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The Winner's Curse and Public Information in Common Value Auctions: Comment

By JAMES C. COX, SAMUEL H. DINKIN, AND VERNON L. SMITH*

The often-cited paper by John H. Kagel and Dan Levin (1986) has had a large influence on the literature concerned with the properties of common value auctions. The low signal, public information experimental design that they introduced continues to be used by researchers.[1] This information environment has also been used to interpret data from related experiments.[2] In addition, both the claimed advantages of the low signal, public information approach to experiments, and Kagel and Levin's original conclusions about public information and the winner's curse, have recently been widely disseminated (Kagel, 1995 pp. 536–60).

We explain that their reported Nash equilibrium bid function for the public information design is in error, which implies that all of their conclusions about the relation between the observed and predicted effects of public information are non sequiturs because they did not follow from existing theory. Our findings have implications for bidding theory and data analysis. They also have implications for experimental methods because they imply that a different approach should be used to study the effects of public information.

I. Theoretical Problems with Low Signal Public Information

Kagel and Levin's (1986) experiments were conducted as follows. The computer draws, but does not announce, the common value of the auctioned item (x_0) from the uniform distribution on $[\underline{x}, \bar{x}]$. Then the signals, x_i, $i = 1, 2 \cdots, n$, are independently drawn from the uniform distribution on $[x_o - \theta, x_o + \theta]$, and each signal is announced privately to each of the n bidders.[3] In addition, in the low signal, public information experiments, the lowest of the n signals (x_L) is announced publicly. This approach continues to be advocated as follows (Kagel, 1995 p. 540):

> There are, however, several methodological advantages to using x_L. First, the RNNE bid function is readily solved for x_L, so that the experimenter continues to have a benchmark model of fully rational behavior against which to compare actual bidding. Second, x_L provides a substantial amount of public information about $x_o \cdots$, while still maintaining an interesting auction.

We will examine the first claim, about the risk-neutral Nash equilibrium (RNNE) bid function.

Kagel and Levin's (1986 p. 902) reported RNNE bid function for signals in $[\underline{x} + \theta, \bar{x} - \theta]$ is

$$(1) \quad b(x_i, x_L, n) = x_L + \frac{n-2}{n-1}\left[\frac{x_i + x_L}{2} - x_L\right] = \frac{n}{2(n-1)} x_L + \frac{n-2}{2(n-1)} x_i .$$

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[1] See Kagel (1995); Kagel et al. (1995); Kagel and Levin (1996).

[2] See Kagel (1995); Levin et al. (1996).

[3] We use the same notation as Kagel and Levin (1986) except that we denote the support of the uniform distribution of signals as $[x_o - \theta, x_o + \theta]$ in place of their denotation as $[x_o - \varepsilon, x_o + \varepsilon]$. This change was necessary because in some of our proofs we follow standard convention in using ε to denote small changes in variables.

We will demonstrate that (1) is not a Nash equilibrium bid function for $n \geq 3$ bidders, and that there does not exist a pure-strategy Nash equilibrium bid function for $n = 2$ bidders, when the lowest signal received by the bidders is public information.

A. *Counterexample for* $n \geq 3$

Bid function (1) implies that the low signal holder always bids an amount equal to her signal (x_L), which gives her an expected payoff of zero for $n \geq 3$.[4] We will show that (1) is *not* a Nash equilibrium bid function because, if all other bidders bid according to (1), then there exists an alternative bid function that gives the low signal holder *positive* expected payoff.

Suppose, contrary to equation (1), that the low signal holder uses the alternative bid function,

$$\beta(x_L, \theta, n) = x_L + \frac{n-2}{2(n-1)}\theta. \tag{2}$$

Note that

$$\beta(x_L, \theta, n) \geq b(x_L, x_L, n), \text{ as } n \geq 2. \tag{3}$$

The maximum-likelihood estimate of the common value of the auctioned item is $(x_L + x_H)/2$, where x_H is the random value of the high signal. Bid function (1) implies that the x_i-inverse image of $\beta(x_L, \theta, n)$ is $x_L + \theta$. Therefore, if all other bidders bid according to (1) then the expected profit to the low signal holder from bidding according to $\beta(x_L, \theta, n)$ is

