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Using Spatial Factor Analysis to Measure Human Development[1]

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Abstract

In this paper, we propose a Bayesian factor analysis model with the purpose of serving as an alternate approach to calculating the UNDP's Human Development Index, as well as providing a general methodology which can be used to augment existing indices or build new ones. In addition to addressing several potential issues of the official HDI, we also estimate an alternative "green HDI" index by adding a new environmental variable, and build a novel MDG index as an example of constructing a new index with a more complex variable structure. Under our methodology, we find the "living standard" dimension provides a greater proportional contribution to human development than it is assigned by the official HDI while the "longevity" dimension provides a lower proportional contribution. The results also show considerable levels of general disagreement when compared to the ranks of the official HDI. We show that incorporating an environmental variable increases the amount of disagreement between model based ranks and the official HDI, but decreases the amount of uncertainty associated with model ranks. In addition, we report the sensitivity of our methods to the choice of functional form and data imputation procedures.

Keywords: Human Development Index, factor analysis

JEL Classification: O15, O57

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1. Introduction

Designed as a ranking system to track global human development, the Human Development Index (HDI) was first introduced in 1990 by the United Nations Development Programme (UNDP) in their now long running series of annual Human Development Reports (HDR). Prior to the HDI's initial publication, GDP, GDP per capita, and GNP had long served as the primary indicators of development for academics, policymakers, and other interested parties; but each lacked something the UNDP saw as vital to fully understanding global development - the human factor. Defined by the first HDR as, "…the process of enlarging people's choices" (UNDP, 1990), human development is simply any method by which nations expand or strengthen their citizens' access to human capital building resources. Based on this notion, the HDI formulates its national ranks using three key indicators which are believed to be connected to a country's human development level: longevity, education, and decency of living standards.[2]

In the years since its introduction, the HDI has come to serve as the standard for government agencies, private industry professionals, development groups, and academic researchers interested in studying and comparing national levels of human development. During a session in 2006, the National Congress of Indonesian Human Development restated their use of HDI as an economic indicator of development outcomes and the satisfaction of basic human living needs (Fattah and Muji, 2012). The government of Ireland also provides more development aid to countries classified as "low human development" by the HDI (O'Neill, 2005; Wolff et al., 2011). In private industry, the pharmaceutical company Merck sells drugs at a significant discount to nearly all of the countries categorized as "low human development" (Petersen and Rother, 2001; Wolff et al., 2011). Additionally, there have been proposals when designing international climate

[2] For a more detailed account of the rationale behind the design of the first HDI, see Anand and Sen (1994).

change policy that each country's HDI ranking should be factored into their reduction obligations for greenhouse gas emissions (Hu, 2009; Wolff et al., 2011). In research, the HDI is widely used as an alternative to other traditional economic indicators when evaluating a nation's relative level of human development (Anand and Ravallion, 1993; Easterlin, 2000). Furthermore, the HDI is not only heavily utilized by economists and other social scientists, but a wide array of academic disciplines including the medical research community.[3]

With the HDI's position as a top index now solidified through time and use, it serves as an advantageous exercise to reevaluate its formulation. When studied critically, the HDI does have a number of potential issues which we seek to address. Each of the three indicators used to calculate the official HDI are assigned deterministic weights relating to the proportional contribution they are *assumed* to provide towards a nation's human development level. Additionally, the HDI does not incorporate a measure of uncertainty in their rankings; implying that each publication of the official HDI can be interpreted as only one of many potential rankings. A considerable number of previous studies have attempted to address these and similar concerns with potential methods to correct for deterministic weights across dimensions (Ravallion, 2012), and lack of uncertainty from measurement error, index structure, and formula volatility (Noorbakhsh, 1998; Morse, 2003a; Wolff et al., 2009). Abayomi and Pizarro (2013) take a Bayesian framework to generate the confidence intervals of the HDI with the goal of incorporating uncertainty by first assuming prior distributions of both the underlying data and variable weights, and then examining the posterior replicates. An even more relevant study to our paper, Hoyland et al. (2012) also adopt a Bayesian factor analysis model; but it differs from our

[3] For instance, the relationship between the HDI and health has extensively been studied in topics such as: cancer (Bray et al., 2012), infant and maternal death (Lee et al., 1997), depressive episodes (Cifuentes et al., 2008), kidney cancer incidents and incident-to-mortality rates (Patel et al., 2012), suicide (Shah, 2009), and prevalence of physical inactivity (Dumith et al., 2011).

methodology in that they allow for correlations among indicators by first assuming correlations among the factor loadings of the HDI's four manifest variables.

This paper adopts a Bayesian factor analysis model which was initially developed to address many of the same concerns present in the material deprivation index (Hogan and Tchernis, 2004).[4] The model assumes an underlying latent variable, a factor representing levels of human development, which is manifested in the observed measures. The factor is influenced by the observed variables, and the strength of this influence is computed strictly from the data as opposed to expert opinion. The results of our model are summarized by computing the posterior distribution of ranks for all countries which are then presented with confidence intervals. This gives a more comprehensive view of a nation's standing relative to its peers given the inherent uncertainty of the estimation process. To further reduce the uncertainty of our measurements, we also include measures of spatial correlation and national population. Spatial correlation is often used in related literature as it allows for the incorporation of potential spillover effects from other factors which are highly correlated with HDI (Eberhardt et al. 2013; Ertur and Koch, 2011; Conley and Ligon, 2002; Keller, 2002).[5]

We illustrate the flexibility of our model to the inclusion of additional data in two ways. First, we add a measure of environmental sustainability to the HDI. Common candidates used as environmental variables are resource consumption, such as a nation's net natural capital stock (Neumayer, 2001; Morse, 2003b), and pollution levels, which we see in prior literature using CO_2 emissions per capita. To construct our "green HDI", we also use CO_2 emissions per capita as

[4] The same model has also been adopted in the measurement of county health rankings for Wisconsin and Texas (Courtemanche et al., 2015).

[5] The spatial dependence of HDI is based on prior literature. Research and development or long-run economic growth, both of which could be correlated with each factor of the index, has the documented potential for international spillovers (Eberhardt et al. 2013; Ertur and Koch, 2011; Conley and Ligon, 2002; Keller, 2002). Additionally, Malczewski (2010) shows that there are statistically significant geographical groups of high and low life expectancies in Poland.

it is recommended by the UNDP for the purposes of international analysis (Fuentes-Nieva and Pereira, 2010). Second, our general method is also easily utilized when trying to construct new indices as well. To exemplify the process of formulating a completely novel index, we construct an "MDG index" using data from the United Nations Millennium Development Goals (MDG).[6] Since the MDG's primary purpose was to track global development progress overtime, it can be interpreted as an alternative measure of human development to the HDI. Given the complex and decentralized nature of the MDG's design, a considerable quantity of prior research also attempts to construct an index summarizing information presented by the MDG's targets (Alkire and Santos, 2010; De Muro et al. 2011; Abayomi and Pizarro, 2013).

2. Methods

Methods of the official HDI

As a precursor to discussing our methods, it is of use to summarize the methodology used by the UNDP to formulate the official HDI. Since 2010, the HDI has constructed its three development indicators using four manifest variables: life expectancy at birth (longevity), mean years of schooling (education), expected years of schooling (education), and purchasing power-adjusted real GNI per capita (living standard). [7]

[6] Established in 2000, the MDG are a set of eight development goals which the United Nations member countries committed to achieve by the year 2015.
[7] Since its introduction in 1990, the HDI has seen several alterations to its formulation. Some changes have been minor, but a considerable overhaul was done in 2010. Prior to 2010, the four variables used to construct HDI were life expectancy at birth (longevity), adult literacy rate (education), combined educational enrollment (education), and purchasing power-adjusted real GDP per capita (living standard). Three normalized indicators (longevity, education, living standard) are calculated from the four variables. A simple average of the three indicators is scaled to range from 0 to 1 to represent the HDI score.

First, the three indicators are derived and normalized using the HDI's four observed variables. These indicators are the Life Expectancy Index (LEI), Education Index (EI), and Income Index (II). Each indicator is constructed using the following method:

$$Life\ Expectancy\ Index(LEI) = \frac{LE - 20}{85 - 20}$$

$$Education\ Index(EI) = \frac{MYSI + EYSI}{2}$$

$$MYSI = \frac{MYS}{15}, \qquad EYSI = \frac{EYS}{18}$$

$$Income\ Index\ (II) = \frac{\ln(GNIpc) - \ln(100)}{\ln\left(75{,}000\right) - \ln(100)}$$

After calculating the three indicators, the indicators' geometric mean is found using the formula below:

$$HDI = \sqrt[3]{LEI * EI * II}$$

With this algorithm, the UNDP is able to guarantee that each HDI score will fall into the range of values from 0 to 1. Following the designation of each nation's raw HDI score, countries are both ranked and categorized into one of the following four development tiers: "very high development" (HDI≥0.8), "high development" (HDI 0.7-0.8), "medium development" (HDI 0.55-0.7), and "low development" (HDI<0.55).

Proposed model

The official HDI presents several potential issues which we seek to address, including: the use of *ad hoc* weightings, no measure of uncertainty in rankings, no measure of spatial correlation between nations, and no consideration for country population differences. We now propose a hierarchical factor analysis model with spatial correlation to correct for each of the problems above.

Prior to adding either spatial correlation or adjusting for population, our basic factor analysis model is specified as:

$$Y_{ij} = \mu_j + \lambda_j \delta_i + \varepsilon_{ij}$$

where Y_{ij} represents the manifest variables, $j = 1, \ldots, J$, of country $i = 1, .., N$; μ_j is the average across countries of manifest variable Y_{ij}; δ_i is the latent factor which represents a country's level of human development, and which also serves as our model-based index; λ_j is the factor loading for variable j, and represents the covariance between the latent development measure, δ_i, and the manifest variable Y_{ij}; and finally $\varepsilon_{ij} \sim N(0, \sigma_j^2)$ is the model's normally distributed idiosyncratic error.

The model assumes each ε_{ij} to be both independently and identically distributed, implying that all manifest variables, Y_{ij}, are correlated with one another only through our latent factor, δ_i. Additionally, the basic factor analysis model assumes factor scores to be normally distributed, $\delta_i \sim N(0, 1)$.

With the basic model now defined, the next step in developing our full model is incorporating spatial correlation. We use a Conditionally Autoregressive model which specifies the relationship between factor scores for both a country, i, and its neighbors. While neighbors can be defined in a number of ways, we use the simplest definition based on adjacency in terms of either a land or maritime connection. We define a set of neighbors for country i as $\mathcal{R}_i$, and specify the conditional distribution of the country's factor score in the following way**:**

$$\delta_i|\delta_j \sim N\left(\sum_{j\in\mathcal{R}_i} \omega\delta_j \ , \nu\right)$$

where ω measures degree of spatial correlation and the conditional variance, ν, is a measure of residual variation.

Primarily, our specification has two attractive properties. First, it intuitively defines the relationship between neighboring countries through the distribution mean of factor scores. More flexible models could include additional levels of dependence through both the conditional mean and conditional variance, but these are not statistically identified within a factor analysis model.[8] Second, by setting the conditional variance such that $\nu = 1$, our conditional specification results in a simple marginal distribution for the vector of factor scores:

$$\delta \sim \mathrm{N}(0, (I - \omega W)^{-1})$$

where W is an $N \times N$ "neighbor matrix" such that $W_{ik} = W_{ki} = 1$ if a country k is adjacent to country i in terms of either land or maritime connections, and $W_{ik} = 0$ if otherwise. Additionally, $W_{ii} = 0$. It is also important to note that since the variance matrix of δ is a full matrix under this specification, all countries are correlated with one another even if they do not share a common border.

[8] For a more detailed discussion of this, see Hogan and Tchernis (2004).

For our last step of model development, we introduce population sizes into both the inverse variance of the error terms and factor scores. The intuition is that *a priori* we are less uncertain regarding the amount of noise in the manifest variables and factor scores of countries with larger populations compared to countries with smaller populations.

