The Effects of Mathematical Modeling Instruction on Precalculus Students' Performance and Attitudes Toward Rational Functions

Solomon A. Betanga

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THE EFFECTS OF MATHEMATICAL MODELING INSTRUCTION ON PRECALCULUS STUDENTS’ PERFORMANCE AND ATTITUDES TOWARD RATIONAL FUNCTIONS

by

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Under the direction of Dr. Iman C. Chahine

ABSTRACT

According to Blum (2011), mathematical modelling is the translation between the real world and mathematics and from mathematics back to the real world. Blum and other studies Nourallah and Farzad (2012) for example, have indicated that this process of alternating between reality and mathematics during mathematical activities has impacts on students’ mathematical knowledge.

This study investigated the effects of mathematical modeling instruction on precalculus students’ performance in a Rational Function Exam (RFE) and their attitudes toward rational functions. It was an exploratory embedded single case study design using both quantitative and qualitative methods. A sample of 54 precalculus students enrolled in two sections of precalculus at a local college in one major southern city of the United States was used for this study. The two precalculus sections were purposefully selected from five sections, with 24 students in the treatment group and 30 students in the comparison group.

Quantitatively, participants completed a pre-post Rational Function Exam (RFE) and an Attitude Toward Mathematic Inventory (ATMI) survey (Tapia & Marsh, 2004) before and after the study. Qualitative techniques were employed to determine the type and cognitive complexity of representations. These qualitative methods included interviews, a questionnaire, artifacts of students’ work and the researcher’s memos. The interviews and questionnaire
responses were used to gather demographic and in-depth information about students’ experiences with the method of instruction. ANCOVA and reliability analysis were used to analyze quantitative data while coding (Saldaña, 2013) was used to analyze qualitative data.

Quantitative analysis results using ANCOVA showed a statistically significant difference \( (p < 0.001) \) between the posttest mean score on the RFE of the treatment group and the mean posttest score of the comparison group. The ANCOVA results also showed a statistically significant difference \( (p = 0.004) \) between the ATMI mean posttest score of the treatment group and that of the comparison group.

Qualitative data analysis of the artifacts, interviews, researcher’s memos and the questionnaire by coding revealed three important themes describing the effects of modeling instruction on students’ types and cognitive complexity of representations of rational functions:
1) Students tend to have positive views of rational functions and display engaging and immersed attitudes towards learning mathematics in a modeling instructional setting, 2) teacher’s guidance during modeling instruction tend to help students’ mathematical representations of functions and real-world scenarios & 3) mathematical modeling instruction tend to foster critical thinking and conceptual understanding of rational functions, increasing students’ representations capabilities and cognitive complexities.

These results suggest that mathematical modeling instruction had positive effects on students' learning and understanding of rational function concepts, their attitudes towards learning rational functions and the cognitive complexity of their representations of functions.

INDEX WORDS: Keywords

DEDICATION

I thank the Lord Almighty for making it possible for me to get this level of education. This work is dedicated to my late father Augustine Betanga, my mother Rose Awungngia and my late uncle Christopher (Kitts) Mbeboh. This Ph.D. achievement would not have been realized without their support and encouragement. May their souls (dad and uncle Kitts) rest in perfect peace, Amen.

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<td>ATMI</td>
<td>Attitude towards Mathematics Inventory</td>
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<td>Analysis of covariance</td>
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CHAPTER 1

Introduction

Statement of the Problem

Research studies (Cangelosi et al., 2013; Yee & Lam, 2008; Nair, 2010; Datson, 2009; Bardini et al., 2014 etc.) indicate that student have a hard time dealing with rational functions. A rational function is a ratio of two polynomial functions. This means that both the numerator and denominator are polynomial functions, with the denominator different from zero. For example, if the function \( R(x) \) is a rational function, the \( R(x) = \frac{f(x)}{g(x)}, g(x) \neq 0 \), where \( f(x) \) and \( g(x) \) are polynomials functions. These are functions taught in precalculus classes to both mathematics and non-mathematics major students to prepare them for advance courses and careers.

Cangelosi et al. (2013) indicated that college students enrolled in college algebra and calculus have misconceptions and make errors with the concept of negative exponential expressions. Negative exponential expressions are rational functions which many students do not believe so. For example, \((3x + 5)^{-1}\) is a negative exponential expression which is a rational function of the form \( \frac{1}{3x+5}, 3x + 5 \neq 0 \). Yee and Lam (2008) reported that many pre-university students made many errors in the integration of rational functions which they attributed to students’ week algebraic skills. Nair (2010) points out that some high school and college students have an incomplete conception of rational functions, asymptotes, limits and continuity which often becomes a challenge for their understanding of other mathematical concepts. Nair also indicates that some students think that rational functions are rational numbers and some think that a rational function has a number in the denominator instead of a
variable. Datson (2009) showed that some students have misconceptions with the concepts of domain and zeros of rational functions.

Bardini et al. (2014) found that both high school and college students have misconceptions with mathematical concepts including the concept of a function which plays a vital role in the understanding of further mathematics sections including calculus and algebra. The Bardini et al. (2014) study found that many beginning undergraduate students master skills without any conceptual understanding. The study also showed that out of 383 student participants, only 62.8% of the students could define and give an appropriate description of a function, only 41.8% could tell whether a given graph or rule represented a function and up to 15% could not make the connection between function graphs and tables of values.

The 2015 report of the National Assessment of Educational Progress (NAEP) shows that only 37% of students scored at or above 163 on the NAEP mathematics scale (0 – 300), which is the indicator for college mathematics preparedness. The same report also indicates a decline in the average mathematics score of 12 graders compared to the results in 2013. The average mathematics score for 12 graders was 150 in 2015 compared to 152 in 2013. In the same report, the mathematics results for Black and Latino students was low. Only 7 percent of Blacks and 12 percent of Latinos scored at or above proficiency level. The 2017 NAEP report also show a similar trend in 2015 with no significant change in mathematics scores. Twelve graders and college students face enormous challenges in mathematics, especially when dealing with mathematical problem solving involving rational functions.

Another report in 2015 from the Program of the International Student Assessment (PISA) indicates that students from Singapore, China, Estonia, Hong Kong, Slovenia, Japan, Korea, Finland, New Zealand, Australia, Canada and Germany continue to outperform students
from United States. According to the 2015 PISA results, the United States scored 470 points in mathematics below the international average score of 490, with Singapore having the highest score of 564 points. This same report shows a decline in the average three-year trend score of 2 points for American students.

A third and the latest 2015 report of the Trends in International Mathematics and Science Study (TIMSS) is not so different from those of the PISA and the NAEP for the United States. The 2015 TIMSS results show East Asian countries (Singapore, Korea, Chinese Taipei, Hong Kong SAR and Japan) widening their mathematics achievement gap by 48 points ahead of the United States at the twelfth position. In fact, the Center for Education indicates that Globally, US is 21st and 26th in Science and Mathematics respectively.

Precalculus students need to have a firm grasp of important concepts of rational functions, from solving rational equations, rational inequalities, finding domains, asymptotes, to a full analysis of rational functions, to be successful in the course as well as subsequent mathematics courses including calculus.

Given these challenges faced by college students in mathematics and particularly rational functions, according to the Center of Education and Workforce and the National Science Foundation (NSF), there is a shortage of American students graduating from K-12, Colleges and Universities equipped with the skills to go into STEM careers such as Engineering, Medicine, Science, Technology that require them to think critically outside the box and collaborate to solve different societal problems.

There is therefore, need for student-centered instructional strategies in these institutions of learning such as mathematical modeling that could help reverse this negative trend on students’ mathematics achievement at the same time help them understand the world around
them using mathematics. According to (Blum, 2011), mathematical modeling is a translation between the real world and mathematics in both directions that “is meant to contribute to various mathematical competencies and appropriate attitudes towards mathematics and has the potential of helping students understand the world around them and have a true picture of mathematics” (p.19). Despite the positive impacts of mathematical modeling according, research on mathematical modeling with rational functions is limited or almost non-existence. This study will provide college, undergraduate students and teachers a research based instructional strategy (mathematical modeling) that they can employ in the teaching learning of rational functions, while adding to the existing literature in mathematics.

**Purpose and Rationale of the Study**

This study investigated effects of mathematical modeling instruction on Precalculus students’ performance and attitude toward rational functions. Specifically, the purpose of the study was to find out if there is a statistically significant difference in Precalculus students’ performance as measured by a score on a Rational Function Exam (RFE) between Precalculus students who received instruction through mathematical modeling and Precalculus students who received instruction through lecturing. A second purpose was to find out if there is a statistically significant difference in attitude toward rational functions between Precalculus students who received instruction through mathematical modeling and counterparts who received instruction through lecturing. Furthermore, the study explored the nature of the effect of mathematical modeling instruction on the types and cognitive complexity of representations used by Precalculus students on rational functions.

The rationale for this was to provide the students with a learning approach that focusses on critically thinking, interpreting and validation results, when presented with real world
scenarios or problems. Precalculus students are going into careers like engineering, nurses, medicine, science etc. where there will be presented with difficult and complicated situations such as those involving rational functions. Problems like these require conceptual understanding of the situation and their ability to rigorously and critically think through and solve these complicated problems. Such skills are acquired and developed through mathematical modeling instruction not the traditional lecturing instruction. Furthermore, the lack of any research on modeling with rational functions was a motivating factor for this study.

**Research Questions**

The purpose of this study was to investigate the effects of mathematical modeling instruction on Precalculus students’ performance and attitude toward rational functions. The following research questions will guide this investigation:

1. What is the effect of mathematical modeling instruction on Precalculus students’ performance as measured by a score on a Rational Function Exam (RFE) and attitudes toward rational functions?
2. What is the nature of the effect of mathematical modeling instruction on the types and cognitive complexity of representations used by Precalculus students on rational functions?

**Null and Alternative Hypotheses**

H$_{01}$: There is no statistically significant difference in Precalculus students’ performance as measured by a score on a rational function exam (RFE) between Precalculus students who receive instruction through mathematical modeling and Precalculus students who receive instruction through lecturing.
Hₐ: There is a statistically significant difference in Precalculus students' performance as measured by a score on a Rational Function Exam (RFE) between Precalculus students who receive instruction through mathematical modeling and Precalculus students who receive instruction through lecturing.

H₀₂: There is no statistically significant difference in attitude toward rational functions between Precalculus students who receive instruction through mathematical modeling and Precalculus students who receive instruction through lecturing.

Hₐ: There is a statistically significant difference in attitude toward rational functions between Precalculus students who receive instruction through mathematical modeling and Precalculus students who receive instruction through lecturing.

Definitions of Terms

Mathematical Modeling

Blum (2011) defines mathematical modeling as a translation between the real world (reality) and mathematics in both directions.

Mathematical Model

Blum (2011) defines a mathematical model as equations that result from the transformation of the real model through mathematization.

Performance

Performance in this study is the students’ score on a Rational Function Exam (RFE).

Attitudes

Gökyürek (2016) defines attitude as the positive or negative response of an individual toward a certain object, a situation or an event. He considers attitudes to be changeable and transferable, meaning that a positive attitude can be transformed to a negative attitude and vice versa. This study is adopting this definition.
Representations

Fennel (2006) defines representations as the process of using models (manipulative materials, graphs, diagrams, and symbols) to organize record and communicate mathematical ideas. This study will be adopting this definition.

Cognitive Complexity

Robinson (2001) defines cognitive complexity as “the processing demands of tasks and the availability of relevant knowledge” (p.28).

Theoretical Framework

This study is grounded in the Blum (2011) modeling cycle framework which is the educational or pedagogical perspective of mathematical modeling whose main idea is to integrate mathematical modeling into the teaching and learning of mathematics. According to Blum (2011), mathematical modelling is the translation between reality and mathematics and from mathematics back to reality. Blum believes that enormous mathematical knowledge as well as mathematical and modeling competencies is gained through this process. Much of Blum’s research work is focused on analyzing the cognitive aspects of students’ work when they are engaged in mathematical modeling.

The rationale for using the Blum (2011) framework in this study was the fact that it focused on students’ behavior or attitudes, their actions and their representation of the mathematical model from the situation model during the modeling process, which could further explain students’ achievement in modeling and mathematics. These are the variables that this study was out to investigate. Furthermore, the framework was broken down into smaller and simpler steps, thus making it easier to examine closely the behavior and thought (cognitive) processes of the students and teachers when they are engaged in solving problems through
mathematical modeling. It was equally a tool that facilitated a close examination of the different stages of the mathematical modeling process. Precisely, this framework facilitated the description, the interpretation and the explanation of what goes on in the minds of students and teachers during a modeling activity. According to Blum and Ferri (2009), the modeling cycle is very instrumental in the cognitive analysis of a modeling task. This modeling cycle was therefore helpful in designing the modeling activities for this study as I referred to different stages involved in the process.

![Mathematical modeling cycle](image)

*Figure 1. Mathematical modeling cycle. Adopted from Blum (2011, p.18).*

The Blum (2011) modeling cycle framework begins and ends with a real-world problem (situation problem), comprises of seven stages in the modeling process. It is based on the idea that mathematical knowledge is gained through a translation between the real world and mathematics and from mathematics to the real world. Blum illustrates this using ‘Giant’s shoes’ and the ‘filling up’ tasks. According to Blomhøj (2008), the role of the modeling cycle as an educational perspective is for “designing and analyzing tasks with respect to intensions for students’ learning” (p.11). It is also used for defining mathematical modeling competency as a
learning goal. Blum and others in this view, consider mathematical modeling as a means of learning and acquiring mathematical knowledge.

**Nature of Acquiring Mathematical Knowledge from the Framework**

The Blum (2011) framework is the educational or pedagogical view that considers mathematical modeling as a necessary tool to help students acquire mathematical knowledge. The framework begins with a mathematical task that the students are expected to understand and look for mathematical relationships that match the situation. As the students establish these mathematical connections at each step of the modeling cycle, mathematical knowledge is acquired. Deal (2015) also indicated that there is a connection between mathematical modeling and algebraic reasoning which occurs during the last five stages of the modeling cycle through mathematization. This framework was therefore employed as a tool to analyze and understand students’ knowledge or learning as they navigated through the different stages of the mathematical modeling process to solve real world problem situations. I will now describe the process of acquiring mathematical knowledge at the different stages of the modeling cycle.

Figure 1 above shows the steps involved in the modeling process which Blum refers to them as sub-competencies. The modeling cycle by Weiner Blum shows the relationship between the real-world and mathematics and vice versa.

The first step of the seven-step modeling cycle begins with the construction of the situation model from the real-world problem. The construction of this situation model according to Blum is a demonstration of the understanding of the context of the real-world problem statement. Imm and Lorber (2013) pointed out that understanding the problem context in the modeling process is crucial to connecting mathematical knowledge to the real-world knowledge. At this stage, the modular is trying to make sense of the problem situation. Deal
(2015), reported that Blum and Leiss (2007) considered the construction stage to be where the problem situation is represented in terms of pictures and diagrams to try to understand the problem. Ferri (2006) considered the situation model as “the mental representation of the situation (MRS) given in the problem because this best describes the internal processes (mental picture) of an individual after or while reading the complex modeling task” (p.87). According to Ferri, the most important phase in the modeling process as pointed out by Blum and Leiss is the situation model because everyone in the modeling process most go through it and because it is where understanding of the problem takes place as there is the transition between the real situation and the situation model. In terms of students’ modeling competencies at this stage, Blum and Greefrath (2016) indicate that the students at this level construct their own mental model from a given problem and thus formulate an understanding of the problem.

The second stage of the cycle deals with simplifying and structuring the situation model making it more accurate and precise, producing the real model of the situation. This is where the variables are defined, and the assumptions and relationships are made very clearly. Through simplification and restructuring, the modeling process begins to move from the real-world to the mathematical world, where mathematizing begins. In terms of students’ modeling competencies at this stage, Blum and Greefrath (2016) indicate that students at this level are identifying relevant and irrelevant information from a real problem.

The third stage of the modeling cycle is mathematization. According to Blum (2011), mathematization enables the transformation of the real model to a mathematical model made up of equations. All the relevant information of the real model (e.g. data, relations, concepts etc.) is isolated and put into mathematical statements at level. The mathematical operations in the real model are performed leading to the production of the mathematical model (equations).
According to Yilmaz and Dede (2016), mathematization competencies include identifying assumptions, identifying variables based on assumptions and constructing mathematical models based on the relationship among the identified variables. In terms of students’ mathematical believes, Blomhøj (2008) indicated that “during mathematization and interpretation, the students’ mathematical beliefs can be unveiled” (p.6). Students translate specific, simplified real situations into mathematical models (e.g., terms, equations, figures, diagrams, and functions (Blum & Greefrath, 2016).

The fourth stage of the modeling cycle (working mathematically) deals with solving the mathematical problem to obtain the mathematical results. Here, the necessary calculations are made to solve the equations (s). These mathematical results are then interpreted in the context of the real-world to produce real results during the fifth stage of the modeling process.

During the fifth stage, which is interpretation, Blum and Greefrath (2016) indicate that students relate results obtained from manipulation within the model to the real situation and thus obtain real results. This is an indicator of the students’ modeling competency at this level.

At the sixth stage, these real-world results are validated to see if they are consistent with the mathematical model. The Students according to Blum and Greefrath (2016) judge the real results obtained in terms of plausibility. Validating the model here means checking whether the model does what it is meant to do in the real world. If the real results are not valid, meaning if there are some limitations of the mathematical model, then there is some revision to the model resulting to a restart of the modeling cycle, where the modular takes a second look at the real-world problem statement, revise the assumptions and proceed to solving the problem. As the process continuous, if the mathematical results and real results are valid, then the seventh (last stage) of the modeling cycle is completed. This is where the modeling results are exposed or
published to others. Czocher (2017) indicated that this last step is also known as the communication stage in other theoretical models. At this exposing stage, the students relate the results obtained in the situational model to the real situation, and thus obtain an answer to the problem (Blum & Greefrath, 2016).

