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A Simulation Study Comparing Two Methods Of Evaluating Differential Test Functioning (DTF): DFIT and the Mantel-Haenszel/Liu-Agresti Variance

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ACCEPTANCE

This dissertation, A SIMULATION STUDY COMPARING TWO METHODS OF EVALUATING DIFFERENTIAL TEST FUNCTIONING (DTF): DFIT AND THE MANTEL-HAENSZEL/LIU-AGRESTI VARIANCE, by CHARLES VINCENT HUNTER, JR., was prepared under the direction of the candidate's Dissertation Advisory Committee. It is accepted by the committee members in partial fulfillment of the requirements for the degree Doctor of Philosophy in the College of Education, Georgia State University.

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A SIMULATION STUDY COMPARING TWO METHODS OF EVALUATING
DIFFERENTIAL TEST FUNCTIONING (DTF): DFIT AND THE MANTEL-
HAENZSEL/LIU-AGRESTI VARIANCE

by

CHARLES VINCENT HUNTER, JR

Under the Direction of T. C. Oshima

ABSTRACT

This study uses simulated data to compare two methods of calculating Differential Test Functioning (DTF): Raju's DFIT, a parametric method that measures the squared difference between two Test Characteristic Curves (Raju, van der Linden & Fleer, 1995), and a variance estimator based on the Mantel-Haenszel/Liu-Agresti method, a non-parametric method enabled in the DIFAS (Penfield, 2005) program.

Most research has been done on Differential Item Functioning (DIF; Pae & Park, 2006), and theory and empirical studies indicate that DTF is the summation of DIF in a test (Donovan, Drasgow & Probst; 2000, Ellis & Mead, 2000; Nandakumar, 1993). Perhaps because of this, measurement of DTF is under-investigated. A number of reasons can be given why the study of DTF is important. From a statistical viewpoint, items, when compared to tests, are small and unreliable samples (Gierl, Bisanz, Bisanz, Boughton, & Khaliq, 2001). As an aggregate measure of DIF, DTF can present an overall view of the effect of differential functioning, even when no

single item exhibits significant DIF (Shealy & Stout, 1993b). Decisions about examinees are made at the test level, not the item level (Ellis & Raju, 2003; Jones, 2000; Pae & Park, 2006; Roznowski & Reith, 1999; Zumbo, 2003).

Overall both methods performed as expected with some exceptions. DTF tended to increase with DIF magnitude and with sample size. The MH/LA method generally showed greater rates of DTF than DFIT. It was also especially sensitive to group distribution differences (impact) identifying it as DTF where DFIT did not. An empirical cutoff value seemed to work as a method of determining statistical significance for the MH/LA method. Plots of the MH/LA DTF indicator showed a tendency towards an F-distribution for equal Reference and focal group sizes, and a normal distribution for unequal sample sizes. Areas for future research are identified.

INDEX WORDS: DTF, Differential Test Functioning, DFIT, Mantel-Haenszel

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DEDICATION

To the memory of my Uncle,

William Joseph Rycroft, Sr.,

Who said to me,

“Vince, you need to get a Ph.D. We need a doctor in the family,
and I finally realized that none of my children are going to do it.”

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List of Abbreviations

1PL	One Parameter Logistic Model
2PL	Two Parameter Logistic Model
3PL	Three Parameter Logistic Model
AERA	American Educational Research Association
APA	American Psychological Association
CDIF	Compensatory Differential Item Functioning
CFA	Confirmatory Factor Analysis
CTT	Classical Test Theory
DF	Differential Functioning
DBF	Differential Bundle Functioning
DFIT	Differential Functioning of Items and Tests
DIF	Differential Item Functioning
DIFAS	Differential Item Functioning Analysis System
DTF	Differential Test Functioning
EFA	Exploratory Factor Analysis
ETS	Educational Testing Service
ICC	Item Characteristic Curve
IRF	Item Response Function
IRT	Item Response Theory
LOR	Log Odds Ratio
MH	Mantel-Haenszel Method
MH/LA	Mantel-Haenszel/Liu-Agresti Method

MML	Marginal Maximum Likelihood
NCDIF	Non-Compensatory Differential Item Functioning
NCME	National Council on Measurement in Education
OCI	Observed Conditional Invariance
REM	Random Effects Model
RMSEA	Root Mean Squared Error of Approximation
RMWSD	Root Mean Weighted Squared Difference
SE	Standard Error of Measurement
SIBTEST	Simultaneous Item Bias Test
UCI	Unobserved Conditional Invariance

CHAPTER 1

INTRODUCTION

Ensuring that tests are valid for their intended purposes is a major concern of measurement theory. Modern measurement theory developed as a response to social concerns about the fairness of tests and their application (Rudner, Getson, & Knight, 1980) in areas of employment (Drasgow, 1987) and education, but it is equally necessary in the areas of psychological and medical assessment (Cameron, Crawford, Lawton, & Reid, 2013; Van den Broeck, Bastiaansen, Rossi, Dierckx, & De Clercq, 2013).