The final model, in vector notation, is now presented as:

$$Y|\delta \sim N(\mu + \Lambda\delta, M^{-1} \otimes \Sigma)$$

$$\delta \sim \mathrm{N}(0, M^{-\frac{1}{2}} \boldsymbol{\psi} M^{-\frac{1}{2}})$$

where Y is a vector of Y_{ij}'s stacked over j and then i; $\Lambda = I_N \otimes \lambda,$ with I_N as an $N \times N$ identity matrix, $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_J)'$ and $\otimes$ denotes a Kronecker product; Σ is a diagonal matrix with σ_j^2 as the diagonal elements, and 0's as the off-diagonal elements; $\boldsymbol{\psi} = (I - \omega W)^{-1}$; M is an $N \times N$ matrix with country populations $m_1, m_2, \ldots, m_N$ along the diagonal and 0's elsewhere.

To complete the model we also specify the prior distribution of our parameters. We use a set of conjugate, but non-informative, priors which simplify the derivation of the posterior distributions without providing much information. This implies that the posterior distributions are informed primarily from the data and not the prior distribution assumptions. We delegate the details of this to Appendix I.

Following Hogan and Tchernis (2004), we work with the variance stabilizing square root transformation of the original variables, such that $Y_{ij} = (S_{ij})^{\frac{1}{2}}$, where S_{ij}'s are the HDI's nontransformed variables.[9] This implies that $var(Y_{ij})$ is inversely proportionate to the country's population, m_i (Cressie and Chan, 1989; Hogan and Tchernis, 2004).

Our model is estimated using Markov Chain Monte Carlo (MCMC) methods, specifically Metropolis-Hastings with Gibbs Sampler. The method's primary goal is to produce a summary of the distribution of ranks for each of country. At each iteration of the sampler, for which we run 4,000 total iterations after the convergence phase of 500 iterations, we rank the draws from the posterior distribution of the factor scores. This allows us to produce samples from the posterior distribution of the countries' ranks. A more detailed description of the estimation process can be found in Appendix I.

Our Bayesian methodology can be seen as an improvement over the methodology of official HDI in several respects. First, our model based ranks are a function of the weighted manifest variables. This implies that the weights are informed by the data as opposed to expert opinion. Second, we are able to provide a summary of uncertainty through our ranking distributions. Third, our rank for each country is informed by data for both the specific country and any potential spillover effects from neighboring countries using spatial correlation. Finally, we incorporate additional information contained in a country's population, resulting in *a priori* lower uncertainty for more populous nations. Even though our model provides a flexible structure for the estimation of country ranks, there are a number of potential sensitivity issues which we also address in Section 5.

Using the methods outlined in this section, we calculate three sets of ranks: ranks using only data from the official HDI, ranks for our green HDI which combines official HDI data and an

[9] S_{ij} is already in a "per-capita" form (e.g. GNI per capita, population mean years of schooling, etc.).

environmental dimension, and the ranks for our MDG index which uses a comprehensive set of variables found in the MDG data. The next section explains the sources for our data as well as information regarding any variable selection.

3. Data

Data for model based HDI

For data pertaining to official HDI variables, we utilize the data used to construct 2010's official HDI. The data for each of the 195 countries are publicly available on the UNDP's website.[10] Of the full dataset we collected, 8 of the 195 countries are excluded from our estimation due to missing data as they are also removed from the estimation of official HDI. The four manifest variables used to calculate official HDI are: years of life expectancy at birth, mean years of schooling for adults, the expected years of schooling for children, and GNI per capita. For our measure of spatial correlation, we use both land and maritime borders to construct the "neighbor matrix" *W*. Country population measures for 2010 are gathered from the World Bank's total population midyear estimates.[11]

Since our model's ability to add information from new variables without assuming their effect *a priori* is perhaps its greatest strength, we exemplify this contribution through our estimation of a green HDI which includes an environmental variable not found in the official HDI. The purpose of including an environmental variable is to account for a nation's environmental sustainability with respect to their human development factors. We use CO_2 emissions per capita

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[10] The data was downloaded on 06/01/2016 from http://hdr.undp.org/en/data.
[11] World Bank Total Population Data: http://data.worldbank.org/indicator/SP.POP.TOTL?page=1

(hereafter CO_2) for the purposes of this paper. One common hurdle prior research has encountered when adding CO_2 to their models is the inherently uncertain relationship between carbon emissions and development levels.[12] Unlike previous studies (Fuentes-Nieva and Pereira, 2010; Bravo, 2014) which rely on assumptions regarding the effect of CO_2 on human development, our method uses only the data to inform the model about this relationship, with the sign of the factor loading communicating whether CO_2 contributes positively to a nation's human development level or not. National CO_2 emissions per capita data for 2010 are collected from the 2014 Human Development Report (UNDP and Malik, 2014).

Data for constructing the Millennium Development Goals index

While we show the potential for our model to estimate and add variables to an existing index, our method also applies to the creation of new and more complex indices as well. We illustrate this by designing a novel index for measuring human development using the United Nation's Millennium Development Goals (MDG). The MDG includes 8 broad primary goals with a total of 80 indicator variables used to track their progress. Due to the large number of MDG variables we choose to include in the estimation, our model has an inherent advantage in that we are able to skip the deterministic assignment of factor weights *a priori*, as they are a direct product of our model's estimation. We can also ignore assumptions regarding variable groupings, allowing us to avoid a high quantity of extra correlation parameters. Using our model, correlations between variables, regardless of their dimensions, are captured solely by the spatial correlation structure embedded in the latent factor.

[12] The functional form of our environmental variable is addressed more extensively in Section 5.

Data for each MDG variable is collected directly from the United Nation Development Program.[13] While primary target data is available for 234 countries and comparable areas, there is a considerable quantity of missing observations in the UNDP's dataset. With this in mind, of the 80 potential MDG indicator variables available to us we select the 12 which have the most complete data across countries to serve as our MDG index's manifest variables.[14] Comparing datasets across time, we also find 2010 to be the year with the most complete collection of data for the greatest number of countries. To help ensure accurate post-analysis comparisons between the HDI and our new MDG index, we restrict the selection of observations for our MDG data to the same 187 countries ranked by the official HDI.

After selecting our manifest variables, we impute values for the missing MDG data using two separate methods. The first round of imputation is a naïve imputation process for which the variables are imputed in order from those with the highest to the lowest number of non-missing observations. Table 1 presents summary statistics for the 12 manifest variables both before and after the naïve imputation. As it shows, the number of missing observations among variables varies considerably, and the change in variable means and standard deviations following the naïve imputation is relatively low.

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[13] Millennium Development Goals Indicators: http://mdgs.un.org/unsd/mdg/Data.aspx

[14] The 12 selected indicators are: (1) "maternal morality ratio per 100,000 live births" (MMR), (2) "children under five mortality rate per 1,000 live births" (U5MR), (3) "population undernourished, percentage"(PU), (4) "total net enrolment ratio in primary education, both sexes"(NER), (5) "gender parity index in primary level enrolment" (GPI), (6) "tuberculosis prevalence rate per 100,000 population (mid-point)" (TB), (7) "proportion of the population using improved drinking water sources" (WATER), (8) "people living with HIV, 15-49 years old, percentage" (HIV), (9) "carbon dioxide emissions (CO2), metric tons of CO2 per capita (CDIAC)" (CO2), (10) "fixed-telephone subscriptions per 100 inhabitants" (TELE), (11) "employment-to-population ratio, both sexes, percentage" (ETP), and (12) "adolescent birth rate, per 1,000 women" (ABR).

Table 1. Summary Statistics of MDG Indicators

	Before Imputation			After Naïve Imputation		
Variable	**Obs**	**Mean**	**St.D**	**Obs**	**Mean**	**St.D**
TELE	187	18.80	17.66	187	18.80	17.66
TB	186	157.37	190.40	187	156.56	190.21
U5MR	185	38.80	40.77	187	38.44	40.70
WATER	181	86.93	15.53	187	87.03	15.54
CO2	181	4.71	6.41	187	4.79	6.40
MMR	178	176.83	233.37	187	169.65	229.95
PU	162	12.20	10.53	187	12.22	10.49
GPI	149	0.97	0.06	187	0.97	0.06
NER	119	92.41	9.49	187	91.72	9.89
HIV	114	2.38	4.92	187	1.85	4.27
ETP	108	54.77	10.55	187	55.07	9.99
ABR	97	37.62	36.68	187	50.72	44.86

The specific technique used for our naïve imputation is a "univariate imputation using predictive mean matching" (PMM). PMM is a combination of the ordinary least squares (OLS) regression prediction and the nearest-neighbor imputation methods. First, PMM produces linear predictions for all data, missing and observed, using a traditional OLS regression. These predicted values are then compared to one another across observations. For each missing observations, the imputed value used is the value of the *non-missing observation* which has the closest predicted value to that of the missing observation, known as the missing observation's "nearest neighbor". By using PMM, we honor existing bounds in the non-imputed portion of the data while also preserving the observed data's distribution (Little, 1988). All PMM imputation procedures are done using Stata's PMM syntax.

The second of our two imputations comes from the posterior imputation process embedded in our model. The posterior imputation replaces the naïvely imputed values with observations sampled from the distributions of missing data. This allows us to take potential uncertainty inherent in the missing data into better account (Rubin, 1976; Little and Rubin, 2002; Daniels and Hogan, 2008). We address the posterior imputation more fully, along with the sensitivity of our results to the choice of imputation process, in Section 5.

4. Results

Model based ranks vs. official HDI ranks

The rankings of official HDI fail to account for either uncertainty, spatial correlation, or population. Alternatively, our index ranks are estimated in terms of distributions, which provide a measure of uncertainty. Since factor weightings are different between our model-based index and the official HDI, there must be some differences between the posterior mean ranks and the official HDI ranks. We compare the two rankings, including the information for the 99% confidence interval of the posterior ranks, in Figure 1.

For Figure 1 and subsequent figures of the same layout, the dashed grid lines partition the 0%-20% (1^{st}), 20%-40% (2^{nd}), 40%-60% (3^{rd}), 60%-80% (4^{th}), and 80%-100% (5^{th}) quintiles of ranks respectively. The solid dots show the locations of both posterior mean ranks and official HDI ranks. Solid horizontal lines across each dot represent the 99% confidence interval for each country's posterior model based rank. The numbers in Figure 1 correspond to individual country identifiers, which are assigned alphabetically and can be referenced in Appendix II.

It is immediately apparent that our model's rankings harbor a considerable level of uncertainty for some countries, with several confidence intervals even reaching across quintiles. Interestingly, this uncertainty persists in various degrees along the entire spectrum of ranks as opposed to being prevalent in only certain categories of development. As an example, Bhutan, a low development level country, has a posterior 99% confidence interval of (141, 164), implying that their rank could fall into either the 4th or the 5th quintile. Similar results are also found for more highly developed nations like Kuwait, which has a posterior 99% confidence interval of (10, 45), implying that its rank could fall into either the 1st or 2nd quintiles. While Bhutan and Kuwait represent the most extreme examples, it is not uncommon for nations to be categorized into different quintiles given their confidence intervals.

The relationship between the rank of the country and the amount of uncertainty is an inverted U-shape, with levels of uncertainty decreasing for the most and least developed countries. This is due to a number of factors. First, countries ranked and the top (bottom) have the highest (lowest) values for each manifest variable. Second, these often tend to be the most populous countries. Third, they are also closer to one another on average geographically, leading to a reduction in uncertainty through spatial smoothing. Finally, this is also due to the truncation of variable values from both below or above for the most and least developed countries.

Another feature of Figure 1 is that it shows the discordance between our model-based ranks and those of the official HDI. The greater the distance between solid dots and the 45^o line, the greater the disagreement between our model-based ranks and the ranks of official HDI. For only 9 countries are our model-based and official HDI ranks the same. For 87 countries, the absolute value of difference between the two ranks is less than 5. For 54 countries however, the absolute value of difference is larger than 10.

