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How do people play against Nash opponents in games which have a mixed strategy equilibrium?

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Abstract

We examine experimentally how humans behave when they, unbeknownst to them, play against a computer which implements its part of a mixed strategy Nash equilibrium. We consider two games, one zero-sum and another unprofitable with a pure minimax strategy. A minority of subjects' play was consistent with their Nash equilibrium strategy. But a larger percentage of subjects' play was more consistent with different models of play: equiprobable play for the zero-sum game, and the minimax strategy in the non-profitable game.

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1 Introduction

How do humans play in normal form games against opponents who play their part of a strictly mixed strategy Nash equilibrium? If we view Nash equilibrium as a positive theory, we predict humans will play their parts of a Nash equilibrium strategy profile. However, a Nash equilibrium in strictly mixed strategies has some attributes which undermine our confidence in this prediction. First, a strictly mixed strategy Nash equilibrium of a normal from game is weak – namely a player will have more than one action that is a best response to his opponent's equilibrium strategy. Another reason is that a player's mixed strategy doesn't depend on his own payoff function but rather his opponent's payoff function. In other words, a player's equilibrium strategy is calculated as to make other players indifferent between different actions. Not surprisingly, previous experimental studies have shown that mixed strategy equilibria are less likely to be played than pure strategy equilibria. This study provides an incisive test of how self enforcing a mixed strategy Nash equilibrium is.

We report an experiment in which each subject repeatedly plays a simple 2×2 normal form game for which the unique Nash equilibrium is in mixed strategies. In all sessions, each subject is paired against a computerized algorithm which plays its part of the Nash equilibrium profile. Two different games are used in this study, with each subject playing only one. One game is zero-sum and the other is unprofitable (the minimax and Nash equilibrium solutions are distinct but yield the same expected payoff for each player). By having the computer play its part of Nash equilibrium, we have provided an ideal environment to see if the mixed strategy Nash equilibrium is self enforcing with the subject adopting his part of the profile.

We present three hypotheses to evaluate human behavior when facing an opponent following its mixed strategy Nash equilibrium. The first hypothesis is that human subjects will adopt their part of the mixed strategy Nash equilibrium. For most subjects we reject this hypothesis. The second hypothesis is that subjects will adopt an equiprobable mixed strategy. There is support for this hypothesis in the aggregated data across subjects in the zero-sum game; however, at the individual level there is too much heterogeneity to conclude the subjects are all choosing the equiprobable mixed strategy. The third hypothesis is that subjects who play the unprofitable game will play a pure strategy minimax strategy rather than their part of the mixed strategy Nash equilibrium. We find some support for this hypothesis.

Our current study builds upon the results of several past studies. There is an extensivre literature examining the behavior of human subjects playing zero-sum games which have a unique Nash equilibrium in mixed strategies. Many studies reach the conclusions that subjects do not play according to the Nash Equilibrium mixture (Brown and Rosenthal (1990), Rosenthal, Shachat, and Walker (2003), and Shachat (2002)) and that play does not generally converge over time to the Nash equilibrium (Mookherjee and Sopher (1994) and Erev and Roth (1998)). In contrast, some studies argue that while one can statistically reject the precise predictions of a mixed strategy Nash equilibrium, the equilibrium predictions do a satisfactory job of qualitatively describing how subjects play (O'Neill (1987) and Binmore, Swierzbinski, and Proulx (2001)). Several studies have taken our approach of having a human subject play the same type of zero-sum games against a computer implementing a mixed Nash equilibrium strategy (Lieberman (1962), Messick (1967), and Fox (1972)).

These studies have found that the human play does not conform to the Nash equilibrium strategies.[1] Another set of studies have shown that in non-zero sum games which have a unique Nash equilibrium in mixed strategies, subjects' play depends significantly on structure of the magnitude of their own potential game payoffs (Ochs (1995), McKelvey, Palfrey, and Weber (2000) and Willinger and Noussair (2003)). In this present study, we consider one type of own payoff function effect by comparing the behavior in a zero-sum game to that in an unprofitable game. Morgan and Sefton (2002) found that in 3×3 unprofitable games, neither Nash equilibrium nor minimax accurately described play in their experiment.

We proceed by presenting the two games and their theoretical solutions in the next section. In the third section we describe our experimental design. In section four we present our data analysis and evaluate our hypotheses. Finally, we provide some concluding remarks.

