Observational Constraints on the Solar Dynamo and the Hunt for Precursors to Solar Flares

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ABSTRACT

While it is believed magnetic spots on solar-like stars originate in their convection zones, constraining the exact location of their generation has not been hitherto possible. Based on theoretical magnetohydrodynamic considerations and an analysis of helioseismic data on solar torsional oscillation, here we demonstrate that dynamic Lorentz forces and an equivalent Lenz’s law for magnetic induction conspire together to reveal where sunspots – strong magnetic field concentrations observed on the Sun’s surface – originate. Our results illuminate physical processes at the heart of solar-stellar magnetic cycles and indicate that stellar magnetic spots are generated in the lower half of their convection zones.

The Sun’s axisymmetric flows - differential rotation and meridional flow, govern the dynamics of the solar magnetic cycle and serve as vital inputs for numerical models of the solar cycle. A variety of methods are used to measure these flows, each with its own strengths and weaknesses. Measurements based on cross-correlating images of the surface magnetic field (magnetograms) have been made since the 1970s. Measuring these flows precisely with
this method requires advanced numerical techniques which are capable of detecting movements of less than the pixel size in images of the Sun. We have identified several systematic errors which influence previous measurements of these flows and propose numerical techniques which can minimize these errors. Our analysis of magnetograms from the *Michelson Doppler Imager* (MDI) on the ESA/NASA *Solar and Heliospheric Observatory* (SOHO) and *Helioseismic and Magnetic Imager* (HMI) on the NASA *Solar Dynamics Observatory* (SDO) shows long-term variations in the meridional flow and differential rotation from 1996 to 2019 which has implications for solar cycle prediction.

We also introduce and make openly accessible a comprehensive, multivariate time series (MVTS) dataset extracted from solar photospheric vector magnetograms in the Spaceweather HMI Active Region Patch (SHARP) data series obtained from HMI onboard SDO. Our dataset includes a cross-checked NOAA solar flare catalog which immediately facilitates solar flare prediction efforts, for the first time using time series in a detailed, quantitative manner. We discuss methods used for data collection, cleaning and pre-processing of the active region and flare data; and we further describe a novel data integration and sampling methodology. This dataset covers 4,075 MVTS of active regions between May 2010 and August 2018 matched to over 10,000 flare reports. Potential directions toward expansion of the time series, either “horizontally” – by adding more prediction specific parameters, or “vertically” – by generalizing it in order to predict other solar eruptions, are also indicated. The purpose of this dataset is two fold. First, it serves as a mine for precursors to solar flares and it also serves as a benchmark which facilitates the comparison of different solar flare prediction algorithms with both operational (research-to-operations), and basic research (operations-to-research), benefits potentially following in the future.

**INDEX WORDS:** Sun, Sunspots, Space weather, Torsional oscillation, Meridional flow
OBSERVATIONAL CONSTRAINTS ON THE SOLAR DYNAMO AND THE HUNT FOR PRECURSORS TO SOLAR FLARES

by

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DEDICATION

I dedicate this dissertation to my parents (Sushil Mahajan and Smita Mahajan), my wife Radhika Mahajan and to the source of energy in living beings on Earth: the Sun.

O! Lord Surya (Sun), ruler of the world, you are the remover of all diseases, the repository of peace. I bow to you. Please bless your devotees with long life, health, and wealth.

∼Surya Mantra
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As a young boy from the town of Aurangabad in India, I journeyed to the holy city of Varanasi in my quest for education. I begin with thanking the Universe for arranging a spectacular total solar eclipse on the day of my registration at the Indian Institute of Technology, Banaras Hindu University. Mysteriously, I ended up pursuing Heliophysics by the time I graduated.

From the crowded streets of Varanasi I came to, the city in a forest, Atlanta for my doctoral studies. Over the past five years, a number of people have helped me learn critical thinking skills necessary to work towards a career in scientific research.

Dr. Dibyendu Nandi nurtured my scientific curiosity in the early stages of my career, always encouraging me to perform back of the envelope estimations to judge the feasibility of my ideas. My PhD supervisor Prof. Petrus Martens always gave me the freedom to work on what I found interesting, and taught me to focus on the physical implications of any kind of data analysis.

I am grateful to the NASA Advanced Supercomputing Division for hosting me during the summer of 2016 as an intern and to Dr. David H. Hathaway who supervised me during this period. Other than concepts related to solar physics, he taught me golf at the Moffett field golf course and the importance of maintaining a journal to document my progress in a research project.

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# Table of Contents

List of Tables .................................................................................................................. x

List of Figures ................................................................................................................ xi

1 Introduction to the Solar Dynamo ........................................................................ 1
  1.1 Overview .................................................................................................................. 1
  1.2 The Solar Cycle ....................................................................................................... 3
  1.3 Basic Equations ....................................................................................................... 6
    1.3.1 Magnetic Reynolds Number ............................................................................ 9
    1.3.2 Magnetic Pressure and Plasma $\beta$ ................................................................ 10
  1.4 Differential Rotation .............................................................................................. 11
  1.5 Hale’s Polarity Law ................................................................................................. 13
  1.6 Joy’s Law ................................................................................................................. 15
  1.7 Parker’s Hypothesis of the Origin of Sunspots ...................................................... 15
  1.8 Meridional Circulation and the Babcock Leighton $\alpha$-effect ............................. 17

2 Lorentz Force Variations Reveal Origin of Sunspots in the Deep Solar Interior .... 20
  2.1 Background ............................................................................................................ 20
  2.2 Analysis Of Helioseismic Inversions Of Differential Rotation ......................... 25
  2.3 Conclusion .............................................................................................................. 33

3.1 Background ................................................................. 36
3.2 Data .......................................................................... 40
   3.2.1 MDI/HMI Magnetograms ............................................. 40
   3.2.2 Synthetic Magnetograms for Testing ............................... 40
3.3 Improvements to the Correlation Tracking Algorithm ............... 42
   3.3.1 Latitude-Longitude interpolation used for Heliographic projection . 42
   3.3.2 Choosing block sizes and changing search area with latitude .... 45
   3.3.3 Peak-locking ................................................................. 47
   3.3.4 Discovery and Correction of the Center-to-limb effect .......... 48
3.4 Results ..................................................................... 53
   3.4.1 Meridional Flow ............................................................ 53
   3.4.2 Torsional Oscillation ..................................................... 55
   3.4.3 Properties of constant shift error $\Delta$ and time-lag dependent $\delta$ . 56
3.5 Conclusion .................................................................. 58

4 SWAN: a Machine Learning Ready Space Weather ANalytics Dataset for the Prediction of Solar Flares .................................................. 62
4.1 Background & Summary .................................................. 62
4.2 Methods .................................................................... 64
   4.2.1 Solar Flare Reports ...................................................... 64
   4.2.2 SHARP Data and Magnetic Field Parameters ..................... 69
   4.2.3 Flare Integration with SHARP data ................................ 73
   4.2.4 Task-based Dataset Generation ..................................... 74
   4.2.5 Extending the Datasets ................................................. 78
   4.2.6 Code availability ......................................................... 79
4.3 Technical Validation .................................................... 79
4.4 Data Records ............................................................... 79
<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>5  Concluding Remarks</td>
<td>82</td>
</tr>
<tr>
<td>Bibliography</td>
<td>85</td>
</tr>
<tr>
<td>B  Spatiotemporal Co-occurrence Analysis of Flare Reports from GOES and AIA</td>
<td>93</td>
</tr>
<tr>
<td>C  Combining data from XRS on several GOES satellites</td>
<td>97</td>
</tr>
<tr>
<td>D  Example Task-based Data Generation Scenario</td>
<td>99</td>
</tr>
</tbody>
</table>
List of Tables

3.1 Correlation coefficient between the oversampled and high resolution image when different interpolation algorithms are used to oversample a low resolution image into a high resolution image of a feature on a synthetic magnetogram. 45

3.2 Search area at solar disk center as a function of time-lag is shown here chosen so that velocities upto $\pm 400 m/s$ can be detected. The time unit for MDI is 96 minutes and for HMI is one hour. The longitudinal search area at the equator is equal to the values in this table and varies with a factor of the secant of the latitude. ......................... 45

4.1 Computed magnetic field parameters. Parameters in red are described in Bobra & Couvidat (2015), but not available in SHARP headers. .......... 71

4.2 Summary and categorization of the time series parameters in our data. Individual parameters in blue have categorical values, while the ones in red are spatial parameters. All remaining have continuous values. ................. 80

C.1 Dates and times when the primary and/or secondary satellite designations were changed by NOAA for XRS data. Note that this table has been reproduced only for the time period 2010-05-01 to 2018-09-01, the duration of our dataset. Data courtesy: Dr. Janet Machol. ......................... 98
List of Figures

1.1 Sunspot cycle and butterfly diagram. (Image courtesy: Dr. David Hathaway) ............................................. 4

1.2 Radial magnetic field on the solar surface (shown in colorscale) averaged over all longitudes is plotted versus latitude and time. This is called a magnetic butterfly diagram. This plot shows the flipping of polar magnetic field after every sunspot cycle. The equatorward migration of sunspot belt is also evident. (Image courtesy: Dr. David Hathaway) ............................................. 5

1.3 An overview of the different layers of the Sun: the three major interior zones are the core, the radiative zone, and the convective zone. Photosphere represents the visible surface of the Sun on which sunspots and granules are seen. Chromosphere is the lower atmosphere of the Sun and Corona is its outer atmosphere. Flares, prominences and solar wind are typically found in the solar corona (Image courtesy: Kelvin Ma). ............................................. 6

1.4 The rotation rate in the solar interior derived from global helioseismology (in \(nHz\)) shows differential rotation. These measurements were derived from Global Oscillations Network Group (GONG) data reduced by Antia & Basu (2011). ............................................. 11

1.5 An illustration of Parker’s \(\Omega\)-effect. A dipolar magnetic field line (black) passing through the solar interior is advected by differential rotation (indicated by blue arrows with length proportional to rotation rate) and wrapped around the Sun to create a clockwise toroidal field in the northern hemisphere and an anti-clockwise toroidal field in the southern hemisphere. ............................................. 13
1.6 A typical magnetogram image of the Sun taken during one of its active phases (year 2012). White color represents positive magnetic field (directed towards the camera) and black color represents negative magnetic field (directed away from the camera). Black polarity is leading in the northern hemisphere while white polarity is leading in the southern hemisphere as the solar surface rotates from left to right. The leading and following polarities switch places every 11 years in a rhythmic fashion (Image Credit: Helioseismic & Magnetic Imager (HMI) and www.helioviewer.org).

1.7 An illustration of a magnetic flux tube rising through the solar convection zone and piercing the photosphere at two locations creating a bipolar magnetic region (BMR). Coriolis force acts on flux tubes when they are rising through the convection zone creating the tilt leading to Joy’s law. The rotation of the Sun is towards the right (\(\hat{\phi}\)).

1.8 The direction of meridional flow in the \(r-\theta\) plane. The flow has been observed towards the poles on the surface in both hemispheres, and it is believed that there exists a return flow towards the equator deep inside the convection zone.

1.9 An illustration of the Babcock Leighton alpha effect. Some of the magnetic flux from leading polarities in both hemispheres cancels across the equator due to diffusion overcoming meridional flow, while the flux from following polarities is transported towards the poles by meridional flow.

2.1 The solid circles represent 0.70\(R_{\odot}\) and 1.00\(R_{\odot}\) (the bounding surfaces of our volume integrals) whereas the dashed circle represents the location of the tachocline. The bending of a poloidal field by differential rotation denoted by blue arrows is demonstrated here. Such a field line would exert magnetic tension force in a manner shown by the red arrows.
2.2 Time series of the azimuthal rotation velocity at three different depths and nine different latitudes from GONG data are shown here with 1σ error bars. The latitude is indicated in the left column whereas the depth is indicated in the top row. The bottom panel in every column shows the smoothed monthly sunspot number (SSN) (SILSO World Data Center 1995-2017) for comparison. The dashed magenta lines mark the timings of solar minimum. The black dashed lines show that the basal rotation rate at high latitudes, mean rotation rate at latitudes 25° – 35° and the ceiling rotation rate at low latitudes is consistent across sunspot cycles.

2.3 Same as Fig. 2.2 obtained from MDI-HMI data.

2.4 Background: The net force per unit volume $\rho \frac{\partial v_\phi}{\partial t}$ in the azimuthal direction inside the Sun is shown at several depths from polynomial fits to GONG data. Overplotted: Locations of sunspots on the solar surface from the Royal Greenwich Observatory (RGO) database are shown by green dots. The magenta lines mark the timings of solar minimum and maximum and the transparent gray regions cover areas with poor signal to noise ratio and high uncertainties.

2.5 Same as Fig. 2.4 showing the net azimuthal force in MDI-HMI data.

2.6 The net force per unit volume $\rho \frac{\partial v_\phi}{\partial t}$ in the azimuthal direction inside the Sun shown at several time stamps from polynomial fits to GONG data. Locations of sunspots on the solar surface from the Royal Greenwich Observatory (RGO) database are shown by green circles on the surface. The arrows in panels with time 2007.12 and 2008.30 years mark the location at 0.82$R_\odot$ where the signs of solar cycle 24 first appear in the data.

2.7 Same as Fig. 2.6 showing the trends in MDI-HMI data. The arrows in panels with time 2007.07 and 2008.26 years mark the location at 0.82$R_\odot$ where the signs of solar cycle 24 first appear.
3.1 A synthetic solar magnetogram with 1000 positive polarity magnetic features produced by AMC and sampled by the CCD emulator. The magnetic field shown here is in Gauss. .......................... 41

3.2 The performance of various interpolation algorithms used in projection routines. The first two panels in the top row show the heliographic projection of a well resolved 2D Gaussian magnetic feature analytically placed on a synthetic magnetogram and sampled at different resolutions. The third panel in the first row and all panels in the second row show the same magnetic feature when different interpolation algorithms are used to reconstruct the higher resolution projection of the feature by oversampling its low resolution image. In the third and fourth rows, corresponding plots for a feature which is barely resolved in the high-resolution image and unresolved in the low resolution image are shown. .......................... 43

3.3 Blocks selected for tracking from a HMI magnetogram (in Gauss) projected onto a uniform grid in latitude-longitude are marked in red. The horizontal extent stretches from $-90^\circ$ on the left to $+90^\circ$ longitude on the right and the vertical extent from the south pole to north pole. The yellow line indicates the solar equator. .......................... 46

3.4 Left: A histogram of all measurements of meridional flow velocity from Carrington rotation 2098 at 8 hr time-lag shows peak-locking at 18 m/s intervals. Moreover, the measurements in the original algorithm were clipped due to a small search area. Right: Corresponding histogram of meridional flow measurements from our improved algorithm mitigates peak locking and captures the full range of the velocities making individual measurements more accurate. 48
3.5 A comparison of tracking results from signed magnetograms (top row) and unsigned magnetograms (middle row) for Carrington rotation 2098 are shown here. The bottom row shows tracking results from synthetic data created with a noise level of 10 Gauss (FWHM). The first column in each row shows the residual rotation rate measured at one hour time-lag obtained after removing the average rotation rate measured at each latitude, while the second column shows the residual rotation rate measured at the equator at several time-lags. The third column in each row shows the residual meridional flow speed measured at one hour time-lag obtained after removing the average meridional flow speed measured at each latitude, while the fourth column shows the latitudinal variation of meridional flow speed at several time-lags.

3.6 A sample magnetic field pattern with a positive and a negative feature is shown on the left with solid line and its mean value is marked by the dashed line. The same pattern’s unsigned value and its mean is shown on the right.

3.7 The panels from left to right show the baseline meridional flow velocity, constant shift $\Delta_{MF}$ and the flow change parameter $\delta_{MF}$ derived from combinations of measurements at different time-lags as in Eqs. 5 to 12 and averaged over one year from May 2010 to April 2011 for HMI. Their agreement supports our assumption that $\Delta$, $\delta$ and the baseline flow are independent of time-lag.

3.8 The baseline flow velocity, $\Delta$ and $\delta$ averaged over one year (from May 2010 to April 2011) and over different combinations of time-lags are shown here. The top row shows the baseline flow velocity (in $m/s$), middle row shows $\Delta$ (in $km$) and $\delta$ (in $m/s$ per hour) is shown in the bottom row. The first and third columns show the residuals after removing the average quantity at each latitude. The second column shows the longitudinal dependence of the quantity at the solar equator and the fourth column shows the latitudinal dependence of the quantity.
3.9 The top row shows latitude-time plot of the baseline meridional flow and torsional oscillation obtained from over 23 years of measurements from MDI and HMI data in units of m/s. The corresponding plots for $\Delta$ (in km) and $\delta$ (in $m/s$ per hour) are shown in the second and the third row respectively. The boundaries of active latitudes are marked in black using the Royal Greenwich Observatory sunspot records. The bottom row shows the scaled sunspot number (in black) and the fitting coefficients of associated Legendre polynomial of order 1, degree 2 to the meridional flow profile and of order 1, degree 1 to the torsional oscillation with $2\sigma$ errors.

4.1 The block diagram of dataset generation process with principal procedures of flare cleaning (in red), MVTS generation and flare integration (in blue), and machine learning-ready dataset creation (in orange). (Image courtesy: Dr. Berkay Aydin)

4.2 X-ray flux from GOES15 satellite on 2011-02-14 and 2011-02-15 along with occurrences of large flares, annotated with magnitudes. The GOES flare classification is also provided in minor y-axis. Dashed red lines indicate the flares’ peak time. During these two days, background X-ray flux was high, making it difficult to identify small flares. (Image courtesy: Dr. Berkay Aydin)

4.3 The number of flares after cleaning procedures per each flare class. Blue bars show the “trusted” flares with a valid location, red bars show the flares with no implicit or explicit location information, purple bars show the flares that could not be validated using an AIA flare report. (Image courtesy: Dr. Berkay Aydin)
4.4 (a) Scatter plot of flare latitudes (in degrees, helioprojective) and peak times between May 2010 and September 2018. Flare locations are approximated when originally not available. (b) The spatial distribution of M- and X-class flares. (c) The spatial distribution of C-class flares. (Image courtesy: Dr. Berkay Aydin) ................................. 68

4.5 Example slicing and labeling of time series. (Image courtesy: Dr. Berkay Aydin) ................................................................. 76

B.1 Overview of our flare cleaning and enhancement procedures resulting in two sets of flare reports: (1) validated flares and (2) flares without location or without an AIA flare correspondence. (Image courtesy: Dr. Berkay Aydin) .............................. 93

B.2 The distribution of distances between the closest AIA and GOES flares, which temporally overlap. The distance measurement is Euclidean over Helioprojective-Cartesian (HPC) coordinates. (Image courtesy: Dr. Berkay Aydin) .............. 95

B.3 The spatiotemporal and spatial distribution of the invalidated GOES flares.
(Image courtesy: Dr. Berkay Aydin) ......................................................... 96

C.1 (a): The blackout durations in GOES XRS data from primary satellites show a biannual periodicity. Total downtime was 1.43%. (b): After filling some of the gaps in primary GOES XRS data with available data from secondary satellites, the total downtime reduced to 0.80%. ................................. 98

D.1 Coverage of different folds (partitions) in our example scenario. The number of MVTS in each partition, number of flare annotations in each partition, and number of large flares occurred during these intervals are also illustrated. (Image courtesy: Dr. Berkay Aydin) ................................. 100
D.2 An overview of slicing, labeling and filtering. (a) Number of instances removed due to the lack of sufficient (at least 90%) magnetic field information. (b) Number of non-flaring (NF) instances filtered due to the invalidated flares or lack of high quality X-ray data. (c) Number of flaring and non-flaring instances created for each partition. (Image courtesy: Dr. Berkay Aydin)

D.3 The number of instances in each partition after climatology-based undersampling. (Image courtesy: Dr. Berkay Aydin)
Chapter 1

Introduction to the Solar Dynamo

1.1 Overview

One of the main streams of astrophysics has been the physics of our courtyard star. It is light and life for us and we would not exist without it. In addition to being the star closest to us, it is also the most dynamical object in the solar system as it burps out large masses of superheated plasma known as Coronal Mass Ejections (CMEs) and releases short bursts of high energy X-ray radiation known as solar flares form time to time. The Sun makes its presence felt in the whole solar system through its expanding solar corona (the outermost layer) which comprises of the solar wind: a continuous stream of plasma particles emanating from the Sun filling up interplanetary space within the boundaries of the solar system.