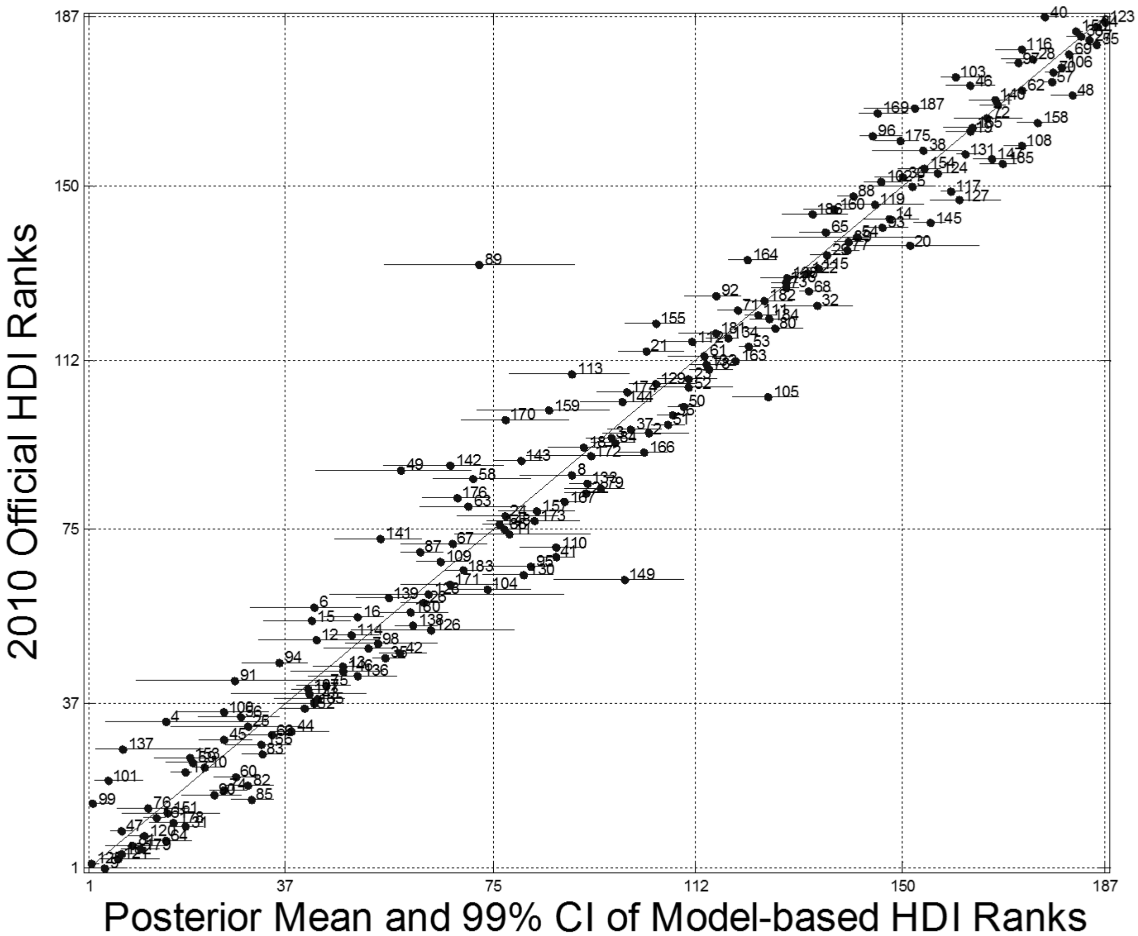


Figure 1. Posterior Mean and 99% CI of Model-based HDI Ranks vs. Official HDI Ranks

Discordance between model based and official HDI ranks

Table 2(a) shows the ten countries which have the largest differences between their official HDI rankings and their rankings as determined by our model. As an example, Mongolia is ranked 109 in the 2010 official HDI; but is assigned a posterior mean rank of 89 by our model with a 99% confidence interval of (78, 100). Therefore, Mongolia's posterior confidence interval fails to even cover the range of its official HDI rank. It is reasonable to conclude from our results that the official HDI underestimates Mongolia's level of human development. Alternatively, Mexico, which has an official HDI rank of 71, has posterior mean rank of 87 in our model with a 99% confidence interval of (80, 92). So, in an opposite pattern to Mongolia, the official HDI seems to very much overestimate the human development level of Mexico given our findings. Since the majority of these highly discordant countries have relatively small populations, we also present the seven nations with large populations (over five million) which also have a difference-in-ranks between their model based and official HDI rankings larger than 10 in Table 2(b).

The most plausible reason behind these large discordances in rank is the difference in factor weights between the official HDI and our model based index. As we discuss in the following section, our model based index assigns a greater proportional contribution to the "living standard" indicator but a lower proportional contribution to the "longevity" indicator; implying that countries with either outstanding or dismal performance in these two dimensions see a considerable amount of movement between the two indices.

Table 2 (a). Ten Countries with the Largest Differences in Ranks Between Official HDI and Model-based HDI

Country	Ranks		Manifest variable			
	HDI	Model-based	LE	MYS	EYS	GNIpc
Kiribati	133	73 (55, 90)	67.9	7.8	12.3	2465
Seychelles	64	99 (86, 110)	72.7	9.4	13.1	21027
Dominica	88	58 (43, 71)	77.4	7.7	12.7	9433
Tonga	99	77 (69, 89)	72.2	9.4	14.7	5117
Saint Lucia	89	67 (55, 77)	74.4	8.3	12.6	10515
Maldives	104	125 (119, 131)	76.8	5.8	12.7	9112
Togo	166	145 (142, 151)	55.5	5.3	11.7	1063
Qatar	27	7 (2, 22)	78	8.9	13.8	126332
Mongolia	109	89 (78, 100)	66.8	8.3	14.6	5880
Saint Kitts and Nevis	73	54 (46, 62)	72.9	8.4	12.9	20137

Table 2 (b). Countries with Differences in Ranks over 10 and Larger-populations (>5M)

Country	Ranks		Manifest variable			
	HDI	Model-based	LE	MYS	EYS	GNIpc
Pakistan	147	160 (155, 168)	66.1	4.6	7.4	4381
Japan	16	31 (27, 35)	83.1	11.5	15.1	35508
Mexico	71	87 (80, 92)	76.8	8.3	12.7	15173
Iran	84	95 (88, 99)	73.1	7.8	13.1	14073
Thailand	92	103 (98, 107)	73.9	7.3	13.1	12270
Congo (DRC)	187	176 (175, 178)	49	3.1	9.1	386
South Africa	120	105 (99, 110)	54.5	9.6	13.1	11379

Squared correlation coefficients

Due to differences in methodology, there is no simple way to compare the estimated contributions of each manifest variable to the official HDI or the latent factor in our model. To provide a general measure of comparability, we follow Ravallion (2012) which suggests the marginal weights of each variable be calculated as the partial derivative of the official HDI with respect to each variable.[15] Following this, we can therefore obtain the marginal weights of each variable in the official HDI by regressing standardized HDI scores on standardized manifest variables.

To summarize the contribution of each variable to the latent factor in our model, we apply the methods of Hogan and Tchernis (2004) and present normalized "squared correlation coefficients". The "squared correlation coefficient" for each variable j is specified as:

$$\rho_j^2 = \frac{\lambda_j^2}{\lambda_j^2 + \sigma_j^2}$$

Each correlation coefficient is the proportion of variation in the manifest variable, j, that is explained by the latent human development factor. In Table 3, we compare the normalized marginal weights for each manifest variable of the official HDI to the normalized "squared correlation coefficients" produced by our model-based index.

Table 3. Comparison of HDI Weights and Normalized Squared Correlations ρ^2

Variable	HDI Weights (95% CI)	ρ^2(95% CI)
Life Expectancy at Birth	0.33 (0.32, 0.34)	0.19 (0.17, 0.20)
Mean Years of Schooling	0.29 (0.29, 0.29)	0.27 (0.27, 0.28)
Expected Years of Schooling	0.23 (0.22, 0.24)	0.27 (0.27, 0.28)
GNI per capita	0.15 (0.14, 0.15)	0.26 (0.26, 0.27)

[15] For a more detailed overview, see Ravallion (2012).

With respect to our results, we find the "longevity" dimension offers a lower contribution to a country's human development level than the weights of official HDI would suggest. Our model also attributes a much greater contribution to the "living standard" dimension when compared to official HDI. Additionally, while the official HDI assigns a greater proportional contribution to "mean years of schooling" than "expected years of schooling", our model assigns identical levels of contribution to both variables of the "education" dimension.

The most and least developed countries

One of the HDI's primary purposes is to identify the countries with both the highest and lowest levels of human development. Distinguishing countries with best practices establishes role models for other nations; while identifying the least developed countries has significant implications for nations with lower levels of human development. Since comparing the relative performance of nations is so important, it again becomes a potential concern that the official HDI offers only a single rank for each country as opposed to a plausible range of values. This can be especially detrimental to countries which border the poorest rankings of human development, as it may disqualify them from participating in beneficial international assistance programs should their official HDI rank fall outside of a program's bounds. Since our method produces distributions of ranks, we are able to estimate and assign probabilities for each country to be among the least or most developed.

In Figure 2 we estimate the probability of countries being among the top 10 most developed countries using our model, along with their official HDI rankings. Of the 187 total countries, 17 have non-zero probabilities of being included in our model's "Top 10". Additionally, for these

17 countries, 8 are not among the "Top 10" according to their official HDI ranks. Alternatively, Canada, which is in the official HDI's "Top 10", has zero probability of being included in the "Top 10" of our model. In Figure 3 we present the probability of countries being among the 10 least developed countries using our model, along with their official HDI rankings. Of the 13 countries which have non-zero probabilities associated with being included in our model's "Bottom 10", 5 are not listed among the "Bottom 10" according to official HDI. Mozambique and Burundi, both of which are members of the official HDI's "Bottom 10", have zero probability of being in the "Bottom 10" produced by our model. The Democratic Republic of Congo also has a relatively low probability of being included in our model's "Bottom 10", despite having the second lowest level of human development according to official HDI.