2 The Games

The first game we employ is a zero-sum asymmetric matching pennies game introduced by Rosenthal et al. (2003). The game is called Pursue-Evade because the Row player "captures" points from the Column play on a match, and the Column player avoids a loss when he avoids a match. In the game each player can move either Left or Right. The normal form representation of the game is given in Figure 1. The game has a unique Nash equilibrium in which each player chooses Left with probability two-thirds. In equilibrium, Row's expected payoff is 2/3 and Column's expected payoff is -2/3.

		Column Player	
		Left	Right
Row Player	Left	1 , -1	0 , 0
	Right	0 , 0	2 , -2

Figure 1: The Pursue-Evade Game

The second game we employ is a unprofitable game introduced by Shachat and Swarthout (2004) referred to as Gamble-Safe. Each player has a Gamble action (Left for each player) from which he receives a payoff of either two or zero and a Safe action (Right for each player) which guarantees a payoff of one. The normal form representation of the game is given in Figure 2. This game has a unique Nash equilibrium in which each player chooses his Left action with probability one-half, and his expected equilibrium payoff is one. Notice that this game is not constant-sum; therefore the minimax solution need not coincide with the Nash equilibrium. In this game, Right is the minimax strategy for both players and it guarantees a payoff of one. A game for which minimax and Nash equilibrium solutions differ but generate

[1] These studies substantially differ from ours as they informed subjects they were playing against a computer but not what strategy the computer is playing. Also, some of the experiments did not provide perfect information regarding the payoff functions of the games.

Row Player \ Column Player	Left	Right
Left	2 , 0	0 , 1
Right	1 , 2	1 , 1

Figure 2: The Gamble-Safe Game

the same expected payoff is called an unprofitable game. Aumann (1985) argues that the Nash equilibrium prediction is not a plausible prediction in such a game because to achieve the Nash level payoff by playing the Nash equilibrium strategy requires assuming unnecessary risk. For example, If the Row player plays his Nash equilibrium strategy while the Column player adopts his minimax strategy Right, then the Row player's expected payoff is one-half. This aspect makes the Gamble-Safe game a more challenging for the Nash equilibrium solution concept than does the zero-sum Pursue-Evade game.

3 Experimental Design

We conducted the experiment sessions in the Economic Science Laboratory at the University of Arizona and the IBM T.J. Watson Research Center. We report results from six sessions, using a total of 60 undergraduate students. Each session contained between 8 and 16 subjects. The subjects were evenly divided between the Pursue-Evade and the Gamble-Safe game treatments. Half of the subjects were assigned as Row players, and the other half were assigned as Column players. Each subject was seated at a computer workstation such that no subject could observe another subject's screen. Subjects first read instructions detailing how to enter decisions and how earnings were determined. Then, 200 repetitions of the game were played. For the Pursue-Evade game, each Column subject was initially endowed with a balance of 250 tokens, while a Row subject began with no tokens: each token was valued at 10 cents. Each subject's total earnings consisted of either a $5 (University of Arizona) or a $15 (T.J. Watson Research Center) show-up payment plus his token balance after the 200th repetition. No Column subjects went bankrupt.

At the beginning of each repetition, a subject saw a graphical representation of the game on the screen. Each Column subject's game display was transformed so that he appeared to be a Row player. Thus, each subject selected an action by clicking on a row, and then confirmed his selection. After the repetition was complete, each subject saw the outcome highlighted on the game display, as well as a text message stating both players' actions and his own earnings for that repetition. Finally, at all times a subject's current token balance and a history of past play were displayed. The history consisted of an ordered list with each row displaying the repetition number, the actions selected by both players, and the subject's payoffs from the specific repetition.

In each session, a Row subject and a Column subject played against each other for the first twenty-three repetitions of the game. Then, unbeknownst to the human pair, they stopped playing against each other and for the remainder of the experiment they each played against a computer that implemented the Nash equilibrium strategy.[2]

We adopted a simple technique to make the transition from human versus human to human versus Nash equilibrium seamless from the subjects' perspective. From period twenty-four onward, each human pair had no further interaction except for the timing of how action choices were revealed. Specifically, although the computers generated their action choices instantly, the computers didn't reveal their choices until both humans had selected their actions. This protocol preserved the natural timing rhythm established by the humans in the first twenty-three stage games and helped mask the fact the subjects were playing against a computer. Given the structure of the games and the design of the experiments, there are some natural hypotheses we will test in the next section.

