4-1-2008

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Experimental Economics Center Working Paper 2008 - 04

Georgia State University

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By James C. Cox, Vjollca Sadiraj, Bodo Vogt, and Utteeyo Dasgupta[1]

Theories of decision under risk that model risk averse behavior with decreasing marginal utility of money have previously been critiqued with calibration analyses. This paper introduces a dual calibration critique that applies to decision theories that represent risk aversion with nonlinear transformation of probabilities or nonlinear transformation of payoffs or both types of transformations. The dual calibration critique makes clear how plausibility problems with theories of decision under risk are fundamental. Testable calibration propositions are derived that apply to dual theory of expected utility, cumulative prospect theory, rank dependent utility theory, and expected utility theory. Heretofore, calibration critiques have been based on thought experiments. This paper reports real experiments that provide data on the empirical relevance of the calibration critique to evaluating the plausibility of theories of decision under risk.

Keywords: Decision Theory, Risk, Calibration, Experiments

1. Introduction

Much literature during the last 25 years has focused on differences between alternative theories of decision making for risky environments. In contrast, we focus on the fundamental problems that they have in common.

Prominent decision theories explain risk-avoiding behavior with models that incorporate (a) concave transformation of money payoffs (expected utility theory) or (b) convex transformation of decumulative probabilities (dual theory of expected utility) or (c) both payoff and probability transformations (rank dependent utility theory and cumulative prospect theory). We identify patterns of risk aversion for which all of these theories have implausible implications that follow from calibration propositions.

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[1] *This is a revision and extension of our 2005 working paper titled "On the Empirical Plausibility of Theories of Risk Aversion." The present paper incorporates a third experiment (this one on probability transformations). We are grateful to Glenn W. Harrison, Peter P. Wakker, and Nathanial T. Wilcox for helpful comments and suggestions. Financial support was provided by the National Science Foundation (grant numbers DUE-0226344, DUE-0622534, and IIS-0630805)*

Previous calibration literature focuses on implausible implications that can follow from concave transformation of money payoffs (or "decreasing marginal utility of money"). We provide dual calibrations that focus on implications of nonlinear transformation of probabilities as well as nonlinear transformation of payoffs. The first calibration proposition identifies a pattern of small-stakes risk aversion that has implausible large-stakes risk aversion implications for dual theory of expected utility but not for expected utility theory. In contrast, the second calibration proposition reports a different pattern that has implausible implications for expected utility theory but not for the dual theory of expected utility. Each calibration proposition has a corollary that implies implausible risk aversion for rank dependent utility theory and cumulative prospect theory. The dual corollaries reveal that theories that model risk aversion with nonlinear transformations of both probabilities and payoffs are subject to both types of calibration critique. Since prominent theories model risk-avoiding behavior with nonlinear transformation of payoffs or nonlinear transformation of probabilities or both types of nonlinear transformation, our analysis shows how all such theories can be subjected to calibration critique.

Calibrations of the implications of nonlinear transformations of payoffs are based on suppositions about patterns of small-stakes risk aversion exhibited by individual agents. Previous literature has not reported empirical tests of such suppositions; hence the empirical relevance of extant calibrations of payoff transformation theories has been unknown. The present paper reports three experiments designed to shed light on the empirical validity of the small-stakes risk aversion suppositions underlying both the nonlinear payoff transformation and nonlinear probability transformation propositions and corollaries reported herein, and thereby on the relevance of these calibrations to evaluating the empirical plausibility of theories of decision under risk. Each experimental design also generates data with testable implications for theories with the other type of nonlinear (money payoff or probability) transformation as well as implications for expected value theory (that transforms neither money payoffs nor probabilities).

2. Calibration of Probability Transformations

The dual theory of expected utility (Yaari, 1987) models risk aversion solely with convex transformation of decumulative probabilities and has a utility functional that is always linear in money payoffs as a consequence of the dual independence axiom. Cumulative prospect theory (Tversky and Kahneman, 1992) and rank dependent utility theory (Quiggin, 1993) represent risk preferences with nonlinear transformations of both probabilities and payoffs. We report a calibration proposition for the dual theory and a corollary that applies to cumulative prospect theory and rank dependent utility theory.

2.1. Calibration of Patterns of Risk Aversion for Dual Theory of Expected Utility

We follow Yaari's (1987) convention in writing the probability transformation function $f(\cdot)$ for dual theory using decumulative probabilities. Let L denote a lottery that pays amounts of money y_j with probabilities p_j, $j = 1, 2, \cdots, n$, where $y_j \geq y_{j-1}$ for all j. The dual expected utility of the lottery L is

(1) $$U_{DU}(L) = \sum_{j=1}^{n}[f(\sum_{k=j}^{n} p_k) - f(\sum_{k=j+1}^{n} p_k)]y_j .$$

If the agent is risk averse then the function $f(\cdot)$ is convex. If the agent is risk neutral then the probability transformation function $f(\cdot)$ is linear and functional (1) represents the expected value of the lottery.

We start with an example that illustrates the large stakes risk aversion implications for dual theory of a specific pattern of small stakes risk aversion. For simplicity, the example assumes global convexity of the probability transformation function, while the following proposition does not. Consider lotteries of the following type: $2n-1$ pairs of lotteries indexed by $i = 1, 2, \cdots, 2n-1$. Lottery A_i in pair i pays amounts of money 40 or 0 with probabilities $i/2n$ and $1-i/2n$, and has expected value $40i/2n$. Lottery B_i in pair i is constructed from A_i by transferring probability $1/2n$ from each of the "tail" payoffs of 40 and 0 to a middling payoff of 10. This produces a lottery that pays 40 or 10 or 0 with probabilities $(i-1)/2n$ and $1/n$ and $1-(i+1)/2n$, with expected value

$(40i-20)/2n$. If B_i is preferred to A_i, for all $i=1,2,\cdots,2n-1$, then according to the dual theory $U_{DU}(B_i)\geq U_{DU}(A_i)$, which together with statement (1) imply

(2) $$40f((i-1)/2n)+10[f((i+1)/2n)-f((i-1)/2n)]\geq 40f(i/2n),$$

for $i=1,2,\cdots,2n-1$. Inequality (2) can be rewritten as

(3) $$f((1+i)/2n)-f(i/2n)\geq(30/10)\big[f(i/2n)-f((i-1)/2n)\big],\ i=1,\ldots,2n-1,$$

which with notation $k=i/2n$, is simply

(4) $$f(k+1/2n)-f(k)\geq 3\big[f(k)-f(k-1/2n)\big].$$

Differentiability and convexity of the function $f(\cdot)$ imply that the expression on the left hand side of inequality (4) is not larger than $f'(k+1/2n)/2n$ whereas the expression inside the square brackets on the right hand side is not smaller than $f'(k-1/2n)/2n$; hence $f'(k+1/2n)\geq 3f'(k-1/2n)$. The last inequality and iteration for positive integers t, such that $2t\leq 2n-i$, imply

(5) $$f'(i/2n+2t/2n)\geq 3^t f'(i/2n).$$

Inequality (5) provides intuition for calibration of probability transformation functions: it informs us that the postulated pattern of risk aversion implies that the slope of the probability transformation function $f(\cdot)$ at points $1/n$ apart increases exponentially. Exponential increase in transformed *decumulative* probability of money payoffs implies that, relative to small payoffs, large payoffs receive very low (transformed-probability) weight in the dual theory functional (1), which implies the types of implausible large-stakes risk aversion described in the examples following the proposition.

Proposition 1 presents a general calibration result for dual theory probability transformations; the proposition does not assume global convexity. Let $\{y_3,p_3;y_2,p_2;y_1\}$ denote a three-outcome lottery that pays positive amounts: y_3 with probability p_3; y_2 with probability p_2; and y_1 with probability $1-p_2-p_3$. As above, we use the convention $y_j\geq y_{j-1}$ for all j. Let $\{y_2,p;y_1\}$ denote a binary lottery that pays the larger amount of money y_2 with probability p and the smaller positive amount of money y_1 with probability $1-p$. We consider the $2n-1$ pairs of lotteries

$B_i = \{cx, (i-1)/2n; x, 1/n; 0\}$ and $A_i = \{cx, i/2n; 0\}$. The highest payoff cx in a B_i lottery is assumed to be more than twice the middle payoff x; that is, $c > 2$. Define $K(t,n) = 1 + \sum_{j=1}^{n} (t-1)^j \Big/ \sum_{i=1}^{n} (t-1)^{1-i}$. Let $\succsim$ and $\succ$, respectively, indicate weak and strong preference. Let N denote the set of positive integers.

Proposition 1 (calibration for dual theory of expected utility)*. Let* $n \in N$ and $c > 2$ be given. If
P(1*) $\{cx, (i-1)/2n; x, 1/n; 0\} \succsim \{cx, i/2n; 0\}$, for all $i = 1, 2, \cdots, 2n-1$, then
$z \succ \{zK(c,n), 0.5; 0\}$, for all $z > 0$.

Proof: see appendix A.1.

Note that, for $c > 2$, $K(c,n) \to \infty$ as $n \to \infty$. Therefore, the larger is the value of n, the more extreme are the implications from the calibration. This implies that for any K, as big as one chooses, there exists a large enough n such that preference for the three outcome lottery B_i over the two outcome lottery A_i, for all integers $i = 1, 2, \cdots, 2n-1$, implies a preference for z for sure over the risky lottery $\{zK, 0.5; 0\}$ for all $z > 0$.

Some implications of Proposition 1 are reported in Table 1. For example, let the payoffs be $cx = 210$ and $x = 100$. Suppose that the agent rejects a lottery with payoffs [210,0] and probabilities [*i*/100,1-*i*/100] in favor of a lottery with payoffs [210,100,0] with probabilities [(*i*-1)/100,2/100,1-(*i*+1)/100] for all *i* = 1,…99. Application of Proposition 1 with *c* = 2.1, *x* = 100 and *n* = 50 tells us that for this pattern the dual theory predicts that the agent prefers 100 for sure to a lottery that pays 11,830 or 0 with even odds, as reported in the First DU Calibration column and third row of Table 1. The Third DU Calibration column of Table 1 reports calibrations for the small stakes risk aversion pattern that involves the lotteries $\{40, i/2n; 0\}$ and $\{40, (i-1)/2n; 10, 1/n; 0\}$ used in the illustrative example above. If the pattern of rejection is true for all *i* = 1,…,19 (as in the second row of the table) then the prediction is that the agent prefers 100 for sure to a lottery that pays 5.9 million (that is, 0.59

$\times 10^7$) or 0 with even odds. The Second DU Calibration column reports results for cx = 250 and x = 100.

