Indicators of Mathematics Skill Acquisition in Children with Mild Intellectual Disability: Phonological Awareness, Naming Speed, and Vocabulary Knowledge

Matthew E. Foster
Georgia State University

Follow this and additional works at: https://scholarworks.gsu.edu/psych_theses

Recommended Citation
https://scholarworks.gsu.edu/psych_theses/91

This Thesis is brought to you for free and open access by the Department of Psychology at ScholarWorks @ Georgia State University. It has been accepted for inclusion in Psychology Theses by an authorized administrator of ScholarWorks @ Georgia State University. For more information, please contact scholarworks@gsu.edu.
INDICATORS OF MATHEMATICS SKILL ACQUISITION IN CHILDREN WITH MILD INTELLECTUAL DISABILITY: PHONOLOGICAL AWARENESS, NAMING SPEED, AND VOCABULARY KNOWLEDGE

by

MATTHEW EDWARD FOSTER

Under the Direction of Rose A. Sevcik

ABSTRACT

Deficiencies in mathematics skill constrain students’ educational achievement and subsequently, their employment outcomes. This study included 265 school-identified students with mild intellectual disabilities. The research questions investigated the extent to which phonological awareness, color naming speed, and vocabulary knowledge, was related to mathematics skill after controlling for grade level via regression analyses. Further, the mediating effects of expressive vocabulary on the relationship between receptive vocabulary and mathematics skill as well as the indirect effect of receptive vocabulary knowledge on mathematics skill through expressive vocabulary were examined. The findings indicated that after controlling for grade level, phonological awareness, naming speed, and vocabulary knowledge were significantly related to mathematics skill. The mediating effects of expressive vocabulary as well as the indirect effects of receptive vocabulary knowledge on mathematics skill were also significant.

INDEX WORDS: Phonological awareness, Naming speed, Receptive vocabulary, Expressive vocabulary, Mathematics skill, Math difficulties, Mild intellectual disabilities
INDICATORS OF MATHEMATICS SKILL ACQUISITION IN CHILDREN WITH MILD INTELLECTUAL DISABILITY: PHONOLOGICAL AWARENESS, NAMING SPEED, AND VOCABULARY KNOWLEDGE

by

MATTHEW EDWARD FOSTER

A Thesis Submitted in Partial Fulfillment of the Requirements for the Degree of Master of Arts in the College of Arts and Sciences Georgia State University 2011
INDICATORS OF MATHEMATICS SKILL ACQUISITION IN CHILDREN WITH MILD INTELLECTUAL DISABILITY: PHONOLOGICAL AWARENESS, NAMING SPEED, AND VOCABULARY KNOWLEDGE

by

MATTHEW EDWARD FOSTER

Committee Chair: Rose A. Sevcik
Committee: MaryAnn Romski
Robin Morris

Electronic Version Approved:
Office of Graduate Studies
College of Arts and Sciences
Georgia State University
December 2011
DEDICATION

This thesis is dedicated to my parents, Walter and Deborah Foster. Thank you for supporting me in all of my endeavors, as well as for your continued love and encouragement. Without both of you, this thesis would have never been accomplished. I love you both.
ACKNOWLEDGEMENTS

I would like to acknowledge the guidance and support that my advisor, Rose A. Sevcik, provided throughout the creation of this thesis. Her guidance and assistance made this manuscript possible. Rose, thank you for helping me achieve this milestone.

I would also like to acknowledge that this thesis was supported in part by grant number H324K040007, awarded by the Institute for Educational Sciences, Department Of Education, to Dr. Rose Sevcik, “Evaluating the effectiveness of reading interventions for students with mild mental retardation”.
TABLE OF CONTENTS

ACKNOWLEDGEMENTS ..................................................................................................................v

LIST OF TABLES ............................................................................................................................ viii

LIST OF FIGURES .......................................................................................................................... ix

1 INTRODUCTION .............................................................................................................................1
   1.1 Phonological Awareness ...........................................................................................................2
   1.2 Naming Speed ..........................................................................................................................2
   1.3 Vocabulary Knowledge ..........................................................................................................2
   1.4 Mathematics Skill ...................................................................................................................3
   1.5 Mathematics Difficulties ........................................................................................................6
   1.6 Relationship between Phonological Awareness (PA) and Computation Skills in Typically Developing Children ..........................................................................................................................9
   1.7 Mathematics Research and Children with Learning and Language Disabilities ......................10
   1.8 Mathematics Research and Children with Reading Disabilities .........................................12
   1.9 Children with Mild Intellectual Disabilities ............................................................................14
   1.10 Purposed Study .......................................................................................................................15

2 METHOD .......................................................................................................................................18
   2.1 Participants .............................................................................................................................18
   2.2 Measures ................................................................................................................................20
   2.3 Procedure ...............................................................................................................................28
   2.4 Data Analysis ..........................................................................................................................29

3 RESULTS .......................................................................................................................................32
<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.1</td>
<td>Descriptive Statistics</td>
<td>32</td>
</tr>
<tr>
<td>3.2</td>
<td>Investigating the Relationships between Phonological Awareness,......</td>
<td>37</td>
</tr>
<tr>
<td></td>
<td>Naming Speed, and Vocabulary Knowledge, and Mathematics Skill</td>
<td></td>
</tr>
<tr>
<td>3.3</td>
<td>Factor Analysis</td>
<td>37</td>
</tr>
<tr>
<td>3.4</td>
<td>Zero-order Correlations</td>
<td>39</td>
</tr>
<tr>
<td>3.5</td>
<td>Regression Analyses</td>
<td>40</td>
</tr>
<tr>
<td>4</td>
<td>DISCUSSION</td>
<td>47</td>
</tr>
<tr>
<td>4.1</td>
<td>Limitations of the Current Thesis</td>
<td>50</td>
</tr>
<tr>
<td>4.2</td>
<td>Future Directions</td>
<td>51</td>
</tr>
<tr>
<td>4.3</td>
<td>Conclusion</td>
<td>53</td>
</tr>
<tr>
<td>5</td>
<td>REFERENCES</td>
<td>54</td>
</tr>
</tbody>
</table>
LIST OF TABLES

Table 1: Chronological Age and School Reported Grade Distribution of the Participant……..19

Table 2: Descriptive Statistics for the CTOPP, PPVT, EVT, and KeyMath-R Subtests at…….33
the Baseline Time Point

Table 3: Innercorrelations of KeyMath-R subtests.........................................................38

Table 4: KeyMath-R Factor Loadings and Communalities.............................................39

Table 5: Zero-order Correlations.................................................................................40

Table 6: Regression Analysis with Phonological Awareness Predicting Mathematics Skill….41

Table 7: Regression Analysis with Color Naming Speed Predicting Mathematics Skill…….42

Table 8: Regression Analysis with Vocabulary Knowledge Predicting Mathematics Skill…..43

Table 9: Regression Analysis with Phonological Awareness and Vocabulary Knowledge….44
Predicting Mathematics Skill

Table 10: Direct and Indirect Effects of Receptive Vocabulary Knowledge on Mathematics…46
Skill through Expressive Vocabulary Knowledge
LIST OF FIGURES

Figure 1: Mediation and Indirect Effects Diagram……………………………………………18

Figure 2: Numeration Means and Standard Deviations for the Current Study and the……….34
KeyMath-R Standardization Sample

Figure 3: Geometry Means and Standard Deviations for the Current Study and the…………35
KeyMath-R Standardization Sample

Figure 4: Addition Means and Standard Deviations for the Current Study and the ………….35
KeyMath-R Standardization Sample

Figure 5: Subtraction Means and Standard Deviations for Current Study and the……………36
KeyMath-R Standardization Sample

Figure 6: Measurement Means and Standard Deviations for Current Study and the ………36
KeyMath-R Standardization Sample

Figure 7: Time/Money Means and Standard Deviations for Current Study and the ………37
KeyMath-R Standardization Sample

Figure 8: Mediation and Indirect Effects Analysis……………………………………………46
1 INTRODUCTION

Deficiencies in mathematics skills constrain students’ educational achievement and subsequently their employment outcomes. For example, after controlling for years of schooling, vocational training, region of the United States, and race/ethnicity, Rivera-Batiz (1991) found that basic mathematics skills and their application to everyday problems were major factors accounting for the likelihood of full-time employment among adult men and women ages 21 to 25 years. Additional evidence comes from studies examining the correlations between individuals’ scores on the Armed Forces Qualifications Test (which includes arithmetic reasoning and numerical operations), and their yearly earnings and likelihood of employment. These studies (Berlin & Sum, 1987; Sum, Harrington, & Goedicke, 1986) identified high scores on the Armed Forces Qualifications Test as being positively correlated with employment and an individual’s yearly earnings. Given the relationship between mathematics skills and employment, an important next step is to determine indicators related to mathematics acquisition and subsequently, mathematics difficulties (MD). To date, researchers have identified indicators predictive of mathematics skill by studying the relationships between early arithmetic computation skills and phonological processing abilities (Torgesen, Wagner, & Rashotte, 2001) and vocabulary knowledge (Fazio, 1999), as well as by investigating the relationships between general mathematics skills (e.g., numeration, addition, subtraction, geometry, measurement, and time/money) and phonological awareness skill and naming speed (Wise et al., 2008). Continuing research efforts concerned with the identification of indicators related to mathematics acquisition can serve to inform mathematics intervention efforts and subsequently, mathematics proficiency across children, as well as potentially improving future employment outcomes.
1.1 Phonological Awareness

Phonological awareness (PA), naming speed, vocabulary knowledge, and mathematics skill grow substantially during early childhood. The first skill, PA, refers to the awareness of, and access to, the sound structure of oral language (Wagner & Torgesen, 1987), which begins prior to school entry as children develop awareness of syllables and rhyme (MacLean, Bryant, & Bradley, 1987). Then, during kindergarten and first grade, when children are taught to read, their skill at making judgments about small phonological units such as suffixes and prefixes of words as well as phonemes emerges (Fox & Routh, 1975; Treiman & Zukowski, 1996).

1.2 Naming Speed

Naming speed has been conceptualized as the amalgamation of perceptual, attentional, articulatory, and lexical retrieval processes with higher order cognitive and linguistic processes (Wolf, Bowers, & Biddle, 2000). In typically developing children, naming speed becomes faster, more deliberate, and requiring less cognitive resources as children progress through early childhood. Typically developing children achieve a level of automaticity in regard to naming colors, numbers, and objects (Lovett, Steinbach, & Frijters, 2000).

1.3 Vocabulary Knowledge

In addition to phonological awareness and naming speed, vocabulary knowledge also has been identified as important to early reading, especially reading comprehension (Perfetti, 2010; McKeown, Beck, Omanson, & Perfetti, 1983). The development of vocabulary knowledge begins at birth. On average, children say their first word by 12 months of age. Between 2 and 2½ years of age, the vocabulary of children expands to include labels for attributes of objects (i.e., modifiers) such as size and color (e.g., “big”, “green”) as well as possession (e.g., “my toy”, “daddy key”; Nelson, 1976). Between 3 and 5 years of age, children begin to master temporal
terms (“today” vs. “yesterday”, “now” vs. “then”; Stevenson & Pollitt, 1987). Finally, by 6 years of age, the average vocabulary of typically developing children has reached 10,000 words (Bloom, 1998) and by 11 years (the end of elementary school), vocabulary exceeds 40,000 words (Anglin, 1993). Students with small vocabularies will experience difficulty understanding reading material compared to their same-age peers with larger vocabularies (Hoff, 2009).

