5-13-2021

Jackknife Empirical Likelihood Methods for the Cox Regression Model

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by

LAUREN DRINKARD

Under the Direction of Yichuan Zhao, PhD

ABSTRACT

In the thesis, we consider the Cox regression model. We develop the jackknife empirical likelihood (JEL), adjusted jackknife empirical likelihood (AJEL), mean jackknife empirical likelihood (MJEL), transformed jackknife empirical likelihood (TJEL) and (TAJEL) transformed adjusted jackknife empirical likelihood for the inference about the regression parameters. Additionally, the adjusted empirical likelihood (AEL), mean empirical likelihood (MEL), transformed empirical likelihood (TEL) and transformed adjusted empirical likelihood (TAEL) methods are developed. We compare methods under different distributions in terms of the coverage probability and average length of confidence interval for the regression parameter with simulation studies and three real data sets. The simulation analyses indicate that the MJEL, AJEL, and TAJEL methods are the best performing JEL methods while the MEL method was the best performing EL method. The real data analyses yielded results consistent with the simulation studies.

INDEX WORDS: Cox regression model, Jackknife empirical likelihood, Adjusted jackknife empirical likelihood, Mean jackknife empirical likelihood, Transformed jackknife empirical likelihood, Transformed adjusted jackknife empirical likelihood, Wilk's theorem, Adjusted empirical likelihood, Mean empirical likelihood, Transformed empirical likelihood, Transformed adjusted empirical likelihood

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A Thesis Submitted in Partial Fulfillment of the Requirements for the Degree of

Master of Science

in the College of Arts and Sciences

Georgia State University

2021

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May 2021

DEDICATION

To all those who use their education to benefit the world.

ACKNOWLEDGEMENTS

I express my gratitude to the spirits who gifted me with the support of many people. My earnest gratitude to my advisor Professor Yichuan Zhao, who is an uplifting, patient, and insightful person. This thesis was a delight to accomplish due to his guidance, which has been instrumental for the duration of this thesis. His never-ending words of encouragement, intelligence, and patience are highly remarkable traits I hope to acquire and exude throughout my life.

I am grateful to the other committee members for their valuable advice. I am also thankful to all my professors both in the Department of Mathematics and Statistics and the School of Public Health, GSU. Without their support, I would not be able to fulfil the requirements of the graduate program or make the connections between the two fields of study that inspired me to pursue my chosen career path.

Additionally, I want to thank Ali Jinnah for the help on R coding and who took the time to elucidate concepts vital to meeting the demands of this research. I also appreciate my course mates for their ability to create a welcoming environment. I would like to thank Iyanouluwa Ayodele for proofreading and for making my educational experience a happy, loving, and thriving place. Our ability to assist, push, and intrigue each other made for an exceptional learning experience.

Furthermore, I would like to thank my senior cat for spending her life playing, loving, and growing with me, and my kitten for entrusting her health, happiness and life with me. I am exceptionally grateful to my parents, brother, sister-in-law, friends, and lovely partner Daniel for all their support and joy throughout my life. My family and friends have always been the backbone of my education. Their continued dedication to being my teachers, motivators, and supporters has shaped me in accomplishing every milestone I have hit in my life. I love you all.

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LIST OF ABBREVIATIONS

- CI - Confidence Interval
- NA - Normal Approximation
- EL - Empirical Likelihood
- MEL - Mean Empirical Likelihood
- LLR - Log Likelihood Ratio
- JEL - Jackknife Empirical Likelihood
- AEL - Adjusted Empirical Likelihood
- TEL - Transformed Empirical Likelihood
- TAEL - Transformed Adjusted Empirical Likelihood
- AJEL - Adjusted Jackknife Empirical Likelihood
- MJEL - Mean Jackknife Empirical Likelihood
- TJEL - Transformed Jackknife Empirical Likelihood
- TAJEL - Transformed Adjusted Jackknife Empirical Likelihood

CHAPTER 1

INTRODUCTION

1.1 General introduction

The field of survival analysis is a vastly utilized in Biostatistics. Survival analysis is often used to monitor survival times in clinical trials. These clinical trials often consist of participants that are subject to random censoring and failure times. The Cox regression model is widely used model survival analysis. The Cox's regression model enables us to quantify the relationship between failure times and a set of explanatory variables. Cox (1972) introduced the partial likelihood to make an inference for the true value $\boldsymbol{\beta}_0$. The Cox proportional hazard model by Cox (1972) is given as follows:

$$\lambda_0(t|Z) = \lambda_0(t)exp(\boldsymbol{\beta}_0^{\tau} Z),$$

where $\lambda_0(t)$, denotes an unknown baseline hazard function, $\boldsymbol{\beta}_0$ is a vector of regression coefficients and Z is a p-dimensional covariate vector.

1.2 The review of normal approximation

We want to review the Cox's regression model and the basic results from large sample theory. This will be useful to understand the EL method, our proposed JEL method and thus our proposed variations of the JEL method. We will establish the normal approximation (NA) method.

1.3 The review of empirical likelihood

The empirical likelihood (EL) was originally introduced by Owen (1988, 1990) for the mean vector. Owen (1990) showed that the Wilk's theorem (Wilks 1938) holds under mild conditions. The EL method is a nonparametric method that outperforms the parametric approach Jayasinghe and Zeephongsekul (2012) introduced. The EL method allows us to use likelihood methods to construct confidence regions without assuming a parametric family for the data. The regions are invariant under transformations and tend to show improved performance over the confidence regions obtained from the NA method when the sample size is small (Chen and Keilegom (2009)). Qin and Jing (2001) attempted to apply empirical likelihood on Cox regression model with incorrectly assuming that the baseline hazard function $\lambda_0(t)$ is known. To remedy this issue, a plug-in EL method for the Cox regression model was proposed in Zhao and Jinnah (2012) by estimating the cumulative hazard function $\Lambda_0(t)$ instead of assuming that the baseline hazard function $\lambda_0(t)$ is known.

The empirical likelihood confidence regions generally suffer from under-coverage. Under-coverage issue occurs when the coverage probability is noticeably lower than the nominal level (e.g., Owen (2001)). This problem of under-coverage for the EL method occurs considerably in small sample sizes. The problem can be traced to the rate at which the original empirical log-likelihood ratio statistic converges to the limiting chi-square random variable, and the convex hull restraint that confines the confidence region to a bounded region in a parameter space (Tsao, 2013). The EL method can be studied in many fields, Wang and Jing (2001) used EL methods to study a class of functionals of survival functions, Zhou (2005) employed the EL method on the accelerated failure time model and Yu et al. (2009) studied it on a censored median model.

1.4 The review of jackknife empirical likelihood

The jackknife empirical likelihood (JEL) method was first introduced by Thomas and Grunkemeier (1975) in a paper utilizing variations of empirical likelihood-based confidence interval construction for survival data analysis. JEL method was proposed and proved to be effective when dealing with complicated statistics (Jing et al., 2009) and proven to be simple to implement, making the JEL method the often-preferred method over the EL method. The JEL method involves implementing jackknife pseudo-values into EL methods. The JEL method has been used in many settings such as in comparing two correlated Gini indices with the goal of reducing computational cost according to Alemdjrodo and Zhao (2019). Gong et al. (2010) proposed a smoothed JEL for the receiver operating characteristic curve, which is used to enhance computational efficiency. We propose a jackknife empirical likelihood to make an inference on the confidence region of $\boldsymbol{\beta}$ for the Cox model.

1.5 The review of adjusted jackknife empirical likelihood

Chen et al. (2008) proposed the adjusted empirical likelihood (AEL) to solve problems created by the non-existence of solutions when computing the empirical likelihood. Notably, the adjusted empirical likelihood contains the desired asymptotic properties that the empirical likelihood holds. Notably, the EL method is affected by the low precision of the chi-square distribution for small sample sizes. The EL method suffers from being effected if the dimension of the accompanying estimating function is high, but the AEL method improves upon the defect according to Liu and Chen (2010). Utilizing the AEL methods of Zhao, Meng, and Yang (2016) proposed an adjusted JEL. This method results in a reduction of error rates of the proposed jackknife empirical likelihood ratio. Zhao et al. (2015) and Chen and Ning (2016) developed an adjusted jackknife empirical likelihood (AJEL) method. The AJEL method combines the AEL and JEL methods. The AJEL method decreases the error rates of the jackknife empirical likelihood ratio. This thesis will employ the adjusted jackknife empirical likelihood (AJEL) to get a confidence region for $\boldsymbol{\beta}$.

1.6 The review of mean jackknife empirical likelihood

To overcome low levels of accuracy of the EL method for small sample sizes and multidimensional situations, Liang et al. (2019) presented the approach known as the mean empirical likelihood (MEL) method. The MEL method employs pairwise-mean data. The MJEL method will combine the MEL and JEL methods. We will explore the mean jackknife empirical likelihood (MJEL) to get a confidence region for $\boldsymbol{\beta}$ for the Cox regression model.

1.7 The review of transformed jackknife empirical likelihood

The EL method suffers an under-coverage problem that can be very apparent in small sample sizes (Owen 2001). Jing et al. (2017) propose the Transformed Empirical Likelihood (TEL) to improve upon the under-coverage issues. The TEL improves the coverage probability for the EL method for applications. In this thesis, we propose applying the TEL and JEL to form a transformed jackknife empirical likelihood (TJEL) method of obtaining a confidence region for $\boldsymbol{\beta}$.

1.8 The review of transformed adjusted jackknife empirical likelihood

We propose the new method of transformed adjusted jackknife empirical likelihood (TAJEL) which is a combination of the TJEL and AJEL methods. Incorporating the two methods into one, we strive to get the benefits of the two methods, i.e., the improvements from the TJEL vs. JEL and AJEL vs. JEL. We will employ TAJEL to get an estimate of the confidence intervals of $\boldsymbol{\beta}$, the regression parameter in the Cox regression model.

1.9 Purpose of the study

In this thesis, we review the normal approximation (NA), empirical likelihood (EL) inference procedure for $\boldsymbol{\beta}$ in the Cox regression model. Then we propose jackknife empirical likelihood (JEL), adjusted jackknife empirical likelihood (AJEL), and propose mean jackknife empirical likelihood (MJEL), transformed jackknife empirical likelihood (TJEL) and transformed adjusted jackknife empirical likelihood (TAJEL) for the inference of regression parameter $\boldsymbol{\beta}$. In addition, we develop AEL, MEL, TEL, and TAEL based on EL. Then, we construct the confidence interval and calculate the length of the confidence intervals. We will evaluate these methods in terms of the coverage probability and the average length of the confidence interval for $\boldsymbol{\beta}$.

The thesis is organized as follows. In Chapter 2, we first review NA and EL methods and then study alternative EL methods and construct confidence intervals for $\boldsymbol{\beta}$ using AEL, MEL, TEL, and TAEL in the Cox regression model. In Chapter 3, we combine jackknife and EL together, to form the jackknife empirical likelihood method (JEL). Then we propose confidence intervals for $\boldsymbol{\beta}$ in the Cox regression model using JEL, AJEL, MJEL, TJEL, TAJEL methods. In Chapter 4, we conduct an extensive simulation study. In Chapter 5, we apply the proposed methods to three real data sets. In Chapter 6, we make a conclusion for the proposed methodology.

CHAPTER 2

EL METHODOLOGY

In this chapter, we propose adjusted empirical likelihood, mean empirical likelihood, transformed empirical likelihood, and transformed adjusted empirical likelihood, for the interval estimate of confidence regions of $\boldsymbol{\beta}$.

2.1 Normal approximation confidence region for $\boldsymbol{\beta}$

In this section, we review the Cox regression model and the normal approximation method for the inference of regression parameter $\boldsymbol{\beta}$. We adopt the same notations as in Zhao and Jinnah (2012). We denote t_i's as the failure times and c_i's as the censoring variables. Also, we denote $x_i = min(t_i, c_i)$ and $\delta_i = I(x_i = t_i)$ where $I(\cdot)$ denotes the indicator function. As previously stated, Z is a p-dimensional covariate vector. Next, we assume (t_i, c_i, Z_i^{τ})'s are $i.i.d$ for $i = 1, ..., n$. Also, given Z_i we assume the t_i and c_i, the failure times and censoring variables respectively, are conditionally independent. To make an inference for $\boldsymbol{\beta}_0$, Cox (1975) suggested the partial likelihood function, denoted $L(\boldsymbol{\beta})$, given as follows,

$$L(\boldsymbol{\beta}) = \prod_{i=1,...,n;x_i \leq T} \left[\frac{exp(\boldsymbol{\beta}^{\tau} Z_i)}{\sum_j^n exp(\boldsymbol{\beta}^{\tau} Z_j) I(x_j \geq x_i)} \right]^{\delta_i}$$

where T satisfies $P(x_i \geq T) > 0$. Denote $U(\boldsymbol{\beta})$ as the partial likelihood score function. $U(\boldsymbol{\beta})$ is generated by calculating the first derivatives of $\log L(\boldsymbol{\beta})$. We denote $\hat{\boldsymbol{\beta}}$ as the maximum partial likelihood estimator that is obtained by solving $U(\boldsymbol{\beta}) = 0$. It is known that the partial likelihood score function is as follows:

$$U(\boldsymbol{\beta}) = \sum_{i=1}^{n} \int_0^T \left(Z_i - \frac{\hat{\alpha}_1(t, \boldsymbol{\beta})}{\hat{\alpha}_0(t, \boldsymbol{\beta})} \right) dN_i(t)$$

where $\hat{\alpha}_0(t,\boldsymbol{\beta}) = n^{-1}\sum_i exp(\boldsymbol{\beta}^\tau Z_i)I(x_i \geq t)$, $\hat{\alpha}_1(t,\boldsymbol{\beta}) = n^{-1}\sum_i Z_i exp(\boldsymbol{\beta}^\tau Z_i)I(x_i \geq t)$ and $N_i(t) = I(x_i \leq t, \delta_i = 1)$. It is shown that (cf., Anderson and Gill, 1982), $n^{1/2}(\hat{\boldsymbol{\beta}} - \boldsymbol{\beta}_0) \xrightarrow{L} N(0, I(\boldsymbol{\beta}_0)^{-1})$, where $I(\boldsymbol{\beta}_0) = \lim_{n\to\infty}\hat{I}(\boldsymbol{\beta}_0)$ and

$$\hat{I}(\boldsymbol{\beta}) = n^{-1}\sum_{i=1}^{n}\delta_i\left(\frac{\hat{\alpha}_2(x_i,\boldsymbol{\beta})}{\hat{\alpha}_0(x_i,\boldsymbol{\beta})} - \left(\frac{\hat{\alpha}_2(x_i,\boldsymbol{\beta})}{\hat{\alpha}_0(x_i,\boldsymbol{\beta})}\right)^{\otimes 2}\right)$$

is the information matrix with $a^{\otimes 2} = aa^\tau$ and $\hat{\alpha}_2(t,\boldsymbol{\beta}) = n^{-1}\sum_i Z_i^{\otimes 2} exp(\boldsymbol{\beta}^\tau Z_i)I(x_i \geq t)$.

