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Estimation of the Economic Benefits of Urban Water Supply and Sewerage Projects in Developing Countries

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INTERNATIONAL BANK FOR RECONSTRUCTION AND DEVELOPMENT

ESTIMATION OF THE ECONOMIC BENEFITS OF WATER SUPPLY AND SEWERAGE PROJECTS

Metropolitan and Regional Research Center

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Maxwell School of Citizenship and Public Affairs
Syracuse University

October 1973
ESTIMATION OF THE ECONOMIC BENEFITS OF WATER SUPPLY AND SEWERAGE PROJECTS

Roy W. Bahl
Stephen Coelen
Jeremy J. Warford

Prepared for the Public Utilities Department of the International Bank for Reconstruction and Development by the Metropolitan and Regional Research Center of the Maxwell School of Syracuse University and the Syracuse University Research Corporation

October 1973
FOREWORD

This research was initiated in the Public Utilities Department of the World Bank in response to a serious problem of investment choice—the measurement of benefits for water supply and sewerage projects. The research was carried out in the Metropolitan and Regional Research Center of the Maxwell School of Citizenship and Public Affairs of Syracuse University under a grant from the International Bank for Reconstruction and Development to the Syracuse University Research Corporation.

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The authors are indebted particularly to Mr. Paul O'Farrell for his help in assembling the basic data for the Kuala Lumpur case study. Miss Andrea Latchem edited the manuscript and acted as administrative officer for the project. Miss Latchem and Miss Joan Panagakis typed the final manuscript.

Roy W. Bahl
Director
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It is generally acknowledged that the fundamental problem encountered in cost-benefit analysis is that, in many sectors of activity, benefit measurement is either difficult or impossible. This, coupled with the need to observe income-distributional or non-economic constraints, means that economic rates of return cannot be relied upon entirely to determine intersectoral allocation of funds. Moreover, the difficulties of benefit measurement vary considerably between sectors, and, in particular, it is alleged that water supply and sewerage projects fare relatively badly according to such tests.

A precise scientific allocation of funds between sectors will, therefore, not be achieved, even if we were to succeed in identifying the economic worth of water and sewerage investments in a partial equilibrium context. However, investment decisions in this and in other sectors competing for funds should be improved as a result of such analysis. Even if "judgment" or social and political pressure is to some degree inevitable in determining public expenditure decisions, it is clearly true that these decisions are likely to be improved, if their economic consequences are made explicit, and the range of choice narrowed. The object of this study is to see whether this modest goal is feasible for water supply and
sewerage projects.

A basic problem in project evaluation is to measure the total amount that beneficiaries would be willing to pay for a project rather than do without it altogether. This represents the economic benefit stemming from the project, and consists of two elements: the amount that people actually do pay plus the "consumers' surplus" that is obtained. In practice, it is rare that the total benefit derived from consumption of the output of public utilities can be established, and this is particularly applicable to water supply and sewerage projects. The necessary data are rarely available, and, even if it can be predicted accurately, the financial rate of return obtained by the utility, which represents just that part of the benefit measured by the amount that people actually do pay, underestimates the economic rate of return.

This follows from several facts. The first is that water undertakings are natural monopolies, one effect of which is that the demand curve for their output tends to be inelastic. Second, the lumpiness of investment in the water supply field means that output cannot be increased by tiny increments so that the price paid by existing consumers cannot be expected to capture total consumers' surplus, the extreme and most relevant case being the connection of new consumers to the system. Both facts point to the contrast with the situation relating to agricultural projects. There, total benefits can be quantified simply by multiplying incremental yields by world prices. Roughly, the existence of competitive market conditions (implying demand elasticity) implies an equality of economic benefits and ex-subsidy revenues.
Third, the mystique that is often attached to water supply has frequently prevented water authorities from raising charges sufficiently to provide adequate information on consumers' willingness to pay. Of course, where water undertakings are financed by some form of taxation unrelated to consumption (i.e., metering does not exist), the pricing mechanism is ruled out, by definition, as a source of information concerning economic benefits. Fourth, there are probably significant "external" benefits resulting from the supply of potable water, so that willingness to pay for a service, even if it could be determined, would tend to underestimate benefits.

The relationship between the benefits of sewerage projects and the financial return of the enterprise concerned is even more tenuous. This even applies where recipients of the joint service of water supply and sewage collection facilities are charged according to the amount of water they consume. Willingness to pay a given price represents a minimum evaluation of water supply plus its convenient disposal; the value placed upon the joint service is therefore an unsatisfactory indication of the value placed upon the individual elements of it. It is not possible, for example, to use the difference in the price paid for water and that paid for water plus sewage disposal as a measure of the value of the latter. Similarly, financial rates of return are irrelevant in determining benefits where a sewage authority is financed, say, by property taxes. In fact, it is the rate of return which often determines property taxes. Finally, a particularly large proportion of the benefits from sewage facilities accrues to parties other than those who pay for them.

An important consequence stems from the use of financial returns as a
surrogate for economic returns. This is that benefit-cost ratios are, for water and sewerage projects of given economic net worth, likely to be underestimated relative to those calculated for other important sectors, which are, in varying degrees, better able to estimate consumers' surplus. Competition for funds between sectors according to economic rates of return is therefore not carried out on an equal footing.

This can be illustrated by comparing water supply and sewerage projects with highway projects. In the latter case, a financial rate of return is not usually a meaningful measure of project benefits either. However, the bulk of the consumers' surplus resulting from these projects can often be estimated, for many of them are by their nature cost-saving. Thus, since a saving of $X is equivalent to the maximum amount that a rational beneficiary would pay for it, it provides us with our definition of total benefits.

However, even transportation projects face problems when quantitative and qualitative differences are involved; for example, the treatment of additional traffic generated by a project cannot be evaluated so easily. The theoretical position is that where services provided by the public project are identical in quality and quantity to those that would otherwise be privately supplied, cost-savings provide an adequate measure of benefit. Normally, of course, there are qualitative differences (e.g., a piped water supply may be preferable to purchase from a street trader on grounds of convenience and water quality), as well as differences in the quantity of water supplied. However, even where such differences are present, an alternative cost calculation may be of some help in providing information to rank projects by need.
In practice, cost-savings have rarely been sufficient to demonstrate the economic justification of water supply or sewerage projects financed by the World Bank. Basically, this is because the area of overlap between the quantity and quality of the private alternative and that of the Bank-financed project is usually so small. An exception to this was the industrial water supply component of the Ibar, Yugoslavia, multi-purpose project. This was unique because industrial consumers had indicated their intention to proceed with their own source of supply, the yield of which was similar to that produced by the Ibar scheme.

More generally, appraisals often refer to the savings obtained because people no longer have to travel to obtain water supplies, or because private sellers of water apparently incur such large costs. These references are basically of a qualitative nature, however, and much the same applies to the benefits of sewage disposal projects. In particular, the savings in costs of maintaining and emptying septic tanks are often referred to in appraisal reports, but have not yet been used as a justification of a sewerage project. (The Sao Paulo appraisal showed a 6 percent economic rate of return by this method, and it is unlikely that anything much higher than this could be achieved.)

On occasion, methods of benefit estimation other than financial rates of return or cost savings have been used. Thus, land value enhancement has been used in four projects: Sao Paulo river pollution control, Bucaramanga erosion control, Mexico city flood control and Lahore water supply and drainage. However, the circumstances surrounding these schemes were exceptional, and the approaches used cannot readily be adapted to suit
the appraisal of more routine projects.

Benefits to public health have rarely been quantified, although Bucaramanga again offers an exception to this rule, along with the Addis Ababa water supply project. These were heroic efforts to improve project analysis, and it is easy, though not quite fair, to criticize them. More frequently, vague references are made to the health implications of water and sewerage projects, data limitations preventing quantitative analysis of these effects from being incorporated into a cost-benefit calculation.

II. Property Values and Economic Benefits

The foregoing provides a flavor of the kind of problems associated with the economic evaluation of water supply and sewerage projects, and it is probably fair to say that the economic benefits of these projects have never been estimated in a sophisticated way. This is somewhat less of a problem where we are dealing with relatively small additions to the amount of consumption or quality of service available to existing consumers who are currently receiving a supply or service twenty-four hours per day throughout the year, i.e., the normal situation in developed countries. In the countries with which the Bank deals, however, a typical decision to be made is whether or not to provide piped water supplies and an efficient drainage system to people who have never had such facilities before.

These investments frequently result in a dramatic improvement in the well-being of the households connected to the systems. Consumers may no longer have to walk long distances to obtain supplies from communal taps,
or, worse still, from ditches, ponds or rivers; they may no longer have to put up with being surrounded by filth and excreta, which may now be quickly and efficiently disposed of. Aesthetics, health and convenience should all be improved by such measures.

As a consequence of this, one might reasonably expect that householders would place a value upon being connected to piped supplies and sewers that is well in excess of the amount actually paid for these services. This implies that, other things being equal, the value of a house that obtains the services would be greater than that of one that does not. In contrast to the water/sewerage "market", which does not work very well because there is not a host of suppliers able and willing to compete effectively with the water/sewerage undertaking, the housing market may consist of large numbers of competitive buyers and sellers, and, where public regulation is minimal, there is opportunity for the effective willingness of people to pay for water and sewerage facilities to be revealed.

To illustrate, suppose an individual is willing to pay up to an additional $100 for a house because it has piped water supply. This suggests that implicitly or explicitly he estimates the present worth to him of the benefits thereby derived, and from this deducts the present worth of the water charges he expects to pay as a result of having a connection. The difference is $100, which represents the consumers' surplus (i.e., the difference between the maximum amount that he would pay if he had to, and the amount that he actually does pay).

In developing such a method for benefit measurement in water/sewer project analysis, two problems must be dealt with. The first is to outline
the market conditions necessary for house sales to reveal the full benefits of infrastructural improvements. The second is the statistical problems of measurement. These are discussed in turn below.

III. The Housing Market

The object of the study is to make use of the relatively competitive nature of the housing market to determine benefits. This implies that the analysis is particularly appropriate for developing countries, where installation of water and sewerage facilities often takes place after houses have been occupied. In developed countries, on the other hand, these facilities are normally installed prior to construction; changes in land prices are therefore typically the outcome of bilateral monopoly bargaining, so that the resulting price has little welfare significance.

Clearly, land markets rarely conform precisely to the textbook ideal of perfect competition, and to the extent that they diverge from the ideal, they become less useful for our purposes. However, aside from areas in which there is a large degree of public intervention, usually in the form of public housing or rent control, this market represents a considerable improvement on the use of revenue from water sales as a benefit measure. The main question to be faced, therefore, is whether or not increasing values in the housing market correctly capture the effects of the increase in demand for improved properties. In other words, is the increased consumer's

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1 The terms, land market, housing market and property market, are used interchangeably, since virtually all increments in valuation are perceived to be attributable to the land itself.
surplus in the water market transferred to an equivalent shift in the area under the property market demand curve? And, if so, under what conditions are expenditures on properties also increased by this amount?

The first step in answering these questions requires consideration of the nature of the housing market. Since the impact of the infrastructural improvements will be an increase in demand for the houses affected, the resulting price increase will not be a perfect indicator of benefit unless the following two conditions hold (or are compensatory):

(a) the slope of the demand curve does not change; and
(b) the supply of housing is perfectly inelastic.

Consider the implications of conditions (a) and (b). First, if the demand schedule for housing in the project area changes, one might expect it to become relatively more inelastic—there are fewer good substitutes now that the house has piped-in water. That is, it would take a greater price increase to bid an individual away from a house with water than it would when the same house did not have public water supply. The implications of such a change in the slope of the demand schedule are that consumer surplus will increase, i.e., the difference between the individual's and the market's valuation of the property will increase. This would mean that the sum of revenues from water sales and the property value increment due to a new water supply underestimates the true benefits of a water project. As to the necessary supply condition, the presence of elasticity means that, as the price of properties in the affected area is bid up, property owners in the area are induced to offer a greater amount of housing, which will tend to cause land value increments to understate the increase
in net benefits. If either the supply or the demand conditions in this simple framework are violated, therefore, the sum of land value increments and revenue from water sales will not be a perfectly accurate measure of the true benefits of a water supply.

Unfortunately, there is, in practice, no way of determining the magnitude of such violations. As to the supply conditions, however, the application of the method to areas with relatively high building densities, stable zoning regulations, and so on, might be necessary in order to identify project benefits fully. Moreover, as indicated, it is likely that any shortcomings due to the absence of the stated demand and supply conditions will tend to produce a conservative estimate of benefits, but the method will still provide a better measure of benefits than revenue from water sales alone.

A general theoretical issue arises from the fact that the consumers' surplus triangle on a conventional demand curve is unsuitable as a measure of welfare because real income does not remain constant as prices change. This can be ignored for small-scale expenditures, but not, presumably, for housing expenditures, which account for a fairly high proportion of consumers' budgets. However, in the present context, this does not cause too much trouble, because, rather than trying to measure consumers' surplus by estimating demand curves, the statistical analysis gives the measure direct and, in so doing, incorporates the appropriate "income

2 This is borne out by the experience of the empirical studies described below.

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effects.  

IV. Specific Complications

Broadly speaking, empirical testing makes use of a "project area," which is the site of the investment, and a "control area," used to help isolate project effects from all other factors that might influence land values. Regression analysis is used to estimate the impact of the project on property values, as evidenced by sales in both areas over a number of years. Before outlining the details of the measurement technique employed, some specific complications are discussed.

The Problem of Anticipation

Frequently, property values rise in anticipation of a promised improvement. Where this happens in a project area, the increase in property values after the investment will tend to understate the true benefits. Ideally, one should take account of this, but it would, of

3It has also been suggested to the authors that it seems inappropriate to use this analysis of demand for things (houses) of which each consumer buys but one. The alternative statement of the problem would be:

all people already in houses with water value the latter at > (X – T), where T is transaction cost of changing house;

all people buying houses with water rather than houses without it value it at > X;

all people already in houses without water value it at < (X + T);

all people buying houses without water value it < X.

It is the last two of these four inferences which are relevant to the benefits of providing water to those houses currently without it.
course, be complicated, particularly if the control area is adjacent, and properties are also increasing in value in expectation that project area improvements will be extended to them.

Property Taxes

In most types of local taxation systems, the impact of the improvements will raise the property tax base. That is, the increment in property values will itself be subject to a tax. Consequently, if land values would have risen by $X in the absence of a tax, they will rise by something less than $X in the presence of a tax. It can be said, roughly, that the total benefit of an improvement should be achieved by the addition of the increase in property values to the additional tax liability. Problems of standardization arise where different areas are subject to different tax systems or levels.

Externalities

"External" effects arise where there is a divergence between the private and social costs and benefits of a project. In the present context, problems may arise in that the supply of water or sewerage facilities to property X may be to the advantage of property Y, but potential purchasers of X would not take this into account in deciding how much they are willing to pay for the property. Externalities may arise where the health of people living in areas adjacent to the project area improves as a result of the improvement in the health of project area residents, or where aesthetic nuisances are prevented from spreading to other areas. To the extent that these effects are associated with water and sewerage investments, use of the technique employed here will tend to underestimate
benefits. Note, however, that externalities within the project area should be accounted for by use of property value enhancement as a measure of benefits.

The presence of externalities creates a problem in selecting a control area. Since the control and project areas should be as similar as possible, there are grounds for selecting contiguous areas. However, since the control area should be free of project-related externalities, there is a case for choosing more distant properties for this purpose. The resultant choice of a control area must therefore compromise between the alternatives. Any choice may be criticized on the grounds either that it is not similar enough to the project area or that it is subject to some externalities.

**Information Problems**

Use of individuals' willingness to pay as a measure of benefits runs into the problem that individuals may not be the best judge of their own welfare because they may not be in possession of all the relevant facts. This appears to be particularly true in the present context: a frequently voiced complaint is that beneficiaries of water and sewerage projects in developing countries do not appreciate or make the best use of these facilities because they do not realize their worth. If we take the impact of such investments on their health as an example, however, it is easy to sympathize with them. No one really understands the precise relationship between provision of these services and the health of beneficiaries, and it is extremely unlikely that the impact on property values will ever reflect these benefits with precision.

**Site Improvements**

A property served by a public water supply or sewerage system for
the first time may only be able to utilize these services fully, if
improvements, such as the addition of a bathroom, are added. The benefits
of these improvements will be capitalized and reflected in land values.
While attributable to the project, the resultant increases will overstate
net benefits, if a correction is not made for the costs of the improvements.
Since it can be assumed that competition in the home improvement industry
is fairly keen, subtraction of the capitalized value of improvement costs
from the property value increment should be sufficient adjustment.

Zoning

Water projects and, to a larger extent, sewer projects, may have the
effect of changing institutional zoning requirements. Thus, a sewer
project may permit a district to become zoned for higher residential
density which in turn may result in an increase in land values. Where
this takes place, it will rarely be possible to determine the extent to
which land values rise as a result of the sewerage investment or as a
source of information on individuals' willingness to pay for infra-
structural investments.

Effects on Property Values outside the Project Area

A number of other attempts have been made recently to determine the
extent to which property values reflect environmental conditions, notable
among which are studies relating to air pollution and urban categories.
A feature of these analyses have been a preoccupation with the possibility
that an increase (decrease) in demand for housing in area A as a result
of an environmental change may be offset, at least partially, by a resultant
decrease (increase) in an adjacent area B. Controversy has centered over
the welfare implications of such a possibility, the question at issue
normally being whether an increase in property values in Area A alone can
be taken as a measure of social benefit or whether the fall in values in
B should be subtracted\textsuperscript{4} in order to derive the appropriate net effect.
Although this is an issue that is rarely raised except when land values
are being used to estimate economic benefits, it is clearly one that is
generally applicable to cost-benefit analyses of all kinds. Thus, any
project which changes the pattern of consumer expenditures, e.g., a power
project which results in additional electricity consumption, and a
reduction, say, in transportation expenditure, has a theoretically
equivalent effect. However, it is generally agreed that, in practice,
this constitutes a serious problem only where the project concerned is
so large that the whole constellation of prices throughout the economy
is affected.

To illustrate, consider a water and sewerage project which can be
seen as increasing the supply of improved land, the demand for improved
land remaining unchanged. Theoretically, this might be expected to have
three kinds of effect. First, there would be an increase in the value of
the land that is not improved. Second, there would tend to be a reduction
in the average price of improved land in the economy, as more is supplied.
Third, since improved and unimproved land are to some degree substitutable,

\textsuperscript{4}Or even added, as one theory suggests.

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the fall in price of improved land will tend to result in a fall in demand for unimproved land, which in turn will tend to depress its price. 5

Whether or not this creates a real problem depends largely upon the size of the project. If it is large enough to affect all land prices throughout the economy, it will be difficult to draw any conclusions about the cost and benefits of the project. However, municipal water supply and sewerage schemes are rarely of this magnitude, so, with one reservation, we can rely upon partial equilibrium analysis to estimate benefits. In practice, the increase in supply of improved land brought about by any one project will normally be negligible in relation to the total improved land in the economy, and even if it is not, the close substitutability between improved and unimproved land suggests that the demand curve for the former is elastic. This implies that the impact on the average price of improved, and in consequence unimproved, land should also be negligible.

Now to the reservation. Although the implication of the foregoing is that the relationship between the amount of newly improved and the total amount of improved land in the economy is the crucial factor, it is conceivable that a project could increase the supply of improved land in a particular city by a large proportion, bringing about some reduction in the price of unimproved land in the city, even though, in economy-wide terms, the addition to the stock of improved land is insignificant.

5 Clearly, the increase in supply of improved land is physically equivalent to a reduction in supply of unimproved land, but it can safely be assumed that the impact on prices is proportionately far greater in the former market than in the latter. For all practical purposes, therefore, all that needs to be discussed is the impact of the project on prices in the improved land market, and, via this, the impact on the price of unimproved land.
Clearly, if the concern here with the impact of the project is from the standpoint of a municipal, rather than a national, authority, this kind of result would be of great interest.

However, such a state of affairs would imply a high degree of factor immobility, preventing the initial impact on the non-project area from being overcome by immigration. This would not only be inconsistent with the assumption that project area values might rise, which also calls for factor mobility, but, in the context of developing countries in particular, is probably unrealistic. In practice, the general prevalence of rural-urban drift suggests that the theoretical problem of dealing with offsetting price changes in non-project areas can be ignored. Indeed, the conclusion is that the assumptions made in the present analysis--including those about factor mobility--are perfectly consistent with those normally made in the cost-benefit studies.