Although the Sun is farther than a hundred million kilometers from Earth, sudden bursts of superheated plasma or X-ray radiation from the solar surface can cause geomagnetic storms in the polar regions on Earth. One of the affects of these storms is the bombardment of the Earth’s atmosphere and magnetosphere with highly energetic particles some of which ionize or excite the gases in Earth’s atmosphere at high altitudes. In a desperate attempt to regain their ground state, these excited gases emit red, purple and green light creating one of the most aesthetic events of astrophysical origin: auroras (aurora borealis in the northern hemisphere and aurora australis in the southern). Auroras look like multicolored curtains dancing with the wind in the night sky. Stronger geomagnetic storms have the potential to create auroras at lower latitudes on Earth and can also interfere with communication
satellites in space and power grids on the ground. Geomagnetic storms also have a history of interference with telegraph lines, rendering them useless in many cases.

A recent account, which demonstrated the potential of geomagnetic storms, was the storm on March 13, 1989 (Odenwald 2007). On Friday, March 10 1989, a powerful solar flare accompanied by a Coronal Mass Ejection (CME) weighing about a billion tonnes was ejected by the Sun. While X-rays from a solar flare travel at the speed of light, the CME was hurled towards the Earth at a million miles per hour. The radiation from the flare immediately caused short-wave radio interference in Europe while the CME was still on its way. The CME struck on Sunday evening (March 12) causing havoc in the Earth’s magnetic field and creating spectacular auroras seen as far south as Florida and Cuba. The geomagnetic disturbances in this storm induced strong currents in the North American region and found a weak spot in the power grid of Quebec, Canada at 2:44 am on Monday, March 13 following which the city plunged into a 12-hour long blackout crippling its refrigeration, heating, transport and communication systems. Moreover, NASA’s TDRS-1 satellite also registered over 250 anomalies during the storm due to the bombardment of highly energetic particles messing up its onboard circuitry. The cost of this storm was evaluated at 13.2 billion Canadian dollars (Boteler et al. 1998). If such an event were to take place today, the losses incurred would be much more since today’s cities are vastly more dependent on electricity than ever before.

CMEs and solar flares, which are a subset of space weather events, have been linked to the presence of intense magnetic fields as rigorous studies have shown that magnetic reconnection is the prime physical process behind these explosive bursts. Although the entire solar surface is weakly magnetic, Hale (1908) discovered that the strongest magnetic fields (of the order of 1000 Gauss) are found in sunspots. Pulling together long-term sunspot records after the discovery of sunspots by Galileo (1611), Schwabe (1844) reported that sunspots appear and disappear with a periodicity of about 10 years in distinct sunspot cycles with their strength varying from cycle to cycle. We now know that the probability of the occurrence of CMEs and/or solar flares is higher where there is a complex group of sunspots on the solar surface.
This implies that the solar magnetic field is the driver of space weather in our solar system
and in order to understand and possibly predict space weather events, we need to understand
the physical processes responsible for the sunspot cycle and the global evolution of the Sun’s
magnetic field.

Understanding solar activity and developing the capability of making reliable predictions
of solar-related events is at the forefront of research today. Keeping this perspective in mind,
this work is aimed at constraining theoretical models of the solar magnetic dynamo so as to
explain some of the unsolved mysteries of the Sun like the nature and predictability of the
solar magnetic cycle.

1.2 The Solar Cycle

Galileo started recording sunspot numbers in the 17th century, and in the following
years, other astronomers kept his quest alive. It was not until the year 1844, that a German
astronomer Heinrich Schwabe (Schwabe 1844) noticed what he thought was a 10-year peri-
odicity in the sunspot numbers. Today we know from the measurement of periods of over
23 cycles that they have a period close to 11 years. This periodicity is clearly visible in Fig.
1.1, which shows the sunspot numbers as well as sunspot area coverage as a function of time
from May, 1874 to February, 2019.

The nature of sunspots was unknown till 1908, when Hale (1908) observed Zeeman split-
ting in the light coming from sunspots on the Sun and concluded that sunspots are highly
magnetic structures on the solar surface with magnetic fields of the order of 1000 Gauss.
Another major breakthrough came in 1919, when Hale et al. (1919) proposed that the Sun
has a magnetic cycle that comprises of two sunspot cycles. This discovery will be discussed
in detail in section 1.5.

It is well known today, that the Sun has a dipolar magnetic field that flips its orien-
tation every 11 years with a 90° phase lag with the sunspot cycle (Hathaway 2010). A
typical sunspot cycle starts with a strong dipolar magnetic field while there are hardly
Figure 1.1: Sunspot cycle and butterfly diagram. (Image courtesy: Dr. David Hathaway)
Figure 1.2: Radial magnetic field on the solar surface (shown in colorscale) averaged over all longitudes is plotted versus latitude and time. This is called a magnetic butterfly diagram. This plot shows the flipping of polar magnetic field after every sunspot cycle. The equatorward migration of sunspot belt is also evident. (Image courtesy: Dr. David Hathaway)

any sunspots. As time progresses, the number of sunspots increases while the dipolar field strength decreases gradually, until it becomes zero during the sunspot maximum. As the cycle progresses further, the sunspot number starts to decline and a dipolar magnetic field of opposite polarity starts building up and as the sunspot cycle reaches its end, the new dipolar field is fully formed. Thus, one sunspot cycle corresponds to one half of the solar magnetic cycle, with the full cycle having a period of roughly 22 years.

The flipping of the dipolar magnetic field can be seen in Fig. 1.2 which has been constructed from magnetograms (images with value of magnetic field measured in each pixel) of the solar surface (courtesy of Dr. David Hathaway). It is also evident from Figs. 1.1 and 1.2 that sunspots appear at higher latitudes of around 40° in the initial stages of the sunspot cycle, and gradually start appearing at lower latitudes as the cycle progresses. Thus, plotting the locations of sunspots in a latitude versus time graph gives a plot that looks like the wings of a butterfly. Such plots are commonly known as “butterfly diagrams”.

It is believed that the solar magnetic cycle originates in the convectively unstable layers of the Sun termed as the solar convection zone (SCZ) (Parker 1955a; Charbonneau 2010) which stretches from 0.70\(R_\odot\) to its surface (1.00\(R_\odot\)). The energy produced by nuclear fusion in the Sun’s core (within 0.25\(R_\odot\) from its center) travels predominantly in the form of radiation through its interior from 0.25\(R_\odot\) until 0.70\(R_\odot\). This intermediate zone is called the radiation
Figure 1.3: An overview of the different layers of the Sun: the three major interior zones are the core, the radiative zone, and the convective zone. Photosphere represents the visible surface of the Sun on which sunspots and granules are seen. Chromosphere is the lower atmosphere of the Sun and Corona is its outer atmosphere. Flares, prominences and solar wind are typically found in the solar corona (Image courtesy: Kelvin Ma).

zone. Above $0.70R_\odot$, the opacity of the plasma makes convection the most efficient mode of energy transport giving rise to large scale convective motions with a turnover time of approximately two months. These convective motions, along with the rotation of the Sun play a major role in the solar magnetic cycle. An illustration of different layers of the Sun is shown in Fig. 1.3.

The following sections in this chapter cover some of the basic theoretical concepts and ground breaking discoveries that played a crucial role in enhancing our understanding of the inner workings of the solar dynamo.

1.3 Basic Equations

We begin with Maxwell’s equations, and simplify these equations with approximations applicable to the solar plasma:

$$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon} \approx 0$$

(1.1)
\[ \nabla \cdot \vec{B} = 0 \quad (1.2) \]
\[ \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad (1.3) \]
\[ \nabla \times \vec{B} = \mu \vec{J} + \mu \epsilon \frac{\partial \vec{E}}{\partial t} \approx \mu \vec{J}. \quad (1.4) \]

Here, \( \vec{E} \) represents the electric field while \( \vec{B} \) represents the magnetic field of plasma with charge density \( \rho \), permittivity \( \epsilon \), magnetic permeability \( \mu \) and current density \( \vec{J} \). Plasmas are quasi-neutral superheated gases in which positive and negative charges are separate but still live in harmony. Any small scale concentration of positive charges gets instantly shielded by negative charges surrounding it and vice-versa. This phenomenon is called Debye shielding and is responsible for the fact that large scale charge separation cannot be sustained in a plasma over length scales greater than the Debye length. The scale of Debye length in the solar convection zone is of the order of a few millimeters, while we will be dealing with fluid flows over the whole convection zone that has the length scales of the order of the radius of the Sun. It is thus justified to assume that charge density in the solar convection zone remains zero everywhere (Eq. 1.1).

In all the above equations, \( \nabla \) is of the order of \( \frac{1}{L} \), where \( L \) is the typical length scale of the system. Thus, from Faraday’s law we have,

\[ \nabla \times \vec{E} \sim \frac{E}{L} \quad \text{and} \quad \frac{\partial \vec{B}}{\partial t} \sim \frac{B}{T} \Rightarrow E = \frac{BL}{T} \quad (1.5) \]

Similarly, Eq. 1.4 gives:

\[ \nabla \times \vec{B} \sim \frac{B}{L} , \quad \mu \vec{J} \sim \frac{B}{L} , \quad \frac{1}{c^2} \frac{\partial E}{\partial t} \sim \frac{1}{c^2 \frac{B}{T^2}} \sim \frac{v^2 B}{c^2 L} \quad (1.6) \]

\[ \therefore \frac{B}{L} = \frac{B}{L} + \frac{v^2 B}{c^2 L}. \quad (1.7) \]
The plasma velocities \( v \) in the solar convection zone are very low compared to the speed of light. Therefore, we can ignore the second term in Eq. 1.7 which is the displacement current term in Eq. 1.4.

The solar plasma is a highly conducting fluid. Therefore the relationship between the electric field \( \vec{E} \) and current density \( \vec{J} \) can be expressed by modified Ohm’s law as:

\[
\vec{J} = \sigma (\vec{E} + \vec{v} \times \vec{B}) \quad (1.8)
\]

where \( \sigma \) is the conductivity of the plasma. Thus, the curl of \( \vec{E} \) can be written using Ohm’s law as:

\[
\nabla \times \vec{E} = \nabla \times \left( \frac{\vec{J}}{\sigma} - \vec{v} \times \vec{B} \right) \quad (1.9)
\]

Using Eq. 1.3, we have

\[
\frac{\partial \vec{B}}{\partial t} = \nabla \times \left( \vec{v} \times \vec{B} - \frac{\vec{J}}{\sigma} \right) \quad (1.10)
\]

Substituting the value of \( \vec{J} \) from Eq. 1.4, we get

\[
\frac{\partial \vec{B}}{\partial t} = \nabla \times \vec{v} \times \vec{B} - \frac{1}{\mu \sigma} \nabla \times \nabla \times \vec{B} \quad (1.11)
\]

assuming \( \sigma \) is constant in space. Using the vector identity

\[
\nabla \times \nabla \times \vec{B} = \nabla (\nabla \cdot \vec{B}) - \nabla^2 B \quad (1.12)
\]

and Eq. 1.2, we get

\[
\frac{\partial \vec{B}}{\partial t} = \nabla \times \vec{v} \times \vec{B} + \eta \nabla^2 B \quad (1.13)
\]

where \( \eta = \frac{1}{\mu \sigma} \) is called the magnetic diffusivity. Eq. 1.13 is commonly referred to as “the induction equation”. This is the basic equation on which any astrophysical magnetic dynamo system is based.
1.3.1 Magnetic Reynolds Number

The first term on the right hand side of Eq. 1.13 is commonly known as the advection term. This term models two major components: 1) the transport of magnetic field advected by plasma flows 2) the amplification of seed magnetic fields (small magnetic fields that act as seeds for amplification) in the presence of shearing flows. Thus, the advection term also serves as a source term for magnetic field.

The second term on the right hand side of Eq. 1.13 is simply a diffusion term that acts as a dissipative term. In a magnetic system with both source and sink terms, the dominance of one over the other is decided by the magnetic Reynolds number ($R_m$), where $R_m$ is the ratio of the source term to the sink term. An order of magnitude calculation of the magnetic Reynolds number yields:

$$R_m = \frac{V B / L}{\eta B / L^2} = \frac{V L}{\eta} \quad (1.14)$$

where $V$, $B$ and $L$ are the typical velocity, magnetic field and length scales of the system. In astrophysical systems with $R_m >> 1$, diffusivity is very low and the dynamics are dominated by advection which allows for the amplification of magnetic field in the presence of a shear in velocity. Whereas, for $R_m << 1$ diffusion is dominant and does not let the advection term amplify the magnetic fields significantly. For the case $R_m << 1$, we can estimate (order of magnitude) the diffusion time scale as

$$\frac{B}{\tau} = \frac{\eta B}{L^2} \quad \Rightarrow \quad \tau = \frac{L^2}{\eta} \quad (1.15)$$

where $\tau$ is the diffusion time scale. In the convective zone of the Sun, the highly turbulent convection contributes to the diffusion of concentrated magnetic flux tubes by shredding them. This implies that the diffusivity in the convective zone is higher and the diffusion time scale is lower compared to the radiation zone where convection is absent. Therefore, one way to store magnetic fields over the time scale of the solar cycle is to amplify them in
the convection zone which has a significant shear in velocity and store them in the radiative
zone where the diffusion time scale is longer. Whether this is what happens inside the Sun,
however, is not known.

1.3.2 Magnetic Pressure and Plasma $\beta$

For a plasma, the Lorentz force can be written as:

$$\vec{F} = \vec{J} \times \vec{B} = \frac{1}{\mu} (\vec{\nabla} \times \vec{B}) \times \vec{B}$$

(1.16)

and can be split into two components as:

$$\vec{F} = \frac{1}{\mu} (\vec{B}, \vec{\nabla}) \vec{B} - \frac{1}{2\mu} \nabla (B^2).$$

(1.17)

The first component in the above equation is the magnetic tension force that tries to
straighten curved or twisted magnetic field lines. This force is directed against curvature
and is analogous to the restoring elastic force in a rubber band. The magnetic pressure term
denotes the pressure associated with stored magnetic energy in magnetic fields. Magnetic
pressure is quantitatively equal to the magnetic energy density ($\frac{B^2}{2\mu}$) in magnetic fields and
acts as an augmentation to gas pressure. If magnetic fields are stored in a cylindrical volume
of fluid and are directed along its axis, magnetic pressure acts outwards on the walls of the
cylindrical volume, and always tends to push out the magnetic field.

The behavior of plasma in an astrophysical system is decided by a parameter called the
plasma $\beta$, which is the ratio of gas pressure (or plasma pressure) to the magnetic pressure
in the system

$$\beta = \frac{P_g}{P_m}.$$  

(1.18)

When $\beta \gg 1$, gas pressure dominates over the system, and governs the way the magnetic
fields move and stretch. Magnetic fields, in this case, can be considered to be “frozen” in
Figure 1.4: The rotation rate in the solar interior derived from global helioseismology (in nHz) shows differential rotation. These measurements were derived from Global Oscillations Network Group (GONG) data reduced by Antia & Basu (2011).

A high β plasma as they move along with the motion of the plasma. An example of this scenario is the solar interior where the gas pressure dominates over the magnetic pressure and hence magnetic fields move with the plasma. Thus, sunspots which are highly magnetic in nature appear to be moving with the plasma near the solar surface and were first used by Carrington (1859) for measuring the solar rotation rate.

If β << 1, then magnetic pressure is dominant over gas pressure and governs the dynamics of the system. The solar wind is an example of low β plasma, where the magnetic fields govern how the plasma moves.

1.4 Differential Rotation

Richard Carrington (1859) tracked the motion of sunspots on the solar surface and discovered that the velocity of sunspots on the solar surface varies with their latitude. He published his study claiming that the Sun rotates differentially, and that its rotation rate is fastest
near the equator and it gradually decreases with increasing latitude away from the equator. Today, through much more precise helioseismic measurements shown in Fig. 1.4 (Antia & Basu 2011), we know that the Sun’s azimuthal (\( \phi \)) rotation velocity varies with latitude as well as depth inside the Sun.

In the radiative zone, the solar plasma rotates with nearly constant frequency around its rotation axis. Above \( 0.7R_\odot \), the rotation frequency is greater at low latitudes than at high latitudes. This means there is a latitudinal as well as radial shear in the azimuthal rotation rate which can amplify a seed magnetic field present in the plasma through the source (first) term of Eq. 1.13. The total shear in rotation rate is maximum in a thin layer called the tachocline which has been shown to be around \( 0.713R_\odot \) by Christensen-Dalsgaard et al. (1991) and Basu & Antia (1997). This layer is about \( 0.05R_\odot \) thick, and it has been widely speculated that this is the layer where magnetic field is amplified to create sunspots primarily due to the presence of high radial shear in differential rotation.

Apart from high radial shear in the tachocline, the latitudinal shear in azimuthal rotation rate present throughout the bulk of the solar convection zone can also act on the dipolar field (which is in the \( r-\theta \) plane) of the Sun to produce the magnetic field in sunspots. Consider a dipolar solar magnetic field as shown in Fig. 1.5. Due to high plasma \( \beta \) in the solar interior, the magnetic field is coupled with plasma motion and it moves and twists along the plasma. Due to the latitudinal shear in rotation rate, the plasma near the equator completes one rotation around the Sun in about 25 days whereas plasma near the poles takes 33 days to complete one full rotation. This differential rotation will bend the magnetic flux tubes which are oriented initially in the poloidal direction and will start wrapping them around the Sun in the toroidal (azimuthal) direction. The poloidal field thus can be converted into toroidal field by differential rotation and can also be amplified in the process. It should be noted that positive magnetic field at the North pole (as in Fig. 1.5.) produces negative (clockwise) toroidal field in the northern hemisphere and the converse is true for the southern
hemi-sphere. This process of solar magnetic field amplification due to shear in differential rotation was first proposed by Parker (1955b) and is now known as Parker’s Ω-effect.