1.4 Mathematics Skill

As with vocabulary knowledge, the development of mathematics skill also begins in infancy. Knowledge of cardinality and ordinality emerge, and set the foundation for later mathematics development. Cardinality refers to the understanding of absolute numerical size and has been argued as being observed in infants as young as 6 months via discrimination tasks involving small numbers. Discrimination tasks (as described by Starkey, Spelke, & Gelman, 1990) utilized preferential looking procedures and were based on findings from Spelke (1976), which suggested that infants attend preferentially to visible objects that correspond in number to an accompanying series of sounds. It was hypothesized that if infants detected the number of items in visible and audible displays, they should look at the display of objects that matches in number of items and sequence of sounds. Results from Spelke (1976) and others (Antell & Keating, 1983; Starkey et al., 1990; Van Loosbroek, & Smitsman, 1990) suggested that infants attend longer to numerically corresponding displays, and that infants discriminated one object from two, and two objects from three. Wynn (1992) also used the preferential looking paradigm to study simple addition and subtraction in infants (e.g., 3+1=4). Results of Wynn’s (1992) study suggested that infants recognized the consequences of adding and subtracting small numbers of objects. Together, these studies indicate that cardinality develops early in life.
Ordinality refers to understanding relational properties or magnitudes of numbers and like cardinality, begins to develop in infancy, around 10 months of age. Feigenson, Carey, and Hauser (2002) demonstrated that 10 and 12 month old infants understand the basic ordinal concepts of more and less when the number of objects involves small numbers (e.g., 1, 2, and 3). By 4 and 5 years of age, children can extend their understanding of ordinality to larger sets and to values that are close together (Siegler & Robinson, 1982). For example, 5 year old children can consistently answer questions such as, “Which is more, 6 oranges or 4 oranges?”

At 3 to 4 years of age, typically developing children become proficient in another skill essential for the development of subsequent mathematics skill, counting (Siegler, & Alibali, 2005). Counting is guided by five principles which most children master by 5 years of age: (1) the one-to-one principle: assigning one and only one number word to each object in a group of objects being counted; (2) the stable order principle: always assigning the numbers in the same order when counting; (3) the cardinal principle: when counting, the last number indicates the total number of objects in the group being counted; (4) the order irrelevance principle: the order in which objects from a group being counted are irrelevant; (5) the abstraction principle: principles one through four apply to any set or group of objects being counted (Gelman & Gallistel, 1978).

When children enter elementary school they use various strategies to solve single-digit computation problems. In addition to retrieval, children solve computation problems by counting on their fingers as well as by supporting finger counting with verbal counting. The two most commonly used strategies have been termed sum (or counting all) and min (or counting on; Fuson, 1982; Groen & Parkman, 1972). The sum procedure typically emerges prior to the min procedure, when students have minimal experience with arithmetic computation problems. The
sum procedure involves counting both addends from 1. For example, a child would count 1, 2, 3, 4, 5, 6, 7, to solve 4 + 3. The sum procedure is a strategy that is often characteristic of children in first grade. In contrast, the min procedure involves stating the larger addend and then counting a number of times equal to the smaller addend. For example, a child would count 4, 5, 6, 7, to solve 4 + 3. The min procedure is a strategy that is often characteristic of children in second grade (Carpenter & Moser, 1984; Fuson, 1990). An additional strategy, the max procedure, is used when a child states the value of the smaller of two addends and then counts up to the larger addend. For example, a child would say 3, 4, 5, 6, 7, to solve 4 + 3.

Accompanying the changes in procedures and strategies used as children gain experience with arithmetic computation problems are improvements in the speed and efficacy in which children solve arithmetic computation problems. Improvement in speed and efficacy with solving arithmetic computation problems typically occurs between the ages of 6 and 9 years. This pattern of development has been found to be consistent across children and observed among children in Europe, North America and East Asia (Fuscon & Kwon, 1992; Geary, Bow-Thomas, Fan, Siegler, 1993; Lemaire & Seigler, 1995; Naito & Miura, 2001). Improved efficacy related to strategies and counting knowledge enables typically developing children to move on to more challenging arithmetic calculations such as adding two, three-digit numbers in column form (e.g., 243 + 136; but in column form). Arithmetic calculations of two, three-digit numbers is a skill that typically corresponds with the onset of third grade (Ginsburg, 1997).

During third grade, the demands for automatic retrieval of rote material increases as instructional demands grow and children shift from learning counting procedures to learning number facts (Mercer & Miller, 1992). The vocabulary of mathematics then shifts from the utilization of basic relational terms such as “add” and “subtract,” to the utilization of complex
interrelated terms such “divisor” and “least common denominator” (Geary, 1994). By upper elementary school (3rd through 5th grade), children are learning an interrelated network of mathematics knowledge that includes: a) conceptual (and vocabulary) knowledge, which refers to the language of mathematics and specialized terms (e.g., multiplication and division symbols); b) procedural knowledge, such as the rules of mathematics (e.g., aligning numbers correctly when solving problems involving decimal points); and c) declarative knowledge, which refers to memorized material (e.g., counting past 1000 & basic arithmetic facts; Goldman & Hasselbring, 1997). In short, as children progress through elementary school, the demands for automatic retrieval of previously learned strategies, procedures, and number facts increase. Subsequently, the vocabulary demands that accompany the child’s growing mathematics skills also increase.

1.5 Mathematics Difficulties (MD)

To date, research on understanding indicators related to and predictive of MD, is emerging. Gersten et al. (2005) outlined several deficits associated with MD. These deficits included low mastery of, and fluency with, retrieval of arithmetic combinations, slow digit naming speeds, inefficient and immature counting strategies, weak number sense, and impaired nonverbal working memory. Additional indicators such as PA (i.e., blending words, elision, detection of rhyme & alliteration, detection of phonemes, phonological memory rate, and rate of access to phonological name codes from long-term memory) also have been identified as predictive of mathematics skill acquisition (Bryant, MacLean, Bradley, & Crossland, 1990; Hecht, Torgesen, Wagner, & Rashotte, 2001; and Wise et al., 2008). Converging evidence supports the hypothesis that PA is a core indicator predictive of mathematics skill development and deficient in children with MD. However, measuring this relationship in children from additional special populations can bolster this argument. In addition, the relationships between
other possible indicators of mathematics skill such as naming speed (Wise et al., 2008) and vocabulary knowledge (Fazio, 1999) have not been studied to the same extent as the relationship between PA and mathematics skill has. Doing so may provide a more comprehensive understanding of mathematics skill acquisition and of the challenges that children with MD experience.

An additional related area of concern for mathematics research is clearly and consistently identifying how children with MD are conceptualized and defined. This task is critical to the field of mathematics research since the relationships between PA, naming speed, and vocabulary knowledge (as well as other indicators) with mathematics skill may be different in children who meet more stringent definitions for MD compared to children who meet less stringent definitions for MD. Conceptualizations of children with MD have included: (1) students performing one grade level below their expected grade (Russell & Ginsburg, 1984); (2) school-identified students who are two grades below their expected grade (Parmar, Cawley, & Frazita, 1996); (3) students scoring below the 35th percentile on standardized tests (Hanich, Jordan, Kaplan, & Dick, 2001); and (4) students participating in Chapter 1 services (mean study scores corresponded to percentile ranks of 24.6 in 1st grade and 40.4 in 2nd grade; Geary, 1990). Fuchs, Fuchs, and Prentice (2004) defined risk for MD as performance below the 25th percentile, which has been employed for designating disability risk for kindergarten and early elementary school students (Torgesen & Bryant, 1994).

Past research has suggested the prevalence of MD to be between 4% and 7% within school age children (Badian, 1983; Gross-Tsur, Manor, & Shalev, 1996; Kosc, 1974; and Lewis, Hitch, & Walker, 1993). However, it has been argued that prevalence estimates of MD in the school-age population are of limited utility because: (1) most research has focused on arithmetic
facts, not higher level processes and content; (2) estimates of MD may be high due to the lack of sound prevention in the primary grades; (3) research studies do not generally examine the severity of MD as a function of how the disability is defined; and (4) operational definitions of MD have varied across studies (Fuchs, Compton, Fuchs, Paulsen, Bryant, & Hamlett, 2005).

Despite the limitations related to conceptualizing MD and determining prevalence, some studies have identified difficulties that students with MD encounter throughout their school experience. For example, students with MD have been described as having trouble executing strategies to support solving of basic arithmetic computation problems and as having difficulty retrieving correct answers from memory (Geary, 1994; Geary, Hamson, & Hoard, 2000). In first grade, children with MD use immature counting procedures (e.g., the sum procedure), execute strategies slowly and inaccurately, and rarely use retrieval accurately. In second grade, these children often improve with regard to the counting procedures and strategies they utilize. However, they continue to have difficulty retrieving correct answers from memory (both at that point in time and in later years; Geary, 1990; Geary & Brown, 1991; Goldman, Pellegrino, & Mertz, 1988; Jordan, Levine, & Huttenlocher, 1995).

As children with MD progress through school, they continue to experience problems with skills that build on basic arithmetic computation (Hanich, Jordan, Kaplan, & Dick, 2001; Zawaiza & Gerber, 1993; Zentall & Ferkis, 1993). A few of the most common problems include understanding the inversion principle [i.e., the idea that adding and subtracting the same number leaves the original quantity unchanged (e.g., $5 + 8 - 8 = ?$); Klein & Bisanz, 2000; Vilette, 2002] and understanding the principle of mathematical equality [i.e., understanding that the equal sign means that values on each side of it, represent the same quantity (e.g., $4 + 5 = 3 + 6$); Perry, Church, & Goldin-Meadow, 1988; Rittle-Johnson, & Alibali, 1999]. Other common problems
include, understanding the concepts associated with solving multi-digit problems (e.g., 409 – 136 = ?; Brown & Burton, 1978) as well as understanding the relation between symbols and magnitude in regard to fractions (Silver, 1983) and decimals (Ellis, Klahr, & Siegler, 1993; Resnick, Nesher, Leonard, Magone, Omanson, & Peled, 1989).

1.6 Relationship between Phonological Awareness (PA) and Computation Skills in Typically Developing Children

Bryant, MacLean, Bradley, and Crossland (1990) investigated the relationships between PA skills (i.e., detection of rhyme and alliteration, and detection of phonemes) and basic computation skills (e.g., addition, subtraction, multiplication, division) in a longitudinal study, with 64 typically developing middle class children. The average age when the first measure was taken was 4 years 7 months and their mean IQ was 110.94 (SD = 12.33). The average age when the last measure was taken was 6 years 7 months and the mean IQ at this measurement point was 111.84 (SD = 16.29). Basic mathematics skill was included as an outcome measure in order to test the hypothesis that phonological tasks and success in reading rely on children breaking words down into their constituent sounds in order to read. It also was hypothesized that measures of phonological awareness would be specifically related to reading, not mathematics. Results of fixed order multiple regressions analysis suggested that alliteration oddity at 4 years 7 months, and detection of phonemes at 5 years 11 months, were predictive of children’s computation, spelling, and reading skills at 6 years 7 months of age. This finding suggests that PA is related to the development of early skill in mathematics computation.

