A Compartmentalized Mathematical Models of Normal and Failing Mouse Cardiac Myocytes

Tesfaye Negash Asfaw

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A Compartmentalized Mathematical Models of Normal and Failing Mouse Cardiac Myocytes

by

Tesfaye Asfaw

Under the Direction of Vladimir Bondarenko, Ph.D.

A Dissertation Submitted in Partial Fulfillment of the Requirements for the Degree of

Doctor of Philosophy

in the College of Arts and Sciences

Georgia State University

2022
ABSTRACT

Atrial fibrillations and heart failure are among the leading cardiovascular diseases in the world. Understanding the development and progression of these diseases requires a thorough knowledge of the electrophysiological mechanisms in a healthy and diseased cardiac myocyte. This goal can be achieved by using mathematical modeling along with experimental investigations. Here, we developed two new comprehensive mathematical models of the mouse atrial and ventricular myocytes. The first one is a novel compartmentalized mathematical model of mouse atrial myocytes. This model combines the action potential, $[\text{Ca}^{2+}]_i$ dynamics, and $\beta$-adrenergic signaling cascade for a subpopulation of right atrial myocytes with a developed transverse-axial tubule system. The model consists of three compartments related to $\beta$-adrenergic signaling (caveolae, extracaveolae, and cytosol) and employs local control of $\text{Ca}^{2+}$-release. It also simulates the mechanisms of action potential generation and describes atrial-specific $\text{Ca}^{2+}$ handling and frequency dependences of the action potential and $[\text{Ca}^{2+}]_i$ transients. The model showed that the T-type $\text{Ca}^{2+}$ current significantly affects the later stage of the action potential with little effect on $[\text{Ca}^{2+}]_i$ transients. Blocking the small-conductance $\text{Ca}^{2+}$-activated $\text{K}^+$ current leads to the prolongation of the action potential at high intracellular $\text{Ca}^{2+}$ concentrations. Simulation results obtained from the atrial cell model were compared to those from ventricular myocytes. The developed model presents a valuable tool for studying complex electrical properties in the mouse atria and could be applied to understand atrial physiology and arrhythmogenesis. The second model is a novel compartmentalized mathematical model of failing mouse ventricular myocytes after TAC procedure. The model effectively describes the cell geometry, action potentials, $[\text{Ca}^{2+}]_i$ transients, and $\beta_1$- and $\beta_2$-adrenergic signaling in the failing cells. Simulation results obtained from a failing cells’ model were compared to those from the normal ventricular myocytes. Exploration
of the model revealed the sarcoplasmic reticulum Ca\(^{2+}\) load mechanisms in failing ventricular myocytes. We also described a proarrhythmic behavior of Ca\(^{2+}\) dynamics upon stimulation with isoproterenol and mechanisms of the proarrhythmic behavior suppression. The developed model can be used to explain the existing experimental data on failing mouse ventricular myocytes and make experimentally testable predictions of the failing myocyte behavior.

INDEX WORDS: Atrial myocytes, T-type Ca\(^{2+}\) current, Ventricular myocytes, TAC, \(\beta\)-adrenergic, Heart failure, Late sodium current.
A Compartmentalized Mathematical Models of Normal and Failing Mouse Cardiac Myocytes

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Georgia State University
May 2022
DEDICATION

This dissertation is dedicated to my family and friends, who have been a continuous source of encouragement and support.
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<tbody>
<tr>
<td>AC</td>
<td>Adenylyl cyclase</td>
</tr>
<tr>
<td>AMP</td>
<td>Adenosine monophosphate</td>
</tr>
<tr>
<td>AP</td>
<td>Action potential</td>
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<tr>
<td>APD</td>
<td>Action potential duration</td>
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<tr>
<td>AV</td>
<td>Atrioventricular</td>
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<tr>
<td>β-AR</td>
<td>Beta-adrenergic</td>
</tr>
<tr>
<td>CaM</td>
<td>Calmodulin</td>
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<tr>
<td>CaMKII</td>
<td>Ca^{2+}/calmodulin-dependent protein kinase II</td>
</tr>
<tr>
<td>cAMP</td>
<td>Cyclic adenosine monophosphate</td>
</tr>
<tr>
<td>Casq2</td>
<td>Calsequestrin 2</td>
</tr>
<tr>
<td>GRK2</td>
<td>G Protein-Coupled Receptor Kinase 2</td>
</tr>
<tr>
<td>HF</td>
<td>Heart failure</td>
</tr>
<tr>
<td>JSR</td>
<td>Junctional Sarcoplasmic Reticulum</td>
</tr>
<tr>
<td>LV</td>
<td>Left ventricle</td>
</tr>
<tr>
<td>Nav</td>
<td>Voltage-gated sodium channel</td>
</tr>
<tr>
<td>PDE</td>
<td>Phosphodiesterase</td>
</tr>
<tr>
<td>PKA</td>
<td>Protein kinase A</td>
</tr>
<tr>
<td>PKI</td>
<td>Protein kinase inhibitor</td>
</tr>
<tr>
<td>PLM</td>
<td>phospholemman</td>
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<tr>
<td>PLN</td>
<td>Phospholamban</td>
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<tr>
<td>PP1</td>
<td>Protein phosphatase I</td>
</tr>
<tr>
<td>RA</td>
<td>Right atria</td>
</tr>
<tr>
<td>Abbreviation</td>
<td>Description</td>
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<tr>
<td>--------------</td>
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<tr>
<td>RyR</td>
<td>Ryanodine receptors</td>
</tr>
<tr>
<td>SA</td>
<td>Sinoatrial</td>
</tr>
<tr>
<td>SERCA</td>
<td>Sarcoplasmic reticulum Ca(^{2+})ATPase</td>
</tr>
<tr>
<td>SR</td>
<td>Sarcoplasmic reticulum</td>
</tr>
<tr>
<td>TAC</td>
<td>Transverse aortic constriction</td>
</tr>
<tr>
<td>TATS</td>
<td>Transverse-axial tubular system</td>
</tr>
<tr>
<td>TnC</td>
<td>Troponin C</td>
</tr>
<tr>
<td>TNF</td>
<td>Tumor necrosis factor</td>
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<td>TnI</td>
<td>Troponin I</td>
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<td>TnT</td>
<td>Troponin T</td>
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1 INTRODUCTION

1.1 Structure and Electrophysiology of Cardiac Cells

1.1.1 Heart Function

The electrical system of the heart controls the mechanical functioning of the heart. The heart creates electrical impulses (action potentials) and controls the impulses’ route via a specific conduction pathway. Myocardial electrical activity is responsible for generating action potentials in individual cardiac cells. The normal coordinated electrical functioning of the whole heart is readily detected in surface electrocardiograms. The functioning of the heart depends on action potential (AP) generation and propagation, followed by relaxation and a period of refractororiness until the next impulse is generated. A cardiac impulse is generated by the sinoatrial (SA) node located in the right atrium (Figure 1.1). The SA node establishes the basic rhythm of the heartbeat and is called the pacemaker of the heart (Klabunde, 2012; Baranchuk et al., 2021).

Figure 1.1 Anatomy of the cardiac conduction system and the path of the action potential propagation

The electrical impulse generated at the SA node passes into the atria and is transmitted to another mass of specialized cells, the atrioventricular (AV) node, located between the atria and
ventricles. The electrical stimulus from the SA node that reaches the AV node is briefly delayed, which gives the atria time to contract and empty into the ventricles before ventricular contraction occurs. Once the atria are empty of blood, the valves between the atria and ventricles close. At this point, the atria begin to refill, and the electrical stimulus travels down into a large bundle of specialized tissue, the Bundle of His, which conducts it down to the ventricles through the Purkinje fibers. The Purkinje fibers carry the electrical impulse from both the left and right bundle branch to the myocardium of the ventricles. The electric signal that arrives at the ventricle ultimately causes the muscle tissue of the ventricles to contract to eject blood out of the heart (Harmrell, 2018).

The heart’s effective pumping action requires precise coordination of the myocardial contractions (millions of cells), which is accomplished via the action potential propagation through the heart.

### 1.1.2 Action Potential and Ionic Currents in Normal Hearts

An elementary and distinctive property of any excitable cell is its action potential profile. The propagation of cardiac action potential (AP) provides the coupling of the electrical and mechanical functions of the heart, which is essential for normal heart operations. The normal orderly contraction of the atria and ventricles requires the rapid and synchronous activation of groups of cardiac cells (Grant, 2009).

The action potential of myocardial cells is characterized by an initial rapid upstroke followed by a plateau of maintained depolarization and the final repolarization. The duration of the action potential determines the time course of the repolarization of the cell. The generation, duration, and shape of the action potential are the results of the selective permeability of ion
channels distributed on the cell membrane. During the action potential, the permeability of ion channels changes, and each ion moves down its electrochemical gradients.

![SA Node Atrium AV Node Purkinje Fiber](image)

![Endocardium Midmyocardium Epicardium](image)

**Figure 1.2 Typical action potential waveforms corresponding to the different regions of the heart.**

The AP is shaped by its underlying ionic currents and varies depending on the species and specific region in the heart, including the sinoatrial node, atria, atrioventricular node, His-Purkinje system, and ventricles (Nerbonne & Kass, 2005; Rahm et al., 2018). The shapes of action potentials in different heart regions are distinct (Figure 1.2). The SA and AV node’s action potentials are brief and activate slowly. The atrial and ventricular action potentials activate fast and are longer. Action potential shape and duration are also species-specific because of differences in the sets of underlying inward and outward ionic currents (Nattel et al., 2007; Liu et al., 2016).

The 5 phases of the typical normal action potential for large mammals are shown in Figure 1.3. (Nerbonne, 2004).

**Phase 4:** Resting potential (-80 to -70 mV). The resting potential in a healthy cardiac myocyte is mainly determined by the inwardly rectifying potassium currents ($I_{K1}$).

**Phase 0:** Rapid depolarization phase. Voltage-gated Na$^+$ channels open to produce the inward fast Na$^+$ current that depolarizes the cell.
Phase 1: Rapid repolarization phase. This phase sets the potential for the next phase of the action potential. Voltage-gated Na\(^+\) channels inactivate, and some voltage-gated K\(^+\) channels (like the transient outward K\(^+\) currents) begin to open.

Phase 2: A plateau phase. This phase marks the entry of Ca\(^{2+}\) into the cell. K\(^+\) efflux continues and is counter-balanced by Ca\(^{2+}\) influx. It is the longest phase and is unique among excitable cells.

Phase 3: Final rapid repolarization. This phase restores the membrane potential to resting value. Voltage-gated Ca\(^{2+}\) channels close, and delayed rectifier voltage-gated K\(^+\) channels open.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{action_potential.png}
\caption{A schematic of typical action potential for human ventricular myocyte.}
\end{figure}

In smaller mammals, like mice and rats, the shape of the action potential and the set of repolarization ionic currents are quite different (Figure 1.4). As in large mammals, the AP starts from phase 4 (resting potential of about -80 to -70 mV). Then it is followed by Phase 0, AP upstroke, generated by the activation of the fast Na\(^+\) current. Phase 1 (initial repolarization) is predominantly defined by the rapidly recovering transient outward K\(^+\) current \(I_{Kto,f}\). Then the action
potential demonstrates a decaying quasi-plateau (that corresponds to Phase 2 and Phase 3 in the large mammals) defined by competition of the inward Ca$^{2+}$ and outward K$^+$ currents. As a result, the murine action potential shows a continuing repolarization phase rather than a distinctive plateau. Finally, the quasi-plateau transfers to phase 4, resting potential.

![Figure 1.4](image-url)

**Figure 1.4 A schematic of typical action potentials for mouse ventricular myocyte.**

In general, the duration of cardiac action potentials increases with body size. The action potential duration for mouse ventricles is about 50 ms (Danik et al., 2002), compared to 250 ms in humans (Edvardsson et al., 1984). The shape of the AP is further characterized by measuring its duration at different levels of repolarizations, 25%, 50%, 75%, and 90%. These AP durations are commonly called APD$_{25}$, APD$_{50}$, APD$_{75}$, and APD$_{90}$. In this dissertation, we will be using APD$_{25}$, APD$_{50}$, APD$_{75}$, and APD$_{90}$ to characterize mouse atrial and ventricular APs.

### 1.1.3 Structural and Electrical Remodeling in Heart Failure

Heart failure (HF) is one of the leading cardiovascular diseases in the world. HF has been defined as “a complex syndrome that results from any structural or functional impairment of
ventricular filling or ejection of blood” (Yancy et al., 2013). Understanding the development and progression of HF requires an in-depth knowledge of the electrophysiological mechanisms in a healthy and diseased cardiac myocyte. Common denominators of heart failure include ion channel remodeling, cellular uncoupling, altered calcium homeostasis, and extracellular matrix changes. Most of the functional changes in a failing heart result from ion channel remodeling in myocytes, i.e., changes in the expression and function of proteins involved in the electrical activity of cardiac myocytes (Coronel et al., 2013).

1.1.3.1 Structural Remodeling

In failing hearts, cardiac myocytes are increased in size (Konstam et al., 2011). This leads to changes in t-tubules and dyadic space structure. It was found experimentally that the volume density and the surface area of junctional sarcoplasmic reticulum (SR) and those of SR-coupled t-tubules were decreased in failing heart cells (Wu et al., 2012). Moreover, the spatial span of individual t-tubule–SR junctions was markedly reduced in failing heart cells (Wu et al., 2012). Some RyRs move away from T-tubules or lose contact with L-type Ca\(^2+\) channels and do not participate in Ca\(^2+\)-induced Ca\(^2+\) release (Song et al., 2006; van Oort et al., 2011). These structural changes impair Ca\(^2+\)-induced Ca\(^2+\)-release, which is responsible for myocyte contraction.

1.1.3.2 Electrical Remodeling

In addition to structural remodeling, failing myocytes demonstrate significant electrical remodeling. The cardiac action potential is generated by the opening and closing of membrane-embedded ion channels, permeable to different ions. The ion channel composition of each cardiac cell contributes to the formation and characteristics of the cardiac action potential that is important for the propagation of electric signals (Bartos et al., 2015). Moreover, ion channel complexes form
the basis for excitation-contraction coupling, including calcium-induced calcium release and mechanical contraction.

Several ion channels are remodeled during the development of heart failure. They were studied in multiple heart failure models that have been developed to assess specific ion channels functions in different animal species (Houser et al., 2012).

**Na⁺ Currents**

The cardiac myocytes depend on a rapid influx of sodium ions (Na⁺) to generate an action potential. The influx of Na⁺ allows the depolarization of myocytes and propagation of the action potential through the myocardial tissues.

Voltage-gated cardiac Na⁺ (Nav) channels open rapidly upon membrane depolarization (Figure 1.3, Figure 1.4) and underlie the rapidly rising phases of the action potentials in excitable cells. Voltage-gated sodium channel Nav1.5 is the main sodium channel expressed in myocardial cells. The SCN5A gene, found on chromosome 3p21, encodes the Naᵥ1.5 channel. Although the properties of the voltage-gated Na⁺ channels expressed in different cardiac cells are similar, these channels' biophysical and pharmacological properties are distinct from Nav channels expressed in other excitable cells, such as neurons and skeletal muscle (Nerbonne & Kass, 2005).

Nav1.5 channels activate at a membrane voltage of about -55 mV, and the activation of these channels is sharply voltage-dependent. The inactivation of Naᵥ channels is also voltage-dependent. The sodium current (I_Na) magnitude is determined by the density of the Naᵥ channels on the plasma membrane and the channels’ opening probability (Dong et al., 2020).

In HF, peak I_Na has been shown to be reduced by posttranslational reduction of the alpha subunit Naᵥ1.5 and other posttranslational modifications (Ufret Vincenty et al. 2001).
Late (Persistent) Na\(^+\) current

In addition to the rapidly opening, persistent openings of cardiac Nav channels are occasionally observed at depolarized membrane potentials and action potential plateau (Figure 1.3). The inward Na\(^+\) current through open Na\(_{\text{v}}\) channels during the action potential plateau (phase 2) will counter the effects of the increased K\(^+\) efflux, thereby slowing or delaying repolarization and increasing action potential duration. The late sodium current \(I_{\text{Na,L}}\) of a normal myocyte is generally small, less than 0.1\% of the peak \(I_{\text{Na}}\), but it may be much higher (more than three-fold) in a diseased myocyte (Kistamás et al., 2021; Berecki et al., 2006). Therefore, abnormalities in the inactivation of Na\(^+\) channels lead to a persistent late sodium current that causes the prolongation of APD and early afterdepolarizations (EADs) (Ufret-Vincenty et al., 2001; Yu et al., 2018).

The late Na\(^+\) current \(I_{\text{Na,L}}\) is increased in several models of HF (Undrovinas et al., 2010; Maltsev & Undrovinas, 2008; Valdivia et al., 2005). Increased late \(I_{\text{Na}}\) opposes repolarizing currents and lengthens the AP. An increase in late \(I_{\text{Na}}\) in pathological states also increases Na\(^+\) loading adequately to raise intracellular Na\(^+\). Increased intracellular Na\(^+\) will cause increased intracellular Ca\(^{2+}\) because less energy is available in the Na\(^+\) gradient to extrude Ca\(^{2+}\) through Na\(^+\)/Ca\(^{2+}\) exchange (Bers, 2002; Pourrier et al., 2014).

K\(^+\) Currents

In cardiomyocytes, potassium channels are the most abundant and important ion channels dictating the cell’s repolarization. They regulate the resting membrane potential, the frequency of pacemaker cells, and the shape and duration of the cardiac action potential. The driving force for K\(^+\) efflux is high during the plateau phase of the action potential in ventricular and atrial myocytes. As the Ca\(^{2+}\) channels inactivate, the outward K\(^+\) currents predominate, resulting in repolarization, bringing the membrane voltage back to its resting potential (Grandi et al., 2017).
There are diverse K⁺ channel types with distinct kinetic and voltage-dependent properties. Cardiac K⁺ currents can be distinguished based on differences in their functional and pharmacological properties. They include the rapidly activating and inactivating transient outward current (\(I_{Kto}\)), the ultra-rapidly activating (\(I_{Kur}\)), rapidly activating (\(I_{Kr}\)), and slow (\(I_{Ks}\)) components of the delayed rectifier, and the inward rectifier (\(I_{K1}\)). The activation time constants of the currents change in a quite large interval, from fractions of a millisecond to hundreds of milliseconds (Tamargo et al., 2004). The inactivation time constants of K⁺ currents also have a large range, from tens milliseconds to seconds (Xu & Nerbonne, 1999; Zhou et al., 2003).

Voltage-gated K⁺ currents in myocytes from different species and heart regions have the same time- and voltage-dependent properties. This suggests that the same (or very similar) molecular entities contribute to the generation of each of the various types of voltage-gated K⁺ channels in different heart regions and species. However, the kinetics of repolarization varies significantly with the cardiac region and species reflecting the variations in relative levels of K⁺ channel expressions. This heterogeneity contributes considerably to the observed regional and species-wide differences in action potential forms. Changes in the properties or the functional expression of voltage-gated K⁺ channels, as occurs in various myocardial diseases, can, therefore, have notable effects on action potential waveforms, propagation, and rhythmicity (Nerbonne & Kass, 2005).

Experimental studies show an increase in the whole-cell membrane capacitance (\(C_m\)) and a decrease in the \(I_{Kto,f}\), \(I_{K,slow}\), \(I_{Kss}\), and \(I_{K1}\) densities in Transverse Aortic Constriction (TAC) as compared to sham LV myocytes. In addition, a decrease in the amplitude of \(I_{K,slow}\), and an increase in the amplitude of \(I_{Kss}\) was reported in several TAC models (Marionneau et al., 2008; Tamayo et al., 2020). The inward rectifying potassium current (\(I_{K1}\)) stabilizes the resting membrane potential
and is responsible for shaping the action potential’s initial depolarization and final repolarization phases. Studies indicate a decrease in the inwardly rectifying K\(^+\) currents \(I_{K1}\) in TAC LV myocytes (Marionneau et al., 2008; Nattel, 2008; Näbauer & Kääb, 1998).

**Ca\(^{2+}\) Currents**

In cardiac muscle, two types of Ca\(^{2+}\) channels, the L-type (low threshold type) and T-type (transient-type), transport Ca\(^{2+}\) into the cells. The L-type channels are found in all cardiac cell types. The T-type channel is found principally in the pacemaker, atrial, and Purkinje cells (Grant, 2009).

The L-type cardiac calcium channel Ca\(_{v}1.2\) plays a critical role in excitation-contraction coupling and action potential duration. The Ca\(^{2+}\) ions entering the cardiomyocytes through the Ca\(_{v}1.2\) channels shape the ventricular action potential plateau phase. The L-type Ca\(^{2+}\) channels are also required for contraction by inducing calcium release from the sarcoplasmic reticulum. A small influx of Ca\(^{2+}\) through Ca\(_{v}1.2\) triggers the ryanodine receptor (RyR) opening and the release of Ca\(^{2+}\) ions from the SR. Phosphorylation of RyR channels increases their permeability to calcium and increases the contractility of their respective cardiac myocytes (Shaw & Colecraft, 2013). In addition, the Ca\(^{2+}\) ion channel properties can be regulated by drugs and toxins that bind the channel pore, the voltage sensor, or the C terminus.

In general, there are no significant functional abnormalities in the basic properties of \(I_{CaL}\) in myocytes isolated from failing human and mouse hearts (Kamp & He, 2002). Available data indicate that \(I_{CaL}\) density may be increased in mild hypertrophy but unchanged in moderate degrees of hypertrophy and at the early stages of heart failure (Toischer et al., 2010; Li et al., 2013; Tamayo et al., 2020).
1.1.4 Calcium Handling in Heart Failure

The influx of Ca\(^{2+}\) into the cytosol as Ca\(^{2+}\) current (\(I_{Ca}\)) and the release of Ca\(^{2+}\) from the sarcoplasmic reticulum are central to the excitation-contraction coupling (Fig. 1.5). The triggering Ca\(^{2+}\) current (\(I_{Ca}\)) is generated by the L-type Ca\(^{2+}\) channels expressed in the transverse tubules (T-tubules) and is initiated by membrane depolarization. The excitation-contraction coupling is achieved by increasing cytosolic [Ca\(^{2+}\)]\(_i\) levels from nanomolar (100 nmol/L) to micromolar (10 \(\mu\)mol/L) (Walker and Spinale, 1999). Precise regulation of Ca\(^{2+}\) is vital for the proper myocyte function (Bers, 2000). Depolarization of the myocyte triggers the entry of small amounts of Ca\(^{2+}\) through the L-type Ca\(^{2+}\) channels located on the cell membrane, which in turn prompts SR Ca\(^{2+}\) release through ryanodine receptors, a process known as calcium-induced calcium release. The rapid rise in cytosolic levels activates Ca\(^{2+}\)–troponin-C interactions and induces sarcomere contraction. For relaxation to occur, Ca\(^{2+}\) must dissociate from troponin C, which requires a reduction in cytosolic Ca\(^{2+}\). The activity of the sarcoplasmic reticulum Ca\(^{2+}\)-ATPase (SERCA), an ATP-dependent calcium pump, returns cytosolic Ca\(^{2+}\) into the SR to restore the background Ca\(^{2+}\) level. In addition to the SERCA pump, Ca\(^{2+}\) extrudes the cytosol via the sarcolemmal Na\(^+\)/Ca\(^{2+}\) exchanger (NCX) and Ca\(^{2+}\) pump.

The two major factors determining the rate, strength, and duration of myocyte contraction are the amount and rate of Ca\(^{2+}\) delivered to and removed from the cytoplasm. The SR Ca\(^{2+}\) load is essential in determining the amount of SR Ca\(^{2+}\) released in response to each inward Ca\(^{2+}\) current.
Alterations in myocyte ion currents and Ca\(^{2+}\) handling proteins can contribute to heart failure by causing structural remodeling and ventricular hypertrophy. Studies indicate that SR Ca\(^{2+}\) load is reduced in heart failure (Hobai & O'Rourke, 2001; Lindner et al., 1998). Moreover, the Ca\(^{2+}\) transient from failing hearts exhibits a reduced rate of Ca\(^{2+}\) removal from the cytosol (Tamayo et al., 2020). A failing human heart also shows increased resting intracellular Ca\(^{2+}\) level, leading to diastolic dysfunction (Beuckelmann et al., 1992; Eisner et al., 2020).

**Sarcoplasmic Reticulum Ca\(^{2+}\)-ATPase (SERCA)**

To restore the myocardial relaxation, Ca\(^{2+}\) is mainly returned to intracellular stores by the cardiac isoform sarcoplasmic reticulum Ca\(^{2+}\) ATPase 2a (SERCA2a) and into the extracellular space by the Na\(^+\)/Ca\(^{2+}\) exchanger. In humans and other bigger animals, 70-80\% of the systolic Ca\(^{2+}\) is removed from the cytosol by SERCA and 20-30\% by Na\(^+\)/Ca\(^{2+}\) exchanger (Frank et al., 2003).
In contrast, SERCA plays a more predominant role in rat and mouse cardiac myocytes with an approximately 90% contribution to removing Ca\(^{2+}\) and about 7-10% by Na\(^+\)/Ca\(^{2+}\) exchanger. Upon sarcoplasmic resequestration, Ca\(^{2+}\) binds mainly to the SR Ca\(^{2+}\) storage protein, calsequestrin, and other Ca\(^{2+}\)-binding proteins, such as calreticulin and the histidine-rich-binding proteins (Bers, 2002).

In both experimental and clinical heart failure, SERCA2a expression is significantly reduced, leading to abnormal Ca\(^{2+}\) handling and impaired contractility. Inefficient SR Ca\(^{2+}\) uptake in myocytes is associated with a decrease in the expression and activity of the SERCA2a (Lipskaia et al., 2013; del Monte & Hajjar, 2008).

**Phospholamban (PLN)**

PLN is an SR protein that regulates the activity of SERCA. In the dephosphorylated form, PLN decreases the apparent Ca\(^{2+}\)-affinity of SERCA2a. The inhibitory effect of PLN on SERCA is removed because of the dissociation of the PLN/SERCA2a complex upon phosphorylation of PLN through \(\beta\)-adrenergic stimulation and enhanced cyclic adenosine monophosphate (cAMP) - dependent protein kinase A (PKA) activity (Frank & Kranias, 2000; Maier & Bers, 2007). The PLN levels show no change in a mouse failing myocyte (Vinet et al., 2012).

**Calsequestrin**

Calsequestrin is the major calcium-binding protein in the sarcoplasmic reticulum. It is the most abundant Ca\(^{2+}\) binding protein in the junctional SR of cardiac muscle. Experiments on mice show that removing the cardiac variant calsequestrin (Casq2) causes a steep increase in SR Ca\(^{2+}\) release supporting the regulatory effect of Casq2. An increase in \([\text{Ca}^{2+}]_{\text{SR}}\) also increases RyR open probability resulting in a much higher likelihood of SR Ca\(^{2+}\) release. (Lukyanenko et al. 1996;
Gyorke & Gyorke, 1998; Györke et al., 2009). Experimental data on failing mouse ventricular myocytes show no change in the expression of calsequestrin (Vinet et al., 2012).

1.1.5 The β-adrenergic Pathway

The β-adrenergic receptors (β-ARs) are G-protein coupled receptors located on the sarcolemma. Two major subtypes of β-ARs (β₁-AR, β₂-AR) have been identified in cardiac cells. Each subtype has different affinities for different ligands, which allow independent activation of each subtype. While β₂-AR is coupled to both Gₛ and Gᵢ proteins in cardiomyocytes, β₁ is associated only with Gₛ proteins. The β₁ subtype is the most prominent in cardiac cells. In normal human cardiac myocytes, there is approximately a 4:1 ratio of β₁-AR to β₂-AR (de Lucia & Koch, 2018; Lohse et al., 2003; Zhu et al., 2011). In mice, the β₁-AR to β₂-AR ratio is 2:1 (Hilal-Dandan et al., 2000).

By stimulating the β-adrenergic pathways, the sympathetic nervous system can significantly enhance the Ca²⁺ circulation mechanisms and hence the excitation and contraction of cardiomyocytes. In the cardiomyocyte’s sarcolemma, agonists such as epinephrine and norepinephrine bind to β₁- or β₂-adrenergic receptors, ultimately leading to the phosphorylation of Nav1.5, Cav1.2, RyR, PLN, and other proteins. The binding of agonists to the β-adrenergic receptors activates adenylyl cyclase (AC), resulting in cAMP production from the hydrolysis of ATP. The cAMP acts as a second messenger to activate protein kinase A (cAMP-dependent protein kinase), which can phosphorylate different targets within the cell (Wachter et al., 2012; Lohse et al., 2003; Woo & Xiao, 2012).

PKA plays multiple roles in heart function regulation, including contraction, metabolism, ion fluxes, and gene transcription. PKA is a protein comprised of two catalytic (PKA-C) and two regulatory (PKA-R) subunits (Wachter & Gilbert, 2012). The binding of cAMP to the PKA-R
subunits induces the release of PKA-C, allowing the phosphorylation of many target proteins involved in the excitation-contraction (E–C) coupling mechanism, such as cardiac troponin I (cTnI), cardiac myosin binding protein C (cMyBPC), phospholamban (PLB), L-type calcium channel, phosphodiesterase (PDE), cAMP-response element-binding protein (CREB), and the ryanodine receptors to modify their function. PKA phosphorylation increases the RyR and SERCA activities, causing a faster opening and closing of the channels. The simultaneous increase of RyR and SERCA activities from PKA phosphorylated PLN results in overall hastened SR Ca²⁺ release and uptake (Marks, 2013; Najafi et al., 2016).

Thus, the net effect of β-adrenergic stimulation is to increase the overall amplitude of Ca²⁺ transients by accelerating the cardiomyocyte's Ca²⁺ channels and pumps, Ca²⁺ transport activities. Isoproterenol, a β-AR agonist, increases the strength of the cardiac contraction by increasing the level of myocardial cAMP. cAMP levels are tightly regulated by the cyclic nucleotide degrading enzymes phosphodiesterases (PDEs). PDEs break down cyclic nucleotide phosphodiesterases, such as cAMP and cGMP. PDE isoforms, PDE4 and PDE8, are specific for cAMP degradation, while PDE1, PDE2, and PDE3 can hydrolyze cAMP and cGMP (Najafi et al., 2016).

The activation of the inhibitory G-protein (G_i-protein) inhibits the β-adrenergic signaling system. Therefore, the stimulation of this pathway inactivates PKA, decreases Ca²⁺ entry into the cell, and Ca²⁺ release by the sarcoplasmic reticulum (Klabunde, 2012).

Researches suggest that the β-adrenergic receptor system plays a major role in heart failure. Studies indicate a reduction of the β₁-AR density on the plasma membrane from a failing heart by about 50%. In contrast, β₂-AR shows no such change in the failing heart. (Madamanchi, 2007; Bristow et al., 1989; Woo & Xiao, 2012)
1.2 Mathematical Models of Mouse Atrial Myocytes

Mathematical modeling of cardiac myocytes and tissues is complementary to the experimental investigations and a potent tool for studying cardiac electrical activity, intracellular Ca\(^{2+}\) dynamics, protein signaling networks, pathological remodeling, and arrhythmogenesis. Multiple mathematical models have been developed to describe experimentally observed behavior of cardiac myocytes from different heart regions, such as ventricles, atria, sino-atrial node, Purkinje fibers, and others (Bondarenko et al., 2004; Demir et al., 1994; Jafri et al., 1998; Nygren et al., 1998; Vaidyanathan et al., 2013). The majority of the mathematical models are species-specific; they describe the behavior of cardiac cells from the human, dog, rabbit, guinea pig, rat, and mouse hearts.

Prevailing fractions of mathematical models have been developed for ventricular myocytes, which allowed the simulations of action potential and [Ca\(^{2+}\)]\(_i\) transients. These models have also simulated the frequency dependence and role of AP and [Ca\(^{2+}\)]\(_i\) transients in pro-arrhythmic activity (see, for example, (Bondarenko et al., 2004; Grandi et al., 2010; Jafri et al., 1998; Luo & Rudy, 1994)). Less attention has been given to the atrial cardiomyocytes, perhaps, due to less severe consequences of atrial arrhythmias and/or because of the more complex atrial structure and dynamics. However, recent experimental focus has turned to the heart atria and atrial cells (Heijman et al., 2016), which in turn demanded comprehensive models of atrial myocytes.

More than 300 computational studies have used mathematical models for the atrial myocyte of different species to simulate atrial electrophysiology, Ca\(^{2+}\) dynamics, arrhythmias, and therapeutic interventions (Heijman et al., 2016). Initially, they focused on a different shape of atrial AP as compared to that in ventricular myocytes and were developed for various species, including rabbits, dogs, and humans (Courtemanche et al., 1998; Hilgemann & Noble, 1987; Nygren et al.,
1998; Ramirez et al., 2000). Later modeling studies have been devoted to the differences in Ca\(^{2+}\) dynamics between atrial and ventricular myocytes (see (Heijman et al., 2016) for a comprehensive review), including compartmentalized Ca\(^{2+}\) signaling and coherent local control of Ca\(^{2+}\)-induced Ca\(^{2+}\) release (Jafri et al., 1998). The modeling of atrial myocytes is also complicated by their complex structure. Atria consists of both myocytes with transverse-axial tubule system (TATS) and those that do not have TATS (Glukhov, Balycheva, et al., 2015; Lenaerts et al., 2009). In addition, atrial and ventricular myocytes demonstrate different expressions of Ca\(^{2+}\) handling proteins (Gaborit et al., 2007; Lüss et al., 1999), which also need to be considered in mathematical modeling.

