to multiple unit discriminative auctions. Experimental tests of the multiple unit model strongly support the interpretation that bidders bid as if they were heterogeneous and risk averse (we reject the hypothesis of homogeneous agents in 24 of 28 experimental groups).

This constant relative risk-averse (CRR) model assumes that (a) each agent i chooses b_i to maximize $EU(b_i) = (v_i - b_i)^r G_i(b_i)$, where $G_i(b_i)$ is the probability that b_i is the highest of N bids; (b) agent expectations are rational, $G_i(b_i) = G(b_i)$; (c) each v_i in any auction is drawn independently from the constant density on $[0, \bar{v}]$; (d) the N agents are drawn from a population with some distribution $\Phi(r_i)$ on the characteristic $r_i \in (0, 1]$. For single unit auctions, these assumptions imply the inverse equilibrium bid function

$$(1) \quad v_i = (N - 1 + r_i) b_i / (N - 1),$$

for all $b_i \in [0, \bar{b}]$,

where $\bar{b} = (N - 1)\bar{v}/N$ is the maximum bid that would be made by a risk-neutral agent. (The solution for $b_i > \bar{b}$ has no closed form.) Hence, if any two of N bidders (i, j) have

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distinct *CRR*A parameters (r_i, r_j), a prediction of the model is that in a sequence of auctions $t = 1, 2, \dots, T$, the observed bids will identify a distinct homogeneous linear bid function for each bidder whose slope will reveal each bidder's *CRR*A utility parameter.

We have conducted and reported two direct tests of the above *CRR*A model and its multiple unit generalization. The first test applies the following property of *CRR*A utility: a scalar change, α , in payoffs has no effect on expected *CRR*A utility-maximizing decisions. This is seen in (a) above if we express expected utility in the form, $EU(b_i) = [\alpha(v_i - b_i)]^r G(b_i)$, where $\alpha(v_i - b_i)$ is the outcome in U.S. currency for the winning bidder. Since α affects only the scale of utility it has no effect on the bid function (1). We report paired comparison experiments in which $\alpha = \$1$ in the control experiments and $\alpha = \$3$ in the paired treatment experiments. There is no significant difference in the outcomes between paired experiments. Similarly, if \bar{v} is increased, say tripled, this model predicts the same scalar increase in v_i , b_i and $v_i - b_i$; that is, in (a) we can write expected utility in the form $EU(\mu_i) = [\bar{v}(v_i - \mu_i)]^r G(\mu_i)$, where $\mu_i = b_i/\bar{v}$, $v_i = v_i/\bar{v}$. Hence a scalar change in the v_i has no effect on normalized bids. We have reported comparison experiments (multiple units) in which \bar{v} (and each v_i) is tripled. The effect is to triple average bids, and the conclusions based on $\alpha = \$1$ are not altered when $\alpha = \$3$.

Since *CRR*A utility is the only utility function with these scalar invariance properties, these experiments provide important independent support for the theory beyond the earlier *ex post* analysis showing that observed bids are consistent with the assumption that agents are risk averse and heterogeneous. The new tests for scalar effects were motivated a priori by the theory.

Two well-known "logical" objections to all *CRR*A utility models are a recurring part of the conventional wisdom connected with expected utility theory (*EUT*), although these objections are devoid of observational support: (A) "*CRR*A utility is unacceptable as it implies that absolute risk aversion grows without bound as $v - b$ approaches zero";

(B) "The *CRR*A bidding model admits of a tractable solution only if initial wealth is (or can be normalized on) zero."

The a priorist objection (A) asserts that any *EUT* model can be "tested" by examining its absolute risk-averse implications, and that behavior near some boundary (zero) is a crucial test of any hypothesis. This is like arguing (without resorting to observation) that the inverse square law of attraction is falsified, since the force of attraction goes to infinity as the distance between masses approaches zero! For the vast majority of subjects, when v is near zero, the ratio of bid to value is similar to that for large values of v ; that is, one observes no peculiarity in bidding behavior near zero, which is predicted by equation (1) based on *CRR*A utility. A small minority of subjects bid either zero or their value, at low values of v . This "throw away" bid phenomenon can be interpreted as the result of payoffs being so low that it is not worth the trouble of a "serious" bid. Since it is not clear what is "optimal" when payoffs are at *epsilon* levels, other theories such as random or erratic behavior should not be discounted, just as in particle theory (which is disciplined by data) other theories take over in the small.