4 Data Analysis

A useful starting point is to inspect each subject's action choice frequencies versus the realized action choice frequencies generated by his computerized Nash equilibrium opponent. We present this view of the data for those subjects who were Pursue-Evade Row players in Figure 3, Pursue-Evade Column players in Figure 4, Gamble-Safe Row Players in Figure 5, and Gamble-Safe Column players in Figure 6. In each of these figures, the x-axis is the proportion of Left play for the Column player and the y-axis is the proportion of Left play for the Row player. Within each of these figures is a collection of arrows. Each arrow is a summary of play for a single human-computer pair. The origin of the arrow is located at the joint frequency of Left play in stage games twenty-four through one hundred, and the tip of the arrowhead is located at the joint frequency of Left play in the final one hundred stage games. These arrows show the adjustments subjects make from the first half of stage games to the second half.

In Figure 3, we see in the Pursue-Evade game that the Human Row subjects' frequency of Left play is contained within the 50 to 70 percent range and its difficult to ascertain whether Nash equilibrium or equi-probable play better describes the data. For the Human Column subjects in the same game, Figure 4 shows greater heterogeneity of play; there is a cluster of observations around 50 percent and also some subjects who play Left or Right nearly exclusively. In Figure 5, we see that Human Row subjects in the Gamble-Safe game appear on average to play the Gamble action less than 50 percent, but the range of left frequencies is quite broad covering a range of 0 to 70 percent. For the Human Column subjects in this game, Figure 6 suggests that there is a bias towards the minimax strategy as almost all frequencies of Left are below 50 percent. We will now evaluate our hypotheses.

Hypothesis 1 *Subjects will adopt their part of the Nash equilibrium.*

We first test this in the aggregate by pooling the last 100 rounds within each of the game and player types, and then conducting a z-test that the proportion of Left play is 67 percent

[2]This initial phase of human versus human play is a byproduct of the fact that this data was originally collected as part of another study presented in Shachat and Swarthout (2008).

for the Pursue-Evade sessions, and 50 percent for Gamble-Safe sessions. The results of these hypothesis tests are presented in Table 1. The results for the Gamble-Safe game are found in the last two columns of the first row, and the results for the Pursue-Evade game are found in the first two numerical columns of the second row. Nash Equilibrium is strongly rejected in each case. However, the figures of joint play reviewed above suggest a fair amount of heterogeneity in subject play, so we use a chi-square test to evaluate whether each subject is using the same mixed strategy. The results are given in the third row of Table 1. Except in the case of Row Pursue-Evade subjects, we reject homogeneity of play. This suggests testing our hypotheses separately on each individual subject.

We next use the binomial test to determine whether each subject is playing his Nash frequency of Left. For the Pursue-Evade game, the mixed-strategy equilibrium prediction for Left is two-thirds. We present these results in Table 2. For each subject we present his frequency of Left play during rounds 101 through 200. A two-tailed test at the 95 percent level of confidence gives us critical regions of less than 58 and more than 76 Left plays for the final one hundred rounds of play. We reject the Nash proportion of Left play for six out of fifteen Row subjects and for fourteen out of fifteen Column subjects. In Table 2 we boldface these rejections and in the last row of Table 1 we report these total number of rejections.

We conduct a similar set of tests for the Gamble-Safe game where the Nash proportion is 50 percent. In this case, the critical regions of the binomial test for the final 100 rounds are less than forty and more than sixty Left plays. The results of these tests are given in Table 3 where we boldface the rejections, and we report the number of rejections in row four of Table 1. Here the Nash hypothesis is rejected for eleven out of fifteen Row subjects and ten out of fifteen Column Subjects. In total 41 of our 60 subjects behave in a manner inconsistent with the predictions of mixed-strategy equilibrium. We will now move to consider other possible descriptions of play.