Most of the mathematical atrial cell models have been developed for human myocytes (Courtemanche et al., 1998; Koivumäki et al., 2011; Grandi et al., 2011; Nygren et al., 1998). There also exist detailed models for rabbits (Hilgemann & Noble, 1987; Lindblad, 1996), bullfrogs (Rasmusson et al., 1990), and canine (Kneller et al., 2002; Ramirez et al., 2000) atrial myocytes. For mice, an atrial AP model by Davis et al. (2014) was modified from a ventricular cell model (Bondarenko et al., 2004), where the ventricular ion currents were adjusted to reproduce atrial AP. However, the model by Davis et al. (2014) did not include atrial-specific ionic currents, namely the T-type Ca\(^{2+}\) current (\(I_{CaT}\)) and the small-conductance Ca\(^{2+}\)-activated K\(^+\) current (\(I_{K,Ca}\)), and lacked the vital differences in Ca\(^{2+}\) dynamics between atrial and ventricular myocytes.

In this study, we developed a comprehensive compartmentalized mathematical model of mouse atrial myocytes. The model was designed for a subpopulation of right atrial myocytes with developed TATS that was observed experimentally in a substantial fraction (23-84%) of mouse atrial myocytes (Yue et al., 2017). Importantly, our new model 1) has a revised cell geometry, 2) includes atrial-specific ion currents \(I_{CaT}\) and \(I_{K,Ca}\), and 3) formulates atrial AP, Ca\(^{2+}\) dynamics, and
\[ \beta \text{-adrenergic signaling system in a compartmentalized manner which is based on a rigorous review of the latest experimental studies on mouse atrial myocytes (Fig. 2.1). We also reformulated the outward K}^+ \text{ currents by separating } I_{Kur} \text{ into } I_{K,\text{slow}1}, \text{ and } I_{K,\text{slow}2} \text{ due to their differential response to } \beta \text{-adrenergic stimulation and made several other model adjustments to fit experimental data. The resulting model was able to generate, with high accuracy, the mouse’s right atrial AP, [Ca}^{2+}]_i \text{ and [Na}^+]_i \text{ dynamics, and their frequency dependences. It also recapitulates voltage-clamp experiments for the } I_{Ca,T} \text{ and the } I_{K,Ca} \text{ currents and estimate their impact on AP and [Ca}^{2+}]_i \text{ generation at baseline and under } \beta \text{-adrenergic stimulation. We also showed the differences in the AP, [Ca}^{2+}]_i \text{ transients, and Ca}^{2+} \text{ fluxes in mouse atrial and ventricular myocytes models. Finally, a sensitivity analysis of the model was performed to demonstrate the stability of the model with respect to perturbations due to the cell-to-cell variability of the ionic currents within the mouse right atria.} \]

1.3 Mathematical Models of Heart Failure

Heart failure is a complex cardiac disease that manifests in a significantly reduced heart fractional shortening and ejection fraction, generally due to decreased intracellular calcium transients ([Ca}^{2+}]_i) and myocyte contractions. Heart failure is considered as the final common pathway of many cardiac pathologies (Coronel et al., 2013). At the cellular level, heart failure progression results in structural and functional remodeling that affects cell structure, properties of ionic currents and transporters, and cellular signaling systems (Tomaselli & Marban, 1999; Nattel et al., 2007). The failing cardiac myocytes increase in size, which is accompanied by adaptive and maladaptive changes in t-tubules structure, sarcoplasmic reticulum size, dyadic space, and contractile protein expression (Schaper et al., 2002; Li et al., 2013). In addition, cardiac myocytes from failing hearts exhibit the prolongation of the action potential, resulting from the destruction of the delicate balance between the depolarizing and repolarizing currents. In most cases, failing
myocytes show a decrease in the fast Na\(^+\) current \((I_{Na})\), the transient outward K\(^+\) currents \((I_{Kto,f} \text{ and } I_{Kto,s})\), the time-independent inwardly rectifying K\(^+\) current \((I_{K1})\), and the slow delayed-rectifier K\(^+\) current \((I_{Ks})\); no change in the rapid delayed-rectifier K\(^+\) current \((I_{Kr})\) and the L-type Ca\(^2+\) current \((I_{CaL})\); and an increase in the late Na\(^+\) current \((I_{Nat})\) and the Na\(^+\)/Ca\(^2+\) exchanger current \((I_{NaCa})\) \((\text{Tomaselli & Marban, 1999; Nattel et al., 2007; Rivaud et al. 2017})\). Further, the Ca\(^2+\) dynamics was found to be impaired, in the failing myocytes, due to a reduction in the function of the SERCA2a pump, an increase in the functions of the Na\(^+\)/Ca\(^2+\) exchanger, and an increase in Ca\(^2+\) leak from the sarcoplasmic reticulum \((\text{Nattel et al., 2007})\). Finally, the modified β₁- and β₂-adrenergic signaling in the failing hearts resulted in blunted response to adrenergic stimulation \((\text{Woo & Xiao, 2012})\).

Multiple experimental models of heart failure have been investigated in different species, including humans \((\text{Zaragoza et al., 2011; Riehle & Bauersachs, 2019; Spannbauer et al., 2019})\). The mouse is the most studied species used in heart failure investigations. Several models of heart failure have been developed for mice that include genetic modifications \((\text{van Oort et al., 2011})\), myocardial infarction \((\text{Yoo et al., 2009})\), and transverse aortic constriction (TAC) \((\text{Li et al., 2013; Ljubojevic-Holzer et al., 2020})\), etc. Specifically, experimental models of mouse heart failure using TAC demonstrate structural remodeling, the changes in properties of ionic currents and transporters, and cellular signaling systems similar to those seen in larger animals and humans \((\text{Nienaber et al., 2003; Fabritz et al., 2010; Li et al., 2013; Bryant et al., 2018; Ljubojevic-Holzer et al., 2020})\).

In addition to the common features of heart failure, such as action potential prolongation, reduced [Ca\(^{2+}\)]; transient, reduced sarcoplasmic reticulum Ca\(^{2+}\) load, reduced transient outward K\(^+\) currents \((I_{Kto,f} \text{ and/or } I_{Kto,s})\), and increased \(I_{NaCa}\), experimental investigations also resulted in several
controversial findings (Tomaselli & Marban, 1999; Nattel et al., 2007). The data on modifications of the SERCA pump demonstrates a decrease in function and mRNA expression but a reduction or no change in corresponding protein expression (Nattel et al., 2007). This finding is closely related to the alterations of phospholamban expression, in which mRNA is reduced, but protein expression demonstrates a reduction or no change. There are also different interpretations of the role of the SERCA pump, the Na+/Ca²⁺ exchanger, and the ryanodine receptor leak in the reduction of the SR Ca²⁺ load (Bers et al., 2003).

Several mathematical models have been developed to describe failing cardiomyocytes in different species (see (Gomez et al., 2015) for review) to address the issues encountered in the experimental investigations of failing hearts. The first mathematical model of the failing myocytes was developed by Priebe and Beuckelmann for the human ventricle (1998). In their model, the authors considered both normal and failing human ventricular myocytes. In the failing myocytes, they changed several model parameters as compared to the normal myocytes: reduced the transient outward K⁺ current, the inward-rectifier K⁺ current, the Na⁺-K⁺ pump, and SERCA pump function, but increased the Na⁺/Ca²⁺ exchanger function and Ca²⁺ leak, and did not change the delayed rectifier currents $I_{Kr}$ and $I_{Ks}$ and cell geometry. The model simulated prolongation of the action potential and a reduction of [Ca²⁺] transient, which have been observed experimentally. In addition, Priebe and Beuckelmann have shown larger susceptibility of the failing human ventricular myocytes to early afterdepolarizations (EAD) (Priebe and Beuckelmann, 1998). A similar approach, with some changes in the model's current parameters, has been applied in the mathematical model for the failing canine ventricular myocytes, developed by Winslow et al. (1999). They simulated the prolongation of the action potential and the reduction of [Ca²⁺] transient in the failing myocytes and estimated the differences in the Na⁺/Ca²⁺ exchanger and
SERCA pump function between normal and failing myocytes. Yet another mathematical model of failing rabbit ventricular myocyte has been explored by Shannon et al. (2005). It was found that the [Ca\textsuperscript{2+}]\textsubscript{i} transient and the SR load were reduced in the failing myocytes. In addition, failing rabbit ventricular myocytes had more depolarized resting potential and were more susceptible to delayed afterdepolarizations (DAD). A mathematical model of failing mouse ventricular myocytes overexpressing TNF-\(\alpha\) has also been investigated by Petkova-Kirova et al. (2012). The model simulated action potential prolongation and [Ca\textsuperscript{2+}]\textsubscript{i} transient reduction that resulted in the instability of the action potential and Ca\textsuperscript{2+} dynamics in the failing myocytes at lower stimulation frequencies. At the cardiac tissues level, the heart failure model by Petkova-Kirova et al. (2012) demonstrated higher susceptibility to the action potential block, Ca\textsuperscript{2+} alternans, and re-entry.

More recently, attention was paid to the structural changes and their possible significant effects in mathematical models of failing cardiac cells (Polakova & Sobie, 2013). It was suggested by Polakova & Sobie (2013) that the models should consider the changes in t-tubule and dyadic space structure that occurs upon the development of heart failure, which plays an important role in Ca\textsuperscript{2+} homeostasis. Later, the role of re-distribution of the L-type Ca\textsuperscript{2+} channels on the development of EADs in failing human ventricular myocytes was modeled by Sanchez-Alonso et al. (2016). It was demonstrated that redistribution of the L-type Ca\textsuperscript{2+} channels from t-tubules to crest regions in the failing myocytes promoted EADs. In addition, sensitivity analysis of the human failing myocyte model (Mora et al., 2017) revealed the important contribution of the SERCA pump dysfunction to the altered Ca\textsuperscript{2+} homeostasis, with the Na\textsuperscript{+}/Ca\textsuperscript{2+} exchanger and other Ca\textsuperscript{2+} cycling proteins also playing a significant role.

Finally, a compartmentalized mathematical model of human failing ventricular myocytes that includes modifications of the \(\beta\)-adrenergic signaling system in addition to alteration of the
ionic currents and transporters has been developed (Mora et al., 2021). The model simulated multiple features of the human failing ventricular cells, such as the prolongation of the action potential, reduced [Ca^{2+}]_i transients, and blunted response of the failing myocytes to adrenergic stimulation.

While multiple mathematical models of the failing ventricular myocytes for several species have been developed, they did not consider the increase in the myocyte size that was consistently observed in the failing myocytes. In addition, despite multiple experimental heart failure models developed for mice, there is no comprehensive mathematical model of the failing mouse ventricular myocytes that could be used to interpret the observed experimental data. In this study, we developed a comprehensive, experimentally verified model of the failing mouse ventricular myocytes. It is specifically developed for the mathematical description of failing mouse ventricular myocytes data obtained after transverse aortic constriction, and those experimental data were used for model verification. As a basis, we used mathematical models of mouse ventricular myocytes developed previously (Bondarenko et al., 2004; Petkova-Kirova et al., 2012; Bondarenko, 2014; Rozier and Bondarenko, 2017). In the new model, we redesigned the cells geometry based on the experimental findings from the failing mouse ventricular myocytes, reformulated the set of outward K^+ currents by separating I_{kur} into I_{K,slow1} and I_{K,slow2} due to their differential response to β-adrenergic stimulation, added late Na^+ current due to its possible important role in heart failure, and made several other model adjustments to fit the experimental data. The resulting model was able to generate, with high accuracy, the mouse’s left ventricular AP, [Ca^{2+}]_i, and [Na^+]_i dynamics and their frequency dependences under normal and diseased conditions. We also showed the differences in the AP, [Ca^{2+}]_i transients, and Ca^{2+} and Na^+ fluxes in the models of normal and failing mouse ventricular myocytes. The model revealed the major players in the sarcoplasmic
reticulum Ca\(^{2+}\) load in failing ventricular myocytes. We also described a proarrhythmic behavior of Ca\(^{2+}\) dynamics upon stimulation with isoproterenol and mechanisms of the proarrhythmic behavior suppression. Finally, we performed a sensitivity analysis of the model.
2 A COMPARTMENTALIZED MATHEMATICAL MODEL OF MOUSE ATRIAL MYOCYTES

2.1 Materials and Methods

2.1.1 Model Development

Our mathematical model of mouse atrial myocytes is based on the previously published models of mouse ventricular myocytes (Bondarenko, 2014; Bondarenko, 2004; Petkova-Kirova et al., 2012; Rozier and Bondarenko, 2017). Recent experimental findings from mouse atria and other species revealed myocytes both with and without TATS (Brandenburg et al., 2018; Yue et al., 2017). Here, we developed a model for a subpopulation of right atrial myocytes with a developed transverse-axial tubule system. Our model assumptions are based on the comprehensive experimental study by Yue et al. (2017), who found that about 23% of atrial myocytes possessed TATS with a well-organized spacing of transverse tubules, similar to those in mouse ventricular myocytes. The authors also found that 162 out of 193 atrial myocytes (84%) contained identifiable TATS with varying degrees of TATS organization. In addition, Yue et al. (2017) have demonstrated “near-synchronous” Ca$^{2+}$ transients in atrial myocytes (61 atrial cells, 6 mice), without significant differences between time-to-peaks of Ca$^{2+}$ transient in the subsarcolemmal space, in the cell center, or the global (cellular) Ca$^{2+}$ transient. Therefore in our modeling, we assumed that the Ca$^{2+}$-induced Ca$^{2+}$ release mechanism for a subpopulation of mouse atrial myocytes with developed TATS, which represent somewhere between 23% and 84% of all mouse atrial myocytes, is similar to or close to that described for mouse ventricular myocytes.
Figure 2.1 A schematic representation of the mouse right atrial myocyte. The cell consists of three compartments (caveolae, extracaveolae, and cytosol) related to the combined β₁- and β₂-adrenergic signaling systems. The subspace volume ($V_{ss}$) is localized in the extracaveolar compartment. The biochemical portions of the combined β₁- and β₂-adrenergic signaling systems are the β₁-adrenergic receptors ($\beta_1$-AR), the β₂-adrenergic receptors ($\beta_2$-AR), the α-subunit of stimulatory $G$-protein ($G_{s\alpha}$), the α-subunit of inhibitory $G$-protein ($G_{i\alpha}$), the βγ-subunit of $G_s$ and $G_i$ ($G_{\beta\gamma}$), the adenylyl cyclases of type 5/6 or 4/7 (AC5/6 or AC4/7, respectively), the phosphodiesterases of type 2, 3, or 4 (PDE2, PDE3, or PDE4, respectively), the cyclic AMP (cAMP), regulatory (R) and catalytic (C) subunits of protein kinase A holoenzyme, the protein kinase A inhibitor (PKI), the G-protein-coupled receptor kinase of type 2 (GRK2), the protein phosphatases of type 1 and 2A (PP1 and PP2A, respectively), the inhibitor-1 (I-1). Targets of the combined β₁- and β₂-adrenergic signaling systems are in the caveolae (including the fast $Na^+$ current ($I_{Na}$), the $L$-type $Ca^{2+}$ current ($I_{Ca,L,cav}$), the $T$-type $Ca^{2+}$ current ($I_{Ca,T,cav}$), the sarcolemmal $Ca^{2+}$ pump ($I_{p(Ca)}$, also known as plasmalemmal $Ca^{2+}$-ATPase, PMCA), the $Na^+/Ca^{2+}$
exchanger (INaCa), the rapid delayed rectifier K⁺ current (IKr), the non-inactivating steady-state voltage-activated K⁺ current (IKss), and the Ca²⁺ and Na⁺ background currents (ICab and INab), which are not affected by the combined β1- and β2-adrenergic signaling systems. The Ca²⁺ fluxes are uptake of Ca²⁺ from the cytosol to the network sarcoplasmic reticulum (NSR) (Ju) by the SERCA pump and Ca²⁺ release from the junctional sarcoplasmic reticulum (JSR) (Jrel) through the RyRs. [Ca²⁺]i, [Na⁺]i, and [K⁺]i are the intracellular Ca²⁺, Na⁺, and K⁺ concentrations in the caveolae, extracaveolae, and cytosol; [Ca²⁺]o, [Na⁺]o, and [K⁺]o are the extracellular Ca²⁺, Na⁺, and K⁺ concentrations. Modified from (Bondarenko, 2014).

We also made several principal changes in our mouse ventricular myocyte model to ensure that the new model gives a comprehensive description of mouse atrial electrophysiology, which is currently unavailable. We changed the cell geometry and added two major ionic currents: T-type Ca²⁺ current, ICa,T, and small-conductance Ca²⁺-activated K⁺ current, IK,Ca. The magnitude of the rapid delayed rectifier current, IKr, was increased by a factor of 10 as compared to the mouse ventricular myocyte, according to the experimental data by Nakamura et al. (2010). However, there was no marked contribution to the AP shape in simulations, as it was also observed in Vaidyanathan et al. (2013). We also changed the formulation of the time-independent K⁺ current, IK1, to fit the experimental data from mouse atrial myocytes. The ultra-rapidly activating K⁺ current, IKur, was reformulated to include two components, IK,sow1 (encoded by Kv1.5 channels and also referred to as IKur component) and IK,sow2 (encoded by Kv2.1 channels) that demonstrate different responses to the activation of β1- and β2-adrenergic signaling systems (Kodirov et al., 2004; Wilson et al., 1994; Zhou et al., 2003; Zhou et al., 2012). The magnitudes of the L-type Ca²⁺ current, ICa,L, and several K⁺ currents, were changed to fit the experimental values obtained from mouse atrial myocyte. The following differential equation describes the resulting model’s action potential:

\[
\frac{dV}{dt} = -\frac{1}{C_m}(ICaL + ICaT + IP(Ca) + INaCa + ICab + INa + INab + INaK + IKto,f + IK1 + IK,sow1 + IK,sow2 + IKss + IK,Ca + IKr + ICi,Ca - I_{stim}),
\]
where $I_{\text{CaL}}$ is the L-type Ca\textsuperscript{2+} current, $I_{\text{CaT}}$ is the T-type Ca\textsuperscript{2+} current, $I_{p(\text{Ca})}$ is the sarcolemmal Ca\textsuperscript{2+} pump, $I_{\text{NaCa}}$ is the current from the Na\textsuperscript{+}/Ca\textsuperscript{2+} exchanger, $I_{\text{Cab}}$ is the Ca\textsuperscript{2+} background current, $I_{\text{Na}}$ is the fast Na\textsuperscript{+} current, $I_{\text{Nab}}$ is the Na\textsuperscript{+} background current, $I_{\text{NaK}}$ is the current of the Na\textsuperscript{+}-K\textsuperscript{+} pump, $I_{\text{Kto,f}}$ is the rapidly recovering transient outward K\textsuperscript{+} current, $I_{\text{K1}}$ is the time-independent K\textsuperscript{+} current, $I_{\text{K,slow1}}$ is the rapidly activating, slowly inactivating K\textsuperscript{+} current (encoded by K\textsubscript{v}1.5; 4-aminopyridine-sensitive; also named as $I_{\text{Kur}}$, the ultra-rapidly activating delayed rectifier K\textsuperscript{+} current), $I_{\text{K,slow2}}$ is the rapidly activating, slowly inactivating K\textsuperscript{+} current (encoded by K\textsubscript{v}2.1; TEA-sensitive), $I_{\text{Kss}}$ is the noninactivating steady-state voltage-activated K\textsuperscript{+} current, $I_{\text{K,Ca}}$ is the small-conductance Ca\textsuperscript{2+}-activated K\textsuperscript{+} current, $I_{\text{Kr}}$ is the rapid delayed rectifier K\textsuperscript{+} current, $I_{\text{Cl,Ca}}$ is the Ca\textsuperscript{2+}-activated chloride current, and $I_{\text{stim}}$ is the stimulus current.

2.1.2 Cell Geometry and Cell Compartments

Structurally, mouse atrial myocytes differ from ventricular myocytes by the cell size and the volume of subcellular compartments. To deal with these differences, the atrial myocyte’s length and diameter were set to 110 µm and 16 µm, respectively, according to the experimental data by Brandenburg et al. (2016), which resulted in cell volume of $V_{\text{cell}} = 22.1200 \times 10^{-6}$ µl. The geometric surface area of atrial myocytes was calculated to be equal to $0.5931 \times 10^{-4}$ cm\textsuperscript{2}, and it was the same as their capacitive surface area, in contrast to ventricular myocytes, where capacitive surface area is about 40-50% larger than their geometric surface area (Giles et al., 1988; Hume et al., 1985).

Mouse atrial myocytes also show different sizes of intracellular compartments as a percentage of the total cell volume compared to ventricular myocytes (Forbes et al., 1990). Mitochondria and the sarcoplasmic reticulum (SR) occupy $V_{\text{mito}} = 24.73 \pm 0.61\%$ and $V_{\text{SR}} = 12.15 \pm 0.44\%$ of the right atrial myocytes, respectively, leaving the myoplasm volume of 63.12%
(or $V_{myo} = 13.9600 \times 10^{-6} \mu l$). The network SR and junctional SR constitute 11.54% and 0.61% of the cell volume, respectively, yielding $V_{NSR} = 2.5523 \times 10^{-6} \mu l$ and $V_{JSR} = 0.1349 \times 10^{-6} \mu l$. We used these data for our model cell.

To minimize the number of intracellular compartments without losing significant functionality of the model, we introduced three major compartments: caveolae (cav), extracaveolae (ecav), and cytosol (cyt) (Fig. 2.1). The caveolar compartment represents the subsarcolemmal space associated with caveolae microdomains, which represents the cholesterol-rich invaginations of the plasma membrane and includes caveolin-3 scaffolding protein. The extracaveolar compartment represents the subsarcolemmal space associated with cholesterol-rich lipid rafts that do not include caveolin-3. The cytosolic compartment represents the bulk cytoplasmic compartment associated with the remaining cell membrane (Agarwal et al., 2018; Iancu et al., 2007). In our model, TATS is considered as part of the sarcolemma with the presence of the three compartments mentioned above.

The ionic currents are distributed among different compartments, as found experimentally (see (Bondarenko et al., 2004) and references therein for details). The caveolar compartment contains a fraction of the L-type ($I_{CaL, cav}$) and the T-type ($I_{CaT, cav}$) Ca$^{2+}$ currents, the fast Na$^+$ current ($I_{Na}$), the small-conductance Ca$^{2+}$-activated K$^+$ current ($I_{K, Ca}$), the current of the Na$^+$-K$^+$ pump ($I_{NaK}$), and the time-independent K$^+$ current ($I_{K1}$). The extracaveolar compartment includes a fraction of the L-type Ca$^{2+}$ current ($I_{CaL, ecav}$), the rapidly recovering transient outward K$^+$ current ($I_{Kto,f}$), the rapidly activating, slowly inactivating K$^+$ current ($I_{K, slow1}$, encoded by Kv1.5; 4-aminopyridine-sensitive), the rapidly activating, slowly inactivating K$^+$ current ($I_{K, slow2}$, encoded by Kv2.1; TEA-sensitive), the noninactivating steady-state voltage-activated K$^+$ current ($I_{Kss}$), the rapid delayed rectifier K$^+$ current ($I_{Kr}$), the sarcolemmal Ca$^{2+}$ pump ($I_{p(Ca)}$), the current form the
Na+/Ca2+ exchanger ($I_{NaCa}$), and the Ca2+-activated chloride current ($I_{Cl, Ca}$). Finally, the cytosolic compartment contains a fraction of the T-type Ca2+ current ($I_{CaT, cyt}$), the Ca2+ background current ($I_{CaB}$), and the Na+ background current ($I_{NaB}$).

The model also describes compartmentalized Ca2+ dynamics, including local control of Ca2+-induced Ca2+ release (Fig. 2.1). Ca2+ ions that enter through the extracaveolar fraction of L-type Ca2+ channels are localized within the subspace volume ($V_{ss}$, also called “dyadic space”) and cause Ca2+ release from the junctional sarcoplasmic reticulum (JSR). Then Ca2+ ions from $V_{ss}$ diffuse to the rest of the cell volume, formatting a raising part of intracellular [Ca2+]i transient. Ca2+ ions are pumped back to the network sarcoplasmic reticulum (NSR) by the SERCA pump ($J_{up}$) and extruded outside of the cell by the Na+/Ca2+ exchanger and the sarcolemmal Ca2+ pump, resulting in a decline of [Ca2+]i transient.

**2.1.3 T-type Ca2+ Current**

In contrast to ventricular myocytes, Mouse atrial myocytes possess significant $I_{CaT}$ (Curran et al., 2015). Its maximum magnitude reaches $\pm 0.56 \pm 0.11$ pA/pF (Curran et al., 2015), which makes a small, but a noticeable contribution to the total Ca2+ entry to the cell in addition to $I_{CaL}$ (magnitude is $\sim 2$ pA/pF (Curran et al., 2015; Lomax et al., 2003). It has been shown that Cav3.1 and Ca3.2 channel proteins that represent the molecular basis for $I_{CaT}$, are co-localized with caveolar scaffolding protein caveolin-3, suggesting their possible localization in the caveolae compartment (Markandeya et al., 2011). On the other hand, the overexpression of the ion channels responsible for $I_{CaT}$ demonstrated their localization in both caveolar (68%) and t-tubules (32%) compartments (Jaleel et al., 2008). Jaleel et al. (2008) have also shown that even a larger Ca2+ entry produced by $I_{CaT}$ in mouse ventricular myocytes overexpressing Ca3.1, resulted in a much
smaller Ca\textsuperscript{2+} release from the SR, suggesting their primary localization outside the dyadic space in the cytosolic compartment. This observation is in line with the experimental data by Sipido et al. (1998), who have demonstrated a small contribution of \( I_{Ca,T} \) to the SR Ca\textsuperscript{2+} release in guinea pig ventricular myocytes.

For our mouse atrial myocyte model, we adopted a Markov model of the T-type Ca\textsuperscript{2+} channel developed by Serrano et al. (1999). The model consists of five closed states (C\(_0\)-C\(_4\)), five closed-inactivated states (I\(_0\)-I\(_4\)), an open state O, and an open-inactivated state I\(_0\) (Fig. 2.2A). This model reflects the independent activation of four-channel subunits, where 0 to 4 subunits are activated in C\(_0\) to C\(_4\) states, respectively. The forward and backward rate constants (\( k_{CaTpv} \) and \( k_{CaTmv} \)) and the backward rate constant \( k_{CaTmo} \) from open state O to closed state C\(_4\) are voltage-dependent. Forward rate constant \( k_{CaTpo} \) is rate-limiting and voltage-independent. Inactivated states are allosterically coupled to both the closed states C\(_0\)-C\(_4\) and the open state O. Two allosteric factors \( f_{CaT} \) and \( h_{CaT} \) regulate the coupling of the inactivated states to the closed states (Serrano et al., 1999). Several rate constants were modified from the Serrano et al. (1999) model to fit the experimental data on \( I_{Ca,T} \) (See Appendix A1). Unlike the L-type Ca\textsuperscript{2+} channels, in which inactivation is both Ca\textsuperscript{2+} and voltage-dependent, \( I_{Ca,T} \) inactivation depends only on transmembrane voltage (Vassort et al., 2006).
Figure 2.2 The T-type Ca\(^{2+}\) channel’s Markov model and current kinetics. 

(A) The model consists of five closed states (C0-C4), five closed-inactivated states (I0-I4), an open state O, and open-inactivated state IO. Transition rates \(k_{\text{Ca}T\text{pv}}\), \(k_{\text{Ca}T\text{mv}}\), and \(k_{\text{Ca}T\text{mo}}\) are voltage-dependent. **(B)** Current-voltage dependences (I-V) for \(I_{\text{CaT}}\). Simulation data are shown by a solid black line. Experimental data by Li et al. (2012), Satin and Cribbs (2000), and Zhou and Lipsius (1994) are shown by filled circles, unfilled circles, and filled triangles, respectively. **(C)** Steady-state inactivation relationships (SSI) and \(G/G_{\text{max}}\) of \(I_{\text{CaT}}\). Simulation data for SSI and \(G/G_{\text{max}}\) are shown by solid and dashed lines, respectively. Experimental data on SSI by Talavera et al. (2003), Li et al. (2012), Satin and Cribbs (2000), and Zhou and Lipsius (1994) are shown by filled circles, squares, diamonds, and triangles, respectively. Experimental data on \(G/G_{\text{max}}\) by Talavera et al. (2003), Li et al. (2012), Satin and Cribbs (2000), and Zhou and Lipsius (1994) are shown by
unfilled circles, squares, diamonds, and triangles, respectively. In (B) and (C), simulation data were obtained by two-pulse protocol: a 500 ms pulses P1 from −100 to +30 mV applied from the holding potential −100 mV in 10-mV steps and followed by a second pulse P2 to −20 mV for 100 ms. (D) Deactivation kinetics and time-to-peak currents of ICa,T. Simulation data for deactivation time constant and time-to-peak currents are shown by solid and dashed lines, respectively. Experimental data for deactivation kinetics were obtained by Satin and Cribbs (2000) (filled circles) and Talavera et al. (2004) (filled squares). Experimental data on time-to-peak current were obtained by Talavera et al. (2003) (unfilled squares). For simulation of time-to-peak current, we used the same protocol as in (B) and (C). Simulation data on the deactivation time constant were obtained by the following two-pulse protocol: the pulse P1 was set to −20 mV for 3 ms and followed by a second pulse that ranged between −120 and −50 mV for 200 ms in steps of 10 mV (holding potential was −100 mV). (E) Inactivation and recovery from inactivation kinetics for ICa,T. Simulation data for inactivation and recovery time constants are shown by dashed and solid lines, respectively. Experimental data on inactivation kinetics by Satin and Cribbs (2000) and Talavera et al. (2004) are shown by unfilled circles and squares, respectively. Experimental data on recovery from inactivation kinetics by Satin and Cribbs (2000) are shown by filled circles. Inactivation kinetics were obtained by the same voltage-clamp protocol as in (B) and (C). Recovery from inactivation kinetics was obtained from a two-pulse protocol: pulses to 0 mV (P1) for 1000 ms from holding potential −100 mV were followed by a second 100-ms depolarization (P2) to 0 mV with a variable time gap. Interpulse voltage varied from −120 to −80 mV.

Simulation and experimental properties of ICa,T under different experimental protocols are shown in Fig. 2.2, B-E. The model properly describes current-voltage relationships (I-V) (Fig. 2.2B), steady-state inactivation relationships and G/Gmax (Fig. 2.2C), time-to-peak current and deactivation kinetics (Fig. 2.2D), as well as inactivation and recovery from inactivation kinetics (Fig. 2.2E). The simulated maximum magnitude of ICa,T = −0.56 pA/pF, which is equal to the experimental value of −0.56±0.11 pA/pF (Curran et al., 2015). In our model, we also implemented the effects of β-adrenergic stimulation of ICa,T. Figure 2.3A shows the experimental time course of the magnitude of ICa,T upon the application of 1 µM isoproterenol reported by Li et al. (2012). Similar time behavior was obtained from our simulations, where dotted, dashed, and solid lines plot time courses of phosphorylated fractions of caveolar ICa,T (ICaT,cav), tubular ICa,T (ICaT,cyt), and averaged phosphorylated fraction of the cellular ICa,T. Our model was able to reproduce the magnitude of increase of ICa,T upon 10-minute exposure to 1 µM isoproterenol obtained experimentally by Li et al. (2012), as shown in Fig. 2.3B.
Figure 2.3 The effects of isoproterenol on T-type Ca$^{2+}$ current ($I_{Ca,T}$).

(A) Simulated time courses of the phosphorylated fraction of $I_{Ca,T,cav}$ and $I_{Ca,T,cyt}$, as well as averaged phosphorylated fraction of $I_{Ca,T}$, are shown by dotted, dashed, and solid lines, respectively, in response to the application of 1 µM isoproterenol. The experimental time course of $I_{Ca,T}$ increase upon stimulation with 1 µM isoproterenol obtained by Li et al. (2012) is shown by unfilled circles. (B) Simulated and experimental data (obtained by Li et al. (2012)) on $I_{Ca,T}$ magnitude in control and after 10 minutes exposure to 1 µM isoproterenol are shown by black and gray bars, respectively.