1.5 Hale’s Polarity Law

In 1919, George Ellery Hale at Mt. Wilson Observatory discovered that most sunspots appear in pairs having opposite magnetic polarities (Hale et al. 1919). He observed that sunspots of a particular polarity lead in the direction of rotation, while the other polarity sunspots follow. This can be seen in Fig. 1.6, that negative polarity (black) sunspots are leading in the Northern hemisphere while the positive polarity (white) sunspots are leading in the Southern hemisphere. This pattern of leading and following polarities in both hemispheres switches in about 11 years, i.e., after every sunspot cycle. George Hale, therefore concluded that the complete solar magnetic cycle has a period of 22 years considering both the polarities of magnetic field.
Figure 1.6: A typical magnetogram image of the Sun taken during one of its active phases (year 2012). White color represents positive magnetic field (directed towards the camera) and black color represents negative magnetic field (directed away from the camera). Black polarity is leading in the northern hemisphere while white polarity is leading in the southern hemisphere as the solar surface rotates from left to right. The leading and following polarities switch places every 11 years in a rhythmic fashion (Image Credit: Helioseismic & Magnetic Imager (HMI) and www.helioviewer.org).
1.6 Joy’s Law

A pair of sunspots with opposite polarity flux is called a Bipolar Magnetic Region (BMR). Alfred Harrison Joy found that the axis of BMRs, or the line joining the opposite polarity sunspots in a BMR, show systematic tilts with respect to the East-West direction (Hale et al. 1919). In both hemispheres, the leading polarity sunspot is at a slightly lower latitude than the following polarity sunspot. Moreover, he discerned that the tilt of BMRs is positively correlated to their latitude of emergence. Early in the sunspot cycle, BMRs appear at higher latitudes, with higher tilts (Fig. 1.2) and as cycle progresses, they start appearing at lower latitudes with lower and lower tilts. This relationship between the tilt of BMRs and their latitude of emergence is called Joy’s law in his honor.

1.7 Parker’s Hypothesis of the Origin of Sunspots

Now, consider a cylindrical volume of plasma in the solar convection zone in which there exists some magnetic field aligned in the same direction while the magnetic field outside this cylinder is negligible. We call this cylindrical volume a magnetic flux tube. The magnetic field inside the flux tube exerts some magnetic pressure on the walls of the tube. Let us assume that the pressure of plasma outside the flux tube is $P_e$ and the plasma pressure inside the flux tube is $P_i$. For the flux tube to be in hydrodynamic equilibrium, total internal pressure must be equal to total external pressure. Thus:

$$P_e = P_i + \frac{B^2}{2\mu}. \quad (1.19)$$

The pressure of a gas (or plasma) is related to its density by the equation:

$$P = \frac{\rho RT}{m} \quad (1.20)$$
where, $R$ is the gas constant, $T$ is the temperature of the plasma and $m$ is its molecular mass. Thus, rewriting Eq. 1.19 in terms of density yields:

$$\rho_e = \rho_i + \frac{B^2 m}{2\mu RT} \tag{1.21}$$

which means that the plasma density inside the flux tube is less than the plasma density outside (since $\frac{B^2}{2\mu RT} > 0$). Thus, in the presence of a gravitational field, the flux tube will experience a force of buoyancy due to Archimedes’ principle. This force is represented by:

$$F_B = \Delta \rho V g \ \hat{r}. \tag{1.22}$$

Strong magnetic flux tubes deep inside the solar convection zone can thus become buoyant and tend to rise up to the solar surface (Parker 1955a).

Hale’s polarity law (section 1.5), Joy’s law (section 1.6) and magnetic buoyancy paved the way to an ingenious suggestion by Parker (1955b) that toroidal fields produced deep inside the convection zone (due to the action of differential rotation) could become magnetically buoyant and rise to the surface to produce a pair of sunspots which exhibit magnetic fields of the opposite polarity (Fig. 1.7) as one of them marks the location where magnetic field is pointed away from the surface and the other marks the location where magnetic field is pointed into the solar surface. This has been the most widely used theory for explaining the formation of sunspots and it marked the birth of the solar dynamo theory. Much later, D’Silva & Choudhuri (1993) further solidified Parker’s ideas by showing that magnetic flux tubes rising through the solar convection zone subject to Coriolis force can explain the systematic Joy’s law tilts.

According to Parker (1955b), the poloidal field could be converted to toroidal field by differential rotation and amplified in the process. Once enough toroidal field has been produced in a flux tube, the tube can rise up to the surface through magnetic buoyancy and present itself in the form of sunspots. There was only one major piece of the solar dynamo
theory that was missing after Parker’s exploits in 1955. It was the failure of understanding how pairs of sunspots, which contain zero net flux, can reverse the Sun’s dipolar field.

1.8 Meridional Circulation and the Babcock Leighton $\alpha$-effect

Meridional circulation, a flow pattern in the radius-latitude plane, inevitably occurs in systems that have turbulent convection in a rotating shell. Meridional circulation in the solar convection zone is a flow that is directed towards the poles on the surface, and therefore must return towards the equator in the deeper layers (see Fig. 1.8). Meridional flow on the solar surface has been observed (Komm et al. 1993; Hathaway 1996; Basu & Antia 2000), but the depth profile of the equatorward return meridional flow in the deeper layers has not been confirmed beyond doubt (Jackiewicz et al. 2015).

Meridional flow on the surface is crucial for creating the Sun’s dipolar field from the radial field in sunspots. Let us say we have a couple of pairs of sunspots that appear in the northern solar hemisphere as shown in Fig. 1.9, and similar pairs but with inverse magnetic
Figure 1.8: The direction of meridional flow in the $r-\theta$ plane. The flow has been observed towards the poles on the surface in both hemispheres, and it is believed that there exists a return flow towards the equator deep inside the convection zone.

Figure 1.9: An illustration of the Babcock Leighton alpha effect. Some of the magnetic flux from leading polarities in both hemispheres cancels across the equator due to diffusion overcoming meridional flow, while the flux from following polarities is transported towards the poles by meridional flow.
configuration in the southern hemisphere. The nature of meridional flow is such that its flow speed is zero near the equator, and the poleward flow speed rises gradually as we move to higher latitudes, peaks at mid-latitudes around 35°-40° latitude and then declines again to near zero at latitudes higher than 75°.

Most sunspots in a cycle appear below 40° latitude, and they appear with a systematic tilt. As a result, the meridional flow transports the following polarity flux (which is at higher latitude) towards the poles faster than it transports the flux in the leading polarity. This creates some separation in the fluxes of the two polarities in a pair of sunspots. Some of the flux in sunspots still cancels within the BMR itself due to diffusion, but some of it is separated. Some of the leading polarity’s flux cancels with the flux from the leading polarity in the other hemisphere due to diffusion across the equator which overcomes the weak meridional flow in that region. Since flux is conserved, and the total flux in all sunspot eruptions is zero (as they always appear in pairs), the amount of flux cancelled between leading polarities across the equator is equal to the amount of opposite polarity flux that is transported towards the poles by meridional flow. This mechanism of creation of polar field from systematically tilted sunspots was proposed by Babcock (1961) and Leighton (1969) and is known as the Babcock-Leighton α-effect.

With this, all major parts of solar dynamo theory are in place. Outstanding questions, however, still remain: 1) where inside the Sun does Parker’s Ω-effect operate? Is it in the tachocline as suggested by Nandy & Choudhuri (2002) or much closer to surface as suggested by Brandenburg (2005)? 2) What is the nature of variations in meridional flow over time-scales of the solar cycle and how does this affect the solar cycle? 3) Are there precursors to solar flares and CMEs? Can we predict such events in advance?

We have tackled the first of these questions in chapter 2, the second question in chapter 3, the third question in chapter 4 with concluding remarks in chapter 5.
Chapter 2

Lorentz Force Variations Reveal Origin of Sunspots in the Deep Solar Interior

2.1 Background

In 1611, soon after the invention of the telescope, Galileo himself started observing sunspots unaware of their magnetic nature or the physical process(es) which bring them into existence. More than three centuries later, in 1955, Parker (Parker 1955b) proposed a mechanism for induction of the toroidal ($\phi$) component of the magnetic field from the poloidal ($r - \theta$) component due to stretching by solar differential rotation – a mechanism which is termed as the $\Omega$-effect. While Parker also postulated that toroidal fields rise up due to magnetic buoyancy and erupt through the surface forming bipolar sunspots (Parker 1955a) the exact location where the sunspot-forming toroidal fields form remained observationally unconstrained and highly debated (Nandy & Choudhuri 2002; Brandenburg 2005; Charbonneau 2010).

Solar and stellar magnetic cycles which are manifested in magnetic spots are believed to originate via a magnetohydrodynamic (MHD) dynamo mechanism that recycles the toroidal and poloidal components of the magnetic field relying on plasma flows in stellar interiors (for a review see Charbonneau (2010)). The magnetic field and plasma flow field in such a non-linear MHD system is governed by the equations

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{v} \times \vec{B} - \eta \nabla \times \vec{B}).$$

(2.1)

$$\rho \left( \frac{\partial \vec{v}}{\partial t} - \vec{v} \times (\nabla \times \vec{v}) \right) = -\nabla \left( P + U_g \right) - \rho \frac{\nu^2}{2} \nabla \times (\vec{J} \times \vec{B}) + \Psi$$

(2.2)
Eq. 2.1 is the magnetic induction equation which models the evolution of magnetic field \( \vec{B} \) in a velocity field \( \vec{v} \) and a turbulent magnetic diffusivity \( \eta_t \) while Eq. 2.2 ensures the conservation of momentum in a plasma with density \( \rho \), gravitational potential \( U_g \), pressure \( P \) and current density \( \vec{J} \). In Eq. 2.2 the term \( \Psi \) represents a combination of viscous, convective and any external forces. In Eq. 2.1 the first term on the right hand side can amplify a seed poloidal field \( B_p \) and create a toroidal field in the presence of shear in the rotation rate \( \Omega \). This change in magnetic field in turn modifies the velocity field through the Lorentz force (the \( \vec{J} \times \vec{B} \) term in Eq. 2.2). We can convert both equations to units of work done (energy) per unit time by taking the dot product of the first equation with \( \vec{B}/\mu \) and of the second equation with \( \vec{v} \) (where \( \mu \) is the magnetic permeability of solar plasma) and integrating them over a closed volume \( V \) with the prescription given in Chapter IV of Chandrasekhar (1961) (see complete proof in Appendix A). By doing so we establish the time evolution equations for the total magnetic and plasma kinetic energies, given by

\[
\frac{\partial E_m}{\partial t} = - \int_V \vec{v} \cdot (\vec{J} \times \vec{B}) dV - \int_V \frac{J^2}{\sigma} dV - \frac{1}{\mu} \int_S (\vec{E} \times \vec{B}) \cdot d\vec{S} \quad (2.3)
\]

\[
\frac{\partial (K.E)}{\partial t} \approx \int_V \left( \rho \frac{\partial v^2}{2 \partial t} \right) dV = - \int_V \vec{v} \cdot \nabla \left( P + U_g \right) dV - \int_V \vec{v} \cdot \left( \frac{\rho}{2} \nabla v^2 \right) dV + \int_V \vec{v} \cdot (\vec{J} \times \vec{B}) dV + \int_V \vec{v} \cdot \Psi dV \quad (2.4)
\]

where \( E_m \) is the total magnetic energy in the closed volume \( \int_V \frac{B^2}{2\mu} dV \), \( \sigma \) is the conductivity of the plasma, \( \vec{E} \) is the electric field, \( d\vec{S} \) is a surface element of the volume \( V \) and \( K.E. \) is the total kinetic energy in the volume. We note that Eq. 2.3 is Poynting’s theorem for plasmas and the first term on its right hand side (RHS) also appears with an opposite sign in Eq. 2.4. This term is the net work done per unit time by the Lorentz force and represents a transfer of energy between the two equations. In Eq. 2.3, the second term on the RHS represents the loss of magnetic energy due to Joule heating whereas the third term represents the Poynting flux of electromagnetic energy entering or leaving the volume through
its bounding surfaces. In a volume which envelopes the entire convection zone of the Sun 
\((0.70R_\odot - 1.00R_\odot)\), we can assume that there is no Poynting flux leaving or entering from 
its bottom boundary if the magnetic field in the radiative zone is negligible. We also know 
that the magnetic energy reaching the Sun’s surface from the interior drives activity in its 
atmosphere. This implies that the Poynting flux term represents magnetic energy escaping 
our volume through the surface. Since both Joule heating and Poynting flux terms can only 
act as a sink for magnetic energy, the work done by Lorentz force has to be the source 
of magnetic energy in the solar convection zone (SCZ). The Lorentz force, therefore, must 
extract energy from terms contributing to the evolution of plasma kinetic energy in Eq. 2.4 
and convert it into magnetic energy.

These equations also show that when Lorentz force \((\vec{J} \times \vec{B})\) operates in the direction 
of the velocity field it extracts magnetic energy and transforms it into some of the terms 
in Eq. 2.4 whereas when the Lorentz force operates in a direction opposite to the velocity 
field, it extracts kinetic and potential energy from terms in Eq. 2.4 and transforms it into 
magnetic energy. It is, therefore, the work done by Lorentz forces which drive solar-stellar 
magnetic cycles. This is a general conclusion and is valid both in the presence and absence 
of turbulence.

Our analysis makes it evident that in order to locate where magnetic energy is produced 
within the Sun, one needs to locate the spatial domain where Lorentz forces operate. To 
achieve this, we need to make certain reasonable assumptions. We make the mean field 
approximation (implying velocity and magnetic fields are averages over solar convective turn 
over time scales) and assume that the Sun is an axisymmetric MHD system.

The Lorentz force can be decomposed into two components

\[
\vec{J} \times \vec{B} = -\nabla \left( \frac{B^2}{2\mu} \right) + \left( \vec{B} \cdot \nabla \right) \vec{B} \mu
\]  

(2.5)
where the first term represents the gradient of magnetic pressure and the second term represents magnetic tension force which resists the bending and twisting of magnetic field lines.

Let us analyze only the azimuthal component of Lorentz force. This component comprises completely of the magnetic tension force as the gradient of magnetic pressure vanishes if we assume axisymmetry ($\frac{\partial}{\partial \phi} = 0$). Thus, to locate where the $\Omega$-effect operates to produce the toroidal component of the magnetic field, we need to find signatures of magnetic tension forces in the azimuthal direction.

The magnetic tension force can be visualized as a force which acts to reduce any curvature and twist in the magnetic field lines. Let us say a solar magnetic cycle begins with a poloidal magnetic field which is dipolar in nature and pointed from south to north inside the solar interior (i.e., clockwise). The Sun’s equator rotates faster than its poles. If the $\Omega$-effect indeed operates in the solar interior, this field line will be bent by the action of differential rotation in the convection zone as shown in Fig. 2.1 and it will exert magnetic tension in the direction of the red arrows: against rotation at low latitudes near the equator and along rotation at high latitudes. If we analyze the azimuthal component of Eq. 2.2, the $\nabla (P + U_g)$ and $\frac{\rho}{2} \nabla v^2$ terms vanish due to the assumption of axis-symmetry. Guerrero et al. (2016) analyzed the contribution of all remaining terms from Eq. 2.2 in 3D full MHD simulations and found that the magnetic tension force was the dominant term which modifies differential rotation and creates torsional oscillation in the solar interior. Taken together, these considerations suggest that the manifestation of this force could thus be found in measurements of differential rotation conducted over the course of a solar cycle.

Based on our analysis we conclude that wherever the $\Omega$-effect is operating inside the Sun, magnetic tension forces would modify the differential rotation in the following fashion: 1) the rotation rate at low latitudes should decelerate when sunspot producing magnetic field is being amplified in the solar interior (during the rising phase of sunspot cycle) as magnetic tension will oppose rotation there; 2) conversely, the rotation rate at high latitudes should accelerate during the rising phase of the solar cycle; 3) the division between decelerating
Figure 2.1: The solid circles represent $0.70R_\odot$ and $1.00R_\odot$ (the bounding surfaces of our volume integrals) whereas the dashed circle represents the location of the tachocline. The bending of a poloidal field by differential rotation denoted by blue arrows is demonstrated here. Such a field line would exert magnetic tension force in a manner shown by the red arrows.

region at low latitudes and accelerating region in rotation rate at high latitudes should coincide with locations of sunspots on the solar surface and 4) the rate of acceleration (deceleration) in rotation rate in the solar interior should be proportional to the magnetic field there and hence the strength of the sunspot cycle. Interestingly, we note that postulates 1) and 2) above may be considered as an equivalent of Lenz’s law for solar-stellar MHD dynamo mechanism: An induced magnetic field opposes the process of its induction i.e. the shear in plasma differential rotation.

Below we provide observational evidence supporting our theoretical conclusions and demonstrate that time varying Lorentz forces and the Lenz’s law equivalent for dynamo action combine together to reveal where the sunspot forming toroidal fields originate in the Sun’s interior.
2.2 Analysis Of Helioseismic Inversions Of Differential Rotation

The advent of Helioseismology over the past four decades has made it possible to infer the differential rotation of the solar interior by measuring the travel times of acoustic waves observed on its surface through Doppler images. Such observations are regularly conducted by the ground based Global Oscillations Network Group (GONG, operational from 1995 to present) (Harvey et al. 1996) and space based Michelson Doppler Imager (MDI, operational from 1995 to 2011) (Scherrer et al. 1995) as well as Helioseismic Magnetic Imager (HMI, operational from 2010 to present) (Schou et al. 2012). We obtain one set of measurements of differential rotation from GONG data reduced by Antia & Basu (2011) for the time period 7 May 1995 to 21 June 2017 and another set of differential rotation measurements by combining data from MDI (1995-2011) and HMI (2010-2018) available from the Joint Science Operations Center (www.jsoc.stanford.edu) which was reduced by Larson & Schou (2015).

The first data set from GONG data consists of differential rotation derived from two dimensional regularized least squares (2dRLS) inversion of GONG data provided by Antia & Basu (2011) over the time period June 11, 1995 to July 9, 2017. These consist of rotation rates in $nHz$ measured between $0.50R_\odot$ and $1.00R_\odot$ in intervals of $0.02R_\odot$. These rotation inversions assume axis-symmetry and hemispheric symmetry across the equator so that information from the North and South hemispheres of the Sun is convolved into one and evaluated at a resolution of two degrees in latitude. The 2dRLS results consist of 224 sets of meridional ($r-\theta$) profiles of rotation rate with a cadence of 36 days where each set is constructed from a moving Gaussian temporal average with a full width at half maximum of 108 days. The rotation rates obtained from this data are shown in Fig. 2.2 at certain locations in the solar convection zone.

The second set of 2dRLS inversion of rotation rates was obtained from the Joint Science Operations Center (www.jsoc.stanford.edu)(Larson & Schou 2015). This set of rotation
Figure 2.2: Time series of the azimuthal rotation velocity at three different depths and nine different latitudes from GONG data are shown here with $1\sigma$ error bars. The latitude is indicated in the left column whereas the depth is indicated in the top row. The bottom panel in every column shows the smoothed monthly sunspot number (SSN) (SILSO World Data Center 1995-2017) for comparison. The dashed magenta lines mark the timings of solar minimum. The black dashed lines show that the basal rotation rate at high latitudes, mean rotation rate at latitudes $25^\circ - 35^\circ$ and the ceiling rotation rate at low latitudes is consistent across sunspot cycles.
Figure 2.3: Same as Fig. 2.2 obtained from MDI-HMI data.
Inversions is derived from 72 day non-overlapping chunks of measurements all over the solar convection zone. The 2dRLS inversions of rotation rate from MDI (May 1, 1996 to February 12, 2011) were extracted from the data series `mdi.vw_V_sht_2drls_asym` and for HMI (April 30, 2010 to March 14, 2019) were extracted from data series `hmi.V_sht_2drls_asym`. These measurements are available on a non-uniform radial grid which has a variable resolution with depth (resolution is $0.000085R_\odot$ at surface and $0.0101R_\odot$ at $0.70R_\odot$) and a fixed latitudinal resolution of $1.875^\circ$ making the resolution higher than GONG data everywhere in the convection zone. This data is available in three different versions which were derived by using different trade-off values for regularization (6, 18 & 36). We analyzed all three versions and find that the trends we discuss in this chapter are consistent in all three versions. In order to compare this data set with GONG data, we interpolated these measurements using a biharmonic spline interpolation algorithm on the radius-latitude grid of the GONG data. Measurements from the two instruments (MDI & HMI) were integrated using their overlap period for cross-calibration.