2.2. Calibration of Patterns of Risk Aversion for Rank Dependent Utility Theory and Cumulative Prospect Theory

Calibration of patterns of risk aversion for theories that transform only probabilities can be extended to theories that transform both payoffs and probabilities, such as rank dependent utility theory (Quiggin,1993) and cumulative prospect theory (Tversky and Kahneman, 1992). The utility functional for rank dependent utility theory incorporates a transformation function $\upsilon(\cdot)$ for money payoffs. We write the functional with transformation function $h(\cdot)$ for decumulative probabilities.[2] In that case, this theory represents preferences over a lottery L with a functional of the form

$$(6) \qquad U_{RD}(L) = \sum_{j=1}^{n} [h(\sum_{k=j}^{n} p_k) - h(\sum_{k=j+1}^{n} p_k)] \cdot \upsilon(y_j) \cdot$$

Cumulative prospect theory transforms both probabilities and payoffs differently for losses than for gains. However, for the specific lotteries considered in this section, cumulative prospect theory does not differ from rank dependent utility theory.

The following corollary to Proposition 1 applies to rank dependent utility theory and cumulative prospect theory. The proposition for dual theory, with functional that is linear in payoffs, incorporates the assumption that $c > 2$, which implies that the highest payoff cx in a B_i lottery is more than twice the amount of the middle payoff x in the lottery. In the corollary, we restate the assumption for money payoff transformations $\upsilon(\cdot)$, that can be nonlinear, as $\upsilon(cx)/\upsilon(x) > 2$. Let $\upsilon^{-1}(\cdot)$ be the inverse function of $\upsilon(\cdot)$. One has:

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[2] *In the original version of rank dependent utility theory, Quiggin (1993) wrote functionals with transformation functions for cumulative probabilities. The two representation conventions are logically equivalent.*

Corollary 1 (calibration for cumulative prospect theory and rank dependent utility theory). Suppose that condition P(1*) is satisfied and that $\upsilon(cx)/\upsilon(x) > 2$. Then $z \succ \{\upsilon^{-1}(\upsilon(z)K(\upsilon(cx)/\upsilon(x), n)), 0.5; 0\}$, for all $z > 0$.

Proof: see appendix A.1.

Specification of a money transformation function $\upsilon(\cdot)$ implies pairs of values of c and x that satisfy the condition $\upsilon(cx)/\upsilon(x) > 2$ and can be used to illustrate implications of Corollary 1. For example, the right-most column in Table 1 reports calibrations for rank dependent utility theory (RD) and cumulative prospect theory (PT) using the money transformation function $\upsilon(y) = y^{0.88}$ (Tversky and Kahneman, 1992, p. 311). With this money transformation function, lottery payoffs $cx = 40$ and $x = 10$ satisfy the condition. As reported in the third row of the right-most column, if the agent rejects a lottery with payoffs [40,0] and probabilities [*i*/100,1-*i*/100] in favor of a lottery with payoffs [40,10,0] with probabilities [(*i*-1)/100,2/100,1-(*i*+1)/100], for all *i* = 1,…99, then calibration for cumulative prospect theory and rank dependent utility theory implies that 100 for sure is preferred to the even odds lottery that pays 0.29×10^{24} or 0.

3. Calibration of Payoff Transformations

Expected utility theory represents risk aversion solely with concave transformation of money payoffs and has a utility functional that is always linear in probabilities as a consequence of the independence axiom. As noted above, cumulative prospect theory and rank dependent utility theory represent risk preferences by nonlinear transformations of both probabilities and payoffs. We report a calibration proposition for expected utility theory and a second corollary that applies to cumulative prospect theory and rank dependent utility theory.[3]

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[3] *Previous calibration analyses for theories with nonlinear transformations of payoffs are reported in Rabin (2000), Neilson (2001), Cox and Sadiraj (2006), and Rubinstein (2006).*

3.1. Calibration of Patterns of Risk Aversion for Expected Utility Theory

Let w denote a given amount for the agent's initial wealth. The expected utility of the binary lottery $\{y_2, p; y_1\}$ is

(7) $\quad U(\{y_2, p; y_1\}) = p\varphi(y_2) + (1-p)\varphi(y_1)$,

where $\varphi(y)$ represents: $u_{TW}(w+y)$ for the expected utility of terminal wealth model; $u_I(y)$ for the expected utility of income model; or $u_{W\&I}(w,y)$ for the expected utility of initial wealth and income model.[4]

We now explain the large stakes risk aversion implications of postulated patterns of small stakes risk aversion for expected utility theory. These implications hold for all three expected utility models. Consider pairs of certain payoffs in amounts $x+a$ and binary lotteries $\{x+b, p; x\}$. Each binary lottery is assumed to have higher expected value than its paired certain payoff; that is, $pb > a$. Suppose that an agent prefers the certain payoff $x+a$ to the lottery $\{x+b, p; x\}$, for all $x \in [m, M]$, $M > m \geq 0$. Such a pattern of risk aversion implies exponentially decreasing marginal utility of lottery payoff y (Cox and Sadiraj, 2006, pgs. 48-49), which is the intuition for calibrations of money payoff transformation functions that imply implausible risk aversion for expected utility theory.

Proposition 2 states a concavity calibration result for expected utility theory. Let $\langle x \rangle$ denote the largest integer smaller than x and define $r(t) = [(1-t)/t] \times [a/(b-a)]$. One has:

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[4] *These three expected utility models are discussed in detail in Cox and Sadiraj (2006).*

Proposition 2 (calibration for expected utility theory). Let positive numbers a, b and $p \in (0,1)$ be given such that $pb > a$. Suppose that

(P.2*) $x + a \succsim \{x + b, p; x\}$, for all integers $x \in [m, M]$, $M > m \geq 0$.

If $\varphi(\cdot)$ is (weakly) concave then for $q = r(p)$ and all $z \in [m + b\ln(q - qp)/\ln q, M]$,

$z \succ \{G, p; m\}$ for all G such that

(*) $\quad G < M + b(2q-1)/(1-q) + Aq^{-\langle (M-m)/b \rangle}$,

where $A = \left((b/a - 1)q^2(1 - q^{\langle (z-m)/b \rangle}) - q^{\langle (z-m)/b \rangle}\right) b/(1-q)$.

Proof: See appendix A.2.

Note that for any given m and z, the third term on the right hand side of inequality (*) increases geometrically in M because $q < 1$ (which follows from $pb > a$). This implies that for any amount of gain G, as big as one chooses, there exists a large enough interval in which preference for $x + a$ over a risky lottery $\{x + b, 0.5; x\}$, for all integers x from the interval $[m, M]$, implies a preference for z for sure to the risky lottery $\{G, 0.5; m\}$. We use inequality (*) in Proposition 2 to construct the illustrative examples in Table 2.

Suppose that an agent prefers the certain amount of income $x + 100$ to the lottery $\{x + 210, 0.5; x\}$, for all integers $x \in [900, M]$, where values of M are given in the "Rejection Intervals" column of Table 2. In that case all three expected utility (of terminal wealth, income, and initial wealth and income) models predict that the agent prefers receiving the amount of income 3,000 for sure to a risky lottery $\{G, 0.5; 900\}$, where the values of G are given in the "First EU Calibration" column of Table 2. For example, if $[m, M] = [900, 50000]$ then $G = 0.1 \times 10^{13}$ for all three expected utility models. According to the entry in the "Second EU Calibration" column and M = 30,000 row of Table 2, expected utility theory predicts that if an agent prefers certain payoff in amount $x + 100$ to lottery $\{x + 250, 0.5; x\}$, for all integers x between 900 and 30,000, then such an agent will prefer 3,000 for sure to the 50/50 lottery with positive outcomes of 900 or 0.12×10^{24}.

3.2. Calibration of Patterns of Risk Aversion for Rank Dependent Utility Theory and Cumulative Prospect Theory

In the case of binary lotteries, the utility functional for rank dependent utility theory can be written as

(8) $U_{RD}(\{y_2, p; y_1\}) = h(p)\upsilon(y_2) + [1-h(p)]\upsilon(y_1)$,

given the convention of transforming decumulative probabilities. For the binary lotteries with all positive payoffs considered here, utility functional (8) also represents preferences for the original version of cumulative prospect theory with zero-income reference point (Tversky and Kahneman, 1992).

The following corollary to Proposition 2 applies to rank dependent utility theory and cumulative prospect theory with zero-income reference point. The proposition for expected utility theory, with functional that is linear in probabilities, incorporates the assumption that $pb > a$, which implies that the expected value of the lottery $\{x+b, p; x\}$ is larger than the amount of the certain payoff $x+a$. In the corollary, we restate the assumption for probability transformation functions $h(\cdot)$, that can be nonlinear, as $h(p)b > a$.

Corollary 2 (calibration for cumulative prospect theory and rank dependent utility theory). Let positive numbers a, b and $p \in (0,1)$ be given such that $h(p)b > a$. Suppose that statement (P.2*) is satisfied. If $\upsilon(\cdot)$ is (weakly) concave then for $q = r(h(p))$ and for all $z \in [m + b\ln(q - qh(p))/\ln q, M]$, $z \succ \{G, p; m\}$ for all G that satisfy inequality (*) in Proposition 2.

Proof: See appendix A.2.

Calibration with the lottery $\{x+210, 0.5; x\}$ and sure payoff $x+100$, in one of the examples considered above, has no implications for cumulative prospect theory or rank dependent utility theory if one uses probability transformation functions from the literature. For cumulative prospect theory, Tversky and Kahneman's (1992, p.300) probability weighting function for binary lotteries with non-negative payoffs is $w^+(0.5) = 0.42$. For rank dependent utility theory, Quiggin's (1993, p.52) probability transformation function $1-q(p)$ for binary lotteries has $q(0.5)=0.58$. In our notation, let $h(0.5)=w^{+}(0.5)=1-q(0.5)=0.42$. With this value, $210h(0.5)<100$; therefore Corollary 2 does not apply. However, preference for the certain payoff $x+100$ over the lottery $\{x+250, 0.5; x\}$ satisfies the assumption in Corollary 2 because $250h(0.5)>100.$ As shown in the First PT & RD Calibration column and M = 30,000 row of Table 2, rank dependent utility theory and cumulative prospect theory with $h(0.5)=0.42$ imply that an agent will prefer 3,000 for sure to the 50/50 lottery with positive outcomes 900 or 0.46 $\times 10^{7}$.

As a final illustrative example, suppose that an agent prefers certain payoff in amount $x+20$ to the lottery $\{x+50, 0.5; x\}$, for all integers x between 900 and 6,000. Again, this pattern satisfies the assumption in Corollary 2 because $50h(0.5)>20$. According to the entry in the Third EU Calibration column and M =6,000 row of Table 2, expected utility theory predicts that an agent who rejects these lotteries will prefer 3,000 for sure to the lottery with 0.5 probabilities of gaining 900 or 0.4 $\times 10^{20}$. As shown in the right-most column of Table 2, cumulative prospect theory and rank dependent utility theory imply than an agent who rejects these same lotteries will prefer 3,000 for sure to the lottery with 0.5 probabilities of gaining 900 or 0.29 $\times 10^{7}$.

