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Weber, Shafir, and Blais (2004) advocate use of the coefficient of variation (CV) as a measure of risk sensitivity and apply CV in a meta-analysis of data for risky choices by humans and animals. We critically re-examine the CV measure as either a normative or descriptive criterion for decision under risk. CV fails as a normative criterion because it violates first order stochastic dominance. Whether or not CV succeeds as a descriptive criterion depends on its consistency or inconsistency with data from experiments designed to test its distinctive properties. We report an experiment with human subjects motivated by salient monetary payoffs. The data are inconsistent with the hypothesis that the CVs of risky lotteries are a significant determinant of subjects' choices between the lotteries and certain payoffs.

Weber, Shafir, and Blais (2004) report a meta analysis of data for human and animal decision making under risk that uses the coefficient of variation (CV) as a measure of risk sensitivity. They argue that the coefficient of variation (CV), defined as the ratio of standard deviation (SD) to expected value (EV), is a good predictor of animal and human "risk sensitivity." Weber, et al. (2004, pgs. 432 – 434) review results from the analysis of animal data in Shafir (2000) in which the proportion of animals choosing certain (sure thing) rewards rather than risky rewards is regressed on the CV of risky rewards. For comparison, Weber, et al. (pg. 434) report results from another regression in which the proportion of sure thing choices is regressed on the EV of risky rewards. They conclude that: "…neither SD nor EV predicts risk sensitivity in isolation. Their ratio, however, in the form of the CV does so very well."

Weber, et al. (eq. 1, p. 433) write the utility of an option with random return X as a linear function of the utility of its expected value, $u(\mathrm{EV}(X))$ and its risk $\mathrm{R}(X)$. They note that, for a quadratic utility function, the measure of risk is the variance, $\mathrm{Var}(X)$. They argue for use of the dimension-invariant $\mathrm{CV}(X)$, instead of $\mathrm{Var}(X)$ or $\mathrm{SD}(X)$, in predicting risk sensitivity in humans and lower animals.

We critically re-examine the Weber, et al. CV measure of risk sensitivity. We ask whether CV can provide either a credible normative criterion for decision under risk or a descriptive criterion of decision making in risky environments.

The CV measure only provides a measure of risk sensitivity that is distinct from SD when it is applied to risks with distinct EVs. If application of CV were to be restricted to comparisons across choice alternatives with the same EV then it would produce a measure that was identical to SD except for an irrelevant scale factor. Thus we consider using CV as a measure of riskiness across gambles with distinct EVs. Consider two risky choice options with random returns, X and Y. If for individual j, $u_j(\mathrm{EV}(X)) - b_j\mathrm{CV}(X) < u_j(\mathrm{EV}(Y)) - b_j\mathrm{CV}(Y)$, then option Y is preferred to option X by individual j according to the CV measure of risk sensitivity (Weber, et al., pgs. 433, 443).

We analyze CV as a criterion for decision under risk. We first ask whether choosing among risky alternatives according to their ordering by CV would promote the self-interest of the decision maker. In other words, we ask whether CV provides a (credible) normative criterion for decision under risk that is consistent with monotonicity of preferences, as in the criterion of first-order stochastic dominance (Hadar and Russell, 1969; Rothschild and Stiglitz, 1970). Subsequently, we report an experiment, with financially motivated human decision makers, designed to provide a direct test for consistency of subjects' decisions under risk with use of CV as a decision criterion.

Can the CV Measure Provide a Normative Criterion for Decision under Risk?

We here ask whether the CV measure of risk sensitivity can provide a credible normative criterion for decision under risk. We show that the CV measure cannot provide a credible normative criterion for choosing between non-degenerate lotteries because it is inconsistent with first-order stochastic dominance (FOSD).[1] Consistency with FOSD is the most basic requirement for rational choice over risky alternatives because it follows from positive monotonicity of the agent's objective function in the reward medium. An animal that "prefers" to increase its fitness should make risky foraging choices that are consistent with FOSD.[2] A human who prefers more money to less should make financial choices that are consistent with FOSD.[3]

