Semantic, Executive, and Visuospatial Abilities in Mathematical Reasoning of Referred College Students

Robin Morris
*Georgia State University*, robinmorris@gsu.edu

Mary K. Morris
*Georgia State University*

Paul Cirino
*University of Houston*, pcirino@uh.edu

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Semantic, Executive, and Visuospatial Abilities in Mathematical Reasoning of Referred College Students

Paul T. Cirino  
Texas Institute for Measurement, Evaluation, and Statistics (TIMES); University of Houston

Mary K. Morris  
Robin D. Morris  
Georgia State University

Semantic retrieval (SR) and executive-procedural (EP), but not visuospatial (VS) skills, have been found to be uniquely predictive of mathematical calculation skills in a sample of clinically referred college students. This study set out to cross-validate these results in an independent sample of clinically referred college students (N = 337) as well as extend them by examination of the contributions of these cognitive domains to math reasoning skills. Results indicate that these cognitive domains were able to predict 30% of the variance in calculation skills and 50% of the variance in math reasoning; however, in both cases, only the domains of semantic retrieval and visuospatial skill contributed uniquely. Differences between studies, and the lack of unique contribution of the EP domain to either type of math skill, may be due to measurement and sampling differences, the degree of shared relations among domains, and the choice of measures that represent the EP domain. Implications and future directions are explored.

Keywords: math reasoning; executive; visuospatial; retrieval; neuropsychology

In 1993, Geary reviewed cognitive, neuropsychological, and genetic studies of Math Disorders (MD) and high-lighted three areas of manifest difficulty in this population. The first difficulty is with the representation, storage, and retrieval of information from long-term semantic memory (Semantic Retrieval, or SR), which may present as weak, slow, or incorrect use of math facts. The second difficulty is with computational strategies and procedural knowledge (Executive Procedural, or EP, in the current study), which are typified by problems such as improperly following an algorithm, failing to generate an appropriate problem-solving strategy, implementing an incorrect procedure, incorrectly estimating answers, and inattention to relevant information. Deficits in this area also are consistent with several recent studies (Bull & Johnston, 1997; Gathercole & Pickering, 2000a, 2000b; McLean & Hitch, 1999; Passolunghi & Siegel, 2001) that have identified working memory or other executive difficulties in children who experience difficulty with math. The third difficulty is with visuospatial (VS) skill, which includes problems with rotation, place value, and decimal and column alignment; in addition, these VS skills may be explicitly necessary for certain math functions such as geometry.

Geary’s (1993) review focused on those factors related to deficits in arithmetic skills in children and in individuals who were neurological patients rather than adults across the range of math abilities; in addition, many individual studies typically focus on only one of the cognitive domains (SR, EP, or VS) rather than considering all of them simultaneously. Cirino, Morris, and Morris (2002), however, assessed adults across the range of calculation abilities and examined all three domains simultaneously. Neuropsychological (NP) and intellectual (IQ) measures were utilized to derive structural equation factors (in LISREL) representing each of the domains (SR, EP, and VS, broadly defined) proposed by Geary (1993). Two of the three latent constructs (EP and SR)
contributed independent predictive variance for calculation abilities. The VS domain was not found to be predictive of calculation skills once EP and SR constructs were included but did account for significant proportions of variance when entered first into regression models.

Whereas the Cirino et al. (2002) study examined only calculation skills, a similar systematic investigation of mathematical reasoning abilities has not been undertaken in adults across the range of abilities. Clearly, these skills are different at a manifest level, suggesting possible differences in the degree to which specific cognitive skills may be predictive of performance for them. For example, on the Woodcock Johnson–Revised (WJ–R) Tests of Achievement (Woodcock & Johnson, 1990), the Calculations subtest has several types of items: those that involve straightforward addition, subtraction, multiplication, and division; items that also involve decimals and fractions or other complex processes; items involving algebra or higher level math such as calculus; and some items assessing geometric or trigonometric skill. In contrast, on the WJ-R Applied Problems subtest, all items are presented in a story format and many problems deal with time and money concepts, several of which require direct semantic fact retrieval. In addition, visuospatial functions such as geometry or trigonometry appear to be directly tapped by several questions. These content differences, which are also likely apparent on other measures of computation and mathematical reasoning, suggest that the latter may require a greater emphasis on language skills (the SR domain) and visual representations (the VS domain) than do measures of calculation. Both types of measures appear to require EP skills to a similar degree, although the specific skills needed are likely to be different across the different types of math problems. For example, in arithmetic calculation, skills such as selecting an appropriate procedure and following an algorithm correctly are likely to be the EP skills most in demand; on the other hand, estimation skills and ignoring irrelevant information are relevant EP skills for solving word problems (Marzocchi, Lucangeli, De Meo, Fini, & Cornoldi, 2002).