4. Experimental Design Issues

The calibration propositions and corollaries demonstrate that prominent theories of decision under risk may have implausible implications. But such a calibration critique of decision theory has unknown empirical relevance in the absence of data that provide support for the "calibration patterns" of risk aversion that are postulated in the propositions and corollaries. We next discuss issues that arise in designing experiments with these calibration patterns. The issues differ with the supposition underlying a calibration and the type of theory of risk aversion the calibration applies to.

4.1 Power vs. Credibility with Probability Calibration Experiments

Table 1 illustrates the relationship between the scale of payoffs in the lotteries (x), the ratio of high and middle payoffs in the risky lottery (c), and the difference between probabilities in adjacent terms in the calibration (determined by the value of n in $\frac{i}{2n}-\frac{i-1}{2n}$). The design problem for probability transformation function calibration experiments is inherent in the need to have a fine enough partition of the [0,1] interval for the calibration in Proposition 1 to lead to the implication of implausible risk aversion in the large (if the risk aversion supposition underlying the calibration has empirical validity).

There are two problems with big values of the partition parameter n. First, a subject's decisions may involve trivial financial risk because the differences between all the moments of the distributions of payoffs for the two outcome lottery $\{cx, i/2n; 0\}$ and the three outcome lottery $\{cx, (i-1)/2n; x, 1/n, 0\}$ become insignificant as n increases. For example, if $n = 500$, c = 4, and x = \$25, then the lotteries are $\{\$100, i/1000; 0\}$ and $\{\$100, (i-1)/1000; \$25, 1/500, 0\}$. In that case, the difference between expected values of the two-outcome and three-outcome lotteries is 5 cents (for all i). For the same n, c, and x values, the difference between standard deviations of payoffs for the two-outcome and three-outcome lotteries, at i = 500, is 4 cents. Second, for large n the adjacent probabilities differ by $1/2n$ and the subject's decision task is to make $2n$ choices. For example, for $n = 500$ adjacent probabilities differ by 0.001 and the subjects' decision task is to make 1,000 choices. In such a case, the subjects would not be sensitive to the probability differences and the payoffs would arguably not dominate decision costs because of the huge number of choices needing to be made. In contrast, if the length of each subinterval is $1/10$ (i.e. $n = 5$) then the difference in expected payoffs between the two-outcome and three-outcome lotteries is \$5 for the above values $cx = \$100$ and $x = \$25$, and for i = 5 the difference in standard deviations is \$4.17; furthermore, the subjects' decision task is to make only 10 choices. The calibration implications of n = 5 are less spectacular than for n = 500, as shown in Table 1, but the resulting experimental design can credibly be implemented.

4.2 Affordability vs. Credibility with Payoff Calibration Experiments

Table 2 illustrates the relationship between the size of the interval $[m, M]$ in the left-most column, used in the supposition underlying a payoff transformation function calibration, and the size of the high gain G in the result reported in the other columns of the table. Payoff transformation function calibration experiments involve tradeoffs between what is affordable and what is credible, as we shall next explain.

As an example, suppose one were to consider implementing an experiment in which subjects were asked to choose between a certain amount of money $\$x + \$100$ for sure and the binary lottery $\{\$x + \$210, 0.5; \$x\}$ for all x between *m* = \$900 and *M* = \$350,000. Suppose the subject always chooses the certain amount $\$x + \$100$ and that one of the subject's decisions is selected randomly for payoff. Then the expected payoff to a single subject would exceed \$175,000. With a sample size of 30 subjects, the expected payoff to subjects would exceed \$5 million, which would clearly be unaffordable. But why use payoffs denominated in U.S. dollars? Proposition 2 is dimension invariant. Thus, instead of interpreting the figures in Table 2 as dollars, they could be interpreted as dollars divided by 10,000; in that case the example experiment would cost about \$500 for subject payments and clearly be affordable. So what is the source of the difficulty? The source of the difficult tradeoff for experimental design becomes clear from close scrutiny of Proposition 2: the unit of measure for *m* and *M* is the same as that for the amounts *a* and *b* in the certain payoffs and binary lotteries (see statement (*) in Proposition 2). If the unit of measure for *m* and *M* is \$1/10,000 then the unit of measure for *a* and *b* is the same (or else the calibration doesn't apply); in that case the certain payoff becomes $\$0.0001x + \$0.01$ and the binary lottery becomes $\{\$0.0001x + \$0.021, 0.5; \$0.0001x\}$, which involves only trivial financial risk of about one cent.

The design problem for concavity calibration experiments with money payoffs is inherent in the need to calibrate over an $[m, M]$ interval of sufficient length for the calibrations in Proposition 2 and Corollary 2 to lead to the implication of implausible risk aversion in the large (if the supposition underlying the calibration has empirical validity). There is no way to avoid this problem; the design of any experiment on payoff transformation function calibration will reflect a tradeoff between affordability of the payoffs and credibility of the incentives.

5. Magdeburg Experiment with Probability Transformation Theories

An experiment with probability transformation theories was conducted at the MAX-Lab of the Otto-von-Guericke-University of Magdeburg in February 2007. There were in total 32 subjects. Data for two subjects are unusable because of some unrecorded responses by the subjects. In this experiment, all payoffs were denominated in euros (€).

5.1 Experimental Design

Subjects were asked to make choices in each of the nine rows shown in Table 3. Row number j, for $j = 1, 2, ..., 9$, presented a choice between (a) a lottery that paid €0 with probability $j/10$ and €40 with probability $1 - j/10$ and (b) a lottery that paid €0 with probability $(j-1)/10$, €10 with probability $2/10$, and €40 with probability $1-[(j-1+2)/10]$.[5] In each row, a subject was asked to choose among option A (the two outcome lottery), option B (the three outcome lottery), and option I (indifference). The subjects were presented with the instructions at the beginning of the session where the payment protocol of selecting one of the nine rows randomly for money payoff (by drawing a ball from a bingo cage in the presence of the subjects) was clearly explained to the subjects in the instructions as well as orally. The instructions also explained that if a subject chose option I then the experimenter would flip a coin in front of the subject to select option A or B for him or her (if that row was randomly selected for payoff). It was also explained that payoff from the chosen lottery would be determined by drawing a ball from a bingo cage in the presence of the subject.[6] Appendix B.1 provides more information on the experiment protocol.

5.2 Implications of the Data for Dual Theory of Expected Utility

In all tests for the presence of choice patterns that imply implausible risk aversion in the large, we aggregate choices of option B with choices of option I (indifference) because the "if" statement in

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[5] *The lotteries in Table 3 correspond to the example in section 2.1 when n = 5 and row j in the table represents term 10 – i in the example.*

[6] *Subject instructions in English for all experiments reported in this paper are available on http://excen.gsu.edu.jccox.*

P(1*) in Proposition 1 involves *weak* preference for B over A. The aggregation of options B and I is represented as B*. We begin with a conservative test for incidence in the data of patterns of choices that, according to Proposition 1, imply implausible risk aversion in the large with dual theory of expected utility. The conservative test procedure admits no errors in observed choices; it simply counts the number of subjects whose choices exactly fit patterns that imply implausible risk aversion in the large. Subsequently, we apply a model that allows errors in observed choices.

As reported above, there are usable data for 30 subjects. For eight subjects who switched at most once from option A to B*, and who chose B* from row 4 (or earlier) through row 9 in Table 3, the dual theory predictions are reported in the top four rows of Table 4.[7] Data for these eight subjects support the conclusion that the revealed small stakes risk aversion in the experiment implies large stakes risk aversion for which $\{z, p; 0\} \succ \{kz, 0.5; 0\}$, where k takes values 9, 27, 81 and 244 whereas p takes values 0.7, 0.8, 0.9 and 1 respectively. In other words, a lottery that pays z with probability p and pays 0 with probability $1-p$ is preferred to a lottery that pays kz or 0 with probability 0.5. Furthermore, the indicated preference holds for all positive values of z. For example, the row 3 entry in Table 4 reports that the risk aversion revealed by subject 17 implies (from setting $z = 4{,}000$) that he or she would prefer the lottery that pays 4,000 with probability 0.9 and pays 0 with probability 0.1 to the lottery that pays 324,000 (since $k = 81$) or 0 with probabilities of 0.5. Alternatively, note that the row 3 entry implies a preference for the lottery that pays 100 with probability 0.9 and 0 with probability 0.1 over the 50/50 lottery that pays 8,100 or 0. The data for subject 7 (see row 4) implies that he or she prefers the lottery that pays 4,000 for sure to the lottery that pays 976,000 (since $k = 244$) or 0 with probabilities of 0.5. The implied aversions to large stakes risks for the 8 subjects reported in the top four rows of Table 4 are implausible.

Another five subjects who chose option B in row 1 and either option B or I in row 9 reveal risk preferences that can be calibrated, as follows. We assume that if an individual switches from option B

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[7] *It is straightforward to extend the proof in appendix A.1 to apply to the case of "six or more adjacent choices of B*" as follows: in statement (a.5), replace 1 - f(0.5) with f(p) – f(0.5), where p = 0.7, 0.8, 0.9 or1, depending on the row in Table 4, and then complete the remaining steps in the proof using the parameters cx = 40 and x = 10.*

choices to an option A choice, and then back to option B, that the individual is less risk averse but not locally risk preferring at the switch row. In that case, data for these five subjects support the conclusion that an individual is predicted by dual theory to prefer a certain payoff in amount z to playing a 50/50 lottery with payoffs of 0 or kz reported in the bottom 5 rows of Table 10, where z is any positive amount. For example, the data for subject 11 support the conclusion that he or she would prefer 4,000 for sure to the 50/50 lottery with payoffs of 0 or 396,000. The implied aversions to large stakes risks for the five subjects reported in the five bottom rows of Table 4 are implausible .

This conservative procedure for data analysis leads to the conclusion that 43 percent (or 13/30) of the subjects' choice patterns imply implausible risk aversion in the large for dual theory. This counting procedure is quite conservative in that it only includes choice patterns that exactly match one of the calibration patterns.