While modern measurement (Item Response Theory) focuses on the individual test item as the unit of study, decisions about individual persons are made based on test level data (Wang & Russell, 2005; Ellis & Raju, 2003; Jones, 2000; Pae & Park, 2006; Roznowski & Reith, 1999; Russell, 2005). Test functioning may be conceived of as the aggregate of item functioning (Rudner, Getson, & Knight, 1980; Shealy & Stout, 1993a). Therefore, studying Differential Test Functioning (DTF) to determine how best to control test validity is both appropriate and worthwhile. Using simulated data, this present study compares two methods of calculating DTF: DFIT by (Raju, van der Linden and Fler, 1995), and the Mantel-Haenszel/Liu Agresti Variance method (Penfield & Algina, 2006) with the purposes of understanding better how each functions, and how they relate to each other.

Background and Research Questions

The modern study of measurement grew out of the science of psychology that began in the 1880's. To defend their work from critics in the physical sciences and philosophy, psychologists began to quantify their findings. As psychology enlarged its scope from physical to mental phenomena, the need arose to measure what could not be directly observed. The unobservable mental phenomena (generically called constructs) were measured by observable behaviors that were theorized to express them. For example, knowledge of mathematics is observed by performance on a mathematics test (Crocker & Algina, 1986).

Measurement theory is concerned with the quantification of observable behaviors that express the existence of non-observable mental skills and conditions. An integral part of measuring behaviors is the degree to which measurements are done correctly and with precision. An imperfect measurement produces measurement error that must be taken into account when performing statistical analyses. This is done in Classical Test Theory (CTT) with the definition of a measurement as the sum of the true score plus measurement error (Crocker & Algina, 1986):

$$X = T + E$$

Beyond this, Crocker and Algina (1986) also discuss issues of reliability and validity. Reliability is defined as the production of consistent scores by a test from one administration to another. That is, test administrators (teachers, researchers) will get similar results from a test when it is administered to similar groups of examinees under similar conditions. Measurement error negatively affects test reliability. Error may be divided into random error and systematic error. Random error is sporadic and varies from one test administration to the next. It may be caused by such things as ambient temperature or lighting. Systematic error is consistent by examinee (or group of examinees) among all test administrations. It may be caused by such

things as hearing impairment or personality traits. Systematic error that occurs over a demographic group may be related to test validity and issues of differential functioning (DF) and multidimensionality (see also Camilli, 1993). Kunnan (1990) states that tests with large numbers of DF items are potentially unreliable, meaning that the tests also would be invalid for all examinees.

Validity deals with the appropriateness of a test for evaluating an examinee for a given purpose. It is generally studied as content validity, criterion validity, and construct validity. Content validity relates to a test's being an adequate representation of a given area of knowledge. Criterion validity relates to using a test to measure performance where a direct measurement of performance is not easily available. These tests are often used in a selection process as predictors of expected performance, such as in college or on a job. Construct validity relates to measuring unobservable or intangible things generally dealt with in psychology and education, such as intelligence and personality traits. Messick (1995) asserts that validity needs to be understood as pertaining to the meaning of test scores, not to the test itself. Overall it is an evaluation of how appropriate decisions based on test scores are.

A fourth aspect of validity, fairness, rose to prominence beginning in the 1960's. Fairness is a cluster of concepts, not strictly related, that center around the relation between tests and test takers. Four of the main uses of the term fairness are a lack of statistical bias, a test process that is equitable for all examinees, similar outcomes for all demographic groups of examinees, and an opportunity to learn the material of the test by all examinees (AERA, APA, & NCME, 1999). The perception that all groups should have similar results on tests was an early driver of the focus on fairness. Drasgow (1987) discusses two of these cases relating to standardized tests from the 1980's. In both cases, the parties settled on future tests having minimal (proportions of

.05 to .15) differences in pass rates between different demographic groups. While this is a politically acceptable resolution, it is not defensible scientifically, because it confounds Differential Functioning (DF) with Impact.

Impact is the group difference on the construct being measured where the difference is relevant to the testing purpose (Clauser & Mazor, 1998; Drasgow, 1987; Millsap & Everson, 1993). DF is the currently preferred term for statistical bias, because of the popular understanding of bias as prejudicial intent against groups of people. DF occurs when test items consistently produce different results for different demographic groups which are not related to the purpose of the test.