These sets of differential rotation measurements can be visualized as three dimensional data of rotational velocity in radius ($r$), colatitude ($\theta$) and time ($t$) as they also assume axis-symmetry ($\partial/\partial\phi = 0$). Modulation in differential rotation of the Sun with a period of around 11 years was first noted by Howard & Labonte (1980) and it is now known as torsional oscillation. This modulation is evident in the timeseries of rotation rate from GONG data in Fig. 2.2 and from MDI-HMI data in Fig. 2.3 at several locations in the solar convection zone.

In order to analyze long-term acceleration and deceleration trends in these data sets over time scales of the solar cycle, we first smooth all the data sets along the time dimension with local regression using weighted linear least squares and a second degree polynomial model over a moving window of two years to mitigate annual variations. We then fit polynomials of degree six to the smoothed data in the time dimension to avoid small scale fluctuations and reveal the long-term trends in data over time scales of the solar cycle. Fitting polynomials
instead of sinusoidal functions with fixed amplitude allows accounting of solar cycle variations
in the amplitude of torsional oscillation (our data spans more than one solar cycle). We note
that our results are robust and do not vary with the variation of smoothing or polynomial
fitting procedures.

The fitted torsional oscillation data is converted into measurements of net force per unit
volume in the azimuthal direction \( (\rho \frac{\partial v_\phi}{\partial t}) \) using the density profile from a standard solar model
(Bahcall et al. 2001). A positive azimuthal force corresponds to acceleration in rotation rate
while a negative force corresponds to deceleration in rotation rate at any location. This force
is plotted in Fig. 2.4 from GONG data and Fig. 2.5 from MDI & HMI data. Shaded regions
in these figures show areas where the signal to noise ratio is too low to have confidence in
the acceleration or deceleration trends.

Our first and second theoretical conjecture are clearly confirmed in the observations
spanning 0.76\( R_\odot \) to 0.94\( R_\odot \) as we see acceleration above the latitudinal belt where sunspots
appear (green dots) and deceleration below this belt during the rising phase of sunspot cycle
23. The trends in cycle 24 are too weak to be significant in the range 0.76\( R_\odot \) and 0.80\( R_\odot \).
During the rising phase of cycle 23, the overall latitudinal shear in differential rotation
at these depths decrease. Post sunspot maximum of cycle 23, these trends reverse as the
high latitudes decelerate while the low latitudes accelerate to restore the latitudinal shear
in differential rotation. Our third expectation that the locations of sunspots on the solar
surface (denoted by green dots) should lie at the interface between high latitude acceleration
and low latitude deceleration regions is also seen in these figures; in particular, we note that
the interface propagates towards the equator along with sunspot eruption latitudes in the
range 0.76\( R_\odot \) to 0.84\( R_\odot \) during solar cycle 23.

We observe that the amplitude of this force also increases with increasing depth inside the
Sun likely due to the increase in plasma density which implies that the forces below 0.84\( R_\odot \)
dominate over the forces in the upper half of the convection zone. Above 0.84\( R_\odot \), sunspot
locations deviate from our theoretical conjecture, and appear in regions of deceleration in
Figure 2.4: *Background:* The net force per unit volume $\rho \frac{\partial v}{\partial t}$ in the azimuthal direction inside the Sun is shown at several depths from polynomial fits to GONG data. *Overplotted:* Locations of sunspots on the solar surface from the Royal Greenwich Observatory (RGO) database are shown by green dots. The magenta lines mark the timings of solar minimum and maximum and the transparent gray regions cover areas with poor signal to noise ratio and high uncertainties.
Figure 2.5: Same as Fig. 2.4 showing the net azimuthal force in MDI-HMI data.
the rotation rate. This effect is independently postulated to be due to the thermal shadow effect (Spruit 2003) – a mechanism localized near the solar surface above $0.90R_\odot$ (Rempel 2006). We focus on the deeper layers between $0.76R_\odot$ and $0.84R_\odot$ for the remainder of this chapter, where we find signatures of the time-varying Lorentz forces.

The time series of the raw data (prior to smoothing and polynomial fitting) shown in Figs. 2.2 & 2.3 presents evidence in support of our fourth expectation: the acceleration (deceleration) in rotation rate at high (low) latitudes during the rising phase of a sunspot cycle is proportional to the amplitude of the corresponding cycle. Moreover, we find that the average rotation rate is consistent across the two sunspot cycles only between $25^\circ$ and $35^\circ$ latitudes. At other latitudes, the basal (lowest) rotation rate at high latitudes and the ceiling (highest) rotation rate at low latitudes are consistent across the two sunspot cycles. This implies that the temporal average of the differential rotation profile of the Sun is not its baseline rotation and one has to carefully construct the baseline differential rotation profile by measuring the basal rotation rate at high latitudes, ceiling rotation rate at low latitudes and the mean rotation rate at mid latitudes.

This has implications for solar dynamo models as the temporally averaged differential rotation profile is widely used in these models as the baseline profile. Our analysis suggests that the baseline rotation profile is the profile with maximum latitudinal shear whereas assuming the temporally averaged rotation rate as the baseline rotation rate underestimates the latitudinal shear in differential rotation.

Figs. 2.6 & 2.7 show the azimuthal force in the meridional plane ($r - \theta$) as a function of time and the locations of sunspots on the surface (green circles). This figure shows distinct blobs of acceleration-deceleration pairs propagating towards the equator synchronized with sunspot locations on the surface. Along with that, a widespread acceleration at high latitudes and deceleration at low latitudes can be seen during the rising phases (1997-2001 and 2010-2014) with a key difference: this pattern is not very pronounced (in terms of area coverage
and strength) during solar cycle 24. This is in keeping with the fact that cycle 24 turned out to be a weak cycle.

This confirms that the overall latitudinal shear in rotation decreases in time during the rising phase of sunspot cycles and suggests that the decrease is proportional to the strength of the corresponding cycle. The panels corresponding to 2007.12 years in Fig. 2.6 and 2007.07 years in Fig. 2.7 show the earliest signs of solar cycle 24 at a depth of $0.82R_{\odot}$, between $30^\circ - 60^\circ$ latitude for GONG and $25^\circ - 60^\circ$ for MDI-HMI data (marked by arrows). Even though the first sunspots of cycle 24 appear in 2008 we cannot be sure that these signatures precede the appearance of sunspots because we used a biannual smoothing before fitting polynomials to the data. It is clear, however, that the acceleration at high-latitude and deceleration at low latitude pattern is well formed in the next panel in the respective figures and coincides with the appearance of the first sunspots of cycle 24.

2.3 Conclusion

Through our analysis of the magnetic induction equation and the equation for conservation of momentum in MHD systems we first show that the work done by Lorentz forces is the source of magnetic energy which drives solar-stellar magnetic cycles. In an axis-symmetric mean field scenario, the azimuthal component of Lorentz force is due to magnetic tension exerted by the poloidal field lines being wrapped around the Sun by differential rotation (Parker’s $\Omega$-effect). Based on our theoretical analysis we predict four diverse signatures of the Lorentz force and Lenz’s law associated with the magnetic field induction process.

Our analysis of helioseismic observations of solar torsional oscillation profile in the Sun’s interior confirms our theoretical conjectures. The observational data constrains the location of the expected Lorentz force variations (relative to the appearance of sunspots) to within $0.76R_{\odot}$ and $0.84R_{\odot}$ revealing the depths at which the sunspot producing magnetic fields originate. The range may further extend below $0.76R_{\odot}$ but the signal to noise ratio below $0.76R_{\odot}$ is too low to draw any significant conclusions. Nonetheless, we firmly establish that
Figure 2.6: The net force per unit volume $\rho \frac{\partial v}{\partial t}$ in the azimuthal direction inside the Sun shown at several time stamps from polynomial fits to GONG data. Locations of sunspots on the solar surface from the Royal Greenwich Observatory (RGO) database are shown by green circles on the surface. The arrows in panels with time 2007.12 and 2008.30 years mark the location at 0.82$R_\odot$ where the signs of solar cycle 24 first appear in the data.

the toroidal fields that produce sunspots originate predominantly in the lower half of the solar convection zone and not in the near-surface layers. This leads to the conclusion that the dynamo $\Omega$-effect in solar-like stars – that is at the heart of solar-stellar magnetic cycles – operates deep within stellar interior.

We note that the rotation rate at high latitudes between 0.76$R_\odot$ and 0.84$R_\odot$ returns back to its basal rotation rate at cycle minima, while the rotation rate at low latitudes returns back to its maximum at minima. This implies that during the declining phase of a sunspot cycle leading up to the minima, the latitudinal shear that is depleted during the rising phase is replenished back to its original level. The processes responsible for this restoration are possibly reliant on the flux of thermal energy that sustains solar-stellar convection.
Figure 2.7: Same as Fig. 2.6 showing the trends in MDI-HMI data. The arrows in panels with time 2007.07 and 2008.26 years mark the location at 0.82\(R_\odot\) where the signs of solar cycle 24 first appear.
Chapter 3

Improved Measurements of Long-term Variations in Solar Surface Flows

3.1 Background

The Sun’s axisymmetric flows, differential rotation (DR) and meridional flow (MF) play key roles in virtually all models of the Sun’s magnetic dynamo (Charbonneau 2010). The differential rotation (variation in rotation rate with both latitude and depth) stretches the radial and latitudinal components of the magnetic field in the longitudinal direction - thereby increasing the field strength and changing the field direction. The meridional flow transports the radial and longitudinal components of the magnetic field in the latitudinal direction - thereby building up the polar fields and annihilating oppositely directed fields on either side of the equator. Thus, measurements of the meridional flow are important for the prediction of the build up of polar fields which in turn are good predictors for the next solar cycle (Muñoz-Jaramillo et al. 2013).

Measurements of these axisymmetric flows have been made with a wide variety of methods. Carrington (1859) measured the positions of sunspots to determine the latitudinal variation in rotation rate through the limited range of sunspot latitudes. Since those earliest measurements using the only known tracer at the time (sunspots), many more measurements have been made using a variety of tracers and/or methods. Tracers include: sunspots, magnetic features (network magnetic elements), Doppler features (supergranules), and intensity features (granules). Measurement methods (beside feature tracking) include direct Doppler (spectrographic) and helioseismic measurements. Beck (2000) has compared and contrasted
these various methods. Each has its own set of advantages and disadvantages. This is particularly true for measurements of the meridional flow, primarily due to the intrinsic low amplitude of this global flow.

Sunspot tracking is limited in both time and space. Sunspots only appear in the lower latitudes and at times, don’t appear at all. Most other methods have much better coverage in space (latitude) and time but present other problems.

Direct Doppler measurements of the meridional flow (Hathaway et al. 1996; Ulrich 2010) require an accurate determination of the convective blue shift signal and its dependence on local magnetic fields. This signal is stronger than the meridional flow signal and is similar in spatial characteristics - making the meridional flow measurement via its direct Doppler signal very difficult.

Feature tracking the Doppler features (supergranules) has an advantage in having a strong signal near the limb - and hence high latitudes, but suffers from systematic changes in the features due to line-of-sight effects. Hathaway et al. (2006) showed that these projection effects make the supergranule pattern appear to rotate faster than it actually does. The impact of these line-of-sight effects on meridional flow measurements has not been adequately investigated.

Helioseismology also has difficulty near the limb, but more importantly, exhibits a systematic center-to-limb shift in the apparent acoustic field. This effect comprises of a pseudo flow away from the disk center which increases as a function of center to limb distance and interferes with measurements of the meridional flow. Zhao et al. (2012) accounted for it by measuring the rotation rate as a function of longitude from the central meridian at the equator and subtracting it from meridional flow measurements assuming that the effect is symmetric on the solar disk.

Feature tracking the photometric intensity features (granules) requires high spatial and temporal resolution which have only been available recently. These features are difficult to track near the limb and their short lifetimes make accurate measurements of slow flows, such
as the meridional flow, quite difficult. This kind of tracking shows a “shrinking” Sun effect (an apparent flow toward disk center) with an amplitude of $\sim 1000 \, m \, s^{-1}$ at a heliocentric angle of 60°. Löptien et al. (2016) investigated this effect and showed that in large parts it originates from the apparent asymmetry of granulation due to radiative transfer effects when observing at a viewing angle. However, their model of the “shrinking Sun” effect was not accurate enough to measure the much weaker meridional flow. They also pointed out that the shrinking Sun effect was not exactly symmetric across the central meridian.

Feature tracking the weak (network) magnetic features has the advantages of good coverage over the visible disk (but with the same difficulty near the limb as with granules and acoustic waves) coupled with long lifetimes that are advantageous for meridional flow measurements. Komm et al. (1993) used this method on magnetograms separated by a day in time. By averaging their daily measurement over two year intervals they acquired enough accuracy to show a systematic variation in the meridional flow with solar cycle phase. Hathaway & Rightmire (2010) used a variant of their method on magnetograms obtained with SOHO/MDI (Scherrer et al. 1995) at eight-hour intervals. The sheer number of observations allowed for accurate measurements over individual 27-day solar rotations. Dikpati et al. (2010) suggested that these measurements were compromised by a systematic effect due to supergranule diffusion. They argued that this diffusion would produce an apparent outflow from magnetic flux concentrations in active regions and tested their argument using a purely diffusive model with a diffuse magnetic field. Hathaway & Rightmire (2011) investigated this possibility with a far more realistic model - magnetic elements transported by evolving supergranule flows - and found that this systematic effect was too small to be measured. They further noted that the nature of the meridional flow variations with solar cycle phase produced inflows toward active latitudes, not outflows.

Until now, a systematic center-to-limb effect was not reported in tracking network magnetic elements making it the most promising method for measuring the meridional flow. We, in this chapter describe the discovery of a center-to-limb effect similar to the systematic flow
away from disk center seen with helioseismic methods which compromises feature tracking measurements from magnetograms reported by Lamb (2017) and Imada & Fujiyama (2018) and to some extent Hathaway & Rightmire (2010).

Here we describe our measurements of the meridional flow using a feature tracking method on the weak magnetic features. Our original intent was to incorporate the daily magnetograms from NSO/Kitt Peak to extend the meridional flow measurements to cover four solar cycles. We discovered early on that several improvements to the existing methods were needed.

Testing our tracking algorithm with synthetic magnetograms revealed the need to improve the interpolation algorithm used for projection of magnetograms onto a Heliographic (uniformly spaced latitude-longitude) grid. Histograms of the meridional flow velocity measurements indicated: 1) the need to broaden the search for the correlation maximum to larger distances at high latitudes so as not to truncate the normal distribution (and thereby influence the value of the average), 2) the need to obtain multiple measurements in longitude with larger correlation windows at higher latitude so as to cover similar areas on the surface of the Sun, and 3) the need to more accurately determine the position of the correlation maximum with fractional pixel accuracy.

Furthermore, from making measurements at different time-lags (from a day down to an hour) we discovered a systematic shift in the pattern away from disk center - nearly identical in form to the effect found for the acoustic oscillations (Zhao et al. 2012). This shift is in the form of an offset away from disk center that manifests itself over the course of about an hour. When this offset is divided by the time difference to get a velocity it results in large velocities at short time lags and proportionally smaller velocities as the time-lag increases. This sort of behavior was seen previously by Lamb (2017) (in the form of faster meridional flow with a near linear dependence on latitude for short-lived features) and by Imada & Fujiyama (2018) (in the form of a faster meridional flow speed for weaker magnetic elements).
In the following sections we describe the data, our measurement method, and the improvements we made. We characterize the systematic signal and describe our method for accounting for, and removing it from our measurements without relying on the assumption of symmetry. We end with a synopsis of our improved measurement of the meridional flow and torsional oscillation profile as a function of time through the last two solar cycles.

3.2 Data

3.2.1 MDI/HMI Magnetograms

We obtained full disk (1024×1024 pixel) line-of-sight magnetograms from the Michelson Doppler Imager (MDI) (Scherrer et al. 1995) every 96 minutes and full disk (4096×4096 pixel) line-of-sight magnetograms from the Helioseismic and Magnetic Imager (HMI) (Scherrer et al. 2012) every hour. The MDI (HMI) magnetograms are obtained at a 60-second (45-second) cadence and averaged over 24 minutes (12 minutes) to reduce the 5-minute oscillation signal.

These magnetograms are created from polarization measurements along the profiles of magnetically sensitive photospheric absorption lines (the Ni I 676.8 nm line for MDI and the Fe I 617.3 nm line for HMI). We use the line-of-sight magnetic measurements from both instruments and project them onto grids uniformly spaced in longitude and latitude. The methods used in projecting the data onto these grids were tested for accuracy as described in section 3.3.1.

3.2.2 Synthetic Magnetograms for Testing

In order to calibrate the performance of any tracking algorithm, a “ground-truth” dataset with known movements of features is needed. For this purpose, we developed an algorithm which allows us to create synthetic magnetograms of the Sun by placing magnetic features at predetermined locations with analytical precision.
Given the central latitude ($\lambda_1$) and longitude ($\phi_1$) of a magnetic feature, its 2D Gaussian magnetic field distribution is modelled as in Eqs. 3.1 & 3.2 below:

$$B(\lambda, \phi) = Ae^{-\frac{d^2}{w}} \cos(\rho + \eta)$$  \hspace{1cm} (3.1)

$$d = 2R_\odot \sin^{-1} \sqrt{\sin^2 \left(\frac{\lambda - \lambda_1}{2}\right) + \cos(\lambda_1) \cos(\lambda) \sin^2 \left(\frac{\phi - \phi_1}{2}\right)}$$  \hspace{1cm} (3.2)

where $A$ is the amplitude i.e. the maximum magnetic field of the feature, $\phi$ is the longitude, $\rho$ is the heliocentric angle from disk center, $\eta$ is the angle between the line-of-sight and the line joining the observer to the center of the Sun, $w$ is the width of the Gaussian and $d$ is the great circle distance between the location of the peak of the feature ($\lambda_1, \phi_1$) and an evaluation point ($\lambda, \phi$).

We call this algorithm the “Analytical Magnetogram Creator (AMC)” and its output is an analytical function which describes the magnetic field on the surface of the synthetic
Sun. Note here that being analytical functions, magnetograms created by AMC have infinite resolution as they can be sampled at any desired resolution.

To create MDI/HMI-like magnetograms from the output of AMC, we created a Charge-coupled Device (CCD) emulator which emulates a camera placed at a given distance from the Sun, with a prescribed plate scale (resolution), P angle (position angle of the solar rotation axis relative to the “vertical”), B angle (tilt of the solar rotation axis toward or away from the observer) and coordinates of the center of the solar disk. It numerically integrates the magnetic flux within the boundaries of every image pixel by dividing each pixel into 100 sub-pixels and evaluating the magnetic flux density in the total area of the pixel which is equivalent to the measured magnetic field.