V. Empirical Approach

The measurement problems associated with estimating the property value effects of water-sewerage investments are clearly enormous. Most important is the overriding issue of how to separate the effects of the particular investment being studied from the effects of the myriad of other factors which influence land values. The model developed here deals with these problems by using a "control area" approach. Property values in the "project" area are compared with property values in a similar "control" area over a period during which the water-sewerage
project in question is built. The object is to estimate the difference in the increase in property values in the two areas during the period.

It is clearly important that the selected control area should be similar in most relevant respects to the project area, i.e., with respect to housing and population density; income; age, area and value of property; ethnic mix; and location within the metropolitan area. The control area in each of the case studies undertaken here was chosen on the basis of such similarities, in consequence of which it was assumed that land values in the control area would react identically to those in the project area to the influence of any exogenous factor. For example, the improvement of bus services or the building of a new plant in the immediate area would have to have the same effects on property values in the control and project areas, if the results are not to be distorted. Only during the period when the investment takes place should the time trend in project and control area property values diverge. It is this divergence which we wish to measure.

VI. Statistical Method

The basic data to be used in such an analysis are recorded sales values from property transfer records. However, properties sell at

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6 The statistical method described here is appropriate to the data available in the Nairobi case study described below. Two small modifications to this method were necessitated in the Kuala Lumpur study, where less data were available. (See Chapter Five.)

7 An alternative to property values as the unit of analysis would be rents (imputed or nominal). This was rejected primarily because appropriate data were not available.
irregular intervals with the result that, over a study period of any given number of years, sales value data are available for different properties in different years. To illustrate the problem, assume that sales value data were recorded for N properties in the control and project areas over a period of t years. It would be possible to compute a rate of growth in property value between any two years, for any property which sold between those two years. These rates of growth might be arrayed in the following type of table:

<table>
<thead>
<tr>
<th>Property Number</th>
<th>Combinations of Years</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( \text{ro}<em>{12} \text{ ro}</em>{13} \text{ ro}<em>{1t} \text{ ro}</em>{23} \text{ ro}<em>{2t} \text{ ro}</em>{t-1,t} )</td>
</tr>
<tr>
<td>2</td>
<td>( \text{ro}<em>{12} \text{ ro}</em>{13} \text{ ro}<em>{1t} \text{ ro}</em>{23} \text{ ro}<em>{2t} \text{ ro}</em>{t-1,t} )</td>
</tr>
<tr>
<td>3</td>
<td>( \text{ro}<em>{12} \text{ ro}</em>{13} \text{ ro}<em>{1t} \text{ ro}</em>{23} \text{ ro}<em>{2t} \text{ ro}</em>{t-1,t} )</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>K</td>
<td>...</td>
</tr>
<tr>
<td>K+1</td>
<td>...</td>
</tr>
<tr>
<td>K+2</td>
<td>...</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>N</td>
<td>( \text{ro}<em>{12} \text{ ro}</em>{13} \text{ ro}<em>{1t} \text{ ro}</em>{23} \text{ ro}<em>{2t} \text{ ro}</em>{t-1,t} )</td>
</tr>
</tbody>
</table>
where rog\textsubscript{12} is the compound rate of growth in property value between years 1 and 2, and rog\textsubscript{2t} the rate of growth between year 2 and the year t (the last year included in the study). If the first K rows in the table represent properties in the project area, and if rows K+1 to N represent properties in the control area, and if property sold every year so that every rog\textsubscript{ij} could be calculated (i.e., every cell could be filled), the estimation of the property value effects of the investment would be straightforward. That is, the difference between the average rates of growth in the project and control areas around the "investment year" would yield the property value effect of the investment. In practice, the fact that all properties do not sell every year complicates the estimation process considerably. It becomes necessary to use those data on sales values which are available to impute values for the missing rog\textsubscript{ij}, i.e., to estimate on the basis of existing data on rates of growth in value what the rate of growth in value would have been had each property sold every year. The comparison between project and control area property value growth rates may then proceed using the mixture of observed and imputed growth rates.

The total procedure requires eight steps. First, the observed rate of growth between any two years when the property sold is calculated. If a property sold in years 1, 7 and 12 of, say, 20 years studied, then growth rates in value may be computed between years 1 and 7, 1 and 12, and 7 and 12. Second, the growth rates between every combination of two years for which data are available are averaged separately for the control and project areas. For example, if five control area properties sold in both
years 2 and 5 and had annual percent growth rates in value of 4, 5, 5, 5, and 6 respectively, then these are averaged to show a 5 percent growth rate for the control area between years 2 and 5. Computed in this fashion, the result of the second step is one rog (for the control and project areas separately) for each pair of years in which some property sold.

The third step involves adjustment of these average growth rates for underlying differences between the control and project areas which may affect the rate of growth in property values. Specifically, a linear regression is used (separately for the project and control areas) to determine the systematic relationship between differences in observed rates of growth in value between pairs of years, and the corresponding differences in the age of housing, the area of housing, the area of the property, and in the extent of building activity on vacant lots. The residuals from this regression are thought to be "clean" estimates of control and project area differences in rates of land value growth.

To this point, the analysis only uses data on the rate of growth between years in which sales actually took place, and there are no observed rates of growth for those pairs of years in which some property did not sell. The fourth step uses those data which are available—the residuals described above—to impute rates of growth for all possible pairs of contiguous years. Since, for example, the rate of growth between years 2 and 5 is some product of the rates of growth between 2 and 3, 3 and 4, and 4 and 5, it follows that, if the growth rates between some pairs of these years are known, the others may be calculated or statistically estimated. This is done for every pair of contiguous years, separately.
for the project and control areas. By using the rog in the first year in the study as a base and assigning it a value of 100, every rog may be transformed into an index number, again separately for the project and control areas. At the end of this step there is one index number for every year in the study. For example, if 12 years were being studied, there would be index numbers for the rate of growth between 1 and 2, 2 and 3, 3 and 4, .... 11 and 12.

The fifth step is to determine the time trend in these index numbers for the control area. This is done by simply regressing the rog indices against a time dummy variable, e.g., for the value index of year 1, the independent variable takes on a value of one, for year 2 a value of two, etc. In step six, this time trend is applied to the project area indices in order to clear the project area of all cyclical and secular trends--since it is assumed that the control and project areas are subject to the same kinds of cyclical and secular influences. Using this time trend, a set of residuals may be calculated for the project area, each residual being the difference between the actual project area index and the index estimated on the basis of the control area time trends.

If the water-sewerage investment has no effect on property values, the mean of these residuals will be equal for the period before and after the project, but if there is a property value effect, the mean residual will be greater after the year of the investment. The seventh step involves measuring the difference in the mean residuals before and after the investment year. The eighth and final step involves translating the difference in these mean residuals into land value terms.
Steps 5 through 8 might be clarified with a hypothetical graphical analysis. The index numbers for each contiguous-year rate of growth are plotted for the control area as in Figure I-1(a), the same kind of scatter diagram for the project area indexes being shown in Figure I-1(b). For the control area, a least squares linear regression of the index is fitted against the years (where 1-2 assumes a value of 1), and the resulting equation is plotted in Figure I-1(a). Now superimpose the control area equation on the project area scatter diagram, as in Figure I-1(b). Calculate the algebraic mean of the residuals in the project area before the project year (i.e., the $a$ values), and the algebraic mean after the project year (i.e., the $b$ values). If the investment has no effect on property values, the project area scatter points would be distributed randomly about the control area equation both before and after the project year—there would be no significant difference in the mean of the residuals. The graphical example, of course, shows a positive effect on property values, i.e., the residuals are greater after the project year.

VII. Nairobi Case Study: Water Supply and Sewerage

The theoretical and empirical model described above was tested first in the City of Nairobi on both a water supply and a sewerage project. A three-stage sewerage project in an area known as the Upper Hill-Kilimani Estates was selected for study. The first phase of the project sewered the northeast portion of the area in 1960-61. The second stage sewered the southern half in 1961-62. The third stage, currently under
FIGURE I-1(a): CONTROL AREA

\[ I_c = d_c + \beta_c Y \]

FIGURE I-1(b): PROJECT AREA

\[ I_p = a_c + \beta_c Y \]
construction, will sewer the remaining area to the north and west during this and next year (i.e., 1971-1972). Those portions of the area that were not undergoing a particular change in any given year were selected as control areas, i.e., in studying the land value effects of the second (1961-62) phase, the third phase (not sewered until 1971-72) was used.

In the sewerage case study areas, the sewers have generally replaced either conservancy tanks or septic tanks. Here, benefits are: (1) public maintenance of the system replacing private maintenance; (2) greater sanitation and more healthful treatment of waste; (3) increased dependability (especially in rainy weather); and (4) relaxed conditions on subdivision of property.

The water supply project selected for study involved the extension of public mains to an area known as Spring Valley Estate. The control areas used are known as Barton Estate and Upper Parklands Estate. Barton, Spring Valley, and Upper Parklands are contiguous areas northwest of the city center, lying within the new city boundaries. Barton, Spring Valley, and Upper Parklands are homogeneous areas. Upper Parklands was the first to receive city water and provides a control on the other two areas. Upper Parklands is the only one of the areas that was included in the old city—that being the reason for its receiving city water supply first. The Barton area is still largely without city water supply. Spring Valley was given public supply during the study period in 1964.

In the case of the water supply project, the benefits from the public reticulation system are to be judged in comparison with private borehole-distribution systems partially augmented by public water carried from
center city kiosks. In most cases where the private water is supplemented with public water carried from the city, the additional public water is used for drinking and the private water is used for non-drinking purposes. The benefits are, therefore, mainly: (1) better water quality; (2) increased dependability; and (3) greater convenience. It is the property value changes induced by these factors that we attempt to quantify in this study.

All areas, either in the water or sewerage study, were thought to be relatively free from public control of rent levels or of property transaction values. This does not imply that the government was not a landowner in these parts of the city. In fact, several plots were owned by the Kenyan government as well as by other national governments. Government land holdings in this area, however, were not used to subsidize housing costs, but were used mainly as reserves for future government projects. The relative freedom of operation of the property market is a major reason why this analysis has concentrated on the northern and western areas of the city. 8

The basic data were collected on property transaction values for the

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8In the eastern and southern sections of the city, landowning by government was much different than in the west. There, the government ownership was to provide subsidized, rent-controlled, low-income housing. To the large-scale extent that the government was involved in property sales in these areas, even the value of privately owned land would be held somewhat below true market value. Determination of true market values in these areas would, at best, be an extrapolated forecast from an analysis, using western data to fit a property value determination model. It was considered that a direct analysis of the western areas would be more valid, statistically. Consequently, no areas in the eastern sector were chosen for this part of the analysis.
1950-1971 period, and on the characteristics of the properties including age of house, land area, and plinth area of house. In total, these (and other) data were collected for 1188 properties in the study areas. These data, on-site visits, and sample survey data obtained from the Nairobi Urban Study Group were used to ascertain the similarity of the control and project areas selected. These tests, though cursory, did verify the similarity between the control and project areas selected.

The empirical analysis was carried out as described in the preceding section. However, at the point where the time series data are adjusted for secular and cyclical influences (See Figures I-1(a) and (b) above.), three alternative specifications of the estimating equation were introduced. Hence, there is a range of three estimates for the increase in property values which results from the water supply or sewerage investment.

For the water supply investment, the results showed estimates of property value increase which range from 35 percent to 62 percent, corresponding to a range of from 27,100 to 48,000 shillings. These seemingly high estimates are in part due to the fact that this area was annexed by the city at approximately the same time that the city water system was extended to the project area. The effects of incorporation and investment in water are irrevocably bonded together in these estimates so that it is not only the water investment, but also the other benefits of annexation (or expectations of such benefits) that are measured. Clearly, these results are likely to overestimate the benefits of the water supply investment.

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9 Based on an average sales price for water area properties in 1964. The average price from sales data is 77,500 Kenyan shillings.
improvement per se. However, the results show strong positive effects on property values, and interviews with public officials in Nairobi made it clear that the single most important change which took place during the period in question was the extension of city water to the area. Therefore, while perhaps not as "clean" as they need be, these estimates show a marked effect of water supply improvements on land values.

The two sewerage projects studied occurred in 1960-61 and 1961-62, and involved the replacement of rather inefficient conservancy tanks. It was estimated that the 1960-61 project was associated with an increase in property values of between 20 percent and 45 percent (11,399 to 25,400 shillings), the range for the 1961-62 project being 25 percent to 48 percent (15,800 to 30,336 shillings). Unfortunately, a difficulty similar to that encountered in the estimation of water supply benefits arose, in that the introduction of a sewerage system was accompanied on each occasion by a zoning change in the affected area. The introduction of a sewerage system allowed higher density zoning, which would undoubtedly increase property values. The impact of the sewerage investment would, therefore, be comprised of two elements: first, the intrinsic value to existing occupiers of land because they can now dispose of waste water more efficiently and, second, the potential gains to be derived from subdivision of properties. There seems to be no way to disentangle these effects statistically.

The results for Nairobi, therefore, did not provide the exact  

10 Property values used here are averages around the time of the project.
information necessary to judge the value placed upon connection to improved water supply and sewerage systems. Moreover, much information was collected which proved to be of no use in the analysis. The Kuala Lumpur case study, which followed, took advantage of these lessons, and was more economical in terms of time and data collection, and avoided the problem of boundary changes and zoning by selection of areas which were not subject to problems in these respects. However, other problems were encountered in using the Kuala Lumpur data, which also precluded the possibility of drawing specific conclusions regarding the impact of environmental improvements on property values.

VIII. Kuala Lumpur Case Study: Sewerage Only

Unlike Nairobi, the analysis in Kuala Lumpur was restricted to sewerage investments, since public water supply had been available for virtually all of the population for a number of years. Areas presently sewered are mainly in the city center. Sewerage extensions for which data were collected occurred during the period 1959-69, with the largest extensions into the project areas taking place between 1965 and 1968.

Unsewered and sewered areas differ dramatically in terms of disposal methods. Properties in sewered areas are served by the most modern and convenient sewage disposal facilities. Homes are connected to a fully reticulated system of sewer pipes which permit conventional plumbing systems in bathroom, kitchen, and laundry areas. Disposal is immediate. Unsewered areas largely utilize ground dumping and bucket systems where
liquid wastes are poured off onto local disposal areas or are stored in containers on the property, to be collected by sewage disposal trucks.

Two project areas and one control area were selected for study. These areas are shown in Figure V-1 of Chapter Five. As in Nairobi, the project and control areas were carefully selected so as to exclude any properties that were used for governmentally subsidized, low-income housing, or that would be affected noticeably by tourism. Several areas were rejected from the study for these reasons.

Compared with the statistical estimation procedure described in Chapter Three for the Nairobi analysis, one major change was adopted for the Kuala Lumpur case study. This modification is required because fewer data observations are available, and it involves the statistical treatment of an individual observation.

With the Nairobi data, the first step in the actual estimation involved an attempt to explain the rate of growth for any pair of years. This rate of growth was constructed as the average for all those properties selling in each of the two years. The Nairobi method utilized average rates of growth because of the large number of observations available, and because the intent is to explain average property value behavior. In Kuala Lumpur, the areas selected were much smaller—in terms of number of properties—and, consequently, the number of observations was much less. As a result, with Kuala Lumpur data, averaging was not performed and the regression was run on the actual rate of growth for individual properties.

One serious complication occurred in the compilation of data for the control area in Kuala Lumpur. On the basis of the estimation procedure
used in the Nairobi study, it was determined that 250 properties with two or more sales would represent an acceptable number of data elements. With this data objective, field work began; however, during the data collection stage, it was impossible to obtain a good estimate of how many such properties were contained in the project and control areas.

Data were drawn from three different sources—the sales data from records of the Malay State Office of Lands and Mines, the property record (characteristics) data from the Kuala Lumpur City Valuer's Office and also from the Malay State Office of Lands and Mines (but from different source books), and the sewerage hookup data from the Kuala Lumpur Sewer Department. Because of the sheer size of the data accumulation, an on-site check was maintained to assure matchup among the three data sets, i.e., to assure that an adequate amount of sales data and other necessary data for individual properties could be drawn from the sample areas. Since data were exhaustively collected from three distinct geographical areas (Over 800 properties were examined.) for each of the data requirements, it was assumed that a good matchup would occur. However, upon return to the United States, and after eliminating "bad sales data,"¹¹ it was found that significantly smaller matchup had occurred than had been anticipated. This was due in large part to incompleteness of local records. Consequently, for some properties, only sales data were available; for other properties, only property characteristic data could be gathered; for yet other properties, only hookup data could be recorded. In fact, by assigning an

¹¹Sales which were inexplicably low, those which were affected by subdivision, amalgamation, etc.

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observation to either project or control groups through use of property registration numbers, and by lumping the two project areas together, it was found that only 190 complete observations (of dual sales) existed for the project area and 93 observations existed for the control area.

Empirical estimation of the model is possible in spite of the relatively small number of observations available, if the empirical method is adjusted so that it is applicable to the smaller data sets. However, upon further computer evaluation of the data, at the stage of constructing property value indices for the control area, more difficulties were encountered in the data set. Specifically, of all the observations containing dual sales for the control area, very few contained a prior sale in any year before 1961. Thus, it was impossible to obtain adequate information on pre-1961 rates of growth and property indices for the control area. This was such a systematic data weakness that the physical records relating to Kuala Lumpur were re-evaluated. None were miscoded or deleted from the computer stored data, but, upon further investigation, the source of error was determined. The control area prior to 1961 was composed of larger properties which were subdivided, after 1961, into many smaller properties. This problem suggests that, before future similar case studies are undertaken, a complete and historical record of land market conditions must be made available to investigators.

By convention, when this problem of inadequate earlier-year data occurred in the Nairobi study, a procedure was arbitrarily adopted to assign to the missing year's rate of growth a value equal to that of the succeeding year's rate of growth. This appeared to work satisfactorily in
Nairobi because, even with many study groups, observations were sufficient to provide nearly all, or all but one or two, of the requisite rates of growth. This procedure was consequently written into the estimating format. When this procedure was applied to such a serious problem as encountered in Kuala Lumpur, however, the rates of growth for each year from 1949 to 1961 became equal by assumption, and the analysis was clearly biased by the assumption. If, for example, the 1961 control growth rate was very low, then all growth rates from 1949 to 1961 were assigned very low values. When the control and project areas were compared in subsequent empirical steps, the control area consequently seemed to jump to a higher rate of growth at about the time of the project (post-1961) relative to the project area. The result was to obtain a negative effect of the project on property values; in fact, a drop of 30 percent was estimated. Further examination of data on property transactions provided a reasonable explanation of this result.

The existence of a large-scale subdivision around the year 1961 in one isolated geographical area might be expected to tend, if many immediate sales occurred, to depress the potential sales values in that geographical market. The subdivision certainly had the effect of flooding the market with properties, and, since price is a decreasing function of such outward supply shifts, land values would be below that level which would have existed, had there been relatively fewer sales. After the initial flood of properties, however, (say post-1964) land values were found to have increased not only because of normal market forces and exogenous forces, but also because of the slower rate at which properties
in the area were put on the market. The rates of growth on these dual (pre- and post-1964 sales) therefore appeared very high, but certainly at least part of their high value was attributable to the subdivision flood effect.

When a control area subject to the above influences is compared to a project area in the manner in which the empirical methodology of the present study intends, the influence of the water/sewerage project will necessarily be underestimated. That is, the control area will be perceived to have had a higher growth rate over the project data than the project area. The ultimate result, if such an effect is large enough, is to estimate that a water/sewerage project reduces property values. For Kuala Lumpur, this is the result which was empirically estimated. While this could be interpreted as implying that residents actually paid more for water and sewerage than they were intrinsically worth to them, this result is discounted. Not only has the Kuala Lumpur study suffered from lack of data, but also it is subject to the large subdivision influence. The negative results subsequently reported are totally discounted and are attributed to empirical problems.

IX. Implications for Future Results

The research reported in this study is meant to be preliminary. There is no pretense that the theoretical and empirical problems have all been solved, nor is it suggested that the statistical results derived for Nairobi or Kuala Lumpur are of direct policy relevance. Still, there are
lessons here, about the reasonableness of the approach in general and about the prerequisites to further studies of this kind in particular.