Finally, this framework provides some implications for teaching mathematical modeling, which was helpful in the design and the teaching of the lessons for this study. These lessons included encouraged students to work actively and independently in creating their own knowledge of the situation, while guiding them during the process when the need arises. Also, fostering and encouraging different meta-cognitive activities such as reflecting on their solutions.

**The Framework as a Lens into this Study**

The Blum (2011) modeling cycle is a pedagogical perspective of modeling which argues forcefully for the inclusion mathematical modeling in the teaching of mathematics. It is a conceptual framework with the purpose of developing students’ understanding of mathematical concepts as well as the modeling process (Greefrath & Vorhölter, 2016). According to Blum, the seven steps of the modeling cycle (constructing, simplifying, mathematizing, working mathematically, interpreting, validating and exposing) represent the steps the students will go through as they solve a mathematical modeling problem or task. In this study, the modeling cycle was used to analyze and understand students’ work at every stage of the modeling process. This framework afforded the opportunity to clearly see and describe what the students are doing, how they are thinking, their difficulties as the move from one step of the modeling process to the other. According to Czocher (2017), mathematical cycles allow a focus on cognition and a means
for understanding how to trace individuals ‘thinking even though other perspectives for studying the students’ mathematical learning during the mathematical modeling process do exists.

Leong (2012) indicated that modeling cycles can also be used as a tool for assessing modeling tasks. Haines and Crouch (2013) indicated that a modeling cycle provides an opportunity for researchers to describe students’ behavior within the modeling cycle, and by so doing, they can gain insight into the processes deployed by students when they are faced with real world problems. At every stage of the modeling cycle, it was possible to evaluate different modeling sub-competencies and hence the mathematical competencies of the students.

Significance of the Study

Theoretical Significance

Theoretical findings from this study could add to the literature of previously conducted studies in mathematical modeling (Blum & Niss, 1989; Niss, Blum & Huntly, 1991; Blum et al. 2002; Blum & Leiss, 2005; Blum & Leiss, 2007; Blum & Leiβ 2006; Blum & Leiβ 2007; Blum & Ferri, 2009; Blum, 2011; Nourallah & Farzad, 2012 etc.). Furthermore, the focus of this study on rational functions and modeling instruction, an area of limited or no research is of unique importance, particularly for teaching undergraduate algebra.

Practical Significance

Practically, this study could provide insight on students’ learning and the teachers’ ways of teaching rational functions. I argue that using mathematical modeling, students will be more engaged in learning meaningful connections between the real world and mathematics, instead of the usual lecturing approach to the learning of mathematics. Furthermore, mathematical modeling helps students to have a better understanding of the world, supports mathematical learning including motivation, concept formation, comprehension, retaining, promotes
appropriate attitudes towards mathematics and makes mathematics learning meaningful by revealing the true picture of mathematics to students (Blum, 2011). The study could give teachers a new approach (mathematical modeling) to teaching rational functions. As a teacher, this brings new perspective and strategy to the teaching of mathematics and an alternative approach to guiding students while maintaining a balance between their independence and guidance as they create their own knowledge.

Finally, this study could have societal, cultural and scientific benefits as well. Since mathematical modeling deals with real world situations, according to Blum (2002), the real world are things concerning nature, society or culture, including subjects at all levels, scholarly and scientific disciplines other than mathematics. Stacey (2015) points out that the use of the real-world context is an essential part of teaching mathematics for functional purposes and motivation of the students.
CHAPTER 2

Literature Review

The literature review is divided into four sections. The first part of the review will deal with mathematical modeling. Under mathematical modeling, mathematical models which are bi-products of the modeling process will be discussed followed by representations of these mathematical models. I will then follow closely with a discussion of the role of teachers in mathematical modeling. A distinction between mathematical modeling, lecturing and problem solving will be highlighted. Potential impacts/benefits of mathematical modeling on the teaching and learning of mathematics. A review of the gaps in the literature in mathematical modeling will then follow. The final section will be used to highlight and address the main methodologies from literature that have been used to study mathematical modeling.

Mathematical Modeling

Blum (2011) defines mathematical modeling as a process involving the translation between mathematics and the real world in both directions. Blum’s conception of the modeling process is cyclic (modeling cycle) and he believes that as the students go through the modeling process (transitioning between reality and mathematics) trying to resolve a mathematical task or activity, enormous mathematical knowledge is gained. Blum also believes that a particularly helpful tool for cognitive analysis of the modeling task is the modeling cycle (Blum, 2011; Blum & Leiß, 2007). He considers mathematical modeling as a means of teaching mathematics and he calls for effective ways of teaching mathematical modeling (Blum, 2009) which includes having a good modeling task, encouraging students to apply multiple problem-solving techniques, to have knowledge of multiple intervention strategies and adequately support the students in the modeling process.
Blum (2011) indicates that the mathematical modeling process (modeling cycle) involves seven steps to transition from the real world to the mathematical world and vice-versa.

The first step modeling process (modeling cycle) begins with the construction of the situation model from the real-world problem. The construction of this situation model is a demonstration of the understanding of the context of the real-world problem statement. At this stage, the modular is trying to make sense of the problem situation.

The second stage of the modeling process is the simplifying and structuring of the situation model making it more accurate and precise, produces real model of the situation. Mathematization is the third stage in the modeling cycle. Mathematization enables the translation of the real model to a mathematical model made up of equations (Blum, 2011).

The fourth stage of the modeling cycle (working mathematically) deals with solving the mathematical problem to obtain the mathematical results. The fifth step is interpretation of results. The sixth step is validating the results and the seventh step is to expose or publish the results if they are valid.

Studies on mathematical modeling show the existence of different versions of the modeling cycles by different authors depending on the details and the stages envisage by these authors (Blum & Niss, 1991; Blum & LeiB, 2007; Blum, 2011; Blomhøj, 2003).

Other definitions of mathematical modeling exist. According to Frejd (2011), many definitions exist in mathematical modeling depending on the modeling perspective adopted. Lesh et al. (2013) define mathematical modeling as a process of developing a purposeful mathematical description or interpretation of a problem-solving situation. Czocher (2017) used the quadruplet \{S, Q, M, R\} to define mathematical modeling as “a process of rendering a real-world problem, Q, as a mathematical problem that can be answered through the analysis of
those mathematical statements M. The process creates a relation R mapping the objects and relationships of the situation S to the mathematical entities M” (p.130). Meyer (2012) defines mathematical modeling as “an attempt to describe some parts of the real world in mathematical terms” (p. 1). Dundar et al. (2012) considered mathematical modeling to be the conversion of real-life situations to mathematical or the conversion from mathematics to real-life situations that are believable. Confrey and Maloney (2007) also consider mathematical modeling to be the process of bringing inquiry, reasoning and mathematical structures to transform and solve indeterminate problem situations, leading to the creation of mathematical models. Despite the existence these varied definitions, a common theme of mathematical modeling among them is the relationship between real-life and mathematics which can make a huge impact on students’ attitudes towards mathematics and ultimately their success in mathematics.

The literature in mathematical modeling further suggests the existence of different perspectives both in the national and international arena. The studies (Aztekin, & Şener, 2015; Blomhøj, 2008; Kaiser & Sriraman, 2006; Greefrath & Vorhölter, 2016) provide mathematical modeling perspectives which include 1) the realistic (pragmatic) and applied modeling perspective with a focus on solving real and authentic problems in industry and science, 2) the pedagogical (educational) modeling perspective which is process-related (modeling cycle) and its visualization, as well as content-related goals. Here, modeling is a vehicle for teaching of mathematics, 3) the socio-critical modeling perspective with the focus of critically examining the role of mathematics and mathematical models in society, 4) the cognitive modeling perspective which is focused on scientific goals trying to analyze and understand the cognitive procedures during modeling 5) the epistemological or theoretical modeling perspective which has theory-
oriented goals & 6) the contextual modeling perspective, which is subject-oriented with the central goal of solving word problems.

Additionally, Niss (2012) highlights the existence of two different views of mathematical models and modeling in the teaching and learning of mathematics: (1) The idea that mathematics is for applications, models and modeling and (2) the idea that the learning of mathematics is for applications, models and modeling. Erbas et al. (2014) echoed similar ideas about modeling in mathematical education, arguing for modeling as a purpose for teaching mathematics and the view that modeling is a means to teach mathematics. In the modeling as a purpose for teaching mathematics perspective, they argue that mathematical modeling is the basic competency or requirement and the reason for teaching mathematics to ensure that the students have the necessary tools to be able to solve real world problems in mathematics and other are fields of study.

Mathematical modeling is not without challenges for some students as they make connections between reality and mathematics (Blum, 2011). Blum says it so because of the cognitive demands of the modeling tasks since modeling has connections with other mathematical competencies such as reading, communicating, designing and applying problem-solving strategies.

**Mathematical Models and Representations**

Mathematical models are produced through the process of mathematical modeling. Blum (2011), indicates that a mathematical model is the outcome of mathematization which is the transformation of the real model into a mathematical model (made up of equations and variable). Meyer (2012) defines a model as “an object or concept that is used to represent something else. It is reality scaled down to a form we can comprehend” (p. 2). Meyer considers a mathematical
model as a model consisting of constants, variables, functions, equations, inequalities. Dym (2004) also considers a mathematical model as a mathematical representation of the behavior of real devices and objects.

A mathematical modular for a context is a person who introduces from scratch a mathematical model into that context (Niss, 2012). Niss says that, unlike mathematical modeling where mathematical models are created from scratch by a modular, application of mathematics occurs when a mathematical model is already present in a context created by someone else. A person who investigates or assesses such a model is called a model analyst (Niss, 2012). Li et al. (2004) are cited Meyer (1985) for highlighting six criteria to be used to evaluate the goodness of a mathematical model including: accuracy, correct assumptions, precision, robustness, generality and usefulness.

Representations of a mathematical model, which is a point of focus for this present study is crucial in students’ understanding of the problem situation. Bostic (2011) indicates that the representation of a mathematical model influences the procedure that is used to solve the problem, which further affects the derivations from the analysis of the mathematical model. Mathematical models, which are created through mathematical modeling, can have different representations, which may include graphs, equations and tables (Blum, 2011). Fennel (2006) also adds that models can be represented by manipulative materials, graphs, diagrams, and symbols. He considers representations as an important part of lesson planning for teachers. Furthermore, Ainsworth (2014) shows that multiple representations of functions by students have positive effects on students’ mathematics achievement. According to Ainsworth, learners can gain deeper understanding when they abstract over multiple representations to achieve insight into the nature of the representations and the domains.
The National Council of Teachers of Mathematics (NCTM. 2000) states, “Representation is central to the study of mathematics. Students can develop and deepen their understanding of mathematical concepts and relationships as they create, compare, and use various representations. Representations such as physical objects, drawings, charts, graphs and symbols also help students communicate their thinking” (p.280). The National Council of Teachers of Mathematics (NCTM) process standards for mathematics on representation recommends the use of representations to model and interpret physical, social and mathematical phenomena.

**Teacher’s Role in Mathematical Modeling**

For mathematical modeling and the modeling process to be successfully implemented in the classroom, teachers need to know what they are doing. According to Blum (2011), teachers are indispensable in students’ mathematics learning. Blum suggests the following principles for teachers who want to teach mathematical modeling: 1) The criteria for quality teaching should be considered when teaching modeling, teachers should find a permanent balance between students’ independence and their guidance by their flexibility and adaptive interventions, 2) teachers should use a broad tasks spectrum for teaching and assessments that cover different topics, context, competencies and cognitive levels, 3) teachers should support students’ individual modeling routes and encourage multiple solutions & 4) teachers ought to foster enough student strategies for solving modeling tasks and stimulate different meta-cognitive activities like reflection on solution processes and on similarities between different situations and contexts.

Mathematical modeling is relatively new to many teachers. As such, teachers need professional development to understand the modeling process. Gould (2013) found that many teachers have misconceptions of the mathematical modeling process and need guidance to help them understand the modeling process. If the teachers are not well grounded with the
mathematical modeling process, then the students will be completely lost. Temur (2012) indicated that prospective mathematics teachers had difficulties in teaching mathematical modeling because of lack of experience and training. Huson (2016) pointed out that teachers are key in implementing the standards, but resources to help them teach modeling are not well developed. Huson also found that teachers considered modeling to be engaging but had challenges at some steps of the modeling process especially at the early stage. Furthermore, Huson recommends more training and resources for teachers to help them understand how to implement all steps of the modeling cycle in their classrooms. Another study by Wolf (2013) explored teachers’ concerns with mathematical modeling in the common core standards and the results showed that teachers were willing to carry out mathematical modeling practices in their classrooms but had many concerns about time, material and adequate preparation with professional development. According to Hiltrimartin et al., (2018), many teachers do not understand that mathematical modeling should come from real world scenarios and requires making choices and assumptions.

**Mathematical Modeling, Lecturing and Problem Solving**

West (2013), indicates that while students in the traditional college algebra classrooms where lecturing is prevalent spend a good amount of time solving for the variables in equations and inequalities, finding zeros, x-intercepts and y-intercept, students in mathematical modeling classrooms, approach mathematics holistically with students spending time learning how to collect, analyze and apply data from real-life situations with the use of technology. According to West, mathematical modeling instruction is highly student-centered enabling the students to be engaged and active in the classroom, unlike students in the traditional settings (lecturing) who are very passive and do not play an active role in the classroom because the teacher is in control.
of all aspects of the learning. Also, in the traditional setting, problems are less rigorous and there is little or no collaboration among the students to solve problems. The students rely on the teacher for the structure and content of the course.

Furthermore, Smith (2013) points out that in the reformed classroom, multiple problem-solving techniques are used, and the teacher is more concerned with the most efficient way of solving problems, which is not the case in the traditional setting. Smith also indicated that unlike reformed instructional approaches, which make use of multiple representations such as tables, graphs, pictures, symbols and writing, there is frequent use of procedural algebraic techniques to solve problems in traditional-lecturing instructional classrooms.

For mathematical modeling to be well implemented in mathematics classrooms, mathematics teachers should distinguish between mathematical modeling and problem solving. Sole (2013) indicated that mathematics educators and curriculum developers have difficulties distinguishing between a textbook problem, mathematical modeling and problem-solving exercises. He highlighted six differences between mathematical modeling, problem solving and textbook in terms of how rigorous they are in modeling, essential and non-essential variables, number of approaches or techniques used to solve the problem which are wider in mathematical modeling than in problem solving, differences in mathematical model creation, context of the problem and validating results.

**Potential Impacts of Mathematical Modeling**

Blum (2011) indicates that there are potential benefits of mathematical modeling to students which include: (1) helping students to understand the world around them, (2) supporting mathematics learning (motivation, concept formation, comprehension and retaining), (3) contributing in developing different mathematical competencies and attitudes, (4) making
mathematics more meaningful and (5) enable students to have a complete picture of mathematics. Similarly, mathematical modeling has been shown to have positive impact on students’ attitudes towards mathematics (Wethall, 2011).

Nourallah and Farzad, (2012) show that mathematical modeling at university level has positive impacts on students’ problem-solving abilities. Similarly, Sokolowski (2015) used the meta-analytic technique to investigate the effects of mathematical modeling on students’ mathematical knowledge acquisition at the high school and college levels. The study results showed that modeling helps students with the understanding and application of mathematics.

Other studies (Mubeen et al., 2013; Mensah et al., 2013; Pawl et al., 2009; Prasad et al., 2014) indicate that students who are taught mathematics through mathematical modeling tend to have positive attitudes towards mathematics, hence positive outcomes on students’ mathematical achievement. According to Popham (2005), students’ attitudes toward a subject can lead to academic achievement. Teachers, knowledge about students’ attitudes toward a discipline that they teach is crucial because such information can assist them modify their instructional strategies to better reach the students.

Also, Saha (2014) says that to educate students, more emphasis should be placed on developing positive attitude and analytic thinking skills in solving mathematical problems rather than giving students ready-made problem-solving hints. Mensah et al. (2013) indicate that teachers’ positive attitudes, radiate confidence in students making them to develop positive attitude toward the learning of mathematics.

Furthermore, mathematics education currently emphasizes engaging students in mathematical modeling to understand problems of everyday life and society (Lesh & Zawojewski, 2007; Sharma, 2013; Vorhölter, Kaiser & Borromeo Ferri, 2014). Vorhölter et al.
(2014) highlight the fact that unlike what goes on in the traditional classrooms where students are learning mathematical concepts and procedures only to pass examinations and forget them after the exams are over, mathematical modeling will offer the students more than just passing the examinations by showing them how mathematics will be used in their daily lives. This strong support for mathematical modeling as an instructional method is gaining worldwide attention as evident by the participation of about 30 countries around the world including the top mathematics achieving countries including Singapore, China, Japan, Australia and Germany at the 2009 14th International Conference on the Teaching of Mathematical Modeling and Applications (ICTMA-14) in Germany (Kaiser, Blum, Ferri, & Stillman, 2011).

Dasher and Shahbari (2015) also indicate that engaging students in modeling activities helps them learn mathematics in a meaningful way. I believe that if rational functions are considered as mathematical models of real-life situations, which students can relate to, students may be motivated to learn and understand mathematical concepts. Kaiser and Schwarz (2006) indicate that “mathematics should deal with examples from which students understand the relevance of mathematics in everyday life, in the environment, in the sciences, and examples from which the students acquire the competencies to enable them to solve real mathematics problems, those of everyday life, the environment and the sciences” (p.196).