CTT had difficulty dealing with these problems. The principal difficulty of CTT is that test characteristics and examinees characteristics are confounded. They can only be interpreted jointly: this set of examinees with this set of test questions. If either set is changed, then the interpretation changes. This makes it difficult to compare different versions of a test, and to predict how any examinee will perform on a given test. As a way out of this difficulty, Item Response Theory (IRT) became the dominant measurement theory. IRT provides the theoretical basis for item and examinee-level measurement vs. test and group-level measurement. Said differently, under the assumptions of IRT, the probability that any given examinee will answer correctly any given question can be estimated, but the assumptions of CTT allow only estimating the probability of the proportion of correct answers on a test from a given group of examinees. Thus, under IRT, item traits are independent of the test takers, and examinee scores can be described independently of the set of test items (Bock, 1997; Hambleton, Swaminathan & Rogers, 1990).

In contrast to Classical Test Theory which focuses on the test as a whole, IRT takes the individual test question, or item, as its basic unit of study. It is this focus on the item that enables practitioners to measure the difficulty, and other characteristics, of individual items. Thus, it becomes possible to describe examinees by the difficulty of the questions that they can successfully answer. Bock (1997) points out that this description is most appropriately called “proficiency” in educational settings, although the literature generally refers to it as ability.

Items, however, are generally not used in isolation. They are compiled into groups measuring constructs of interest. These groups may be divided into sub-tests and tests. There are two kinds of sub-test groupings. Testlets are groups of test items grouped contiguously and associated with a common probe, often a reading passage. Bundles are groups of items that have a common organizing theme, but do not necessarily appear together or refer to a common probe (McCarty, Oshima, & Raju, 2007). Items and DF can be studied both in isolation, and at all levels of aggregation. This study focuses on the entire test, and different methods of measuring DTF.

When looking at DF at the test level, both the distribution of DF over the items, and the summative nature of DF must be considered. The distribution of DF over the items derives from the fact that each item is measured for DIF separately. Because of this, tests may exist where all DIF favors the same group of examinees. However, it is also possible that some items may show DIF favoring one group of examinees, while other items show DIF favoring other groups of examinees. DIF that systematically favors a single group of examinees is called “unbalanced” or unidirectional DIF, whereas, DIF that does not systematically favor either group is called “balanced” or bidirectional DIF (Gierl, Gotzman, & Boughton, 2004; Oshima, Raju, & Flowers, 1997). The summative nature of DF is expressed in the concepts of amplification and

cancellation (Nandakumar, 1993). Amplification refers to the fact that multiple items each exhibiting non-significant DIF, may, when aggregated at the test level, exhibit significant DF. Cancellation is the opposite of amplification. Significant DIF may exist in different items favoring different demographic groups, but, when aggregated at the test level, DF is not detectable because the DF favoring one group offset that favoring the other group. Obviously, balanced DIF would emphasize cancellation effects, while unbalanced DIF would emphasize amplification effects.

This study looks at two methods of assessing DTF under conditions of impact, and differential functioning. Under differential functioning, it looks at cancellation and amplification effects on DTF under balanced and unbalanced DIF. The two methods of assessing DTF are Differential Functioning of Items and Tests (DFIT; Raju, van der Linden, and Fleer [1995]), and the Mantel-Haenszel/Liu-Agresti variance method as implemented in DIFAS (Penfield, 2007). The methods are explained in the literature review.

CHAPTER 2

LITERATURE REVIEW

Introduction to methods of evaluating Differential Test Functioning (DTF)

Measuring performance, both past and potential, is a fact of contemporary life. From the beginning of a child's formal schooling through the end of a person's work career, measurement is an on-going process. Modern measurement theory has developed, as a response to social concerns about the fairness of the testing instruments used, and their application (Rudner, Getson, & Knight, 1980), and to legal and ethical imperatives to avoid bias (Gierl, Bisanz, Bisanz, Boughton, & Khaliq, 2001). Unfair tests, that is, tests that inhibit a person's rights and opportunities, are, in the words of Cronbach (1988), "inherently disputable" (p. 6). They suffer not only the weakness of social disapprobation, but are also weak in scientific validity (Millsap & Everson, 1993; Rudner, et al.). The current consensus on standards for fairness is published in the *Standards for Educational and Psychological Testing* of the American Educational Research Association, American Psychological Association, and National Council on Measurement in Education (AERA, APA, & NCME, 1999). In the context of measurement, the standard of bias-free tests requires special attention.

