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POLARIZATION ROTATION STUDY OF MICROWAVE INDUCED MAGNETORESISTANCE OSCILLATIONS IN THE GAAS/ALGAAS 2D SYSTEM

by

HAN-CHUN LIU

Under the Direction of Ramesh Mani, Ph.D

ABSTRACT

Previous studies have demonstrated the sensitivity of the amplitude of the microwave radiation-induced magnetoresistance oscillations to the microwave polarization. These studies have also shown that there exists a phase shift in the linear polarization angle dependence. But the physical origin of this phase shift is still unclear. Therefore, the first part of this dissertation analyzes the phase shift by averaging over other small contributions, when those contributions are smaller than experimental uncertainties. The analysis indicates nontrivial frequency dependence of the phase shift. The second part of the dissertation continues the study of the phase shift and the results suggest that the specimen exhibits only one preferred radiation orientation for different Hall-bar sections. The third part of the dissertation summarizes our study of the Hall and longitudinal resistance oscillations induced by microwave frequency and dc bias at low filling factors. Here, the phase of these resistance oscillations depends on the contact pair on the device, and the period of oscillations appears to be inversely proportional to radiation frequency.
INDEX WORDS: GaAs/AlGaAs heterostructures, Two-dimensional electron system, Magneto-transport, Microwave radiation, Microwave linear polarization, phase shift, dc bias, magnetoresistance oscillations, Shubnikov de Haas oscillations
Polarization rotation study of microwave induced magnetoresistance oscillations in the GaAs/AlGaAs 2D system

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HAN-CHUN LIU

A Dissertation Submitted in Partial Fulfillment of the Requirements for the Degree of Doctor of Philosophy in the College of Arts and Sciences Georgia State University

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by

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Georgia State University
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DEDICATION

To God, my family, my wife Sarah, and myself ten years ago.
I would like to thank my advisor, Dr. Ramesh Mani, for providing such a free research atmosphere and teaching assorted research-related techniques in terms of hardware skills, design of experiment, background knowledge, etc. Outside the research, I also have gained some helpful life experience from him. Then, I would also like to thank my committee members, Dr. Vadym Apalkov, Dr. Alexander Kozhanov, and Dr. Unil Perera, for their time and the guidance. I am grateful to former lab members, Dr. Aruna Ramanayaka and Dr. Tianyu Ye, for the training and their help in everything. Plus, I am thankful to current lab members, especially to Zhuo Wang and Rasanga Samaraweera that we enrolled into Georgia State University together, for encouraging and helping each other to overcome the hard times. In addition, I want to specially thank workshop manager Peter Walker, staff Dwayne Torres and Samuel Mayberry for their helps of the design and fabrication of mechanical and electronic pieces. Also, I want to acknowledge our funding agencies, Army Research Office and Department of Energy, for their financial support. Finally, I would like to thank God, my parents, my brother, my wife Sarah, my teachers, and my friends for everything that supported me to achieve this milestone.
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Chapter 1

INTRODUCTION

This dissertation comprises seven chapters. The theoretical and experimental backgrounds of the research are introduced and described in Chapter 2 and 3. My research is mainly divided into two major aspects in three chapters. The first major aspect is the analysis of angular phase shift observed in radiation polarization angle dependence in microwave-induced magnetoresistance oscillations in Chapter 4 and 5. The second major aspect concerns the quantum oscillations occurring owing to external ac and dc excitations.

In Chapter 2, I will start with the introduction of two-dimensional electron system, including size quantization, Landau quantization, and the formation of two-dimensional electron system in GaAs/AlGaAs heterostructure. And then, Hall effect is introduced since electron behavior is hugely influenced by magnetic field. The concepts of diagonal and Hall voltage are created because of Hall effect. After that, I introduce the quantized Hall effects, which are observed in two-dimensional electron system. Here, I will discuss the most two famous integer and fractional quantum Hall effects.

In Chapter 3, the measurements of microwave-induced magnetoresistance oscillations and the phenomenon of sensitivity to linear polarization are described. The linear polarization sensitivity of microwave-induced magnetoresistance is the main topic of the first research aspect. And then, numerous theoretical approaches for microwave-induced magnetoresistance oscillations are reported.

In Chapter 4, first, we determine experimental uncertainties with the help of the power detector prior to the measurements. Then, we analyze the phase shift ruling out the experimental uncertainties by averaging over the phase shift obtained from opposite sides of Hall-bar device. The results indicate nontrivial microwave frequency dependence of the phase shift over $32 \leq f \leq 44 \text{ GHz}$. 
In Chapter 5, continuing the work of Chapter 4, the phase shifts extracted from different Hall-bar sections are studied over a quasi-continuous microwave frequency band over $36 \leq f \leq 40 \text{ GHz}$. Meanwhile, \textit{in-situ} measurement of incident radiation polarization orientation is carried out by a carbon sensor near the sample to determine the experimental uncertainties. By averaging over the phase shift over the frequency band to reduce the influence of experimental uncertainties, remarkably, the average phase shifts obtained from various Hall-bar sections are nearly the same.

In Chapter 6, we discuss the observation of magnetic field-periodic Hall and longitudinal resistance oscillations excited by \textit{ac}, i.e., radiation, and \textit{dc} bias at high magnetic field. We find that the relative phase is either zero degrees or one-hundred-eighty degrees between various Hall and longitudinal resistance oscillations.

Finally, in Chapter 7, the conclusions of all the research works as well as future prospective experiments are presented.
Chapter 2

QUANTUM HALL EFFECTS

2.1 Background

In 1879, Edwin Herbert Hall found that in a planar conductor with its normal oriented along the z-direction, when a current is applied in the x-direction, and a magnetic field is applied in the z-direction, a voltage appears along the y-direction. This phenomenon is well-known as the Hall effect [1] and the voltage in \( y \) direction is called the Hall voltage. The Hall effect provides a method to determine the dominant carriers in a semiconductor. Nearly one hundred years later, a new remarkable phenomenon was discovered by K. von Klitzing in 1980. This was the discovery of integer quantum Hall effect (IQHE) [2–4]. When a two-dimensional electron system (2DES) is subject to strong magnetic fields at low temperature, plateaus in the Hall resistance that went together with vanishing longitudinal resistance were observed, and the plateaus of Hall resistance were reported at \( h/Ne^2 \), with \( N \) integer. Interestingly, the fine structure constant, \( e^2/\hbar c \), can be determined precisely from such experiment. Thus, K. von Klitzing won 1985 Nobel Prize in Physics because of this finding. After that, many interesting phenomena were reported, such as the fractional quantum Hall effect (FQHE) and the Wigner crystal. FQHE was the term used to describe the discovery by D. Tsui, H. Störmer and A. Gossard in 1982 [5]. Similar to IQHE, the vanishing longitudinal resistance went together with new plateaus in the Hall resistance. Next year, R. Laughlin proposed variational ground-state and excited-state wave functions to describe FQHE [6]. In 1998, R. Laughlin, H. Störmer, and D. Tsui were awarded Nobel Prize in Physics. Wigner crystal [7, 8] is the ground state of the ideal FQHE in the infinite magnetic field limit, i.e. an ordered array of electrons. A Wigner crystal on the surface of helium was reported by Grimes and Adams in 1979 [9]. According to Laughlin [4], quantum Hall effects can be considered as a macroscopic quantum phenomena in terms of a single
electron or quasi-particle in Landau levels in the presence of magnetic fields, lattice, electric fields and the interaction between these factors.

2.2 Two Dimensional Electron System

2.2.1 Background

A 2-dimensional electron system (2DES) is a system of electrons strictly confined to a 2D plane. Simply speaking, electrons are mobile on $xy$ plane but quantized in $z$ direction, which results in energy level splitting along $z$ direction. Similarly, a two-dimensional hole system is a system of holes confined to a 2D plane.

The study of 2DES was initially carried out in a $n$-channel Si metal-oxide-semiconductor field-effect-transistor (MOSFET) [10]; see Figure 2.1. Here, insulating SiO$_2$ separates Si from a metal gate. When a positive bias is applied through the metal gate, holes are driven away from the Si/SiO$_2$ interface. Once the threshold voltage breaks, it produces a narrow inversion layer of electrons at the Si/SiO$_2$ interface, which connects source and drain as shown in Figure 2.2. Normally, the thickness of the inversion layer is of the order of 3-10 nm. The electrons are tightly confined at the Si/SiO$_2$ interface.

![Figure 2.1. Schematic diagram of an $n$-channel MOSFET.](image)

Other common methods to realize 2DES are rectangular quantum wells and heterostructure. The idea is to utilize the difference between conduction bands of two semiconductor materials to confine electrons at the interface. One of the advantages of this method is that
Figure 2.2. Energy levels of MOSFET for the case (a) without gate bias and (b) with gate bias $V_G$. When $V_G$ is greater than threshold voltage, the inversion layer establishes and 2DES is at the interface of Si/SiO$_2$. 
high mobility 2DES is easily to be achieved in quantum wells and heterostructure [11]. Recently, 2DES is also discovered in a new kind of oxide heterostructure, e.g., ZnO/MgZnO. Although it has two insulating oxides on both side, 2DES is still formed at the interface [12]. For the studies reported in this dissertation, the specimens are fabricated from GaAs/AlGaAs heterostructure. Therefore, semiconductor heterostructure will be discussed more in Section 2.2.4.

2DES can be established on the surface of a material as well. For example, electrons can move freely on the surface of liquid helium, but still cling to liquid helium [13]. Another popular material is topological insulators [14], e.g. Bi$_2$Te$_3$. It is a material where the interior acts like an insulator but it has conducting states on the surface. Hence, free electrons are restrained to the surface.

Recently, a new material has been developed that is atomically thin material sheet, such as graphene [15]. Graphene is an one-atom-level carbon sheet and 2DEG or 2DHG can be adjusted in terms of applying a gate bias or doping chemically. In such a structure, electrons are confined to 2D material plane.

2.2.2 Size and energy quantization

We suppose that an electron is confined in a simple square well of width $w$ along $z$ direction with the infinite potential energy $V(z)$. According to fundamental quantum mechanics, the $z$ component of eigenfunction and associated eigenvalue for one electron is [16]

$$\phi_\alpha(z) = \left(\frac{2}{w}\right)^{1/2}\sin\left[\frac{(\alpha + 1)\pi z}{w}\right]$$

$$\varepsilon_\alpha = \frac{\hbar^2(\alpha + 1)^2\pi^2}{2m^*w^2}$$

(2.1)

(2.2)

where $\alpha$ is the quantum number with $\alpha = 0$ is ground state and $m^*$ is the effective mass. Considering $x$ and $y$ directions, the total energy for one electron in a $z$-direction confined space is

$$E = \varepsilon_\alpha + \frac{\hbar^2k^2}{2m^*}$$

(2.3)
and $k^2 = k_x^2 + k_y^2$. $\varepsilon_\alpha$ is the discrete electron energy level owing to size quantization and $\hbar^2 k^2 / 2m^*$ is the electron momentum on $xy$ plane. The density of state (DOE) of 2DES due to size quantization effect in subband $\alpha$ is sketched in Figure 2.3. DOE increases from lower to higher energy subband.

![Figure 2.3. Schematic diagram of 2DES density of state due to size energy quantization effect. DOS increases with higher quantized energy level.](image)

Many factors influence the observation of energy quantization. Size confinement is just one of them. For example, if thermal energy is greater than energy splitting, i.e., $k_B T \gg \varepsilon_{\alpha+1} - \varepsilon_\alpha$, then energy quantization is negligible with respect to thermal energy. Or in a real 2D system, electron scattering happens because of defects, impurities, etc, that can be described by relaxation time, $\tau$. According to Heisenberg uncertainty principal, $\Delta E \sim \hbar / 2\tau$. Once $\Delta E \gg \varepsilon_{\alpha+1} - \varepsilon_\alpha$, i.e., $\tau \ll 1$, the occurrence of scattering would diminish quantization effect. Therefore, low-temperature environment, high-quality specimen, small layer thickness, and proper carrier concentration are general requirements to achieve size quantization effect.

### 2.2.3 Landau quantization

Here, we consider a situation that 2DES is under an external magnetic field along $z$ direction, i.e., $B = (0, 0, B)$, which is perpendicular to the surface of 2DES. In this case,
electrons make cyclotron motion. The Hamiltonian of the electron can be written as [17]

\[ H = \frac{1}{2m^*} \left[ (\mp i\hbar \partial_x + eA_x)^2 + (\mp i\hbar \partial_y + eA_y)^2 \right] \]

(2.4)

where \( A_k \) is the vector potential of electromagnetic field. Then we define covariant momentum

\[ P_x \equiv -i\hbar \partial_x + eA_x, \quad P_y \equiv -i\hbar \partial_y + eA_y \]

(2.5)

and the guiding-center coordinate

\[ X \equiv x + \frac{1}{eB} P_y, \quad Y \equiv y - \frac{1}{eB} P_x \]

(2.6)

The guiding center \((X,Y)\) and the covariant momentum \((P_x,P_y)\) are independent variables because they obey

\[ [X,Y] = -i\ell_B^2, \quad [P_x, P_y] = \frac{\hbar^2}{\ell_B^2} \]

(2.7)

\[ [X, P_x] = [X, P_y] = [Y, P_x] = [Y, P_y] = 0 \]

(2.8)

where \( \ell_B = \sqrt{\hbar/eB} \) is magnetic length or characteristic size of an electron orbit. Since \( X \) and \( Y \) do not commute, \( [X,Y] = -i\ell_B^2 \), \( 2\pi\ell_B^2 \) is the most accurate area that the electron position can be determined. Thus, two pairs of operators can be defined in terms of these variables,

\[ a \equiv \ell_B \sqrt{\frac{\ell}{2\hbar}}(P_x + iP_y), \quad a^\dagger \equiv \ell_B \sqrt{\frac{\ell}{2\hbar}}(P_x - iP_y) \]

(2.9)

\[ b \equiv \frac{1}{\sqrt{2\ell_B}}(X - iY), \quad b^\dagger \equiv \frac{1}{\sqrt{2\ell_B}}(X + iY) \]

(2.10)

Here, \( a, b \) are annihilation operators and \( a^\dagger, b^\dagger \) are creation operators. They are two independent harmonic oscillators. Then the Hamiltonian (2.4) becomes

\[ H = (a^\dagger a + aa^\dagger) \frac{\hbar \omega_c}{2} = (a^\dagger a + \frac{1}{2}) \hbar \omega_c \]

(2.11)
with the cyclotron frequency, \( \omega_c = eB/m^* \). This is exactly the Hamiltonian of quantum harmonic oscillators and the eigenvalue is

\[
\varepsilon_N = (N + \frac{1}{2})\hbar\omega_c
\]  

(2.12)

They are Landau levels and \( N = 0, 1, 2, \cdots \) is the quantum number for Landau levels. In addition, while electron spin degree is involved, using Zeeman Hamiltonian

\[
H_z = -g^*\mu_B \vec{B} \cdot \vec{s}
\]

(2.13)

where \( g^* \) is the effective Lande g-factor, \( \mu_B = e\hbar/2m^* \) is Bohr magneton. Then the eigenvalues are

\[
\varepsilon_z = \pm \frac{1}{2}g^*\mu_B B
\]

(2.14)

where minus sign for spin up, \( s = 1/2 \), and plus sign for spin down, \( s = -1/2 \). As a result, the final energy spectrum including size quantization, Landau quantization, Zeeman effect, and classic kinetic energy becomes

\[
E_f = \varepsilon_\alpha + \varepsilon_N + \varepsilon_z
\]

\[
= \frac{\hbar^2(\alpha + 1)^2\pi^2}{2m^*w^2} + (N + \frac{1}{2})\hbar\omega_c + \frac{1}{2}g^*\mu_B B
\]

(2.15) \hspace{1cm} (2.16)

The DOS of 2DES ground state, \( \varepsilon_0 \), without external \( B \) can be written as

\[
D(E) = 2 \times \frac{1}{(2\pi)^2} \times 2\pi|k|\frac{d|k|}{dE} = \frac{m^*}{\pi\hbar^2}
\]

(2.17)

with \( dE = \hbar^2|k|d|k| \); shown in Figure 2.4(a) \[18\]. Note that, typically, the energy separation due to size confinement is considerably greater than Fermi energy, \( E_F \). Hence, here, we only take into account ground state when \( B \) is applied. Figure 2.4(b) shows Landau level splitting when \( B \) is applied and spin is not taken into account. Each Landau level separates by \( \hbar\omega_c \).
Figure 2.4. Schematic diagram of 2DES density when (a) without $B$ (b) with $B$ but $g^* = 0$ and (c) with $B$ and $g^* \neq 0$. Without $B$, DOS of ground state energy is continuous and constant; see panel (a). With $B$ but without considering spin degree, continuous DOS becomes discrete Landau levels with energy separation of $\hbar \omega_c$; see panel (b). Take into account electron spin, each Landau level splits into two subbands with half degeneracy of Landau levels that ignore electron spin; see panel (c).
The degeneracy of Landau levels become

\[ D(E) = 2 \times \frac{1}{2\pi \ell_B^2} = \frac{2eB}{h} \]  

(Eq.(2.12)). The degeneracy of Landau levels become derived by the definition of \(2\pi \ell_B^2\). When electron spin is taken into account, Zeeman effect can be observed in Landau quantization; see Figure 2.4(c). Based on Eq.(2.14), the spin-up (↑) and spin-down (↓) Landau levels have energy separation of \(g^*\mu_B B\) and the degeneracy of spin-split Landau levels is \(eB/h\), which is half of degeneracy of non spin-split Landau levels.