Let an individual's preferences over lotteries that involve gains be characterized by some $b > 0$ and some positively monotonic function $u(\cdot)$ over outcomes. Let positive numbers $\ell, k,$ and $n > 2$ be given. Let L_n denote the binary lottery that yields payoff of ℓ / p with probability p and payoff of $n\ell/(1-p)$ with probability $1-p$. Similarly, let L_{n+k} denote the binary lottery that yields payoff of ℓ / p with probability p and payoff of $(n+k)\ell/(1-p)$ with probability $1-p$. We show that there exists $p \in [0.5, 1)$ such that the individual prefers binary lottery L_n to lottery L_{n+k} according to the CV criterion. Note that the only difference between lotteries L_n and L_{n+k} is that the high outcome in L_{n+k} is $k\ell/(1-p)$ larger than the high outcome in L_n. Yet lottery L_n is preferred to L_{n+k} according to CV model.

To show the result start by noting that the CV of lottery L_n is given by

$$(1) \qquad CV = \sqrt{\frac{\ell^2 / p + n^2\ell^2 / (1-p)}{(n+1)^2 \ell^2} - 1} = \frac{\left| p - 1/\left(n+1\right)\right|}{\sqrt{p(1-p)}}.$$

It can be easily checked that for $p > 1/(n+1)$ (which follows from $p \in [0.5, 1)$, and $n > 2$), the CV of gambles L_n is *increasing* in n. According to the CV measure of risk sensitivity (Weber, et al., pgs. 433, 443), option L_n is preferred to option L_{n+k} if

$$(2) \qquad u(EV_{L_n}) - bCV_{L_n} > u(EV_{L_{n+k}}) - bCV_{L_{n+k}}.$$

Substituting expressions for EV and CV yields

$$(3) \qquad u(EV_{L_n}) - bCV_{L_n} = u((n+1)\ell) - b\frac{\left| p - 1/\left(n+1\right)\right|}{\sqrt{p(1-p)}}.$$

and

$$(4) \qquad u(EV_{L_{n+k}}) - bCV_{L_{n+k}} = u((n+1+k)\ell) - b\frac{\left| p - 1/\left(n+1+k\right)\right|}{\sqrt{p(1-p)}}$$

Therefore, in statement (2), we get that L_n is preferred to option L_{n+k} if:

$$(5) \qquad \begin{aligned} 0 &< u((n+1)\ell) - u((n+1+k)\ell) - b\left[CV_{L_n} - CV_{L_{n+k}}\right] \\ &= -\Delta u - \frac{b}{\sqrt{p(1-p)}}\left(\frac{1}{n+k+1} - \frac{1}{n+1}\right) \end{aligned}$$

Hence, statement (2) is equivalent to

$$(6) \qquad \frac{bk}{(n+1)\left(n+1+k\right)\Delta u} > \sqrt{p(1-p)}$$

For any given n, k, $u(\cdot)$, and b, the left hand side of the last inequality is a finite number, say C. The right-hand side has limit 0 for p close to 1, so, we can always find a p such that $\sqrt{p(1-p)}$ is less than C. Therefore, L_n is preferred to option L_{n+k} for some p. This is inconsistent with positive monotonicity of preferences.

As an example, for any given $b > 0, \ell = 1, n = 2$ and $k = 1$ normalize $u(\cdot)$ such that $u(4) - u(3) = 2bk/(n+1)(n+k+1) = b/6$. Take $p = 1/2$. Then, statement (6) is satisfied. Consider option *A*, a binary lottery that pays 2 with probability 1/2 and 4 with probability 1/2. Let option *B* be the binary lottery with 1/2 probabilities of payoffs of 2 and 6. It is easiest to think of these numbers as referring to amounts of money but that interpretation is not necessary for our argument; the payoffs could, instead, be based on a measure of foraging yield. If the payoffs are amounts of money then whether they are numbers of dollars or millions of dollars or some other unit of account is irrelevant to the argument. Note that $EV_A = 3, EV_B = 4, CV_A = 1/3,$ and $CV_B = 1/2.$ According to the CV measure of risk sensitivity (Weber, et al., pgs. 433, 443):

(7) $$U(X_A) - U(X_B) = u(3) - b/3 - u(4) + b/2.$$

Note that $U(X_A) - U(X_B) > 0$ for $b > 6[u(4) - u(3)];$ so, in that case, the agent prefers option A to option B. But this conclusion is inconsistent with the self-interest of any agent who is not satiated in the reward medium because the only difference between the two gambles is that the high payoff equals 4 in option A and 6 in option B.