Evidence for the role of EP skills in math reasoning has been found in several studies of children with difficulty in math (e.g., Bull & Scerif, 2001; Sikora, Haley, Edwards, & Butler, 2002). Both of these studies found that measures of executive function and working memory contributed to performance on a composite measure, which involved components of applied math reasoning but did not separate these from calculation. In one portion of a study by Barnes et al. (2002), children with hydrocephalus were administered six subtests of the KeyMath Test–Revised and subtests of Information (as a measure of general knowledge), Block Design (visual–spatial), and Digit Span (short-term and working memory) were utilized as predictors. These predictors accounted for between 22% (Division) to 54% (Estimation) of the variance in math skills, although all three predictors made significant contributions only for Estimation. Floyd, Evans, and McGrew (2003), utilizing data from the standardization sample of the Woodcock Johnson–III (WJ–III), found that clusters of the Cattell–Horn–Carroll (CHC) theory of cognitive abilities (McGrew, 1997) were predictive of both calculation and applied math clusters in the school-age years. The CHC clusters with the strongest relation to math were Comprehension–Knowledge (Gc) and several clusters related to executive processes (Fluid Reasoning [Gf], Short–Term Memory [Gsm], Processing Speed [Gs]), and a clinical scale of Working Memory that overlaps considerably with Gsm). Contributions to both types of math skills were similar, although Gf and Gc clusters were more highly related to math reasoning than to calculation, whereas the opposite pattern was found for the Gs cluster. These CHC factors were similarly predictive of mathematics performance in a series of studies utilizing the WJ-R (Hale, Fiorello, Kavanaugh, Hoepner, & Gaither, 2001; McGrew, Flanagan, Keith, & Vanderwood, 1997; Williams, McCallum, & Reed, 1996).

The studies reviewed above clearly contribute to our knowledge of the cognitive components of math reasoning skills, at least for children and adolescents, but differences in sample composition, the cognitive areas examined (and the constructs they represent), and the math outcome variables utilized make it difficult to integrate these findings and assess their concordance with information provided by Geary (1993). For example, although the Floyd et al. (2003) and related studies specifically examined the cognitive components of calculation and applied math reasoning, calculation included not only the Calculations subtest of the WJ–III but also the Math Fluency subtest, which may have increased relations with the Gs cluster. Relatled, applied included not only the Applied Problems subtest but also the Quantitative Concepts subtest, which may have increased
relations with Gsm (given the requirements of holding information), Gc (given its direct-retrieval requirements), and Gs (given its timed nature) clusters. In addition, although these studies do not find evidence for relations with visual spatial abilities (Gv cluster), the Gv cluster includes spatial manipulation and visual memory but not other features of visuo-perceptual-motor skills frequently assessed, such as analysis and synthesis, visual-motor skills, figure-ground differentiation, and visual discrimination. Therefore, further examination of the domains in Geary’s (1993) model is likely to add valuable information to the knowledge base on the prediction of applied mathematical reasoning skills.

The present study assessed the role of SR, EP, and VS factors in predicting math performance in college students referred for learning difficulties. We utilized similar predictor measures as Cirino et al. (2002), although we combined clinical neuropsychological and intellectual measures to derive these factors. We hypothesized that the measurement model would map onto the domains suggested by Geary (1993). We hypothesized that all three domains would be significantly related to basic mechanical calculation abilities but that only the SR and EP domains would predict unique variance, consistent with prior results (Cirino et al., 2002). In addition, we hypothesized that each of the three domains would be significant predictors of math reasoning skills and predict unique variance and that the overall predictive power of the domains would be greater for math reasoning relative to calculation skills. We anticipated that the SR and VS domains in particular would account for this greater variance in math reasoning given the increased linguistic and visual-spatial demands of this type of task. The results of this study will extend current literature because there is no study of adults that simultaneously assesses the domains of SR, EP, and VS in predicting applied mathematical reasoning. These results may be compared to what is known regarding the prediction of applied mathematical reasoning in children and provide convergent validity for a cognitive model of
component math skills by comparison with Floyd and colleagues results, which utilized the CHC theory of intellectual functioning (e.g., Floyd et al., 2003; Hale et al., 2001; McGrew et al., 1997). This study also quasi-replicates and extends the results of Cirino et al. (2002) through the use of a similar conceptual model, a sample that is similar in type but completely nonoverlapping, and an exploration of applied mathematical reasoning in addition and in relation to calculation skills.