We next consider error-rate analysis to infer statistical conclusions about the proportion of subjects in this experiment who made choices that reveal underlying preferences that are subject to large stakes risk aversion calibration according to Proposition 1.[8] As explained above, there are two relevant categories of choices for each decision task, option A and the aggregation of options B and I, which we represent as B*. We record choice patterns for subjects with sequences of nine letters. The order in which we write the symbols A and B* corresponds to the order of the rows in Table 3 from top to bottom. For example, the pattern [A,B*,B*,A,B*,B*,B*,B*,A] would indicate that a subject chose option A in rows 1, 4, and 9 and chose option B or option I in all other rows. Given the parametric configuration of our experiment, the theory would predict clearly implausible risk aversion in the large for any pattern of choices with B* in seven or more adjacent rows from the bottom of Table 3 because this would imply that $\{z,0.8;0\} \succ \{27z,0.5;0\}$ for all $z > 0$.[9] With nine choice tasks (or rows in Table 3) and two relevant feasible choices for each task (A or B*), there are in total

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[8] *We are grateful to Nathaniel Wilcox for generous advice about this approach to data analysis and for supplying SAS code.*

[9] *Straightforward extension of the proof in appendix A.1 to apply to the case of "seven or more adjacent choices of B*" is done as follows: in statement (a.5), replace 1-f(0.5) with f(0.8) – f(0.5) and then complete the remaining steps in the proof.*

512 choice patterns but only four patterns that would clearly imply implausible risk aversion in the large. These four patterns are reported in the right column of Table 5.

Choice probabilities are assumed to deviate from 1 or 0 by a constant error rate ε that is the same for all subjects and all decision tasks, as in Harless and Camerer (1994). Thus if B* is preferred to A then Prob(choose B*) = $1-\varepsilon$ and if B* is *not* preferred to A then Prob(choose B*) = ε, where $\varepsilon < 0.5$. The error rate model postulates that a subject with real preferences for B* (respectively A) over A (respectively B*) in all nine decision rows could nevertheless be observed to have chosen the other option in some rows. For example, the model assumes that a subject with underlying preferences [B*,B*,B*,B*,B*,B*,B*,B*,B*] could, instead, choose a different pattern such as [B*,B*,A,A,B*,B*,B*,B*,B*], an event with probability $(1-\varepsilon)^2\varepsilon^2(1-\varepsilon)^5$.

Consider the four "alternative stochastic types" in the right column of Table 5, that clearly imply implausible risk aversion in the large for dual theory, and their mirror images in the left ("null stochastic types") column that are *not* subject to the calibration. Using the error rate model, we estimate the fraction of subjects with choice patterns consistent with the "alternative stochastic types." The maximum likelihood point estimate of the proportion of subjects whose preferences are characterized by the four alternative patterns is 0.86, with Wald 90 percent confidence interval (0.73, 0.99). In this way, we conclude that the percentage of the subjects in the probability transformation experiment who made choices for which dual theory implies implausible large stakes risk aversion is at least 73 percent and as high as 99 percent. The maximum likelihood point estimate of the error rate is 0.216 and the log likelihood is -163.86.

Next, consider another model that includes alternative stochastic types with B* entries in the bottom six decision rows of Table 3 and for which the prediction is $\{z,0.9;0\} \succ \{9z,0.5;0\}$ or $k=9$. The total number of null and alternative patterns then doubles, from eight (considered above) to 16. For this model, the point estimate of the proportion of subjects with preferences that are subject to the calibration critique is 0.85, with Wald 90-percent confidence interval (0.73, 0.98). The maximum likelihood point estimate of the error rate (0.14) is lower than for the eight-type model and the log likelihood value (-136.05) is larger. The likelihood ratio test rejects the model with eight patterns in favor of the one with 16 patterns.

5.3 Implications of the Data for Cumulative Prospect Theory and Rank Dependent Utility Theory

Corollary 1 applies to cumulative prospect theory and rank dependent utility theory. Using the notation in functional (6) and Corollary 1, the choice patterns included in “alternative stochastic types” in Table 5 have known calibration implications for these theories so long as the condition $\upsilon(cx)/\upsilon(x) > 2$ is satisfied. In the experiment, $cx = 40$ and $x = 10$. The condition $\upsilon(40)/\upsilon(10) > 2$ is satisfied by the value function in Tversky and Kahneman (1992, p. 311), hence similar conclusions to those stated for dual theory of expected utility in section 5.2 apply here as well (although the *k*-values will here depend on the ratio $\upsilon(40)/\upsilon(10)$).

5.4 Implications of the Data for Expected Utility Theory

Using the expected utility functional in statement (7), it is straightforward to show that expected utility theory implies that an agent’s preference for option A or option B in any row of Table 3 depends only on whether $\varphi(10)$ is larger or smaller than $0.5\varphi(0) + 0.5\varphi(40)$. This comparison is the same for all rows in Table 3; therefore there is no concavity calibration implication for expected utility theory from this experiment. However, expected utility theory can be tested with data from the experiment because the theory predicts that an agent will always choose the same option. Data from the experiment reveal that 28 out of the 30 (or 94 percent) of the subjects in the experiment made choices that were inconsistent with this prediction. A probit panel regression of individual choices supports the conclusion that the prediction “always choose the same option” cannot account for observed behavior in this experiment. This prediction implies that the estimated coefficient for the variable “row” (corresponding to the row in Table 3) should be insignificant. On the contrary the probit panel regression with random effects reports a significant row effect; the estimated coefficient is 0.23 with standard error 0.03 and p-value 0.00.

The error rate model can be used to address the question whether the two choice patterns consistent with expected utility theory are as consistent with the data as the eight choice patterns in Table 5. Applying the error rate model to the two expected utility patterns [A,A,A,A,A,A,A,A,A] and [B*,B*,B*,B*,B*,B*,B*,B*,B*] yields a point estimate for the error rate of 0.33 with log likelihood -

177.57. As reported above, the log likelihood for the eight choice patterns in Table 5 is -163.86. The likelihood ratio test rejects the two-pattern expected utility model in favor of the eight-pattern model at 5 percent significance level.

5.5 Implications of the Data for Expected Value Theory

In row j of Table 3, the expected value of option A is €($40-4j$) while the expected value of option B is €($38-4j$). Hence a risk neutral agent will choose option A over option B in all rows. The data show that 28 out of 30 (or 93%) of the subjects made choices inconsistent with this implication of risk neutral preferences. The above probit regression test of expected utility theory also implies rejection of the testable implication of expected value theory. The error rate model can be used to address the question whether the one choice pattern consistent with expected value theory is as consistent with the data as are the eight choice patterns in Table 5. Applying the error rate model to the one expected value choice pattern [A,A,A,A,A,A,A,A,A] yields log likelihood -187.15. The likelihood ratio test rejects expected value theory in favor of the eight pattern model in Table 5 at 1 percent significance.

6. Calcutta Experiment with Payoff Transformation Theories

An experiment with money payoff transformation theories was conducted at the Indian Statistical Institute in Calcutta during the summer of 2004. The subjects were resident students at the institute. Two sessions were run, each with 15 subjects. In this experiment, all payoffs were denominated in Indian rupees.

6.1 Experimental Design

In each experiment session a subject was asked to perform two tasks. For the first task, subjects were asked to make six choices between a certain amount of money x rupees + 20 rupees and a binary lottery {x rupees + 50 rupees, 0.5; x rupees} for values of x from the set {100, 1K, 2K, 4K, 5K, 6K}, where K = 1,000. Subjects were asked to choose among option A (the risky lottery), option B (the certain amount of money), and option I (indifference). The alternatives given to the subjects are presented in Table 6. The second task was completion of a questionnaire including questions about amounts and sources of income. Appendix B.2 contains detailed information on the protocol of this experiment.

6.2 Economic Significance of the Certain Incomes and Lottery Risks

The exchange rate between the Indian rupee and the U.S. dollar at the time the Calcutta experiment was run was about 42 to 1. This exchange rate can be used to convert the rupee payoffs discussed above into dollars. Doing that would not provide very relevant information for judging the economic significance to the subjects of the certain payoffs and risks involved in the Calcutta experiment because there are good reasons for predicting that none of the subjects would convert their rupee payoffs into dollars and spend them in U.S. markets. Better information on the economic significance of the payoffs to subjects is provided by comparing the rupee payoffs in the experiment to rupee-denominated monthly stipends of the student subjects and rupee-denominated prices of commodities available for purchase by students residing in Calcutta.

The student subjects' incomes were in the form of scholarships that paid stipends of 1,200-1,500 rupees per month in addition to the standard tuition waiver that each received. This means that the highest certain payoff used in the experiment (6,000 rupees) was equal to four or five months' stipend for the subjects. The daily rate of pay for the students was 40 – 50 rupees. This means that the size of the risk involved in the lotteries (the difference between the high and low payoffs) was greater than or equal to a full day's pay.

A sample of commodity prices in Calcutta at the time of the experiment (summer 2004) is reported in Table 7. Prices of food items are reported in number of rupees per kilogram. There are 2.205 pounds per kilogram and 16 ounces in a pound, hence there are 35.28 ounces per kilogram. The U.S. Department of Agriculture's food pyramid guide defines a "serving" of meat, poultry, or fish as consisting of 2 – 3 ounces. This implies that there are about 15 servings in a kilogram of these food items. As reported in Table 7, for example, we observed prices for poultry of 45 – 50 rupees per kilogram. This implies that the size of the risk involved in the lotteries (50 rupees) was equivalent to 15 servings of poultry. The price of a moderate quality restaurant meal was 15 – 35 rupees per person. This implies that the 50 rupee risk in the experiment lotteries was the equivalent of about 1.5 – 3 moderate quality restaurant meals. The observed prices for local bus tickets were 3 – 4.5 rupees per ticket. This implies that the 50 rupee risk in the experiment lotteries was the equivalent of about 14 bus tickets.

6.3 Implications of the Data for Expected Utility Theory, Rank Dependent Utility Theory, and Original Cumulative Prospect Theory

The "if" statement in P(2*) in Proposition 2 involves weak preference for option B over option A. Therefore, in all tests for the presence of choice patterns that imply implausible risk aversion in the large, we aggregate choices of option B with choices of option I (indifference) and denote the aggregated choice category as B*. We begin with a conservative test for incidence in the data of patterns of choices that, according to Corollary 2, imply implausible risk aversion in the large with expected utility theory, cumulative prospect theory, and rank dependent utility theory. The conservative test procedure admits no errors in observed choices; it simply counts the number of subjects whose choices exactly fit patterns that imply implausible risk aversion in the large. Subsequently, we apply a model that allows errors in observed choices.

There are data for 30 subjects. Nine subjects never rejected a risky lottery whereas five subjects rejected the risky lottery only in the first decision task. Eight out of 30 (or 27 percent of the) subjects revealed an interval of risk aversion with length at least 3.9K. The expected utility theory, rank dependent utility theory, and cumulative prospect theory calibration implications for these individuals are reported in Table 8. Figures reported in the table show implausible risk aversion calibration implications for these eight individuals.