This demonstrates that CV cannot provide a credible normative criterion for decision under risk because, by violating first-order stochastic dominance, it would require in some contexts that animals make choices that are inconsistent with increasing fitness and humans make choices that are inconsistent with preferring more money to less.

Can the CV Measure Provide a Descriptive Model for Decision under Risk?

Although the CV measure of risk sensitivity is inconsistent with fitness-maximizing and wealth-maximizing decision making under risk, perhaps it is nevertheless consistent with how agents actually make decisions. Weber, et al. estimate parameters that fit SD and CV measures to data from many experiments and conclude that CV fits the data better. We ask a different question than parameter fitting. We design an experiment on choice under risk that provides a direct test for the effects of CV on subjects' willingness to take risks.

Experimental Design

The decision task is to choose between two options in each of six decision pages. One of the six decision pages of each subject is randomly chosen for money payoff by rolling a die in the presence of the subject. On each decision page, a subject is asked to circle A, B, or I, thereby indicating a choice of option A or option B or indifference. Subjects are told that if they circle I then the experimenter will choose option A or option B for them. Subject instructions and decision pages (or response forms) are included in the appendix. The decision pages are given to each subject in an independently drawn random order. Subjects have access to all six decision pages while making each decision.

Table 1 reports the six decision tasks in the experiment. In Task 1, for example, a subject is asked to choose between \$7 for sure and the binary lottery that pays \$2.50 if ball 1 is drawn from the bingo cage or pays \$7.50 if ball 2, 3,…,9, or 10 is drawn. Table 1 reports the EV and CV of option B in each of decision Tasks 1 – 6. Note that the EV of option B in each task equals the sure payoff in option A of that task. The CV measure increases monotonically with decision task number, from a low of 0.21 for Task 1 to a high of 2.50 for Task 6. Therefore, if the CV measure is consistent with subjects' decision making under risk then their frequency of choosing the sure amount of payoff will increase monotonically with the task number in Table 1 since we are in a gain domain (i.e. b > 0) and

(8) $$u(A) - bCV_A - u(B) + bCV_B = bCV_B.$$

<u>Data from the Experiment</u>

Fifty subjects participated in the experiment. Figure 1 shows graphs of the proportions of subjects who chose the safe option and the CVs of the lotteries in Tasks 1 – 6 in the experiment. CV is monotonically increasing with task number. In contrast, the proportion of subjects who chose the sure thing shows no clear relationship to CV. The correlation coefficient is 0.143 according to the Spearman rank correlation test. This test reports a p-value of 0.787, so the null hypothesis that proportions of choices of the sure thing and CVs of the risky lotteries are independent is *not* rejected.

The Spearman test and the graphs in Figure 1 use aggregate data. Perhaps the effects of subject heterogeneity mask the effects of CV on individuals' willingness to take risks. The Model I Probit column of Table 2 reports results from a random effects probit analysis of the data in which observations for each individual subject comprise a "group" of observations.[4] The explanatory variables are CV of the risky lotteries and 15 measures of subjects' individual characteristics. These 15 measures are provided by subjects' responses on the questionnaire included in the appendix. The p-value for the chi-square test of significance of the "regression" is 0.024. The coefficients of two of the explanatory variables are significant at 1% while four more are significant at 10%. The p-value of the coefficient for CV is 0.141; hence the coefficient is insignificantly different from 0.

The Model II Probit column of Table 2 reports a probit analysis that includes all of the variables in Model I plus the EV of the lotteries. The coefficients for EV and CV are insignificant. Coefficients for two subject characteristics are significant at 1% while three others are significant at 10%. Significance tests for Probit coefficients are based on normally distributed errors.