2.1.4 Small-Conductance Ca$^{2+}$-Activated K$^+$ Current

Mouse atrial myocytes are shown to express the small-conductance Ca$^{2+}$-activated K$^+$ current, $I_{K, Ca}$, which may contribute to AP shape (Li et al., 2009). This current is activated by intracellular Ca$^{2+}$, and most of its gating characteristics are voltage-independent (Hirschberg et al., 1998). Experimental data by Zhang et al. (2018) demonstrated that the small-conductance Ca$^{2+}$-activated K$^+$ channels are localized quite close to one fraction of the L-type Ca$^{2+}$ channels, at a distance of ~100 nm, which we assume belong to the caveolar compartment. However, they are relatively far from the second fraction of the L-type Ca$^{2+}$ channels, at a distance of ~400-500 nm, which we assume are in the extracaveolar compartment. Similarly, the surface small-conductance Ca$^{2+}$-activated K$^+$ channels are localized relatively far from the ryanodine receptors, at distances of ~400 nm, which are in the extracaveolar compartment. We thus can suggest that the small-conductance Ca$^{2+}$-activated K$^+$ channels are localized outside of the dyadic space of cardiac
myocytes. In addition, half-activation Ca\textsuperscript{2+} concentration for \( I_{K,Ca} \) is around 0.23-0.74 \( \mu \text{M} \) Ca\textsuperscript{2+} (Hirschberg et al., 1998; Kennedy et al., 2017), which is much smaller than the Ca\textsuperscript{2+} concentration in dyadic space (tens of \( \mu \text{M} \)). To get a gradual response from \( I_{K,Ca} \), the corresponding channels have to be localized quite far from the dyadic space, where the Ca\textsuperscript{2+} concentration declines to less than 1 \( \mu \text{M} \). Further, experimental data demonstrated that the small-conductance Ca\textsuperscript{2+}-activated K\textsuperscript{+} channels colocalize with caveolin-1 in different cell types (Balut et al., 2012), and caveolin-1 and caveolin-3 may colocalize in mouse and rat atrial myocytes (Volonte et al., 2008). These experimental facts suggest a possible localization of the small-conductance Ca\textsuperscript{2+}-activated K\textsuperscript{+} channels in the caveolae compartment, which, therefore, was implemented in our mathematical model of mouse atrial myocytes.

We employed the Hirschberg et al. (1998) mathematical model of the \( I_{K,Ca} \), with several adjustments to fit the experimental data (Fig. 2.4A). The model consists of four closed states (C\textsubscript{1}-C\textsubscript{4}) and two open states (O\textsubscript{1} and O\textsubscript{2}). Forward rate constants (\( k_{K,Ca1} \), \( k_{K,Ca2} \), and \( k_{K,Ca3} \)) are Ca\textsuperscript{2+}-dependent, and the other rate constants are voltage- and Ca\textsuperscript{2+}-independent. \( I_{K,Ca} \) current is defined by the equation:

\[
I_{K,Ca} = g_{K,Ca}(V)(O_1 + O_2),
\]

where O\textsubscript{1} and O\textsubscript{2} are the probabilities of finding a channel in open states, \( g_{K,Ca}(V) \) is a third-order polynomial function (see Appendix A2).
**Figure 2.4** Small-conductance Ca\(^{2+}\)-activated K\(^{+}\) current.

(A) Markov model of the small-conductance Ca\(^{2+}\)-activated K\(^{+}\) current (I\(_{\text{K,Ca}}\)). The model consists of four closed states (C\(_1\)-C\(_4\)) and two open states (O\(_1\) and O\(_2\)). Forward rate constants (k\(_{\text{KCaf1}}\), k\(_{\text{KCaf2}}\), and k\(_{\text{KCaf3}}\)) are Ca\(^{2+}\)-dependent, and the other rate constants are constant. (B) I\(_{\text{K,Ca}}\) as a function of voltage at [Ca\(^{2+}\)]\(_i\) = 0.9 µM. Simulation data are shown by a solid line; experimental data by Yu et al. (2012) are shown by unfilled circles. (C) Open probability of the small-conductance Ca\(^{2+}\)-activated K\(^{+}\) channel as a function of [Ca\(^{2+}\)]\(_i\). Simulation data are shown by a solid line; experimental data by Hirschberg et al. (1998) are shown by filled and unfilled circles. (D) Time course of the small-conductance Ca\(^{2+}\)-activated K\(^{+}\) channel phosphorylation at relatively low concentrations of PKA catalytic subunit (0.032184 µM). Simulation data are shown by a solid line; experimental data by Ren et al. (2006) are shown by filled and unfilled circles. (E) Phosphorylated fraction of the small-conductance Ca\(^{2+}\)-activated K\(^{+}\) channels. Experimental data by Ren et al. (2006) are obtained with 50 µM forskolin together with 100 µM of phosphodiesterase inhibitor Ro-201724. Simulation data are obtained at the maximal concentrations of the catalytic subunits of PKA.
The simulated current-voltage relationships for $I_{K,\text{Ca}}$ are shown in Fig. 2.4B by a solid line. Simulation fits well with the experimental data from mouse atrial myocytes obtained by Yu et al. (2012). $I_{K,\text{Ca}}$ current is virtually absent during diastole and is activated by Ca$^{2+}$ entry into the cell during systole. The simulated channel’s open probability as a function of intracellular Ca$^{2+}$ is shown in Fig. 2.4C. Symbols plot corresponding experimental data from Hirschberg et al. (1998). It is seen that the half-activation Ca$^{2+}$ concentration for $I_{K,\text{Ca}}$ is within the range from 0.2 to 1.0 µM Ca$^{2+}$, suggesting the current activation by intracellular [Ca$^{2+}]_i$ transients.

The small-conductance Ca$^{2+}$-activated K$^+$ channels can be stimulated by both $\beta_1$- and $\beta_2$-adrenergic signaling systems via the channels’ phosphorylation. When phosphorylated, the channels are removed from the membrane and do not participate in the conduction of trans-sarcolemmal current. Simulated and experimental time courses of the phosphorylation of the small-conductance Ca$^{2+}$-activated K$^+$ channel at relatively weak stimulation with 0.032184 µM of the catalytic subunit of protein kinase A (PKA) are shown in Fig. 2.4D. It took about 1 hour to obtain a saturation level of phosphorylation, which is relatively small, fractions of a percent. Stronger stimulation by 50 µM forskolin (a non-specific stimulator of adenylyl cyclases) in the presence of 100 µM phosphodiesterase inhibitor Ro-201724 produced a significant effect by removing $\sim$35% of the channel from the surface within 10 minutes (Ren et al., 2006). We also simulated this intervention by the maximum activation of PKA in our model, which resulted in a 26.7% phosphorylation level after 10-minute stimulation, which is close to the experimental finding by Ren et al. (2006).

### 2.1.5 Other Model Adjustments

First, experimental data shows a significantly smaller density of $I_{\text{Ca,L}}$ in mouse atrial myocytes as compared to ventricular myocytes. Therefore, we reduced $I_{\text{Ca,L}}$ conductance by a
factor of 3.35 to fit the experimental data by Lomax et al. (2003). Second, the magnitude of the fast-recovering transient outward K⁺ current \( I_{K_{to,f}} \) was decreased by a factor \(~2.73\) from that in the ventricular myocyte model as observed experimentally (Bao et al., 2016; Bondarenko, 2004; Lomax et al., 2003; Petkova-Kirova et al., 2012).

Third, we modified the description of the ultra-rapidly activating K⁺ current, \( I_{K_{ur}} \), also called the rapidly activating, slowly inactivating K⁺ current (\( I_{K_{slow}} \)). In the mathematical model by Bondarenko et al. (2004) and Bondarenko (2014), \( I_{K_{ur}} \) includes K⁺ currents carried by both Kv1.5 and Kv2.1 ion channels, which have a different response to β-adrenergic stimulation. An experimental study by Li et al. (1996) demonstrated a 35% increase in \( I_{K_{ur}} \) in human atrial myocytes upon stimulation with 1 µM isoproterenol. Additional experiments with an application of 200 µM of 4-aminopyridine, a Kv1.5 inhibitor, with and without 1 µM isoproterenol, have shown that the predominant portion of \( I_{K_{ur}} \) in the human atria is Kv1.5 channels. This finding is also supported by the experimental data on the expression of Kv1.5 and Kv2.1 in the human atria, where the expression level of Kv2.1 is about 1% of Kv1.5 (Gaborit et al., 2007). Electrophysiological recordings also show that the magnitude of the remaining component of \( I_{K_{ur}} \) in the human atria is about \(~15\%\) (Brandt et al., 2000). In mice, K⁺ currents encoded by both Kv1.5 and Kv2.1 have approximately equal magnitudes (Kodirov et al., 2004; London et al., 2001). The effect of PKA on Kv2.1 is quite small, as it was shown experimentally (Wilson et al., 1994; Zhou et al., 2012). Therefore, we included two currents: the rapidly activating, slowly inactivating K⁺ current, \( I_{K_{slow1}} \) (encoded by Kv1.5; 4-aminopyridine-sensitive; also named as \( I_{K_{ur}} \)), and the rapidly activating, slowly inactivating K⁺ current, \( I_{K_{slow2}} \) (encoded by Kv2.1; TEA-sensitive) (Fig. 2.5A).

Only \( I_{K_{slow1}} \) is affected by isoproterenol in our mathematical model. The magnitude of \( I_{K_{slow1}} \) plus
$I_{K,slow2}$ was decreased by a factor of 3, based on the data by Xu et al. (1999), which show that $I_{K,slow}$ ($I_{K,slow1}$ plus $I_{K,slow2}$) in mouse ventricular myocytes is by a factor 3 smaller than in atrial myocytes.

**Figure 2.5** Current traces for $K$ currents and current-voltage relationships for the time-independent $K^+$ current ($I_{K1}$).

(A) Current traces of $I_{Kto,f}$, $I_{K,slow1}$, and $I_{K,slow2}$ elicited by a 200-ms depolarization pulse to +40 mV applied to mouse atrial myocytes from the holding potential −80 mV at 100 ms. Simulations for control and after 10-minute exposure to 1 µM isoproterenol are shown by solid and dashed lines, respectively. (B) Current-voltage relationships for the time-independent $K^+$ current $I_{K1}$ in mouse atrial and ventricular myocytes. Solid and dashed lines show simulated data for $I_{K1}$ for atrial and ventricular myocytes, respectively. Experimental data for atrial (Lomax et al., 2003) and ventricular (Petkova-Kirova et al., 2012) are plotted by filled and unfilled circles, respectively.

Forth, in the atrial myocyte model, the magnitude of the non-inactivating steady-state $K^+$ current $I_{Kss}$ was reduced by about 43% from that in the ventricular myocyte model (Xu et al., 1999). In addition, since mouse atrial myocytes possess a relatively large rapid delayed rectifier current, in our model, it was increased accordingly by a factor of 10 as in the ventricular myocyte model (Bondarenko et al., 2004; Nakamura et al., 2010). Finally, we increased the maximum conductance of the time-independent $K^+$ current $I_{K1}$ by a factor of 1.7 to fit the experimental data for mouse right atrial myocytes by Lomax et al. (2003) (Fig. 2.5B). We also plotted the data on $I_{K1}$
obtained from mouse ventricular myocytes (Petkova-Kirova et al., 2012) (Fig. 2.5B) for comparison.

We also have made the required adjustments to the Ca$^{2+}$ handling. Experimental data shows that the SR volume and SR Ca$^{2+}$ concentration are larger in mouse atrial versus ventricular myocytes. In particular, the SR Ca$^{2+}$ concentration is twice larger in atrial cells (Brandenburg et al., 2016). In addition, the Ca$^{2+}$ entry through the L- and T-type Ca$^{2+}$ channels in atrial cells is by a factor $\sim$3 smaller than that in ventricular cells due to the relatively small current magnitudes (Bondarenko, 2014; Lomax et al., 2003). This requires a reduction of the Na$^+$/Ca$^{2+}$ exchanger and Na$^+$/K$^+$ pump function in atrial cells. Experimental data from human hearts shows that Na$^+$/K$^+$ pump activity in the atrium is $\sim 50\%$ of the ventricular activity, and the abundance of Na$^+$/K$^+$-ATPase in the atrium is $\sim65$-$70\%$ of the ventricular expression in the human and guinea pig hearts (McDonald et al., 2000; Wang et al., 1996). Therefore, we multiplied ventricular $I_{NaK}^{max}$ by a factor of 0.5. However, the experimental data on the expression of the Na$^+$/Ca$^{2+}$ exchanger in mouse atrial vs. ventricular myocytes shows similar levels (Groenke et al., 2013), so we did not change the magnitude of k$NaCa$ in the atrial myocyte model. We also reduced the amount of calsequestrin in the atrial cell by 40$\%$ of the ventricular value, increased the SR Ca$^{2+}$ maximum pumping rate by 2-fold, and reduced the half-saturation constant for the SR Ca$^{2+}$ pump by a factor of 0.9 to account for a smaller amount of phospholamban, according to Lüss et al. (1999). Several other adjustments were made to obtain proper equilibrium intracellular Ca$^{2+}$, Na$^+$, and K$^+$ concentrations (see Appendix A and Table 2.1).
Table 2.1 Differences between the model of mouse atrial myocytes presented in this paper and the model of mouse ventricular myocytes.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Definition</th>
<th>Ventricular cell model (6,79)</th>
<th>Atrial cell model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_{cap}$</td>
<td>Capacitive membrane area</td>
<td>$1.5340 \times 10^{-4}$ cm$^2$</td>
<td>$0.5931 \times 10^{-4}$ cm$^2$</td>
</tr>
<tr>
<td>$V_{cell}$</td>
<td>Cell volume</td>
<td>$38.00 \times 10^{-6}$ µl</td>
<td>$22.12 \times 10^{-6}$ µl</td>
</tr>
<tr>
<td>$V_{cyt}$</td>
<td>Cytosolic volume</td>
<td>$25.84 \times 10^{-6}$ µl</td>
<td>$13.96 \times 10^{-6}$ µl</td>
</tr>
<tr>
<td>$V_{JSR}$</td>
<td>Junctional SR volume</td>
<td>$0.1200 \times 10^{-6}$ µl</td>
<td>$0.1349 \times 10^{-6}$ µl</td>
</tr>
<tr>
<td>$V_{NSR}$</td>
<td>Network SR volume</td>
<td>$2.098 \times 10^{-6}$ µl</td>
<td>$2.5523 \times 10^{-6}$ µl</td>
</tr>
<tr>
<td>$V_{ss}$</td>
<td>Subspace volume</td>
<td>$1.485 \times 10^{-9}$ µl</td>
<td>$2.970 \times 10^{-9}$ µl</td>
</tr>
<tr>
<td>$I_{Cal,\text{max}}$</td>
<td>Normalization constant for the L-type Ca$^{2+}$ current</td>
<td>7.0 pA/pF</td>
<td>2.0 pA/pF</td>
</tr>
<tr>
<td>$v_1$</td>
<td>Maximum RyR channel Ca$^{2+}$ permeability</td>
<td>$4,500$ s$^{-1}$</td>
<td>$405$ s$^{-1}$</td>
</tr>
<tr>
<td>$v_2$</td>
<td>Ca$^{2+}$ leak rate constant from the NSR</td>
<td>$0.01740$ s$^{-1}$</td>
<td>$0.05220$ s$^{-1}$</td>
</tr>
<tr>
<td>$v_3$</td>
<td>SR Ca$^{2+}$-ATPase maximum pump rate</td>
<td>$3.0600 \times 10^{-4}$ µMs$^{-1}$</td>
<td>$8.0478 \times 10^{-4}$ µMs$^{-1}$</td>
</tr>
<tr>
<td>$K_{m,\text{up}}^{np}$</td>
<td>Half-saturation constant for SR Ca$^{2+}$-ATPase pump (non-phosphorylated)</td>
<td>0.41 µM</td>
<td>0.3097 µM</td>
</tr>
<tr>
<td>$K_{m,\text{up}}^{p}$</td>
<td>Half-saturation constant for SR Ca$^{2+}$-ATPase pump (phosphorylated)</td>
<td>0.31 µM</td>
<td>0.20375 µM</td>
</tr>
<tr>
<td>$\tau_{xfer}$</td>
<td>Time constant for transfer from subspace to myoplasm</td>
<td>0.0080 s</td>
<td>0.0104 s</td>
</tr>
<tr>
<td>$G_{Cal}$</td>
<td>Specific maximum conductivity for L-type Ca$^{2+}$ channel (non-phosphorylated)</td>
<td>$0.3772$ mS/µF</td>
<td>$0.1127$ mS/µF</td>
</tr>
<tr>
<td>$G_{Cal,p}$</td>
<td>Specific maximum conductivity for L-type Ca$^{2+}$ channel (phosphorylated)</td>
<td>$0.7875$ mS/µF</td>
<td>$0.2351$ mS/µF</td>
</tr>
<tr>
<td>$[CSQN]_{\text{tot}}$</td>
<td>Total junctional SR calsequestrin concentration</td>
<td>$15,000$ µM</td>
<td>$9,000$ µM</td>
</tr>
<tr>
<td>$I_{\text{max}}^{\text{NaK}}$</td>
<td>Maximum Na$^+$/K$^+$ pump current</td>
<td>4.0 pA/pF</td>
<td>2.0 pA/pF</td>
</tr>
</tbody>
</table>
Experimental data shows that the level of the expression and the ratio of $\beta_1$- and $\beta_2$-adrenergic receptors in atrial cells is quite similar to those reported in ventricular myocytes (Brodde, 1991). Similar expressions were observed in atrial and ventricular myocytes for adenylyl cyclases (Brandenburg et al., 2016) and phosphodiesterases (Hua et al., 2012). Therefore, we used

<table>
<thead>
<tr>
<th>$I_{\text{p(Ca)}}$</th>
<th>Maximum Ca$^{2+}$ pump current</th>
<th>0.051 pA/pF</th>
<th>0.153 pA/pF</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G_{\text{Cab}}$</td>
<td>Maximum background Ca$^{2+}$ current conductance</td>
<td>0.000284 mS/µF</td>
<td>0.000213 mS/µF</td>
</tr>
<tr>
<td>$G_{K_{\text{to,f}}}$</td>
<td>Maximum conductance for the rapidly inactivating transient outward K$^+$ current (non-phosphorylated)</td>
<td>0.3846 mS/µF</td>
<td>0.14067 mS/µF</td>
</tr>
<tr>
<td>$G_{K_{\text{to,f,p}}}$</td>
<td>Maximum conductance for the rapidly inactivating transient outward K$^+$ current (phosphorylated)</td>
<td>0.3846 mS/µF</td>
<td>0.086231 mS/µF</td>
</tr>
<tr>
<td>$G_{K_{\text{slow1}}}$</td>
<td>Maximum conductance for the rapidly activating, slowly inactivating K$^+$ current (encoded by $K_{v1.5}$, non-phosphorylated)</td>
<td>0.3424 mS/µF</td>
<td>0.05766 mS/µF</td>
</tr>
<tr>
<td>$G_{K_{\text{slow1,p}}}$</td>
<td>Maximum conductance for the rapidly activating, slowly inactivating K$^+$ current (encoded by $K_{v1.5}$, phosphorylated)</td>
<td>0.53307 mS/µF</td>
<td>0.07496 mS/µF</td>
</tr>
<tr>
<td>$G_{K_{\text{slow2}}}$</td>
<td>Maximum conductance for the rapidly activating, slowly inactivating K$^+$ current (encoded by $K_{v2.1}$, non-phosphorylated)</td>
<td>Current absent</td>
<td>0.05766 mS/µF</td>
</tr>
<tr>
<td>$G_{K_{\text{ss}}}$</td>
<td>Maximum conductance for the noninactivating steady-state K$^+$ current</td>
<td>0.0611 mS/µF</td>
<td>0.0428 mS/µF</td>
</tr>
<tr>
<td>$G_{K_{r}}$</td>
<td>Maximum conductance for the rapid delayed-rectifier K$^+$ current</td>
<td>0.078 mS/µF</td>
<td>0.780 mS/µF</td>
</tr>
<tr>
<td>$G_{K_{1}}$</td>
<td>Maximum conductance for the time-independent K$^+$ current</td>
<td>0.27 mS/µF</td>
<td>0.459 mS/µF</td>
</tr>
</tbody>
</table>

2.1.6 $\beta_1$- and $\beta_2$-Adrenergic Signaling Systems

Experimental data shows that the level of the expression and the ratio of $\beta_1$- and $\beta_2$-adrenergic receptors in atrial cells is quite similar to those reported in ventricular myocytes (Brodde, 1991). Similar expressions were observed in atrial and ventricular myocytes for adenylyl cyclases (Brandenburg et al., 2016) and phosphodiesterases (Hua et al., 2012). Therefore, we used
the same model for β₁- and β₂-adrenergic signaling systems as for ventricular cells (Rozier & Bondarenko, 2017).

2.1.7 Simulation Methods

The model equations were solved by a fourth-order Runge-Kutta method, with two different time steps. A relatively small time step of 0.000002 ms was used during a 15-millisecond interval after the initiation of the stimulus current; for all other times, we used the time step 0.0001 ms. The cellular behavior was simulated without electrical stimulation with a time step of 0.1 ms. The model equations were implemented in FORTRAN 90. All simulations were performed on a single processor under SUSE Linux 11 on a Dell Precision Workstation T3500 with a six-core Intel Xeon CPU W3670 (3.2 GHz, 12 GB RAM). The model was developed for a room temperature of 25°C (T = 298ºK). Initial conditions were obtained by running the program code without electrical stimulations for about 10,000 seconds to ensure quasi-steady-state. APs and [Ca²⁺]ᵢ transients were initiated by a stimulus current (Istim = 80 pA/pF, τstim = 1 ms) with the frequency 1 Hz (electrical stimulation).
2.2 Results

2.2.1 Mouse Atrial Action Potential and Ionic Currents

Experimental data shows that the APs in mouse atrial and ventricular myocytes have different shapes due to different sets of repolarization currents (Hua et al., 2015; Jansen et al., 2018; Lomax et al., 2003; Xu et al., 1999). Some of the experimental data for APD20, APD50, APD70, and APD90 are shown in Table 2.2. There are also differences between the APs from the left and right atria (Jansen et al., 2018; Lomax et al., 2003). In general, the atrial AP is more prolonged than the ventricular AP and has additional ionic currents that affect repolarization such as the T-type Ca2+ current and small-conductance Ca2+ activated K+ current (Curran et al., 2015; Li et al., 2009; Xu et al., 1999). Stimulation of the β-adrenergic signaling system with isoproterenol prolongs both atrial and ventricular mouse APs (Hua et al., 2015; Tong et al., 2006; Wu et al., 2002).

Table 2.2 Experimental data on APDs in right atrial myocytes (ms).

<table>
<thead>
<tr>
<th></th>
<th>APD20</th>
<th>APD50</th>
<th>APD70</th>
<th>APD90</th>
</tr>
</thead>
<tbody>
<tr>
<td>Control</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lomax et al. (2003)</td>
<td>11±2</td>
<td></td>
<td>44±5.5</td>
<td></td>
</tr>
<tr>
<td>Hua et al. (2015)</td>
<td>7.0–11.8</td>
<td>18.4–22.0</td>
<td>45.5–49.6</td>
<td></td>
</tr>
<tr>
<td>Jansen et al. (2018)</td>
<td>1.7±0.2</td>
<td>10.3±1.4</td>
<td>23.7±3.6</td>
<td>54.6±6.3</td>
</tr>
<tr>
<td>10 nM Iso</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hua et al. (2015)</td>
<td>12.1–16.5</td>
<td>27.6–32.7</td>
<td>58.0–63.6</td>
<td></td>
</tr>
</tbody>
</table>

Our mathematical model was able to reproduce the shape of the atrial AP and APDs found experimentally at different levels of repolarization in the right atrial mouse myocytes and the effects of 1 µM isoproterenol on the APD (Fig. 2.6, Table 2.3). We also compared our simulations with those made for apical ventricular myocytes (Fig. 2.6, Table 2.3). Simulated APD25 for right atrial myocytes is 2.83 ms, slightly longer than APD20 = 1.7±0.2 ms for right atrial myocytes by
Jansen et al. (2018), but it is comparable to APD$_{25} = 2.8\pm0.6$ ms for atrial myocytes from Xu et al. (1999). Simulated APD$_{50}$ is 6.21 ms, which is comparable to the experimental data ~7.0-11.8 ms (Hua et al., 2015; Jansen et al., 2018; Lomax et al., 2003) for right atrial cells. Simulated APD$_{90} = 44.39$ ms is also close to the experimental values of 44±5.5 ms (Lomax et al., 2003), 45.5-49.6 ms (Hua et al., 2015), and 54.6±6.3 ms (Jansen et al., 2018).

**Figure 2.6** Simulated mouse action potentials in atrial and ventricular myocytes and underlying ionic currents under different physiological conditions.  
(A) Simulated mouse atrial action potentials for control (solid line) and after a 5 minutes exposure to 1 µM isoproterenol (dashed line).  
(B) Simulated mouse ventricular action potentials for control (solid line) and after a 5 minutes exposure to 1 µM isoproterenol (dashed line).  
(C) Simulated major ionic currents underlying mouse atrial action potential in control.  
(D) Simulated major ionic currents underlying mouse ventricular action potential in control.  
(E) Simulated major ionic currents underlying mouse atrial action potential after a 5 minutes exposure to 1 µM isoproterenol.
Simulated major ionic currents underlying mouse ventricular action potential after a 5 minutes exposure to 1 μM isoproterenol. In (A-F), action potentials and ionic currents are obtained after 300 s stimulation with 1 Hz.

If we compare APD$_{25}$, APD$_{50}$, APD$_{75}$, and APD$_{90}$ for the right atrial versus apical ventricular myocytes, we see that the APD at all levels of repolarizations is longer for atrial myocytes (Table 2.3). The difference ranges between 66.5% for APD$_{25}$ and 188.0% for APD$_{75}$. These could be explained by the differences in major repolarization currents for atrial and ventricular myocytes obtained from simulations (Figs. 2.6C and 2.6D). Mouse atrial myocytes have about twice smaller repolarizing, rapidly recovering transient outward K$^+$ current, $I_{K\text{to},f}$, compared to ventricular myocytes. The magnitude of the sum of the currents $I_{K\text{slow}1}$ and $I_{K\text{slow}2}$ in atrial cells is much smaller than the magnitude of $I_{Kur}$ in ventricular cells. The difference in outward K$^+$ currents is partially compensated by the difference in the inward L-type Ca$^{2+}$ current, which is much smaller in the atrial myocytes, resulting in more prolonged APDs in atrial versus ventricular cells. Notice that the contribution of the T-type Ca$^{2+}$ current to atrial APD is quite small as compared to the L-type Ca$^{2+}$ current (Fig. 2.6C).

**Table 2.3 Comparison of the atrial and ventricular mouse action potential durations (ms).**

<table>
<thead>
<tr>
<th></th>
<th>Atrial myocyte</th>
<th>Ventricular myocyte</th>
<th>APD prolongation (%) Atrial vs. Ventricular</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Control</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>APD$_{25}$</td>
<td>2.83</td>
<td>1.70</td>
<td>66.5</td>
</tr>
<tr>
<td>APD$_{50}$</td>
<td>6.21</td>
<td>3.19</td>
<td>94.7</td>
</tr>
<tr>
<td>APD$_{75}$</td>
<td>26.89</td>
<td>9.34</td>
<td>188</td>
</tr>
<tr>
<td>APD$_{90}$</td>
<td>44.39</td>
<td>26.16</td>
<td>69.7</td>
</tr>
<tr>
<td><strong>1 μM Iso</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>APD$_{25}$</td>
<td>3.47</td>
<td>1.84</td>
<td>88.6</td>
</tr>
<tr>
<td>APD$_{50}$</td>
<td>8.91</td>
<td>3.80</td>
<td>134</td>
</tr>
<tr>
<td>APD$_{75}$</td>
<td>31.72</td>
<td>11.12</td>
<td>184</td>
</tr>
<tr>
<td>APD$_{90}$</td>
<td>48.48</td>
<td>29.97</td>
<td>61.8</td>
</tr>
</tbody>
</table>
The application of 1 µM isoproterenol (5 minutes exposure) resulted in a prolongation of both atrial and ventricular APs in simulations, which is similar to the values observed experimentally (Figs. 2.6A and 2.6B). Table 2.3 shows that atrial APD₉₀ was prolonged from 2.83 to 3.47 ms (22.6% lengthening), APD₅₀ - from 6.21 to 8.91 ms (43.5%), and APD₉₀ - from 44.39 to 48.48 ms (9.2%). APD prolongations are comparable to those obtained after an application of 10 nM isoproterenol on mouse right atrial myocytes (Hua et al., 2015). The average experimental APD₅₀ prolongation was 53%, and the average APD₉₀ prolongation was 28%. Note that, although the experimental data is for a 10 nM isoproterenol concentration, it can still provide quite valid numbers for comparison to the simulation results. Because, as we demonstrated previously, mouse APDs do not change significantly within isoproterenol concentrations from 10 nM to 1 µM (Grinshpon and Bondarenko, 2016). Significantly smaller APD prolongations were observed in ventricular myocytes (Table 2.3). APD₂₅ was prolonged from 1.7 to 1.84 ms (by 8.2%), APD₅₀ - from 3.19 to 3.8 ms (by 19.1%), and APD₉₀ - from 26.16 to 29.97 ms (by 14.6%). At all levels of repolarization, atrial APDs are more prolonged than ventricular after exposure to a 1 µM isoproterenol.

### 2.2.2 Ca²⁺ Dynamics in Mouse Atrial Myocytes

Simulated [Ca²⁺]ᵢ transients and integral Ca²⁺ fluxes in mouse right atrial myocytes without and with an application of 1 µM isoproterenol are shown in Fig. 2.7. Similar simulation data for mouse ventricular myocytes were also plotted in Fig. 2.7. In control, the peak [Ca²⁺]ᵢ transient in atrial myocytes is equal to 0.4657 µM, which is a little less than the peak [Ca²⁺]ᵢ transient in ventricular myocytes (0.5294 µM). Simulated time-to-peak for [Ca²⁺]ᵢ transients in mouse atrial cell is 24.1 ms, which is larger than the time-to-peak of [Ca²⁺]ᵢ transients in mouse ventricular cell (17.1 ms) and comparable to those observed experimentally by Mancarella et al. (2008; 23±2 ms),
Glukhov, Kalyanasundaram, et al. (2015; ~30 ms), and Yue et al. (2017; 28.3±1.2 ms). The simulations are in line with the experimental data by Babu et al. (2007), where [Ca\textsuperscript{2+}]\textsubscript{i} transient in atrial myocytes was close to those in ventricular cells. Nevertheless, the simulated Ca\textsuperscript{2+} SR load is about 2 times larger in mouse atrial versus ventricular myocytes (1946 µM vs. 890 µM, respectively), which is close to the experimental Ca\textsuperscript{2+} SR loads (10.5±0.2 A.U. versus 5.85±0.20 A.U., for atrial and ventricular cells, respectively (Brandenburg et al., 2016)).

**Figure 2.7** Simulated [Ca\textsuperscript{2+}]\textsubscript{i} transients and Ca\textsuperscript{2+} fluxes in mouse atrial and ventricular myocytes under different physiological conditions. (A) Simulated mouse atrial [Ca\textsuperscript{2+}]\textsubscript{i} transients for control (solid line) and after application of 1 µM isoproterenol (dashed line). (B) Simulated mouse ventricular [Ca\textsuperscript{2+}]\textsubscript{i} transients for control (solid line) and after application of 1 µM isoproterenol (dashed line). (C) Simulated Ca\textsuperscript{2+} influxes in mouse atrial myocytes under control conditions. (D) Simulated Ca\textsuperscript{2+} influxes in mouse ventricular myocytes under control conditions. (E) Simulated Ca\textsuperscript{2+} influxes in mouse atrial myocytes after a
5-minute exposure to 1 µM isoproterenol. (F) Simulated Ca\(^{2+}\) influxes in mouse ventricular myocytes after a 5-minute exposure to 1 µM isoproterenol. In (A-F), [Ca\(^{2+}\)]\(_i\) transients and Ca\(^{2+}\) fluxes are obtained after 300 s stimulation with 1 Hz.

Application of 1 µM isoproterenol increases simulated [Ca\(^{2+}\)]\(_i\) transients in both atrial (1.33 µM, increase by a factor 2.85) and ventricular (2.03 µM, increase by a factor 3.83) myocytes, with the larger increase in ventricular cells (Figs. 2.7A and 2.7B). This result is qualitatively similar to the data by Babu et al. (2007), where the application of 1 µM isoproterenol increased [Ca\(^{2+}\)]\(_i\) transients by factors of 1.5 and 1.64, respectively. However, it should be noted that the experimentally observed increase in [Ca\(^{2+}\)]\(_i\) transients in mouse ventricular myocytes after application of 1 µM isoproterenol, as observed by others, ranges from a factor of 2 to 5 (data collected in (Bondarenko, 2014) and by a factor 3.1 in mouse atrial myocytes (Li et al., 2005). So we adjusted the atrial model parameters until we got an increase by a factor 2.85 in atrial cells, which is slightly less than 3.83 for ventricular cells.

We also simulated integral Ca\(^{2+}\) fluxes in atrial and ventricular myocytes. Simulations show a larger integral Ca\(^{2+}\) release flux in mouse atrial (48.28 µM) versus ventricular (35.04 µM) myocytes in control (Figs. 2.7C and 2.7D). However, the triggering Ca\(^{2+}\) influx by the \(I_{\text{Ca,L}}\) is larger in mouse ventricular (1.57 µM) as compared to mouse atrial (0.51 µM) cells. Additional Ca\(^{2+}\) entry through the T-type Ca\(^{2+}\) channels (0.18 µM) increases the total triggering Ca\(^{2+}\) influx to 0.69 µM, which is by a factor of 2.3 less than the triggering Ca\(^{2+}\) influx in mouse ventricular cells. We suggest that the larger Ca\(^{2+}\) release flux in mouse atrial cells is mainly due to the larger SR Ca\(^{2+}\) load and smaller atrial myocyte volume (Brandenburg et al., 2016), which create a larger pro-arrhythmic substrate in mouse atrial vs. ventricular cells. Simulated Ca\(^{2+}\) extrusion by the Na\(^+\)/Ca\(^{2+}\) exchanger is larger in mouse ventricular myocytes (3.12 µM) as compared to mouse atrial myocytes (1.27 µM) due to a larger trans-sarcolemmal Ca\(^{2+}\) entry. Application of 1 µM
isoproterenol increases Ca\(^{2+}\) entry through the L-type Ca\(^{2+}\) channels both in mouse atrial (0.94 µM) and ventricular (3.29 µM) myocytes, as well as integral Ca\(^{2+}\) release fluxes in mouse atrial (80.32 µM) and ventricular (61.16 µM) myocytes (Figs. 2.7E and 2.7F). The integral Ca\(^{2+}\) influxes and Ca\(^{2+}\) release fluxes increase by a factor of 2 after isoproterenol application.