Concerning objection (B), we have been quite explicit from the beginning in referring to $v_i - b_i$ as the monetary *income* from an auction. This is because we accept the findings of a vast literature going back at least to Markowitz, and corroborated by Mosteller and Nogee, Davidson, Suppes and Siegel, Edwards, Kahneman and Tversky, Binswanger, and others (see Mark Machina, 1982, for numerous references). Generally this literature supports the relative invariance of risk-taking decision behavior with initial wealth (the "Markowitz hypothesis" of a horizontally shifting utility of wealth). Also, this literature does not find support for constant absolute risk aversion (*CARA*) (Machina, p. 285). *The prize to which EUT applies* (wealth, income etc.) is a hypothesis separate from the axioms of *EUT* which do not define that prize.

Various extensions of the original Vickrey model and of Holt's identical bidders risk-averse model are contained in the literature.

Paul Milgrom and Robert Weber (1982) consider the effect of information. However, given the robust experimental result that bidders bid as if they are heterogeneous and risk averse, these extensions (for first price and Dutch auctions) are based on behavioral assumptions already shown to be inconsistent with the data. Eric Maskin and John Riley (1984) offer a potentially fruitful extension based on the assumption of a one-parameter utility function $u(-b_i, \theta_i)$, where b_i is i 's bid. If $\theta_i = v_i$, we have the case of identical bidders with differing private values, but with the bidding commodity medium distinct from the commodity item being auctioned. If $\theta_i = r_i$ (any risk parameter), we have heterogeneous risk-averse bidders, but, implicitly, all must place the same value on the auctioned item. Thus the minimum generalization requires a utility function of the form $u(v_i - b_i, r_i)$ to capture both taste and risk attitude diversity. Cox and Smith (1984) develop an equilibrium bidding model for a utility function of the form $u(\theta_1 - b, \theta_{-1})$, where (θ_1, θ_{-1}) is an M vector of characteristics.

II. Models of Control for the Effect of Risk Aversion

We interpret the observation that subjects bid in excess of the predictions of the Vickrey model as due to heterogeneous risk-averse agents, and have used this *interpretation* to develop an improved model. Subsequently this model was found to be consistent with the scalar invariance tests described above. Now we ask whether direct methods might be applied to examine this risk-averse interpretation of the data. Other interpretations are possible. We might assume in place of (a) that agents choose bids to maximize $EU(b_i) = (v_i - b_i)G_i(b_i)$, and instead of (b), that $G_i(b_i) = [G(b_i)]^{1/r_i}$, where $1/r_i$ is now a characteristic of bidder i that transforms the objective probability of winning, $G(b_i)$, into a subjective probability of winning, $[G(b_i)]^{1/r_i}$. This subjective expected value (*SEV*) model is prominent in psychology (see Machina, pp. 290–91). It abandons Muthian rational expectations, but the resulting model yields a bid function identical with (1), and the two theories are observa-

tionally equivalent on the basis of all experimental tests to date. The methodological point is that the parameter r_i is not observable; it is a construct based on an interpretation of what is driving behavior, and other interpretations are potentially admissible. We have adopted the heterogeneous risk-averse interpretation because it is an integral part of the traditional *EUT*, while the alternative is thought to be “*ad hoc*.” This does not mean that *EUT* is “true,” but that it appears that there is not yet a sufficient basis for the scientific community to abandon *EUT*.

We propose two payoff manipulation models which, based on *EUT*, should have a determinate effect as interpreted in terms of risk aversion. If these models are “correct,” and our interpretation that subjects are risk averse is correct, the new data should be consistent with these predictions.

Model I. In a first price auction, if subject i wins, suppose that instead of paying $(v_i - b_i)$ dollars to i we pay $a(v_i - b_i)^2$ dollars, $a > 0$. In the *CRRRA* model it is seen that the problem now is to maximize $EU(b_i) = [a(v_i - b_i)^2]^{r_i} G(b_i)$ and equation (1) becomes $v_i = (N - 1 + 2r_i)b_i / (N - 1)$, for all $b_i \in [0, \hat{b}]$, where $\hat{b} = (N - 1)\bar{v} / (N + 1)$. Thus if an individual's personal measure of *CRRRA* is $1 - r_i$, under the payoff transformation of Model I, that individual will behave “as if” the *CRRRA* measure had changed to $1 - 2r_i$. This equation provides strong quantitative predictions of the effect of the transformation. A weaker qualitative prediction is that the individual will bid less under the transformation.