METHOD

Participants and Procedures

Three hundred thirty-seven college students who were referred for an evaluation because they were experiencing academic difficulty at a 2- or 4-year state college or university comprised the sample. The present sample was obtained from the same clinical setting as the Cirino et al. (2002) study but the participants were nonoverlapping. Each participant received a comprehensive examination that investigated intellectual, academic, cognitive, and socioemotional functioning. For the purposes of this study, only a subset of those measures (similar to those utilized in the Cirino et al., 2002, study) were used for data analysis. The mean age of the participants was 24.1 ($SD = 7.7$), and the mean Wechsler Adult Intelligence Scale–III (WAIS–III) Full Scale IQ score was 102.47 ($SD = 12.7$). The mean performance on the WJ-R Calculations subtest was 101.22 ($SD = 16.3$) and on the WJ-R Applied Problems subtest was 96.3 ($SD = 13.3$). Further descriptive information on participants is provided in Table 1.

Measures

WJ-R Psychoeducational Battery math subtests. The WJ-R Calculations and Applied Problems subtests were chosen as measures of math skill. The WJ-R is a well-standardized instrument with good reliability and validity (McGrew, Werder, & Woodcock, 1991; Woodcock & Mather, 1990). The Calculations subtest requires simple addition, subtraction, multiplication, and division; computation with fractions and decimals; algebra; and other related skills. The Applied Problems subtest requires an individual to listen and read a question asking for a specific math operation; concepts involved include time and money, fractions, division, geometry, and some questions that involve ignoring irrelevant detail. Age-normed standard scores for the number of correct math problems completed on each subtest were utilized as dependent measures for all analyses.

Domain measures (semantic retrieval, executive-procedural, visuospatial). In a previous study (Cirino et al., 2002), several measures hypothesized to represent each domain were chosen from an assessment battery using either only neuropsychological measures or only subtests of the WAIS-R. In that study, either set of measures produced good model fits; therefore, they were combined and chosen to represent the domains in the present study.

Five measures were initially chosen to represent the SR domain: the Boston Naming Test (BNT; Kaplan, Goodglass, & Weintraub, 1983); the Peabody Picture Vocabulary Test–III (PPVT–III; Dunn & Dunn, 1981); and the Information, Vocabulary, and Comprehension subtests of the WAIS–III (Wechsler, 1997). These measures were chosen given their emphasis on retrieval of previously learned information. The BNT requires confrontation naming of pencil-and-paper drawings of common and uncommon objects. The PPVT–III is a measure of receptive vocabulary that requires pointing to a picture representation of a spoken word from among distractors. Information requires recalling factual knowledge of increasing difficulty, Vocabulary involves providing verbal definitions to verbal and visually presented words, and Comprehension requires verbal expression of knowledge of what to do in practical social situations. The unit of analysis was the age-normed standard score.

Six measures were initially chosen to comprise the EP domain: the Trailmaking Test Part B (Reitan & Wolfson, 1985); the Visual Search and Attention Test (VSAT; Trenerry, Crosson, DeBoe, &
Leber, 1990); the Verbal Fluency Test (Spreen & Benton, 1969); and the Picture Arrangement, Digit Span, and Digit Symbol subtests of the WAIS-III (Wechsler, 1997). These measures were chosen given their emphasis on attention, sequencing, and working memory, although ideally measures focusing on planning and problem solving could have been added if available on most participants. The Trailmaking Test Part B requires the alternating sequencing of 13 letters and 13 numbers in a speeded format. The VSAT is a speeded scanning task that requires crossing out of identified targets from among perceptually similar distractors. The Verbal Fluency Task requires the speeded spontaneous production of words that begin with a given letter in 1 min, with a total score generated across all three trials. Picture Arrangement requires the sequencing of cards that describe social or practical situations within a time limit, Digit Span requires the recall of an increasing sequence of digits in forward and reverse order, and Digit Symbol requires the transcription of marks associated with numbers to a random sequence of numbers within a time limit. As with the SR domain, age-normed standard scores were utilized in the analyses.

Seven measures were initially chosen to represent VS: Visual Discrimination; Figure Ground; and Closure sub- tests of the Test of Visual Perceptual Skills—Upper Level (TVPS-UL; Gardner, 1992a); the Test of Visual Motor Skills—Upper Level (TVMS-UL; Gardner, 1992b); and the Block Design, Matrix Reasoning, and Picture Completion subtests of the WAIS-III (Wechsler, 1997). These measures were chosen given their emphasis on visual processing, with varying degrees of perceptual, spatial, motor, and reasoning skills. Visual Discrimination involves the matching of a target figure to one of five perceptually similar figures, Figure Ground requires the participant to find a target stimulus hidden within one of five perceptually similar dis- tractors, and Closure requires the perceptual completion of geometric figures from among similar distractors. The TVMS-UL requires the drawing of successively more complex geometric figures. Block Design measures involves the viewing of two-dimensional visual designs and the subsequent construction of a three dimensional model of the picture under time constraints. Matrix Reasoning involves the identification of abstract stimuli from among distractors that best completes a geometric or other pattern. Picture Completion involves the identification of missing parts from objects within a time limit. Standard scores were again utilized for analyses.