The asymptotic $100(1-\alpha)\%$ confidence region for $\boldsymbol{\beta}$ based on the normal approximation method (cf. Zhao and Jinnah (2012)) is

$$R_1 = \{\boldsymbol{\beta} : n(\hat{\boldsymbol{\beta}} - \boldsymbol{\beta})^\tau \hat{I}(\hat{\boldsymbol{\beta}})(\hat{\boldsymbol{\beta}} - \boldsymbol{\beta}) \leq X_p^2(\alpha)\},$$

where $\chi_p^2(\alpha)$ is the $(1-\alpha)$-th quantile of the chi-square distribution with p degrees of freedom.

2.2 Empirical likelihood confidence region for $\boldsymbol{\beta}$

In this section, we review EL inference method for the Cox model the same as in Zhao and Jinnah (2012). For completeness, we provide details as follows. We must propose a new $W_{n,i}$ that does not involve the unknown $\lambda_0(s)$ and enables us to implement the plug-in EL method proposed. It is known that $U(\boldsymbol{\beta})$ can be re-expressed as

$$U(\boldsymbol{\beta}) = \sum_{i=1}^{n}\int_0^T\left(Z_i - \frac{\hat{\alpha}_1(t,\boldsymbol{\beta}_0)}{\hat{\alpha}_0(t,\boldsymbol{\beta}_0)}\right)dM_i(t),$$

with $EU(\boldsymbol{\beta}_0) = 0$ for the true $\boldsymbol{\beta}_0$, when $M_i(t) = N_i(t) - \int_0^t exp(\boldsymbol{\beta}_0^t Z_i)I(x_i \geq s)\lambda_0(s)ds$, $i = 1, ..., n$ are martingales. The baseline hazard function $\lambda_0(s)$ is unspecified in the Cox's regression model and thus estimation is required. Therefore, we replace $\lambda_0(s)ds$ with the estimated equivalent $d\hat{\Lambda}_0(s,\boldsymbol{\beta}_0)$ in the above equation, where

$$\hat{\Lambda}_0(t,\boldsymbol{\beta}_0) = \sum_{i=1}^{n}\int_0^t \frac{dN_i(s)}{\sum_{j=1}^{n} I(x_j \geq s)exp(\boldsymbol{\beta}_0^\tau Z_j)}.$$

The score function satisfies

$$U(\boldsymbol{\beta}_0) = \sum_{i=1}^{n} \int_0^T \left(Z_i - \frac{\hat{\alpha}_1(t, \boldsymbol{\beta}_0)}{\hat{\alpha}_0(t, \boldsymbol{\beta}_0)} \right) d\hat{M}_i(t)$$

where $\hat{M}_i(t) = N_i(t) - \int_0^t exp(\boldsymbol{\beta}_0^\tau Z_i I(x_i \geq s) d\hat{\Lambda}_0(s, \boldsymbol{\beta}_0)$. Then, as Hjort et al. (2009) set the precedence, we adopt the plug-in EL method. Let $\mathbf{p} = (p_1, ..., p_n)$ be a probability vector such that $\sum_{i=1}^n p_i = 1$ and $p_i \geq 0$ for all i. Then, for $1 \leq i \leq n$, let

$$W_{n,i}(\boldsymbol{\beta}_0) = \int_0^T \left(Z_i - \frac{\hat{\alpha}_1(t, \boldsymbol{\beta}_0)}{\hat{\alpha}_0(t, \boldsymbol{\beta}_0)} \right) d\hat{M}_i(t).$$

We note that $\prod_{i=1}^n p_i$, subject to $\sum_{i=1}^n p_i = 1$, achieves its maximum, denoted n^{-n}, at $p_i = n^{-1}$. Therefore, the EL ratio with plug-in estimate, when evaluated at true parameter $\boldsymbol{\beta}_0$, is the following:

$$R(\boldsymbol{\beta}_0) = \sup\{\prod_{i=1}^n np_i : \sum_{i=1}^n p_i = 1, \sum_{i=1}^n p_i W_{n,i}(\boldsymbol{\beta}_0) = 0, p_i \geq 0\}.$$

Employing Lagrange multipliers, we find that the $L(\boldsymbol{\beta}_0)$ is maximized. Thus, we conclude

$$R(\boldsymbol{\beta}_0) = \prod_{i=1}^n \left(1 + \lambda^\tau W_{n,i}(\boldsymbol{\beta}_0)\right)^{-1},$$

where $\boldsymbol{\lambda} = (\lambda_1, ..., \lambda_p)^\tau$ is the solution to the equation

$$\frac{1}{n} \sum_{i=1}^n \frac{W_{n,i}(\boldsymbol{\beta}_0)}{1 + \lambda^\tau W_{n,i}(\boldsymbol{\beta}_0)} = 0.$$

We put $\hat{l}(\boldsymbol{\beta}) = -2\log R(\boldsymbol{\beta})$. It is known that $W_{n,i}$ involves the estimate of $\lambda_0(t)$. The $W_{n,i}(\boldsymbol{\beta}_0)$'s are asymptotically $i.i.d.$ and the limiting distribution of the EL ratio $\hat{l}(\boldsymbol{\beta})$ is a standard χ_p^2 distribution. We establish the Wilk's theorem.

Theorem 2.1 (Zhao and Jinnah, 2012) Assume $\lambda_0(t)$ is continuous and that the covariate vector Z_i is bounded. Then, the statistic $\hat{l}(\boldsymbol{\beta}_0) \xrightarrow{\mathcal{D}} \chi_p^2$, as $n \to \infty$.

Thus, an asymptotic $100(1-\alpha)\%$ empirical likelihood confidence regions for $\boldsymbol{\beta}$ is as follows:

$$R_2 = \{\boldsymbol{\beta} : \hat{l}(\boldsymbol{\beta}) \leq \chi^2_p(\alpha)\},$$

where $\chi^2_p(\alpha)$ is the $(1-\alpha)$-th quantile of the chi-square distribution with p degrees of freedom.

We can use profile empirical likelihood by profiling out nuisance parameters from the full EL. Define $\boldsymbol{\beta}_0 = ((\boldsymbol{\beta}_0^{(1)})', (\boldsymbol{\beta}_0^{(2)})')'$. Suppose that we want to construct EL confidence regions for a q-dimensional $(q < p)$ subvector $\boldsymbol{\beta}_0^{(1)}$. This can be done by profiling out $\boldsymbol{\beta}^{(2)}$ from the full EL. Then the profile EL ratio for $\boldsymbol{\beta}^{(1)}$ is

$$l_{profile}(\boldsymbol{\beta}^{(1)}) = \min_{\boldsymbol{\beta}^{(2)}} l((\boldsymbol{\beta}^{(1)})', (\boldsymbol{\beta}^{(2)})').$$

We established Wilk's theorem for $l_{profile}(\boldsymbol{\beta}^{(1)})$ as follows:

Theorem 2.2 Under the regular conditions, as $n \to \infty$,

$$l_{profile}(\boldsymbol{\beta}_0^{(1)}) \xrightarrow{\mathcal{D}} \chi^2_q,$$

where χ^2_q is a standard chi-square distribution with q degrees of freedom.

Based on Theorem 2.2, we construct the asymptotic $100(1-\alpha)\%$ empirical likelihood confidence regions for $\boldsymbol{\beta}_0^{(1)}$ is as follows:

$$R^{EL}_{profile}(\alpha) = \{\boldsymbol{\beta}^{(1)} : l_{profile}(\boldsymbol{\beta}^{(1)}) \leq \chi^2_q(\alpha)\},$$

where $\chi^2_q(\alpha)$ is the $(1-\alpha)$-th quantile of the chi-square distribution with q degrees of freedom.

2.3 Adjusted empirical likelihood confidence region for $\boldsymbol{\beta}$

The EL method suffers from the convex hull problem which refers to zero not being guaranteed to be in the convex hull. To improve the coverage probability of the empirical likelihood methods, Chen et al. (2008) proposed the adjusted empirical likelihood (AEL) method. The AEL method involves adding one artificial data point into the data set and then applying the EL method on the new data set. The AEL function for $\boldsymbol{\beta}$ is as follows:

$$h_i^{ad}(\boldsymbol{\beta}) = \hat{W}_{n,i}(\boldsymbol{\beta}), i = 1, ..., n,$$

where $h_{n+1}^{ad}(\boldsymbol{\beta}) = -a_n \bar{h}^{ad}(\boldsymbol{\beta}) = -a_n \sum_{i=1}^{n} \hat{W}_{n,i}(\boldsymbol{\beta})/n$ and a_n is a constant number,

$$a_n = \max\left(1, \log(n)/2\right).$$

Thus, with the $(n+1)$ data points we have the adjusted empirical likelihood ratio function, evaluated at $\boldsymbol{\beta}$ as

$$R^{ad}(\boldsymbol{\beta}) = \sup\left\{\prod_{i=1}^{n+1}(n+1)p_i : \sum_{i=1}^{n+1} p_i W_{n,i}^{ad}(\boldsymbol{\beta}) = 0, \sum_{i=1}^{n+1} p_i = 1, p_i \geq 0, i = 1, 2, \ldots, n+1\right\}.$$

Therefore, the adjusted empirical log-likelihood ratio at $\boldsymbol{\beta}$ is

$$\begin{aligned} l^{ad}(\boldsymbol{\beta}) &= -2\log R^{ad}(\boldsymbol{\beta}) \\ &= 2\sum_{i=1}^{n+1} \log\{1 + \lambda W_{n,i}^{ad}(\boldsymbol{\beta})\}, \end{aligned}$$

where λ satisfies the following equation:

$$f(\lambda) = \sum_{i=1}^{n+1} \frac{W_{n,i}^{ad}(\boldsymbol{\beta})}{1 + \lambda W_{n,i}^{ad}(\boldsymbol{\beta})} = 0.$$

We establish the following Wilk's Theorem for $l^{ad}(\boldsymbol{\beta}_0)$. **Theorem 2.3** Assume that the regularity conditions hold. As $n \to \infty$,

$$l^{ad}(\boldsymbol{\beta}_0) \xrightarrow{\mathfrak{D}} \chi_p^2,$$

where χ_p^2 is a standard chi-square distribution with p degrees of freedom.

Based on Theorem 2.3, we can construct the asymptotic $100(1-\alpha)\%$ adjusted empirical likelihood confidence region for $\boldsymbol{\beta}_0$ as

$$R_3(\alpha) = \{\boldsymbol{\beta} : l^{ad}(\boldsymbol{\beta}) \leq \chi^2_p(\alpha)\},$$

where $\chi^2_p(\alpha)$ is the upper α-quantile of χ^2_p.

Similar to before, we derive the Wilks' Theorem for the profile AEL ratio in Theorem 2.4.

Theorem 2.4 Under the regularity conditions given in the Appendix, as $n \to \infty$,

$$l^{ad}_{\text{profile}}(\boldsymbol{\beta}^{(1)}_0) \xrightarrow{\mathfrak{D}} \chi^2_q.$$

Based on Theorem 2.4, we can construct the asymptotic $100(1-\alpha)\%$ adjusted empirical likelihood confidence region for $\boldsymbol{\beta}^{(1)}_0$ as

$$R^{AEL}_{\text{profile}}(\alpha) = \Big\{\boldsymbol{\beta}^{(1)} : l^{ad}_{\text{profile}}(\boldsymbol{\beta}^{(1)}) \leq \chi^2_q(\alpha)\Big\},$$

where $\chi^2_q(\alpha)$ is the $(1-\alpha)$ quantile of χ^2_q.

2.4 Mean empirical likelihood confidence region for $\boldsymbol{\beta}$

To improve upon the EL confidence regions for small sample sizes, Liang et al. (2019) introduced the mean empirical likelihood (MEL). For this thesis, we define the mean empirical likelihood (MEL) pseudo values as,

$$S^{EL}_i(\boldsymbol{\beta}) = \frac{W_{n,j}(\boldsymbol{\beta}) + W_{n,k}(\boldsymbol{\beta})}{2}, i = 1, ..., N, 1 \leq j \leq k \leq n,$$

where $N = n(n+1)/2$.

Thus, the mean empirical likelihood ratio at $\boldsymbol{\beta}$ is,

$$R^{MEL}(\boldsymbol{\beta}) = \max\Big\{\prod_{i=1}^{N} Np_i : \sum_{i=1}^{N} p_i = 1, \sum_{i=1}^{N} p_i S^{EL}_i(\boldsymbol{\beta}) = 0, p_i \geq 0\Big\}.$$

Now we have the mean empirical log-likelihood ratio as follows,

$$\frac{-2\log R^{MEL}(\boldsymbol{\beta})}{n+1} = \frac{2}{n+1}\sum_{i=1}^{N}\log\{1+\lambda S_i^{EL}(\boldsymbol{\beta})\},$$

where λ satisfies the equation

$$f(\lambda) = \sum_{i=1}^{N}\frac{S_i^{EL}(\boldsymbol{\beta})}{1+\lambda S_i^{EL}(\boldsymbol{\beta})} = 0.$$

Denote $\boldsymbol{\beta}_0$ as the true value of $\boldsymbol{\beta}$. Employing the method by Liang et al. (2019), we establish the following theorem.

Theorem 2.5 Assume the regularity conditions. Denote $\boldsymbol{\beta}_0$ as the true parameter value of $\boldsymbol{\beta}$. When $n \to \infty$, $\frac{-2\log R^{MEL}(\boldsymbol{\beta}_0)}{n+1}$ converges in distribution to χ_p^2.

The $100(1-\alpha)\%$ MEL confidence region for $\boldsymbol{\beta}$ is constructed as follows:

$$R_4 = \Big\{\boldsymbol{\beta} : \frac{-2\log R^{MEL}(\boldsymbol{\beta})}{n+1} \leq \chi_p^2(\alpha)\Big\}.$$

We establish the Wilk's Theorem for the profile MEL ratio.