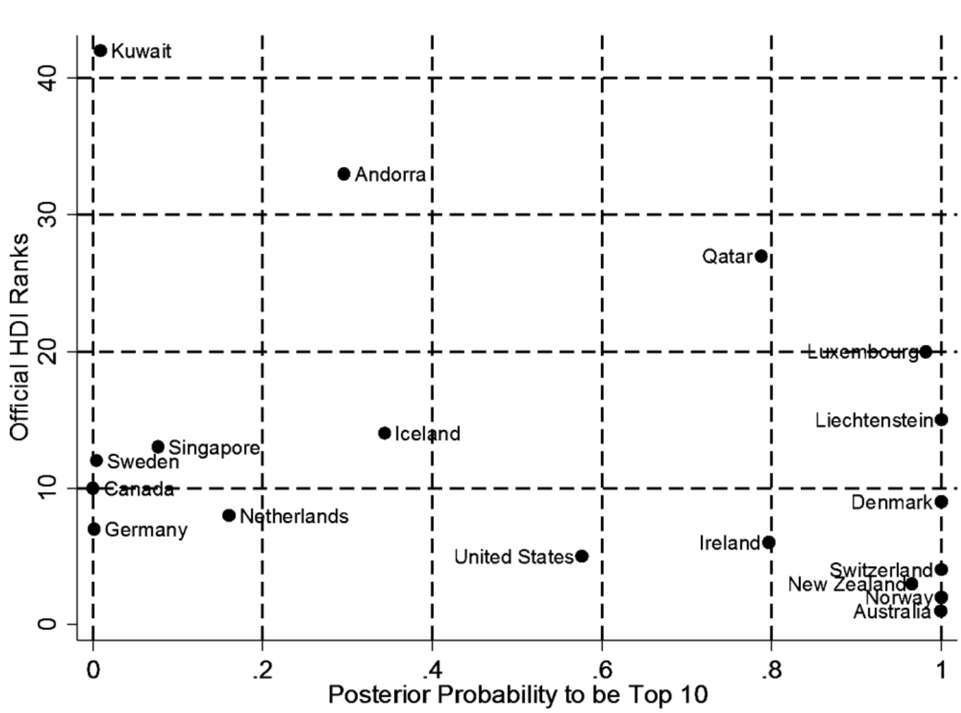


Figure 2. The Probability to be Model-based "Top 10" vs. Official HDI Ranks

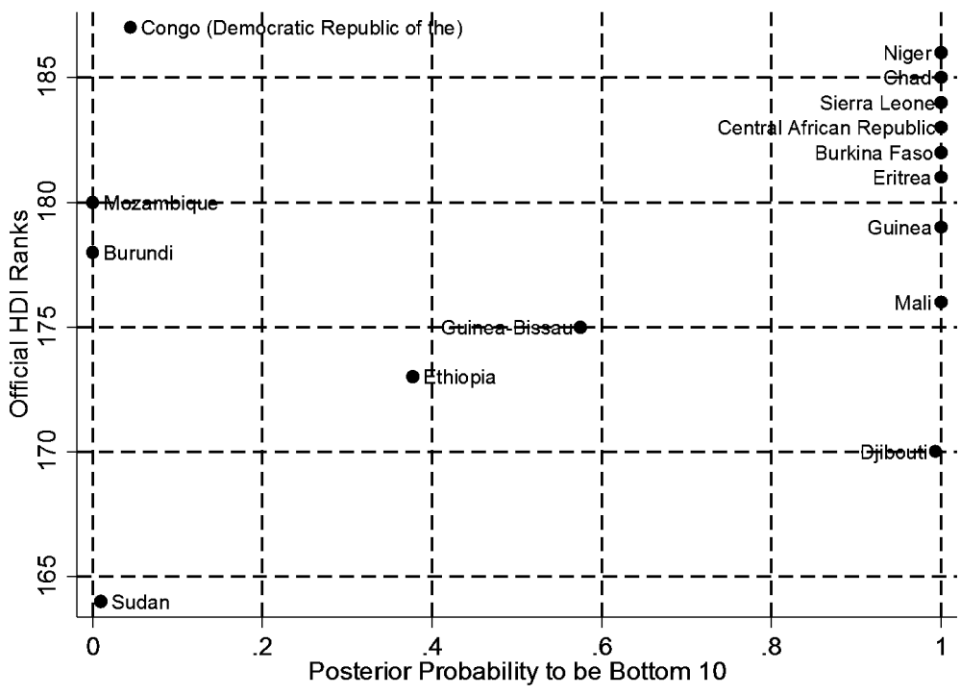


Figure 3. The Probability to be Model-based "Bottom 10" vs. Official HDI Ranks

Incorporation of an environmental indicator

We incorporate CO_2 to construct a model of HDI with an additional indicator representing a country's environmental stewardship and emissions level. We present the posterior ranks of our green HDI with those of the 2010 official HDI in Figure 4. The inclusion of CO_2 shifts the posterior ranks of some countries and presents slightly more discordance between the ranks of our model and the ranks of official HDI compared to the results of our model without CO_2. Without CO_2 the sum of absolute differences between the ranks of our model and those of the official HDI is 1335.9. After including CO_2, the sum of absolute differences increases to 1463.9, implying a 10% increase in discordance between our model's results and the ranks of official

HDI when using CO_2.[16] Comparing the results of our model with and without CO_2 to one another, we find the sum of absolute differences to be 1038.99.[17] This implies that while there is a level of disagreement between the ranks of our model under different specifications, it is lower than the level of discordance for either model when compared to the official HDI.

Table 4 shows the specific changes in the "squared correlation coefficients" between the rankings of our model with and without CO_2. While "education" remains the dominant indicator of human development, the proportional contribution attributed to CO_2 is similar to that of the other variables. The contribution of "health" also becomes smaller after adding the environmental dimension, which is likely a result of CO_2 capturing certain health issues associated with a country's pollution level. The addition of CO_2 also leads to considerable movements in rank for several oil producing nations. As example, Trinidad and Tobago and Kuwait, which are ranked 63 and 42 respectively by official HDI, both see dramatic improvements in rank under our green HDI model; moving to posterior mean rankings of 30.69 and 4.86 respectively.

Another interesting difference between the rankings of our model with and without CO_2 is that the ranks of our green HDI are estimated with less uncertainty than the ranks of our original model. This can be seen by comparing the confidence intervals shown in Figure 4 to those of Figure 1. To confirm this, we also sum the standard deviations of the posterior ranks for each country produced by our model with and without CO_2. After including CO_2, the sum of standard deviations of our posterior ranks decreases from 397.9 to 352.9, an 11% decline, indicating less

[16] The average absolute difference in ranks between the official HDI and our model increases from 7.1 to 7.8 after including CO_2.
[17] The average absolute difference in ranks between our model with and without CO_2 is 5.6.

uncertainty.[18] One possible reason for this is that we simply accept the positive correlation between CO_2 and the official HDI score. Since we let the factor loading produced by our model decide the direction of its contribution to human development, CO_2 is incorporated "as is" without alteration to its direction or functional form. The sensitivity of our model to functional form changes of CO_2 is explored in Section 5.

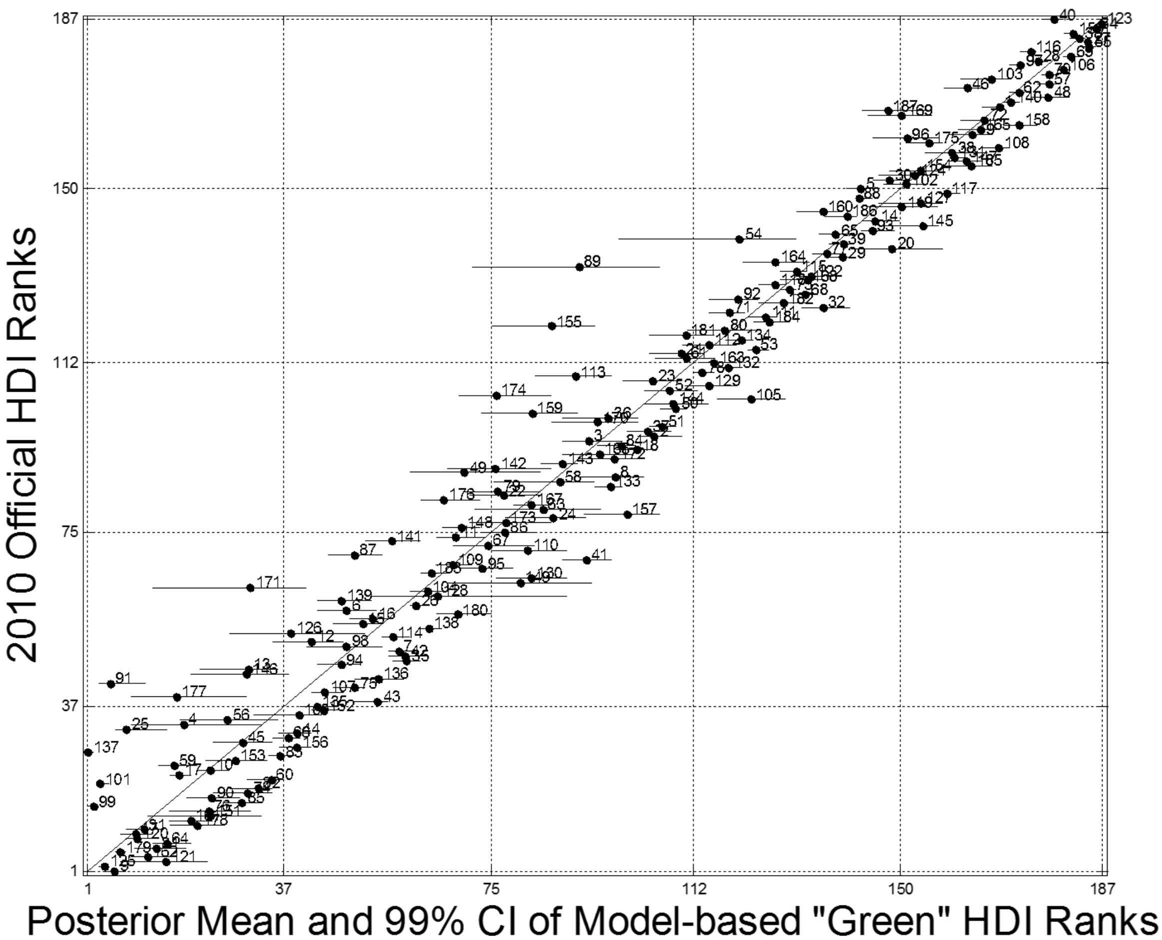


Figure 4. Posterior Mean and 99% CI of Model-based "Green" HDI Ranks vs. Official HDI Ranks

[18] The average standard deviation in posterior country rank decreases from 2.13 to 1.89.

Table 4. Comparison of Normalized Squared Correlations ρ^2 without CO2 and with CO2

Variable	Model-based Index without CO_2		Model-based Index With CO_2	
	ρ^2	95% CI	ρ^2	95% CI
Life Expectancy at Birth	0.19	(0.17, 0.20)	0.15	(0.14, 0.16)
Mean Years of Schooling	0.27	(0.27, 0.28)	0.22	(0.21, 0.22)
Expected Years of Schooling	0.27	(0.27, 0.28)	0.21	(0.21, 0.21)
GNI per capita	0.26	(0.26, 0.27)	0.22	(0.22, 0.23)
CO_2 Emissions per capita	-	-	0.20	(0.20, 0.20)

Results for MDG index

Initially we construct our MDG index using a naïve imputation process to estimate any missing data. We also formulate the index using a posterior imputation method, the results for which are covered in Section 5. In Figure 5, we compare the posterior mean ranks of our MDG index with the ranks of official HDI using the naïvely imputed data. Figure 5 affirms a positive association between the ranks of our "MDG index" and those of the official HDI.[19]

As the MDG index includes both a higher number of variables and variables which are not incorporated into the official HDI, it does intuitively lead to a greater level of discordance than the results obtained from our alternative formulations of the HDI itself. The sum of absolute differences between the ranks of our MDG index and the official HDI is 2230.[20] Referencing the top-right corner of Figure 5 for a visual example of the discordance between the two indices, Equatorial Guinea, Congo, Zambia, and Kenya, none of which fall into the lowest development quintile of official HDI, are all located in the lowest development quintile of our MDG rankings.

[19] The correlation between the posterior mean ranks of our MDG index and the ranks of official HDI is 0.95.
[20] The average absolute difference in ranks between the official HDI and MDG index is 11.9.

Therefore, the official HDI is likely overestimating the development levels of these countries with respect to the findings of our MDG index. Alternatively, with reference to the bottom-left corner of Figure 5, Brunei and Lithuania are both ranked outside of the most developed quintile of our posterior MDG ranks while they are included in the most developed quintile of the official HDI. It is therefore possible that the official HDI overestimates the development level of these countries given our findings. We also find the total level of uncertainty produced by our MDG index to be lower than our estimations of HDI and green HDI, with a sum standard deviations of 264, corresponding to an average standard deviation in ranks of 1.4.

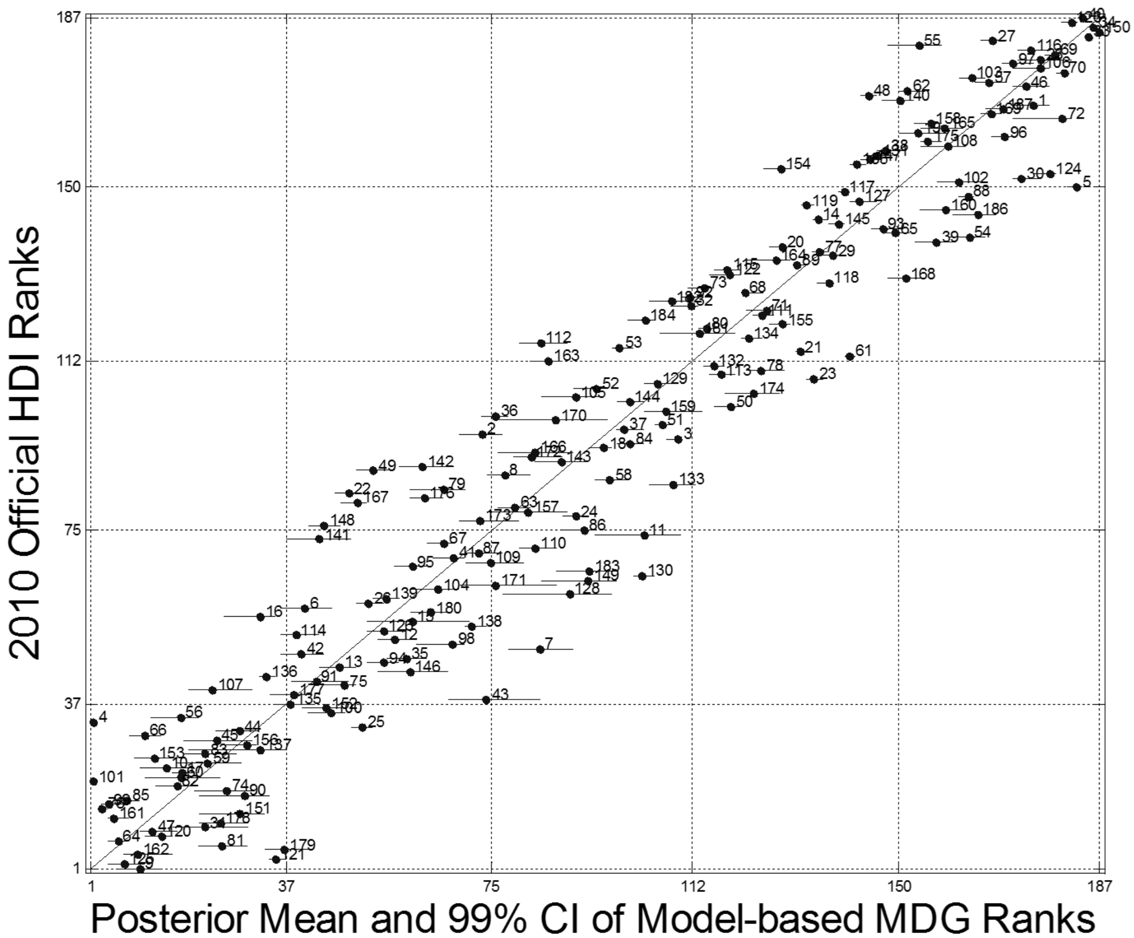


Figure 5. Posterior Mean and 99% CI of Model-based MDG Ranks vs. Official HDI Ranks