Analyses Overview

A structural equation modeling (SEM) framework was utilized. First, the measurement model (confirmatory factor analysis describing how the three latent domains are identified by the observed variables) was tested and finalized. Next, the structural model was tested, adding math criterion variables to the model and examining relation- ships among latent domains and their unique and combined relations to math skill. These two steps (measurement model, structural model) are common in SEM (Byrne, 1998); details on SEM also are available in Schumacker and Lomax (2004). Analyses were conducted in MPLUS v. 2.13 (Muthén & Muthén, 1998-2001) utilizing the covariance matrix of the data and a maximum likelihood approach. Model comparisons restricted different pairs of correlations to be equal, with difference in fit of these nested models evaluated. To determine unique contribution, Cholesky factor decomposition was utilized, which is a mathematical procedure analogous to hierarchical regression within a structural model framework without affecting model fit (de Jong, 2000); this factorization is based on the pattern of intercorrelations and orthogonally describes the overlap among predictors.
TABLE 2
Descriptive Information and Results of Confirmatory Factor Analysis (N = 337)

<table>
<thead>
<tr>
<th>Measure</th>
<th>M</th>
<th>SD</th>
<th>SR</th>
<th>EP</th>
<th>VS</th>
</tr>
</thead>
<tbody>
<tr>
<td>WAIS-III* Vocabulary</td>
<td>106.83</td>
<td>14.1</td>
<td>92</td>
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<tr>
<td>WAIS-III Information</td>
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<td>WAIS-III Comprehension</td>
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<tr>
<td>Boston Naming Test</td>
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<tr>
<td>PPVT-III</td>
<td>104.24</td>
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<td>Verbal Fluency</td>
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<td>14.3</td>
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<td>36</td>
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<tr>
<td>Trailmaking Test, Part B</td>
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<tr>
<td>VSAT total score</td>
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<td>WAIS-III Digit Span</td>
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<tr>
<td>TVPS-UL Closure</td>
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<tr>
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<tr>
<td>TVPS-UL Figure Ground</td>
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<td>24.6</td>
<td>67</td>
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<tr>
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<tr>
<td>WAIS-III Block Design</td>
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<th>Name of Fit Index</th>
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<tr>
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<td>Fit ratio (chi-square/df)</td>
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<td>Root mean square error of approximation (RMSEA)</td>
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<td>Standardized root mean square residual (SRMSR)</td>
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<td>Comparative Fit Index (CFI)</td>
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<td>Tucker-Lewis Index (TLI)</td>
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NOTE: Factor loadings are fully standardized path coefficients. SR = Semantic Retrieval Domain; EP = Executive/Procedural Domain; VS = Visuospatial Domain; WAIS-III = Wechsler Adult Intelligence Scale–III; PPVT-III = Peabody Picture Vocabulary Test–III; VSAT = Visual Search and Attention Test; TVMS-UL = Test of Visual Motor Skills–Upper Level; TVPS-UL = Test of Visual Perceptual Skills–Upper Level. All scores are standard scores. N = 337. The chi-square value is significant (p < .00001). a. WAIS-III subtest scores are expressed in standard score units.
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RESULTS

Measurement

Initial model identification for the three latent variables (SR, EP, and VS) was based on loadings in the Cirino et al. (2002) study, although because the present study utilized some different measures and combined the earlier intellectual and neuropsychological models, slight modifications were made. Specifically, within the SR domain, a path was added from the Verbal Fluency measure (in addition to the path from Verbal Fluency to EP); this measure shares much in common with both EP and SR domains because it requires both executive skills (fluency, inhibition, rule-following, and is timed) as well as semantic skills (vocabulary, retrieval). The path from WAIS-III Picture Arrangement to the VS domain also was suggested and was consistent with this measure’s placement on the Performance IQ scale of the WAIS-III (Wechsler, 1997); adding this path, however, gave rise to a small, negative loading (−0.07) for this measure in the EP domain, and this (original) path was deleted. No other modifications were made. Multiple fit indices were available to evaluate the model (see Byrne, 1998; Muthén & Muthén, 1998-2001, for a discussion of measures). Based on several commonly used measures of fit (e.g., Fit Ratio, root mean square error of approximation, root mean square residual), the final model adequately fit the data. Results are provided in Table 2.
Relations Among Domains and Their Prediction of Mathematical Skill

A full model was tested, which included the observed math criterion variables, and produced latent correlations among the three domains and the two mathematical sub-tests (for which latent variables were created from these single measures); these relationships appear in Table 3. The correlations provided in Table 3 indicate that all three domains were significantly related to both Calculation and Applied Problems individually (all ps < .0001).