In a related study, Hecht, Torgesen, Wagner, and Rashotte (2001) investigated the influence of phonological processing abilities (e.g., phonological memory, rate of access to phonological name codes from long-term memory, and PA) on growth in computation skills
(e.g., addition and subtraction) through confirmatory factor analysis followed by hierarchical regression analysis. Participants included 201 second through fifth grade students (7 years 6 months to 11 years 2 months) with estimated verbal IQ scores in the average range (e.g., 90 to 110). Two research questions examined by Hecht et al. (2001) pertinent to the current thesis were: (1) the extent to which phonological processing abilities were uniquely related to growth in computation skills; and (2) the extent to which phonological processing abilities accounted for the associations between individual differences in reading and computation skills. Results suggested that phonological memory rate, rate of access to phonological name codes from long-term memory, and PA were significantly related to growth in computation skills. Further, individual differences in phonological processing abilities uniquely explained growth in computation skills during the second to third grade time interval.

1.7 Mathematics Research and Children with Learning and Language Disabilities

Past research involving children with learning disabilities has revealed significant relationships between skills predictive of reading development and skills predictive of mathematics development. Cawley and Miller (2001) examined the mathematics performance of 220 children 8 to 17 years of age, who were diagnosed with a learning disability by their local school district. Performance levels by age and mean percentage correct for each mathematics subtest (i.e., arithmetic calculations & applied problems) on the Woodcock-Johnson Psycho-Educational Battery (WJ; Woodcock & Johnson, 1977) were obtained, as were IQ (i.e., verbal, performance, & full scale) scores from the Wechsler Intelligence Scale for Children-Revised (WISC-R; Wechsler, 1974). Intercorrelations of the WJ and the WISC-R were investigated. Significant relationships were found between several subtests of the WJ: (1) mathematics calculations and letter-word ID; (2) mathematics calculations and word attack; and (3)
mathematics calculations and comprehension. Moreover, regression analysis revealed that letter-word ID was predictive of mathematics calculations for children in the 14 to 17 year old group, and that letter-word ID accounted for 10% of the unique variance in mathematics calculations.

Additional research (Fazio, 1999) also supports the argument that factors predictive of reading development are predictive of mathematics development. Fazio conducted a 5-year follow-up on mathematics calculation skills in 32, 9 and 10 year old children. Of the participants, 10 children had been diagnosed with specific language impairment as preschool children; 11 children were described as typically developing and served as the age comparison group. Inclusion of these 11 children allowed Fazio to investigate the role of language deficits in learning mathematics. The third and final group included 11 additional children that were younger (8 years old) and who served as language-matched peers. All children were from low-income families, and were readministered the Columbia Mental Maturity Scale (CMMS; Burgemeister, Blum, & Lorge, 1972), the Clinical Evaluation of Language Fundamentals-Revised (CELF-R; Semel, Wiig, & Secord, 1987), and the mathematics computation (addition and subtraction problems) and number recall (recitation of number series) subtests of the Kaufman Assessment Battery for Children (Kaufman & Kaufman, 1983) in order to document current language and cognitive performance.

Fazio (1999) found significant positive correlations between mean language scores and number recall, mathematics facts, and mathematics computation. Mean language scores were computed by taking the mean of the students’ expressive and receptive language subtest scores, as measured by the CELF-R. These findings suggested that as mean vocabulary improved, children gained greater facility with recalling numbers, learning mathematics facts, and performing basic calculations. Fazio also identified one significant negative correlation between
mean language scores and speed of calculation indicating that as students’ mean language skills improved, the time required to complete computation problems decreased. Although Fazio’s results are correlational, they do support the hypothesis that vocabulary knowledge and mathematics skills are related.

1.8 Mathematics Research and Children with Reading Disabilities

Wise et al., (2008) investigated the relationships between both naming speed (e.g., letters, numbers, objects, & alternating) and PA skills (i.e., blending words and elision) with mathematics skills (e.g., numeration, geometry, addition, subtraction, measurement, and time/money) in 114 second and third grade students (ages ranged from 6 years 9 months to 9 years 1 month) with reading disabilities and children with reading disabilities who were at risk for MD. Students were referred by their public elementary school teachers due to difficulties in learning how to read and were included if their Kaufman-Brief Intelligence Test (K-BIT; Kaufman & Kaufman, 1990) composite score was 70 or above and their reading skills were equal to or less than a standard score of 85 on the Woodcock Reading Mastery Test-R (WRMT-R; Woodcock, 1987). In order to determine the consequences of using different cutoff levels to classify children with reading disability as at risk for MD, Wise et al. (2008) first evaluated students with a performance below the 25th percentile on the KeyMath-Revised Diagnostic Inventory (Connolly, 1988). Then students whose performance was below the 15th percentile on the KeyMath-R were investigated. Using the 25th percentile as the cutoff, 60 students’ evidenced reading disability (RD) only, while 54 demonstrated concurrent difficulties in both reading and mathematics. Using the 15th percentile, 75 children evidenced RD, while 39 students evidenced concomitant difficulties in both reading and mathematics.
In evaluating the significance of PA and naming speed in predicting mathematics skills, hierarchical regression analyses were run with the entire sample (n=114). When PA was entered first into the regression model (naming speed was entered second), PA accounted for a significant amount of unique variance in each of the mathematics subtest scores. Further, naming speed accounted for a significant amount of unique variance in all mathematics outcomes, except geometry. When the order of predictor variables was reversed and naming speed was entered into the regression analysis first, it accounted for unique variance in each mathematic subtest score except geometry. In this model, PA skills accounted for a significant amount of additional variance in all mathematics domains. In explaining these results, Wise et al. (2008) concluded that PA, as compared to naming speed, was a more robust indicator of mathematics skill.

Together, the previously discussed studies provide evidence that skills related to early reading development are not only related to early mathematics skill (e.g., word attack, letter word-ID, comprehension, and vocabulary knowledge), but predictive of early mathematics skill (e.g., elision, detection of phonemes, detection of alliteration, phonological memory rate, and rate of access to phonological name codes from long-term memory). In accounting for the previous findings, researchers argue that the representation of number in mathematics, and the representation of lexical and semantic information in reading, both draw on similar, if not related processing networks (Bull & Johnson, 1997; Geary, 1993; Rourke & Conway, 1997). To illustrate, consider a child solving a basic computation problem. First, the terms and the operator must be translated into a speech-based code (Campbell, 1998; Dehaene, 1992). Routinely children translate alphanumeric symbols (i.e., the numerical representation of the number; e.g., 4) into verbal representations when solving such problems (Geary, Bow-Thomas, Liu & Siegler, 1996; Logie, Gilhooly, & Wynn, 1994; Miura, Okamoto, Vlahovic-Stetic, Kim, & Han, 1999).
The child must then choose a strategy to process relevant phonological information. For example, when presented with the problem $4 + 3$, the child may retrieve a phonetically based answer code such as “four plus three equals seven,” from long-term memory (Geary, Hoard, & Hamson, 1999; Siegler & Shipley, 1995). If the child does not have an answer stored in long-term memory, and as result, direct retrieval fails, he or she may rely on a back-up strategy that utilizes counting (e.g., min, max, etc.). As with retrieving a phonologically based answer from long-term memory, strategies that require counting also utilize phonological processing as the child articulates phonological name codes that correspond to the counted numbers (Geary, 1993; Logie & Baddeley, 1987). Therefore, it can be hypothesized that both, early reading and early mathematics skill development are supported by shared or closely related general cognitive systems (e.g., phonological processing) that facilitate the acquisition and understanding of symbolic activity.

The previously cited studies indicate that PA, naming speed, and vocabulary knowledge are related to mathematics skill and suggest that there are links between children’s early reading competencies and concurrent mathematical development. Therefore, one next step is to investigate the relationship between mathematics skill and PA, naming speed, and vocabulary knowledge, as well as the magnitude of these relationships.

1.9 Children with Mild Intellectual Disabilities

To date, children with mild intellectual disabilities have not been included in research seeking to identify and understand indicators of mathematics skill acquisition and MD. Including children with mild intellectual disabilities will inform the field of mathematics research by extending the current knowledge base to an additional special population as well as strengthen our understanding of the developmental precursors to mathematics skill development.
in this population of students. By definition, children with mild intellectual disabilities evidence below average IQ (<70 to 50-55), exhibit deficits in adaptive functioning (e.g., self-help, daily living skills, etc.) and often have difficulties with oral language learning (Abbeduto, 2003), as well as subsequent written language (Sevcik, 2005). According to the Centers for Disease Control and Prevention (2010), intellectual disabilities are the most common developmental disability (Centers for Disease Control and Prevention, 2010) with mild intellectual disability making up 75% to 80% of all children diagnosed with intellectual disabilities (Glass, Christiansen, & Christiansen, 1982). Despite evidence indicating that mild intellectual disability is the most common developmental disability, research related to mathematics development concerning children with mild intellectual disability is sparse. The few published research studies that have included this group of children are primarily related to teaching and learning, and are limited by (a) small sample size (i.e., less than four participants); (b) failure to randomly assign students to study conditions; and (c) limited scope. The scope of studies that include children with mild intellectual disabilities have focused on instruction related to teaching students how to count money (Cihak & Grim, 2008; Stith & Fishbein, 1996), learning mathematics facts (Bouck et al., 2009; Geurts, 2006; Hayter, Scott, McLaughlin, & Weber, 2007; Zisimopoulos, 2010) and mathematics strategies (Creekmore & Creekmoore, 1983).

1.10 Proposed Research

The primary goal of this thesis and the following research questions is to further our understanding of mathematics skill development by identifying indicators of mathematics skill and subsequently, identifying causal deficit(s) related to MD. According to Chiappe (2005), identifying causal deficits of MD is the primary task related to this field of research. This task may be difficult due to the complexity (Geary, Hamson, & Hoard, 2000) and breadth of
mathematics skills. However, it is of utmost importance to subsequent research efforts related to the development of readiness tests and intervention efforts. Therefore, this study investigated the relationships between phonological awareness (PA), naming speed, receptive and expressive vocabulary knowledge, and mathematics skills (i.e., KeyMath-R subtests: numeration, geometry, addition, subtraction, measurement, and time/money) in children with mild intellectual disabilities.

In order to gain a better understanding of the relationships between mathematics skills and PA, naming speed, and vocabulary knowledge, the following research questions were addressed. Research question 1: Are students’ PA scores related to their mathematics scores? It was hypothesized that the measures of PA (elision and blending words subtests of the CTOPP; Wagner, Torgesen & Rashotte, 1999) would be positively related to mathematics skill development as measured by the KeyMath-R. That is, as students’ scores on elision and blending words improved, students’ mathematics scores also would improve. This prediction is supported by previous research (Hecht et al., 2001; Wise et al., 2008) which indicates that children utilize phonological memory and phonological names codes when reading, as well as when solving mathematics problems. For example, when performing basic calculations, a child must accurately and efficiently access operators that are represented by phonological name codes and stored in phonological memory. Therefore, as the child’s proficiency in accessing stored phonological representations improves, solving mathematics problems also improves.

Research question 2: Is naming speed related to mathematics skill in students with mild intellectual disabilities? It was predicted that as naming speed improves, mathematics skill also would improve. This is based on the argument that fluent rapid naming requires efficient retrieval of phonological information from long-term memory (Wagner et al., 1999). In relation
to reading, improved timing on naming tasks has been argued as necessary for the development of fluent reading (Wolf, 2000). Further, students who are able to rapidly and accurately retrieve phonological name codes from memory are thought to have additional attentional resources to allocate towards higher-level problem solving. Therefore, it was hypothesized that as with reading, improved timing on naming speed tasks would be related to increased proficiency in solving mathematics problems and subsequently, improved mathematics scores.

