8-7-2012

The Impact of Manipulatives on Students' Performance on Money Word Problems

Jessica Luke
Georgia State University

ACCEPTANCE

This dissertation, THE IMPACT OF MANIPULATIVES ON STUDENTS' PERFORMANCE ON MONEY WORD PROBLEMS, by JAYE K. LUKE, was prepared under the direction of the candidate's Dissertation Advisory Committee. It is accepted by the committee members in partial fulfillment of the requirements for the degree of Doctor of Philosophy in the College of Education, Georgia State University.

The Dissertation Advisory Committee and the student's Department Chair, as representatives of the faculty, certify that this dissertation has met all standards of excellence and scholarship as determined by the faculty. The Dean of the College of Education concurs.

____
Daphne Greenberg, Ph.D.
Committee Co-Chair

____
Laura D. Fredrick, Ph.D.
Committee Co-Chair

____
Nannette Commander, Ph.D.
Committee Member

____
Phill Gagne, Ph.D.
Committee Member

____
Date

____
Amy Lederberg, Ph.D.
Interim Chair, Department of Educational Psychology and Special Education

____
R.W. Kamphaus, Ph.D.
Dean and Distinguished Research Professor
College of Education

AUTHOR'S STATEMENT

By presenting this dissertation as a partial fulfillment of the requirements for the advanced degree from Georgia State University, I agree that the library of Georgia State University shall make it available for inspection and circulation in accordance with its regulations governing materials of this type. I agree that permission to quote, to copy from, or to publish this dissertation may be granted by the professor under whose direction it was written, by the College of Education's director of graduate studies and research, or by me. Such quoting, copying, or publishing must be solely for scholarly purposes and will not involve potential financial gain. It is understood that any copying from or publication of this dissertation which involves potential financial gain will not be allowed without my written permission.

____

Jaye K. Luke

NOTICE TO BORROWERS

All dissertations deposited in the Georgia State University library must be used in accordance with the stipulations prescribed by the author in the preceding statement. The author of this dissertation is:

Jaye K. Luke
730 Henson Ct.
Marina, CA 93933

The directors of this dissertation are:

Dr. Daphne Greenberg
Department of Educational Psychology and Special Education
College of Education
Georgia State University
Atlanta, GA 30303 – 3083

Dr. Laura D. Fredrick
Department of Educational Psychology and Special Education
College of Education
Georgia State University
Atlanta, GA 30303-3083

VITA

Jaye K. Luke

ADDRESS: 730 Henson Ct.
Marina, CA 93933

EDUCATION:

Ph.D.	2012	Georgia State University Educational Psychology
M.S.	2003	University of Kansas Special Education
B.S.E.	2001	University of Kansas Education

PROFESSIONAL EXPERIENCE

2008-Present	Language & Literacy Fellow Department of Educational Psychology and Special Education Georgia State University, Atlanta, GA
2007-2008	Graduate Research Assistant Department of Educational Psychology and Special Education Georgia State University, Atlanta, GA

PUBLICATIONS

Dollar, C. A., Fredrick, L. D., Alberto, P., & Luke, J. K. (2012). Using simultaneous Prompting to teach individual living and leisure skills to adults with severe intellectual disabilities. *Research in Developmental Disabilities, 33,* 189-195.

Fredrick, L. D., Luke, J. K., Gama, R., Dollar, C. A., & Alberto, P. A., (in preparation) The Impact of density of instruction on acquisition rate of reading skills for students with intellectual disabilities.

Luke, J. K., & Fredrick, L. D., (in preparation) The impact of sight-word instruction on oral language in students with intellectual disabilities.

SCHOLARLY PRESENTATIONS

Fredrick, L. D., Luke, J. K., Alberto, P. A., Dollar, C. A., Gama, R (2012, May). *Density of instruction for an integrated literacy curriculum*. Poster presentation for the annual 2012 Applied Behavior Analysis International Association, Seattle, WA.

Dollar, C. A., Fredrick, L. D., Alberto, P., & Luke, J. K. (2012, May). *Using simultaneous prompting to teach individual living and leisure skills to adults with severe intellectual disabilities*. Poster presentation for the annual 2012 Applied Behavior Analysis International Association, Seattle, WA.

Luke, J. K., Fredrick, L. D., Davis, D. H., Dollar, C. A., Waugh, R. E., & Alberto, P. A. (2011, May). *An integrated literacy program for students with moderate to severe intellectual disabilities*. Symposium presentation for the annual 2011 Applied Behavior Analysis International Association, Denver, CO.

Luke, J. K., Fredrick, L. D., Waugh, R. E., Davis, D. H., Dollar, C. A., & Alberto, P. A. (2011, May). *The impact of sight-word instruction on oral language in students with intellectual disabilities*. Poster presentation for the annual 2011 Applied Behavior Analysis International Association, Denver, CO.

Luke, J. K., & Luke, R. C. (2011, March). *If and how Facebook can be used in the classroom*. Paper presentation for the Georgia State University GTA Pedagogy Conference, Atlanta, GA.

Luke, J. K., & Fredrick, L. D., (2010, May) *Women seeking women: Teaching social skills to improve speed-dating behaviors*. Poster presentation for the annual 2010 Applied Behavior Analysis International Association, San Antonio, TX.

Luke, J. K., Davis, D. H., Waugh, R. E., & Gama, R., (2009, March) *Impact of language activities embedded in components of integrated literacy curriculum for students with moderate and severe disabilities*. Paper presentation for the annual 2009 Georgia Reading Association conference, Atlanta, GA.

Luke, J. K., Fredrick, L. D., & Dispenza, F. (2009, May) *Teaching social skills to Improve college students' speed-dating behaviors*. Poster presentation for the 2009 Applied Behavior Analysis International Association, Phoenix, AZ.

ABSTRACT

THE IMPACT OF MANIPULATIVES ON STUDENTS' PERFORMANCE
ON MONEY WORD PROBLEMS
by
Jaye K. Luke

Numeracy skills are needed for daily living. For example, time management and budgeting are tasks that adults face on a frequent basis. Instruction for numeracy skills begins early and continues throughout childhood. Obtaining numeracy skills is difficult for some students. For example, there may be an inadequate fit between the student's knowledge and the design of the instruction, the student may be unable to select an appropriate strategy for solving the problem, or the student may have a learning disability. Students with a learning disability comprise approximately 40% of identified children with disabilities who receive special education services (U.S. Department of Education, 2005).

The National Council of Teachers of Mathematics helps teachers mediate the difficulties students may have in math. The council recommends problem solving and representation with physical objects as a teaching method. Chapter 1 presents a literature review on children with a learning disability, the use of manipulatives, and problem solving. The literature review indicates that children with a learning disability are poor problem solvers, but that further research is needed to investigate best instructional strategies. Chapter 2 presents a study on the impact of manipulatives on the accurate completion of money word problems. Three populations were included: adults who struggle with numeracy (n = 20), children with a learning disability (n = 20), and children who are typically developing (n = 23). Participants were administered a measure of 10 money word problems and were asked to solve them without the use of manipulatives. Participants were then randomly assigned to one of two groups: perceptually rich and perceptually bland manipulatives. Results indicate that none of the participants performed better with manipulatives than they performed without manipulatives. There was an interaction of Condition x Type of participant with the participants with a learning disability in the bland condition performing significantly worse than the other participants. More research is warranted to understand the impact of manipulative use in mathematics instruction for adults who struggle with numeracy, children with a learning disability, and children who are typically developing.

THE IMPACT OF MANIPULATIVES ON STUDENTS' PERFORMANCE
ON MONEY WORD PROBLEMS

by

Jaye K. Luke

A Dissertation

Presented in Partial Fulfillment of Requirements for the
Degree of
Doctor of Philosophy
in
Educational Psychology
in
the Department of Educational Psychology and Special Education
in
the College of Education
Georgia State University

Atlanta, GA
2012

ACKNOWLEDGEMENTS

This page is my inadequate attempt to express the gratitude that I feel towards the individuals associated with this academic degree.

I would like to thank Ryan Luke for his support during this process. To illustrate how much I love and appreciate him, I would promise to get Sofia Vergara to play me if “our story” was turned into a movie. Thanks to my brother Lee who continuously had my back and answered my late-night phone calls.

I was extremely lucky to have co-chairs for my dissertation! Dr. Greenberg and Dr. Fredrick have been true gifts to my educational and personal growth and I will never be able to repay them for their hard work and guidance. Dr. Gagne and Dr. Commander have been valued committee members and are the epitome of calm, cool, and collected; I hope to be that for my future students.

I would like to extend my gratitude to Chad, Dawn, Des, Lizzy, Rebecca, and Sheral for helping me to collect data and making the required early morning OTP trips. Ryan Hall was a recruitment rock star! I hope that when her time comes to find participants for her dissertation, she has someone as dedicated and wonderful as herself. Dr. McNeil, whose 2009 study I replicated, was extremely helpful with any questions I had regarding the methodology of her study, and I would like to thank her too.

Additionally, my thanks to all the schools and adult literacy programs for allowing me to recruit participants.

Lastly, thank you to all my participants, without you, the study could not have happened!

TABLE OF CONTENTS

ABBREVIATIONS

CRA	Concrete-representational-abstract
CSA	Concrete-semiconcrete-abstract
IOA	Interobserver Agreement
NAAL	National Assessment of Adult Literacy
NCLD	National Joint Committee on Learning Disabilities
NCTM	National Council for Teaching Mathematics
SBTI	Schema-based transfer instruction

CHAPTER 1

THE USE OF PROBLEM SOLVING AND PHYSICAL REPRESENTATION FOR TEACHING NUMERACY SKILLS TO STUDENTS WITH LEARNING DISABILITIES

Introduction

Numeracy pervades many adult activities and is important for success in today's society (Condelli et al., 2006). For example, calculating a tip, time management, balancing a budget, and figuring out the cost of sale items are tasks that adults face on a frequent basis and illustrate the need for numeracy skills to manage diverse quantitative situations (Gal, 2000). In addition, many jobs require some level of proficiency in mathematics, and for some jobs it is the essence of the worker's tasks (Patton, Cronin, Bassett, & Koppel, 1997). Inadequate numeracy skills may impact an individual's daily living.

Due to the importance of numeracy skills, mathematics instruction to enhance the development of mathematical proficiency starts very early in preschool and elementary school. From preschool onward, children learn abstract mathematical concepts and principles in addition to procedures and facts through various types of instruction. In spite of this focus on mathematical instruction throughout school, adults frequently lack the skills necessary for solving the multistep problems they encounter in the context of everyday situations and work settings (Dossey, Mullis, Lindquist, & Chambers, 1988). A lack of functional math skills in adults suggests there is a lack of mastery of math skills while in school.

There are a variety of reasons an individual may not master math skills while in school. A student may not receive adequate, strategic, and sequenced instruction (Cawley, Fizmaurice, Shaw, Kahn, & Bates, 1978), or there may be a poor fit between the design of the instruction and student characteristics. Alternatively, the student may learn the skills but not when to apply them. These reasons may be compounded for children with a learning disability. Learning disabilities impact approximately 6%-14% of society (Barberesi, Katusic, Colligan, Weaver, & Jacobson, 2005). The National Joint Committee on Learning Disabilities (NCLD, 1991) provides the following definition of learning disabilities:

> Learning disabilities is a general term that refers to a heterogeneous group of disorders manifested by significant difficulties in the acquisition and use of listening, speaking, reading, writing, reasoning, or mathematical skills. These disorders are intrinsic to the individual, are presumed to be due to a central nervous system dysfunction, and may occur across the life span. Problems in self-regulatory behaviours, social perception, and social interaction may exist with learning disabilities but do not by themselves constitute a learning disability. Although a learning disability may occur concomitantly with other disabilities or with extrinsic influences, it would not be a result of those conditions or influences. (NCLD, 1991, p. 20)

Children with a learning disability struggle from an early age and throughout their schooling with working memory, procedures, and numerical concepts (Geary, 2004). For example, young children with a math learning disability may have trouble with counting, memorizing numbers, and organizing items in a logical way. School-aged children with a math learning disability may encounter problems with retrieving math facts, problem solving, and math vocabulary (Miller & Mercer, 1997). Teenagers and adults with a math learning disability may have difficulty with estimating time, estimating costs, and finding different approaches to one problem (National Joint Committee on Learning Disabilities [NCLD] Editorial Staff, 2010).

Fortunately, there are a variety of strategies supported by the literature to address poor numeracy skills. Most of these strategies fall into three dominant categories: a direct instruction model, self-instructional strategy training, and assisted performance. The direct instruction model typically is used for task-specific strategies. This instruction is often scripted and students are taught a specific behavioural sequence. The self-instructional strategy training reminds students of the current task. The assisted performance model attempts to engage cognitive and metacognitive processes. The teacher models the task just slightly above what the learner is independently capable of doing. Additionally, the teacher may use coaching, fading, questions, and explanation to make the task explicit (Goldman, 1989). The goal of all of these strategies is to make the problem clear and concise. Making the problem clear and comprehending the question is essential to gaining numeracy skills. Children with a learning disability struggle with understanding what the question is asking and can benefit from the above recommended strategies, as well as from those found in Miller, Butler, and Lee's (1998) meta-analysis. Miller et al. (1998) examined the research conducted between 1988 and 1997 and found the following teaching strategies to be effective for teaching mathematical concepts to students with a learning disability: constant time delay, manipulative devices, direct instruction, strategy instruction, lecture pause, goal structure, self-regulation, computational synthesis, problem-solving instruction, and peer tutoring. Of these, the National Council of Teachers of Mathematics (NCTM) stresses problem solving and manipulative devices/representation.

The NCTM is an organization created to help teachers mediate students' difficulties in learning math (NCTM, 1989). The council recommends many process standards for the teacher to use during instruction. Process standards are endorsed methods for teaching the NCTM content standards. This paper will focus on the combination of two of the process standards: the development of problem-solving skills and representing ideas with physical objects (NCTM, 2000). Teaching content through problem solving and physical representation allows students to have access to two strategies for learning math content.

There is mounting evidence that problem solving is a powerful and effective vehicle for learning mathematical concepts (Reusser, 1995; Van De Walle, Karp, Bay-Williams, 2009). This focus on problem solving is in agreement with mathematics education reformists who encourage the understanding of mathematics and the development of problem-solving skills rather than the memorization of algorithms. However, many students with a learning disability receive specialized math instruction that typically does not include instruction during which students can construct their own meaning or connect a relationship to the physical world (Schifter & Fosnot, 1993). Instead it emphasizes the teacher telling the students the appropriate algorithm to use to solve the problem.

Effective teachers guide their students' learning (Fuson & Briars, 1990) by selecting a strategy that matches the content. Recently, the NCTM (2012) reported two trends for the successful development of conceptual understanding. The first trend for the development of conceptual understanding includes instruction that makes mathematical relationships explicit. Secondly, students should wrestle with important ideas. That is, students should use effort to make sense of the concepts and relationships. Effective teachers are capable of making math relationships explicit and providing activities that allow students to grapple with important ideas.