Theorem 2.6 Under the regular conditions as $n \to \infty$,

$$l_{profile}^{MEL}(\boldsymbol{\beta}_0^{(1)}) \xrightarrow{\mathcal{D}} \chi_q^2.$$

Based on Theorem 2.6, we construct the asymptotic $100(1-\alpha)\%$ mean empirical likelihood confidence regions for $\boldsymbol{\beta}_0^{(1)}$ as follows:

$$R_{profile}^{MEL}(\boldsymbol{\beta}^{(1)}) = \Big\{\frac{-2\log R_{profile}^{MEL}(\boldsymbol{\beta}^{(1)})}{n+1} \leq \chi_q^2(\alpha)\Big\}.$$

2.5 Transformed empirical likelihood confidence region for β

We will use a simple transformation of the EL to improve upon the original EL method. Recall that the original empirical log-likelihood ratio is denoted as $R(\boldsymbol{\beta})$. For a constant

$\gamma\epsilon[0,1]$, we define

$$g_t(R(\boldsymbol{\beta});\gamma) = R(\boldsymbol{\beta}) \times \max\{1 - R(\boldsymbol{\beta})/n, 1-\gamma\}.$$

We will refer to $g_t(R(\theta);\gamma)$ as the truncated quadratic transformation of $R(\boldsymbol{\beta})$. Jing et al. (2017) set $\gamma = 1/2$. Then, we have the transformed empirical log-likelihood ratio, $R^{TEL}(\boldsymbol{\beta})$ as

$$\begin{aligned} R^{TEL}(\boldsymbol{\beta}) &= g_t(R(\boldsymbol{\beta}); \gamma = 1/2) \\ &= R(\boldsymbol{\beta}) * \max\{1 - R(\boldsymbol{\beta})/n, 1/2\}. \end{aligned}$$

We have,

$$R^{TEL}(\boldsymbol{\beta}) = \begin{cases} R(\boldsymbol{\beta})[1 - R(\boldsymbol{\beta})/n] & R(\boldsymbol{\beta}) \leq n/2 \\ R(\boldsymbol{\beta})/2 & R(\boldsymbol{\beta}) > n/2. \end{cases}$$

The TEL shares the same asymptotic properties with the EL. See Jing et al. (2017) for further explanation.

Theorem 2.7 Assume that the regularity conditions hold. When $n \to \infty$, $-2\log R^{TEL}(\boldsymbol{\beta}_0) \xrightarrow{\mathcal{D}} \chi^2_p$.

The asymptotic $100(1-\alpha)\%$ TEL confidence region is as follows,

$$R_5 : \{\boldsymbol{\beta} : -2\log R^{TEL}(\boldsymbol{\beta}) \leq \chi^2_p(\alpha)\}.$$

Similar to before, we derive the Wilk's Theorem for the profile TEL ratio in Theorem 2.8.

Theorem 2.8 Under the regular conditions as $n \to \infty$,

$$l^{TEL}_{profile}(\boldsymbol{\beta}^{(1)}_0) \xrightarrow{\mathcal{D}} \chi^2_q.$$

Based on Theorem 2.8, we construct the asymptotic $100(1-\alpha)\%$ transformed empirical likelihood confidence regions for $\boldsymbol{\beta}_0^{(1)}$ as follows:

$$R_{profile}^{TEL}(\boldsymbol{\beta}^{(1)}) = \{\boldsymbol{\beta}^{(1)} : l_{profile}^{TEL}(\boldsymbol{\beta}^{(1)}) \leq \chi_q^2(\alpha)\}.$$

2.6 Transformed adjusted empirical likelihood confidence region for $\boldsymbol{\beta}$

In this section, we propose transformed adjusted empirical likelihood (TAEL) as a new method. TAEL is a combination of the AEL and TEL methods. We set $g_i = g_i(y_i; \boldsymbol{\beta})$ for $i = 1, ..., n$. For any $\boldsymbol{\beta}$ and for some positive a_n. By default, Chen et al. (2008) set a_n =log(n)/2. We define

$$g_{n+1} = \frac{-a_n}{n}\overline{g}_n,$$

where $\overline{g}_n = n^{-1}\sum_{i=1}^n g_i$. Thus, $R(\boldsymbol{\beta})$ can be re-defined as

$$R_{TEL}^*(\boldsymbol{\beta}) = \max\sum_{i=1}^{n+1}\log\{(n+1)p_i\},$$

subject to the constraints

$$\sum_{i=1}^{n+1} p_i = 1, \sum_{i=1}^{n+1} p_i g(Y_i, \boldsymbol{\beta}) = 0, p_i \geq 0, i = 1, ..., n+1.$$

For a constant $\gamma\epsilon[0, 1]$, we define

$$g_t^*(R^*(\boldsymbol{\beta}); \gamma) = R^*(\boldsymbol{\beta}) * \max\{1 - R^*(\boldsymbol{\beta})/(n+1), 1-\gamma\}.$$

Thus, $g_t^*(R^*(\boldsymbol{\beta}); \gamma)$ is a truncated quadratic transformation of $R^*(\boldsymbol{\beta})$ and as Jing et al. (2017) set the precedent, we will set the default value of $\gamma = 1/2$. Thus, we define the transformed adjusted empirical log-likelihood ratio, $R_{TEL}^*(\boldsymbol{\beta})$ as follows,

$$\begin{aligned} R_{TEL}^*(\boldsymbol{\beta}) &= g_t^*(R(\boldsymbol{\beta}); \gamma = 1/2) \\ &= R^*(\boldsymbol{\beta}) * \max\{1 - R^*(\boldsymbol{\beta})/(n+1), 1/2\}. \end{aligned}$$

We have,

$$R^*_{TEL}(\boldsymbol{\beta}) = \begin{cases} R^*(\boldsymbol{\beta})[1 - R^*(\boldsymbol{\beta})/(n+1)] & R^*(\boldsymbol{\beta}) \leq (n+1)/2 \\ R^*(\boldsymbol{\beta})/2 & R^*(\boldsymbol{\beta}) > (n+1)/2. \end{cases}$$

Theorem 2.9 Assume the regularity conditions hold. Denote $\boldsymbol{\beta}_0$ as the true parameter value. When $n \to \infty$, $-2\log R^*_t(\boldsymbol{\beta}_0)$ converges in distribution with χ^2_p.

The $100(1-\alpha)\%$ TAEL confidence region for $\boldsymbol{\beta}$ is constructed as follows:

$$R_6 : \{\boldsymbol{\beta} : -2\log R^*_{TEL}(\boldsymbol{\beta}) \leq \chi^2_p(\alpha)\}.$$

We derive the Wilk's Theorem for the profile TAEL ratio in Theorem 2.10.

Theorem 2.10 Under the regular conditions, as $n \to \infty$, we have

$$l^{TAEL}_{profile}(\boldsymbol{\beta}^{(1)}_0) \xrightarrow{\mathcal{D}} \chi^2_q.$$

Based on Theorem 2.10, we construct the asymptotic $100(1-\alpha)\%$ transformed adjusted empirical likelihood confidence regions for $\boldsymbol{\beta}^{(1)}_0$ as follows:

$$R^{TAEL}_{profile}(\boldsymbol{\beta}^{(1)}) = \left\{ l^{TAEL}_{profile}(\boldsymbol{\beta}^{(1)}) \leq \chi^2_q(\alpha) \right\}.$$

CHAPTER 3

JEL METHODOLOGY

In this chapter, we apply the jackknife empirical likelihood method (cf. Jing et al., 2009) to the Cox regression model. The jackknife empirical likelihood, adjusted jackknife empirical likelihood, mean jackknife empirical likelihood, transformed jackknife empirical likelihood, transformed adjusted jackknife empirical likelihood are proposed for the interval estimate of $\boldsymbol{\beta}$.

3.1 Jackknife empirical likelihood confidence region for $\boldsymbol{\beta}$

In this section, we propose a jackknife empirical likelihood method to make an inference on $\boldsymbol{\beta}$. The JEL method's foundation is the process of creating a sample mean from the statistic of interest based on jackknife pseudo-values (Jing et al. (2009)). Note $Z_i, i = 1, ..., n$ is bounded in the Cox regression model. Recall that $U(\boldsymbol{\beta})$ is the score function of $\boldsymbol{\beta}$.

$$
\begin{aligned}
U(\boldsymbol{\beta}) &= \frac{1}{n}\sum_{i=1}^{n}\int_0^T \left(Z_i - \frac{\hat{\alpha}_1(t,\boldsymbol{\beta}_0)}{\hat{\alpha}_0(t,\boldsymbol{\beta}_0)}\right) dN_i(t) \\
&= T(Z_1, ..., Z_n, \boldsymbol{\beta}).
\end{aligned}
$$

$$
\hat{W}_{jel,i}(\boldsymbol{\beta}) = n * U(\boldsymbol{\beta}) - (n-1) * U_{n-1}^{-i}(\boldsymbol{\beta}), i = 1, ..., n,
$$

where $U_{n-1}^{-i}(\boldsymbol{\beta}) := T(Z_1, ..., Z_{i-1}, Z_{i+1}, ..., Z_n, \boldsymbol{\beta})$ is computed on the $n-1$ samples formed from the original data set by deleting the ith observation. The jackknife estimator $\hat{T}_{nj}(\boldsymbol{\beta})$ is the average of all of the pseudo-values,

$$
\hat{T}_{nj}(\boldsymbol{\beta}) = \frac{1}{n}\sum_{i=1}^{n}\hat{W}_{jel,i}(\boldsymbol{\beta}).
$$

The jackknife empirical likelihood ratio at $\boldsymbol{\beta}$ is defined as

$$R(\boldsymbol{\beta}) = \max\Big\{ \prod_{i=1}^{n} np_i : \sum_{i=1}^{n} p_i \hat{W}_{jel,i}(\boldsymbol{\beta}) = 0, \sum_{i=1}^{n} p_i = 1, p_i \geq 0 \Big\}.$$

Using Lagrange multipliers we can solve for the jackknife empirical log-likelihood ratio,

$$-2\text{log}R(\boldsymbol{\beta}) = 2 \sum_{i=1}^{n} \text{log}\{1 + \lambda \hat{W}_{jel,i}(\boldsymbol{\beta})\},$$

where λ satisfies the following equation

$$f(\lambda) = \frac{1}{n} \sum_{i=1}^{n} \frac{\hat{W}_{jel,i}(\boldsymbol{\beta})}{1 + \lambda \hat{W}_{jel,i}(\boldsymbol{\beta})} = 0.$$

Then, we establish the Wilk's Theorem and use it to construct a confidence region for $\boldsymbol{\beta}$.

Theorem 3.1 Assume the regularity conditions hold. When $n \to \infty$, $-2\text{log}R(\boldsymbol{\beta}_0) \xrightarrow{\mathcal{D}} \chi^2_p$.

The $100(1-\alpha)\%$ JEL confidence region for $\boldsymbol{\beta}$ is constructed as follows:

$$R_7 = \{\boldsymbol{\beta} : -2\text{log}R(\boldsymbol{\beta}) \leq \chi^2_p(\alpha)\}.$$

If we are interested in only one component of the regression parameter, we can tackle the nuisance parameter by profiling the empirical likelihood. Define $\boldsymbol{\beta} = (\boldsymbol{\beta}^{(1)'}, \boldsymbol{\beta}^{(2)'})^T$, where $\boldsymbol{\beta}^{(1)} \in R^q$ and $\boldsymbol{\beta}^{(2)} \in R^{p-q}$. This is similar to Yang and Zhao (2012), where we define the profile JEL ratio and log-likelihood ratio as follows:

$$R^*(\boldsymbol{\beta}^{(1)}) = \max_{\boldsymbol{\beta}^{(2)}} R(\boldsymbol{\beta})$$

and

$$l^*(\boldsymbol{\beta}^{(1)}) = -2\text{log}R^*(\boldsymbol{\beta}^{(1)}).$$

We derive the Wilk's theorem for the profile JEL ratio.

Theorem 3.2 Under the regular conditions, as $n \to \infty$,

$$l_{profile}(\boldsymbol{\beta}_0^{(1)}) \xrightarrow{\mathcal{D}} \chi_q^2.$$

Based on Theorem 3.2, we construct the asymptotic $100(1-\alpha)\%$ jackknife empirical likelihood confidence regions for $\boldsymbol{\beta}_0^{(1)}$ as follows:

$$R_{profile}^{JEL}(\alpha) = \{\boldsymbol{\beta}^{(1)} : -2\log R^*(\boldsymbol{\beta}_1) \leq \chi_q^2(\alpha)\}.$$

3.2 Adjusted jackknife empirical likelihood confidence region for $\boldsymbol{\beta}$

The adjusted empirical likelihood (AJEL) was proposed by Chen et al. (2008). The AJEL is an improvement on the original method according to Zhen and Yu (2013). The AJEL method can avoid the convex hull restrictions that coincide with the JEL method. The AJEL function for $\boldsymbol{\beta}$ is as follows:

$$h_i^{ad}(\boldsymbol{\beta}) = \hat{W}_{jel,i}(\boldsymbol{\beta}), i = 1, ..., n,$$

where $h_{n+1}^{ad}(\boldsymbol{\beta}) = -a_n \bar{h}^{ad}(\boldsymbol{\beta}) = -a_n \sum_{i=1}^{n} \hat{W}_{jel,i}(\boldsymbol{\beta})/n$ and a_n is a constant number,

$$a_n = \max\left(1, \log(n)/2\right).$$

Thus, with the $(n+1)$ data points, the adjusted jackknife empirical likelihood ratio at $\boldsymbol{\beta}$ is

$$R^{adj}(\boldsymbol{\beta}) = \max\left\{ \prod_{i=1}^{n+1} (n+1)\, p_i, \sum_{i=1}^{n+1} p_i = 1, \sum_{i=1}^{n+1} p_i h_i^{ad}(\boldsymbol{\beta}) = 0, p_i \geq 0 \right\}.$$

Employing Lagrange multipliers method, we obtain the adjusted jackknife empirical log-likelihood ratio,

$$-2\log R^{adj}(\boldsymbol{\beta}) = 2\sum_{i=1}^{n+1} \log\{1 + \lambda h_i^{ad}(\boldsymbol{\beta})\},$$

where λ satisfies the following equation,

$$f(\lambda) = \frac{1}{n+1}\sum_{i=1}^{n+1}\frac{h_i^{ad}(\boldsymbol{\beta})}{1+\lambda h_i^{ad}(\boldsymbol{\beta})} = 0.$$

Incorporating the approaches of Chen at el. (2008) and Jing et al. (2009), we have that as $n \to \infty$ the Wilk's theorem holds.

Theorem 3.3 Assume the regularity conditions. Denote $\boldsymbol{\beta}_0$ as the true parameter value. When $n \to \infty$, $-2\log R^{adj}(\boldsymbol{\beta}_0) \xrightarrow{\mathcal{D}} \chi_q^2$.

The $100(1-\alpha)\%$ AJEL confidence region for $\boldsymbol{\beta}$ is constructed as follows:

$$R_8 = \{\boldsymbol{\beta} : -2\log R^{adj}(\boldsymbol{\beta}) \leq \chi_p^2(\alpha)\}.$$

Similar to before, we derive the Wilk's Theorem for the profile AJEL ratio.