We use the error rate model to draw statistical conclusions from these data. Recall that this type of analysis takes into account that a subject with real preferences for option B* rather than option A in all six rows could nevertheless be observed to have chosen B* in five (or fewer) out of six rows. That is, the model assumes that a subject with real underlying preferences such as [B*,B*,B*,B*,B*,B*] could, instead, choose a different pattern, say [B*,B*,B*,A,B*,B*], an event with probability $(1-\varepsilon)^3\varepsilon(1-\varepsilon)^2$, where ε is an error rate.

The main empirical question is whether a significant fraction of subjects in this experiment have underlying preferences for which the theory predicts implausible risk aversion with large stakes risky lotteries. We here define large stakes risk aversion as "implausible" if it involves predictions of the type reported in Table 8, for example the prediction (in the middle row and right column) that an agent would prefer 3,000 for sure to the 50/50 lottery that pays 2,000 or 400,000. Given the parametric configuration of our experiment, expected utility theory, cumulative prospect theory and rank dependent utility theory would, according to Corollary 2, imply implausible risk aversion for large stakes risks for any pattern of choices of option B* rather than option A in at least four adjacent rows in Table 6. We consider two models. Model I consists of the choice patterns in the top three rows of Table 9. The alternative stochastic types in the top three rows include all choice patterns containing at least four adjacent B* entries, including the last entry, and at most one switch between the alternatives A and B*. The restriction to only one switch does not allow patterns of choices inconsistent with non-monotonic risk aversion in the certain payoff amount x. The alternative patterns imply implausible risk aversion following from calibrations of the type derived in Proposition 2 and Corollary 2. The mirror image patterns reported in the first three rows of the null stochastic types column present choice patterns that are not subject to the above calibrations. Model I assumes that underlying real preferences are one of these six types, and we ask the question what fraction of the population is of the three alternative types.

The maximum likelihood point estimate of the proportion of subjects whose preferences are characterized by the three alternative patterns in Model I is 0.397, with Wald 90 percent confidence interval (0.196, 0.598). In this way, we conclude that the percentage of subjects in the Calcutta payoff transformation experiment who made choices for which expected utility theory, cumulative prospect theory and rank dependent utility theory imply implausible large stakes risk aversion is at least 19.6 percent and as high as 59.8 percent. The maximum likelihood point estimate of the error rate is 0.18 and the log likelihood is -109.47.

Next consider Model II that includes all 16 choice patterns in Table 9. Unlike in Model I, the stochastic types in Model II *do* include some patterns with more than one switch between A and B* (that is, risk aversion that is non-monotonic in the certain payoff x). The alternative stochastic types of Model 2 include choice patterns with at least four adjacent B* entries in any position in the sequence of six choices. The point estimate of the proportion of subjects whose choices are characterized by the eight alternative stochastic types in Model II is 0.495, with Wald 90 percent confidence interval (0.289, 0.702). Therefore, this estimation implies that the percentage of subjects for whom the three theories imply implausible large stakes risk aversion is at least 28.9 percent and at most 70.2 percent. The maximum likelihood point estimate of the error rate for Model II is 0.107 and the log likelihood is -101.75. The likelihood ratio test rejects Model II in favor of Model I at five percent significance. In this way, the data provide more support for the relatively parsimonious Model I, with point estimate of 39.7 percent of subjects for whom the three theories imply implausible large stakes risk aversion.

6.4 Implications of the Data for Dual Theory and Prospect Theory with Editing of Reference Points

The utility functional in statement (1) for dual theory gives us the necessary and sufficient condition for rejection of the risky lottery $\{x+50, 0.5; x\}$, in favor of $x+20$ for sure, for some specific value of x in the Calcutta experiment:

(9) $$x+20 > f(0.5)\times[x+50]+[f(1)-f(0.5)]\times x,$$

where again f is the dual theory transformation function for decumulative probabilities. Since in the dual theory $f(0)=0$ and $f(1)=1$, statement (9) implies $f(0.5) < 20/50$, which is independent of the value of x. Therefore, dual theory implies that an agent will reject the risky lottery $\{x+50, 0.5; x\}$ for all positive value of x if and only if he does so for one positive value of x. Hence dual theory predicts that a subject will choose the same option in every row of Table 6.

In their development of cumulative prospect theory, Kahneman and Tversky (1992) dropped some of the elements of the original ("non-cumulative") version of prospect theory (Kahneman and Tversky, 1979). One element of the original version of prospect theory, known as "editing," can be described as follows. In comparing two prospects, an individual is said to look for common amounts in the payoffs, to disregard (or "edit") those common amounts, and then compare the remaining distinct payoff terms in order to construct a preference ordering of the prospects. Some recent applications of cumulative prospect theory (Kőszegi and Rabin, 2006, 2007; Schmidt, Starmer, and Sugden, forthcoming) have reintroduced editing in the form of reference point payoffs that differ from the zero-payoff reference point used by Tversky and Kahneman (1992). Non-zero reference points have implications for application of cumulative prospect theory to our experiments. For example, the concavity calibration in Corollary 2 is based on the supposition that an agent prefers the certain amount $x+a$ to the lottery $\{x+b, p; x\}$ for all $x \in [m, M]$. But x is a common amount in the certain payoff, $x+a$ and both possible payoffs in the lottery $\{x+b, p; x\}$. If this common (or "reference point") amount x is edited, that is eliminated from all payoffs, then all comparisons are between the certain amount a and the single lottery $\{b, p; 0\}$ and there remains no interval $[m, M]$ over which to calibrate. In this way, editing of reference point payoffs can immunize prospect theory to critique by calibration of payoff transformation functions (Wakker, 2005).[10]

Editing of common amounts x from payoffs does not immunize prospect theory to being tested with data from payoff transformation experiments, as can be seen by applying editing to the lotteries and certain payoffs used in the Calcutta experiment. If we perform editing by subtracting from all payoffs in each row of Table 6 the amounts that set the lower lottery payoff in all rows in the option A column equal to 0 then the resulting comparison in every row is between the lottery $\{50, 0.5; 0\}$ and the certain payoff 20. Alternatively, if we perform editing by subtracting from all payoffs in each row the amounts that set the certain payoff in the option B column equal to 0 then the resulting comparison in every row is between the lottery $\{30, 0.5; -20\}$ and the certain payoff 0. Whichever way editing is applied it has the same implication: that an agent will view all rows in Table 6 as involving exactly the same choice and hence make the same decision. Therefore, this version of prospect theory has the same testable implication as dual theory of expected utility for data from the Calcutta experiment.

Both of these two theories predict that an agent will reject the risky lottery $\{x+50, 0.5; x\}$ for one positive value of $x+20$ if and only if he does so for all positive values of $x+20$. The data reveal that 77 percent of the subjects made choices that are inconsistent with this prediction. A probit panel

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[10] *Editing of reference point payoffs does not immunize prospect theory to critique by calibration of probability transformation functions, as in Corollary 1.*

regression of the data supports the conclusion that these two models cannot account for observed behavior in this experiment. The prediction is that the estimated coefficient for the row should not be significant. On the contrary, the probit panel regression with individual-subject random effects yields a parameter estimate of -0.12 for the row number variable` that is significantly different from 0 at 6 percent significance level. Furthermore, the only choice patterns consistent with these two theories are [A,A,A,A,A,A] and [B*,B*,B*,B*,B*,B*]. Maximum likelihood estimation of the error rate model with only these two types yields an error rate of 0.25 and a log likelihood of -114.51. The log likelihood ratio test rejects this model at 5% significance level in favor of Model I, reported in the previous section, that allows for six stochastic types. Recall that Model I is consistent with the original version of cumulative prospect theory with constant, zero-income reference point. Therefore, one interesting interpretation of this test is that it implies rejection of the variable reference point version of prospect theory in favor of the original version with zero-income reference point.

6.5 Implications of the Data for Expected Value Theory

The choice faced by a subject in a row of Table 6 is between an option A lottery $\{x+50, 0.5; x\}$ and an option B certain amount $x+20$. Since the expected value of an option A lottery is $x+25$, a risk neutral agent will prefer option A to option B in every row of the table. The data show that 80 percent (or 24/30) of the subjects made choices inconsistent with the prediction "always choose option A."

The above probit regression test for dual theory and cumulative prospect theory with variable reference point also implies rejection of the testable implication of expected value theory. The error rate model can be used to address the question whether the one choice pattern consistent with expected value theory is as consistent with the data as are the six choice patterns in Model I. Estimation of the error rate model for the one expected value choice pattern [A,A,A,A,A,A] yields log likelihood -122.88. The likelihood ratio test rejects the implication of expected value theory in favor of Model I at 1 percent significance.

7. Magdeburg Contingent Payoff (Casino) Experiment with Payoff Transformation Theories

An experiment was conducted in Magdeburg in the winter of 2004 with contingent money payoffs. This experiment was conducted at the MAX-Lab of the Otto-von-Guericke-University of Magdeburg and the Magdeburg Casino. The subjects were adults who were older than typical students. All payoffs were denominated in euros (€).

7.1. Experimental Design

There were two sessions, one with 20 subjects and the other with 22. The experiment had three parts consisting of Step 1, Step 2, and a questionnaire. In Step 1 all subjects were asked to choose between option A, a risky lottery $\{€x, 0.5; €y\}$ in which $y = x + 210$, and option B, the certain amount of money, $€z$, where $z = x + 100$ and where z took values from {3*K*, 9*K*, 50*K*, 70*K*, 90*K*, 110*K*} and *K* = 1,000. These six decision tasks are reported in Table 10.[11] Step 2 involved bets by an experimenter on an American roulette wheel at the Magdeburg Casino, the realization of which determined whether the euro payments determined in Step 1 were made in real euros.

Step 1 took place in the MAX-Lab in Magdeburg and lasted about 45 minutes. The participants made their decisions in well separated cubicles. First the instructions of Step 1 and the choices were given to the subjects. After the subjects completed Step 1 they got a questionnaire. After having completed the questionnaire they received the instructions for Step 2. In Step 1 the participants were told that whether their payoffs would be hypothetical or real depended on a condition which would be described later in Step 2; they were not informed of use of the Casino until they had completed their Step 1 decisions.

In Step 2 the payoff procedure was described. After the instructions for Step 2 were read by the participants, they were given the opportunity to change the decisions they made in Step 1. No one changed his or her decisions. Money payoff to a subject was conditional on an experimenter winning a

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[11] *There was an additional, different type of decision that subjects were asked to make in a task 7. This task was implemented incorrectly in one of the two sessions so we do not use the data. The subsequent implementation of task 7 could not affect the subjects' decisions in tasks 1-6 because they had completed their responses to tasks 1-6 before they were shown task 7. Task 7 data are available upon request to the authors.*

gamble in the casino. Based on conditional rationality, all choices had the same chance to become real and the condition should not influence decisions.