Papageorgiou (2009) points out that students engaged in mathematical modeling activities express positive views of the modeling process and are pleased that such activities are connected to real world unlike what they do in their traditional classes. Ellington (2005) show that modeling-based instruction has a positive effect on students. The results of Ellington’s study show that students have higher success rate, perform better in common exams, and do slightly better in a subsequent business and mathematics application course compared to the College
Algebra students in the traditional instructional setting. Niss (2012) highlights the fact that mathematical models and modeling are always needed either implicitly or explicitly whenever mathematics is applied to issues, problems, situations, and contexts in domains outside of mathematics. Czocher (2017) point out that when mathematical modeling principles are emphasized in traditionally taught differential equations course, there is a statistically significant effect on students’ learning.

Through mathematical modeling, mathematics is used to describe, predict, understand and prescribe the reality we live in (Blomhoj & Kjeldsen, 2007). Kertil and Gurel (2016) consider mathematical modeling as a bridge to the STEM education. They believe that mathematical modeling applications provide students with important local conceptual developments and meaningful learning of basic mathematical ideas in real situations. Modeling-based mathematics instruction has a positive impact on the students’ conception of the average rate of change and their first semester grade in the mathematics course (Doerr et al., 2014). Bahmaei (2013) indicates that mathematical modeling instruction has greater effect on students’ problem-solving abilities compared to that of students in the traditional classroom environment.

Wedelin and Adawi (2014) show that a good number of students who take mathematical modeling courses show impressive changes in their abilities to think mathematically and they also express satisfaction with the mathematical modeling course, noting that mathematical modeling is an important course in education.

Though mathematical modeling may have positive impacts on the teaching and learning of mathematics, Freeman (2014) showed that students faced challenges when resolving mathematical modeling problems because they did not have model development competencies. Similarly, Blum (2011) highlighted the fact that students around the world have difficulties
with modeling tasks as shown by the PISA reports, due to the cognitive complexities of the modeling tasks.

**Gaps in the Literature**

Gaps in the literature on mathematical modeling exist in content, methodology, strategies and frameworks. In the content area, research on rational functions, rational function models and modeling as well as the teaching and learning of rational functions is very limited as compared to research on other function models such as linear, polynomial, exponential and logarithmic models. Furthermore, research in mathematical modeling is heavily focused in the development and understanding of scientific, engineering, medical and technological models of some real-world phenomena (Diekmann et al., 2013; León et al., 2008; Magnus et al., 2013; Richard et al., 2014), but not much is invested towards studying students’ performance or achievement in mathematics at the undergraduate level.

Because of this heavy focus on scientific models, it also creates a gap in the theoretical framework as well. Such research studies therefore approach mathematical modeling through the lens of the realistic (pragmatic) and applied modeling perspective with a focus on solving real and authentic problems in industry and science (Kaiser, 2005; Pollak, 1968; Kaiser & Schwarz, 2006). There is therefore limited research in the pedagogical (educational) modeling perspective with the focus on process-related (modeling cycle) and content-related goals (Blum, 2011). The mathematical modeling methodologies for studying the mathematical content are therefore limited to a few qualitative and quantitative methods and some case studies. Tao and Hu (2001) point out that there are few publications on theoretical properties and practical aspects of rational function models. Freeman (2014) highlights the fact that there are very few research studies on the effects of mathematical modeling on community college mathematics courses. He however
points out the existence of research on the value and efficacy of mathematical modeling in elementary, secondary and some undergraduate courses. He equally notes the absence of research on issues related to mathematical modeling in college mathematics courses such as the modeling process challenges, effective modeling activities, assessment of mathematical modeling and the students’ perception of modeling as well as their behavior towards modeling.

**Common Methodologies in the Literature**

A review of the literature in mathematical modeling reveal a growing list of researchers have used mixed methodologies involving both quantitative and qualitative methods for the data collection, data analyses (Coacher, 2017; Freeman, 2014). Some researchers however have used purely quantitative methods or purely qualitative methods. Doerr et al. (2014) used a quasi-experimental methodology in their study. Ellington (2005), used purely quantitative methods to investigate the effects of a modeling-based college algebra course on students’ achievement. Dedrick et al. (2009) indicate in a methodological literature review of 99 articles in 13 peer review journals that most studies are non-experimental and used non-probabilistic samples. Their review also indicate that many studies do not report enough information for the readers to be able to critique the reported analysis.

Aztekin and Şener (2015) employed two content analysis techniques as methodology for their study. Celik (2017) examined mathematical modeling studies done in Turkey between 2004 and 2015 and results indicated that most of the studies were qualitative with predominantly purposeful sampling methods used to collect the data. The research design for this study used content analysis technique. Sokolowski (2015) used the meta-analytic technique to investigate the effects of mathematical modeling on students’ mathematical
knowledge acquisition at the high school and college levels. The study results showed that modeling help students with the understanding and application of the mathematical concepts.

Prasad and Rao (2014) used a one-way ANOVA to investigate the differences between positive and negative attitudes toward mathematics for 573 secondary school students. They found that there were significant differences between them. Their conclusion was that students want to understand mathematics, but a lack of understanding makes students to have negative attitudes towards mathematics.

Wilkins and Ma (2003) used hierarchical linear modeling methods to model variations in students’ rate of change with variables associated with students’ characteristics, instructional experiences, the environment, variables that affect change at different levels of secondary schools and variables for the different affective domains (attitudes and beliefs about mathematics).

**Summary of the Literature Review**

This study investigated the effects of mathematical modeling as instructional strategy on Precalculus students’ achievement, representations and attitudes towards rational functions. The declining trend in the mathematics achievement of American students as indicated by the TIMSS, PISA and NEAP reports and other research studies compared to other countries (Singapore, Finland, Germany, China, Korea), calls for student - centered instructional methods including mathematical modeling. Mathematical modeling has been shown to have some impact on students’ mathematics’ achievement (Mubeen et al., 2013; Pawl et al., 2009).

Despite the contributions of these studies to the literature on mathematical modeling, many of them have been focused on other functions like linear, quadratic, exponential functions, with little or no attention directed towards rational functions. Also, many
mathematical modeling studies have been concentrated at the elementary and secondary levels with very few on college and undergraduate level mathematics. Furthermore, studies on modeling have largely focused on the pragmatic perspective of mathematical modeling (Kaiser & Schwarz, 2006), whose goal is to solve real world problems and build mathematical models for science and engineering purposes. Very few studies have focused on the pedagogical perspective (Blum, 2011) of modeling that is considered the student’s vehicle for learning and understanding mathematics. Gaps have therefore, been created in the literature on mathematical modeling in terms of the content, methodology, strategies and frameworks (Tao & Hu, 2001; Freeman, 2014). This study seeks to bridge these gaps in the literature, while contributing to the already existing one in mathematics and mathematical modeling.
CHAPTER 3

Methodology

In this chapter, I present a route map of how the study was carried out. This include (a) the research design, (b) the research setting, (c) the participants and sampling techniques, (d) the data collection techniques (quantitative and qualitative), (e) the procedure used, (f) the data analysis techniques (quantitative and qualitative), (g) the data management plan (h) the researcher’s role in the study, (I) the limitations and finally (j) a summary of the methodology.

The purpose of this study was to investigate the effects of mathematical modeling instruction on Precalculus students’ performance and attitude toward rational functions. The following research questions guided the investigation:

1. What is the effect of mathematical modeling instruction on Precalculus students’ performance as measured by a score on a Rational Function Exam (RFE) and attitudes toward rational functions?

2. What is the nature of the effect of mathematical modeling instruction on the types and cognitive complexity of representations used by Precalculus students on rational functions?

Research Design

An exploratory embedded single case study design with both quantitative and qualitative methods was employed. According to Yin (2014), a case study is “an empirical inquiry that investigates a contemporary phenomenon in depth and within its real-life context especially when the boundaries between the phenomenon and context are not evident” (p.16). According to Yin (2014), a single case study is the best choice when studying just a single group such as a group of people. The single case here is a group of precalculus students. He distinguishes a case
study from an experiment by pointing out that an experiment intentionally separates a phenomenon from its context, making it possible to only work with a few variables. Yin (2014) describes a case study as covering contextual conditions that are believed to be relevant to the phenomenon being studied. This study is thus in line with Yin’s view of a case study in the sense that it was an in-depth investigation of a contemporary issue in this case, the effects of mathematical modeling instruction on Precalculus students within a given real-life context.

The rationale for this case study was, therefore, in line with conditions outlined by Yin (2014) for using a case study, which include the nature of research questions, the extent of researcher’s role and the extent to which the study is concerned about contemporary issues. This case study was a single case with embedded units of analysis. Yin further indicates that single case involves intensive data collection at the same site by a team of investigators. To him, a single case is analogous to a single experiment. He points out five rationales for a single case study: (a) when it represents the critical case in testing a well-formulated theory, b) where the case is an extreme or unique case (c) when the case is representative or typical with the objective of capturing the circumstances and conditions of an everyday situation d) when the case is revelatory and (e) when the case is longitudinal (studying the same single case at two different points in time).

According to Baxter and Jack (2008), Yin puts case studies into three categories: explanatory, exploratory and descriptive. He considers a case study to be exploratory when it is used to explore situations in which the intervention being evaluated has no clear, single set of outcomes. The intervention used in this study is mathematical modeling instruction which does not have a clear single set of outcomes. Also, the study was exploratory based on the research question guiding this investigation (Yin, 2014). This study employed quantitative and qualitative
techniques. Yin highlights the fact that a case study enables the researcher to gather data from multiple sources to support the research thesis to guard against construct validity. Data sources for this study were interviews, the researcher’s memos, a questionnaire, artifacts of students’ work on the pretest and posttest and a pretest posttest RFE and ATMI survey. According to Hancock and Algozzine (2015), multiple methods are often used when doing a case study research. To them, the relationship between the design and the method is fundamental to conducting a successful investigation.

![Figure 2. Research design-Two groups, random assignment, Pre-test, Post-test.](image)

**Research Setting**

This study was carried out at a local college in one major southern city of the United States. The typical student population at this college is diverse with majority white. On average, Black students are second to Whites in terms of population, followed by Hispanics. The least student population is the Asian. It is a four-year college institution with students graduating with bachelor’s degrees and associate degrees in both the School of Arts and the School of Science as well as professional degree (e.g. nursing) and has a teacher certification program in the School of Education. The college offers courses in many major disciplines including Biology, Chemistry, Mathematics, Education, Physics, Engineering, and Registered Nursing. Many students planning
to take up careers in the nursing, engineering, Biology, business fields are required to register and obtain at least a ‘C’ grade in Precalculus. The college offers weekend and online classes. The graduation rate for minority students is low compared to their White counterparts. About 3 in 5 students here use financial aids to cover their tuition and other school expenses. There is one main campus with other associated campuses at different locations in the state. The school participates in different sporting events and competitions around the state and beyond.

**Participants and Sampling Techniques**

The study sample included 54 students enrolled into two precalculus sections based on their availability (See Table1). These two precalculus sections (24 students in the treatment and 30 students in the comparison) out of five sections were chosen after consultation with the classroom teachers ensure their readiness teach these two sections using the two instructional methods. Selection of the sample was therefore accomplished using purposeful sampling technique in which the classes were selected based on whether the teachers of these classes were willing and available to implement mathematical modeling instruction and the traditional instruction in their classes.

Assignment of precalculus sections into treatment and comparison groups was random even though the students in each section were non-randomly placed in the groups depending on their availability during the semester. Demographic information in Table 1 below from the students about their gender, ethnicity and their major (STEM and non-STEM) was obtained using a questionnaire. Participation in this study was entirely voluntary and the students’ consent was sought to participate. The teachers who taught the two groups were contacted prior to the start of the study. After they agreed to participate, the first meeting with the teachers and I was held to discuss the modalities for the study and after that I met with the teachers individually
once a week to discuss the implementation of the instructional methods in their respective classes.

Table 1

**Demographic Data**

<table>
<thead>
<tr>
<th>Group</th>
<th>Male</th>
<th>Female</th>
<th>White</th>
<th>Black</th>
<th>Asian</th>
<th>Hispanic</th>
<th>Other</th>
<th>Stem</th>
<th>Non-Stem</th>
</tr>
</thead>
<tbody>
<tr>
<td>Comp</td>
<td>11</td>
<td>19</td>
<td>17</td>
<td>8</td>
<td>0</td>
<td>2</td>
<td>3</td>
<td>18</td>
<td>12</td>
</tr>
<tr>
<td>Treat</td>
<td>12</td>
<td>12</td>
<td>11</td>
<td>6</td>
<td>0</td>
<td>2</td>
<td>5</td>
<td>16</td>
<td>8</td>
</tr>
<tr>
<td>Total</td>
<td>23</td>
<td>31</td>
<td>28</td>
<td>14</td>
<td>0</td>
<td>4</td>
<td>8</td>
<td>34</td>
<td>20</td>
</tr>
</tbody>
</table>

*Note. N= 54; Comp =Comparison; Treat = Treatment*

Table 1 shows that, forty three percent \(n=23\) were males fifty seven percent \(n=31\) were females. The number of white participants was twice that of blacks. Whites were 52%, blacks 26% and Hispanics made up 7%. Fifteen percent of participants identified themselves as other (mixed race, Caucasians etc.). There were no Asians. Thirty-four participants (63%) were STEM majors and twenty (37%) were non-STEM majors. Sixteen (47%) of the STEM participants were in the treatment group and eighteen (53%) in the comparison group.

Participants in both the treatment class and the comparison class completed the same pre-test and a post-test on Rational Functions (RFE), and the Attitude Toward Mathematics Inventory (ATMI) survey and a questionnaire. Artifacts of student work on the pretest and posttest, was collected from both the treatment and groups the comparison groups.

**Data Collection Techniques**

Both quantitative and qualitative data techniques (instruments) were used for data collection. Two quantitative (pretest-posttest on RFE and pretest-posttest on ATMI) and three
qualitative instruments (questionnaire and artifacts and interviews) were used for data collection. Quantitative techniques involved the use of a pretest- posttest covering important concepts of rational functions at the beginning and at the end of the course.

Qualitative techniques were interviews, artifacts (students’ work sheets) on the pre-posttests and a questionnaire. The questionnaire was used to collect demographic information as well as their experiences with rational functions before and after their participation in this study. Interviews were used to collect more detailed and in-depth information about the students’ thoughts and experiences with rational functions. Interviews were also used to follow up on students’ responses on the questionnaires and their performance on the quantitative posttests.

Robinson (2016) cites Yin (2014) for providing four major principles for data collection which are a) using multiple sources of data, b) creating a data management plan, c) maintaining a chain of evidence and d) ensuring that the data is safe. Yin (2014) also indicates that a case study can be both quantitative where data is numeric and qualitative where data is non-numeric.

Quantitative instruments.

Pretest-posttest. Quantitative data was collected from the pretest, posttest items on the RFE and the ATMI (Tapia & Marsh, 2004) survey after the students were exposed to mathematical modeling instruction on rational functions. There were 12 test items on the RFE, covering specific content areas of representations, equations, inequalities, domain and range, zeros, asymptotes, and context driven problems as shown on Table 2 below.

Attitude towards mathematics inventory (ATMI) Likert scale survey (Tapia & Marsh, 2004). The survey is made up of 40-item Likert scale with four subscales (self-confidence, value, enjoyment and motivation. A confirmatory factor analysis of the ATMI (Majeed et al. 2013) showed that this scale has a high reliability Cronbach’s alpha of 0.963.
Students in both the treatment and control groups completed the ATMI survey (Tapia & Marsh, 2004) before and after the study, to determine the effect (if any) of the intervention (mathematical modeling instruction) on students’ attitudes towards mathematics, hence rational functions.

Table 2

*Rational Function Concepts on the RFE and Number of Items per Concept*

<table>
<thead>
<tr>
<th>Concept</th>
<th>Objective</th>
<th>Number of items</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Rational function models</td>
<td>Students should be able to create rational function models (graphs, tables,</td>
<td>4</td>
</tr>
<tr>
<td>(representations)</td>
<td>equations) of real-world phenomena. They should be able to transform from</td>
<td></td>
</tr>
<tr>
<td></td>
<td>one model or representation to another</td>
<td></td>
</tr>
<tr>
<td>2. Rational equations</td>
<td>Students should be able to solve rational function equations.</td>
<td>1</td>
</tr>
<tr>
<td>3. Rational Inequalities</td>
<td>Students should be able to solve rational function inequalities</td>
<td>1</td>
</tr>
<tr>
<td>4. Rational function operations</td>
<td>Students should be able to find the sum, difference, product and quotient</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>of rational functions</td>
<td></td>
</tr>
<tr>
<td>5. Domain and range of rational</td>
<td>Students should be able to find domain and range of rational functions</td>
<td>1</td>
</tr>
<tr>
<td>functions</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6. Zeros of Rational functions</td>
<td>Students should be able to find the zeros of rational functions</td>
<td>1</td>
</tr>
<tr>
<td>7. Asymptotes</td>
<td>Students should be able to find the vertical and horizontal asymptotes of</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>rational functions</td>
<td></td>
</tr>
<tr>
<td>8. Context driven problem</td>
<td>Students should be able to solve context driven problems</td>
<td>1</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>12</td>
</tr>
</tbody>
</table>
**Qualitative instruments.** To provide answers to the research question two about the types and level of cognitive complexity of precalculus students’ representation of rational functions, data from interviews, students’ artifacts, questionnaire and researcher’s memos were collected and qualitatively analyzed using coding through a web-based application Dedoose.

**Questionnaire.** To gather information on students’ experiences and thoughts with the instructional method used. Questionnaire were given at the end of the study.

**Artifacts.** To explore students’ representations (written work, tables, graphs and equations) of functions and mathematical ideas and to make sense of the quantitative findings, a thorough review of the students’ solutions on the pre-post RFE tests were examined.