Manipulatives, or concrete representations of an item are often used in mathematical instruction and may be called a "hands-on-approach" strategy. The NCTM (1989) promotes this teaching strategy and teachers are encouraged to use manipulatives to teach abstract (McNeil & Jarvin, 2007) and other mathematical concepts (Parmar & Cawley, 1997). Behrend (2003) prepared a report for the NCTM and reported an example of a hands-on approach in which the students were given a word problem, allowed time to think about the problem, and then asked to solve the problem using counters. After solving the problem, the students shared their strategy. The first problem was: “There are 3 bags of marbles. Each bag has 4 marbles in it. How many marbles are there all together?” In the report, one participant’s experience was described as follows. Once the problem was posed, Cal stated that he did not know how to solve it. He put 3 counters down, took 4 more counters and then counted all 7 of the counters. After hearing his answer, the teacher prompted, “Does that tell us how many marbles there are? Do you want me to reread the question? The problem was reread and Cal said, “Oh, that’s easy” and placed 4 counters in 3 areas and counted a total of 13. His friend said there were 12, so Cal recounted and agreed that the correct answer was 12. The other problems were solved in a similar manner. It was reported that students were able to justify their answers to others and had opportunities to make sense of the problems. This example of instruction with manipulatives illustrates the benefits of using them in very small groups and with step-by-step guidance from a teacher.

Problem Solving

Problem solving can promote students’ conceptual understanding and foster their ability to reason and communicate mathematically (Cai & Lester, 2010). Problem solving is the application of knowledge to solve a novel problem (Carnine, 1997). When teaching through problem solving, learning occurs while attempting to solve the problem (NCTM, 2012). Good problem solvers can estimate, predict, draw conclusions, evaluate, and use information effectively (Bley & Thornton, 2001). Students with a math learning disability experience several deficits that make it difficult for them to succeed in problem solving (Hanich, Jordan, Kaplan, & Dick, 2001; Jitendra & Xin, 1997; Maccini & Gagnon, 2002; Miller & Mercer, 1993; Swanson & Bebe-Frankenberger, 2004; Xin & Jitendra, 1999). When confronted with a math problem, students with a math learning disability may not readily be able to retrieve the necessary math facts and this could lead to an inconsistency in correct calculations. Additionally, they may misperceive what the question is asking or select an ineffective strategy for solving the problem (Torgeson & Kail, 1980).

To gain the needed skills for problem solving, students must have many occasions to practice (NCTM, 1980, 1989). Word problems are probably the most common form of problem solving seen in classrooms and provide many opportunities for students to practice. However, it has been suggested that the typical word problems students solve in classrooms are not challenging enough (Cai & Lester, 2010). Teachers are encouraged to increase the challenge of word problems by providing problem-solving exercises that represent problems students may encounter in real life (NCTM, 1980, 1989).

There are multiple advantages to problem solving in a real-life context. Problems presented with a real-life context are unique as the problem can be organically tied to the authentic needs and instincts of the solver (Kilpatrick, 2003). Problems in a real-life context provide cues that can "prime" students to select an effective problem solving strategy (Schliemann & Carraher, 2002) and make connections between school mathematics and students' informal knowledge (Peterson et al., 1991 as cited in Goldman & Hasselbring, 1997). Students may benefit if problems are presented in a real-life context because they are less likely to treat the problems as artificial ''games'' that bear little relation to life outside of school (Palm, 2008; Wyndhamn & Saljo, 1997). Using realistic contexts can make problems more engaging for students (Boaler, 1994) and help the teachers evaluate the ability to generalize problem-solving skills to a similar setting. Permitting students to solve problems in realistic contexts may make the problems seem more authentic, emphasizing the practical application in real-world settings (Brown, McNeil, &

Glenberg, 2009). Authentic tasks tend to provide more freedom to demonstrate competencies (Fook & Sidhu, 2010; Pellegrino, Chudowsky, & Glaser, 2001) and may provide motivation for the learners (Boaler, 1994; Peacock, 1997). A key finding regarding problems presented in real-life context is demonstrated by research completed with Brazilian children.

Carraher, Carraher, and Schliemann (1987) investigated the effect of three different contexts on problem solving for 8-13 year old Brazilian children who were low achieving students. Each student was interviewed/assessed in a one-on-one setting and allowed to answer the math questions orally or with pencil and paper. The problems were presented orally in the context of (a) computational problems, (b) simulated store situations, and (c) embedded word problems. In the computational problem condition, children were presented with a problem that did not have any reference to real-world objects or events (e.g., 8 + 16=?), in the simulated store context, the child played the role of the storeowner and the researcher played the role of the customer (e.g., One ring costs *8* cruzeiros. One piece of candy costs *16* cruzeiros. I want one of each. How much do I have to pay you?). In the embedded word-problem context, the problems were presented in a more traditional word-problem format (e.g., Mark went to see a movie. He spent *8* cruzeiros for the bus and *16* for the movie ticket. How much did he spend altogether?). All of the problems included the same number combinations, but in three different contexts. Each child completed 10 problems for each context and had difficulty with all of the problems. However, they had the most difficulty with the computational problems (as indicated by an average of 3.8 problems solved correctly). They provided a similar number of correct responses for both the simulated store problems (mean = 5.7) and the word problems (mean = 5.6). While the students had difficulty with all problem types, they performed better on the problems that included real-life contexts and because of this the researchers hypothesized that these real-life contexts may have allowed the students to use real-world knowledge.

Accessing real-world knowledge is easily promoted when problems are presented in a real-life context. For example, Jill is presented with the following problem regarding her pizza delivery route. " Each Friday night, you deliver a pizza to the house at 5th and Mission. The pizza place is at 1st and Frank. If you keep an average of 22 mph, how long does it take you to deliver the pizza?" Jill responds with two answers: a) 15 minutes if she takes the path suggested by the app on her phone and b) 12 minutes if she takes the alley behind the coffee shop to avoid traffic. In this example, Jill accessed her real-world knowledge similar to the way students in Bottge's (1999) research accessed real-world knowledge.

Using a pre/post design, Bottge (1999) recruited 8th graders in remedial math classes and divided them into two groups. One group was the contextualized word-problem group; the second group was the traditional word-problem group and served as the control group. Both groups received 10 days of instruction and five of the seven steps of Montague's (1997) cognitive strategy for solving problems: paraphrase, hypothesize, estimate, compute, and check. The contextualized word-problem group received 10 days of instruction that included two video-based, contextualized math problems, *The 8th Caller* and *Bart's Pet Project*. The videos included purchasing items and solving problems (e.g., building a cage for their pet). The problems were vague (i.e., what-if questions) compared to standard word problems. For example, What if the cage plans had called for a side that was ¼ inch longer? The students worked in groups of two or three, were asked to identify pertinent information, to formulate hypotheses about how the information could be used, and to test their hypotheses. The control group received instruction with traditional single and multistep word problems. These word problems paralleled the content of *Bart's Pet Project* and were written by the researcher. However, while their word problems were parallel to the contextualized groups; their problems were not presented within a context and looked like typical word problems found in a basal math textbook. For example, Carlos bought a candy bar for $2.27 and a drink for $1.72. He gave the cashier $5.00, how much change did he receive?

Both of Bottge's (1999) groups were asked to complete three tests: computation, word problems, and contextualized word problems. The computation test included 18 items that measured students' abilities to add and subtract fractions. The word problem test included 18 items that measured students' abilities to solve single-step and multistep word problems about linear measurement. The contextualized word-problem test included purchasing a small pet and building a home for the pet and was based on an 8-minute video *Bart's Pet Project*. The students were asked to add and subtract money and fractions, and to convert simple measurement equivalents. The students in the contextualized condition outperformed students in the traditional word-problem condition on both the word-problem test and the contextualized word-problem test. However, the students in the control group outperformed the students in the experimental group in the computation test. Due to the performance of the participants in the contextualized condition, the researchers concluded that providing word problems within in a real-life context is a strategy to help students be successful. Real-life problems can be complicated as there is extraneous information, missing information, and outside influences. To solve real-life problems students must focus on the necessary components of the problem.

Schema-based transfer instruction helps students identify the necessary components of a problem by explicitly teaching transfer features. Fuchs, Fuchs, Finelli, Courey, and Hamlett (2004) focused on schema-based transfer instruction (SBTI) which explicitly teaches transfer features that change problems in superficial ways to make them appear novel even though they still require known solution strategies. Some transfer features used in this study included: different formats, different vocabulary, irrelevant information, and different questions. The study included 351 students representative of schools in the area (e.g., English language learners, students with special needs, etc.,). The students were randomly assigned by classroom to one of three conditions: a control condition (basal and district curriculum), SBTI (regular SBTI), or expanded SBTI (SBTI plus expanded transfer features). All conditions included 16-weeks of instruction and were similar in amount of instructional time.

All students were exposed to four different problem types. The four problem types included: “shopping list" problems (e.g., Joe needs supplies for the science project. He needs 2 batteries, 3 wires, and 1 board. Batteries cost $4 each, wires cost $2 each, and boards cost $6 each. How much money does he need to buy supplies?); "half" problems (e.g., Marcy will buy 14 baseball cards. She'll give her brother half the cards. How many cards will Marcy have?); "buying bag" problems (e.g., Jose needs 32 party hats for his party. Party hats come in bags of 4. How many bags of party hats does Jose need?); and "pictograph problems (e.g., Mary keeps track of the number of chores she does on this chart [pictograph is shown with label; each picture represents 3 chores]. She also took her grandmother to the market 3 times last week. How many chores has Mary done?).

The control condition received instruction from the basal text. During instruction the students focused on one of the problem types at a time and the concepts underlying the problem type. In addition, a prescribed set of problem-solution rules was taught with explicit steps for creating a solution to the problem. There was no attempt to broaden students’ schemas. However, in comparison to the other two groups, these students received more practice in applying the problem-solution rules and had a greater emphasis on computational requirements.

The participants in the SBTI condition were taught a 6-unit curriculum. The first unit taught basic problem-solving strategies. Units 2-5 included the four problem types. In each unit the students were taught how to apply the transfer features to the relevant problem type, using problems that had varied cover stories, quantities, and one transfer feature per problem. The teachers taught the meaning of transfer and three transfer features (i.e., format, vocabulary, and question) that changed the problem without altering its type or solution. Fuchs et al. (2004) examples included "For the holidays, Anita will buy 1 CD for each of her friends. CDs come in bags with 3 CDs in each bag. How many bags will she buy for her 13 friends?" Changing the transfer feature of format, that question becomes, "For the holidays, Anita will buy 1 CD for each of her friends. The sign at the store reads like this: !!!!CDs for sale, 3 per bag!!! How many bags will she buy her 13 friends? Or, if the transfer feature of vocabulary had been altered, the problem could read like this "For the holidays, Anita will buy 1 CD for each of her friends. CDs come in wrappers with 3 CDs in each wrapper. How many wrappers will she buy for her 13 friends." To change the transfer question feature, the question could have looked like this, " For the holidays, Anita has $30 to spend on CDs. She was going to buy 1 CD for each of her friends. CDs are in bags with 3 CDs in each bag. Each bag is $5. How much money will she need for buying CDs for her 13 friends? In unit 6, the teachers reviewed the three transfer features. Participants were encouraged to look for opportunities during the day where they might transfer skills to problems requiring the same solution methods.

The expanded SBTI condition was identical to the SBTI condition except that a day of instruction per unit was devoted to the incorporation of three additional problem types. These problems had three more challenging superficial problem features: irrelevant information, combining problem types, and mixing of the superficial problem features. Students were taught that real-life situations include more information than most of the problems at school and that extraneous information makes a problem more difficult to solve.

Two forms of the Transfer 4 measure (pre and posttest) were administered to all students in all conditions. The measure was formatted to appear as a commercial test and included a narrative with a real-life problem. The narrative included both irrelevant and needed information as is typical in a real-life problem. On the posttest for the Transfer 4, participants in the expanded SBTI condition outperformed the participants in the SBTI condition who outperformed the participants in the control condition. This indicates that on the Transfer 4 measure, the expanded SBTI is beneficial for problem solving. Six percent of the participants in this study had a learning disability and the results for those students paralleled the findings for the larger sample.

In teaching through problem solving, learning takes place during the process of attempting to solve the problems in which relevant math concepts and skills are embedded (Lester & Charles, 2003; Schoen & Charles, 2003). For students to be successful problem solvers, they must have multiple sessions of practice. Providing problem-solving opportunities and presenting them in a real-life context are of great benefit to helping students gain problem-solving skills. Additionally, problem-solving skills also can be enhanced through the instruction of transfer features.

Manipulatives

When people solve problems in real-life contexts they often have objects to manipulate. Objects that are directly manipulated are called manipulatives (Sowell, 1989). To help students solve problems, many classrooms have a variety of manipulatives that may include Cuisenaire rods, Geoboards, counters, and classroom money kits with bills and coins similar to real currency. Additionally, numerous classrooms have educational games that use manipulatives as part of the game. For example, Payday, High Ho Cherry O, and Monopoly, games that build and require math skills, all use manipulatives (Laury & Holt, 1997).

The use of manipulatives is based on a constructivist approach to math. Constructivism includes individuals acting upon environmental stimuli and assimilating new experiences by altering an existing schema to construct knowledge (Ormrod, 2011). Many theorists propose that children construct abstract concepts through interaction with concrete objects in the environment (Bruner, 1966; Montessori, 1917; Piaget, 1952; Skemp, 1987). For example, Maria Montessori developed rods to teach length. These earlier theorists hypothesized that interactions with concrete objects provide a basis for abstract thought, serve as an opportunity for experience, and result in greater understanding.

Bruner (1966) theorized students needed to progress through three stages of representation to learn: enactive, iconic, and symbolic. In the enactive stage, children begin to develop understanding through actively manipulating objects. In the iconic stage, children begin to make mental images of something and no longer need the object in their visual field. Finally, in the symbolic stage, children begin to use abstract ideas to represent the world. Bruner's theory of intellectual development is the foundation for the CRA (concrete-representational-abstract) strategy for teaching mathematical concepts (Witzel, Riccomini, & Schneider, 2008). It may also be called the CSA (concrete-semiconcrete-abstract) strategy. The CRA strategy is an instructional method that has been used with students with disabilities and allows the teacher to select the manipulative he/she thinks will be most beneficial for the student's learning

The CRA sequence is a three-part instructional strategy that permits students to build on previous instruction. Instruction begins with the concrete phase and includes the use of manipulatives to illustrate the mathematical concept. The representational phase may involve drawing pictures to help solve a problem, and while concrete materials are not provided during this phase students are guided to bridge the learning from the concrete phase to this new phase. Eventually, in the abstract phase the students use symbols to illustrate the mathematical concept. There is evidence that the CRA sequence is effective for teaching students with a learning disability to learn mathematical concepts such as fractions (Jordan, Miller, & Mercer, 1999), algebra (Maccini & Hughes, 1997), place value (Peterson, Mercer, & O'Shea, 1988), and basic math facts (Mercer & Miller, 1992).