Bias can be defined as measurement inaccuracy that occurs systematically in a test (Millsap & Everson, 1993). More precisely, bias is a different probability of attaining a correct answer by members of one group than by members of another group, where the reasons for this

difference are unexpected and unrelated to the purpose of the test (Clauser & Mazor, 1998; Raju & Ellis, 2003). Mathematically this may be expressed in the dichotomous answer model as

$$P_i(+|\theta_s, G_a) \neq P_i(+|\theta_s, G_b) \quad (1)$$

where P_i is the probability of an answer for item i , $+$ is a correct answer, θ_s is the ability level for examinee s , and G_x are the different groups (Raju & Ellis).

Groups may, however, have different probabilities of attaining a correct score for reasons that are relevant to test purposes. A lack of bias, then, does not mean that different groups will have equal scores. Group differences where bias does not exist are referred to as impact (Clauser & Mazor, 1998; Ellis & Raju, 2003; Millsap & Everson, 1993). Shealy and Stout (1993a) specify that impact occurs from different distributions of the ability trait among the different groups. Because of this contrast between impact and bias, the preferred terminology for group differences in testing is differential functioning (DF; Hambleton, et al., 1991; Holland & Thayer, 1988).

Millsap and Everson (1993) state that the presence of DF indicates an incomplete understanding of the latent construct that the test is measuring. That is, after different groups have been matched on the ability that the test purports to measure, differences between group performances depend on another, unidentified ability. DF thus indicates the presence of multidimensionality in the test instrument (Gierl, et al., 2004; Shealy & Stout, 1993b). That is, the item is measuring more than one construct or dimension. Zhou, Gierl, and Tan (2006) define dimension as any trait that can affect the probability of a correct answer for a test item.

Where the test purpose is to measure a single primary dimension, the other dimensions are nuisance traits (Clauser & Mazor, 1998; Shealy & Stout, 1993b). However, Oshima and Miller (1992) point out that by itself multidimensionality does not produce DF. Where in the

distributions of the nuisance traits are the same among the various group, DF does not occur. Also, intentional multidimensionality that is not identified before analysis can be misidentified as DF (Zhou, et al., 2006).

Where DF does exist, researchers have different approaches towards resolving the differential functioning. Some make the assumption that identifying and removing items that exhibit DF will reduce or eliminate DF at the test level (Raju & Ellis, 2003). Other researchers hold that it is incumbent upon the test developer/researcher to evaluate the studied item and determine whether the differences measure something relevant to the test purpose or irrelevant to it, because removal of items, without first doing this evaluation, does not automatically create a fair test, or ensure the test's appropriate use, and runs the risk of letting the statistics drive the analysis (Clauser & Mazor, 1998; Rudner, et al., 1980; Gierl, et al., 2001). From a similar viewpoint, still other researchers hold that removing DF items may make for a weaker test, because the variance that produces DIF is likely to come from a variety of sources, and removing DF items may make that variance stronger in relation to the variance of the studied construct, leading to an overall weaker test (Roznowski & Reith, 1999).

Differential functioning exists at two levels: the individual item level and the aggregate (test or bundle) level (Rudner, et al., 1980). Item level DF is referred to as differential item functioning (DIF), and test level DF is referred to as differential test functioning (DTF). Item and test level DF seem to be related, with DTF being the sum of DIF. Raju and colleagues (1995) conceptualize DIF as Compensatory DIF, which sums to DTF, and Non-Compensatory DIF, which does not sum. As a sum, DIF that favors different groups in different items may offset at the test level. This is called cancellation (Flora, Curran, Hussong, & Edwards, 2008; Nandakumar, 1993; Rudner, et al., 1980; Takala, & Kaftandjieva, 2000). Nandakumar points out

that, conversely, multiple items with small DIF favoring the same group can add up to significant DTF, a process called amplification. Research using confirmatory factor analysis gives inconsistent evidence of DIF at the test level. Pae and Park (2006) indicate that DIF may be carried to the test level (so if DIF exists, assume that DTF exists), because DIF does not cancel at the test level.¹ Zumbo (2003), who also used factor analysis, indicates that cancelled DIF does not appear at the test (scale) level.

Most research to date has been done on DIF (Pae & Park, 2006), and theory and empirical studies indicate that DTF is the summation of DIF in a test. Perhaps because of this, measurement of DTF is under-investigated. However, a number of reasons can be given why the study of DTF is important. From a statistical viewpoint, items, when compared to tests, are small and unreliable samples (Gierl, et al., 2001). Because DTF is an aggregate measure of DIF, DTF can present an overall view of the effect of differential functioning, even when no single item exhibits significant DF (Shealy & Stout, 1993a). Where a test is stratified using a variable (such as the total score) that is internal to the test, DIF in one item can affect the estimate of DIF in other items in the same stratification. In this case, DTF can give an index of the potential effect of the stratifying variable on DF (Penfield & Algina, 2006). From a non-statistical, yet eminently practical viewpoint, decisions about examinees (e.g., school or job promotion, professional certification) are made at the test level, not the item level (Ellis & Raju, 2003; Jones, 2000; Pae & Park, 2006; Roznowski & Reith, 1999; Russell, 2005), therefore studying DTF to determine how best to control test validity is both appropriate and worthwhile. Also, because of the expense and time required for creating test items, being able to use already created items saves time, and financial and human resources (Bergson, Gershon, & Brown, 1993; Bolt & Stout, 1996).