The notion that certain types of public investment benefits are revealed in land value increments, and that such benefits are at least roughly measurable, is plausible. Economic theory would suggest that the conditions under which land value increments roughly account for consumer welfare increases are largely met in the case of the urban water/sewer projects in less-developed countries studied here. Apart from instances where there is direct government intervention in the housing market, there is reason to expect that these conditions are present in most developing countries. That land value increments are measurable is subject to question, but support for the position that they are may be offered in two ways. First, a land value increment approach is already being used in some less-developed countries to estimate project benefits, e.g., land adjustment in Korea, valorization in Colombia, and town planning schemes in India. Second, this research makes a modest effort at the difficult measurement question by identifying a large body of usable data and by using this data in a systematic way to estimate land value increases.

This exercise, through its shortcomings, also suggests a number of ways in which further research of this type may be strengthened. One is the devotion of more detailed study to the possible project and control areas so as to insure adequate data availability and to guard against the existence of exogenous influences which may distort the results. A second is to investigate the possibilities of using smaller samples and the possibility of tapping information sources such as real estate agents in
assembling land value estimates. Many other suggestions for modification of the research design are to be found in the body of the report.
CHAPTER TWO

SOME THEORETICAL ISSUES

I. Introduction

The premise of this study is that investments in public sewerage and water facilities will be reflected largely in increased land values. The next section introduces the basic model, beginning with simplifying linearity assumptions, and proceeding to the general case. The remainder of the chapter deals with the most difficult empirical and theoretical problems to be overcome, namely, those concerning the treatment of repercussions on property values outside the area that is improved. Relevant literature is examined, and a practical approach to the problem defined.

II. The Basic Model

Use of the property market as an indication of the value of water and sewerage projects rest on the same basic assumptions used in public utility pricing theory. These include the assumption that consumers are

1 Throughout this paper, the terms, water market and sewerage market, are frequently substituted for each other. This affects the theory only in a few places. Where it does, a distinction between the markets will be drawn.
relatively well informed, that prices in general represent reasonable measures of social costs and benefits, that there are no externalities, and that the marginal utility of money is constant. Where appropriate, these assumptions will be relaxed as we proceed.

Let us begin by supposing that only one property in a city is given a public water supply, and the benefit from that water supply is given to the $i^{th}$ individual as $a_i$. The expenditure made by individual $i$ on the quantity of water that returns $a_i$ worth of benefits to him is given as $b_i$. The $i^{th}$ individual would be willing to pay $a_i - b_i$ more for that property because it contained a public water supply than he would pay, if the same property had no public water supply. $a_i - b_i$ represents the maximum amount by which the individual would be willing to bid up the price of the property. Whether the price is actually bid up by $a_i - b_i$ depends upon whether benefits from the investment are fully capitalized into property values.

The connection between the improvement and land values must be shown through a shifting demand curve. The relation between the shift in demand and the improvement is made explicit by introducing the psychic and real income effects due to the investment into the utility function. Demand, derived from the utility function, can then be related to the initial investment. Presuming that an investment does not increase quantities of land consumed (supply inelastic), these arguments indicate at least one plausible assumption. From above, $a_i - b_i$ is the amount by which an individual, $i$, is willing to bid up property values. Given the usual interpretation of the area under a demand curve as the
willingness-to-pay area, $a_p - b_p$ should represent the increased area under the demand curve—integrated over the interval $0$ to $q_E$, the (unchanged) equilibrium quantity.

A Graphical Illustration

Suppose the market for property looks as pictured in Figure II-1. The crucial characteristics are a linear demand curve and a perfectly inelastic supply curve. The original demand curve is given by $D$. Supply is given by $S$. The area $P_EBq_EO$ is the pre-investment expenditure on properties. The total willingness-to-pay for these properties is given as $ABq_EO$. Consumers' surplus is the difference, $ABP_E = ABq_EO - P_EBq_EO$.

Assume that, after a water/sewerage investment, there is a parallel shift in demand to $D'$. The area, $A'B'BA$, is, by definition, equivalent to benefits due to the investment that are not captured by sales in the water/sewerage markets themselves. That is, $A'B'BA$ is definitionally equivalent to the increase in willingness-to-pay area. Thus, $A'B'BA = A'B'q_EO - ABq_EO$. The question becomes one of trying to show that the additional consumer surplus ($A'B'BA$) is equivalent to the increase in expenditures on the property. The increase in the expenditures is determined by $\Delta P \cdot q_E$ (since $q_E$ is constant). $\Delta P$ is, of course, the change in equilibrium price: $\Delta P = P'_E - P_E$. The total change in expenditures is given by the area of $P'_EB'_B$.

It is necessary to show that $P'_EB'_B$ is equivalent to $A'B'BA$. From the simplest theorems of plane geometry, it can be shown that $\theta = \theta'$, $B'P'_E = BP_E$, and the intersections of $BP_E$ and $B'P'_E$ with $OA$ and $OA'$ respectively...
FIGURE II-1

THE MARKET FOR PROPERTY
tively are right angles. These are sufficient conditions to guarantee that the areas $A'B'P'E$ and $ABP_E$ are equal. Construction was made so that:

$$A'B'BP'E - A'B'P'E = B'BP'E - ABP_E.$$ 

It is evident that $A'B'BP'E - ABP_E = A'B'BA$ by construction. Finally, by the equality between $A'B'P_E$ and $ABP_E$, the required result that $B'BP_E = A'B'BA$ is guaranteed. For a case with these special conditions, the change in expenditures on land values fully measures the benefits of the water/sewerage investment which are not accounted for by direct expenditures on water/sewerage itself, i.e., the increase in price exactly exhausts the increase in consumer surplus.

**An Algebraic Solution**

The assumption of a linear demand curve is not crucial to the argument. Assume that the property demand function is in the form of:

$$q_d = g(q_d)$$

and that the supply is still given by a constant:

$$q_s = k$$

Equating supply and demand, equilibrium price is derived:

$$P_E = f_1 [g(q_d), k]$$

The consumers' surplus is:

$$C = \left[ \int_0^{q_d} g(q_d) - P_E \right] k$$

These are the conditions that exist prior to investing in the water supply/sewerage facilities on these properties. Introducing an investment, the demand curve for properties is assumed to shift outward. The new demand curve is:

$$q_d = h(q_d)$$
Similarly, the new consumers' surplus is now measured:

\[ C' = \int h(q_d) - P_E^k \] (6)

The new equilibrium price level is defined as:

\[ P_E^* = f_2(h(q_d), k) \] (7)

The increase in consumers' surplus between the two periods, \( C' - C = \Delta C \), is:

\[ \Delta C = \int h(q_d) - P_E^k - \left( \int g(q_d) - P_E^k \right) \]

\[ \Delta C = \int h(q_d) - \int g(q_d) - \Delta P_E^k \] (8)

The increase in net benefits, \( \Delta B \), of the water/sewerage project has, by assumption, been given as the increase in area under the demand curve.

That is, the increase in benefits is the difference in the integrals:

\[ \Delta B = \int h(q_d) - \int g(q_d) \] (9)

Substituting (9) into (8):

\[ \Delta C = \Delta B - \Delta P_E^k \] (10)

The change in expenditures on land, given by \( \Delta P_E^k \), will only equal the change in true benefits in this case, if the change in consumers' surplus is zero, e.g., when the supply curve is perfectly inelastic and the new demand curve is parallel to the old.

III. Effects on Property Values Elsewhere

The St. Louis Air Pollution Study

Probably the best known empirical study in this area is by Ridker
and Henning\textsuperscript{2} on the effect of air pollution on land values in St. Louis.

Their main conclusion is that:

...if the sulfation levels to which any single-family dwelling unit is exposed were to drop by 0.25 mg./100cm\textsuperscript{2}/day, the value of that property could be expected to rise by at least $83 and more likely closer at $245. Using the latter figure and assuming the sulfation levels are 0.25 mg., but in no case below 0.49 mg. (taken as the background level), the total increase in property values for the St. Louis Standard Metropolitan Statistical Area could be as much as $82,790,000.

The Ridker-Henning model has been readapted several times for air pollution studies.\textsuperscript{3} However, only one alternative model exists--that of Strotz,\textsuperscript{4} which is discussed below. A number of instructive criticisms have, however, been leveled at the Ridker-Henning approach by other authors.

Freeman\textsuperscript{5} is critical of the assumptions that one might implicitly

\begin{itemize}
\end{itemize}
assign to the empirical model used by Ridker and Henning. First, he suggests that we cannot worry about whether the land value changes measure benefits exhaustively, when we cannot even determine whether, or to what extent, demand or supply factors are at work:

In any urban area this relationship (property value-air quality) is the result of the interaction between the availability of land with different levels of air quality (supply factor) and tastes and preferences, other prices, income, and its distribution (demand factors). For any given set of demand factors different supply factors will lead to different patterns of property values and different regression results.6

Moreover, the pollution abatement project effectively changes the supply conditions under which the relationship between property values and air quality was originally estimated. Freeman continues:

This equation only purports to explain the variation in mean property values among observations. The air pollution coefficient can be used to predict the difference in property values between two properties within a system under ceteris paribus conditions, and these conditions must include no change in air quality over other land in the system. But the regression equation cannot be used to predict the general pattern of property values or changes in the value of any given property when the pattern of air quality over the whole urban area has changed.7

Edel makes a similar criticism,8 as follows:

6 Ibid., p. 415.
7 Ibid.
If pollution in all districts of St. Louis were reduced to the 0.49 mg. "background" level, demand for space in the formerly most polluted neighborhoods would certainly increase. But this increase might come at the expense of demand in census tracts that formerly had a unique advantage in low pollution levels. The $82,790,000 estimate assumes that demand will increase in the newly cleaned areas to equal demand elsewhere now, without the balancing effect of demand reduction. It is therefore almost certainly an overestimate.

Edel's analysis correctly points to the fact that, while the estimate of $82,790,000 may or may not be a correct assessment of the value or benefits of a pollution abatement project, it is doubtful if such a change in overall property values would in fact be revealed. However, as long as each property would increase in value by $245 if pollution levels were reduced, it makes little difference whether the increase in price really occurs or not. It is only the willingness to pay the additional amount that matters. Edel also concludes that the difference between net benefits of $82,790,000 and the actual change in land values determines the distributional effects of the investment. Thus:

Notwithstanding the error in this prediction of property value changes, Ridker and Henning may be correct in their estimation of an $82,790,000 value for the removal of pollution. If the $245 that is now bid for the advantage of a less polluted house represents the average of the values that all consumers would place on clean air, then multiplying this by the number of households should give the total benefit of cleaner air. The difference between this figure and the lower change in property values would be a consumers' surplus accruing to the families whose housing costs fell. Total land price changes would then be an inadequate measure of the benefits of an air cleanup, although the comparison of prices between polluted and non-polluted areas would allow a measure of the benefits.
Another way of saying this is that extrapolation of the increase in property values in an area that is subject to environmental improvement to the whole area may (1) overstate the total increase in property values for the whole area, because prices outside the project area might fall, but (2) still be a measure of value, the reduction in values elsewhere simply being a transfer item, or redistributional effect. Strotz, however, goes further, as outlined below.

The Strotz Model

Suppose that a pollution abatement project has the effect of increasing land values by $1,000,000 in the project area, and of decreasing values by $700,000 in the non-project area. Is the net benefit $1,000,000--or $300,000? Or, indeed, is it $1,700,000, as Strotz' model suggests?

We begin by outlining the Strotz model, demonstrating that although it is logically correct if the initial assumptions are accepted, the assumptions themselves are unrealistic. The Strotz model is summarized below by listing (1) the variables employed, (2) assumptions, and (3) method of algebraic manipulation.

1. Variables
   
a. \( u^i \) - the utility function of \( i^{th} \) individual \( i=1, \ldots, I \)
b. \( n^i \) - quantity of land held in the north by \( i \)
c. \( s^i \) - quantity of land held in the south by \( i \)
d. \( x^i \) - quantity of bread (a unit good--money) consumed by \( i \)
e. \( \alpha \) - an index of relative attractiveness of land in the north versus land in the south. As \( \alpha \) increases, northern land vis-à-vis southern land becomes more attractive.
f. \( p \) - price of \( x \)
g. \( \Pi \) - profits (returned to the I individuals) of the bread exchange
h. \( \rho \) - profits (also returned) of the real estate firm
i. \( r_n, r_s \) - rental rates of northern and southern land, respectively
j. \( p_x \) - selling price of bread—normalized to 1
k. \( N, S \) - fixed amounts of land in north and south, respectively

2. **Assumptions**

a. Air pollution affects every part of a region equally.
b. \( N = S \).
c. A pollution abatement project changes in northern half of town for the better so that there is a shift in demand, causing rents to increase by $1,000,000 in the north and decrease by $700,000 in the south.
d. There are no moving costs.
e. Each person occupies some land in both the north and south (to assure convexity in the indifference surfaces).
f. Each person has an identical quantity, \( \bar{x} \), of bread at his disposal.
g. No land is owned, but is rented from the real estate company.
h. Each person receives equal shares, \( \Pi \) and \( \rho \), of profits from the bread exchange and the real estate company, respectively.
i. Each individual maximizes his utility subject to the budget constraint.
j. The bread exchange buys bread at price, \( p \), sells it at a normalized price, \( 1 \); \( r_n \) is the rental rate of land in the north, \( r_s \) is the rental rate in the south.

k. \( a \) is assumed to vary, exogenously, with the spatially differentiated impact of public investments.

l. The change in welfare, \( W \), given a change in \( \alpha \), is defined as a weighted sum of the individual changes:

\[
\frac{dW}{d\alpha} = \sum_{i=1}^{I} \frac{1}{i} \frac{du_i}{d\alpha} ;
\]

\( u^i_x \) is the partial derivative of \( u^i \) with respect to \( x \).

In a cardinal sense, this represents the marginal utility of money.

3. Methodology

a. From the Lagrangean of the utility function and budget constraint, maximize with respect to \( s^i, n^i, \) and \( x^i \).

b. Differentiate \( u^i \) totally with respect to \( \alpha \).

c. Differentiate totally with respect to \( \alpha \) those extremum conditions taken above in (a) for \( n^i \) and \( s^i \). Subtract the result for \( s^i \) from that for \( n^i \) and divide this by \( u^i_x \). This gives the result that:

\[
\frac{dr_n}{d\alpha} - \frac{dr_s}{d\alpha} = r_n.
\]

d. Divide the results of (b) by \( u^i_x \), substitute the 1st order conditions of (a) into this result and sum over \( i \).
e. Substituting the result from (c) into that from (d) after utilizing the conditions (by assumption) that:

\[ \frac{\partial n^i}{\partial a} = \frac{\partial s^i}{\partial a} = \frac{\partial x^i}{\partial a} = 0 \]

and that \( N = S \), the conclusion is obtained that:

\[ \frac{dW}{da} = \frac{dr_nN}{da} - \frac{dr_sS}{da}, \]

i.e., the net welfare effect in the Strotz example is $1,700,000, derived by adding the $1,000,000 increase in land values in the north to the (positive value of) $700,000 decrease in values in the south.

The weakness of the Strotz model is to be found in three of the assumptions listed above, which are particularly restrictive. The first of these is that the total quantities of land in the north (the project area) and the south (the non-project area), and bread (which represents all other goods) are fixed, and distributed evenly among residents. The second is that both the bread exchange and the real estate company make zero profit, implying that there is no aggregate substitution between bread and land. The third is that individuals do not own land, all of which belongs to the real estate company, which returns all receipts of rent back to the residents in the form of profits from the rent company activities.

The foregoing infers that the welfare function is homogeneous of degree zero in uniform increases or decreases in rent to all individuals. Suppose, for example, all rents are raised by \( $r \). The increase in total rent payments is given as \( $rI \), the product of the increased average rent
and the number of renters. Profits of the real estate company increase by $ri also. These profits are then distributed back to the residents so that everyone's income increases by $ri/I, leaving each resident just able to pay the additional rent, with no income left over to increase his level of welfare. If the rent on the land increased and the residents owned their own land, on the other hand, this surely would be perceived as a net welfare gain, because the value of the land which they own is a determinant of their stock of wealth, which may later be traded in to obtain a greater flow of future goods and services. It is the present value of this increase in land values which will be reflected in increased welfare. These effects will be similar, but in reverse, for a uniform decrease in property rental rates.

The Strotz model result is correct, if one accepts the assumptions that form the basis of the model. Since there are no substitution effects and no wealth effects, welfare changes are easily assessed. Land in the south has not deteriorated in quality. If this land is used as a kind of numeraire, it is clear that the people judge the northern land to be a net $1,700,000 better than southern land. This is represented by the change in the relative values of northern and southern properties.

The model may be made more realistic by (1) permitting substitution effects and (2) allowing some sort of wealth effect to operate in the utility function. This latter consideration may be approached by either of two alternative methods. The problem may first be considered a dynamic programing one in which residents attempt to maximize the stream of utility over time. With this approach, increasing property values
might be reflected in allowing the budget constraint to shift outward over time. The second method would be to introduce land values directly into the utility functions. Parameters in the utility functions would presumably transform these values into present values using the individual's personal rate of discount.

The rest of this section will attempt to modify the Strotz model, using the second alternative above and admitting land values directly into the utility function. Individual rates of discounting remain implicit rather than explicit. The model is also altered by allowing substitution effects. An additional change is made that involves the parameter, a, which assesses the relative attractiveness of northern versus southern land. Parameter a is, in fact, often under the control of society, and may be changed, for example, by a group of citizens combining together in a civic town beautification project. On a more complex level, a group of residents may combine, borrow capital, and invest large sums of money into transportation, pollution abatement, water/sewerage or other types of projects. The benefits of these projects are both psychic and real. They can clearly have a marked effect on local land values. If, however, a is considered to be varied by a community to increase its welfare, then the cost of improving a must also be included in the model. If it were not, there would be no optimal and equilibrium solution since increasing a would amount to a costless way of increasing welfare, W. Consequently, we shall treat a as a control variable and place the cost of increasing a, $p_a$, into the budget constraint. Finally, in investigating the welfare effect of a shift in a, the model which we develop here will include both
benefits and costs of a shift in \( a \). This will become evident as these terms are derived out of the consumers' equilibrium.

There are two ways to include land values in the utility function. Since \( n^i \) and \( s^i \), the quantities of land owned in north and south by the \( i \)th individual, are already included in the argument list, land values can be incorporated into the utility function simply by adding the northern and southern rental rates, \( r_n \) and \( r_s \), to the arguments of the function. This technique implies that the functional form should be cognizant that utility is derived from quantities of land consumed and the values (not simply unit prices—\( r_n \) and \( r_s \)) which the land represents. An alternative approach is to include \( r_n n^i \) and \( r_s s^i \) among the utility function's argument list. These will directly reflect total property values and will be arguments in addition to the \( n^i \) and \( s^i \) already included. The choice between these methods should be arbitrary. The latter approach has been arbitrarily selected for the model that this section presents.

The foregoing arguments lead to a conceptual utility function and budget constraint which eventually leads to the following Lagrangean:

\[
L^i = u^i (n^i, s^i, x^i, r_n n^i, r_s s^i) + \lambda (M-x^i - r_n n^i - r_s s^i - p_a/\lambda) \tag{11}
\]

It is our assumption that \( M \) (money income) is fixed and exogenously given to consumer \( i \). This is contrary to the Strotz assumption, and is included in order to avoid dealing with a general equilibrium model, and also to avoid suggesting, as Strotz does, that \( M \) is derived by receipts from the bread and land exchanges. The bread and land exchanges are, in fact, dropped from our theory.
The first order conditions for utility maximization are:

\[ \frac{\partial u^i}{\partial \alpha} = r_n \lambda_i \]  
(12)

\[ u^i_\alpha = r_s \lambda_i \]  
(13)

\[ u^i_x = \lambda_i \]  
(14)

\[ u^i_r n = \lambda_i \]  
(15)

\[ n u^i_{\alpha n} = p_a / \lambda_i \]  
(16)

where the symbolism is read as, for example, \( u^i_{\alpha n} = \partial u^i / \partial \alpha \). The total derivative of \( u^i \) with respect to \( \alpha \) in the utility function is given by:

\[ \frac{du^i}{d\alpha} = \left( \frac{\partial u^i}{\partial \alpha} + r_n u^i_{rn} \right) \frac{dn}{d\alpha} + \left( u^i_\alpha + r_s u^i_r s \right) \frac{ds}{d\alpha} + n u^i_{\alpha n} \]

\[ + u^i_x \frac{dx}{d\alpha} + n u^i_r n \frac{dr_n}{d\alpha} + s i u^i_r s \frac{dr_s}{d\alpha}. \]  
(17)

The ultimate goal of this analysis, of course, is finally to relate a change in \( \alpha \) to some welfare change. Since it will also be instructive to keep the analysis similar to Strotz's for comparison, we shall employ the Strotz welfare formula on both counts. This defines welfare changes with respect to \( \alpha \) as:

\[ \frac{dw}{d\alpha} = \sum_{i=1}^{I} \left( \frac{1}{u^i_x} \frac{du^i}{d\alpha} \right). \]  
(18)
Consequently, in order to transform equation (17) into equation (18), the results in (17) are multiplied by \((1/\lambda_i)\) and summed over \(i\).\(^9\)

\[
\frac{dw}{d\alpha} = \sum \left\{ \left( \frac{\sum i \lambda_i + \sum i \lambda_r \lambda_i n_i}{u_i + \sum u_i} \right) \frac{dn_i}{d\alpha} + \left( \frac{\sum i \lambda_i + \sum i \lambda_s \lambda_i s_i}{u_i + \sum u_i} \right) \frac{ds_i}{d\alpha} + \frac{n_i u_i}{u_i} \right\}
\]

Much of this is simplified by applying the first order conditions given by equations (12) through (16). By applying the 1st order conditions at this point, the interpretation of \(dw/d\alpha\) is that it represents the welfare effects of a change (or disturbance) in \(\alpha\) around the equilibrium values of the model.