### 2.2.3 The Effects of the T-type Ca\(^{2+}\) Current on Atrial Action Potential and Ca\(^{2+}\) Dynamics

Experimental measurements of the \(I_{\text{Ca,T}}\) in cardiac cells is quite difficult due to its significant overlap with the \(I_{\text{Ca,L}}\). The \(I_{\text{Ca,T}}\) was found in both atrial and ventricular myocytes of different species and in sino-atrial nodal cells (for a review, see (Vassort et al., 2006)). \(I_{\text{Ca,T}}\) was also found in mouse atrial myocytes (Curran et al., 2015). Experimental data also demonstrated that the \(I_{\text{Ca,T}}\) affects pacemaking activity of sinoatrial nodal cells, as well as causes prolongation of the AP in the mouse Purkinje cells (Vaidyanathan et al., 2013) or, if expressed in pathological conditions, in mouse ventricular myocytes (Jaleel et al., 2008). There are different experimental data on the role of the \(I_{\text{Ca,T}}\) in triggering Ca\(^{2+}\)-induced Ca\(^{2+}\)-release from the SR. Some experimental data show the minor role of the T-type Ca\(^{2+}\) current in triggering Ca\(^{2+}\)-releases (Jaleel et al., 2008; Sipido et al., 1998), while other data demonstrates that the \(I_{\text{Ca,T}}\) can be responsible for a substantial part of Ca\(^{2+}\) release (Kitchens et al., 2003).

To investigate the role of the T- and L-type Ca\(^{2+}\) currents in shaping the mouse right atrial AP and its effects on \([\text{Ca}^{2+}]_i\) transients, we performed simulations under different physiological conditions (Fig. 2.8). Figure 2.8A demonstrates the effects of the \(I_{\text{Ca,T}}\) block on the APD. Note that despite the small magnitude of the \(I_{\text{Ca,T}}\), ~0.5 pA/pF, it makes a significant contribution to the shape of the AP. It can be seen from the figure that the effect is mostly on the later stage of repolarization, where the inhibition of \(I_{\text{CaT}}\) shortens APD\(_{50}\), APD\(_{75}\), and APD\(_{90}\) by 2.5%, 18%, and
14%, respectively. A similar effect is seen after $I_{Ca,T}$ block in the presence of 1 µM isoproterenol, where APD$_{75}$ and APD$_{90}$ are shortened by 19% and 15%, respectively. The $I_{Ca,T}$ block also affects peak $[Ca^{2+}]_i$ transient, which decreases by ~13% in control and by ~15% in the presence of 1 µM isoproterenol (Fig. 2.8B). A similar minor effect of the $I_{Ca,T}$ block on $[Ca^{2+}]_i$ transient is observed experimentally by Sipido et al. (1998) in guinea pig ventricular myocytes.

**Figure 2.8** Simulated atrial action potential and $[Ca^{2+}]_i$ transient under different physiological conditions.

(A) Simulated action potentials in control (solid black line), after application of 1 µM isoproterenol (dashed black line), upon $I_{Ca,T}$ block (solid blue lines), upon $I_{Ca,T}$ block, and after application of 1 µM isoproterenol (dashed blue line). (B) Simulated $[Ca^{2+}]_i$ transient in control (solid black line), after application of 1 µM isoproterenol (dashed black line), upon $I_{Ca,T}$ block (solid blue lines), upon $I_{Ca,T}$ block, and after application of 1 µM isoproterenol (dashed blue line).
(C) Simulated action potentials in control (solid black line), upon $I_{Ca,L}$ block (solid red lines), and upon $I_{Ca,L}$ block and a 25-fold increase of the control $I_{Ca,T}$ magnitude (solid green line). (D) Simulated $[Ca^{2+}]_i$ transient in control (solid black line), upon $I_{Ca,L}$ block (solid red lines), and upon $I_{Ca,L}$ block and a 25-fold increase of the control $I_{Ca,T}$ magnitude (solid green line). (E) Simulated action potentials in control (solid black line) and after hypothetical movement of $I_{Ca,T}$ to subspace volume (solid blue line). (F) Simulated $[Ca^{2+}]_i$ transient in control (solid black line) and after hypothetical movement of $I_{Ca,T}$ to subspace volume (solid blue line).

It would also be interesting to see how the $I_{Ca,L}$ block affects the shape of the mouse atrial AP. Figure 2.8C shows the effect is stronger than that from the $I_{Ca,T}$ block. The $I_{Ca,L}$ block decreases the APD at 50%, 75%, and 90% repolarization by 17%, 27%, and 16%, respectively. It also essentially eliminates $[Ca^{2+}]_i$ transient (Fig. 2.8D). This simulation demonstrates that the $I_{Ca,L}$ is mostly responsible for the $Ca^{2+}$-induced $Ca^{2+}$ release in mouse atrial myocytes. A similar effect is observed in the experiments with guinea pig ventricular myocytes, where the $I_{Ca,L}$ block basically eliminated $[Ca^{2+}]_i$ transient (Sipido et al., 1998).

We also simulated the effects of overexpression of $I_{Ca,T}$ in mouse atrial myocytes when the $I_{Ca,L}$ is blocked (Fig. 2.8C). In this simulation, we increased the magnitude of the $I_{Ca,T}$ by a factor of 25 from its control value. An increase in the magnitude of $I_{Ca,T}$ is then found to dramatically prolong the mouse’s atrial AP duration (by about 3-fold for APD$_{50}$ and APD$_{75}$ and by about 2-fold for APD$_{90}$). Note that a similarly dramatic increase in APDs was observed in mouse ventricular myocytes with similar overexpression of $I_{Ca,T}$ (Lipp & Niggli, 1994). Our simulations also demonstrated a noticeable but much smaller $[Ca^{2+}]_i$ transient (green line in Fig. 2.8D), which was generated in mouse atrial myocytes overexpressing $I_{Ca,T}$ and at the $I_{Ca,L}$ block, as compared to the control cells (black line in Fig. 2.8D).

Finally, we tested a hypothesis that the localization of T-type $Ca^{2+}$ channels in the subspace volume (in the extracaveolar domain, Fig. 2.1) can trigger larger $[Ca^{2+}]_i$ transients compared to their natural localization in the caveolar and cytosolic domains. Figures 2.8E and 2.8F show the
results of simulations for control (solid black lines) and for relocated T-type Ca\(^{2+}\) channels (solid blue lines), which are virtually identical. We consider that a smaller magnitude of \(I_{\text{CaT}}\) as compared to \(I_{\text{CaL}}\) is responsible for this behavior. This results in a significantly smaller Ca\(^{2+}\) entry through the T-type Ca\(^{2+}\) channels to the subspace volume as compared to the L-type Ca\(^{2+}\) channels. Our simulations agree with the findings of Bovo et al. (2011) on the non-linear (threshold-like) dependence of Ca\(^{2+}\) exit from the SR through ryanodine receptors. The latter suggests a much smaller or no Ca\(^{2+}\) release from the SR at relatively small values of \([\text{Ca}^{2+}]_i\), which could be achieved by the Ca\(^{2+}\) entry through T-type Ca\(^{2+}\) channels as compared to the L-type Ca\(^{2+}\) channels.

Thus, our simulations show that both the T- and L-type Ca\(^{2+}\) currents significantly affect the shape of the mouse atrial AP. When the T- and L-type Ca\(^{2+}\) currents are available, or only the L-type Ca\(^{2+}\) current is available, we obtained Ca\(^{2+}\)-induced Ca\(^{2+}\) release and normal \([\text{Ca}^{2+}]_i\) transients. When there is only the T-type Ca\(^{2+}\) current, Ca\(^{2+}\) release from the SR is not happening.

### 2.2.4 The Effects of the Small-Conductance K\(^+\) Current on Atrial Action Potential and Ca\(^{2+}\) Dynamics

Experimental data demonstrates that the small-conductance K\(^+\) current \(I_{\text{Ca,K}}\) can play a role in shaping mouse atrial AP (Hancock et al., 2015; Xu et al., 2003; Yi et al., 2015; Zhang et al., 2014). Block of \(I_{\text{Ca,K}}\) with apamin resulted in AP prolongation. However, a marked impact of \(I_{\text{Ca,K}}\) was observed in mouse atrial myocytes at relatively high intracellular \([\text{Ca}^{2+}]_i\) concentrations equal to 1 µM. On the other hand, experimental observations with atrial myocytes from other species (dog, rat) under normal physiological conditions do not show any noticeable contribution of \(I_{\text{Ca,K}}\) to AP morphology (Nagy et al., 2009).

Therefore, we simulated the effects of the small-conductance K\(^+\) current \(I_{\text{Ca,K}}\) on the mouse atrial AP and \([\text{Ca}^{2+}]_i\) transients under different physiological conditions. Figure 2.9A shows that the AP under control conditions was not affected by \(I_{\text{Ca,K}}\), as the block of this current does not
change the shape of AP. The effect is noticeable under stimulation of the β-adrenergic signaling system with 1 µM isoproterenol and only in the later phase of repolarization (Fig. 2.9A). APD<sub>75</sub> and APD<sub>90</sub> prolongations upon $I_{Ca,K}$ block was only ~3-4%, which are quite difficult to measure experimentally. The mechanism of this prolongation is shown in Fig. 2.9D, where $I_{Ca,K}$ was much larger upon application of 1 µM isoproterenol (black dashed line) compared to the control conditions (solid black line). In this respect, our simulations confirm the results of Nagy et al. (2009) that there is no effect of $I_{Ca,K}$ block on the atrial AP under normal physiological conditions.

**Figure 2.9** Simulated atrial action potential and Ca<sup>2+</sup>-activated K<sup>+</sup> current ($I_{K,Ca}$) under different physiological conditions.

(A) Simulated action potentials with normal [Ca<sup>2+</sup>]<sub>i</sub> transient in control (solid black line), after application of 1 µM isoproterenol (dashed black line), upon $I_{K,Ca}$ block (solid blue lines), upon $I_{K,Ca}$ block and after application of 1 µM isoproterenol (dashed blue line).

(B) Simulated action potentials with normal [Ca<sup>2+</sup>]<sub>i</sub> transient in control (solid black line) and after application of 1 µM isoproterenol (dashed black line); simulated action potentials with high [Ca<sup>2+</sup>]<sub>i</sub> = 1 µM in control (solid red lines) and after application of 1 µM isoproterenol (dashed red line).

(C) Simulated action potentials with [Ca<sup>2+</sup>]<sub>i</sub> = 1 µM in control (solid red lines), after application of 1 µM isoproterenol (dashed red line), upon $I_{K,Ca}$ block (solid blue lines), and upon $I_{K,Ca}$ block, and after application of
Simulations also demonstrated a slight shortening of the atrial AP under high intercellular calcium transient ([Ca$^{2+}$]$_i$ = 1 µM) conditions compared to control (Fig. 2.9B). APD$_{25}$ was reduced by 5%, APD$_{50}$ – by 7.4%, and APD$_{90}$ – by 7.6%. The reduction was attributable to the larger magnitude of $I_{Ca,K}$ under high [Ca$^{2+}$]$_i$ conditions (Fig. 2.9D).

However, the strongest effect of $I_{Ca,K}$ block on the mouse atrial AP was obtained for higher [Ca$^{2+}$]$_i$ (Fig. 2.9C). Without the application of isoproterenol, the effects were predominantly on APD$_{75}$ and APD$_{90}$, which were prolonged by 10% and 12%, respectively. The results of simulations are in agreement with the experimental data by Hancock et al. (2015), where APD$_{90}$ was prolonged by 10% after $I_{Ca,K}$ block. They are also in line with the data by Yi et al. (2015), where significant prolongation was observed for APD$_{90}$ (by 22%), but not for APD$_{50}$. We also obtained a larger prolongation of the APD after applying 1 µM isoproterenol, which enhanced the effect of $I_{Ca,K}$ block, resulting in prolongation of APD$_{50}$, APD$_{75}$, and APD$_{90}$ by 14%, 16%, and 10%, respectively. We did not find the corresponding experimental data for comparison.

The effect of $I_{Ca,K}$ block on [Ca$^{2+}$]$_i$ transients was simulated both in control and after application of 1 µM isoproterenol under normal physiological conditions. In both cases, the effect was negligible, less than 1%.

### 2.2.5 Frequency Dependences of the AP and [Ca$^{2+}$]$_i$ Transients

We also investigated the frequency dependences of the APDs at different levels of repolarization and the [Ca$^{2+}$]$_i$ transients without and with the application of 1 µM isoproterenol. Figure 2.10A shows that APD$_{25}$ and APD$_{50}$ moderately increased and APD$_{90}$ moderately decreased at higher pacing frequencies. These behaviors sustain after the application of 1 µM isoproterenol,
which prolongs APDs at all levels of repolarization. The weak frequency dependence of simulated APDs is in line with the experimental data by Lu et al. (2007), where no significant frequency dependence of APD$_{50}$ and APD$_{90}$ was observed in the range from 0.1 to 10 Hz. Another experimental paper (Hua et al., 2015) also suggests APD prolongation after the application of isoproterenol which supports our simulation result.

**Figure 2.10** Simulated atrial action potential durations (APDs) and [Ca$^{2+}$]$_i$ transients as functions of stimulation frequency. (A) Simulated APD$_{25}$ (circles), APD$_{50}$ (squares), and APD$_{90}$ (diamonds) as a function of stimulation frequency in control (closed symbols) and after 5-minute exposure to 1 µM isoproterenol (open symbols). (B) Simulated diastolic (closed symbols) and systolic (open symbols) [Ca$^{2+}$]$_i$ transients as a function of stimulation frequency in control (circles) and after 5-minute exposure to 1 µM isoproterenol (squares).

Our simulations show an increase in both diastolic and systolic [Ca$^{2+}$]$_i$ concentrations with stimulation frequency in control (Fig. 2.10B). We suggest that this increase is due to the larger Ca$^{2+}$ influx mainly through the L-type Ca$^{2+}$ channels with the stimulation frequency per unit time, which Ca$^{2+}$ removal by the Na$^+$/Ca$^{2+}$ exchanger does not completely compensate. After an application of 1 µM isoproterenol, the dependence becomes biphasic. It includes an increasing part at the lower stimulation frequencies and a weakly decreasing part at the higher frequencies. We suggest the same mechanism of frequency dependence for [Ca$^{2+}$]$_i$ transients without isoproterenol. However, we need to take into account a quite large increase in the L-type Ca$^{2+}$ current that leads to about a 2-3-fold increase in [Ca$^{2+}$]$_i$ transient, resulting in the faster Ca$^{2+}$ removal by the Na$^+$/Ca$^{2+}$
exchanger at higher frequencies. At smaller stimulation frequencies, the Na+/Ca\(^{2+}\) exchanger cannot compensate Ca\(^{2+}\) influx through the L-type Ca\(^{2+}\) channels due to a relatively small systolic [Ca\(^{2+}\)]\(_i\) concentration. As the systolic [Ca\(^{2+}\)]\(_i\) concentration increases with the stimulation frequency, more Ca\(^{2+}\) is extruded by the Na+/Ca\(^{2+}\) exchanger. For example, after the application of 1 µM isoproterenol, the peak I\(_{CaL}\) increases by ~4%, and peak I\(_{NaCa}\) increases by ~44% when stimulation frequency changes from 0.5 to 4 Hz.

### 2.2.6 Sensitivity Analysis

In order to define how cell-to-cell variability of the ionic current expression affects the AP and [Ca\(^{2+}\)]\(_i\) transients in mouse atrial myocytes without and with β-adrenergic stimulation, we performed a sensitivity analysis (Fig. 2.11). For this purpose, we first increased and then decreased the conductance of each of the model currents by 20% as compared to the baseline model (the magnitude of changes was similar to that used by Sobie (2009) and Kernik et al. (2019)). We then stimulated the resulting atrial model cell for 300 s with a basic cycle length 1000 ms (\(I_{stim} = 80\) pA/pF, \(\tau_{stim} = 1\) ms) and calculated APD\(_{25}\), APD\(_{50}\), APD\(_{90}\), AP amplitude, and [Ca\(^{2+}\)]\(_i\) transients in control and after application of 1 µM isoproterenol. The AP and [Ca\(^{2+}\)]\(_i\) characteristics were compared to the model with baseline parameters in control and after application of 1 µM isoproterenol.
Figure 2.11  Sensitivity analysis.
Changes of action potential (AP) durations at 25% (Panels A and F), 50% (Panels B and G), and 90% repolarizations (Panels C and H), AP amplitude (Panels D and I), and [Ca$^{2+}$], transient (Panels E and J) between the two consecutive stimuli (the 301$^\text{st}$ and 300$^\text{th}$ pacing beats) at 1 Hz. We increased and decreased the magnitude of one of several ionic currents by 20%, and the model cell was paced with 300 stimuli. The changes were calculated as the differences between AP durations, AP amplitudes, and [Ca$^{2+}$], transients during the 301$^\text{st}$ and 300$^\text{th}$ beats. Simulations were performed without isoproterenol (Control, Panels A-E) and after a 5-minute application of 1 μM isoproterenol (Panels F-J). Sensitivity analysis is performed with respect to 20% increase (black bars) and 20% decrease (gray bars) of the L-type Ca$^{2+}$ current (ica,l), the T-type Ca$^{2+}$ current (icat), the sarcolemmal Ca$^{2+}$ pump current (ipca), the Na$^+$/Ca$^{2+}$ exchanger current (inaca), the Ca$^{2+}$ background current (icab), the fast Na$^+$ current (ina), the Na$^+$ background
current (inab), the Na\textsuperscript{+}-K\textsuperscript{+} pump current (inak), the rapidly recovering transient outward K\textsuperscript{+} current (iktof), the time-independent K\textsuperscript{+} current (ik1), the rapidly activating, slowly inactivating K\textsuperscript{+} current (encoded by K\textsubscript{v}1.5; 4-aminopyridine-sensitive; ikslow1), the rapidly activating, slowly inactivating K\textsuperscript{+} current (encoded by K\textsubscript{v}2.1; TEA-sensitive; ikslow2), the noninactivating steady-state voltage-activated K\textsuperscript{+} current (ikss), the small-conductance Ca\textsuperscript{2+}-activated K\textsuperscript{+} current (ikca), the rapid delayed rectifier K\textsuperscript{+} current (ikr), and the Ca\textsuperscript{2+}-activated chloride current (iclca). No changes are shown for the simulations without 20\% perturbation of the model parameters (c).

In control, the major player for APD\textsubscript{25} and APD\textsubscript{50} is the rapidly recovering transient outward K\textsuperscript{+} current, $I_{Kto,f}$ (Fig. 2.11, A and B). Three other currents, the L-type Ca\textsuperscript{2+} current, $I_{CaL}$, and two rapidly activating, slowly inactivating K\textsuperscript{+} currents, $I_{K,slow1}$ and $I_{K,slow2}$, also made significant contributions to the change in APD\textsubscript{25} and APD\textsubscript{50}. The K\textsuperscript{+} currents tend to shorten, while $I_{CaL}$ tends to prolong APD\textsubscript{25} and APD\textsubscript{50}. The regulation of APD\textsubscript{90} is more complex. Eight major ionic currents affect APD\textsubscript{90} (Fig. 2.11C). An increase in the T-type Ca\textsuperscript{2+} current, $I_{CaT}$, the fast Na\textsuperscript{+} current, $I_{Na}$, the background Na\textsuperscript{+} current, $I_{Nab}$, and the Na\textsuperscript{+}/Ca\textsuperscript{2+} exchanger current, $I_{NaCa}$, tend to prolong APD\textsubscript{90}, while the time-independent K\textsuperscript{+} current, $I_{K1}$, two rapidly activating, slowly inactivating K\textsuperscript{+} currents, $I_{K,slow1}$ and $I_{K,slow2}$, and the noninactivating steady-state voltage-activated K\textsuperscript{+} current, $I_{Kss}$, tend to shorten APD\textsubscript{90}. In control, AP amplitude is affected by four major currents (Fig. 2.11D). Three currents, $I_{Na}$, $I_{NaK}$, and $I_{K1}$, increase, while $I_{Nab}$ decreases AP amplitude. The major contributing factor to $[Ca^{2+}]_i$ transient is the L-type Ca\textsuperscript{2+} current (Fig. 2.11E). Four other currents, the Na\textsuperscript{+}/Ca\textsuperscript{2+} exchanger current, $I_{NaCa}$, the background Ca\textsuperscript{2+} current, $I_{Ca}$, the Na\textsuperscript{+}/K\textsuperscript{+} pump current, $I_{NaK}$, and the background Na\textsuperscript{+} current, $I_{Nab}$, also made some noticeable contributions. $[Ca^{2+}]_i$ increased with an increase in $I_{Ca}$ and $I_{Na}$ and decreased with an increase in $I_{NaCa}$ and $I_{NaK}$.

Upon application of 1 µM isoproterenol, APD\textsubscript{25} and APD\textsubscript{50} were affected by the same four major currents, $I_{CaL}$, $I_{Kto,f}$, $I_{K,slow1}$, and $I_{K,slow2}$, but in different proportions. For APD\textsubscript{25}, the effect of $I_{Kto,f}$ was not changed, but the contribution of $I_{CaL}$, $I_{K,slow1}$, and $I_{K,slow2}$ increased (Fig. 2.11F). In APD\textsubscript{50}, the role of $I_{Kto,f}$ decreased, but $I_{CaL}$, $I_{K,slow1}$, and $I_{K,slow2}$ increased their effects (Fig. 2.11G).
Nine currents affect APD\(_{90}\) after the application of 1 µM isoproterenol (Fig. 2.11H). Among them, eight currents have similar action as in control, and one additional current, \(I_{p(Ca)}\), also increased the tendency to shorten APD\(_{90}\). The major players and their proportional contributions that affect AP amplitude and \([Ca^{2+}]_i\) transient after stimulation of the \(\beta\)-adrenergic signaling system are the same as in control (Fig. 2.11, I and J).

2.3 Discussion

This dissertation introduces a novel and comprehensive compartmentalized mathematical model that simulates APs from the mouse right atrial myocytes with developed TATS. The model includes three compartments (caveolae, extracaveolae, and cytosol) related to \(\beta_1\)- and \(\beta_2\)-adrenergic signaling systems, as well as compartmentalized Ca\(^{2+}\) dynamics that describe local control of Ca\(^{2+}\)-induced Ca\(^{2+}\) release (Fig. 2.1). Using this model, we described several differences in APs and Ca\(^{2+}\) dynamics between atrial and ventricular myocytes and the role of atrial-specific currents \(I_{CaT}\) and \(I_{K,Ca}\) in shaping the AP and \([Ca^{2+}]_i\) transients. Finally, we performed a sensitivity analysis to demonstrate the variability of the AP and \([Ca^{2+}]_i\) transients due to the cell-to-cell variability of ionic currents.

2.3.1 Differences in AP and Ca\(^{2+}\) Dynamics Between Atrial and Ventricular Myocytes

Experimental investigations demonstrated significant differences in the mechanism of AP and \([Ca^{2+}]_i\) transient generation between atrial and ventricular myocytes (Ng et al., 2010). Specifically, ventricular myocytes are larger in size, have, in general, a more prolonged AP and a different set of repolarizing currents, and generate larger \([Ca^{2+}]_i\) transients and contraction force. For example, in the rabbit heart, atrial AP has a triangular shape, while ventricular AP has a prominent depolarization plateau (Giles & Imaizumi, 1988). In addition, atrial AP has a shorter duration, a larger transient outward K\(^+\) current, and a larger Ca\(^{2+}\)-dependent K\(^+\) current (Giles &
In humans, atrial AP also has a larger transient outward K⁺ current, smaller Na⁺/Ca²⁺ exchanger and Na⁺-K⁺ pump currents, a smaller \( I_{K1} \) current, and a decreased availability of the ultra-rapidly activating K⁺ current (which is absent in human ventricular myocytes) (Grandi et al., 2011). Therefore, mathematical models have been developed both for atrial and ventricular myocytes for these and other species to simulate these differences (Lindblad et al., 1996; Nygren et al., 1998; Ramirez et al., 2000; Shannon et al., 2004; ten Tusscher et al., 2004; Winslow et al., 1999).

Similar to other species, there are differences between mouse atrial and ventricular APs due to the differences in the cell size, magnitude of the L-type Ca²⁺ current, expression of Na⁺-K⁺ and SERCA pumps, calsequestrin, and phospholamban, as well as atrial-specific ionic currents, \( I_{CaT} \) and \( I_{K,Ca} \) (Curran et al., 2015; Li et al., 2009). As a result, mouse atrial myocytes have a longer APD and possess smaller \([Ca^{2+}]_i\) transients.

Our model incorporates all of the AP differences between atrial and ventricular mouse myocytes mentioned above. The model also considers differences in geometries and expression of Ca²⁺ handling proteins. As a result, the model was able to reproduce the much higher SR Ca²⁺ load found experimentally by Brandenburg et al. (2016) in mouse atrial versus ventricular myocytes. It should be noted that such differences in the SR load between atrial and ventricular myocytes are not unique for mice. Similar differences were found in the rat (Walden et al., 2009) and fish hearts (Haverinen & Vornanen, 2009). Our model also predicted higher Ca²⁺ fluxes inside the atrial myocytes than ventricular cells, which future experiments can verify.

The model also simulated experimental findings on the effects of β-adrenergic stimulation by isoproterenol, including APD prolongation (Fig. 2.6 and Tables 2.2-2.3). Simulated APD₅₀ prolongation (≈43.5%) was close to that observed experimentally (≈53%) (Hua et al., 2015). On
the other hand, simulated APD$_{90}$ prolongation (~9.2%) was somewhat smaller than experimental values (~28%) (Hua et al., 2015). The effect of β-adrenergic stimulation in mouse atrial myocytes was similar to that in ventricular cells, which also led to AP prolongation (Fig. 2.6). Simulations with the application of 1 µM isoproterenol to the mouse atrial myocytes resulted in 2.85 fold larger [Ca$^{2+}$]$_i$ transients compared to unstimulated cells. This value is within the range that was observed in experiments (1.5 to 3.1-fold increase) (Babu et al., 2007; Li et al., 2005). The model also closely simulated an experimentally found increase in the magnitude of the T-type Ca$^{2+}$ current upon stimulation of β-adrenoceptors (Fig. 2.3; (Li et al., 2012)). It also reproduced the time course of phosphorylation and the fraction of phosphorylated channels of the small-conductance Ca$^{2+}$-activated K$^+$ current (Fig. 2.4).

2.3.2 The T-Tubular System in Atrial Myocytes

Our model was designed specifically for right atrial myocytes with developed TATS. The presence and functional role of the atrial t-tubule system has been debatable for a long time. However, recent experimental studies clearly showed the functional presence of both transversal and axial tubules (i.e., TATS) in the atria of various species, including mice, rats, rabbits, sheep, pigs, horses, and humans (Brandenburg et al., 2016; Brandenburg et al., 2018; Dibb et al., 2009; Frisk et al., 2014; Glukhov, Balycheva, et al., 2015; Richards et al., 2011). It was shown that TATS is likely heterogeneously expressed throughout the atria: whole atria TATS staining indicates that myocytes with TATS seem to be localized within the muscle bundles of the atrial appendages (Glukhov, Balycheva, et al., 2015). In general, atrial TATS is sparse and less regular when compared with the well-organized t-tubule system in ventricular myocytes as assessed both in situ (Wei et al., 2010) and in vitro (Smyrnias et al., 2010). Moreover, a dense network of axial components may significantly contribute to the synchronization of subcellular Ca$^{2+}$ release.
compensating for the irregular organization of atrial TATS (Brandenburg et al., 2016; Brandenburg et al., 2018; Yue et al., 2017). As the first step in modeling mouse atrial myocytes, we have chosen myocytes located within the right atrial appendage and possessed a developed TATS. We did not discriminate axial tubules from transversal ones and considered TATS as a part of the sarcolemma membrane with a similar distribution to the three compartments used in the model. Modeling atrial myocytes that do not have TATS and where subcellular Ca$^2+$ release relies on Ca$^2+$ diffusion from the submembrane regions towards the cell interior would require a different computational approach (Koivumäki et al., 2011).

2.3.3 The Role of the T-type Ca$^2+$ Current in Shaping AP and Ca$^2+$ Dynamics

The T-type Ca$^2+$ current in cardiac cells was found in 1985 (Bean, 1985; Nilius et al., 1985). It was present in the whole heart during the embryonic stage but mainly remained in sinoatrial and atrioventricular nodal cells and in atrial and Purkinje cells in adult hearts (Li et al., 2018; Vassort et al., 2006). The T-type Ca$^2+$ current has a smaller amplitude than the L-type Ca$^2+$ current and is activated at more hyperpolarized voltages than the L-type Ca$^2+$ current. For a long time, it was considered that the T-type Ca$^2+$ current plays a minor role in cardiac repolarization and Ca$^2+$ dynamics. It was also difficult to study the properties of the T-type Ca$^2+$ current due to the difficulty of its separation from the L-type Ca$^2+$ current (Vassort et al., 2006).

The T-type Ca$^2+$ current plays multiple physiological roles in the heart. It was shown that $I_{CaT}$ is expressed in pacemaker myocytes, including the sino-atrial and atrioventricular nodes and Purkinje fibers, and thus might be involved in heart rhythm regulation in mice and other species (Mesirca et al., 2015). Li et al. (2018) have shown that β-adrenergic stimulation affects the heart rhythm by increasing $I_{CaT}$ magnitude in sino-atrial nodal cells. In embryonic hearts, $I_{CaT}$ may play
a role in sustaining ventricular automaticity together with the hyperpolarization-activated inward current $I_f$ (Niwa et al., 2004).

Our simulations showed that $I_{CaT}$ affects the later phase of the mouse atrial AP repolarization (Fig. 2.8A). We found that the block of $I_{CaT}$ resulted in the shortening of $APD_{75}$ and $APD_{90}$ by 18% and 14%, respectively. Simulation of the overexpression of the T-type Ca$^{2+}$ channels resulted in a dramatic prolongation of $APD_{75}$ and $APD_{90}$ by 3-fold and 2-fold, respectively. These results agree with the experimental observation of APD prolongation in mouse ventricular myocytes overexpressing T-type Ca$^{2+}$ channels (Jaleel et al., 2008).

The late phase of cardiac repolarization is an important phase as it is mostly responsible for the APD and the length and variation of the refractory period in cardiac cells and tissues (Trenor et al., 2017). APD affects the degree of inactivation of the voltage-activated Na$^+$, Ca$^{2+}$, and K$^+$ currents in cardiac cells, as they have different time constants of inactivation. This results in different recovery times of these channels, from inactivation to participation in the generation of subsequent APs. The T-type Ca$^{2+}$ current, affecting this phase, plays a significant role in regulating $APD_{75}$ and $APD_{90}$. Prolongation of the AP due to the increased magnitude of $I_{CaT}$ provides an antiarrhythmic effect for re-entrant arrhythmias. At the same time, the increased expression of $I_{CaT}$ increases susceptibility to triggered arrhythmias through early afterdepolarizations. For example, $I_{CaT}$, while absent in healthy adult ventricles, was detected in ventricular cells of several animal models of cardiac hypertrophy with pronounced re-entrant and triggered arrhythmias, where it can play both pro- and antiarrhythmic roles (Martínez et al., 1999; Nerbonne & Kass, 2005).

There are some controversies in the interpretation of the role of the T-type Ca$^{2+}$ current in triggering Ca$^{2+}$-induced Ca$^{2+}$ release. It was found that $I_{CaT}$ provides an additional Ca$^{2+}$ influx into the cardiac cell (Sipido et al., 1998). However, it is quite unclear whether it triggers Ca$^{2+}$ release
from the SR. Sipido et al. (1998) found that the T-type Ca\(^{2+}\) current can trigger Ca\(^{2+}\) release in guinea pig ventricular myocytes. However, the T-type Ca\(^{2+}\) current is less efficient than the L-type Ca\(^{2+}\) current in triggering Ca\(^{2+}\) release. On the other hand, the Na\(^+\)/Ca\(^{2+}\) exchanger was proposed as a trigger of Ca\(^{2+}\) release (Lipp & Niggli, 1994). In addition, the localization of the T-type Ca\(^{2+}\) channels apart from the Ca\(^{2+}\) release channels favors a minor role of the T-type Ca\(^{2+}\) current in Ca\(^{2+}\) release (Jaleel et al., 2008; Markandeya et al., 2011).

Using the presented mathematical model, we investigated the role of \(I_{\text{CaT}}\) in triggering Ca\(^{2+}\) release in mouse atrial myocytes. Although \(I_{\text{CaT}}\) contributed to \([\text{Ca}^{2+}]_i\) transient, we found that it did not produce Ca\(^{2+}\)-induced Ca\(^{2+}\) release when \(I_{\text{CaL}}\) was blocked. In addition, we have shown that the hypothetical re-localization of the T-type Ca\(^{2+}\) channels to dyadic space did not change significantly \([\text{Ca}^{2+}]_i\) transient.