Features placed on synthetic magnetograms of the Sun can be moved precisely by a prescribed amount with insignificant numerical errors (verified empirically) because the magnetic field values come from an exact analytical formula. Such synthetic magnetograms of the Sun enable the accurate calibration of our correlation tracking algorithm. One such synthetic magnetogram generated by placing 4000 magnetic features randomly on the solar disk is shown in Fig. 3.1.

3.3 Improvements to the Correlation Tracking Algorithm

3.3.1 Latitude-Longitude interpolation used for Heliographic projection

Most correlation tracking studies on solar data project the solar images onto a two dimensional plane before the movements of features are tracked. We choose the equirectangular Heliographic projection (uniformly spaced in latitude and longitude) for our task following the prescription of Hathaway & Rightmire (2010). It is extremely important to minimize uncertainty in projected locations of features and distortion in their shape to maximize the accuracy of correlation tracking. For this purpose, we have comprehensively tested our Heliographic projection algorithm using the AMC and the CCD emulator.
Figure 3.2: The performance of various interpolation algorithms used in projection routines. The first two panels in the top row show the heliographic projection of a well resolved 2D Gaussian magnetic feature analytically placed on a synthetic magnetogram and sampled at different resolutions. The third panel in the first row and all panels in the second row show the same magnetic feature when different interpolation algorithms are used to reconstruct the higher resolution projection of the feature by oversampling its low resolution image. In the third and fourth rows, corresponding plots for a feature which is barely resolved in the high-resolution image and unresolved in the low resolution image are shown.
In Heliographic projection algorithms, it is common to interpolate solar images using a bicubic or bicubic spline interpolant (Liang et al. 2018) which operates on the uniformly spaced image pixel grid. These algorithms, however, incorrectly assume that the physical information in each pixel is equally spaced in physical distance as well. To evaluate the accuracy of these interpolation techniques, we performed the following experiments. In two separate experiments two magnetic features, one well resolved (FWHM = 5.33 Mm) and one barely resolved (FWHM = 1.78 Mm), were placed at 60° latitude and 60° longitude using the AMC and each of them were then sampled by the CCD emulator at two different resolutions: 4096 × 4096 (high) and 1024 × 1024 (low). Both bicubic and bicubic spline interpolation were then used to oversample the low resolution magnetograms in order to reconstruct their high resolution counterparts. We found that interpolation over the image pixel grid distorts the oversampled magnetograms as shown in Fig. 3.2 with the smaller feature suffering more distortion. Correlation between the oversampled and the high-resolution magnetogram listed in Table 3.1 was used as a measure of accuracy.

To minimize distortion in the projection algorithm, we adopted the biharmonic spline interpolation algorithm of Sandwell (1987) and employed it on the non-uniformly spaced (λ, φ cos(λ)) coordinates of the centers of image pixels. This interpolation algorithm relies on data values in neighboring pixels and their physical distance from the target location. Two variants of this algorithm were used, one which utilizes data from a 3 × 3 pixel area and another which uses data from a 4 × 4 pixel area surrounding the target location which is in between the second and the third pixel in both dimensions. All interpolation methods magnified the barely resolved feature (Fig. 3.2) more than the well resolved feature. Bicubic and bicubic spline interpolation completely changed alignment of the barely resolved feature while they only distorted the shape of the well resolved feature. Biharmonic spline interpolation utilizing data from 4 × 4 neighboring pixels outperformed all other methods as is suggested by the higher correlation coefficient between the oversampled and the high-resolution images.
Table 3.1: Correlation coefficient between the oversampled and high resolution image when different interpolation algorithms are used to oversample a low resolution image into a high resolution image of a feature on a synthetic magnetogram.

<table>
<thead>
<tr>
<th>Interpolation method</th>
<th>Correlation (well resolved feature)</th>
<th>Correlation (barely resolved feature)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bicubic (pixels)</td>
<td>0.9604</td>
<td>0.7881</td>
</tr>
<tr>
<td>Bicubic spline (pixels)</td>
<td>0.9664</td>
<td>0.7802</td>
</tr>
<tr>
<td>Biharmonic spline 3 × 3 (lat-long)</td>
<td>0.9763</td>
<td>0.8380</td>
</tr>
<tr>
<td>Biharmonic spline 4 × 4 (lat-long)</td>
<td>0.9833</td>
<td>0.8463</td>
</tr>
</tbody>
</table>

Table 3.2: Search area at solar disk center as a function of time-lag is shown here chosen so that velocities upto ±400 m/s can be detected. The time unit for MDI is 96 minutes and for HMI is one hour. The longitudinal search area at the equator is equal to the values in this table and varies with a factor of the secant of the latitude.

<table>
<thead>
<tr>
<th>No. of time units</th>
<th>HMI search area</th>
<th>MDI search area</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>±0.132°</td>
<td>±0.352°</td>
</tr>
<tr>
<td>2</td>
<td>±0.308°</td>
<td>±0.352°</td>
</tr>
<tr>
<td>4</td>
<td>±0.659°</td>
<td>±0.528°</td>
</tr>
<tr>
<td>8</td>
<td>±1.362°</td>
<td>±1.231°</td>
</tr>
</tbody>
</table>

In our correlation tracking algorithm, each magnetogram is projected onto a uniformly spaced latitude-longitude grid (−90° to +90° in both directions) with number of grid points equivalent to the dimension of the magnetogram itself. So, MDI magnetograms are projected onto a 1024 × 1024 and HMI magnetograms on a 4096 × 4096 heliographic grid.

### 3.3.2 Choosing block sizes and changing search area with latitude

Hathaway & Rightmire (2010) used thin longitudinal strips to measure meridional flow in line-of-sight magnetograms. This approach, however, does not provide us any information about longitudinal variation in the measurements. We have chosen to split the projected magnetogram into rectangular blocks in latitude-longitude such that each block is large enough to contain at least one supergranule (≈ 30 Mm). Each of these blocks has a latitudinal extent of 4.44° so that there are 40 blocks covering all latitudes. To avoid tracking a block
Figure 3.3: Blocks selected for tracking from a HMI magnetogram (in Gauss) projected onto a uniform grid in latitude-longitude are marked in red. The horizontal extent stretches from $-90^\circ$ on the left to $+90^\circ$ longitude on the right and the vertical extent from the south pole to north pole. The yellow line indicates the solar equator.
all the way to the limb of the Sun, all blocks were chosen within 60° longitude from the central meridian and their longitudinal extent was 8.93° at the equator. Tracking such blocks provides us less latitudinal resolution than thin longitudinal strips, but we gain longitudinal information in flow patterns. To keep the physical area inside each block constant, their longitudinal extent was varied as a function of latitude.

Each of the blocks from one magnetogram (mag1) was cross-correlated within a search area around their expected location (estimated by the average differential rotation) in another magnetogram (mag2) a certain time apart. This process was done both forward (select a block in mag1 and search around its expected location in mag2) and backwards (select a block in mag2 and search around its expected location in mag1) in time to double the number of measurements and minimize any systematic errors. This search area was varied with time-lag between successive magnetograms and a factor of secant of the heliocentric latitude of block center (see Table 3.2) in order to keep the search area constant in physical units. The search area was chosen so that it will encompass the distribution of velocities beyond three standard deviations. In post-processing, we filter out values beyond three standard deviations for each block. Calculating the average velocity without doing this underestimates the velocities by biasing the measurements towards the center of the search area.

To track the network magnetic elements, any 9×9 pixel area in projected HMI or 3×3 pixel area in projected MDI with an average magnetic field in excess of 150 Gauss was masked.

### 3.3.3 Peak-locking

Hathaway & Rightmire (2010) found the location of the maximum correlation by fitting a parabola near the peak of the correlation coefficient to measure the displacement of each block within the time-lag. Such a parabolic fit has an accuracy of 0.1 pixel shift. A histogram of all measurements of meridional flow velocities at a time-lag of eight hours is shown in Fig. 3.4a. We found that a simple parabolic (or 2D Gaussian) fit to the spatial distribution of
correlation coefficient results in a phenomenon known as peak-locking in the field of particle image velocimetry (Huang et al. 1997; Westerweel 1997; Chen & Katz 2005). We get more measurements at integer pixel shifts (equivalent to $\approx 18 \text{ m/s}$ at HMI resolution for 8 hr time-lag) than at fractional pixel shifts. This phenomenon appears to be known in the solar-physics community but has not been specifically addressed. We resolved this problem by taking the result from the parabolic shift as an estimate, interpolating the reference block to shift it by this fractional pixel estimate and iterating this procedure until we find a zero shift. Implementing such an iterative procedure improved our histogram of meridional flow measurements and increased the accuracy of tracking five times to 0.02 pixel shift.

3.3.4 Discovery and Correction of the Center-to-limb effect

Flow measurements from correlation tracking performed on signed magnetic field at several time-lags averaged over Carrington rotation 2098 are shown in the top row of Fig. 3.5. These measurements indicate that the east limb of the Sun is rotating $\sim 100 \text{ m/s}$ slower than the west limb of the Sun at one hour time-lag. This difference in rotation rate between the east and west limb decreases as a function of time-lag. The meridional flow measurements
Tracking On Signed Magnetograms

Tracking On Unsigned Magnetograms

Tracking On Synthetic Magnetograms

Figure 3.5: A comparison of tracking results from signed magnetograms (top row) and unsigned magnetograms (middle row) for Carrington rotation 2098 are shown here. The bottom row shows tracking results from synthetic data created with a noise level of 10 Gauss (FWHM). The first column in each row shows the residual rotation rate measured at one hour time-lag obtained after removing the average rotation rate measured at each latitude, while the second column shows the residual rotation rate measured at the equator at several time-lags. The third column in each row shows the residual meridional flow speed measured at one hour time-lag obtained after removing the average meridional flow speed measured at each latitude, while the fourth column shows the latitudinal variation of meridional flow speed at several time-lags.

Figure 3.6: A sample magnetic field pattern with a positive and a negative feature is shown on the left with solid line and its mean value is marked by the dashed line. The same pattern’s unsigned value and its mean is shown on the right.
after removal of the longitudinally averaged measurement at each latitude do not show any limb to limb variation in the longitudinal direction. The meridional flow profiles, however, vary significantly as a function of time-lag. This pattern of variation with time-lag was also seen when the algorithm of Hathaway & Rightmire (2010) was used for tracking. This combination of variation in meridional flow and the east-west asymmetry in rotation rate with time-lag is reminiscent of the center-to-limb variation reported by Zhao et al. (2012). It appears like a nearly instantaneous shift away from disk center which contributes less to velocity measurements at longer time-lags.

Very similar variation in meridional flow measured from one month of data was reported by Lamb (2017) from tracking performed on individual magnetic features with varying lifetimes: 2 hr, 10 hr and 20 hr. This variation in meridional flow profile as a function of feature lifetimes was attributed to short-lived flows and apparent motion due to feature-feature interactions, but the longitudinal variation of the flow velocities was not analyzed. The fact that we see unrealistic longitudinal variation in rotation rate rules out short-lived flows as the culprit and indicates that the source of this discrepancy must be a systematic error because the Sun cannot know which meridian is centered in HMI magnetograms and cannot adjust its rotation rate accordingly.

Interestingly, this systematic center-to-limb effect reduces significantly (by ∼50%) when unsigned magnetograms are used for tracking as is shown in the second row of Fig. 3.5. The third row of Fig. 3.5 shows the corresponding patterns measured in synthetic magnetograms suggesting that these effects do not originate from our projection or tracking algorithm. In order to understand why tracking the unsigned magnetic field improves the measurements, we must take a closer look at the dynamics of the solar photosphere.

Small scale magnetic features on the photosphere of the Sun tend to accumulate near converging flows and tend to be absent at locations where the plasma flow is diverging. Thus, they accumulate near the boundaries of supergranules where plasma is seeping below the photosphere driving a converging flow on the photosphere whereas they tend to not be
found in the central areas of supergranules where plasma is rising to the photosphere thereby driving a diverging flow pattern (Simon & Leighton 1964). When we use the signed value of magnetic field for correlation tracking, we track both the positive and negative magnetic features without giving any consideration to places where they are not found. Thus, the correlation tracking methodology works well when magnetic features do not change their magnetic polarity, live long enough to be seen in consecutive magnetograms and appear along the boundaries of the same supergranules.

Let us consider a simple one dimensional example of signed magnetic field with two opposite polarity features as shown in the left panel of Fig. 3.6. We use the correlation coefficient in Eq. 3.3

\[
C = \frac{\Sigma (x - \mu_x)(y - \mu_y)}{\sqrt{\Sigma (x - \mu_x)^2} \sqrt{\Sigma (y - \mu_y)^2}}.
\] (3.3)

to correlate this magnetogram with a similar pattern which has translated a bit at a later time where \(x\) and \(y\) are the magnetograms at different times with averages \(\mu_x\) and \(\mu_y\). In this case, the places where \(B \approx 0\) do not affect the correlation coefficient. However, when we use the unsigned value shown in the right panel of Fig. 3.6 for tracking, both the positive, negative features and the gap between them \((B \approx 0)\) contribute towards the correlation coefficient because the mean unsigned magnetic field is non-zero. Thus, using unsigned value for tracking will work even if a feature switches its polarity or it is replaced by a feature with opposite polarity as long as it appears along the boundary of the same supergranule where plasma flow is converging so that the large scale network is tracked.

Even though correlation tracking on unsigned magnetograms show improved performance, some residual variation of meridional flow persists and rotation rate still shows some asymmetry in the east-west direction. To correct for this, we make certain simplifying assumptions about composition of the residual variations:

- \(\Delta\) component: A constant shift seen at any time-lag defined as in Eq. 3.4.
• δ component: A quantity with the dimensions of acceleration which accounts for changes in flow speed as a function of time-lag. (We assume δ to be constant with respect to time-lag and thus the variation in flow velocity has to be linear with time-lag.)

Using the definitions above, we can rewrite the displacement, $D_{\Delta t}$, detected by correlation tracking at time-lag, $\Delta t$, as

$$D_{\Delta t} = (v + \delta \Delta t)\Delta t + \Delta$$

so that $\delta$ and $\Delta$ can be determined via the expressions

$$\delta_{1-2-4} = (D_4 - 3D_2 + 2D_1)/6$$

$$\delta_{2-4-8} = (D_8 - 3D_4 + 2D_2)/24$$

$$\Delta_{1-2} = (D_1 - D_2)/1 + 2\delta$$

$$\Delta_{2-4} = (D_2 - D_4)/2 + 8\delta$$

$$\Delta_{4-8} = (D_4 - D_8)/4 + 32\delta.$$  \(3.5\) \(3.6\) \(3.7\) \(3.8\) \(3.9\)

Finally, we can derive “baseline” velocity profiles

$$v_{1-2} = (D_2 - D_1)/1 - 3\delta$$

$$v_{2-4} = (D_4 - D_2)/2 - 6\delta$$

$$v_{4-8} = (D_8 - D_4)/4 - 12\delta.$$  \(3.10\) \(3.11\) \(3.12\)

The baseline velocity profiles, $v$, time-lag dependency parameter, $\delta$, and constant shift, $\Delta$, for different combinations of time-lags are derived for the meridional flow and differential rotation during each Carrington rotation. Their one year temporal averages obtained from
Figure 3.7: The panels from left to right show the baseline meridional flow velocity, constant shift $\Delta_{MF}$ and the flow change parameter $\delta_{MF}$ derived from combinations of measurements at different time-lags as in Eqs. 5 to 12 and averaged over one year from May 2010 to April 2011 for HMI. Their agreement supports our assumption that $\Delta$, $\delta$ and the baseline flow are independent of time-lag.

different combinations of time-lags are shown in Fig. 3.7. Their agreement validates our assumption that the baseline flow, $\Delta$ and $\delta$ are independent of time-lag. One year averages of the derived baseline flow profiles, $\delta$ and $\Delta$ averaged over different combinations of time-lags are shown in Fig. 3.8. Amazingly, the systematic center-to-limb error is automatically captured by $\Delta$ in both meridional flow and rotation rate measurements. As $\Delta$ varies as a function of position on the solar disk and the latitude at the center of the solar disk varies annually, we see annual variations in $\Delta$ in Fig. 3.9. If the $\Delta$ component was not removed, these annual variations would contaminate the flow velocity measurements. Time-lag dependency parameter $\delta$ however does not show any unrealistic dependence on longitude in Fig. 3.8 and thus is probably not a systematic error but rather an indication of different meridional flow for longer lived features. The interpretation and properties of $\Delta$ and $\delta$ are further discussed in section 3.4.3.

3.4 Results

3.4.1 Meridional Flow

The baseline meridional flow velocity derived from MDI and HMI during their overlap period from May 20, 2010 to December 24, 2010 was nearly identical and no calibration was required. The footprint of solar active regions is clearly visible in the meridional flow panel of Fig.
Figure 3.8: The baseline flow velocity, $\Delta$ and $\delta$ averaged over one year (from May 2010 to April 2011) and over different combinations of time-lags are shown here. The top row shows the baseline flow velocity (in $m/s$), middle row shows $\Delta$ (in $km$) and $\delta$ (in $m/s$ per hour) is shown in the bottom row. The first and third columns show the residuals after removing the average quantity at each latitude. The second column shows the longitudinal dependence of the quantity at the solar equator and the fourth column shows the latitudinal dependence of the quantity.
3.9 with meridional flow being slower (by around 5m/s) at mid-latitudes during the sunspot maximum. This variation is represented in the fitting coefficient of the associated Legendre polynomial of degree 2, order 1 to the meridional flow profile shown in the left panel of the last row of Fig. 3.9. Note that this is remarkably similar to the fitting coefficients reported by Hathaway & Rightmire (2010) in spite of all improvements we have made and suggests that the amplitude of meridional flow was lowest near the sunspot maximum of solar cycle 23. The top left panel of Fig. 3.9, however, captures the variations in meridional flow in greater detail. The fact that meridional flow outside the active region belts (marked by black lines) also varies supports the argument of Hathaway & Rightmire (2011), that these measurements of meridional flow variation are not due to the interference of turbulent diffusion around active regions.

### 3.4.2 Torsional Oscillation

The baseline rotation rate measured in MDI magnetograms during the overlap period also was nearly identical to the one measured from HMI magnetograms within 50° latitude from the solar equator and no calibration was required. At latitudes 50° to 65°, flow profiles obtained from HMI were much less noisy compared to MDI. Interestingly, the differences between the raw measurements \( D_{\Delta t} \) from MDI and HMI were captured by the \( \Delta \) error making the derived baseline velocity profiles and \( \delta \) profiles consistent across instruments. We subtracted the average rotation rate during the HMI era to derive the torsional oscillation profile (deviations from the average rotation rate) in Fig. 3.9. This measured torsional oscillation pattern is remarkably similar to the one obtained from helioseismic techniques (Howe et al. 2000; Antia & Basu 2000; Howe 2009) and shows the equatorward propagating branches of faster and slower than average rotation rate in great detail while the signal to noise ratio is not high enough to see the high-latitude poleward propagating branches clearly during the MDI era but it is sufficient enough to see them during the HMI era from May 2010 onwards. The equatorward propagating branch with faster than average rotation rate
which the active regions of solar cycle 25 (expected to start from year 2020) are expected to follow is very clearly visible before the end of solar cycle 24 around 40° latitude from the year 2018 onwards and its origin can be traced back to the year 2011 around 60° latitude.