Research question 3: Are students’ vocabulary knowledge scores related to their mathematics scores? It was hypothesized that students’ Peabody Picture Vocabulary Test (PPVT-III; Dunn & Dunn, 1997) and Expressive Vocabulary Test (EVT; Williams, 1997) scores would predict their KeyMath-R mathematics scores. As students’ receptive and expressive vocabulary knowledge increases in depth and breadth, they may be better able to understand and apply relational terms such as, *smallest, largest, add, subtract, more, less, longest, shortest, difference, same,* etc. Inversely, it is possible that students who have lower vocabulary knowledge scores are less able to understand and apply, relational and interrelated terms to mathematics problems. Thus, the development of mathematics skills in children who have weaker vocabulary knowledge may be less mature, compared to children who exhibit vocabulary knowledge of greater breadth and depth.

Research question 4: Are the effects of students’ receptive vocabulary knowledge scores on their mathematics scores mediated by the effect of their expressive vocabulary knowledge scores? Further, are there indirect effects of receptive vocabulary knowledge on mathematics skill? That is, is receptive vocabulary knowledge related to expressive vocabulary knowledge, which in turn, is related to mathematics skill? Figure 1 below, displays this relationship.
It was thought that expressive vocabulary knowledge would mediate the effects of receptive vocabulary knowledge on mathematics scores and that receptive vocabulary knowledge scores would be indirectly related to mathematics scores through its relationship with expressive vocabulary scores. These relationships were hypothesized for two reasons. First, the mediating effects were based on the hypothesis that as students gain a better understanding of relational terms as earlier discussed their efficacy with applying relational terms to mathematics problems should improve. Second, support for an indirect effect, is a hypothesis that is grounded in theories of language development. Specifically, before children can use vocabulary (expressively), they must be able to understand vocabulary (receptively).

2 METHOD

2.1 Participants

Participants included 265 students who participated in a reading intervention efficacy project (Sevcik, 2005) evaluating the effectiveness of two reading interventions with a mathematics contrast condition for elementary school children with mild intellectual disabilities. Students were from 12 public elementary schools in the Atlanta metropolitan area. These schools were diverse with respect to socio-economic status and race/ethnicity. Of the schools, six were Title 1 schools. In regard to participation in free/reduced lunch programs, data were
available for 223 of the 265 participants. Of the 223 participants, 74.9% (167) did participate in free/reduced lunch programs.

Of the participants, 63.8% (169) were male, 36.2% (96) were female. Race/ethnicity of the participants was as follows: Asian, 2.3% (6); Black/African American, 56.6% (150); Hispanic, 16.2% (43); White/Caucasian, 20.0% (54); and Multi Racial, 4.9% (13). The mean chronological age of the participants was 8.87 years (range= 6.6-12 years; see Table 1). The students mean age equivalent as measured by the Peabody Picture Vocabulary Test III (Dunn & Dunn, 1997) was 4.73 years (range=1.09-11.04 years). In respect to grade level, 81 students were in 2nd grade, 58 were in 3rd grade, 68 were in 4th grade, and 54 were in 5th grade. Further, information related to participation in speech-language services was available for 225 of the 265 participants. Of the 225 participants, 72.9% (164) were recipients of speech-language services.

Table 1

<table>
<thead>
<tr>
<th>Chronological age distribution of participants</th>
<th>Frequency</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>80-months</td>
<td>1</td>
<td>.4</td>
</tr>
<tr>
<td>84- to 95-months</td>
<td>50</td>
<td>18.9</td>
</tr>
<tr>
<td>96- to 107-months</td>
<td>69</td>
<td>26.2</td>
</tr>
<tr>
<td>108- to 119-months</td>
<td>58</td>
<td>21.8</td>
</tr>
<tr>
<td>120- to 131-months</td>
<td>58</td>
<td>21.8</td>
</tr>
<tr>
<td>132- to 143-months</td>
<td>26</td>
<td>9.9</td>
</tr>
<tr>
<td>144- to 147-months</td>
<td>3</td>
<td>1.2</td>
</tr>
<tr>
<td>Total</td>
<td>265</td>
<td>100</td>
</tr>
</tbody>
</table>
All participants were assessed for mild intellectual disability by their local school districts and subsequently, met the state criteria and district eligibility for mild intellectual disability (i.e., IQ between <70-50 with concomitant deficits in adaptive behavior). Inclusionary criteria of the larger study included measured IQ (<70-50) and poor or no reading skills (below the 10th percentile on standardized reading measures). However, in order to authentically represent eligibility decisions made within the public school system, all students who received special education services under mild intellectual disability had the opportunity to participate in the study. This resulted in an increased range of student IQ scores (range=37-90, n=209), with 13 students scoring below and 47 students scoring above, the mild intellectual disability IQ range.

In addition to the previous inclusion criteria, students were excluded if they did not speak English, had a history of hearing impairment (<25 dB at 500+Hz bilaterally), a history of uncorrected visual impairment (<20/40), and/or had serious emotional/psychiatric disturbance (e.g., major depression, psychosis) as evidenced by parent reports. Written consent was provided by a parent or guardian and student assent was verbally provided by each child prior to participation in the study.

2.2 Measures

All students were given an extensive battery of standardized and experimental tests at up to four time points (e.g., pretest, mid-point, post-test, and follow-up) in order to describe growth in skills related to academic achievement. Analyses pertinent to the current research questions utilized data from the pretest time point, which occurred during the fall of the school year. The four standardized assessments concerning the current research questions are: (1) the Peabody Picture Vocabulary Test III (PPVT-III; Dunn & Dunn, 1997); (2) the Expressive Vocabulary Test (EVT; Williams, 1997); (3) the Comprehensive Test of Phonological Processing (CTOPP;
Wagner, Torgesen & Rashotte, 1999); and (4) the Key Math-Revised Diagnostic Inventory (KeyMath-R; Connolly, 1988).

**Peabody Picture Vocabulary Test III (PPVT-III)**

The PPVT-III is an untimed, individually administered, norm-referenced test designed to assess an individual’s receptive vocabulary acquisition. The 204 items on the PPVT-III do not require any reading on the part of the examinee and are divided into 17 sets of 12 items. The PPVT-III is administered in easel format, with the examiner showing the examinee a series of plates on which four pictures are displayed. The examiner reads a stimulus word for each plate and the examinee points to the picture which best represents the given stimulus word. For example, the examiner would say, “*Touch sleeping.*” The participant’s responsibility is to touch one of the four pictures on the plate that matches the stimulus word, “*Sleeping.*” Administration began with the first item and was discontinued when the student made eight or more errors from a set of 12 administered items.

According to the examiner’s manual, the standardization process of the PPVT-III was careful to include proportions of individuals from each ethnicity, age, education level, exceptionalities (e.g., special education categories: children with learning disabilities-5.5%, children with speech impairment-2.3%, children with intellectual disability-1.2%, children with hearing impairment-0.13%, children identified as gifted and talented-2.9%), and geographic area which were representative of the U.S. school population. The PPVT-III is a reliable screening and testing device. For form IIIA, the mean alpha reliability for children ages 7 to 11 years was .95; split-half reliability was .94; and the standard error of measurement was 3.7. Test-retest reliability coefficients were calculated for 4 age groups (2 years, 6 months-5 years, 11 months; 6 years, 0 months-10 years, 11 months; 12 years, 0 months-17 years, 11 months; & 26 years, 0
months-57 years, 11 months), with a one-month interval between administrations. All reliability coefficients were in the .90’s.

In terms of content validity, the PPVT-III examiner’s manual argued that it measures what it intends to, hearing vocabulary for single standard English words. Evidence of construct validity of the PPVT-III was supported by research and reports, such as Wechsler (1974), Wechsler (1991), and Elliott (1983). Internal validity of the PPVT-III was supported through growth curve analysis (Dunn & Dunn, 1997). According to Dunn and Dunn (1997), in growth curve analysis, for a test item to be included, the percentage of participants that respond correctly to that item must gradually increase for each successive age group. In meeting this criterion, the PPVT test items are reflective of age differentiation (i.e., steady increases in mean raw scores), with each age having more correctly answered items than previous age groups.

The PPVT-III was used for this study because, (1) it is widely used in research and schools; (2) it complements the EVT and when combined, yields a more comprehensive picture of students vocabulary knowledge than when using either the PPVT-III or EVT in isolation; and (3) the PPVT-III was conormed with the EVT (Dunn & Dunn, 1997). Conorming allows for direct comparisons between receptive and expressive vocabulary knowledge as well as the ability to combine scores into a composite vocabulary knowledge score.

The Expressive Vocabulary Test (EVT)

The EVT is an untimed, individually administered, norm-referenced test of expressive vocabulary knowledge. It does not require any reading on the part of the examinee and is made up of 190 items ordered by difficulty. The first 38 items are labels and the following 152 items are synonyms. The EVT is administered in easel format. At the beginning of the EVT examinees are shown individual colored pictures and are asked to name them. For example, the
examiner would say, “What do you see?” and the examinee should respond, “cup.” At the more advanced level, examinees are again shown individual colored pictures, but are now asked to provide a one-word synonym to label them. For example, the examiner would say, “What’s another word for jet?” and the examinee would respond, “Airplane.” As with the PPVT-III, administration began with the first item. Test administration was discontinued when the student responded incorrectly on five consecutive responses. Uses of the EVT include: screening expressive language problems, screening preschool children, measuring word retrieval (if used in conjunction with the PPVT-III), understanding reading difficulties, and monitoring expressive vocabulary growth.

Like the PPVT-III, during the standardization process, the EVT included proportions of individuals from each ethnicity, age, education level, exceptionalities (e.g., special education categories: children with learning disabilities-5.5%, children with speech impairments-2.3%, children with intellectually disablity-1.2%, children with hearing impairments-0.13%, children identified as gifted and talented-2.9%), and geographic area which were representative of the U.S. school population. The alpha reliabilities were determined for 25 standardization groups based on age. Mean alpha reliability for children ages 7-11 years was .95 (range=.93-.96). Mean split-half correlation for children ages 7-11 years, which was corrected by the Spearman-Brown formula for full test length, was .88 (range=.85-.91). The EVT’s mean standard error of measurement for children ages 7-11 years was 5.12 standard score points. Test-retest reliability was calculated for the same four age groups as the PPVT-III (e.g., 2 years, 6 months-5 years, 11 months; 6 years, 0 months-10 years, 11 months; 12 years, 0 months-17 years, 11 months; & 26 years, 0 months-57 years, 11 months), with a mean interval time between administrations of 42 days. All reliability coefficients ranged from .77-.90.
In terms of content validity, the vocabulary included in the EVT was selected on the basis of usage. “Words of high or moderately high frequency that could be acquired through common life experiences were included; words that required specialized knowledge were not” (Williams, 1997, p. 69). Construct validity of the EVT was supported by the developmental progression reflected in word frequency data, as well as the correlation with the PPVT-III (Form A, \( r = .76 \), Form B, \( r = .77 \)).

The EVT was used for this study because, (1) it is widely used in research and school; (2) it complements the PPVT-III and when combined, yields a more comprehensive picture of students vocabulary knowledge compared to using either, the PPVT-III or EVT in isolation; and (3) the EVT was conormed with the PPVT-III (Dunn & Dunn, 1997) which allows for direct comparisons between receptive and expressive vocabulary knowledge as well as the ability to combine scores into a composite vocabulary knowledge score.