¹ Pae and Park (2006) bring out an important point. Balanced DIF does not “remove” DIF. It only masks it at the test level (DTF). If items exhibiting DIF are removed carelessly, then the entire test can exhibit DTF, and no longer be considered “fair” overall.

Therefore, studying DTF to determine how best to control test validity is both appropriate and worthwhile.

Related to this is the predictive value of a test. That is, differential functioning is related to the psychometric properties of a test, but using a test to select persons for a position based on the results of the test is qualitatively different in that it seeks to determine how well a person will perform in the future based on the results of the test. Whereas DF analysis may detect statistically significant differences between groups (caused, in part by the large sample sizes used in IRT analysis), the practical importance of these differences is often nil. To use DTF in the prediction process requires the use of an effect size (Stark, Chernyshenko, & Drasgow, 2004).

Measurement of differential functioning. The analysis of differential functioning is a part of the larger task of assessing the reliability and validity of test scores. The researcher must, therefore, keep in mind the purpose of the test and choose the method of analysis that is best suited to the data while accomplishing the intended purpose, keeping in mind that all analysis techniques have problems and limitations as well as strengths (Clauser & Mazor, 1998; Roznowski & Reith, 1999; Rudner, et al., 1980).

While much work currently is being done on test items that are intentionally multidimensional (Ackerman, 1996; Ackerman, Gierl, & Walker, 2003; Oshima, et al., 1997; Yao & Schwarz, 2006), traditionally tests and items are considered unidimensional. They measure only one construct. In practice, this is seldom, if ever, achieved. Differential functioning then becomes, not a quality that exists or does not exist, but a quality that exists in different degrees in different items. The amount of differential functioning detected may vary with the analysis used (Rudner, et al., 1980). An example of this would be trying to detect non-uniform DIF with the Mantel-Haenszel (MH) technique, which is not sensitive to non-uniform DIF

(Millsap & Everson, 1993; Raju & Ellis, 2003), as opposed to the two-parameter logistic regression model, which was developed to detect non-uniform DIF (Hambleton, et al., 1991).

Differential functioning detection and analysis methods work with multiple groups of examinees: a base group (in bias studies usually the group suspected of benefiting from bias, or in standardization studies, the group with more stable scores), also called the reference group; and one or more study groups, also called focal groups (Dorans & Kulick, 1986). Differential functioning is measured with two broad classes of statistics: parametric and non-parametric. Millsap & Everson (1993) refer to them respectively as unobserved conditional invariance (UCI) and observed conditional invariance (OCI) based on their use of an unobserved logical construct to segregate examinees into ability groups, or an observed construct, such as raw test score. IRT methods of DF measurement fall in the UCI group because they work by using an unobserved ability parameter derived for each of the test groups to measure DF. Some methods, such as SIBTEST, use elements from both of these two general groups: an observed Z-score like the non-parametric methods, and adjusting group means before comparing them like the parametric methods (Millsap & Everson). Wainer (1993), who uses a different classification system, points out that classification schema are somewhat arbitrary. For example, the Mantel-Haenszel non-parametric model is closely related to the Rasch (a special case of the one-parameter logistic model; Takala & Kaftandjieva, 2000; Weeks, 2010) model, and the standardization model is conceptually similar to models that compare item response functions (Shealy & Stout, 1993b).

Methods of studying DTF

One is tempted to look for the “best” method to detect DF that will function well in all situations. Such a method does not exist. Rather, each method has its own shortcomings and advantages (Anastasi & Urbina, 1997). For example, analysis using the Rasch or the Mantel-

Haenszel methods cannot detect crossing DF, but these methods can be used effectively on smaller sample sizes than are required by IRT methods. IRT methods can detect crossing DF but generally need large sample sizes to function well (Ferne & Rupp, 2007; Lai, Teresi, & Gershon, 2005).