3.4.3 Properties of constant shift error $\Delta$ and time-lag dependent $\delta$

The constant error $\Delta$ which captures the center-to-limb effect is defined in Eq. 3.4 and can be re-written as

$$\lim_{\Delta t \to 0} D_{\Delta t} = \Delta$$

which means it is a constant shift away from disk center which would be there even if we correlate magnetograms with near zero time-lag. When this constant shift is divided by time-lag, it dominates the measured velocities at small time-lags. For HMI, we find a constant shift of $\approx 60km$ around 60° from disk center in both the latitudinal and longitudinal displacements (see Fig. 3.8). This implies a velocity of $\approx 1,333m/s$ which agrees with the speed shown in Fig. 4 of Löptien et al. (2016) whereas the velocity contribution from this constant shift is 16.6$m/s$ at 1 hr and 2.1$m/s$ at 8 hr time-lags. While the contribution from the constant shift error decreases as a function of time-lag, the contribution from time-lag dependency parameter $\delta$ increases due to the $\Delta t^2$ factor.

The footprint of active regions is clearly visible in the long-term measurements of $\Delta_{MF}$ and $\Delta_{DR}$ in Fig. 3.9 suggesting that this systematic error is higher in the quiet Sun regions than in the vicinity of active regions. This explains why Lamb (2017) measured a very high meridional flow for weak features with life times up to two hours which primarily are found in quiet Sun regions. Löptien et al. (2016) also suggested that the effect may depend on the magnetic field which agrees with our observations here. This constant shift error may also be the cause of deviation between the actual flow measurements and the fitted functional form of meridional flow profile shown in Fig. 3 of Imada & Fujiyama (2018).
Moreover, both $\Delta_{MF}$ and $\Delta_{DR}$ are drastically different ($\sim 50\%$) in flows derived from HMI data compared to flows from MDI magnetograms. This may be due to its dependence on some property of the different spectral lines which are observed to create the magnetograms e.g.: their height of formation in the solar atmosphere (Löptien et al. 2016).

The time-lag dependency parameter, on the other hand accounts for variation in flow velocity with time-lag so that the effective flow velocity at any time-lag ($v'_{\Delta t}$) is slightly different from the baseline flow velocity as

$$v'_{\Delta t} = v_{\Delta t} + \delta_{\Delta t}. \quad (3.14)$$

The patterns of time-lag dependency parameter in meridional flow $\delta_{MF}$ and rotation rate $\delta_{DR}$ shown in Fig. 3.9 do not show any abrupt change between the two instruments. $\delta_{MF}$ hovers around $\sim 0.6 \text{ m/s per hour}$ with a sign opposite to that of the mean meridional flow in each hemisphere indicating that the meridional flow slows down as a function of time-lag. Whereas $\delta_{DR}$ is in the range 0.5 to 2.0 $\text{ m/s per hour}$ over the solar disk indicating that the rotation rate increases with time-lag. The one year average of $\delta_{MF}$ and $\delta_{DR}$ plotted in Fig. 3.8 does not show any unrealistic longitudinal variation and appears to be axisymmetric. Therefore, there is no reason to suspect that $\delta_{MF}$ and $\delta_{DR}$ are some kind of systematic errors.

We speculate that $\delta_{DR}$ accounts for the increase of rotation rate with increase in time-lag because at longer time-lags the measurements are biased towards tracking larger, stronger magnetic features with longer lifetimes which are anchored deeper inside the Sun and hence move with the velocities of the deeper layers as suggested by Hathaway (2012). The behavior of $\delta_{DR}$ also agrees with helioseismic inversions of rotation rate which show a near surface shear layer where the rotation rate increases as we look deeper below the photosphere (Antia et al. 2008).
On the other hand, for $\delta_{MF}$, our measurements suggest that meridional flow decreases with depth below the photosphere. This was first noted by Hathaway (2012) in the meridional motions of supergranules which indicated that the poleward meridional flow seen at the photosphere decreases with depth across the surface shear layer and becomes equatorward at the base of this layer. This has also been confirmed by helioseismic studies (Zhao et al. 2013; Jackiewicz et al. 2015; Chen & Zhao 2017). Even though the flow velocities for features with low magnetic field strength derived by Imada & Fujiyama (2018) may be affected by the constant shift error, their general conclusion (rotation rate increases with field strength and depth; meridional flow decreases with field strength and depth) agrees with ours.

In the light of our speculation, the visibility of the footprint of active regions in the pattern of $\delta_{MF}$ and $\delta_{DR}$ then suggests that the difference between the velocities of the deeper layers and the photosphere is less in the presence of high magnetic field strength features. This, again, is explained by the fact that stronger magnetic features which are rooted deeper below the photosphere but appear at the photosphere to move with the velocity of the deeper layers in which they are anchored, thus reducing the difference in the velocity between the photosphere and deeper layers in their neighborhood.

If our speculation of the meaning of $\delta_{DR}$ and $\delta_{MF}$ is correct then the implied meaning of the baseline flow velocities is that this is the velocity with which the weakest magnetic features move and the stronger magnetic features (predominantly tracked at longer time-lags) move with $v'_{\Delta t}$ (Eq. 3.14).

### 3.5 Conclusion

We have described here several improvements to the correlation tracking methodology of Hathaway & Rightmire (2010) some of which are generally applicable to correlation tracking on any spherical object. We have demonstrated how using the biharmonic spline interpolation algorithm of Sandwell (1987) for projection preserves the shapes of features compared to bicubic or bicubic spline interpolation on an image pixel grid (section 3.3.1). We have
Figure 3.9: The top row shows latitude-time plot of the baseline meridional flow and torsional oscillation obtained from over 23 years of measurements from MDI and HMI data in units of $m/s$. The corresponding plots for $\Delta$ (in $km$) and $\delta$ (in $m/s$ per hour) are shown in the second and the third row respectively. The boundaries of active latitudes are marked in black using the Royal Greenwich Observatory sunspot records. The bottom row shows the scaled sunspot number (in black) and the fitting coefficients of associated Legendre polynomial of order 1, degree 2 to the meridional flow profile and of order 1, degree 1 to the torsional oscillation with $2\sigma$ errors.
increased the search area and block size with latitude to preserve the area inside each block (section 3.3.2) and we have discovered and corrected peak-locking in correlation tracking measurements (section 3.3.3). These improvements have increased the accuracy of individual tracking measurements five fold from 0.1 pixel shift to 0.02 pixel shift.

Along with this, we have shown that previous measurements from correlation tracking performed on line-of-sight magnetograms were affected by a center-to-limb effect in the form of a shift away from disk center similar to that seen in helioseismic measurements (Zhao et al. 2012). This constant (independent of time lag between correlated images) shift, ∆, contributes heavily to velocities measured at short time-lags (where predominantly short-lived, weak magnetic features are tracked) as compared to longer time-lags (where predominantly long-living, stronger magnetic features are tracked). This resulted in Lamb (2017) measuring a faster meridional flow for features with lifetimes of two hours as opposed to features with lifetimes over 20 hours and this also explains the deviation between the measured flow velocities and the functional fit of Imada & Fujiyama (2018). We have shown that this constant error (∆) is a systematic error because it is non-axisymmetric and that it reduces by ∼50% when unsigned magnetograms are used for tracking.

We have proposed a novel method for the removal of the remaining systematic center-to-limb effect by utilizing correlation tracking measurements at several time-lags. The shift measured at each time-lag can be decomposed into a baseline velocity profile, v, the center-to-limb shift, ∆, and a time-lag dependency parameter, δ. This technique does not assume any symmetry in the center-to-limb error and yet the ∆ we obtained extracts most of the non-axisymmetric component out of the measured shifts. Moreover, the baseline flow profiles and the profiles of ∆ and δ obtained from different combinations of time-lags agree within their errors and validate our assumption that their profiles are independent of time-lag. Moreover, only the systematic error (∆) is significantly different between MDI and HMI measurements whereas the baseline flow profiles and profiles for δ are nearly identical during the overlap period and do not require cross-calibration.
Finally, we have described the refined measurements of photospheric flows and discussed their temporal variations. We have also proposed an interpretation for the meaning of the time-lag dependency parameter $\delta$ which is informed by helioseismic inversions of flow velocities in the solar interior. Our measured torsional oscillation profile agrees with those measured through helioseismology and the temporal variation in meridional flow agrees with the measurements of Hathaway & Rightmire (2010) and Komm et al. (1993). These measurements will inform studies on surface flux transport, dynamics of the near surface shear layer and solar dynamo models.
Chapter 4

SWAN: a Machine Learning Ready Space Weather ANalytics Dataset for the Prediction of Solar Flares

4.1 Background & Summary

Solar flares and Coronal Mass Ejections (CMEs) (Benz 2008; Howard 2011; Martens & Angryk 2018) are events occurring in the solar corona and heliosphere that can have a major negative impact on our technology dependent society (National Science and Technology Council 2015). A flare is a sudden brightening by orders of magnitude in Extreme Ultra-Violet (EUV), X-ray and, for large events, gamma-ray emission from a small area on the Sun, lasting from a few minutes to hours.

Energetic electromagnetic radiation and particles from solar flares and eruptions are filtered out by Earth’s atmosphere but they pose a hazard to astronauts and sensitive equipment in space. A strong enough CME can induce currents in the Earth’s atmosphere and large networks of conductive materials such as power grids, leading to surges, tripping and melting of transformers. A 2008 report by the National Research Council concluded that a solar superstorm, similar to the 1857 Carrington event (Carrington 1859), could cripple the entire US power grid for months and lead to an economic damage of 1 to 2 trillion dollars (National Research Council 2009).

In response, the White House released the National Space Weather Strategy and measures aimed at predicting and mitigating the effects of solar eruptive activity. The roadmap suggest prediction by machine learning as one method to be pursued. Key for this approach
is to produce benchmark datasets for testing flare prediction algorithms, as mentioned in Dickinson & Murtagh (2015) and Nita et al. (2018). Our benchmark dataset called *Space Weather ANalytics* (SWAN) is intended as a testbed for solar physicists or machine learning practitioners to test new ideas and methods on a cleaned, integrated and readily available dataset comprising validated information from multiple sources.

SWAN mainly relies on Spaceweather HMI Active Region Patches (SHARPs) (Bobra et al. 2014) available from the Joint Science Operations Center (JSOC). This data product stems from solar vector magnetograms obtained by the Helioseismic Magnetic Imager (HMI) (Schou et al. 2012) onboard the Solar Dynamics Observatory (SDO) (Pesnell 2015). HMI observes the Sun 24/7 and provides magnetic field information on the Sun’s surface. Since the cause of a solar flare is the sudden release of magnetic energy in the solar corona, it makes sense to use the solar magnetic field for their prediction (Falconer et al. 2011; Bobra & Couvidat 2015). However, much of the HMI data is irrelevant for large flare prediction as flares are known to originate from active regions; namely, areas of high concentration of magnetic flux. Thus, SHARPs were created (Bobra et al. 2014) – a data pipeline which identifies and tracks active regions on the Sun, crops these regions of interest and provides trimmed vector magnetic field maps and metadata containing various physical parameters.

The information about the flares, however, is missing from SHARPs. The National Oceanic and Atmospheric Administration (NOAA) operates Geostationary Operational Environmental Satellites (GOES) which have X-ray and particle detectors onboard. Since 1975 the GOES satellites have been detecting solar flares and a catalog of all detected flares is available from NOAA (Space Weather Prediction Center (SWPC) 2018). These flares are broadly classified logarithmically via their peak X-ray flux as A, B, C, M and X. The GOES flare catalog contains the time, GOES class, peak X-ray flux, a spatial location on the solar disk, and NOAA active region (AR) number, where available. In addition, Solar Region Summary (SRS) product provides daily data on ARs, such as location, sunspot classification, etc.
Since the launch of SDO in 2010, flares are automatically detected in images from the Atmospheric Imaging Assembly (AIA) onboard SDO and are regularly reported to the Heliospheric Events Knowledgebase (HEK), a metadata repository for solar physics (Hurlburt et al. 2010; Martens et al. 2012). Using both the NOAA ARs and AIA flare reports enabled us to verify the missing locations of flares in the GOES catalog. This enhanced and cross-checked reports were integrated to our dataset. Successful flare predictions via machine learning models trained and tested on this dataset intend to (1) tackle a central problem in space weather forecasting and (2) help identify physical mechanisms pertaining, or even giving rise, to solar flares. This dataset envisions to become a valuable resource for an unbiased comparison between results from various flare prediction algorithms. To date, discrepancies in skill score values between different machine learning methods cannot be attributed unambiguously to the dataset or the quality of the methods at hand.

4.2 Methods

Creating benchmark datasets for solar flare prediction based on magnetic maps of the Sun’s surface is a three-fold problem. First, solar flare reports from GOES need to be cleaned, with conflicting information resolved. Second, solar flare reports need to be matched with solar magnetic data. This can be done by either utilizing available NOAA AR numbers, if matched to HARP numbers or using a spatiotemporal overlap procedure between the occurrence time and location of a flare and the bounding box of a SHARP region at a given time. Finally, sampling biases need to be eliminated when creating labeled datasets, which we can use to train machine learning models on. We demonstrate a schematic overview of overall MVTS dataset generation in Fig. 4.1.

4.2.1 Solar Flare Reports

In NOAA/GOES observations (Hanser & Sellers 1996), when a sudden, yet persistent, increase is detected in the XRS data, the event is flagged as a likely flare. Its peak X-ray flux
is used to categorize the event into A-, B-, C-, M- or X-classes on a logarithmic scale. The background X-ray radiation emitted by the Sun is usually at the level of A- or B-class flares, making it difficult to capture all A- and B-class flares. C-, M- and X-class flares, on the other hand, are seldom missed. We present an example GOES X-ray flux series annotated with occurrences of flares on 2011-02-14 and 2011-02-15 in Fig. 4.2. As data from XRS has no spatial information, GOES uses Solar X-ray Imager (SXI) cameras capturing X-ray images of the Sun in 0.6 to 6 nm wavelength bands to pin-point each flare location (Hill et al. 2005). This spatiotemporal pin-pointing of solar flares allows NOAA’s Space Weather Prediction Center (SWPC) to find the active region responsible for the flare. Nonetheless, the GOES catalog is not perfect: the locations and NOAA active region numbers for many B-, C- and even a few M-class flares are missing. Our aim is to create a set of clean, cross-validated

![Block diagram of dataset generation process](image-url)

Figure 4.1: The block diagram of dataset generation process with principal procedures of flare cleaning (in red), MVTS generation and flare integration (in blue), and machine learning-ready dataset creation (in orange). (Image courtesy: Dr. Berk Aydin)
flare reports; therefore, we integrated the centroid locations of NOAA ARs to GOES flare
reports without an explicit location, and later cross-checked these locations with AIA flares.

**Data Acquisition:** We considered the GOES flare catalog as our primary source and used
AIA flare reports and NOAA ARs to enhance and clean the data. Both GOES and AIA flare
reports were downloaded using SunPy modules (Mumford et al. 2015). We also downloaded
the 1-minute averaged GOES X-ray flux (0.1 to 0.8 nm) time series available from NOAA
(Space Weather Prediction Center (SWPC) 2018). The X-ray flux is used to detect the
flares. Additionally, we downloaded the NOAA AR data, which consist of locations of active
regions’ centroids reported daily by NOAA SRS (Space Weather Prediction Center (SWPC)
2018).

In the period, of interest, namely 2010-05-01 to 2018-09-01, there are records of 14,367
GOES flares, of which 50 X-, 742 M-, 7,754 C- and 5,821 A- or B-class events. We also

![Figure 4.2: X-ray flux from GOES15 satellite on 2011-02-14 and 2011-02-15 along with
occurrences of large flares, annotated with magnitudes. The GOES flare classification is also
provided in minor y-axis. Dashed red lines indicate the flares’ peak time. During these two
days, background X-ray flux was high, making it difficult to identify small flares. (Image
courtesy: Dr. Berkay Aydin)
downloaded 105,867 AIA flare reports from "SSW Latest Events" and "Flare Detective" recognition modules. AIA reports are known to have duplicates in several cases. The downloaded GOES flare reports have the following attributes: start time, peak time, end time, NOAA active region number, GOES class (not available for AIA flares from Flare Detective), and point location (latitude and longitude in heliographic coordinates (degrees)).

Data Cleaning for GOES Flares: Our first step of data cleaning was removing the GOES flares without a valid NOAA AR number or a valid spatial location. A total of 2,345 flare events (34 M-, 911 C-, and 1,400 A- and B-class) had no location information associated to them, either in terms of coordinates or in terms of NOAA AR number; and therefore they were removed. Next, for those flares having a NOAA AR number, but not a valid location (7,025 out of the remaining 12,022), we augmented the active region centroid location. Note that NOAA AR locations are reported daily, thus, we have interpolated the location of the active region at the peak time of a particular flare using the known differential rotation profile of the Sun (LaBonte et al. 2007). After that, we cross-validated the locations of the GOES flares using the AIA flares. In a nutshell, for each GOES flare report, we checked

![Flare Class Distribution](chart.png)

Figure 4.3: The number of flares after cleaning procedures per each flare class. Blue bars show the “trusted” flares with a valid location, red bars show the flares with no implicit or explicit location information, purple bars show the flares that could not be validated using an AIA flare report. (Image courtesy: Dr. Berkay Aydin)
if there are any temporally overlapping (based on start and end times) AIA flares within \( \sim 163 \) arcsec (\( \sim 118 \) Mm) of the GOES flare, and marked the GOES flares that have an AIA flare correspondence as validated flares. We set the \( \sim 163 \) arcsec threshold using an outlier analysis detailed in the Appendix B. At the end of this step, we removed another 1,148 flare reports (14 M-class, 524 C-class, and 610 A- or B-class).

![Flare occurrence and spatial distribution](image.png)

**Figure 4.4:** (a) Scatter plot of flare latitudes (in degrees, helioprojective) and peak times between May 2010 and September 2018. Flare locations are approximated when originally not available. (b) The spatial distribution of M- and X-class flares. (c) The spatial distribution of C-class flares. (Image courtesy: Dr. Berkay Aydin)
**Resulting Flare Reports:** After initial cleaning, location augmentation, and cross-validation, we have a total of 10,874 flare reports. The breakdown of these flare reports can be seen in Fig. 4.3, including the number of flare reports discarded after each of our cleaning steps. There are 50 X-class, 694 M-class, 6,319 C-class, and 3,811 A- or B-class flares. The cleaning procedure did not remove any X-class flares, while it removed \(\sim 6.5\%\) of the initial M-class flare reports (\(\sim 4.6\%\) did not have any location information, while \(\sim 1.9\%\) could not be validated with a nearby AIA flare). The scatter plot of flare latitudes and peak times is presented in Fig. 4.4.a. Additionally, we present the spatial distribution of M- and X-class flares in Fig. 4.4.b and C-class flares in Fig. 4.4.c. A more detailed discussion for the cleaning and cross-validation of flare reports can be found in the Appendix B.