The payoff contingency was implemented in the following way. For each participant the experimenter placed €90 on the number 19 on one of the (four American) roulette wheels at the Magdeburg Casino. The probability that this bet wins is 1/38. If the bet wins, it pays 35 to 1. If the first bet won, then the experimenter would bet all of the winnings on the number 23. If both the first and second bet won, then the payoff would be €(35 × 35 × 90) = €110,250, which would provide enough money to make it feasible to pay any of the amounts involved in the Step 1 decision tasks.[12] If the casino bets placed for a subject paid off, one of that subject's Step 1 decisions would be paid in real euros, otherwise no choice would be paid. The decision that would be paid would be selected randomly by drawing a ball from an urn containing balls with numbers 1 to 7. The number on the ball would determine the decision task to be paid. If a subject had chosen indifference then a coin flip would determine whether the certain amount was paid or the lottery would be played. We informed the participants that any money resulting from casino bets that was not paid (because the subject's decision randomly selected for payoff involved amounts less than €110,250) would be used for subject payments in other experiments. Some more details of the protocol are explained in appendix B.3.

This experiment has no implications for rank dependent utility theory or cumulative prospect theory with a probability transformation function such that $h(0.5) < 0.476$ (for example, $h(0.5) = 0.42$, as in Kahneman and Tversky, 1992) since for such values of $h(0.5)$ the assumption $h(p)b > a$ of Corollary 2 is not satisfied by the lottery with $b = 210$ and $a = 100$. The data do have implications for expected utility theory because the assumption $pb > a$ in Proposition 2 is satisfied.

7.2 Implications of the Data for Expected Utility Theory

There were in total 42 subjects in this experiment. Eleven subjects never rejected a risky lottery. There were 20 subjects who revealed an interval of risk aversion with length at least 40*K*. Proposition 2

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[12] *It is not clear, a priori, that subjects perceive large contingent payoffs, involving such low probabilities, differently than hypothetical payoffs.*

implies implausible large-stakes risk aversion for these 20 individuals, as reported in Table 11. Entries in Table 11 are interpreted as follows. Row 1, for example, reports that seven subjects (NOBS = 7) rejected the risky lottery $\{x-100, 0.5; x+110\}$ in favor of the certain payoff x, or reported indifference, for all values of $x+100$ between 3K and 110K. According to Proposition 2, expected utility theory implies that an agent with these risk preferences would also prefer the certain payoff 9K (= 3K+ 6K) to the 50/50 lottery that pays 3K or 0.21×10^{25}. The fourth row of Table 11 reports that eight subjects chose the certain payoff or indifference for all values of $x+100$ between 50K and 110K According to Proposition 2, expected utility theory implies that an agent with these risk preferences would prefer the certain payoff 56K (= 50K+6K) to the 50/50 lottery that pays 50K or 0.11×10^{16}.

We apply error rate models to these data, as follows. Given the large interval for calibration, [3K,110K] in this experiment, three or more adjacent choices of B* can imply implausible risk aversion according to Proposition 2. Therefore, the (single-switch) Model I null pattern used here includes the choice patterns in the right column and top three rows of Table 9 plus the pattern [A,A,A,B*,B*,B*]`. Maximum likelihood estimation for this model yields a point estimate of the proportion of subjects with choice patterns included in alternative stochastic types of 0.538 and Wald 90 percent confidence interval (0.392, 0.683). In this way, we conclude that at least 39.2 percent and at most 68.3 percent of the subjects made choices for which expected utility theory implies implausible large stakes risk aversion. The estimated error rate for this model is 0.057 and the log likelihood is -112.99.

Since with these data only three adjacent choices of $B*$ can imply implausible large stakes risk aversion, the version of Model II (that allows more than one switch between A and B*) includes more than the 16 patterns in Table 9. This version of Model II includes 34 patterns. The estimated proportion of subjects with the 17 alternative patterns in this version of Model II is 0.538 with estimated Wald 90 percent confidence interval (0.384, 0.692). The estimated error rate for Model II is 0.031 and the log likelihood is -107.16. For these data, the log likelihood ratio test fails to reject Model I in favor of Model II at 5 percent significance; hence the data provide support for use of the relatively parsimonious Model I with point estimate of 53.8 percent of the subjects for whom expected utility theory implies implausible large stakes risk aversion.

7.3 Implications of the Data for Dual Theory and Prospect Theory with Editing of Reference Points

Section 6.4 contains explanations that dual theory and prospect theory with editing of reference points imply that a subject will make the same choice of the sure thing or the lottery in all responses in the Calcutta experiment. Similar arguments show that these theories imply that a subject will make the same choice in all responses in the Magdeburg Casino experiment. The data reveal that 24 out of 42 (or 57 percent) of the subjects made choices inconsistent with this prediction. If a subject were to choose the same option in all rows, as predicted by dual theory and prospect theory with variable reference point, then the subject's choices will be the same in all rows of Table 11 and hence invariant with the amount of the certain payoff. This implies that the row number variable will be insignificant in a probit regression with the data. A probit panel regression with individual-subject random effects yields a parameter estimate of 0.33 for the row variable that is significantly different from 0 at 1 percent significance level.

The only choice patterns consistent with these two theories are [A,A,A,A,A,A] and [B*,B*,B*,B*,B*,B*]. Maximum likelihood estimation of the error rate model with only these two types yields an error rate 0.21 and a log likelihood of -151.36. The likelihood ratio test rejects this two-type model, at 5% significance level, in favor of the four row version of Model I, reported in the previous section, that allows for eight stochastic types. Recall that the aggregation of null and alternative patterns in Model I is consistent with the original version of cumulative prospect theory with constant, zero-income reference point. Therefore, the variable reference point version of prospect theory is again rejected in favor of the original version with zero-income reference point.

7.4 Implications of the Data for Expected Value Theory

The choice faced by a subject in a row of Table 10 is between an option A lottery with expected value $x+105$ and an option B certain payoff $x+100$. Therefore, a risk neutral agent will prefer option A in every row. The data show that 32 out of the 42 (or 76 percent) of the subjects made choices inconsistent with this implication of risk neutrality. The above probit regression test for dual theory and cumulative prospect theory with variable reference point also implies rejection of the testable implication of expected value theory. The error rate model can be used to address the question whether the one choice pattern consistent with expected value theory is as consistent with the data as are the choice patterns in Model I. Estimation of the error rate model for the one expected value choice pattern [A,A,A,A,A,A] yields log likelihood -174.67. The likelihood ratio test rejects this implication of expected value theory in favor of Model I reported above at 1 percent significance.

8. Is There a Plausible Decision Theory for Risky Environments?

Prominent theories of decision making for risky environments model individuals' risk averse preferences over lotteries with nonlinear transformation of payoffs or nonlinear transformation of probabilities or transformations of both payoffs and probabilities. Previous calibration literature has focused on the possibly-implausible implications of modeling risk aversion with nonlinear transformation of payoffs. This paper provides a dual critique that focuses on implications of nonlinear transformation of probabilities as well as nonlinear transformation of payoffs. Previous literature has offered no data supporting empirical relevance of supposed patterns of risk aversion that have calibration implications. This paper provides data from two distinct types of experiments with designs that incorporate two different patterns of risk aversion that, respectively, have implications for theories that transform probabilities and theories that transform payoffs. The two types of supposed patterns of risk aversion and their implications are stated in the two propositions and their corollaries.

Proposition 1 derives calibration implications of modeling risk aversion solely with nonlinear transformation of probabilities, as in dual theory of expected utility. The pattern of risk aversion that is postulated in Proposition 1 has no calibration implications for expected utility theory (the theory that is dual to Yarri's (1987) dual theory of expected utlility). Corollary 1 extends this type of calibration to theories, such as rank dependent utility theory and cumulative prospect theory, that model risk-avoiding behavior with nonlinear transformations of both probabilities and payoffs. Proposition 2 derives implications of modeling risk aversion solely with nonlinear transformation of payoffs, as in expected utility theory. The pattern of risk aversion that is postulated in Proposition 2 has no calibration implications for dual theory of expected utility. Corollary 2 extends this type of calibration to theories that incorporate nonlinear transformations of both payoffs and probabilities, for example rank dependent utility theory and cumulative prospect theory.

We report an experiment run in Magdeburg, Germany with euro payoffs. Data from this experiment and the probability transformation calibration in Proposition 1 support the conclusion that a large proportion of the subjects exhibit patterns of risk aversion that have implausible implications for risk aversion in the large according to dual theory of expected utility theory. Data from the experiment and Corollary 1 lead to the conclusion of implausible large stakes risk aversion for rank dependent utility theory and cumulative prospect theory with (money transformation or) value functions that satisfy the condition in the corollary, for example ones similar to that in Tversky and Kahneman (1992).[13] The design of this experiment has no *calibration* implication for theories that do not transform probabilities, but such theories can still be tested with data from the experiment. Testable implications for expected utility theory and expected value theory are inconsistent with data for large proportions of the subjects.

We report an experiment run in Calcutta, India with rupee payoffs. Data from this experiment and payoff transformation calibrations in Proposition 2 and Corollary 2 support the conclusion that a significant proportion of the subjects exhibit patterns of risk aversion that have implausible implications if one models their behavior with expected utility theory, rank dependent utility theory, or cumulative prospect theory with zero-income reference point. Although prospect theory with "editing" of variable reference point payoffs can be immunized to problems from payoff transformation calibration, this theory can still be tested with data from the experiment. The testable implication of variable reference point editing is the same as for dual theory of expected utility; this implication is inconsistent with data for most of the subjects. The data are also mainly inconsistent with the testable implication of expected value theory.

Finally, we report an experiment run in Magdeburg with contingent payoffs of large amounts of euros. Data from this experiment and Proposition 2 imply implausible risk aversion in the large for about half of the subjects if one models their behavior with expected utility theory. Data for a majority of subjects in this experiment are also inconsistent with testable predictions of dual theory of expected

[13] *Estimated value functions differ across empirical applications of cumulative prospect theory. Corollary 1 provides guidance for experimental design for any given value function.*

utility and prospect theory with editing of variable reference point payoffs. In addition, data for a majority of subjects are inconsistent with the testable implication of expected value theory.

The two types of calibration propositions and their corollaries show how theories of decision under risk can have implausible implications regardless of whether they model risk-avoiding preferences with nonlinear transformations of payoffs, nonlinear transformation of probabilities, or both types of transformations. The experiment data provide support for the suppositions that underlie the calibrations. Further empirical testing is clearly needed. But the limited data now available provide support for empirical validity of risk aversion patterns underlying the calibrations. Accordingly, we conclude that the answer to the question about whether there exists a plausible theory for decision under risk may be “no.”