$$(4)\quad E[\pi(\beta(x_L,\theta,n))] = \int_{-\infty}^{+\infty}\int_{-\infty}^{x_L+\theta} [\tfrac{1}{2}x_L + \tfrac{1}{2}x_H - \beta(x_L,\theta,n)] \times f(x_L, x_H)\, dx_H\, dx_L$$

where $f(x_L, x_H)$ is the joint density function for the high and low signals on the domain of Kagel and Levin's bid function (1):[5]

$$(5)\quad f(x_L, x_H) = n(n-1)\left(\frac{1}{2\theta}\right)^n (x_H - x_L)^{n-2} \times I_{(x_o-\theta, x_o+\theta)}(x_L) I_{(x_L, x_o+\theta)}(x_H).$$

Substitution from (2) and (5) into (4) permits us to rewrite $E[\pi(\beta(x_L, \theta, n))]$ as

$$(6)\quad E[\pi(\beta(x_L,\theta,n))] = \int_{x_o-\theta}^{x_o}\int_{x_L}^{x_L+\theta}\left[\frac{1}{2}(x_H - x_L) - \frac{n-2}{2(n-1)}\theta\right] n(n-1) \times \left(\frac{1}{2\theta}\right)^n (x_H - x_L)^{n-2}\, dx_H\, dx_L$$
$$+ \int_{x_o}^{x_o+\theta}\int_{x_L}^{x_o+\theta}\left[\frac{1}{2}(x_H - x_L) - \frac{n-2}{2(n-1)}\theta\right] n(n-1) \times \left(\frac{1}{2\theta}\right)^n (x_H - x_L)^{n-2}\, dx_H\, dx_L.$$

Integration of (6) yields

$$(7)\quad E[\pi(\beta(x_L,\theta,n))] = \left(\frac{1}{2}\right)^{n-1} \frac{1}{(n-1)(n+1)}\theta > 0.$$

[4] With all bidders using bid function (1) and $n \geq 3$, the low signal holder never submits the high bid except in the degenerate case where all of the signals equal x_L. The expected common value equals x_L in the degenerate case, which occurs with probability zero.

[5] The domain is $[x_o - \theta, x_o + \theta]$. In equation (5), $I_{(a,b)}(z)$ denotes an indicator function: $I_{(a,b)}(z) = 1$, for $z \in (a, b)$, and $I_{(a,b)}(z) = 0$, for $z \notin (a, b)$.

Therefore, if all of the other bidders bid according to Kagel and Levin's bid function (1), then the low signal holder would prefer to reply with the different bid function, $\beta(x_L, \theta, n)$. Therefore, bid function (1) is *not* a Nash equilibrium bid function.[6]

Kagel and Levin focused on the role that public information might play in a common value auction, particularly its effect on seller's revenue and the winner's curse. They tested six hypotheses, three of which involved predicted effects of public information. They also concluded that "... public information ... failed to raise revenues by the predicted amount, even in markets without a winner's curse" (Kagel and Levin, 1986 p. 913). But we have shown that the bid function they offer as a RNNE bid function for public information (in addition to private signals) is not, in fact, an equilibrium bid function. Therefore, the conclusions that they draw about the relation between the observed and predicted effects of public information do not follow from the bidding theory in their article.

The preceding counterexample demonstrates that the reported equilibrium bid function is wrong, but it does not convey an intuition about the nature of the problem. A clear understanding of the source of the inherent problems with the low signal approach is needed in order to understand the implications of our comment for bidding theory, data analysis, and experimental methods. The following nonexistence proof provides the essential insight. We present the nonexistence proof for the special case of two bidders because of its simplicity and clarity.