**Interviews.** To gather more detailed in-depth information about their experiences with mathematical modeling and rational functions and as a follow up to students’ responses on the questionnaires, 4 students (2 from the treatment and 2 from the comparison group) were interviewed based on their pretests and posttests scores. The interviews were conducted outside the regular class time based in the participants’ availability.

**Researcher’s Memos.** Although formal observation protocols of instruction were not put in place, I did however make informal visits to the teachers’ classrooms once a week during which memos about the instruction were taken. These memos were about the student-teacher interactions, students’ engagement and behavior, teaching strategy, the type of problems and examples given to students to work on during class. The memos were used for discussions with the teachers during our meetings to ensure proper
implementation of instructional strategy. These memos also gave me a true picture of what was going in the two classrooms.

Attrition Rate

According to the What Works Clearinghouse (WWC), an initiative of the U.S. Department of Education’s Institute of Education Sciences, attrition is the loss of sample during a study for a variety of reasons. Similarly, Amico (2009) considers attrition as the loss of randomly assigned participants’ data which can introduce bias in the external validity. Thus, the lower the attrition rate the lower the threat to external validity of a study. Fifty-seven students consented to participate in this study and took the pretest, but three students did not take the RFE and ATMI posttests (2 students from the treatment and 1 student from the comparison group). The three students had withdrawn from the course before the posttests were administered. This resulted in an attrition rate of 5.3% and a 0% differential attrition rate. Six students (3 from each group) selected to be interviewed based on their test scores. Four of the six students volunteered to be interviewed. The other two students (one from each group) were absent on scheduled dates for the interviews resulting to an overall attrition rate of 33.3% and a 0% differential attrition rate for interviews. According to Lewis (2013), overall attrition is the combined attrition rate in the treatment and the control groups while differential attrition is the difference between the attrition rate in the treatment group and that in the control group. The WWC standards for overall attrition rates of below 40% and differential attrition rates below 2% are acceptable levels of bias under both the liberal and conservative assumptions.

Procedure

The study was conducted over a period of 5 weeks. Before collecting data, I obtained an approval from the college by submitting a research request application detailing my research
proposal to the office in charge of research at the college. An Institutional Review Board (IRB) approval from Georgia State University was also obtained before beginning data collection for this study. Once the approvals to conduct the study were obtained, I began to contact teachers to see those who were willing and prepared to teach the two precalculus sections (treatment and comparison groups). After securing the teachers’ participation, I began meetings with them to brief them on the purpose of the study and the how the study was to be carried out. I continuously met with the teachers individually once a week until the end of the study. First, I visited their classes to recruit the students and ask for their consent to be part of the study since participation was entirely voluntary. One precalculus section ($n=24$ students) was randomly assigned to the treatment group while the other section ($n=30$ students) was used as the comparison group.

To ensure proper implementation of the instructional methods, I met with the teachers to discuss the procedures and agreed upon prior to the start of the study. My meetings with the teacher of the treatment group were focused on incorporate mathematical modeling strategies in the classroom and providing extra resources, including different problem types and project activities that align with the content of the course syllabus to implement in teaching, using modelling techniques. Two 30-45 minutes training and discussion sessions were conducted with the teacher of the treatment group during the first week of the study to ensure proper implementation of the intervention.

Both groups took the same, pre-post RFE and ATMI survey before and at the end of the study (see Appendix A for RFE items). The data collected from the pretests and posttests was quantitatively analyzed using a statistical software ANCOVA while coding was used to analyzed qualitative data from the interviews, questionnaires, researchers’ memos and artifacts of
students’ work. The duration of the study was five weeks, starting with the recruitment of participants, meeting with instructors, data collection and data analysis.

Table 3 below provides a list of content covered during the intervention and a weekly timeline of implementation.

Table 3

*Unit Objectives, Related Activity and Timeline*

<table>
<thead>
<tr>
<th>Concept</th>
<th>Objective</th>
<th>Timeline</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Rational function models</td>
<td>Students should be able to create rational function models (graphs, tables, equations) of real-world phenomena. They should be able to translate from one models to another</td>
<td>Week 1</td>
</tr>
<tr>
<td>2. Rational Functions operations</td>
<td>Students should be able to find the sum, difference, product and quotient of rational functions.</td>
<td>Week 2</td>
</tr>
<tr>
<td>3. Rational Function equation and inequalities</td>
<td>Students should be able to solve rational function inequalities and equations</td>
<td>Week 3</td>
</tr>
<tr>
<td>4. Domain, range, zeros, horizontal and vertical asymptotes of rational functions</td>
<td>Students should be able to find domain and range, vertical and horizontal asymptotes</td>
<td>Week 4</td>
</tr>
<tr>
<td>5. Review, posttests interviews and questionnaire</td>
<td>Students review, posttests, questionnaire and interviews</td>
<td>Week 5</td>
</tr>
</tbody>
</table>

A questionnaire was given before and after the study to collect demographic information and to understand how the students felt after learning rational functions through the given instructional method (mathematical modeling or lecturing).
Four students (2 from the treatment group and 2 from the comparison group) were interviewed to gather more information about their experiences with mathematical modeling and rational functions and to follow up on their questionnaire responses. Table 4 below shows the data design techniques, data collection instruments and the data analysis techniques for this study.

Table 4

*Data Collection Procedure*

<table>
<thead>
<tr>
<th>Research questions</th>
<th>Design technique</th>
<th>Data collection instruments</th>
<th>Data analysis technique</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) What is the effect of mathematical modeling instruction on Precalculus students’ performance as measured by a score on a Rational Function Exam (RFE) and attitudes toward rational functions?</td>
<td>Quantitative</td>
<td>RFE- Pre/post</td>
<td>ANCOVA</td>
</tr>
<tr>
<td>2) What is the nature of the effect of mathematical modeling instruction on the types and cognitive complexity of representations used by Precalculus students on rational functions?</td>
<td>Qualitative</td>
<td>Artifacts</td>
<td>Coding protocol by Saldaña (2013)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Interviews</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Questionnaire</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Researcher’s memos</td>
<td></td>
</tr>
</tbody>
</table>

**Differences between the treatment and comparison groups.** In the comparison class, students \((n = 30)\) received regular instruction on rational functions which was done through lecturing, where the teacher was in control of the class explaining rational function concepts to students on the board. Context was not the central focus. As noted in my memos, there were few student-teacher interactions as well as student-student interactions. The teacher regularly put
notes on the board while the students copy from the board. Very few students asked questions and during my visits, I did not observe students solving problems on the board. There was less collaboration, little or no discussions and reflection on solutions. The teacher gave a step by step approach to solving problems, followed by examples on the board for the students to copy. Many examples of problems assigned both on the board and from the textbook here involve direct use of the formulas and were mostly computational in nature. There were neither real-world application problems nor thought-provoking type problems.

Students in the treatment group \( (n = 24) \) were taught rational functions using mathematical modeling instructional strategies where mathematical modeling principles were emphasized. Rational functions were contextualized in this class as my memos indicate. For example, rational functions were viewed as models of some real-world phenomena and real-life situation problems were transformed into mathematical models to help solve the problem situation. The teacher approached brought in a more holistic approach to solve problems on rational functions. This involved making connections with the real-world, mathematics and other subjects, reflecting on their solutions, collaborating and socially interacting with other students in the group to make sense of the problem situations. The work load for both groups was the same. However, the differences between the groups were the instructional methods, teacher prior teaching experiences, the class meeting time, the number of students in each group, number of meeting sessions per week.

The instructors in both classrooms were veteran teachers of mathematics with over 20 years of teaching. The treatment group instructor is an assistant professor of mathematics with a Master of Science degree in Actuarial Science. She has 10 years of teaching high school mathematics and 13 years of teaching undergraduate mathematics including mathematical
modeling. The comparison group instructor is an associate professor of mathematics with a Ph.D. in mathematics with over 30 years of both undergraduate and graduate levels teaching of mathematics. Table 5 below summarizes the differences between the two groups for this study.

Table 5

*Differences between the Treatment and the Comparison Groups*

<table>
<thead>
<tr>
<th>Group</th>
<th>Instructional method</th>
<th>Teacher’s teaching experiences</th>
<th>Number of students</th>
<th>Class meeting time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treatment</td>
<td>Mathematical modeling involving real-word problems, reflection, validation of solutions, collaboration and making connections between subjects, the real-world and mathematics. Brainstorming of ideas</td>
<td>M.Sc. Actuarial Science, 10 years of high school and 13 years of undergraduate mathematics and mathematical modeling teaching</td>
<td>24</td>
<td>Class met twice per week on Tuesdays and Thursdays for a total of 3 hours and 20 minutes, from 12:30pm to 2:10pm</td>
</tr>
<tr>
<td>Comparison</td>
<td>Traditional lecturing Students listen and copy notes from the teacher, Little or no collaboration and brainstorming of ideas</td>
<td>Ph.D. Mathematics, 30+ years of undergraduate and graduate mathematics teaching</td>
<td>30</td>
<td>Class met three times per week on Mondays, Wednesdays and Fridays for a total of 3 hours and 30 minutes, from 8:00am to 9:10am</td>
</tr>
</tbody>
</table>

Before and after the two groups were taught using the different instructional methods, all the students completed an ATMI Likert scale attitude survey.

A questionnaire was given before and after the study to collect demographic information and to understand how the students felt after learning rational functions through the given instructional method (mathematical modeling or lecturing).
Four students (2 from the treatment group and 2 from the comparison group) were interviewed to gather more information about their experiences with mathematical modeling and rational functions and to follow up on their questionnaire responses.

**Fidelity of Implementation.**

Triangulation with self-reporting and assistance was used to ensure fidelity of implementation of the procedures. To ensure that the intervention (mathematical modeling instruction) was implemented correctly as a method of instruction for rational functions throughout the study, I visited the teacher’s classroom once a week to observe the instruction. The teacher also self-reported what went on in the classroom after every lesson and during our regular meetings. Students’ questionnaire and interview responses related to the instructional method were also used to assess fidelity of implementation of the instruction.

**Data Management**

All the data (quantitative and qualitative) collected for this study either digital or paper was stored in secured locations (keyed and locked cabinets). The data collected was coded to ensure non-identification of participants. Data from the pretest and the post-test was collected in the regular classroom stored in a lock and keyed cabinet by the classroom teacher. All Artifacts of students’ work on the pretest post-test were collected in the regular classroom stored in a lock and keyed cabinet by the classroom teacher. Questionnaire responses were collected in the regular classroom stored in a lock and keyed cabinet by the classroom teacher.

Both the primary investigator and I had access and transportation of the information. Institutions that ensured that this study was correctly carried also had access to your information. They are the GSU Institutional Review Board, the Office of Human Research Protection (OHRP) and the University Research Services and Administration (URSA) office at GSU.
Data Analysis

**Unit of analysis.** In this study, my unit of analysis (case) is precalculus students. Yin (2009) considers the unit of analysis in a study to be the case and that it is related to the way the initial research question(s) is defined. It is what the researcher is trying to analyze in a study, which could be an individual, a process, a program or even differences between organizations.

**Quantitative data analysis.** To provide answers to the research question one about the effect of mathematical modeling instruction on Precalculus students’ performance, data from the pre-posttest RFE and the ATMI was collected and analyzed using a one-way ANCOVA. ANCOVA is a statistical technique used to test the main and interactive effects of categorical independent variables on a continuous dependent variable, while controlling the effects of a continuous confounding variable called the covariate. In this study the independent variable was the instructional method (modeling and traditional) while the dependent variable was the posttest. The pretest was the covariate that is being controlled. ANCOVA was appropriate for this study due to the presence of the covariate pretest whose effect on the dependent variable could be controlled by ANCOVA to increase the power of the results. By controlling the effects of the pretests, this helped to put students in both the groups on the same ability before the intervention. ANCOVA was also used to compare means of posttest scores on ATMI while Cronbach’s was used to measure the internal consistency of the RFE and the ATMI instruments.

**Qualitative data analysis.**

**Coding.** According to Saldaña (2013), “coding is just one way of analyzing qualitative data and not the only way” (p.2). The data can be interview transcripts; field notes observations, journals, artifacts, email correspondences, photographs, videos etc. He points out that “a code in qualitative inquiry is most often a word or short phrase that symbolically assigns
a summative, salient, essence-capturing, and/or evocative attribute for a portion of language-based or visual data” (Saldaña 2013, p.3). Coding and recoding was achieved through Dedoose’s 2017 web-based application.

Saldaña (2013) also indicated that “qualitative codes are essence-capturing and essential elements of research, that, when clustered together according to similarity and regularity, they actively facilitate the development of categories and thus analysis of their connections” (p. 8). The data was coded to identify the different categories, concepts and themes. According to Saldaña, a theme is an outcome of coding, categorization or analytic reflection and coding is a cyclical process. Saldaña explains that, the first cycle is rarely perfect. The second, third, fourth cycles etc. of recoding has the responsibilities to further manage, filter, highlight and focus the salient features of qualitative data record for generating categories, themes, concepts, meaning and building theory.

Qualitative data analysis with exploratory techniques was used to analyze the data collected. The qualitative question of this study was answered by collecting and analyzing data from artifacts (students’ work sheets), students’ interviews, the questionnaire and the researcher’s memos. Artifacts of students’ solutions on the RFE helped to identify students’ representations of rational functions. The representations of rational function models (graphs, tables, equations and written) of students from each group were analyzed by looking at the quality of students’ work based on the strategy and accuracy used to translate from one representation of the rational function to the other.

To codify according to Saldaña (2013), is to apply and reapply codes to qualitative data. He further distinguishes codes and themes by arguing against the recommendations of some researchers that one should initially code for themes. He says that a theme is not something that
is coded, but an outcome of coding, categorization or analytic reflection. The number of codes one generates depends on many contextual factors including the nature of one’s data, the coding method as well as how detailed one wants to be. The coding scheme in Table 6 below shows how students’ representations were coded.

Table 6

_Coding Protocol for Students’ Representations_

<table>
<thead>
<tr>
<th>Representation</th>
<th>Category</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Table</td>
<td>A</td>
<td>Any chart or table that is used to organize data</td>
</tr>
<tr>
<td>2. written</td>
<td>B</td>
<td>Any word or phrase used to represent any thought, numbers or mathematical idea</td>
</tr>
<tr>
<td>3. Equation</td>
<td>C</td>
<td>Any expression that consist of numbers or symbol or both</td>
</tr>
<tr>
<td>4. Graph</td>
<td>D</td>
<td>A pictorial representation numbers, value or real-world scenario</td>
</tr>
<tr>
<td>5. Mixed</td>
<td>E</td>
<td>Use of table, graph and equation and verbal</td>
</tr>
</tbody>
</table>

**Validity and Reliability**

**Internal Validity.** This is a measure of the soundness of the research. Lewis (2013) cited Shadish et al. (2002) for indicating that a study has internal validity when the causal relationship between two variables is properly demonstrated. Internal validity is making sure that with the research done right? It is related to the number of confounding variables in the experiment. The lower the confounding variables the higher the internal validity, which is expected. To ensure the validity of the study, I worked closely with the Primary Investigator of this research as well as
the teachers to ensure that all procedures are well implemented to reduce confounding variables and increase the validity of the study.

**External Validity.** This deals with the study findings to see if the study results can be generalized to other persons and settings (Lewis, 2013). According to Yin (2014, p. 47), to have a high-quality case study design means responding to four tests. The tests include, (1) construct validity, which means identifying the correct operational measures for the concepts being studied; (2) internal validity, which means to seek to establish a causal relationship, whereby certain conditions are believed to lead to other conditions, as distinguished from spurious relationships; (3) external validity, which means defining the domain to which a study findings can be generalized; and (4) reliability, which means to demonstrate that data collection procedures for the study can be repeated with the same outcome. Yin (2014) also indicates that construct validity can be achieved through collecting multiple sources of evidence, establishing a chain of evidence.

Saltkind (2009) indicates that “the reliability (or consistency) and validity (or the does-what-it-should qualities) of a measurement instrument are essential because the absence of these qualities could explain why you act incorrectly in accepting or rejecting your research hypothesis” (p.109). He further says that reliability and validity are a researcher’s first line of defense against incorrect conclusions, “if the instrument fails, then everything else down the line fails as well.” While reliability happens when a test measures the same thing repeatedly resulting in the same outcomes, validity is concerned with making sure the test or the instrument measures what it was intended to measure. Validity is about the results of a test not about the test itself.

Robinson (2016) quoted Trochim (2006) for arguing that “in a single case design, there are 4 threats to internal validity concerning pre/post test data which are history, maturation,
testing and instrumentation” (p. 74). Steps were taken in this study to protect against these threats. The pre-posttest (RFE) was tested for internal reliability using Cronbach’s alpha.

For the reliability of the ATMI survey, the instrument shows a reliability Cronbach alpha of 0.963 (Tapia, 1996). According to Tapia and Marsh (2004), a factor analysis of the 40-item scale consisting of four subscales (self-confidence, value, enjoyment and motivation) showed good internal reliability and indicated stability over time of the test retest. The data also showed a high level of reliability of the subscales.

Confidentiality and Ethics

Robinson (2016) highlighted the fact that Yin (2014) talks of using precaution to collect, use and store the data. Keeping participant information secured and ensuring the safety of subjects is crucial in a research study and this study was not different. All the confidentiality and ethics rules governing research studies were upheld during this study. To ensure that the rights of participants were not violated during this study, all personal information of participants was confidentially kept and not used. The participants’ name and other important information was not revealed using codes throughout the study.