One non-equilibrium model of behavior that offers a reasonable alternative to Nash Equilibrium play is equiprobable play: subjects play each action with equal frequency since the expected payoff is the same for both actions. Of course, for the Gamble-Safe game, the Nash equilibrium strategy and equal probability play both yield the same prediction. However, in the Pursue-Evade game, Nash equilibrium is distinct from equal probable play. For those subjects who play the Pursue-Evade Game we will test the following hypothesis:

Hypothesis 2 *Subjects will adopt an equiprobable mixed strategy in the Pursue-Evade game.*

To test for equiprobable play in the Pursue-Evade game, we pool the data for the last 100 periods for each Row and Column subject, and see that a z-test rejects the hypothesis that the proportion of Left and Right play are equal (see the first row of Table 1). We also conduct a Binomial test on each subject's choices at the 95 percent level of confidence to evaluate individual equiprobable play, and we reject eight out of fifteen Row subjects and seven out of fifteen Column subjects. These individual rejections are indicated by the underlined proportions of Left play presented in Table 2. We conclude that one-half of the Pursue-Evade subjects are behaving consistent with equal-probable play.

An implication of both the Nash Equilibrium and equiprobable play hypotheses is that the subjects' choices are generated from a time independent draws from a fixed distribution. For each of our 60 subjects we conduct a non-parametric runs test for serial independence in the last 100 plays of the game. We reject serial independence for twenty-six of the sixty subjects. These rejections are identified by bold-faced entries of columns four and seven of Tables 2 and 3. Interestingly, twenty-two of these rejections come from a negative Z-value below the rejection threshold. These rejections come from too few runs, evidence that there is positive serial correlation not negative serial correlation.

Next we consider the minimax model of behavior as an alternative to mixed-strategy equilibrium play in the Gamble-Safe game. Since the Gamble-Safe game is unprofitable, we have a minimax strategy in this game which is distinct from mixed-strategy equilibrium play.

Hypothesis 3 *Subjects will play a pure minimax strategy in the Gamble-Safe game.*

For the Gamble-Safe game, the minimax strategy Right is the natural alternative to Nash Play. For each subject we conduct a one-tailed binomial test at the 95 percent level of confidence for each subject's last 100 decisions to see whether Left is played statistically significantly less than 50 percent. While this is not the most extreme test of minimax play, we nonetheless view it as an appropriate threshold test to determine whether play deviates away from mixed strategy equilibrium play in the direction of minimax play. The critical region for this test is less than 38. The results of the test are reported Table 3 where rejections are underlined, and in row five of Table 1. For six of the Row subjects and ten of the Column subjects we reject Nash play of 50 percent Left in favor of the alternative of less than 50 percent. So minimax does attract play but not exclusively.

5 Conclusions

In this study we have examined the self enforcing nature of a Nash Equilibrium when it's unique and in mixed strategies. Our experiment is particularly well suited for this purpose, as each subject plays against a computer which generates actions according to its mixed strategy equilibrium strategy. However, because the timing of each stage game is tied to when a pair of humans have made their choices, it's not apparent to each subject that his opponent is a computer. Our study employs two games. One is a zero-sum game for which the minimax and Nash equilibrium solutions are the same. The other is a non-profitable game which has a mixed strategy Nash equilibrium and a pure minimax strategy profile, and thus provides a tougher challenge for the Nash equilibrium hypothesis.

In the constant sum Pursue-Evade game we found that we can reject Nash equilibrium play for half of the Row subjects and all but one of the Column subjects. We also find that we can reject equiprobable play for about half of both Row and Column subjects. Moreover we find that, despite all subjects playing against the same equilibrium strategy, there is significant heterogeneity across subjects. In the nonprofitable Gamble-Safe game, we reject the Nash equilibrium (and equiprobable) model for two-thirds of both the Row and Column players. Further, we see many of the human Column players selecting Left well below fifty percent of the time, indicating a tendency towards the minimax safe strategy.

The general failure of a mixed strategy equilibrium to be self enforcing has significant consequences. For example our study suggests that auditors are likely to falter in adopting optimal monitoring frequencies which is likely to lead to greater losses or overexpenditure on monitoring. Also, firms facing monopolistic competition are not likely to optimally mix their pricing strategies and can be exploited.