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Table 1. Calibrations for Probability Transformations

$$100 \succ \{G, 0.5; 0\}$$

Rejection Intervals	First DU Calibration (cx = 210, x=100)	Second DU Calibration* (cx = 250, x=100)	Third DU Calibration (cx = 40, x=10)	PT & RD Calibration with υ(y) = y$^{0.88}$ (cx = 40, x=10)
N	***G***	***G***	***G***	***G***
5	260	859	24,400	14,229
10	350	586	0.59×10^{7}	0.19×10^{7}
50	11,830	0.63×10^{11}	0.71×10^{27}	0.29×10^{24}
100	0.13×10^{7}	0.40×10^{20}	0.51×10^{51}	0.86×10^{45}
200	0.18×10^{11}	0.16×10^{38}	0.26×10^{99}	0.74×10^{88}
500	0.49×10^{23}	0.11×10^{91}	0.36×10^{242}	0.48×10^{217}

*figures of G reported in this column are the same for a risk aversion pattern with cx=50 and x=20

Table 2. Calibrations for Payoff Transformations

$$3,000 \succ \{G, 0.5; 900\}$$

Rejection Intervals [900,*M*]	**First EU Calibration (b=210, a=100)**	**Second EU Calibration (b=250, a=100)**	**First PT & RD Calibration (b=250, a=100)**	**Third EU Calibration (b=50, a=20)**	**Second PT & RD Calibration (b=50, a=20)**
M	***G***	***G***	***G***	***G***	***G***
5000	8,000	301,000	8,000	0.12×10^{17}	564,000
6000	10,000	0.15×10^{7}	10,000	0.4×10^{20}	0.29×10^{7}
8000	15,000	0.38×10^{8}	13,000	0.44×10^{27}	0.79×10^{8}
10000	24,000	0.98×10^{9}	18,000	0.49×10^{34}	0.21×10^{10}
30000	0.11×10^{9}	0.12×10^{24}	0.46×10^{7}	0.13×10^{105}	0.5×10^{24}
50000	0.1×10^{13}	0.14×10^{38}	0.34×10^{10}	0.37×10^{175}	0.1×10^{39}

Table 3. Choice Alternatives in the Magdeburg Probability Transformation Experiment

Row	**Option A**		**Option B**			**My Choice**
	0 Euro	**40 Euros**	**0 Euro**	**10 Euros**	**40 Euros**	
1	1/10	9/10	0/10	2/10	8/10	A B I
2	2/10	8/10	1/10	2/10	7/10	A B I
3	3/10	7/10	2/10	2/10	6/10	A B I
4	4/10	6/10	3/10	2/10	5/10	A B I
5	5/10	5/10	4/10	2/10	4/10	A B I
6	6/10	4/10	5/10	2/10	3/10	A B I
7	7/10	3/10	6/10	2/10	2/10	A B I
8	8/10	2/10	7/10	2/10	1/10	A B I
9	9/10	1/10	8/10	2/10	0/10	A B I

Table 4. Large-Stakes Risk Aversion Implied by Probability Calibration for Magdeburg Subjects

Subject	Dual Theory Predictions
1, 8, 9, 13, 21	$\{z, 0.7; 0\} \succ \{9z, 0.5; 0\}$
22	$\{z, 0.8; 0\} \succ \{27z, 0.5; 0\}$
17	$\{z, 0.9; 0\} \succ \{81z, 0.5; 0\}$
7	$\{z, 1; 0\} \succ \{244z, 0.5; 0\}$
4	$\{z, 1; 0\} \succ \{15z, 0.5; 0\}$
5	$\{z, 1; 0\} \succ \{19z, 0.5; 0\}$
11	$\{z, 1; 0\} \succ \{99z, 0.5; 0\}$
26	$\{z, 1; 0\} \succ \{17z, 0.5; 0\}$
30	$\{z, 1; 0\} \succ \{82z, 0.5; 0\}$

Table 5. Error Rate Model for Probability Calibration

Null Stochastic Types	Alternative Stochastic Types
[A,A,A,A,A,A,A,A,A]	[B*,B*,B*,B*,B*,B*,B*,B,*B*]
[B*,A,A,A,A,A,A,A,A]	[A,B*,B*,B*,B*,B*,B*,B*,B*]
[A,B*,A,A,A,A,A,A,A]	[B*,A,B*,B*,B*,B*,B*,B*,B*]
[B*,B*,A,A,A,A,A,A,A]	[A,A,B*,B*,B*,B*,B*,B*,B*]

Table 6. Choice Alternatives in the Calcutta Experiment

Option A	Option B	My Choice
80 or 130	100	A B I
980 or 1030	1000	A B I
1980 or 2030	2000	A B I
3980 or 4030	4000	A B I
4980 or 5030	5000	A B I
5980 or 6030	6000	A B I

Table 7. Calcutta Price Survey Data

Commonly used items for day-to-day living in Calcutta	Average Price range in Rupees
Food Items*	
Poultry	45-50
Fish	25-50
Red meat	150
Potatoes	7
Onions	10-12
Tomatoes	8-10
Carrots	8-10
Rice	11
Lentils	30
Public Transport	
Buses	3-4.5/ticket
Local trains	5-10/ticket
Eating out	
Average restaurants	15-35/person
Expensive restaurants	65-100/person
Five-star hotels/restaurants	500-1000/person

* Prices are in rupees per kilogram; 1 kilogram = 2.205 pounds

Table 8. Large-Stakes Risk Aversion Implied by Payoff Calibration for Calcutta Subjects

NOBS (30)	**Observed Rejection Intervals (m, M)**	**G Values for EU** $m+1K \succ_{EU} \{m,0.5;G\}$	**G Values for CPT & RDEU** $m+1K \succ_{CP,RD} \{m,0.5;G\}$
2	(1K, 5K)	0.12×10^{17}	0.399×10^{6}
4	(2K, 6K)	0.12×10^{17}	0.4×10^{6}
2	(100, 4K)	0.54×10^{16}	0.338×10^{6}

Table 9. Error Rate Model for Payoff Calibration

Model	**Null Stochastic Types**	**Alternative Stochastic Types**
Model I	[A,A,A,A,A,A]	[B*,B*,B*,B*,B*,B*]
	[B*,A,A,A,A,A}	[A,B*,B*,B*,B*,B*]
	[B*,B*,A,A,A,A]	[A,A,B*,B*,B*,B*]
Additional Patterns in Model II	[A,B*A,A,A,A]	[B*,A,B*,B*,B*,B*]
	[A,A,A,A,B*,B*]	[B*,B*,B*,B*,A,A]
	[A,A,A,A,B*,A]	[B*,B*,B*,B*,A,B*]
	[A,A,A,A,A,B*]	[B*,B*,B*,B*,B*,A]
	[B*,A,A,A,A,B*]	[A,B*,B*,B*,B*,A]

Table 10. Choice Alternatives in the Magdeburg Payoff Transformation Experiment

Option A	Option B	My Choice
2,900 or 3,110	3,000	A B I
8,900 or 9,110	9,000	A B I
49,900 or 50,110	5,0000	A B I
69,900 or 70,110	70,000	A B I
89,900 or 90,110	90,000	A B I
109,900 or 110,110	110,000	A B I

Table 11. Large-Stakes Risk Aversion Implied by Payoff Transformations for Magdeburg Subjects

NOBS	Observed Rejection Intervals (m, M)	G Values for EU $m+6K \succ_{EU} \{m,0.5;G\}$
7	(3K, 110K)	$0.21\text{x}10^{25}$
1	(3K, 90K)	$0.24\text{x}10^{21}$
1	(3K, 50K)	$0.3\text{x}10^{13}$
8	(50K, 110K)	$0.11\text{x}10^{16}$
1	(50K, 90K)	$0.13\text{x}10^{12}$
2	(70K, 110K)	$0.13\text{x}10^{12}$

Appendix A. Proofs of Propositions and Corollaries

A.1 Proof of Proposition 1 and Corollary 1

General result 1. Let a decision theory D represent preferences over lotteries L with "utility functional"

(a.i) $$U(L)=\sum_{j=1}^{n}[f(\sum_{k=j}^{n}p_k)-f(\sum_{k=j+1}^{n}p_k)]v(y_j)$$

where $f(.)$ is the transformation of decumulative probabilities whereas $v(.)$ is the money transformation function. Suppose that

(a.ii) $\{cx,(i-1)/2n;x,1/n;0\}\succsim\{cx,i/2n;0\}$, for all $i=1,2,\cdots,2n-1$, and

(a.iii) $v(cx)/v(x)>2$

Using notation $C\equiv v(cx)/v(x)$ we show that getting z for sure is preferred to getting $v^{-1}(v(z)K(C,n))$ or zero with even odds, where the function $K(.,.)$ is in section 2.1.

PROOF. To simplify notation, let $\delta=1/2n$. First note that, according to theory D, $\{cx,(i-1)\delta;x,2\delta;0\}\succsim\{cx,i\delta;0\}$, for all $i=1,2,\cdots,2n-1$ implies

(a.1) $$v(x)f((1+i)\delta)+\left[v(cx)-v(x)\right]f((i-1)\delta)\geq v(cx)f(i\delta),\ i=1,\ldots,2n-1$$

which is equivalent to

(a.2) $$f((1+i)\delta)-f(i\delta)\geq(C-1)\left[f(i\delta)-f((i-1)\delta)\right],\ i=1,\ldots,2n-1$$

Writing inequality (a.2) for $i+k(=1,\ldots,2n)$ and applying it $k-1$ other times one has

$$\begin{aligned}f((i+k)\delta)-f((i+k-1)\delta)&\geq(C-1)\left[f((i+k-1)\delta)-f((i+k-2)\delta)\right]\geq\ldots\\&\geq(C-1)^k\left[f(i\delta)-f((i-1)\delta)\right]\end{aligned}$$

which generalizes as

(a.3) $$f(j\delta)-f((j-1)\delta)\geq(C-1)^{j-i}\left[f(i\delta)-f((i-1)\delta)\right],\ j=i,\ldots,2n$$

Next, if we show that

(a.4) $$f(0.5)\leq\left[f(0.5)-f(0.5-\delta)\right]\sum_{i=1}^{n}\left(\frac{1}{C-1}\right)^{i-1}$$ and

(a.5) $$1-f(0.5)\geq\left[f(0.5)-f(0.5-\delta)\right]\sum_{j=1}^{n}(C-1)^j$$

then we are done since inequalities (a.4) and (a.5) imply

$$\frac{1-f(0.5)}{\sum_{j=1}^{n}(C-1)^j}\geq f(0.5)-f(0.5-\delta)\geq\frac{f(0.5)}{\sum_{i=1}^{n}(C-1)^{1-i}},$$

and therefore $1 \geq f(0.5)\left[1+\sum_{j=1}^{n}(C-1)^{j} / \sum_{i=1}^{n}(C-1)^{1-i}\right]$. For any given z multiply both sides with $v(z)$ and note that the last inequality implies that $v(z) \geq f(0.5)v(z)K(C,n)$. That is, getting z for sure is preferred to getting $v^{-1}(v(z)K(C,n))$ or zero with even odds.