Trustworthiness

The quantitative data collection methods and analysis presented above were conducted with all the ethical considerations in mind. For the qualitative data collection and analysis, Cope (2014) highlighted five criteria for establishing the trustworthiness of qualitative research which include (1) credibility - the truth of the data or research findings, (2) dependability- the consistency of the research findings under similar conditions by a different researcher, (3) confirmability- the ability to demonstrate that the data represents the students’ viewpoints and not those of the researcher, (4) transferability – the application of findings to other settings or
groups and (5) authenticity – the ability and extend to which the researcher expresses the emotions and feelings of participants’ experiences in a faithful manner. These criteria were met using triangulation of multiple data sources involving the test scores, students’ artifacts, interview responses in the students’ own words, the researchers’ memos, questionnaire responses, and the self-reports from teachers.

**Researcher’s subjectivity**

I am currently a lecturer of mathematics at the same college where this study was conducted. As a student in a mathematics classroom and a mathematics instructor for more than 15 years, I have realized the importance of introducing and creating mathematics classroom activities in the real-world context and giving the students the opportunity and time to explore these real-world phenomena under the guidance of the teacher. The students’ active participation in developing their competencies as they think through these problems and formulate strategies to solving them, plays a vital role in their success in mathematics, especially when dealing with problems and concepts that students appear to struggle with such as rational functions. I believe that teachers should engage the students through different instructional strategies including mathematical modeling which I am investigating its effects on precalculus students’ performance in this study.

As the researcher also, I oversaw the overall data collection process which included the recruitment of participants (teachers and students), provided the students with consent forms before participation. I ensured that all the data collection techniques and instruments were properly implemented including administering the pre-posttest (RFE), the ATMI survey and the research questionnaire. With input from some precalculus instructors at the college, including the teacher participants in this study, I created the rational function exam using the precalculus
common final exam questions as a guide and template to ensure the validity of the questions in the RFE. I met with the teachers of both groups once a week for 30-35 minutes to discuss the memos progress of instruction in their classes. The memos written during my unofficial visits to the teacher’s classes were also discussed during my meetings with the instructors. After the data collection, I proceeded to analyzing the data from the assessments to determine whether there was a significant difference between the performance and attitudes of precalculus students who were taught rational functions through mathematical modeling and those who were taught rational functions through the traditional lecturing method. I equally ensured that the data collected for this study was kept confidential and in safe locations to maintain privacy of participants involved in the study.

**Potential Limitations**

The study had several potential limitations, which should be taken into considerations when looking at the results.

1. The fact that the different sections of the precalculus (rational functions) were taught by different instructors may or may not have had the teacher effect on the outcome of this study.

2. The sample size was affected by subject attrition as participants eventually dropped out of the study due to withdrawal from the course before the posttests. The attrition rate for both the RFE and ATMI was 5.3% and 33.3% for the interviews.

3. Cognitive complexity is a psychological variable that can have different meanings or definitions and hence not easy to measure or quantify
CHAPTER 4

Data Analysis and Results

The purpose of this study was to investigate the effects of mathematical modeling instruction on precalculus students’ performance and attitude toward rational functions. The intervention for this study was mathematical modeling as an instructional method. The design was exploratory embedded single case, with both quantitative and qualitative methods employed to collect and analyze the data. The following research questions and hypotheses were used for the investigation:

1. What is the effect of mathematical modeling instruction on Precalculus students’ performance as measured by a score on a Rational Function Exam (RFE) and attitudes toward rational functions?

2. What is the nature of the effect of mathematical modeling instruction on the types and cognitive complexity of representations used by Precalculus students on rational functions?

The null hypotheses were as follows:

$H_01$: There is no statistically significant difference in Precalculus students’ performance as measured by a score on a rational function exam (RFE) between Precalculus students who receive instruction through mathematical modeling and Precalculus students who receive instruction through lecturing.

$H_02$: There is no statistically significant difference in attitude toward rational functions between Precalculus students who receive instruction through mathematical modeling and Precalculus students who receive instruction through lecturing.
The study employed both quantitative and qualitative data techniques for data collection. Quantitative techniques included a pretest-posttest RFE covering important concepts of rational functions and a pretest-posttest ATMI survey. Qualitative techniques were interviews, artifacts on the posttests and a questionnaire. Quantitative data was analyzed using a one-way ANCOVA and Cronbach’s alpha for reliability analysis while qualitative data was analyzed using coding protocols according to (Saldaña, 2013) through Dedoose’s 2017 web-based application.

**Quantitative Data Analysis**

To provide answers to the research question one about the effect of mathematical modeling instruction on Precalculus students’ performance and attitudes towards rational functions, data from the pre-posttest RFE and the ATMI was analyzed using a one-way ANCOVA. The first null hypotheses for question one (H₀₁) which states that there is no significant difference between the average performance score on a rational function exam (RFE) between the treatment and comparison groups was tested using a one-way ANCOVA. ANCOVA was also used to test the second null hypothesis that there is no statistically significant difference in attitude toward rational functions between the treatment and comparison groups. Before using a one-way ANCOVA, the data was tested for the assumptions to ensure that the data was appropriate for use with a one-way ANCOVA.

**Reliability Analysis.** Before using ATMI survey instrument, a reliability analysis was conducted using Cronbach’s alpha to ensure the internal consistency of the instrument. Cortina (1993) points out that a Cronbach’s alpha level greater than 0.70 is acceptable for the reliability of the instrument. Table 7 below summarizes the Cronbach alpha value calculated for the pretest and posttest in both the treatment and comparison groups, showing that the ATMI
instrument was a reliable instrument for this study given that the Cronbach’s alpha was greater than .70 for both the pretest and posttest in both groups.

Table 7

Reliability Analysis of ATMI Instrument

<table>
<thead>
<tr>
<th></th>
<th>Pretest-Comparison</th>
<th>Posttest-Comparison</th>
<th>Pretest-Treatment</th>
<th>Posttest-Treatment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cronbach’s alpha</td>
<td>.74</td>
<td>.78</td>
<td>.71</td>
<td>.72</td>
</tr>
</tbody>
</table>

Testing the null hypothesis for RFE using ANCOVA-H₀₁

Table 8

Descriptive Statistics for RFE Posttest – Dependent Variable

<table>
<thead>
<tr>
<th>Group</th>
<th>M(SD)</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treatment</td>
<td>45.54 (16.14)</td>
<td>24</td>
</tr>
<tr>
<td>Comparison</td>
<td>21.23 (11.71)</td>
<td>30</td>
</tr>
<tr>
<td>Total</td>
<td>32.04 (18.35)</td>
<td>54</td>
</tr>
</tbody>
</table>

*Note. M = mean. SD = standard deviation

Testing the Outliers’ Assumptions for ANCOVA on the RFE

Figure 3. Box plot of RFE posttest data.

Figure 3 above shows that there were no outliers in the RFE posttest data of the treatment and comparison groups. Therefore, the outlier assumption was met.
Testing the Assumption for Equality (Homogeneity) of Error Variances

The Levene's Test of equality of error variances was conducted on the Posttest which showed a non-statistically significance value of 0.62 ($p > .05$). This indicated that there was no significant difference between the variances of the posttest score of the treatment and comparison group. Therefore, the equal variances assumption was met.

Testing the Normality Assumptions for ANCOVA on the RFE

Table 9

Tests of Normality of RFE Posttest Data

<table>
<thead>
<tr>
<th>Group</th>
<th>Kolmogorov-Smirnov</th>
<th>Shapiro-Wilk</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Statistics</td>
<td>$df$</td>
</tr>
<tr>
<td>Posttest</td>
<td>Treatment</td>
<td>.11</td>
</tr>
<tr>
<td></td>
<td>Comparison</td>
<td>.11</td>
</tr>
</tbody>
</table>

Note. *$p < .05$ indicates significance

Table 9 shows a non-statistically significant Shapiro-Wilk value ($p > .05$) for the normality test in both the treatment and comparison groups. This means that the posttest data for both groups were normally distributed. The normality assumption of data was met.

Testing the Assumption of Linearity between the Covariate RFE Pretest and Posttest

![Figure 4](image-url). Assumption of linearity between the covariate (RFE pretest) and RFE- posttest.
Table 10

Estimated Marginal RFE Means- Dependent Variable: Posttest

<table>
<thead>
<tr>
<th>Group</th>
<th>M</th>
<th>Std. Error</th>
<th>LL</th>
<th>UL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treatment</td>
<td>45.90</td>
<td>2.05</td>
<td>41.79</td>
<td>50.01</td>
</tr>
<tr>
<td>Comparison</td>
<td>20.94</td>
<td>1.83</td>
<td>17.27</td>
<td>24.62</td>
</tr>
</tbody>
</table>

Note. \( M \)=mean; \( LL \)= lower limit; \( UL \)= upper limit.

Table 10 shows an estimated marginal (adjusted mean) posttest mean of 45.9 for the treatment group and 20.9 for the comparison group, when the pretest scores were controlled as covariate. The adjusted posttest mean for the treatment group was 25 more than that of the comparison group.

Analysis of Covariance

Table 11

ANCOVA Results for RFE-Test of Between Subjects-Effects: Dependent Variable- Posttest

<table>
<thead>
<tr>
<th>Source</th>
<th>df</th>
<th>Type III SS</th>
<th>MS</th>
<th>F</th>
<th>P</th>
<th>Partial Eta square</th>
</tr>
</thead>
<tbody>
<tr>
<td>Corrected Model</td>
<td>2</td>
<td>12715.23</td>
<td>6357.61</td>
<td>63.27</td>
<td>.000</td>
<td>.713</td>
</tr>
<tr>
<td>Intercept</td>
<td>1</td>
<td>3642.85</td>
<td>3642.85</td>
<td>36.25</td>
<td>.000</td>
<td>.415</td>
</tr>
<tr>
<td>Pretest</td>
<td>1</td>
<td>4836.63</td>
<td>4836.63</td>
<td>48.13</td>
<td>.000</td>
<td>.486</td>
</tr>
<tr>
<td>Group</td>
<td>1</td>
<td>8292.48</td>
<td>8292.48</td>
<td>82.53</td>
<td>.000*</td>
<td>.618</td>
</tr>
<tr>
<td>Error</td>
<td>51</td>
<td>5124.70</td>
<td>100.48</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>54</td>
<td>73264.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Corrected Total</td>
<td>53</td>
<td>17839.93</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note. *\( p < .05 \) indicates significance; \( SS \)= sum of squares; \( MS \)= mean square

Table 11 above shows a statistically significant group difference \( (p < .001) \), eta square = .618. Therefore, the null hypothesis \( (H_0) \) was rejected indicating that there was a statistically significant difference in Precalculus students’ average performance in a Rational Function Exam (RFE) between Precalculus students who receive instruction through mathematical modeling (\( M \))
= 45.54, $SD = 16.14$) and Precalculus students who receive instruction through the traditional lecturing approach ($M = 21.21$, $SD = 11.71$). This test results were consistent with the data collected.

**Testing the Null Hypothesis for ATMI Using ANCOVA- H₀2**

Table 12

<table>
<thead>
<tr>
<th>Group</th>
<th>$M(SD)$</th>
<th>$N$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treatment</td>
<td>129.67 (12.40)</td>
<td>24</td>
</tr>
<tr>
<td>Comparison</td>
<td>120.54 (12.00)</td>
<td>30</td>
</tr>
<tr>
<td>Total</td>
<td>124.75 (12.91)</td>
<td>54</td>
</tr>
</tbody>
</table>

*Note. $M =$ mean. $SD =$ standard deviation*

Table 12 shows the average ATMI posttest score of the treatment group to be 129.67 greater than that of the comparison group with an average score of 120.54. This gives a mean difference of 9.13.

**Testing the Outliers’ Assumptions for ANCOVA on the ATMI**

![Box plot of ATMI posttest data](image)

*Figure 5. Box plot of ATMI posttest data.*

Figure 5 shows that there are no outliers in the ATMI posttest data of the treatment and comparison groups. Therefore, the outlier assumption was met.
Testing the Assumption of Equality (Homogeneity) of Error of Variances

The Levene's Test of equality of error variances was conducted on the ATMI Posttest which showed a non-statistically significance value of 0.30 ($p>.05$). This indicated that there was no significant difference between the variances of the ATMI posttest score of the treatment and comparison groups. Therefore, the equal variances assumption was met.

Table 13

<table>
<thead>
<tr>
<th>Tests of Normality of ATMI Posttest</th>
<th>Kolmogorov-Smirnov</th>
<th>Shapiro-Wilk</th>
</tr>
</thead>
<tbody>
<tr>
<td>Group</td>
<td>Statistics</td>
<td>$df$</td>
</tr>
<tr>
<td>Posttest</td>
<td>Treatment</td>
<td>.11</td>
</tr>
<tr>
<td></td>
<td>Comparison</td>
<td>.11</td>
</tr>
</tbody>
</table>

*Note. *$p < .05$ indicates significance*

Table 13 shows a non-statistically significant Shapiro-Wilk value ($p>.05$) for the normality test in both the treatment and comparison groups. This means that the posttest data for both treatment and comparison groups was normally distributed. The normality was met.

Testing the Assumption of Linearity between the ATMI Covariate Pretest and Posttest

*Figure 6. Assumption of linearity between the covariate (ATMI-pretest and posttest.*
Figure 6 shows that the assumption of linearity between the covariate (pretest) and the dependent Variable (posttest) for each Level of the Independent Variable was met.

Table 14

*Estimated Marginal ATMI Means- Dependent Variable: Posttest*

<table>
<thead>
<tr>
<th>Group</th>
<th>M</th>
<th>Std. Error</th>
<th>LL</th>
<th>UL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treatment</td>
<td>129.29</td>
<td>2.07</td>
<td>125.13</td>
<td>133.44</td>
</tr>
<tr>
<td>Comparison</td>
<td>120.86</td>
<td>1.91</td>
<td>117.02</td>
<td>124.71</td>
</tr>
</tbody>
</table>

*Note.* M = mean; LL = lower limit; UL = upper limit.

Table 14 shows and estimated marginal (adjusted mean) ATMI posttest mean of 129.29 for the treatment group and 120.86 for the comparison group, when the pretest scores were controlled as covariate. This indicates an adjusted mean difference of 8.43 between the treatment and the comparison groups.

**Analysis of Covariance**

Table 15

*ANCOVA Results for ATMI-Test of Between Subjects-Effects: Dependent Variable- Posttest*

<table>
<thead>
<tr>
<th>Source</th>
<th>df</th>
<th>Type III SS</th>
<th>MS</th>
<th>F</th>
<th>P</th>
<th>Partial Eta Square</th>
</tr>
</thead>
<tbody>
<tr>
<td>Corrected Model</td>
<td>2</td>
<td>3491.85</td>
<td>1745.92</td>
<td>17.08</td>
<td>.000</td>
<td>.411</td>
</tr>
<tr>
<td>Intercept</td>
<td>1</td>
<td>1610.21</td>
<td>1610.21</td>
<td>15.75</td>
<td>.000</td>
<td>.243</td>
</tr>
<tr>
<td>Pretest</td>
<td>1</td>
<td>2414.39</td>
<td>2414.39</td>
<td>23.61</td>
<td>.000</td>
<td>.325</td>
</tr>
<tr>
<td>Group</td>
<td>1</td>
<td>914.29</td>
<td>914.29</td>
<td>8.94</td>
<td>.004*</td>
<td>.154</td>
</tr>
<tr>
<td>Error</td>
<td>51</td>
<td>5009.91</td>
<td>102.24</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>54</td>
<td>817755.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Corrected Total</td>
<td>53</td>
<td>8501.75</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Note.* *p* < .05 indicates significance; SS = sum of squares; MS = mean square

Table 15 shows a statistically significant group difference $F (1, 51) = 8.94$, ($p = .004$), Eta square = .154. Therefore, the null hypothesis ($H_0$) was rejected indicating that there was a
statistically significant difference in Precalculus students’ average attitude score on an ATMI survey between those who received instruction through mathematical modeling ($M = 129.62, SD = 12.40$) and those who receive instruction through the traditional lecturing ($M = 120.54, SD = 12.00$). This test results were consistent with the data collected.

**Qualitative Data Analysis**

Qualitative data for this study was collected from students’ artifacts, interviews, questionnaire and researcher’s memos. The data was coded through Dedoose’s 2017 web-based application guided by Saldaña (2013) coding protocols. Figure 7 below shows the coding process from codes to categories to concepts and themes according to Saldana (2013) that was adopted for this study.

![Figure 7](image)

*Figure 7. A code-to-theory model for qualitative inquiry. Adopted from Saldana (2013, p. 13).*

According to Saldaña (2013), the first cycle is hardly perfect but the second, third, fourth cycles etc. manages and filters the qualitative data generating categories, themes, concepts,
meaning and building theory. He considers a theme as an outcome of coding and categorization or analytic reflection.

Dedoose’s 2017 web-based application which was then used to analyze the data, has the ability of integrating qualitative data with quantitative data analyzing the data in great depths, producing codes, categories and themes. Cates (2018) indicates that Dedoose allows for qualitative data to be coded through traditional qualitative analysis methods and linking the data to force-choice quantitative data. Figure 8 below shows the qualitative data analysis process involved.

**Figure 8.** Diagram depicting qualitative data analysis conducted in the study.

In the first coding cycle, the data was the raw qualitative data from the four sources (interviews, artifacts, and questionnaire and research memos). As such, the first codes were the first striking ideas in the data. The following 45 codes were noted during this first round: (1) Graphs, (2) encouragement, (3) difficulties, (4) helpful methods, (5) equations (6) real word application problems, (7) confidence, (8) rational functions are hard, (9) Challenging word problems, (10) prior knowledge, (11) teaching method, (12) teacher guidance, (13) struggle to understand, (14) asymptotes (15) motivation, (16) expectations, (17) domain of functions (18)reflecting on problems, (19) engaging, (20) better understanding after intervention, (21)
struggling, (22) procedure, (23) concentration, (24) focused, (25) importance of the lesson, (26) collaboration, (27) students’ attitude on different problems, (28) attitudes toward graphs, (29) modeling instruction, (30) attitudes towards equations, (31) conceptual understanding, (32) practice many problems, (33) critical thinking, (34) optimistic, (35) misconceptions, (36) reluctant to give up (37) passionate, (38) lacks basic algebraic skills, (39) high level thinking skills, (40) progress, (41) successful, (42) accuracy and procedure on representing a function as graph or equation, (43) attitudes toward word problems, (44) intercepts of functions, (45) Range.