Model II. Instead of paying $(v_i - b_i)$ dollars to the high bidder, suppose we pay the winner $(v_i - b_i)$ unit lottery tickets. The individual then participates in a lottery in which he/she receives x_1 dollars in U.S. currency with probability $(v_i - b_i)/\bar{v}$ and x_2 dollars ($x_1 > x_2$) with probability $1 - (v_i - b_i)/\bar{v}$. Suppose further that the $N - 1$ low bidders in the auction all receive x_2 dollars. Since the probability of x_1 is linearly increasing in $(v_i - b_i)$, if *EUT* applies to individual behavior this procedure will cause the individual to bid “as if” risk neutral (Alvin Roth and Michael Malouf, 1979; Joyce Berg et al., 1984). To see this, note that bidder i 's deci-

sion problem is to

$$(2) \quad \max_{b_i} \left[G_i(b_i) \left(u_i(x_1) \left(\frac{v_i - b_i}{\bar{v}} \right) + u_i(x_2) \left(1 - \frac{v_i - b_i}{\bar{v}} \right) \right) + (1 - G_i(b_i)) u_i(x_2) \right] \\ = \left[\frac{u_i(x_1) - u_i(x_2)}{\bar{v}} \right] \\ \times \max_{b_i} [(v_i - b_i) G_i(b_i)] + u_i(x_2),$$

which is formally equivalent to Vickrey, giving $v_i = Nb_i / (N - 1)$, if $G_i(b_i) = G(b_i)$.

Berg et al. have generalized this procedure to induce any prespecified preferences, which they proceed to test experimentally using *CARA* and constant absolute risk-preferring (*CARP*) preferences. They provide a qualitative test by soliciting responses to a choice between two bets, *A* and *B*, with the property that the induced *CARA* function implies that *A* is preferred to *B* while the *CARP* function predicts that *B* is preferred to *A*. They report that significantly more than half (88.3 percent) of the choices correspond to the predictions. They also elicit minimum selling prices for bets from the two groups and compare these with the calculated certainty equivalents of the bets. The observed prices reported by the subjects are then compared with those predicted to provide a quantitative test of their model. For both groups they reject the null hypothesis of no relationship between the *average* observed prices and those predicted. However, the average prices from the risk-averse group tended to be systematically biased above the predicted certainty equivalent. Also, the variance of the observed prices was high.

This is encouraging in that it provides evidence favoring the gross predictive implications of inducing known preferences on subjects. The procedure is potentially important in enabling one to (a) control for risk aversion where other aspects of behavior are the primary focus of the investigation (Roth and Malouf), or (b) induce known risk pref-

erences in a market whose behavior is hypothesized to be driven by risk aversion.

To our knowledge this promising procedure has not been test-calibrated in a market context; that is, used to induce particular preferences in a market which yields predictions *interpretable* in preference terms. High price auctions allow one to do this based on the Vickrey risk-neutral special case.

III. Experiments and Results: Model I, Quadratic Transformation

Twelve subjects participated in three sessions each consisting of 4 bidders. Each session consisted of a baseline sequence (*EIB*) of 20 auctions (12 in session 1) in which each subject was paid one cent for each *PLATO* experimental cent earned,

$$(3) \quad (\text{cash cents}) = (\text{PLATO cents}),$$

followed by a transformation sequence (*EIT*) of 20 auctions in which for each auction cash earnings were calculated using

$$(4) \quad (\text{cash cents}) = 0.02 (\text{PLATO cents})^2.$$

In the *PLATO* instructions for *EIT*, tables and graphs are used to inform the subjects of the payoff implications of this transformation. After the first three sessions were completed, four of the subjects were recruited for a fourth retest session consisting of 20 transformation auctions.

Our initial approach to comparing bidding behavior in *EIT* and *EIB* was twofold. First, we ask whether the mean normalized bid of a subject differs in a transformation experiment from that in a baseline experiment. Since the value realizations from the uniform distribution will differ in the two experiments, if *i* bids b_i^* when the realized value is v_i^* , we normalize the bid by subtracting the risk-neutral Vickrey bid, $b_n(v_i^*)$. Thus, for each *i* we compute the difference $D_i = b_i^* - b_n(v_i^*)$ for each auction, giving a set of differences $\{D_i^B\}$ in *EIB* and a set $\{D_i^T\}$ in *EIT*. The means \bar{D}_i^B were positive for all subjects, indicating that all were risk averse in the baseline sequence. Also, $\bar{D}_i^T > 0$ for all

i , as each subject continues to exhibit risk aversion under the transformation. Furthermore, $\bar{D}_i^B - \bar{D}_i^T$ is positive for eight of the twelve subjects indicating, as predicted, a shift toward less risk-averse behavior with the transformation. In the retest, session 4, all four subjects bid lower in *EIT* than in *EIB* (one subject had bid higher in the earlier session).