From equation (19) this change is given as:

\[
\frac{dw}{d\alpha} = \sum \frac{\Sigma (\sum \frac{dn_i}{d\alpha} + \sum \frac{ds_i}{d\alpha} + \sum \frac{pa/I}{d\alpha})}{d\alpha} + \sum \frac{\Sigma (\sum \frac{dx_i}{d\alpha} + \sum \frac{dr_n}{d\alpha} + \sum \frac{dr_s}{d\alpha} + \sum \frac{n_s}{d\alpha})}{d\alpha}
\]

The first two terms are equal to zero by the fact that total land available in north and south is constant. That is, \(\Sigma (dn_i/d\alpha) = \Sigma (ds_i/d\alpha) = 0\).

\(^9\)The effect of dividing by \(u_i\) which in the 1st order condition equals \(\lambda_i\), is not made clear in Strotz. However, as Harberger specifies, this transformation of putting \(u_i/\lambda_i\) standardizes the utility functions for individuals so that they may be aggregated in a common, standardized unit. See: Arnold C. Harberger, "Three Basic Postulates for Applied Welfare Economics: An Interpretive Essay," Journal of Economic Literature, Vol. IX (September 1971).
The third term sums to the total costs of the project. The fourth term is the substitution effect on other goods in the model; or alternatively, it may be judged as the effective net increase or decrease in expenditures for all land in the model. This is a possible interpretation by virtue of $x$ representing either all other goods or money. The last two terms are the net land value changes in the north and south respectively. Rewriting (2) to reflect these interpretations:

$$\frac{dw}{da} = pa + \sum dx + \Delta r_n + \Delta r_s$$  \hspace{1cm} (21)

This is, as it should be for any cost-benefit framework, an implicit comparison of costs (the first term) and benefits (the last three terms). Given the shape of the utility functions, one can determine whether an investment is warranted, i.e., benefits $\geq$ costs. Failing in this knowledge, ex post observations on land values may provide useful information on the relationship between costs and benefits.

The result in (21) is significantly different from that derived in the original Strotz model. This translated into the present notation is:

$$\frac{dw}{da} = \Delta r_n - \Delta r_s$$  \hspace{1cm} (22)

The major differences are:

1. Strotz's model does not treat $a$ as a control variable and comes to

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10 This is from: Strotz, "The Use of Land Rent Changes," p. 7, equation (26).
a final welfare effect that specifically excludes project costs which are
given in the result here, equation (21), as \( p_{a} \).

2. Equation (21) specifies the possibility of a substitution effect
in the term \( \Sigma(dx^{2}/da) \). The Strotz model cannot show such an effect because
of the strong assumptions which it draws.

3. While equation (21) shows a plus sign between the land value
effects for north and south, the Strotz model, equation (22), shows a
negative sign.

The last of these differences implies a crucial change in the conclusions.
Assuming, as Strotz does, that \( \Delta r_{n} = +$1,000,000 \) and \( \Delta r_{s} = -$700,000 \),
his model does, indeed, give benefits at $1,700,000. The model
developed in our analysis, however, would give the result of $300,000
plus a substitution term. Our conclusion is therefore that, if factors
are free to move in and out of the area (north plus south), the
substitution term will, subject to the conditions outlined in Chapter One,
be approximately \( +$700,000 \), so that the net effect on property values
is \( +$1,000,000 \). Strotz's assumptions about factor mobility\(^{11}\) and
property ownership provide a result that is intuitively and empirically
unacceptable; in practice, the sensible approach is to ignore changes in
price outside the project area. This is done in the present analysis.

\(^{11}\)The issue of factor mobility and return to land, labor and capital
is further discussed in Appendix I.
CHAPTER THREE

ESTIMATION METHOD

I. Introduction

The purpose of this chapter is to describe the method used to estimate the effects of the water/sewerage investments studied here on property values.\(^1\) The procedure to be used appears quite simple on the surface. Property values in the "project" area will be compared with property values in a similar "control" area over a period which includes installation of the water/sewerage project in question. The difference in property value increase observed during the relevant period is approximately the result desired here. There are, however, two relatively serious measurement problems to be dealt with in this context. The first is the adjustment of the data for differences between the control and project areas which may distort the results. The second has to do with the development of a technique for the sheer handling of such a mass of data, i.e., comparing control and project area property value increase when value observations are available for individual parcels for different years in the series. The empirical method which has been developed to deal with these problems is presented in some detail below.

\(^1\)This method was followed exactly for Nairobi. Minor adjustments were made in the Kuala Lumpur study where less data were available.
While the empirical method is developed here on a step-by-step basis, with consideration of each assumption involved, a summary of the technique is presented in the concluding section. Also included in this chapter are a section which examines the basic rationale for the model, and a section which compares this empirical technique to others which have been used in land value studies.

II. A "Control Area" Approach

The theoretical model presented above suggests that an investment will have immediate property value effects in those areas where properties are intrinsically changed by the investment. Such an area is called a "project" area. In other areas where properties are not intrinsically affected by the investment, there is no general equilibrium land value reaction to the investment. That is, the investment in water/sewerage in the project area has no effect on land value in these areas. When chosen on a basis of similarity with the project area, these non-affected areas are designated here as "control" areas.

The choosing of a "control" area which bears appropriate similarity to the project, or study area, poses considerable difficulty. However, such similarity is of great importance because of the need to cancel out as many exogenous influences as possible without direct adjustments in the model. With respect to the objectives here, the similarity sought has to do with the mean and variance of variables such as housing and population density, income, age and value of housing, ethnic mix, and

III-2
location within the metropolitan area. Choosing areas with these similarities ought to enable us to avoid direct analysis of many of the basic factors affecting land value growth.

On the basis of similarity in these variables, the selection of control areas was made. The control area is expected to react identically (to the project area) in response to any change other than the investment itself. The control area information generally lends itself as a correction for long- and short-run secular and cyclical trends in the project area data, i.e., except for periods when the project area has been influenced by the investment, the project and control areas should have identical time trends in property values. During the periods when the investment affects property values, the time trends on project and control area property values diverge. It is this divergence which we wish to measure.

III. A Regression-Property Value Index Method

Data on property transactions in the control and project areas may be collected on a number of properties, say N, over a time period of length T.

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2 This implies, for example, an insignificant Chow or Gujarati test when data are fit to both project and control areas. The Chow test is described in: Gregory C. Chow, "Tests of Equality Between Sets of Coefficients in Two Linear Regressions," *Econometrica*, Vol. 28 (July 1960), 591-605. A modified version of this test is given in: Damodar Gujarati, "Use of Dummy Variables in Testing for Equality Between Sets of Coefficients in Two Linear Regressions: A Note," *The American Statistician* (February 1970), 50-52.
A typical property may have been sold once, twice or several times over this period. The problem is how to use these data to obtain information on property values and the way in which they change over time. Consider the \( N \times T \) matrix of property value (pv) observations which would result, if every property were considered sold in every year under discussion.

\[
\begin{bmatrix}
  PV_{11} & PV_{12} & \cdots & PV_{1T} \\
  PV_{21} & PV_{22} & \cdots & PV_{2T} \\
  \vdots & \vdots & \ddots & \vdots \\
  PV_{N1} & PV_{N2} & \cdots & PV_{NT}
\end{bmatrix}
\]

(1)

In reality, since properties do not sell every year, most of the observations in the sales values, i.e., the cells, in (1) will be missing. There is considerable variance for individual properties in the amount of information they provide about value increases— that is, in the number of cells in each row for which data are available. Many properties will not have sold at all and will therefore yield no information on property value changes, while other properties may have sold only once and will therefore yield only limited information. In fact, the model developed here considers only those properties which have sold two or more times. That is, any of the \( N \) rows with less than two non-zero cells is dropped from the analysis, and the intent is to impute from the remaining information a value for each remaining cell.

As the first step in this process, price relatives are constructed as the ratio of each pair of sales, with the newer sales value in the numerator
and the earlier sales value in the denominator. If all possible price relatives were calculated for a particular property that has sold M times, there would be a total of \( M(M-1)/2 \) price relatives.\(^3\) If those relatives are calculated so that each compares a newer price to an older price, but also so that a price appears in the denominator only once and in the numerator only once, or in numerator or denominator only once, but not in both, it would be possible to construct a maximum of \( M-1 \) relatives. This constraint results in \( ((M/2)-1) \) fewer relatives than would the calculation of all possible relatives and, in a sense, some information is lost—but problems of correlated error terms which would result from using all possible price relatives are avoided.

Price relatives relating sales values of property \( p \) for years \( i \) and \( j \) (\( P_{ij} \)) are calculated over the \( T \) years for all \( N \) properties as follows:

\[
P_{ij} = \frac{PV_j}{PV_i} \quad \text{with } i < j
\]

(2)

The next step involves converting these relatives into the rate of growth between any two years \( i \) and \( j \) (\( ROG_{ij} \)) in the following way:

\[
ROG_{ij} = \frac{1}{j-i-1} \left( P_{ij} - 1 \right)
\]

(3)

The form of equation (3) insures that \( ROG_{ij} \) is the average annual compounded rate of growth on property \( p \) between years \( i \) and \( j \). Since our interest lies

---

\(^3\)This method will produce more price relatives than property value observations for \( M > 3 \); it will produce an equal number for \( M = 3 \) and fewer for \( M < 3 \).
in the average or overall growth rate in the project and control areas, rather than in the value growth of individual properties, the third step of the statistical procedure is to obtain an average rate of growth (ROG) of all properties in some area which may be either a control or a project area. This average is obtained from:

\[
\overline{\text{ROG}}_{i,j} = \frac{\sum_{p=1}^{N_k} \text{ROG}_{i,j}}{N_k}
\]  

(4)

where, again, \(\overline{\text{ROG}}_{i,j}\) is the average rate of growth in value of properties in the relevant area, between the years \(i\) and \(j\). For a time span of \(T\) years, \(\frac{T(T-1)}{2}\) possible rates of growth may be calculated. The variation in \(\overline{\text{ROG}}_{i,j}\) (i.e., the variation in the growth rate in land values between pairs of years) might be attributable to the variation in certain exogenous variables associated with the \(p\) properties which sold in \(i\) and \(j\). This variance is unwanted for the purposes at hand and might be eliminated by 'residualizing' the \(\overline{\text{ROG}}_{i,j}\).

These exogenous factors exist in this control area approach because it is not possible to select control and project areas which are exactly similar in every respect. The exogenous factors for which adjustment is made here are:

1. the age of the house, measured as the average age of all houses which sold in years \(i\) and \(j\). The average age is taken as the year \((j+i)/2\). This variable is referred to as \(\overline{\text{AGE}}_{i,j}\).

---

\(\text{The first and second steps of the procedure are given by equations (2) and (3) respectively. Later, we shall discuss deleting certain steps from the model, and examine their empirical effect.}\)
2. the land area of the property, calculated as the average for all properties sold in i and j. This variable is referred to as \( \text{ARP}_{i,j} \).

3. the plinth area of the house, calculated as the average for all properties sold in i and j. This variable is referred to as \( \text{ARH}_{i,j} \).

4. the percent of properties selling both in i and in j, that were vacant in i, but on which a house was built between i and j. This variable is referred to as \( \%\text{BLT}_{i,j} \).

In fact, since the age of housing, and areas of housing and property are variables that explain property values, the level of property value may be used as a proxy for these three exogenous factors. Accordingly, the \( \text{ROG}_{i,j} \) variable is adjusted here using only two variables:

1. the initial level of property values, \( \text{PV}_{i,j} \), averaged over all of the properties in the relevant area that sold in both i and j; and

2. the percent of properties selling both in i and j, that were vacant in i, but on which a house was built between i and j (\( \%\text{BLT}_{i,j} \)).

The adjustment in \( \text{ROG}_{i,j} \) is made by first estimating \( \hat{a} \), \( \hat{b} \), and \( \hat{c} \) from equation (5), using ordinary least squares:

\[
\text{ROG}_{i,j} = \hat{a} + \hat{b} \text{PV}_{i,j} + \hat{c} \%\text{BLT}_{i,j} + \hat{v}_{i,j}
\]  

(5)

The residuals, or adjusted growth rate (\( \hat{\text{ROG}}_{i,j} \)), may be defined as

\[
\hat{v}_{i,j} = \hat{\text{ROG}}_{i,j} - \text{ROG}_{i,j} - \hat{a} - \hat{b} \text{PV}_{i,j} - \hat{c} \%\text{BLT}_{i,j}
\]  

(6)

These residuals represent that part of the growth in value between any two years i and j which is not a function of the exogenous factors introduced above.
Still, there remains the problem that many \( \overline{\text{ROG}_{i,j}} \) are blank because of the absence of sales value data. It now remains to estimate values for these blanks. Accordingly, the sixth step in this procedure involves first obtaining a best fit for individual year rates of growth (noted as \( \text{rog}_\alpha \) for year \( \alpha \)) against the residual average compounded annual rates of growth between years \( i \) and \( j \), i.e., \( \overline{\text{ROG}_{i,j}} \). The following relation is expected to hold with the usual assumptions of log normality in the distribution of the error terms, mean of 1 and zero covariance:

\[
(1 + \overline{\text{ROG}_{i,j}})^{j-i} = \varepsilon_{i,j} \prod_{\alpha=i}^{j-1} (1 + \text{rog}_\alpha)^{1} \quad (7)
\]

Equation (7), of course, cannot be statistically fit directly because the number of terms on the right side will vary with choice of \( i \) and \( j \). However, the character of the equation is left unchanged, if we add more terms to its right side:

\[
(1 + \overline{\text{ROG}_{i,j}})^{j-i} = \varepsilon_{i,j} \prod_{\alpha=1}^{i-1} (1 + \text{rog}_\alpha) \prod_{\alpha=i}^{j-1} (1 + \text{rog}_\alpha) \prod_{\alpha=i}^{1} (1 + \text{rog}_\alpha)^{0} \quad (8)
\]

Linear regression techniques may still not yet be employed, and hence logs of both sides are taken:

\[
(j-i) \log_e (1 + \overline{\text{ROG}_{i,j}}) = \sum_{\alpha=1}^{i-1} 0 \log_e (1 + \text{rog}_\alpha) + \sum_{\alpha=i}^{j-1} 1 \log_e (1 + \text{rog}_\alpha) + \sum_{\alpha=i}^{T} \log_e (1 + \text{rog}_\alpha) + \sum_{\alpha=j}^{T} \log_e \varepsilon_{i,j} \quad (9)
\]

\[\text{The reason that the products extend from 1 to } T \text{ is that } i \text{ and } j \text{ are constrained so that } 1 \leq i < j \leq T.\]
Letting \( b_\alpha = \log_e (1 + \text{Rog}_\alpha) \), equation (9) may be rewritten using Kronecker deltas in the following terms:

\[
(j-i) \log_e (1 + \text{Rog}) = \sum_{\alpha=1}^{T} b_\alpha \delta_\alpha + \log_e \varepsilon_{i,j}
\]

where:

\[ \delta_\alpha = 1 \text{ if } i \leq \alpha < j, \]
\[ \delta_\alpha = 0 \text{ otherwise} \]

Equation (10) is suitable for linear multiple regression analysis. However, \( \delta_T \) will always equal zero by the constraints in \( i \) and \( j \), \( 1 \leq i < j \leq T \), and the definition of \( \delta_\alpha \) in equation (10). Therefore, if \( \delta_T \) is included in the regression, the matrix of independent variable values will be singular and the necessary inversion of the cross products matrix cannot be obtained in order to estimate the \( b_\alpha \)'s. Therefore, we drop the last term in the summation in equation (10), i.e., the observation showing the growth rate in the last year of data. Finally, the sixth step in the statistical procedure is completed by estimating the regression equation:

---

6It is now possible to justify a statement made earlier without proof. That is, using all possible relatives that may be constructed when a property has sold three or more times guarantees that the error terms will be correlated. Consider the model as it is expressed in equation (7) above. We shall apply this to a property with three sales, one each in years \( i \), \( j \), and \( l \), with \( i < j < l \). Equation (7) may be expressly written for this property and three plausible relatives that are constructed from it:

\[
(1 + \text{Rog}_{i,j})^{j-i} = \prod_{\alpha=i}^{j-1} (1 + \text{Rog}_\alpha) \tag{12a}
\]

\[
(1 + \text{Rog}_{j,l})^{l-j} = \prod_{\alpha=j}^{l-1} (1 + \text{Rog}_\alpha) \tag{12b}
\]
\[ (j-i) \log_e (1 + \text{ROG}_{ij}) = \sum_{\alpha=1}^{T-1} \delta_{\alpha} \log \epsilon_{ij} + \log \epsilon_{ij} \]  \hspace{1cm} (11)

where: \( \delta_{\alpha} = 1 \) if \( i \leq \alpha < j \)
\( \delta_{\alpha} = 0 \) otherwise

It is true by definition that:
\[ (1 + \text{ROG}_{il})^{l-i} = \prod_{\alpha=i}^{l-1} (1 + \text{ROG}_{\alpha}) \]  \hspace{1cm} (12c)

Using equation (13), we may express equation (12c) as:
\[ (1 + \text{ROG}_{il})^{l-i} (1 + \text{ROG}_{jl})^{l-j} = (1 + \text{ROG}_{il})^{l-i} \]  \hspace{1cm} (13)

Substituting equation (12c) into equation (15), the dependence among errors is shown:
\[ \epsilon_{il} = \epsilon_{ij} \epsilon_{jl} \]  \hspace{1cm} (16)

Hence, as implied above, we do not utilize all possible relatives, but only a portion of them, such that any one sale will never appear more than twice in a calculation, once in the denominator and once in the numerator of a relative.
In this example demonstrating the dependence among error terms, it is implied that $PV_{pi}$ is used twice in the denominator, once in constructing $ROG_{ij}$ and once in constructing $ROG_{il}$.

The seventh step in the statistical procedure is to take the estimated $\hat{b}_\alpha$ from equation (11) and transform them into property value indices for each of the $T$ years, in each of the $k$ areas. It should be noted that this is possible since the statistical procedure outlined above is carried out separately for each of the different project and control areas. This transformation into property value indices is done as follows:

$$k\text{Index}_\alpha = 100 \sum_{\delta=1}^{\alpha-1} e^{\hat{b}_\delta}$$

where the value of the index in year 1 is taken to be 100. From equation (17), a property value index for each of $T$ years for each project and each control area is obtained. The index for the control area ($\text{Index}_{\alpha}^{con}$) is then adjusted for a time trend in equation (18), where $r$ represents the time dummy variable:

$$\text{Index}_{\alpha}^{con} = \hat{r} + \hat{a}$$

The parameters estimated in equation (18) are then used to residualize the project area index, $\text{Index}_{\alpha}^{proj}$; this step effectively clears the project area index for all secular and cyclical trends—since it is assumed that the control and project areas are subject to the same kinds of cyclical and secular influences. The residual index for project areas may now be
calculated as:

$$\hat{\text{Index}}_{\text{proj}} = \text{Index}_{\text{proj}} - \text{Index}_{\text{con}} = \hat{\text{Index}}_{\text{proj}} - \hat{r} - \hat{s}$$  \hspace{1cm} (19)

At this point, all effects on property values in the project area, save that of the investment itself, have been removed. Therefore, the statistical procedure is completed by testing for differences in $\hat{\text{Index}}_{\text{proj}}$ before and after the period in which an investment on water/sewerage was implemented. This testing is easily handled with an analysis of variance or a means (and associated significant difference between means) test. An alternative, which is equivalent to the means test, is to use a dummy variable regression with one independent dummy variable. This is accomplished:

$$\hat{\text{Index}}_{\text{proj}} = \hat{m} + \hat{n} \delta_{\text{proj}}$$  \hspace{1cm} (20)

where $\delta_{\text{proj}} = 1$ if $\alpha = $ project period when the investment was implemented

= 0 otherwise.