**Comprehensive Test of Phonological Processing (CTOPP)**

The CTOPP is an individually administered, norm-referenced test that is used with individuals 5-25 years of age, and is comprised of 13 subtests in three areas of phonology: phonological awareness, phonological memory, and rapid naming. The three subtests used in this thesis were elision, blending words, and color naming speed. The elision subtest measures the ability to delete sounds from spoken words in order to create new words (e.g., “Say bold.” “Now say bold without saying b.” The final student response should be, “old.”), where as the blending words subtest measures an individual’s ability to synthesize sounds into words (e.g., “can-dy” requires the response “candy”). The color naming speed subtest measures the speed at which an individual can name a series of different colored blocks.
The administration of the elision subtest began with the first item and was discontinued when a student missed three consecutive items after items 1 through 4 had been administered. As with the elision subtest, the blending words subtest also began with the first item and was discontinued when a student missed three consecutive items after items 1 through 5 had been administered. Finally, the color naming speed subtest is made up of two pages, each with four rows of nine alternating colored blocks. Examinees were instructed to say the names of the colors as fast as they could and were timed.

The CTOPP norming sample included 1,656 individuals who resided in 30 states. According to the examiner’s manual, the norming sample was representative of the U.S. population in 1997. Coefficient alpha was used to estimate the reliability of all CTOPP subtests (except the color naming speed subtest which used alternate-form coefficients). The mean coefficient alpha for the elision subtest for children 7-11 years of age was .89 (range=.86-.91). Coefficient alpha for the subgroup of individuals with learning disabilities on the Elision subtest was .91. The mean coefficient alpha for the blending words subtest for children 7-11 years of age was .84 (range=.83-.87). Coefficient alpha for the subgroup of individuals with speech/language disabilities on the blending words subtest was .93.

For children ages 5-7 years of age, the test-retest reliability for both the elision and blending words subtest was .88. For children ages 8-17 years of age, the test-retest reliability for the elision subtest was .79, and for the blending words subtest, .72.

Content validity of the CTOPP was based on an experimental task that has been used to study phonological processing in the published literature (Wagner et al., 1987; Wagner & Torgesen, 1987; Wagner, Torgesen, Laughon, Simmons, & Rashotte, 1993). Following the identification of content, classical item analysis, item response theory analyses, and confirmatory
factor analyses were conducted. Construct validity of the CTOPP (e.g., phonological awareness, phonological memory, and rapid naming) was confirmed through factor analysis, age differentiation (i.e., constructs were strongly correlated with age), and group differentiation (i.e., the scores of individuals from different ethnic/cultural and disability groups cluster together). The examiner’s manual also argues for construct validity of the CTOPP by stating that items on particular subtests are highly correlated with the total score of that subtest. Furthermore, the authors supported their argument related to construct validity by providing empirical evidence, which indicated that training effects for phonological awareness are more evident than training effects for phonological memory or rapid naming following a phonological awareness intervention. The CTOPP was used for this study because of its utility in documenting the development or growth of phonological processing skills and for its inclusion of individuals with learning disabilities and speech/language disabilities within the norming process.

*The KeyMath-Revised Diagnostic Inventory of Essential Mathematics (KeyMath-R)*

The KeyMath-R (Connolly, 1998) is an individually administered, norm-referenced test, which is an update from its original and can be used for instructional planning, comparison of students to one another, evaluating educational progress, and curriculum evaluation. Its 258 items are divided into three areas. The first area, basic concepts, comprises three sub-sections: numeration, rational numbers, and geometry. The second area, operations, consists of addition, subtraction, multiplication, division, and mental computation. The third area, applications, consists of measurement, time/money, estimation, interpretation of data, and problem solving. The KeyMath-R is administered from an easel and each stimulus page is displayed to the student, one at a time. The examiner asks the examinee to respond to one or two questions based on the stimulus page. For example, in the numeration subtest, the examiner will ask, “How many sheep
are in the picture?” The examinee should respond, “Three.” Written computation is only permitted on a few subtests within the operations area (e.g., addition, subtraction, multiplication, and division). For example, item seven of the addition subtest presents the arithmetic computation problem “26 + 50” in column format. The examinee should provide the written response, “76.” Administration of all subtests began with the first item and was discontinued following three consecutive incorrect responses.

According to the examiner’s manual, the KeyMath-R was normed nationally based on the latest U.S. Census reports available at the time (Connolly, 1998) and the norming sample was stratified by geographic region, as well as within geographic region by grade (kindergarten through 9th grade), sex, socioeconomic level, race, and parents’ level of educational achievement. Information regarding stratification by disability was not reported. Test reliability of the Key Math-R was estimated using split-half reliability coefficients, which were obtained by correlating odd and even items for each subtest. The following are split-half reliability coefficients for the fall administration of the measure for children 7-11 years of age. The split-half reliability coefficients for the numeration subtest ranged from .81-.90 (mean=.85); the geometry subtest ranged from .77-.82 (mean=.79); the addition subtest ranged from .40-.84 (mean=.70); the subtraction subtest ranged from .64-.89 (mean=.79); the measurement subtest ranged from .57-.82 (mean=.74); finally, the time/money subtest ranged from .73-.93 (mean=.86).

According to the examiner’s manual, the items of the KeyMath-R represent content from kindergarten through ninth grade and were created with support from nationally recognized experts in the field of mathematics. Construct validity of the KeyMath-R is supported by evidence that the mean performance level increases with each consecutive grade level. Thus, the test measures a developmentally sequenced progression of fundamental knowledge and skills in
mathematics. The KeyMath-R’s internal consistency is supported through correlations of individual subtests within their given area compared to subtests from other areas. The examiner’s manual provides intercorrelations of each subtest for kindergartners, second-graders, fourth-graders, sixth-graders, and eighth-graders, for the spring standardization sample. The intercorrelation for the numeration and geometry subtest for second-graders was .45, for fourth-graders, .46. The intercorrelation for the addition and subtraction subtest for second-graders was .58, for fourth-graders, .51. The last intercorrelation pertinent to the current study was for the measurement and time/money subtests. For second-graders this correlation was .65, and for fourth-graders, .72.

The KeyMath-R was used for this study because it is widely used in research and education settings; and was based on a comprehensive domain-referenced scope and sequence that identified hierarchies of concepts and skills across 13 threads of content which are measured by separate subtests. Further, according to the KeyMath-R scoring instructions, subtest scores can be combined to form composite scores. For example, addition, subtraction, multiplication, division, and mental computation subtest scores can be summed into a composite score. The six subtests of interest to the current study are numeration, geometry, addition, subtraction, measurement, and time/money.

2.3 Procedure

The PPVT-III, EVT, CTOPP, and KeyMath-R were individually administered to students at the beginning of the school year in areas within their local school that were relatively free of distractions. Typically used rooms included non-occupied classrooms and offices. Test administration for each assessment began with the first item and was complete when students reached the ceiling specific to each assessment.
2.4 Data Analysis

Students’ scores were left in raw form in order to avoid floor effects and to increase variability. To ensure accuracy and quality control of data, all data were entered into SPSS 18 using a double entry procedure with two independently working researchers. Crosschecks between the two entries were run to determine potential inconsistencies. If an inconsistency was found, the original test protocol was referenced, the data corrected, and cross checks run again. This process was continued for all data until no inconsistencies were found.

SPSS 18 was also utilized for all analyses. Prior to investigation of regression analyses using mean centered variables, correlations, descriptive statistics, and regression assumptions were checked (e.g., linearity, heteroskedasticity, and normality of residuals) by comparing the linear fit line to the lowess line in partial plots, examining residual plots of the dependent variable and independent variable, as well as examining normal q-q plots. In order to increase power and reduce multicollinearity in the outcome variables, a factor analysis with principle axis factoring was performed for the KeyMath-R subtests (numeration, geometry, addition, subtraction, measurement, and time/money). During this process, a regression coefficient was saved that represented each participant’s score for the underlying latent mathematics variable.

However, prior to investigating the regression analyses, a zero-order correlation matrix was run and included the math factor score as well as scores concerning the variables of interest (i.e., phonological awareness, naming speed, and vocabulary knowledge). Further, socioeconomic status as measured by the Four-Factor HollingsHead Scale (total family score) was included because the participants were from schools with diverse student bodies (see page 18 and 19). Finally, participant’s grade level, were included within the zero-order correlation
matrix, as they were thought to influence mathematics skill development and subsequently, would need to be controlled for when investigating all regression analyses.

The students were from a diverse range of schools and to understand whether their PA scores were related to math factor scores, standard regression analyses were run. In order to control for experience, participants’ grade was entered into the regression analysis first. Then, mean centered measures of PA (i.e., elision and blending words subtests) were simultaneously entered as predictor variables, with participants’ math factor score included as the dependent variable. It was hypothesized that participants who had stronger PA skills had stronger mathematics skill. This hypothesis was based on the rationale that students retrieve phonological name codes when solving mathematics problems. Therefore, as students PA skills improved, mathematics skill was hypothesized to improve.

Next, to examine the relationship between color naming speed and math factor scores, standard regression analyses were run. Again, students’ grade level was entered first. Color naming speed reflected participants’ task speed, and was entered into the regression analysis second, predicting a linear relationship with math factor scores. It was thought that as students naming speed improved, increased attentional resources would be available to devote towards solving mathematics problems resulting in improved math factor scores.

In order to determine if students’ vocabulary knowledge (receptive and expressive) scores were related to math factor scores, students’ grade level was entered into step one of the regression analysis. Mean centered receptive vocabulary knowledge scores as measured by the PPVT and mean centered expressive vocabulary knowledge scores as measured by the EVT were entered second. The criterion variable was participants’ math factor scores. It was predicted that receptive and expressive vocabulary knowledge would be positively related to
math factor scores. This prediction was based on the hypothesis that, as participants’ receptive and expressive vocabulary knowledge improved their skill at understanding and applying relational terms to solving mathematics problems also would improve.

In considering the mediating effects of expressive vocabulary knowledge on the relationship between receptive vocabulary knowledge and mathematics skill, and to examine the indirect effects of receptive vocabulary via expressive vocabulary, bootstrap analysis estimating the indirect, direct, and total effects were run. The predictor variable, receptive vocabulary knowledge scores and the mediating variable, expressive vocabulary knowledge scores were mean centered. Next, 1000 bootstrap samples, with 95% bias corrected confidence intervals were estimated using a script version of the indirect macro described in Preacher and Hayes (2008). According to MacKinnon, Lockwood, and Williams (2004), bootstrap confidence intervals adjusted for bias exhibits higher levels of statistical power than the normal theory approach which involves: (1) regressing the dependent variable on the independent variable; (2) regressing the mediator on the independent variable; and (3) regressing the dependent variable on the mediator and the independent variable simultaneously.

It was thought that the direct effects of receptive vocabulary knowledge and expressive vocabulary knowledge would be significantly related to math factor scores. Therefore, the total effects of vocabulary knowledge on math factor scores also would be significant. Further, it also was predicted that the indirect and mediating effects of receptive vocabulary knowledge on math factor scores would be significant. These conclusions were based on the rationale that as students improve their skill at understanding vocabulary, their skill at applying vocabulary within mathematics problems also would improve, and result in improved mathematic outcomes.
In sum, this hypothesis predicts that as receptive vocabulary knowledge improves, expressive vocabulary knowledge improves. In turn, this results in improved mathematics scores.