Three principal approaches to studying DF have been developed. These are factor analysis, methods based on CTT using traditional statistical methods, and methods based on IRT (Magis & Facon, 2012). The first two approaches are generally non-parametric methods that are not based on IRT, whereas IRT methods generally require the calculation of parameters to estimate the various characteristics (such as difficulty and discrimination) of the items (Hambleton, Swaminathan, & Rogers, 1991). The Mantel-Haenszel method is an example of a traditional statistical method, and DFIT, which requires the calculation of item parameters, is an IRT method. SIBTEST (Shealy & Stout, 1993a) is a bridge between CTT and IRT in that it uses traditional non-parametric statistical mathematics with an IRT approach to analysis.

Factor analysis is generally performed using one of the standard programs (e.g., LISREL, Jöreskog & Sörbom, 1993) that analyze covariance matrices. Variance analysis has been operationalized by Penfield (2005, 2007) using an estimator based on the Mantel-Haenszel/ Liu-Agresti (MH/LA) method, and published in the DIFAS program. SIBTEST (Shealy & Stout, 1993a) is both a method and a computer program that produces a standardized measure of differential functioning. DFIT (Raju, et al., 1995) is both a method and a computer program that measures the squared difference between two Test Characteristic Curves.

Factor analysis. In a measurement context, factor analysis is performed at the test level rather than at the item level that then is summed to the test level (Pae & Park, 2006; Zumbo, 2003). Factor analysis takes the data of the groups (whether persons or tests) to be compared,

and, based on the factors used, evaluates how close the factor matrices are between the groups. As an example, one assumes that gender and ethnicity are related to test results. A confirmatory factor analysis would evaluate how closely these two variables are associated with the level of test scores. Zumbo used factor analysis in the context of comparing two different versions of the same test, an example of measurement equivalence; that is, whether a test has the same meaning for different groups. Measurement equivalence (DTF) is measured as the differences between the correlation matrices of the different groups being examined.

Factor analysis works by taking a group of observed variables (say, the items on a test) and tries to relate them to a smaller set of “unobserved” variables (latent or explanatory factors; say the gender and ethnicity of the test takers) that can explain the variations in correlations among the observed variables. There are two main types of factor analysis: Exploratory Factor Analysis (EFA), and Confirmatory Factor Analysis (CFA; Lance & Vandenberg, 2002). If the factors need to be identified, then an Exploratory Factor Analysis may be used to identify a set of factors for later confirmation (Zumbo, 2003). However, if a set of explanatory factors associated with measurement outcomes has already been identified before hand, then Confirmatory Factor Analysis is used to evaluate how closely that set of factors match outcomes (Jöreskog & Sörbom, 1993). There are two major differences between EFA and CFA that come from their different purposes. In EFA with its focus on finding relationships, correlations between all observed and latent variables are calculated, and model fit is not a big concern. In CFA, which is trying to verify propositions, only observed and latent variables that are hypothesized to have a relationship are allowed to co-vary, and model-data fit is of great importance (Lance & Vandenberg). CFA is the principal method used in DTF analysis.

The explanation that the factors give is determined by partialing out the correlations among the observed variables. Jöreskog and Sörbom (1993) describe this process using the following model.

$$x_i = \lambda_{i1}\xi_1 + \dots + \lambda_{in}\xi_n + \delta_i \quad (2)$$

where x_i is the i^{th} observed variable, ξ_n is the n^{th} factor, λ_{in} is the factor loading of observed variable i onto factor ξ_n , and δ_i represents factors unique to x_i . Two items need more explanation.

If x_i does not depend on ξ_n , then $\lambda_{in} = 0$. The unique part, δ_i , of the observed variable has two pieces: a factor specific to that variable, and error measurement, which are confounded in most designs (Jöreskog & Sörbom, 1993; Lance & Vandenberg, 2002). That is, Factor Analysis looks for invariance (also called measurement equivalence) in the factors, with the tests for this being model fit (e.g., the Root Mean Squared Error of Approximation, RMSEA) and χ^2 tests of invariance on factor loadings, and variance and covariance matrices (Pae & Park, 2006). Two major model fit indices identified by Lance and Vandenberg are the least squares difference

$$F_{LS} = tr[S - \hat{\Sigma}(\theta)^2] \quad (3)$$

where tr is the trace of a matrix, and the function minimizes the squared differences (S) between the sample data and the covariance matrices ($\hat{\Sigma}(\theta)^2$); and the maximum likelihood function

$$F_{ML} = \ln|\hat{\Sigma}(\theta)| + tr[S\hat{\Sigma}(\theta)^{-1}] - \ln|S| - p \quad (4)$$

where $\ln|A|$ is the base e log of matrix A, p is the number of observed variables, and the function minimizes differences between variances between the sample (S) and covariance ($\hat{\Sigma}(\theta)$) matrices (Lance & Vandenberg, 2002).