Prior to 1990 there was little empirical evidence of the effectiveness of the CRA strategy. To test the effectiveness of the CRA strategy, Mercer and Miller (1992) field-tested the *Strategic Math Series* using the CRA strategy with 40 students with special needs. Instruction for place value (n = 20) and basic math facts (n = 20) included scripted lessons and instruction. Each instructional session lasted 30 minutes. The lessons included three concrete lessons, three representational lessons, and three abstract lessons. The lessons included the following instructional sequence, the teacher provided an advanced organizer, described and modeled the problem for the students, conducted guided practice, gave independent practice time, and provided feedback. Each student was given a pretest, a posttest, and a retention test. The retention test was given 5 to 10 days after the conclusion of instruction. There was a mean increase of 70% from pretest to posttest. There was an average of 90% retention. These results led to the inclusion of the CRA sequence in the *Strategic Math Series* curriculum for teaching place value and basic math facts. Additionally, it initiated the empirical research for the CRA strategy.

Butler, Miller, Crehan, Babbitt, and Pierce (2003) examined the impact of the CRA instructional sequence on mastery of fraction concepts and procedures with middle school students (aged 11-15) with disabilities (e.g., learning disability, attention deficit disorder, emotional disability). Twenty-six students formed the CRA group and 24 students comprised the RA (representational-abstract) group. Again, all students received sequenced instruction over 10 lessons and the instruction followed the format presented in the *Strategic Math Series*. That is, the teachers provided an advance organizer, modeling, guided and independent practice, and feedback. The CRA group received concrete manipulatives for the first three lessons. Lessons 4-6 focused on concept development with representational drawings. Lesson 7 introduced the algorithm needed to compute equivalent fractions. Lessons 8-10 allowed students to practice the application of the algorithm to abstract problems. The RA group received the same amount of instructional time with the same instructional sequence as the CRA group. However, in lessons 1-3, the students used drawings representing the concepts being taught. Lessons 4-10 were the same as for the CRA group. Both groups of students improved on the five subtests used to measure fraction concepts and procedures suggesting there was no particular advantage to beginning the instruction with manipulatives.

Selecting the appropriate manipulative for instruction is crucial for learning. For example, counters (of different colors) can be used for addition, subtraction, and fractions. Undoubtedly, some manipulatives can be used for instruction for several mathematical concepts and some are better suited for specific mathematical concepts. Marsh and Cooke (1996) found Cuisenaire rods were beneficial for selecting the correct operation for a word problem.

Using a multiple baseline across participants design, Marsh and Cooke (1996) tested the impact of using Cuisenaire rods to set up word problems. They recruited third graders with a history of low achievement and provided instruction on how to use Cuisenaire rods to select the correct operation for a word problem. During baseline all students received group instruction using the verbal five-step plan SOLVE (Enright & Beattie, 1989). The SOLVE strategy includes studying the problem, organizing the problem, lining up the data, verifying the problem, and evaluating the match. Once a student was placed into intervention he/she was given 20 minutes of instruction similar to baseline and an additional 20 minutes of instruction on how to use the Cuisenaire rods to select the correct operation for a word problem. Compared to baseline, students exhibited immediate and sustained improvement on probes administered during treatment. The researchers concluded that Cuisenaire rods (along with the SOLVE strategy) were beneficial for selecting the correct operation for a word problem.

The Geoboard is another manipulative that is effective for solving word problems. Using a multiple baseline across participants design Cass, Cates, Smith, and Jackson (2003) presented problems requiring the calculation of area and perimeter to 13 to 16 year old students with a learning disability. The students used paper and a Geoboard. All students received an intervention that included modeling, prompting/guided practice, independent practice, and manipulative training to help them solve area and perimeter problems. After the intervention all of the students increased their percentage of correctly solved problems for the area and perimeter questions. The authors suggested that this study demonstrated that teenage students with a learning disability can improve their math performance with the use of manipulatives because of the functional relation between the percent of correctly solved problems and the intervention. However, this intervention was an instructional package that included other strategies and not just manipulatives. That is, the intervention had four components. The students' success could be based on any one of those four components or on a combination of any number of those components. It would be erroneous to assume that it was the manipulatives, and the manipulatives alone, that helped the students solve the problems.

Both Cuisenaire rods and Geoboards have existed for decades, but have little empirical research regarding their impact on student achievement. A more recently developed manipulative is the Rekenrek. Tournaki, Bae, and Karakes (2008) taught first grade students with a learning disability to use the Rekenrek to solve addition and subtraction problems. The Rekenrek has two strings attached to supports and each string has five white balls and five red balls. The students were randomly assigned to one of three groups. All three groups completed the pretest measure that included addition and subtraction questions with numbers from zero to 20. Group One received normal classroom instruction plus fifteen, 30-minute individual sessions with the Rekenrek across three weeks. Group Two received normal classroom instruction plus fifteen, 30-minute individual sessions *without* the Rekenrek across three weeks. Group Three received only normal classroom instruction. The results from the posttest, similar in format/content to the pretest, indicated that the students in Group One (received instruction with Rekenrek) performed at a higher level than the other two groups.

Many teachers of the learning disabled are encouraged to use manipulatives and manipulatives intuitively make sense (McNeil & Jarvin, 2007). Additionally, it is reasonable to assume that when money is the manipulative that is made available to students they would perform better because their real-life knowledge was cued (Hiebert & Wearne, 1996; McNeil, Uttal, Jarvin, & Sternberg, 2009). However manipulatives may not always be beneficial for students with a learning disability when learning abstract mathematical concepts (Amaya, Uttal, O'Doghtery, Liu, & DeLoache, 2008; Fennema, 1972; Moody, Bausell, & Abell, 1971; Petersen & McNeil, 2008).

Dual Representation

One hypothesized explanation of why manipulatives may not always be beneficial is that manipulatives do not necessarily carry meanings of insight for everyone (Moyer, 2001). That is, the physical apparatus does not offer an unmediated mathematical experience, as the apparatus does not contain nor generate mathematics (Pimm, 1994). Instead, a manipulative is a symbol. The primary function of symbols is to free us from the here and now (Werner, 1963). A symbol is an entity that someone intends to stand for something other than itself (DeLoache, 1995). Learners may struggle with using symbolically-represented information to solve the problems.

Representational insight is the basic realization of the existence of a symbol-referent relation (DeLoache, 1995) and one must detect and mentally represent the relation between the symbol and its referent (DeLoache, 2000). Gaining representational insight does not occur in a continuous and predictable pattern but rather depends on the particular stimuli and situation (DeLoache, 2000). To have representational insight, one must understand the intention of the symbol maker. If one cannot understand the intention of the symbol maker, it is possible the individual has an issue with dual representation. For example, if a learner is given a map and asked to find a location and the learner does not understand that the blue parts of the map symbolize water, the learner does not have representational insight for that map.

To recognize dual representation is to understand an object's dual reality, that is, to represent both the concrete and abstract nature of the symbol simultaneously (DeLoache, 1987). There are three obstacles to dual representation: (a) nontransparent mapping between the manipulatives and the concepts or procedure they symbolize (b) an individual's limited cognitive resources (c) an individual's resistance to change (McNeil & Jarvin, 2007). Dual representation is not automatic because manipulatives are not transparent. For example, the reeling of a fishing pole line can be predicted by a linear function and directly manipulated (i.e., a person can cast and reel the line). However, the linear function may not be automatically inferred. Instead the student's attention may be drawn to the fishing pole as an object or activity, rather than to the mathematical relationship (linearity) it is intended to demonstrate.

Additionally, recognizing dual representation is resource intensive. When children interact with manipulatives their cognitive resources may be devoted to manipulating and representing the object, leaving few cognitive resources for other tasks (e.g., the problem to be solved, math facts). Finally, a third obstacle to dual representation is resistance to change. This is apparent when an individual is unable to see or understand an object in a new light. For example, if a student is used to cookies as a snack, he/she may struggle with using them as counters during math instruction.

McNeil et al. (2009) examined dual representation by asking fourth and sixth grade students to solve word problems that involved money. The money problems were a subset of a paper-pencil assessment designed to assess students' understanding of concepts and procedures targeted by the NCTM for grades 3-5 (e.g., Charles buys a gumball for $.20 and a candy bar for $1.15. If he gives the cashier $5.00, how much change does he get back?) Students were randomly assigned to one of two conditions. In the first condition, students were given a stack of bills (major denominations up to $50) and a small plastic bag containing pennies, nickels, dimes, and quarters. The bills and coins looked similar to United States currency. Students were told that they should use the money to help them solve the problems. However, the students were not instructed to use the money in any certain manner. In the control condition, students did not receive bills and coins and were asked to solve the same problems. Students in the control condition solved more problems correctly suggesting the concrete objects designed to give a concrete representation and activate students' real-world knowledge may have hindered performance on mathematics problems. Additionally, the participants in the control condition were not required to use additional cognitive resources that the participants in the experimental condition may have used. Additional cognitive resources would have not been needed for the control group because they were not given manipulatives and would not have needed to dually represent an item.

McNeil et al. (2009) extended the above research by including an additional condition. Fifth grade students were randomly assigned to one of three conditions (a) a perceptually rich condition (i.e., the bills and coins had a similar appearance to US currency), (b) a bland condition (i.e., the bills and coins were black type-on-white paper that had no extraneous perceptual details), and (c) a control condition that did not receive any bills or coins. The students who received the perceptually rich bills and coins made more errors than the students who received the bland bills and coins or the students who did not receive any bills and coins. The perceptually rich stimuli seemed to interfere with successful completion of the task. The researchers hypothesized that students had difficulty with dual representation. For example, they may have focused on the interesting details of the stimuli, failing to see the money as a tool to help them and instead viewing it as something they used to buy items. The researchers reported the use of the manipulatives may lead to a trade-off between real-life knowledge and school-learned algorithms. That is, the manipulatives may have activated real-life knowledge but it may have interfered with a student's understanding of a problem.

A potential way to mediate dual representation is to examine how manipulatives impact performance. Once empirically-based guidelines are established it would be important to train teachers how to effectively use manipulatives (e.g., the step-by-step guidance used in the CRA method). Research by Moyer (2001) supports the need for teacher training since teachers reported that manipulatives were fun and rewarding but they did not see the value in using them as tools for math. For manipulatives to be helpful, it is essential for teachers to make the connection between the concrete and abstract concepts because simply providing manipulatives does not guarantee success.

Conclusion

Research has indicated that problem solving may be especially difficult for students with a learning disability (Hanich et al., 2001; Jitendra & Xin, 1997; Swanson & Bebe-Frankenberger, 2004). This is unfortunate because students with mild disabilities will live independent and productive lives, and therefore mathematical problem-solving skills are essential for them (Wilson & Sindelar, 1991). There have been attempts to help students become more proficient in mathematics through the use of authentic problems for problem solving (Boaler, 1994; Brown et al., 2009) and by giving students a chance to solve problems within a real-life context. Manipulatives also have been introduced as a way to make abstract math more concrete (NCTM, 2000; Parmar & Cawley, 1997). However, while the literature is mixed regarding the use of manipulatives for the general population of students (Amaya et al., 2008; Fennema, 1972; Moody et al., 1971; Petersen & McNeil, 2008; Stein & Bovalino, 2001), there has been some research that supports the use of manipulatives with students with a learning disability (Marsh & Cooke, 1996; Tournaki et al., 2008). To establish clear guidelines for manipulative use and benefits for students with a learning disability, additional research is needed (Funkhouser, 1995; Marsh & Cook, 2006; Tournaki et al., 2008).

Future research is recommended to explore the following types of questions: What are the crucial elements of instruction for teaching problem solving to this population? Are there other NCTM process methods, in addition to the use of manipulatives, that could be used in combination with problem solving? With what types of questions are students more likely to use manipulatives to help them find a solution to a problem? Can all math concepts be effectively taught using manipulatives? Answers to these questions will help identify appropriate mathematical instructional situations for the use of manipulatives with children who have a learning disability.

References

Amaya, M. M., Uttal, D. H., O'Doghtery, K. D. Liu, L. L., & DeLoache, J. S. Two-digit subtraction: Examining the link between concrete and abstract representations of knowledge. Manuscript under revision.

Barbersi, W. J., Katusic, S. K., Colligan, R. C., Weaver, A. L., & Jacobson, S. J. (2005). Math learning disorder: Incidence in a population-based cohort, 1976-1982, Rochester, Minn. *Ambulatory Pediatrics, 5,* 281-289. Retrieved from mdecgateway.org.

Behrend, J. L. (2003). Learning-disabled students make sense of mathematics. *Teaching Children Mathematics, 9,* 269-279. Retrieved from http://ezproxy.gsu.edu/login?url=http://search.proquest.com.ezproxy.gsu.edu/docview/214134288?accountid=11226

Bley, N. S., & Thornton, C. A. (2001). *Teaching mathematics to students with learning disabilities*. Austin, TX. Pro-Ed.

Boaler, J. (1994). When do girls prefer football to fashion? An analysis of female underachievement in relation to "realistic" mathematics contexts, *British Educational Research Journal, 20*, 551-564. Retrieved from www.tandf.co.uk/journals/titles/00131881.asp

Bottge, B. A. (1999). Effects of contextualized math instruction on problems solving of average and below average achieving students. *The Journal of Special Education, 33,* 91-92. doi: 10.1177/00224669990300202

Brown, M. C., McNeil, N. M., & Glenberg, A. M. (2009). Using concreteness in education real problems, real solutions. *Child Development Perspectives, 3*, 160-164. doi: 10.1111/j.1750-8606.2009.00098.x

Bruner, J. S. (1966). *Toward a theory of instruction*. London, England: Oxford University Press.

Butler, F. M., Miller, S. P., Crehan, K., Babbit, B., & Pierce, T. (2003). Fraction instruction for students with math disabilities: Comparing two teaching sequences. *Learning Disabilities Research and Practice, 18,* 99-111.doi: 10.1111/11540-5826.00066

Cai, J., & Lester, F. (2010). *Why is teaching with problem solving important to student learning?* (Problem Solving Research Brief). Reston, VA: National Council of Teachers of Mathematics.

Carnine, D. (1997). Instructional design for mathematics for students with learning Disabilities. *Journal of Learning Disabilities, 30*, 130-141. doi: 10.1177/002221949703000201

Carraher, T. N., Carraher, D. W., & Schliemann, A. D. (1987). Written and oral mathematics. *Journal for Research in Mathematics Education*, *18*, 83-97.

Cass, M., Cates, D., Smith, M., & Jackson, C. (2003). Effects of manipulative instruction solving area and perimeter problems by students with learning disabilities. *Learning Disabilities Research & Practice, 18,* 112-120. doi: 10.1111/1540-5826.00067

Cawley, J. F., Fitzmaurice, A. M., Shaw, R. A., Kahn, H., & Bates, H. III (1978). Mathematics and learning disabled youth: The upper grade levels. *Learning Disability Quarterly, 1*(4), 37-52. Retrieved from http://www.jstor.org/stable/1510976

Condelli L., Safford-Ramus, K., Sherman, R., Coben, D., Gal, I., & Hector-Mason, A. (2006). *A review of the literature in adult numeracy: Research and conceptual issues*. Washington, DC: American Institutes for Research.