Selected fit indices for the full model (with all three latent domains and two math outcomes) appear in Table 4 as Full Model (Model 1). Table 4 also provides the results of alternative models constraining various correlations to be equal. Because these models are nested, their fit can be compared, with better fitting models having lower values for $\chi^2$ (which can be tested using a $\chi^2$ difference test) as well as lower values of associated criteria. As shown in Table 4, Model 1 was a better fit to the data relative to Models 2, 3, 5, 6, 8, 9, 11, and 12, implying that those pairs of correlations identified (listed in Table 3) were unequal.

Thus, among the three domains, the relation between SR and EP was lower than that of SR and VS (Model 2) and also was lower than that of EP with VS (Model 3), but the relation between EP and VS was equal to that of SR and VS ($p > .05$, Model 4 v. Model 1). For the relation of the latent domains to Calculation, these were greater for SR (Model 5) and VS (Model 6) relative to the EP domain, but SR and VS domains did not differ from one another ($p > .05$, Model 7 v. Model 1). For the relation of the latent domains to Applied Problems, these were again greater for SR (Model 8) and VS (Model 9), relative to the EP domain, but again SR and VS domains did not differ from one another ($p > .05$, Model 10 v. Model 1). Finally, the correlation of both SR and VS with Applied Problems was greater than their correlations with Calculations (SR, Model 11; VS, Model 12). The correlations of EP with Calculations and with Applied Problems were equivalent ($p > .05$, Model 13 v. Model 1).

The primary results were the combined and unique contributions of the three domains to the two math skills, which are presented in Table 5. For Calculation, the three domains together predicted 30% of the variance in these scores. The SR ($R^2_\Delta = .06$) and VS ($R^2_\Delta = .04$) domains each contributed significant unique variance to Calculations over the others, with $\beta$ weights of similar size. The EP domain, in contrast, did not contribute unique variance ($p > .05$). For Applied Problems, the three domains together predicted 50% of the variance in these scores. The overall pattern of unique contributions was similar to that of Calculations, with SR ($R^2_\Delta = .07$) and VS ($R^2_\Delta = .13$).
domains again predicting unique variance in Applied Problems, with a larger β weight for VS relative to SR; again, the EP domain did not contribute unique variance (p > .05). The general similarity in the pattern of contribution across the two mathematical skills is likely related to their high intercorrelation (r = .78, p < .0001). However, Table 5 also suggests that the unique contribution of VS to Applied Problems (R2∆ = .13) was stronger than the unique contribution of VS to Calculation (R2∆ = .04). The unique contributions of SR to both types of math skill were similar (Calculation R2∆ = .06; Applied Problems R2∆ = .07). EP did not contribute unique variance in either model (both R2∆ < .004) considering the other domains.

Follow-Up Analyses

Because of the differences between the results of the current study relative to that of Cirino et al. (2002; a larger proportion of variance accounted for, a decreased EP contribution and increased VS contribution), data from the earlier study were reanalyzed utilizing the measurement model of the current study. The current model provided an adequate fit to the previous sample’s data, χ2(128) = 270.422, p < .0001, Fit Ratio = 2.11, root mean square error of approximation (RMSEA) = .061, standardized root mean square residual (SRMSR) = .055. When the Calculation subtest was included in this model, the result also provided an adequate fit, χ2(143) = 296.817, p < .0001, Fit Ratio = 2.08, RMSEA = .060, SRMSR = .054. Correlations among the three domains were similar to those reported in Table 3 (SR with EP = .41, SR with VS = .61, EP with VS = .58) and the three domains together were significantly predictive of Calculations, R2 = .26. SR (β = .186, R2∆ = .03, t = 3.155, p < .001) and EP (β = .266, R2∆ = .07, t = 3.878, p < .0001) contributed significant unique variance to the prediction of Calculation but VS did not (β = .035, R2∆ = .001, t < 1, p > .05). Thus, the pattern of unique contributions was more similar to Cirino et al. (2002) than to that of the current study.
<table>
<thead>
<tr>
<th>Domain</th>
<th>$\beta$</th>
<th>$R^2 \Delta$</th>
<th>$t$</th>
<th>$p &lt;$</th>
<th>Total Unique $R^2$</th>
<th>Total $R^2$</th>
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<tr>
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<td>.125</td>
<td>6.81</td>
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</table>