### 4.2.2 SHARP Data and Magnetic Field Parameters

A sequence of active region patches in a given SHARP series can be seen as a time series of vector magnetic field and photospheric intensity maps of one or more solar active regions with a cadence of 12 minutes. Each SHARP series is labeled with a unique identifier, \textsc{harpnum}. The number of observations in a series of SHARPs varies depending on how long that particular active region was visible on the Sun.

There are two types of SHARP data series available from JSOC: the *definitive* and the *near real-time* (\textit{NRT}). The NRT series is more useful for space weather forecasting in an operational context as it is ordinarily processed within three hours of acquiring new data. However, the NRT dataset pipeline changes the bounding box size of SHARPs as it evolves and assigns different identifiers to active regions before they merge or after they split.

The definitive series, however, is processed after observing an active region for its entire rotation across the earthward solar hemisphere. A bounding box which can encompass active regions within a SHARP when they occupied the largest area is chosen and remains fixed in the definitive series. Active regions, which merge or split, are also tracked as a single big active region in the definitive series. The higher data quality and consistency makes the
definitive series a better option for creating benchmark datasets for enhancing the physical understanding of the space weather phenomena and their possible links to the photospheric magnetic field, including the identification and optimization of solar flare predictors.

SHARP series are available in two coordinates: native CCD and Lambert cylindrical equal area (CEA). In the CEA projection, the vector magnetic field is decomposed into radial ($r$), westward ($\phi$), and southward ($\theta$) components. This projection is very convenient to calculate various area dependent quantities such as the total area of the active region, its magnetic flux, etc. For our dataset, we have used the raster data in the definitive series mapped to CEA projection with 720 seconds cadence (hmi.Sharp_cea_720s). Provided that this dataset leads to results on improved performance of certain flare predictors, the next meaningful step would be the creation of an NRT dataset for the pre-operational testing of prediction algorithms. Any performance discrepancies between the NRT- and the definitive-based flare forecasting should then be attributed to caveats and shortcomings of the NRT dataset.

**Magnetic Field Parameters:** From the hmi.Sharp_cea_720s data series, we derived several physical parameters using the vector magnetic field data, which could potentially be important to analyze and forecast solar flares and coronal mass ejections. Cui et. al. (Cui et al. 2006, 2007) reported a strong correlation between magnetic field properties and occurrence of solar flares. Other studies have demonstrated that including magnetic field properties in flare forecasting models improves their prediction performance (Bobra & Couvidat 2015; Yu et al. 2010; Ahmed et al. 2013).

We emphasize that, to the best of our knowledge, most of the earlier studies on extreme space weather event prediction did not consider these magnetic field parameters as time series. Instead, forecasting relied on cross-sectional, or point-in-time (snapshot) parameter values. There are a few exceptions: Gallagher et al. (Gallagher et al. 2002), Falconer et al. (Falconer et al. 2014), and Leka et al. (Leka et al. 2018) used the flaring rate or previous flaring in an active region, while McCloskey et al. (McCloskey et al. 2018) considered the
Table 4.1: Computed magnetic field parameters. Parameters in red are described in Bobra & Couvidat (2015), but not available in SHARP headers.

<table>
<thead>
<tr>
<th>Magnetic Field Parameters from Bobra &amp; Couvidat (2015)</th>
<th>Description</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABSNJZH</td>
<td>Absolute value of the net current helicity in G2/m</td>
<td>$H_{c_{abs}} \propto</td>
</tr>
<tr>
<td>EPSX</td>
<td>Sum of X-component of normalized Lorentz force</td>
<td>$\delta F_x \propto \frac{\sum B_x B_z}{\sum B_x^2}$</td>
</tr>
<tr>
<td>EPSY</td>
<td>Sum of Y-component of normalized Lorentz force</td>
<td>$\delta F_y \propto -\frac{\sum B_y B_z}{\sum B_y^2}$</td>
</tr>
<tr>
<td>EPSZ</td>
<td>Sum of Z-component of normalized Lorentz force</td>
<td>$\delta F_z \propto \frac{\sum (B_x^2 + B_y^2 - B_z^2)}{\sum B_x^2}$</td>
</tr>
<tr>
<td>MEANALP</td>
<td>Mean twist parameter</td>
<td>$\alpha_{total} \propto \frac{1}{N} \sum \frac{B_z}{B_z^2}$</td>
</tr>
<tr>
<td>MEANGAM</td>
<td>Mean inclination angle</td>
<td>$\bar{\gamma} = \frac{1}{N} \sum \arctan \left( \frac{B_y}{B_x} \right)$</td>
</tr>
<tr>
<td>MEANGBH</td>
<td>Mean value of the horizontal field gradient</td>
<td>$\nabla B_h = \frac{1}{N} \sum \sqrt{ \left( \frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} \right)^2}$</td>
</tr>
<tr>
<td>MEANGBT</td>
<td>Mean value of the total field gradient</td>
<td>$\nabla B_t = \frac{1}{N} \sum \sqrt{ \left( \frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} \right)^2}$</td>
</tr>
<tr>
<td>MEANJZD</td>
<td>Mean vertical current density</td>
<td>$J_z \propto \frac{1}{N} \sum \left( \frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y} \right)$</td>
</tr>
<tr>
<td>MEANJZH</td>
<td>Mean current helicity</td>
<td>$H_{c_{total}} \propto</td>
</tr>
<tr>
<td>MEANPOT</td>
<td>Mean photospheric excess magnetic energy density</td>
<td>$\bar{\rho} \propto \frac{1}{N} \sum \left( \vec{B}<em>{\text{Obs}} - \vec{B}</em>{\text{Pot}} \right)^2$</td>
</tr>
<tr>
<td>MEANSHR</td>
<td>Mean shear angle</td>
<td>$\bar{\Gamma} = \frac{1}{N} \sum \arccos \left( \frac{\vec{B}<em>{\text{Obs}} \cdot \vec{B}</em>{\text{Pot}}}{</td>
</tr>
<tr>
<td>R_VALUE</td>
<td>Total unsigned flux around high gradient polarity inversion lines using the $B_r$ component</td>
<td>$\Phi = \sum</td>
</tr>
<tr>
<td>SAVNCPP</td>
<td>Sum of the absolute value of the net current per polarity</td>
<td>$J_{\text{sum}} \propto \frac{1}{N} \sum</td>
</tr>
<tr>
<td>SHRGT45</td>
<td>Area with shear angle greater than 45 degrees</td>
<td>Area with $\text{Shear}&gt;45^\circ$</td>
</tr>
<tr>
<td>TOTBSQ</td>
<td>Total magnitude of Lorentz force</td>
<td>$F \propto \sum B^2$</td>
</tr>
<tr>
<td>TOTFX</td>
<td>Sum of X-component of Lorentz force</td>
<td>$F_x \propto \sum B_x B_y dA$</td>
</tr>
<tr>
<td>TOTFY</td>
<td>Sum of Y-component of Lorentz force</td>
<td>$F_y \propto \sum B_x B_y dA$</td>
</tr>
<tr>
<td>TOTFZ</td>
<td>Sum of Z-component of Lorentz force</td>
<td>$F_z \propto \sum (B_x^2 + B_y^2 - B_z^2) dA$</td>
</tr>
<tr>
<td>TOTPOT</td>
<td>Total photospheric magnetic energy density</td>
<td>$\rho_{\text{pot}} \propto \sum \left( \vec{B}<em>{\text{Obs}} - \vec{B}</em>{\text{Pot}} \right)^2 dA$</td>
</tr>
<tr>
<td>TOTUSJH</td>
<td>Total unsigned current helicity</td>
<td>$H_{c_{total}} \propto \sum</td>
</tr>
<tr>
<td>TOTUSJZ</td>
<td>Total unsigned vertical current</td>
<td>$J_{\text{total}} = \sum</td>
</tr>
<tr>
<td>USFLUX</td>
<td>Total unsigned flux in Maxwells</td>
<td>$\Phi = \sum</td>
</tr>
</tbody>
</table>

evolution of sunspot characteristics as a flare predictor. We, on the other hand, will derive a set of magnetic field parameters from the individual SHARPs and transform them into multivariate time series (MVTS) of magnetic field parameters.

A number of physically important magnetic parameters for the purpose of solar flare prediction have been listed by (Bobra & Couvidat 2015) and are reproduced in Table 4.1. While most of these parameters are available in the header information of the SHARP data series, the ones highlighted in red are not. For the generation of these parameters, we
used five segments from the SHARP series that are: $B_r$ (radial component of the magnetic field), $B_\theta$ (southward/azimuthal component of the magnetic field), $B_\phi$ (westward/toroidal component of the magnetic field), $CONF\_DISAMB$ (confidence map of magnetic field disambiguation), and $BITMAP$ (active region boundary). Using these segments as inputs to our magnetic field parameter calculation module (Basodi et al. 2017), we generated the time series of magnetic field parameters listed in Table 4.1. The parameters available in SHARP headers were also independently calculated for maintainability and we compared them against the SHARP keyword values for correctness. In addition, we re-calculated the $R\_VALUE$ parameter with the prescription of (Schrijver 2007) with one modification: the calculation was performed on the $B_r$ component instead of the line-of-sight component of the magnetic field to reduce projection artifacts. In the end, we have a multivariate time series of parameters, each calculated from an individual SHARP. The length of the time series is based on the length of the SHARP magnetogram series.

**Cleaning the MVTS:** The cleaning steps we used for our MVTS account for empty SHARPs, location-based filtering, and missing values. Firstly, we remove the empty SHARPs, which possibly resulted from merging active regions. After cleaning empty SHARPs, we generated 4,075 MVTS files. Furthermore, $\sim 8.32\%$ of timestamps were missing in the SHARP data and these were restored with null values to maintain the fixed cadence of 12 minutes.

To warn about severe projection effects and the low signal-to-noise ratio for magnetic field measurements near the limb of the Sun, while still allowing the interested researchers to perform limb-to-limb analyses, we added a boolean flag, $TMFI$ (trusted magnetic field information) in our MVTS data, reserved for (1) SHARPs with longitudinal locations within 70 degrees from the solar central meridian and (2) SHARP Quality index obtained from headers. This warning does not remove any records from our MVTS data. To assign the $TMFI$ flag, we use the data quality flag ($QUALITY$), the Carrington longitude of the SHARP area ($CRVAL1$) and Carrington longitude at solar disk center ($CRLN\_OBS$) attributes from the SHARP header files. If any observation at a time step falls within the exclusion area,
|CRVAL1-CRLN_OBS| > 70.0 or its quality index value is greater than or equal to 65536, the record’s TMFI field is set to False.

4.2.3 Flare Integration with SHARP data

The flare reports have three temporal attributes (start, peak and end times) and two spatial attributes, which are the explicit point location and implicit NOAA AR number. Moreover, as the SHARP detection module identifies smaller active regions and reorganizes the reports in definitive series, HARPNUMs (identifiers of SHARP series) do not show a one to one correspondence with NOAA AR numbers. There are some SHARP series, which are not mapped to any NOAA ARs, while some others are mapped to multiple NOAA ARs. Due to these inconsistencies in both SHARPs and flare reports, we apply two flare integration procedures based on (1) NOAA AR numbers and (2) location attributes. In a nutshell, for every MVTS, we find the associated flares and create eight additional time series of flare history parameters (for B-, C-, M-, and X-class flares using NOAA AR numbers or locations). The history series signify the identifier, magnitude, and, when available, NOAA AR number of the flares. The flare annotations are inserted to the closest timestamp to peak times.

Using NOAA active region numbers: First, we find the NOAA AR number(s) that correspond to the HARPNUM of our series and search the flare reports based only on NOAA ARs. Using these NOAA AR number(s), we create NOAA AR number-based flare history series for B-, C-, M-, and X-class flares separately. If there are no flares for a particular NOAA AR number or if the resulting subset of flares did not occur during the lifespan of the respective SHARP series, no flares are integrated.

Using location attributes: For each bounding box in the spatiotemporal trajectory of active regions (obtained using LAT_MIN, LON_MIN, LAT_MAX, and LON_MAX keywords of SHARP headers), we perform a spatiotemporal search on the flare reports. We initially perform a temporal search on the flares that occurred during the lifespan of the SHARP series. Next, for each flare report, we check if its spatial location is within the expanded
bounding box of the SHARP region at its peak time. After that, we have a list of flares, which spatiotemporally overlap with the SHARP series and we use these series to create location-based flare history series for B-, C-, M-, and X-class flares.

**X-ray flux integration:** In addition to flare history parameters, we integrated the 1-minute averaged X-ray flux data to our MVTS. As the cadence of X-ray flux data (1 minute) and our MVTS (12 minutes) is different, we integrated the maximum X-ray flux during a 12-minute interval centered around the timestamps of records in MVTS. Additionally, we created an XRQUALITY flag to identify blackouts, which shows how many of X-ray recordings in a particular 12-minute interval are of high quality. It should be noted that while flare reports are specific to particular active regions, X-ray flux is measured from the entire Sun. Details of the construction of a continuous time series from X-ray flux measurements by combining data from several GOES satellites and its integration into our MVTS data are discussed in Appendix C.

### 4.2.4 Task-based Dataset Generation

Our main data product is annotated multivariate time series from active regions. In addition to that, we now present a methodology for creating machine learning-ready timeseries datasets and provide source code for generating them. The knowledge discovery process starts with determining the data mining task, and the entire process of data handling and preparation should be tailored for the task at hand. Supervised machine learning tasks can be loosely separated into two parts based on the characteristics of the target variables: classification (if the target variable is discrete or categorical) or regression (if the target variable is continuous). For the task of dataset generation we will concentrate on a classification task based on discrete flare labels.

An important step towards accelerating machine learning-based solar physics analyses is providing sampled benchmark datasets that are cleaned, partitioned, properly sliced and la-
beled, and consistently balanced. We have already discussed the applied cleaning procedures and will review the partitioning, slicing, labeling, and balancing procedures next.

**Partitioning:** The first step in creating a machine learning application is to determine the task, and therefore to specify the target classes. Target classes are determined using a flare intensity threshold list. For a common binary classification schema, where M- or X-class flares are considered flaring and others are considered non-flaring, target class specification will only get [M1.0] as the threshold list. For creating a 4-class classification schema (e.g., B-class or lower, C-class, M-class, and X-class), we can use [C1.0, M1.0, X1.0] as the threshold list. Different threshold lists can be produced for different needs.

In machine learning applications, the creation of validation datasets is usually performed by holding out parts of datasets once or more times, as in the stratified random sampling or k-fold cross-validation schemata. Here, we will suggest another, similar, though more robust, strategy for solar physics applications dictated by the scarcity of major flare events and the likely correlations between different time series segments stemming from the same MVTS—time-segmented stratification. It is important to remember that large flares (M- or X-class), which are the most commonly targeted in predictive analyses, are scarce. In our dataset, we have 694 M-class flares and only 50 X-class flares. Among 4,075 MVTS, only 27 contain X-class flares and 178 have M-class flares. 3,275 MVTS do not have any flares (including B- or C-class flares).

Our stratification method separates the dataset into time intervals (partitions) covering ideally very similar numbers of large flare reports. This is to balance the number of minority class events in each partition, while keeping the ratio between minority (large flares) and majority (no large flares) classes similar. For example, if we have 450 M- or X-class flares integrated into MVTS in the dataset, a 5-fold separation aims to create 5 time intervals each covering roughly 90 M- and X-class flare reports. This way, we can (1) have non-overlapping time segments in each partition, so that the training and testing samples rely on different
Observation window length: $T_{obs} = 8\tau$
Latency: $L = 4\tau$
Prediction window length: $T_{pred} = 6\tau$

Label is set as the magnitude of the largest flare during the prediction window (e.g., M1.6, C7.0) or NF if no flares are reported.
MVTS and (2) preserve the ratio between the minority class among different folds as much as possible.

**Slicing and Labeling:** The next step after partitioning the dataset is to methodically slice and label them based on a desired prediction scenario. To achieve that, we employ the observation window, latency, and prediction window concepts. We use the observation window length \((T_{\text{obs}})\) to determine the duration of time series slices we want to obtain. To label each of these slices, we need to determine a target time interval (i.e., prediction window), which we derive from the latency \((L)\) and prediction window length \((T_{\text{pred}})\) parameters. Latency represents the lag between the end of observation window and the start of prediction window, while prediction window length signifies the validity of prediction or forecast.

For a time series slice starting at \(t_i\), the observation window corresponds to the slice at \([t_i, (t_i + T_{\text{obs}})]\). The prediction window corresponds to the slice at \([(t_i + T_{\text{obs}} + L), (t_i + T_{\text{obs}} + L + T_{\text{pred}})]\). Each of these instances (slices) are then labeled with the flare class of the largest flare that occurred during its respective prediction window. A schematic overview of how we perform an example scenario of slicing and labeling for a MVTS can be seen in Fig. 4.5.

Though optional, another important step is ensuring the quality of the individual slices and their labels. There are three factors that may impact the quality. First one is the lack of trusted magnetic field information, and the quality of the individual records in slices can be checked using the TMFI parameter. Second one is the lack of high quality X-ray flux data. The slices whose prediction window coincides with a prolonged period of unavailable or low quality X-ray flux data should be eliminated, as there can be missed flare reports during these intervals. This can be checked using the XRQUALITY parameter. Third one is the invalidated flare reports, which can result in mislabelings primarily for non-flaring slices whose prediction window coincides with the large invalidated flares’ peak times. We provide these invalidated flare reports as an addendum to our dataset.

**Undersampling with flare climatology:** The last step in our dataset generation procedure is adjusting the class distributions of the instances in each partition. Despite different
frequencies of large flares during different parts of solar cycle, the representation of instances from minority class (usually M- or X-class flares) should be consistently proportional among each partitioned fold. To achieve this consistency, we will use the flare climatology, which essentially shows the probability of flare occurrences (for different classes of flares) for any prediction window interval. For the sake of example, let us investigate a case where we aim to predict the occurrence of M- or X-class flares within 24 hours. We found that the daily climatology for large flares (≥ M1.0) in our datasets is ~13.5%, which means the probability of at least one large flare occurring at any given 24-hour period is 13.5%. Thus, to create a climatology-preserving undersampled dataset, we should preserve the ratio between minority and majority classes, that is: 13.6% to 86.5% respectively, which roughly creates a 1:6.4 class imbalance ratio.

We provide an example task-based data generation scenario for a predicting the occurrence of M- and X-class flares in Appendix D.

4.2.5 Extending the Datasets

While the dataset generation procedures described throughout this work provide a framework for the prediction of solar flares, we can also provide further information on possible directions to extend and improve the dataset. We present two methods of extension, coined “horizontal” and “vertical” for properly distinguishing between them.

A horizontal extension would be the addition of more time series variables (parameters) to our dataset. These parameters can bring new dimensions to our original dataset and help shed more and complementary light on flare prediction. Possible new dimensions can include the number of nearby solar events such as sigmoids, filaments, eruptions and the distance of the flare to them.

A vertical extension would be the integration with other resources for predictive analysis of space weather. These extensions, similar to flare reports, could be annotated to enhance
the predictive ability of the datasets. Examples of additional resources are CMEs, filament eruptions, or SEP events.

4.2.6 Code availability

Our open-source repositories for MVTS generation, task-based sampling, and validation is available on Bitbucket\(^1\). Interested parties might get involved in the development and extensions of the benchmark dataset at will.