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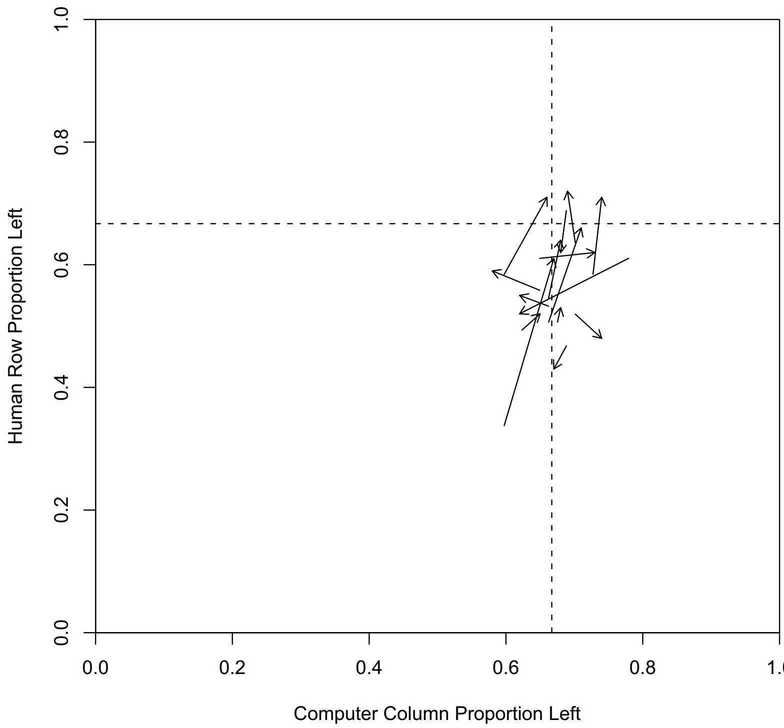
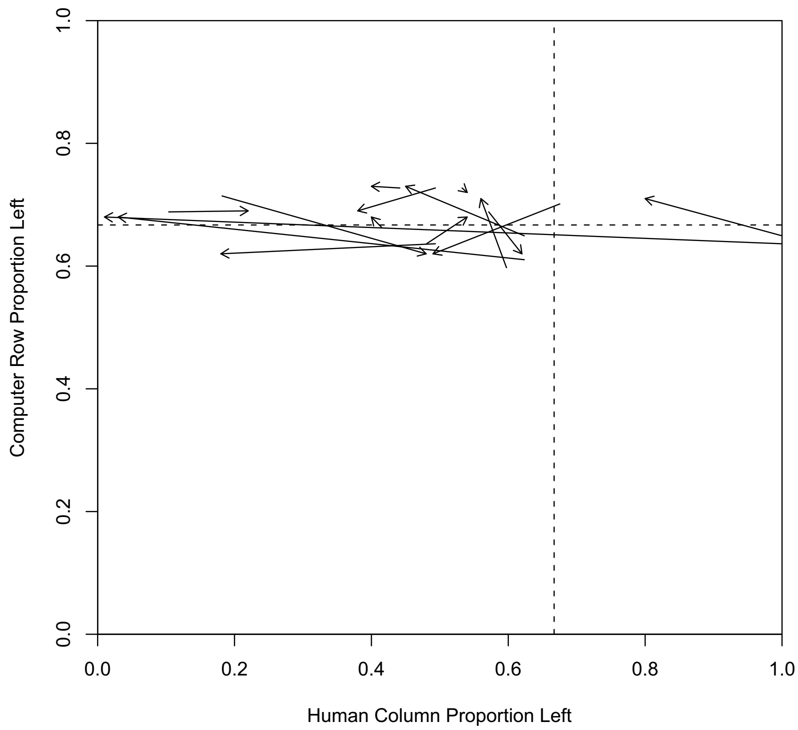