However, this apparently good support for Model I could not withstand deeper examination. The payoffs in (3) and (4) imply that a bid for a profit of less than 50 yields a lower return under the transformation than in the baseline, and vice versa for a bid with potential profit in excess of 50. Aware of this, in advance of the experiments we had conjectured that a “satisficer” might bid relatively lower (higher) in *EIT* at profit levels below (above) 50, as a means of maintaining *EIB* performance in *EIT*. A closer examination of individual bids revealed that this effect was strong, contrary to the predictions of Model I.

IV. Experiments and Results: Model II, Lottery Payoffs

Twelve new subjects participated in three sessions, each consisting of 4 bidders and two parts. The first part was a sequence of 20 baseline experiments (*E2B*) with each subject paid one cent in cash for each cent earned in the experiment. The second part consisted of a sequence of 20 auctions (*E2L*) in which subjects in effect earned one lottery ticket for each cent won in an auction. Eight of the twelve subjects were then recruited for two retest sessions, 4 and 5, consisting of 20 auctions with the lottery payoff. The lottery operated as follows: A box containing 1000 tickets, numbered consecutively, was displayed to the subjects. The high bidder in each auction was assigned ticket numbers in an amount equal to the bidders experimental profit in cents. Thus, if the winning bidder had a value of \$8 and bid \$6 when $\bar{v} = \$10$, she might be assigned the lottery numbers 1–200. If the ticket she then drew was in the range 1–200, she received a cash payoff of \$7.50. Otherwise, she received \$0.25. All losing bidders received \$0.25 in cash.

Ten of the twelve subjects bid *significantly* “as if” risk averse in *E2B*, but, contrary to Model II, only one of these subjects was induced to bid as if risk neutral in *E2L*. It appears that Model II has one chance in ten of making a correct strong form prediction. This result was not changed for any subject in the retest sessions 4 and 5. A weak form prediction of Model II is that the difference between baseline and lottery mean bids, $\bar{D}_i^B - \bar{D}_i^L$, is positive indicating a shift toward risk neutrality. Only six of twelve subjects were consistent with this prediction.

V. Conclusions of the Calibration Experiments

Model I, applying a quadratic payoff transformation, predicts a doubling of the “as if” *CRRA* parameter r_i , or, more weakly, a shift in the direction of lower bids (less risk-averse bidding). The experimental results belie this prediction. A close examination of individual bidding suggests that subjects bid less only when the profit potential is below the 50 cent “break-even” level. Above this 50 cent potential profit level, subjects tend to bid relatively higher. This can be interpreted as a type of “satisficing” behavior in which subjects attempt to do at least as well under the quadratic transformation as in the baseline experiments. Does this test invalidate the *CRRA* model of bidding? No; literally, it questions the *conjunction* of the *CRRA* model with the transformation of Model I. Since the *CRRA* model has performed well in previous tests, Model I should be the immediate focus of deeper examination. In particular, the results suggest the need for a change in design that would eliminate the break-even 50 cent profit defined by the intersection of the baseline and quadratic payoff functions. The predicted result is that the hypothesized satisficing effect will be eliminated. Of course, the theory asserts that behavior should not be affected by this artifact, but one would like to know if the theory does better when the artifact is removed. After all, these are not calculating agents, and it may not be difficult to introduce perceptual distortions that alter equilibrium behavior.

Model II predicts risk-neutral bidding for any subject showing risk-averse bidding in a baseline experiment. Since nine of ten subjects bid significantly above the risk-neutral bid function under both lottery and monetary payoffs, these results do not support the predictions of Model II. Given the generally supportive results of earlier direct tests of the bidding model, the predictive failure of Model II can be interpreted as providing (indirect) evidence against the compound lottery axiom of *EUT* that is essential in Model II. Furthermore, these results may have implications for other research programs that must postulate the behavioral validity of the lottery procedure as a conditional in experimental tests of models that require risk attitude of agents to be controlled.

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