Theorem 3.4 Under the regular conditions, as $n \to \infty$,

$$l_{profile}^{adj}(\boldsymbol{\beta}_0^{(1)}) \xrightarrow{\mathcal{D}} \chi_q^2.$$

Based on Theorem 3.4, we construct the asymptotic $100(1-\alpha)\%$ adjusted jackknife empirical likelihood confidence regions for $\boldsymbol{\beta}_0^{(1)}$ as follows:

$$R_{profile}^{adj}(\alpha) = \{\boldsymbol{\beta}^{(1)} : l_{profile}^{ad}(\boldsymbol{\beta}^{(1)}) \leq \chi_q^2(\alpha)\}.$$

3.3 Mean jackknife empirical likelihood confidence region for $\boldsymbol{\beta}$

For this thesis, we define the mean jackknife empirical likelihood (MJEL) pseudo value as,

$$S_i(\boldsymbol{\beta}) = \frac{\hat{W}_{jel;j}(\boldsymbol{\beta}) + \hat{W}_{jel;k}(\boldsymbol{\beta})}{2}, i = 1, ..., N, 1 \leq j \leq k \leq n,$$

where $N = n(n+1)/2$.

Thus, the mean jackknife empirical likelihood ratio at $\boldsymbol{\beta}$ is,

$$R^{mjel} = \max\Big\{\prod_{i=1}^{N} Np_i : \sum_{i=1}^{N} p_i = 1, \sum_{i=1}^{N} p_i S_i(\boldsymbol{\beta}) = 0, p_i \geq 0\Big\}.$$

Now we have the mean jackknife empirical log-likelihood ratio as follows,

$$\frac{-2\text{log}R^{mjel}(\boldsymbol{\beta})}{n+1} = \frac{2}{n+1}\sum_{i=1}^{N} \log\{1+\lambda S_i(\boldsymbol{\beta})\},$$

where λ satisfies the equation

$$f(\lambda) = \sum_{i=1}^{N} \frac{S_i(\boldsymbol{\beta})}{1+\lambda S_i(\boldsymbol{\beta})} = 0.$$

Denote $\boldsymbol{\beta}_0$ as the true value of $\boldsymbol{\beta}$. Employing the method by Liang et al. (2019), we establish the following theorem.

Theorem 3.5 Assume the regularity conditions. When $n \to \infty$, $-2\text{log}R^{mjel}(\boldsymbol{\beta}_0)/(n+1)$ converges in distribution to χ^2_p.

The $100(1-\alpha)\%$ MJEL confidence region for $\boldsymbol{\beta}$ is constructed as follows:

$$R_9 = \Big\{\boldsymbol{\beta} : \frac{-2\text{log}R^{mjel}(\boldsymbol{\beta})}{n+1} \leq \chi^2_p(\alpha)\Big\}.$$

We derive the Wilk's Theorem for the profile MJEL ratio in Theorem 3.6.

Theorem 3.6 Under the regular conditions as $n \to \infty$,

$$l^{mjel}_{profile}(\boldsymbol{\beta}^{(1)}_0) \xrightarrow{\mathcal{D}} \chi^2_q.$$

Based on Theorem 3.6, we construct the asymptotic $100(1-\alpha)\%$ mean jackknife empirical likelihood confidence regions for $\boldsymbol{\beta}_0^{(1)}$ as follows:

$$R_{profile}^{mjel}(\boldsymbol{\beta}^{(1)}) = \Big\{\frac{-2\log R_{profile}^{mjel}(\boldsymbol{\beta}^{(1)})}{n+1} \leq \chi_q^2(\alpha)\Big\}.$$

3.4 Transformed jackknife empirical likelihood confidence region for $\boldsymbol{\beta}$

Jing et al. (2017) proposed a simple transformed EL to improve the coverage probability of the original EL method. Recall that the original empirical log-likelihood ratio is denoted as $R(\boldsymbol{\beta})$. For a constant $\gamma\epsilon[0,1]$, we define

$$g_t(R(\boldsymbol{\beta});\gamma) = R(\boldsymbol{\beta}) \times \max\{1 - R(\boldsymbol{\beta})/n, 1-\gamma\}.$$

We will refer to $g_t(R(\theta);\gamma)$ as the truncated quadratic transformation of $R(\boldsymbol{\beta})$. Jing et al. (2017) set $\gamma = 1/2$. Then, we have the transformed empirical log-likelihood ratio, $R_t(\boldsymbol{\beta})$ as

$$\begin{aligned} R_t(\boldsymbol{\beta}) &= g_t(R(\boldsymbol{\beta}); \gamma = 1/2) \\ &= R(\boldsymbol{\beta}) * \max\{1 - R(\boldsymbol{\beta})/n, 1/2\}. \end{aligned}$$

We have,

$$R_t(\boldsymbol{\beta}) = \begin{cases} R(\boldsymbol{\beta})[1 - R(\boldsymbol{\beta})/n] & R(\boldsymbol{\beta}) \leq n/2 \\ R(\boldsymbol{\beta})/2 & R(\boldsymbol{\beta}) > n/2. \end{cases}$$

The TEL shares the same asymptotic properties with the EL. See Jing et al. (2017) for further explanation.

Theorem 3.7 Assume that the regularity conditions hold. When $n \to \infty$, $-2\log R_t(\boldsymbol{\beta}_0) \xrightarrow{\mathcal{D}} \chi_p^2$.

The asymptotic $100(1-\alpha)\%$ TJEL confidence region is as follows,

$$R_{10} : \{\boldsymbol{\beta} : -2\log R_t(\boldsymbol{\beta}) \leq \chi_p^2(\alpha)\}.$$

Similarly, we derive the Wilk's Theorem for the profile TJEL ratio in Theorem 3.8.

Theorem 3.8 Under the regular conditions as $n \to \infty$,

$$l^{TJEL}_{profile}(\boldsymbol{\beta}^{(1)}_0) \xrightarrow{\mathcal{D}} \chi^2_q.$$

Based on Theorem 3.8, we construct the asymptotic $100(1-\alpha)\%$ transformed jackknife empirical likelihood confidence regions for $\boldsymbol{\beta}^{(1)}_0$ as follows:

$$R^{TJEL}_{profile}(\boldsymbol{\beta}^{(1)}) = \{\boldsymbol{\beta}^{(1)} : l^{TJEL}_{profile}(\boldsymbol{\beta}^{(1)}) \leq \chi^2_q(\alpha)\}.$$

3.5 Transformed adjusted jackknife empirical likelihood confidence region for $\boldsymbol{\beta}$

In this section, we propose transformed adjusted jackknife empirical likelihood (TAJEL) as a new method. TAJEL is a combination of the AJEL and TJEL methods. We set $g_i = g_i(y_i; \boldsymbol{\beta})$ for $i = 1, ..., n$. For any $\boldsymbol{\beta}$ and for some positive a_n. By default, Chen et al. (2008) set a_n =log$(n)/2$. We define

$$g_{n+1} = \frac{-a_n}{n}\overline{g}_n,$$

where $\overline{g}_n = n^{-1}\sum_{i=1}^n g_i$. Thus, $R(\boldsymbol{\beta})$ can be re-defined as

$$R^*_t(\boldsymbol{\beta}) = \max \sum_{i=1}^{n+1} \log\{(n+1)p_i\},$$

subject to the constraints

$$\sum_{i=1}^{n+1} p_i = 1, \sum_{i=1}^{n+1} p_i g(Y_i, \boldsymbol{\beta}) = 0, p_i \geq 0, i = 1, ..., n+1.$$

For a constant $\gamma\epsilon[0,1]$, we define

$$g^*_t(R^*(\boldsymbol{\beta}); \gamma) = R^*(\boldsymbol{\beta}) * \max\{1 - R^*(\boldsymbol{\beta})/(n+1), 1-\gamma\}.$$

Thus, $g_t^*(R^*(\boldsymbol{\beta});\gamma)$ is a truncated quadratic transformation of $R^*(\boldsymbol{\beta})$. Jing et al. (2017) sets the default value of $\gamma = 1/2$. Thus, we define the transformed adjusted jackknife empirical log-likelihood ratio, $R_t^*(\boldsymbol{\beta})$ as follows,

$$\begin{aligned} R_t^*(\boldsymbol{\beta}) &= g_t^*(R(\boldsymbol{\beta});\gamma = 1/2) \\ &= R^*(\boldsymbol{\beta}) * \max\{1 - R^*(\boldsymbol{\beta})/(n+1), 1/2\}. \end{aligned}$$

We have,

$$R_t^*(\boldsymbol{\beta}) = \begin{cases} R^*(\boldsymbol{\beta})[1 - R^*(\boldsymbol{\beta})/(n+1)] & R^*(\boldsymbol{\beta}) \leq (n+1)/2 \\ R^*(\boldsymbol{\beta})/2 & R^*(\boldsymbol{\beta}) > (n+1)/2. \end{cases}$$

Theorem 3.9 Assume the regularity conditions hold. When $n \to \infty$, $-2\log R_t^*(\boldsymbol{\beta}_0)$ converges in distribution with χ_p^2.

The $100(1-\alpha)\%$ TAJEL confidence region for $\boldsymbol{\beta}$ is constructed as follows:

$$R_{11} : \{\boldsymbol{\beta} : -2\log R_t^*(\boldsymbol{\beta}) \leq \chi_p^2(\alpha)\}.$$

We derive the Wilk's Theorem for the profile TAJEL ratio.

Theorem 3.10 Under the regular conditions, as $n \to \infty$, we have

$$l_{profile}^{TAJEL}(\boldsymbol{\beta}_0^{(1)}) \xrightarrow{\mathcal{D}} \chi_q^2.$$

We construct the asymptotic $100(1-\alpha)\%$ transformed adjusted jackknife empirical likelihood confidence regions for $\boldsymbol{\beta}_0^{(1)}$ as follows:

$$R_{profile}^{TAJEL}(\boldsymbol{\beta}^{(1)}) = \left\{ l_{profile}^{TAJEL}(\boldsymbol{\beta}^{(1)}) \leq \chi_q^2(\alpha) \right\}.$$

CHAPTER 4

SIMULATION STUDY

In this chapter, we report the proposed NA, EL, JEL, AJEL, MJEL, TJEL and TAJEL confidence regions by performing simulation studies. We want to compare the performance of the JEL, AJEL, MJEL, TJEL, TAJEL, AEL, MEL, TEL and TAEL methods in relation to the NA and EL methods.

4.1 Coverage probabilities

For the simulation studies, we will work with two-dimensional covariates. The model will be the Cox regression model:

$$\lambda(t|Z) = \lambda_0(t) exp(\boldsymbol{\beta_0} Z),$$

with $\boldsymbol{\beta_0} = (1,1)^t$, $\lambda_0(t) = 1$ corresponds to the hazard function of the standard exponential distribution, $\lambda_0(t) = \frac{(1/t)\phi(\ln t)}{\Phi(-\ln t)}$ corresponds to the hazard function of the log-normal distribution where mean $\mu = 0$ and variance $\sigma^2 = 1$, and $\lambda_0(t) = 2\sqrt{t}$ corresponds to the hazard function of the Weibull distribution with scale parameter $\frac{4}{3}$ and shape parameter $\frac{3}{2}$, respectively. Censoring rate (CR) is chosen to be 10%, 40%, and 70%. The proportion of simulated censored data is in the range of ± 0.05 of the CR. Sample size n is chosen to be 50, 100, 150, 200. This simulated data is generated with $1,000$ repetitions.

The estimated coverage probabilities for the NA, EL, JEL, AJEL, MJEL, TJEL, TAJEL, AEL, MEL, TEL and TAEL methods are the proportions of the data set that satisfy the inequalities R_1, R_2, R_3, R_4, R_5, R_6, R_7, R_8, R_9, R_{10}, and R_{11} respectively. To calculate the confidence region for the various methods the same simulated data is used. Confidence regions are created with the upper time limit $T = 5$, where all the observed failure times or censored times are less than or equal to T. The nominal confidence level $1 - \alpha$ is 0.95.

We profile the nuisance parameter β_2 to get CI for β_1. The coverage probabilities with the different combinations for sample size, censoring rate, and confidence levels are reported in Tables 4.1 and 4.4 for the Weibull distribution, respectively. Tables 4.2 and 4.5 report the results for the Exponential distribution and Tables 4.3 and 4.6 report the results for the Log-normal distribution, respectively.

4.2 Comparison of coverage accuracy and average interval lengths

Analyzing the tables we can compare the coverage probabilities under the various distributions. We expect the coverage probability to be better for EL and JEL methods than for the NA method. We can compare the average interval length under the various distributions. It is known that the lower average interval length is better. The general expectation is that an increase in censoring rate results in the worse coverage. We expect the interval length will decrease as the sample size increases for each censoring rate.

In Table 4.1 we can analyze the EL methods for the Weibull distribution and see that the MEL, TEL, and TAEL methods have over coverage. Table 4.2 shows that the NA, EL, and TAEL methods have the most instances of over coverage for the Exponential distribution The TAEL method has the longest lengths in this table. Notably, all methods, other than the EL method, have over coverage for a censoring rate of 70 %. The MEL method appears to be the closest to the nominal level making it the best performing method for the Exponential distribution. The Log-normal distribution EL results are reported in Table 4.3. Here we see the the TEL method has the shortest lengths and lowest coverage probabilities. The TAEL method appears to perform best for small sample sizes. MEL appears to perform the best.