A concern in using factor analysis is the determination of model fit. Most goodness of fit indices have a Chi-square distribution. The common maximum likelihood fit function is

distributed as Chi-square only for large sample sizes. Because Chi-square is greatly influenced by sample size, even trivial differences are likely to appear as significant. CFA is also very dependent on model specification. The researcher must understand the theory driving the model in order to specify it well (Lance & Vandenberg, 2002).

Variance Analysis. The analysis of the variance of DIF was sparked by an observation of Rubin (1988) that, even though DTF were small, there could be wide variability among the various DIF indices for the various items. He suggested that this variability be measured. Following this, Longford, Holland, & Thayer (1993) developed an iterative non-parametric measure of DTF. Camilli and Penfield (1997), on this basis, developed a variance estimator of global unsigned DTF. They took the log odds ratio (LOR), as an index of differential functioning, and its standard error from the Mantel-Haenszel statistic (Holland & Thayer, 1988; Mantel & Haenszel, 1959) to develop a non-iterative, non-parametric measure of DTF for dichotomous items. This was generalized to the polytomous, and mixed dichotomous-polytomous cases by Penfield and Algina (2006) using the Liu-Agresti generalization of the Mantel-Haenszel statistic. Following Camilli and Penfield's (1997) format, I briefly discuss the Logistic IRT model, the Mantel-Haenszel procedure, Longford, et al.'s procedure, and then Camilli and Penfield's procedure, before I discuss Penfield and Algina's generalization.

The three parameter logistic (3PL) model for DIF expresses the probability of a correct response as

$$P_{Gi}(1|\theta) = c_i + (1 - c_i) \frac{\exp(Da_i(\theta - b_i))}{1 + \exp(Da_i(\theta - b_i))} \quad (5)$$

where the left hand side of the equation, $\equiv P_G$, is the probability of a member of group G, who has an ability level θ , providing a correct answer to item i . On the right hand side of the equation c_i is the guessing parameter for item i , a_i is the discrimination (used to separate examinees into

ability groups) parameter for item i , b_i is the difficulty (the point where half of the examinees answer the item incorrectly and half answer it correctly, i.e., the mean) parameter for item i , and D is a scaling factor, generally set to 1.7 to make the logistic Item Characteristic Curve (ICC) approximate the normal ICC (Hambleton, et al., 1991; Raju, 1988).

This formula may be expanded to show the presence of DIF as a shift in the ICC

$$P_{Gi}(1|\theta) = c_i + (1 - c_i) \frac{\exp(Da_i(\theta - b_i) + G\xi_i)}{1 + \exp(Da(\theta - b) + G\xi_i)} \quad (6)$$

In this model G is an indicator of group membership (0 for the focal group, 1 for the reference group), and ξ_i is the amount of DIF for item i . When ξ_i is positive, DIF is against the focal group; negative, against the reference group; and zero indicates no DIF (Camilli & Penfield, 1997; Penfield & Algina, 2006). The other symbols are as defined for equation (5).

$G\xi_i$ may be indexed by the log-odds ratio (LOR). Thus, for a 1PL model (where the c parameter = 0, and the a parameters (discrimination) are the same for both groups), for a given ability level

$$LOR = \lambda(\theta) = \ln \left[\frac{P_R(\theta)/Q_R(\theta)}{P_F(\theta)/Q_F(\theta)} \right] \quad (7)$$

Where θ is the ability level, P is the probability of a correct response, Q is the probability of an incorrect response, R is the reference group, and F is the focal group (Camilli & Penfield, 1997).

This leads to a digression on the Mantel-Haenszel LOR, which is the basis for Camilli and Penfield's model.

Mantel and Haenszel (1959) developed a model of relative risk given certain risk factors. Using a non-iterative 2x2 contingency table (see Table 1), they show that the expected value of any cell in the table can be tested as a χ^2 with one degree of freedom, calculated as the ratio of a squared deviation from the expected value of the cell to its variance, where the variance is defined as the marginal totals divided by the squared total times the total minus one

$$V(A_i) = \frac{N_{1i}N_{2i}M_{1i}M_{2i}}{T_i^2(T_i-1)}, \quad (8)$$

where V is the variance, A represents the studied cell of the contingency table, N represents the row totals, M represents the column totals, and T represents the over all total.

Table 1
Two by Two Contingency Table at Level J of 1 – J levels

		Score		
		Correct	Incorrect	Total
Group	Reference	A	B	n _R
	Focal	C	D	n _F
Total		m ₁	m ₀	T

The expected value of a cell is then defined as

$$E(A_i) = \frac{N_{1i}M_{1i}}{T_i}, \quad (9)$$

where the symbols are as defined for equation (8), and $(T_i - 1)$ is a population correction factor.