4.3 Technical Validation

Our technical validation can be summarized in two parts: (1) we compared the magnetic field parameters we calculated with the ones provided in SHARP headers, and (2) we cross-validated the flare reports we obtained from GOES with the AIA flares. Details of the latter are thoroughly explained in Appendix B. Our analyses regarding the magnetic field parameters show that our calculations are accurate and minimally differ from the values reported in SHARP headers. To perform these analyses, we recorded the differences (in ratio) between the parameter values we calculated and the ones available in SHARP headers. Our comparisons show that \(\sim 96.6\%\) of our calculated values do not differ more than 1\% and 98.1\% of the values do not differ more than 2\%. Most of the departures (90\%) between our and JSOC/SHARP estimations correspond to the SHRGT45 parameter (Table 4.1).

4.4 Data Records

As described throughout this chapter, a major portion of the Multi-Variate Time Series (MVTS) data in our benchmark Space Weather ANalytics (SWAN) dataset is derived from the SHARP data series covering the period between 2010-05-01 to 2018-08-31. Each MVTS in SWAN consists of 46 series. We categorized these parameters into four groups and demon-

\(^1\)DMLab Flare Prediction project available on https://bitbucket.org/account/user/gsudmlab/projects/FP
Table 4.2: Summary and categorization of the time series parameters in our data. Individual parameters in blue have categorical values, while the ones in red are spatial parameters. All remaining have continuous values.

<table>
<thead>
<tr>
<th>Parameter Category</th>
<th>Time and Location</th>
<th>Magnetic Field Parameters (Table 4.1)</th>
<th>Flare History Parameters</th>
<th>Quality</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>ABSNZH</td>
<td>EPSX</td>
<td>EPSY</td>
</tr>
<tr>
<td>Individual Parameters</td>
<td>TIMESTAMP</td>
<td>MEANGBH</td>
<td>MEANGBT</td>
<td>MEANJBZ</td>
</tr>
<tr>
<td></td>
<td>LAT_MIN</td>
<td>MEANJZH</td>
<td>MEANPOT</td>
<td>MEANJSR</td>
</tr>
<tr>
<td></td>
<td>LON_MIN</td>
<td>NOAA_AR</td>
<td>SHRTG15</td>
<td>TOTBSQ</td>
</tr>
<tr>
<td></td>
<td>LAT_MAX</td>
<td>TOTFX</td>
<td>TOTFY</td>
<td>TOTFZ</td>
</tr>
</tbody>
</table>

1 Location parameters, i.e., LAT_MIN, LON_MIN, LAT_MAX, LON_MAX, show the extended bounding box of locations of active region patches. NOAA_AR series signifies the corresponding NOAA active region number, when available.

2 Flare history parameters show the number of associated flares occurring at a particular time. BFLARE, CFLARE, MFLARE, XFLARE series signify the flare counts (of particular classes of flares) integrated using NOAA active region numbers, while BFLARE_LOC, CFLARE_LOC, MFLARE_LOC, XFLARE_LOC series are flares integrated using location attributes.

3 The flare label series (e.g., CFLARE_LABEL or XFLARE_LABEL_LOC) are stored as annotations in the form of JSON objects, shown as follows:

```json
{
    "magnitude" : [GOES class of the flare],
    "id" : [flare identifier],
    "NOAA_AR" : [associated NOAA active region number if available]
}
```

4 XRAY_FLUX series signifies the X-ray flux (from 1-8 Angstrom), while XRQUALITY is the quality flag showing its quality.

stratified the individual parameters in each group in Table 4.2. The time and location parameters include timestamp and bounding box information, as well as the corresponding NOAA active region number demonstrating the implicit location of active regions. The quality parameters include magnetic field and X-ray quality information along with the TMFI flag described in Sec. 4.2.2. Two large groups of parameters are magnetic field and flare history parameters. Details on magnetic field parameters are demonstrated in Table 4.1. The flare history parameters signify the number of associated flares occurring during a particular time interval. We described the flare integration procedures in Sec. 4.2.3.
Our data records are available at http://www.solarmagnetism.org/swan.html along with usage notes. In total, we have 4,075 MVTS in our SWAN benchmark dataset stored in CSV format. The filenames correspond to the HARPNUM of the SHARP series.
Chapter 5

Concluding Remarks

The results in this thesis can broadly be classified into two categories: 1) constraining models of the solar dynamo using observational inferences (chapters 2 & 3) 2) creating a machine learning dataset in order to facilitate the hunt for precursors to solar flares and Coronal Mass Ejections (CMEs).

Ever since Parker (1955b) laid down the foundations of solar dynamo theory by proposing the Ω-effect and magnetic buoyancy, the hunt has been on for finding conclusive evidence in support of his ideas from helioseismic observations of the solar interior (Antia et al. 2000, 2003; Chou & Serebryanskiy 2005). In chapter 2, we first theoretically prove that the source of magnetic energy in the solar convection zone is the work done by Lorentz force and that the Lorentz force acts as a mediator between the kinetic and magnetic energy reservoirs of the plasma. Under reasonable assumptions, we postulate four distinct patterns of variation in rotation rate of the solar interior which would directly result as a consequence of Lorentz force (and Parker’s Ω-effect). We then analyze the long-term trends over time scales of the sunspot cycle in measurements of rotation rate of the solar interior from three different instruments (GONG, MDI & HMI) and find that our postulated patterns are clearly evident at $0.76R_\odot$ to $0.84R_\odot$ in the lower half of the convection zone. While these patterns may also exist below $0.76R_\odot$, large uncertainties in data below $0.76R_\odot$ do not allow us to make any inferences in deeper layers. We also find that the temporal average of rotation rate in the solar interior is consistent across sunspot cycles only at mid-latitudes ($25^\circ - 35^\circ$) whereas, the basal rotation rate at high-latitudes and the ceiling rotation rate at low-latitudes are
consistent across sunspot cycles. This presents an added challenge in deriving the baseline rotation rate of the solar interior.

Another pillar of the solar dynamo is the Babcock-Leighton $\alpha$-effect (Babcock 1961; Leighton 1969) which is governed by the meridional flow on the solar surface. The weak amplitude ($\sim 15 \text{ m/s}$) of this poleward flow makes it very difficult to measure. The precision required to measure meridional flow on the solar surface with an accuracy of $1 \text{ m/s}$ or better is analogous to tracking the movement of an object moving in the state of Hawaii with a displacement equivalent to the width of a human hair in one second while you are stationed 4000 miles away in Atlanta, Georgia. To achieve this level of accuracy one has to very carefully calibrate one’s tracking algorithm and remove any systematic errors contaminating the measurements. In chapter 3 we find and fix several such systematic errors in the local correlation tracking algorithm of Hathaway & Rightmire (2010) which tracks the large scale network of magnetic features in solar magnetograms, thereby increasing the accuracy of the algorithm five fold. We also show that the center-to-limb effect we discovered in our correlation tracking measurements contaminates the flow measurements of Lamb (2017) and Imada & Fujiyama (2018) and develop a robust methodology for the removal of this systematic effect with minimal assumptions leading to the realization that the differences in flow measurements from MDI and HMI are all in the systematic error. After removal of the systematic error, flow velocities derived from MDI and HMI magnetograms (a total of over five terabytes of data) match within errorbars during their overlap period and do not require cross-calibration. This is very encouraging and increases our confidence in these measurements. Finally, we present the measurements of rotation rate and meridional flow over the course of two solar cycles (1996-2019) which shows that meridional flow was faster ($\approx 20 \text{ m/s}$) around the sunspot minimum of 2007-2009 and slower ($\approx 15 \text{ m/s}$) during the sunspot maxima of 2002 and 2013. These measurements of meridional flow will serve as vital inputs to solar dynamo models, surface flux transport models and coronal evolution models.
Other than scientific curiosity, one of the main motivations to study the evolution of the solar magnetic field is the threat posed by solar flares and Coronal Mass Ejections (CMEs). The difficulties associated with prediction of solar flares are analogous to those associated with prediction of earthquakes. We know where the fault lines (high magnetic gradient polarity inversion lines in complex active regions) are, which gives us a fair idea about where the next solar flares are going to occur. The extremely difficult part is predicting the timing, magnitude and the number of flares. The Space-weather Active Region Patches (SHARPs) are a high quality dataset derived from HMI data and have more than eight terabytes of information. They also require a lot of post-processing, integration with reports of solar flares from multiple sources and integration with X-ray flux measured by GOES satellites to make them suitable for use in the hunt for precursors to solar flares. In chapter 4, we describe the methodology we adopted to extract magnetic parameters from SHARPs and to integrate them with flare reports as well as GOES X-ray flux measurements. Utmost care was taken in the curation of this benchmark dataset to include relevant quality flags for SHARPs, flare reports and X-ray flux data. Furthermore, we have shown how this benchmark dataset can be tailored to analyze the temporal evolution of various magnetic parameters by slicing the data into several observation, latency and prediction windows. In order to train, test and validate machine learning models on this dataset we also propose different task based partitioning methods which conserve the climatology of flares or the number of strong flares in each partition. This benchmark dataset is presently the most comprehensive, easily accessible dataset which is tailored towards prediction of solar flares. It will serve as a testbed and a means to compare the performance of different machine learning models for the prediction of solar flares in the near future and will facilitate data mining, pattern recognition studies aimed at finding precursors to solar flares.
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Appendix A

Proof of the Global Magnetic Energy Equation (Eq. 2.3)

The following MHD equations and vector identities will be used in this derivation:

\[ \text{curl}(\vec{B}) = \mu \vec{J} \quad \text{(A.1)} \]

\[ \vec{J} = \sigma(\vec{E} + \vec{v} \times \vec{B}) \quad \text{(A.2)} \]

\[ \eta_t = \frac{1}{\mu \sigma} \quad \text{(A.3)} \]

\[ \int \limits_{V} \vec{X} \cdot \text{curl}(\vec{Y}) dV = \int \limits_{V} \vec{Y} \cdot \text{curl}(\vec{X}) dV - \int \limits_{S} (\vec{X} \times \vec{Y}) \cdot d\vec{S} \quad \text{(A.4)} \]

\[ \vec{X} \cdot (\vec{Y} \times \vec{Z}) = -\vec{Y} \cdot (\vec{X} \times \vec{Z}) \quad \text{(A.5)} \]

where all notations stand for the same quantities as in the main text of this paper. The dot product of \( \frac{\vec{B}}{\mu} \) with Eq. 2.1 is

\[ \int \limits_{V} \frac{\partial (\frac{\vec{B}^2}{2\mu})}{\partial t} dV = \frac{1}{\mu} \int \limits_{V} \vec{B} \cdot \text{curl}(\vec{v} \times \vec{B}) dV - \frac{1}{\mu} \int \limits_{V} \vec{B} \cdot \text{curl}(\eta_t \text{curl}(\vec{B})) dV \quad \text{(A.6)} \]

which can be expanded using Eq. A.4 as
Using Eqs. A.1, A.2 & A.3 we get

\[ \int \nabla \cdot (B^2) \, dV = \frac{1}{\mu} \int (\vec{v} \times \vec{B}) \cdot \text{curl} (\vec{B}) \, dV - \frac{1}{\mu} \int (\vec{B} \times (\vec{v} \times \vec{B})) \cdot \vec{dS} \]

\[ - \frac{1}{\mu} \int V \eta \text{curl} (\vec{B}) \cdot \text{curl} (\vec{B}) \, dV + \frac{1}{\mu} \int S \eta (\vec{B} \times \text{curl} (\vec{B})) \cdot \vec{dS}. \]  

(A.7)

and we get Eq. 2.3 which is Poynting’s theorem in its integral form using the vector identity in Eq. A.5

\[ \int \nabla \cdot (B^2) \, dV = \left[ J \cdot (\vec{v} \times \vec{B}) \right] \, dV - \frac{1}{\mu} \int S (\vec{B} \times (\vec{v} \times \vec{B})) \cdot \vec{dS} - \int V \frac{J^2}{\sigma} \, dV + \frac{1}{\mu} \int S (\vec{E} \times (\vec{v} \times \vec{B})) \cdot \vec{dS}. \]  

(A.8)

\[ \int \nabla \cdot (B^2) \, dV = \left[ \vec{v} \cdot (\vec{J} \times \vec{B}) \right] \, dV - \int V \frac{J^2}{\sigma} \, dV - \frac{1}{\mu} \int S (\vec{E} \times \vec{B}) \cdot \vec{dS}. \]  

(A.9)
Appendix B

Spatiotemporal Co-occurrence Analysis of Flare Reports from GOES and AIA

We schematically show our flare cleaning and enhancement procedures in Fig. B.1. The first step is to filter the flares without location information. The second step is a data enhancement process, where we augment the locations of the associated NOAA active regions to flares. The last step is a verification step, where we check the locations of the GOES flare reports with AIA flare reports.

![Diagram showing the cleaning and enhancement procedures](image.png)

**Figure B.1:** Overview of our flare cleaning and enhancement procedures resulting in two sets of flare reports: (1) validated flares and (2) flares without location or without an AIA flare correspondence. (Image courtesy: Dr. Berkay Aydin)

To perform the last verification step, we downloaded the AIA flare reports, reported by Flare Detective and SSW Latest Events feature recognition modules from Heliophysics Event Knowledgebase (13,546 flare reports were downloaded from SSW Latest Events and
91,879 from Flare Detective). The flare reports from both of these modules are referred to as AIA flares. The SSW Latest Events module reports the magnitudes of the flares, while the Flare Detective does not. It is also important to note that there are occasional outages from both of these modules. In addition, both AIA and GOES flare reports include spatial and temporal attributes, namely flare locations, start and end times.

Our methodology for cross verification can be summarized as follows. To verify the locations of GOES flares, we find a subset of AIA flares, which temporally overlap (based on the start and end times) with the GOES flares that have an implicit or explicit location information. Then, we match the GOES flare with the spatially closest and temporally overlapping AIA flare, and record the distance between them. Note that the distance is calculated as the Euclidean distance over the Helioprojective-Cartesian (HPC) coordinates (in arcsec). In Fig. B.2, we demonstrate the distribution of distances between the closest AIA and GOES flares. Median and mean distances between the closest AIA and GOES flares is \(~25.31\) and \(~69.37\) arcsec, respectively. Here, we decided to use 95th percentile as the threshold point for outliers, which is \(~163.18\) arcsec. Then, we filtered all the GOES flares that does not have a temporally overlapping AIA flare within 163.18 arcsec.

In addition to the distribution of minimum distances between temporally overlapping AIA and GOES flares, we also demonstrate the spatial distributions and latitudes of invalidated flares in Fig. B.3.(a) and Fig. B.3.(b), respectively. In total, there are 1,148 invalidated flares, and majority of them are B-class (610) and low C-class flares, where 494 of 524 C-class flares are \(\leq C5.0\). From Fig. B.3.(a), we can see that a large number of flares, whose interpolated locations falls beyond the solar limb (\(\pm 90\) degrees) are invalidated. Additionally, from Fig. B.3.(b), we can observe that there are clusters of flares in certain dates, which corresponds to the AIA outages.
Figure B.2: The distribution of distances between the closest AIA and GOES flares, which temporally overlap. The distance measurement is Euclidean over Helioprojective-Cartesian (HPC) coordinates. (Image courtesy: Dr. Berkay Aydin)
Figure B.3: The spatiotemporal and spatial distribution of the invalidated GOES flares. (Image courtesy: Dr. Berkay Aydin)
Appendix C

Combining data from XRS on several GOES satellites

NOAA operates the constellation of GOES satellites, many of which have an X-ray sensor (XRS) onboard. The first GOES to have an XRS capable of continuous monitoring was GOES-5 and since then, many GOES satellites have been used as the primary and the secondary source of X-ray flux data as shown in Table C.1. Being in geostationary orbit, the Earth or the Moon intercepts the line of sight between each of these satellites and the Sun every six months at slightly different times. The start time plotted against the duration of data gaps from GOES primary satellite data are shown in Fig. C.1.(a). The X-ray data from primary satellites has a downtime of 1.43%. During these downtimes, data from the secondary satellites was used to fill the missing values which reduced the downtime to 0.80% and the remaining gaps are shown in Fig. C.1.(b).
Figure C.1: (a): The blackout durations in GOES XRS data from primary satellites show a biannual periodicity. Total downtime was 1.43%. (b): After filling some of the gaps in primary GOES XRS data with available data from secondary satellites, the total downtime reduced to 0.80%.

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Table C.1: Dates and times when the primary and/or secondary satellite designations were changed by NOAA for XRS data. Note that this table has been reproduced only for the time period 2010-05-01 to 2018-09-01, the duration of our dataset. Data courtesy: Dr. Janet Machol.
Appendix D

Example Task-based Data Generation Scenario

In this part, we describe in detail a possible use of our benchmark dataset. The task in this example is to predict the occurrence of large flares (> M1.0) within 24 hours using 12 hours of time series observations with no latency period.

Task specification and partitioning

Here, we have two discrete class labels: Flaring (FL), which includes M- and X-class flares and (2) Non-flaring (NF), which includes C-/B-class and flare-quiet intervals. Based on this, we created a 5-fold time-segmented cross-validation schema. MVTS are partitioned into 5 folds covering non-overlapping time segments, as depicted in Fig. D.1.

Slicing, Labeling and Filtering

The observation window and prediction window for the specified task is 12 hours and 24 hours, respectively ($T_{obs} = 12h$ and $T_{pred} = 24h$). Latency will be set to 0 hours. Each MVTS is sliced using a 12 hour sliding window with 1 hour step size. Each of the sliced time series instances is then labeled as the class of the largest flare that occurred in the next 24 hours. Then, to ensure the quality of instances, we first filtered if more than 10% of parameters (during observation period) in an instance have trusted magnetic field information (using TMFI parameter). Then, if the slice is labeled as NF and if more than 10% of X-ray flux data is low quality in its prediction window (XRQUALITY is set 0), we removed these instances. Lastly, we filtered the NF instances if there exists an invalid large flare (i.e., a flare report,
Figure D.1: Coverage of different folds (partitions) in our example scenario. The number of MVTS in each partition, number of flare annotations in each partition, and number of large flares occurred during these intervals are also illustrated. (Image courtesy: Dr. Berkay Aydin)

removed because either it has no location information or it cannot be validated using the AIA flares) occurring during its prediction window.

Based on this, in total, we created 6,352 flaring and 363,422 non-flaring instances. We filtered 150,523 instances due to insufficient trusted magnetic field information, 1,542 non-flaring instances due to lack of X-ray flux data, and 5,953 non-flaring instances due to invalidated flares. We demonstrate the details of the number of instances created and filtered in each partition in Fig D.2.

**Undersampling**

Using our flare reports, we determined the following flare climatology statistics for 24-hour prediction window scenario: 13.5% for M or X-class and 86.5% for no major flares. Based on this, we applied undersampling to the partitions of our dataset. The instance counts of the final data can be seen in a Figure D.3. The climatology-undersampled and time-segmented dataset is also available as an addendum to our dataset and available at [http://www.solarmagnetism.org/swan.html](http://www.solarmagnetism.org/swan.html).
Figure D.2: An overview of slicing, labeling and filtering. (a) Number of instances removed due to the lack of sufficient (at least 90%) magnetic field information. (b) Number of non-flaring (NF) instances filtered due to the invalidated flares or lack of high quality X-ray data. (c) Number of flaring and non-flaring instances created for each partition. (Image courtesy: Dr. Berkay Aydin)
Figure D.3: The number of instances in each partition after climatology-based undersampling. (Image courtesy: Dr. Berkay Aydin)