5. Sensitivity Analysis

In this section we explore the sensitivity of our results with regards to three aspects of the data. First, we address the sensitivity to choices of functional form by comparing results using GNIpc vs. ln(GNIpc), which is used in calculating the official HDI. Second, we compare the results using CO2 vs. ln(CO2). Finally, we address the sensitivity to the choice of methods used to impute the missing data of our MDG index.

Calculating HDI using the logarithm of GNI per capita

To account for the diminishing effect of income on development, we use the natural logarithm of GNI per capita as an alternative measure in our model based HDI. Recall that the official HDI also uses the logarithm of GNIpc to calculate their Income Index. Table 5 compares the normalized "squared correlation coefficients" of our model using both GNI per capita and ln(GNIpc), along with the factor weightings of official HDI. We can see that there are no substantial changes in the squared correlation coefficient due to the change in the functional form of GNIpc.

Figure 6 presents our model based ranks using ln(GNIpc) versus the ranks of official HDI. Looking between Figure 6 and Figure 1, we see that changing the functional form of GNI per capita does not substantially alter the posterior ranks of our model. However, comparing the ranks of our model and the official HDI, the sum of absolute differences between the official HDI and our model based index using ln(GNIpc) decreases from 1337 to 986, a decline of 26%.[21] This is most likely due to the fact that ln(GNIpc) is the functional form specification used to calculate the official HDI's Income Index. The discordance between the results of our model using GNI per capita and ln(GNIpc) is 640, which is 35% smaller than the discordance between the ranks of official HDI and our model-based index using ln(GNIpc).[22] While the discordance between the results of our model and the official HDI decreases when using ln(GNIpc), the

[21] The average absolute difference in rank between official HDI and our model is 5.8 when using ln(GNI), comparing to 7.1 when using GNI per capita.
[22] The average absolute difference in rank between our model with GNI per capita and ln(GNI) is 3.4.

posterior ranks' sum of standard deviations increases from 398 to 436, a 10% increase. This implies that the total amount of uncertainty in our model increases with the use of ln(GNIpc).

Table 5. Comparison of Normalized Squared Correlations $\rho^2(95\%\ CI)$

Variable	HDI Weights (95% CI)	ρ^2 (95% CI)	
		GNI per capita	$\ln(\text{GNI per capita})$
Life Expectancy at Birth	0.33 (0.32, 0.34)	0.19 (0.17, 0.20)	0.20 (0.19, 0.21)
Mean Years of Schooling	0.29 (0.29, 0.29)	0.27 (0.27, 0.28)	0.26 (0.26, 0.26)
Expected Years of Schooling	0.23 (0.22, 0.24)	0.27 (0.27, 0.28)	0.26 (0.26, 0.26)
GNI per capita	0.15 (0.14, 0.15)	0.26 (0.26, 0.27)	0.28 (0.27, 0.29)

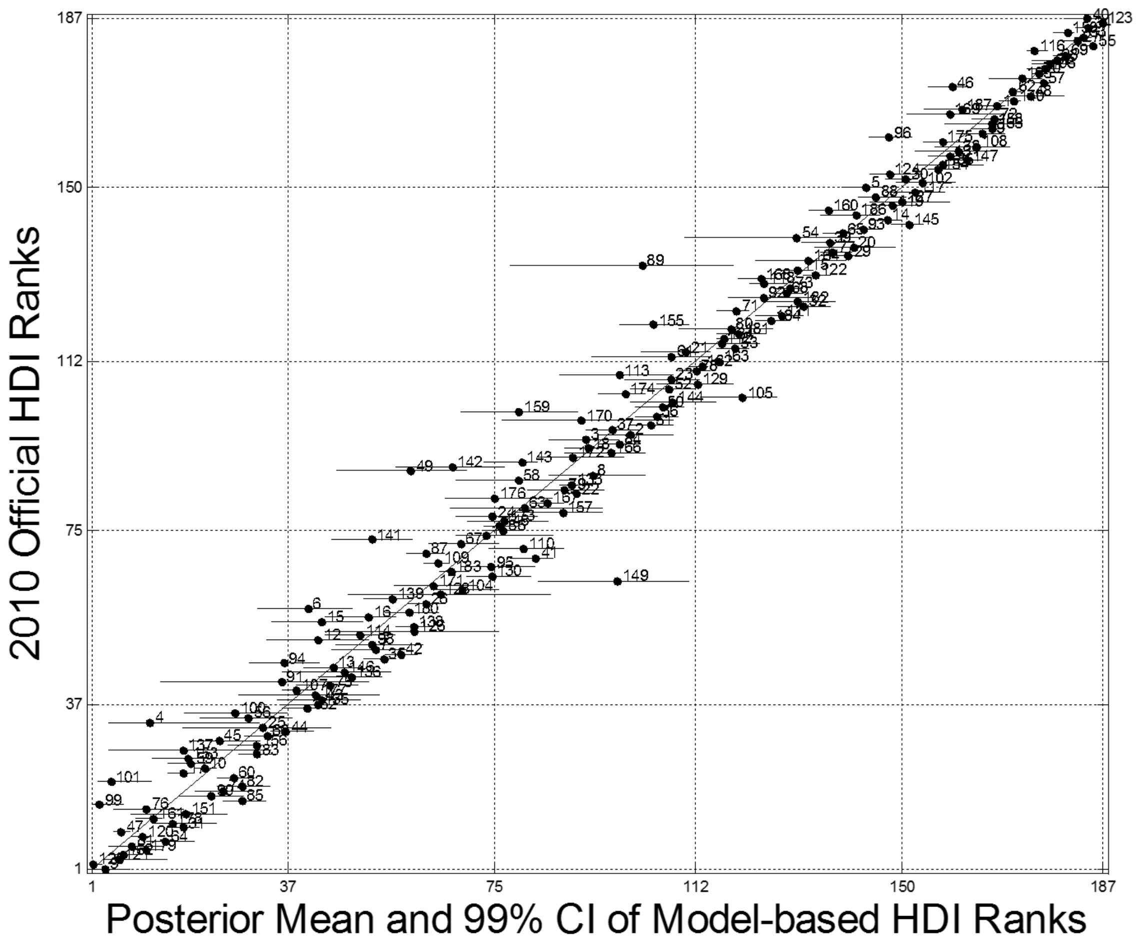


Figure 6. Posterior Mean and 99% CI of Model-based Ranks vs. Official HDI Ranks Using ln(GNI per capita)

Calculating green HDI with the logarithm of CO_2

We observe a positive association between CO_2 level and country specific human development level, as measured by official HDI score. Figure 7(a) shows the relationship between official HDI score and CO_2 in 2010 for the 187 countries of our sample. Figure 7(b) shows the same relationship, except delineated by the official HDI's development level categories.[23] For countries in the categories of "low development", "medium development" and "high development", the relationship between CO_2 and official HDI ranking is apparently positive. It is only for countries in the "very high development" category that the association become seemingly insignificant.[24]

While the relationship between CO_2 and human development level is positive, it is decidedly non-linear. To account for this, we also formulate our green HDI ranks using the natural logarithm of CO_2. Table 6 compares the normalized "squared correlation coefficients" using each possible functional form combination of both GNI per capita and CO_2. The small variation in variable contributions between estimations further substantiates our claim that the logarithmic functional form does little to alter the factor weightings of our model-based index. Figures A1, A2 and A3 in Appendix III display the ranks of our green HDI versus those of the official HDI using the specifications in columns (2), (3), and (4) of Table 6. When compared to Figure 6, alteration to the functional form of CO_2 seems to have a negligible effect on the level of discordance between our green HDI and official HDI for each of the three estimations.

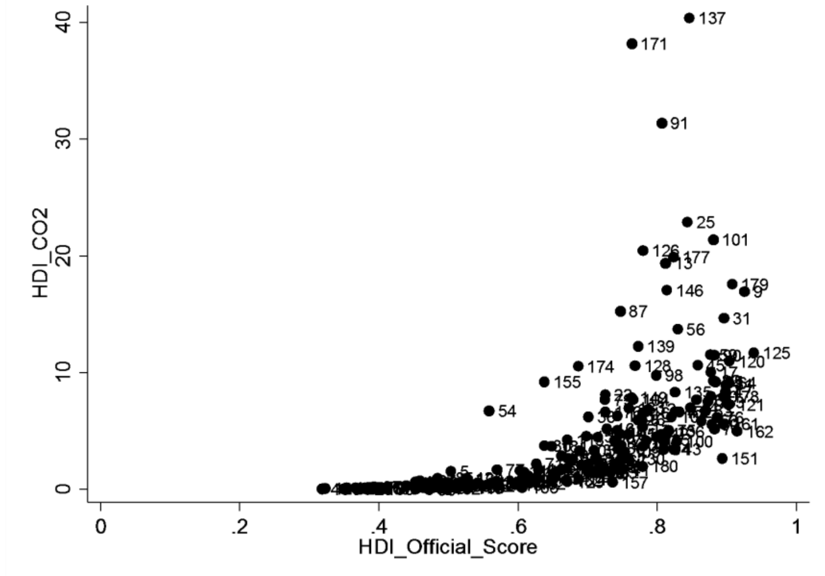
As discussed in the previous section, using the logarithm of GNI per capita results in a considerable decrease in discordance between the ranks of our model and the official HDI. Using the logarithm of GNI per capita in our green HDI also decreases the level of discordance between our model and the official HDI, resulting in a decrease in the sum of absolute differences in rank from 1464 to 1334, a decline of 9%.[25] The sum of absolute differences

[23] "very high development" (HDI≥0.8), "high development" (HDI 0.7-0.8), "medium development" (HDI 0.55-0.7), and "low development" (HDI<0.55)
[24] The correlation coefficients between CO2 emissions and official HDI are 0.50, 0.31, 0.37, and -0.06 for countries of "low development", "medium development", "high development" and "very high development", respectively.
[25] The average of absolute differences in rank decreases from 7.8 to 7.1.