Recoding the initial codes, and continuing through four recoding cycles, resulted to more refined codes as redundant words were removed, and others combined into categories and concepts and then to themes. Table 16 below shows emerging codes, concepts, and categories.

Table 16

<table>
<thead>
<tr>
<th>Codes</th>
<th>Concept representation</th>
<th>Categories</th>
</tr>
</thead>
<tbody>
<tr>
<td>Critical thinking, reflection,</td>
<td>Attitudes towards learning</td>
<td>Students’ perception of mathematical modeling instruction</td>
</tr>
<tr>
<td>Confidence, engagement,</td>
<td>mathematics</td>
<td></td>
</tr>
<tr>
<td>motivated, passionate,</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Attempts challenging word</td>
<td></td>
<td></td>
</tr>
<tr>
<td>problems, misconceptions,</td>
<td></td>
<td></td>
</tr>
<tr>
<td>struggles to understand,</td>
<td></td>
<td></td>
</tr>
<tr>
<td>lacks basic algebraic skills,</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Chart, data organizer</td>
<td>Table</td>
<td></td>
</tr>
<tr>
<td>Any word, phrase, any thought,</td>
<td>Written</td>
<td>Students’ representations and cognitive complexity</td>
</tr>
<tr>
<td>A mathematical idea</td>
<td></td>
<td></td>
</tr>
<tr>
<td>An expression consisting of</td>
<td></td>
<td></td>
</tr>
<tr>
<td>numbers or symbol or both</td>
<td>Equation</td>
<td></td>
</tr>
<tr>
<td>A pictorial representation</td>
<td></td>
<td></td>
</tr>
<tr>
<td>numbers, value or real-world</td>
<td></td>
<td></td>
</tr>
<tr>
<td>scenario</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Applies more than one</td>
<td></td>
<td></td>
</tr>
<tr>
<td>representation,</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Multiple representation</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Categories

Two main categories (students’ perception of mathematical modeling instruction and students’ representations and cognitive complexity) as shown in Table 16, were developed to provide answers to the qualitative research question 2 on the students’ representations. The Students’ perception of mathematical modeling instruction category encompassed concepts and codes of patterns related to student’s behavior and attitudes towards learning mathematics, specifically rational functions. The students’ representations and cognitive complexity category was a grouping of codes and concepts of students’ representation of rational functions as reflected in their artifacts, interviews, questionnaire and researcher’s memos.

Emerging Qualitative Findings

From codes, categories and concepts and repeated examination of these data sources, three themes emerged (one from the students’ perception of mathematical modeling instruction category and 2 from the students’ representations and cognitive complexity category). From the students’ perception of mathematical modeling instruction category, the theme that emerged was that students tend to have positive views of rational functions and display engaging and immersed attitudes towards learning mathematics in a modeling instructional setting.

From the students’ representations and cognitive complexity category the themes that emerged were: 1) Teacher’s guidance during modeling instruction tend to help students’ mathematical representations of functions and real-world scenarios & 2) mathematical modeling instruction tend to foster critical thinking and conceptual understanding of rational functions, increasing students’ representations capabilities and cognitive complexities. The next section is a description of each of the four themes in detail with excerpts of students’ artifacts, interviews, questionnaire and researcher’s memos.
Students’ Perception of Mathematical Modeling Instruction Category

Theme 1: Students tend to have positive views of rational functions and display engaging and immersed attitudes towards learning mathematics in a modeling instructional setting. From the interview responses of 4 students (2 from the treatment group and 2 from the comparison group), three of the students indicated that they had a better understanding of rational functions than they did before the instruction on rational function. One student from the comparison group indicated that she still did not like rational functions. Similarly, when asked in the questionnaire to describe their feelings after the lessons on rational functions, 19 students (79.2%) of the students in the treatment class stated that the felt better compared to 14 students (46.7%) in the comparison class (see Table 17 below).

Table 17
Sample Students’ Questionnaire Responses on how they Felt After Instruction

<table>
<thead>
<tr>
<th>Question</th>
<th>Treatment group</th>
<th>Comparison group</th>
</tr>
</thead>
<tbody>
<tr>
<td>How do you feel about rational functions now after the lessons you just received?</td>
<td>5 Comfortable</td>
<td>2 I am still a bit confused</td>
</tr>
<tr>
<td></td>
<td>16 I feel that I understand Rational functions better now than originally learned</td>
<td>22 I still do not like the topic and now I am a little more confused.</td>
</tr>
<tr>
<td></td>
<td>15 Pretty okay, I understand the basics</td>
<td>10 Still do not like them. They are difficult</td>
</tr>
<tr>
<td></td>
<td>8 Better, just need to study more</td>
<td>11 Still need more help</td>
</tr>
<tr>
<td></td>
<td>3 Ok</td>
<td>5 They are still difficult to understand</td>
</tr>
</tbody>
</table>
The students were given pseudonyms in the form of ID numbers to protect their privacy. Figures 9, 10, 11, and 12 below are excerpts of the students’ responses.

**Figure 9.** Response of student 2 in comparison group.

**Figure 10.** Response of student 22 in comparison group.

**Figure 11.** Response of student 16 in Treatment group.
Further indication of student’s feelings about rational functions after mathematical modeling was noted on students’ responses to the ATMI survey. On specific items on the survey, when asked in Item 37 whether the students were comfortable expressing their own ideas on how to look for a solution to a difficult problem in mathematics, 70.8% strongly agreed in the treatment class while only 15 (50%) strongly agreed. The response was similar in item 38 when the students were asked if they were comfortable answering questions in a mathematics class. In the treatment group 83.3% strongly agreed compared to 47.7% in the comparison group. Further examination of all the responses on ATMI on issues related to the value of mathematics, engagement and confidence and motivation was done. Their responses showed that a higher percentage of students in the treatment strongly agreed (score of 5) on the issues of motivation, the value of mathematics, engagement and confidence. For example, on the survey statement of students having a lot of self-confidence with mathematics (see ATMI in Appendix B- item 17), 20.8% of the students strongly agreed in the treatment class compared to only 6.7% of the students in the comparison class. On the issue of enjoying doing mathematics in school (item 24), 8.3% strongly agreed in the treatment group compared to only 3.3% of students in the comparison group. According to Tapia and Marsh (2004), the ATMI survey is a 40-item
inventory Likert scale type survey. The items are grouped into subscales: Self-confidence items (9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 40), value items (1, 2, 4, 5, 6, 7, 8, 35, 36, 39), enjoyment items (3, 24, 25, 26, 27, 29, 30, 31, 37, 38) and motivation items (23, 28, 32, 33, 34). Table 18 below summarizes the percentage of students’ responses in both groups when on issues of self-confidence, motivation, value mathematics and enjoyment.

On the issue of valuing mathematics, students in the comparison group scored higher (34.5%) compared to those in the treatment group (31.4%) even though they were less motivated and had less confidence in mathematics than those in the treatment group.

Table 18

Percentage of Students’ Responses on the ATMI-Survey on Related Issues

<table>
<thead>
<tr>
<th>Issue</th>
<th>Survey items</th>
<th>% of students’ score</th>
<th>Treatment group</th>
<th>Comparison group</th>
</tr>
</thead>
<tbody>
<tr>
<td>Self-confidence</td>
<td>9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 40</td>
<td>35.2</td>
<td>33</td>
<td></td>
</tr>
<tr>
<td>Value</td>
<td>1, 2, 4, 5, 6, 7, 8, 35, 36, 39</td>
<td>31.4</td>
<td>34.5</td>
<td></td>
</tr>
<tr>
<td>Enjoyment</td>
<td>3, 24, 25, 26, 27, 29, 30, 31, 37, 38</td>
<td>25.8</td>
<td>20.6</td>
<td></td>
</tr>
<tr>
<td>Motivation</td>
<td>23, 28, 32, 33, 34</td>
<td>13.4</td>
<td>10.9</td>
<td></td>
</tr>
</tbody>
</table>

Table 18 shows that the percentage of students’ scores was higher in the treatment group in all categories except the value of mathematics. As such, an engaging and motivating
instructional strategies such as mathematical modeling could play an important role in the way students feel and learn mathematics as seen in the case of students in the treatment group.

To further illustrate students’ immersed attitudes in a modeling environment, and to add depth to the quantitative results, I closely examined the students’ written solutions on each item on the RFE in both groups to identify the errors or misconceptions made in solving the problems. This was done to determine whether the student met the learning objectives outlined in Table 3 above. According to Blum (2011) the modeling process (cycle) begins with understanding the problem situation and being able to construct the context of the situation. This is also true with any mathematical problem-solving strategy which begins by the understanding of the situation at hand before trying to solve the problem. If a student does not demonstrate understanding of the problem situation, it becomes clear that the student is not going to employ the right strategy to solve the problem. Therefore, to analyze the students’ solutions on the RFE, I checked for understanding of the problem situation as the first step. I also focused on the problem-solving strategy and the accuracy of the final answer. I noticed that majority of the students in the comparison group did not fully understand the problems and did not use the right strategy nor earn full credits (above 50% of credits) on the problem. As a result, the did not earn full or partial credits.

Table 19 below shows the percentage of students who correctly or partially solved the different problem items on the rational function posttest. As indicated in the table, the students in the treatment group clearly out performed their counterparts in the comparison group in representations of rational functions, solving rational equations and inequalities, finding zeros (x-intercepts), asymptotes and solving context driven problems. Students who understood these concepts performed better on the exam as results indicate.
Table 19

**Percentage of Students with Partial/Full Credits Scores on the RFE Items**

<table>
<thead>
<tr>
<th>Concept</th>
<th>Objective</th>
<th># of items</th>
<th>Item # on RFE (see Appendix A)</th>
<th>% of students with partial or full credit score (met objective)</th>
<th>Treatment group</th>
<th>Comparison group</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rational function models</td>
<td>Students can represent functions in multiple ways</td>
<td>4</td>
<td>2d</td>
<td>8.3</td>
<td>6.7</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>3</td>
<td>45.8</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>4a</td>
<td>12.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>4b</td>
<td>12.5</td>
<td>13.3</td>
<td></td>
</tr>
<tr>
<td>Rational equations</td>
<td>Students can solve rational equations</td>
<td>1</td>
<td>1c</td>
<td>66.7</td>
<td>6.7</td>
<td></td>
</tr>
<tr>
<td>Rational Inequalities</td>
<td>Students can solve rational inequalities</td>
<td>1</td>
<td>1d</td>
<td>66.7</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Rational function operations</td>
<td>Students can subtract and add rational functions</td>
<td>2</td>
<td>1a</td>
<td>62.5</td>
<td>3.3</td>
<td></td>
</tr>
<tr>
<td>Domain and range</td>
<td>Students can find domain and range</td>
<td>1</td>
<td>1b</td>
<td>58.3</td>
<td>27</td>
<td></td>
</tr>
<tr>
<td>Zeros</td>
<td>Students can find zeros</td>
<td>1</td>
<td>2a</td>
<td>46</td>
<td>37</td>
<td></td>
</tr>
<tr>
<td>Asymptotes</td>
<td>Students can find asymptotes</td>
<td>1</td>
<td>2c</td>
<td>71</td>
<td>27</td>
<td></td>
</tr>
<tr>
<td>Context driven problem</td>
<td>Students can solve problems in context</td>
<td>1</td>
<td>2b</td>
<td>83.3</td>
<td>33.3</td>
<td></td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td></td>
<td></td>
<td>5</td>
<td>50</td>
<td>40</td>
<td></td>
</tr>
</tbody>
</table>

**Students’ Representations and Cognitive Complexity Category**

According to Robinson (2001), cognitive complexity is “the processing demands of tasks and the availability of relevant knowledge” (p.28). Five concepts of representations of functions were put into this category (Table, written, equation, graph and multiple representations). After five cycles of coding and recoding, two main themes emerged from this category:
Theme 2: Teacher’s guidance during modeling instruction tend to help students’ mathematical representations of functions and real-world scenarios. Interview responses from students showed that teacher guidance played a vital role in their understanding of the concepts and motivation. Specifically, on context driven problems requiring students to understand the problem situation and provide a route map or representation of the situation mathematically. Student 25 from the treatment group for example, indicated during the interview when asked to describe the aspects of the instructions that were helpful, she said some of the real-world application problems during instruction were hard, but he did not give up, thanks to guidance and support from the teacher. Also, when she was asked to rate her level of satisfaction of the instruction on a scale from 1 to 5, 5 being extremely satisfied, she gave a 5. Both 2 students interviewed group stated that they were extremely satisfied with the way the instruction was handled by the teacher. According to Mensah et al. (2013) teachers’ positive attitudes, radiate confidence in students making them to develop positive attitude toward the learning of mathematics.

As indicated previously, interviews were used to follow up students’ responses on the questionnaire questions to get a better understanding of the students thinking. This was the response of student 25 to the questionnaire question, when she was asked to describe the aspects of the instruction that were helpful to her. She wrote: “I learned how to find asymptotes – this was new to me. I also have more confidence finding the x-y intercepts of an equation. Seeing the teacher work problems and explain is helpful to me.”

The explanation from the teacher helped her understand how to find asymptotes, x, and y-intercepts which are prerequisite concepts for graphical representations.
Figure 13. Response of student 25 on questionnaire question.

Here is an excerpt of my interview with student 25 on the same question she had on the questionnaire. Speaker 1 is the interviewer (myself) and speaker 2 was the participant (student 25)

Speaker 1: Describe the aspect of the instruction that you found helpful to you.

Speaker 2: He went step by step on the board through the problems and how to graph it.

Speaker 1: Okay.

Speaker 2: You know, didn't bounce all over and whatnot.

Speaker 1: Okay. So, what about a real-world application problem, were they helpful to you?

Speaker 2: Yeah, he talked about oceanography and how it applies to science majors and how you'll find it after college.

Speaker 1: Okay. And so, describe the things that you liked about the lesson.

Speaker 2: I'd have to say again, how it goes step by step through the problems.

Speaker 1: Okay.
Speaker 2: He gave a lot of examples.

Speaker 1: Lot of examples. And you had to work some of these problems by yourself, on your own?

Speaker 2: Yes.

**Theme 3: Mathematical modeling instruction tend to foster critical thinking and conceptual understanding of rational functions, increasing students’ representations capabilities and cognitive complexities.** Zooming more further into the students’ solutions, an in-depth examination of students’ artifact on the RFE pre and posttest revealed important information and difference in conceptual understanding between the treatment and the comparison groups. To check for conceptual understanding of the concepts, I examine the solutions of two students (Student 5 from the treatment group and student 6 from the comparison group) on item 2d which involved representing an equation of a rational function in graphical format. The solution of student #5 in the treatment group who score 29% in the pretest on RFE, scored a 73% on the posttest after the intervention showed that the student was had good knowledge of the concepts and was well prepared for the exam than the fellow student #6 in the comparison group. Students were not allowed to use their real names for confidentiality and privacy purposes. They were given identification numbers (pseudonyms) which was used throughout the study.

Presented below, are some of the misconceptions and misunderstanding in the steps taken by two students (student 5 from treatment group and student 6 from the comparison group) to solve problem item 2d involving representations on the RFE. Item 2d required students to give a graphical representation of the rational function equation

\[ f(x) = \frac{(x - 2)(x + 3)}{(x - 1)(x + 2)(x - 5)} \] by hand without a graphing aid.
The steps to solve this problem involves knowledge of the following concepts: 1) Domain and range item 2a, 2) intercepts (x and y) in item 2c, 3) asymptotes (horizontal, vertical and oblique) in item 2b & 4) end behavior.

1) The domain of this function is \( \{x | x \neq 1, -2, 5\} \) and Range is \( \{y | y \neq 0\} \)

2) The x-intercept is where the function intersects with the x axis found by setting y to 0 and solving for x. The y-intercept is where the function intersects with the y-axis found by setting x to 0. The x-intercept or zeros are at x = 2 and x = -3 which are the points (2, 0) and (-3, 0). The y intercept is \( f(0) = -6/10 = -3/5 \)

3) The vertical asymptotes are the lines x = 1, x = -2 and x = 5 where the function is undefined in the domain. The horizontal asymptotes are found by looking at the limit of the function as x goes to infinity. If the degree of the numerator is smaller than that of the denominator, the line \( y = 0 \) is the horizontal asymptotes. That is the case with this function.

4) End behavior involves looking at the behavior of x as y goes to infinity and the behavior of y as x goes to infinity.

5) The resulting graph of \( f(x) \) above should look like what we have below.

---

*Figure 14. Graph of \( f(x) \) in RFE item 2d.*
Table 20 below compares how well the two students solved item 2d.

Table 20

*Comparison of Misconceptions between Student 5 and Student 6 on Item 2d of RFE*

<table>
<thead>
<tr>
<th>Concept Knowledge</th>
<th>Student 5 of treatment group</th>
<th>Student 6 of comparison group</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Domain and range of function f(x)</td>
<td>Stated domain correctly, earned all 5 points</td>
<td>Stated domain correctly Did not give the range. Earned 2.5/5 partial credit</td>
</tr>
<tr>
<td>2. intercepts (x and y)</td>
<td>Found the intercepts (x and y) and earned all 5 points</td>
<td>Found only the x-intercept and earned 2.5/5</td>
</tr>
<tr>
<td>3. Asymptotes Horizontal and vertical</td>
<td>Found both asymptotes and earned all 5 points</td>
<td>Found both asymptotes and earned all 5 points</td>
</tr>
<tr>
<td>4. End behavior</td>
<td>Showed some knowledge on graph</td>
<td>No knowledge shown on graph</td>
</tr>
<tr>
<td>5. Graphing the function f(x)</td>
<td>Had some knowledge of graphing f(x) and earned partial credits 2/5</td>
<td>Had no knowledge of graphing f(x) and earned no credits</td>
</tr>
</tbody>
</table>

Table 20 shows that student 5 from the treatment group had a more conceptual understanding of rational functions than the counterpart in the comparison group and could easily represent this rational function equation in graphical format (See appendix G).