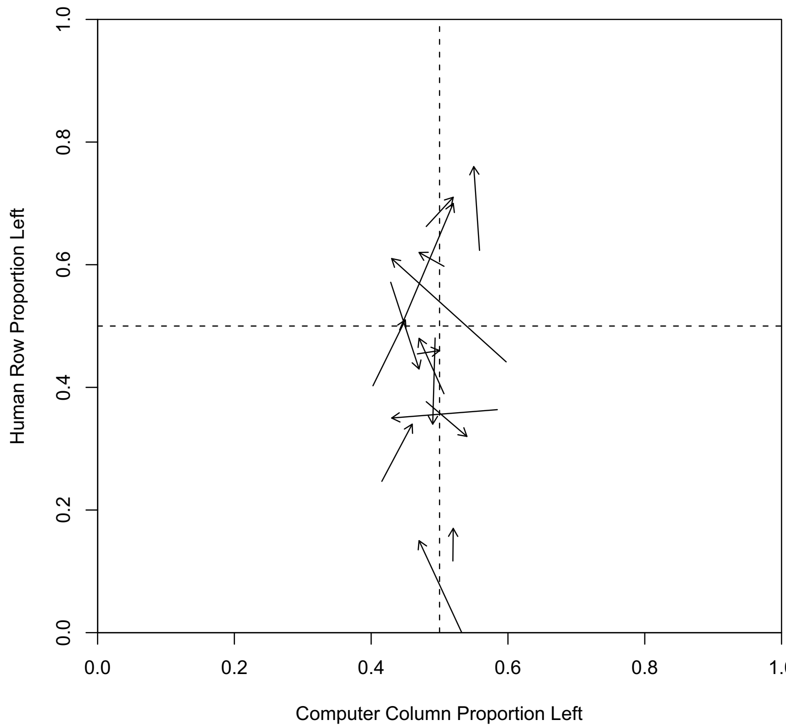
Notes: Each arrow represents a subject-computer pair. The tail of the arrow is the joint frequency of Left play in rounds 24-100, and the head is the joint frequency of Left play in rounds 101-200. The dashed lines denote the Nash equilibrium mixture of 2/3 Left.

Figure 3: Pursue-Evade Joint Left Frequencies of Human Row Players vs. Computer NE



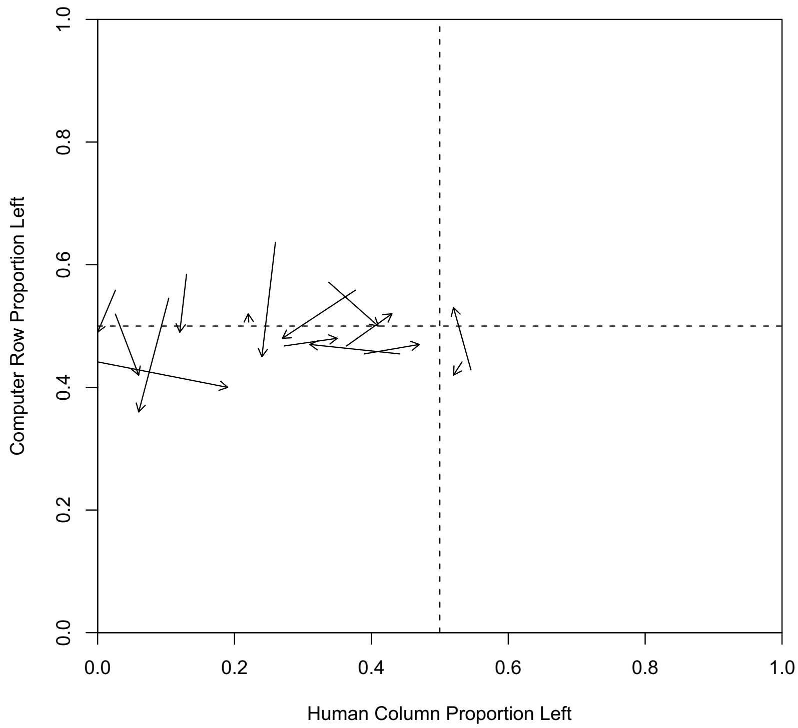
Notes: Each arrow represents a subject-computer pair. The tail of the arrow is the joint frequency of Left play in rounds 24-100, and the head is the joint frequency of Left play in rounds 101-200. The dashed lines denote the Nash equilibrium mixture of 2/3 Left.

Figure 4: Pursue-Evade Joint Left Frequencies of Human Column Players vs. Computer NE



Notes: Each arrow represents a subject-computer pair. The tail of the arrow is the joint frequency of Left play in rounds 24-100, and the head is the joint frequency of Left play in rounds 101-200. The dashed lines denote the Nash equilibrium mixture of 1/2 Left.

Figure 5: Gamble-Safe Joint Left Frequencies of Human Row Players vs. Computer NE



Notes: Each arrow represents a subject-computer pair. The tail of the arrow is the joint frequency of Left play in rounds 24-100, and the head is the joint frequency of Left play in rounds 101-200. The dashed lines denote the Nash equilibrium mixture of 1/2 Left.