In Table 4.4 we can see that all of the JEL methods have instances of over coverage. MJEL and AJEL methods appear to have the least over coverage and thus seem the best performing methods of the Weibull distribution. In Tables 4.5 and 4.6 we can see the results for the Exponential distribution and Log-normal distribution JEL methods, respectively. Overall, the results for the Exponential distribution appear more nominal than the Log-normal distribution. For the Exponential distribution, MJEL and TAJEL methods have the

Table (4.1) Coverage probabilities and average interval lengths for EL methods with weibull distribution

			CL =95%					
n	CR		NA	EL	AEL	MEL	TEL	$TAEL$
50	10	β_1	.936 (2.207)	.922 (2.076)	.936 (2.174)	.937 (2.208)	.937 (2.184)	.947 (2.285)
		β_2	.939 (2.198)	.905 (2.096)	.901 (2.354)	.917 (2.355)	.914 (2.327)	.924 (2.438)
100	10	β_1	.945 (1.517)	.944 (1.472)	.947 (1.510)	.947 (1.526)	.947 (1.504)	.949 (1.542)
		β_2	.949 (1.516)	.935 (1.475)	.913 (1.555)	.925 (1.551)	.923 (1.527)	.929 (1.566)
150	10	β_1	.950 (1.223)	.946 (1.205)	.951 (1.227)	.953 (1.236)	.950 (1.221)	.920 (1.249)
		β_2	.939 (1.224)	.939 (1.206)	.917 (1.244)	.921 (1.242)	.918 (1.227)	.920 (1.249)
200	10	β_1	.939 (1.055)	.936 (1.046)	.937 (1.061)	.938 (1.067)	.937 (1.056)	.938 (1.071)
		β_2	.944 (1.055)	.928 (1.044)	.920 (1.070)	.920 (1.068)	.920 (1.057)	.923 (1.072)
50	40	β_1	.941 (2.441)	.933 (2.340)	.945 (2.452)	.947 (2.487)	.945 (2.463)	.951 (2.579)
		β_2	.930 (2.445)	.910 (2.332)	.943 (2.375)	.951 (1.913)	.950 (2.742)	.959 (2.883)
100	40	β_1	.945 (1.654)	.946 (1.631)	.956 (1.673)	.958 (1.690)	.955 (1.666)	.957 (1.709)
		β_2	.939 (1.641)	.923 (1.619)	.941 (1.368)	.949 (1.698)	.956 (1.715)	.961 (1.761)
150	40	β_1	.957 (1.332)	.956 (1.326)	.959 (1.350)	.962 (1.359)	.959 (1.344)	.964 (1.369)
		β_2	.955 (1.329)	.941 (1.306)	.939 (1.359)	.952 (1.303)	.960 (1.439)	.965 (1.374)
200	40	β_1	.954 (1.149)	.953 (1.142)	.956 (1.158)	.958 (1.162)	.956 (1.153)	.959 (1.170)
		β_2	.939 (1.144)	.935 (1.122)	.944 (1.128)	.957 (1.161)	.948 (1.148)	.941 (1.165)
50	70	β_1	.959 (3.140)	.958 (3.268)	.965 (3.432)	.964 (3.485)	.965 (3.451)	.972 (3.627)
		β_2	.956 (3.187)	.927 (3.186)	.944 (3.013)	.944 (3.060)	.951 (2.954)	.961 (3.430)
100	70	β_1	.963 (2.121)	.966 (2.179)	.969 (2.235)	.969 (2.240)	.968 (2.226)	.970 (2.284)
		β_2	.951 (2.110)	.930 (2.105)	.943 (2.251)	.949 (2.103)	.952 (2.083)	.946 (2.198)
150	70	β_1	.959 (1.695)	.961 (1.732)	.963 (1.764)	.962 (1.764)	.961 (1.756)	.967 (1.788)
		β_2	.943 (1.691)	.931 (1.670)	.952 (1.793)	.963 (1.771)	.948 (1.248)	.951 (1.504)
200	70	β_1	.955 (1.459)	.957 (1.483)	.959 (1.504)	.960 (1.501)	.958 (1.498)	.959 (1.519)
		β_2	.940 (1.453)	.949 (1.421)	.961 (1.544)	.961 (1.508)	.954 (1.394)	.944 (1.589)

Table (4.2) Coverage probabilities and average interval lengths for EL methods with exponential distribution

			CL =95%					
n	CR		NA	EL	AEL	MEL	TEL	$TAEL$
50	10	β_1	.948 (2.194)	.937 (2.105)	.944 (2.205)	.945 (2.241)	.946 (2.214)	.952 (2.317)
		β_2	.946 (2.200)	.918 (2.114)	.904 (2.295)	.928 (2.296)	.928 (2.272)	.940 (2.379)
100	10	β_1	.952 (1.496)	.948 (1.468)	.954 (1.506)	.956 (1.522)	.953 (1.500)	.961 (1.538)
		β_2	.956 (1.502)	.933 (1.480)	.920 (1.493)	.936 (1.500)	.933 (1.476)	.939 (1.514)
150	10	β_1	.948 (1.218)	.942 (1.207)	.945 (1.228)	.946 (1.238)	.915 (1.217)	.946 (1.245)
		β_2	.947 (1.212)	.939 (1.210)	.905 (1.215)	.915 (1.217)	.916 (1.204)	.921 (1.225)
200	10	β_1	.943 (1.046)	.943 (1.041)	.946 (1.055)	.950 (1.061)	.946 (1.051)	.947 (1.066)
		β_2	.943 (1.046)	.939 (1.041)	.921 (1.039)	.923 (1.042)	.921 (1.031)	.927 (1.046)
50	40	β_1	.929 (2.432)	.917 (2.408)	.934 (2.524)	.936 (2.555)	.934 (2.534)	.944 (2.655)
		β_2	.956 (2.411)	.906 (2.414)	.935 (2.766)	.934 (2.705)	.934 (2.692)	.937 (2.838)
100	40	β_1	.953 (1.633)	.945 (1.640)	.955 (1.682)	.956 (1.694)	.951 (1.676)	.961 (1.718)
		β_2	.947 (1.625)	.940 (1.636)	.952 (1.666)	.948 (1.650)	.941 (1.636)	.953 (1.679)
150	40	β_1	.952 (1.322)	.941 (1.334)	.946 (1.358)	.948 (1.363)	.946 (1.352)	.953 (1.376)
		β_2	.951 (1.312)	.936 (1.328)	.940 (1.300)	.940 (1.299)	.939 (1.290)	.943 (1.314)
200	40	β_1	.952 (1.133)	.945 (1.151)	.952 (1.167)	.950 (1.170)	.947 (1.162)	.953 (1.179)
		β_2	.947 (1.134)	.938 (1.141)	.947 (1.103)	.946 (1.090)	.945 (1.087)	.947 (1.103)
50	70	β_1	.951 (2.852)	.942 (2.982)	.952 (3.128)	.957 (3.155)	.953 (3.143)	.964 (3.298)
		β_2	.951 (2.852)	.933 (2.949)	.949 (3.010)	.960 (3.973)	.952 (3.025)	.965 (3.594)
100	70	β_1	.952 (1.943)	.948 (2.006)	.955 (2.057)	.955 (2.057)	.952 (2.049)	.956 (2.102)
		β_2	.949 (1.943)	.938 (1.978)	.957 (2.544)	.958 (2.466)	.954 (2.472)	.959 (2.545)
150	70	β_1	.946 (1.569)	.947 (1.610)	.951 (1.639)	.949 (1.636)	.949 (1.631)	.955 (1.661)
		β_2	.951 (1.564)	.935 (1.587)	.953 (1.841)	.950 (1.801)	.951 (1.803)	.957 (1.838)
200	70	β_1	.947 (1.356)	.943 (1.391)	.945 (1.411)	.945 (1.406)	.944 (1.404)	.946 (1.425)
		β_2	.953 (1.351)	.934 (1.369)	.949 (1.530)	.947 (1.497)	.947 (1.499)	.948 (1.521)

Table (4.3) Coverage probabilities and average interval lengths for EL methods with log-normal distribution

			CL =95%					
n	CR		NA	EL	AEL	MEL	TEL	$TAEL$
50	10	β_1	.938 (2.150)	.919 (2.051)	.930 (2.149)	.935 (2.180)	.933 (2.157)	.943 (2.257)
		β_2	.942 (2.148)	.905 (2.073)	.905 (2.214)	.934 (2.229)	.932 (2.200)	.941 (2.304)
100	10	β_1	.934 (1.469)	.937 (1.441)	.943 (1.478)	.947 (1.496)	.943 (1.472)	.947 (1.510)
		β_2	.927 (1.468)	.924 (1.457)	.913 (1.471)	.926 (1.480)	.921 (1.456)	.928 (1.493)
150	10	β_1	.953 (1.176)	.946 (1.168)	.949 (1.189)	.948 (1.199)	.948 (1.184)	.951 (1.205)
		β_2	.945 (1.176)	.944 (1.172)	.934 (1.178)	.945 (1.183)	.940 (1.167)	.949 (1.188)
200	10	β_1	.951 (1.018)	.947 (1.011)	.950 (1.025)	.950 (1.031)	.949 (1.021)	.952 (1.036)
		β_2	.944 (1.017)	.941 (1.014)	.921 (1.012)	.931 (1.015)	.928 (1.005)	.934 (1.019)
50	40	β_1	.950 (2.145)	.934 (2.080)	.946 (2.179)	.950 (2.212)	.947 (2.188)	.954 (2.289)
		β_2	.941 (2.145)	.924 (2.101)	.933 (2.144)	.939 (2.241)	.938 (2.212)	.940 (2.317)
100	40	β_1	.943 (1.479)	.937 (1.460)	.943 (1.498)	.945 (1.513)	.943 (1.492)	.951 (1.530)
		β_2	.934 (1.482)	.931 (1.469)	.932 (1.477)	.933 (1.481)	.930 (1.459)	.936 (1.496)
150	40	β_1	.944 (1.198)	.943 (1.192)	.946 (1.214)	.949 (1.223)	.944 (1.208)	.950 (1.230)
		β_2	.959 (1.199)	.937 (1.198)	.934 (1.167)	.936 (1.171)	.935 (1.158)	.938 (1.178)
200	40	β_1	.947 (1.034)	.941 (1.034)	.946 (1.048)	.946 (1.053)	.945 (1.044)	.947 (1.059)
		β_2	.935 (1.032)	.936 (1.035)	.934 (1.011)	.934 (1.012)	.932 (1.003)	.934 (1.017)
50	70	β_1	.930 (2.225)	.924 (2.218)	.932 (2.325)	.935 (2.358)	.932 (2.334)	.939 (2.443)
		β_2	.943 (2.222)	.905 (2.233)	.931 (2.323)	.930 (2.312)	.929 (2.291)	.933 (2.402)
100	70	β_1	.960 (1.527)	.954 (1.543)	.960 (1.583)	.963 (1.595)	.958 (1.576)	.962 (1.616)
		β_2	.955 (1.527)	.944 (1.551)	.943 (1.467)	.942 (1.460)	.940 (1.448)	.948 (1.485)
150	70	β_1	.942 (1.235)	.945 (1.259)	.948 (1.282)	.948 (1.288)	.947 (1.276)	.950 (1.299)
		β_2	.941 (1.623)	.938 (1.258)	.935 (1.157)	.935 (1.155)	.934 (1.145)	.937 (1.166)
200	70	β_1	.951 (1.066)	.950 (1.083)	.955 (1.099)	.954 (1.102)	.954 (1.094)	.957 (1.109)
		β_2	.943 (1.064)	.947 (1.082)	.942 (0.993)	.942 (0.991)	.941 (0.984)	.943 (0.998)

Table (4.4) Coverage probabilities and average interval lengths for JEL methods with weibull distribution

			CL =95%						
n	CR		NA	EL	JEL	$AJEL$	$MJEL$	$TJEL$	$TAJEL$
50	10	β_1	.935 (2.214)	.926 (2.095)	.923 (2.205)	.934 (2.310)	.932 (2.355)	.934 (2.321)	.942 (2.420)
		β_2	.949 (2.202)	.903 (2.104)	.931 (2.234)	.939 (2.453)	.941 (2.511)	.939 (2.467)	.949 (2.587)
100	10	β_1	.936 (1.522)	.937 (1.483)	.926 (1.522)	.934 (1.561)	.935 (1.584)	.933 (1.555)	.940 (1.595)
		β_2	.930 (1.516)	.917 (1.485)	.921 (1.513)	.923 (1.595)	.927 (1.622)	.923 (1.589)	.927 (1.629)
150	10	β_1	.950 (1.229)	.945 (1.207)	.951 (1.227)	.956 (1.249)	.957 (1.264)	.955 (1.244)	.956 (1.266)
		β_2	.940 (1.227)	.937 (1.204)	.932(1.224)	.938 (1.270)	.937 (1.286)	.935 (1.265)	.940 (1.288)
200	10	β_1	.955 (1.057)	.953 (1.044)	.949 (1.056)	.951 (1.070)	.953 (1.079)	.950 (1.066)	.953 (1.081)
		β_2	.948 (1.055)	.946 (1.040)	.941 (1.030)	.943 (1.082)	.942 (1.092)	.942 (1.078)	.953 (1.081)
50	40	β_1	.941 (2.447)	.916 (2.364)	.914 (2.505)	.924 (2.626)	.924 (2.672)	.924 (2.638)	.933 (2.764)
		β_2	.941 (2.447)	.900 (2.349)	.929 (2.508)	.939 (2.820)	.942 (2.887)	.941 (2.837)	.952 (2.979)
100	40	β_1	.952 (1.652)	.944 (1.626)	.938 (1.692)	.942 (1.736)	.943 (1.757)	.941 (1.729)	.945 (1.774)
		β_2	.952 (1.648)	.930 (1.620)	.943 (1.710)	.950 (1.785)	.951 (1.813)	.950 (1.779)	.954 (1.825)
150	40	β_1	.959 (1.336)	.955 (1.324)	.945 (1.364)	.948 (1.388)	.950 (1.403)	.947 (1.382)	.954 (1.407)
		β_2	.953 (1.330)	.931 (1.301)	.953 (1.334)	.960 (1.414)	.957 (1.430)	.958 (1.408)	.961 (1.433)
200	40	β_1	.943 (1.150)	.948 (1.145)	.945 (1.171)	.948 (1.188)	.950 (1.196)	.948 (1.183)	.948 (1.200)
		β_2	.937 (1.144)	.926 (1.121)	.931 (1.135)	.936 (1.203)	.950 (1.1960)	.935 (1.198)	.937 (1.215)
50	70	β_1	.949 (3.230)	.945 (3.355)	.934 (3.717)	.944 (3.913)	.946 (3.970)	.944 (3.932)	.954 (4.139)
		β_2	.940 (3.214)	.916 (3.245)	.921 (3.792)	.928 (5.891)	.93.2 (5.906)	.929 (6.319)	.941 (6.720)
100	70	β_1	.967 (2.114)	.969 (2.171)	.954 (2.357)	.966 (2.420)	.963 (2.432)	.964 (2.409)	.970 (2.472)
		β_2	.944 (2.101)	.943 (2.107)	.935 (2.340)	.941 (2.542)	.943 (2.560)	.941 (2.531)	.946 (2.598)
150	70	β_1	.968 (1.697)	.967 (1.736)	.953 (1.868)	.956 (1.903)	.955 (1.908)	.954 (1.894)	.958 (1.928)
		β_2	.945 (1.688)	.933 (1.665)	.935 (1.843)	.939 (1.959)	.939 (1.966)	.938 (1.950)	.943 (1.986)
200	70	β_1	.963 (1.460)	.963 (1.483)	.944 (1.594)	.944 (1.616)	.945 (1.619)	.944 (1.609)	.945 (1.632)
		β_2	.946 (1.457)	.931 (1.426)	.940 (1.573)	.946 (1.654)	.944 (1.658)	.942 (1.646)	.951 (1.670)