The expectation, given the theoretical model, is that $\hat{n} > 0$, i.e., that there will be a significant increase in the property value index as a result of the investment.

IV. Rationale for the Model

The model described above is, for the most part, a fairly straightforward application of statistics. However, the sixth step, described in equations (7) through (11), is less straightforward and deserves some
additional discussion.

Consider the matrix of rates of growth in property values as described above. Suppose, for example, that the rates of growth between the \( i \) and \( j \) years are calculated and ordered in an upper triangular portion of a matrix; various values of \( i \) are represented by rows, values of \( j \), by columns:

\[
\begin{array}{cccc}
\text{ROG}_{12} & \text{ROG}_{13} & \text{ROG}_{14} & \text{ROG}_{1T} \\
\text{ROG}_{23} & \text{ROG}_{24} & \text{ROG}_{2T} & \\
\text{ROG}_{34} & & \text{ROG}_{3T} & \\
& & & \\
& \ddots & & \text{ROG}_{T-1T} \\
\end{array}
\]

(21)

On a surface inspection, it appears that only the \( \text{ROG}_{12} \) element gives information on the rate of growth between period 1 and 2. However, \( \text{ROG}_{13} \) also gives information, if \( \text{ROG}_{23} \) is known. Similarly, the entire first row of the matrix of (21) gives information; the last element, for example, \( \text{ROG}_{1T} \) gives information, if \( \text{ROG}_{23} \text{ROG}_{34} \ldots \text{ROG}_{T-1'T} \) are known, or if \( \text{ROG}_{2T} \) is known.

The same information is available on \( \text{ROG}_{23} \). \( \text{ROG}_{23} \) gives direct information on the true value, but other sources give additional information. \( \text{ROG}_{24} \) will yield an estimate, if \( \text{ROG}_{34} \) is known, \( \text{ROG}_{13} \), if \( \text{ROG}_{12} \) is known. In general, each rate of growth, \( \text{ROG}_\alpha \), can be estimated from observed values on \( \text{ROG}_{i,j} \), as long as \( i \leq \alpha < j \). This leads to a symmetrical amount of
information on each of the estimated ROG's: \( \text{ROG}_{12} \text{ ROG}_{23} \ldots \text{ROG}_{\frac{T}{2}, \frac{T+2}{2}} \)

\( \ldots \text{ROG}_{\frac{T-2}{2}, \frac{T-1}{2}} \text{ ROG}_{\frac{T-1}{2}, T} \). For even valued \( T \), the amounts of information available to estimate each of the ROG's up to the middle term, \( \text{ROG}_{\frac{T}{2}, \frac{T+2}{2}} \), increase and then decrease to the last term \( \text{ROG}_{\frac{T-1}{2}, T} \). For odd valued \( T \), there will be two middle terms, \( \text{ROG}_{\frac{T-1}{2}, \frac{T+1}{2}} \) and \( \text{ROG}_{\frac{T+1}{2}, \frac{T+3}{2}} \), to which information becomes maximum.

In order to use that information which is available to estimate values for all blank cells in (21), we use a regression approach which, in addition to providing a familiar conceptual device, provides a minimum variance (linear) estimate of the individual year rate of growth.

The fact that different amounts of information are available to estimate the individual year rates of growth (i.e., the rog's) is taken into account in the analysis. For example, there are \( T-1 \) calculated bits of evidence on which to base the estimate of rog_1, although only one of the \( T-1 \) ROG's on which rog_1 is based is direct evidence.\(^7\) The other \( T-2 \) ROG's can be used in conjunction with the remaining \( (T-2)(T-1)/2 \) ROG's that are observed. rog_2 is based upon \( 2(T-2) \) bits of information; again, only one of these represents direct information and the other ROG's can be used in conjunction with the information from the rest of the observed ROG's. In general,

\(^7\)This is ROG_{12}.
\[ \hat{\beta}_\alpha \] will be based upon \( \alpha(T-\alpha) \) bits of information.\(^8\) One would initially suspect that this would produce the least reliable results at the extremes (high and low) of the estimated index. On the contrary, the extremes and middle\(^9\) are both estimated equally well. The regression suggested in equation (11) will produce estimates of the \( \hat{\beta}_\alpha \) which will have an identically equal standard error of estimates for the \( \hat{\beta}_\alpha \)'s. This, of course, is what really should be expected when it is considered that the variance and covariances of the estimated coefficients are given by:

\[ \text{var}(b) = (X'X)^{-1} \sigma^2 \]

where:
- \( X \) is a \( T(T-1)/2 \) by \( (T-1) \) matrix of 0's and 1's, defined by the \( \delta_\alpha \) in equation (11), above
- \( \sigma^2 \) is the true variance of \( \log_\varphi V'_{ij} \) from equation (11), above.

The standard error of estimate of the \( q^{th} \) coefficient, \( \hat{\beta}_q \), is given as the square root of the \( q^{th} \) diagonal element of var(b). This is determined solely by the number of years, \( T \), and the variance of the error terms.

\[ 8 \text{In support of the statement that the maximum number of information bits is available in the middle of the time period, } 1...T, \text{ it can be shown that } d\alpha(T-\alpha)/da = T-2\alpha. \text{ Setting this to zero, we obtain that a maximum occurs at } \alpha=T/2. \text{ The second derivative is negative, insuring a maximum rather than a minimum.} \]

\[ 9 \text{Assuming that all of the } (T-1)T/2 \text{ cells of data in the observation matrix (21) above are filled, this will be true. If some data are missing for } \text{ROC}_{ij}, \text{ these conditions are approached, but do not hold exactly.} \]

\[ 10 \text{This is given by any standard text on regression analysis. See, for example: } \text{N.R. Draper and H. Smith, Applied Regression Analysis} (\text{New York: John Wiley and Sons, 1966}), \text{ p. 61.} \]

\[ 11 \text{This implicitly eliminates a constant term from the regression suggested in equation (11). If the constant term were included, a row containing elements all equal to one would have to be added to the } X \text{ matrix.} \]
The variance of the error term is a scalar constant. The result $X'X$ gives the following $(T-1)$ by $(T-1)$ matrix:

$$X'X = \begin{bmatrix}
T-1 & T-2 & T-3 & \ldots & 2 & 1 \\
T-2 & 2(T-2) & 2(T-3) & \ldots & 4 & 2 \\
T-3 & 2(T-3) & 3(T-3) & \ldots & 6 & 3 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
2 & 4 & 6 & \ldots & 2(T-2) & (T-2) \\
1 & 2 & 3 & \ldots & (T-2) & (T-1)
\end{bmatrix}$$

(23)

where the $i^{th}$, $j^{th}$ element is

$$\varepsilon_{i,j} = j(T-i) \text{ for } i \geq j,$$ and

$$\varepsilon_{j,i} = \varepsilon_{i,j},$$

so that the matrix is symmetrical.

$X'X$ is a very special matrix, symmetric about the principal and subordinate axes. It may also be expressed in its non-reduced form:

$$X'X = \begin{bmatrix}
1(T-1) & 1(T-2) & 1(T-3) & \ldots & 1(T-(T-2)) & 1(T-(T-1)) \\
1(T-2) & 2(T-2) & 2(T-3) & \ldots & 2(T-(T-2)) & 2(T-(T-1)) \\
1(T-3) & 2(T-3) & 3(T-3) & \ldots & 3(T-(T-2)) & 3(T-(T-1)) \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
1(T-(T-2)) & 2(T-(T-2)) & 3(T-(T-2)) & \ldots & (T-2)(T-(T-2)) & (T-2)(T-(T-1)) \\
1(T-(T-1)) & 2(T-(T-1)) & 3(T-(T-1)) & \ldots & (T-2)(T-(T-1)) & (T-1)(T-(T-1))
\end{bmatrix}$$

(24)

Equation (24) is merely an expression of the condition that $\varepsilon_{i,j} = j(T-i)$ for $i \geq j$ and $\varepsilon_{i,j} = i(T-j)$ for $i \geq j$. The matrix $X'X$ has a determinant which
is very easy to calculate. This is true because it can be observed that, as the relevant multiple of the 1st column is added to each of the subsequent columns, the determinant can easily be changed into the following form:

\[
|X^*X| = \begin{vmatrix} 1(T-1) & M_{11} & M_{12} & \ldots & M_{1(T-2)} \\
1(T-2) & 0 & M_{22} & \ldots & M_{2(T-2)} \\
& \ddots & \ddots & \ddots & \ddots \\
& & 1(T-(T-1)) & 0 & 0 & \ldots & 0 \end{vmatrix}
\]  

which yields, by expansion with cofactors and properties of triangular matrices and determinants, to:

\[
|X^*X| = (-1)^T \left[ 1(T-(T-1)) \right] \prod_{Y=1}^{T-2} M_{YY}
\]  

Applying the principle behind equations (25) and (26) and the definitions of \( \epsilon_{ij} \) for \( i \geq j \) and \( j \geq i \), we may obtain the determinant of \( X^*X \):

\[
|X^*X| = (-1)^T \left[ 1(T-(T-1)) \right] \prod_{i=1}^{T-2} i (T-(i+1)) -(i+1)(T-1)
\]  

\[
|X^*X| = (-1)^T \prod_{i=1}^{T-2} i - i^2 - i -it + i^2 -T + i
\]  

\[
|X^*X| = (-1)^T \prod_{i=1}^{T-2} (-T) = (-1)^T(-T)^{T-2} = (T)^{T-2}
\]  

This implies that the determinant is also given as the value of \( T \) raised

\[12\text{ We also make use of the fact that, along the crucial diagonal, for example, in (25) the } M_{i,i} \text{ diagonal, the matrix designates these elements in (24) as } \epsilon_{ij} \text{ with } j=i+1. \]
to the $T-2$ power.

It is now possible to identify any element in the inverse, $(X^\top X)^{-1}$. This is given for the $i, j$th element of the inverse as the product of the reciprocal of the determinant of $X^\top X$ and the cofactor of the $j, i$th element in $X^\top X$. We are specifically interested in the elements along the main diagonal, i.e., where $i = j$. The same principle that was employed in obtaining equations (27)-(29) is used here. For all diagonal elements but the first, the cofactor may be examined as:

$$
\text{cofactor } (X^\top X)_{\gamma\gamma} = (-1)^{T-1} [1-((T-(T-1))))\prod_{i=1}^{T-2} \left[ i(T-(i+1))-(i+1)(T-i) \right] \\
\quad \quad \rightarrow \prod_{i=\gamma}^{T-2} \left[ i(T-(i+1))-(i+1)(T-i) \right] \\
\quad \quad \rightarrow (\gamma-1)(T-(\gamma+1))-(\gamma+1)(T-(\gamma-1))]
$$

(30)

$$
= (-1)^{T-1} \left[\prod_{i=1}^{T-2} i(T-1)^2-i-1T+i \right] \\
\quad \rightarrow \prod_{i=\gamma}^{T-2} i(T-1)^2-i-1T+i \\
\quad \rightarrow (\gamma-1)(T-\gamma+1)-\gamma T-1\gamma^2-\gamma-T+1)
$$

(31)

$$
= (-1)^{T-1} \left\{\prod_{i=1}^{T-2} (-T) \right\} \{-2T\} \\
\quad \rightarrow \prod_{i=\gamma}^{T-2} (-T) \{-2T\} \\
\quad \rightarrow (-1)^{T-1} (-T)^{-4} (-2T) = (-1)^{T-1} 2(-T)^{-3} = 2(T)^{-3}
$$

(32)

The cofactor for the 1st diagonal element is similarly shown to be equal

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to $2(T)^{T-3}$. The proof proceeds by dividing the second column of the matrix $X'X$ (the first column of the cofactor) by 2 and then "triangularizing" the determinant to be evaluated. These results then yield the diagonal results of the inverse, $(X'X)^{-1}$, by dividing the constant cofactor by the determinant of $X'X$. The elements along the diagonal of the inverse are given to be identically equal to each other and to depend solely on $T$. They are:

$$
(X'X)^{-1}_{\gamma\gamma} = 2(T)^{T-3}/(T)^{T-2} = 2/T \quad (34)
$$

The techniques that have so far been employed show that the inverse, $(X'X)^{-1}$, has a constant element along its principal diagonal and that this element is given to be $2/T$. This implies that each of the coefficients in the index will have a standard error of estimate which is equivalent to each of the standard errors of estimate for the other index coefficients. This is given by:

$$
\text{Standard error of estimate } \hat{\beta}_{\alpha} = (X'X)^{-1}_{\alpha\alpha} \sigma^2 = \frac{2\sigma^2}{T} \quad (35)
$$

The same techniques which we have employed thus far can generalize information about the off-diagonal elements of $(X'X)^{-1}$. Certain modifications are required in technique. First, different relationships between the $i$ and $j$ must be substituted as we diverge from the main diagonal where $i=j$. Second, by dropping out an off-diagonal element in evaluating the element in the inverse by cofactors, it is not directly possible to obtain a form such as (25). The form obtained in evaluating off-diagonal elements is more generally of the form that requires evaluation of a cofactor which has
zeros everywhere below the main diagonal except for elements $e_{y+1,y}$ for $j \leq y < i$. These elements that lie below the main diagonal of the cofactor may be put to zero (without changing the value of the cofactor) by appropriate column addition within the cofactor. The result of these operations proves that, for any $i, j$ combinations where the absolute difference between them is greater than or equal to one, the cofactor becomes zero. Hence, elements off the main diagonal of $(X'X)^{-1}$ by more than one row or column are shown to be zero. Elements removed by only one row or column from the main diagonal are $(-1)^{T-2} (-T)^{T-3}/(T)^{T-2} = (-1)^{T-1}/(T)$. This implies that the $(X'X)^{-1}$ matrix looks like:

\[
\begin{bmatrix}
2 & a & 0 & 0 & \ldots & 0 & 0 \\
-1 & a & 0 & 0 & \ldots & 0 & 0 \\
0 & a & 2 & a & \ldots & 0 & 0 \\
0 & 0 & a & 2 & \ldots & 0 & 0 \\
0 & 0 & 0 & 0 & \ldots & 2 & a \\
0 & 0 & 0 & 0 & \ldots & a & 2 \\
\end{bmatrix}
\]

\[\left(\frac{1}{T}\right)\]

where $a = (-1)^{T-1}$

\[\left(\frac{1}{T}\right)\]

\[T-1\times(T-1)\]

---

13 Numerical examples of this inverse have been calculated for the cases of $T=4$ and $T=5$. These support and exemplify the result given in the text.

For $T=4$, $(X'X)^{-1}$ is:

\[
(X'X)^{-1} = \begin{bmatrix}
1/2 & -1/4 & 0 \\
-1/4 & 1/2 & -1/4 \\
0 & -1/4 & 1/2 \\
\end{bmatrix}
\]

For $T=5$, $(X'X)^{-1}$ is:

\[
(X'X)^{-1} = \begin{bmatrix}
2/5 & 1/5 & 0 & 0 \\
1/5 & 2/5 & 1/5 & 0 \\
0 & 1/5 & 2/5 & 1/5 \\
0 & 0 & 1/5 & 2/5 \\
\end{bmatrix}
\]

III-20
The covariance between any two consecutive pairs of regression coefficients, $b_\alpha$ and $b_{\alpha+1}$ in equation (11) is equal to $(-1)^{T-1} \sigma^2 / T$. The covariance between any two non-consecutive pairs of coefficients, $b_\alpha$ and $b_{\alpha+y}$ ($y>1$), is zero. This implies that if $b_\alpha$ is incorrectly estimated, so also will $b_{\alpha+1}$ and since $b_{\alpha+1}$ is estimated incorrectly so also will be $b_{\alpha+2}$, etc. We should expect this type of phenomenon with the model that we are using. That is, the model fits the product of several estimated, individual rates of growth to a given, observed average annual compounded rate of growth. If an estimate for one year is high, it should be expected that estimates for other years will be correspondingly lower.

The covariance that is expected among the estimated regression parameters is caused by peculiarities of the specific data matrix of 0's and 1's used in estimating the regression. These peculiarities are also reflected in the correlation matrix generated from the data on the independent variables. Of course, as Christ well states, the two peculiar phenomena of the variance–covariance matrix of estimated parameters and of the correlation matrix are related:

A principal cause of high positive or negative covariance between two estimators such as $p_1$ and $p_2$ is high correlation between the corresponding independent variables $z_1$ and $z_2$. ...high covariances among estimated parameters in an equation can be reduced by taking large samples.¹⁴

This type of relationship between samples and sizes of covariances is observed in the existence of $\frac{2}{T}$ in the diagonal and $(-T)^{-1}/T$ in the off-diagonal terms of the variance-covariance matrix. The peculiarities of the independent variable correlation matrix are easily observed from the multiple symmetry that exists for the T=22 case. The correlation matrix is illustrated in Table III-1, where the column variables Y01-Y21 represent the (T-1) independent variables.

Finally, it may be shown that each of the (T-1) regression coefficients is simply the weighted sum of the T(T-1)/2 observations on the average annual compounded rates of growth. We have the vector of estimated coefficients given, as for any multiple, linear, ordinary least squares model:

$$\hat{B} = (X'X)^{-1} X'Y$$  \hspace{1cm} (37)

We know from above what $(X'X)^{-1}$ looks like. $X'Y$ is also relatively easy to describe:

$$X'Y = \begin{bmatrix}
T & 1 \\
\Sigma & \Sigma \\
j=2 & \alpha=1 \\
\vdots & \vdots \\
T & 1 \\
\Sigma & \Sigma \\
j=T-1 & \alpha=1 \\
\vdots & \vdots \\
T & 1 \\
\Sigma & \Sigma \\
j=T & \alpha=1 \\
\end{bmatrix}
$$  \hspace{1cm} (38)

III-22
### TABLE III-1

**CORRELATIONS AMONG THE INDEPENDENT VARIABLES**

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\(^1\)All values are expressed without decimals, which may be thought to precede the numbers in each cell of the matrix here. The matrix is, of course, symmetrical and only the upper triangular portion is shown.
This will be a \((t-1)\times 1\) matrix of summations of the values of the dependent variable. Premultiplying \(X'Y\) by \((X'X)^{-1}\) gives a typical element of the result vector as:

\[
\hat{b}_i = \sum_{\Pi=i-1}^{i+1} \psi_{\Pi} \sum_{j=\Pi+1}^{T} \sum_{\alpha=1}^{T} (j-\alpha) \log_e (1+\text{ROG}_{\alpha j})
\]

(39)

where:

\[
\psi_{\Pi} = (-1)^{T-1}/T \quad \text{for } \Pi = i = 1 \text{ and } \Pi = i+1
\]

except for \(i=1\) when \(\psi_o = 0\)

and for \(i = T-1\) when \(\psi_T = 0\)

\[
\psi_i = 2/T \quad \text{for } \Pi = i
\]

From the form of the calculation of each regression coefficient, it is quite clear that the coefficients are weighted averages of the observed data on rates of growth. As claimed earlier, it is also quite clear that certain of the coefficients are based upon more data than others. For example, it is evident in equation (39) that \(\hat{b}_1\) is based upon the weighted average of \(2(T-2) + T-1\) terms. The estimate of \(\hat{b}_2\) is based upon \(3(T-3) + 2(T-2) + T-1\) terms. From this, it is easily generalized that \(\hat{b}_\alpha\) is based upon \(\alpha+1(T-(\alpha+1)) + \alpha(T-\alpha) + \alpha-1(T-(\alpha-1)) = 3\alpha(T-\alpha)+2\) terms in its calculation.\(^{15}\)

By using a regression approach, we obtain a best linear weighted index, which minimizes the variance of the errors in explaining any observed rates of growth between years \(i\) and \(j\) by estimated rates of growth for each

\(^{15}\)On the surface, this disagrees with the statement above (p.14). "In general, \(\text{ROG}_\alpha\) will be based upon \(\alpha(T-\alpha)\) bits of information." There, however, we counted each of the \(\text{ROG}_{ij}\)'s that gave information on \(\text{ROG}_\alpha\) without counting any twice. Here, we count the number of \(\text{ROG}_{ij}\)'s that are weighted in estimation of \(\text{ROG}_\alpha\) and several are double counted. In fact, \(3\alpha(T-\alpha)+2-\alpha (T-\alpha)=2\alpha(T-\alpha)+2\) are counted more than once.
year between years i and j-1.