### 2.3.4 The Role of \(I_{\text{K,Ca}}\) Current in Generating AP and Ca\(^{2+}\) Dynamics

The small conductance Ca\(^{2+}\)-dependent K\(^+\) current, \(I_{\text{K,Ca}}\), is present in both atrial and ventricular myocytes (Tuteja et al., 2005). Some studies demonstrated that \(I_{\text{K,Ca}}\) contributes to cardiac repolarization under high Ca\(^{2+}\) conditions when the block of \(I_{\text{K,Ca}}\) prolongs APD (Xu et al., 2003). Other studies demonstrated no effect of this current on the AP shape both in atrial and ventricular myocytes of different species under normal physiological conditions (Nagy et al., 2009).

To address this issue, we simulated the effects of \(I_{\text{K,Ca}}\) block on repolarization of the mouse atrial AP (mouse ventricular myocytes do not express a significant amount of SK channels) under normal and high Ca\(^{2+}\) conditions. Our data indicate that there is no effect of \(I_{\text{K,Ca}}\) block on mouse atrial AP under normal physiological conditions (Fig. 2.9A). However, some effect was noticed upon stimulation of the β-adrenergic signaling system with 1 μM isoproterenol, where a small
APD prolongation was observed (Fig. 2.9A). Even a larger APD prolongation was obtained in simulations of $I_{K,Ca}$ block at high Ca$^{2+}$ conditions (Fig. 2.9C). This allowed us to conclude that $I_{K,Ca}$ block can prolong APD and potentially be both pro-arrhythmic (for early afterdepolarization-induced arrhythmias) and anti-arrhythmic (for reentrant arrhythmias).

While no significant effects of $I_{K,Ca}$ block were observed in atrial myocytes under normal physiological conditions, a connection between $I_{K,Ca}$ expression, and atrial fibrillation were identified in several species (see review by Diness et al. (2015)). Experimental findings show that both up- and downregulation of $I_{K,Ca}$ can lead to atrial fibrillation. Diness et al. (2015) have shown that inhibition of $I_{K,Ca}$ resulted in the termination of atrial fibrillation in guinea pigs, rabbits, and rat hearts. On the other hand, knock out of $I_{K,Ca}$ has been shown to be pro-arrhythmic (Li et al., 2009). Further studies are necessary to investigate the role of $I_{K,Ca}$ current in mechanisms of atrial arrhythmias.

2.3.5 Sensitivity Analysis: On Mice and Men

To evaluate the model performance for multiple atrial cells, in which the conductance of ionic currents can vary from cell to cell, we performed a sensitivity analysis. In these simulations, we increased and decreased each of the model ionic currents by 20% and investigated the parameters of the resulting APs and [Ca$^{2+}$]$_i$ transients. A similar analysis was performed by Sobie using the Kurata et al.’s model of the human ventricular myocytes (Kurata et al., 2005; Sobie, 2009) and by Kernik et al. for the human-induced pluripotent stem cell-derived cardiomyocytes (iPSC-CMs) (Kernik et al., 2019). In addition, Grinshpon and Bondarenko (2016) performed a sensitivity analysis with a 5% increase in the conductance of major ionic currents for the model of mouse ventricular myocytes.
Our mouse atrial mathematical model simulations illuminated major players affecting APD\textsubscript{25} and APD\textsubscript{50}: $I_{K\text{to},f}$, $I_{K\text{slow1}}$, $I_{K\text{slow2}}$, and $I_{CaL}$. These currents are also the major contributing factors to APD\textsubscript{25} and APD\textsubscript{50} in mouse ventricular myocytes (Grinshpon and Bondarenko, 2016), where $I_{K\text{ur}}$ stands for $I_{K\text{slow1}}$ and $I_{K\text{slow2}}$. For APD\textsubscript{90}, the major contributing currents only partially overlap between mouse atrial and ventricular myocytes. The similar major players are $I_{K1}$ and $I_{K\text{ur}}$ (or sum of $I_{K\text{slow1}}$ and $I_{K\text{slow2}}$ in mouse atrial myocytes) and, to a lesser extent, $I_{Na\text{Ca}}$. However, in mouse ventricular myocytes, an additional contribution to APD\textsubscript{90} comes mostly from $I_{CaL}$ and $I_{NaK}$, while in mouse atrial myocytes $I_{CaT}$, $I_{Ks}$, $I_{Na}$, and $I_{Na\text{b}}$ contribute the most. Interestingly, $I_{CaL}$, $I_{K1}$, and $I_{Kr}$ are the major APD\textsubscript{90} affecting currents in iPSC-CMs, and two of them overlap with mouse ventricular myocytes (Grinshpon and Bondarenko, 2016; Kernik et al., 2019). $I_{CaL}$ and $I_{K1}$ currents were also among the major players in the Kurata et al. (2005) model of human ventricular myocytes, together with $I_{Kr}$, $I_{Ks}$, $I_{Na\text{Ca}}$, and $I_{NaK}$ currents (Sobie, 2009). Taken together, we can observe similarities in the major role of $I_{K1}$ in APD\textsubscript{90} both for mice and men, regardless of the myocyte type (atrial, ventricular). In mouse atrial myocytes, the effect of $I_{CaL}$ is replaced by $I_{CaT}$, but $I_{CaL}$ is still one of the major players in mice and men ventricular myocytes. It can also be suggested that the role of $I_{Kr}$ in the human ventricular myocytes is similar to the role of $I_{K\text{ur}}$ in the mouse ventricular myocyte model (or the sum of $I_{K\text{slow1}}$ and $I_{K\text{slow2}}$ in the mouse atrial cell model).

In both mouse’s atrial and ventricular myocytes, AP amplitude depends mainly on the four ionic currents, $I_{Na}$, $I_{Na\text{b}}$, $I_{K1}$, and $I_{NaK}$ (this study and Grinshpon and Bondarenko (2016)). Two of these currents, $I_{Na}$ and $I_{K1}$, also provided significant contributions to the changes in AP amplitude for iPSC-CMs (Kernik et al., 2019).

Finally, our sensitivity analysis demonstrated the major contributing currents to the change in [Ca\textsuperscript{2+}]	extsubscript{i} in mouse atrial myocytes. The outstanding contributor is the L-type Ca\textsuperscript{2+} current (Fig.
2.11), which is also the outstanding contributor for the models of a mouse (Grinshpon & Bondarenko, 2016) and human (Sobie, 2009) ventricular myocytes. Other major contributing currents for the mouse atrial myocyte model are $I_{\text{NaCa}}$, $I_{\text{Cab}}$, $I_{\text{NaK}}$, and $I_{\text{NaB}}$. When compared to the Kurata et al. human ventricular cell model (Kurata et al., 2005; Sobie, 2009), the currents contributing to $[\text{Ca}^{2+}]_i$ change are essentially the same as for the mouse atrial myocytes (i.e., $I_{\text{NaCa}}$, $I_{\text{NaK}}$, and $I_{\text{Cab}}$). This result gives an interesting conclusion about the similarity in the regulation of $[\text{Ca}^{2+}]_i$ in the mouse atrial and human ventricular myocytes.

### 2.3.6 Model Limitations

While we developed a quite comprehensive mathematical model of mouse atrial myocytes that describes a significant amount of experimental data well, it has several limitations. First, the model was developed for right atrial myocytes with the developed TATS and cannot be applicable to atrial myocytes without TATS. Second, the model has a limited number of compartments related to $\text{Ca}^{2+}$ dynamics (dyadic space, bulk cytosol, and sarcoplasmic reticulum) and β-adrenergic signaling system (caveolae, extracaveolae, cytosol). Therefore, it does not account for the effects of submembrane $\text{Ca}^{2+}$ concentration in detail. At the same time, the model takes them into account through the “effective” parameters of the ionic currents and transporters. Third, not all model parameters were measured directly in experiments and thus were adjusted to fit the experimental data. Fourth, the model consists of several compartments described by ordinary differential equations; it does not include spatial effects that require partial differential equations.
3 MATHEMATICAL MODEL OF MOUSE VENTRICULAR MYOCYTES WITH HEART FAILURE

3.1 Materials and Methods

To develop a mathematical model of the failing ventricular myocytes, we used experimental data from the mouse ventricular myocytes after a procedure of TAC when the heart failure was evident by a significant change in ejection fraction to ~40% or heart fractional shortening to ~20% (Nie et al., 2019; Ljubojevic-Holzer et al., 2020). In most cases, it takes eight weeks or more for the mouse heart to develop heart failure after TAC.

3.1.1 Control Model Modifications

We modified a previously published compartmentalized mathematical model for ventricular myocytes from mice (Rozier & Bondarenko, 2017), which is based on the previously published models of mouse ventricular myocytes (Bondarenko et al., 2004; Petkova-Kirova et al., 2012; Bondarenko, 2014; Rozier & Bondarenko, 2017). The schematic drawing of the model is shown in Fig. 3.1.

First, the ultra-rapidly activating K+ current, \( I_{kur} \), was reformulated to include two components, \( I_{K,slow1} \) (encoded by \( K_v1.5 \) channels and also referred to as \( I_{kur} \) component), and \( I_{K,slow2} \) (encoded by \( K_v2.1 \) channels). The currents \( I_{K,slow1} \) and \( I_{K,slow2} \) demonstrate a different response to the activation of \( \beta_1 \)- and \( \beta_2 \)-adrenergic signaling systems (Kodirov et al., 2004; Wilson et al., 1994; Zhou et al., 2003; Zhou et al., 2012; Asfaw et al., 2020) (Appendix B). These currents have approximately equal amplitudes in mouse ventricular myocytes (Zhou et al., 2003). The effect of PKA on \( K_v2.1 \) is quite small, as it was shown experimentally (Wilson et al., 1994; Zhou et al., 2012). Therefore, only \( I_{K,slow1} \) is affected by isoproterenol in our revised mathematical model. The response of the model currents \( I_{K,slow1} \) and \( I_{K,slow2} \) to a 200 ms depolarization pulse from a holding
potential –80 mV to +40 mV is shown in Fig. 3.2A. In the simulated results, we see an increase in $I_{K_{slow1}}$ magnitude upon the application of 1 μM isoproterenol, while $I_{K_{slow2}}$ is unchanged.

**Figure 3.1** A schematic representation of the mouse ventricular myocyte.

The cell consists of three compartments (caveolae, extracaveolae, and cytosol) related to the combined $\beta_1$- and $\beta_2$-adrenergic signaling systems. The subspace volume ($V_{ss}$) is localized in the extracaveolar compartment. The biochemical portions of the combined $\beta_1$- and $\beta_2$-adrenergic signaling systems are the $\beta_1$-adrenergic receptors ($\beta_1$-AR), the $\beta_2$-adrenergic receptors ($\beta_2$-AR), the $\alpha$-subunit of stimulatory $G$-protein ($G_{s\alpha}$), the $\alpha$-subunit of inhibitory $G$-protein ($G_{i\alpha}$), the $\beta\gamma$-subunit of $G_s$ and $G_i$ ($G_{s\beta\gamma}$), the adenylyl cyclases (AC5/6 or AC4/7, respectively), the phosphodiesterase (PDE2, PDE3, or PDE4, respectively), the cyclic AMP (cAMP), regulatory (R) and catalytic (C) subunits of protein kinase A, the protein kinase A inhibitor (PKI), the $G$-protein-coupled receptor kinase (GRK2), the protein phosphatases (PP1 and PP2A, respectively), the inhibitor-1 (I-1). Targets of the combined $\beta_1$- and $\beta_2$-adrenergic signaling systems are in the caveolae (including the fast Na$^+$ current ($I_{Na}$), the late Na$^+$ current ($I_{NaL}$), the L-type Ca$^{2+}$ current ($I_{CaL}$), the Na$^+$/K$^+$ pump ($I_{NaK}$), regulated by phospholemman (PLM), phosphodiesterases PDE2-PDE4, and the time-independent K$^+$ current ($I_{K_{indep}}$)), the extracaveolae (including the L-type Ca$^{2+}$ current ($I_{CaL,ecav}$, two equal components, one inside $V_{ss}$, one outside $V_{ss}$), the rapidly recovering transient outward K$^+$ current ($I_{K_{to,f}}$), the rapidly activating, slowly inactivating K$^+$ current ($I_{K_{slow1}}$, encoded by $K_{1.5}$; 4-AP-sensitive), the rapidly activating, slowly inactivating K$^+$ current ($I_{K_{slow2}}$, encoded by $K_{2.1}$; TEA-sensitive), ryanodine receptors (RyRs), and
phosphodiesterases (PDE2, PDE4)), and cytosol (including phospholamban (PLB), troponin I (TnI), and phosphodiesterases PDE2-PDE4). Stimulatory links are shown by black arrows and inhibitory links are shown by red dashed lines with balls of effectors. Other transmembrane currents are the sarcolemmal Ca2+ pump (I_{p(Ca)}, also known as plasmalemmal Ca2+-ATPase, PMCA), the Na+/Ca2+ exchanger (I_{NaCa}), the rapid delayed rectifier K+ current (I_{Kd}), the non-inactivating steady-state voltage-activated K+ current (I_{Kss}), and the Ca2+ and Na+ background currents (I_{Cab} and I_{Nab}), which are not affected by the combined β1- and β2-adrenergic signaling systems. The Ca2+ fluxes are uptake of Ca2+ from the cytosol to the network sarcoplasmic reticulum (NSR) (J_{up}) by the SERCA pump, Ca2+ release from the junctional sarcoplasmic reticulum (JSR) (J_{rel}) through the RyRs, Ca2+ flux from the subspace volume to the cytosol (J_{xfer}), and Ca2+ leak from the SR to the cytosol (J_{leak}). [Ca2+]i, [Na+]i, and [K+]i are the intracellular Ca2+, Na+, and K+ concentrations in the caveolae, extracaveolae, and cytosol; [Ca2+]o, [Na+]o, and [K+]o are the extracellular Ca2+, Na+, and K+ concentrations. Proteins with characteristics that are modified in the mouse failing ventricular myocyte are shown in green. Modified from (Bondarenko, 2014).

Second, we added a dependence of the sarcoplasmic reticulum Ca2+ leak on the stimulation level of the β-adrenergic signaling system (Appendix B). Our simulations showed similar dependences of the concentration of catalytic subunit of PKA in the extracaveolar compartments ([C]_{ecav}) and the intracellular [Ca2+]i, transients, which reflects the SR Ca2+ load on the isoproterenol concentration. Therefore, we set a linear dependence of the leak rate ν2 on [C]_{ecav} as

$$ν2 = (1.2701 + 3.06045[C]_{ecav}) \cdot 10^{-2} \text{ (s}^{-1})$$

The numerical coefficients in this dependence were adjusted to fit the experimental data on the SR Ca2+ load without and with the application of the saturating isoproterenol concentration of 1 μM (Fig. 3.2B). It can be seen from Fig. 3.2B that the previously published model (Rozier & Bondarenko, 2017) gave a much higher increase in the SR Ca2+ load upon application of 1 μM isoproterenol as compared to the modified model, which agrees with the typical experimental data (Kashimura et al., 2010; Liu et al., 2011).
Third, we incorporated the late Na\(^+\) current, I\(_{\text{NaL}}\), in the model due to its possible important role in the failing ventricular myocytes. We assume that I\(_{\text{NaL}}\) is produced by the cardiac Na\(^+\) channel isoform (Na\(_V\)1.5), operating in special gating modes (Maltsev & Undrovinas, 2008).

Figure 3.3 demonstrates the experimental normalized current-voltage (Fig. 3.3A) and steady-state
inactivation (Fig. 3.3B) relationships, as well as voltage dependence of normalized conductance $G/G_{\text{max}}$ (Fig. 3.3B). We adjusted the Hund-Rudy model of $I_{\text{NaL}}$ (Hund & Rudy, 2004) (Table 3.1) to fit the experimental data for mice, which are shown in Fig. 3.3A-B. Because of the low accuracy of the measurements of the magnitude of $I_{\text{NaL}}$, we used the average from five experimental data to define the model conductance of the current in control and the average of two experimental data on the effects of saturating concentrations of isoproterenol (Fig. 3.3C). Finally, our model fits the kinetics of $I_{\text{NaL}}$ upon application of isoproterenol, as shown in Fig. 3.3D.

**Figure 3.3** The late Na⁺ current $I_{\text{NaL}}$ in mouse ventricular myocytes.
Normalized current-voltage relationships. Experimental data shown by symbols with error bars were obtained by Signore et al. (2015) for mice and by Undrovinas et al. (2013) for dogs. Simulated data are shown by solid lines (data are obtained by two-pulse protocol, holding potential is $-140 \text{ mV}$, first pulse duration is 2000 ms for voltages from $-140$ to $+50 \text{ mV}$ in 10 mV steps, the second pulse duration is 100 ms at a voltage $-30 \text{ mV}$).

Steady-state inactivation and normalized conductance $G/G_{\text{max}}$. Experimental data shown by symbols with error bars were obtained by Philippaert et al. (2021) and Signore et al. (2015) for mice and by Undrovinas et al. (2013) for dogs. Filled and unfilled symbols plot experimental data for steady-state inactivation and normalized conductance $G/G_{\text{max}}$, respectively. Simulation data for steady-state inactivation and normalized conductance $G/G_{\text{max}}$, are shown by the solid and dashed lines, respectively.

The maximum amplitudes of the late Na$^+$ current. Experimental data for control by Signore et al. (2015), Rivaud et al. (2018), Philippaert et al. (2021), Fabritz et al. (2010), El Refaey et al. (2019), and averaged experimental data are shown by error bars; simulation data for control is shown by a bar without errors. Experimental data after the application of isoproterenol (I$\text{o}$) by Fabritz et al. (2010) (1 $\mu\text{M}$ isoproterenol) and El Refaey et al. (2019) (100 nM isoproterenol) and the averaged experimental data are shown by error bars. Simulation data after the application of 1 $\mu\text{M}$ isoproterenol is shown by a bar without errors bars.

Time course of the activation of the fast Na$^+$ current upon application of 0.1 $\mu\text{M}$ isoproterenol. Experimental data from Matsuda et al. (1993) obtained for the normalized peak I$_{\text{Na}}$ in rabbit ventricular myocytes is shown by closed circles. Data is obtained with 40-ms pulses from a holding potential of $-100 \text{ mV}$ to $-30 \text{ mV}$ at a stimulation frequency of 0.2 Hz. A solid line shows the time course of simulated data on relative $I_{\text{Na}}$ phosphorylation upon application of 0.1 $\mu\text{M}$ isoproterenol.

These modifications did not significantly change model action potential and $[\text{Ca}^{2+}]_i$ transient in control but significantly changed their response to isoproterenol (Fig. 3.2C-D). The action potential demonstrated larger prolongation by the revised control model, which fits better with the typical experimental data after isoproterenol application (Tong et al., 2006; Wang et al., 2008). The revised model also demonstrates a smaller increase in $[\text{Ca}^{2+}]_i$ transient when compared to the previous model by Rozier & Bondarenko (2017) due to the higher $\text{Ca}^{2+}$ leak from the SR and smaller SR load after isoproterenol application. These observations are consistent with most experimental findings (see, for example, Kashimura et al., 2010; Liu et al., 2011; Wang et al., 2008).
**Table 3.1** Differences between the models of mouse ventricular myocytes from control (sham) hearts and the hearts with heart failure after TAC.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Definition</th>
<th>Control</th>
<th>Heart failure</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V^{cell}$</td>
<td>Cell volume</td>
<td>38.00×10⁻⁶ µl</td>
<td>68.40×10⁻⁶ µl</td>
</tr>
<tr>
<td>$V^{cyt}$</td>
<td>Cytosolic volume</td>
<td>25.84×10⁻⁶ µl</td>
<td>47.71×10⁻⁶ µl</td>
</tr>
<tr>
<td>$V_{JSR}$</td>
<td>Junctional SR volume</td>
<td>0.1200×10⁻⁶ µl</td>
<td>0.1512×10⁻⁶ µl</td>
</tr>
<tr>
<td>$V_{NSR}$</td>
<td>Network SR volume</td>
<td>2.098×10⁻⁶ µl</td>
<td>2.6435×10⁻⁶ µl</td>
</tr>
<tr>
<td>$V_{ss}$</td>
<td>Subspace volume</td>
<td>1.485×10⁻⁹ µl</td>
<td>1.8700×10⁻⁹ µl</td>
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<tr>
<td>$A_{cap}$</td>
<td>Capacitive membrane area</td>
<td>1.534×10⁻⁴ cm²</td>
<td>2.1282×10⁻⁴ cm²</td>
</tr>
<tr>
<td>$v_1$</td>
<td>Maximum RyR channel Ca²⁺ permeability</td>
<td>4,500 s⁻¹</td>
<td>5,625 s⁻¹</td>
</tr>
<tr>
<td>$v_3$</td>
<td>SR Ca²⁺-ATPase maximum pump rate</td>
<td>3.0600×10⁻⁴ µM s⁻¹</td>
<td>2.3562×10⁻⁴ µM s⁻¹</td>
</tr>
<tr>
<td>$\tau_{xfer}$</td>
<td>The time constant for transfer from subspace to the cytosol</td>
<td>8.0 ms</td>
<td>16.0 ms</td>
</tr>
<tr>
<td>$k_a^+$</td>
<td>RyR P₁C₁ – P₀₁ rate constant</td>
<td>6.075 µM⁻⁴ s⁻¹</td>
<td>1.215 µM⁻⁴ s⁻¹</td>
</tr>
<tr>
<td>$k_b^+$</td>
<td>RyR P₀₁ – P₀₂ rate constant</td>
<td>4.05 µM⁻³ s⁻¹</td>
<td>0.81 µM⁻³ s⁻¹</td>
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<tr>
<td>$k_c^+$</td>
<td>RyR P₀₁ – Pₐ₂ rate constant</td>
<td>9.0 s⁻¹</td>
<td>1.8 s⁻¹</td>
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<tr>
<td>$k_{cp}^+$</td>
<td>RyR P₀₁p – Pₐ₂p rate constant</td>
<td>50$k_c^+$</td>
<td>5$k_c^+$</td>
</tr>
<tr>
<td>$k_{cp}^-$</td>
<td>RyR Pₐ₂p – P₀₁p rate constant</td>
<td>30$k_c^-$</td>
<td>3$k_c^-$</td>
</tr>
<tr>
<td>$G_{Na}$</td>
<td>Specific maximum conductivity for the fast Na⁺ channel (non-phosphorylated)</td>
<td>14.4 mS/µF</td>
<td>10.08 mS/µF</td>
</tr>
<tr>
<td>$G_{Nap}$</td>
<td>Specific maximum conductivity for the fast Na$^+$ channel (phosphorylated)</td>
<td>18.0 mS/µF</td>
<td>12.6 mS/µF</td>
</tr>
<tr>
<td>$G_{Kto,f}$</td>
<td>Specific maximum conductivity for the rapidly inactivating transient outward K$^+$ current (non-phosphorylated)</td>
<td>0.3846 pA/pF</td>
<td>0.2115 pA/pF</td>
</tr>
<tr>
<td>$G_{Kto,fp}$</td>
<td>Specific maximum conductivity for the rapidly inactivating transient outward K$^+$ current (phosphorylated)</td>
<td>0.3846 pA/pF</td>
<td>0.2115 pA/pF</td>
</tr>
<tr>
<td>$G_{K,slow1}$</td>
<td>Maximum conductance for the rapidly activating, slowly inactivating K$^+$ current (encoded by Kv1.5, non-phosphorylated)</td>
<td>0.1712 mS/µF</td>
<td>0.0856 mS/µF</td>
</tr>
<tr>
<td>$G_{K,slow1p}$</td>
<td>Maximum conductance for the rapidly activating, slowly inactivating K$^+$ current (encoded by Kv1.5, phosphorylated)</td>
<td>0.26654 mS/µF</td>
<td>0.1333 mS/µF</td>
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<tr>
<td>$G_{K,slow2}$</td>
<td>Maximum conductance for the rapidly activating, slowly inactivating K$^+$ current (encoded by Kv2.1, non-phosphorylated)</td>
<td>0.1712 mS/µF</td>
<td>0.0856 mS/µF</td>
</tr>
<tr>
<td>$G_{Kss}$</td>
<td>Maximum conductance for the noninactivating steady-state K$^+$ current</td>
<td>0.0611 mS/µF</td>
<td>0.04583 mS/µF</td>
</tr>
<tr>
<td>$G_{K1}$</td>
<td>Maximum conductance for the time-independent K$^+$ current</td>
<td>0.27 mS/µF</td>
<td>0.2025 mS/µF</td>
</tr>
<tr>
<td>$G_{Cab}$</td>
<td>Maximum background Ca$^{2+}$ current conductance</td>
<td>0.000284 mS/µF</td>
<td>0.0005112 mS/µF</td>
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<tr>
<td>$G_{NaL}$</td>
<td>Maximum conductance for the late Na$^+$ current (non-phosphorylated)</td>
<td>0.01451 mS/µF</td>
<td>0.020314 mS/µF</td>
</tr>
<tr>
<td>$G_{NaLp}$</td>
<td>Maximum conductance for the late Na$^+$ current (phosphorylated)</td>
<td>0.03101 mS/µF</td>
<td>0.043414 mS/µF</td>
</tr>
</tbody>
</table>
### Myocyte Geometry

Experimental data showed that TAC resulted in a significant change in the cell geometry, specifically, increased myocyte size when the heart progressed to heart failure (Rivaud et al., 2017; Bryant et al., 2018; Tamayo et al., 2020). This increase was measured either directly by confocal microscope (Bryant et al., 2018) or by measurements of the cell capacitance assuming linear dependence between the cell volume and cell capacitance (Rivaud et al., 2017; Tamayo et al., 2020). The average data from these measurements gave the failing myocyte volume increase by a factor of ~1.8 compared to the sham myocyte.

Progression to heart failure also led to myocyte dyadic space structure and content changes. The lengths of junctions in failing ventricular myocytes were decreased by 30%, and the volume

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value 1</th>
<th>Value 2</th>
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<tbody>
<tr>
<td>$\tau_{NaL}$</td>
<td>Inactivation time constant for the late Na$^+$ current</td>
<td>0.6 s</td>
<td>0.6 s</td>
</tr>
<tr>
<td>$k_{NaCa}$</td>
<td>Scaling factor for Na$^+$/Ca$^{2+}$ exchanger</td>
<td>275 pA/pF</td>
<td>495 pA/pF</td>
</tr>
<tr>
<td>$[R_{\beta_1}]_{tot}$</td>
<td>Total $\beta_1$-adrenoceptor concentration</td>
<td>0.0103 $\mu$M</td>
<td>0.00515 $\mu$M</td>
</tr>
<tr>
<td>$f_{cav}^{\beta_2}$</td>
<td>Fraction of $\beta_2$-adrenoceptors located in caveolae</td>
<td>0.99</td>
<td>0.5</td>
</tr>
<tr>
<td>$k_{GRK2^+}$</td>
<td>Rate of GRK2 phosphorylation of $\beta_1$- and $\beta_2$-adrenoceptors</td>
<td>0.000243 s$^{-1}$</td>
<td>0.0005589 s$^{-1}$</td>
</tr>
<tr>
<td>$k_{GRK2^-}$</td>
<td>Rate of GRK2 dephosphorylation of $\beta_1$- and $\beta_2$-adrenoceptors</td>
<td>0.0002025 s$^{-1}$</td>
<td>0.00046575 s$^{-1}$</td>
</tr>
<tr>
<td>$[AC]_{tot}$</td>
<td>Total cellular AC concentration</td>
<td>0.02622 $\mu$M</td>
<td>0.02079 $\mu$M</td>
</tr>
<tr>
<td>$f_{AC56,AC47}$</td>
<td>Fraction of AC that is of type 5 or 6</td>
<td>0.74</td>
<td>0.672</td>
</tr>
<tr>
<td>$[PDE4]_{tot}$</td>
<td>Total cellular concentration of PDE4</td>
<td>0.026687 $\mu$M</td>
<td>0.0133435 $\mu$M</td>
</tr>
</tbody>
</table>
density of junctional SR was reduced by 31.6% (Wu et al., 2012). Similar data were obtained by Li et al. (2013), where the volume density of t-tubules coupled to JSR was decreased by ~30%, and the lengths of junctions in failing ventricular myocytes were decreased by ~25%. However, this decrease is not accompanied by the change in the distance between RyRs and t-tubules (van Oort et al., 2011). The experimental data also demonstrated that the volume density of mitochondria did not change in the failing myocytes compared to the control (LaRocca et al., 2020).

Therefore, in our model, we increased myocyte size by a factor of 1.8, from $38.0 \cdot 10^{-6} \mu l$ to $68.4 \cdot 10^{-6} \mu l$. In this case, the capacitive area, which is twice larger than the geometric area, is increased by a factor of 1.4 only, from $1.534 \cdot 10^{-4} \text{ cm}^2$ to $2.1282 \cdot 10^{-4} \text{ cm}^2$. As a 30% volume density reduction of junctions occurs in the cells that increase in volume by a factor of 1.8, the volumes of junctional SR and subspace volume $V_{ss}$ were increased by a factor of 1.26 in failing myocytes. We also increased the volume of network SR by a factor of 1.26. While mitochondria volume density did not change in the failing myocytes, its volume increased by a factor of 1.8. This resulted in an increase in cytosolic volume by a factor $\sim 1.84635$, from $25.84 \cdot 10^{-6} \mu l$ to $47.71 \cdot 10^{-6} \mu l$. The model geometries for control and failing ventricular myocytes are shown in Table 3.1.

### 3.1.3 Redistribution in Microdomains

Experimental data demonstrate changes in the myocyte geometry and the redistribution of the cellular proteins, specifically, the L-type Ca$^{2+}$ channels and β2-adrenergic receptors, in the failing myocytes as compared to control.

Consider first the L-type Ca$^{2+}$ channels. Experimental microstructure and physiological data on the localization of the L-type Ca$^{2+}$ channels in atrial and ventricular myocytes
demonstrated that they moved outside of the dyadic space when heart failure developed (Wu et al., 2012; Glukhov, Balycheva, et al., 2015; Bryant et al., 2018). However, the majority of experiments did not show a decrease in the total cellular I_{CaL} in either rat or mouse failing myocytes (van Oort et al., 2011; Wu et al., 2012; Li et al., 2013; Bryant et al., 2015; Tamayo et al., 2020). They demonstrated mainly the re-distribution of I_{CaL} from t-tubules to the surface. Based on these observations, we did not change the total cellular I_{CaL} in our model for failing myocytes, but we re-distributed I_{CaL} leaving 50% in the dyadic space and moving 50% to extracaveolae domain outside the dyadic space (Fig. 3.1).

Further, we considered the re-distribution of β2-adrenergic receptors (β2-ARs). Experimental data (Nikolaev et al., 2010; Wright et al., 2014) with ventricular myocytes demonstrated that most of β2-ARs in the healthy mouse and rat hearts are localized in the t-tubules (caveolae domain in our model). However, in the failing hearts, β2-ARs are re-distributed to the cell crest (extracaveolae in our model) with approximately equal densities in t-tubules and cell crest. In addition, the application of PTX did not change cAMP generation from β2-ARs in the crest region, indicating that G_{i} proteins are not re-distributed from t-tubules to the crest. Therefore, in our model, we re-distributed β2-ARs to have 50% in caveolae and 50% in extracaveolae and did not change the distribution of G_{i} proteins.

3.1.4 Ion Current Modifications

In the model, we reduced the fast Na\(^+\) current (normalized to the cell capacitance) in failing myocytes by 30% compared to control (sham) myocytes, according to the experimental data by Rivaud et al. (2017) in mouse heart failure after 6 weeks TAC. Experimental data also showed that the channel gating (activation, normalized conductance G/G_{max}, steady-state inactivation, and recovery from inactivation) was not changed. In the experiments, the late Na\(^+\) current was
increased by \(~40\%\) in failing myocytes compared to sham (Glynn et al., 2015; Philippaert et al., 2021). Therefore, we increased \(I_{\text{NaL}}\) in the failing myocytes model by \(40\%\).

The L-type \(\text{Ca}^{2+}\) current \(I_{\text{CaL}}\) was not changed after TAC in the failing myocytes, but we re-distributed \(I_{\text{CaL}}\) in the extracaveolae compartment to allow 50% of the current to enter into dyadic space \((V_{ss})\) and 50% of the current into extracaveolae compartment, but outside the dyadic space (Bryant et al., 2018).

The rapidly recovering transient outward \(\text{K}^+\) current \(I_{\text{Kto,f}}\) was found to be reduced in failing ventricular myocytes by \(45\%\) compared to sham cells (Tamayo et al., 2020). Experimental data from sham mice and mice after 9 weeks of TAC demonstrated a decrease in the sum of \(I_{\text{K,slow1}}\) and \(I_{\text{K,slow2}}\) by \(~50\%\) (Tamayo et al., 2020). In the model, we decreased the conductance of both currents by \(50\%\) as well. According to the experimental data, the steady-state non-inactivating \(\text{K}^+\) current \(I_{\text{Kss}}\) was reduced only by \(25\%\) in TAC mice (Tamayo et al., 2020). We also reduced the time-independent \(\text{K}^+\) current, \(I_{\text{K1}}\), by \(25\%\) to fit the experimental data from sham and TAC mouse ventricular myocytes (Marionneau et al., 2008). However, the data on \(I_{\text{K1}}\) by Marionneau et al. (2008) was obtained only after one week of TAC, and no data is available for the longer time after TAC.