NOTE: $\beta$ = fully standardized path coefficient for domain when entered last in Cholesky factorization; $R^2 \Delta$ = unique proportion of variance accounted for by a given domain; considering the other domains; $t = t$ test of path coefficient; $p <$ = probability of $t$ test value; total unique $R^2 = $ sum of unique variances attributed to the three domains; total $R^2 = $ total amount of variance accounted for by all three domains together.
Further analyses across samples indicated that six variables correlated differentially with the Calculations subtest (utilizing z score comparisons for independent correlations) in the current sample relative to that of Cirino et al. (2002). Five measures (Visual Discrimination and Closure subtests of the TVPS-UL and Comprehension, Information, and Matrix Reasoning subtests of the WAIS) correlated more strongly in the current study (Mdn $r = .42$) relative to the earlier study (Mdn $r = .23$). Conversely, only one measure (The Trailmaking Test) had a significantly lower correlation in the current sample ($r = .23$) relative to the earlier study ($r = .37$). Whereas the intellectual subtests changed normative standards across studies from the WAIS-R to the WAIS-III, reasons for the differences in correlations of the Trailmaking Test and TVPS-UL subtests to Calculations are less clear. Other significant differences ($\chi^2$ test, $p < .05$) across samples were that lower proportions of students in the present study relative to Cirino et al. (2002) met criteria for a mood and/or anxiety disorder (27% to 45%) or other psychiatric or medical disorders (12% to 22%); however, a greater
proportion of students in the present study met clinic criteria for a math learning disability (MD, 14% to 9%) or for a reading learning disability (RD, 41% to 29%). The representation of sex, ethnicity, age, and Attention Deficit Hyperactivity Disorder (ADHD) was generally similar across the two samples.

DISCUSSION

The purpose of the present study was to provide cross-validation of a measurement model of the three cognitive domains (SR, EP, and VS) described by Geary (1993) and empirically validated by Cirino et al. (2002) for referred college students and to examine the ability of these domains to predict not only calculation but also math reasoning skills. Measurement models created with a sample independent from that of Cirino et al. (2002) showed an adequate fit to the data, as hypothesized. The current study combined the neuropsychological and intellectual tasks into a single model based on the results of Cirino et al. (2002), which suggested that the chosen neuropsychological and intellectual tasks could be used to represent the three constructs of interest. The current-fit indices (Table 2) supported this approach, although some minor differences in the loadings of observed variables onto latent factors were found (i.e., Verbal Fluency on both EP and SR factors and WAIS-III Picture Arrangement on the VS domain rather than the EP domain); however, neither change is particularly surprising and may be related to the constraints imposed in the earlier study, which utilized two independent models. In all, the three domains were significantly predictive of both calculation (30%) and math reasoning (50%) performances, also as hypothesized, although EP was not predictive of either type of performance when all three cognitive domains were simultaneously considered.

Prediction of Calculation

We hypothesized that the three cognitive domains would be predictive of Calculation skills to a similar degree as in the Cirino et al. (2002) study. The cognitive domains in the present study predicted 30% of the variance in Calculation skills, which was greater than the 17% or 18% predicted by the models in the earlier study. Few other studies specifically examine the prediction of calculation skills alone, but in one study (of children with learning disabilities), Hale et al. (2001) found that 40% of the variance in math computation was accounted for by including 6 CHC clusters scores derived from 12 subtests of the WISC-III as predictors. The most important predictor was clearly Gq (the Arithmetic subtest), which was specifically excluded from the present study because of its similarity to the criterion measure given that it would be the only predictor that explicitly involves the completion of math problems. Although each of the three domains was significantly related to computation skill (p < .0001), we also hypothesized that the SR and EP domains would be unique predictors of calculation, similar to the findings of Cirino et al. (2002); however, in the present study, SR and VS were the only unique predictors and EP no longer contributed significant independent variance.

Differences between current findings and those of Cirino et al. (2002) with regard to the latent factors’ inter-relationships to calculations performance likely are not due to the modified measurement model in the current study (e.g., the fact that intellectual and neuropsychological models were combined into one, the fact that the Picture Arrangement subtest loaded on the VS factor instead of the EP factor). This was examined directly with a reanalysis of the Cirino et al. (2002) data with the present measurement model, which produced results similar to the earlier study rather than those of the current one in terms of the unique relative contributions of the three domains. Although there were no differences in terms of level of performance across samples, there was a larger standard deviation, more skewness, and less kurtosis in the present sample relative to the earlier sample; however, the measurement model of the current study predicted Calculations to approximately the same degree in both samples (26% and 30%). There were differences between the patterns of unique contributions of the domains to Calculations in the present study relative to Cirino et al. (2002), although both utilized essentially the same type of measures. Three interrelated explanations for differences in these unique contributions include measurement changes, shared variance, and differences in sample characteristics.
First, measurement changes include test version and normative differences for numerous measures (e.g., the WAIS-III instead of the WAIS-R, the PPVT-3 instead of the PPVT-R, and Verbal Fluency norms), which may have altered the nature of the latent construct derived from the observed variables. Some of these changed measures showed differences in the magnitude of their relationship to computation, although other unchanged measures also varied in their relation to computation so these changes appear to be unlikely explanations for the pattern of unique prediction.