Since the degeneracy of spin-split Landau levels depends on \(B\), when \(B\) increases, to keep entire degeneracy of 2DES constant, \(E_F\) has to move so that fewer and fewer Landau levels are occupied. Thus, the number of occupied spin-split Landau levels at a given \(B\) is defined as

\[ \nu = \frac{n}{eB/h} = \frac{nh}{eB} \]  

where \(n\) is carrier density and \(\nu\) is called filling factor.

### 2.2.4 GaAs/AlGaAs heterostructure

In GaAs/AlGaAs heterostructure, generally, 2DES forms at the interface with Si atoms doped in AlGaAs with certain distance away from the interface; shown in Figure 2.5(a). The system is called modulation-doped heterostructure [11, 19]. Figure 2.5(b) exhibits the thermal-equilibrium energy diagram of GaAs/AlGaAs heterostructure. In the process of attaining equilibrium, it is energetically favorable for the electrons from AlGaAs to fall into the GaAs well. The charge redistribution builds an electrostatic field and bends the band edges until Fermi level becomes constant across the structure. The formation of the electric field also limits the amount of electrons from wide-bandgap material (AlGaAs) to narrow-bandgap material (GaAs). In such a case, 2DES or 2DHS is able to be realized in terms of modulation-doped technique by doping either donors (\(n\) type) or acceptors (\(p\) type). However, the existence of donor ions substantially reduces the electron mobility owing to large scattering rates. To solve this issue, the impurities are embedded spatially
separated from the interface by $\sim 100 \text{ nm}$. This is called spacer (Figure 2.5(b)). As a result, the electron-impurity scattering rate dramatically drops due to sufficient spacer and high electron mobility ($> 10^7 \text{ cm}^2\text{V}^{-1}\text{s}^{-1}$) can be achieved. Further, in an ideal 2DES, an electron gas is completely restricted on a 2D plane. Yet, practically, electrons are confined in a finite layer $\sim 10 \text{ nm}$.

Figure 2.5. (a) The illustration of GaAs/AlGaAs heterostructure. 2DES forms at the interface and Si atoms is usually doped in AlGaAs material with certain distance. (b) The energy diagram of GaAs/AlGaAs heterostructure after thermal equilibrium. During equilibrium, electrons (black dots) transfer to lower conduction band and accumulate on the narrow-bandgap side near the interface.
2.3 Hall Effect and the Drude Model

If a planar metal or semiconductor including mostly electrons is subject to an $x$-direction current and an $z$-direction magnetic field, due to Lorentz force [20]

$$\vec{F} = -e\vec{v} \times \vec{B} \quad (2.20)$$

where $e$ is the electron charge and $\vec{v}$ is the velocity of electrons. A voltage appears in the $y$ direction and the $y$-direction voltage is called Hall voltage. Based on Newton’s law of motion,

$$\vec{F} = m^* \frac{d\vec{v}}{dt} = \frac{d\vec{p}}{dt} = \hbar \frac{d\vec{k}}{dt} \quad (2.21)$$

Substituting Eq.(2.21) into Eq.(2.20), it becomes

$$\hbar \frac{d\vec{k}}{dt} = -e(\vec{E} + \vec{v} \times \vec{B}) \quad (2.22)$$

When scattering is taken into account, the electron motion in a finite life time, $\tau$, at $B = 0$ is

$$m^* \vec{v} = h\vec{k} = -e\vec{E}\tau \quad (2.23)$$

Thus, the electron velocity due to scattering can be written as

$$\vec{v} = -\frac{e\vec{E}\tau}{m^*} \quad (2.24)$$

where $\tau$ is called relaxation time. Therefore, the force on the electron is

$$m^*(\frac{d\vec{v}}{dt} + \frac{\vec{v}}{\tau}) = -e(\vec{E} + \vec{v} \times \vec{B}) \quad (2.25)$$

We take the steady state with $d\vec{v}/dt = 0$ so that

$$\vec{v} = -\frac{e\tau}{m^*}(\vec{E} + \vec{v} \times \vec{B}) \quad (2.26)$$
or

\[ v_x = -\frac{e\tau}{m^*} E_x - \omega_c \tau v_y \]  
\[ v_y = -\frac{e\tau}{m^*} E_y + \omega_c \tau v_x \]  

where cyclotron frequency \( \omega_c = eB/m^* \). To solve Eq.(2.27) and (2.28), they yield

\[ v_x = \frac{1}{1 + \omega_c^2 \tau^2} \left( -\frac{e\tau}{m^*} E_x + \frac{e\omega_c \tau^2}{m^*} E_y \right) \]  
\[ v_y = \frac{1}{1 + \omega_c^2 \tau^2} \left( -\frac{e\tau}{m^*} E_y - \frac{e\omega_c \tau^2}{m^*} E_x \right) \]

Since current density is \( j = -nev = \sigma E \), where \( n \) is electron density, the conductivity tensor is as

\[ \sigma = \frac{\sigma_0}{1 + \omega_c^2 \tau^2} \begin{pmatrix} 1 & -\omega_c \tau \\ \omega_c \tau & 1 \end{pmatrix} \]  

with \( \sigma_0 = ne^2 \tau/m^* \), the conductivity in the absence of \( B \). Here, \( \sigma_{xy} = -\sigma_{yx} \) means the reversal of \( E_y \) direction, which also can be reversed by tuning \( B \) direction, i.e. \( \sigma_{xy}(B) = \sigma_{yx}(-B) \). Similarly, by the definition of resistivity \( \rho = E/j \),

\[ \rho = \frac{1}{\sigma_0} \begin{pmatrix} 1 & \omega_c \tau \\ -\omega_c \tau & 1 \end{pmatrix} \]  

Eq.(2.31) and (2.32) directly leads to

\[ \rho_{xx} = \frac{\sigma_{xx}}{\sigma_{xx}^2 + \sigma_{xy}^2}, \quad \rho_{xy} = \frac{\sigma_{xy}}{\sigma_{xx}^2 + \sigma_{xy}^2} \]  

When electrons keep being deflected owing to Lorentz force, the accumulation of electrons creates an transverse field (also called Hall field) \( E_y \) along \( y \) direction. Eventually, electric force will balance Lorentz force and no more electrons accumulate [21]. At equilib-
rium state, i.e., $j_y = 0$, Eq.(2.27) becomes

$$\rho(B) = \frac{E_x}{j_x} = \frac{1}{\sigma_0}$$

(2.34)

where $\rho(B)$ is magneto-resistivity, and it is surprisingly $B$-field independent. Since $E_y$ balances Lorentz force, which is proportional to $B$ and $j_x$, a quantity is defined as

$$R_H = \frac{E_y}{Bj_x}$$

(2.35)

where $R_H$ is known as Hall coefficient. Then setting $j_y = 0$ in Eq.(2.28),

$$E_y = -\frac{1}{ne}Bj_x$$

(2.36)

Therefore, the Hall coefficient is

$$R_H = -\frac{1}{ne}$$

(2.37)

The results strikingly suggest that $R_H$ depends on no parameters of materials but the carrier density, and the genre of carriers can be determined by sign of Hall coefficient. Note that, in the absence of $B$, $\vec{j}$ is collinear with $\vec{E}$. However, when $B$ is applied, due to Lorentz force, $\vec{j}$ is at an angle, $\theta$, to $\vec{E}$. This angle is called Hall angle, which can be described as $\tan\theta = \omega_c \tau$.

2.4 Integer Quantum Hall Effect

2.4.1 The measurement

In Section 2.2.3, we mentioned that if a 2DES is subject to a perpendicular $B$, the ground state energy splits into Landau levels. Figure 2.6(b) shows a typical plot that appears in the literature showing quantum Hall transport in a GaAs/AlGaAs device in the millikelvin range. Although the measured quantities are resistances, the plot shows resistivities. The transformation of resistances to resistivities assumes uniform bulk current flow. Under
quantum Hall conditions, this aspect is questionable especially if one subscribes to edge current model where the current flows along the edges, bypassing the bulk [22, 23]. Figure 2.6 shows: (a) plateaus in $\rho_{xy}$ where it remains constant as $B$ varies correspond to vanishing $\rho_{xx}$. (b) $\rho_{xx}$ exhibits a genre of quantum oscillations. They are Shubnikov de Haas oscillations (SdHOs). (c) The interesting aspect is that, in experiment, the Hall resistance is quantized in units of $h/e^2$. The mechanism of IQHE can be explained in light of Landau quantization and Laughlin’s gauge argument theory [4].

![Figure 2.6](image)

Figure 2.6. (a) The schematic diagram of quantum Hall effect measurement. 2DES is put in a perpendicular $B$ with a current, $I$, flowing. The longitudinal voltage, $V_{xx}$, and Hall voltage, $V_{xy}$, are measured. Here, the separation between $V_{xx}$ probes is $L$ and $V_{xy}$ probes is $W$. (b) Sketch of the IQHE. The longitudinal, $\rho_{xx}$, and Hall, $\rho_{xy}$, resistivity are plotted as a function of magnetic fields in GaAs/AlGaAs heterostructures in the milli-kelvin range. Courtesy of D. R. Leadley, Warwick University 1997

In an ideal 2DES, i.e., in the absence of impurities, Landau levels are represented by delta functions. However, in a real 2DES, when impurities exist, Landau levels broaden.
Figure 2.7 shows Landau levels when impurities are taken into account, Landau levels consist of two types of states: extended states (white regions) and localized states (gray regions) [4, 24]. The extended states are near the core of original Landau levels and responsible for the current transport. When the occupancy of extended states remains unchanged upon changing magnetic field or filling factor, the IQHE is observed. Localized states do not carry current but they act as an electron reservoirs. When $E_F$ is in between Landau levels, the DOS of extended states is increased with increasing $B$.

![Diagram of Landau levels showing extended and localized states](image)

Figure 2.7. The sketch of broadened Landau levels due to impurities. The extended states (white regions) carry current and the localized states (gray regions) provides localized electrons to fill up extended states when $E_F$ is in the Landau level gap.

Figure 2.8 qualitatively illustrates the extent of Landau level splitting as $B$ increases. First, in the absence of $B$, only size quantization is taken into account. The lowest subband is occupied upto the Fermi energy, $E_F$; in panel (a). When a small $B$ is applied, Landau quantization starts in effect, but Landau levels still severely overlap each other, as shown in panel (b). At this point, IQHE is still obscure. Once $B$ is strong enough to split Landau levels completely, IQHE starts showing up due to the discussion associated with Figure 2.7. Panel (c) is the scenario that when $E_F$ is crossing the extended states of a Landau level, $\rho_{xx}$ oscillates and $\rho_{xy}$ increases linearly. Fewer Landau levels are occupied as $B$ increases. Panel (d) represents the point when $\rho_{xx}$ vanishes and $\rho_{xy}$ exhibits plateau. It is when $E_F$ lies in the localized states in between Landau levels. According to Pauli exclusion principle, electron
Figure 2.8. The illustrations of Landau level splitting as $B$ increasing. (a) Without $B$, only ground state energy due to size confinement is considered. (b) Landau levels start splitting with a small external $B$, but they still overlap each other. (c) Landau levels separate completely as $B$ is strong enough. It shows the moment that $E_F$ is crossing a Landau level. (d) As $B$ keeps increasing, the separation between Landau levels becomes wider. When $E_F$ is in between Landau levels, vanishing $\rho_{xx}$ and plateaus in $\rho_{xy}$ are observed.
transition from one state to another state is forbidden. Therefore, \( \sigma_{xx} = 0 \) and \( \sigma_{xy} = Ne^2/h \). Then, based on Eq. (2.33), we can derive \( \rho_{xx} = 0 \) and \( \rho_{xy} = h/Ne^2 \).

### 2.4.2 Laughlin’s Gedanken experiment

In 1981, Laughlin proposed a Gedanken experiment to explain the quantization of the Hall conductivity [4, 16, 24, 25]. In his thought experiment, Laughlin assumed that the end current contacts of the Hall bar are irrelevant to the Hall measurement [4, 23]. Thus he cut out the ends of the Hall bar which include the current contacts and wrapped the Hall bar into a cylinder assuming the applicability of periodic boundary conditions in the QHE measurement [23, 26]. Figure 2.9 demonstrates a 2DES surface bent into a cylinder with a constant \( B \) perpendicular to the cylinder surface at all points. The radius of cylinder is \( R \). A persistent current, \( I_y \), is assumed to flow around the cylinder under quantum Hall conditions [4]. A solenoid is placed along the axis of cylinder and a magnetic flux, \( \phi_x \), is generated inside the solenoid. Note that, none of \( \phi_x \) exist on the surface of the cylinder.

Figure 2.9. The diagram of Laughlin’s Gedanken experiment. 2DES is confined on the surface of a cylinder with radius \( R \). An external \( B \) is perpendicular to the surface, and a current, \( I_y \), flows around the cylinder under quantum Hall conditions. A solenoid is placed at the center of the cylinder with magnetic flux \( \phi_x \).
Laughlin related the total current carried around the loop to the potential drop from one edge to another by relating the current to the adiabatic derivative of the total energy of the system with respect to the magnetic flux through the system. He suggested that, by gauge invariance, adding flux results in a net transfer of $n$ electrons from one edge to another leading to the relation $I = ne^2V/h$. The quantum Hall effect was thus viewed as a quantum effect that is related to the persistence of currents in the 2D system [4].

2.4.3 Dual simultaneous quantized Hall effects in Mani’s geometry

![Figure 2.10](image)

Figure 2.10. (a) Hall effect measurement in the Hall bar configuration. (b) The configuration of Hall bar with a hole. The Hall resistance is determined by the injected current and the Hall voltage, i.e., $R_{xy} = V_{D,F}/I_{A,B}$. (c) The Hall effect measurement of the anti-Hall bar configuration. The Hall resistance may be determined from the interior injected current and the interior Hall voltage, i.e., $R_{xy} = V_{4,6}/I_{1,2}$. (d) A superimposing configurations produces an anti-Hall bar inside a Hall bar. Two boundary current injected into the specimen simultaneously leads to two Hall resistance from exterior boundary, $V_{D,F}/I_{A,B}$, and interior boundary, $V_{4,6}/I_{1,2}$. After Mani et al. (1994) [29].

To examine Lauglin’s Gedanken experiment, in ca. 1993, Mani et al. [22, 23, 26–40] invented a novel dual-current technique in a double-boundary geometry, including an anti-Hall
Figure 2.11. The measurements of quantized Hall effect carried out at low temperature and high magnetic field. The inset shows the illustration of the anti-Hall bar and Hall bar geometry. The results of Hall effect $V_{3,5}$ and $V_{C,E}$ with $I_{1,2} = 25 \, nA$ and $-25 \leq I_{A,B} \leq 0 \, nA$ are depicted in panel (a) and (b). After Mani et al. (1996) [30].
Figure 2.10(a) shows a usual Hall effect measurement in a the Hall configuration. The Hall resistance, $R_{xy}$, is defined in terms of the applied current, $I_{A,B}$, and the external Hall effect, $V_{D,F}$, i.e., $R_{xy} = V_{D,F}/I_{A,B}$. Inserting a hole in the specimen does not change the measurement. Thus the configuration of Figure 2.10(b) also yields a valid Hall effect and QHE measurement as in the simply connected specimen (panel(a)). Turning the configuration of Figure 2.10(b) inside-out results in the configuration of Figure 2.10 (c). Here, a Hall resistance within the internal boundary can be acquired in terms of injected current, $I_{1,2}$, and the Hall effect,$V_{4,6}$, as well, i.e, $R_{xy} = V_{4,6}/I_{1,2}$; see Figure 2.10(c) [22,23,26–40]. As the interior arrangement is the counterpart of the exterior configuration, one might identify this type of Hall effect measurements as Hall measurements in the “anti-Hall bar” configuration. Figure 2.10(d) exhibits the experiment includes the anti-Hall configuration within the standard Hall setup, with two currents injected into the specimen. In such experimental configuration, two Hall effects can be measured simultaneously, one each from the external and the internal boundaries [22,23,26–40].