Experimental data showed that the expression of the protein responsible for the \(\text{Na}^+/\text{Ca}^{2+}\) exchanger was increased by \(~90\pm30\%\) after eight weeks of TAC (Lu et al., 2011). We increased \(I_{\text{NaCa}}\) by \(80\%\) in our failing myocyte model to keep the change within the experimental error and to fit the SR \(\text{Ca}^{2+}\) load data. We also increased the background \(\text{Ca}^{2+}\) current \(I_{\text{Cab}}\) by a factor of 1.8 in the failing cells to compensate for the \(\text{Ca}^{2+}\) loss due to an increased function of \(I_{\text{NaCa}}\) by the same factor in failing cardiomyocytes and to keep a similar diastolic \([\text{Ca}^{2+}]_i\) concentration, as it was observed in most experimental data of the failing hearts with reduced ejection fraction (Eisner et
al., 2020). No significant changes were observed in $I_{\text{NaK}}$ pump in the experiments (Wansapura et al., 2011; Correll et al., 2014); therefore, we did not change the current in the failing myocyte model.

### 3.1.5 Modifications of $\text{Ca}^{2+}$ Handling Proteins

Experimental data with mice shows that heart failure leads mostly to modification of the expression of SERCA pump, which is decreased on average to 70.2±8.2% in mice with heart failure as compared to sham mice (Ito et al., 2000; Lu et al., 2011; Vinet et al., 2012; Tamayo et al., 2020). Ventricular myocytes from TAC mice also demonstrate increased $\text{Ca}^{2+}$ leak from the SR by ~30% (van Oort, Respress, et al., 2010); however, the author claimed that the change was not significant. The phospholamban, calsequestrin, and ryanodine receptors expressions do not significantly change in TAC mice (Kho et al., 2011; Vinet et al., 2012). Collectively, these changes resulted in a reduction of the SR content on average to 83±16% as compared to sham mice (Bryant et al., 2018; Nie et al., 2019; Tamayo et al., 2020) and decreased $[\text{Ca}^{2+}]_{i}$ transients on average to 72.8±7.0% (experimental data from different sources varied from 65% to 87% decrease) (Li et al., 2013; Nie et al., 2019; Cai et al., 2019; Tamayo et al., 2020; Ljubojevic-Holzer et al., 2020; Chen et al., 2020).

To simulate these changes in $\text{Ca}^{2+}$ handling proteins in failing myocytes, we decreased the SR $\text{Ca}^{2+}$ maximum pump rate in failing myocytes to 77% of its control value to stay within the experimental range (Ito et al., 2000; Lu et al., 2011; Vinet et al., 2012; Tamayo et al., 2020). $\text{Ca}^{2+}$ SR leak rate constant was increased by a factor of 1.15 for the heart failure model to account for the intermediate value between 0% and 30% observed by van Oort, Respress, et al. (2010). We also changed several rate constants responsible for RyR gating and the time constant for $\text{Ca}^{2+}$
transfer from subspace to myoplasm $\tau_{\text{xfer}}$ to simulate delayed Ca$^{2+}$ release (see Table 3.1), as was observed in the experiments (Ljubojevic-Holzer et al., 2020).

### 3.1.6 Modifications of β-adrenergic Signaling System

Experimental data on the mouse hearts and the hearts of other species demonstrate a decrease in the expression of $\beta_1$-adrenoceptors but no significant change in the expression of $\beta_2$-adrenoceptors in the failing hearts (Pelá et al., 1990; Ungerer et al., 1993; Yoo et al., 2009; Norman, 2016). We used the data by (Esposito et al., 2002; Nienaber et al., 2003; Tachibana et al., 2005) to evaluate a decrease in the expression of $\beta_1$-adrenoceptors in the failing mouse hearts after TAC. However, these data presented only a decrease in the total $\beta_1$- and $\beta_2$-adrenoceptor densities. Therefore, for estimation purposes, we assumed that the density of $\beta_1$-adrenoceptors is two-fold higher than the $\beta_2$-adrenoceptor density in control (Hilal-Dandan et al., 2000), and heart failure does not change the density of $\beta_2$-adrenoceptors. Our estimations from the data by (Esposito et al., 2002; Nienaber et al., 2003; Tachibana et al., 2005) showed that the density of $\beta_1$-adrenoceptors should decrease to 51.5% of the control value, which is consistent with a percentage of decrease in other species (Pelá et al., 1990; Ungerer et al., 1993). Therefore, in the failing mouse ventricular myocytes model, we decreased the density of $\beta_1$-adrenoceptors by 2-fold and did not change the density of $\beta_2$-adrenoceptors. We also did not change the density of $G_s$ and $G_i$ proteins following the data by (Feldman et al., 1988) for humans and by (Norman 2016) for mice.

Experimental data on adenylyl cyclase activity in mice showed a decrease in the activities of AC5/6 in the failing hearts to 72±13% without application of isoproterenol and to 60±12% after application of 1 μM isoproterenol as compared to control myocytes (Esposito et al., 2002; Nienaber et al., 2003; Tachibana et al., 2005). Experimental data on PDE activities showed that only PDE4 decreased activity in the mouse failing hearts by 2-fold after eight weeks of TAC (Sprenger et al.,...
The activities of PDE2 and PDE3 did not change significantly. In addition, the activity of GRK2 increased in the failing mouse hearts by 2.3-fold as compared to control (Esposito et al., 2002). The activities of PKA, PP1 and PP2A did not change after TAC (Perino et al., 2011; Grote-Wessels et al., 2008). We used these numbers for the model development.

First, we reduced AC5/6 activity in failing myocytes to 72% of its control value, which reduced total AC activity and fraction of AC5/6 (see Table 3.1). We also reduced PDE4 concentration by 2-fold and increased GRK2 activity by 2.3-fold.

The following differential equation describes the resulting model for the AP:

\[
\frac{dV}{dt} = -\frac{1}{C_m} (I_{CaL} + I_{p(Ca)} + I_{NaCa} + I_{Cab} + I_{Na} + I_{NaL} + I_{NaB} + I_{NaK} + I_{Kto,f} + I_{K1} \\
+ I_{K,slow1} + I_{K,slow2} + I_{Kss} + I_{Kr} + I_{CL, Ca} - I_{stim})
\]

where \( I_{CaL} \) is the L-type Ca\(^{2+} \) current, \( I_{p(Ca)} \) is the sarcolemmal Ca\(^{2+} \) pump, \( I_{NaCa} \) is the current from the Na\(^{+}\)/Ca\(^{2+} \) exchanger, \( I_{Cab} \) is the Ca\(^{2+} \) background current, \( I_{Na} \) is the fast Na\(^{+} \) current, \( I_{NaL} \) is the late Na\(^{+} \) current, \( I_{NaB} \) is the Na\(^{+} \) background current, \( I_{NaK} \) is the current of the Na\(^{+}\)-K\(^{+} \) pump, \( I_{Kto,f} \) is the rapidly recovering transient outward K\(^{+} \) current, \( I_{K1} \) is the time-independent K\(^{+} \) current, \( I_{K,slow1} \) is the rapidly activating, slowly inactivating K\(^{+} \) current (encoded by K\(_v\)1.5; 4-aminopyridine-sensitive; also named as \( I_{Kur} \), the ultra-rapidly activating delayed rectifier K\(^{+} \) current), \( I_{K,slow2} \) is the rapidly activating, slowly inactivating K\(^{+} \) current (encoded by K\(_v\)2.1; TEA-sensitive), \( I_{Kss} \) is the non-inactivating steady-state voltage-activated K\(^{+} \) current, \( I_{Kr} \) is the rapid delayed rectifier K\(^{+} \) current, \( I_{CL, Ca} \) is the Ca\(^{2+}\)-activated chloride current, and \( I_{stim} \) is the stimulus current.
3.1.7 Simulation Methods

The model equations were solved by a fourth-order Runge-Kutta method, with two different time steps. A relatively small time step of 0.000002 ms was used during a 15-millisecond interval after the initiation of the stimulus current; for all other times, we used the time step 0.0001 ms. Simulation of the cellular behavior without electrical stimulation was performed with a time step of 0.1 ms. The model equations were implemented in FORTRAN 90. All simulations were performed on a single processor under SUSE Linux 11 on a Dell Precision Workstation T3500 with a six-core Intel Xeon CPU W3670 (3.2 GHz, 12 GB RAM) or on the ARCTIC supercomputer cluster at Georgia State University. The model was developed for a room temperature of 25ºC (T = 298ºK). Initial conditions were obtained by running the program code without electrical stimulations for about 10,000 seconds to ensure quasi-steady-state. APs and [Ca²⁺]ᵢ transients were initiated by a stimulus current (\( I_{\text{stim}} = 80 \text{ pA/pF, } \tau_{\text{stim}} = 1 \text{ ms} \)) with different frequencies from 0.25 to 5 Hz (electrical stimulation).

3.2 Results

In this part of the dissertation, we developed a new compartmentalized mathematical model of failing mouse ventricular myocytes after TAC and compared model behavior with the corresponding model for control conditions.

3.2.1 Adenylyl Cyclase Activity in Control and Failing Mouse Ventricular Myocytes

First, we simulated adenylyl cyclase activities in control and failing mouse ventricular myocytes (Fig. 3.4). It is seen from Fig. 3.4A that the failing myocytes demonstrate a decrease in the cellular AC activity as compared to control at all isoproterenol concentrations, from 0.1 nM to 10 μM. We also compared the ratio of the cellular AC activities for failing to control myocytes (\( AC_{\text{HF}}/AC_{\text{Control}} \)) without and with isoproterenol application. Without isoproterenol, the ratio
ACHF/AC_{Control} was 0.67 and decreased to $\sim$0.48 at 10-100 μM isoproterenol. Comparison of the simulated ratio ACHF/AC_{Control} with the experimental data for control and after application of 100 μM isoproterenol in Fig. 3.4B shows a good agreement within the experimental errors. We need to notice that the stimulation of the combined β₁- and β₂-adrenergic systems in failing myocytes demonstrate a reduced response to the adrenergic stimulation, as compared to control myocytes, which can also reduce the effects on different protein kinase A targets.

**Figure 3.4 Adenylyl cyclase dependency on isoproterenol.**

(A) Adenylyl cyclase activity as a function of isoproterenol. Experimental data on AC activity (in pmol/mg/min) in mouse hearts and ventricular myocytes obtained after 10-minutes exposure to isoproterenol are shown by unfilled circles (Tepe & Liggett, 1999) and filled circles (Lemire et al., 1998). The solid and dashed lines show corresponding simulated AC activity at different concentrations of isoproterenol upon activation of both β₁-ARs and β₂-ARs in control and failing ventricular myocytes, respectively. (B) The effects of isoproterenol on adenylyl cyclase activity in control and failing myocytes. Experimental data on the ratio of AC activity in myocardial membranes from failing to control myocytes are obtained after exposure to 100 μM isoproterenol and are shown by bars with error bars (Tachibana, 2005; Nienaber, 2003; Esposito, 2002). The corresponding simulated data are shown by light gray bars when both β₁-ARs and β₂-ARs are activated.

3.2.2 **Compartmentalized cAMP and PKA Dynamics**

One of the interesting results of our mathematical model is an increased role of β₂-adrenoceptors in the failing ventricular myocytes. To demonstrate the differences in the roles of β-adrenergic signaling systems in control and failing myocytes, we simulated compartmentalized
cAMP and PKA dynamics (Figs. 3.5 and 3.6). The β₁- and β₂-adrenergic systems were stimulated with 1 μM isoproterenol together and individually.

Figure 3.5 cAMP dynamics in mouse ventricular myocytes. Simulated cAMP concentrations are shown as functions of time in the caveolar (A) extracaveolar (B), and cytosolic compartments (C), as well as in the whole cell (D). Simulations were performed for six cases: activation of both β₁-ARs and β₂-ARs in control, activation of β₁-ARs alone in control, activation of β₂-ARs alone in control, activation of both β₁-ARs and β₂-ARs in heart failure (hf), activation of β₁-ARs alone in heart failure (hf), activation of β₂-ARs alone in heart failure (hf). Data in (A-D) are obtained upon application of 1 μM isoproterenol.

In the caveolar compartment, stimulation of the combined β₁- and β₂-adrenergic systems in the failing myocytes resulted in a smaller cAMP transient compared to control (Fig. 3.5A). Stimulation of only the β₁-adrenergic system in the failing myocytes led to a dramatic decrease in cAMP production due to the reduction in β₁-AR concentration. In contrast, a similar decrease in
β2-ARs in the caveolar compartment due to their translocation to the extracaveolae compartment demonstrated a smaller decline. In the extracaveolar compartment, stimulation of the combined β1- and β2-adrenergic systems and β1-adrenergic system alone does not show a remarkable difference in cAMP production both in failing and control myocytes (Fig. 3.5B). However, cAMP production in failing myocytes is greater than in control for both cases. There is also a difference in cAMP production between the failing and control myocytes upon stimulation of the β2-adrenergic system alone. Figure 3.5B shows that the failing myocytes produced a significantly larger cAMP transient than control myocytes due to a much weaker connection of β2-ARs to inhibitory G protein, Gi, in the extracaveolar compartment. Therefore, translocation of β2-ARs to the extracaveolar compartment increases their contribution to β-adrenergic signaling in failing myocytes. Finally, in the cytosolic compartment, the cAMP transient is shown to be much smaller in the failing myocytes compared to control upon stimulation of the combined β1- and β2-adrenergic system or β1-adrenergic system alone (Fig. 3.5C). Stimulating the β2-adrenergic system alone does not produce a marked cAMP increase in the cytosolic compartment. Similar data were obtained for the whole-cell model (Fig. 3.5D).

The behavior of the catalytic subunit of PKA is similar to that of cAMP (Fig. 3.6). Stimulation of the combined β1- and β2-adrenergic signaling systems or β1-adrenergic signaling system alone in the control cell leads to a similar, large increase of PKA activity in the caveolar and extracaveolar compartments but a smaller increase in the cytosol and whole cardiac cell (Fig. 3.6A-D, red and blue solid lines). However, in failing ventricular myocytes, stimulation of the combined β1- and β2-adrenergic signaling systems in the caveolae resulted in a larger PKA activity than the stimulation of the β1-adrenergic signaling system alone (Fig. 3.6A). Stimulation of β2-ARs alone in the failing myocytes demonstrated a smaller decrease in PKA activity than in control (Fig.
In the extracaveolar compartment, a higher PKA activity was obtained upon stimulation of the combined $\beta_1$- and $\beta_2$-adrenergic signaling systems or $\beta_1$-adrenergic signaling system alone in the failing myocytes compared to control cells (Fig. 3.6B). Stimulation of $\beta_2$-ARs alone produces quite a significant effect on PKA activity in the extracaveolar compartment of the failing cell. In contrast, the control myocytes did not show a considerable change in their PKA activity (Fig. 3.6B, cyan lines). Stimulation of the combined $\beta_1$- and $\beta_2$-adrenergic signaling systems or $\beta_1$-adrenergic signaling system alone in the cytosol and the whole cell of control myocytes show higher PKA activities than those for the failing myocytes. In contrast, stimulation of $\beta_2$-ARs alone does not markedly affect the background PKA activity in both failing and control cells (Fig. 3.6C, D).

**Figure 3.6** PKA catalytic subunit dynamics in mouse ventricular myocytes.
Simulated PKA catalytic subunit concentrations are shown as functions of time in the caveolar (A), extracaveolar (B), and cytosolic compartments (C), as well as in the whole cell (D). Simulations were performed for six cases: activation of both $\beta_1$-ARs and $\beta_2$-ARs in control,
activation of β₁-ARs alone in control, activation of β₂-ARs alone in control, activation of both β₁-ARs and β₂-ARs in heart failure (hf), activation of β₁-ARs alone in heart failure (hf), activation of β₂-ARs alone in heart failure (hf). Data in (A-D) are obtained upon application of 1 µM isoproterenol.

3.2.3 Mouse Failing and Control Ventricular Action Potentials, Ca²⁺ Dynamics, and Ionic Currents

Experimental data shows that the APs in mouse control and failing ventricular myocytes have different durations due to the differences in the magnitudes of repolarization currents (Toischer et al., 2013; Glynn et al., 2015; Mohamed et al., 2018; Bartoli et al., 2020). Some of the experimental data for APD₂₀, APD₅₀, APD₇₅, and APD₉₀ are shown in Table 3.2. This data shows that the APs from the failing myocytes are more prolonged than those from control cells.

Our mathematical model reproduced the experimentally found AP durations at different levels of repolarization in control and failing mouse myocytes. Simulated APD₂₅ for control myocytes is 1.71 ms which is slightly longer than the APD₂₀ = 1.44±0.08 ms from the experimental data by Bartoli et al. (Bartoli et al., 2020). Simulated APD₅₀ is 3.25 ms, which is comparable to the experimental data ~4.0-5.0 ms (Glynn et al., 2015; Bartoli et al., 2020) for control cells. Simulated APD₉₀ = 28.86 ms is also close to the experimental values of 28.0 ms (range from 20 to 47 ms) (Glynn et al., 2015) and 34.5±6.0 ms (Bartoli et al., 2020) but somewhat shorter than the data by Toischer et al. (58±9 ms (Toischer et al., 2013)) and Mohamed et al. (60±5 ms (Mohamed et al., 2018)). Simulated APD₂₅ for failing myocytes is equal to 2.44 ms, which is similar to APD₂₀ = 2.89±1.0 ms from the experimental data by Bartoli et al. (Bartoli et al., 2020). Simulated APD₅₀ is 5.65 ms, slightly less than the experimental data ~8-38 ms (Glynn et al., 2015; Bartoli et al., 2020) for the failing cells. Simulated APD₉₀ = 86.82 ms is also within the experimental range from 55 ms to 138 ms (Toischer et al., 2013; Glynn et al., 2015; Mohamed et al., 2018; Bartoli et al., 2020) (See Table 3.2).
Table 3.2 Experimental data on APDs in mouse control and failing ventricular myocytes (ms).

<table>
<thead>
<tr>
<th></th>
<th>APD&lt;sub&gt;20&lt;/sub&gt;</th>
<th>APD&lt;sub&gt;50&lt;/sub&gt;</th>
<th>APD&lt;sub&gt;75&lt;/sub&gt;</th>
<th>APD&lt;sub&gt;90&lt;/sub&gt;</th>
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<tr>
<td><strong>Control</strong></td>
<td></td>
<td></td>
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<tr>
<td>Toischer et al. (2013) 1 Hz</td>
<td></td>
<td></td>
<td></td>
<td>58±9</td>
</tr>
<tr>
<td>Mohamed et al. (2018)</td>
<td></td>
<td></td>
<td></td>
<td>60±5</td>
</tr>
<tr>
<td>Glynn et al. (2015)</td>
<td></td>
<td>4.0 (2.5-10)</td>
<td></td>
<td>28.0 (20-47)</td>
</tr>
<tr>
<td>Bartoli et al., (2020)</td>
<td>1.44±0.08</td>
<td>5.02±0.47</td>
<td></td>
<td>34.5±6.0</td>
</tr>
<tr>
<td><strong>Heart Failure</strong></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Toischer et al. (2013)</td>
<td></td>
<td></td>
<td></td>
<td>138±28</td>
</tr>
<tr>
<td>Mohamed et al. (2018)</td>
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<td></td>
<td>130±15</td>
</tr>
<tr>
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<td></td>
<td>22.0 (8-38)</td>
<td>50.0 (18-112)</td>
<td>55.0 (30-110)</td>
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<tr>
<td>Bartoli et al., (2020)</td>
<td>2.89±1.0</td>
<td>11.89±4.26</td>
<td></td>
<td>76.77±9.84</td>
</tr>
</tbody>
</table>

If we compare APD<sub>25</sub>, APD<sub>50</sub>, APD<sub>75</sub>, and APD<sub>90</sub> for simulated control and failing ventricular myocytes, we see that the AP durations at all levels of repolarizations are longer for the failing myocytes (Table 3.3). The difference ranges between 42.9% for APD<sub>25</sub> and 200.9% for APD<sub>90</sub>. These differences are explained by the differences in the major repolarization currents for control and failing ventricular myocytes obtained from the simulations (Figs. 3.7C and 3.7D). Failing myocytes have about twice smaller repolarizing rapidly recovering transient outward K<sup>+</sup> current, $I_{K_{to,f}}$, than control ventricular myocytes. The magnitudes of $I_{K_{slow}}$ and $I_{K_{slow2}}$ in failing cells are also much smaller compared to those in control myocytes. The difference in outward K<sup>+</sup> currents is partially compensated by the difference in the inward fast Na<sup>+</sup> current, which is smaller in the failing myocytes due to a larger inactivation at more depolarized resting potential, but this current affects only the initial stage of repolarization. The later repolarization stage is mainly affected by the precise balance of the time-independent K<sup>+</sup> current $I_{K1}$, the Na<sup>+</sup>/Ca<sup>2+</sup> exchanger current $I_{NaCa}$, the Na<sup>+</sup>-K<sup>+</sup> pump current $I_{NaK}$, the late Na<sup>+</sup> current $I_{NaL}$, and the background currents $I_{Cab}$ and $I_{Nab}$ (Fig. 3.7E and 3.7F). The major differences between control and failing myocytes are
that the outward phase of $I_{K1}$ is activated at later times, and the $I_{NaL}$ is more prolonged in the failing cells.

Table 3.3 Comparison of the simulated mouse ventricular action potential durations from control and failing hearts (ms).

<table>
<thead>
<tr>
<th></th>
<th>Control myocyte</th>
<th>Failing myocyte</th>
<th>APD prolongation (%) Failing vs. Control</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>No Iso</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>APD$_{25}$</td>
<td>1.71</td>
<td>2.44</td>
<td>42.7</td>
</tr>
<tr>
<td>APD$_{50}$</td>
<td>3.25</td>
<td>5.65</td>
<td>73.8</td>
</tr>
<tr>
<td>APD$_{75}$</td>
<td>10.06</td>
<td>19.48</td>
<td>93.6</td>
</tr>
<tr>
<td>APD$_{90}$</td>
<td>28.86</td>
<td>86.82</td>
<td>200.8</td>
</tr>
<tr>
<td><strong>1 µM Iso</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>APD$_{25}$</td>
<td>1.93</td>
<td>2.94</td>
<td>52.3</td>
</tr>
<tr>
<td>APD$_{50}$</td>
<td>4.40</td>
<td>8.50</td>
<td>93.2</td>
</tr>
<tr>
<td>APD$_{75}$</td>
<td>14.16</td>
<td>25.55</td>
<td>80.4</td>
</tr>
<tr>
<td>APD$_{90}$</td>
<td>37.32</td>
<td>88.73</td>
<td>137.8</td>
</tr>
</tbody>
</table>

Simulations also show that the magnitude of [$Ca^{2+}$]$_i$ transient is greater in control myocytes as compared to failing cells (Fig. 3.7B), which is also observed experimentally (Tamayo et al., 2020; Bartoli et al., 2020; Li et al., 2013). The simulated ratio of the peak [$Ca^{2+}$]$_i$ for the failing and control myocytes is equal to 0.68, which compares well to the experimentally obtained average data of 0.73±0.07 (Tamayo et al., 2020; Li et al., 2013; Ljubojevic-Holzer et al., 2020; Cai et al., 2019; Chen et al., 2020; Nie et al., 2019).
Figure 3.7 Mouse action potentials, [Ca^{2+}]_i transients, and underlying major ionic currents for control and failing mouse ventricular myocytes. (A) Simulated mouse action potentials for control (solid line) and failing (dashed line) ventricular myocytes. (B) Simulated [Ca^{2+}]_i transients for control (solid line) and failing (dashed line) ventricular myocytes. (C, E) Simulated major ionic currents underlying control mouse action potential. (D, F) Simulated major ionic currents underlying failing mouse action potential. In all panels, action potentials, [Ca^{2+}]_i transients, and ionic currents are shown after 300 s stimulation with 1 Hz.

The application of 1 µM isoproterenol (a 5-minute exposure) resulted in the prolongation of both control and failing APs in simulations (Figs. 3.7A and 3.8A). The failing myocytes APD_{25}
prolonged from 2.44 to 2.94 ms (20.8% lengthening), APD$_{50}$ - from 5.65 to 8.50 ms (50.4%), and APD$_{90}$ - from 86.82 to 88.73 ms (2.2%) (Table 3.3). After the application of 1 µM isoproterenol, significantly smaller APD prolongations were observed in control ventricular myocytes at earlier stages of repolarization (Table 3.3). However, the later stages of repolarization were more significantly affected by isoproterenol in control myocytes than in the failing. In control myocytes, APD$_{25}$ prolonged from 1.71 to 1.93 ms (by 12.9%), APD$_{50}$ - from 3.25 to 4.40 ms (by 35.4%), and APD$_{90}$ - from 28.86 to 37.32 ms (by 29.3%). APDs of failing myocytes, at all levels of repolarization, are more prolonged than in control after exposure to 1 µM isoproterenol. The role of underlying ionic currents in the repolarization of the control and failing myocytes is similar to that without the application of isoproterenol (Fig. 3.8C-3.8F).

Simulations demonstrate an increase in the magnitude of [Ca$^{2+}$]$_{i}$ transient both in control and failing myocytes after the application of 1 µM isoproterenol (Fig. 3.7B and 3.8B), which is also observed experimentally (Ginsburg & Bers 2004; Bing et al., 1991). In control cells, [Ca$^{2+}$]$_{i}$ transient increased by 3.27 fold, while in failing cells, the increase was only by 1.93 fold. A smaller increase of [Ca$^{2+}$]$_{i}$ in the failing cells demonstrates a reduced response to β-adrenergic stimulation than the control myocytes.
Figure 3.8 Mouse action potentials, $[Ca^{2+}]_i$ transients, and underlying major ionic currents for control and failing mouse ventricular myocytes after application of 1 μM isoproterenol.  
(A) Simulated mouse action potentials for control (solid line) and failing (dashed line) ventricular myocytes.  
(B) Simulated $[Ca^{2+}]_i$ transients for control (solid line) and failing (dashed line) ventricular myocytes.  
(C, E) Simulated major ionic currents underlying control mouse action potential.  
(D, F) Simulated major ionic currents underlying failing mouse action potential.  
In all panels, action potentials, $[Ca^{2+}]_i$ transients, and ionic currents are shown after 300 s stimulation with 1 Hz. 1 μM isoproterenol is applied at time $t = 0$ s.
3.2.4 Integrated Ca\textsuperscript{2+} and Na\textsuperscript{+} Fluxes in Mouse Failing and Control Myocytes

Simulated integral Ca\textsuperscript{2+} fluxes in mouse control and failing ventricular myocytes without and with the application of 1 μM isoproterenol are shown in Fig. 3.9 and Table 3.4. Without isoproterenol, simulations show larger integral Ca\textsuperscript{2+} release flux in control (35.63 μM) versus failing (24.30 μM) myocytes (Figs. 3.9A and 3.9B). However, the triggering Ca\textsuperscript{2+} influx by the $I_{Ca,L}$ is only 12% larger in control (1.60 μM) as compared to failing (1.41 μM) cells. We suggest that the larger Ca\textsuperscript{2+} release flux in mouse control cells is mainly due to the larger SR Ca\textsuperscript{2+} load compared to the failing myocytes (894 μM and 599 μM for control and failing myocytes, respectively). Simulated Ca\textsuperscript{2+} extrusion by Na\textsuperscript{+}/Ca\textsuperscript{2+} exchanger is higher in failing mouse ventricular myocytes (3.54 μM) than in control myocytes (3.15 μM). However, this difference is quite small, considering that the expression of the Na\textsuperscript{+}/Ca\textsuperscript{2+} exchanger has increased by 1.8-fold (in the model, parameter $k_{NaCa}$ was increased from 275 pA/pF to 495 pA/pF). Application of 1 μM isoproterenol increases Ca\textsuperscript{2+} entry through the L-type Ca\textsuperscript{2+} channels both in mouse control (3.47 μM) and failing (2.87 μM) myocytes, as well as integral Ca\textsuperscript{2+} release fluxes in control (50.82 μM) and failing (30.03 μM) myocytes (Figs. 3.9C and 3.9D). It is apparent that the failing myocytes demonstrate a weaker response to β-adrenergic stimulation in terms of the released Ca\textsuperscript{2+} from the SR. The amount of Ca\textsuperscript{2+} released from the SR was increased by 1.43-fold and 1.24-fold for control and failing cells, respectively. Nevertheless, we need to note that the difference is not dramatically large, suggesting that our mathematical model simulated a relatively early stage of heart failure after 6-8 weeks of TAC.
Figure 3.9 Simulations of the integrated Ca\textsuperscript{2+} fluxes (influxes) in isolated control and failing mouse ventricular myocytes during one cardiac cycle. 

(A, C) Simulated Ca\textsuperscript{2+} influxes for control myocytes. (B, D) Simulated Ca\textsuperscript{2+} influxes for the failing myocytes. Simulations without and with 1 μM isoproterenol are shown in (A, B) and (C, D), respectively. Here, $J_{rel}$ is the Ca\textsuperscript{2+} release from the SR; $J_{CaL}$ is the Ca\textsuperscript{2+} entering the cell through the L-type Ca\textsuperscript{2+} channels; $J_{up} - J_{leak}$ is the uptake of Ca\textsuperscript{2+} from the cytosol to the network SR with subtracted Ca\textsuperscript{2+} leak from the SR to the cytosol; $J_{NaCa}$ is the Ca\textsuperscript{2+} outflux from the cytosol through the Na\textsuperscript{+}/Ca\textsuperscript{2+} exchanger, and $J_{pCa}$ is the Ca\textsuperscript{2+} outflux through the sarcolemmal Ca\textsuperscript{2+} pump. Ca\textsuperscript{2+} fluxes are shown after 300 s of stimulation with 1 Hz. 1 μM isoproterenol is applied at time $t = 0$ s.
Table 3.4 Integrated Ca\(^{2+}\) fluxes in mouse control and failing ventricular myocytes

<table>
<thead>
<tr>
<th>Flux (μM)</th>
<th>Control</th>
<th>HF</th>
</tr>
</thead>
<tbody>
<tr>
<td>J(_{rel})</td>
<td>35.63</td>
<td>24.30</td>
</tr>
<tr>
<td>J(_{CaL})</td>
<td>1.60</td>
<td>1.41</td>
</tr>
<tr>
<td>J(<em>{up} - J</em>{Leak})</td>
<td>35.63</td>
<td>24.30</td>
</tr>
<tr>
<td>J(_{NaCa})</td>
<td>3.15</td>
<td>3.54</td>
</tr>
<tr>
<td>J(_{pCa})</td>
<td>0.177</td>
<td>0.1204</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Flux (μM)</th>
<th>Control</th>
<th>HF</th>
</tr>
</thead>
<tbody>
<tr>
<td>J(_{rel})</td>
<td>50.82</td>
<td>30.03</td>
</tr>
<tr>
<td>J(_{CaL})</td>
<td>3.47</td>
<td>2.87</td>
</tr>
<tr>
<td>J(<em>{up} - J</em>{Leak})</td>
<td>50.8185</td>
<td>30.0271</td>
</tr>
<tr>
<td>J(_{NaCa})</td>
<td>4.8594</td>
<td>4.9174</td>
</tr>
<tr>
<td>J(_{pCa})</td>
<td>0.301</td>
<td>0.189</td>
</tr>
</tbody>
</table>

We also simulated integrated Na\(^+\) fluxes in mouse control and failing ventricular myocytes (Fig. 3.10). Simulations showed that the integrated Na\(^+\) influx through the fast Na\(^+\) channels is much larger in control myocytes (3.89 μM) versus failing myocytes (2.05 μM) due to a smaller I\(_{Na}\) in the failing cells. This difference is compensated in parts by the larger integrated Na\(^+\) influx by the Na\(^+\)/Ca\(^{2+}\) exchanger (10.62 μM vs. 9.45 μM in failing vs. control cells, respectively) and the late Na\(^+\) current (0.42 μM vs. 0.29 μM in failing vs. control cells, respectively) in the failing myocytes. However, the major competitive fluxes in both control and failing cells are the Na\(^+\) influx by the Na\(^+\) background current and the Na\(^+\) outflux by the Na\(^+\)-K\(^+\) pump. Simulated Na\(^+\) entries by the background Na\(^+\) currents are 46.54 μM and 32.90 μM for control and failing myocytes, respectively; Na\(^+\) outfluxes by the Na\(^+\)-K\(^+\) pump are 60.00 μM and 45.63 μM for control and failing myocytes, respectively.
Figure 3.10 Simulations of the integrated Na⁺ fluxes (influxes) in isolated control and failing mouse ventricular myocytes during one cardiac cycle.  

(A, C) Simulated Na⁺ influxes for control myocytes. (B, D) Simulated Na⁺ influxes for the failing myocytes. Simulations without and with 1 μM isoproterenol are shown in (A, B) and (C, D), respectively. Here, \( J_{Na^{+}} \) is the Na⁺ influx through the channels responsible for the fast Na⁺ current; \( J_{Na^{+}} \) is the background Na⁺ influx; \( J_{NaCa} \) is the Na⁺ influx through the Na⁺/Ca²⁺ exchanger; \( J_{NaK} \) is the Na⁺ outflux through the Na⁺-K⁺ pump. Na⁺ fluxes are shown after 300 s of stimulation with 1 Hz. 1 μM isoproterenol is applied at time \( t = 0 \) s. 