Table (4.5) Coverage probabilities and average interval lengths for JEL methods with exponential distribution

			CL =95%						
n	CR		NA	EL	JEL	$AJEL$	$MJEL$	$TJEL$	$TAJEL$
50	10	β_1	.944 (2.194)	.926 (2.099)	.911 (2.290)	.921 (2.399)	.927 (2.450)	.922 (2.411)	.935 (2.525)
		β_2	.944 (2.190)	.909 (2.109)	.922 (2.316)	.931 (2.535)	.932 (2.599)	.931 (2.550)	.941 (2.674)
100	10	β_1	.944 (1.504)	.927 (1.479)	.927 (1.577)	.934 (1.618)	.936 (1.644)	.933 (1.612)	.939 (1.653)
		β_2	.936 (1.504)	.919 (1.485)	.926 (1.559)	.929 (1.656)	.933 (1.687)	.929 (1.650)	.939 (1.691)
150	10	β_1	.953 (1.216)	.950 (1.202)	.943 (1.263)	.946 (1.286)	.948 (1.301)	.944 (1.281)	.951 (1.304)
		β_2	.951 (1.212)	.947 (1.208)	.946 (1.233)	.950 (1.304)	.951 (1.321)	.948 (1.298)	.953 (1.322)
200	10	β_1	.938 (1.049)	.940 (1.043)	.937 (1.092)	.939 (1.107)	.938 (1.117)	.938 (1.103)	.940 (1.118)
		β_2	.941 (1.045)	.933 (1.043)	.948 (1.089)	.950 (1.127)	.952 (1.139)	.949 (1.122)	.951 (1.138)
50	40	β_1	.946 (2.430)	.932 (2.412)	.906 (2.751)	.926 (2.885)	.924 (2.945)	.927 (2.899)	.940 (3.039)
		β_2	.934 (2.433)	.913 (2.401)	.906 (2.751)	.918 (3.131)	.923 (3.366)	.918 (3.153)	.928 (3.328)
100	40	β_1	.944 (1.627)	.945 (1.648)	.935 (1.839)	.942 (1.887)	.943 (1.917)	.941 (1.880)	.947 (1.929)
		β_2	.955 (1.627)	.933 (1.649)	.939 (1.889)	.946 (1.917)	.945 (1.947)	.944 (1.910)	.952 (1.959)
150	40	β_1	.953 (1.316)	.956 (1.328)	.939 (1.473)	.945 (1.500)	.944 (1.518)	.942 (1.494)	.951 (1.521)
		β_2	.933 (1.311)	.945 (1.320)	.932 (1.461)	.933 (1.522)	.936 (1.538)	.933 (1.515)	.939 (1.543)
200	40	β_1	.938 (1.136)	.933 (1.154)	.937 (1.272)	.940 (1.290)	.944 (1.300)	.939 (1.285)	.943 (1.303)
		β_2	.949 (1.135)	.933 (1.145)	.937 (1.260)	.943 (1.303)	.944 (1.315)	.942 (1.298)	.945 (1.316)
50	70	β_1	.947 (2.865)	.938 (3.000)	.917 (2.946)	.934 (3.726)	.935 (3.785)	.936 (3.743)	.945 (3.937)
		β_2	.944 (2.867)	.917 (2.946)	.925 (3.612)	.935 (4.616)	.936 (4.716)	.937 (4.643)	.944 (4.938)
100	70	β_1	.956 (1.943)	.958 (2.013)	.939 (2.324)	.943 (2.386)	.943 (2.404)	.942 (2.376)	.952 (2.438)
		β_2	.934 (1.932)	.944 (1.983)	.929 (2.341)	.937 (2.513)	.937 (2.537)	.936 (2.503)	.942 (2.570)
150	70	β_1	.944 (1.571)	.946 (1.614)	.925 (1.855)	.931 (1.889)	.932 (1.899)	.931 (1.880)	.932 (1.915)
		β_2	.948 (1.566)	.937 (1.588)	.946 (1.827)	.948 (1.940)	.951 (1.949)	.948 (1.931)	.952 (1.966)
200	70	β_1	.942 (1.353)	.945 (1.385)	.927 (1.589)	.931 (1.612)	.931 (1.617)	.930 (1.605)	.936 (1.628)
		β_2	.947 (1.350)	.936 (1.366)	.945 (1.592)	.948 (1.639)	.946 (1.641)	.947 (1.629)	.948 (1.653)

Table (4.6) Coverage probabilities and average interval lengths for JEL methods with log-normal distribution

			CL =95%						
n	CR		NA	EL	JEL	$AJEL$	$MJEL$	$TJEL$	$TAJEL$
50	10	β_1	.940 (2.145)	.925 (2.037)	.904 (2.255)	.912 (2.362)	.917 (2.412)	.913 (2.374)	.927 (2.485)
		β_2	.931 (2.145)	.903 (2.042)	.901 (2.207)	.915 (2.488)	.917 (2.550)	.916 (2.504)	.928 (2.626)
100	10	β_1	.946 (1.466)	.946 (1.434)	.937 (1.544)	.944 (1.584)	.946 (1.613)	.944 (1.579)	.948 (1.619)
		β_2	.939 (1.465)	.939 (1.448)	.921 (1.558)	.927 (1.618)	.934 (1.647)	.926 (1.612)	.936 (1.653)
150	10	β_1	.952 (1.178)	.951 (1.165)	.939 (1.232)	.943 (1.254)	.946 (1.271)	.943 (1.249)	.948 (1.271)
		β_2	.947 (1.184)	.943 (1.171)	.948 (1.219)	.952 (1.264)	.954 (1.282)	.952 (1.259)	.954 (1.281)
200	10	β_1	.949 (1.018)	.944 (1.011)	.937 (1.062)	.945 (1.077)	.946 (1.087)	.943 (1.073)	.946 (1.088)
		β_2	.946 (1.017)	.938 (1.014)	.927 (1.065)	.932 (1.081)	.930 (1.092)	.929 (1.077)	.933 (1.092)
50	40	β_1	.919 (2.135)	.913 (2.080)	.891 (2.333)	.905 (2.444)	.909 (2.501)	.907 (2.458)	.917 (2.574)
		β_2	.931 (2.142)	.900 (2.099)	.898 (2.347)	.909 (2.582)	.908 (2.664)	.909 (2.598)	.917 (2.725)
100	40	β_1	.941 (1.478)	.936 (1.471)	.922 (1.622)	.924 (1.664)	.928 (1.696)	.924 (1.658)	.929 (1.700)
		β_2	.948 (1.476)	.928 (1.483)	.926 (1.648)	.930 (1.712)	.932 (1.749)	.930 (1.706)	.935 (1.750)
150	40	β_1	.939 (1.197)	.927 (1.198)	.917 (1.311)	.921 (1.335)	.941 (1.377)	.920 (1.330)	.926 (1.354)
		β_2	.950 (1.195)	.921 (1.203)	.939 (1.274)	.941 (1.377)	.943 (1.399)	.941 (1.371)	.946 (1.396)
200	40	β_1	.943 (1.030)	.941 (1.038)	.937 (1.137)	.938 (1.153)	.938 (1.166)	.937 (1.148)	.940 (1.164)
		β_2	.950 (1.030)	.934 (1.039)	.931 (1.141)	.933 (1.156)	.937 (1.168)	.933 (1.152)	.935 (1.168)
50	70	β_1	.941 (2.225)	.934 (2.215)	.911 (2.590)	.917 (2.714)	.918 (2.775)	.917 (2.729)	.924 (2.860)
		β_2	.941 (2.206)	.919 (2.236)	.904 (2.585)	.917 (2.978)	.920 (3.078)	.918 (2.998)	.928 (3.152)
100	70	β_1	.947 (1.528)	.939 (1.546)	.918 (1.800)	.925 (1.846)	.926 (1.882)	.925 (1.840)	.929 (1.887)
		β_2	.950 (1.526)	.932 (1.550)	.917 (1.803)	.922 (1.873)	.924 (1.906)	.921 (1.867)	.928 (1.915)
150	70	β_1	.956 (1.236)	.951 (1.255)	.914 (1.449)	.925 (1.476)	.928 (1.496)	.924 (1.469)	.930 (1.496)
		β_2	.936 (1.232)	.949 (1.257)	.908 (1.458)	.912 (1.490)	.914 (1.510)	.911 (1.483)	.917 (1.510)
200	70	β_1	.940 (1.067)	.943 (1.085)	.918 (1.246)	.921 (1.263)	.921 (1.275)	.918 (1.258)	.926 (1.276)
		β_2	.946 (1.066)	.938 (1.081)	.911 (1.267)	.918 (1.287)	.917 (1.301)	.915 (1.281)	.919 (1.300)

longest, and very similar average lengths. MJEL and TAJEL methods outperform the AJEL method, which outperforms the JEL method. However, MJEL and TAJEL have some over coverage. The TJEL method has no over coverage and performs nicely for the Exponential distribution. For small sample sizes for the Log-normal distribution the JEL method has under coverage issue. Overall for the Log-normal distribution the EL method has the shortest average lengths.

CHAPTER 5

REAL DATA ANALYSIS

In the real data analysis, we utilized data sets with small sample sizes to illustrate the proposed methods.

The NA, EL, JEL, AJEL, MJEL, TJEL, TAJEL, AEL, MEL, TEL and TAEL methods were separately applied to analyze the data sets. Then, we calculated the confidence interval length and the confidence interval bounds of the point estimate of β at confidence level 95%.

5.1 Larynx data set

The tables show the length of 95% confidence intervals. The data set that is used in the real data analysis will be the larynx data set where Kardaun (1983) reports on 90 male patients with laryngeal cancer were studied. The study tracked survival times, patient's age, year of diagnosis, and disease stage was noted. We will analyze the real data set at the multiple times points. The covariates β_1, β_2 and β_3 are stage 2, stage 3, and stage 4, respectively. The covariate β_4 is patient age and β_5 is an interaction between patient age and β_1. The length of 95% confidence intervals will be taken at three values of the time variable, 4, 6, and 10, respectively.

5.2 Myeloma data set

The data set that is used in the second real data analysis will be the Myeloma data set that studies factors involving 65 patients that are treated with alkylating agents reported by Krall et al. (1975). 48 of the patients died during the study and 17 survived. The death variable indicates whether the patient was alive or dead at the of end the study. The variables thought to be related to survival are logarithm of blood urea nitrogen at diagnosis (logBUN) and hemoglobin at diagnosis (HGB), as the covariate variables β_1 and β_2, respectively.

Table (5.1) The confidence interval (length) for the EL methods with the larynx data set

		CL =95%		
$method$		$time = 4$	$time = 6$	$time = 10$
NA	β_1	(0.261, 1.374) 1.113	(-1.193, -0.171) 1.022	(-1.186, -0.176) 1.010
	β_2	(-1.218, -0.038) 1.180	(0.0850, 1.208) 1.123	(0.047, 1.173) 1.126
	β_3	(1.186, 2.532) 1.346	(1.025, 2.397) 1.372	(1.060, 2.426) 1.366
	β_4	(5.645, 10.351) 4.706	(5.455, 10.052) 4.597	(5.046, 9.366) 4.320
	β_5	(-2.157, 1.418) 3.575	(-1.588, 2.004) 3.592	(-1.427, 2.172) 3.599
EL	β_1	(-1.116, -0.524) 0.592	(-1.228, -0.645) 0.583	(-0.950, -0.525) 0.425
	β_2	(0.570, 1.797) 1.227	(0.588, 1.707) 1.119	(0.731, 1.653) 0.922
	β_3	(2.810, 4.462) 1.652	(2.658, 4.150) 1.492	(2.408, 3.71) 1.302
	β_4	(-1.072, 4.487) 5.559	(-0.124, 5.033) 5.157	(0.545, 4.916) 4.371
	β_5	(-1.418, -0.157) 1.261	(-1.532, -0.289) 1.243	(-1.115, -0.224) 0.891
AEL	β_1	(-1.126, -0.518) 0.608	(-1.237, -0.639) 0.598	(-0.957, -0.520) 0.437
	β_2	(0.549, 1.812) 1.263	(0.569, 1.721) 1.152	(0.716, 1.665) 0.949
	β_3	(2.656, 4.864) 2.208	(2.511, 4.474) 1.963	(2.271, 3.958) 1.687
	β_4	(2.656, 4.573) 1.917	(-0.188, 5.114) 5.302	(0.491, 4.983) 4.492
	β_5	(-0.997, -0.402) 0.595	(-1.115, -0.529) 0.586	(-0.846, -0.414) 0.432
MEL	β_1	(-1.339, -0.423) 0.916	(-1.449, -0.546) 0.903	(-1.094, -0.447) 0.647
	β_2	(0.414, 1.804) 1.390	(0.449, 1.713) 1.264	(0.623, 1.658) 1.035
	β_3	(2.454, 4.844) 2.39	(2.316, 4.458) 2.142	(2.087, 3.947) 1.860
	β_4	(-1.666, 4.562) 6.228	(-0.659, 5.104) 5.763	(0.093, 4.975) 4.882
	β_5	(-1.159, -0.301) 0.858	(-1.275, -0.429) 0.846	(-0.954, -0.336) 0.618
TEL	β_1	(-1.125, -0.518) 0.607	(-1.236, -0.639) 0.597	(-0.956, -0.520) 0.436
	β_2	(0.554, 1.810) 1.256	(0.574, 1.719) 1.145	(0.719, 1.663) 0.944
	β_3	(2.663, 4.859) 2.196	(2.517, 4.470) 1.953	(2.276, 3.955) 1.679
	β_4	(-1.128, 4.564) 5.692	(-0.175, 5.105) 5.280	(0.502, 4.976) 4.474
	β_5	(-0.996, -0.403) 0.593	(-1.114, -0.530) 0.584	(-0.846, -0.415) 0.431
$TAEL$	β_1	(-1.135, -0.511) 0.624	(-1.246, -0.632) 0.614	(-0.962, -0.515) 0.447
	β_2	(0.533, 1.825) 1.292	(0.555, 1.732) 1.177	(0.705, 1.675) 0.970
	β_3	(2.632, 4.897) 2.265	(2.488, 4.500) 2.012	(2.249, 3.978) 1.729
	β_4	(-1.197, 4.654) 5.851	(-0.238, 5.190) 5.428	(0.449, 5.047) 4.598
	β_5	(-1.007, -0.397) 0.610	(-1.124, -0.524) 0.600	(-0.852, -0.410) 0.442

Table (5.2) The confidence interval (length) for the JEL methods with the larynx data set