To show inequality (a.4) note that 0.5 = $n\delta$ and that

$$f(0.5)=\sum_{i=1}^{n}\left[f(i\delta)-f((i-1)\delta)\right] \leq \left[f(n\delta)-f((n-1)\delta)\right]\sum_{i=1}^{n}\left(\frac{1}{C-1}\right)^{i-1}$$
$$=\left[f(0.5)-f(0.5-\delta)\right]\sum_{i=1}^{n}\left(\frac{1}{C-1}\right)^{i-1}$$

where the inequality follows from inequality (a.3). Similarly, inequality (a.5) follows from

$$1-f(0.5)=\sum_{j=n+1}^{2n}\left[f(j\delta)-f((j-1)\delta)\right] \geq \left[f((n+1)\delta)-f(n\delta)\right]\sum_{j=1}^{n}(C-1)^{j-1}$$
$$\geq \left[f(0.5)-f(0.5-\delta)\right]\sum_{j=1}^{n}(C-1)^{j}$$

Proof of Proposition 1 (dual theory of expected utility).

In dual expected utility theory $v(z)=z$. If c > 2 then $v(cz)/v(z)=c>2$ and therefore the general result 1 applies for this particular $v(z)=z$; hence z for sure is preferred to getting $zK(c,n)$ or zero with even odds.

Proof of Corollary 1 (cumulative prospect theory and rank dependent utility theory).

It is a straightforward application of the general result 1 for $v(z)=\upsilon(z)$.

A.2. Proof of Proposition 2 and Corollary 2

General result 2. Let a decision theory D with "utility functional" U in statement (a.i) be given. We assume here that v is (weakly) concave. Suppose that

(a.iv) $x+a \succsim \{x+b, p; x\}$ for all $x \in (m,M)$, $m>0$, and

(a.v) $bf(p)>a$.

We show that for all $z \in \left(m+b+b\ln(1-f(p))/\ln q, M\right)$, $z \succ \{G,p;m\}$ for all G that satisfy inequality (*) in Proposition 2 with $q=r(f(p))$.

PROOF. Let N be the largest integer smaller than (M-m)/b. This assumption and the definition of N imply

(a.6) $\quad v(x+a) \geq (1-f(p))v(x)+f(p)v(x+b)$, for all $x \in (m, m+Nb)$.

First we show that (a.6) and concavity of v imply that for all $y \in (m, m+Nb)$

(a.7) $\quad v'(y+jb) \leq q^j v'(y)$, for all $j \in \Psi_y$,

where $\Psi_y = \{ j \in \mathbb{N} \mid y+(j-1)b \in (m, m+Nb) \}$ and $q = (1/f(p)-1)/(b/a-1)$

Next let K be the largest integer smaller than $(z-m)/b$, and J be the smallest integer larger than $(\overline{G}-m)/b-K$ where $\overline{G}$ is the expression on the right hand side of inequality (*) in the statement of Proposition 2. We show that

(a.8) $\quad v(m+Kb) \geq f(p)v(m+(K+J)b)+(1-f(p))v(m).$

This completes the proof since all G that satisfy inequality (*) also satisfy $G < m+(K+J)b,$ which together with (a.8) and the definition of K imply $v(z) > f(p)v(G)+(1-f(p))v(m).$

To derive (a.7), first write $v(x+a) = f(p)v(x+a)+(1-f(p))v(x+a)$, next rewrite (a.6) with $x = y$, and finally group together terms with factors $f(p)$ and $1-f(p)$ on opposite sides of the inequality (a.6) to get

(a.9) $\quad (1-f(p))\left[v(y+a)-v(y)\right] \geq f(p)\left[v(y+b)-v(y+a)\right]$, $\forall y \in (m, m+Nb)$.

Inequalities $\left[v(y+b)-v(y+a)\right]/(b-a) \geq v'(y+b)$ and $\left[v(y+a)-v(y)\right]/a \leq v'(y),$ (both following from the weak concavity of v), inequality (a.9) and notation q imply

(a.10) $\quad v'(y+b) \leq \left(\dfrac{1-f(p)}{f(p)} \dfrac{a}{b-a} \right) v'(y) = qv'(y)$, $\forall y \in (m, m+Nb)$.

Iteration of inequality (a.10) j times, for $j \in \Psi_y$, gives inequalities that together imply statement (a.7):

$$v'(y+jb) \leq qv'(y+(j-1)b) \leq \ldots \leq q^j v'(y).$$

To show statement (a.8), let y denote $m+Kb$ and note that if $J+K > N$ then

$$v(y+Jb)-v(y)=\sum_{j=0}^{J-1}\left[v(y+(j+1)b)-v(y+jb)\right]$$

(a.11) $$\le b\left[(J-N+K)v'\big(y+(N-K)b\big)+\sum_{j=0}^{N-K-1}v'(y+jb)\right]$$

$$\le bv'(y)\left[q^{N-K}(J-N+K)+\sum_{j=0}^{N-K-1}q^{j}\right]$$

(In (a.11) the first inequality follows from (weak) concavity of φ and $J+K>N$ whereas the second one follows from statement (a.7).) If however $J+K\le N$ then one has

(a.11’) $$v(z+Jb)-v(y)\le b\sum_{j=0}^{J-1}v'(y+jb)\le bv'(y)\frac{1-q^{J}}{1-q}$$

Similarly, one can show that

(a.12) $$v(y)-v(y-bK)\ge bv'(y)\sum_{k=0}^{K-1}\frac{1}{q^{k}}$$

Hence, in case of $J+K>N$, (a.11) and (a.12) imply that a sufficient condition for (a.8) is

(a.13) $$(1-f(p))\sum_{k=0}^{K-1}\frac{1}{q^{k}}\ge f(p)\left[q^{N-K}(J-N+K)+\sum_{j=0}^{N-K-1}q^{j}\right]$$

Substitute $\sum_{j=0}^{N-K-1}q^{j}=\frac{1-q^{N-K}}{1-q}$, and $\sum_{k=0}^{K-1}\frac{1}{q^{k}}=\frac{q^{1-K}-q}{1-q}$ in (a.13) to get

(a.14) $$J\le N-K+\frac{1}{q^{N-K}}\left(\frac{1-f(p)}{f(p)}\frac{q^{-K}-1}{1-q}q-\frac{1-q^{N-K}}{1-q}\right)=N-K+\frac{1}{1-q}+\frac{A}{b}q^{-N}$$

The last inequality is true since

$$J\le(\overline{G}-m)/b-K+1=\left(M+b(2q-1)/(1-q)+Aq^{-N}-m\right)/b-K+1$$
$$\le\left(m+bN+bq/(1-q)+Aq^{-N}-m\right)/b-K+1$$
$$=N-K+1/(1-q)+q^{-N}A/b$$

Finally, if $J+K\le N$, (a.11’) and (a.12) imply that a sufficient condition for (a.8) is

(a.15) $$\left(q^{-K}-1\right)q>f(p)/(1-f(p))\ .$$

Note that definition of K and $z\in\left(m+b+b\ln(1-f(p))/\ln q,M\right)$ imply $q^{K-1}<1-f(p)$, hence (a.15) is satisfied.

Proof of Proposition 2 (expected utility theory).

It is a straightforward application of the general result 2 for $f(p) = p$ and $v(z) = u(z)$.

Corollary 2 (cumulative prospect theory and rank dependent utility theory).

It is a straightforward application of the general result 2 for $f(p) = h(p)$ and $v(z) = \upsilon(z)$

Appendix B. Additional Details of the Experiment Protocols

B.1 Magdeburg Experiment for Probability Transformation Theories

Once the subjects finished reading the instructions they were asked to mark their choices on the response form and write the ID number/letter that they had picked up at the beginning of the experiment on top of each sheet. After all subjects were done with their decisions, task 2 was given to them, which consisted of filling out a questionnaire. Again the subjects were asked to write the number/letter on the questionnaires that they had picked up. A subject's responses were identified only by an identification code that that was the subject's private information in order to protect their privacy with respect to answers on the questionnaire. At the end of the two tasks, the experimenter went to an adjoining room and called each of the students privately for payment. For each subject, a ball was drawn from a bingo cage containing balls numbered 1,2,…,9 to decide the relevant decision row and a ball was drawn from another bingo cage to determine the lottery payoff.

B.2 Calcutta Experiment for Payoff Transformation Theories

Each subject was asked to pick up a sheet of paper with either a number or a letter written on it. The subjects were presented with the instructions at the beginning of the session where the payment protocol of selecting one of the six tables randomly for money payoff (by rolling a six-sided die in the presence of the subject) was clearly explained to the subjects in the instructions as well as orally. The instructions also clarified that if they marked option I then the experimenter would flip a coin in front of the subject to choose between options A and B for him (if that decision was randomly selected for payoff). It was also clarified that if the subject chose the risky lottery in the selected decision task, then the lottery payment would be determined by flipping a coin in the presence of the subject.

Once the subjects finished reading the instructions they were given six sheets of paper, each containing one of the rows from Table 6, and were asked to mark their choices for each table and write the number/letter that they had picked up at the beginning of the experiment on top of each sheet. After all subjects were done with their decisions, task 2 was given to them, which consisted of filling out an income survey questionnaire. Again the subjects were asked to write the number/letter on the wealth questionnaires that they had picked up. A subject's responses were identified only by an identification code that that was the subject's private information in order to protect their privacy with respect to answers on the questionnaire. At the end of the two tasks, the experimenter went to an adjoining room and called each of the students privately for payment. For each subject, a die was rolled to decide the relevant payoff table. Further, if the subjects had marked the risky alternative in the selected table then a convention of paying the lower amount if the head came up and the higher amount if tails came up was announced to the student subject and incorporated. The student was asked to leave the questionnaire in a separate pile in order to protect privacy of responses.

B.3 Magdeburg Contingent Payoff (Casino) Experiment for Payoff Transformation Theories

After step 1 was finished the questionnaire was handed out to the participants. Every participant could choose whether to answer the questionnaire or not. She was paid 10 euros if she answered it. Since all participants could only be identified by a code the answers to the questionnaire could not be attributed to a personally-identifiable individual, but only to the choices 1-7 she made. All participants filled out the questionnaire.

In Step 2 we selected three subjects randomly (in the presence of all of the subjects) to accompany the experimenter to the casino and verify that he bet the money as described above. After the visit to the casino, the experimenter and the three participants returned to the university and informed the remaining subjects about the results. If a participant would have won, we would have drawn the balls from an urn afterwards and correctly performed the coin flip. Step 2 was executed some hours later, on the same day as Step 1, after the casino opened. (As it turned out, none of the bets placed on a roulette wheel in the casino paid off.)