Stimulation of β-adrenergic receptors increased the integrated Na⁺ influx through the fast Na⁺ channels to 4.21 μM in control versus 2.39 μM in failing myocytes; by the Na⁺/Ca²⁺ exchanger to 14.58 μM in control versus 14.75 μM in failing myocytes; by the late Na⁺ current to 0.84 μM in control versus to 0.99 μM in failing myocytes. In addition, the Na⁺ entry by the background Na⁺ currents for control and failing myocytes did not change significantly and are equal to 46.92 μM and 33.17 μM, respectively. However, Na⁺ outfluxes by the Na⁺-K⁺ pump increased appreciably to 70.39 μM and 52.85 μM for control and failing myocytes, respectively, due to a larger Na⁺-K⁺ pump activity upon β-adrenergic stimulation (Bondarenko, 2014).
3.2.5 Frequency Dependences of $[Ca^{2+}]_i$ Transients in Failing and Control Myocytes

We also investigated frequency dependences of the $[Ca^{2+}]_i$ transients without and with the application of 1 µM isoproterenol. Our simulations show an increase in diastolic $[Ca^{2+}]_i$ concentrations with stimulation frequency in both control and failing myocytes (Fig. 3.11A and 3.11 B). Without isoproterenol, both control and failing myocytes demonstrate biphasic frequency dependence of the peak $[Ca^{2+}]_i$, which has been observed experimentally (see, for example (Ito et al., 2000). However, the peak $[Ca^{2+}]_i$ concentrations are greater in control as compared to failing myocytes. Comparison of the decrease in normalized $[Ca^{2+}]_i$ transients in failing versus control myocytes between experimental and simulations shows good agreement (Fig. 3.11C). After the application of 1 µM isoproterenol, the dependence becomes different. It includes an increasing part at the lower stimulation frequencies and a weakly decreasing part at the higher frequencies.

Table 3.5 Integrated Na$^+$ fluxes in mouse control and failing ventricular myocytes

<table>
<thead>
<tr>
<th>Na$^+$ Flux (µM)</th>
<th>Control</th>
<th>HF</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>No Iso</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$J_{Nav}$</td>
<td>3.89</td>
<td>2.05</td>
</tr>
<tr>
<td>$J_{Nab}$</td>
<td>46.54</td>
<td>32.90</td>
</tr>
<tr>
<td>$J_{NaCa}$</td>
<td>9.45</td>
<td>10.62</td>
</tr>
<tr>
<td>$J_{NaK}$</td>
<td>60.00</td>
<td>45.63</td>
</tr>
<tr>
<td>$J_{NaL}$</td>
<td>0.29</td>
<td>0.42</td>
</tr>
<tr>
<td><strong>1 µM Iso</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$J_{Nav}$</td>
<td>4.21</td>
<td>2.39</td>
</tr>
<tr>
<td>$J_{Nab}$</td>
<td>46.92</td>
<td>33.17</td>
</tr>
<tr>
<td>$J_{NaCa}$</td>
<td>14.58</td>
<td>14.75</td>
</tr>
<tr>
<td>$J_{NaK}$</td>
<td>70.39</td>
<td>52.85</td>
</tr>
<tr>
<td>$J_{NaL}$</td>
<td>0.84</td>
<td>0.99</td>
</tr>
</tbody>
</table>
We also obtained a much larger increase in the peak $[\text{Ca}^{2+}]_i$ concentrations in control myocytes, which indicated a blunted response to $\beta$-adrenergic stimulation in the failing cells.

**Figure 3.11** Simulations of the frequency dependence of $[\text{Ca}^{2+}]_i$ transients in control and failing mouse ventricular myocytes. 

(A) Simulated diastolic (filled symbols) and systolic (unfilled symbols) $[\text{Ca}^{2+}]_i$ magnitudes without isoproterenol (circles) and after $1 \ \mu\text{M}$ isoproterenol application (triangles) as functions of stimulation frequency for control myocytes. (B) Simulated diastolic (filled symbols) and systolic (unfilled symbols) $[\text{Ca}^{2+}]_i$ magnitudes without isoproterenol (circles) and after $1 \ \mu\text{M}$ isoproterenol application (triangles) as functions of stimulation frequency for failing myocytes. (C) Comparison of the experimental and simulated $[\text{Ca}^{2+}]_i$ transients for control and failing ventricular myocytes. Experimental data are from Li et al. (2013); Nie et al. (2019), Cai et al. (2019), Tamayo et al. (2020), Ljubojevic-Holzer et al. (2020), and Chen et al. (2020). Simulated data are shown after $300 \ \text{s}$ of stimulation with $1 \ \text{Hz}$. 
3.2.6  

**Sensitivity Analysis**

In order to define the major contributing ionic currents to the changes in AP and [Ca$^{2+}$]$_i$ transients in control and failing mouse ventricular myocytes, we performed a sensitivity analysis (Fig. 3.12). For this purpose, we first stimulated the ventricular model cell for 300 s with a basic cycle length 1000 ms ($I_{\text{stim}} = 80$ pA/pF, $\tau_{\text{stim}} = 1$ ms) and then abruptly increased one of the current conductance by 5% at the beginning of the 301$^{\text{st}}$ beat. Then the effect of this abrupt change was estimated on APD$_{25}$, APD$_{50}$, APD$_{90}$, AP amplitude, and [Ca$^{2+}$]$_i$ transient in both control and failing ventricular myocytes.

In control, the major player for APD$_{25}$ and APD$_{50}$ is the rapidly recovering transient outward K$^+$ current, $I_{K\text{to,f}}$ (Fig. 3.12, A and B). Three other currents, the L-type Ca$^{2+}$ current, $I_{\text{CaL}}$, and two rapidly activating, slowly inactivating K$^+$ currents, $I_{K,\text{slow1}}$ and $I_{K,\text{slow2}}$, also made significant contributions to the change in APD$_{25}$ and APD$_{50}$. The K$^+$ currents tend to shorten, while $I_{\text{CaL}}$ tends to prolong APD$_{25}$ and APD$_{50}$. The regulation of APD$_{90}$ is more complex. Eight major ionic currents affect APD$_{90}$ (Fig. 3.12C). The L-type Ca$^{2+}$ current, $I_{\text{CaL}}$, the Na$^+$/Ca$^{2+}$ exchanger current $I_{\text{NaCa}}$, and the background Na$^+$ current, $I_{\text{Nab}}$, tend to prolong APD$_{90}$. In contrast, the Na$^+$-K$^+$ pump current, $I_{\text{NaK}}$, the time-independent K$^+$ current, $I_{K1}$, two rapidly activating, slowly inactivating K$^+$ currents, $I_{K,\text{slow1}}$ and $I_{K,\text{slow2}}$, and the noninactivating steady-state voltage-activated K$^+$ current, $I_{K\text{ss}}$, tend to shorten APD$_{90}$. In control, AP amplitude is affected by four major currents (Fig. 3.12D). Three currents, $I_{\text{Na}}$, $I_{\text{NaK}}$, and $I_{K1}$, increase, while $I_{\text{Nab}}$ decreases AP amplitude. The major contributing factor to [Ca$^{2+}$]$_i$ transient is the L-type Ca$^{2+}$ current (Fig. 3.12E), which increases [Ca$^{2+}$]$_i$. Two rapidly activating, slowly inactivating K$^+$ currents, $I_{K,\text{slow1}}$ and $I_{K,\text{slow2}}$, and the rapidly recovering transient outward K$^+$ current, $I_{K\text{to,f}}$, also made some contribution, probably, through the shortening APDs.
Figure 3.12 Sensitivity analysis.
Changes of action potential (AP) duration at 25% (Panels A and F), 50% (Panels B and G), and 90% repolarization (Panels C and H), AP amplitude (Panels D and I), and $[\text{Ca}^{2+}]_i$ transient (Panels E and J) between the two consecutive stimuli (the 301st and 300th pacing beats) at 1 Hz. We increased the magnitude of one of several ionic currents by 5% during the 301st pacing beat. The changes were calculated as the differences between AP durations, AP amplitudes, and $[\text{Ca}^{2+}]_i$ transients during the 301st and 300th beats. Simulations were performed for control (Control, Panels A-E) and failing (Heart failure, Panels F-J) myocytes. Sensitivity analysis is performed with respect to 5% increase of the L-type $\text{Ca}^{2+}$ current ($I_{\text{CaL}}$), the sarcolemmal $\text{Ca}^{2+}$ pump current ($I_{\text{pCa}}$), the $\text{Na}^+$/Ca$^{2+}$ exchanger current ($I_{\text{NaCa}}$), the Ca$^{2+}$ background current ($I_{\text{Cabr}}$), the fast $\text{Na}^+$
current (ina), the late Na\(^+\) current (INaL), the Na\(^+\) background current (INab), the Na\(^+\)-K\(^+\) pump current (INaK), the rapidly recovering transient outward K\(^+\) current (IKtof), the time-independent K\(^+\) current (IK1), the rapidly activating, slowly inactivating K\(^+\) current (encoded by Kv1.5; 4-aminopyridine-sensitive; ikslow1), the rapidly activating, slowly inactivating K\(^+\) current (encoded by Kv2.1; TEA-sensitive; IKslow2), the noninactivating steady-state voltage-activated K\(^+\) current (IKss), the rapid delayed rectifier K\(^+\) current (IKr), and the Ca\(^2+\)-activated chloride current (IClCa).

No changes are shown for the simulations without 5% perturbation of the model parameters (c).

In the failing ventricular myocytes, APD\(_{25}\) and APD\(_{50}\) were affected by the six major currents, \(I_{CaL}\), \(I_{Na}\), \(I_{Nab}\), \(I_{Kto,f}\), \(I_{K,slow1}\), and \(I_{K,slow2}\), but in different proportions. For APD\(_{25}\), the effect of \(I_{Kto,f}\) and the contributions of \(I_{CaL}\), \(I_{Na}\), \(I_{K,slow1}\), and \(I_{K,slow2}\) have increased compared to control cells (Fig. 3.12F). For APD\(_{50}\), the role of \(I_{Kto,f}\) was also increased along with \(I_{CaL}\), \(I_{Na}\), \(I_{K,slow1}\), and \(I_{K,slow2}\), which increased their effects (Fig. 3.12G). Nine currents affect APD\(_{90}\) of failing myocytes (Fig. 3.12H). The major role was played by the background Na\(^+\) current \(I_{Nab}\), which prolonged APD\(_{90}\), and the Na\(^+\)-K\(^+\) pump current \(I_{NaK}\), which shortened APD\(_{90}\). Seven other currents, \(I_{NaCa}\), \(I_{NaL}\), \(I_{Kto,f}\), \(I_{K1}\), \(I_{K,slow1}\), \(I_{K,slow2}\), and \(I_{Kss}\), have a smaller but still significant contribution to APD\(_{90}\).

The major players and their proportional contributions that affect AP amplitude and [Ca\(^{2+}\)]\(_i\) transient in the failing myocytes are the same as in control (Fig. 3.12, I and J). However, the AP amplitude of the failing myocytes is much more sensitive to these currents than the control (Fig. 3.12, D and I). In contrast, [Ca\(^{2+}\)]\(_i\) transient is less sensitive to the changes in the major contributing currents as compared to control (Fig. 3.12, E and J).

### 3.2.7 Sarcoplasmic Reticulum Ca\(^{2+}\) Load Regulation in Control and Failing Ventricular Myocytes

It is believed that the major factor that regulates myocyte contraction and [Ca\(^{2+}\)]\(_i\) transient is the SR Ca\(^{2+}\) load (Eisner et al., 2017; Eisner et al., 2020). In control myocytes, the SR load is defined by a delicate balance between the SERCA pump function, the ryanodine receptors release and leak, the Na\(^+\)/Ca\(^{2+}\) exchanger function, the Ca\(^{2+}\) influx through the L-type Ca\(^{2+}\) channels, the
Ca\(^{2+}\) buffering, the sarcolemmal Ca\(^{2+}\) pump, and the background Ca\(^{2+}\) influx mechanism. In failing hearts, dysregulation of one or several of these factors results in a smaller SR Ca\(^{2+}\) load and [Ca\(^{2+}\)]\(_i\) transient, as observed experimentally (Bartoli et al., 2020; Chen et al., 2020; Li et al., 2013; Bryant et al., 2018). However, the mechanism of the dysfunction is quite complicated and is different for different heart failure phenotypes (Eisner et al., 2020).

To demonstrate the role of different Ca\(^{2+}\) handling proteins in the SR Ca\(^{2+}\) load in failing mouse ventricular myocytes, we used the base failing myocyte model described above and changed some Ca\(^{2+}\) handling protein functions to their control model values. We did not change the magnitudes of the L-type Ca\(^{2+}\) current, Ca\(^{2+}\) buffering, and the sarcolemmal Ca\(^{2+}\) pump, as they did not change too much in heart failure or did not make a significant contribution; therefore, they were excluded from this investigation.

Figure 3.13 shows the action potentials, the [Ca\(^{2+}\)]\(_i\) transients, and the junctional SR Ca\(^{2+}\) concentrations [Ca\(^{2+}\)]\(_{JSR}\) as functions of time for the base failing myocyte model (red lines) and for the failing myocyte models where one of the Ca\(^{2+}\) handling protein’s model parameters (the SERCA pump, the Na\(^+\)/Ca\(^{2+}\) exchanger, the background Ca\(^{2+}\) current, Ca\(^{2+}\) leak from the SR, and Ca\(^{2+}\) release from the SR) was replaced by the corresponding equivalent control model value. Simulation results show that among those Ca\(^{2+}\) handling proteins, the major effect on the action potential in failing myocytes is caused by the background Ca\(^{2+}\) current I\(_{Cab}\) that has increased and prolonged the action potential in the failing heart (Fig. 3.13A). Other Ca\(^{2+}\) handling proteins do not significantly affect the AP.
Figure 3.13 The role of Ca\textsuperscript{2+} handling proteins in the SR Ca\textsuperscript{2+} load in failing ventricular myocytes. Action potentials (A), [Ca\textsuperscript{2+}]\textsubscript{i} transients (B), and the SR Ca\textsuperscript{2+} load (C) as functions of time for the base failing myocyte model (HF base, red lines), as well as for the models in which the base model parameters were replaced by the control model parameters for the SERCA pump (J\textsubscript{up} control, green lines), the Na\textsuperscript{+}/Ca\textsuperscript{2+} exchanger (I\textsubscript{NaCa} control, blue lines), the background Ca\textsuperscript{2+} current (I\textsubscript{Cab} control, cyan lines), Ca\textsuperscript{2+} leak from the SR (J\textsubscript{leak} control, dark red lines), and Ca\textsuperscript{2+} release from the SR (J\textsubscript{rel} control, dark green lines). Simulated data are shown after 300 s of stimulation with 1 Hz.

The result is quite different for [Ca\textsuperscript{2+}]\textsubscript{i} transients and the JSR Ca\textsuperscript{2+} concentration (Fig. 3.13B and C). The most dramatic changes in [Ca\textsuperscript{2+}]\textsubscript{i} and [Ca\textsuperscript{2+}]\textsubscript{JSR} concentrations are caused by the Na\textsuperscript{+}/Ca\textsuperscript{2+} exchanger and the background Ca\textsuperscript{2+} current when inverted to their control values. The heart failure myocyte model with control I\textsubscript{NaCa} demonstrates a dramatically increased [Ca\textsuperscript{2+}]\textsubscript{i} transient and the JSR Ca\textsuperscript{2+} concentration. This means that the adaptive increase in I\textsubscript{NaCa} in the failing myocyte prevents the cell from Ca\textsuperscript{2+} overload, reduces [Ca\textsuperscript{2+}]\textsubscript{i} transient, and, as a consequence, reduces energy consumption during contraction. On the other hand, a reversion of the background Ca\textsuperscript{2+} current I\textsubscript{Cab} to its control value resulted in a dramatic decrease of [Ca\textsuperscript{2+}]\textsubscript{i}. 
transient and JSR Ca\(^{2+}\) concentration, leading to impaired cell (heart) contraction. Therefore, the failing myocytes require more Ca\(^{2+}\) entry to maintain proper cardiac output. A less dramatic but quite significant role is seen for the SERCA pump. The reversion of \(J_{up}\) to its control value increases \([\text{Ca}^{2+}]_i\) transient by 17% and \([\text{Ca}^{2+}]_{\text{JSR}}\) by 39%; therefore, adaptation of the SERCA pump function serves to prevent Ca\(^{2+}\) overload in the failing myocytes. As seen from the simulations, the sole action of two other Ca\(^{2+}\) handling mechanisms, Ca\(^{2+}\) leak, and Ca\(^{2+}\) release, does not significantly affect \([\text{Ca}^{2+}]_i\) transient and \([\text{Ca}^{2+}]_{\text{JSR}}\).

3.2.8 Mechanism of Proarrhythmic \([\text{Ca}^{2+}]_i\) Transient in Failing Ventricular Myocytes upon Activation of \(\beta\)-adrenergic Signaling

Experimental data showed that the application of isoproterenol to failing mouse hearts made them more susceptible to pro-arrhythmic events and ventricular tachycardia (Cerrone et al., 2007; van Oort, McCauley, et al., 2010; Liu et al., 2011). It was also shown that the proarrhythmic events were related to the dysfunction of genetically modified ryanodine receptors (van Oort, McCauley, et al., 2010; Liu et al., 2011).

To investigate whether a similar mechanism is possible in the failing mouse ventricular myocytes after TAC, we studied the myocyte behavior at a relatively fast 5-Hz pacing without and with the application of 1 μM isoproterenol (Fig. 3.14). In addition, we also studied the effects of \(I_{\text{NaL}}\) and \(\beta_1\)-AR blocks on the myocyte’s activity. Without isoproterenol in control, \([\text{Ca}^{2+}]_i\) transients show clear rising (due to Ca\(^{2+}\) release from the SR and Ca\(^{2+}\) movement from the dyadic space to the cytosol) and decaying (predominantly due to the SERCA pump and \(I_{\text{NaCa}}\) function) phases (Fig. 3.14C). A block of \(\beta_1\)-ARs reduces \([\text{Ca}^{2+}]_i\) transient predominantly due to a reduction of the L-type Ca\(^{2+}\) current and the corresponding Ca\(^{2+}\) entry into the cell; however, block of \(I_{\text{NaL}}\) does not affect \([\text{Ca}^{2+}]_i\) transient. Both the block of \(\beta_1\)-ARs and \(I_{\text{NaL}}\) does not affect action potential
shape (Fig. 3.14A). Upon application of 1 μM isoproterenol in control, \([\text{Ca}^{2+}]_i\) transient demonstrates a perturbation of the decaying phase, which leads to a prolongation of \([\text{Ca}^{2+}]_i\) (Fig. 3.14F). Block of \(I_{NaL}\) does not affect perturbed \([\text{Ca}^{2+}]_i\) transient; however, block of \(\beta_1\)-ARs improves \([\text{Ca}^{2+}]_i\) transient shape (Fig. 3.14F). In addition, the block of \(\beta_1\)-ARs shows a stronger effect on the action potential than the block of \(I_{NaL}\) (Fig. 3.14E). Thus, our simulations demonstrate that the block of \(\beta_1\)-ARs prevents pro-arrhythmic \([\text{Ca}^{2+}]_i\) transient, while the block of \(I_{NaL}\) does not show significant effects.

To reveal the mechanism of pro-arrhythmic \([\text{Ca}^{2+}]_i\) transient upon application of isoproterenol in the failing mouse ventricular myocytes, we investigated the behavior of \(\text{Ca}^{2+}\) concentration in the dyadic space \([\text{Ca}^{2+}]_{ss}\) and the opening probability of ryanodine receptors (Fig. 3.14). Without isoproterenol in control, \([\text{Ca}^{2+}]_{ss}\) demonstrates a relatively sharp raising phase, which is defined by the rate of the RyR opening, and a somewhat slower decaying phase, which is determined by the diffusion of \(\text{Ca}^{2+}\) from the dyadic space to bulk cytosol (Fig. 3.14C). The block of \(I_{NaL}\) does not affect \([\text{Ca}^{2+}]_{ss}\); however, the block of \(\beta_1\)-ARs slightly reduces the peak value of \([\text{Ca}^{2+}]_{ss}\). Upon the application of 1 μM isoproterenol in control (\(I_{NaL}\) blocked), \([\text{Ca}^{2+}]_{ss}\) shows two maxima that prolong the high \(\text{Ca}^{2+}\) concentration phase in the dyadic space (Fig. 3.14G), with a significantly smaller first maximum as compared to \([\text{Ca}^{2+}]_{ss}\) without isoproterenol. The block of \(I_{NaL}\) does not affect perturbed \([\text{Ca}^{2+}]_{ss}\); however, the block of \(\beta_1\)-ARs restores normal behavior of \([\text{Ca}^{2+}]_{ss}\). It seems like the delayed \(\text{Ca}^{2+}\) release from the SR plays a significant role in the pro-arrhythmic activity.
Figure 3.14 Mechanism of proarrhythmic \([\text{Ca}^{2+}]_i\) transient and its suppression in mouse failing ventricular myocytes.

Action potential (A, E), \([\text{Ca}^{2+}]_i\) transient (B, F), dyadic space \(\text{Ca}^{2+}\) concentration \([\text{Ca}^{2+}]_{ss}\) (C, G), and opening probability of RyRs (D, H) as functions of time. Data without and with the application of 1 μM isoproterenol are shown in (A-D) and (E-H), respectively. Simulation for control failing myocytes are shown by red lines, after \(I_{\text{NaL}}\) block – by green lines, and after \(\beta_1\)-ARs block – by blue lines. Simulated data are shown after 300 s of electrical stimulation with 5 Hz.
Indeed, the analysis of the behavior of the RyRs opening probability in these simulations provided a clear explanation of the obtained results. Without isoproterenol in control, ryanodine receptors open very rapidly due to their strong (3rd and 4th power) dependence on [Ca\textsuperscript{2+}] ss (Keizer and Levine, 1996) and then decay slower after a plateau phase (Fig. 14D). Neither block of \(\beta_1\)-ARs nor \(I_{NaL}\) affect the time behavior of RyR opening. After the application of 1 \(\mu\)M isoproterenol in control, ryanodine receptors do not reach the same opening probability as that without isoproterenol due to different gating properties of phosphorylated and non-phosphorylated fractions of RyRs (Fig. 3.14H), which results in a slower receptor closing and subsequent reactivation of the open state. This ultimately prolongs Ca\textsuperscript{2+} release and distorts [Ca\textsuperscript{2+}]\textsubscript{i} transient. A block of \(\beta_1\)-ARs, which dramatically reduces the phosphorylated fraction of RyRs, restores their normal gating and normal Ca\textsuperscript{2+} release.
3.3 Discussion

3.3.1 Compartmentalization, β-adrenergic Regulation, and Action Potential in Normal and Failing Ventricular Myocytes

Experimental data showed that the development of heart failure resulted in a reduction of the concentration of β1-adrenergic receptors in multiple species; however, the concentration of β2-adrenergic receptors was preserved (Baker, 2014). Experimental data also showed that the overexpression of β1-adrenergic receptors led to accelerated heart failure development (Engelhardt et al., 1999), while a moderate overexpression of β2-adrenergic receptors improved cardiac function (Milano et al., 1994). Therefore, it is considered that a reduction of β1-AR concentration is an adaptive mode in the failing hearts to slow down further development of heart failure. In addition, a decrease in adenylyl cyclase activity was also observed in the failing hearts (Tachibana, 2005; Nienaber, 2003; Esposito, 2002). Finally, the failing ventricular myocytes demonstrated re-distribution of β2-ARs from the t-tubules to the rest of the cell (Nikolaev et al., 2010).

Reviewing the available experimental data, we concluded that in failing mouse ventricular myocytes obtained after TAC procedure, the reduction of β1-AR concentration is also ~50%. We also reduced adenylyl cyclase activity to fit the experimental data and moved 50% of β2-ARs from the caveolar to the extracaveolar compartment. The resulting model reproduced adenylyl cyclase activity observed in the mouse failing ventricular myocytes after TAC procedure (Tachibana, 2005; Nienaber, 2003; Esposito, 2002). In addition, our compartmentalized model allowed us to simulate the time behavior of cAMP and PKA with and without the simultaneous stimulation of both β1- and β2-ARs. Simulations using the failing ventricular myocyte model were compared to those for the control model. Simulations reproduced the experimentally observed reduction of adenylyl cyclase activity increase in failing cells upon stimulation with isoproterenol compared to
control (Fig. 3.4). We also found that redistribution of β2-ARs resulted in increased cAMP production and PKA activity in the extracaveolar compartment, leading to more phosphorylation of the ion channels, including the L-type Ca2+ channel, that its conductance was moderately increased. This outcome supports the notion that redistribution of β2-ARs plays a significant role in maintaining β-adrenergic function in failing ventricular myocytes.

Experimental investigations also demonstrated significant differences in the AP between normal and failing ventricular myocytes in different species (Tomaselli & Marban, 1999; Nattel et al., 2007). Specifically, failing ventricular myocytes have, in general, more prolonged AP and reduced K+ repolarization currents and generate smaller [Ca2+]i transient and contraction force than the normal cells. In most species, failing ventricular myocytes possess reduced transient outward K+ currents, \( I_{KTo,f} \) and \( I_{KTo,s} \), and time-independent inward-rectifier current \( I_{K1} \) (Tomaselli & Marban, 1999; Nattel et al., 2007). However, the reduction of the transient outward K+ currents more significantly affects rodent myocytes where \( I_{KTo,f} \) and \( I_{KTo,s} \) are among the primary repolarization currents (Tamayo et al., 2020). In addition, the downregulation of \( I_{K,slow} \) current make remarkable contributions to AP prolongation in rodents (Tamayo et al., 2020). In the larger species, a downregulation of the slow delayed rectifier K+ current \( I_{Ks} \) is the most prominent and consistent change that causes AP prolongation in heart failure (Nattel et al., 2007). Our simulations confirm the experimental findings on AP prolongation in failing mouse ventricular myocytes as compared to the normal cells. We also found that a reduction of \( I_{KTo,f} \), \( I_{K,slow1} \), and \( I_{K,slow2} \) are the major contributing factors to the AP prolongation that can be potentially pro-arrhythmic.
3.3.2 Ca\textsuperscript{2+} Dynamics and Mechanisms of the SR Ca\textsuperscript{2+} Load in Normal and Failing Ventricular Myocytes

In addition to APs, differences in [Ca\textsuperscript{2+}]\textsubscript{i} transients were observed between normal and failing ventricular myocytes (Janse, 2004; Li et al., 2013; Nie et al., 2019). [Ca\textsuperscript{2+}]\textsubscript{i} transients were also observed to be smaller in the failing myocytes compared to the normal cells. In addition, [Ca\textsuperscript{2+}]\textsubscript{i} transients in failing myocytes demonstrate a weaker response to β-adrenergic stimulation (O'Rourke et al., 1999). These differences result from the complex remodeling effects of several Ca\textsuperscript{2+} handling proteins involved in [Ca\textsuperscript{2+}]\textsubscript{i} production and their regulation by the β-adrenergic signaling system. However, the mechanism of the downregulation of the [Ca\textsuperscript{2+}]\textsubscript{i} transients in failing hearts is still a subject of debate (Bers et al., 2003).

It is considered that a reduction of [Ca\textsuperscript{2+}]\textsubscript{i} transients in the failing myocytes is mainly due to a reduction of the SR Ca\textsuperscript{2+} load (Bers et al., 2003). Multiple factors affect the SR Ca\textsuperscript{2+} load, including the SERCA pump, ryanodine receptors, the Na\textsuperscript{+}/Ca\textsuperscript{2+} exchanger, the background Ca\textsuperscript{2+} influx mechanisms, and the sarcolemmal Ca\textsuperscript{2+} pump. There is a balance between these mechanisms in healthy myocytes that maintains a relatively constant SR Ca\textsuperscript{2+} load under normal physiological conditions. However, the progress to a maladaptive change in one or several of these Ca\textsuperscript{2+} handling mechanisms leads to a reduction in the SR Ca\textsuperscript{2+} load. It was found experimentally that heart failure manifests in a decrease in the SERCA pump function (Piacentino et al., 2003; O'Rourke et al., 1999; Pieske et al., 1999), an increase in ryanodine receptor leak (Marx et al., 2000), an increase in the Na\textsuperscript{+}/Ca\textsuperscript{2+} exchanger function (Reinecke et al., 1996; O'Rourke et al., 1999), as well as the background Ca\textsuperscript{2+} influx (Hutchings et al., 2022).

Using our model, we investigated, numerically, the contribution of these major factors to the reduced SR Ca\textsuperscript{2+} load. Our simulations show that one of the major factors is an increased
Na\(^+\)/Ca\(^{2+}\) exchanger function (Fig. 3.13). If we keep the normal magnitude of Na\(^+\)/Ca\(^{2+}\) exchanger outflux in the model of failing myocyte, we obtain a dramatic increase in the SR Ca\(^{2+}\) load and [Ca\(^{2+}\)] transient. Therefore, our simulation results support the importance of a dramatic rise in Na\(^+\)/Ca\(^{2+}\) exchanger function in the failing myocytes to prevent them from Ca\(^{2+}\) overload, and such dramatic upregulation of I\(_{NaCa}\) is consistently observed experimentally in the failing hearts of mice and other species (Bers, Pogwizd, & Schlotthauer, 2002; O'Rourke et al., 1999; Pott et al., 2012).

Further, our simulations of failing myocytes also show a significant reduction in [Ca\(^{2+}\)] transient when we used the normal magnitude of the background Ca\(^{2+}\) current. Therefore, an increase in the background Ca\(^{2+}\) influx is an important mechanism to maintain proper [Ca\(^{2+}\)] transient in the failing myocyte. Our simulations also demonstrated that a reversal of the SERCA pump function to its normal magnitude in the failing myocytes resulted in a relatively modest increase in [Ca\(^{2+}\)] transient and the SR Ca\(^{2+}\) content (Fig. 3.13). While most of the experimental data showed a decrease in function of the SERCA pump in failing hearts, some data showed no change in its function, which points to its somewhat smaller role in the myocyte adaptation in heart failure (Schwinger et al., 1995). Finally, our simulations do not support a significant contribution of the SR Ca\(^{2+}\) leak or the rate of Ca\(^{2+}\) release from the SR into the failing [Ca\(^{2+}\)] transient in mice. However, such mechanisms can be important in other species or in combination with other adaptations of Ca\(^{2+}\) handling protein functions.

### 3.3.3 Pro-arrhythmic Activity in Failing Mouse Ventricular Myocytes

Arrhythmogenic changes in the failing hearts are caused by multiple cardiovascular diseases that include myocardial infarction, congenital heart disease, hypertension, dilated cardiomyopathy, hypertrophic cardiomyopathy, and others (Coronel et al., 2013). These cardiovascular diseases are characterized by electrophysiological remodeling, changes in Ca\(^{2+}\)
dynamics, remodeling of the extracellular matrix, and alteration of the signaling systems, including β-adrenergic signaling. Therefore, it is required to reveal a specific cause of heart failure to treat it efficiently.

It is considered that the triggered activity in the failing myocytes due to the altered Ca\(^{2+}\) dynamics and ionic currents are among the leading causes of ventricular tachycardia at the cellular level in different species (Pogwizd & Bers, 2004; Coronel et al., 2013; Marks, 2013). Specifically, an increased function of \(I_{NaCa}\), increased ryanodine receptor leak, decreased \(I_{K1}\) current, and increased \(I_{NaL}\) current are shown to trigger pro-arrhythmic events (EADs, DADs). In addition, it is suggested, to some extent, that the preserved function of β-adrenergic signaling can also contribute to an increased arrhythmogenesis in heart failure (Pogwizd & Bers, 2004).

This work is focused on heart failure with reduced ejection fraction in mice after TAC, which represents a relatively early stage of heart failure. As an example of the application of our mathematical model, we investigated the role of the late Na\(^+\) current, ryanodine receptor gating, and β-adrenergic signaling in the proarrhythmic events in the failing myocytes. Our simulations showed that the activation of β-adrenergic signaling by 1 μM isoproterenol prolonged AP duration and increased and prolonged \([Ca^{2+}]_i\) transients (Fig. 3.14). Investigation of dyadic space Ca\(^{2+}\) concentration, \([Ca^{2+}]_{ss}\), and ryanodine receptor gating behavior demonstrated the prolongation of Ca\(^{2+}\) release from the SR and a more prolonged time for ryanodine receptors to be in open states. Inhibition of \(I_{NaL}\) did not reverse this maladaptive change. However, \(β_1\)-AR block restored relatively normal \([Ca^{2+}]_i\) transient, \(Ca^{2+}\) release, and ryanodine receptor gating (Fig. 3.14). These simulations suggest that in the mouse model of the early stage of heart failure after TAC, one of the major pro-arrhythmic factors is the instability of the closed state and prolongation of the open states of ryanodine receptors, and the major antiarrhythmic factor is the block of \(β_1\)-adrenergic
receptors. These simulation results are in line with the experimental data, which suggest that increased phosphorylation and destabilization of the closed state of ryanodine receptors are among the major causes of pro-arrhythmic events in heart failure (Marks, 2013).
4 CONCLUSION

The results of this dissertation allow for the following major conclusions:

1. A new comprehensive model for mouse atrial myocytes that accurately described mouse atrial action potential, $\text{Ca}^{2+}$ dynamics, and $\beta_1$- and $\beta_2$-adrenergic signaling systems was developed and investigated.

2. The model showed that the T-type $\text{Ca}^{2+}$ current significantly affects the later stage of the action potential with a little effect on $[\text{Ca}^{2+}]_i$ transients.