		CL =95%		
$method$		$time = 4$	$time = 6$	$time = 10$
NA	β_1	(0.261, 1.374) 1.113	(-1.193, -0.171) 1.022	(-1.186, -0.176) 1.010
	β_2	(-1.218, -0.038) 1.18	(0.0850, 1.208) 1.123	(0.047, 1.173) 1.126
	β_3	(1.186, 2.532) 1.346	(1.025, 2.397) 1.372	(1.060, 2.426) 1.366
	β_4	(5.645, 10.351) 4.706	(5.455, 10.052) 4.597	(5.046, 9.366) 4.320
	β_5	(-2.157, 1.418) 3.575	(-1.588, 2.004) 3.592	(-1.427, 2.172) 3.599
EL	β_1	(-1.116, -0.524) 0.592	(-1.228, -0.645) 0.583	(-0.950, -0.525) 0.425
	β_2	(0.570, 1.797) 1.227	(0.588, 1.707) 1.119	(0.731, 1.653) 0.922
	β_3	(2.810, 4.462) 1.652	(2.658, 4.150) 1.492	(2.408, 3.710) 1.302
	β_4	(-1.072, 4.487) 5.559	(-0.124, 5.033) 5.157	(0.545, 4.916) 4.371
	β_5	(-1.418, -0.157) 1.261	(-1.532, -0.289) 1.243	(-1.115, -0.224) 0.891
JEL	β_1	(-1.284, -0.374) 0.910	(-1.462, -0.459) 1.003	(-1.308, -0.348) 0.960
	β_2	(0.323, 1.525) 1.202	(0.511, 1.609) 1.098	(0.915, 1.941) 1.026
	β_3	(2.548, 4.178) 1.630	(2.651, 4.123) 1.472	(1.631, 3.204) 1.573
	β_4	(-1.454, 4.739) 6.193	(-0.768, 5.578) 6.346	(-0.039, 4.871) 4.910
	β_5	(-1.093, -0.384) 0.709	(-1.218, -0.504) 0.714	(-0.958, -0.314) 0.644
$AJEL$	β_1	(-1.300, -0.364) 0.936	(-1.481, -0.449) 1.032	(-1.184, -0.341) 0.843
	β_2	(0.302, 1.538) 1.236	(0.492, 1.622) 1.130	(0.695, 1.683) 0.988
	β_3	(2.287, 4.264) 1.977	(2.386, 4.271) 1.885	(2.229, 3.992) 1.763
	β_4	(-0.655, 5.405) 6.060	(-0.497, 5.186) 5.683	(-0.147, 5.645) 5.792
	β_5	(-1.107, -0.377) 0.730	(-1.232, -0.497) 0.735	(-0.916, -0.375) 0.541
$MJEL$	β_1	(-1.300, -0.364) 0.936	(-1.471, -0.432) 1.039	(-1.179, -0.319) 0.860
	β_2	0.027 , 1.531) 1.504	(0.251 , 1.614) 1.363	(0.538, 1.675) 1.137
	β_3	(1.857, 4.250) 2.393	(1.985, 4.255) 2.270	(1.910, 3.977) 2.067
	β_4	(-1.582, 4.794) 6.376	(-0.977, 5.604) 6.581	(-0.285, 5.596) 5.881
	β_5	(-1.280, -0.213) 1.067	(-1.416, -0.322) 1.094	(-1.022, -0.268) 0.754
$TJEL$	β_1	(-1.478, -0.436) 1.042	(-1.476, -0.45) 1.026	(-1.324, -0.340) 0.984
	β_2	(0.701, 1.883) 1.182	(0.497, 1.619) 1.122	(0.902, 1.952) 1.050
	β_3	(2.882, 5.052) 2.170	(2.394, 4.266) 1.872	(2.615, 4.537) 1.922
	β_4	(-0.574, 5.311) 5.885	(-0.831, 5.662) 6.490	(-0.099, 4.942) 5.041
	β_5	(-1.097, -0.386) 0.711	(-1.230, -0.499) 0.731	(-0.970, -0.307) 0.663
$TAJEL$	β_1	(-1.504, -0.425) 1.079	(-1.673, -0.447) 1.226	(-1.345, -0.330) 1.015
	β_2	(0.683, 1.899) 1.216	(0.796, 1.959) 1.163	(0.886, 1.966) 1.080
	β_3	(2.855, 5.093) 2.238	(2.869, 5.043) 2.174	(2.586, 4.567) 1.981
	β_4	(-1.537, 4.831) 6.368	(-0.845, 5.678) 6.523	(-0.171, 5.032) 5.203
	β_5	(-1.114, -0.381) 0.733	(-1.266, -0.680) 0.586	(-0.983, -0.506) 0.477

Table (5.3) The confidence interval (length) for the EL methods with the Myeloma data set

		CL =95%			
method		$time = 45$	$time = 55$	$time = 65$	$time = 90$
NA	β_1	(-0.965, 4.750) 5.715	(-0.951, 5.860) 6.811	(-0.955, 5.839) 6.794	(-0.772, 5.686) 6.458
	β_2	(-0.314, 0.115) 0.429	(-0.438, -0.023) 0.415	(-0.379, -0.034) 0.344	(-0.329, 0.140) 0.343
EL	β_1	(0.816, 6.077) 5.261	(-0.931, 5.815) 6.746	(-0.989, 5.745) 6.734	(-0.588, 5.661) 6.249
	β_2	(-0.450, -0.081) 0.369	(-0.379, -0.047) 0.332	(-0.356, -0.036) 0.320	(-0.282, 0.008) 0.290
AEL	β_1	(0.679, 6.156) 5.477	(-1.143, 5.902) 7.045	(-1.200, 5.831) 7.031	(-0.772, 5.744) 6.516
	β_2	(-0.453, -0.081) 0.372	(-0.383, -0.047) 0.336	(-0.359, -0.036) 0.323	(-0.286, 0.008) 0.294
MEL	β_1	(0.650, 6.117) 5.467	(-1.271, 5.860) 7.131	(-1.380, 5.789) 7.169	(-1.019, 5.697) 6.716
	β_2	(-0.457, -0.076) 0.381	(-0.386, -0.042) 0.344	(-0.363, -0.031) 0.332	(-0.289, 0.013) 0.302
TEL	β_1	(0.681, 6.150) 5.469	(-1.147, 5.894) 7.041	(-1.206, 5.824) 7.030	(-0.778, 5.741) 6.519
	β_2	(-0.456, -0.075) 0.381	(-0.384, -0.041) 0.343	(-0.361, -0.030) 0.331	(-0.287, 0.014) 0.301
$TAEL$	β_1	(0.536, 6.230) 5.694	(-1.368, 5.983) 7.351	(-1.428, 5.913) 7.341	(-0.971, 5.826) 6.797
	β_2	(-0.464, -0.068) 0.396	(-0.391, -0.035) 0.356	(-0.367, -0.024) 0.343	(-0.292, 0.020) 0.312

The time variable measures survival time in months from diagnosis. We will analyze the real data set at the multiple time points. The tables show the length of 95% confidence intervals. The length of 95% confidence intervals is taken at four values of the time variable, 45, 55, 65, and 90, respectively.

5.3 BMT data set

The tables show the length of 95% confidence intervals. The data set that will be used in the real data analysis will be the Bone Marrow Transplant (BMT) data set that studies factors involving bone marrow transplant patients. The data set is from Klein and Moeschberger (1997) and has 137 observations. The data set is from a treatments for acute myelocytic leukemia article Copeland et al. (1991). The data is gathered from a multi-center trial of patients that were followed after transplantation until relapse, death, or end of study. The variable death represents whether the death of the patient was observed or was the observation censored. The covariate β_1 is patient age measured in years and β_2 is donor sex. Covariates β_3 and β_4 are patient CMV status and Donor CMV status, respectively. The fifth covariate, β_5, is an indicator for FAB Grade 4 or 5 and AML or not. The time variable

Table (5.4) The confidence interval (length) for the JEL methods with the Myeloma data set

		CL =95%			
$method$		$time = 45$	$time = 55$	$time = 65$	$time = 90$
NA	β_1	(-0.965, 4.750) 5.715	(-0.951, 5.860) 6.811	(-0.955, 5.839) 6.794	(-0.772, 5.686) 6.458
	β_2	(-0.314, 0.115) 0.429	(-0.438, -0.023) 0.415	(-0.379, -0.034) 0.344	(-0.329, 0.140) 0.343
EL	β_1	(0.816, 6.077) 5.261	(-0.931, 5.815) 6.746	(-0.989, 5.745) 6.734	(-0.588, 5.661) 6.249
	β_2	(-0.450, -0.081) 0.369	(-0.379, -0.047) 0.332	(-0.356, -0.036) 0.320	(-0.282, 0.008) 0.290
JEL	β_1	(-0.427, 7.133) 7.560	(-0.378, 6.680) 7.058	(-0.426, 6.513) 6.939	(-0.233, 6.123) 6.356
	β_2	(-0.301, 0.060) 0.361	(-0.299, 0.034) 0.333	(-0.287, 0.039) 0.326	(-0.258, 0.040) 0.298
$AJEL$	β_1	(-0.647, 7.258) 7.905	(-0.581, 6.789) 7.370	(-0.626, 6.619) 7.245	(-0.409, 6.217) 6.626
	β_2	(-0.308, 0.068) 0.376	(-0.305, 0.041) 0.346	(-0.289, 0.039) 0.328	(-0.259, 0.040) 0.290
$MJEL$	β_1	(-0.691, 7.353) 8.044	(-0.691, 7.353) 8.044	(-0.622, 6.873) 7.495	(-0.727, 6.603) 7.330
	β_2	(-0.307, 0.066) 0.373	(-0.307, 0.066) 0.373	(-0.303, 0.039) 0.342	(-0.292, 0.044) 0.336
$TJEL$	β_1	(-0.646, 7.259) 7.905	(-0.580, 6.790) 7.370	(-0.624, 6.620) 7.244	(-0.408, 6.218) 6.626
	β_2	(-0.307, 0.067) 0.374	(-0.304, 0.040) 0.344	(-0.292, 0.045) 0.337	(-0.262, 0.046) 0.308
$TAJEL$	β_1	(-0.877, 7.388) 8.265	(-0.792, 6.904) 7.696	(-0.833, 6.728) 7.561	(-0.593, 6.314) 6.907
	β_2	(-0.314, 0.075) 0.389	(-0.310, 0.047) 0.357	(-0.298, 0.052) 0.350	(-0.267, 0.052) 0.319

measures the time to death or on the study time in days. We will analyze the real data set at the multiple times points. The length of 95% confidence intervals will be taken at three values of the time variable, 1000, 1500, and 2000, respectively.

5.4 Discussion of data set results

In Table 5.1 we can compare the confidence interval length for the EL methods. The MEL method appears to have the longest lengths. The AEL, EL, and TEL methods appear to produce very similar results with EL producing slightly shorter lengths than the AEL and TEL methods. Table 5.2 reports the confidence interval length for the JEL method for the comparison. JEL produces notably longer lengths for β_4 in comparison to EL and NA methods.

In Table 5.3 TAEL has the longest lengths while TEL and AEL are very comparable. Comparing Tables 5.3 and 5.4 for the myeloma data set we can clearly see the JEL methods in Table 5.4 tend to have longer lengths than the EL methods in Table 5.3. In Table 5.4 we

Table (5.5) The confidence interval (length) for the EL methods with the BMT data set

		CL =95%		
$method$		$time = 1000$	$time = 1500$	$time = 2000$
NA	β_1	(-0.119, 13.942) 14.061	(-0.111, 0.079) 0.190	(-0.894, 0.131) 1.025
	β_2	(-2.442, 7.359) 9.801	(-3.482, 7.886) 11.368	(-1.927, 5.016) 6.943
	β_3	(-3.999, -1.724) 2.275	(-1.189, 2.691) 3.881	(-1.111, 2.143) 3.254
	β_4	(-3.001, 18.520) 21.521	(-6.074, 8.398) 14.472	(-0.928, 4.857) 5.786
	β_5	(1.831, 9.897) 8.066	(1.608, 9.421) 7.813	(1.591, 9.019) 7.428
EL	β_1	(-0.076, 12.029) 12.105	(-0.052, 0.058) 0.110	(-0.049, 0.059) 0.108
	β_2	(-1.388, 6.443 7.831	(-1.608, 6.946) 8.554	(-0.854, 3.359) 4.213
	β_3	(-3.541, -1.610) 1.931	(-3.654, -1.551) 2.103	(-3.713, -1.649) 2.064
	β_4	(-0.429, 16.978) 17.407	(-4.628, 7.772) 12.400	(-1.094, 3.445) 4.539
	β_5	(1.729, 9.233) 7.504	(1.591, 8.050) 6.459	(1.575, 7.656) 6.081
AEL	β_1	(-0.131, 12.399) 12.530	(-0.042, 0.052) 0.094	(-0.039, 0.052) 0.091
	β_2	(-1.408, 6.517) 7.925	(-2.292, 7.777) 10.069	(-0.877, 3.509) 4.386
	β_3	(2.077, 3.343) 1.266	(-3.678, -1.527) 2.151	(-3.737, -1.628) 2.109
	β_4	(-7.240, 10.666) 17.906	(1.582, 14.070) 12.488	(-1.162, 3.51) 4.672
	β_5	(1.029, 9.688) 8.659	(-1.562, 5.117) 6.679	(2.403, 7.709) 5.306
MEL	β_1	(-0.394, 13.022) 13.416	(-0.031, 1.179) 1.210	(-0.659, 0.262) 0.921
	β_2	(-1.990, 6.724) 8.714	(3.787, 14.492) 10.705	(-4.524, 0.517) 5.041
	β_3	(2.075, 3.346) 1.271	(-1.685 , 1.006) 2.691	(-5.046, -1.130) 3.916
	β_4	(7.757, 26.136) 18.379	(-31.434, -17.509) 13.924	(-1.170, 3.602) 4.772
	β_5	(1.015, 10.989) 9.974	(0.706, 7.757) 7.051	(-2.403, 3.519) 5.922
TEL	β_1	(-0.233, 12.429) 12.662	(-0.042, 0.052) 0.094	(-0.039, 0.052) 0.091
	β_2	(-1.811, 6.519) 8.330	(1.506, 12.042) 10.536	(-0.872, 3.462) 4.334
	β_3	(-3.172, -1.938) 1.234	(-0.824, 2.174) 2.998	(-0.046, 3.629) 3.675
	β_4	(-2.995, 15.161) 18.156	(-31.143, -17.419) 13.724	(-1.137, 3.49) 4.627
	β_5	(-4.778, 4.340) 9.118	(-1.526, 5.115) 6.641	(2.411, 7.591) 5.180
$TAEL$	β_1	(-0.458, 13.437) 13.895	(-0.033, 1.181) 1.214	(-0.666, 0.268) 0.934
	β_2	(-2.051, 6.831) 8.882	(3.719, 14.492) 10.773	(-1.034, 4.331) 5.365
	β_3	(-1.124, 0.217) 1.341	(1.423, 4.138) 2.715	(-5.057, -1.118) 3.939
	β_4	(7.724, 26.136) 18.412	(-31.748, -17.157) 14.591	(-1.189, 3.743) 4.932
	β_5	(3.719, 14.492) 10.773	(-1.292, 5.995) 7.287	(-2.512, 3.623) 6.135