The inset in Figure 2.11 shows a sketch of the double-boundary Mani-geometry with an anti-Hall bar etched in the inside of a Hall bar. Figure 2.11(a) demonstrates that when the current, $I_{1,2}$, flowing via the interior boundary keeps constant and the current, $I_{A,B}$, applied via the exterior boundary changes, the ordinary and quantized Hall voltage in $V_{3,5}$ in the anti-Hall device does not vary. Namely, the $V_{3,5}$ is insensitive to $I_{A,B}$. On the other hand, the Hall effect $V_{C,E}$ obtained on the Hall bar device shows variations in proportion to $I_{A,B}$; see Figure 2.11(b). These experiments suggest that the Hall resistance should be defined as the Hall effect on a boundary divided by the current injected via the same boundary [22,23].

The study of Hall effects in a doubly connected specimen helps understand the measurement of quantized Hall effect in Laughlin’s Gedanken experiments, which includes a hole as shown in Figure 2.12(a). Figure 2.12 (b) and (c) indicate the suggested configuration of Hall measurements in the anti-Hall bar formation configuration [23]. Here, the superposition property specifies that one might produce dual simultaneous quantized Hall
Figure 2.12. (a) Laughlin’s geometry. (b) A measurement of Hall effect, $V_1$. (c) Another measurement of Hall effect, $V_2$. Based on superposition, the scenario (b) and (c) can exist concurrently. (d) An inter-boundary voltage measurement. (e) An equivalent configuration of panel (d) based on reciprocity theorem. (f) An axial surface cut in the configuration of (d) turns into a Hall bar geometry. After Mani et al. (1996) [30].
effect, i.e., \( V_1/I_1 = h/e^2 \) and \( V_2/I_2 = h/e^2 \). Panel (d) demonstrates an inter boundary voltage measurement with a boundary-injected current. It is topologically different than the standard Hall measurement because the potential probes are on disconnected boundaries. By applying to the reciprocity theorem \([41]\), exchanging current and potential probes along with the reversal of magnetic field is identical to the same measurement; shown in panel (e). This, however, appears to be an undefined four-terminal measurement under steady-state integral filling factors because current may not be injected into the specimen without inducing “breakdown” in this “Corbino” configuration when the diagonal conductivity vanishes. If the configuration is modified so that the cylinder is cut (see panel (f)), the geometry effectively becomes a Hall bar and the four-terminal quantized Hall effect is expected to exist, i.e., \( V_1/I_1 = h/e^2 \). Thus, these experiments show that typical applications of periodic boundary conditions in quantum Hall systems, as in the Laughlin’s gauge argument, are incompatible with the QHE measurement. Cutting off the current contacts and removing the current source also eliminates the current in the quantum Hall system. Then, there is no measurement and no effect \([23]\). If the Hall effect measurement in the cylindrical geometry is specified by Figure 2.12(b) and (c), then the gauge argument does not help to relate the Hall effect to the current via fundamental constants.

### 2.4.4 Shubnikov de Haas oscillations

As mentioned in Section 2.4.1, while IQHE is observed, \( \rho_{xx} \) exhibits a genre of quantum oscillations. These oscillations are called SdHOs. The mechanism of SdHOs is owing to Fermi energy passing through the oscillating DOS of Landau levels. Figure 2.13(a) displays a typical SdHOs in GaAs/AlGaAs heterostructures at low \( B \) field. In the extent of low \( B \), \( B \) slightly splits Landau levels but not strong enough to separate them completely (Figure 2.8(b)). The DOS becomes a continuous sinusoidal function. Thus, SdHOs takes place but are not able to reach \( \rho_{xx} = 0 \). In terms of the definition of filling factor (Eq.(2.19)), we can know that SdHOs are periodic in \( 1/B \) and with frequency \( nh/e \) or \( nh/2e \), depending on if
spin-split is resolved. Figure 2.13(b) shows the plot of SdHOs as a function of $1/B$ and filling factor, $\nu$.

![Graph showing SdHOs as a function of $1/B$.](image)

Figure 2.13. (a) The SdHOs trace vs. $B$ in GaAs/AlGaAs heterostructure at low $B$ field. (b) The SdHOs as a function of $1/B$. $\nu$ is labeled on the top. It exhibits the periodicity in $1/B$.

SdHOs are important features since it includes information of three material parameters: carrier density $n$, effective mass $m^*$, and quantum life time $\tau_q$. First, $n$ can be extracted from the frequency of SdHO in $1/B$. Then, theoretically, SdHOs without background can
be described as [42,43]

$$\Delta \rho_{xx}/\rho_0 = \frac{X_T}{\sinh(X_T)} e^{\pi/\omega_c \tau_q} \cos(2\pi \frac{E_F - \varepsilon_\alpha}{\hbar \omega_c} - \phi), \quad X_T = \frac{2\pi^2 K_B T}{\hbar \omega_c}$$ (2.38)

where $\rho_0$ is the zero-field resistivity, $\phi$ is a phase constant, and $T$ is temperature. Since $X_T$ is dependent on $T$, it suggests that the amplitude of SdHOS are sensitive to temperature. Therefore, $m^*$ can be obtained from thermal damping terms, $X_T$, by conducting temperature-dependence measurements. And by analyzing Dingle factor, $e^{\pi/\omega_c \tau_q}$, the information of $\tau_q$ can be acquired [44].

2.5 Fractional Quantum Hall Effect

2.5.1 Background

FQHE was discovered in the situation of high $B$ and high-quality ($n \sim 1 \times 10^{11} cm^{-2}$ and $\mu \sim 1 \times 10^5 cm^2 V^{-1} s^{-1}$) 2DES [5]. Suppose $B$ is sufficiently high at $T \approx 0$, i.e., filling factor less than one, $\nu < 1$, electrons cannot be localized within the range smaller than magnetic length $\ell_B$, and electron-electron interaction dominates due to the absence of kinetic energy, if disorder can be neglected. However, if $B$ is increased to infinity, i.e., $\nu \ll 1$, $\ell_B$ becomes much smaller the average distance between electrons.

Figure 2.14 shows FQHE of longitudinal resistance, $R_{xx}$, and Hall resistance, $R_{xy}$, as a function of $B$ [45]. Similar to IQHE, Hall resistance is quantized and the plateaus on $R_{xy}$ correspond to vanishing $R_{xx}$. But, remarkably, $\nu$ takes on fractional values in FQHE instead of integers in IQHE. FQHE is observed at the principle filling factors $\nu = 1/q$ and other rational fractional filling factor $\nu = p/q$, where $q$ is odd integer [45]. Yet, some unusual even-denominator filling factors, such as $\nu = 5/2$, are found as well. Due to the fractional filling factors, fractional charges are thought by some to be responsible for elementary excitations in FQHE, e.g., $e/3$ for $\nu = 1/3$ state [46,47]. Note that although the exhibitions of FQHE resemble IQHE’s, the origins are sometimes thought to be completely different. FQHE
state is sometimes thought to be an intrinsically many body and incompressible quantum liquid [17,18,20].

Figure 2.14. The plot of longitudinal resistance, $R_{xx} = V_x/I_x$, and Hall resistance, $R_{xy} = V_y/I_x$, vs. $B$ in 2DES of density $n = 2.33 \times 10^{11} \text{ cm}^{-2}$ at $T = 85 \text{ mK}$. Numbers represent filling factors, $\nu$. It show that the plateaus in quantized $R_{xy}$ correspond to vanishing $R_{xx}$ in both IQHE ($\nu = \text{integer}$) and FQHE ($\nu = p/q$). After Eisenstein et al. (1990) [45].

FQHE is a delicate effect so that high-quality 2DES is required. It is because in FQHE regime, the ground state is segregated from the excited state by a finite energy gap, $\Delta$. Theoretically, $\Delta \sim 0.1e^2/4\pi\varepsilon B$, where $\varepsilon$ is dielectric constant, for $\nu = 1/3$ in an ideal 2DES, i.e., no disorder, zero layer-thickness, and infinitely Landau level separation. In a real 2DES, however, $\Delta$ becomes much smaller than $0.1e^2/4\pi\varepsilon B$ [48] as a result of the existence of disorders, finite layer-thickness, and Landau level overlap.

2.5.2 Fractal geometry model

An interpretation was proposed in 1995 that FQHE constitutes another manifestation of fractal geometry in nature [49]. This approach focused upon constructing the Hall resistance curve, instead of wavefunctions, to understand the FQHE. The first steps of this Hall curve construction are exhibited in Figure 2.15. For instance, the basic Hall curve to $\nu = 1/3$ might be constructed from an experimental trace (‘fractal generator’) showing IQHE upto
\(\nu = 1\); shown in Figure 2.15. Trace (a) in Figure 2.15 is an experimental IQHE trace to \(\nu = 1\). It is the elementary unit to construct FQHE trace. Then, in order to obtain the curve of trace (b), the generator is reflected about \(R_{xy} = 1R_k\) and \(B/B_0 = 1\) and attached to the end of the trace (a), where \(B(\nu = 1) = B_0\). And the sequence to \(\nu = 1/3\) might be achieved by attaching the trace (a) to the end of the trace (b). Thus the trace (c) exhibits the main sequence of FQHE to \(\nu = 1/3\). Similar idea may be applied to higher Hall resistances and \(B/B_0\) by repeatedly attaching sections.

Figure 2.15. (a) The IQHE trace of the Hall resistance as a function of \(B/B_0\), where \(B(\nu = 1) = B_0\) upto \(\nu = 1\). It is an elementary trace to construct FQHE ('fractal generator'). (b) The fractal generator is first reflected about \(R_{xy} = 1R_K\), followed by another reflection about \(B/B_0\), and attached to the end of trace (a). (c) Then the main sequence of FQHE upto \(\nu = 1/3\) may be achieved by attaching trace (a) to the end of the trace (b). After Mani et al. (1996) [49].

In this approach, see Ref. [49], repeated rescaling reconstructions of the Hall curve produce much fine structure including higher order fractions, above and beyond the main sequence FQHE’s. Indeed, hints of some of the sequences proposed in this model, such as those about \(\nu = 3/8\), were reported much later. The interpretation according to this model is that a fractal electronic structure with gaps within the Landau band is induced by a lattice-like electron arrangement, and the Fermi level within these gaps produces FQHE. Furthermore, the lattice-like electron arrangement happens not only under FQHE conditions
but also in the range of \( \nu \) between consecutive FQHE. As a result, FQHE is viewed as a transport response which becomes observable when an electron becomes sensitive to the periodic arrangement of other electrons, and it has been reasoned that FQHE constitutes a fractal originating from a Hofstader type spectrum induced by (local) Wigner crystallization [49].
Chapter 3

ELECTRON MAGNETOTRANSPORT IN 2DES UNDER RADIATION

3.1 Background

Electron transport measurements in 2DES under $B$ have been revealing a number of phenomenal features, such as, IQHE and FQHE. As the quality of 2DES and cryogenic techniques progress, more and more remarkable phenomena have been investigated in past decades. One of the most significant discoveries has been the report by Mani et al. [50] of novel Zero Resistance States, ZRS, in microwave, millimeter wave, and terahertz radiation induced magnetoresistance oscillations. The latter oscillations when they are observed in the microwave band, are called microwave induced magnetoresistance oscillations, MIMOs. Until now, the research of MIMOs is still in progress experimentally [51–92] and theoretically [93–126].

3.2 Microwave-Induced Magneto-transport

3.2.1 The measurement

The transport measurement method for ZRS and MIMOs is almost identical to that used for quantum Hall effect (Figure 2.6(a)). The only difference is that the subject to microwaves for MIMOs. Figure 3.1 shows longitudinal resistance, $R_{xx}$, and Hall resistance, $R_{xy}$, as a function of $B$ up to 10 $T$ when GaAs/AlGaAs 2DES is under microwaves at frequency $f = 103.5$ GHz and temperature $T = 1.3$ K [50]. Clearly, the $R_{xy}$ trace exhibits quantum Hall effect and the $R_{xx}$ trace demonstrates vanishing resistance wherever plateaus occur. In the mean time, some radiation-induced signals take place at $B < 0.4$ $T$ (Figure 3.1 inset), and these oscillations reach ZRS over a broad interval of $B$ values.
Figure 3.1. The plot of $R_{xx}$ and $R_{xy}$ vs. $B$ in the range of $0 \leq B \leq 10 \, T$. The traces exhibit IQHE, FQHE, as well as MIMOs. The inset shows MIMOs occurring at $B < 0.4 \, T$ and reach vanishing resistance over a internal of $B$. After Mani et al. (2002) [50].

Figure 3.2(a) shows a high-detailed $B$ sweep measurement at $-0.4 < B < 0.4 \, T$ of $R_{xx}$ and $R_{xy}$. The blue curve is $R_{xx}$ with microwave excitation and the red one is in the absence of excitation. Apparently, the oscillations of blue trace is induced by microwaves. Further, the $B$ where MIMOs fall onto ZRS can be express as

$$B = \frac{4}{4j + 1} B_f, \quad B_f = \frac{2\pi f m^*}{e}$$

with $j = 1, 2, 3, \cdots$. In contrast, $R_{xy}$ is not affected by microwaves. It means unlike quantum Hall effect, the vanishing $R_{xx}$ is not accompanied by plateaus in $R_{xy}$ in MIMOs region. It is quite unexpected since the vanishing longitudinal resistance is always though of accompanied by quantization of Hall resistance. Panel (b) manifests $R_{xx}$ vs. $B^{-1}/\delta$, where $\delta$ is the oscillatory period in $B^{-1}$. It appears MIMOs is periodic in $B^{-1}$ with a 1/4-cycle phase shift.

The sensitivity of $B$-field position of resistance minima to microwave frequency, $f$, is observed in Figure 3.3. The oscillation minima shift toward higher $B$ with $f$ increasing from 85 to 110 $GHz$. The inset shows the weighted resistance minima, $B^*$, is linearly proportional to $f$ and $dB^*/df = 2.37 \, mT \cdot GHz^{-1}$, where $B^* = \sum_{j=1}^{5} \{(4j + 1)/4\}B_{4/(4j+1)}\}/5$, the first five resistance minima. Figure 3.4(a) manifests the microwave power dependence of MIMOs
Figure 3.2. (a) The sketch of the typical results of MIMOs in $R_{xx}$ (blue trace) and $R_{xy}$ (green trace) compared to regular quantum Hall effect in $R_{xx}$ (red trace). The vanishing $R_{xx}$ happens at $B = [4/(4j + 1)]B_f$, where $B_f = (2\pi fm^* )/e$. (b) The $R_{xx}$ vs. $B^{-1}/\delta$, where $\delta$ is the period in $B^{-1}$. It shows MIMOs is periodic in $B^{-1}$. After Mani et al. (2002) [50].
Figure 3.3. $f$-dependence measurements of MIMOs at $85 < f < 110 \text{GHz}$. The data appears $B^*$ is linear to $f$, where $B^* = \sum_{j=1}^{5} \left\{ \frac{(4j + 1)/4B_4/((4j+1))}{5} \right\}$, the first five resistance minima. After Mani et al. (2002) [50].

Figure 3.4. (a) The power dependence of MIMOs amplitude in $\Delta R_{xx}$, where $\Delta R_{xx}$ is longitudinal resistance without dark signal. The amplitude of MIMOs is extracted from fit formula. (b) The plot of amplitude, $A$, vs. microwave power, $P$. It implies the sub-linear relation between $A$ and $P$. After Mani et al. (2010) [74].
amplitude in $\Delta R_{xx}$ measurements, where $\Delta R_{xx}$ is longitudinal resistance subtracted from dark signal [74]. Then, the amplitude of MIMOs are extracted by fitting formula to the data using

$$\Delta R_{xx} = -A e^{\lambda B} \sin\left(\frac{2\pi F}{B}\right)$$

(3.2)

where $A$ is the amplitude, $\lambda$ is the damping parameter, and $F$ is the $f$-dependence resistance oscillation frequency. Figure 3.4(b) implies sub-linear relationship between amplitude, $A$, and microwave power, $P$, which usually can be represented as $A \sim \sqrt{P}$ [86,91,113].