3 RESULTS

3.1 Descriptive Statistics

Prior to analysis, all data were inspected for accuracy of data entry and missing values. Descriptive statistics for the included variables reported at the baseline time point are displayed in Table 2. Comparison of the loess lines to the linear fit lines suggested that the data met the assumption of linearity. Visual inspection of residual plots and $q-q$ plots suggested that the data met the assumptions of heteroskedasticity and normality. Examination of histograms for each measure suggested significant skewness and/or kurtosis reflecting the excess of student scores near the floor for each measure. To adjust for this characteristic, data were corrected according to Mosteller and Tukey’s (1977) recommendation of adding a small constant, 1, and applying a logarithmic transformation. Visual inspection of the generated histograms, scatterplots, and $q-q$ plots did not suggest that this correction improved the shape of the distribution. Therefore, data were left in raw form for all analyses.
Table 2

Descriptive Statistics for CTOPP, PPVT, EVT, and KeyMath-R subtests at baseline time point

<table>
<thead>
<tr>
<th>Subtest</th>
<th>(\bar{x}(sd))</th>
<th>Range</th>
<th>Skewness(se)</th>
<th>Kurtosis(se)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>CTOPP</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Blending Words</td>
<td>4.68(3.34)</td>
<td>0-13</td>
<td>.13(.15)</td>
<td>-.93(.30)</td>
</tr>
<tr>
<td>Elision</td>
<td>1.82(2.37)</td>
<td>0-12</td>
<td>1.38(.15)</td>
<td>1.36(.30)</td>
</tr>
<tr>
<td>Color Naming</td>
<td>114.49(52.61)</td>
<td>24-386</td>
<td>2.11(.15)</td>
<td>6.65(.30)</td>
</tr>
<tr>
<td><strong>PPVT-III</strong></td>
<td>67.04(22.22)</td>
<td>13-139</td>
<td>.15(.15)</td>
<td>.13(.30)</td>
</tr>
<tr>
<td><strong>EVT</strong></td>
<td>49.39(10.57)</td>
<td>23-94</td>
<td>.70(.15)</td>
<td>1.20(.30)</td>
</tr>
<tr>
<td><strong>KeyMath-R</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Numeration</td>
<td>4.99(2.69)</td>
<td>0-13</td>
<td>1.01(.15)</td>
<td>.98(.30)</td>
</tr>
<tr>
<td>Geometry</td>
<td>3.68(3.09)</td>
<td>0-14</td>
<td>.60(.15)</td>
<td>-.31(.30)</td>
</tr>
<tr>
<td>Addition</td>
<td>3.15(2.76)</td>
<td>0-14</td>
<td>1.25(.15)</td>
<td>1.30(.30)</td>
</tr>
<tr>
<td>Subtraction</td>
<td>1.22(1.57)</td>
<td>0-10</td>
<td>1.74(.15)</td>
<td>3.96(.30)</td>
</tr>
<tr>
<td>Measurement</td>
<td>2.63(2.29)</td>
<td>0-13</td>
<td>1.54(.15)</td>
<td>2.86(.30)</td>
</tr>
<tr>
<td>Time/Money</td>
<td>2.20(2.25)</td>
<td>0-12</td>
<td>1.62(.15)</td>
<td>3.17(.30)</td>
</tr>
</tbody>
</table>

Note: CTOPP (Comprehensive Test of Phonological Processing), PPVT-III (Peabody Picture Vocabulary Test-III), EVT (Expressive Vocabulary Test), KeyMath-R (KeyMath-Revised Diagnostic Inventory)

Comparison of KeyMath-R subtest scores between current sample and standardization sample

The first step in analyzing the data was to compare KeyMath-R subtest scores for the current sample of participants with mild intellectual disabilities with the KeyMath-R subtest scores for the sample of participants included in the KeyMath-R standardization procedures. Initially, intercorrelations for the subtests were going to be compared. However, the information provided in the examiner’s manual was limited. First, only intercorrelations for subtest scaled
scores and total-test standard scores for the spring standardization sample were provided (not raw scores). Second, these intercorrelations were only provided for alternating grade years beginning with kindergarten. However, the examiner’s manual did include means and standard deviations for subtest raw scores. Therefore, in an attempt to compare the distribution of scores found in the current study with that of the KeyMath-R standardization sample, line graphs of the means and standard deviations are displayed below (Figures 2-7).

Figure 2

Numeration means and standard deviations for the current study and the KeyMath-R standardization sample
Figure 3

Geometry means and standard deviations for the current study and the KeyMath-R standardization sample

![Graph showing geometry means and standard deviations](image)

Figure 4

Addition means and standard deviations for the current study and the KeyMath-R standardization sample

![Graph showing addition means and standard deviations](image)
Figure 5
Subtraction means and standard deviations for the current study and the KeyMath-R standardization sample

Figure 6
Measurement means and standard deviations for the current study and the KeyMath-R standardization sample
Figures 2 through 7 clearly indicate that the subtest raw scores for participants in the current study are consistently lower at each grade level compared to the scores of the participants in the KeyMath-R standardization sample. However, it should be noted that the mean subtest scores of students in the current study improved between second and fifth grade. This indicates that students with mild intellectual disabilities do develop mathematics skills and suggests that provided appropriate instruction, improvement in mathematics skills for students with mild intellectual disabilities may be substantially greater than the displayed trends (Figures 2-7).

3.2 **Investigating the Relationships between Phonological Awareness, Naming Speed, and Vocabulary Knowledge, and Mathematics Skills**

3.3 **Factor Analysis**

In order to increase the power of regression analyses, and determine if KeyMath-R subtest measures were measuring the same underlying latent construct, a factor analysis with
principle axis factoring was conducted for participants’ KeyMath-R subtest scores (i.e., numeration, geometry, addition, subtraction, measurement, and time/money). Table 3 displays the subsequent correlation matrix and corresponding standard deviations.

Table 3

<table>
<thead>
<tr>
<th></th>
<th>1.</th>
<th>2.</th>
<th>3.</th>
<th>4.</th>
<th>5.</th>
<th>6.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Numeration</td>
<td>1.0</td>
<td>-----</td>
<td>-----</td>
<td>-----</td>
<td>-----</td>
<td>-----</td>
</tr>
<tr>
<td>Geometry</td>
<td>.55***</td>
<td>1.0</td>
<td>-----</td>
<td>-----</td>
<td>-----</td>
<td>-----</td>
</tr>
<tr>
<td>Addition</td>
<td>.75***</td>
<td>.52***</td>
<td>1.0</td>
<td>-----</td>
<td>-----</td>
<td>-----</td>
</tr>
<tr>
<td>Subtraction</td>
<td>.67***</td>
<td>.52***</td>
<td>.64***</td>
<td>1.0</td>
<td>-----</td>
<td>-----</td>
</tr>
<tr>
<td>Measurement</td>
<td>.65***</td>
<td>.59***</td>
<td>.60***</td>
<td>.57***</td>
<td>1.0</td>
<td>-----</td>
</tr>
<tr>
<td>Time/Money</td>
<td>.73***</td>
<td>.55***</td>
<td>.67***</td>
<td>.70***</td>
<td>.59***</td>
<td>1.0</td>
</tr>
</tbody>
</table>

Standard deviations 2.69  3.09  2.76  1.57  2.29  2.25

Note: two-tailed significance ***p<.001

The Kaiser-Meyer-Olkin measure verified the sampling adequacy for the analysis, KMO = .90 (‘superb’ according to Field, 2009), and anti-image correlation values along the diagonal for individual items were ≥ .87, which is well above the acceptable limit of .5 (Field, 2009).

Bartlett’s test of sphericity $\chi^2 (15) = 963.98$, $p < .001$, indicates that correlations between items were sufficiently large for principal axis factoring. The initial analysis identified one eigenvalue of 3.75 following extraction, which exceeded Kaiser’s criterion of 1.0. In addition, examination of the scree plot justified retaining one factor. Given the large sample size, the convergence of the eigenvalue, the scree plot, and that the factor explained 62.46% of the total variance, one factor was retained and the factor matrix is displayed in Table 4.
Table 4

KeyMath-R factor loadings and communalities

<table>
<thead>
<tr>
<th></th>
<th>Factor 1</th>
<th>h²</th>
</tr>
</thead>
<tbody>
<tr>
<td>Numeration</td>
<td>.87</td>
<td>.76</td>
</tr>
<tr>
<td>Geometry</td>
<td>.67</td>
<td>.45</td>
</tr>
<tr>
<td>Addition</td>
<td>.82</td>
<td>.66</td>
</tr>
<tr>
<td>Subtraction</td>
<td>.79</td>
<td>.62</td>
</tr>
<tr>
<td>Measurement</td>
<td>.75</td>
<td>.56</td>
</tr>
<tr>
<td>Time/Money</td>
<td>.83</td>
<td>.69</td>
</tr>
</tbody>
</table>

Note: h²: communality

3.4 Zero-order Correlations

Missing values were deleted listwise and no outliers were identified, which resulted in a final sample of 265 participants. Variable means, standard deviations, skewness, and kurtosis values were displayed in Table 3. Zero-order correlations between participants’ math factor scores and variables of interest are presented below in Table 5. It is important to note that SES did not significantly correlate with math factor scores (r=-.05). Subsequently, it was not included in the following regression analyses. The other covariate of interest, grade level, was significantly related to math factor scores (r=.49, p<.01) and was included as a covariate in the following regression analysis.
Table 5

Zero-order correlations

<table>
<thead>
<tr>
<th></th>
<th>1.</th>
<th>2.</th>
<th>3.</th>
<th>4.</th>
<th>5.</th>
<th>6.</th>
<th>7.</th>
<th>8.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Grade Level</td>
<td>1.0</td>
<td>-----</td>
<td>-----</td>
<td>-----</td>
<td>-----</td>
<td>-----</td>
<td>-----</td>
<td>-----</td>
</tr>
<tr>
<td>2. SES</td>
<td>.04</td>
<td>1.0</td>
<td>-----</td>
<td>-----</td>
<td>-----</td>
<td>-----</td>
<td>-----</td>
<td>-----</td>
</tr>
<tr>
<td>3. Blending Words</td>
<td>.39**</td>
<td>.13*</td>
<td>1.0</td>
<td>-----</td>
<td>-----</td>
<td>-----</td>
<td>-----</td>
<td>-----</td>
</tr>
<tr>
<td>4. Elision</td>
<td>.36**</td>
<td>.01</td>
<td>.57**</td>
<td>1.0</td>
<td>-----</td>
<td>-----</td>
<td>-----</td>
<td>-----</td>
</tr>
<tr>
<td>5. Color Naming Speed</td>
<td>-.20**</td>
<td>.18**</td>
<td>-.25**</td>
<td>-.27**</td>
<td>1.0</td>
<td>-----</td>
<td>-----</td>
<td>-----</td>
</tr>
<tr>
<td>6. PPVT-III</td>
<td>.43**</td>
<td>.09</td>
<td>.43**</td>
<td>.50**</td>
<td>-.23**</td>
<td>1.0</td>
<td>-----</td>
<td>-----</td>
</tr>
<tr>
<td>7. EVT</td>
<td>.43**</td>
<td>-.01</td>
<td>.47**</td>
<td>.56**</td>
<td>-.27**</td>
<td>.68**</td>
<td>1.0</td>
<td>-----</td>
</tr>
<tr>
<td>8. Math Factor</td>
<td>.49**</td>
<td>-.05</td>
<td>.52**</td>
<td>.64**</td>
<td>-.32**</td>
<td>.67**</td>
<td>.69**</td>
<td>1.0</td>
</tr>
</tbody>
</table>

Note: correlation is significant *p < .05 (two-tailed), **p < .01 (two-tailed); SES (socio economic status, Four-factor Hollingshead scale), PPVT-III (Peabody Picture Vocabulary Test), EVT (Expressive Vocabulary Test); Blending Words, Elision, and Color Naming are subtests of the Comprehensive Test of Phonological Processing

### 3.5 Regression Analyses

To address each research question and determine the relationship between phonological awareness (PA), color naming speed, and vocabulary knowledge scores with math factor scores, standard multiple regression analyses were run using mean centered predictor variables. Grade level was controlled for and entered into step one of all regression analyses. Research question 1 asked, is PA related to mathematics skill? Grade level was entered as a covariate in step one of the regression analysis. Blending words and elision scores were mean centered and simultaneously entered into step two. Table 6 displays the regression analysis summary for research question 1. Blending words and elision scores were significantly related to math factor scores \((B=.05, p<.01; B=.19, p<.001)\). As PA scores improved, math factor scores improved. Further, the corresponding effect size for this analysis was large \((R^2=.28)\) indicating that there is
a strong relationship between PA and mathematics skill. Thus, the hypothesis that PA is related to mathematics skill is supported.