The Model I Logit and Model II Logit columns of Table 2 report logit analyses with significance tests based on errors with logistic distribution. The coefficients on CV in both models, and the coefficient on EV in Model 2, are all insignificant. Significance of coefficients on other variables are similar to that for the probit analysis.

The Spearman test, probit analysis, and logit analysis all support the same conclusion: the CV of risky lotteries does not have a significant effect on decisions by subjects in our experiment. We conclude that the CV measure of risk sensitivity does not provide either a credible normative criterion for decision under risk or a descriptive model consistent with subjects' actual decision making in the risky environment of our experiment.

Endnotes

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1. Let F and G be two cumulative probability distributions of payoffs defined on the set Ω. Strict first order stochastic dominance of F over G is defined by:
(*i*) $F(x) \le G(x)$, for all $x \in \Omega$; and (*ii*) $F(x_k) < G(x_k)$, for at least one $x_k \in \Omega$.

2. Such "preference" can refer either to a deterministic ordering or a probabilistic ordering in which a violation of FOSD occurs when an agent chooses the dominated (risky-choice) alternative more than 50% of the time.

3. In contrast, it is second-order stochastic dominance that provides a well-defined nonparametric measure of riskiness (Hadar and Russell, 1969; Rothschild and Stiglitz, 1970). For example, the cumulative probability distribution of payoffs G is "more risky than" the distribution F if G is a mean-preserving spread of F.

4. Software for probit and logit analysis does not use the seven observations in which subjects chose indifference rather than strict preference for option A or option B (coded as 0 and 1). Hence the parameter estimates are based on 293 observations.

References

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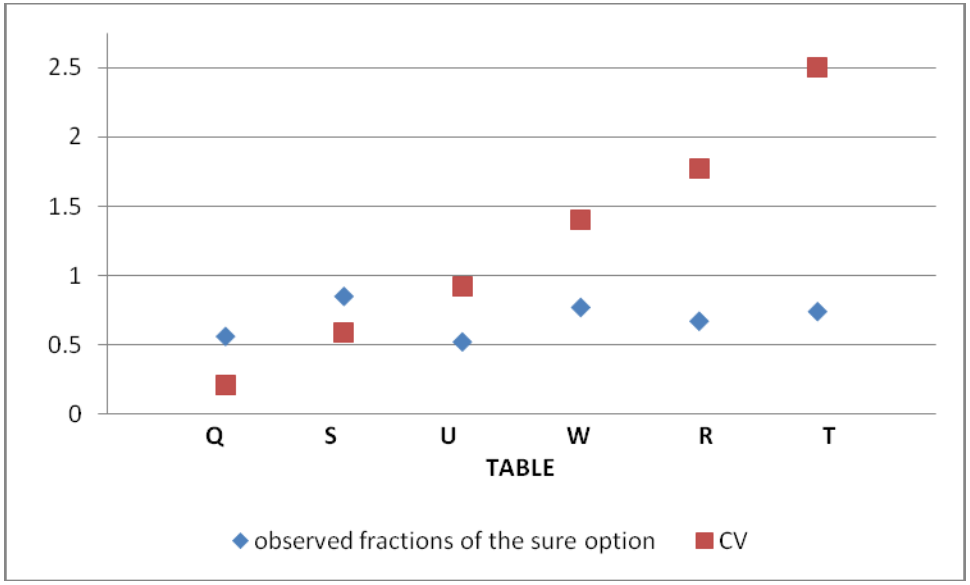

Figure 1. The observed frequencies of the sure option and values of CV across tasks

Table 1. Options A and B in Decision Tasks 1 – 6

Task	***Option A***	***Option B***					
		Low Payoff $	***Low Payoff Balls***	***High Payoff $***	***High Payoff Balls***	***EV***	***CV***
Task 1	*$7*	*$2.50*	*1*	*$7.50*	*2, 3, 4, 5, 6, 7, 8, 9, 10*	*$7*	*0.21*
Task 2	*$27*	*$2.50*	*1,2,3*	*$37.50*	*4, 5, 6, 7, 8, 9, 10*	*$27*	*0.59*
Task 3	*$35*	*$2.50*	*1, 2, 3, 4, 5*	*$67.50*	*6, 7, 8, 9, 10*	*$35*	*0.92*
Task 4	*$31*	*$2.50*	*1, 2, 3, 4, 5, 6, 7*	*$97.50*	*8, 9, 10*	*$31*	*1.40*
Task 5	*$22*	*$2.50*	*1, 2, 3, 4, 5, 6, 7, 8*	*$100.00*	*9, 10*	*$22*	*1.77*
Task 6	*$15*	*$2.50*	*1, 2, 3, 4, 5, 6, 7, 8, 9*	*$127.50*	*10*	*$15*	*2.50*