A further examination of modeling principles as applied by the two students on problem solving again showed a better understanding of the concepts by the student in the treatment group compared to the other in the comparison group. Blum (2011) presents modeling principles in the form of a modeling cycle framework which are the seven stages of the modeling process employed to resolve a problem situation by translating from the real-world situation to mathematics and back. These steps are 1) understanding the situation, 2) simplifying, 3) mathematizing, 4) working mathematically, 5) interpreting the results, 6) validating the results & 7) exposing the results. Table 21 below shows how well the two students (5 and 6) above in the
treatment and comparison groups applied modeling principles as they solved problems on the RFE.

Table 21

*Comparison of Problem-Solving behavior of Student #5 and Student #6 on RFE*

<table>
<thead>
<tr>
<th>Student applied Modeling principles</th>
<th>Item # on RFE</th>
<th>Student #5 in Treatment group</th>
<th>Student #6 in comparison group</th>
</tr>
</thead>
<tbody>
<tr>
<td>Understanding the situation,</td>
<td>Y</td>
<td>Y</td>
<td>N</td>
</tr>
<tr>
<td>Simplifying the situation</td>
<td>Y</td>
<td>Y</td>
<td>N</td>
</tr>
<tr>
<td>Mathematizing</td>
<td>Y</td>
<td>Y</td>
<td>N</td>
</tr>
<tr>
<td>Working mathematically</td>
<td>Y</td>
<td>Y</td>
<td>N</td>
</tr>
<tr>
<td>Interpreting the results</td>
<td>Y</td>
<td>Y</td>
<td>N</td>
</tr>
<tr>
<td>Validating the results</td>
<td>2d</td>
<td>Y</td>
<td>N</td>
</tr>
<tr>
<td>Exposing the results</td>
<td>Y</td>
<td>Y</td>
<td>N</td>
</tr>
<tr>
<td>Understanding the situation,</td>
<td>Y</td>
<td>Y</td>
<td>N</td>
</tr>
<tr>
<td>Simplifying the situation</td>
<td>Y</td>
<td>Y</td>
<td>N</td>
</tr>
<tr>
<td>Mathematizing</td>
<td>Y</td>
<td>Y</td>
<td>N</td>
</tr>
<tr>
<td>Working mathematically</td>
<td>Y</td>
<td>Y</td>
<td>N</td>
</tr>
<tr>
<td>Interpreting the results</td>
<td>Y</td>
<td>Y</td>
<td>N</td>
</tr>
<tr>
<td>Validating the results</td>
<td>Y</td>
<td>Y</td>
<td>N</td>
</tr>
<tr>
<td>Exposing the results</td>
<td>3</td>
<td>Y</td>
<td>N</td>
</tr>
<tr>
<td>Understanding the situation,</td>
<td>Y</td>
<td>Y</td>
<td>N</td>
</tr>
<tr>
<td>Simplifying the situation</td>
<td>Y</td>
<td>Y</td>
<td>N</td>
</tr>
<tr>
<td>Mathematizing</td>
<td>Y</td>
<td>Y</td>
<td>N</td>
</tr>
<tr>
<td>Working mathematically</td>
<td>Y</td>
<td>Y</td>
<td>N</td>
</tr>
<tr>
<td>Interpreting the results</td>
<td>Y</td>
<td>Y</td>
<td>N</td>
</tr>
<tr>
<td>Validating the results</td>
<td>Y</td>
<td>Y</td>
<td>N</td>
</tr>
<tr>
<td>Exposing the results</td>
<td>4a</td>
<td>Y</td>
<td>N</td>
</tr>
<tr>
<td>Understanding the situation,</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>Simplifying the situation</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>Mathematizing</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>Working mathematically</td>
<td>Y</td>
<td>Y</td>
<td>N</td>
</tr>
<tr>
<td>Interpreting the results</td>
<td>4b</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>Validating the results</td>
<td>Y</td>
<td>Y</td>
<td>N</td>
</tr>
<tr>
<td>Exposing the results</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
</tr>
</tbody>
</table>

*Note.* Y indicate the use of modeling principle, N indicate the absence of modeling.
Table 21 suggests that application of modeling principles to problem solving enabled the students in the treatment group to better understand the concepts, understand the problem situation and apply the right strategy to solve the problems including representations. The student in the comparison group on the other hand lacked understanding of the concepts and struggled with representation of functions. This further suggests that modeling instruction could help with students’ representations of functions.

**Summary of Results**

Table 22

<table>
<thead>
<tr>
<th>Research question</th>
<th>Quantitative Findings</th>
<th>Themes emerged</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. <strong>H₀₁</strong></td>
<td>There is statistically significant difference in Precalculus students’ average performance in a Rational Function Exam (RFE) between Precalculus students who receive instruction through mathematical modeling ($M = 45.54$, $SD = 16.14$) and Precalculus students who receive instruction through the traditional lecturing approach ($M = 21.21$, $SD = 11.71$)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>There is a statistically significant difference in Precalculus students’ average attitude score on an ATMI survey between those who received instruction through mathematical modeling ($M = 129.62$, $SD = 12.40$) and those who receive instruction through the traditional lecturing ($M = 120.54$, $SD = 12.00$). This test results were consistent with the data collected.</td>
<td></td>
</tr>
<tr>
<td>2. <strong>H₀₂</strong></td>
<td>1. Students tend to positive views of rational functions and display engaging and immersed attitudes towards learning mathematics in a modeling instructional setting.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2. Teacher’s guidance during modeling instruction tend to help students’ mathematical representations of functions and real-world scenarios.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3. Mathematical modeling instruction tend to foster critical thinking and conceptual understanding of rational functions, increasing students’ representations capabilities and cognitive complexities</td>
<td></td>
</tr>
</tbody>
</table>
CHAPTER 5

Discussion and Recommendations

This section highlights the major findings for the study and situate them within the literature. The implications and recommendations for future research are discussed as well as the interpretation of the research findings within the scope the Blum (2011) modeling framework. A sample of 54 Precalculus students from a local college in the southern United States took part in this study. This exploratory embedded single case study employed both quantitative and qualitative techniques to investigate the effects of mathematical modeling instruction on Precalculus students’ performance and attitude toward rational functions. Two research questions guided the investigation:

1. What is the effect of mathematical modeling instruction on Precalculus students’ performance as measured by a score on a Rational Function Exam (RFE) and attitudes toward rational functions?

2. What is the nature of the effect of mathematical modeling instruction on the types and cognitive complexity of representations used by Precalculus students on rational functions?

Major Findings

Quantitatively, the analysis of the RFE results to provide answers to the first research question indicate that students in the treatment group who were taught rational functions through mathematical modeling instruction performed better on the RFE posttest with a mean score of 45.54 and standard deviation of 16.14, compared to their counterparts in the comparison group with a mean score of 21.21 and standard deviation of 11.71. This resulted in a mean posttest score difference between the two groups of 24.33 in favor of the treatment
group. This suggests that mathematical modeling instruction played an important role in the students’ mastery of rational function concepts such as multiple representations of rational functions, solving rational function equations and inequalities, finding asymptotes of rational functions, finding zeros, carrying out arithmetic operations with rational functions (addition, subtraction, multiplication and division) and resolving real world problems involving rational functions.

Similar quantitative analysis of the ATMI survey results showed that the students in the treatment group who studied rational functions through mathematical modeling scored higher on the ATMI posttest survey with a mean score of 129.67 compared to their counterparts in the comparison group with a mean score of 120.54. This resulted in a mean posttest score difference between the two groups of 9.13 in favor of the treatment group. This again suggests that after the students were taught rational functions through mathematical modeling instruction, they had a more favorable view of rational functions and mathematics than they did prior to the intervention.

Qualitatively, three important themes emerged from the analysis of the artifacts, interviews, the questionnaire and the research memos, that describing the effects of modeling instruction on students’ types and cognitive complexity of representations of rational functions:

1. Students tend to have positive views and display engaging and immersed attitudes towards learning mathematics in a modeling instructional setting.

2. Teacher’s guidance during modeling instruction tend to help students’ mathematical representations of functions and real-world scenarios.
3. Mathematical modeling instruction tends to foster critical thinking and conceptual understanding of rational functions, increasing students’ representations capabilities and cognitive complexities.

**Situating of Findings within the Literature**

The first major finding of this study came from the quantitative data analysis which indicated that mathematical modeling instruction impacted precalculus students’ achievement and their attitudes towards learning of rational functions and mathematics in general. Results of the data analysis showed that there was a statistically significant difference between the mean posttest scores precalculus students who received instruction on rational functions through mathematical modeling and the mean posttest score of their counterparts who were in the traditional lecturing environment. Results also indicated a statistically significant difference between Precalculus students’ attitudes towards rational functions in the modeling instructional classroom and in the traditional lecturing classroom.

A review of the literature on impacts of mathematical modeling instruction on students’ attitudes and achievement (Blum, 2011; Dasher & Shahbari, 2015; Kertil & Gurel, 2016; Mubeen et al., 2013; Mensah et al., 2013; Nourallah & Farzad, 2012; Prasad et al., 2014; Pawl et al., 2009; Papageorgiou, 2009; Saha, 2014; Santos et al., 2015; Wedelin & Adawi, 2014 etc.), shows similar results to those of this study. In some cases, the studies show mathematical modeling instruction as having positive impacts on students’ mathematics achievement and in other cases they show students display positive attitudes towards mathematics under mathematical modeling instructional environment.

Santos et al., (2015) found that mathematical modeling instruction helps reduce students’ mathematical anxiety and had positive effects on students’ mathematics performance.
According to Wethall (2011), students were aware of the positive impacts of mathematical modeling on their learning and they are more willing and able to try new problems and take risks. Blum (2011) pointed out that modeling instruction has the potential of helping students understand world around them, motivating them, changing their attitudes towards mathematics and giving helping them to develop their mathematical competencies. Nourallah and Farzad (2012) also show that university level students display problem-solving capabilities through mathematical modeling. Sokolowski (2015) study results showed that modeling helps students with the understanding and application mathematics. Jackson, Dukerich and Hestenes (2008) pointed out that modeling instruction produces students who engage intelligently in public discourse and debate about scientific and technical matters. Furthermore, studies (Mubeen et al. 2013; Prasad et al., 2014; Mensah et al., 2013; Pawl et al., 2009; Popham, 2005) indicate that students who are taught mathematics through mathematical modeling tend to have positive attitudes towards mathematics, hence positive outcomes on students’ mathematical achievement.

Vorhölter et al. (2014) highlighted the fact that mathematical modeling provides the students more than just passing the examinations by showing them how mathematics is applied in their daily lives. A similar study by Dasher and Shahbari (2015) students learn mathematics in a meaningful way when engaged in mathematical modeling. Papageorgiou (2009) found that students have positive views of the modeling process and are pleased that such activities are connected to real life issues. Ellington (2005) showed that modeling-based instruction has a positive effect on students. Niss (2012) highlighted the fact that mathematical models and modeling are always needed either implicitly or explicitly whenever mathematics is applied to issues, problems, situations, and contexts in domains outside of mathematics. Czocher (2017)
pointed out that emphasizing mathematical modeling principles in traditionally taught
differential equations course had a statistically significant effect on students’ learning.

Doerr et al. (2014) found that modeling-based mathematics instruction had a positive impact on the students’ conception of the average rate of change and their first semester grade in the mathematics course. Bahmaei (2013) indicated that mathematical modeling instruction has greater effect on students’ problem-solving abilities compared to that of students in the traditional classroom environment.

Wedelin and Adawi (2014) show that a good number of students who take mathematical modeling courses show impressive changes in their ability to think mathematically and they also express satisfaction with the mathematical modeling course, noting that mathematical modeling is an important course in education. Akgün (2015) indicated teachers’ approval of mathematical modeling citing their ability to connect to real-life and wanting to implement the teaching method in their future classes.

I argue that mathematical modeling instruction has the potential of keeping students stay engaged and motivated in learning of mathematics, leading to higher mathematics achievement as the results of this study indicate. Students and teachers a like who have challenges when dealing with rational functions and mathematics as studies indicate functions (Cangelosi et al., 2013; Nair, 2010; Datson, 2009 etc.), can benefit from the findings of this study. With a strong support for mathematical modeling as an instructional method gaining worldwide attention as evident by the participation of about 30 countries around the world including the top mathematics achieving countries including Singapore, China, Japan, Australia and Germany at the 2009 14th International Conference on the Teaching of Mathematical Modeling and Applications (ICTMA-14) in Germany (Kaiser, Blum, Ferri, & Stillman, 2011), I have no doubts
that mathematical modeling will continue to have lasting impacts on students’ mathematical knowledge.

Given the fact that mathematics education currently emphasizes engaging students in mathematical modeling instruction to understand problems of everyday life and society (Sharma, 2013; Lesh & Zawojewski, 2007; Vorhölter, Kaiser & Borromeo Ferri, 2014), I am optimistic that through awareness and research findings as this study indicate, teachers can get the necessary training and resources to be able to implement mathematical modeling instructional strategies in their classrooms. I do believe that if rational functions are considered as mathematical models of real-life situations, which students can relate to, students may be motivated to learn and understand mathematical concepts.

The literature also supports emerging themes from the qualitative analysis in this study. On the first theme which indicates that students tend to have positive views and display engaging and immersed attitudes towards learning mathematics in a modeling instructional setting, research studies (Prasad et al., 2014; Mensah et al., 2013) have pointed to a similar conclusion. Popham (2005) indicate that students who are taught mathematics through mathematical modeling tend to have positive attitudes towards mathematics, hence positive outcomes on students’ mathematical understanding and achievement. This means, that engaging students in a mathematics classroom has the potential of producing desirable outcomes in students’ perception of mathematics and help them to better understanding of mathematical concepts such as rational functions. According to Saha (2014), to educate students, more emphasis should be placed on developing positive attitude and analytic thinking skills in solving mathematical problems. Mensah et al. (2013) indicate that teachers’ positive attitudes, radiate confidence in students making them to develop positive attitude toward the learning of mathematics.
The second theme which is the idea that teacher guidance during the modeling process is supported by the literature and is not new. Teacher play a vital role in the modeling process. Some mathematical modeling activities can be challenging especially real-world context problems. As such students rely times on the guidance from the teacher to solve the problem for them, but to give more clarity to the problem. Kirschner, Sweller and Clark (2006) argued against minimal guidance during instruction, indicating that the advantage of guidance during instruction begins to diminish only when the learner has sufficiently prior knowledge to provide what they called “internal” guidance. Wethall (2011) indicated that transfer among mathematical concepts, new problems and contextual situations can occur, but requires guidance from the instructor to become a flexible process. Blum (2011) indicated that the role of teachers irreplaceable, suggesting some principles for teachers of mathematical modeling. He suggested that teachers should find a permanent balance between students’ independence and their guidance through flexibility and adaptive interventions and that teachers should support students’ individual modeling routes and encourage multiple solutions. He also called on teachers to foster enough student strategies for solving modeling tasks and stimulate different meta-cognitive activities like reflection on solution processes and on similarities between different situations and contexts.

Finally, the third theme which deals with the idea that mathematical modeling instruction tend to foster critical thinking and conceptual understanding of rational functions, increasing students’ representations capabilities and cognitive complexities is supported by the literature as well. According to Kertil and Gurel (2016), mathematical modeling is a bridge to the STEM education. They believe that mathematical modeling applications provide students with important local conceptual developments and meaningful learning of basic mathematical ideas in
real situations. Cognitive complexity deals with how well a person perceives and analyzes things, events or information based on how sophisticated their thinking has become. How well information is processed gets better with conceptual understanding. As such, when students have conceptual understanding of representations it does increase their cognitive abilities to process information thereby reinforcing the cognitive complexity of their representations.

Representations according to Seeger, Voight and Werschescio (1998) is “any kind of mental state with a specific content, a mental reproduction of a former mental state, a picture, symbol or sign, symbolic tool one has to learn their language, a something, “in place of” something else.” These definitions of representations and cognitive complexity suggests a linear relationship between them. Therefore, students who are better at multiple representations of functions tend to demonstrate a high level of cognitive complexity in their representations of functions.

**Recommendations for Future Research**

The research findings and discussion of this study are specific to this case study on Precalculus students at this one institution of learning in Southern United States with a limited sample size of 54 students. The limited sample size, according to Cates (2018) contribute to the lack of generalizability. Findings however, suggest important implications in the teaching and learning of rational functions and mathematics and covers a significant gap in the literature. The results may be of interest to students, teachers, mathematical curriculum developers, as well as all those interested in mathematical modeling hoping to help improve the learning experiences of their students.

For future research, I recommend carrying out this study with a larger sample size and in more than one educational institution. The study was carried out on Precalculus students at this college because rational functions were only taught in Precalculus. In some universities and
colleges, rational functions are taught in College Algebra. I also suggest conducting this same study to see the effects of modeling instruction on College Algebra students.

Furthermore, this study’s quantitative findings showed that mathematical modeling had a significant impact on students’ achievement and attitudes towards learning rational functions. The quantitative findings were collaborated with the qualitative inquiry through multiple sources of data including interviews, researcher’s memos, questionnaire and the attitude towards mathematics survey. Though my visits to the teacher’s classrooms were informal, I did collect some valuable information about the instruction in the form of memos. I am therefore suggesting more formal observation as a source of data collection with formal observation protocols put in place. I am also suggesting a study with a larger sample size on interviews and in different institutions as well. Future research may also want to look at the effects of modeling on rational functions on students with different socio-economic status and different ethnicities.