Table 6
Regression analysis with phonological awareness predicting mathematics skill

<table>
<thead>
<tr>
<th></th>
<th>B</th>
<th>SE B</th>
<th>β</th>
<th>T</th>
<th>sr²</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Step 1</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>-1.36</td>
<td>.16</td>
<td>-8.33</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Grade Level</td>
<td>.41***</td>
<td>.05</td>
<td>.48</td>
<td>8.85</td>
<td>.48</td>
</tr>
<tr>
<td><strong>Step 2</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>-.72</td>
<td>.14</td>
<td>-5.16</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Grade Level</td>
<td>.22**</td>
<td>.04</td>
<td>.26</td>
<td>5.45</td>
<td>.24</td>
</tr>
<tr>
<td>Blending Words</td>
<td>.05**</td>
<td>.02</td>
<td>.17</td>
<td>3.09</td>
<td>.14</td>
</tr>
<tr>
<td>Elision</td>
<td>.19***</td>
<td>.02</td>
<td>.47</td>
<td>8.66</td>
<td>.38</td>
</tr>
</tbody>
</table>

Note: $R^2 = .23$ for step 1, $\Delta R^2 = .28$ for step 2, $p < .001$; Significant (two-tailed) **$p < .01$, ***$p < .001$.

For research question 2, is color naming speed related to math factor scores, grade level was again entered into step one of the regression analysis in order to control for the amount of school experience. Participants combined time for form A and form B of the color naming speed subtest was then entered into the second step of the regression analysis. Table 7 displays the regression model summary for research question 2. Results indicated that color naming speed was significantly related to math factor scores ($B=-.004$, $p<.001$) and that, as naming speed decreased (i.e., improved), mathematics skill as reflected in participants’ math factor scores, improved. The effect size for this analysis was small ($R^2=.04$), indicating that color naming speed accounted for less unique variation in mathematics skill compared to the former analysis. This finding may be the result of the KeyMath-R test items not being timed. Subsequently, the
absence of timing when solving KeyMath-R test items may result in failing to accurately
measure the true relationship between naming speed and math factor scores. In turn, failing to
accurately measure the true relationship between naming speed and math factor scores may have
resulted in the effect size being attenuated.

Table 7

| Regression analysis with color naming speed predicting mathematics skill |
|-----------------------------|---|---|---|---|
|                            | B | SE B | β  | T   | sr² |
| **Step 1**                 |   |      |    |     |     |
| Constant                   | -1.34 | .16 | -8.21 |
| Grade Level                | .40*** | .05 | .48 | 8.73 | .48 |
| **Step 2**                 |   |      |    |     |     |
| Constant                   | -.76 | .22 | -3.50 |
| Grade Level                | .36*** | .05 | .43 | 7.86 | .42 |
| Color Naming Speed         | -.004*** | .001 | -.21 | -3.87 | -.21 |

Note: $R^2 = .24$ for step 1, $\Delta R^2 = .04$ for step 2, $p < .001$.
Significance (two-tailed) *$p < .001$.

In investigating research question 3 and determining if receptive vocabulary knowledge
or expressive vocabulary knowledge scores were related to math factor scores, both variables
were mean centered and simultaneously entered into the multiple regression analysis. As
presented in Table 8, receptive vocabulary knowledge and expressive vocabulary knowledge
scores were significantly related to math factor scores ($B=.01, p<.001; B=.04, p<.001$). Further,
as vocabulary knowledge scores improved, math factor scores also improved. The effect size for
this analysis was large and indicated a strong relationship between vocabulary knowledge and
mathematics skill. Thus, the hypothesis that vocabulary knowledge is an indicator of
mathematics skill was supported.
Table 8

Regression analysis with vocabulary knowledge predicting mathematics skill

<table>
<thead>
<tr>
<th></th>
<th>B</th>
<th>SE B</th>
<th>β</th>
<th>t</th>
<th>sr²</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Step 1</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>-1.36</td>
<td>.16</td>
<td>-8.41</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Grade Level</td>
<td>.41***</td>
<td>.05</td>
<td>.48</td>
<td>8.88</td>
<td>.48</td>
</tr>
<tr>
<td><strong>Step 2</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>-.58</td>
<td>.13</td>
<td>-4.47</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Grade Level</td>
<td>.15***</td>
<td>.04</td>
<td>.18</td>
<td>4.10</td>
<td>.16</td>
</tr>
<tr>
<td>Receptive Vocabulary</td>
<td>.01***</td>
<td>.002</td>
<td>.33</td>
<td>5.80</td>
<td>.23</td>
</tr>
<tr>
<td>Expressive Vocabulary</td>
<td>.04***</td>
<td>.01</td>
<td>.41</td>
<td>7.22</td>
<td>.29</td>
</tr>
</tbody>
</table>

Note: $R^2 = .21$ for step 1, $\Delta R^2 = .36$ for step 2, p < .001; Significant (two-tailed) ***p < .001.

Due to the findings from research question 1 and research question 3, a follow-up research question was investigated to determine the robustness of relationships between blending words, elision, receptive and expressive vocabulary knowledge scores, and math factor scores. Grade level was again controlled for and entered into step one of the regression analysis. Then, all four mean centered predictors were simultaneously entered into a multiple regression analysis. The results displayed below in Table 9 indicate that the standardized regression coefficient for each predictor variable was attenuated. However, elision, receptive and expressive vocabulary scores continued to significantly predict math factor scores ($B=.10$, $p<.001$; $B=.01$, $p<.001$; $B=.03$, $p<.001$). In contrast, the effect of blending word scores on math factor scores was substantially attenuated and no longer significantly predicted math factor scores ($B=.02$, $p=.10$).
Finding that blending words scores no longer significantly predicted mathematics skill was the result of receptive and expressive vocabulary knowledge scores accounting for unique variance that blending words scores accounted for in the earlier regression analysis concerning research question 1. Finding that elision, receptive vocabulary knowledge, and expressive vocabulary knowledge continued to significantly predict mathematics skill when entered simultaneously into regression analysis points to the robustness of these indicators in predicting mathematics skill. In sum, blending words was not as robust of an indicator of mathematics skill as compared to elision and vocabulary knowledge scores.

Table 9

Regression analysis with PA and vocabulary knowledge predicting mathematics skill

<table>
<thead>
<tr>
<th></th>
<th>B</th>
<th>SE B</th>
<th>β</th>
<th>t</th>
<th>sr²</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Step 1</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>-1.35</td>
<td>.16</td>
<td>-8.24</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Grade Level</td>
<td>.40***</td>
<td>.05</td>
<td>.48</td>
<td>8.75</td>
<td>.48</td>
</tr>
<tr>
<td><strong>Step 2</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>-3.99</td>
<td>.13</td>
<td>-3.19</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Grade Level</td>
<td>.12**</td>
<td>.04</td>
<td>.14</td>
<td>3.36</td>
<td>.13</td>
</tr>
<tr>
<td>Blending Words</td>
<td>.02</td>
<td>.01</td>
<td>.08</td>
<td>1.63</td>
<td>.06</td>
</tr>
<tr>
<td>Elision</td>
<td>.10***</td>
<td>.02</td>
<td>.26</td>
<td>5.02</td>
<td>.19</td>
</tr>
<tr>
<td>Receptive Vocabulary</td>
<td>.01***</td>
<td>.002</td>
<td>.27</td>
<td>4.98</td>
<td>.19</td>
</tr>
<tr>
<td>Expressive Vocabulary</td>
<td>.03***</td>
<td>.01</td>
<td>.28</td>
<td>4.87</td>
<td>.18</td>
</tr>
</tbody>
</table>

Note: \( R^2 = .23 \) for step 1, \( \Delta R^2 = .64 \) for step 2, \( p < .001 \); Significant (two-tailed) \(*p < .01, \ ***p < .001\).

For research question 4, a macro for SPSS designed by Preacher and Hayes (2008) was used to investigate the mediating effects of expressive vocabulary scores, on the direct effects of receptive vocabulary scores on math factor scores as well as the indirect effects of receptive
vocabulary knowledge on math factor scores via expressive vocabulary knowledge scores.

Receptive vocabulary knowledge was the independent variable; expressive vocabulary knowledge was the mediating variable; and participants’ math factor score was the dependent variable. One thousand bootstrap samples were estimated and bias corrected confidence intervals were requested. Table 10 and Figure 8 display the results of these analyses. Grade level was controlled for and entered into step one of the regression analysis. Path a, the path from receptive to expressive vocabulary knowledge was significant \((B=.30, p<.001)\) indicating that as receptive vocabulary knowledge improved, expressive vocabulary knowledge also improved. This finding is supported by the developmental literature that indicates comprehension vocabulary (i.e., receptive vocabulary knowledge) is acquired earlier and grows more quickly than production vocabulary (i.e., expressive vocabulary knowledge; Benedict, 1979; Goldin-Meadow, Seligman, & Gelman, 1976). Next, as in the previous regression analysis, the direct effects of receptive vocabulary knowledge (path c) and expressive vocabulary knowledge (path b) on mathematics skill were significant \((B=.03, p<.001; B=.04, p<.001)\).

Further, the \(\hat{c}\) path, the path that takes into account the simultaneous regression of mathematics skill on receptive and expressive vocabulary knowledge, was attenuated compared to path c \((B=.01, p<.001)\). Finding that the \(\hat{c}\) pathway was attenuated indicates that expressive vocabulary knowledge mediated the effects of receptive vocabulary knowledge on mathematics skill.

The indirect effect of receptive vocabulary knowledge on math factor scores through expressive vocabulary knowledge was significant \((\text{boot}=.01, \text{Bias corrected}< .0001, \text{CI}=.01-.02)\) and further supports the mediation hypothesis (see Table 10). Finding that the indirect pathway of receptive vocabulary knowledge scores to math factor scores via expressive vocabulary knowledge scores was significant indicates that as receptive vocabulary knowledge scores
improved, expressive vocabulary knowledge scores also improved. In turn, this lead to improved mathematics scores. The findings concerning the mediating effects of expressive vocabulary knowledge and the indirect effects of receptive vocabulary knowledge on mathematics skill via expressive vocabulary knowledge provides evidence that, when solving mathematics problems, more vocabulary knowledge is better, and that the effects of students’ receptive vocabulary knowledge on mathematics skills is conditional on their expressive vocabulary knowledge.