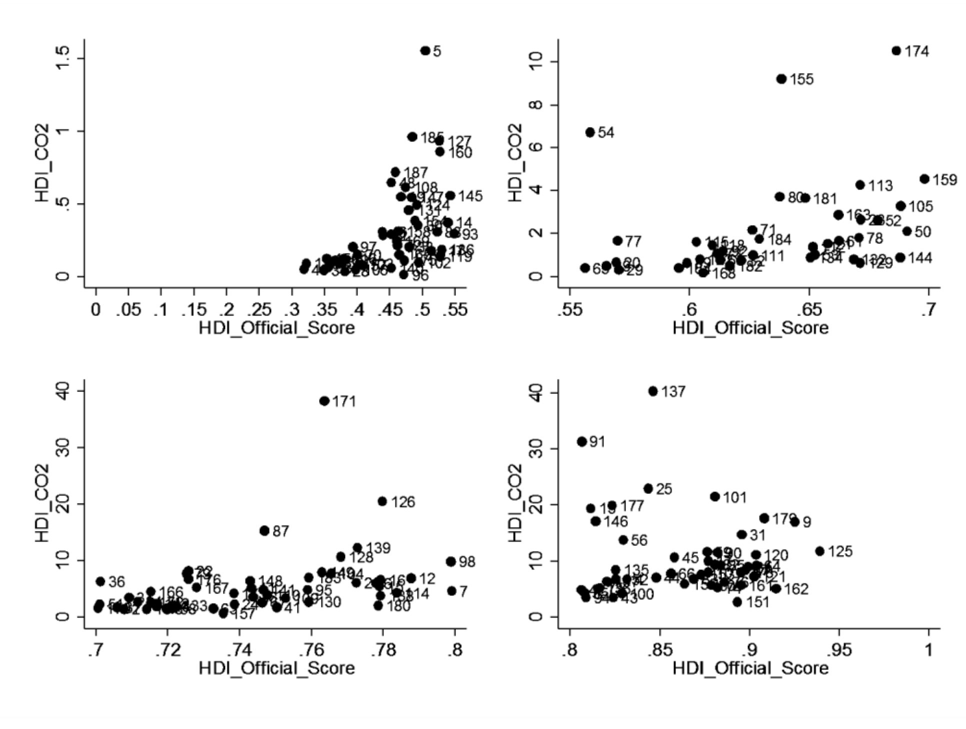
between our green HDI ranks with GNI per capita and ln(GNI) is 572, corresponding to an average difference of 3.1. Using ln(GNI) in our green HDI also leads to an increase in uncertainty compared to our green HDI index with GNI per capita. The sum of standard deviations changes from 353 to 385 when using ln(GNI), an increase of 9%.[26]

We also see that using the logarithm of CO_2 substantially decreases the discordance between the ranks of our model and the official HDI, leading to a decrease in the sum of absolute differences in ranks from 1464 to 1255, a decline of 14%.[27] Comparing the results of our model with and without the logarithm of CO_2, the sum of absolute differences between the two indices is 462, corresponding to an average difference in rank of 2.5. Additionally, the total amount of uncertainty changes very little for our model when using different functional forms of CO_2.[28] Tables showing the measures of discordance and uncertainty between all functional form combinations produced by our model are shown in Appendix IV.

[26] The average standard deviation increases from 1.9 to 2.1.
[27] The average of absolute differences in rank decreases from 7.8 to 6.7.
[28] Compared to using CO_2, the inclusion of the logarithm of CO_2 leads to only a 1% increase in total uncertainty.



(a)



(b)

Figure 7. Relationship between CO_2 and Official HDI for Countries with Different HDI Score in 2010

Table 6. Comparison of Normalized Squared Correlations $\rho^2 (95\%\ CI)$

Variable	GNI per capita (1) CO_2	GNI per capita (2) $\ln(CO_2)$	$\ln$(GNI per capita) (3) CO_2	$\ln$(GNI per capita) (4) $\ln(CO_2)$
Life Expectancy at Birth	0.15 (0.14, 0.16)	0.16 (0.15, 0.17)	0.16 (0.15, 0.17)	0.16 (0.15, 0.17)
Mean Years of Schooling	0.22 (0.21, 0.22)	0.22 (0.21, 0.23)	0.21 (0.21, 0.21)	0.20 (0.20, 0.20)
Expected Years of Schooling	0.21 (0.21, 0.21)	0.22 (0.21, 0.22)	0.20 (0.20, 0.20)	0.20 (0.20, 0.20)
GNI per capita	0.22 (0.22, 0.23)	0.21 (0.21, 0.22)	0.23 (0.23, 0.24)	0.24 (0.23, 0.25)
CO_2 Emissions per capita	0.20 (0.20, 0.20)	0.19 (0.19, 0.20)	0.20 (0.19, 0.20)	0.21 (0.20, 0.21)

Results using posterior imputation

Following the naïve imputation process for the MDG dataset, we next formulate our MDG index using the posterior imputation process built into our model. Figure 8 presents the relationship between the rankings of our MDG index following posterior imputation and the rankings of official HDI. We have shown earlier that there is a substantial amount of data missing from MDG data which were imputed. However, we used these imputed values as data without directly accounting for the uncertainty of imputation. In this section we incorporate the imputation of missing data into the estimation algorithm. Similarly to multiple imputations method, the posterior imputation process also obtains draws from the posterior distribution of the missing values at each iteration of the sampler. We present the results in Figure 8.

While the posterior mean ranks for most countries remains stable, the uncertainty of rankings following posterior imputation appears much larger for some countries when compared to the uncertainty of the naïve imputation results. The more missing values a country has, the more uncertainty it will show following posterior imputation. This leads to countries like Liechtenstein and Hong Kong having extreme confidence intervals compared to the average. Additionally, higher levels of missing data increase the magnitude of separation between a country's naïve and posterior imputation mean ranks.

Formally measuring the amount of discordance between our model under the two imputation processes and the official HDI, we see an increase in the sum of squared differences in rank from 42,812 to 55,533 using posterior imputation, a change of almost 30%. While the sum of squared differences increases considerably following posterior imputation, the sum of absolute differences remains relatively unchanged (a 4% increase from 2230 to 2322).[29] This implies that several outlier countries see a considerable change in rank between the two imputation methods while the general discordance changes a comparably small amount. As for the uncertainty, the sum of standard deviations increases from 264 to 569. Tables showing measures of discordance and uncertainty for the MDG index under both imputation measures can be found in Appendix IV.

[29] The average of absolute differences in rank increases from 11.9 to 12.4.

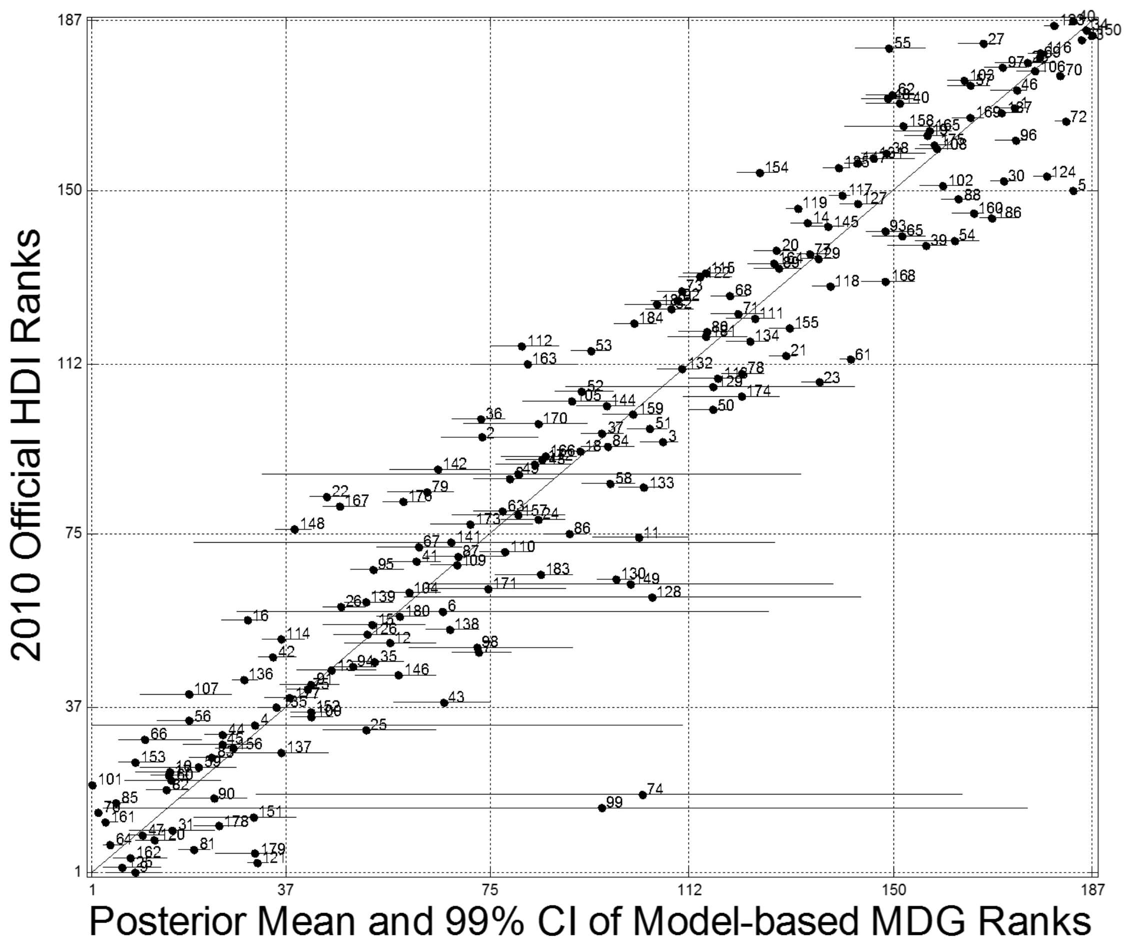


Figure 8. Posterior Mean and 99% CI of MDG Ranks Using Posterior Imputation vs. Official HDI Ranks

6. Conclusion

In this paper, we propose a Bayesian factor analysis model with the purpose of serving as an alternate approach to calculating the UNDP's Human Development Index, as well as providing a general methodology which can be used to augment existing indices or build new ones. We address several potential issues of the official HDI using the following methods. First, our model produces data-driven weights for each manifest variable's contribution to the latent factor of human development. This is in contrast to the *ad hoc* factor weights currently used to calculate the scores of official HDI. Second, our model reports its posterior ranks in terms of distributions, incorporating a measure of uncertainty which is absent from the official HDI's international rankings. Third, we adjust the level of uncertainty by incorporating a measure of spatial correlation between countries while also including country populations in our estimation. Each of these additions helps to improve the precision of our ranking distributions. To then illustrate the potential applications and flexibly of our model, we estimate an alternative green HDI index by adding a new environmental variable, CO_2. Then as an example of constructing a new index with a more complex variable structure, we build a novel MDG index using data from the Millennium Development Goals project.

Under our methodology, we find the "living standard" dimension provides a greater proportional contribution to human development than it is assigned by the official HDI while the "longevity" dimension provides a lower proportional contribution. The results of our model also show considerable levels of general disagreement when compared to the ranks of the official HDI. Countries which are officially categorized into a particular quintile can even arguably be assigned to a different quintile under our estimation. Therefore, the nations constituting a country's "peers" under the rankings of our model can vary widely from the peers observed under the official HDI, leading to a much different picture of a country's relative development level. We also find our incorporation of CO_2 to shift the posterior ranks of some countries substantially, again moving them across quintiles in some cases. It can then be argued that incorporating CO_2 is necessary, as it makes a non-negligible difference in the evaluation of human development level. Additionally, even with the complicated structure of the MDG's indicator variables, we show that our model is able to successfully construct the desired MDG index. Constructing an MDG index exemplifies the adaptive nature of our methodology and provides a blueprint which further researchers can implement to efficiently build indices that may have previously seemed too complex.

Looking to the change in results between different functional forms, we find our model to be more sensitive to functional alterations of GNI per capita than CO_2. The most plausible reason why changing between GNI per capita and its logarithm causes more variation in estimation is because the scale of GNI per capita is much larger than that of CO_2. This implies the values of the variable itself change much more between the two specifications. With this said, the amount of discordance between our model with GNI per capita and the logarithm of GNI per capita is smaller than the level of discordance between either model specification and official HDI. Also, after adding CO_2 our results become much less sensitive to the same functional form changes of GNI per capita. We therefore fail to conclude that our methodology is particularly sensitive to functional form changes. Nonetheless, our model does still allow for the construction of indices and addition of new variables using the minimum number of assumptions, since none are needed for either variable grouping or the direction of relationships between the variables used in the index and the latent factor of interest.