The duration for this study was five weeks. It will be interesting to find out what the results will be for a longer period. I am therefore suggesting an entire semester (3-4) months for future research on mathematical modeling instruction on rational functions or related subject.

**Limitations**

The study had several potential limitations, which were and should be taken into considerations with regards to the findings.

1. The fact that the different sections of the Precalculus (rational functions) were taught by different instructors may or may not have had the teacher effect on the outcome of this study. Different teachers provided instruction for the treatment group and the comparison group.
2. The sample size was affected by subject attrition as participants eventually dropped out for different reasons. The attrition rate for both the RFE and ATMI was 5.3% and 33.3% for the interviews. The research findings may not be generalizable as a case study specific to precalculus students at only one institution of learning in Southern United States with a limited sample size of 54 students. The limited sample size, according to Cates (2018) contribute to the lack of generalizability. The sample size for the interviews was also small. Only four participants were interviewed.

3. Cognitive complexity is a psychological variable that can have different meanings or definitions and hence not easy to measure or quantify.

Implications for the Future

The findings of this study have future implications in the areas of research, methodology and practice.

The practical implications are for teaching and learning of mathematics. This study offers teachers a researched based instructional strategy to be tried in their classrooms to motivate and help students stay engaged and enjoy doing mathematics. Since many high school and undergraduate teachers do not have the necessary skills and training to teach mathematical modeling in their classes, educational institutions will therefore need to invest in professional development to train teachers in modeling strategies so that instruction can be improved to help students especially here in the United States where our students continue to struggle in the STEM fields compared to other countries. According to Blum (2011), the students have a true picture of mathematics with a better understanding of the world around them when engaged in mathematical modeling. The students will not just be learning mathematics to pass an exam, they will be able to understand how to apply their mathematical knowledge to their daily lives.
Furthermore, results of this study also show that mathematical modeling instruction helps students with multiple ways of representations of rational functions and further reinforces their cognitive complexity. Teachers of mathematics now can seize this opportunity to help their students learn how to represent real-life situations in multiple ways knowing that this will help reinforce and strengthen their cognitive complexities.

In terms of research, this study bridges the gap that existed in the literature on mathematical modeling and rational functions. The literature on modeling with other classes of functions (linear, polynomial, exponential etc.) does exist. There is however, little or no research out there in mathematics education on the teaching of mathematical modeling with rational functions.

In research methodology, this study was conducted using mixed methods involving both quantitative and qualitative techniques which future studies can replicate. The advantage of a mixed method study is that it provides an opportunity to validate the study findings from multiple sources of data.

Finally, this study has societal, cultural and scientific benefits as well. College and university graduates are moving out to the society to put their knowledge into practice. Therefore, an instructional method like modeling that prepares students for to deal with real word situations, work collaboratively to solve problems is what our educational institutions should pay close attention to. According to Blum (2002), the real world are things concerning nature, society or culture, including subjects at all levels, scholarly and scientific disciplines other than mathematics. The use of the real-world context is an essential part of teaching mathematics for functional purposes and motivation of the students (Stacey, 2015). I am optimizing about the future given the findings of this study. I believe that when students are
given the right opportunities to develop their own competencies as does in a mathematical modeling instructional environment, students tend to have positive feelings and attitudes about what is being taught and they tend to succeed.

**Conclusion**

Putting both the quantitative and qualitative findings of this study together, the results from the quantitative and qualitative analyses of the data collected indicate that mathematical modeling instruction had positive effects on Precalculus students’ achievement, attitudes towards rational functions as well as the type and level of cognitive complexity of their representations of rational functions. The students who were taught rational functions through mathematical modeling performed better in the RFE posttest, showed positive attitudes toward mathematics from the ATMI survey and displayed a higher ability and confidence level in their representations than their counterparts in the traditional lecturing classroom.

Students in any classroom are there to acquire knowledge through the best means possible to grow, to succeed and to achieve their educational and career goals. They want to be motivated, empowered, guided as well as engaged in the learning process. It is therefore our duty as teachers to continue to improve our skills through education, research, professional development, and practice to provide the students the best learning experiences of their lives. As mathematics instructors and educators in general strive for new researched based strategies of impacting knowledge in their classrooms to ensure that their students are adequately equipped for the job market, it is becoming obvious that some of these strategies have profound impacts on students’ learning more than others.

This study employed both quantitative and qualitative techniques to investigate the effects of mathematical modeling instruction on precalculus students’ performance and attitudes.
towards rational functions. Findings from this studies and others (Blum, 2011; Nourallah & Farzad, 2012; Mubeen et al., 2013; Prasad et al., 2014; Mensah et al., 2013; Pawl et al., 2009; Dasher & Shahbari, 2015; Saha, 2014; Papageorgiou, 2009; Wedelin & Adawi, 2014, Jackson et al., 2008) show that mathematical modeling instruction is certainly one of such instructional strategies that has the potential of fostering conceptual understanding, changing students’ attitudes towards mathematics and helping them stay engaged, motivated and focused in the learning.

This study’s findings resulting from multiple sources of data (interviews, artifacts, research memos, questionnaire the RFE) provide some insight into the teaching and learning of rational functions, closing the gap in the literature in the areas of mathematical modeling instruction, rational functions, students’ achievement and students’ attitudes towards mathematics learning. It is my wish and suggestion that future research studies be conducted on same study with same designed with a larger sample size and in multiple institutional settings.

Finally, one of the themes from this study is that teacher-supported modeling instruction increases students’ cognitive level and types of representations of functions. This is an important finding in the sense that it highlights the key role that teachers play not only in a modeling instructional environment as seen here, but also in other instructional settings in different classrooms. This therefore suggests that teachers have and continue to hold the key to students’ success in any classroom because they determine the instructional strategy through which the students would be best served. An engaging, supportive and empowering instructional method from the teacher would surely leave lasting impressions on students’ learning and success. Mensah et al. (2013) indicate that teachers’ positive attitudes, radiate confidence in students making them to develop positive attitude toward the learning of mathematics.
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Sabella, C. Henderson, & C. Singh (Eds.), *AIP Conference Proceedings 1179*(1), 51-54.


APPENDICES

Appendix A

Pre/posttest - Rational Function Exam (RFE)

Participant Identification Code_________________________________________________

Date: ________________________________________________________________________

Time 40 minutes Show all your work.

1. Given the rational functions 
   \[ f(x) = \frac{x + 1}{x - 3} \] and \[ g(x) = \frac{3}{x + 4} \], find and simplify your solution

   a) \( f(x) - g(x) \)

   b) \( \frac{f(x)}{g(x)} \)

   c) Solve the rational equation \( f(x) = g(x) \)

   d) Solve the rational inequality \( f(x) < 0 \)
2. Given the function \( f(x) = \frac{(x - 2)(x + 3)}{(x - 1)(x + 2)(x - 5)} \)

   a) What is the domain and range of the function \( f(x) \)

   b) Find the vertical, horizontal and oblique asymptotes for the function \( f(x) \) if any.

   c) Find the \( x \) and \( y \) intercepts of the function \( f(x) \)
d) Sketch the graph of the function \( f(x) \) without using technology

3. Write an equation for the rational function with the following characteristics:
Vertical asymptotes at \( x = 5 \) and \( x = -5 \), \( x \) intercepts at \( (2, 0) \) and \( (-1, 0) \), \( y \) intercept at \( (0, 4) \)
4. Given this graph of a rational function \( f \)
   a) Write the equation of the function

   b) Describe the end behavior of the function \( f \) in words

5. A rare species of insect was discovered in the rain forest of Costa Rica. Environmentalists transplant the insect into a protected area. The population of the insect \( t \) months after being transplanted is

   \[ P(t) = \frac{45(1 + 0.6t)}{3 + 0.02t} \]

   a. What was the population when \( t = 0 \)?
   b. What will the population be after 10 years?
   c. When will there be 549 insects?
Appendix B

Attitude Toward Mathematics Inventory adopted from Tapia and Marsh (2004)

Directions: This inventory consists of statements about your attitudes toward mathematics. There are no correct or incorrect responses. Read each item carefully and think about how you feel about each item. Indicate the number that most closely corresponds to how each statement best describes your feelings. Please answer every question.

PLEASE USE THESE RESPONSE CODES:

1 = Strongly Disagree  
2 = Disagree  
3 = Neutral  
4 = Agree  
5 = Strongly Agree

<table>
<thead>
<tr>
<th>No</th>
<th>Statement</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>Mathematics is a very worthwhile and necessary subject.</td>
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<td>2</td>
<td>I want to develop my mathematical skills.</td>
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<td>3</td>
<td>I get a great deal of satisfaction out of solving a mathematics problem involving rational functions.</td>
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<td>4</td>
<td>Mathematics helps develop the mind and teaches a person to think.</td>
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<td>5</td>
<td>Mathematics is important in everyday life.</td>
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<td>6</td>
<td>Mathematics is one of the most important subjects for people to study.</td>
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<td>7</td>
<td>High school courses would be very helpful no matter what I decide to study.</td>
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<td>8</td>
<td>I can think of many ways that I use math outside of school</td>
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<td>9</td>
<td>Mathematics is one of my dreaded subjects.</td>
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<td>10</td>
<td>My mind goes black and I am unable to think clearly when working with mathematics.</td>
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<td>11</td>
<td>Studying mathematics makes me feel nervous.</td>
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<td>12</td>
<td>Mathematics makes me feel uncomfortable.</td>
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<td>13</td>
<td>I am always under a terrible strain in a math class.</td>
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<td>14</td>
<td>When I hear the word mathematics, I have the feeling of dislike.</td>
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<td>15</td>
<td>It makes me nervous to even think about having to do a mathematics problem.</td>
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<td>16</td>
<td>Mathematics does not scare me at all</td>
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<td>17</td>
<td>I have a lot of self-confidence when it comes to mathematics</td>
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<td>18</td>
<td>I am able to solve a mathematics problem without too much difficulty.</td>
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<td>19</td>
<td>I expect to do fairly well in any math class I take.</td>
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<td>20</td>
<td>I am always confused in my mathematics class.</td>
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<td>21</td>
<td>I feel a sense of insecurity when attempting mathematics.</td>
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<tr>
<td>22.</td>
<td>I learn mathematics easily.</td>
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<td>23.</td>
<td>I am confident that I could learn advanced mathematics.</td>
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<td>24.</td>
<td>I have usually enjoyed studying mathematics in school.</td>
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<td>25.</td>
<td>Mathematics is dull and boring.</td>
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<td>26.</td>
<td>I like to solve new problems in mathematics.</td>
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<td>27.</td>
<td>I would prefer to do an assignment in math than to write an essay.</td>
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<td>28.</td>
<td>I would like to avoid using mathematics in college.</td>
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<td>29.</td>
<td>I really like mathematics.</td>
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<td>30.</td>
<td>I am happier in a math class than in any other class.</td>
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<tr>
<td>31.</td>
<td>Mathematics is a very interesting subject.</td>
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<td>32.</td>
<td>I am willing to take more than the required amount of mathematics.</td>
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<td>33.</td>
<td>I plan to take as much mathematics as I can during my education.</td>
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<td>34.</td>
<td>The challenge of math appeals to me.</td>
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<td>35.</td>
<td>I think studying advanced mathematics is useful.</td>
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<td>36.</td>
<td>I believe studying math helps me with problem solving in other areas.</td>
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<td>37.</td>
<td>I am comfortable expressing my own ideas on how to look for solutions to a difficult problem in math.</td>
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<td>38.</td>
<td>I am comfortable answering questions in math class.</td>
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<td>39.</td>
<td>A strong math background could help me in my professional life.</td>
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<td>40.</td>
<td>I believe I am good at solving math problems.</td>
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</table>

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Appendix C

Research Questionnaire

Thanks for providing your candid responses to the following questionnaire. There are 6 questions intended to learn about your experience with the way you have just learned rational functions.

1. What is your ethnicity and gender?
   Ethnicity: _____________________________ Gender____________________

2. What is your major and first language?
   Major: ________________________________ First language________________

3. Have you been taught rational functions before this study? If so, where, when and how was your experience with rational functions then?

4. How do you feel about rational functions now after the lessons you just received?

5. Describe one aspect of the instruction that you find helpful to you.

6. Describe any barriers (if any) that you encountered during this study session
7. On a scale of 1 to 5 rate your level of satisfaction with the way you were taught rational functions, 1 being the least satisfied and 5 being extremely satisfied.

8. Would you recommend a friend or someone to a school that teaches rational functions the way you have been taught? Yes/No. Please explain your answer.

9. Is there anything you would like to say or add?

Thank you for your time.
Appendix D: Interview Protocols

I want to thank you for taking the time to meet with me today for this interview. My name is Solomon Betanga and the purpose of this short semi structured interview is to understand about your learning experience during this 5-weeks study of rational functions. This information will be used in my dissertation study. I will be audio recording this interview and the transcripts will be submitted to you for your review before I use it in my study. Please answer in as more details as you like. This interview should take approximately 20 to 25 minutes.

Please be aware that all your responses will be kept confidential and will only be used for this study. Also, note that you are not obliged to say anything you do not want to, and you may end the interview at any time.

Questions

1. Have you had a lesson on rational functions before this study? If yes, where and when?
2. Describe aspects of the instruction that you found helpful to you.
3. Describe aspects if any of the instruction that you did not like
4. Describe the things you liked about the way the lessons on rational functions were presented to you.
5. Describe any barriers of difficulties (if any) that you encountered during this study session.
6. Would you say that you have more understanding of rational functions now than before? If yes or no, explain you answer.
7. Are you more confident of yourself now to handle rational function problems? If yes or no, explain.
8. On a scale of 1 to 5 rate your level of satisfaction with the way you were taught rational functions, 1 being the least satisfied and 5 being extremely satisfied.

9. Would you recommend a friend or someone to a school that teaches rational functions the way you were taught? Yes/No. Please explain your answer.

10. Is there anything you would like to say or add?

Thank you for your time.
Appendix E

Informed Consent for Students Participants

Title: A Research Study on the Effects of Mathematical Modeling on Precalculus Students’ Performance and Attitudes towards Rational Functions.

Principal Investigator: Dr. Iman Chahine
Student Principal Investigator: Solomon Betanga

Purpose:
The purpose of this research study is to find out if there is a significant difference between the performance of Precalculus students who are taught rational functions through mathematical modeling and those who are taught rational functions through the traditional lecturing method.

You are invited to take part in the study because you are Precalculus students this semester. A total of 60 people will be invited to take part in this research study.

Procedure:
If you decide to take part in this research study, you will complete the following assessments administered by the researcher and the data will be used for this study.

- A pretest and a posttest on Rational Functions.
- A pretest and a posttest on an attitude towards mathematics survey.
- A questionnaire on your thoughts about the instructional method used to teach you rational functions.

Future Research:
Researchers will not use or distribute your data for future research study.

Risks:
In this research study, will not have any more risks than you would in a normal day of life.

Benefits:
This research study is not designed to benefit you personally. Overall, we hope to gain information about the teaching and learning of rational functions.

Alternatives:

If you decide not to take part in this research study, you will be given different problems not related to this research study to work on during this class time.

Voluntary Participation and Withdrawal:
You do not have to be in this research study. If you decide to be part and change your mind, you have the right to drop out at any time. You may skip questions or stop participating at any time.

Confidentiality:
We will keep your records private to the extent allowed by the law. The following people and entities will have access to the information you provide:

- Primary investigator (P.I.) Dr. Iman Chahine and the student P.I. Solomon Betanga.
- GSU Institutional Review Board.

We will use pseudonyms rather than your name on the records. The information you provide will be stored. When we present or publish the results, we will not use your name or other information that may identify you.

Contact Persons:

Contact the Primary Investigator Dr. Iman Chahine at ichahine@gsu.edu and the student primary investigator Solomon Betanga at sbetanga1@student.gsu.edu.

- If you have questions about the research study or your part in it
• If you have questions, concerns, or complaints about the research study

Contact the GSU Office of Human Research Protection at 404-413-3500 or irb@gsu.edu

• If you have questions as a research participant

• If you have questions, concerns, or complaints about the research study

Consent:
You will get a copy of the consent to keep.
If you are willing to volunteer for this study, please sign below:

___________________________________________  ____________________
Name of participant

___________________________________________  ____________________
Signature of participants  Date

___________________________________________  ____________________
Researcher obtaining consent  Date
Appendix F

Recruitment Script to be Read to the Students

Hello students, my name is Solomon Betanga. I am a lecturer of mathematics here at Gordon State College. I am conducting a research study to find out if there is a significant difference between the performance of Precalculus students who are taught rational functions through mathematical modeling and those who are taught rational functions through the traditional lecturing method. You are invited to take part in the study because you are Precalculus students.

If you decide to take part in the study, you will complete the following assessments administered by the researcher and the data will be used for this study.

- A pretest and a posttest on Rational Functions.
- A pretest and a posttest on an attitude towards mathematics survey.
- A questionnaire on your thoughts about the instructional method used to teach you rational functions.

If you decide not to take part in this research study, you will be given different problems not related to this research study to work on during this class time.

Participation in this study is strictly voluntary and your identity will remain confidential during and after the study since you will not be using your real names on any assessment.

If you have any questions and would like to participate in the study, you can ask me when I give you the consent form to indicate whether you want to be part of the research study.

Thank you for your participation.
Appendix G

Artifacts of Students’ Work

Artifact of Student 1 from treatment group

Artifact of Student 24 from treatment group
Artifact of Student 5 from treatment group

Artifact of Student 6 from treatment group

Researchers’ memo