DeLoache, J. S. (2000). Dual representation and young children's use of scale models. *Child Development, 71,* 329-338.doi: 10.1111/1467-8721.ep10772408

DeLoache, J. S. (1995). Early understanding and use of symbols: The model. *Current Directions in Psychological Science, 4,* 109-113.doi: 10.1111/1467-8721.ep10772408

DeLoache, J. S. (1987). Rapid change in the symbolic functioning of very young children. *Science, 238,* 1556-1557. Retrieved from www.sciencemag.org

Dossey, J. A., Mullis, I. V. S., Lindquist, M. M., & Chambers, D. J. (1988). *The mathematics report card. Are you measuring up?* Trends and achievement based on the 1986 national assessment. Princeton, NJ: Educational Testing Service.

Enright, B., & Beattie, J. (1989). Problem solving step by step in math. *Teaching Exceptional Children, 22*, 58-59. Retrieved from http://journals.cec.sped.org/tec

Fennema, E. (1972). Models and mathematics. *Arithmetic Teacher, 19,* 635-640.

Fook, C., & Sidhu, D. (2010). Authentic assessment and pedagogical strategies in higher education. *Journal of Social Science, 6,* 153-161. Retrieved from www.krepublishers.com/02-Journals/JSS/JSS-00-0-000-000-1997-Web/JSS-00-0-000-000-1997-1-Cover.htm

Fuchs, L. S., Fuchs, D., Finelli, R., Courey, S. J., & Hamlett, C. L. (2004). Expanding schema-based transfer instruction to help third graders solve real-life mathematical problems. *American Educational Research Journal, 41*, 419-445. doi: 10.3102/00028312041002419

Funkhouser, C. (1995). Developing number sense and basic computational skills in students with special needs. *School Science and Mathematics, 95,* 236-239. DOI: 10.1111/j.1949-8594.1995.tb15773.x

Fuson, K. C., & Briars, D. J. (1990). Using base-ten blocks learning/teaching approach for first-and second-grade place-value and multidigit addition and subtraction. *Journal for Research in Mathematics Education, 21,* 10-206. Retrieved from webschoolpro.com

Gal, I. *(2000). Adult numeracy: Developmental theory research, practice*. Creskill, NJ: Hampton Press.

Geary, D. C. (2004). Mathematics and learning disabilities. *Journal of Learning Disabilities, 37,* 4-15.doi: 10.11771002221940404037001020

Goldman, S. R. (1989). Strategy instruction in mathematics. *Learning Disability Quarterly, 12(1),* 43-55. Retrieved from http://www.jstor.org/stable/1510251

Goldman, S. R., & Hasselbring, T. S. (1997). Achieving meaningful mathematics literacy for students with learning disabilities. *Journal of Learning Disabilities, 30*, 98-208. doi: 10.1177/002221949703000207

Hanich, L. B., Jordan, N. C., Kaplan, D., & Dick, J. (2001). Performance across different areas on mathematical cognition in children with learning disabilities. *Journal of Educational Psychology, 93*, 615-626. doi: 10.1037//0022-0663.93.3.615

Hiebert, J., & Wearne, P. (1996). Instruction, understanding, and skill in multidigit addition and subtraction. *Cognition and Instruction, 14,* 251-285.

Jitendra, A. K., & Xin, Y. P. (1997). Mathematical word-problem solving instruction for students with mild disabilities and students at risk for failure: A research synthesis. *The Journal of Special Education, 30,* 412-438. Retrieved from www.sed.sagepub.com

Jordan, L., Miller, M. D., & Mercer, C. D. (1999). The effects of concrete to semiconcrete to abstract instruction in the acquisition and retention of fraction concepts and skills. *Learning Disabilities: A Multidisciplinary Journal, 9*(3), 115-122.

Kilpatrick, J. (2003). *A research companion to principles and standards for school mathematics*: Reston, VA: National Council of Teachers of Mathematics.

Laury, S. K., & Holt, C. A. (1997) Classroom games: Voluntary provision of a public good. *Journal of Economic Perspectives, 11,* 209-215.

Lester, F. K., Jr., & Charles, R. I. (Eds.). (2003). *Teaching math through problem polving: PreK-Grade 6*. Reston, VA: National Council for Teaching Mathematics.

Maccini, P., & Gagnon, J. C. (2002). Perceptions and application of NCTM Standards by special education and general education teachers. *Exceptional Children, 68,* 325-344. Retrieved from www.journals.cec.sped.org/

Maccini, P., & Hughes, C. A. (1997). Mathematics interventions for adolescents with learning disabilities. *Learning Disabilities Research & Practice, 12*, 168-176.

Marsh, L. G., & Cooke, N. L. (1996). The effects of using manipulatives in teaching math problem solving to students with learning disabilities, *Learning Disabilities Research & Practice, 11, 58-65.*

McNeil, N. M., & Jarvin, L. (2007). When theories don't add up: Disentangling the manipulatives debate. *Theory Into Practice, 46,* 309-316. doi:10.1080/00405840701593899

McNeil, N. M., Uttal, D. H., Jarvin, L., & Sternberg, R. J. (2009). Should you show me the money? Concrete objects both hurt and help performance on math problems. *Learning and Instruction*, *19*, 171-184. doi: 10.1016/j.learningstruc.2008.03.005

Mercer, C. D., & Miller, S. P. (1992). Teaching students with learning problems in math to acquire, understand, and apply basic math facts. *Remedial and Special Education, 13(3),* 19-35. doi: 10.1177/074193259201300303

Miller, S. P., Butler, F. M., & Lee, K. (1998). Validated practices for teaching mathematics to students with learning disabilities: A review of literature. *Focus on Exceptional Children, 31*, 1-24. Retrieved from ebschohost.com

Miller, S. P., & Mercer, C. D. (1997). Educational aspects of disabilities. *Journal of Learning Disabilities, 30,* 47-56. Retrieved from http://math4lions.com

Miller, S. P., & Mercer, C. D. (1993). Using data to learn about concrete-semiconcrete-abstract instruction for students with math disabilities. *Learning Disabilities Research & Practice, 8,* 89-96. Retrieved from http://www.wiley.com/bwljournal.asp?ref=0938-8982

Montague, M. (1997). Cognitive strategy instruction in mathematics for students with learning disabilities. *Journal of Learning Disabilities, 30,* 164-177.doi: 10.1177/002221949703000204

Montessori, M. (1917). *The advanced Montessori method*. New York, NY: Frederick Stokes Company.

Moody, W. B., Bausell, R., & Abell, R. (1971). The effect of activity-oriented instruction upon original learning, transfer and retention. *Journal for Research in Mathematics Education, 2,* 207-21.

Moyer, P. S. (2001). Are we having fun yet? How teachers use manipulatives to teach mathematics. *Educational Studies in Mathematics*, *47*, 175-197.doi: 10.1023/A:1014596316942

National Council of Teachers of Mathematics (2012). *Process standards for mathematics.* Retrieved from http:///www.nctm.org

National Council of Teachers of Mathematics (2000). *Principles and standards for school mathematics*. Reston, VA: National Council of Teachers of Mathematics.

National Council of Teachers of Mathematics (1989). *Curriculum and evaluation standards for school mathematics*. Reston, VA: National Council of teachers of Mathematics.

National Council of Teachers of Mathematics (1980). *An agenda for action: Recommendations for school mathematics of the 1980s*. Reston, VA: National Council of Teachers of Mathematics.

National Joint Committee on Learning Disabilities. (1991). Learning disabilities: Issues on definition. *Asha, 33,* (Suppl. 5), 18–20.

National Joint Committee on Learning Disabilities Editorial Staff. (2010, December 9). The struggles for individuals with learning disabilities [maintained website]. Retrieved from http://www..org/ld-basics/ld-aamp-language/ld-aamp-math/what-is-dyscalculia)

Ormrod, J. E. (2011). *Educational psychology: Developing learners* (7th ed.). Boston, MA: Allyn & Bacon.

Palm, T. (2008). Impact of authenticity on sense making in word problem solving. *Educational Studies in Mathematics*. *6*(7), *37-58*. doi: 10.1007/s10649-007-9083-3

Parmar, R. S., & Cawley, J. F. (1997). Preparing teachers to teach mathematics to students with learning disabilities. *Journal of Learning Disabilities*, *30,* 188-197. doi: 10.1177/002221949703000206

Patton, J. R., Cronin, M. E., Bassett, D. S., & Koppel, A. E. (1997). A life skills approach to mathematics instruction preparing students with learning disabilities for the real-life mathematical demand of adulthood. *Journal of Learning Disabilities, 30,* 178-187. doi: 10.1177/02221949703000205

Peacock, M. (1997). The effect of authentic materials on the motivated EFL learners. *ELT Journal, 51*, 144-156. doi: 10.1093/elt/51.2.144

Pellegrino, J. W., Chudowsky, N., & Glaser, R. (2001). *Knowing what students know: The science and design of educational assessment*. Washington, DC: National Academic Press.

Petersen, L. A., & McNeil, N. M. (2008). Using perceptually rich objects to help children represent number: Established knowledge counts. In V. Sloutsky, B. Love, & K. McRae (Eds.), *Proceedings of the Thirtieth Annual Conference of the Cognitive Science Society* (p. 1567- 1572). Mahwah, NJ: Erlbaum.

Peterson, S. K., Mercer, C. D., & O'Shea, L. (1988). Teaching learning disability students place value using the concrete to abstract sequence. *Learning Disabilities Research, 4,* 52-56. Retrieved from http://onlinelibrary.wiley.com/journal

Piaget, J. (1952). *The child's concept of number*. New York, NY: Humanities Press.

Pimm, D. (1994). Another psychology of mathematics education. In P. Ernest (Ed.). *Constructing mathematical knowledge: Epistemology and mathematical education* London, England: Psychology Press.

Reusser, K. (August, 1995). The suspension of reality and of sense-making in the culture of school mathematics: The case of word problems. Paper presented at the 6th EARLI conference. Nijmegen.

Reusser, K., & Stebler, R. (1997). Every word problem has a solution—the social rationality of mathematics modeling in schools. *Learning and Instruction, 7, 309-327* Retrieved from http://www.sciencedirect.com

Schifter, D., & Fosnot, C. T. (1993). *Reconstructing mathematic education: Stories of teachers meeting the challenges of reform*. New York, NY: Teachers College Press.

Schliemann, A. D., & Carraher, D. W. (2002). The evolution of mathematical understanding: Everyday versus idealized reasoning. *Developmental Review, 22*, 242-266. doi: 10.1006/drev.2002.0547

Schoen, H. L., & Charles, R. I. (Eds). (2003). *Teaching mathematics through problem solving: Grades 6-12*. Reston, VA: National Council for Teaching Mathematics.

Skemp, R. R. (1987). *The psychology of learning mathematics,* enl.ed. Hillsdale, NJ: Lawrence Erlbaum Associates.

Sowell, E. (1989). Effects of manipulative materials in mathematics instruction. *Journal for Research in Mathematics Education, 20,* 498-505. Retrieved from http://www.jstor.org/pss/749423

Stein, M. K., & Bovalino, J. W. (2001). Manipulatives: One piece of the puzzle. *Mathematics Teaching in Middle School 6*, 356-359. Retrieved from http://search proquest.com

Swanson, H.L., & Bebe-Frankenberger, M. (2004). The relationship between working memory and mathematical problem solving in children at risk and not at risk for math disabilities. *Journal of Education Psychology, 96* , 471-291. doi: 10.1037/022-663.96.3.471

Torgenson, J., & Kail, R. V. (1980). Memory processes in exceptional children. In B Keogh (Ed.), *Advances in special education: Basic constructs and theoretical orientations (Vol. 1)*. Greenwich, CT: Jat Press.

Tournaki, N., Bae, Y. S., & Karekes, J. (2008). Rekenrek: A manipulative used to teach addition and subtraction to students with learning disabilities. *Learning Disabilities: A Contemporary Journal*, *6*(2), 41-59.

U. S. Department of Education, National Center for Education Statistics. (2005) The Condition of Education. Retrieved from http://nces.ed.gov/

Van De Walle, J. A., Karp, K. S., & Bay-Williams, J. M. (2009). *Elementary and middle school mathematics teaching developmentally*. Boston, MA: Allyn & Bacon.

Werner, H. K., B. (1963). *Symbol formation: An organismic-developmental approach to language and the expression of thought*. New York, NY: John Wiley & Sons.

Wilson, C. L., & Sindelar, P. T. (1991). Direct instruction in math word problems: Students with learning disabilities. *Exceptional Children, 57*, 512-519.

Witzel, B. S., Riccomini, P. J., Schneider, E. (2008). Implementing CRA with secondary students with learning disabilities in mathematics. *Intervention in School and Clinic, 43,* 270-276. doi: 10.1177/1053451208314734

Wyndhamn, J., & Saljo, R. (1997). Word problem and mathematic reasoning—A study of children's mastery of reference and meaning in textual realities. *Learning and Instruction,* 7, 361-382. doi: 10.1016/S0959-4752(97)00009-1

Xin, Y. P., & Jitendra, A. (1999). The effects of solving mathematical word problems for students with learning problems. *Journal of Special Education, 32*, 207-225. doi: 10.1177/0022466999032004

CHAPTER 2

THE USE OF PERCEPTUALLY RICH AND PERCEPTUALLY BLAND MANIPULATIVES ON SOLVING MONEY WORD PROBLEMS

Introduction

Acquiring mathematical skills is important in today's society (Condelli et al., 2006; Gal, 2000). Balancing a budget, figuring the cost of sale items, and knowing the amount of money a quarter is worth are all tasks that adults face on a frequent basis. To facilitate the development of mathematical proficiency, mathematics instruction begins at a very young age.

Many educators, theorists, and reformists have influenced today's mathematics instruction. The National Council for Teaching of Mathematics (NCTM) initiated curriculum standards addressing both content and emphasis (NCTM, 1989). These standards helped to guide educators in designing lessons for their pupils. In 2000 the NCTM updated their standards identifying both content and process standards. Problem solving and representation are both recommended methods for acquiring content.

The NCTM standards recognize the need for problem-solving skills within mathematics instruction (NCTM, 2000), and there is strong evidence that problem solving is a powerful and effective vehicle for learning mathematical concepts (Reusser, 1995; Van De Walle, Karp, Bay-Williams, 2009). This focus on problem solving is in agreement with mathematics education reformists who caution that memorizing algorithms is not as valuable as understanding mathematics and developing problem-solving skills.

Problem solving provides real contexts through which learners can develop and learn mathematical ideas and concepts (NCTM, 1989, 1991). The NCTM (1980, 1989) emphasizes that students should have multiple opportunities for solving problems that represent those they may encounter in real life, such as word problems. There are many advantages to problem solving in a real-life context. Problems presented with a real-life context can be organically tied to the authentic needs and instincts of the solver (Kilpatrick, 2003) and can "prime" students to select an effective problem solving strategy (Schliemann & Carraher, 2002). If problems are presented in a real-life context students are less likely to treat the problems as artificial ''games'' that bear little relation to life outside of school (Palm, 2008; Wyndhamn & Saljo, 1997). Using realistic contexts can make problems more engaging for students (Boaler, 1994) and help teachers evaluate students' abilities to generalize the problem-solving skills to a similar setting. Problems with realistic contexts, may seem more authentic, emphasizing the practical application in real-world settings (Brown, McNeil, & Glenberg, 2009).