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Appendix I: Gibbs Sampler Algorithm

Following Hogan and Tchernis (2004), the factor analysis model in our paper is stated in aforementioned hierarchical form as follows:

$$Y|\delta \sim N(\mu + \Lambda\delta, M^{-1} \otimes \Sigma)$$

$$\delta \sim \mathrm{N}(0, M^{-\frac{1}{2}} \boldsymbol{\psi} M^{-\frac{1}{2}})$$

where:

$\mu = [\mu_1, \mu_2, \mu_3, \mu_4]'$;

$\Lambda = I_N \otimes \lambda$, with $\lambda = [\lambda_1, \lambda_2, \lambda_3, \lambda_4]'$;

$\boldsymbol{\psi} = (I - \omega W)^{-1}$;

$\Sigma = diag(\sigma_1^2, \sigma_2^2, \sigma_3^2, \sigma_4^2)$ with all the off-diagonal elements equal to 0.

Therefore, the parameters to estimate are λ, δ, μ, Σ and ω.

Step 1: Sample elements of λ.

Let 1_N be an $N \times 1$ vector with all elements equal to 1. Therefore, for each $\lambda_j, j = 1,2,3,4$, let the estimation equation be $Y_j - 1_N{}'\mu_j = \lambda_j \delta + \varepsilon_j$, where Y_j is the $N \times 1$ vector of manifest variable Y_{ij}, and $\varepsilon_j \sim N(0, \sigma_j^2/M)$. Let the prior distribution be $\lambda_j \sim \mathrm{N}(a, \mathrm{A})$, where $a = 0, A = 1000$.

Hence, the posterior of $\lambda_j$is drawn from conditional distribution $N(b, B)$, where:

$$B = \left(1/A + \delta' M \delta / \sigma_j^2\right)^{-1}$$

$$b = B[a/A + \delta' M\left(Y_j - 1_N\mu_j\right)/\sigma_j^2]$$

As factor loadings, λ_j's are restricted to be positive.

Step 2: Sample δ.

Let the estimation equation be $Y - \mu \otimes 1_N = \Lambda\delta + \varepsilon$, where Y is the $NJ \times 1$ vector of manifest variable Y_{ij} , and $\varepsilon \sim N(0, M^{-1} \otimes \Sigma)$. We have known that the prior distribution is $\delta \sim \mathrm{N}(0, M^{-\frac{1}{2}}\boldsymbol{\psi}M^{-\frac{1}{2}})$.

Hence, the posterior of δ is drawn from conditional distribution $N(d, D)$, where:

$$D = \left[\left(M^{-\frac{1}{2}}\boldsymbol{\psi}M^{-\frac{1}{2}}\right)^{-1} + \Lambda'(M^{-1} \otimes \boldsymbol{\Sigma})^{-\mathbf{1}}\Lambda\right]^{-1}$$

$$d = D[\Lambda'(M^{-1} \otimes \Sigma)^{-\mathbf{1}}(Y - \mu \otimes 1_N)]$$

Step 3: Sample elements of μ.

For each $\mu_j, j = 1,2,3,4$, let the estimation equation be $Y_j - \lambda_j\delta = 1_N\mu_j + \varepsilon_j$. Let the prior distribution be $\mu_j \sim \mathrm{N}(c, \mathrm{C})$, where $c = 0, C = 1000$.

Hence, the posterior of μ_j is drawn from conditional distribution $N(e, E)$, where:

$$E = \left(1/C + 1_N{}'M1_N/\sigma_j^2\right)^{-1}$$

$$e = E[c/C + 1_N{}'M\left(Y_j - \lambda_j\delta\right)/\sigma_j^2]$$

Step 4: Sample elements of Σ.

For each $\sigma_j^2, j = 1,2,3,4$, let the estimation equation be $Y_j = \lambda_j\delta + 1_N\mu_j + \varepsilon_j$. Let the prior distribution be $\sigma_j^2 \sim IG(\alpha_0, \beta_0)$, where $\alpha_0 = 0.001, \beta_0 = 0.001$.

Hence, the posterior of σ_j^2 is drawn from conditional distribution $IG(\alpha_1, \beta_1)$, where:

$$\alpha_1 = \alpha_0 + \frac{N}{2}$$

$$\beta_1 = \left(Y_j - \lambda_j\delta - 1_N\mu_j\right)' M\left(Y_j - \lambda_j\delta - 1_N\mu_j\right) + \beta_0$$

Step 5: Sample ω using a Metropolis-Hasting algorithm.

Let the prior distribution of ω be $\pi(\omega) = N(0{,}1000)I(\xi_1^{-1} < \omega < \xi_N^{-1})$, where ξ_1 and ξ_N denote the minimum and maximum eigenvalues of spatial correlation matrix W. Hence, the target density of ω is $f(\delta|\boldsymbol{\psi}(\omega))\pi(\omega)$, where $f(\delta|\boldsymbol{\psi}(\omega))$ is the kernel of the distribution of δ conditional on $\boldsymbol{\psi} = (I - \omega W)^{-1}$. Let the proposal density be $q(\omega'|\omega) \sim N(\omega, \rho^2)$, so that the candidate ω' is drawn from a random walk equation: $\omega' = \omega + \epsilon$, where $\epsilon \sim N(0, \rho^2)$, and ρ^2 is a tuning parameter. The generated ω is also restricted into the domain $\xi_1^{-1} < \omega < \xi_N^{-1}$.

Therefore, ω' is accepted with probability:

$$\min\{1, \frac{f(\delta|\boldsymbol{\psi}(\omega'))\pi(\omega')q(\omega|\omega')}{f(\delta|\boldsymbol{\psi}(\omega))\pi(\omega)q(\omega'|\omega)}$$

Appendix II: Numbering of Countries

#	**Country**
1	Afghanistan
2	Albania
3	Algeria
4	Andorra
5	Angola
6	Antigua and Barbuda
7	Argentina
8	Armenia
9	Australia
10	Austria
11	Azerbaijan
12	Bahamas
13	Bahrain
14	Bangladesh
15	Barbados
16	Belarus
17	Belgium
18	Belize
19	Benin
20	Bhutan
21	Bolivia (Plurinational State of)
22	Bosnia and Herzegovina
23	Botswana
24	Brazil
25	Brunei Darussalam
26	Bulgaria
27	Burkina Faso
28	Burundi
29	Cambodia
30	Cameroon
31	Canada
32	Cape Verde
33	Central African Republic
34	Chad
35	Chile
36	China
37	Colombia
38	Comoros
39	Congo
40	Congo (Democratic Republic of the)
41	Costa Rica
42	Croatia
43	Cuba
44	Cyprus
45	Czech Republic
46	Côte d'Ivoire
47	Denmark
48	Djibouti
49	Dominica
50	Dominican Republic
51	Ecuador
52	Egypt
53	El Salvador
54	Equatorial Guinea
55	Eritrea
56	Estonia
57	Ethiopia
58	Fiji
59	Finland
60	France
61	Gabon
62	Gambia
63	Georgia
64	Germany
65	Ghana
66	Greece
67	Grenada
68	Guatemala
69	Guinea
70	Guinea-Bissau
71	Guyana
72	Haiti
73	Honduras
74	Hong Kong
75	Hungary

76	Iceland	119	Nepal
77	India	120	Netherlands
78	Indonesia	121	New Zealand
79	Iran (Islamic Republic of)	122	Nicaragua
80	Iraq	123	Niger
81	Ireland	124	Nigeria
82	Israel	125	Norway
83	Italy	126	Oman
84	Jamaica	127	Pakistan
85	Japan	128	Palau
86	Jordan	129	Palestine, State of
87	Kazakhstan	130	Panama
88	Kenya	131	Papua New Guinea
89	Kiribati	132	Paraguay
90	Korea (Republic of)	133	Peru
91	Kuwait	134	Philippines
92	Kyrgyzstan	135	Poland
93	Lao People's Democratic Republic	136	Portugal
94	Latvia	137	Qatar
95	Lebanon	138	Romania
96	Lesotho	139	Russian Federation
97	Liberia	140	Rwanda
98	Libya	141	Saint Kitts and Nevis
99	Liechtenstein	142	Saint Lucia
100	Lithuania	143	Saint Vincent and the Grenadines
101	Luxembourg	144	Samoa
102	Madagascar	145	Sao Tome and Principe
103	Malawi	146	Saudi Arabia
104	Malaysia	147	Senegal
105	Maldives	148	Serbia
106	Mali	149	Seychelles
107	Malta	150	Sierra Leone
108	Mauritania	151	Singapore
109	Mauritius	152	Slovakia
110	Mexico	153	Slovenia
111	Micronesia (Federated States of)	154	Solomon Islands
112	Moldova (Republic of)	155	South Africa
113	Mongolia	156	Spain
114	Montenegro	157	Sri Lanka
115	Morocco	158	Sudan
116	Mozambique	159	Suriname
117	Myanmar	160	Swaziland
118	Namibia	161	Sweden

162	Switzerland
163	Syrian Arab Republic
164	Tajikistan
165	Tanzania (United Republic of)
166	Thailand
167	The former Yugoslav Republic of Macedonia
168	Timor-Leste
169	Togo
170	Tonga
171	Trinidad and Tobago
172	Tunisia
173	Turkey
174	Turkmenistan
175	Uganda
176	Ukraine
177	United Arab Emirates
178	United Kingdom
179	United States
180	Uruguay
181	Uzbekistan
182	Vanuatu
183	Venezuela (Bolivarian Republic of)
184	Viet Nam
185	Yemen
186	Zambia
187	Zimbabwe

Appendix III: Figures of "Green" HDI Using Different Functional Forms

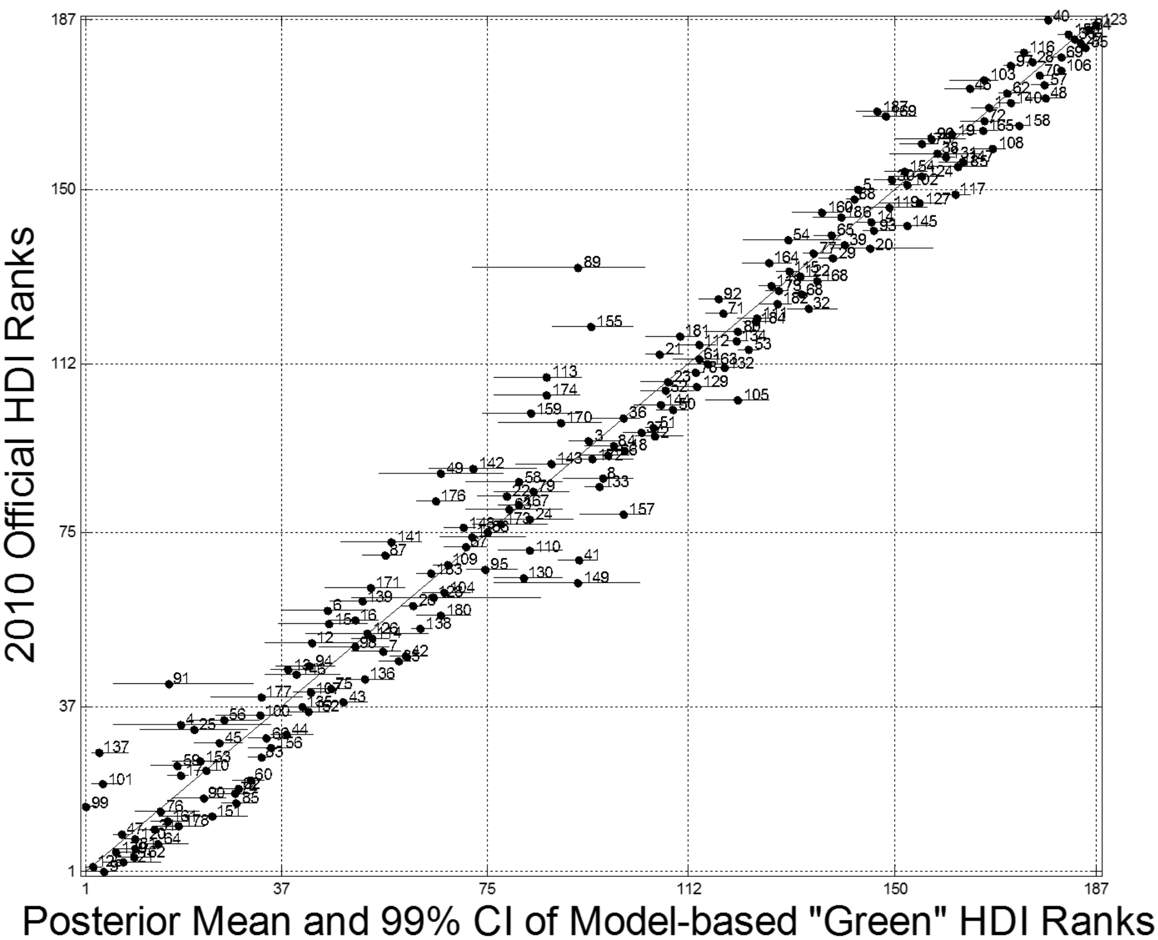

Figure A1. Posterior Mean and 99% CI of Model-based "Green" HDI Ranks vs. Official HDI Ranks Using GNI per capita and ln(CO_2)