V. The Relationship between the Model Presented and Other Models in the Literature

The model that has been presented in the second section and discussed in the third section of this chapter has been independently devised for use on this study. It is similar in many ways to the earlier work of Bailey, Muth and Nourse. In fact, Bailey, Muth and Nourse seem to consider a model similar to that presented in equations (7)-(11) above as an alternative but equivalent way of presenting their analysis:

Alternatively, we could have expressed the model in terms of period-to-period changes in the index number. To do so, let $c_{i,j}=\log \frac{I_j}{I_{j-1}}$ be the log of the relative change in the index from period $j-1$ period to $j$ [sic] and $x_j$ equal +1 if the period $j-1$ to $j$ is included in the interval between initial and final sale and 0 otherwise. It can be shown that the two forms of the model yield identical estimates of the index number for any period....We prefer the form of the model discussed in the text [i.e., their model], however, because it is computationally simpler.

Changing the notation used in Bailey, Muth and Nourse to agree with that used here, we can contrast their methodology to that presented above.

The basic difference is in the frame of reference. As an alternative

---


17 Ibid., p. 935, footnote 2.
to equation (7) above, B-M-N suggest the following relation.

\[ p_{i,j}^R = (B_{j}/B_{.}) p_{i,j}^\mu \]  

(40)

This relates a price relative to the estimated values of a property value index. One obvious difference is in error terms. B-M-N have an error term which is specific not only to a set of year combinations (ij) but also to a property, p. Our model deletes reference to a property so that the error term, \( \varepsilon_{i,j} \), contains only a year (combination) specific error reference.

This results from a difference in preference for treating properties over time. Whether price relatives (or rates of growth) are averaged first and then analyzed or are analyzed without averaging depends upon what one assumes about the errors and possible source of bias for the coefficients. Assumptions of parameter constancies are similar in B-M-N's work and that of the present paper. We assume that the estimated growth rates are constants, regardless of what combination of years they attempt to explain. Constant growth rates, i.e., a deterministic \( r_{\alpha} \), \( 1 \leq \alpha < T \), lead to constant estimates of price index values, which is what B-M-N expect.

Bailey, Muth and Nourse claim that their method constructs a price index by weighting all the observations on price relatives. This is clear in Nourse:

The regression method of computing the index makes each yearly index a weighted average of all sales. An illustration of three properties with two sales each over a three year period demonstrates how the weighting works out.\footnote{Nourse, op.cit., p.38. A more detailed example is given in the B-M-N article. A variation in weighting occurs between the B-M-N system and that presented in this paper (Section C above). This is because we have explicitly averaged over i, j combinations within equation (4). B-M-N, on the other hand, do not average.}
However, in an intuitive sense, the reason for this is not really as clear with the B-M-N presentation as it is in the present form. Here, for example, it is quite reasonable that ROG_{i} will give information on rog_{i}. It is also clear that in order to use this information, we must incorporate such other data as ROG_{i+1} and in general ROG_{j}. That is, b_{a} is constructed as a weighted average of all observed data. In the fourth section of this chapter, equation (39) exemplifies the weighting scheme.

It is possible to compare the two alternative methodologies more closely by mathematical manipulation of B-M-N's equation (40) into our methodology given in equation (7).\text{19}

\[
R_{ij} = \left( \frac{B_j}{B_i} \right) \mu_{ij} \quad (40')
\]

For the same i, j combination, our equation (7) states that

\[
(1 + ROG_{ij})^{j-i} = \varepsilon_{ij} \prod_{a=i}^{j-1} b_{a} \quad (7')
\]

However, it also follows from equation (3) that

\[
(1 + ROG_{ij})^{j-i} = R_{ij} \quad (41)
\]

Hence, we can equate the right hand sides of (7') and (40'):

\text{19} In so doing, we shall assume that B-M-N's model averages over properties of ij combinations. Whether the procedure averages data over the i, j combinations is based upon a priori expectations about the error terms and does not produce the distinguishing feature between the methodologies. We average first because it is expected that this will smooth the data on rates of growth between any i and j pair of years, so that there is less variance to explain. This variance is then explained by averaging the exogenous "quality" variables listed above and used in equation (5). It is suspected that the averaging does not significantly improve or hurt the analysis; however, with large data matrices, the computational efficiency gained is liable to be large.

III-27
It is also defined in (17) that:

\[ B_j = B_i \Pi_{\alpha=1}^{j-1} b_\alpha \]  

(17')

Solving for \( \Pi_{\alpha=1}^{j-1} b_\alpha \) and substituting the result into (42), we obtain:

\[ \varepsilon_{i,j} \left( \frac{B_j}{B_i} \right) = \left( \frac{B_j}{B_i} \right) \mu_{i,j} \]  

(43)

\[ \varepsilon_{i,j} = \mu_{i,j} \]  

(44)

Therefore, the errors, \( \varepsilon_{i,j} \) and \( \mu_{i,j} \), from the two methodologies are equal for all \( i \) and \( j \) (feasible) combinations. As such, the equivalence, suggested by B-M-N, is shown to hold.

The Bailey, Muth and Nourse method has been applied at least five times to data. Each time the original form of the model suggested in the Bailey, Muth and Nourse article has been used. These studies have been in different applications to land value studies. Pendleton applied the procedure in assessing the value of highway accessibility and Bailey, in evaluating racial influences on prices in single-family houses.\(^{20}\) Ridker used the method on air pollution data for St. Louis.\(^{21}\) Nourse


applied the technique to public housing projects in St. Louis. Finally, Dobson, using the model, investigated values in racially changing neighborhoods. The present use both widens the spectrum of applications and applies the alternate formulation of the model.

VI. Interpretation of the Results

The model presented for use in this study may easily be summarized in equation form.

Step 1:

\[ R_{ij}^{j-i} = \frac{PV_{i}^{j}/PV_{i}^{i}}{P} \]

Step 2:

\[ R_{ij}^{j-i} = \frac{1}{P} R_{ij}^{j-i} - 1 \]

Step 3:

\[ ROG_{ij} = \sum_{p=1}^{N_k} \frac{ROG_{ij} / N_k}{Pck} \]

Step 4:

\[ ROG_{ij} = a + b PV_{ij} + c ZBLT_{ij} + v_{ij} \]

22 Nourse, op. cit.

Step 5:
\[
\hat{V}_{ij} = \hat{\text{ROG}}_{ij} = \text{ROG}_{ij} - a \cdot \text{PV}_{ij} + c \cdot \%BLT_{ij}
\]
for data, separately, on each area, \( k \)

Step 6:
\[
(j-i) \log_e (1 + \text{ROG}) = \sum_{a=1}^{T-1} b_a \delta_a + \log_e \hat{e}_{ij}
\]
where
\[
\delta_a = \begin{cases} 
1 & \text{if } i \leq a < j \\
0 & \text{otherwise}
\end{cases}
\]
and
\[
b_a = \log_e (1 + \text{ROG}_a)
\]

Step 7:
\[
\text{Index}_{\text{con}} = 100 \sum_{\delta=1}^{\alpha-1} \hat{e}_\delta
\]
for control area data
\[
\text{Index}_{\text{proj}} = 100 \sum_{\delta=1}^{\alpha-1} \hat{e}_\delta
\]
for project area data

Step 8:
\[
\text{Index}_{\text{con}} = r + s_a
\]

Step 9:
\[
\text{Index}_{\text{proj}} = \text{Index}_{\text{proj}} - r - s_a
\]

Step 10:
\[
\text{Index}_{\text{proj}} = \hat{m} + \hat{n} \delta_{\text{proj}}
\]
where
\[
\delta_{\text{proj}} = \begin{cases} 
1 & \text{if } a \geq \text{project period} \\
0 & \text{otherwise}
\end{cases}
\]
We expect that, if investments in water and sewer have a positive effect on land values, \( \hat{n} \) will be positive in sign. However, in evaluating the effect of the water/sewerage investment, it is useful to know not only the direction of a land value effect, but also the size of the effect; the latter needs to be standardized by some reference. We propose that it should be measured as a certain percentage increase in value for a property. The percentage is expressed as the ratio of increased value to either average market value over the relevant period, or better, as the ratio of the increase to values immediately prior to the investment.

Step 10, in evaluating the level of \( \hat{n} \), measures the increment. In order to compare the increment to values before the investment occurred, say in year \( t' \), we should form the ratio of \( \left( \frac{\hat{n}}{\text{proj}_t \text{Index}_{t'}} \right) \), where \( \text{proj}_t \text{Index}_{t'} \) is obtained from Step 7. The reference is taken from Step 7 rather than from \( \text{proj}_t \text{Index}_{t'} \) from Step 9 because the operation in Step 9 does not affect the size of the increment, \( \hat{n} \), but only clears the index of secular trends and general disturbance noises. If we wish to commute this back into actual monetary values, we could utilize the ratio (percentage) obtained, applying it to the level of land values in real terms given by the data.
CHAPTER FOUR

THE CASE STUDY DESCRIPTION—NAIROBI

I. Introduction

Several residential areas of Nairobi, Kenya, have received water/sewerage facilities over the past twenty years. Many of these areas are also organized around a free, private housing market. That is, the properties are neither government-owned nor rent-controlled. These types of area are suitable as "project" areas in a case study on which the theory and empirical method presented above may be tested. The areas selected for study are located in the northern and western parts of the city, and include both sewerage and water supply projects.

The purpose of this chapter is to describe the case study, including the selection of the study areas, collection of data, and statistical results.

II. The Choice of Project and Control Areas

The first step in selecting study areas involved locating and dating all main extensions on the public reticulation and sewer systems. This was done by inspection of historical maps of the system of mains and planning maps for the project, and by discussions with Nairobi City Council water
engineers. In the process, detailed maps of the public and private water systems were collected. These maps enable the location of any point on the water or sewer systems by road name and/or property land registration numbers. The basic code used on all data was the land registration number.  

A three-stage sewer project in an area known as the Upper Hill-Kilimani Estate district was selected for study. The first phase of the project sewered the northeast portion of the area in 1960-61. The second stage sewered the southern half in 1961-62. The third stage, currently under construction, will sewer the remaining area to the north and west during this and next year (i.e., 1971-72). As control areas for this project, it seemed feasible to use those portions of the area that were not undergoing a particular stage in any given year, i.e., in studying the land value effects of the second (1961-62) phase, the third phase (not sewered until 1971-72) would seem an appropriate control area. However, this approach eventually was not feasible--partly because of the anticipation effect and partly for empirical reasons--and resort was made to a nearby area, which was already sewered, as the control. This area, Upper Parklands, has the additional virtue of also being the control area used in the water supply analysis. The sewerage project areas are shown in Figure IV-1. The control area is shown in Figure IV-2.

---

1 The land registration number consists of two parts--a prefix which is an identification of the general area and an additional number which locates the plot within an area. Upon subdivision, all the smaller plots of the original area are given distinct numbers which can usually identify them as subdivisions of some specific, original plot.
FIGURE IV-1

NAIROBI: SEWER AREAS
FIGURE IV-2

NAIROBI: WATER AND CONTROL AREAS
In the sewerage case study areas, the sewers have generally replaced either conservancy tanks or septic tanks. Here, the benefits are: (1) public maintenance of the system replacing private maintenance; (2) greater sanitation and healthful treatment of wastes; (3) increased dependability (especially in rainy weather); and (4) relaxed conditions on subdivision of property.  

The water supply project selected for study involved the extension of public mains to an area known as Spring Valley Estate. The control areas to be used are known as Barton Estate and Upper Parklands Estate. Barton, Spring Valley, and Upper Parklands are contiguous areas to the northwest of the city center, lying within the new city boundaries, but partly outside the old city boundaries. (See Figure IV-3.) Barton, Spring Valley, and Upper Parklands are homogeneous areas. Upper Parklands was the first to receive city water and provides a control on the other two areas. It is also the only one of the areas that was included in the old city—-that being the reason for its receiving city water supply first. The Barton area is still largely without city water supply. Spring Valley was given public supply during the study period in 1964. These three areas are shown in Figure IV-2.

Finally, a separate analysis of a water supply project was carried out for an area known as Kitisuru. (See Figure IV-2.) Kitisuru is a

2From zoning maps of the City Council of Nairobi, Town Planning Section, it was determined that, for all of the sewerage project area, zoning requirements were decreased from 1/2 acre minimum to 1/4 acre minimum plot areas per residence. That is, it was possible at least to double density through subdivision after sewer ing a project area.
FIGURE IV-3
NAIROBI CASE STUDY

a Sewer I Area
b Sewer II Area
c Sewer III Area
d Water I Area
e Control Area
f Water II Area

NAIROBI

Old City Boundary

New City Boundary

N
separate study because the plots are generally larger than in the other three areas, and because there is only a single main running through the estate. This main was laid to service only one large plot, but passes near several other properties, some of which have not opted to connect to the public supply. The model implies that such a case will still yield increasing property values for all properties located near the main. This is because of the fact that the major investment necessary to bring water to these properties (laying the main) has already been made. Only a small charge for connection service remains in order that these properties may be tied to the city reticulation system. The property values of these plots should have increased by nearly as much as the plot which was already connected. The difference is the connection service charge. The Kitisuru data could provide a test of this hypothesis (although, in fact, too few observations for such a test could be obtained).

In the case study of the water supply project, the benefits from the public reticulation system are to be judged in comparison with private borehole-distribution systems partially augmented by public water carried from center city kiosks. In most cases where the private water is supplemented with public, the additional public water is used for drinking and the private water is used for non-drinking purposes. The benefits are, therefore, mainly: (1) better water quality; ³ (2)

³The improvement in water quality is dramatic, especially in reduction in excessive fluoride content, which, in the private Kitisuru water supplies for example, has been detected as maximum at 9.5 ppm. High fluoride content (cf. a range of 0.2 to 0.4 ppm for public supplies) is extremely detrimental to tooth health. The poorer quality of private water supplies
increased dependability; and (3) greater convenience. It is the property value changes induced by these factors that we attempt to quantify in this study.

All areas, either in the water or in the sewerage study, were thought to be relatively free from public control of rent levels or of property transaction values. This does not imply that the government was not a landowner in these parts of the city. In fact, several plots were owned by the Kenyan government as well as by other national governments. (These were largely for the housing of embassies.) Government land holdings in this area, however, were not used to subsidize housing costs, but were used mainly as reserves for future government projects. The relative freedom of operation of the property market is a major reason why this analysis has concentrated on the northern and western areas of the city.

3 Is also demonstrated by the fact that Kitisuru private supplies have no chlorination or treatment facilities. (This information is available in City Council of Nairobi, "Application for Loan for Development of Water Undertaking," Questionnaire for Water Works Project issued by the IBRD and IDA, pp. XXVII-XXVIII. Two chemists' reports are included in Appendix V, Tables XIII-1 and XIII-2, for comparison between public and private water supply qualities.)

4 These were predominately for future road sites and the size of the individual government holdings was extremely small.

5 In the eastern and southern sections of the city, landowning by government was much different than in the west. There, the government ownership was to provide subsidized, rent-controlled, low-income housing. To the large-scale extent that the government was involved in property sales in these areas, even the value of privately owned land would be held somewhat below true market value. Determination of true market values in these areas would, at best, be an extrapolated forecast from an analysis using western data to fit a property value determination model. It was considered that a direct analysis of the western areas would be more valid, statistically. Consequently, no areas in the eastern sector were chosen for this part of the analysis.

IV-8
III. Empirical Results

Alternative Formulations of the Model

The empirical model to be used here is outlined in some detail in the preceding chapter. The test using Nairobi data will depart from the basic model only insofar as it employs more precise specifications of the estimating equations and methods. It should be noted that alternative formulations of certain of the equations are presented and, therefore, a range of results is derived. Although these alternative specifications do not qualitatively change the model, they do provide a key to the range of results which might be expected and to the next steps necessary in modifying the model.

By using alternative formulations of the model in statistical tests, additional information on the best specification may be obtained. However, this process creates difficulties in assessing the statistical test on the model. In particular, it is alleged by several authors that the probability of committing a type II ($\beta$) error in accepting a false hypothesis increases as the number of alternative specifications fit (to the same set of data) increases.\(^6\) Christ comments:

\begin{quote}
Suppose that the confidence we place in an equation is to be determined by a statistical test of the null hypothesis that the population value of its multiple or partial correlation coefficient is zero. Then the process of trying several alternatives and choosing the one with the highest correlation coefficient will,
\end{quote}


IV-9
on the average, lead to higher observed correlation coefficients when the null hypothesis is true for all our alternatives than will the process of choosing one equation form on a priori grounds. Therefore, the tests of significance will be too likely to reject the null hypothesis, if they are applied to an equation that has been chosen because of having the highest correlation coefficient among a set of alternative equations.7

It would, of course, create problems of the reverse kind as well. If the population value of the multiple or partial correlation coefficients for each of the alternatives is different from zero, by examining all of the alternatives, the probability of finding at least one which accepts the null hypothesis (of zero correlation) increases above the probability of accepting the null hypothesis when only one a priori form is tested. That is, the possibility of a type I (α) error is increased.

These considerations must be consciously entertained as the alternative results are interpreted. They explain away, in part, some of the variation in results that occurs even when only slightly variant forms are used. Nevertheless, the alternative specifications presented here have been fit to data to provide information on the best specification of the model. Confidence testing may be accomplished by choosing the best forms from tests on the Nairobi data and deriving inferences only after these specifications are applied to new data. Since this is at least one of the first attempts to gauge land value reactions to water supply/sewerage investments, this exercise necessarily represents only initial model testing. In this way, the Nairobi test of the model is meant to be

7Ibid., p. 538.
preliminary to any final conclusions.

**Statistical Analysis**

The results presented below are for two sewer project areas and one water supply project area (Spring Valley Estate). Used as controls against these are contiguous areas as described above. Though a number of different formulations of the model were tested, the results presented here are those which conform most closely to the model and which seem most reasonable on an *a priori* basis.

**Spring Valley Water Project**  As noted above, this project extended the city water supply to a relatively high-income area which had been serviced with private borehole supplies which were far inferior to city supplies for drinking purposes. The control area is Upper Parklands, which is a similar area that has had city water service for a long period. The project year was 1964.

Property transactions were recorded for a total of 173 properties in the Spring Valley-Barton area and for 543 properties in the Upper Parklands area. From Step 1 in the empirical procedure, these are translated into price relatives in the project and control areas, and into rates of growth in property values. These individual property rates of growth are then reduced to average rates of growth for each of the project and control areas (Step 3). This results in 150 of a maximum possible 231 observations for the project area and 201 for the control area, with these observations denoted as \( \text{ROG}_{i,j} \).

These average growth rates \( \overline{\text{ROG}}_{i,j} \) are now adjusted by means of the
following regression equations for the project and control areas (Step 5):\footnote{8}

**Control:**

\[
\overline{\text{ROG}}_{ij} = 1.28848543 - 0.00044790 PV_i + 0.26326526 BLT_{ij} \\
- 0.1860251 \Delta \text{TIME}_{ij} + 0.00765485 \Delta \text{TIME}_{ij}^2
\]  

\begin{align*}
\text{(Mult. R = .5939)} \\
\text{(SE Est. = .4870713)} \\
\text{(DF = 196)}
\end{align*}

**Spring Valley:**

\[
\overline{\text{ROG}}_{ij} = 3.05113085 - 0.00176323 PV_i - 0.52173847 BLT_{ij} \\
- 0.30121794 \Delta \text{TIME}_{ij} + 0.01081738 \Delta \text{TIME}_{ij}^2
\]

\begin{align*}
\text{(Mult. R = .5383)} \\
\text{(SE Est. = 1.3194093)} \\
\text{(DF = 145)}
\end{align*}

**Kitisuru:**

\[
\overline{\text{ROG}}_{ij} = 0.26159523 - 0.00001650 PV_i + 0.03572849 BLT_{ij} \\
- 0.03086652 \Delta \text{TIME}_{ij} + 0.00094969 \Delta \text{TIME}_{ij}^2
\]

\begin{align*}
\text{(Mult. R = .7309)} \\
\text{(SE Est. = .0924843)} \\
\text{(DF = 20)}
\end{align*}

\footnote{8}{At this point, the Kitisuru analysis was dropped because of insufficient data.}
where: \( PV_i \) = the average initial property value (i.e., value in year \( i \) of all those properties in a given area which sold in year \( i \) and in year \( j \)).