When people solve problems in real-life contexts they often have objects to manipulate. The NCTM suggests that representing ideas with physical objects is key in mathematical understanding (NCTM, 2000). Teachers are encouraged to use manipulatives or concrete objects that are directly manipulated (Sowell, 1989) to teach mathematical concepts (McNeil & Jarvin, 2007; Parmar & Cawley, 1997). Manipulatives are objects that are directly manipulated (Sowell, 1989). For example, students in some classrooms may use discs to count objects. That is, when they see a problem 3 + 4 =?, they gather three discs and four discs and count them, for a total of seven discs. Many classrooms have a variety of manipulatives that may include Cuisenaire rods (colored rods of differing lengths), Geoboards (a flat piece of wood with nails hammered in at lattice points; Coleman, 1978), counters (any tangible object that is used for counting or other basic math operations), and classroom money kits with bills and coins similar to real currency. Numerous classrooms have educational games that often use manipulatives as part of the game.

For example, Payday, High Ho Cherry O, and Monopoly are all games that use manipulatives and such educational games build and require math skills (Laury & Holt, 1997).

Suydam and Higgins (1976) and Sowell (1989) reviewed the literature on the efficacy of manipulative use in mathematical instruction. In a review of 40 studies in grades 1-8, Suydam and Higgins found that 24 studies favored the use of manipulative materials, 12 studies showed no difference if manipulatives were used, and 4 studies supported non-manipulative materials. Suydam and Higgins concluded that "lessons involving manipulative materials are more likely to produce greater mathematical achievement if the materials are used well" (p 2). In a more current review, Sowell reported findings similar to those reported by Suydam and Higgins. Additionally, Sowell reported that in many of the studies in which the use of manipulatives was successful, the administrators of the experimental conditions had extensive training in teaching with manipulatives. However, Sowell also stated that it was unclear whether there is a grade level or specific amount of instructional time needed before students can benefit from the use of manipulatives.

One hypothesized explanation of why manipulatives may not always be beneficial is that manipulatives do not necessarily carry meanings of insight for everyone (Moyer, 2001). That is, the physical apparatus does not offer an unmediated mathematical experience, as the apparatus does not contain nor generate mathematics (Pimm, 1994). Instead, the manipulative is a symbol. The primary function of symbols is to free us from the here and now (Werner, 1963). A symbol is an entity that someone intends to stand for something other than itself (DeLoache, 1995). Learners may struggle with using symbolically represented information to solve the problems.

Representational insight is the basic realization of the existence of a symbol-referent relation (DeLoache, 1995), one must detect and mentally represent the relation between the symbol and its referent (DeLoache, 2000). Gaining representational insight does not occur in a continuous and predictable pattern but rather depends on the particular stimuli and situation (DeLoache, 2000). To have representational insight, one must understand the intention of the symbol maker. If one cannot understand the intention of the symbol maker, it is possible there is an issue with dual representation. For example, a child's established knowledge of an object could be a hindrance to the successful use of manipulatives (Petersen & McNeil, 2008). If a chocolate chip cookie is used as a counter, the child may not be able to effectively use it to solve a word problem because the child knows the cookie as something one can eat not something one uses to count.

To recognize dual representation is to understand an object's dual reality, that is, to represent both the concrete and abstract nature of the symbol simultaneously (DeLoache, 1987). The dual representation hypothesis predicts that performance will be dependent upon the salience of a symbolic artifact as an object (DeLoache, 2000) . That is, if the symbolic artifact is very similar to the object, performance should not be as affected and it might be easier to recognize a symbol's dual representation.

McNeil, Uttal, Jarvin and Sternberg (2009) examined dual representation by asking fourth and sixth grade students to solve word problems that involved money. The problems were a subset of a paper-pencil assessment designed to assess students' understanding of concepts and procedures targeted by the NCTM for grades 3-5 (e.g., Charles buys a gumball for $.20 and a candy bar for $1.15. If he gives the cashier $5.00, how much change does he get back?) Students were randomly assigned to one of two conditions. In the first condition, students were given a stack of bills (major denominations up to $50) and a small plastic bag containing pennies, nickels, dimes, and quarters. The bills and coins looked similar to United States currency.

Students were told that they should use the money to help them solve the problems. However, the students were not instructed to use the money in any certain manner. In the control condition, students did not receive bills and coins and were asked to solve the same problems. Students in the control condition solved more problems correctly; suggesting the objects designed to give a concrete representation and activate students' real-world knowledge may have hindered performance on mathematics problems.

Some scholars suggest the richness of manipulatives may account for why some students make mistakes when working with manipulatives (McNeil et al. 2009). Using objects, especially ones that stand out from their surroundings, not only attract attention but also stimulate investigation. When a manipulative is brightly colored and unusually textured, it is referred to as a perceptually rich manipulative. Kaminski, Sloutsky, and Heckler (2009) explained that some perceptually rich objects have irrelevant richness, and this can hinder transfer and learning.

A second aspect of the previously described McNeil et al.'s (2009) study, further investigated the impact of the richness of manipulatives. They randomly assigned fifth grade students to one of three conditions: a perceptually rich condition (i.e., the bills and coins had a similar appearance to US currency), a bland condition (i.e., the bills and coins were black type-on-white paper that had no extraneous perceptual details), and a control condition in which the children did not receive any bills or coins. All students were asked to solve paper and pencil word problems that involved money. The students in the control condition performed similarly when compared to the students who received the bland bills and coins. However, the students who received the perceptually rich bills and coins made more errors than the students who received the bland bills and coins or the students who did not receive any bills or coins. The perceptually rich stimuli interfered with successfully completing the task. The researchers hypothesized that students had difficulty with the richness of the manipulatives. In other words, they may have focused on the interesting details of the stimuli. However, the researchers did not asses the students' baseline performance and therefore, it is unknown if the students in the experimental conditions would have been successful without the use of money.

Petersen and McNeil's (2008) study examined the use of perceptually rich manipulatives for counting tasks. The participants were an average age of four years old and were randomly assigned to one of four groups that differed by the type of objects being counted: (a) the object was *not* perceptually rich and students *did not* have an established knowledge of the item (e.g., solid colored plastic discs and solid colored wooden pegs), (b) the object was *not* perceptually rich but students *did* have an established knowledge of the item (e.g., popsicle sticks and colored pencils), (c) the object *was* perceptually rich and students *did not* have an established knowledge of the object (e.g., sparkly hand poms and neon and metallic pinwheels without the stem), and (d) the object *was* perceptually rich and children *did* have an established knowledge of the object (e.g., plastic animals and plastic fruit). The children met individually with the researchers and completed two counting tasks: puppet counting and give-a-number to the puppet. For the puppet counting task students watched a frog puppet count objects in an array. After the puppet was finished counting, the children were asked if the puppet made a mistake. In the give-a-number to the puppet task children were given a pile of 15 objects. The children were asked to give the monkey puppet a specified number of objects. For example, the researcher said, "The monkey wants to play with seven objects. Can you give him seven objects?" For both tasks, results indicated that if the students had the objects that were perceptually rich they were only helpful if the students did not have an established knowledge of the object. If the child had an established knowledge about the object item the perceptual richness negatively impacted the student's performance on the task. Results indicated that the children may have had difficulty utilizing perceptually rich known objects as symbols to help them with the mathematical task (McNeil & Jarvin, 2007; Petersen & McNeil, 2008).

In summary, a review of the literature indicates that activities that use manipulatives may not always be the best way to help students gain abstract mathematical knowledge and generalize learning to new situations (McNeil, et al., 2009; Petersen & McNeil, 2008; Sowell, 1989; Suydam & Higgins, 1976). However, in practice many teachers are encouraged to use manipulatives (NCTM, 2000). The use of manipulatives in mathematical instruction intuitively makes sense, but as indicated by the literature, it may not always be beneficial. In addition, there are some populations for whom there is very little or no research in the area of manipulatives. Two of these populations, children with learning disabilities and adults who struggle with numeracy are the focus of the current study.

Students with a learning disability comprise approximately 40% of identified children with disabilities who receive special education services (U.S. Department of Education, 2005). "Learning disabilities is a general term that refers to a heterogeneous group of disorders manifested by significant difficulties in the acquisition and use of listening, speaking, reading, writing, reasoning or mathematical abilities" (U. S. Department of Education, 1999).

Children with a learning disability take longer to process information and may struggle with many mathematical concepts including fact retrieval, representing numerical information spatially, and keeping information in working memory (Swanson & Jerman, 2006). Many students with a learning disability have difficulty with higher-level mathematics skills such as applied mathematical problems and measurement skills (Carnine, Jones, & Dixon, 1994).

One major area of concern for children with a learning disability is problem solving (Hanich, Jordan, Kaplan, & Dick, 2001; Maccini & Gagnon, 2002; Miller & Mercer, 1993, Jitendra & Xin, 1997; Swanson & Bebe-Frankenberger, 2004; Xin & Jitendra, 1999). Traditional mathematics education for students with a learning disability has emphasized "telling math" (Schifter & Fosnot, 1993) which largely includes the teacher telling students to use algorithms to solve problems. "Telling math" typically does not include instruction during which students can explore multiple possibilities and connect a relationship to the physical world. Therefore, this instruction does not allow students with a learning disability to have ready access to real-world problem solving contexts (Maccini, Mulcahy, & Wilson, 2007).

In addition to "telling math," there are a variety of strategies that can be used to help students with a learning disability be successful in math. Some of these strategies include: problem solving, problem solving within a real-life context, and the use of manipulatives. Unfortunately, there is very little research conducted in this area, and the results show mixed findings.

Bottge's (1999) study exemplifies the use of problem solving, and problem solving within a real-life context as 8^{th} grade remedial math students were placed into two groups and assessed by solving word problems. The first group was instructed with lessons presented in a real-life context while the second group was instructed with lessons in a traditional word problem format. Results indicated that the students in the first group outperformed the second group in contextualized word problem solving, with the author speculating that the use of real-life contextual word problems might be a more effective teaching tool than the use of traditional word problems.

The use of Cuisenaire rods (Marsh & Cooke, 1996) and the Rekenrek (Tournaki, Bae, & Karakes, 2008) has also been suggested for use with this population. Marsh and Cooke (1996) taught third grade students with a history of low achievement how to use Cuisenaire rods to set up math word problems to aid in selecting the correct operation and examined the impact of this strategy in a multiple baseline across participants design. During baseline all students received group instruction using the verbal five-step plan SOLVE (Enright & Beattie, 1989). The SOLVE strategy includes studying the problem, organizing the problem, lining up the data, verifying the problem, and evaluating the match. Once a student was placed into intervention he/she was given 20 minutes of instruction similar to baseline and an additional 20 minutes of instruction on how to use the Cuisenaire rods to select the correct operation for a word problem. Compared to baseline, students exhibited immediate and sustained improvement on probes administered during treatment. The researchers concluded that Cuisenaire rods were beneficial for selecting the correct operation for the word problems in this study.

The Rekenrek was developed in the Netherlands by math curriculum researcher, Adrian Treffers. The Rekenrek consists of 20 beads in two rows of ten, each broken into two sets of five by color. Unlike the abacus, this manipulative is not based on place value columns, instead it is a structure that represents the five fingers on each of our hands and the five toes on each of our feet (Fosnot & Dolk, 2001). Tournaki, Bae, and Karakes (2008) randomly assigned first graders with a learning disability to one of three groups. All three groups completed the pretest measure that included addition and subtraction questions with numbers from zero to 20. Group One received normal classroom instruction plus fifteen, 30-minute individual sessions with the Rekenrek for three weeks. Group Two received normal classroom instruction plus fifteen, 30-minute individual sessions *without* the Rekenrek for three weeks. Group Three received only normal classroom instruction. The results from the posttest, similar in format/content to the pretest, indicated that the students in Group One (received instruction with Rekenrek) performed at a higher level than the students in the other two groups.

Other research does not support the use of manipulatives with children with a learning disability. For example, Miller and Mercer's (1993) participants were students (aged 11-15) who struggled with math. One group of students was randomly assigned to the Concrete-Representational -Abstract (CRA) sequence while another group of students was assigned to an RA sequence (i.e., they were not exposed to the concrete instructional component of the sequence). Both groups received 10 scripted lessons. For the students receiving the CRA sequence, the first 3 lessons included instruction with manipulatives, followed by 3 lessons of representational instruction, and then 4 lessons at the abstract level. For the students receiving the RA sequence, 3 sessions were at the representational and 7 were at the abstract levels. The students in the CRA sequence scored higher than their counterparts on standardized and experimentally-based mathematical test items. However, these differences were not statistically significant. The researchers suggested that future studies should test the efficacy of implementing the CRA instructional sequence for a longer period of time.

Using a multiple baseline across participants design Cass, Cates, Smith, and Jackson (2003), presented problems requiring the calculation of area and perimeter to 13 to 16 year old children with a learning disability. The students used paper and a Geoboard. All students received an intervention that included modeling, prompting/guided practice, independent practice, and manipulative training to help them solve area and perimeter problems. After the intervention all of the students increased their percentage of correctly solved problems for the area and perimeter questions. The authors suggested that this study demonstrates teenage students with a learning disability can improve their math performance with the use of manipulatives because of the functional relation between the percent of correctly solved problems and the intervention. However, this intervention was an instructional package that included four strategies and not just manipulatives (modeling, prompting/guided practice, independent practice, and manipulative training). The students' success could be based on any one of those four components or any combination of the components. It would be erroneous to assume that it was the manipulatives, and the manipulatives alone, that helped the students solve the problems.

The research regarding the use of problem solving and manipulatives for students with a learning disability is very limited. Unfortunately, until effective instructional approaches are developed and tested, many young students who struggle with math will continue to struggle with math (Engelmann, Carnine, & Steely, 1991). More research is desperately needed in this area.

According to the 2003 National Assessment of Adult Literacy (NAAL) (US Department of Education) approximately 55% of adults in the United Stated have basic or below basic numeracy skills. Examples of basic numeracy skills include figuring a tip for a bill or balancing a checkbook. The NAAL is a nationally represented assessment of English literacy among American adults age 16 and older and is sponsored by the National Center for Educational Statistics. Unfortunately, there is very little information about adult learners who struggle with numeracy and their problem-solving capabilities. There is also very little information on mathematical instruction for these individuals.

According to Mikulec and Miller (2011) a good quality instructional task for adult learners has authenticity and involves a real-world problem. This is important because adults choose when learning will be transferable and applicable to a particular facet of their lives (Herr, 1998). Teachers of adults with low numeracy skills have been encouraged to use real-world/authentic problems to develop problem-solving skills (Ginsburg, 2008). The use of real-world problems is important in all academic areas (Purcell-Gates, Degener, Jacobson, & Soler, 2002) and is being used in some classes. A perusal of posts archived in the math and numeracy LINCS discussion list (http://lincs.ed.gov/mailman/listinfo/numeracy/) indicated that a few adult numeracy teachers encourage authentic activities in their classroom. For example, a teacher stated that when he/she blends prior knowledge and real-life examples it helps take the student from what is known to what is unknown (Carro, February 19, 2010). However, other posts are not as affirmative. For instance, a few teachers stated that the use of manipulatives may add a layer of confusion (Jones, February 18, 2010) if the connection between the concrete and the symbol is not solidified. In an additional post Kiefler (February 20, 2010) noted that she used manipulatives (i.e., algebra tiles) to teach algebra and it was a complete disaster. However, when she used the tiles to teach the concept of integers she felt she was more successful.