### 3.2.2 The sensitivity to linear polarization

Another interesting finding of MIMOs is the sensitivity to linear polarization of microwaves. In 2011, Mani et al. [76] reported that the amplitudes of MIMOs are responsive to the relative orientation between microwave electric field and major Hall bar axis, which is assigned to polarization angle $\theta$; shown in Figure 3.5(a). Panel (b) represents the comparison of photo-excited signals between $\theta = 0^\circ$, $\theta = 90^\circ$ at $f = 39$ GHz with $P = 0.63$ mW, and dark signal. The amplitudes of MIMOs are clearly reduced as $\theta$ changes from $0^\circ$ to $90^\circ$.

![Figure 3.5](image)

Figure 3.5. (a) shows the sketch of relative orientation of polarization angle, $\theta$, between microwave electric field and major Hall bar axis. (b) The comparison of amplitudes of MIMOs between $\theta = 0^\circ$, $\theta = 90^\circ$, and the dark signal. The sharp change in amplitude is obtained when changing $\theta$ from $0^\circ$ to $90^\circ$. After Mani et al. [76].
Figure 3.6. (a) The photo-excited and dark traces at the positive $B$ range. The oscillatory extrema are labeled as $P_1$ ($P_2$) and $V_1$ for the first (second) peak and valley respectively. The sinusoidal variations in $R_{xx}$ vs. $\theta$ at (b) $P_1$, (c) $V_1$, and (c) $P_2$ can be fit by a cosine square function (solid line). The extracted $\theta_0$ from the fit appears $f$- and $B$-dependence. After Ramanayaka et al. [78].

Then, Ramanayaka et al. [78] found that the variation in the amplitude of MIMOs exhibits a sinusoidal curve with a continuous change in $0^\circ \leq \theta \leq 360^\circ$ at fixed $B$. Figure 3.6 shows the results of the variation in $R_{xx}$ as a function of $\theta$. A typical MIMOs in the range of positive $B$ at $f = 39 \text{ GHz}$, $P = 0.32 \text{ mW}$, and $\theta = 0^\circ$ with dark signal are shown in panel (a). The oscillatory extrema are assigned to $P_1$ ($P_2$) and $V_1$ for the first (second) peak and valley (panel (a)). Panel (b), (c), and (d) display strong sinusoidal variation in $R_{xx}$ as a function of $\theta$ compared to the dark signal when $B$ is at $P_1$, $V_1$, and $P_2$ respectively. These
sinusoidal curves can be fit to an empirical cosine square function to extract phase shift that

\[ R_{xx} = A \pm C \cos^2(\theta - \theta_0) \]  \hspace{1cm} (3.3)

where \( A \) is the background, \( C \) is the amplitude of cosine square function, \( \theta_0 \) is the phase shift, and plus (minus) sign is for peak (valley). They conclude that \( \theta_0 \) is \( f \)- and \( B \)-dependent, i.e., \( \theta_0 = \theta_0(f, B) \). But the mechanism of \( \theta_0 \) is still unclear.

### 3.3 The Mechanisms

#### 3.3.1 The displacement model

The displacement model has been examined and presented by several groups [94–96]. The fundamental idea is the elastic scattering of inter-Landau-level transition. Figure 3.7 shows the schematic diagram of radiation-induced impurity-assisted current. Landau levels are separated by \( \hbar \omega_c \) and tilted due to applying a \( dc \) bias. When microwaves illuminate 2DES, electrons occupying lower Landau level \( n \) absorb photons with energy \( \hbar \omega \) and jump to higher energy between \( n + 2 \) and \( n + 3 \) Landau levels (Figure 3.7). If disorder is absent, the distribution of electrons over Landau levels does not effect conductivity [93]. Therefore, these photo-excited electrons does not influence the \( dc \) current. In contrast, in the situation of disorder present, the electron-impurity scattering happens. These photo-excited electrons are expelled spatially to either right or left adjacent Landau levels by a distance \( \Delta X \) owing to impurity scattering. If the final DOS toward right exceeds that toward left, thus \( dc \) current decreases. Conversely, the \( dc \) current increases. Consequently, the electron-impurity scattering contributes to \( dc \) conductivity in an oscillatory behavior, which is

\[ \Delta \sigma_{xx} \propto -\sin\left(\frac{2\pi \omega}{\omega_c}\right) \]  \hspace{1cm} (3.4)

Further numerical simulations exhibit the period of oscillations is \( 1/\omega \) and minima are near \( \omega/\omega_c = n + 1/4 \), where \( n \) is integer, which are in agreement with experimental observations.
The model also predicts that the 1/4 phase shift is not universal. It deviates between 0 and 1/2 depending upon disorder and intensity. Yet, the nodes observed in measurements appear at \( n + \frac{1}{4} \) in \( \omega/\omega_c \) and insensitivity to any parameters. Moreover, the simulated resistivity is negative in the vicinity of minima, which corresponds to ZRS.

Figure 3.7. The picture of radiation-induced impurity-assisted current. Landau levels are separated by \( \hbar \omega_c \) and tilted by applying a \( dc \) bias. When microwave is applied, electrons are excited by \( \hbar \omega \). The photo-excited electrons are scattered by impurities toward right of left adjacent Landau levels. If the final DOS to the right is more than that to the left, the \( dc \) current reduces. If vice versa, \( dc \) current enhances. Consequently, electron-impurity scattering contributes to the oscillatory conductivity. After Durst \textit{et al.} [94].

The sensitivity to linearly polarized radiation also can be described by the displacement model. Ryzhii \textit{et al.} indicated that the photoconductivity is related to the orientation of \( ac \) and \( dc \) fields. But in the case of circular-polarization or non-polarized radiation, the photoconductivity remains unchanged [95]. Lei \textit{et al.} further reported the simulations of the sinusoidal variation in longitudinal resistance with regard to \( ac \) and \( dc \) orientation, which is in agreement with experimental results [119].
3.3.2 The nonparabolicity model

The nonparabolicity model was proposed by Koulakov et al. in 2003 [97]. They conclude that a strong variation in longitudinal conductivity, $\sigma_d$, of $dc$ current takes place in the vicinity of cyclotron resonance, i.e., $\omega \approx \omega_c$, for a linear-polarized $ac$ field due to a weak nonparabolicity of the electron spectrum. They assume that $\sigma_d$ is responsive to $ac$ field only if Kohn theorem is violated [93]. Therefore, they consider a model that the Kohn theorem is violated owing to a weak nonparabolicity of the electron spectrum

$$\varepsilon(p) = \frac{p^2}{2m^*}[1 - \frac{p^2}{2m^*E_0}]$$  (3.5)

where $E_0$ is the energy of the order of the band gap. Then, the longitudinal and Hall conductivity can be expressed in terms of the dimensionless parameter $\delta m^*/m^*$. The results suggest that small correction to effective mass due to $ac$ field results in negative $\sigma_d$ but constant Hall conductivity. Note that this model only adapts to linear-polarization $ac$ field, not to a circular polarization. Meanwhile, the sensitivity of $\sigma_d$ to the relative orientation of the $dc$ and $ac$ fields is also predicted. Figure 3.8 demonstrates the longitudinal conductivity, $\sigma_d$, as a function of the angle, $\theta$, where $\theta$ is defined between $dc$ and $ac$ fields. The dashed line is conductivity in the absence of $ac$ field. Clearly, $\sigma_d$ manifests sinusoidal variation compared to dark conductivity with respect to $\theta$. When the intensity of $ac$ field increases, the interval of $\theta$ where $\sigma_d$ is negative becomes wider. However, the model only proves a significant change in $\sigma_d$ in the vicinity of cyclotron resonance. The oscillatory of $\sigma_d$ near the harmonics of cyclotron resonance are not predicted by the model.

3.3.3 The inelastic model

In 2005, Dmitriev et al. reported that the magneto-oscillations in photoconductivity are mainly effected by inelastic scattering, i.e., electron-electron scattering. This is the inelastic model [99, 106]. It states that the inelastic scattering is dominant for oscillatory photoconductivity, i.e., $\tau_{in} \gg \tau_q$, where $\tau_{in}$ is the inelastic relaxation time and $\tau_q$ is the single-
Figure 3.8. The longitudinal conductivity, \( \sigma_d \), vs. the angle between \( ac \) and \( dc \) fields. The dashed line represents the conductivity without \( ac \) field. The simulation exhibits sinusoidal variation of \( \sigma_d \) at \(-\pi \leq \theta \leq \pi\). Plus, while the intensity of \( ac \) field enhances, the negative \( \sigma_d \) regime on \( \theta \) axis becomes wider. After Koulakov et al. [97].

particle relaxation time, which is due to impurity scattering. When the effect of impurity collision due to microwaves is neglected, the dominant effect is caused by a nontrivial energy dependence of the nonequilibrium distribution function, \( f(\varepsilon) \), which is a solution to the stationary kinetic equation

\[
St_w\{f\} + St_{dc}\{f\} = -St_{in}\{f\}
\] (3.6)

where \( St_w\{f\} \) and \( St_{dc}\{f\} \) are microwaves and \( dc \) field effects as impurities are present; \( St_{in}\{f\} \) are the inelastic relaxation. The model assumes the application of a linear-polarization microwave, but the results remain unchanged for a circularly polarized microwave. Then, in the case of overlapping Landau levels, by solving Eq.(3.6), the DOS becomes

\[
\tilde{\nu} = 1 - 2\delta\cos\frac{2\pi\varepsilon}{\omega_c}, \quad \delta = \exp(-\frac{\pi}{\omega_c\tau_q}) \ll 1
\] (3.7)

Here \( \tau_q \ll \tau_{tr} \) (\( \tau_{tr} \) is the transport time) because of high-mobility structures. The solution of \( f(\varepsilon) \) with the first order of \( \delta \) can be written as

\[
f = f_0 + f_{osc} + O(\delta^2), \quad f_{osc} \equiv \delta Re[f_1(\varepsilon)\exp(i\frac{2\pi\varepsilon}{\omega_c})]
\] (3.8)
Figure 3.9 depicts the microwave-induced oscillatory DOS, $\nu(\varepsilon)$, distribution function, $f(\varepsilon)$, and regular Fermi distribution function, $f_T(\varepsilon)$, as a function of $(\varepsilon - \varepsilon_F)/\omega_c$, where $\varepsilon_F$ is Fermi energy. As a result of microwave-induced oscillatory DOS, the oscillatory correction to distribution function is induced as well. Consequently, it generates the oscillations of $dc$ conductivity with periodicity in $1/\omega_c$. In addition, the model predicts the independence of MIMOs upon microwave polarization orientation.

Meanwhile, the model mentions that the temperature-dependent behavior of oscillatory photoconductivity is determined by inelastic relaxation time, $\tau_{in}$. Considering at moderate high $T$, electron-electron collision is the dominant mechanism of inelastic scattering. Although $f_0$ (Eq. (3.8)) diminishes due to electron-phonon scattering, electron-electron scattering at $T \gg \omega_c$ still affects the temperature behavior of oscillations in terms of oscillatory term $f_{osc}$. The calculations suggest that the effect of the oscillatory contribution to photoconductivity is raised with $T^{-2}$ at $T \gg \omega$ and $T^{-1}$ at $T \ll \omega$ as $T$ decreases.

Figure 3.9. The numerical simulations of oscillations of DOS, $\nu(\varepsilon)$, distribution function, $f(\varepsilon)$, and regular Fermi distribution function, $f_T(\varepsilon)$, vs. $(\varepsilon - \varepsilon_F)/\omega_c$. The $1/\omega_c$-periodic oscillatory distribution function leads to oscillations of $dc$ conductivity owing to the oscillatory DOS. After Dmitriev et al. [106].
3.3.4 The radiation-driven electron-orbit model

The foundation of the radiation-driven electron-orbit model is based on solution to an quantum forced harmonic oscillator under the influence of radiation as well as a perturbation for the elastic scattering provided by Inarrea et al. in 2005 [103, 104, 109, 110, 118]. Figure 3.10 demonstrates the electron transport. Panel (a) shows that in the absence of microwaves, electron orbits are fixed. Electrons jump between orbits by a distance \( \Delta X^0 \) owing to elastic scattering. When electron orbits are under microwaves with frequency \( f \), they oscillate back and forth with angular frequency \( \omega \), where \( \omega = 2\pi f \). Panel (b) is the situation that as orbits move backward, electrons need to overcome further distance, i.e., \( \Delta X^{MW} > \Delta X^0 \) to empty electron orbits. As a result, it increases conductivity. On the contrary, if orbits advance forward, shorter distance that electrons need to jump, i.e., \( \Delta X^{MW} < \Delta X^0 \). This case decreases conductivity; see panel (c). Panel (d) describe the occurrence of ZRS. While the intensity of microwaves is strong enough, the extent that orbits advance exceeds the distance that electrons are able to overcome. In this case, all electron orbits are occupied. No electron transition between orbits occurs due to Pauli exclusion. Thus, resistance reaches zero state.

In this semiclassical model, \( \Delta X^{MW} \) is given by

\[
\Delta X^{MW} = \Delta X^0 + \frac{eE_0}{m^*\sqrt{(\omega_c^2 - \omega^2)^2 + \gamma^4}} \cos(\omega\tau) \tag{3.9}
\]

Here, \( E_0 \) is the intensity of microwaves, \( \gamma \) is a damping factor due to electron-phonon interaction, and \( \tau \) is the time that electron takes from initial state to final state. The damping term \( \gamma \) accounts for the temperature dependence of MIMOs. When temperature increases, the prevailing effect of electron-acoustic phonon scattering leads to \( \gamma \) dominant. Therefore, the amplitudes of MIMOs are reduced.

In addition, the model successfully describes the linear-polarization orientation sensitivity of MIMOs [118]. Considering electric field of microwaves in the \( x \) and \( y \) directions, i.e.,
Figure 3.10. The schematic diagrams of electron transport. (a) Without microwaves, electrons jump between fixed orbits by a distance $\Delta X^0$. When orbits are under microwaves, they oscillate. (b) The moment that orbits move backward, electron needs to jump further distance, i.e., $\Delta X^{MW} > \Delta X^0$. The conductivity is increased. (c) Orbits advance forward, shorter distance that electrons need to jump, i.e., $\Delta X^{MW} < \Delta X^0$. Conductivity is decreased. (d) For microwaves with high intensity, all orbits are occupied. Then, ZRS is achieved. After Iñarrea et al. [103].

\[
\vec{E}(t) = E_{0x}\vec{x} + E_{0y}\vec{y}, \text{ where } E_{0x} \text{ and } E_{0y} \text{ are amplitude of microwave field, } \Delta X^{MW} \text{ becomes }
\]

\[
\Delta X^{MW} = \Delta X^0 + \frac{eE_0}{m^* \frac{\omega^2(\omega_c^2 - \omega^2)^2}{\omega^4 \cos^2 \alpha + \omega_c^2 \sin^2 \alpha} + \gamma^4} \cos(\omega \tau) \quad (3.10)
\]

where $\alpha$ is the polarization angle with respect to $x$ direction. Given by Eq.(3.10), if $\gamma > \omega$, $\gamma$ would quench the $\omega$-term. Then, the effect of linear-polarization angle dependence dies away. On the other hand, if $\gamma < \omega$, the $\omega$-term survives. The effect of linear-polarization angle dependence appears. Namely, the dependence of MIMO amplitude on linear polarization angle is only observed in high quality 2DES.

### 3.3.5 The pondermotive force model

The pondermotive force model reported by Mikhailov et al. in 2006 explains the observations of MIMOs and ZRS in terms of pondermotive force induced by the interaction between microwaves and metallic contacts. [108, 116]. Because of the electron density in
the metal is several orders of magnitude more than 2DES, in the near-contact region, the radiation field is screened by metallic contacts instead of 2DES. The screening by metallic contacts causes more effects than by 2DES. Consequently, they assume that electrons near contacts are mainly contributed to radiation-induced oscillations.

In a “contact-2DEG-contact” system (inset in Figure 3.11) that the contacts are considered as a infinite thin 2D layers, the distribution of electric field is the format of functions $\sim \cos(2\pi xn/W)$ in Fourier series by solving Maxwell equations. The numerical simulations of radiation field are shown in Figure 3.11. Panel (a) and (c) are $x$- and $y$-direction fields respectively within the aperture $|x| \leq W/2$. Here, the screening by contacts is only taken into account. The simulations suggest that the $x$-direction field, $E_x$, perpendicular to the contact-2DEG boundary becomes dramatically huge near the boundary relative to incident electric field, $E_0$; see panel (b). The $E_0$ is enhanced hugely along $x$ direction by the metallic contacts in the near-contact region owing to induced charge accumulation near sharp edges of metals. Plus, the near-contact $E_x$ is strongly inhomogeneous and linearly polarized. On the other hand, the field induced by the metallic contacts in $y$ direction, $E_y$, is much weaker than $E_0$ near the contact regime (Panel (c)).