\( BLT_{ij} \) = the percentage of all properties in a given year that have built housing after being sold in year \( i \), but before being sold in year \( j \).

\( \Delta TIME_{ij} \) = the span of elapsed time between years \( i \) and \( j \). This is equal to the value of \( (j-i) \).

\( \Delta TIME_{ij}^2 \) = the squared value of \( \Delta TIME_{ij} \).

and where the adjusted rate of growth (\( \overline{ROG}_{ij} \)) is:

\[
\overline{ROG}_{ij} = \overline{ROG}_{ij} - \overline{ROG}_{ij}.
\] (4)

\( \Delta TIME_{ij} \) and \( \Delta TIME_{ij}^2 \) were used for two reasons. One reason is intuitively that, the longer the time between periods of sales on a property, the more change may have occurred in that property. If there is a systematic bias of positive (or negative) improvements on the property, these variables will remove this effect. The second reason is more a necessity to obtain a good statistical fit. Specifically, the data on property values used in the analysis are in money (not real) terms. Consequently, as longer periods exist between sales, any inflationary pressures that exist will effect an upward bias in the real rate of growth. This implies that the true model that we would like to fit and which is described in Chapter Three will have a form of autocorrelation in the error terms. In the true model, error terms associated with \( \overline{ROG}_{ij} \) when \( i \) and \( j \) are close to each other will be small, and error terms associated with \( \overline{ROG}_{ij} \) when \( i \) and \( j \) are very split in time will be large. This effect can be eliminated.
by using the variables $\Delta \text{TIME}_{ij}$ and $\Delta \text{TIME}_{ij}^2$.

Next (Step 6), the rate of growth in property values for each pair of contiguous years is estimated separately for the control and project areas. (See equation (11) in Chapter Three.) These growth rates are converted into indices and are shown for the project and control areas in Table IV-1.

The next step (Step 7) involves calculating a regression on the control area indices to remove secular and cyclical effects. Three separate equations are fit:

$$\hat{I}_c = 107.46 + .2939a$$  \hspace{1cm} \text{Mult. R} = .0482 \hspace{1cm} (5)$$

$$\hat{I}_c = 107.55 + .3051a - 1.2409D$$  \hspace{1cm} \text{Mult. R} = .0497 \hspace{1cm} (6)$$

$$\hat{I}_c = 155.06 - 11.61a + .5175a^2$$  \hspace{1cm} \text{Mult. R} = .4826 \hspace{1cm} (7)$$

The dummy variable (D) is intended to isolate the reactions to Kenyan independence in the early 1960's. (It should be noted that the use of three alternate forms of this estimating equation assures three estimates of the property value effects of the investment.)

The equations (5) - (7) are fit to the indices for property value growth in the project area. (See Column 3 of Table IV-1.) The final step requires a regression of these residuals against a dummy variable ($\delta$) which equals 1 for years after the project period (1964 in this case) and equals 0 for years before the project period. The estimate of this relationship shows where equations (8), (9) and (10) correspond to the time adjustments implied by (5), (6) and (7) respectively. Equation (8)
TABLE IV-1

PROPERTY VALUE INDICES FOR SPRING VALLEY (PROJECT AREA) AND UPPER PARKLANDS (CONTROL AREA)

<table>
<thead>
<tr>
<th>Year</th>
<th>Property Value Indexes</th>
<th>Residual Index</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Spring Valley</td>
<td>Upper Parklands</td>
</tr>
<tr>
<td>1950</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>1951</td>
<td>97</td>
<td>137</td>
</tr>
<tr>
<td>1952</td>
<td>104</td>
<td>108</td>
</tr>
<tr>
<td>1953</td>
<td>131</td>
<td>177</td>
</tr>
<tr>
<td>1954</td>
<td>117</td>
<td>111</td>
</tr>
<tr>
<td>1955</td>
<td>183</td>
<td>85</td>
</tr>
<tr>
<td>1956</td>
<td>134</td>
<td>88</td>
</tr>
<tr>
<td>1957</td>
<td>104</td>
<td>74</td>
</tr>
<tr>
<td>1958</td>
<td>88</td>
<td>53</td>
</tr>
<tr>
<td>1959</td>
<td>139</td>
<td>43</td>
</tr>
<tr>
<td>1960</td>
<td>143</td>
<td>88</td>
</tr>
<tr>
<td>1961</td>
<td>146</td>
<td>185</td>
</tr>
<tr>
<td>1962</td>
<td>128</td>
<td>69</td>
</tr>
<tr>
<td>1963</td>
<td>150</td>
<td>91</td>
</tr>
<tr>
<td>1964</td>
<td>138</td>
<td>95</td>
</tr>
<tr>
<td>1965</td>
<td>159</td>
<td>84</td>
</tr>
<tr>
<td>1966</td>
<td>254</td>
<td>98</td>
</tr>
<tr>
<td>1967</td>
<td>153</td>
<td>140</td>
</tr>
<tr>
<td>1968</td>
<td>158</td>
<td>158</td>
</tr>
<tr>
<td>1969</td>
<td>162</td>
<td>113</td>
</tr>
<tr>
<td>1970</td>
<td>235</td>
<td>152</td>
</tr>
<tr>
<td>1971</td>
<td>171</td>
<td>122</td>
</tr>
</tbody>
</table>

IV-15
\[ I_p = 61.90 + 27.24\delta \]  
\[ I_p = 54.34 + 33.93\delta \]  
\[ I_p = 49.26 + 17.35\delta \]

may be interpreted as showing that the average value of the index is 27.24 points higher after the project than before the project. Since the forecasted value of the property value index as of the project year was 61.90, the relative increase in land value due to the investment may be estimated as 27.24/61.90, or as 44.01 percent.

Similarly interpreted, the results of equations (9) and (10) imply percent increases of 62.44 and 35.22 respectively. Therefore, by this method, it would be concluded that the effect of introducing the water supply improvement in the Spring Valley area is an increase in property values of between 35 percent and 62 percent.

These estimated value increases seem high, though, in fact, there is no a priori reason to expect any particular quantitative result. However, a "high" result would seem consistent with a major problem in this phase of the analysis, i.e., separating the property value effects of the water supply investment from those of annexation. The Spring Valley area received a water supply in 1964, but was incorporated into the city area in December 1963.\(^{10}\) The two effects of incorporation and investment in

\(^{10}\)J.A. Hurrel, "Technical Appendix No. 4-Industry," Report of the Nairobi Urban Study Group (Nairobi: Urban Study Group, July 1958), p. 1. The "new city area," containing 225.5 square miles, was added to the existing "old city area" of 34.5 square miles in December 1963. Spring Valley and Barton bordered the old city area and were a part of the annexed territory.
water are irrevocably bonded together in such a way that no dummy variable analysis can separate them. Consequently, it is not only the water investment, but also the benefits of annexation—public services, etc.—that are measured. In this light, it is only reasonable that the Spring Valley area should experience a higher increment in land values in 1964 than it would have in the absence of annexation.

The problems inherent in the isolation of these water supply projects benefits illustrate the necessity of choosing a control area properly. Because no data were available on a suitable control area, no manipulation of the data will help isolate the effects of water investment benefits from annexation benefits. However, if a suitable area outside the old city, not receiving water supply projects, could have been found, it could have been used to isolate only water benefits, as it presumably would have been subject to the same annexation benefits as the Spring Valley area. No such area could be found.

Still, the results show strong positive effects on property values, and interviews with public officials in Nairobi made it clear that the single most important change which took place during the period in question was the extension of city water to the area. Therefore, while the estimates are perhaps not as "clean" as they need to be, they do show a marked effect of water supply improvements on land values.

Upper Hill Sewer Projects As described above, the sewer projects analyzed were undertaken in phases in the Upper Hill section of Nairobi. Those portions of the area not undergoing sewerage were first used as the control area, but were eventually rejected because of the anticipation
problem and because of data problems. Consequently, the Upper Parklands area was used as the control.

Since the steps in the analysis are exactly as above, only the results are presented here. Property transactions for 481 properties were recorded for the area. Of these, 150 observations were taken for the sewer projects, with sufficient data to generate at least one pair of sales in every possible combination of two years. Fewer observations would have been generated, if these data had not been available in such quantity. Specifically, for the control area, there were 200 observations, for Sewer I, 70, for Sewer II, 20, and for Sewer III, 70.

The average growth rates ($\overline{ROG}_{ij}$) are now adjusted by means of the following equations:

Sewer I:

$$\overline{ROG}_{ij} = .93863995 - .00037331PV_i - .05165329BLT_{ij}$$
$$- .11472725\Delta TIME_{ij} + .00483815\Delta TIME_{ij}^2$$

(11)

(Mult.R = .5031)
(SE Est. = .3883564)
(DF = 65)

Sewer II:

$$\overline{ROG}_{ij} = .52553518 + .00059442PV_i + 1.03750746BLT_{ij}$$
$$- .13712725\Delta TIME_{ij} + .00431869\Delta TIME_{ij}^2$$

(12)

(Mult.R = .7800)
(SE Est. = .4530830)
(DF = 196)

\(^{11}\) Plans for future sewerage of the area were well known.
Sewer III:

\[ \overline{\text{ROG}}_{ij} = 1.04492292 - 0.00027998PV_{ij} + 0.41938517BLT_{ij} \\
- 0.16520942\Delta\text{TIME}_{ij} + 0.00637923\Delta\text{TIME}_{ij}^2 \]  

(Mult. R = .4463)  
(SE Est. = .6899763)  
(DF = 65)

Following the same procedure as above, these estimates are adjusted for time and for national independence, with the same three specifications as above. The results are described in Table IV-2.

<table>
<thead>
<tr>
<th>Project Area</th>
<th>Percent Increase in Land Value</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \alpha )</td>
</tr>
<tr>
<td>Sewer I</td>
<td>30.02</td>
</tr>
<tr>
<td>Sewer II</td>
<td>36.43</td>
</tr>
<tr>
<td>Sewer III</td>
<td>59.61</td>
</tr>
</tbody>
</table>

These results show that land value increments in response to the sewer project range from an estimated 20.50 percent to 59.61 percent. However, as in the case of the water supply analysis above, there are exogenous influences which affect the increments. That is to say, the
whole of the increment cannot be contributed to the value which area residents place on piped sewerage services. The introduction of piped-in sewerage is accompanied by a lowering of the density zoning of the area and, therefore, leads to a considerable number of subdivisions of land and land increments. Therefore, the estimates derived here incurred this density zoning change and its effect. In fact, it will always be impossible to separate this effect, when density zoning ordinances are affected by the type of sewerage sources provided.
CHAPTER FIVE

THE CASE STUDY DESCRIPTION--KUALA LUMPUR

I. Introduction

Many residential areas of Kuala Lumpur, Malaysia, have public sewerage facilities. These areas seem at least as adaptable and free from government controls as the study areas in Nairobi. Two of the Kuala Lumpur areas, recently sewered and near the central city core, were selected for study. A third area to the northwest of the city was selected as a control. This area appeared to be suitable for this purpose, as it was similar to the project areas in nearly all respects. This chapter describes the Kuala Lumpur case study, indicating the changes made in the statistical method formerly applied to the Nairobi data, and reports the statistical results.

II. The Choice of Project and Control Areas

City and federal agencies are rapidly attempting to complete the sewerage extensions throughout Kuala Lumpur. Sewerage extensions, for which data were collected, occurred during the period 1959-1969. The largest extensions into the data areas took place between 1965 and 1968. Unsewered and sewered areas differ dramatically in their disposal methods. Properties in sewered areas are served by the most modern and
convenient sewage disposal facilities. Homes are connected to a fully reticulated system of sewer pipes which permit conventional plumbing systems in bathroom, kitchen, and laundry areas. Disposal is immediate. Unsewered areas largely utilize ground dumping and bucket systems where liquid wastes are poured off onto local disposal areas and the solid wastes are stored in containers on the property, to be collected by sewage disposal trucks. This provides less than immediate disposal. Benefits obtained by conversion from unsewered to sewered methods obviously include increased sanitation and convenience, and reduced cost.

The first project area lies near the city center, not far from the major hotel area. The area is delineated starting near the Hotel Pavilion and proceeding northeast, splitting the area between Tengkat Tong Shin and Jalan Bukit Bintang, to the intersection of Jalan Berangan and Jalan Beremi, then along Jalan Berangan to Jalan Treacher, and north to Jalan Weld. The project boundary then proceeds west on Jalan Weld and turns south on Lorong Ceylon to Jalan Mesui. From this point, the area loops westward, taking in the majority of properties on both sides of Jalan Hicks, Jalan Bukit Ceylon, and Jalan Ceylon. This westward area is closed at Jalan Sahabat, from where it rejoins the starting point by arching south to include the area on both sides of Tengkat Tong Shin. This area will be referred to as South I. (See Figure V-1.)

The second project area lies further south and east of South I. It includes the properties located around the several streets, Jalan Berunai Selatan, Lorong Berunai Selatan, Jalan Berunai Barat, Jalan Berunai, Jalan Sayor, and a large portion of Jalan Pudu. This area will be referred to
FIGURE V-1

KUALA LUMPUR: SEWER AREAS
as South II. (See Figure V-1.)

The control area does not presently have any facilities for sewerage. It lies northwest of the two project areas. This area is bounded on the southwest by Jalan Ipoh and runs northwest between Jalan Serindit and Jalan Latehan, enclosing properties lying north and east of Jalan Pipit, along the entire stretch of Jalan Pipit to where it rejoins Jalan Ipoh. The area will be called North I. (See Figure V-2.)

As in Nairobi, the project and control areas were carefully selected so as to exclude any properties that were used for governmentally subsidized, low-income housing, several areas being rejected from the study for this reason. Further care was taken in choosing areas that were not affected in any overwhelming manner by tourism, especially in the nearby location of large hotels. Only after much discussion with the Assessor's Office were areas near hotels chosen. *A priori* affirmation was obtained that values in each of the areas would be truly indicative of only the common market interactions.

III. Changes in the Empirical Model

Compared with the statistical estimation procedure described in Chapter Three for the Nairobi data, one change was adopted for the Kuala Lumpur case study. This modification was required because fewer data observations are available; it involves the nature of an individual observation.

With the Nairobi data, the first step in the actual estimation
FIGURE V-2

KUALA LUMPUR: CONTROL AREA
FIGURE V-3
KUALA LUMPUR CASE STUDY

KUALA LUMPUR

a North I
b South I
c South II

Municipal Boundary
involved an attempt to explain the rate of growth for any pair of years. This rate of growth was constructed as the average for all those properties selling in each of the two years. The Nairobi method utilized average rates of growth because of the large number of observations available, and because the intent was to explain average property value behavior.

In Kuala Lumpur, the areas selected were much smaller—in terms of numbers of properties—and, consequently, the number of observations was much smaller as well. As a result, averaging was not performed with the Kuala Lumpur data and the regression was run on the actual rate of growth for individual properties.

One attempted regression in the Nairobi case used, as standardizing variables for the inter-year growth rate in housing value, the age and area of the house, the area of the property, the percent of all those properties used in constructing the average on which a house was built between the two years, and the difference in time between the two years. Another used the initial property value as a substitute for age and area of the house and area of the property. In all cases, these variables were constructed as averages in the same fashion as the dependent variable. The corresponding exogenous or standardizing variables used in the Kuala Lumpur case study were consequently not constructed as averages. This implies, for example, that construction or non-construction of a house on any property between the first and second selling dates (for Kuala Lumpur) became a dummy variable with value equal to one, if a house was built, and equal to zero, if one was not. These modifications provide the first difference in statistical methods between Nairobi and Kuala Lumpur.
IV. Specific Complications with Kuala Lumpur Data

One serious complication occurred in obtaining data for Kuala Lumpur. This was in the collection of data for the control area. On the basis of the estimation procedure used in the Nairobi study, it was determined that 250 properties with two or more sales would represent an acceptable number of data elements. With this objective, field work began; however, it was impossible to obtain a good estimate of how many such properties were contained in the project and control areas.

Data came from three different sources—the sales data from records of the Malay State Office of Lands and Mines, the property record (characteristics) data from the Kuala Lumpur City Valuer's Office and also from the Malay State Office of Lands and Mines (but from different source books), and the sewerage hookup data from the Kuala Lumpur Sewer Department. Because of the sheer size of the data accumulation, an on-site check was maintained to assure matchup among the three data sets, i.e., to assure that an adequate amount of sales data and other necessary data for individual properties could be drawn from the sample areas. Since data were exhaustively collected from three distinct geographical areas (Over 800 properties were examined.) for each of the data requirements, it was assumed that a good matchup would occur. However, upon return to the United States, and after eliminating "bad sales data,"

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1Sales which were inexplicably low, those which were affected by subdivision, amalgamation, etc.
it was found that significantly smaller matchup had occurred than was anticipated. This was due in large part to incompleteness of local records. Consequently, for some properties, only sales data were available; for other properties, only property characteristic data could be gathered; for yet other properties, only hookup data could be recorded. In fact, by assigning an observation to either project or control groups through use of property registration numbers, and by lumping the two project areas together, it was found that only 190 complete observations (of dual sales) existed for the project area and 93 observations for the control area.

Empirical estimation of the model is possible in spite of the relatively small number of observations available, if the empirical method is adjusted so that it is applicable to the smaller data sets. However, upon further computer evaluation of the data, at the stage of constructing property value indices for the control area, more difficulties were encountered in the data set. Specifically, of all the observations containing dual sales for the control area, few contained a prior sale in any year before 1961. Thus, it was not possible to obtain adequate information on pre-1961 rates of growth in property values for the control area. This problem stems from the nature of the control area prior to 1961. It was composed of several large tracts which were subdivided, mostly after 1961, into many smaller properties.

By convention, when this problem of inadequate earlier-year data occurred in the Nairobi study, a procedure was arbitrarily adopted to assign to the missing year's rate of growth a value equal to that of the succeeding year's rate of growth. This appeared to work satisfactorily
in Nairobi because, even with many study groups, observations were sufficient to provide nearly all, or all but one or two, of the requisite rates of growth. This procedure was consequently written into the estimating format. When this procedure was applied to such a serious problem as encountered in Kuala Lumpur, however, the rates of growth for each year from 1949 to 1961 became equal by assumption, and the analysis was clearly biased by the assumption.

If, for example, the 1961 control growth rate was very low, then all growth rates from 1949 to 1961 were assigned very low values. When the control and project areas were compared in subsequent empirical steps, the control area naturally seemed to jump to a higher rate of growth at about the time of the project (post-1961) relative to the project area. The result was to obtain a negative effect of the project on property values. Among the attempts to explain this result was a procedure which eliminated the years 1949-1961 from the project area as they had been eliminated from the control area. This, however, had the effect of also eliminating most of the value of a project-control methodology. To the extent that the control may have been subject to a generally overall higher rate of growth than the project area, the results still showed that the project reduced property values. Further, not enough of the property value index observations (23 at most without elimination of any data) were left after data elimination to provide any ability to correct for these influences.

Still other corrections to the problem were tried. One possible alternative was to assume that the project and control areas grew over the
1949-1961 period at the same rates. Consequently, values were assigned to the control rates of growth in this period. These assigned values were equal to the project area rates of growth. The empirical results still indicated a negative project-property value relationship. Such an equal growth rate assumption is unwarranted; the control and project areas may have been subject to different influences, and not enough data were available to adjust for the differences correctly. Despite our conviction that these results are invalid, they will be presented below.

Again, it should be emphasized that few sales data for the control area prior to 1961 existed because of the large-tract property ownership and, therefore, all initial sales of control properties are probably biased downward. The existence of large-scale subdivision around the year 1961 in one isolated geographical area had the tendency, if many immediate sales occurred, to depress the potential sales values in that geographical market. This subdivision had the effect of flooding the market with properties, and, since price is a decreasing function of such outward supply shifts, land values would be below that level which would have existed, had there been relatively fewer sales. After the initial flood of properties, however, (say post-1964) land values were found to have increased not only because of normal market forces and exogenous forces, but also because of the slower rate at which properties in the area were put on the market. The rate of growth on these dual (pre- and post-1964) sales, consequently, appeared very high. At least part of their high value, however, was attributable to the subdivision flood effect.
When a control area subject to the above influences is compared to a project area in the manner in which the empirical methodology of the present study intends, the influence of the water/sewerage project will necessarily be underestimated. That is, the control area will be perceived to have had a higher growth rate over the project data than the project area. The ultimate result, if such an effect is large enough, is to estimate that a water/sewerage project reduces property values. For Kuala Lumpur, this is the result which was empirically estimated. While, with a strict interpretation, this implies that residents actually paid more for water and sewerage than they were intrinsically worth to them, the result is spurious. Therefore, the negative results subsequently reported may be attributed to empirical problems.