Over half of the adults in the United States have below basic or basic math skills (U. S. Department for Education, 2003). To improve their math skills individuals may attend classes in adult literacy programs. Although it appears that at least in some classes, the adults are exposed to authentic tasks and real-word problems with manipulatives, there are no empirical data to support the efficacy of these practices.

Current Study

This study replicates and extends two studies conducted by McNeil et al. (2009). In their first study, fourth, and sixth grade students who are typically developing (N = 230) were given 10 money word problems to solve. Students were randomly assigned to either the experimental condition in which they were given manipulatives that were perceptually rich (i.e., similar to U. S. currency) or a control condition with no manipulatives. The students in the control condition outperformed the students in the perceptually rich condition, and the authors reported that objects aimed at activating real-world knowledge may hinder performance. To further examine the impact of the manipulatives' perceptual details, they conducted a second study in which they invited fifth grade students (N = 85) to complete the same 10 money word problems. The participants were assigned to one of three conditions: control, perceptually rich (similar to U.S. currency), and perceptually bland (white paper with black writing). Again, the perceptually rich bills hindered performance on the problems as students in the perceptually rich condition solved fewer problems correctly than students in the other two conditions. The number of problems solved correctly by students in the bland condition did not differ significantly from the number of problems solved correctly by students in the control condition. Thus, bland manipulatives did not improve performance, and the perceptual richness of the manipulatives hindered performance on the problems.

The current study on problem solving with manipulatives furthers work by McNeil et al. (2009) in three key ways: a) population, b) definition and measurement of manipulative use, and c) exclusion of a control group. The current study included two populations who historically struggle with numeracy (children with a learning disability, and adults who struggle with numeracy), and children who are typically developing. McNeil et al. estimated the participants' use of manipulatives and included participant behaviors such as looking, touching, and counting. The current study collected interval data on manipulative use, but did not include looking as part of the operational definition. By the request of a school official, a control group was not included in the current study. However, through the screening measure, each participant's baseline performance was established and therefore, it was possible to compare the students against themselves with and without manipulatives.

Rationale

The United States continues to increase the mathematical standards for all learners. According to the NCTM (2000) the "need to understand and ability to use mathematics in everyday life and the workplace has never been greater and will continue to increase" (p. 4). These standards influence all students, including adults with low numeracy skills and children with a learning disability. Students should be taught using strategies that are supported by research.

Real-world problems or materials (e.g., graphs, employee manuals) may activate real-world knowledge and help students solve math word problems (Goonen & Pittman-Shetler, 2010). A variety of strategies are used to help students solve real-world problems including games and manipulatives. The research regarding the use of manipulatives is mixed. While some research indicates that manipulatives are helpful to students when solving word problems (Cass et al., 2003; Marsh & Cooke, 1996), other research indicates that manipulatives are not beneficial for students to use when solving word problems (McNeil et al., 2009; Petersen & McNeil, 2008).

Research Questions

Due to limited previous research, this study was exploratory in nature. The following questions were addressed in this study:

1. Is there an interaction effect between the sample (children who are typically developing, children with a learning disability, and adults who struggle with numeracy) and manipulative conditions (perceptually rich, perceptually bland)?

2. To what extent do children who are typically developing, children with a learning disability, and adults who struggle with numeracy who are all matched for mathematical ability differ on their performance on math word problems?

Methodology

Participants

The current study included participants from three populations: 20 adults who struggled with numeracy, 20 children with a diagnosed learning disability who were receiving specialized math instruction, and 23 children who were typically developing and not receiving specialized math instruction. As indicated by their teachers all participants possessed math performance skills at approximately the 5th grade level and all read at or above the 4th grade equivalency level. In addition, according to the teachers, all participants were proficient in English.

All individuals who participated in this study were recruited from their educational settings in a large city in the Southeastern United States. The adult participants were recruited from four different classrooms in one adult literacy program. The children with a diagnosed learning disability came from four different classrooms in one private school for students with a learning disability. The children who were typically developing were recruited from seven classrooms in three public schools (three classrooms from each of two schools, and one classroom from a third school).

The adults were mainly female (65%) and African American (95%) with 5% reporting “other” for ethnicity. Adults ranged in age from 19 to 75. The children with a learning disability were in grades 5-6, mostly male (70%) and ranged in age from 10-15. Thirty percent of the children were African American, 45% reported their ethnicity as “other,” 10% Hispanic, 10% Caucasian, and 5% declined to report their ethnicity. The children who were typically developing were fifth grade children who ranged in age from 10 to 11 years. They did not have a history of remedial instruction, grade retention, and/or grade acceleration. Most of the participants (70%)

were female, 26% were African American, 26% Caucasian, 17% Hispanic, and 31% reported their ethnicity as “other.”

Procedure

Before recruiting participants, the primary researcher contacted each school’s administrator to explain the purpose of the study and obtain permission for recruiting participants from the school. The administrators identified classrooms that included students who met the inclusion criteria. To meet the inclusion criteria, all participants had to possess 5th grade math skills and a reading level of at least 4th grade. Additionally, the students with a learning disability had to have a diagnosed learning disability and be receiving specialized math instruction. The students who were typically developing were required to have advanced through the grades in the predetermined sequence (i.e., they could not have skipped a grade, or been retained), and they could not be receiving specialized math instruction. The primary researcher discussed the purpose of the research and the space requirements (i.e., a quiet place for participants to complete the required tasks). For the child participants, the teachers distributed the parental permission forms. For the adult participants, the primary researcher distributed the informed consent documents directly to the potential participants.

Once consent and assent forms were collected, the individuals participated in a quiet room in their respective educational settings in small groups of 2-6 individuals for both the screening and experimental procedures. Two to seven days after the participants completed screening, those who answered correctly 10-70% of the items (Gagne, personal communication, June 20, 2011; Palm, 2008) were invited to participate in the experimental condition. Thirty individuals were excluded because they correctly answered fewer than 10% or more than 70% of the items. Specifically, nine adults and 16 children who were typically developing were excluded because they correctly answered more than above 70% of the items. Five children with a learning disability were excluded because they correctly answered fewer than 10% of the items.

Screening condition.

In small groups, the participants were asked to complete 10 money word problems (Appendix A). The participants were told that they could skip any problem, use scratch paper (not a calculator), and could have any problem read to them. They were not provided with manipulatives.

Experimental conditions.

To complete the experimental measure (Appendix B), participants were randomly assigned to one of two conditions (perceptually rich, perceptually bland) by flipping a coin to create two groups of ten for each sample of participants. Due to an on-site confusion, for the children who were typically developing, there were 11 who received the perceptually bland manipulatives and 12 who received the perceptually rich manipulatives.

All participants in each of the experimental conditions were tested separately. That is, the participants in the perceptually rich condition were tested separately from the participants in the perceptually bland condition. All participants first completed a demographic measure (Appendices C & D) that was read aloud by the researcher, and then were provided with the experimental procedure. In the perceptually rich condition, the participants were provided with manipulatives that were perceptually rich (i.e., real United States currency). The participants in the perceptually bland condition were provided with manipulatives that were perceptually bland (i.e., white paper with the monetary amount written in the middle).

The participants received the following instructions: “There are 10 word problems that I would like you to complete. Please use the money in a plastic bag to help you solve the problems. When you are finished, please raise your hand and someone will come and collect the word problems and the money." There was a small subset of participants (n = 6) who were not administered the above directions--three adults and three children who were typically developing. Instead, they received the following instructions, "There are 10 word problems that I would like you to complete, if you would like to use the money in a plastic bag to help you solve the problems, it is available. When you are finished, please raise your hand and someone will come and collect the word problems and the money." None of the 63 participants in the study received any directions about how to use the money to solve the word problems.

Measures.

Demographic measure.

A demographic measure (Appendices C & D) was used to explore gender, age, and ethnicity of participants. In addition, participants were asked about previous experience with games that use money. The games comprised two groups: a) games that primarily used money in their procedures (i.e., Monopoly, Life, Payday) and b) games that did not use money in their procedures (i.e., Candyland, Battleship, Taboo, Scattergories). Participants were also asked about their use of manipulatives in the classroom to help solve word problems.

Screening measure.

The screening measure (Appendix A) was adapted from McNeil et al. (2009) and is a subtest of a paper and pencil assessment used to assess students' understanding of concepts and procedures targeted by the NCTM (2000). It included 10 word problems that required basic mathematical operations. The wording of the measure was slightly altered to ensure that the readability level of the word problems was at a third grade level according the Spache readability formula (Spache, 1953). None of the mathematical concepts on this measure were changed from the McNeil et al. (2009) study.

A second observer independently scored the screening measures of 20% of the participants in each of the participant samples. Interobserver agreement (IOA) was calculated using point-by-point agreement. The number of agreements was divided by the number of agreements plus disagreements and converted to a percent. The IOA for the screening measures for the adults who struggled with numeracy and for the children who were typically developing was 100%. For the students with a learning disability there was 100% agreement except for one child. In this instance, the primary observer incorrectly scored a problem as correct, and therefore for this child, there was 90% agreement. The IOA for the children with a learning disability was 95% (90%-100%).

Experimental measure.

The measure (Appendix B) was in the same format and assessed the same mathematical concepts as the screening measure. When possible, the problems were slightly altered. That is, the name of the individual differed (e.g., John to Jake), the monetary amounts differed (e.g., $1.25 to $1.75), and the contexts differed (e.g., wash cars changed to pulls weeds).

A second observer independently scored the experimental measures of 20% of the participants in each of the participant samples. Interobserver agreement (IOA) was calculated using point-by-point agreement. The number of agreements was divided by the number of agreements plus disagreements and converted to a percent. The IOA for the experimental measures for the adults who struggle with numeracy and for the children who have a learning disability was 100%. For the students who are typically developing there was 100% agreement.

Manipulative usage measure.

This measure (Appendix E) examined the use of the manipulatives during the experimental condition. A research-team member conducted momentary-interval observation checks for 100% of the experimental sessions of all participants to examine if the manipulatives were being used. The observer observed the participants at 10 second intervals. Using the Intervalminder data application (which chimes into headphones to a predetermined time interval), every 10 seconds the observer marked a "+" on the data sheet if the participant was using the manipulative (defined as: behaviors of touching, counting, or lining up the manipulative. It did not include, rolling or spinning the manipulatives), or marked a "0" if the participant was not using a manipulative when observed. IOA was collected for 33% of the experimental sessions using point-by-point agreement. The number of agreements was divided by the number of agreements plus disagreements and was multiplied by 100%. The calculated IOA for the sessions of the adult participants was 100%. The IOA for the students with a learning disability was 97% (90%-100%) and the IOA for the students who were typically developing was 100%.

Teacher questionnaire.

This measure (Appendix F) was used to assess the teachers' use and attitudes towards the use of manipulatives in the classroom.

Treatment integrity.

This measure was used to assess the extent to which the participants (in each condition and group) received the correct instructions. There are two measures (Appendix G & H) because the instructions were slightly altered during the study. Appendix G demonstrates the first set of instructions that were given to six of the participants (3 adults and 3 students who were typically developing) while Appendix H demonstrates the second set of instructions that were given to the remaining participants (n = 57). Treatment Integrity was measured for approximately 20% of all sessions and was calculated at a 100% for all groups and conditions.

Results

Demographic measure.

In addition to gaining information about gender, age, and ethnicity, participants were asked questions regarding in-class use of tools employed to help them solve word problems. The adults reported using the following tools in class to help them solve word problems: calculator (85%), money (45%), drawing a picture (35%), and counters (30%). Thirty percent of the adults reported they had not played any of the listed board games. The remaining 70% percent reported that they had played board games that used money.

The students with a learning disability reported using the following tools in class to help them solve word problems: calculator (70%), money (40%), drawing a picture (30%), and counters (20%). Ninety-five percent of the children with a learning disability reported that they played at least one of the listed board games. Of the children who reported playing at least one of the listed games, 80% reported that they had played a board game that used money.

The students who are typically developing reported the use of the following tools to help them solve word problems in class: drawing a picture (78%), money (74%) calculator (57%) , and counters (44%). All of the students reported they had played at least one of the listed board games, with 91% reporting that they had played a board game that used money.

Manipulative usage.

Fifty percent of the adult participants used the manipulatives at least once during their session, and on average, used the manipulatives during approximately 7% of the observations.

Of the 50% who used the manipulatives, 40% received the perceptually bland manipulative, and 60% received the perceptually rich manipulative. Eighty percent of the children with a learning disability used the manipulative at least once during their session and on average used the manipulatives during approximately 10% of the observations. Of the 80% who used the manipulatives, 43% received the perceptually bland manipulative and 57% received the perceptually rich manipulative. Seventy-four percent of the children who were typically developing used the manipulative at least once during their session, and on average, used the manipulatives during approximately 3% of the observations. Of the 74% who used the manipulative, 47% received the perceptually bland manipulative and 53% received the perceptually rich manipulative.

Teacher questionnaire.

The teachers of the participants were asked to complete a survey about their use and attitude toward the use of manipulatives in the classroom. If a teacher reported he/she used manipulatives he/she was asked to complete the survey. If a teacher reported he/she did not use manipulatives, he/she only had to report why they did not use them and did not have to complete the remainder of the survey. For the teachers who reported they did not use manipulatives for instruction, 50% reported it was due to lack of time and 50% reported “other” as the reason why they did not use manipulatives in classroom instruction.

The adult students were recruited from four different classrooms with three different teachers. However, only two of the three teachers said they used manipulatives for classroom instruction. Therefore, these results will only include the responses from the two teachers who reported using manipulatives. Of the teachers who reported manipulative use, 1 of the teachers used them for instruction 1-2 times a week and the other teacher reported that she used them 3 times a week. Of the two teachers who reported manipulative use, only one of those teachers had received training in using manipulatives in instruction. Both of the teachers reported that their students enjoyed working with manipulatives, with one teacher rating the benefit of manipulatives in math instruction as a 7 and the other as an 8, on a scale of 1 to 10, with 10 being the most beneficial. Both teachers responded that fraction burgers/bars were the manipulatives that were viewed as the most beneficial. A teacher explained that fraction burgers are pieces of material that represent an ingredient in a hamburger and a fraction. For example, the hamburger patty is split into halves or is split into fourths. Fraction bars are bars that are split into fractions. For example, a bar is split into halves or fourths.

The children with a learning disability in this study were recruited from four different classrooms with two different teachers. Both teachers reported that they used manipulatives 1-2 times a week. One of the teachers had received training in manipulative use for instruction. Both teachers reported that the students enjoyed working with manipulatives. Both teachers reported that base 10 blocks were the manipulatives that they found most beneficial and stated that on a scale of 1 to 10, the benefit of using manipulatives was an 8.