Figure 3.11. The illustrations of electric fields in (a) $x$ and (b) $y$ direction within the gap induced by metallic contacts screening incident radiation field. The $x$-direction field, $E_x$, is much larger than incident field, $E_0$, and $y$-direction field, $E_y$, is much smaller than $E_0$ near the contact boundary. (c) The $x$-direction field grows up dramatically near the contact. After Mikhailov et al. [116].
Figure 3.12. The sample geometry of (a) Corbino disk and (b) Hall-bar. The formation of depletion and accumulation areas (solid grey regions) with the help of radiation accounts for the MIMOs and ZRS phenomena. After Mikhailov et al. [116].

Because 2D electrons are mainly affected by the strong, linear-polarization, and inhomogeneous $ac$ field, in light of the linearly polarized electric field $\vec{E} = E_x \cos(\omega t) \vec{x}$, the magnetic field $B = B\vec{z}$, and a weak scattering, the pondermotive force $F_{pm}(x) = -\nabla U_{pm}(x)$, where

$$U_{pm}(x) = \frac{e^2 E_x^2(x)}{8m^*\omega_c} \left( \frac{\omega - \omega_c}{(\omega - \omega_c)^2 + \gamma^2} - \frac{\omega + \omega_c}{(\omega + \omega_c)^2 + \gamma^2} \right)$$

with the momentum relaxation rate $\gamma = e/(m^*\mu)$. In the case of weak scattering, $\gamma \ll \omega, \omega_c$. Here, one important conclusion can be drawn based upon above formula is that $F_{pm}$ may change the direction in the presence of finite $B$. If $\omega < \omega_c$, electrons are attracted to the contacts (low-field areas). In contrast, when $\omega > \omega_c$, electrons are expelled from the contacts (high-field areas). A depletion or accumulation areas are built in the near-contact region at $\omega > \omega_c$ and $\omega < \omega_c$ respectively; see Figure 3.12. Thereby, the depletion or accumulation regions determine the experimental magnetotransport coefficients. It is worth to mention that when a depletion region is formed, the resistance or conductance of the 2DES is suppressed.
4.1 Background

We have discussed the linear-polarization sensitivity of MIMO amplitudes in 3.2.2. Although, at the present time, a number of models have been proposed to explain the origin of MIMOs, the predictions and interpretations in terms of the sensitivity to linearly polarized radiation is still a topic of discussion. In the displacement model, the impurity-assisted photoconductivity is related to the orientation of $ac$ and $dc$ fields \[95,119\]. In the nonparabolicity model, the conductivity are predicted to be sensitive to the angle between $ac$ and $dc$ fields as well, but only near the cyclotron resonance \[97\]. The radiation-driven electron-orbit model predicts the sensitivity to linear-polarization radiation only in the case of high quality 2DES \[118,121\]. Finally, the inelastic model suggests that the conductivity is insensitive to the linear polarization orientation \[106\].

Recently, the observation of linear-polarization sensitivity of MIMOs was reported \[76\]. After that, the study of longitudinal resistance with continuous change in the orientation of $ac$ fields at fixed $B$ are carried out \[78\]. Here, the data trace exhibited a sinusoidal variation in longitudinal resistance at moderate power, following the empirical polarization $R_{xx} = A \pm C \cos^2(\theta - \theta_0)$, where $\theta$ is microwave polarization angle between the microwave polarization and the Hall bar axis, $\theta_0$ is the phase shift, and the plus and minus sign correspond to oscillatory maxima and minima respectively. The fit data showed that $\theta_0$ is $f$- and $B$ dependent, i.e., $\theta_0 = \theta_0(f, B)$, and no systematic changes of $\theta_0$ associated with experimental parameters were reported.
In this chapter, therefore, we analyze the extracted $\theta_0$ over the frequency interval $32 \leq f \leq 50 \, GHz$ by averaging over other contributions that are relative smaller than estimates of experimental uncertainty. The results suggest a nontrivial frequency dependence in phase shift, i.e., $\theta_0 = \theta_0(f)$ [88].

### 4.2 Calibration of Linear Polarization Orientation

Prior to the experiment, the calibration of linear polarization direction is carried out because the polarization orientation has to be defined. Figure 4.1 shows the configuration of the calibration setup. Microwave is generated by a synthesizer and transmitted by a semi-rigid coaxial cable to a probe-coupled antenna inside a rotatable microwave launcher. The $TE_{10}$ mode microwave is excited by the antenna and transported through a circular stainless steel waveguide. The circular waveguide serves to preserve the microwave polarization. A vacuum seal is used to block the waveguide to maintain high vacuum condition when measurements are performed in the cryogenic system. A power detector is placed at the bottom of the waveguide where the specimen is situated in the experiment. Then, the power of microwaves can be read on the power meter. During the calibration, the microwave launcher is turned clockwise from $0^\circ$ to $360^\circ$ with $10^\circ$ intervals. Thus, the power detector will sense various power levels due to the change in polarization orientation.

At the outset of calibration, the antenna and the power detector are aligned; setting polarization $\theta = 0$. Therefore, the maximal response of power detector is anticipated to be at $\theta = 0^\circ$, $180^\circ$, and $360^\circ$. Figure 4.2(a) shows normalized detected power as a function of $\theta$ at $f = 40.8 \, GHz$. As expected, the variation of detected power manifests a sinusoidal trace over $0^\circ \leq \theta \leq 360^\circ$. Since the microwave power is proportional to the square of the electric field, we fit the data to an empirical cosine square function $P = A \pm C\cos^2(\theta - \theta_0)$, where $P$ is the detected power, $A$ is the dark response without microwave, $C$ is the amplitude. Then the phase shift, $\theta_0$, can be extracted by data fit. In the case of $f = 40.8 \, GHz$, the extracted $\theta_0 = 0.2^\circ \pm 0.3^\circ$ represents the maximal power response nearly at $\theta = 0$, in agreement with expectations.
Figure 4.1. The configuration of calibration setup. The $TE_{10}$ mode microwave is excited by the antenna and transmitted through the circular waveguide to be measured by the power detector. By sensing as a function of the launcher rotation angle, the polarization direction can be defined.

Figure 4.2(b) exhibits fit-extracted $\theta_0$ variation with respect to $f$ over $32 \leq f \leq 50$ GHz which indicates $-8^\circ \leq \theta_0 \leq 6^\circ$. Ideally, $\theta_0 = 0$ at each $f$ is expected if the antenna and the power detector are well aligned. Thus, some unavoidable experimental issues, such as minor misalignment and readout errors, result in $14^\circ$ uncertainty of $\theta_0$.

4.3 Comparison of the Phase Shift between Opposite Hall-Bar Sides

The relative orientation between the specimen and microwave field is shown in Figure 4.3. The specimen is a GaAs/AlGaAs heterostructure with 400 $\mu$m-wide Hall bar and alloyed gold-germanium contacts. The specimen is immersed in pumped liquid helium to achieve $T = 1.5 \, K$. The carrier density and the mobility at $1.5 \, K$ are $\sim 2.7 \times 10^{11} \, cm^{-1}$ and $\sim 8 \times 10^6 \, cm^2V^{-1}s^{-1}$ respectively. The probe contacts on two sides of the specimen with length-to-width ratio $L/W = 2$ are measured by utilizing four-terminal lock-in technique with applying a low-frequency current, $I$. In the study, the longitudinal voltages on the right side, $V^R_{xx}$, and on the left side, $V^L_{xx}$, of the device are investigated simultaneously. Finally, the polarization angle, $\theta$, is defined between the Hall bar axis and electric field, and it increases clockwise.
Figure 4.2. (a) The normalized detected power vs. polarization angle at $f = 40.8\ GHz$. $\theta_0$ can be extracted by fitting the cosine square function. It shows $\theta_0 = 0.2^\circ \pm 0.3^\circ$ at $f = 40.8\ GHz$. (b) The plot of $\theta_0$ vs. $f$ indicates $-8 \leq \theta_0 \leq 6^\circ$. It implies $14^\circ$ deviation caused by experimental setup during measurements.
Figure 4.3. The figure of relative orientation between GaAs/AlGaAs heterostructure and microwave field. The longitudinal voltages are measured from the right side, $V_{xx}^R$ and the left side, $V_{xx}^L$ simultaneously. A low-frequency current, $I$, flows through the specimen. The polarization angle, $\theta$, is between the Hall bar axis and electric field.

Before executing polarization-angle-dependence measurements, the $B$ fields of oscillatory extrema have to be determined. Figure 4.4 displays $B$-field sweep over $-0.3 \leq B \leq 0.3 \ T$ of $R_{xx}$ with microwaves $f = 32.5 \ GHz$ (black trace) and without microwaves (red trace) at $\theta = 0$ on both sides of the Hall bar with $R_{xx}^L = V_{xx}^L/I$ (see panel (a)) and $R_{xx}^R = V_{xx}^R/I$ (see panel (b)). Both photo-excited traces exhibit strong MIMOs compared to dark traces. Then, the predominant oscillatory extrema have been labeled as $P1$, $V1$, and $P2$ to represent the first peak, first valley, and the second peak respectively. The sign of “+” and “−” means the direction of $B$ field, i.e., “+” for $0 \leq B \leq 0.3 \ T$ and “−” for $-0.3 \leq B \leq 0 \ T$, and superscript sign of L and R indicates signals obtained from the left and right side of the Hall bar respectively. For example, $P1+L$ represents the first peak of $R_{xx}^L$ trace in the range of $0 \leq B \leq 0.3 \ T$.

Figure 4.5 shows the linear-polarization-angle dependence of $R_{xx}$ at fixed $B$ corresponding to $P1+L$, $V1+L$, $P2+L$, $P1+R$, $V1+R$, and $P2+R$ with and without microwaves at $f = 32.5 \ GHz$. As expected, photo-excited traces vary sinusoidally with $\theta$ and the dark
Figure 4.4. The $B$-field sweep of (a) $R_{xx}^L$ and (b) $R_{xx}^R$ with and without radiations, where $R_{xx}^L = V_{xx}^L/I$ and $R_{xx}^R = V_{xx}^R/I$. Both photo-excited traces appear strong MIMOs compared to dark traces. The predominant oscillatory extrema have been labeled as $P1_{1+}^{L,R}$, $V1_{1+}^{L,R}$, $P1_{1-}^{L,R}$, $P2_{1-}^{L,R}$, $V1_{1-}^{L,R}$, and $P1_{1-}^{L,R}$ (see text).

traces remain constant. The phase shift $\theta_0$ in $R_{xx}$ vs. $\theta$ photo-excited responses are extracted by empirical cosine square function, $R_{xx} = A + C \pm \cos^2(\theta - \theta_0)$, where “+” and “−” sign are for peaks and valleys respectively. To compare $\theta_0$ obtained from opposite sides of the Hall bar, let us first focus on $P1_{1+}^{L}$ (panel (a)) and $P1_{1+}^{R}$ (panel (b)). The fit results manifest $\theta_0^L = 6.2^\circ \pm 0.6^\circ$ for $P1_{1+}^{L}$ and $\theta_0^R = -4.3^\circ \pm 0.5^\circ$ for $P1_{1+}^{R}$. The small standard errors result from well-fit cosine square function to the data. Then, the phase shift difference between $P1_{1+}^{L}$ and $P1_{1+}^{R}$ is $\delta \theta_0 = |\theta_0^R - \theta_0^L| = 10.5^\circ \pm 0.8^\circ$ which is smaller than the uncertainty of measurement $14^\circ$. Likewise, comparing $V1_{1+}^{L}$ (panel (c)) and $V1_{1+}^{R}$ (panel (d)) or $P2_{1+}^{L}$ (panel (e)) and $P2_{1+}^{R}$ (panel (f)) gives $\delta \theta_0 = 1.7^\circ \pm 1.6^\circ$ for $V1_{1+}$ and $\delta \theta_0 = 2.2^\circ \pm 0.5^\circ$ for $P2_{1+}$. The $\delta \theta_0$ are all less than uncertainty of measurement.

Next, similar comparisons are carried out over $-0.3 \leq B \leq 0 \ T$. Figure 4.6 exhibits $R_{xx}$ as a function of $\theta$ with and without microwaves at $f = 32.5 \ GHz$ and at fixed $B$ corresponding to $P1_{1-}$, $V1_{1-}$, $P2_{1-}$, $P1_{1-}^{R}$, $V1_{1-}^{R}$, and $P2_{1-}^{R}$. The phase shift, $\theta_0$, in photo-excited traces of $R_{xx}$ vs. $B$ are extracted from cosine square function and shown in panel (a)-(f). Similarly, we compare $\theta_0$ values between the right and left side of the device. Panel (a) and (b) show $\theta_0^L = -10.5^\circ \pm 1.1^\circ$ for $P1_{1-}$ and $\theta_0^R = -9.3^\circ \pm 1.0^\circ$ for $P1_{1-}^{R}$.
Figure 4.5. The $R_{xx}$ as a function of $\theta$ with and without microwaves at $f = 32.5$ GHz and at fixed $B$ of (a) $P1^{+L}$, (b) $P1^{+R}$, (c) $V1^{+L}$, (d) $V1^{+R}$, (e) $P2^{+L}$, and (f) $P2^{+R}$ respectively. The phase shift, $\theta_0$, are extracted from the cosine square function and the phase shift differences between the right and left side of the device at given oscillatory extrema are smaller than measurement uncertainty $14^\circ$. 
The phase shift difference, $\delta \theta_0 = |\theta_0^R - \theta_0^L| = 1.2^\circ \pm 1.5^\circ$, is still less than uncertainty of measurement $14^\circ$. Likewise, $V1-$ (panel (c) and (d)) and $P2-$ (panel (e) and (f)) lead to $\delta \theta_0 = 0.4^\circ \pm 1.1^\circ$ and $\delta \theta_0 = 6.7^\circ \pm 1.9^\circ$ respectively. As a result, $\delta \theta_0$ for $P1-$, $V1-$, and $P2-$ at $0.3 \leq B \leq 0 \ T$ are within the uncertainty of measurement as well.

Figure 4.6. The $R_{xx}$ as a function of $\theta$ with and without microwaves at $f = 32.5 \ GHz$ and at fixed $B$ of (a) $P1^L$, (b) $P1^R$, (c) $V1^L$, (d) $V1^R$, (e) $P2^L$, and (f) $P2^R$ respectively. The phase shift, $\theta_0$, are extracted from the cosine square function and the phase shift differences between the right and left side of the device at given oscillatory extrema are smaller than measurement uncertainty $14^\circ$. 
4.4 The Phase Shift Analysis at Various Frequencies

We have demonstrated that $\delta \theta_0 = |\theta_0^R - \theta_0^L|$ obtained at oscillatory extrema are within the uncertainty of measurement at $f = 32.5 \text{ GHz}$. To further examine this finding, more $f$ are utilized for $\theta_0$ acquisition. Surprisingly, similar results are obtained for $f = 33.62, 37.56, 39.51, 41.50,$ and $43.30 \text{ GHz}$. Thus, we assume that the fit-extracted $\theta_0$ at given $B$ field acquired from the opposite sides of Hall bar are nearly indistinguishable. As a consequence, we average over $\theta_0^R$ and $\theta_0^L$ to minimize the effect of measurement uncertainty on $\theta_0$, i.e.,

$$\theta_0^{+,-} = (\theta_0^R + \theta_0^L)/2$$

with “+” sign for positive $B$ and “−” sign for negative $B$. By doing so, the $\theta_0^+$ and $\theta_0^-$ are more representative of the corresponding Hall-bar area producing the phase shift. Table 4.1 summarizes calculated $\theta_0^+$ and $\theta_0^-$ at six oscillatory extrema after the measurement errors are reduced. The data suggests that $\theta_0$ values under field reversal for each $f$ are similar and differences are smaller than uncertainty of measurement, e.g., $\theta_0^+(P1) \sim \theta_0^-(P1)$ at $f = 37.56$.