Table 10

Direct and indirect effects of receptive vocabulary knowledge on mathematics skill through expressive vocabulary knowledge

<table>
<thead>
<tr>
<th></th>
<th>B</th>
<th>SE B</th>
<th>t</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grade</td>
<td>.15</td>
<td>.04</td>
<td>4.10</td>
<td>= .0001</td>
</tr>
<tr>
<td>PPVT on EVT (path a)</td>
<td>.30</td>
<td>.02</td>
<td>12.98</td>
<td>&lt; .0001</td>
</tr>
<tr>
<td>EVT on Math Outcomes (path b)</td>
<td>.04</td>
<td>.01</td>
<td>7.22</td>
<td>&lt; .0001</td>
</tr>
<tr>
<td>PPVT on Math Outcome (path c)</td>
<td>.03</td>
<td>.002</td>
<td>12.14</td>
<td>&lt; .0001</td>
</tr>
<tr>
<td>PPVT on Math Outcome (ć path)</td>
<td>.01</td>
<td>.002</td>
<td>5.80</td>
<td>&lt; .0001</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Lower</th>
<th>Upper</th>
</tr>
</thead>
<tbody>
<tr>
<td>Indirect path (ć path)</td>
<td>.001*</td>
<td>.002</td>
</tr>
</tbody>
</table>

* Adjusted R² = .59, F(3, 257) = 125.52, p < .0001; * Bias corrected estimate.

Figure 8

Mediation and indirect effects analysis
4 DISCUSSION

To date, children with mild intellectual disability have not been included in research efforts investigating indicators of mathematics skill. This is the first study to do so and the results indicated that PA, naming speed, and vocabulary knowledge accounted for unique variation in the mathematics skills of children with mild intellectual disability. Further, expressive vocabulary knowledge was found to mediate the effects of receptive vocabulary knowledge on mathematics skill and the indirect effects of receptive vocabulary knowledge on mathematics skill via expressive vocabulary knowledge were significant.

In research question 1, PA scores were related to participants’ math factor scores. Thus, as scores from either PA measure improved, participants’ math factor scores also improved. This finding indicates that children with mild intellectual disabilities rely on PA to solve basic mathematics problems. Similar findings have been found in typically developing children (Bryant et al., 1990; Hecht et al., 2001) and children with reading disabilities, as well as children with reading disabilities who were at risk for mathematics difficulties (Wise et al., 2008).

Given that one method of teaching arithmetic facts in our educational system is by having students orally repeat arithmetic facts to be learned (Robinson, Menchetti, & Torgesen, 2002), finding that PA was significantly related to mathematics skill in children with mild intellectual disability is consistent with findings from other studies that included children from other populations. Oral repetition of arithmetic facts can take the form of whole class choral responding as well as be a component of independent seatwork. When orally repeating arithmetic facts, students memorize a string of phonological information, which includes the arithmetic fact and its solution. For example, the Connecting Math Concepts (SRA/McGraw-Hill, 2003) mathematics curriculum, a curriculum that has supported mathematics learning for
students with and without disabilities, includes activities that require students to orally repeat math facts. Accordingly, as children improve in retrieving phonological information, and as the quality of their phonological representations improve, mathematics skill also improves. Moreover, children with weak phonological representation evidence poor reading as well as poor mathematics skill development (Wise et al., 2008). In sum, PA is significantly related to early mathematics skill beyond the effects of experience as measured by grade level. This finding, as well as those of Bryant et al. (1990), Hecht et al. (2001), and Wise et al. (2008) indicates that early mathematics skill, like reading, is a symbolic activity (Sugiyanto, 1994) that relies on PA skills.

To determine whether color naming speed was related to mathematics skill development, research question 2 was investigated. As anticipated, color naming speed was related to mathematics skill and as naming speed decreased (i.e., the time required to name a series of colors decreased), mathematics performance improved. This finding corroborates that of Wise et al. (2008) who concluded that although naming speed was significantly related to mathematics development, the relationship between naming speed and mathematics skills was not as strong as the relationship between PA and mathematics skills. Further, the corresponding small effect size of this regression analysis suggests that there may be other indicators of mathematics skill that are stronger than naming speed.

For research question 3, PPVT-III (receptive vocabulary knowledge) and EVT (expressive vocabulary knowledge) scores predicted math factor scores such that improvements in vocabulary knowledge were related to improvements in mathematics skill. Further, the corresponding large effect size indicates that vocabulary knowledge accounts for a substantial portion of the variance in mathematics skill. The domain-specific vocabulary needed in
mathematics is complex. Thus, it was expected that improvements in vocabulary knowledge would be related to improvements in mathematics skill. The effects of receptive vocabulary knowledge on mathematics skill may be related to the importance of understanding words within context of a mathematics problem, whereas the effects of expressive vocabulary knowledge on mathematics skill may be related to the importance of being able to apply domain-specific (as well as general vocabulary) to the context of solving a mathematic problem. Another explanation for finding that vocabulary knowledge was significantly related to mathematics skill may be that children with stronger vocabulary knowledge are more likely to have vocabulary knowledge of complex interrelated terms required to solve advanced arithmetic calculations and problems, be more proficient at shifting relational vocabulary to a mathematical context (e.g., “highest number” refers to the larger of two addends, not the number that spatially sits higher on the page; Durkin & Shire, 1987), and/or have the verbal skill to articulate when they understand or are confused with mathematics instruction, compared to children with weaker vocabulary knowledge.

After investigating the individual effects of PA and vocabulary knowledge on mathematics skill, participants’ scores from both measures of PA and both measures of vocabulary knowledge were simultaneously entered into regression analyses. Whereas elision, receptive and expressive vocabulary scores were robust indicators of mathematics skill, blending word scores were not. The 64% of unique variation in mathematics skill that these three variables accounted for, further support the argument that elision, receptive and expressive vocabulary knowledge are robust indicators of mathematics skill compared to blending words. Moreover, finding that blending word scores no longer predicted mathematics skill while controlling for the other three measures indicates that variance in mathematics skill, which was
previously accounted for by the blending words measure, was more strongly related to the vocabulary knowledge measures. This finding underscores the importance of vocabulary knowledge in solving mathematics problems.

With respect to the indirect effects of receptive vocabulary knowledge on mathematics skill via expressive vocabulary knowledge and the mediating effects of expressive vocabulary knowledge on the relationship between receptive vocabulary knowledge and mathematics skill, finding that these relationships were significant suggests that students who have better developed receptive vocabulary knowledge, have a larger expressive vocabulary knowledge lexicon, which is subsequently related to improved mathematics skill. Further, finding that the indirect effects were significant is in line with the developmental literature which indicates receptive vocabulary knowledge is a precursor for expressive vocabulary knowledge (Benedict, 1979; Goldin-Meadow, Seligman, & Gelman, 1976). In the context of solving a mathematics problem, this underscores the importance of students’ understanding vocabulary before expecting them to efficaciously apply potentially novel terms within a mathematics problem. In sum, finding that receptive vocabulary knowledge was indirectly related to mathematics skill through expressive vocabulary knowledge provides a more accurate picture of the relationships between the three variables and that the effect of receptive vocabulary knowledge on mathematics skill is conditional on the student’s corresponding expressive vocabulary knowledge.

4.1 Limitations of the Current Thesis

There are limitations in the current thesis. To begin with, the participants’ range of IQ scores (37 to 90) exceeded both the lower and upper limit that typically define children with mild intellectual disabilities. Heterogeneity of IQ scores for students classified as having mild intellectual disabilities reflects special education placement decisions as well as decisions in
regard to rendering special education services to children in the public education system. Some researchers may view the inclusion of participants with IQ scores that fall outside of the traditional range that defines mild intellectual disabilities as a limitation in the current study because it may have resulted in increased heterogeneity related to IQ. Including students whose scores were outside the range may have also resulted in a small number of students being misclassified.

Next, the data analyzed in the current study were collected at one time point, the fall of the students’ school year. Therefore, time precedence cannot be established and limits statements related to causality. That is, it cannot be determined that the development of the participants’ PA skills, naming speed, or vocabulary knowledge occurred before the development of their mathematics skills.

In addition, this is the first study to investigate the effects of vocabulary knowledge on mathematics skill within a regression model framework. Thus, the extent to which vocabulary knowledge is related to mathematics skill in typically developing children and children from other special populations (i.e., other than mild intellectual disabilities) is unknown and generalizations concerning the effects of vocabulary knowledge on mathematics skill to other populations of children should be made cautiously.

4.2 Future Directions

The results reported in the current thesis included data from measures that were gathered at the same point in time. Therefore, future studies would bolster the current findings by explicitly investigating the causal component between mathematics skill and PA, naming speed, and vocabulary knowledge. For example, does PA measured during the fall of the students’ school year predict their mathematics skill as measured during the spring of that year?
Future studies that include children with mild intellectual disabilities would benefit from having mathematics outcome measures that include timed components. The only mathematics outcome measures utilized in the current study were the six subtests of the KeyMath-R. The KeyMath-R subtests do not require students to respond to test items under timed conditions. The resulting small effect size that corresponded to this regression analysis may be due to the lack of this feature. Assessing participants under timed conditions may more reliably measure the relationship between naming speed and mathematics skill, and be accompanied by a stronger effect size.

The finding that receptive and expressive vocabulary knowledge were significantly related to mathematics skill in children with mild intellectual disabilities extended the findings of Fazio (1999) who identified significant positive correlations between participants' mean vocabulary knowledge scores and their arithmetic calculation subtest score for the Kaufman ABC. One implication of these findings is that instruction designed to explicitly teach content-related vocabulary as well as the application of relational terminology to mathematics problems, may serve to facilitate mathematic skill development. Future studies could benefit from assessing the relationship between participants’ mathematics specific vocabulary and their mathematics skill as well as children’s skill at shifting relational vocabulary from a spatial context to a mathematics context. Finally, the current thesis only includes children that were identified by their local school district as having a mild intellectual disability. Therefore, future studies could benefit from replicating the current findings related to the relationship between students’ vocabulary knowledge and mathematics skill with typically developing children and children from other special populations.
4.3 Conclusion

In conclusion, the findings revealed statistically significant relationships between PA, naming speed, and vocabulary knowledge with mathematics skill. When both measures of PA (blending words and elision) were simultaneously entered into a regression analysis with receptive and expressive vocabulary knowledge, the blending words measure was no longer significantly related to mathematics skill, suggesting that elision, as well as receptive and expressive vocabulary knowledge scores, are more robust indicators of mathematics skill compared to skill in blending words. Further, expressive vocabulary knowledge mediated the relationship between receptive vocabulary knowledge and mathematics skill, suggesting that the effects of receptive vocabulary knowledge on mathematics skill is conditional on students’ expressive vocabulary knowledge. In addition, the indirect effect of receptive vocabulary knowledge on mathematics skill through expressive vocabulary knowledge was significant and indicates that students’ receptive vocabulary is related to their expressive vocabulary knowledge, which in turn, is related to their mathematics skill. Overall, this study indicates that PA, naming speed, and vocabulary knowledge are indicators of mathematics skill for children with mild intellectual disabilities.
5 REFERENCES


longitudinal study of process and concept deficits in children with learning disability.


University, New Haven, CT.


U.S. Census Bureau, Population Division, 2010


Zisimopoulos, D. (2010). Enhancing multiplication performance in students with moderate