Table (5.6) The confidence interval (length) for the JEL methods with the BMT data set

		CL =95%		
$method$		$time = 1000$	$time = 1500$	$time = 2000$
NA	β_1	(-0.119, 13.942) 14.061	(-0.111, 0.079) 0.190	(-0.894, 0.131) 1.025
	β_2	(-2.442, 7.359) 9.801	(-3.482, 7.886) 11.368	(-1.927, 5.016) 6.943
	β_3	(-3.999, -1.724) 2.275	(-1.189, 2.691) 3.881	(-1.111, 2.143) 3.254
	β_4	(-3.001, 18.520) 21.521	(-6.074, 8.398) 14.472	(-0.928, 4.857) 5.786
	β_5	(1.831, 9.897) 8.066	(1.608, 9.421) 7.813	(1.591, 9.019) 7.428
EL	β_1	(-0.076, 12.029) 12.105	(-0.052, 0.058) 0.110	(-0.049, 0.059) 0.108
	β_2	(-1.388, 6.443) 7.831	(-1.608, 6.946) 8.554	(-0.854, 3.359) 4.213
	β_3	(-3.541, -1.610) 1.931	(-3.654, -1.551) 2.103	(-3.713, -1.649) 2.064
	β_4	(-0.429, 16.978) 17.407	(-4.628, 7.772) 12.400	(-1.094, 3.445) 4.539
	β_5	(1.729, 9.233) 7.504	(1.591, 8.050) 6.459	(1.575, 7.656) 6.081
JEL	β_1	(-29.357, -17.800) 11.557	(1.355, 2.282) 0.927	(-0.076, 0.041) 0.117
	β_2	(0.893, 9.593) 8.700	(1.212, 10.365) 9.153	(-0.796, 3.572) 4.368
	β_3	(-3.934, -1.441) 2.493	(-3.768, -1.469) 2.299	(-4.228, -2.053) 2.175
	β_4	(-2.319, 15.928) 18.247	(-30.846, -17.621) 13.224	(-0.587, 4.363) 4.950
	β_5	(-4.765, 4.343) 9.109	(-1.509, 5.008) 6.517	(-0.449, 5.692) 6.141
$AJEL$	β_1	(-30.830, -18.521) 12.309	(0.964, 1.992) 1.028	(-0.579 , 0.330) 0.909
	β_2	(-0.054, 9.233) 9.287	(0.456, 11.462) 11.006	(-0.629, 3.853) 4.482
	β_3	(-3.570, -1.331) 2.239	(-1.203, 2.236) 3.439	(-3.837, -1.561) 2.276
	β_4	(-20.767, -2.262) 18.505	(0.992, 15.796) 14.804	(-0.591, 4.403) 4.994
	β_5	(-3.942, 5.397) 9.341	(-1.369, 6.076) 7.445	(-0.517, 5.780) 6.297
$MJEL$	β_1	(-31.117, -18.448) 12.669	(-0.092, 1.113) 1.205	(-3.231 , -1.862) 1.369
	β_2	(0.656, 11.463) 10.807	(0.448, 11.462) 11.014	(-4.900, 0.523) 5.423
	β_3	(-3.171, -0.425) 2.746	(-0.061, 3.381) 3.442	(-0.046, 3.629) 3.675
	β_4	(6.619, 25.621) 19.002	(-8.837, 6.972) 15.809	(-0.833, 4.970) 5.803
	β_5	(9.482, 20.211) 10.729	(-1.301, 7.124) 8.425	(-1.022, 5.891) 6.913
$TJEL$	β_1	(-30.952, -18.585) 12.366	(-0.104, 1.120) 1.224	(-0.090, 1.036) 1.126
	β_2	(-3.943, 5.397) 9.341	(0.455, 11.462) 11.007	(-0.711, 3.862) 4.573
	β_3	(-3.024, -0.340) 2.684	(-0.121, 3.392) 3.513	(-2.272, 1.302) 3.574
	β_4	(-24.962, -6.310) 18.652	(-0.053, 15.277) 15.33	(-0.662, 4.417) 5.079
	β_5	(0.471, 11.464) 10.993	(-1.411, 6.340) 7.751	(-0.575, 5.814) 6.389
$TAJEL$	β_1	(-4.481, 9.464) 13.945	(-0.119, 1.130) 1.249	(-1.111, 0.292) 1.403
	β_2	(0.649, 11.463) 10.814	(-7.735, 4.124) 11.859	(-1.299, 4.873) 6.172
	β_3	(-3.189, -0.491) 2.698	(-4.316, -0.377) 3.939	(-0.055, 3.642) 3.697
	β_4	(-1.122, 18.836) 19.958	(-8.854, 6.971) 15.825	(-0.928, 4.857) 5.786
	β_5	(0.453, 11.462) 11.009	(-1.980, 7.010) 8.990	(-1.473, 5.008) 6.481

see TAJEL produce similar results for β_2 as the NA method, but longer lengths for β_1 than the NA method.

Tables 5.5 and 5.6 indicate that the NA method tends to have longer lengths in comparison to the other methods for the BMT data set. In Table 5.5 we see that NA and TAEL methods produce similar lengths and the longest lengths in comparison to other methods. MJEL tends to consistently produce longer lengths than AJEL and TJEL, but shorter lengths than TAJEL thus outperforming TAJEL. In Table 5.6 the TAJEL method produces longer lengths in comparison to the other JEL methods.

CHAPTER 6

CONCLUSIONS

In this thesis, we showed the AEL, MEL, TEL, TAEL, JEL, AJEL, MJEL, TJEL, and TAJEL methods could be used in addition to the NA method. The efficiency of the JEL, AJEL, MJEL, TJEL, TAJEL, AEL, MEL, TEL, and TAEL methods were illustrated using the simulated data sets to construct confidence intervals and was compared with the NA-based method in terms of confidence intervals and average lengths. To construct the best possible interval, the coverage probability should be as close as possible to the nominal confidence level while the average length needs to be the shortest. When coverage probabilities of two methods are comparable, we recommend the method that produces shorter C.I. interval lengths.

The simulation studies showed that the MJEL and AJEL methods appear to have the least overcoverage, and thus seem the best performing methods of the other methods for the Weibull distribution. However, for the Exponential distribution the MJEL and TAJEL methods outperform the AJEL method, but have some over coverage. The TJEL method has no over coverage and performs nicely for the Exponential distribution. Among the EL methods the MEL method appears to be the closest to the nominal level making it the best performing method for the various distributions. The real data analysis indicated that the CI lengths observed in real data analysis for different methods is consistent with the observations in simulation studies

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APPENDICES

We assume the following regularity conditions.

Assume that $\lambda_0(t)$ is continuous.

Assume the covariate vector Z_i is bounded, i.e., $P(|Z| < C) = 1$ for some constant C.

Proof of Theorem 2.2 This proof follows similar arguments in Yu et al. (2011) and Yu and Zhao (2019). Let $\beta_0 = ((\beta_0^{(1)})', (\beta_0^{(2)})')$, corresponding to $Z = ((Z_0^{(1)})', (Z_0^{(2)})')$. Define

$$\hat{A}(\beta_0) = \int_0^\tau E[\{Z - \mu_Z(\beta_0, t)\}(Z^{(2)})' \lambda\{H_0(t) + \beta_0' Z\}]Y(t)dH_0(t).$$

As $A(\beta_0)$ is assumed to positive definite, the rank of $\hat{A}(\beta_0)$ is $p - q$. Let $\hat{\beta}_1(\beta_0^{(1)'}) = \arg\inf_{\beta^{(2)}} l((\beta_0^{(1)})', (\beta^{(2)})')'$. Using the arguments in Yu et al. (2011)

$$\sqrt{n}(\hat{\beta}_{11}, \hat{\beta}_0^{(2)'}) = -[\Psi(\beta_0)]^{-1}[\tilde{A}(\beta_0)]'[\Sigma(\beta_0)]^{-1}\frac{1}{\sqrt{n}}\sum_{i=1}^n U_{ni}(\beta_0) + o_p(1),$$

$$\sqrt{n}\theta_2 = 1 - [\Sigma(\beta_0)]^{-1}\hat{A}(\beta_0)[\psi(\beta_0)]^{-1}[\hat{A}(\beta_0]'[\Sigma(\beta_0)]^{-1} - 1\frac{1}{\sqrt{n}}\sum_{i=1}^n U_{n,i}(\beta_0) + o_p(1);$$

where θ_p is the corresponding Lagrange multiplier, and

$$\psi(\beta_0) = [\hat{A}(\beta_0)]'[\sum(\beta_0)]^{-1}\hat{A}(\beta_0).$$

By Taylor expansion, we have

$$\begin{aligned} l_{\text{profile}}(\beta_0^{(1)}) &= \left(\frac{1}{\sqrt{n}}\sum_{i=1}^n U_{ni}(\beta_0)\right)' \left([\Sigma(\beta_0)]^{-1} - [\Sigma(\beta_0)]^{-1}\tilde{A}(\beta_0)[\Psi(\beta_0)]^{-1}[\tilde{A}(\beta_0)]'[\Sigma(\beta_0)]^{-1}\right) \\ &\quad \left(\frac{1}{\sqrt{n}}\sum_{i=1}^n U_{ni}(\beta_0)\right) + o_p(1) \\ &= \left([\Sigma(\beta_0)]^{-1/2}\frac{1}{\sqrt{n}}\sum_{i=1}^n U_{ni}(\beta_0)\right)' S \left([\Sigma(\beta_0)]^{-1/2}\frac{1}{\sqrt{n}}\sum_{i=1}^n U_{ni}(\beta_0)\right) + o_p(1), \end{aligned}$$

where

$$S = I - [\Sigma(\beta_0)]^{-1/2}\tilde{A}(\beta_0)[\Psi(\beta_0)]^{-1}[\tilde{A}(\beta_0)]'[\Sigma(\beta_0)]^{-1/2}$$

is a symmetric idempotent matrix with trace q. Then with under the regularity conditions, if β_0 is the true values of β,

$$\frac{1}{\sqrt{n}}\sum_{i=1}^{n} U_{ni}(\beta_0) \xrightarrow{\mathfrak{D}} N(0, \Sigma(\beta_0)).$$

We have that,

$$[\Sigma(\beta_0)]^{-1/2}\frac{1}{\sqrt{n}}\sum_{i=1}^{n} U_{ni}(\beta_0) \xrightarrow{\mathfrak{D}} N(0, I_{p\times p}).$$

Hence, we prove Theorem 2.6. □

Proof of Theorem 2.9 We want to establish that the results hold for $R^*(\beta)$, so that we can then show that $R^*(\beta_0)$ composes a true transformation of $R^*(\beta_0)$. It is significant to note that $R^*(\beta_0)$ is similar to the AJEL section. Therefore, the results of Chen et al. (2008) hold for $R^*(\beta)$. To begin, we look at the criteria that make a true transformation.

(T_1) $0 \leq R_t^*(\beta_0) \leq R^*(\beta_0)$;

(T_2) $R_t^*(\beta_0)$ must be a monotonically increasing function of $R^*(\beta_0)$;

(T_3) $R_t^*(\beta_0) = R^*(\beta_0) + o_p(1)$;

(T_4) The level τ_1 contour of $R_t^*(\beta_0), \beta_0 : R_t^*(\beta_0) = \tau_1$ must be the same in shape as some level τ_2 contour of $R_t^*(\beta_0), \beta_0 : R_t^*(\beta_0) = \tau_2$ and $R_t^*(\tilde{\beta}_0) < R_t^*(\beta_0)$, for $\tilde{\beta}_0 \neq \beta_0$ for any $\tau_1 \epsilon [0, \infty)$.

We will show that $R_t^*(\beta_0)$ comprises a true transformation that preserves the asymptotic properties of $R_t^*(\beta_0)$. We look at the four criteria of a true transformation. We choose $\gamma = 1/2$.

(T_1) We note that $R^*(\beta_0) \geq 0$. Therefore, $0 < \max\{1 - (R^*(\beta_0)/n, \gamma = 1/2\} \leq 1/2$ and thus $0 \leq R_t^*(\beta_0) \leq R^*(\beta)$.

(T_2) Observe $R_t^*(\beta) = R^*(\beta_0)[1 - R^*(\beta_0)/n]$ when $R^*(\beta_0)\epsilon[0, n/2]$. Then, this is a strictly monotonically increasing function of $R^*(\beta_0)$. We note that $R_t^*(\beta_0) = (R^*(\beta_0)/2)$ when $R^*(\beta_0) > 1/2$. Then, this is a strictly monotonically increasing function of $R^*(\beta_0)$. Over the entire interval of $[0, \infty]$, $R_t^*(\beta_0)$ is continuous. Therefore, $R_t^*(\beta_0)$ continuous strictly monotonically increasing, and non-negative throughout $R^*(\beta_0)\epsilon[0, \infty)$.

(T_3) We note that $R^*(\beta_0) = O_p(1)$ with the limiting distribution of $R^*(\beta_0)$ being a χ_p^2 distribution. Thus, $R^*(\beta_0) \leq n/2$, with the probability tending to unity. Then, for all asymptotic discussion we employ the assumption of $R_t^*(\beta_0) = R^*(\beta_0)[1 - R^*(\beta_0)/n]$. $R^*(\beta_0) = O_p(1)$ coupled with this assumption gives us gives us (T_3).

(T_4) For a level τ_1 contour of the $R_t^*(\beta_0)$, $\beta : R_t^*(\beta_0) = \tau_1$, let $\tau_2 = R_t^{*(-1)}(\tau_1)$, then $\beta : R_t^*(\beta_0) = \tau_1 = \beta_0 : R^*(\beta_0) = \tau_2$. The second part of ($T_4$) stems from the monotonicity of $R_t^*(\beta_0)$ and noting that $R^*(\beta_0)$ is likely to have a unique minimum at some $\hat{\beta}$.

Therefore, $R_t^*(\beta_0)$ fulfills the necessary requirements of a true transformation and we can say $R_t^*(\beta_0)$ constitutes a true transformation preserving the asymptotic properties of $R_t^*(\beta_0)$. Thus, as $n \to \infty$, $-2R_t^*(\beta_0) \xrightarrow{\mathcal{D}} \chi_p^2$. $\square$