B. *Nonexistence for n = 2*

We now show, for $n = 2$ bidders, that there does not exist a pure-strategy Nash equilibrium bid function for common value auctions in which the low signal is public information. Suppose that the low signal holder adopts the pure strategy

$$(8) \qquad b_L = f(x_L)$$

such that $f(x_L) > x_L$ for some values of x_L. Let the high signal holder adopt the pure strategy

$$(9) \quad b_H = \begin{cases} g(x_L, x_H), & \text{for } f(x_L) < \frac{1}{2}x_L + \frac{1}{2}x_H \\ h(x_L, x_H), & \text{for } f(x_L) \geq \frac{1}{2}x_L + \frac{1}{2}x_H. \end{cases}$$

Note that equations (8) and (9) reflect the information asymmetry that is implied by having the low signal be public information: the high signal holder can condition her bid on x_L, and hence on $f(x_L)$, but the low signal holder cannot condition his bid on x_H. An implication of the information asymmetry is that the high signal holder can choose

$$(10) \qquad g(x_L, x_H) = f(x_L) + \varepsilon$$

and

$$(11) \qquad h(x_L, x_H) = f(x_L) - \varepsilon.$$

For $\varepsilon > 0$ sufficiently small, equations (10) and (11) ensure that the high signal holder has a positive expected profit and the low signal holder has a negative expected profit.

In order to have nonnegative expected profit from a pure strategy, the low signal holder would have to choose $f(\cdot)$ such that $f(x_L) \leq x_L$ for all x_L. The high signal holder could then win almost all auctions, and earn positive expected profits, by bidding $f(x_L) + \varepsilon \cdot \phi(x_L, x_H)$, for $\varepsilon > 0$, where

$$(12) \quad \phi(x_L, x_H) = \tfrac{1}{2}x_L + \tfrac{1}{2}x_H - f(x_L).$$

But if $f(\cdot) + \varepsilon \cdot \phi(\cdot, \cdot)$ is a good reply to $f(\cdot)$, then $f(\cdot) + {}^1\!/_2\varepsilon \cdot \phi(\cdot, \cdot)$ is a better reply, $f(\cdot) + {}^1\!/_4\varepsilon \cdot \phi(\cdot, \cdot)$ is an even better reply, and so on. Thus there does not exist a pure-strategy RNNE bid function in this continuous-variables model for the low signal, public information common value auction.[7]

[6] Of course, if the other bidders knew that the low signal holder would bid according to $\beta(x_L, \theta, n)$, then they would prefer *not* to reply with bid function (1).

[7] There also does not exist a pure-strategy Nash equilibrium bid function in a *discrete*-variables model for the experimental design parameters used by Kagel and Levin (1986). The spirit of the proof is similar to the continuous

II. Implications for Common Value Auction Research

The problems that are inherent in the low signal, public information environment have implications for bidding theory, experimental methods, and data analysis.

A. *Implications for Bidding Theory*

The preceding analysis makes clear that the theoretical problem inherent in low signal public information is the information *asymmetry* that it creates: the other bidder(s) know as much about the low signal holder's valuation of the auctioned item as does the low signal holder. We have demonstrated that, with this information asymmetry, there does not exist a pure-strategy Nash equilibrium bid function for $n = 2$. The logic of the nonexistence proof can be extended to any finite n. Thus, the implication of our analysis for bidding theory is that an equilibrium bid function for the low signal, public information environment must involve a *mixed* strategy for the low signal holder. But mixed strategies are very difficult to apply to data, especially data for individual bidders. Thus our analysis can provide methodological guidance for experimental studies in these environments.

B. *Implications for Experimental Methods*

Paul R. Milgrom and Robert J. Weber (1982) demonstrated that public information increases Nash equilibrium bids and the seller's revenue in a model with an exogenously determined number of bidders because it reduces item valuation uncertainty. An example of the type of public information that they modeled is announcement of an additional randomly drawn signal. Unlike the Kagel and Levin low signal environment, this information environment preserves symmetry among the n bidders, each of whom knows his own signal and the value of the $n + 1$st (public) signal, and therefore it is an environment for which there exists a pure-strategy equilibrium bid function. Experimentalists studying the effects of public information are well advised to adopt such a symmetric approach in order to produce data with precise theoretical interpretations. The random signal design also has other advantages, such as variable sample sizes for both public and private information signals. Thus, one can use this approach to study the effects of *more or less* public information; for example, one can compare the effects of randomly drawn public signal samples of size 1 with those of, say, size 5.