Figure 6: Gamble-Safe Joint Left Frequencies of Human Column Players vs. Computer NE

Table 1: Aggregate Hypothesis Tests

Null Hypothesis	Test Type	P-E Row	P-E Column	G-S Row	G-S Column
Aggregated mixed strategy of Left with Probability of 50%	two-tailed z test (p-value)	7.28 (0.00)	-7.23 (0.00)	-2.84 (0.00)	-17.20 (0.00)
aggregated mixed strategy of Left with Probability of 67%	two-tailed z test (p-value)	-5.99 (0.00)	-21.42 (0.00)		
All subjects use same mixed strategy	chi-square test (p-value)	18.14 (0.20)	162.23 (0.00)	108.46 (0.00)	147.71 (0.00)
Individual subject's mixed strategy is 50%	binomal test #rejections at 95% level of confidence	8	7	11	10
Individual subject's mixed strategy is less than 50%	binomal test #rejections at 95% level of confidence			6	10
Individual subject's mixed strategy is 67%	binomal test #rejections at 95% level of confidence	6	14		

Table 2: Pursue-Evade Individual Subject Hypothesis Tests

Session	Row Player Proportion left	Row Player Runs test statistic	Row Player Runs test p-value	Column Player Proportion left	Column Player Runs test statistic	Column Player Runs test p-value
1	0.62	-2.59	**0.01**	**0.40**	-1.26	0.21
2	**0.43**	-5.13	**0.00**	**0.54**	-0.95	0.34
3	0.66	-2.22	**0.03**	**0.18**	-3.26	**0.00**
4	**0.48**	-5.42	**0.00**	**0.56**	-2.50	**0.01**
5	0.61	-9.63	**0.00**	0.62	1.68	0.09
6	0.71	-1.02	0.31	**0.45**	-3.35	**0.00**
7	**0.53**	-0.97	0.33	**0.48**	-4.21	**0.00**
8	0.59	-0.08	0.94	**0.03**	-5.30	**0.00**
9	**0.55**	-0.30	0.76	**0.54**	0.67	0.50
10	**0.52**	0.02	0.99	**0.40**	-3.14	**0.00**
11	0.62	-3.44	**0.00**	**0.01**	0.14	0.89
12	0.64	-3.95	**0.00**	**0.49**	-1.81	0.07
13	0.72	-1.83	0.07	**0.22**	-8.04	**0.00**
14	**0.52**	0.02	0.99	**0.38**	1.90	0.06
15	0.71	-2.98	**0.00**	**0.80**	-0.63	0.53

Notes: A bold face proportion indicates a rejection of the Nash equilibrium proportion at the 5% level of significance. An underlined proportion indicates a rejection of equiprobable proportion at the 5% level of significance. A bold face runs test p-value indicates a rejection of serial independence at the 5% level of significance.

Table 3: Gamble-Safe Individual Subject Hypothesis Tests

	Row Player			Column Player		
Session	Proportion left	Runs test statistic	Runs test p-value	Proportion left	Runs test statistic	Runs test p-value
1	**0.34**	-1.54	0.12	**0.12**	-0.06	0.95
2	**0.76**	-0.69	0.49	**0.22**	0.49	0.62
3	**0.71**	-1.27	0.21	**0.35**	-3.21	**0.00**
4	**0.61**	-2.02	**0.04**	**0.06**	0.67	0.51
5	**0.15**	-7.36	**0.00**	**0.24**	0.14	0.89
6	**0.62**	0.19	0.85	0.52	0.22	0.83
7	0.51	-1.00	0.32	**0.00**	---	---
8	0.48	-3.00	**0.00**	**0.06**	0.67	0.51
9	**0.34**	0.92	0.36	0.43	-2.05	**0.04**
10	0.46	2.69	**0.01**	0.52	0.22	0.83
11	**0.32**	1.04	0.30	0.41	1.79	**0.07**
12	**0.35**	0.55	0.58	0.47	0.64	0.52
13	**0.70**	-2.40	**0.02**	**0.27**	2.45	**0.01**
14	0.43	-2.88	**0.00**	**0.31**	-1.36	0.17
15	**0.17**	-2.95	**0.00**	**0.19**	-1.24	0.21

Notes: A bold face proportion indicates a rejection of the Nash equilibrium, or 50%, proportion at the 5% level of significance. An underlined proportion indicates a rejection of the same test in favor of the alternative that play is less than 50% at the 5% level of significance. A bold-face runs test p-value indicates a rejection of serial independence at the 5% level of significance. Missing values due to inapplicability of test on data with zero variation.