<table>
<thead>
<tr>
<th>$f$ (GHz)</th>
<th>$\theta_0^+(P1)$</th>
<th>$\theta_0^-(P1)$</th>
<th>$\theta_0^+(V1)$</th>
<th>$\theta_0^-(V1)$</th>
<th>$\theta_0^+(P2)$</th>
<th>$\theta_0^-(P2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>32.50</td>
<td>1.0° ± 0.4°</td>
<td>−9.9° ± 0.7°</td>
<td>−17.7° ± 0.8°</td>
<td>−12.6° ± 0.6°</td>
<td>−11.1° ± 0.3°</td>
<td>−6.6° ± 1.0°</td>
</tr>
<tr>
<td>33.62</td>
<td>−12.5° ± 0.6°</td>
<td>−26.8° ± 0.6°</td>
<td>−11.2° ± 0.5°</td>
<td>−17.5° ± 0.9°</td>
<td>−14.0° ± 0.4°</td>
<td>−22.0° ± 0.8°</td>
</tr>
<tr>
<td>37.56</td>
<td>25.1° ± 0.4°</td>
<td>26.7° ± 0.5°</td>
<td>25.0° ± 0.5°</td>
<td>16.7° ± 0.5°</td>
<td>−</td>
<td>−</td>
</tr>
<tr>
<td>39.51</td>
<td>16.3° ± 0.6°</td>
<td>14.3° ± 0.8°</td>
<td>11.7° ± 1.1°</td>
<td>23.7° ± 0.9°</td>
<td>18.4° ± 0.7°</td>
<td>25.1° ± 0.8°</td>
</tr>
<tr>
<td>41.50</td>
<td>−32.5° ± 0.6°</td>
<td>−20.5° ± 0.4°</td>
<td>−23.3° ± 1.0°</td>
<td>−23.8° ± 1.0°</td>
<td>−18.5° ± 1.8°</td>
<td>−14.0° ± 0.6°</td>
</tr>
<tr>
<td>43.30</td>
<td>−11.7° ± 0.3°</td>
<td>−2.7° ± 0.5°</td>
<td>−11.1° ± 0.8°</td>
<td>−3.2° ± 1.9°</td>
<td>−14.3° ± 0.9°</td>
<td>−0.6° ± 0.9°</td>
</tr>
</tbody>
</table>

To determine the $f$-dependence phase shift variation, we average over $\theta_0$ under field reversal at $P1$, $V1$, and $P2$ for each $f$ to reduce the field reversal influence on the phase shift, i.e., $\theta_0^{av.} = (\theta_0^+ + \theta_0^-)/2$. Figure 4.7 illustrates $\theta_0^{av.}$ as a function of $f$ for $P1$, $V1$, and $P2$. The points reveal clustering at each $f$ and the difference between any two $\theta_0^{av.}$ at given $f$ is less than measurement errors $14^\circ$. The results indicate that $\theta_0$ varies systematically with $f$ when minimizing other small contributions to the phase shift.
Figure 4.7. The $\theta_{av}^0$ vs. $f$ for various extrema of MIMOs, where $\theta_{av}^0 = (\theta_0^+ + \theta_0^-)/2$ in order to reduce the small contribution of field reversal to the phase shift. The figure shows clustered $\theta_{av}^0$ at given $f$ and a systematic variation with $f$.

### 4.5 Discussion and Summary

The results exhibited here indicate as follows: (a) The phase shifts extracted from opposite sides of the Hall bar, i.e., $\theta_0^L$ and $\theta_0^R$, by fitting cosine square function to the data at various oscillatory extrema of MIMOs, at a given $f$ show smaller difference than uncertainty of measurement, i.e., $\delta \theta_0 < 14^\circ$. (b) The $\theta_{av}^0$ for various oscillatory extrema demonstrate similar values at a given $f$. Moreover, the point-clusters of $\theta_{av}^0$ exhibit a nontrivial $f$-dependence variation over $32 < f < 44 \text{ GHz}$.

The description of the point (a) agrees with intuitive expectations because the two parallel sides of Hall bar have the same relative orientations with respect to microwave polarization. The mean free path of a GaAs/AlGaAs heterostructures with carrier mobility up to $10^7 \text{ cm}^2 \cdot \text{V}^{-1} \cdot \text{s}^{-1}$ is mm- or sample size scale. In such a highly homogeneous system, the responses of electrons to linear-polarization orientation on the two Hall-bar sides slightly exhibit dissimilarity. In contrast, investigation of a pair of non-parallel Hall-bar sides in correlation with the phase shift might be worth studying in the future.
Point (b) implies that the phase shift is not strongly dependent upon the magnitude of $B$ in the specimen. However, the previous results [78] show the phase shift also depends on $B$ magnitude. The discrepancy could be possibly attributed to the sample quality and defect configuration within the specimen.

According to the balance-equation formulation of their photo-assisted magnetotransport model proposed by Lei et al. [119], they have successfully simulated sinusoidal responses of $R_{xx}$ with respect to $\theta$ and the simulations specify the phase shift is $f$- and $B$-dependence. Plus, they predict that $P1 + (\theta) = P1 - (\pi - \theta)$, $V1 + (\theta) = V1 - (\pi - \theta)$, etc, in an isotropic system. Although, the predictions are not observed in the measurements, it should be noted that extra complexity could be included in real specimen, such as asymmetry, that is not taken into account in the theory. In addition, Iñarrea et al. calculated sinusoidal variation in $R_{xx}$ vs. $\theta$ in light of the radiation-driven electron-orbit model as well, but the phase shift is not expressed in the simulations [121].

Finally, in spite of the fact that the analytic $f$-dependence $\theta_0$ results reported here is not fully realized in experiment and we lack adequate theoretical approaches to interpret the observed phenomena of the phase shift. It is still worth to make a comparison with Faraday rotation in quantum Hall systems. In Faraday rotation, when a linear-polarization radiation penetrate a two-dimensional material subject to an perpendicular magnetic field, the transmitted polarization plane is rotated by an Faraday angle, $\theta_F$, which is related to radiation frequency based on Drude-Lorentz model [127–129]. Here, however, the phase shift $\theta_0$ is observed in the $dc$ response of magnetoresistance under $ac$ excitation. If the scenario of Faraday rotation occurs in 2DES, the phase shift could be qualitatively understood by assuming that the $dc$ response follows the rotated $ac$ excitation polarization. Thus, the phase shift $\theta_0$ could be a manifestation of Faraday angle $\theta_F$. However, the possibility of the Faraday rotation assumption still needs more theoretical works to verify.

To sum up, we perform polarization-angle-dependence measurements at oscillatory extrema of MIMOs and carefully analyze the fit-extracted phase shift $\theta_0$ in sinusoidal response in $R_{xx}$ as a function of $\theta$ by averaging over the contributions relatively smaller than uncer-
tainty of measurement. The analysis demonstrates slight influence of $B$ magnitude on $\theta_0$ for each $f$ and a nontrivial $f$-dependence $\theta_0$ over $32 \leq f \leq 44 \, GHz$. 
Chapter 5

THE PHASE SHIFT EVOLUTION OVER A QUASI-CONTINUOUS FREQUENCY BAND IN LINEAR POLARIZATION ANGLE DEPENDENCE

5.1 Background

In Chapter 4, we have studied the phase shift $\theta_0$ in sinusoidal responses of $R_{xx}$ as a function of $\theta$ at various $f$. By averaging over small contributions less than uncertainty of measurement, the analytic results demonstrate nontrivial $f$-dependence of $\theta_0$ and small influences of $B$ magnitude upon $\theta_0$ at a given $f$ [88]. However, in this approach, the evolution between measured $f$ is unknown. One might wonder that if the phase shift is extremely sensitive to $f$ so that wide variations of $\theta_0$ take place between measured $f$, or if the $\theta_0$ is sensitive to geometry of specimen, e.g., length-to-width ratio $L/W$ of the Hall-bar device. Thereby, in this chapter, we carry out $f$ sweep measurements within $36 \leq f \leq 40 \text{ GHz}$ over $0^\circ \leq \theta \leq 360^\circ$ at a set of oscillatory extrema of MIMOs for Hall-bar sections $L/W = 1$ and 2. Thus, $\theta_0$ extracted from various Hall-bar sections presents the evolution over a continuous $f$ band. The data demonstrates that the $\theta_0$ extracted from the Hall-bar section for $L/W = 1$, i.e., $\theta_{0,1}$, exhibits less $f$ sensitivity than that extracted from the Hall-bar section for $L/W = 2$, i.e., $\theta_{0,2}$. Nonetheless, remarkably, the overall average of $\theta_{0,1}$ is almost identical to that of $\theta_{0,2}$ over $36 \leq f \leq 40 \text{ GHz}$. The fact that the proximity of contacts does not influence the phase shift leads to a disagreement with the expectations of the pondermotive force model.

5.2 Measurement Arrangement

The 200-$\mu$m-wide Hall-bar high-mobility GaAs/AlGaAs heterostructures with carrier density $n_e \sim 3.3 \times 10^{11} \text{ cm}^{-2}$ and mobility $\mu \sim 14 \times 10^6 \text{ cm}^2 \cdot \text{V}^{-1} \cdot \text{s}^{-1}$ at $T = 1.5 \text{ K}$ is measured. The four-terminal lock-in technique is utilized with an ac current $I$ flowing through
the specimen. Linear-polarization radiation illuminates the Hal bar device with a rotatable polarization angle $\theta$ between the Hall bar axis and the electric field increasing clockwise; as in Figure 5.1. The longitudinal voltage, $V_{xx,1}$ and $V_{xx,2}$ are collected simultaneously from the Hall bar sections for $L/W = 1$ and $L/W = 2$ respectively. Even though the calibration of measurement has been carried out by setting $\theta = 0^\circ$ when radiation field is aligned with Hall bar orientation with the help of power detector prior to measurements (see Section 4.2), a carbon sensor with strong negative temperature coefficient $dV_{ABR}/dT \leq 0$, where $V_{ABR}$ is the voltage of the carbon sensor also measured by four-terminal lock-in technique, at liquid helium temperature, is situated next to the specimen for the sake of in-situ measurement of $\theta_0$ and the independent detection of incident radiation rotation at the specimen location [82,83], as indicated in Figure 5.1. The radiation polarization rotation can be detected by the carbon sensor with regard to its preferred axis, i.e., $\theta = 90^\circ$ by sensing the change in heating effect since the maximal heating occurs as radiation polarization is orientated along with the axis of the carbon sensor.

Figure 5.2 exhibits strong MIMOs in longitudinal magnetoresistance, $R_{xx,1}$ and $R_{xx,2}$ as a function of $B$ at $f = 38 \text{ GHz}$, where $R_{xx,1} = V_{xx,1}/I$ and $R_{xx,2} = V_{xx,2}/I$. Similar to Chapter 4, we assign $P1+, V1+, P2+, P2-, V1-, \text{ and } P1-$ to oscillatory extrema. $f$-sweep measurements are carried out at polarization angle extent of $0^\circ \leq \theta \leq 360^\circ$ with $10^\circ$ intervals over $36 \leq f \leq 40 \text{ GHz}$ at six labeled oscillatory extrema. It is well-known that the $B$ filed positions of extremal MIMOs are linearly proportional to $f$ [50]. During the $f$-sweep measurements, therefore, the $B$ positions of extremal MIMOs will vary with $f$. Plus, previous studies have manifested that the phase shift can not only be $f$-dependence but also $B$-dependence. To minimize the influence of $B$ on $\theta_0$, the appropriate $B$ value for each oscillatory extreme is chosen given by $R_{xx}$ vs. $B$ trace at $f = 38 \text{ GHz}$. In accordance with the linear relation between $f$ and $B$, $\sim 4.8 \text{ mT}$ deviation occur in a $2 \text{ GHz}$ band with reference to $38 \text{ GHz}$, which is merely $\sim 5\%$ difference of $B$. Consequently, the influence of $B$ deviation due to the variation in $f$ on $\theta_0$ is negligent in $f$-sweep measurements.
Figure 5.1. The sketch of the relative orientation of the GaAs/AlGaAs heterostructure, the carbon resistor (ABR) and radiation field with rotatable polarization angle $\theta$ with respect to Hall bar axis. The longitudinal voltage $V_{xx,1}$ and $V_{xx,2}$ are measured from Hall bar device for $L/W = 1$ and 2 respectively. $V_{ABR}$ represents the voltage of the carbon sensor which is highly sensitive to heating effect.
Figure 5.2. The MIMOs traces of $R_{xx,1}$ and $R_{xx,2}$ vs. $B$ at $f = 38 \text{ GHz}$. The $B$-field positions of oscillatory extrema, i.e., $P1^+, V1^+, P2^+, P2^-, V1^-$, and $P1^-$, given by $R_{xx}$ vs. $B$ at $f = 38 \text{ GHz}$ are fixed over $36 \leq f \leq 40 \text{ GHz}$ during $f$-sweep measurements.

5.3 Frequency Dependence of Sinusoidal Variation in Longitudinal Resistance

To ensure that the observed $\theta_0$ variations derived from the specimen is not due to uncharacterized polarization rotation resulting from the experimental apparatus, the carbon resistor (ABR) is employed as a radiation polarization rotation detector to determine the polarization orientation near the specimen during the measurements. Since the preferred axis of the carbon resistor is set at $\theta = 90^\circ$, the lowest voltage of the carbon resistor $V_{ABR}$ is at $\theta = 90^\circ$ and the highest $V_{ABR}$ is at $\theta = 0^\circ$ under constant current excitation. Figure 5.3 (a) exhibits normalized $R_{ABR}$ color plot of $f$ vs. $\theta$ over $36 \leq f \leq 40 \text{ GHz}$ with $0^\circ \leq \theta \leq 360^\circ$ at magnetic field corresponding to $V1^-$, where $R_{ABR} = V_{ABR}/I_{ABR}$ (see Figure 5.1). Low resistance is illustrated by blue color and high resistance is illustrated by red color. The color plot depicts high normalized $R_{ABR}$ exhibit roughly at $\theta = 0^\circ$, $180^\circ$, and $360^\circ$ over $36 \leq f \leq 40 \text{ GHz}$. This feature is in agreement with the expectations of polarization calibration. Panel (b) shows the sinusoidal response in normalized $R_{ABR}$ with
θ at \( f = 38.3 \) GHz (dashed black line shown in panel (a)). The data is fit to empirical cosine square formula, \( \text{Nor. } R_{\text{ABR}}(\theta) = A + C\cos^2(\theta - \theta_{0,\text{ABR}}) \), to obtained \( \theta_{0,\text{ABR}} \), and the fit-extracted \( \theta_{0,\text{ABR}} \) within of \( 36 \leq f \leq 40 \) GHz are plot in panel (c). Then, the average \( \theta_{0,\text{ABR}} \) is calculated by averaging over the \( \theta_{0,\text{ABR}} \). It shows the average \( \theta_{0,\text{ABR}} = 5.3^\circ \), very close to \( \theta = 0^\circ \). The small deviation of average \( \theta_{0,\text{ABR}} = 5.3^\circ \) can be attributed to the combined misalignments of the Hall bar, the carbon sensor, and the antenna. Besides, the zigzag feature of normalized \( R_{\text{ABR}} \) color plot with \( f \) indicates the standard deviation of \( \theta_{0,\text{ABR}} \sim 13.0^\circ \) with varying \( f \).

Figure 5.3(d) displays normalized \( R_{xx,1} \) color plot of \( f \) vs. \( \theta \). Like panel (a), the periodic red-blue strips with \( \theta \) implies sinusoidal responses in normalized \( R_{xx,1} \). The cosine square formula, i.e., \( \text{Nor. } R_{xx,1}(\theta) = A - C\cos^2(\theta - \theta_{0,1}) \), fits the experimental data at \( f = 37.3 \) GHz to extract \( \theta_{0,1} \) indicated in panel (e). The \( f \) as a function of \( \theta_{0,1} \) and average \( \theta_{0,1} = 41.8^\circ \) are plotted in panel (f). Likewise, panel (g), (h), and (i) exhibit normalized \( R_{xx,2} \) color plot, sinusoidal curve of normalized \( R_{xx,2} \) vs. \( \theta \), and \( f \) vs. \( \theta_{0,2} \) with average \( \theta_{0,2} = -41.3^\circ \). Here, remarkably, the average \( \theta_{0,1} \) and \( \theta_{0,2} \) are very close to each other. In comparison of average \( \theta_{0,\text{ABR}}, \theta_{0,1}, \) and \( \theta_{0,2} \), they show significant difference between the carbon sensor and the specimen. It implies that \( \theta_{0,1} \) and \( \theta_{0,2} \) reflect characteristic properties of the specimen. The roughly \( f \) independence of \( \theta_{0,1} \) and \( \theta_{0,2} \) suggests \( \theta_0 \) does not change so much with \( f \), even though it does change slightly.