C. *Implications for Data Analysis*

Kagel and Levin analyze the incidence of the winner's curse in their experiments with two measures: (A) profits as a percentage of the RNNE (risk-neutral Nash equilibrium) prediction; and (B) percentage of high bids greater than the expected value of the auctioned item conditional on the high bidder's signal being the highest of n signals. They apply these measures to data from both private signal and public signal auctions. Neither of these measures can be correctly applied to data from their public signal auctions. It is already clear that measure (A) cannot be applied to public signal auction data because their RNNE prediction was derived from an incorrect bid function. Therefore, the measure (A) results reported in Kagel and Levin's Tables 5, 6, and 7 that involve use of their RNNE prediction are incorrect. Measure (B) is also incorrect for public signal auctions, as we shall next explain.

Measure (B) is the difference between the amount of the winning bid (b_W) and the expected value of the auctioned item conditional on the winning bidder's signal (x_i) being the highest (x_H) of the n signals: $b_W - E(x_o | x_i = x_H)$.[8] This is a correct measure for private signal auctions but not for public signal auctions.

case, but requires that $\varepsilon\phi(\cdot) \geq 0.01$. For other choices of experimental design parameters, a pure-strategy Nash equilibrium bid function could exist in a discrete-variables model.

[8] The equation for this measure for the middle part of the signal support is reported in Kagel and Levin [1986 equation (5)]. Equations for all three parts of the signal support are reported in Cox et al. [1996 equations (12)–(17)].

In public signal auctions, each bidder has a sample of size two, not one, from the uniform distribution of signals. Therefore, his maximum-likelihood estimate of the common value of the auctioned item is the midpoint between the two signals (μ_i), not the amount of his private signal that would be the estimate in a private signal auction. The correct measure of the conditional expected value of the auctioned item in public information auctions is $E(x_o | \mu_i = \mu_H)$, the expected value of the auctioned item conditional on one's own *midpoint* being the highest of *n midpoints;* hence the correct measure of the winner's curse is $b_W - E(x_o | \mu_i = \mu_H)$.[9] Therefore, the measure (B) results reported in Kagel and Levin's Table 6 are wrong since they involve an incorrect calculation of the winner's curse.[10]

III. Conclusion

In a widely cited paper, Kagel and Levin (1986) reported tests of six hypotheses, three concerned with the effects of public information and three concerned with the effects of private information on bidding in common value auctions (also see Kagel, 1995 pp. 536–60). We have shown that the low signal, public information bid function underlying their analysis is wrong and, furthermore, that there does not exist a pure-strategy equilibrium bid function for this information environment. The low signal, public information treatment has been introduced in other papers and used to interpret data in related work. Our theoretical analysis implies that this literature needs to be critically reexamined. As we have explained, the alternative random signal(s), symmetric information method for introducing public information is better because it is consistent with pure-strategy equilibrium bidding theory and permits a flexible design in which a researcher can vary the size of the public information sample.

Kagel and Levin also concluded that the winner's curse was ubiquitous in their *private* signal auctions with experienced subjects in which there were more than 3 or 4 bidders. They based this conclusion on calculations of the level of bidder profits and the number of bids greater than conditional expected value. While this is clearly relevant information, it does not tell the whole story because they did not report significance tests. We have conducted *t*-tests and Kolmogorov-Smirnov nonparametric tests and also used a bootstrapping technique to calculate *p*-values for the winner's curse measure that is appropriate for private signal auctions, which is $b_i - E(x_o | x_i = x_H)$. The test results are reported in an Appendix available upon request to the authors. All three of our test procedures reveal a general absence of a *significant* winner's curse in either ''small'' ($n \leq 4$) or ''large'' auctions. We conclude that there is some limited evidence of a winner's curse in the large auctions but that Kagel and Levin greatly overstated their conclusions about it.

[9] Complete derivations of conditional expected values of this type are contained in Cox and Stephen C. Hayne (1998).

[10] A referee, while agreeing that there is a problem with the Kagel and Levin bid function for public information argues that the correct bid function might strengthen their conclusions about seller's revenue. Our point is that their conclusions do not follow from the theory in their paper and we do not speculate on how a correct theory might affect particular conclusions such as the effect of public information on seller's revenue. However we do suggest that the particular Kagel and Levin asymmetric information approach is problematic in that it requires use of an equilibrium involving a mixed strategy. This in turn would call for a fundamentally different approach to analyzing individual subject data.

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