Second, there was significant shared variance among the domains (as they were composed in the present study), and this shared variance was more predictive of math skill than were the unique contributions of the domains. Table 5 indicates that the unique contributions were 34% of the total variance accounted for in computation (e.g., .101/.297) as well as applied math reasoning (.195/.499 = 39%); a similar pattern was apparent in the reanalysis of the Cirino et al. (2002) data for Calculations (.107/.263 = 41%). There is evidence for close relations among EP and VS factors (Cirino, 2002; Gathercole & Pickering, 2000a; Miyake, Friedman, Rettinger, Shah, & Hegarty, 2001); however, as indicated in Table 3, in fact, the VS domain appeared difficult to separate from either the SR domain or the EP domain. Regardless of the pattern of shared versus unique prediction exhibited by the three domains, as noted above, it was still the case that each was significantly related to both types of math skills when examined independently. In light of the pattern of unique versus shared contributions across studies, it was particularly interesting that of the measures to show stronger relations to computation skill in the present study, several were from the VS domain (and the remainder were from the SR domain). Conversely, the only measure to show a weaker relation to computation skill was the Trailmaking Test, which had the highest loading on the EP domain in both studies.

Differences in results between the present study and those of Cirino et al. (2002) also may have arisen from the clinical composition of the samples studied (e.g., the proportion of students with MD, RD, or with a mood, anxiety, or ADHD). Individuals with many of these conditions may experience difficulty in areas related to the EP domain (Cirino, Walker, Wild, & Morris, 2003; Lucey et al., 1997; Paradiso, Lamberty, Garvey, & Robinson, 1997; Purcell, Maruff, Kyrios, & Pantelis, 1998) as well as in other domains or in academic performance. Although individuals with these diagnoses may evidence decreased levels of performance in these areas, the pattern of relationships may or may not also change. However, it is possible that different cognitive profiles may be evident within clinical subgroups, that is, different relationships may exist between latent constructs and math skills (i.e., the covariance within latent constructs and to criterion skills). Although such an investigation was beyond the scope of this study, exploratory multigroup analyses that compared the structural solutions for individuals without neurological disorders but who had RD (n = 101), ADHD (n = 71), or neither of these (n = 107) were conducted; the numbers for these groups differ from those of Table 1 due to comorbidity. Such analyses indicated that overall fit indices of these models were similar to those reported in the Results section. Also, overall proportions of variance accounted for in both computations and applied math reasoning were broadly similar across subgroups, as were many of the latent variable intercorrelations. The primary difference was that for the ADHD subgroup, none of the three latent variables were uniquely predictive of either type of mathematical skill when the other domains were considered, although the n for this subgroup was small for this type of analysis and this subgroup also exhibited the highest intercorrelations among latent domains. Additional work comparing different clinical groups may yield differentially predictive validity results for the three domains.

**Prediction of Math Reasoning**

We hypothesized that all three domains would be predictive of math reasoning performance and that the degree of predictive power would be increased relative to calculation skills, particularly in the SR and VS domains. The cognitive domains accounted for 50% of the variance in math reasoning
skills, which was substantially greater than the 30% for calculations. Few studies specifically examine the degree of prediction of math reasoning skills, although many investigations examine the prediction of composite math scores, which include both computation as well as math reasoning measures (Bull & Johnston, 1997; Bull & Scerif, 2001; Casey, Pezaris, & Nuttall, 1992; Gathercole & Pickering, 2000a; McLean & Hitch, 1999). All of these studies focus on children, and the degree of predictive power for a given variable or set of variables ranges up to 67%, with significant variability according to which and how many variables are included in regression models, with the strongest models those that include other academic measures such as reading (typically as a covariate) in the same model. For example, Casey et al. (1992) found that for boys, a total of 67% of the variance in math performance was accounted for by a mental rotation measure, an achievement composite, and a verbal ability composite; the mental rotation measure alone accounted for 31%, but only 3% above the other composite measures. Similarly, in a sample of 7-year-old children, Bull and Johnston (1997) found that several measures of working memory and a measure of word reading accounted for 58% of the variance in math performance; sequencing and processing speed measures alone accounted for approximately 30%, but less than 10% above the word reading measure. Even considering the difficulty of comparisons across these studies, the ability of the SR, VS, and EP domains to predict math reasoning in this study compare favorably with other investigations of the cognitive contributions to math skills.