Table 2. Probit and Logit Analyses of the Data

	Model I Probit	Model II Probit	Model I Logit	Model II Logit
CV	0.160 (0.141)	0.156 (0.151)	0.250 (0.176)	0.245 (0.180)
EV		0.006 (0.496)		0.010 (0.520)
Study year	0.113 (0.391)	0.110 (0.401)	0.183 (0.412)	0.181 (0.418)
Major category	0.061 (0.480)	0.061 (0.481)	0.108 (0.469)	0.108 (0.472)
age	-0.117*** (0.008)	-0.117*** (0.007)	-0.196*** (0.008)	-0.196*** (0.008)
gender	-0.555* (0.086)	-0.551* (0.089)	-0.912* (0.097)	-0.909* (0.099)
height	0.061* (0.097)	0.061 (0.101)	0.098 (0.120)	0.098 (0.122)
race	0.182 (0.110)	0.184 (0.106)	0.298 (0.123)	0.300 (0.120)
Smoking	0.384 (0.349)	0.382 (0.352)	0.663 (0.335)	0.660 (0.337)
Child nr	0.003 (0.983)	0.002 (0.987)	0.004 (0.985)	0.003 (0.987)
Seek Opportunity	-0.296*** (0.001)	-0.295*** (0.001)	-0.488*** (0.002)	-0.488*** (0.002)
Consequences Free	0.060 (0.341)	0.059 (0.350)	0.103 (0.336)	0.103 (0.340)
Lucky Breaks	0.040 (0.328)	0.040 (0.326)	0.067 (0.323)	0.067 (0.322)
Get Jittery	0.135* (0.015)	0.136* (0.015)	0.223** (0.019)	0.223** (0.018)
Carefully Proceed	-0.043 (0.671)	0.041 (0.682)	-0.060 (0.719)	-0.059 (0.725)
Assess Risk	0.171* (0.051)	0.170* (0.053)	0.273* (0.065)	0.271* (0.066)
Danger Worry free	0.057 (0.352)	0.058 (0.346)	0.096 (0.356)	0.096 (0.353)
Constant	-2.130 (0.456)	-2.217 (0.439)	-3.264 (0.501)	-3.429 (0.480)
Log-likelihood	-160.446	-160.214	-160.881	-160.674
Wald Chi-square	28.98** (0.024)	29.42** (0.031)	26.08* (0.053)	26.39* (0.068)
Nr of observations	293	293	293	293

p-values are reported in parentheses; *p-value < 0.1; **p-value < 0.05; ***p-value < 0.01

Appendix: Subject Instructions, Decision Pages, and Questionnaire

Subject Instructions

You will be paid an amount determined by your decisions in this experiment and the outcomes from rolling a die and drawing a numbered ball from a bingo cage.

The experiment proceeds as follows. First, you choose your preferred option in each of six tables. Second, which of the six tables will be selected for money payoff will be determined by rolling a six-sided die in your presence. The number that ends "up" on the die determines which one of your decision tables pays money.

Note that only one of your six decisions will be selected for money payoff by rolling the die; thus you should decide which option you prefer in each table **independently** of your choice in other tables.

You are asked to choose either option A or option B in each of six tables.

The example below shows the type of choice table that will be included in the experiment.

In Table 1, if you choose Option A, then your payoff is $54 (if this table is selected by rolling the die).

If you choose Option B (and this table is selected by rolling the die) then your payoff is determined by drawing a ball from a bingo cage that contains 10 balls with 10 different numbers. Your payoff from choosing Option B is:

$5 if the bingo cage selects **ball 1, 2, or 3**;
$75 if the bingo cage selects **ball 4, 5, 6, 7, 8, 9, or 10**.