Subsequently, we carried out \( f \)-sweep measurements at \( P1- \). Figure 5.4(a) illustrates normalized \( R_{\text{ABR}} \) color plot of \( f \) vs. \( \theta \). Similar to Figure 5.3(a), the periodic red-blue strips shows high resistance near \( \theta = 0^\circ, 180^\circ, \) and \( 360^\circ \). Figure 5.4(b) indicates sinusoidal response in \( R_{xx,1} \) vs. \( \theta \) at \( f = 38.8 \) GHz. The \( \theta_{0,\text{ABR}} \) is extracted by fitting to cosine square formula, \( \text{Nor. } R_{xx,1}(\theta) = A + C\cos^2(\theta - \theta_{0,1}) \), in the range of \( 36 \leq f \leq 40 \) GHz with the average \( \theta_{0,\text{ABR}} = 13.3^\circ \) are shown in panel (c). Based upon the fact that similar calculated average \( \theta_{0,\text{ABR}} \) values obtained at \( V1- \) and \( P1- \), the polarization hardly changes with \( B \). Panel (d) shows similar periodic strips of normalized \( R_{xx,1} \) color plot of \( f \) vs. \( \theta \). The \( \theta_{0,1} \) at \( P1- \) are determined by fitting cosine square functions; see panel (e). The \( \theta_{0,1} \) trace is depicted in.
Figure 5.3. The $f$-sweep measurements at polarization angle $\theta$ of normalized (a) $R_{ABR}$, (d) $R_{xx,1}$, and (g) $R_{xx,2}$ over $36 \leq f \leq 40$ GHz band with $0^\circ \leq \theta \leq 360^\circ$ at V1−. Panel (b), (e), and (h) illustrate nice sinusoidal variation in resistance with $\theta$ at given $f$. Panel (c), (f), and (i) demonstrate the $\theta_0$ with respect to $f$ and the average of $\theta_0$. Here, $\theta_0$ is extracted by fitting sinusoidal response in normalized resistance at each $f$. Panel (c), (f), and (i) imply a constant average phase shift at V1−.
Figure 5.4. The $f$-sweep measurements at polarization angle $\theta$ of normalized (a) $R_{\text{ABR}}$, (d) $R_{xx,1}$, and (g) $R_{xx,2}$ over $36 \leq f \leq 40$ GHz band with $0^\circ \leq \theta \leq 360^\circ$ at $P1\text{−}$. Panel (b), (e), and (h) illustrate nice sinusoidal variation in resistance with $\theta$ at given $f$. Panel (c), (f), and (i) demonstrate the $\theta_0$ with respect to $f$ and the average of $\theta_0$. Here, $\theta_0$ is extracted by fitting sinusoidal response in normalized resistance at each $f$. Panel (c) and (f) imply $f$-independence of $\theta_{0,\text{ABR}}$ and $\theta_{0,1}$, but not for $\theta_{0,2}$ in panel (i) at $P1\text{−}$.
panel (f) with average $\theta_{0.1} = -41.8^\circ$. In the same way, panel (g) exhibits normalized $R_{xx,2}$ color plot of $f$ vs. $\theta$, but it shows more $f$-dependence in comparison to $R_{xx,1}$. We fit the cosine square formula to the data of normalized $R_{xx,2}$ vs. $\theta_0$ (shown in panel (h)), and plot $\theta_{0.2}$ vs. $f$ with average $\theta_{0.2}$ in panel (i). In comparison to Figure 5.4(f), panel (i) manifests clear $\theta_0$ variation over $36 \leq f \leq 38$ GHz. The standard deviations of $\theta_{0,ABR}$ (11.6$^\circ$), $\theta_{0,1}$ (10.0$^\circ$), and $\theta_{0,2}$ (31.0$^\circ$) within $36 \leq f \leq 40$ GHz reflect $\theta_{0.2}$ is more $f$-sensitive than $\theta_{0,ABR}$ and $\theta_{0.1}$. 

5.4 The Phase Shift Evolution in the Frequency Band

To complete the analysis of the phase shift, $\theta_{0,ABR}$, $\theta_{0.1}$, and $\theta_{0.2}$ at the remaining oscillatory extrema are extracted over $36 \leq f \leq 40$ GHz as well and depicted in Figure 5.5. Panel (a) shows $\theta_{0,ABR}$ as a function of $f$. $\theta_{0,ABR}$ appear compact as clustered-points between $36 \leq f \leq 40$ GHz. Due to the $f$- and $B$-independence features of $\theta_{0,ABR}$, we average over all $\theta_{0,ABR}$ and report an overall average of $\theta_{0,ABR} = 10.6^\circ$. This result confirms that the carbon resistor may be utilized as an in-situ detector of radiation polarization. The expected shift value of 10.6$^\circ$ might result from minor combined misalignment of the Hall bar, the carbon resistor, and the microwave launcher. Figure 5.5(b) and (c) shows $\theta_{0.1}$ and $\theta_{0.2}$ as a function of $f$ with the calculated overall average of $\theta_{0.1}$ and $\theta_{0.2}$. At the first glance, most of the $\theta_{0.1}$ and $\theta_{0.2}$ are located between $-75^\circ \leq \theta_0 \leq -25^\circ$ but $\theta_{0.1}$ appear more clustered than $\theta_{0.2}$. In addition, some $\theta_0$ manifest the property of $f$ dependence, such as, $\theta_{0.2}$ at $P2-$, but some does not, such as, $\theta_{0.1}$ at $P2+$. In order to clarify the reason, we calculate the average$\pm$standard deviation of $\theta_0$ at each oscillatory extrema over $36 \leq f \leq 40$ GHz and list them on table 5.1. The comparison implies that $\theta_{0,ABR}$ and $\theta_{0.1}$ have similar standard deviations at each oscillatory extrema, only except for V1+. Half of $\theta_{0.2}$, however, exhibit greater standard deviations than $\theta_{0,ABR}$. This implies the possibility that $\theta_{0.2}$ exhibits more distinguishable $f$ dependence than that of $\theta_{0.1}$. The overall standard deviations (averaging over all standard deviations of $\theta_0$) of $\theta_{0,ABR} = 13.3^\circ$, $\theta_{0.1} = 14.4^\circ$, and $\theta_{0.2} = 23.7^\circ$ reaffirm
more $f$-sensitivity of $\theta_{0,2}$. Nonetheless, remarkably, the overall averages of $\theta_{0,1} = -42.8^\circ$ and $\theta_{0,2} = -44.9^\circ$ are nearly identical and evidently different from the overall average of $\theta_{0,ABR}$.

Table 5.1. The average ± standard deviation of $\theta_0$ at each oscillatory extrema over $36 \leq f \leq 40 \text{ GHz}$ are calculated. The table indicates $\theta_{0,2}$ has larger standard deviation than $\theta_{0,ABR}$ and $\theta_{0,1}$ over $36 \leq f \leq 40 \text{ GHz}$. Also, the averages of $\theta_{0,1}$ and $\theta_{0,2}$ over all extrema differ significantly from the average of $\theta_{0,ABR}$ over all extrema.

<table>
<thead>
<tr>
<th></th>
<th>$P1-$</th>
<th>$P1+$</th>
<th>$P2-$</th>
<th>$P2+$</th>
<th>$V1-$</th>
<th>$V1+$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_{0,ABR}$</td>
<td>13.3° ± 11.6°</td>
<td>11.4° ± 13.9°</td>
<td>10.7° ± 13.1°</td>
<td>11.5° ± 14.1°</td>
<td>5.3° ± 13.2°</td>
<td>11.2° ± 14.3°</td>
</tr>
<tr>
<td>$\theta_{0,1}$</td>
<td>-32.1° ± 10.2°</td>
<td>-61.3° ± 16.3°</td>
<td>-36.0° ± 8.5°</td>
<td>-41.7° ± 9.7°</td>
<td>-41.8° ± 6.6°</td>
<td>-44.0° ± 35.0°</td>
</tr>
<tr>
<td>$\theta_{0,2}$</td>
<td>-62.5° ± 31.0°</td>
<td>-23.9° ± 41.3°</td>
<td>-18.1° ± 37.2°</td>
<td>-54.4° ± 12.4°</td>
<td>-41.3° ± 13.1°</td>
<td>-69.1° ± 7.3°</td>
</tr>
</tbody>
</table>

5.5 Discussion and Summary

Before the start of measurements, the microwave launcher and the Hall bar axis were oriented. Then, the relative orientation of microwave polarization was determined by the variation of microwave power with the help of the power detector, as in Chapter 4 [88]. The calibration results indicate the standard deviation of incident radiation polarization angle $\sim 8^\circ$. Yet, one might wonder whether the incident polarization angle would be affected near the specimen owing to metallic contacts and bonded gold wires on the surface of the specimen. The carbon sensor in in situ measurements of polarization angle dependence manifests, however, the overall average of $\theta_{0,ABR} = 10.6^\circ$ close to 0 and the standard deviation of $\theta_{0,ABR} = 13.3^\circ$ also within the standard deviation $\sim 8^\circ$ obtained by the power detector before the start of the measurements. This affirms the control of the microwave polarization orientation near the specimen during the experiment. Besides, Figure 5.5(b) and (c) illustrates that most of $\theta_{0,1}$ and $\theta_{0,2}$ are located at $-75^\circ \leq \theta \leq -25^\circ$. Therefore, the preferable radiation orientation at $\theta = -43.9^\circ$ for the Hall bar device is verified by the considerable difference of $\theta_0$ between the carbon sensor ($\theta_{0,ABR}$) and the specimen ($\theta_{0,1}$ and $\theta_{0,2}$). Moreover, the overall average of $\theta_{0,1}$ and $\theta_{0,2}$ reveal nearly identical values. Namely,
Figure 5.5. The phase shift of (a) \( \theta_{0, ABR} \), (b) \( \theta_{0,1} \), and (c) \( \theta_{0,2} \) as a function of \( f \) at each oscillatory extrema, i.e., \( P1- \), \( P1+ \), \( P2- \), \( P2+ \), \( V1- \), and \( V1+ \), with overall average of the phase shift. Panel (a) exhibits small, constant, and clustered \( \theta_{0, ABR} \). Panel (b) and (c) show that \( \theta_{0,2} \) is more \( f \) sensitive than \( \theta_{0,1} \), but nearly identical overall average of \( \theta_{0,1} \) and \( \theta_{0,2} \).
the average $\theta_0$ does not appear to depend on $f$, $B$, and length-to-width ratio of the Hall bar section.

Here, we can compare our results to one of the MIMOs theories: The pondermotive force model. As discussed in Section 3.3.5, the suggestion of the theory is that the incident radiation field is screened by the metallic contacts that will induce a strong linearly polarized electric field near the contacts. The amplitude of near-contact electric field normal to the contact is much greater than that of the incident radiation field. On the other hand, the amplitude of near-contact electric field parallel with the contact is much smaller than that of the incident radiation field. Now, we suppose that the linearly polarized radiation field illuminates the specimen with an arbitrary orientation. The we can decompose the incident radiation field into $\vec{E}_{0x}$ along the Hall bar and $\vec{E}_{0y}$ normal to the Hall bar direction. In accordance with the theory, the probe contacts will induce near-contact electric field $\vec{E}_x$ along the Hall bar and $\vec{E}_y$ normal to the Hall bar. Based on the presumption that $\vec{E}_x \ll \vec{E}_{0x}$ and $\vec{E}_y \gg \vec{E}_{0y}$ in the vicinity of the contacts, the resultant electric field that 2DES responds to near the contacts is $\sim \vec{E}_y$, and away from the contact is $\vec{E}_{0x} + \vec{E}_{0y}$. When the distance between two probe contacts increases, the significance of $\sim \vec{E}_y$ on 2DES section remains constant but the component of $\vec{E}_{0x} + \vec{E}_{0y}$ becomes more dominant. In this scenario, the orientation of the resultant electric field average of the long Hall bar section will approach the $\vec{E}_{0x} + \vec{E}_{0y}$ orientation. Hence, on the basis of the pondermotive force model, the phase shift $\theta_0$ varying with the separation of two probe contacts is expected. However, the experimental results that the average phase shift is independent upon the length-to-width ratio of the Hall bar sections implies a discrepancy between experimental observations and the prediction of the pondermotive force model.

In summary, we determined the incident microwave polarization rotation independently near the specimen with the help of carbon sensor. The minor misalignment of the Hall bar, carbon resistor, and the antenna contributes to the slight $f$ dependence of $\theta_{0, ABR}$. The results indicate that $\theta_{0,1}$ are more clustered with $f$ than $\theta_{0,2}$, but both of them show nearly identical average phase shifts, which are substantially away from $\theta_{0, ABR}$. The features confirms the
existence of a preferable radiation orientation for the specimen and the independence of the average phase shift on $B$, $f$, and length-to-width ratio of the Hall bar sections at oscillatory extrema over $36 \leq f \leq 40 \, GHz$. Yet, the origin of the greater deviations of $\theta_{0,2}$ than that of $\theta_{0,1}$ still needs further study. Finally, the experimental findings of average phase shift independence on the length-to-width ratio of the Hall bar section do not follow the expectations of the pondermotive force model.
HALL AND LONGITUDINAL MAGNETORESISTANCE OSCILLATIONS
INDUCED BY COMBINED AC AND DC EXCITATIONS

6.1 Background

In Chapter 3, we have discussed that the MIMOs are $1/B$-periodic oscillations with $1/4$-cycle phase shift and the oscillatory minima emerge at $B = [4/(4j + 1)]B_f$, where $B_f = 2\pi f m^*/e$, $f$ is the microwave frequency, $m^*$ is the effective electron mass and $j = 1, 2, 3, \ldots$. The MIMOs emerges in the regime of $\omega > \omega_c$, where $\omega$ is microwave radiation and $\omega_c$ is cyclotron frequency. Later on, Kukushkin et al. discovered a new type of $B$-periodic magnetoresistance oscillations induced by microwave irradiation but at low filling factors [130, 131]. In contrast to MIMOs, these $B$-periodic magnetoresistance oscillations occur when $\omega < \omega_c$. The oscillation period was suggested to follow $\Delta B \propto n_e/\omega L$, where $n_e$ is electron density and $L$ is the distance between potential probes along the Hall bar. The oscillations are attributed to the interference of coherently excited edge magnetoplasmons (EMP) in the contact regions [132–134]. Sometime later, Stone et al. found that this sort of $B$-periodic oscillations is also in existence in the range of $\omega \geq \omega_c$, where RIMOs and EMP overlap, but $\Delta B$ is $L$ independent [135].

So far, most of observed magnetoresistance oscillations induced by either microwave radiation or $dc$ bias happen at high filling factors that includes the investigations of the MIMOs strongly affected by $dc$ bias. In this paper, we report the observation of unusual differential Hall magnetoresistance oscillations [136, 137]

\[
R_{xy} = \left( \frac{\partial V_{xy}}{\partial I} \right)_{dc} = \frac{\partial (IR_{xy})}{\partial I} = R_{xy}^0 + I_{dc} \frac{\partial R_{xy}^0}{\partial I} = \left( \frac{V_{xy}}{I} \right)_{ac}
\]  (6.1)
and longitudinal magnetoresistance oscillations

\[ R_{xx} = \left( \frac{\partial V_{xx}}{\partial I} \right)_{dc} = \frac{\partial(IR_{xx})}{\partial I} = R_{xx}^0 + I_{dc} \frac{\partial R_{xx}^0}{\partial I} = (\frac{V_{xx}}{I})_{ac} \] (6.2)

where \( R_{xy}^0 \) and \( R_{xx}^0 \) are Hall resistance and longitudinal resistance without dc bias contributions respectively. \( R_{xy} \) and \( R_{xx} \) are induced by microwave radiation and dc bias, \( I_{dc} \), concurrently at low filling factors from 0.7 \( T \) to 2 \( T \); shown in Figure 6.1. We examine that the \( R_{xy} \) and \( R_{xx} \) oscillatory curves are \( B \)-field periodic and the period remains consistent as \( L \) changes. Therefore, the possibility that the oscillations originate from EMP is excluded in light of experimental results.

Figure 6.1. The sketch of measurement configuration on GaAs/AlGaAs heterostructure. An \( ac \) current, \( I_{ac} \), and a \( dc \) current, \( I_{dc} \), flow through the sample along the Hall bar simultaneously. The Hall voltage, \( V_{xy} \), and longitudinal voltage, \( V_{xx} \), are measured by lock-in amplifiers. During the measurements, the linearly polarized microwave radiation illuminates the sample from the top as well.