As was the case for calculations, although all three domains were predictive of math reasoning skills when examined independently, when examined in the same model, only the SR and VS domains were significant over each of the other domains. The increase in predictive power for math reasoning relative to calculation may be related to greater contributions of SR and VS domains. Such results are consistent with the demand characteristics of the applied math reasoning task utilized in this study, which emphasizes language processing, as well as a substantial number of problems that are presented visually or that require visualization.

The EP Domain

The fact that EP was not significantly predictive of either math calculations or math reasoning beyond the contribution of SR and VS did not support our hypothesis and is inconsistent with numerous studies. The choice of measures that represented the EP domain focused on selective attention, sequencing, and processing speed rather than problem solving, cognitive flexibility, or planning skills, and the inclusion of such measures may have resulted in stronger contributions for this domain and more robust and/or consistent results with the prior study (Cirino et al., 2002). The composition of the current EP domain is a limitation of this study and clearer assessment of working memory (verbal or nonverbal) or problem solving as frequently defined in neuropsychological studies (Bull, Johnston, & Roy, 1999; Bull & Scerif, 2001; Gathercole & Pickering, 2000b; Sikora et al., 2002), or direct assessment of problem-solving strategies and procedural errors in calculation, may be more in line with the Procedural domain as conceptualized by Geary (1993). Thus, the zero-order correlation of math performance to the EP task(s) in the current study was significant for both calculation and applied math reasoning (.31 and .34, respectively), albeit generally lower than correlations between executive skills and math in several of the above studies (range r = .40 to .50).

Conclusions

Measures of SR, VS, and EP were significantly and meaningfully predictive both individually (all ps < .0001) and collectively of both calculations (30%) and math reasoning (50%) to a degree that is similar to investigations in children. Both SR and VS domains were unique predictors of both types of math skill, but their contributions were greater toward math reasoning than to calculation skill. The EP domain did not offer unique predictive variance to either type of math skill, although this was likely a product of measurement and sampling differences, the degree of shared variance among the three domains, and the lack of a more complete assessment of executive skills (e.g., working memory, planning, cognitive
flexibility) rather than it being the case that EP-related skills are unimportant for mathematical performance. Although overall predictive power for both types of math skills was high, considerable variance remains to be explained, particularly for calculations; it is certainly possible that there are additional cognitive domains that are also important (e.g., Floyd et al., 2003).

This study provides a replication of earlier work that examined the relations of semantic, executive, and visuospatial skills to math in adults (Cirino et al., 2002) and also provides an extension of this work to applied mathematical reasoning, including how it relates to calculation skill. In combination, these two studies provide support for a model that hypothesizes that SR, EP, and VS all contribute to calculation and applied mathematical reasoning skill, although differences between studies exemplify the level of complexity needed to interpret the unique contributions of the three domains. Further investigations of the cognitive correlates of math ability are necessary to more comprehensively assess the unique core cognitive contributions to mathematical performance. These include the study of (a) different math subskills (e.g., arithmetic, algebra, geometry); (b) nonclinical populations as well as in disorders such as MD and/or RD, where comorbidities are common (Alarcon, DeFries, Light, & Pennington, 1997; Badian, 1999; Hein, Bzufka, & Neumarker, 2000) and where different patterns have been identified (Jordan, Kaplan, & Hanich, 2002); and (c) different ages, for example, a current focus in children is the identification of mathematical difficulties via precursor skills (Gersten, Jordan, & Flojo, 2005).

REFERENCES


Paul T. Cirino is a research assistant professor in the Department of Psychology and the Texas Institute for Measurement, Evaluation, and Statistics (TIMES) at the University of Houston. He is a developmental neuropsychologist whose interests include disorders of math and reading, executive function, and measurement.

Mary K. Morris is an associate professor in the Department of Psychology at Georgia State University. She is also director of the Regents Center for Learning Disorders at Georgia State University. She is a developmental neuropsychologist whose interests focus on the cognitive and psychosocial correlates of developmental disorders of learning and attention in children and adults and acquired brain injury.

Robin D. Morris is the vice president for research and regents professor in the Department of Psychology and the Department of Educational Psychology and Special Education at Georgia State University. He is a developmental neuropsychologist whose interests include acquired brain injury and the neurodevelopmental correlates of reading and language in children and adults.