You indicate your choice in the third column. If you prefer Option A to Option B then circle A. If you prefer Option B to Option A then circle B. In case you are indifferent between the two options, you circle I. If I is circled then the experimenter will choose Option A or Option B for you.

Table 1

Option **A**	Option **B**	My Choice
$54	**$5** if ball from **{1, 2, 3}** or **$75** if ball from **{4, 5, 6, 7, 8, 9, 10}**	**A B I**

Decision Pages

The response forms used in the experiment consist of six decision pages. Each decision page contains one of the following tables, summary instructions for the choice task in the table, and explanation of the lotteries on that page. Full text of the decision pages is available upon request to the authors. The decision pages were given to individual subjects in independently drawn random order. Subjects had access to all six decision pages while making their decisions on every page. One decision page for each subject was independently, randomly selected for money payoff.

Table Q

Option **A**	Option **B**	My Choice
$7	**$2.50** if ball from **{1}** or **$7.50** if ball from **{2,3,4,5,6,7,8,9,10}**	**A B I**

Table S

Option **A**	Option **B**	My Choice
$27	**$2.50** if ball from **{1, 2, 3}** or **$37.50** if ball from **{4, 5, 6, 7, 8, 9, 10}**	**A B I**

Table U

Option **A**	Option **B**	My Choice
$35	**$2.50** if ball from **{1, 2, 3, 4, 5}** or **$67.50** if ball from **{ 6, 7, 8, 9, 10}**	**A B I**

Table W

Option **A**	Option **B**	My Choice
$31	**$2.50** if ball from **{ 1, 2, 3, 4, 5, 6, 7 }** or **$97.50** if ball from **{ 8, 9, 10 }**	**A B I**

Table R

Option **A**	Option **B**	My Choice
$22	**$2.50** if ball from **{ 1, 2, 3, 4, 5, 6, 7,8 }** or **$100.00** if ball from **{ 9, 10 }**	**A B I**

Table T

Option **A**	Option **B**	My Choice
$15	**$2.50** if ball from **{ 1, 2, 3, 4, 5, 6, 7, 8, 9}** or **$127.50** if ball from **{ 10 }**	**A B I**

Questionnaire (with anonymous, coded responses)

Thank you very much for participating in our decision experiment. We would like to ask you a few questions. Your privacy is protected because your name will not appear on this questionnaire or on your decision tables.

Information about you:

1. What year are you in school? Freshman ____ Sophomore ____ Junior ____ Senior ____ Grad. ____
2. What is your intended or declared major ? ____________________________
3. What is your age? ______
4. What is your gender? Female ____ Male _____
5. What is your race? Asian _____ Black/African American _____ White _____ Other ____

   Prefer not to respond _____
6. What is your height? Feet ____ Inches ____
7. Are you a smoker? Yes ____ No ____
8. What is your birth order? Oldest child in your family? ____ Youngest child? ____ Middle child? ____

General Questions: Please answer the following questions on a scale of 1-10, where 1 is **Strongly Disagree** and 10 is **Strongly Agree.** Please **circle** the number that represents your best answer.

1. I seek opportunities for doing things that I never did before.

1 2 3 4 5 6 7 8 9 10

Strongly disagree Strongly agree

2. I don't worry about the consequences of what I do.

1 2 3 4 5 6 7 8 9 10

Strongly disagree Strongly agree

3. I never get lucky breaks.

1 2 3 4 5 6 7 8 9 10

Strongly disagree Strongly agree

4. I frequently get jittery and worry about things.

1 2 3 4 5 6 7 8 9 10

Strongly disagree Strongly agree

5. I proceed with care in most endeavors.

1 2 3 4 5 6 7 8 9 10

Strongly disagree Strongly agree

6. I always assess the prospects and risks before starting a new activity.

1 2 3 4 5 6 7 8 9 10

Strongly disagree Strongly agree

7. I tend to do dangerous things without adequate precautions.

1 2 3 4 5 6 7 8 9 10

Strongly disagree Strongly agree