V. Statistical Results

As indicated above, the Kuala Lumpur model specification was modified from that used in Nairobi. The modification, however, was kept to a minimum and was only undertaken to get around data problems and to improve the consistency of the model. As in the Nairobi case, many more structural relationships were estimated than will be reported here. These were exploratory attempts to find procedures that would give reliable results. All failed to eliminate the serious problems encountered in the study.

The first regression procedures combined Steps 1 and 2 of the Nairobi method. These regressions were run on individual property data observations. The procedures simultaneously extracted an index for the
yearly rates of growth (ROG) and the effects of road investments (RD), building (BLT), varying initial property value levels (INIT PV), and the change in time between sales (ΔTIME and ΔTIME²). The regressions were run on both project and control areas independently. The ROG dummies in Table V-1 for each year between 1949 and 1970 are to be interpreted, as suggested in Chapter Three, as the log of the estimated rate of growth for each year.

This procedure has combined the first two regressions performed in the Nairobi study into one regression. It has done so on the basis that the first two Nairobi regression steps do not necessarily have to be sequential. Differences in the practice will only result from correlations between the rate of growth dummies and the independent variables, INIT PV, RD, BLT, ΔTIME, and ΔTIME². No clear-cut preference exists between choosing to combine or separate the regressions. In addition, a model was run separating the two regressions, yielding nearly identical results to those shown above. Bailey, Muth, and Nourse² take the combined approach. It is reported here, however, only for simplicity and brevity.

The rates of growth and property value indices for control and project areas were then calculated from the results of Table V-1. These procedures are described in Chapter Three. The project index was called \( \text{proj}_{t} \) and the control index, \( \text{con}_{t} \). These were used so as to standardize the project values with the control values and to isolate

### TABLE V-1

**REGRESSION RESULTS**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>STD Error</th>
<th>Coefficient</th>
<th>STD Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>INIT PV</td>
<td>- .0007</td>
<td>.0001</td>
<td>- .0018</td>
<td>.0003</td>
</tr>
<tr>
<td>RD</td>
<td>- .1465</td>
<td>.1318</td>
<td>- .2484</td>
<td>.1224</td>
</tr>
<tr>
<td>BLT</td>
<td>.0233</td>
<td>.0647</td>
<td>.2851</td>
<td>.0953</td>
</tr>
<tr>
<td>ΔTIME</td>
<td>.1587</td>
<td>.0822</td>
<td>.0075</td>
<td>.0962</td>
</tr>
<tr>
<td>ΔTIME²</td>
<td>- .0003</td>
<td>.0009</td>
<td>.0059</td>
<td>.0057</td>
</tr>
<tr>
<td>ROG 1949</td>
<td>.1303</td>
<td>.1495</td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td>ROG 1950</td>
<td>- .1118</td>
<td>.1259</td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td>ROG 1951</td>
<td>.0603</td>
<td>.1462</td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td>ROG 1952</td>
<td>- .2641</td>
<td>.1653</td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td>ROG 1953</td>
<td>- .2663</td>
<td>.1639</td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td>ROG 1954</td>
<td>- .1312</td>
<td>.1610</td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td>ROG 1955</td>
<td>- .1421</td>
<td>.1189</td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td>ROG 1956</td>
<td>- .2082</td>
<td>.1307</td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td>ROG 1957</td>
<td>- .0382</td>
<td>.1396</td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td>ROG 1958</td>
<td>- .2109</td>
<td>.1389</td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td>ROG 1959</td>
<td>- .2307</td>
<td>.1480</td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td>ROG 1960</td>
<td>- .2078</td>
<td>.1234</td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td>ROG 1961</td>
<td>- .1677</td>
<td>.1545</td>
<td>- .1096</td>
<td>.1430</td>
</tr>
<tr>
<td>ROG 1962</td>
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<td>.1408</td>
<td>- .1390</td>
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</tr>
<tr>
<td>ROG 1963</td>
<td>- .1776</td>
<td>.1139</td>
<td>- .0435</td>
<td>.1158</td>
</tr>
<tr>
<td>ROG 1964</td>
<td>- .1206</td>
<td>.1602</td>
<td>- .1430</td>
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</tr>
<tr>
<td>ROG 1965</td>
<td>.1106</td>
<td>.1809</td>
<td>.1150</td>
<td>.1712</td>
</tr>
<tr>
<td>ROG 1966</td>
<td>- .3380</td>
<td>.1867</td>
<td>.0242</td>
<td>.1432</td>
</tr>
<tr>
<td>ROG 1967</td>
<td>.0876</td>
<td>.1646</td>
<td>- .2685</td>
<td>.1791</td>
</tr>
<tr>
<td>ROG 1968</td>
<td>- .2687</td>
<td>.1722</td>
<td>.0547</td>
<td>.1862</td>
</tr>
<tr>
<td>ROG 1969</td>
<td>- .3630</td>
<td>.1838</td>
<td>- .0187</td>
<td>.1514</td>
</tr>
<tr>
<td>ROG 1970</td>
<td>.1029</td>
<td>.1515</td>
<td>- .0595</td>
<td>.1432</td>
</tr>
<tr>
<td>CONSTANT</td>
<td>.3860</td>
<td>.0849</td>
<td>.5709</td>
<td>.1411</td>
</tr>
</tbody>
</table>

*Assumed to be equal to the project rates of growth reported in same row. These could not be estimated because of missing data. (See the discussion in Section IV above.)*
the 1964 project effect. Consequently, a dummy variable taking on values of 1 after 1964 and 0 before 1964 was constructed, called Proj, and incorporated into the regression. The results were:

\[
\text{proj} \cdot \text{Index}_t = -0.5472 + 1.0006 \text{con} \cdot \text{Index}_t - 6.0856 \text{Proj}_t
\]  

The standard error of the parameter estimates are included in parentheses.

It may be seen that the regression indicated a negative project effect. In percentage terms, this represented a fall in value of 31.37 percent for properties in the sewerage project areas from their 1963 value. As reported above in Section IV, this result is not believed to be indicative of the true project effects. It may also be noted that the good fit and near 1.0 estimate of the \( \text{con} \cdot \text{Index}_t \) parameter is largely biased by the assumption that the control rates of growth were equal to the project rates of growth from 1949 to 1961.

No alternative statistical specifications provided any significantly different results from those reported here. If they had, those results would most likely be attributable more to spurious fit than to any realistic explanation. The data problems involved with the Kuala Lumpur study prohibit any further statistical investigation.
The Factor Mobility Assumption

As indicated in Chapter One, the crucial assumption is about factor mobility. The relevant one is, of course, labor, it being assumed that capital movements are free within national boundaries. If labor is mobile, we can safely assume that changes in property values outside the project area can be ignored. The following discussion of the problems of dealing with migration are, therefore, primarily of theoretical interest.

Let us begin by considering the map of an area as drawn in Figure 1. Assume, for example, that area A is the inner city, B consists of suburbs, and area C is rural. Further, area A is the project area, and areas B and C are non-investment areas. The boundaries between the

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1 This is consistent with the assumption of supply inelasticity for housing in any particular area.
areas are uniquely determined. The boundary between A and B is where direct investment effects stop. The boundary between B and C is at the point at which data collection on empirical variables stopped so that area C is outside the data set. Area B may be referred to as the control area, against which the rate of change in property values in the project area is compared. Migration from B to A augments the demand for property in area A, and decreases demand for property in area B. It is plausible that a similar shift occurs between areas C and A, but this would be with less intensity, since the two areas are further apart than are areas B and A. We shall rather arbitrarily assume that the relative shift in demand from C to A because of an investment project is small enough to be ignored. However, migration from C to A and from C to B does occur. This is evident empirically in this hypothetical example, because of A and B's role as central metropolitan and suburban areas and hence (by assumption) because they have better opportunities, which grow at faster rates, than are available in C.

As a result of the project, land values are expected to rise in A and fall in B, ceteris paribus. However, given immigration from C, this is not necessarily the result which will be observed. If C is treated in a fashion that defers to its much larger size relative to B and A, and consumers' equilibrium exists before and after the investment, then it is reasonable to expect that land values in B will remain unchanged. That is, of course, the assumption we are making.

The major theoretical problem in dealing with migration is the need to draw boundaries around the cross-sectional area on which data is
collected and the model is estimated. This, in turn, results from the continued inability of regional economics to utilize fully the two (or three) dimensions which are the variables of the spatial framework in econometric models.\(^2\) Data and models are often collected and fit by using point representations as average values of a distribution over areal contiguous sections. In the present model, this simplifies the interpretation of the empirical results since only an average effect on land values need be considered. By use of a dummy variable analysis, which will be explained below, these results are the average increase of an investment area property relative to a non-investment area property.

This simplicity is a compelling reason to prefer a discretely spatial type of model over a continuously spatial model. Furthermore, models that represent functions over space in a continuous way lose some of the

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\(^2\) One innovative exception to this is a truly spatial econometric model given in: William Warantz, _Toward a Geography of Prices: A Study in Geo-Econometrics_ (Philadelphia: University of Pennsylvania Press, 1959). This, however, is not applicable in the present situation. A theoretical model that could be adapted to the needs of this section on migration is presented in: Martin J. Beckmann, "On the Distribution of Urban Rent and Residential Density," _Journal of Economic Theory_, Vol. 1 (March 1969), 60-67; and in a correction of the Beckmann article in: Jerome K. Delson, "Correction on the Boundary Conditions in Beckmann's Model on Urban Rent and Residential Density," _Journal of Economic Theory_, Vol 2 (September 1970), 314-318. However, while this model can handle the migration problem, it cannot, as Freeman correctly assessed, be readily adapted to treat spatially differentiated investments. See: A. Myrick Freeman III, "Air Pollution and Property Values: A Methodological Comment," _Review of Economics and Statistics_, Vol. 73 (December 1971), 415-416. Further, it is not directly amenable to empirical testing. Consequently, we are left with the problem as described in the text.

-iii-
contact with reality that a model can obtain when it refers to different properties of different sizes, all with different values, but which are all also nearly at the same distance from the area's gravity center (CBD). An integrative model that uses some continuous and some discrete properties might produce the most meaningful results. The following model is not estimated, is not compatible with models presented in earlier sections, and is meant only to present a possible prototype.

Consider land values to be explained by the \( j \) variables (on \( i \) properties in time \( t \)), \( X_{ijt} \). Also included is a dummy, \( D_{it} \), that equals one, if \( i \) in year \( t \) is, or has been (in prior \( t \)), affected by an investment. Also consider that each \( i \) has a set of cartesian points which locates the property and determines the distance of the property from the gravity center. Order the observations according to the rank of each of the \( i \) properties so that the first has the smallest distance to the gravity center and the last has the largest distance. Assume further that the migratory pattern is as shown in Figure 1. What we really need is to isolate the general equilibrium increase or decrease in relative demand for land in one area as in another. Also assume, to make matters simpler, that the only cause of migration is the investment itself and that the decay function is logarithmic. One potential equation system to measure the shifting relative demands for land is given by:

\[ 3 \]

\( B_j \) is the vector of coefficients of length \( j \). \( X_{ijt} \) is a matrix of observations on variables that is either \( i \) by \( j \)--\( t \) constant--or for varying \( t \), a matrix of \( i \) times \( t \) by \( j \).
\[ L_{it} = a + X_{jit} B_j + cD_{it} + dDist_{it} + d\overline{Dist}_{i-1t} + \ldots + d^j Dist_{i-lt} + \ldots \quad (1) \]

Lagging the result on the $i--not t$:

\[ L_{i-lt} = a + X_{jit-1} B_j + cD_{i-1t} + dDist_{i-1t} + d\overline{Dist}_{i-2t} + \ldots + d^j Dist_{i-l-1t} + \ldots \quad (2) \]

Multiply by $k$ and subtract equation (2) from equation (1):

\[ L_{it} - kL_{i-lt} = (a - ka) + X_{jit} B_j - kX_{jit-1} B_j + cD_{it} - ckD_{i-1t} + dDist_{it} \quad (3) \]

While this equation system still has overidentification of the $B_j$ vector, it begins to describe an alternative approach to that actually taken. This will not be pursued. However, it develops a model free from the problems discussed above in this section.

**Returns to Factors**

One of the most unexpected but useful advantages of using a land values method to investigate water/sewage benefits is its ability to partition the benefits into returns to labor and capital. A further benefit of using this method is obtained by its ability to describe the division of benefits by mobilities of labor and capital. Capital mobility is generally indicated by the relative slope of the supply curve. Labor mobility is indicated by the relative displacement of the demand curves following an improvement to land in one area and no
improvement in other areas.

That capital mobility is reflected in the slope of the supply curve comes from the way that quantity units in the property market are defined. Previously, we had defined quantity in the units of individual, residential properties that are available from a specified finite area of land. Both Edel and Grieson subscribe to this concept of defining quantity units:

> The amount of usable space that can be provided within an area is not fixed, as is total land area, but is expansible, by construction of higher buildings, or in our case, subdivision. Expansion of this intensive margin takes place at increasing costs. Therefore, ... there will be a supply curve such as that in Figure I. [His Figure I proceeds to show a relatively elastic supply curve.]

The increasing costs of expanding supply obviously requires a mobile capital stock that can move among areas to permit subdivisions and construction of higher buildings. Edel obviously assumes that capital is mobile in his representation of supply. If capital were totally immobile so that it was not available to expand supply of usable space, the supply function would necessarily approach perfect inelasticity.

Labor immobility is, on the other hand, reflected in the demand side of the residential property market. Assuming that, within an area, demand is given as some function with a negative slope, improvements of land quality within that area would normally call forth some net increase

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in area under the demand curve of the improved area. However, if the labor, i.e., people, is so immobile that it cannot move from unimproved to improved areas,\(^5\) there will be no increase in demand for the improved property. This will be true except for the presumably small effect of land for bread types of substitution mentioned above. The result, obviously, is that smaller demand shifts occur with more immobile populations, and larger\(^6\) shifts with more mobile populations.

A typical water/sewerage investment will have the following effect on the residential property market. Consider Figure 2:

FIGURE 2

THE MARKET FOR PROPERTY, AGAIN

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\(^5\)This will occur, for example, if the improved area is one in which discrimination is practiced. Residents of the improved area refuse to allow others (effectively) to register their demands for the improved properties. That is, everyone knows that no one will be allowed to move into the improved area.

\(^6\)It is reasonable that the size of the demand shift will have an upward limit of a rational, full information reaction to the improvement.
S is the supply, D and $D'$ are the original and new demand curves. By convention, from the original equilibrium we may accept the following areas:

- $q_{EO}$ - total expenditures
- $EPEB$ - consumers' surplus
- $q_{ECO}$ - actual costs of providing the residential property
- $EPEC$ - producers' surplus.

A similar derivation may be made of the four areas in the new equilibrium conditions. Finally, the changes that occur in these areas as the equilibrium shifts can be determined:

- $P_{E'}F q_{E'}q E P_{E}$ - change in total expenditures
- $EPEB - FP E' A$ - change in consumers' surplus
- $q_{E'}F E q_{E}$ - change in actual costs of providing the residential property
- $P_{E'}F E P_{E}$ - change in producers' surplus.

These areas are derived from fairly general supply and demand curves. Although much of the above (for example, the third section of this part) has specifically warned that the total welfare effects and increasing land values cannot be strictly compared, we may gain some insight by comparing the areas now.

The increase in producers' surplus represents one portion of benefits. It is, in fact, wholly coterminous with a portion of the area that represents increased expenditures--our welfare measure from before. This portion has wide acceptance as being the return to capital, and indeed it is. This is justified on the basis that it costs $q_{E'}q_{F}E q_{E}$.
more to provide the additional land $q_{E'} - q_{E''}$. However, the additional land is sold at a gross amount, $q_{E'} F E q_{E''} + P_{E'} F E P_{E''}$. This results in a net profit, return to capital, of $P_{E'} F E P_{E''}$. The other portion of benefits, $q_{E'} F E q_{E''}$, is divided into returns to labor and returns to land. The returns to labor are that portion of increased expenditures, $q_{E'} F E q_{E''}$, that matches the increased consumers' surplus. Any residual, in case the increased consumers' surplus and the increased expenditure area $q_{E'} F E q_{E''}$ do not match exactly, is attributed (for lack of a better term) as a return to land.

As previously noted, the supply and demand curves and shift in the demand curve in Figure 2 is only one of a number of possible curves that might have been drawn. It is instructive to investigate the way that the areas of producers' and consumers' surplus vary as different mobility assumptions are put into the diagram.

Consider, first, immobile capital, which implies that the supply curve is relatively inelastic. (This essentially brings us back to where we started in showing full capitalization of benefits. (See Figure II-1 in Chapter Two.) This case is redrawn in Figure 3.

All benefits are reflected in increased producers' surplus, and, if there is any increase in consumers' surplus, it must have been derived from a reduction or decrease in returns to land. When capital is immobile, then most of the returns of the investment are (paradoxically) to capital. This may be explained away as being a monopoly profit to that capital which does exist in the area.
FIGURE 3
IMMOBILE CAPITAL IN THE PROPERTY MARKET

Price A

B

P\_E

P\_E'

S

F

D'

D

q\_E, q\_E'

Quantity
The other polar case of complete capital mobility may similarly be represented. This is done in Figure 4.

FIGURE 4

MOBILE CAPITAL IN THE PROPERTY MARKET

This case, quite naturally, gives the opposite result of the first. There is no increase in producers' surplus—indeed, there is no producers' surplus before or after the investment. Consumers' surplus has increased by area, ABEF. Revenue, also, has increased; this by area, $q_E^E\times P_E^E$. The conclusions which one must draw are obvious. All returns are to labor except to the extent that the areas ABEF and
\( q_E^{E} \neq q_E^{E'} \) are not equal. The residual return, as always, is said to accrue to land.

The first two cases have dealt with capital mobility. The conclusions generated there are dependent upon labor mobility; in drawing the conclusions which we have drawn, it was necessary to assume that labor was mobile. Labor mobility is the more important of the two mobilities. Regardless of capital mobility, if labor is totally immobile, only one conclusion may be reached. That is that all returns are to land. Conversely, however, if labor is mobile, capital mobility is of utmost importance to the conclusion.

Therefore, we shall consider finally the case where labor is immobile to the extent that the investment cannot elicit a response of shifting the demand curve outward. This case, shown in Figure 5, shows no effect on land values.

**FIGURE 5**
IMMOBILE LABOR IN THE PROPERTY MARKET

![Diagram showing IMMOBILE LABOR IN THE PROPERTY MARKET](image-url)
There is evidently no shift in land values, consumers' surplus, or producers' surplus. Consequently, if the investment does have some positive benefits, these must be shown as a return to land.

For developing countries, on which the case studies here report, much can be said about the mobilities of capital and labor. Labor is particularly mobile in many urban areas of underdeveloped countries. Although there have been, in past years, some tribal barriers to mobility, these posit the existence of several separate and distinct markets. However, within each there has been excellent mobility, probably due to chronic excess demands for housing and overcrowding. This mobility manifests itself in high turnover rates. For Nairobi, for example, the average turnover on middle- and higher-income residential area properties is approximately one sale per six years. It is quite possible that, with sufficient empirical experience, this turnover rate may eventually provide a rule-of-thumb gauge for the relative mobility of labor, the consequent division of returns to the factors, and the extent of benefit capitalization. That is, this might become, because of its simplicity, a crucial variable to employ in a forecasting model.

Capital, similarly, appears quite mobile in the Nairobi case study. Evidence for this conclusion is gathered by examining the rate of subdivision as prices increase. The greater the rate of subdivision, the greater the mobility of capital— or so it will be assumed. The existence of any subdivision at all is sufficient to suggest capital mobility. Unfortunately, however, little relative empirical work has
been done in this area and, consequently, the evidence gathered on Nairobi is without a reference numeraire on which it may be judged relatively.

7 The same lament holds for the evidence provided on labor mobility. The turnover rate of one per six years appears high, but we are unfamiliar with other work that would suggest that this is, indeed, high.
APPENDIX II
BIBLIOGRAPHY


__________. "Minutes of Proceedings of October, 1970" (of the Council and of the several Committees thereof), Vol. 38, No. 3.


"Discrepancies between Highest and Best Use and Market Value in Property Assessments." Source unknown.


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iv


Williams, D.C., Jr., and Daniel, Donnie L. "The Impact of Reservoirs on Land Values: A Case Study." Water Resources Research Institute, Mississippi State University.