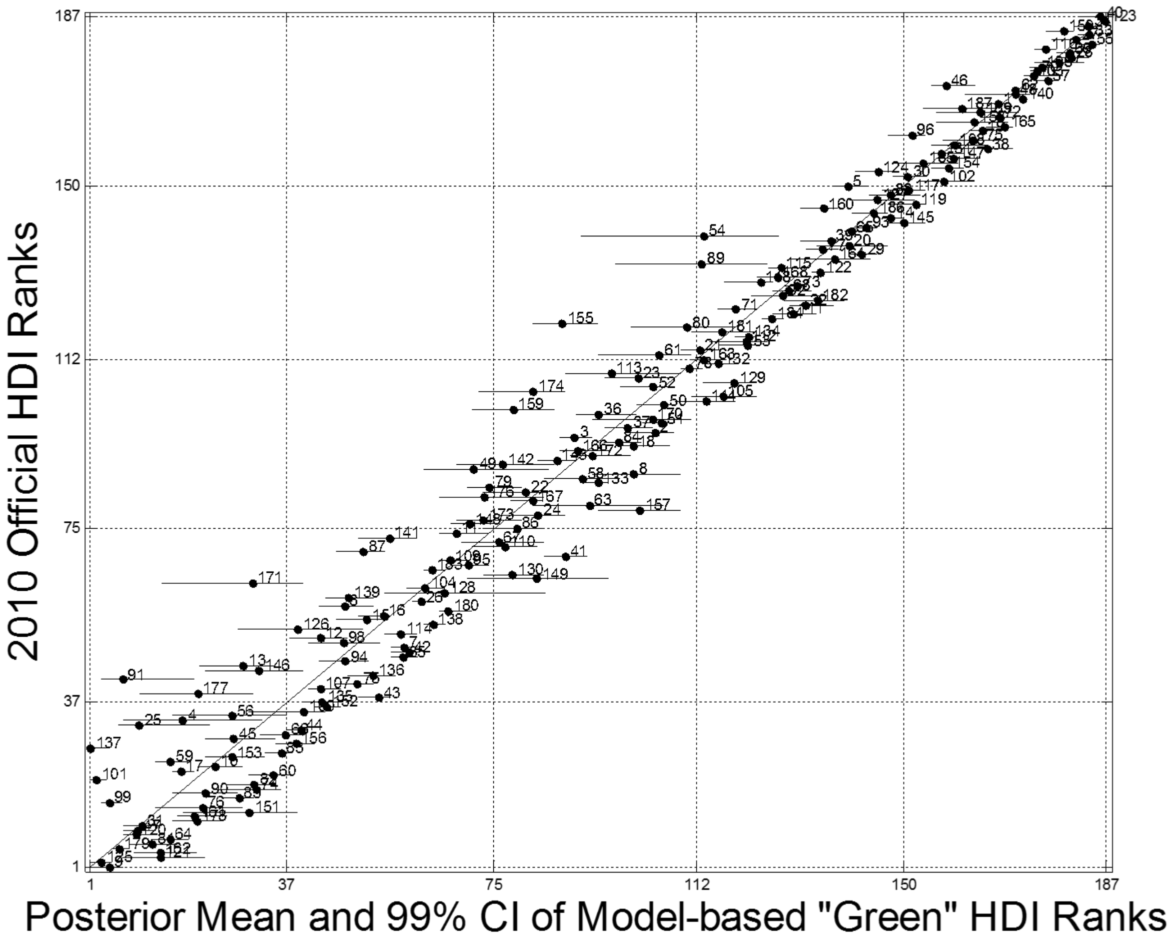

Figure A2. Posterior Mean and 99% CI of Model-based "Green" HDI Ranks

vs. Official HDI Ranks Using ln(GNI per capita) and CO_2

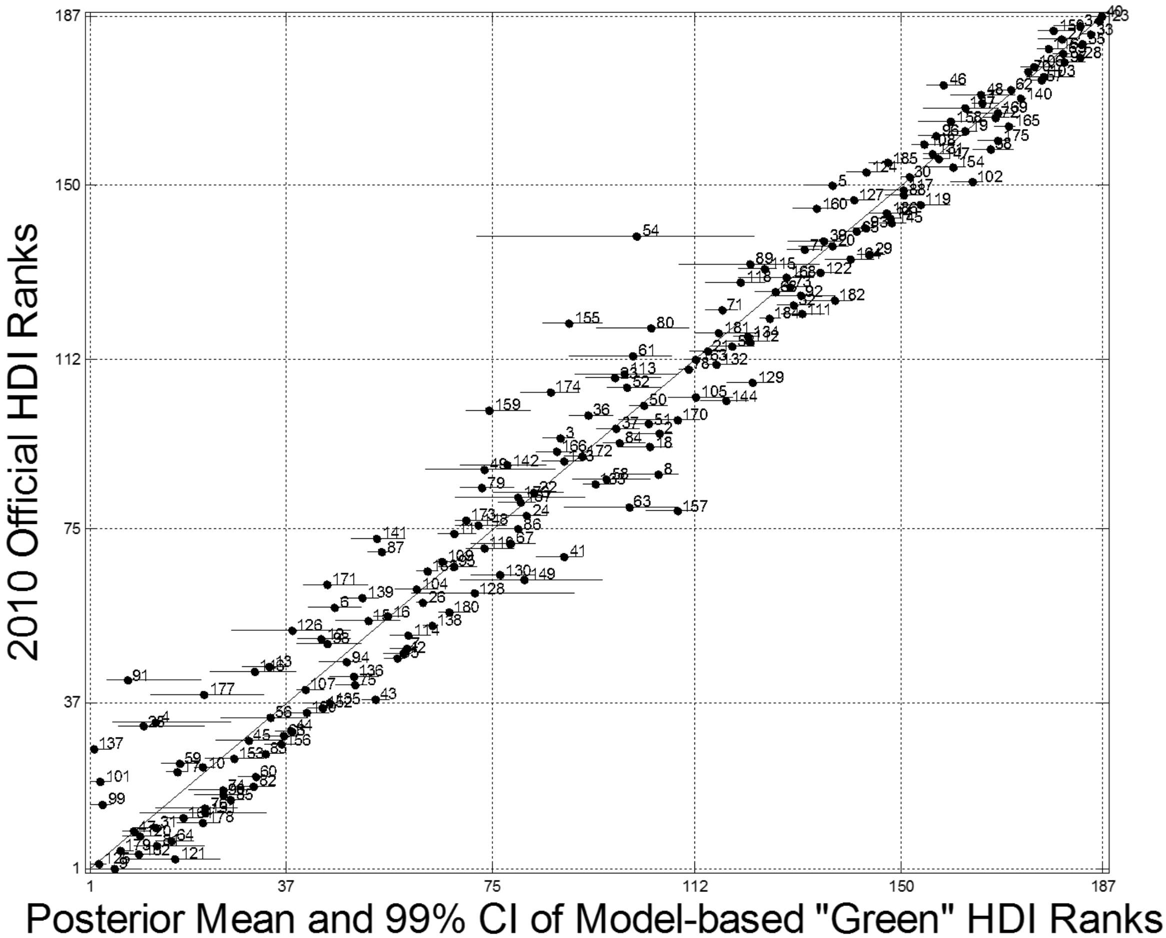

Figure A3. Posterior Mean and 99% CI of Model-based "Green" HDI Ranks

vs. Official HDI Ranks Using ln(GNI per capita) and ln(CO_2)

Appendix IV: Table of Discordance and Uncertainty

Table A1. Discordance Squared Differences

Sum of Squared Differences	**Official HDI**	**Basic 1** (GNIpc without CO2)	**Green 1** (GNIpc + CO2)	**Green 4** (lnGNIpc + lnCO2)
Basic 1 (GNIpc without CO2)	18851			
Basic 2 (lnGNIpc without CO2)	10511	4698		
Green 1 (GNIpc + CO2)	21048	12104		
Green 2 (lnGNIpc + CO2)	17633		3650	1283
Green 3 (GNIpc + lnCO2)	15777		3036	10157
Green 4 (lnGNIpc + lnCO2)	17877			
MDG 1 (Naïve Imp.)	42812			
MDG 2 (Posteior Imp.)	55533			
Average Squared Differences	**Official HDI**	**Basic 1** (GNIpc without CO2)	**Green 1** (GNIpc + CO2)	**Green 4** (lnGNIpc + lnCO2)
Basic 1 (GNIpc without CO2)	100.8			
Basic 2 (lnGNIpc without CO2)	56.2	25.1		
Green 1 (GNIpc + CO2)	112.6	64.7		
Green 2 (lnGNIpc + CO2)	94.3		19.5	6.9
Green 3 (GNIpc + lnCO2)	84.4		16.2	54.3
Green 4 (lnGNIpc + lnCO2)	95.6			
MDG 1 (Naïve Imp.)	228.9			
MDG 2 (Posteior Imp.)	297.0			

Note: "Basic" indicates the model based index with the four variables used by the official HDI. "Green" indicates the model based index adding the environment variable CO2. "MDG" indicates the model based MDG index.

Table A2. Discordance - Absolute Differences

Sum of Absolute Differences	**Official HDI**	**Basic 1** (GNIpc without CO2)	**Green 1** (GNIpc + CO2)	**Green 4** (lnGNIpc + lnCO2)
Basic 1 (GNIpc without CO2)	1337			
Basic 2 (lnGNIpc without CO2)	986	640		
Green 1 (GNIpc + CO2)	1464	1039		
Green 2 (lnGNIpc + CO2)	1334		572	350
Green 3 (GNIpc + lnCO2)	1255		462	1022
Green 4 (lnGNIpc + lnCO2)	1366			
MDG 1 (Naïve Imp.)	2230			
MDG 2 (Posteior Imp.)	2322			

Average Absolute Differences	**Official HDI**	**Basic 1** (GNIpc without CO2)	**Green 1** (GNIpc + CO2)	**Green 4** (lnGNIpc + lnCO2)
Basic 1 (GNIpc without CO2)	7.1			
Basic 2 (lnGNIpc without CO2)	5.3	3.4		
Green 1 (GNIpc + CO2)	7.8	5.6		
Green 2 (lnGNIpc + CO2)	7.1		3.1	1.9
Green 3 (GNIpc + lnCO2)	6.7		2.5	5.5
Green 4 (lnGNIpc + lnCO2)	7.3			
MDG 1 (Naïve Imp.)	11.9			
MDG 2 (Posteior Imp.)	12.4			

Note: "Basic" indicates the model based index with the four variables used by the official HDI. "Green" indicates the model based index adding the environment variable CO2. "MDG" indicates the model based MDG index.

Table A3. Uncertainty – Standard Deviations

Models	Sum of Standard Deviations	Average Standard Deviations
Basic 1 (GNIpc without CO2)	398	2.13
Basic 2 (lnGNIpc without CO2)	436	2.33
Green 1 (GNIpc + CO2)	353	1.89
Green 2 (lnGNIpc + CO2)	385	2.06
Green 3 (GNIpc + lnCO2)	356	1.90
Green 4 (lnGNIpc + lnCO2)	360	1.93
MDG 1 (Naïve Imp.)	264	1.41
MDG 2 (Posteior Imp.)	569	3.04

Note: “Basic” indicates the model based index with the four variables used by the official HDI. “Green” indicates the model based index adding the environment variable CO2. “MDG” indicates the model based MDG index.