6.2 Differential Magnetoresistance Oscillations at Low Filling Factors

Figure 6.2(a) shows the comparisons of Hall magnetoresistance, \( R_{xy} \), and longitudinal magnetoresistance, \( R_{xx} \), between photo-excited and dark curves with \( I_{dc} = 0 \mu A \) at \( f = 58GHz \). Here the dark \( R_{xy} \) is offset for the sake of clarity. The photo-excited curves with dark subtraction, i.e., \( \Delta R = R(\text{photo-}excited) - R(\text{dark}) \), are shown in Figure 6.2(b). In the
Figure 6.2. (a) The comparison of Hall magnetoresistance, $R_{xy}$, and longitudinal magnetoresistance, $R_{xx}$, between photo-excited and dark curves at $f = 58 \text{ GHz}$ when $I_{dc} = 0 \mu A$. The dark $R_{xy}$ is offset for the sake of clarity. The characteristic field of cyclotron resonance is labeled as $B_f$. (b) The photo-excited curves with dark subtraction, i.e., $\Delta R = R(\text{photo-excited}) - R(\text{dark})$, implies a noise-dominating $\Delta R_{xy}$ near 0 and unexpected small oscillations of $\Delta R_{xx}$ up to $1 \text{T}$. (c) The comparison of differential Hall magnetoresistance, $R_{xy}$, and differential longitudinal magnetoresistance, $R_{xx}$, between photo-excited and dark curves at $f = 58 \text{ GHz}$ and $I_{dc} = 30 \mu A$. It is clear that as applying an $I_{dc} = 30 \mu A$, the photo-excited $R_{xy}$ starts exhibiting small oscillations at high $B$. Meanwhile, the photo-excited SdH oscillations split into high-frequency oscillations. (d) The differential Hall magnetoresistance difference, $\Delta R_{xy}$, and differential longitudinal magnetoresistance difference, $\Delta R_{xx}$, between photo-excited and dark curves. The results demonstrate that the maximum (minimum) of $\Delta R_{xy}$ oscillations correspond to minimum (maximum) of $\Delta R_{xx}$ oscillations.
absence of an *dc* bias, the \( R_{xy} \) barely shows difference between photo-excited and dark curves. The photo-excited \( R_{xx} \) exhibits small oscillations comparing to dark \( R_{xx} \) curve below 1 T, and the Shubnikov-de Haas (SdH) oscillations are slightly suppressed by microwaves. The characteristic field of cyclotron resonance is labeled as \( B_f = 2\pi f m^* / e \), where \( f \) is microwave frequency, \( m^* \) is effective mass, \( e \) is the electron charge. However, when an \( I_{dc} = 30 \mu A \) is applied, the photo-excited \( R_{xy} \) shows evident small oscillations in comparison to dark \( R_{xy} \) at high \( B \) field. Meanwhile, photo-excited SdH oscillations split into high-frequency oscillations; see Figure 6.2(c). Again, the dark \( R_{xy} \) is offset for the sake of clarity. The demonstrations of the Hall and longitudinal differential magnetoresistance with background subtraction \( \Delta R \) vs. \( B \) with \( I_{dc} = 30 \mu A \) at \( f = 58 \text{GHz} \) are shown in Figure 6.2(d). The panel suggests that the \( \Delta R_{xy} \) and \( \Delta R_{xx} \) oscillations start emerging near 0.25 T and gradually become noticeable up to 2 T. The maximum (minimum) of \( \Delta R_{xy} \) oscillations is correspondent with the minimum (maximum) of \( \Delta R_{xx} \). Note that the \( B \) field that \( R_{xy} \) and \( R_{xx} \) oscillations are observed is higher than \( B_f \).

### 6.3 Phase Comparison between Hall and Longitudinal Resistance Oscillations

Next, we collect various Hall and longitudinal magnetoresistance from every pair of potential contacts. For the sake of clarity, we label each potential probe with numbers; see in Figure 6.3.

![Figure 6.3](image)

Figure 6.3. The illustration shows the configuration of the sample with labeled potential probes. Various Hall and longitudinal voltages are obtained from each pair of potential contacts.
Figure 6.4. The comparison of $\Delta R_{xy}$ derived from different parts of Hall bar at (a) $f = 49$ GHz and (b) $f = 31$ GHz with $I_{dc} = 30 \, \mu A$ demonstrate that $\Delta R_{xy}^{14}$, $\Delta R_{xy}^{25}$, and $\Delta R_{xy}^{36}$ are in phase. $\Delta R_{xx}$ extracted from various Hall bar sections at (c) $f = 49$ GHz and (d) $f = 31$ GHz with $I_{dc} = 30 \, \mu A$ indicate that $\Delta R_{xx}^{23}$ is out of phase with respect to $\Delta R_{xx}^{56}$ and $\Delta R_{xx}^{12}$. But the period of oscillations is independent upon the length between two probe contacts. The results also suggest the increase in period as $f$ reduces.
Figure 6.4 exhibits a variety of $\Delta R$ oscillations from the original $V$ signals measured from different Hall bar sections. $\Delta R_{xy}^{36}$ is obtained from, for example, the signal of $V_{xy}^{36}$ that is measured between the probe contacts 3 and 6; see Figure 6.3. The length-to-width ratios $(W/L)$ of the Hall bar sections between contacts $1-2$ ($4-5$) and $2-3$ ($5-6$) are 1 and 2 respectively. Panel (a) and (b) illustrate $\Delta R_{xy}$ obtained from different segments of Hall bar device with $I_{dc} = 30 \mu A$ at $f = 49 \text{ GHz}$ and $31 \text{ GHz}$. They show that $\Delta R_{xy}^{14}$, $\Delta R_{xy}^{25}$, and $\Delta R_{xy}^{36}$ possess similar period and stay in phase. Different $\Delta R_{xx}$ oscillations obtained from different Hall bar sections induced by $f = 49 \text{ GHz}$ and $31 \text{ GHz}$ with $I_{dc} = 30 \mu A$ are shown in Figure 6.4(c) and (d) respectively. The results indicate that $\Delta R_{xx}^{23}$ has a nearly $180^\circ$ phase shift in regard to $\Delta R_{xx}^{56}$ and $\Delta R_{xx}^{12}$. However, similar period of oscillations are observed on these three curves at the specified $f$ which means the period are independent upon the separation distance between two probe contacts. The results also suggest that the periods of $R_{xx}$ and $R_{xy}$ oscillations, which are almost identical, increases as $f$ decreases.

### 6.4 Periodicity in Magnetic Field and Power Dependence

To demonstrate the periodicity, the oscillatory maximum of $R_{xy}$ and $R_{xx}$ are assigned to integer index and minimum to half integer index. The plots of index as a function of $B$ position of $R_{xy}$ and $R_{xx}$ at $I_{dc} = 30 \mu A$ with $f = 31, 40, 46, \text{ and } 58 \text{ GHz}$ are exhibited in Figure 6.5(a) and (b), which show linear relationship. The linearity confirms that $R_{xy}$ and $R_{xx}$ oscillations are $B$-periodic. Panel (c) illustrates the period, $\Delta B$, is linear to $1/f$. It affirms that the period of oscillations is inversely linear to microwave frequency.

Figure 6.6 reveals the $P$ dependence of $\Delta R_{xy}$, in panel (a), and $\Delta R_{xx}$, in panel (b), vs. $B$ at $f = 58 \text{ GHz}$ and $I_{dc} = 30 \mu A$. Three labeled $P$ signify microwave source power. It is obvious that both $\Delta R_{xy}$ and $\Delta R_{xx}$ oscillation amplitudes are enhanced by increasing $P$. A closer investigation indicates that the $B$ positions of oscillatory extrema move toward higher $B$ as $P$ increases. Panel (c) and (d) exhibit the amplitudes of specified oscillatory maximum (labeled as an asterisk) of $\Delta R_{xy}$ and $\Delta R_{xx}$ as a function of $P$ at $f = 58 \text{ GHz}$ and $I_{dc} = 30 \mu A$. The data illustrate an non-linear relation between amplitude and $P$. A well-fit power
Figure 6.5. The plots of index vs. $B$, where oscillatory maximum are assigned to integer index and minimum to half integer index, of (a) $R_{xy}$ and (b) $R_{xx}$ at $I_{dc} = 30 \, \mu A$ with $f = 31$, 40, 46, and 58 GHz shows linear relationship. The results confirm $B$-periodic feature of oscillations. (c) The linear relationship between $\Delta B$ and $1/f$ suggests that the period is inversely proportional to microwave frequency.
Figure 6.6. The $P$ dependence of (a) $\Delta R_{xy}$ and (b) $\Delta R_{xx}$ vs. $B$ at $I_{dc} = 30 \mu A$ and $f = 58 \, GHz$ indicate that the amplitude of oscillations become substantial as $P$ increasing. Meanwhile, the change in $B$ positions of oscillatory extrema with different $P$ is observed. The labeled $P$ is defined as the $P$ out of microwave source. The amplitudes of specified oscillatory maximum (labeled as an asterisk) of (c) $\Delta R_{xy}$ and (d) $\Delta R_{xx}$ as a function of $P$ appear non-linear relationship. The power law function, $\Delta R \propto P^\alpha$, is well-fit to experimental data. The extracted $\alpha$ indicate sub-linear relationship between amplitude and $P$ for both $\Delta R_{xy}$ and $\Delta R_{xx}$ oscillations.
law function, $\Delta R \propto P^\alpha$, is applied to experimental data to extract $\alpha$ that most are situated between 0.4 and 0.6. It specifies that the amplitude of both $\Delta R_{xy}$ and $\Delta R_{xx}$ oscillatory extrema manifests sub-linear relationship with $P$.

### 6.5 Discussion and Summary

The oscillations presented here seemingly resemble the ones discussed in Ref. [130] that are induced by microwave radiation and investigated at low filling factors as well. However, there remains three major dissimilarities between each other. (a) We observe the oscillations when moderate radiation power and $dc$ bias are applied simultaneously. (b) In additional to oscillations of longitudinal magnetoresistance, the Hall magnetoresistance oscillations are also reported here, which have not been reported before. (c) The period of $\Delta R_{xx}$ oscillations obtained from $W/L = 1$ and 2 are nearly identical. Recent theory claims the origin of this type of radiation-induced oscillations emerging at either low or high filling factors could be ascribed to EMP model based on the period dependence on microwave frequency, electron density, and distance between potential contacts. Yet, the point (c) that the independence of period on potential probe distance seems in conflict with the presumptions of EMP. The similar phenomenon was observed in Ref. [135] which suggests that the long decay length of EMP modes can propagate along the whole edge around the sample due to high-mobility sample. However, in quantum perspective, the EMP can be treated as a spatial variation of current propagating along the edge where the Hall charge accumulates and rearranging the charge by stretching and compressing [138]. Therefore, the EMP is not able to circulate the entire sample edge, and the EMP model is not sufficient for the interpretation of point (b) and (c).

In order to have a preliminary realization, we analyze the polarities of the electric field from the oscillating resistances, $\Delta R_{xy}$ and $\Delta R_{xx}$; shown in Figure 6.7. According to Figure 6.4, the group (G1) of $\Delta R_{xy}^{14}$, $\Delta R_{xy}^{25}$, $\Delta R_{xy}^{36}$, $\Delta R_{xx}^{12}$, and $\Delta R_{xx}^{56}$ are in phase and have a $180^\circ$ phase shift with regard to the group (G2) of $\Delta R_{xx}^{45}$, and $\Delta R_{xx}^{23}$ that are in phase. Figure 6.7(a) demonstrates the situation that when G1 reaches the minimum of $\Delta R$ and G2 reaches
Figure 6.7. Schematic diagram of electric field polarity for observed effects. (a) As $B = B_1$, the scenario that $\Delta R_{xy}^{14}$, $\Delta R_{xy}^{25}$, $\Delta R_{xy}^{36}$, $\Delta R_{xx}^{12}$, and $\Delta R_{xx}^{56}$ are maximum, and $\Delta R_{xx}^{45}$ and $\Delta R_{xx}^{23}$ are minimum is achieved. (b) As $B = B_2$, the scenario that $\Delta R_{xy}^{14}$, $\Delta R_{xy}^{25}$, $\Delta R_{xy}^{36}$, $\Delta R_{xx}^{12}$, and $\Delta R_{xx}^{56}$ are minimum, and $\Delta R_{xx}^{45}$ and $\Delta R_{xx}^{23}$ are maximum is achieved. In these two scenarios, the polarities of the electric field are represented by arrows.
the maximum at $B_1$ field. The arrow direction represents the polarity of the electric field. We find that the electric potential reveals higher (lower) on the bottom (top) edge of the specimen, and the middle region of the specimen manifests higher (lower) electric potential than two ends on the bottom (top) of the specimen. On the other hand, as $B = B_2$ at which $G_1$ reaches the maximum of $\Delta R$ and $G_2$ reaches the minimum; see panel (b), the polarities of the electric field reverses, i.e., the high (low) electric potential appears on the top (bottom) edge of the specimen, and the middle area of the specimen shows lower (higher) electric potential than two ends on the bottom (top) of the specimen. The inhomogeneity of the electric potential might be due to the gradient of the microwave electric fields. As the microwave power increases, the more intense inhomogeneity happens. But the reversal of the polarity of so far is still not clear.

In summary, we have observed a combined microwave- and dc bias-induced Hall and longitudinal magnetoresistance oscillations with periodicity in $B$ at low filling factors in GaAs/AlGaAs 2D electron system. The Hall and longitudinal magnetoresistance oscillations reveal similar period at given microwave frequency but either in phase or out of phase based on the section of Hall bar device. The amplitude of these oscillations is sub-linear to microwave power.
CONCLUSIONS

The dissertation mainly focuses on three aspects of low-temperature electron magneto-transport in GaAs/AlGaAs heterostructure Hall-bar device under external excitation.

In the study of the frequency dependence of the phase shift in linear-polarization sensitivity of microwave-induced magnetoresistance oscillations, the experimental uncertainties are determined by a power detector before measurements. Then, the data indicates that the phase shifts extracted by fitting the $R_{xx}$ vs. $\theta$ traces to the empiric cosine square formula, $R_{xx} = A \pm C\cos^2(\theta - \theta_0)$, from the opposite sides of the Hall-bar device are similar compared to the experimental uncertainties. Thus, by average over the phase shifts extracted from two sides of the specimen, the contributions of experimental uncertainties to the phase shift are reduced. Similar analytic methods are applied to other microwave frequencies over $32 \leq f \leq 44 \text{GHz}$. The results suggest nontrivial frequency dependence of the phase shift over the frequency band.

This dissertation also examined the phase shift evolution over a quasi-continuous frequency band. In this work, instead of utilizing the power detector to determine the experimental uncertainties, a carbon resistor sensor is placed near the sample for the determination of the uncertainties in in-situ measurements. The carbon sensor shows small measurement uncertainties $\leq 10^\circ$ over $36 \leq f \leq 40 \text{GHz}$. But the sample exhibits distinguishable average phase shifts $\sim 44^\circ$ obtained from length-width ratio, $L/W = 1$ and $L/W = 2$, that excludes the possibility that the phase shift is caused by the random reflective microwaves due to metallic contacts and gold wires. Remarkably, the average phase shift obtained from $L/W = 1$ and $L/W = 2$ are nearly identical. It confirms that the preferred polarization orientation for the Hall bar device is length-to-width ratio independent. The experimental results reflect the disagreement with the expectations of the pondermotive model.
The last aspect examined in the dissertation concerns Hall and longitudinal magnetoresistance oscillations induced by combined \textit{ac} and \textit{dc} excitations. By applying the \textit{ac}, i.e., microwave radiation, and \textit{dc} bias, the quantum oscillations take place at low-filling factors, i.e., high magnetic field. These quantum oscillations are magnetic field periodic and the relative phase between each potential contact pair is either in phase or out of phase. Further, the period of these quantum oscillations is inversely proportional to microwave frequency. However, the further studies are still needed to clarify the physics behind it.

Such magneto-transport in GaAs/AlGaAs heterostructure under external excitations still have numerous unsolved mysteries. For example, in linear polarization sensitivity of MIMOs, what is the factor that determines the preferred polarization orientation for GaAs/AlGaAs heterostructure? Further, in Chapter 5, the results show that the phase shift obtained from $L/W = 1$ is more cluster than that of $L/W = 2$. Does it imply that the fluctuations of the phase shift depend upon the $L/W$ or just upon the potential contact distance? In the unconventional quantum oscillation project, the origin of the oscillations is still unknown. These interesting puzzles still need to be solved by carrying out more creative experiments in the future.
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