The children who were typically developing were recruited from seven different classrooms with seven different teachers. However, only four of the seven teachers said they used manipulatives in classroom instruction. Three of these teachers used them fewer than 1-2 times a week, and one used them 1-2 times a week. Three of the teachers had received training in the use of manipulatives in instruction. All four of the teachers reported that students enjoyed working with manipulatives. Each teacher reported a different manipulative as most beneficial; fraction burgers/bars (1 teacher), base 10 blocks (1 teacher), Geoboard (1 teacher) and shapes (1 teacher). On a scale of 1 to 10, each teacher gave a different rating (7, 8, 9, 10).

Research questions results.

This study focused on two research questions, 1) Is there an interaction effect between the participant types (adults, children with a learning disability, children who were typically developing) and manipulative conditions (perceptually rich, perceptually bland)? 2) To what extent do children who are typically developing, children with a learning disability, and adults who struggle with numeracy who are all matched for mathematical ability differ on their performance on math word problems? To address these questions, a 2 (manipulative condition) x 3 (type of participant) ANOVA was conducted.

Results indicated that the difference between the scores on the screening measure of the adults ($M = 5.30$, $SD = 1.80$), and of the students who were typically developing ($M = 5.35$, $SD = 1.61$) was not statistically significant at the .05-level, $t(38.46) = -.091$, $p = .928$ and the difference on the screening measure scores between adults and students with a learning disability ($M = 4.30$, $SD = 6.27$) was not statistically significant at the .05-level, $t(22.13) = .685$, $p = .501$. Additionally, the difference on the screening measure scores between the students who were typically developing and the students with a learning disability was not statistically significant at the .05-level, $t(21.18) = -.726$, $p = 476$.

However, on the experimental measures, a significant interaction of Condition x Type of participant was present ($F(2,62) = 3.816$, $p < .05$). According to the post hoc S-N-K, in the perceptually bland condition, there was no significant difference in performance between the adults ($M = 6.2$, $SD = 1.61$) and the children who were typically developing ($M = 4.92$, $SD = 2.10$). However, in the perceptually bland condition, the adults and the children who were typically developing, performed significantly better than the children with a learning disability ($M = 1.90$, $SD = 1.37$). In the perceptually rich condition, there was no significant difference among the adults (M = 4.30, SD = 2.40), the children with a learning disability (M = 3.30, SD = 1.76), and the children who were typically developing (M = 5.55, SD = 2.33).

T-tests were conducted to compare the difference between the groups and the type of manipulatives they received. With α = .05, the difference in performance between the adults in the perceptually rich condition (M = 4.30, SD = 2.40) and the adults in the perceptually bland condition (M = 6.20, SD = 1.61) was not statistically significant, t(15.765) = -2.072, p = .055, d = .926 . With α = .05, the difference in performance between the children with a learning disability in the perceptually rich condition (M = 3.30, SD = 1.76) and the perceptually bland condition (M = 1.90, SD = 1.37) was not statistically significant, t(16.950) = 1.980, p = .064; the effect size for this analysis was d = .885 With α = .05, the difference in performance between the children who were typically developing in the perceptually rich condition (M = 5.55, SD = 2.33) and in the perceptually bland condition (M = 4.92, SD = 2.10) was not statistically significant, t(20.238) = .675, p = .507. The effect size for this analysis was d = .282

To examine the participants' performance on their screening measure and their experimental measure (regardless of type of manipulative received), dependent-sample t-tests were used. For the adult participants, with an α = .05, the mean on the screening measure was 5.30 (SD = 1.80), the mean on the experimental measure was 5.25 (SD = 2.22). No significant difference from screening measure to experimental measure was found (t(19)=.121, p > .05). For the children with a learning disability, the mean on the screening measure was 4.30 (SD = 6.27), the mean on the experimental measure was 2.60 (SD = 1.69). No significant difference from screening measure to experimental measure was found (t(19)= 1.700, p > .05). For the children who are typically developing, the mean on the screening measure was 5.35 (SD = 1.61), the mean on the experimental measure was 5.22 (*SD* = 2.19). No significant difference from screening measure to experimental measure was found ($t(19)=.340$, $p > .05$).

Conclusions

The purpose of the current study was to examine the impact of manipulatives (perceptually rich and perceptually bland) on money word problem solving in adults who struggle with numeracy, children who have a learning disability, and children who were typically developing. The current study was a replication of McNeil et al.'s study (2009) with three key differences including participant population, definition and measurement of manipulative use, and established baseline performance. Results of this study support McNeil et al.'s findings that when compared to screening performance levels (manipulatives were not present), the presentation of manipulatives did not increase participants' performance levels. In other words, results from this study indicate that regardless of the type of manipulative condition, none of the participants performed better than they performed during the screening condition (no manipulative). However, while McNeil et al. found that perceptually rich manipulatives hindered performance, participants in this study were neither hindered nor helped by the perceptually rich manipulatives.

There was an interaction of Condition x Type of participant, with the participants with a learning disability in the bland condition performing significantly worse than the other participants. This was the only statistically significant finding among the simple effects tests. Due to the relatively low power for those tests, another difference among the groups is notable. In the perceptually bland group, the adults performed better than students who were typically developing by 1.28 points; in the perceptually rich group, students who were typically developing outperformed adults by 1.25 points. This has two implications for future research.

First, holding the magnitiude of the effect constant, a modest increase in sample size would render these differences significant and this would contribute further to the Condition x Type interaction. Second, that the differences were similar in magnitude but opposite in direction means that inspection of only the main effect of Type would lead to the conclusion that the two groups were essentially identical, differing by only 0.03 points in the present study. Researchers should seek to improve upon the statistical power of the present study and be mindful of the potential for a complex interaction effect which could be lost if attending only to basic main effects.

McNeil et al. (2009) found their child participants who were typically developing were impacted by the use of manipulatives, with the children receiving the perceptually rich bills performing more poorly than the children receiving the perceptually bland bills, or the children who did not receive any manipulatives. The results of this current study do not confirm these findings. It is possible that McNeil et al's participants used the manipulatives more frequently than the participants in this study. McNeil et al. estimated their participants' manipulative use at 71%. That is, 71% of their participants were seen interacting with the bills and coins on a consistent basis ("consistent basis" is not described by the authors). However, observations in this study indicated that the participants hardly used the manipulatives. Only 50% of the adults, 80% of the children with a learning disability, and 74% of the children who were typically developing touched the money at least once. More telling, the manipulatives were not touched during most of the observations, with adults using the manipulatives during approximately 7% of the observations, the children with a learning disability only 10% of the observations, and the children who were typically developing 3% of the observations. Therefore, it cannot be known in this study whether using the manipulatives would have helped the participants or not. In hindsight, it would have been helpful to ask the participants about why they used or did not use the manipulatives.

Both the children with a learning disability and the adults reported the calculator as the tool they most frequently used in class to help them solve word problems. While money was reported as the second most frequently used manipulative, none of the teachers of these two struggling populations reported its use in instruction. It can be assumed that the students reported the use of money to help them solve word problems as something they had used in a previous class. Of interest is a comment reported by a teacher of some of the adult students who reported that some of her students characterized manipulatives as "babyish." Additionally, one adult reported that she did not use cash--she always used her debit card. It is possible that the parents of the participants with a learning disability also do not use cash for most interactions, but instead use a debit card or write a check. Finally, although 74% of the children who were typically developing reported using money to help them solve problems in class, compared to the other participants, they had the fewest interactions with the manipulatives. In addition, the teachers of the children who were typically developing did not report using money as manipulatives in the classroom. For all three populations, it would have been interesting to include follow-up questions addressing which grades they used each of the manipulatives in and how often they use cash on a weekly basis.

Eight of the teachers (2 teacher for adults, 2 teachers for children with a learning disability, 4 teachers of children who were typically developing) reported that they use manipulatives in their mathematical instruction. It would have been interesting to ask teachers to report how they used the manipulatives in instruction. For the teachers who reported they did not use manipulatives for instruction, 50% reported it was due to lack of time. As Moyer (2001) explained, this makes sense, as additional time may be required for using manipulatives in instruction. Fifty percent reported “other” as the reason they did not use manipulatives in classroom instruction. Teachers were not questioned further, but it is possible that their reasons could have included the view of manipulatives as “just for fun” or that they do not see the effectiveness of using manipulatives. Additionally, Moyer (2001) explained that compared to other instructional strategies, manipulatives may have an inferior connotation.

Of special interest is that although the NCTM (2012) encourages teachers to use manipulatives, of the eight teachers who reported use of manipulatives during instruction, only five (1 teacher of adults, 1 teacher of students with a learning disability, and 3 teachers of students who were typically developing) reported they had received training on how to incorporate manipulatives in instruction. Even with training, it should be noted there is a growing body of research, that the use of concrete materials alone does not guarantee successful acquisition of mathematical concepts (Brown, McNeil, & Glenberg, 2009). More research needs to be conducted to explore the efficacy behind the use of manipulatives (NCTM, 2012). If the body of research shows that manipulative use is beneficial, then teachers will need specific training in manipulative use. Knowing how to effectively incorporate problem solving and manipulative use into a math curriculum is not inherent in being an effective math teacher (NCTM, 2012).

Students can gain some experience with numeracy through games (Booker, 2000). In this study, the children who were typically developing reported more experiences with the listed games, but they did not score better than the other two groups, and they used the manipulatives the least amount. Therefore, the participants may not have made the connection between the money word problems presented by the researcher and the money problems they would have experienced in the games of Monopoly, Payday, or Life (Palm, 2008; Wyndhamn & Saljo, 1997).

In the perceptually bland condition, the children with a learning disability performed poorer than adults who struggle with numeracy and the children who are typically developing. Additional research is needed to understand this finding, as this is the first known study to investigate the difference between the impact of perceptually rich and perceptually bland manipulatives on the money word performance of children with a learning disability. In addition, it is clear that much more research is needed in the general area of manipulative use and money word problems. Such an effort is important because numeracy skills are needed on a frequent basis in daily adult living (Condelli et al. 2006; Gal, 2001). Over half of the adults in the United States do not have these skills (US Department of Education, 2003). To help adults who struggle with numeracy, children with a learning disability, and children who are typically developing gain the numeracy skills that they currently and in the future will need, their instruction should be guided by evidence-based instructional strategies.

References

Boaler, J. (1994). When do girls prefer football to fashion? An analysis of female underachievement in relation to "realistic" mathematics contexts, *British Educational Research Journal, 20*, 551-564. Retrieved from www.tandf.co.uk/journals/titles/00131881.asp

Booker, G. (2000). *The maths game: Using instructional games to teach mathematics*. Wellington, New Zealand: New Zealand Council for Educational Research.

Bottge, B. A. (1999). Effects of contextualized math instruction on problems solving of average and below average achieving students. *The Journal of Special Education, 33*, 91-92. doi: 10.1177/002246699903300202

Brown, M. C., McNeil, N. M., & Glenberg, A. M. (2009). Using concreteness in education real problems, real solutions. *Child Development Perspectives, 3*, 160-164. doi: 10.1111/j.1750-8606.2009.00098.x

Carnine, D., Jones, E. D., & Dixon, R. (1994). Mathematics: Educational tools for diverse learners. *School Psychology Review, 23*, 406-427.

Carro, M. (2010, February 19). Re: Manipulatives (Electronic mailing list message). Retrieved from http://lincs.ed.gov/lincs/discussions/subscribe_all.html

Cass, M., Cates, D., Smith, M., & Jackson, C. (2003). Effects of manipulative instruction solving area and perimeter problems by students with learning disabilities. *Learning Disabilities Research & Practice, 18,* 112-120. doi: 10.1111/1540-5826.00067

Coleman, D. B. (1978). Stretch: A Geoboard game. *Mathematics Magazine, 51*, 49-54. Retrieved from http://www.jstor.org/stable/2689646

Condelli L., Safford-Ramus, K., Sherman, R., Coben, D., Gal, I., & Hector-Mason, A. (2006). *A review of the literature in adult numeracy: Research and conceptual issues*. Washington, DC: American Institutes for Research.

DeLoache, J. S. (2000). Dual representation and young children's use of scale models. *Child Development, 71,* 329-338.doi: 10.1111/1467-8721.ep10772408

DeLoache, J. S. (1995). Early understanding and use of symbols: The model. *Current Directions in Psychological Science, 4,* 109-113.doi: 10.1111/1467-8721.ep10772408

DeLoache, J. S. (1987). Rapid change in the symbolic functioning of very young children. *Science, 238,* 1556-1557. Retrieved from www.sciencemag.org

Englemann, S., Carnine, D. W., & Steely, D. (1992). Making connections in mathematics. In D. Carnine & E. Kameenui (Eds.). *Teaching higher order thinking to all students* (pp. 75-106). Austin, TX: PRO-ED.

Enright, B., & Beattie, J. (1989). Problem solving step by step in math. *Teaching Exceptional Children, 22,* 58-59.

Fosnot, C. T., & Dolk, M. (2001). *Addition and subtraction facts on the horizon*. In V. Merecki & L. Peake (Eds.), *Young mathematicians at work: Vol. 1. Constructing number sense, addition and subtraction* (pp. 97-113). Portsmouth, NH: Heinemann.

Gal, I. (2000). *Adult numeracy: Developmental theory research, practice*. Creskill, NJ: Hampton Press.

Ginsburg, F. (2008). Designing instruction with the components of numeracy in mind, *Focus on Basics World Education Connecting Research and Practice, 9a, I 1-49*.

Goonen, B., & Pittman-Shetler, S. (2010). Going beyond basic part II Writing and Reading. Retrieved from http://farwestgreat.org

Hanich, L. B., Jordan, N. C., Kaplan, D., & Dick, J. (2001). Performance across different areas on mathematical cognition in children with learning disabilities. *Journal of Educational Psychology, 93*, 615-626. doi: 10.1037//0022-0663.93.3.615

Herr, L. (1998). Learning outside the box. In T. Reiff (Ed.). *The Pennsylvania ABLE administrators handbook 1998 edition: A guide for administrators of programs providing adult basic and literacy education services in Pennsylvania*. Harrisburg, PA: Pennsylvania Department of Education.

Jitendra, A. K., & Xin, Y. P. (1997). Mathematical word-problem solving instruction for students with mild disabilities and students at risk for failure: A research synthesis. *The Journal of Special Education, 30,* 412-438. Retrieved from www.sed.sagepub.com

Jones, S. (2010, February 18).). Re: Manipulatives (Electronic mailing list message). Retrieved from http://lincs.ed.gov/lincs/discussions/subscribe_all.html

Kaminski, J. A., Sloutsky, V. M., & Heckler, A. (2009). Transfer of mathematical knowledge: The portability of generic instantiations. *Child Development Perspectives, 3,* 151-155.

Kiefler, J. (2010, February 20). Re: Manipulatives (Electronic mailing list message). Retrieved from http://lincs.ed.gov/lincs/discussions/subscribe_all.html

Kilpatrick, J. (2003). A research companion to principles and standards for school mathematics: National Council of Teachers of Mathematics.