3. The block of the small-conductance $\text{Ca}^{2+}$-activated $\text{K}^+$ current leads to a prolongation of the action potential at high intracellular $\text{Ca}^{2+}$ concentration.

4. A new comprehensive model for the action potential, $\text{Ca}^{2+}$ dynamics, and $\beta_1$- and $\beta_2$-adrenergic signaling systems in failing mouse ventricular myocytes was developed, investigated, and compared to the normal mouse ventricular myocytes.

5. The mathematical model revealed the mechanisms of the SR $\text{Ca}^{2+}$ load in failing mouse ventricular myocytes.

6. The mathematical model also disclosed the mechanisms of pro-arrhythmic events in the failing mouse ventricular myocytes and the mechanisms of the pro-arrhythmic event suppression.

The results of the second chapter of the dissertation have been published in (Asfaw et al., 2020). The results of the third chapter of the dissertation are included in the paper that will be submitted to the physiological journal (Asfaw & Bondarenko, pending publication). The related topic of modeling human cardiac sodium channel was a subject of investigation for MS Thesis by Tesfaye Asfaw and has been published in the journal of membrane biology (Asfaw & Bondarenko, 2019).
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Zhou, J., Kodirov, S., Murata, M., Buckett, P. D., Nerbonne, J. M., & Koren, G. (2003). Regional upregulation of Kv2.1-encoded current, IK,slow2, in Kv1DN mice is abolished by


### APPENDICES

**Appendix A: Atrial Model Parameters and Equations**

#### Appendix A.1 T-type Ca\(^{2+}\) Current (\(I_{CaT}\))

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Definition</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(k_{CaTpw})</td>
<td>Forward activation rate constant between closed states</td>
<td>9,500(\exp(V/15.0)) (\text{s}^{-1})</td>
</tr>
<tr>
<td>(k_{CaTmv})</td>
<td>Backward activation rate constant between closed states</td>
<td>32.0(\exp(-V/18.0)) (\text{s}^{-1})</td>
</tr>
<tr>
<td>(k_{CaTpo})</td>
<td>Forward activation rate constant between closed and open states</td>
<td>4,000 (\text{s}^{-1})</td>
</tr>
<tr>
<td>(k_{CaTmo})</td>
<td>Backward activation rate constant between open and closed states</td>
<td>25.0(\exp(-V/34.0)) (\text{s}^{-1})</td>
</tr>
<tr>
<td>(k_{CaTpi})</td>
<td>Forward inactivation rate constant</td>
<td>70.0 (\text{s}^{-1})</td>
</tr>
<tr>
<td>(k_{CaTmi})</td>
<td>Backward inactivation rate constant</td>
<td>0.98 (\text{s}^{-1})</td>
</tr>
<tr>
<td>(f_{CaT})</td>
<td>Allosteric factor for inactivation</td>
<td>0.2</td>
</tr>
<tr>
<td>(h_{CaT})</td>
<td>Allosteric factor for recovery from inactivation</td>
<td>0.5</td>
</tr>
<tr>
<td>(G_{CaT})</td>
<td>Specific maximum conductivity for T-type Ca(^{2+}) channel (non-phosphorylated)</td>
<td>0.01275 mS/(\mu)F</td>
</tr>
<tr>
<td>(G_{CaTp})</td>
<td>Specific maximum conductivity for T-type Ca(^{2+}) channel (phosphorylated)</td>
<td>0.02545 mS/(\mu)F</td>
</tr>
<tr>
<td>(f_{CaT}^{cav})</td>
<td>Fraction of the T-type Ca(^{2+}) channels located in caveolae</td>
<td>0.68</td>
</tr>
<tr>
<td>(E_{CaT,\text{rev}})</td>
<td>Reversal potential of the T-type Ca(^{2+}) current</td>
<td>35.0 mV</td>
</tr>
<tr>
<td>(k_{KCaT_{PKA}})</td>
<td>Phosphorylation rate of the T-type Ca(^{2+}) channel by PKA</td>
<td>0.174 (\mu\text{M}^{-1} \text{s}^{-1})</td>
</tr>
<tr>
<td>(k_{KCaT_{PP}})</td>
<td>Dephosphorylation rate of the T-type Ca(^{2+}) channel by PP1 and PP2A</td>
<td>0.0466 (\mu\text{M}^{-1} \text{s}^{-1})</td>
</tr>
<tr>
<td>(K_{KCaT_{PKA}})</td>
<td>Relative affinity of the T-type Ca(^{2+}) channel for PKA</td>
<td>5.0</td>
</tr>
<tr>
<td>(K_{KCaT_{PP}})</td>
<td>Relative affinity of the T-type Ca(^{2+}) channel for PP1 and PP2A</td>
<td>0.1</td>
</tr>
</tbody>
</table>

\[
I_{CaT} = I_{CaT}^{cav} + I_{CaT}^{cyt}
\]  

(A.1.1)
\[ I_{\text{cat}}^{\text{cav}} = f_{\text{cat}}^{\text{cav}} (G_{\text{cat}} (1 - f_{\text{cat},p}^{\text{cav}}) + G_{\text{catp}} f_{\text{cat},p}^{\text{cav}}) O_{\text{cat}} (V - E_{\text{cat,rev}}) \]  
(A. 1.2)  

\[ I_{\text{cat}}^\text{c} = (1 - f_{\text{cat}}^\text{c}) (G_{\text{cat}} (1 - f_{\text{cat},p}^\text{c}) + G_{\text{catp}} f_{\text{cat},p}^\text{c}) O_{\text{cat}} (V - E_{\text{cat,rev}}) \]  
(A. 1.3)  

\[ \frac{dC_{0,\text{cat}}}{dt} = k_{\text{catmv}} C_{1,\text{cat}} - 4k_{\text{catpv}} C_{0,\text{cat}} + \frac{k_{\text{catmi}}}{h_{\text{cat}}} I_{0,\text{cat}} - k_{\text{catp}} f_{\text{cat}}^3 C_{0,\text{cat}} \]  
(A. 1.4)  

\[ \frac{dC_{1,\text{cat}}}{dt} = 4k_{\text{catpv}} C_{0,\text{cat}} - k_{\text{catmv}} C_{1,\text{cat}} + 2k_{\text{catmv}} C_{2,\text{cat}} - 3k_{\text{catpv}} C_{1,\text{cat}} + \frac{k_{\text{catmi}}}{h_{\text{cat}}} I_{1,\text{cat}} \]  
\[- k_{\text{catp}} f_{\text{cat}}^2 C_{1,\text{cat}} \]  
(A. 1.5)  

\[ \frac{dC_{2,\text{cat}}}{dt} = 3k_{\text{catpv}} C_{1,\text{cat}} - 2k_{\text{catmv}} C_{2,\text{cat}} + 3k_{\text{catmv}} C_{3,\text{cat}} - 2k_{\text{catpv}} C_{2,\text{cat}} + \frac{k_{\text{catmi}}}{h_{\text{cat}}} I_{2,\text{cat}} \]  
\[- k_{\text{catp}} f_{\text{cat}} C_{2,\text{cat}} \]  
(A. 1.6)  

\[ \frac{dC_{3,\text{cat}}}{dt} = 2k_{\text{catpv}} C_{2,\text{cat}} - 3k_{\text{catmv}} C_{3,\text{cat}} + 4k_{\text{catmv}} C_{4,\text{cat}} - k_{\text{catpv}} C_{3,\text{cat}} + k_{\text{catmi}} I_{3,\text{cat}} \]  
\[- k_{\text{catp}} C_{3,\text{cat}} \]  
(A. 1.7)  

\[ \frac{dC_{4,\text{cat}}}{dt} = k_{\text{catpv}} C_{3,\text{cat}} - 4k_{\text{catmv}} C_{4,\text{cat}} + k_{\text{catmo}} O_{\text{cat}} - k_{\text{catpo}} C_{4,\text{cat}} + k_{\text{catmi}} I_{4,\text{cat}} \]  
\[- k_{\text{catp}} C_{4,\text{cat}} \]  
(A. 1.8)  

\[ \frac{dO_{\text{cat}}}{dt} = k_{\text{catpo}} C_{4,\text{cat}} - k_{\text{catmo}} O_{\text{cat}} + k_{\text{catmi}} I_{0,\text{cat}} - k_{\text{catp}} O_{\text{cat}} \]  
(A. 1.9)  

\[ \frac{dl_{0,\text{cat}}}{dt} = k_{\text{catmv}} h_{\text{cat}} l_{1,\text{cat}} - \frac{4k_{\text{catpv}}}{f_{\text{cat}}} l_{0,\text{cat}} + k_{\text{catp}} f_{\text{cat}}^3 C_{0,\text{cat}} - \frac{k_{\text{catmi}}}{h_{\text{cat}}} I_{0,\text{cat}} \]  
(A. 1.10)  

\[ \frac{dl_{1,\text{cat}}}{dt} = \frac{4k_{\text{catpv}}}{f_{\text{cat}}} l_{0,\text{cat}} - k_{\text{catmv}} h_{\text{cat}} l_{1,\text{cat}} + 2k_{\text{catmv}} h_{\text{cat}} l_{2,\text{cat}} - \frac{3k_{\text{catpv}}}{f_{\text{cat}}} l_{1,\text{cat}} \]  
\[+ k_{\text{catp}} f_{\text{cat}}^2 C_{1,\text{cat}} - \frac{k_{\text{catmi}}}{h_{\text{cat}}} I_{1,\text{cat}} \]  
(A. 1.11)  

\[ \frac{dl_{2,\text{cat}}}{dt} = \frac{3k_{\text{catpv}}}{f_{\text{cat}}} l_{1,\text{cat}} - 2k_{\text{catmv}} h_{\text{cat}} l_{2,\text{cat}} + 3k_{\text{catmv}} h_{\text{cat}} l_{3,\text{cat}} - \frac{2k_{\text{catpv}}}{f_{\text{cat}}} l_{2,\text{cat}} \]  
\[+ k_{\text{catp}} f_{\text{cat}} C_{2,\text{cat}} - \frac{k_{\text{catmi}}}{h_{\text{cat}}} I_{2,\text{cat}} \]  
(A. 1.12)
\[
\frac{dI_{3,\text{CaT}}}{dt} = \frac{2k_{\text{CatPv}}}{f_{\text{Cat}}}(I_{2,\text{CaT}} - 3k_{\text{Catmv}}h_{\text{Cat}}I_{3,\text{CaT}} + 4k_{\text{Catmv}}I_{4,\text{CaT}} - k_{\text{CatPv}}I_{3,\text{CaT}} + k_{\text{CatPl}}C_{3,\text{CaT}})
- k_{\text{Catmi}}I_{3,\text{CaT}}
\]
\[
(A.1.11)
\]
\[
\frac{dI_{4,\text{CaT}}}{dt} = k_{\text{CatPv}}I_{3,\text{CaT}} - 4k_{\text{Catmv}}I_{4,\text{CaT}} + k_{\text{Catmo}}I_{0,\text{CaT}} - k_{\text{Catpo}}I_{4,\text{CaT}} + k_{\text{CatPl}}C_{4,\text{CaT}}
- k_{\text{Catmi}}I_{4,\text{CaT}}
\]
\[
(A.1.14)
\]
\[
\frac{dI_{0,\text{CaT}}}{dt} = k_{\text{Catpo}}I_{4,\text{CaT}} - k_{\text{Catmo}}I_{0,\text{CaT}} + k_{\text{CatPl}}O_{\text{CaT}} - k_{\text{Catmi}}I_{0,\text{CaT}}
\]
\[
(A.1.15)
\]
\[
\frac{df_{\text{CatP}}^{cav}}{dt} = k_{\text{ICat}_{,\text{PPA}}}(1 - f_{\text{CatP}}^{cav})[C]^{cav}_{\text{CaT}} - k_{\text{ICaT}_{,\text{PPA}}} f_{\text{CatP}}^{cav}[PP]^{cav}_{\text{CaT}} + f_{\text{CatP}}^{cav}
\]
\[
(A.1.16)
\]
\[
\frac{df_{\text{CatP}}^{cyt}}{dt} = k_{\text{ICat}_{,\text{PPA}}}(1 - f_{\text{CatP}}^{cyt})[C]^{cyt}_{\text{CaT}} - k_{\text{ICaT}_{,\text{PPA}}} f_{\text{CatP}}^{cyt}([PP2A]^{cyt}_{f} + [PP1]^{cyt}_{f})
\]
\[
(A.1.17)
\]

**Appendix A.2 Small-conductance Ca\textsuperscript{2+}-activated K\textsuperscript{+} Current (I_{K,\text{Ca}})**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Definition</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(k_{KCaf1})</td>
<td>Forward activation rate constant between closed states</td>
<td>120.0[Ca\textsuperscript{2+}], s\textsuperscript{-1}</td>
</tr>
<tr>
<td>(k_{KCaf2})</td>
<td>Forward activation rate constant between closed states</td>
<td>96.0[Ca\textsuperscript{2+}], s\textsuperscript{-1}</td>
</tr>
<tr>
<td>(k_{KCaf3})</td>
<td>Forward activation rate constant between closed states</td>
<td>48.0[Ca\textsuperscript{2+}], s\textsuperscript{-1}</td>
</tr>
<tr>
<td>(k_{KCab1})</td>
<td>Backward activation rate constant between closed states</td>
<td>80.0 s\textsuperscript{-1}</td>
</tr>
<tr>
<td>(k_{KCab2})</td>
<td>Backward activation rate constant between closed states</td>
<td>80.0 s\textsuperscript{-1}</td>
</tr>
<tr>
<td>(k_{KCab3})</td>
<td>Backward activation rate constant between closed states</td>
<td>200.0 s\textsuperscript{-1}</td>
</tr>
<tr>
<td>(k_{KCab1})</td>
<td>Forward activation rate constant between closed and open states</td>
<td>160.0 s\textsuperscript{-1}</td>
</tr>
<tr>
<td>(k_{KCab2})</td>
<td>Forward activation rate constant between closed and open states</td>
<td>1200.0 s\textsuperscript{-1}</td>
</tr>
<tr>
<td>( k_{KCab1} )</td>
<td>Backward activation rate constant between closed and open states</td>
<td>1000.0 s(^{-1})</td>
</tr>
<tr>
<td>-----------------</td>
<td>-------------------------------------------------</td>
<td>----------------</td>
</tr>
<tr>
<td>( k_{KCab2} )</td>
<td>Backward activation rate constant between closed and open states</td>
<td>100.0 s(^{-1})</td>
</tr>
<tr>
<td>( k_{KCa_PKA} )</td>
<td>Phosphorylation rate of the small-conductance Ca(^{2+})-activated K(^+) channel by PKA</td>
<td>0.001479 µM(^{-1}) s(^{-1})</td>
</tr>
<tr>
<td>( k_{KCa_PP} )</td>
<td>Dephosphorylation rate of the small-conductance Ca(^{2+})-activated K(^+) channel by PP1 and PP2A</td>
<td>0.00029125 µM(^{-1}) s(^{-1})</td>
</tr>
<tr>
<td>( K_{KCa_PKA} )</td>
<td>Relative affinity of the small-conductance Ca(^{2+})-activated K(^+) channel for PKA</td>
<td>2.5</td>
</tr>
<tr>
<td>( K_{KCa_PP} )</td>
<td>Relative affinity of the small-conductance Ca(^{2+})-activated K(^+) channel for PP1 and PP2A</td>
<td>0.01</td>
</tr>
</tbody>
</table>

\[
I_{K,\text{Ca}} = G_{K,\text{Ca}}(V)(O_{1K,\text{Ca}} + O_{2K,\text{Ca}})(1 - f_{K,\text{Cap}}^{\text{cav}}) \quad \text{(A.2.1)}
\]

\[
G_{K,\text{Ca}}(V) = 4.4 \cdot 10^{-6}V^3 + 4.0 \cdot 10^{-4}V^2 + 0.0164V + 0.7488 \left(\frac{mS}{\mu F}\right) \quad \text{(A.2.2)}
\]

\[
\frac{dC_{1,\text{Ca}}}{dt} = k_{KCab1}C_{2,\text{Ca}} - k_{KCa f1}C_{1,\text{Ca}} \quad \text{(A.2.3)}
\]

\[
\frac{dC_{2,\text{Ca}}}{dt} = k_{KCa f1}C_{1,\text{Ca}} - k_{KCab1}C_{2,\text{Ca}} + k_{KCab2}C_{3,\text{Ca}} - k_{KCa f2}C_{2,\text{Ca}} \quad \text{(A.2.4)}
\]

\[
\frac{dC_{3,\text{Ca}}}{dt} = k_{KCa f2}C_{2,\text{Ca}} - k_{KCab2}C_{3,\text{Ca}} + k_{KCab3}C_{4,\text{Ca}} - k_{KCa f3}C_{3,\text{Ca}} + k_{KCab0}O_{1\text{Ca}} - k_{KCa f01}C_{3,\text{Ca}} \quad \text{(A.2.5)}
\]

\[
\frac{dC_{4,\text{Ca}}}{dt} = k_{KCa f3}C_{3,\text{Ca}} - k_{KCab3}C_{4,\text{Ca}} + k_{KCab0}O_{2\text{Ca}} - k_{KCa f02}C_{4,\text{Ca}} \quad \text{(A.2.6)}
\]

\[
\frac{dO_{1,\text{Ca}}}{dt} = k_{KCa f01}C_{3,\text{Ca}} - k_{KCab0}O_{1\text{Ca}} \quad \text{(A.2.7)}
\]

\[
\frac{dO_{2,\text{Ca}}}{dt} = k_{KCa f02}C_{4,\text{Ca}} - k_{KCab0}O_{2\text{Ca}} \quad \text{(A.2.8)}
\]

\[
\frac{df_{K,\text{Cap}}^{\text{cav}}}{dt} = k_{KCa\_PKA} \left(1 - f_{K,\text{Cap}}^{\text{cav}}\right) \frac{(C)^{\text{cav}}}{K_{KCa\_PKA} + (1 - f_{K,\text{Cap}}^{\text{cav}})} - k_{KCa PP} \frac{f_{K,\text{Cap}}^{\text{cav}}[PP]^{\text{cav}}}{K_{KCa PP} + f_{K,\text{Cap}}^{\text{cav}}} \quad \text{(A.2.9)}
\]
Appendix A.3 Rapidly activating, slowly inactivating K⁺ current encoded by Kv1.5

\((I_{K,\text{slow1}})\)

<table>
<thead>
<tr>
<th>Parameter (k_{K,\text{slow1,PKA}})</th>
<th>Definition</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(k_{K,\text{slow1,PKA}})</td>
<td>Phosphorylation rate of the rapidly activating, slowly inactivating K⁺ channel by PKA</td>
<td>(6.9537 \times 10^{-3} , \mu M^{-1} , s^{-1})</td>
</tr>
<tr>
<td>(k_{K,\text{slow1,PP}})</td>
<td>Dephosphorylation rate of the rapidly activating, slowly inactivating K⁺ channel by PP1</td>
<td>(3.170 \times 10^{-2} , \mu M^{-1} , s^{-1})</td>
</tr>
<tr>
<td>(K_{K,\text{slow1,PKA}})</td>
<td>Relative affinity of the rapidly activating, slowly inactivating K⁺ channel for PKA</td>
<td>0.138115</td>
</tr>
<tr>
<td>(K_{K,\text{slow1,PP}})</td>
<td>Relative affinity of the rapidly activating, slowly inactivating K⁺ channel for PP1</td>
<td>0.23310</td>
</tr>
</tbody>
</table>

\[
I_{K,\text{slow1}} = \left( G_{K,\text{slow1}} k_{K,\text{slow1}}^{ecav} + G_{K,\text{slow1,p}} (1 - f_{K,\text{slow1}}^{ecav}) \right) a_{K,\text{slow1}} i_{K,\text{slow1}} (V - E_K) \quad (A.3.1)
\]

\[
E_K = \frac{RT}{F} \ln \left( \frac{[K^+]_0}{[K^+]_i} \right) \quad (A.3.2)
\]

\[
\frac{df_{K,\text{slow1}}^{ecav}}{dt} = \frac{k_{K,\text{slow1,PP}}}{K_{K,\text{slow1,PP}} + (1 - f_{K,\text{slow1}}^{ecav})} \left(1 - f_{K,\text{slow1}}^{ecav} \right) [PP1]^{ecav} = \frac{k_{K,\text{slow1,PKA}} f_{K,\text{slow1}}^{ecav} [C]^{ecav}}{K_{K,\text{slow1,PKA}} + f_{K,\text{slow1}}^{ecav}} \quad (A.3.3)
\]

\[
\frac{da_{K,\text{slow1}}}{dt} = \frac{a_{ss} - a_{K,\text{slow1}}}{\tau_{aK,\text{slow1}}} \quad (A.3.4)
\]

\[
\frac{di_{K,\text{slow1}}}{dt} = \frac{i_{ss} - i_{K,\text{slow1}}}{\tau_{iK,\text{slow1}}} \quad (A.3.5)
\]

\[
a_{ss} = \frac{1}{1 + e^{-(V+22.5)/7.7}} \quad (A.3.6)
\]

\[
i_{ss} = \frac{1}{1 + e^{(V+45.2)/5.7}} \quad (A.3.7)
\]

\[
\tau_{aK,\text{slow1}} = \frac{6.1}{e^{0.0629(V+40.0)} + e^{-0.0629(V+40.0)}} + 2.058 \quad (A.3.8)
\]
\[ \tau_{iK,slow1} = 803.0 - \frac{18.0}{1 + e^{\frac{V+45.2}{5.7}}} \]  \hspace{1cm} (A.3.9)

**Appendix A.4 Rapidly Activating, Slowly Inactivating K⁺ Current Encoded by Kv2.1**

\[ I_{K,slow2} = G_{K,slow2}a_{K,slow2}i_{K,slow2}(V - E_K) \]  \hspace{1cm} (A.4.1)
\[ a_{K,slow2} = a_{K,slow1} \]  \hspace{1cm} (A.4.2)
\[ \frac{di_{K,slow2}}{dt} = \frac{i_{ss} - i_{K,slow2}}{\tau_{iK,slow2}} \]  \hspace{1cm} (A.4.3)
\[ \tau_{iK,slow1} = 5334.0 - \frac{4912.0}{1 + e^{\frac{V+45.2}{5.7}}} \]  \hspace{1cm} (A.4.4)

**Appendix A.5 Ionic Concentrations [Ca²⁺]ᵢ and [K⁺]ᵢ**

\[ \frac{d[Ca^{2+}]_i}{dt} = B_i \left\{ I_{\text{leak}} + I_{xfer} - I_{\text{up}} - I_{\text{trpn}} - (I_{Ca} - 2I_{Na}Ca + I_{p(Ca)} + I_{Cav}^\text{Ca}) \\
  + I_{CaT} \frac{A_{cap}C_m}{2V_iF} \right\} \]  \hspace{1cm} (A.5.1)
\[ \frac{d[K^+]_i}{dt} = -\left( I_{K,slow1} + I_{K,slow2} + I_{K,ss} + I_{K1} + I_{Kr} + I_{K,Ca} - 2I_{NaK} \\
  - I_{stim} \right) \frac{A_{cap}C_m}{V_iF} \]  \hspace{1cm} (A.5.2)
Appendix B: Heart Failure Model Parameters and Equations

The mathematical model of the combined β1- and β2-adrenergic signaling system in failing mouse ventricular myocytes consists of the model equations (Appendix B.1)-(Appendix B.6) and model parameters presented in this Appendix plus model equations (A.64)-(A.301) and model parameters from (Bondarenko 2014) and model equations (A.1)-(A125) and model parameters from (Rozier & Bondarenko, 2017).

Appendix B.1 Rapidly Activating, Slowly Inactivating K⁺ Current Encoded by Kv1.5

\( I_{K,slow1} \)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Definition</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( k_{Kslow1,PKA} )</td>
<td>Phosphorylation rate of the rapidly activating, slowly inactivating K⁺ channel by PKA</td>
<td>( 6.9537 \times 10^{-3} \ \mu M^{-1} \ \text{s}^{-1} )</td>
</tr>
<tr>
<td>( k_{Kslow1,PP} )</td>
<td>Dephosphorylation rate of the rapidly activating, slowly inactivating K⁺ channel by PP1</td>
<td>( 3.170 \times 10^{-2} \ \mu M^{-1} \ \text{s}^{-1} )</td>
</tr>
<tr>
<td>( K_{Kslow1,PKA} )</td>
<td>Relative affinity of the rapidly activating, slowly inactivating K⁺ channel for PKA</td>
<td>0.138115</td>
</tr>
<tr>
<td>( K_{Kslow1,PP} )</td>
<td>Relative affinity of the rapidly activating, slowly inactivating K⁺ channel for PP1</td>
<td>0.23310</td>
</tr>
</tbody>
</table>

\[
I_{K,slow1} = \left( G_{K,slow1} f_{K,slow1}^{\text{eCaV}} + G_{K,slow1p}(1 - f_{K,slow1}^{\text{eCaV}}) \right) a_{K,slow1} i_{K,slow1}(V - E_K) \quad (B.1.1)
\]

\[
E_K = \frac{RT}{F} \ln \left( \frac{[K^+]_0}{[K^+]_i} \right) \quad (B.1.2)
\]

\[
\frac{df_{K,slow1}^{\text{eCaV}}}{dt} = k_{Kslow1,PP} \frac{(1 - f_{K,slow1}^{\text{eCaV}})[PP1]^{\text{eCaV}}}{K_{Kslow1,PP} + (1 - f_{K,slow1}^{\text{eCaV}})} - k_{Kslow1,PKA} \frac{f_{K,slow1}^{\text{eCaV}}[C]^{\text{eCaV}}}{K_{Kslow1,PKA} + f_{K,slow1}^{\text{eCaV}}} \quad (B.1.3)
\]

\[
\frac{da_{K,slow1}}{dt} = \frac{a_{ss} - a_{K,slow1}}{\tau_{a_{K,slow1}}} \quad (B.1.4)
\]
\[
\frac{d i_{K,\text{slow}1}}{dt} = \frac{i_{ss} - i_{K,\text{slow}1}}{\tau_{iK,\text{slow}1}} \quad (B.1.5)
\]

\[
a_{ss} = \frac{1}{1 + e^{-(V+22.5)/7.7}} \quad (B.1.6)
\]

\[
i_{ss} = \frac{1}{1 + e^{(V+45.2)/5.7}} \quad (B.1.7)
\]

\[
\tau_{a_{K,\text{slow}1}} = \frac{6.1}{e^{0.0629(V+40.0)} + e^{-0.0629(V+40.0)}} + 2.058 \quad (B.1.8)
\]

\[
\tau_{i_{K,\text{slow}1}} = 803.0 - \frac{18.0}{1 + e^{V+45.2}} \quad (B.1.9)
\]

**Appendix B.2 Rapidly Activating, Slowly Inactivating K\(^+\) Current Encoded by K\(_{2.1}\)**

\[(I_{K,\text{slow}2})\]

\[
I_{K,\text{slow}2} = G_{K,\text{slow}2} a_{K,\text{slow}2} i_{K,\text{slow}2}(V - E_K) \quad (B.2.1)
\]

\[
a_{K,\text{slow}2} = a_{K,\text{slow}1} \quad (B.2.2)
\]

\[
\frac{d i_{K,\text{slow}2}}{dt} = \frac{i_{ss} - i_{K,\text{slow}2}}{\tau_{iK,\text{slow}2}} \quad (B.2.3)
\]

\[
\tau_{i_{K,\text{slow}1}} = 5334.0 - \frac{4912.0}{1 + e^{V+45.2}} \quad (B.2.4)
\]

**Appendix B.3 Late Na\(^+\) current I\(_{Na,L}\)**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Definition</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(k_{INA_L,PKA})</td>
<td>Phosphorylation rate of the late Na(^+) channel by PKA</td>
<td>(2.520 \times 10^{-2} , \mu M^{-1} s^{-1})</td>
</tr>
<tr>
<td>(k_{INA_L,PP})</td>
<td>Dephosphorylation rate of the late Na(^+) channel by PP1</td>
<td>(5.941 \times 10^{-2} , \mu M^{-1} s^{-1})</td>
</tr>
<tr>
<td>(K_{INA_L,PKA})</td>
<td>Relative affinity of the late Na(^+) channel for PKA</td>
<td>0.0549415</td>
</tr>
<tr>
<td>(K_{Na_L,PP})</td>
<td>Relative affinity of the late Na(^+) channel for PP1</td>
<td>0.78605</td>
</tr>
</tbody>
</table>
\[ I_{NaL} = (G_{NaL}(1 - f_{INaL}^{ca}) + G_{NaLP} f_{INaL}^{ca}) \alpha_{NaL}^3 h_{NaL}(V - E_{NaL}) \] (B.3.1)

\[ E_{NaL} = \frac{RT}{F} \ln \left( \frac{[Na^+]_o}{[Na^+]_i} \right) \] (B.3.2)

\[ \alpha_{mNaL} = \frac{0.32(V + 47.13)}{1 - \exp(-0.1(V + 47.13))} \] (B.3.3)

\[ \beta_{mNaL} = 0.08 \exp\left(-\frac{V}{11.0}\right) \] (B.3.4)

\[ h_{NaL}^\infty = \frac{1}{1 + \exp((V + 91)/6.1)} \] (B.3.5)

\[ \frac{d\alpha_{NaL}}{dt} = \alpha_{mNaL}(1 - \alpha_{NaL}) - \beta_{mNaL} \alpha_{NaL} \] (B.3.6)

\[ \frac{dh_{NaL}}{dt} = \frac{h_{NaL}^\infty - h_{NaL}}{\tau_{NaL}} \] (B.3.7)

\[ \frac{df_{INaL}^{ca}}{dt} = k_{INaL,PKA} \frac{(1 - f_{INaL}^{ca})[C]^{ca}}{K_{INaL,PKA} + (1 - f_{INaL}^{ca})} - k_{INaL,PP} \frac{f_{INaL}^{ca}[P]^{ca}}{K_{INaL,PP} + f_{INaL}^{ca}} \] (B.3.8)

**Appendix B.4 Ryanodine Receptors**

\[ \frac{dP_{RYR}}{dt} = -0.04P_{RYR} - 0.1 \frac{I_{CaL}}{I_{CaL,max}} e^{-\frac{(V+5.0)^2}{648.0}} \] (Control) (B.4.1a)

\[ \frac{dP_{RYR}}{dt} = -0.008P_{RYR} - 0.07 \frac{I_{CaL}}{I_{CaL,max}} e^{-\frac{(V+5.0)^2}{648.0}} \] (Heart failure) (B.4.1b)

**Appendix B.5 Ca^{2+} Leak Rate Constant from the NSR**

\[ \nu_2 = (1.2701 + 3.6045[C]^{eca}) \cdot 10^{-5} \text{ ms}^{-1} \] (Control) (B.5.1a)

\[ \nu_2 = 1.15 \cdot (1.2701 + 3.6045[C]^{eca}) \cdot 10^{-5} \text{ ms}^{-1} \] (Heart failure) (B.5.1b)
Appendix B.6 Ionic Concentrations

\[ \frac{d[Ca^{2+}]_{li}}{dt} = B_t \left\{ I_{\text{leak}} + I_{\text{fer}} - I_{\text{up}} - I_{\text{trpn}} - (I_{\text{Ca}{b}} - 2I_{NaCa} + I_{p(Ca)}) 
+ \left( I_{CaL} \frac{A_{cap} C_m}{2V_{cyt} F} \right) \right\} \quad \text{(Control)} \tag{B.6.1a} \]

\[ \frac{d[Ca^{2+}]_{li}}{dt} = B_t \left\{ I_{\text{leak}} + I_{\text{fer}} - I_{\text{up}} - I_{\text{trpn}} - (I_{\text{Ca}{b}} - 2I_{NaCa} + I_{p(Ca)} + I_{CaL}^{\text{cav}}) 
+ 0.5I_{CaL}^{\text{cav}} \frac{A_{cap} C_m}{2V_{cyt} F} \right\} \quad \text{(Heart failure)} \tag{B.6.1b} \]

\[ \frac{d[Ca^{2+}]_{ss}}{dt} = B_{ss} \left\{ J_{\text{rel}} \frac{V_{JSR}}{V_{ss}} - I_{\text{fer}} \frac{V_{cyt}}{V_{ss}} - I_{CaL}^{\text{cav}} \frac{A_{cap} C_m}{2V_{ss} F} \right\} \quad \text{(Control)} \tag{B.6.2a} \]

\[ \frac{d[Ca^{2+}]_{ss}}{dt} = B_{ss} \left\{ J_{\text{rel}} \frac{V_{JSR}}{V_{ss}} - I_{\text{fer}} \frac{V_{cyt}}{V_{ss}} - 0.5I_{CaL}^{\text{cav}} \frac{A_{cap} C_m}{2V_{ss} F} \right\} \quad \text{(Heart failure)} \tag{B.6.2b} \]

\[ \frac{d[K^+]_{li}}{dt} = -\left( I_{K_{to,f}} + I_{K_{slow1}} + I_{K_{slow2}} + I_{K_{SS}} + I_{K1} + I_{Kr} - 2I_{NaK} - I_{stim} \right) \frac{A_{cap} C_m}{V_{cyt} F} \tag{B.6.3} \]

\[ \frac{d[Na^+]_{li}}{dt} = -\left( I_{Na} + I_{NaL} + I_{NaB} + 3I_{NaCa} + 3I_{NaK} \right) \frac{A_{cap} C_m}{V_{cyt} F} \tag{B.6.4} \]