Laury, S. K., & Holt, C. A. (1997). Classroom games: Voluntary provision of a public good. *Journal of Economic Perspectives, 11,* 209-215

Maccini, P., & Gagnon, J. C. (2002). Perceptions and application of NCTM Standards by special education and general education teachers. *Exceptional Children, 68,* 325-344. Retrieved from www.journals.cec.sped.org/

Maccini, P., Mulcahy, C. A., & Wilson, M. G. (2007). A follow-up of mathematics interventions for secondary students with learning disabilities. *Learning Disabilities Research & Practice, 22*, 58-74. doi: 10.1111/j.1540-5826.2007.00231.x

Marsh, L. G., & Cooke, N. L. (1996). The effects of using manipulatives in teaching math problem solving to students with learning disabilities, *Learning Disabilities Research & Practice, 11, 58-65.*

McNeil, N. M., & Jarvin, L. (2007). When theories don't add up: Disentangling the manipulatives debate. *Theory into Practice, 46,* 309-316. doi:10.1080/00405840701593899

McNeil, N. M., Uttal, D. H., Jarvin, L., & Sternberg, R. J. (2009). Should you show me the money? Concrete objects both hurt and help performance on math problems. *Learning and Instruction*, *19*, 171-184. doi: 10.1016/j.learningstruc.2008.03.005

Mercer, C. D., & Miller, S. P. (1992). Teaching students with learning problems in math to acquire, understand, and apply basic math facts. *Remedial and Special Education, 13*(3), 19-35.doi: 10.11771074193259

Mikulec, E., & Miller, P. C. (2011). Using project-based instruction to meet foreign language standards. *The Clearing House, 84,* 81-86.doi: 10.1080/00098655.2010.516779

Miller, S. P., & Mercer, C. D. (1993). Using data to learn about concrete-representational-abstract instruction for students with math disabilities. *Learning Disabilities Research & Practice, 8,* 89-96. Retrieved from http://www.wiley.com/bwljournal.asp?ref=0938-8982

Moyer, P. S. (2001). Are we having fun yet? How teachers use manipulatives to teach mathematics. *Educational Studies in Mathematics*, *47*, 175-197.doi: 10.1023/A:1014596316942

National Council of Teachers of Mathematics (2012). *Process standards for school mathematics*. Reston, VA: National Council of Teachers of Mathematics.

National Council of Teachers of Mathematics (2000). *Principles and standards for school mathematics*. Reston, VA: National Council of Teachers of Mathematics.

National Council of Teachers of Mathematics (1991). *Professional standards for teaching mathematics*. Reston, VA: National Council of Teachers of Mathematics.

National Council of Teachers of Mathematics (1989). *Curriculum and evaluation standards for school mathematics*. Reston, VA: National Council of Teachers of Mathematics.

National Council of Teachers of Mathematics (1980). *An agenda for action: Recommendations for school mathematics of the 1980s*. Reston, VA: National Council of Teachers of Mathematics.

Palm, T. (2008) Impact of authenticity on sense making in word problem solving. *Educational Studies in Mathematics*. *6*(7*), 37-58.* doi: 10.1007/s10649-007-9083-3

Parmar, R. S., & Cawley, J. F. (1997). Preparing teachers to teach mathematics to students with learning disabilities. *Journal of Learning Disabilities*, *30,* 188-197. doi: 10.1177/002221949703000206

Petersen, L. A., & McNeil, N. M. (2008). Using perceptually rich objects to help children represent number: Established knowledge counts. In V. Sloutsky, B. Love, & K. McRae (Eds.), *Proceedings of the Thirtieth Annual Conference of the Cognitive Science Society* (p. 1567- 1572). Mahwah, NJ: Erlbaum.

Pimm, D. (1994). Another psychology of mathematics education. In P. Ernest (Ed.). *Constructing mathematical knowledge: Epistemology and mathematical education* London, England: Psychology Press.

Purcell-Gates, V., Degener, S. L., Jacobson, E., & Soler, M. (2002). Impact of authentic adult literacy instruction on adult literacy practices. *Reading Research Quarterly, 137,* 70-92. Retrieved from http://www.jstor.org/stable/748322 .

Reusser, K. (August, 1995). The suspension of reality and of sense-making in the culture of school mathematics: The case of word problems. Paper presented at the 6th EARLI conference. Nijmegen.

Schifter, D., & Fosnot, C. T. (1993). *Reconstructing mathematic education: Stories of teachers meeting the challenges of reform*. New York, NY: Teachers College Press.

Schliemann, A. D., & Carraher, D. W., (2002). The evolution of mathematical understanding: Everyday versus idealized reasoning. *Developmental Review, 22*, 242-266.doi: 10.1006/drev.2002.0547

Spache, G. (1953). "A New Readability Formula for Primary-Grade Reading Materials". *The Elementary School Journal, 53,* 410–413. doi:10.1086/458513. JSTOR 998915.

Sowell, E. (1989). Effects of manipulative materials in mathematics instruction. *Journal for Research in Mathematics Education, 20,* 498-505. Retrieved from http://www.jstor.org/pss/749423

Swanson, H.L., & Bebe-Frankenberger, M. (2004). The relationship between working memory and mathematical problem solving in children at risk and not at risk for math disabilities. *Journal of Education Psychology, 96*, 471-291. doi: 10.1037/022-663.96.3.471

Swanson, H. L., & Jerman, O. (2006). Math Disabilities: A Selective Meta-Analysis of the Literature. *Review of Educational Research, 76,* 249–274. doi: 10.3102/00346543076002249

Suydam, M. N., & Higgins, J. L. (1976) *Review and Synthesis of Studies of Activity Based Approaches to Mathematics Teaching*. Final Report, NIE Contract No. 400-75-0063, September 1976. (Also available from ERIC Information Analysis Center for Science, Mathematics and Environmental Education, Columbus, Ohio.)

Tournaki, N., Bae, Y. S., & Karekes, J. (2008). Rekenrek: A manipulative used to teach addition and subtraction to students with learning disabilities. *Learning*

Disabilities: A Contemporary Journal, *6*(2), 41-59.

U. S. Department of Education (1999). *Assistance to States for the Education of Children With Disabilities and the Early Intervention Program for Infants and Toddlers With Disabilities; Final Regulations* (34 CFR Parts 3003 & 303) Washington, DC: Government Printing Office.

U. S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, (1992). National Adult Literacy Survey.

U. S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, (2003). National Assessment of Adult Literacy.

U. S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, (2005). The Condition of Education.

Van De Walle, J. A., Karp, K. S., & Bay-Williams, J. M. (2009). *Elementary and middle school mathematics teaching developmentally*. Boston, MA: Allyn & Bacon.

Werner, H. K., B. (1963). *Symbol formation: An organismic-developmental approach to language and the expression of thought*. New York, NY: John Wiley and Sons.

Wyndhamn, J. & Saljo, R. (1997). Word problem and mathematic reasoning—A study of children's mastery of reference and meaning in textual realities. *Learning and Instruction,* 7, 361-382. doi: 10.1016/S0959-4752(97)00009-1

Xin, Y. P., & Jitendra, A. (1999). The effects of solving mathematical-word problems for students with learning problems. *Journal of Special Education, 32*, 207-225. doi: 10.1177/002246699903200402

APPENDIXES

APPENDIX A

Screening Measure

1. Maggie wants to save money to buy a new T.V. She gets $15.00 at the end of each week. The T.V costs $45.99. How many weeks will it take her to save the money she needs if she saves all her money each week?

2. John wants money so he mows the lawn. His dad says he can be paid weekly or daily. John's father said he would either pay him $1.25 a week or pay him in the following way for a week: On Monday, he would give him 1 cent; on Tuesday, 2 cents; on Wednesday 4 cents; on Thursday, 8 cents, and so on through Sunday. Which choice should John choose so that he can get more money?

3. Mike is at the store to buy gum. He is trying to decide between buying a 6-pack of gum for $4.80 or an 18-pack for $12.60. Which is cheaper taking into account how much gum he gets for his money?

4. In Emily and Rick's family, some children get paid each week, and others get paid each month. Emily gets $15.00 a week, and her brother gets $50.00 per month. Do they both get the same amount of money each month? (1 month = 4 weeks)

5. Will's mom and dad are having a big party. They are expecting many guests, so they have decided to ask a cook to help them prepare the food. The cook tells them it will cost $7.50 per person. If they order for 12 people, what will the total cost be?

6. Chad buys a ball for $.20 and a candy bar for $1.15. If he gives the cashier $5.00, how much change does he get back?

7. Mr. and Mrs. Jones paid $7.00 for tickets for themselves and their three kids. Considering the following prices, which game did they buy tickets for?

    SOFTBALL: Adults $2.00-Children $1.00

    BASKETBALL: Adults $3.00-Children $1.00

    SOCCER: Adults $5.00-Children $2.50

8. Dan and Elle want to buy a birthday cake for their friend. Dan has $6.00 and can spend ¼ of his money. Elle has $10.00 and can spend ¼. How much can they spend altogether?

9. Pat and Jim are going shopping for their mom at the bakery. They were told to buy 18 rolls, 12 hot dog buns, and 2 loaves of rye bread. How much will it cost them if the following prices apply?

    - White bread loaves- 2 for $1.25
    - Rye bread loaves 2 for $1.35
    - Rolls- 6 for $1.00
    - Hamburger buns- 6 for $0.85
    - Hot dog buns—6 for $0.69

10. You decide to wash cars to earn money. Each month you decide to save part of your money. If you saved $2.00 from your car washes on January 1, $4.00 from your car washes on February 1, $6.00 from your car washes on March 1 , and so on, how much money would you save in 6 months?

APPENDIX B

Experimental Measure

1. Chan buys an apple for $.40 and a drink for $1.75. If he gives the cashier $5.00, how much change does he get back?

2. Mim is at the store to buy gum. He is trying to decide between buying a 6-pack of gum for $2.40 or an 18-pack for $10.20. Which is cheaper taking into account how much gum he gets for his money?

3. Jake wants money so he mows the lawn. His dad says he can be paid weekly or daily. Jake's dad said he would either pay him $2.15 a week or pay him in the following way for a week: On Monday, he would give him 1 cent; on Tuesday, 2 cents; on Wednesday 4 cents; on Thursday, 8 cents, and so on through Sunday. Which choice should John choose so that he can get more money?

4. James wants to save money to buy a new bike. He gets $13.00 at the end of each week. The bike costs $85.92. How many weeks will it take him to save the money he needs if he saves all his money each week?

5. Mr. and Mrs. Jimenez paid $9.00 for tickets for themselves and their three kids. Considering the following prices, which game did they buy tickets for?

SOFTBALL: Adults $2.00-Children $1.00

BASKETBALL: Adults $3.00-Children $1.00

SOCCER: Adults $5.00-Children $2.50

6. In Emile and Rico's family, some children get paid each week, and others get paid each month. Emile gets $20.00 a week, and his brother gets $55.00 per month. Do they both get the same amount of money each month? (1 month = 4 weeks)

7. Bob decides to pull weeds to earn money. Each month he decides to save part of his money. If he saved $1.00 from his weed pulling on January 1, $3.00 from his weed pulling on February 1, $5.00 from his weed pulling on March 1, and so on, how much money would he save in 6 months?

8. Wynn's mom and dad are having a big party. They are expecting many guests, so the have decided to ask a cook to help them prepare food. The cook tells them it will cost $6.50 per person. If they order for 11 people, what will the total cost be?

9. Drake and Elin want to buy a gift for their friend. Drake has $6.00 and can spend ¼ of his money. Elin has $10.00 and can spend ¼. How much can they spend altogether?

10. Pam and Jon are going shopping for their mom at the bakery. They were told to buy 18 rolls, 12 hot dog buns, and 2 loaves of rye bread. How much will it cost them if the following prices apply?

- White bread loaves- 2 for $2.25
- Rye bread loaves 2 for $2.75
- Rolls- 6 for $2.00
- Hamburger buns- 6 for $0.87
- Hot dog buns—6 for $0.69

APPENDIX C

Child Demographic Measure

Are you a ?

Boy Girl SKIP

How old are you? ________

Ethnicity

Asian

African American

Hispanic or Latino

Caucasian

American Indian/Alaska Native

Not mentioned

Decline to say

Have you ever played any of the following games?

Candyland

Monopoly

Life

Taboo

Payday

Battleship

Scattergories

In class, I have used money to help me solve a word problem. Yes No

In class, I have used counters to help me solve a word problem. Yes No

In class, I have used a calculator to help me solve word a problem. Yes No

In class, I have drawn a picture to help me solve a word problem. Yes No

APPENDIX D

Adult Demographic Measure

Are you a ?

Man Woman SKIP

What age are you? __________

OR

What year were you born? __________

Ethnicity

Asian

African American

Hispanic or Latino

Caucasian

American Indian/Alaska Native

Not mentioned

Decline to say

Have you ever played any of the following games?

Candyland

Monopoly

Life

Taboo

Payday

Battleship

Scattergories

In class, I have used money to help me solve a word problem. Yes No

In class, I have used counters to help me solve a word problem. Yes No

In class, I have used a calculator to help me solve a word problem. Yes No

In class, I have drawn a picture to help me solve a word problem. Yes No

APPENDIX E

Manipulative Usage Measure

Use of manipulative is defined as: behaviors of touching, counting, or lining up the manipulative

Use of manipulative does not include:

Spinning or rolling of the manipulative

Start Time ________________

"+" = behavior observed

End Time ________________ "0" = behavior *not* observed

	ID #	ID #	ID #
Interval			
1			
2			
3			
4			
5			
6			
7			
8			

9			
10			
11			
12			
13			
14			

Note. The data sheet had intervals1-308.

APPENDIX F

Teacher Questionnaire

1. Do you use manipulatives (concrete objects) in your class for mathematical instruction?

   Yes No

   If yes, on Average, how often do you use manipulatives (concrete objects) in your classroom?

   1-2 times a week 3 times a week 4 or more times a week

   If no, what are some reasons you do not use manipulatives in your class? **

   not enough time not available to you lack of training other

**If you answered "No" to question 1 this is the last item you need to complete on this survey

2. Have you had any training for using manipulatives in the classroom?

   Yes No

3.What manipulatives do you use in your classroom?

   Money Counters Geoboards base10 blocks Other

4. On a scale of 1 to 10 with one not at all beneficial and 10 very beneficial, how beneficial do you think the use of manipulatives is in the classroom?

   1 2 3 4 5 6 7 8 9 10

5. Name the manipulatives you think are the most beneficial in your class.

6. Do you think your students enjoy working with manipulatives?

   Yes No

APPENDIX G

Treatment integrity checklist for first set of directions

(3 adults and 3 children who are typical)

1. The participants are greeted and thanked for coming. Y N

2. Each participant receives the manipulatives. Y N

3. The directive of There are 10 word problems that I would like you to complete. If you would like to use the money in a plastic bag to help you solve the problems, it is available. When you are finished, please raise your hand and someone will come and collect the word problems and money. Y N

,

4. All of the materials are collected. Y N

APPENDIX H

Treatment integrity checklist for second set of directions

(17 adults, 20 students with LD, and 20 students who are typically developing)

1. The participants are greeted and thanked for coming. Y N

2. Each participant receives the manipulatives. Y N

3. The directive of There are 10 word problems that I would like you to complete. Please use the money in the bag to help you solve the problems. When you are finished, please raise your hand and someone will come and collect the word problems and money. Y N

4. All of the materials are collected. Y N