The overall relative risk is

$$R = \frac{\sum \frac{A_i D_i}{T_i}}{\sum \frac{B_i C_i}{T_i}}, \quad (10)$$

where A represents positive outcomes for group 1, B represents negative outcomes for group 1, C represents positive outcomes for group 2, D represents negative outcomes for group 2 (see Table 1), and the χ^2 with a single degree of freedom is

$$\frac{\left(|A_i D_i - B_i C_i| - \frac{1}{2}\right)^2}{V(A_i)} \quad (11)$$

where i is an item, A, B, C and D are as defined for equation (10), and $V(A_i)$ is as defined for equation (8).

Holland and Thayer (1988) generalize the Mantel-Haenszel χ^2 procedure to education, substituting ability level for risk factor, highlighting that the table works at k levels of the ability level, and noting that the overall significance test proposed by Mantel and Haenszel (1959) is a common odds ratio that exists on a scale of 0 to ∞ with 1 being the null hypothesis of no DIF. They describe the odds ratio as

$$\hat{\alpha}_{MH} = \frac{\sum \frac{A_j D_j}{T_j}}{\sum \frac{B_j C_j}{T_j}} \quad (12)$$

which is the same as Mantel and Haenszel's risk ratio. They define this ratio as the average odds that a member of the reference group has an equal score to a member of the focal group at the same level of ability.

Camilli and Penfield (1997) emphasize that each test item exists at $1, \dots, J$ levels of ability. They re-write the formula to explicitly indicate the levels, and clarify that cells A and B represent the probability of correct responses (P), and C and D represent the probability of incorrect responses (Q). Thus for score level j , the odds ratio is

$$a_j = \frac{P_{Rj}/Q_{Rj}}{P_{Fj}/Q_{Fj}} = \frac{P_{Rj}Q_{Fj}}{P_{Fj}Q_{Rj}} \quad (13)$$

Using the Mantel-Haenszel statistic as presented by Holland and Thayer (1988; equation [12]) the Mantel-Haenszel log odds ratio is

$$\hat{\psi} = \ln(\hat{\alpha}_{MH}) \quad (14)$$

Longford, et al. (1993) developed a measure of global DTF based on the Mantel-Haenszel log odds ratio as a random effects model:

$$-2.35\hat{\psi} = y_{ik} = \mu + \xi_i + \varepsilon_{ik} \quad (15)$$

where -2.35 is a scaling factor to make the Mantel-Haenszel statistic equal the Educational Testing Service (ETS) difficulty (delta) scale; i is item $1, \dots, I$, and k is test administration $1, \dots$

, K ; the constant μ is the unknown average DIF; the variables ξ and ε represent a DIF factor and measurement error. The variance of the measurement error is assumed to be zero over many samples. The variable ξ represents item deviation from μ and is assumed $\sim N(0, \tau^2)$. DTF is the mean square of τ^2 .

Camilli and Penfield (1997) proposed calculating τ^2 as

$$\hat{\tau}^2 = \frac{\sum_{i=1}^I (\hat{\psi}_i - \hat{\mu})^2 - \sum_{i=1}^I s_i^2}{I} \quad (16)$$

where I is the number of items in the test; $\hat{\psi}$ is the MH LOR; μ is the mean of ψ ; and s^2 is the error variance of ψ . The variance estimator of τ^2 may be found by assuming an equal (that is, pooled) error variance for all items. This allows the following transformation

$$\hat{\tau}^2 + s_p^2 = \frac{\sum_{i=1}^I (\hat{\psi}_i - \hat{\mu})^2}{I} \quad (17)$$

that produces the estimated error variance of τ^2 by taking a second derivative of the right side of the preceding equation within the maximum likelihood context:

$$\widehat{Var}(\hat{\tau}^2) = 2 \left[\frac{\sum_{i=1}^I (\hat{\psi}_i - \hat{\mu})^2}{I} \right]^2 \quad (18)$$

Weighted versions of the τ^2 estimator and the variance estimator that adjust for the equal variance assumption are

$$\hat{\tau}^2 = \frac{\sum_{i=1}^I w_i^2 (\hat{\psi}_i - \hat{\mu})^2 - \sum_{i=1}^I w_i}{\sum_{i=1}^I w_i^2} \quad (19)$$

$$\widehat{Var}(\hat{\tau}^2) = 2 [\sum_{i=1}^I v_i^{2v}]^{-1} \quad (20)$$

where $w_i = s_i^{-2}$ and $v_i = (s_i^2 + \hat{\tau}^2)^{-1}$.

Penfield and Algina (2006) extended this model to polytomous data. Where dichotomous data have two response options (0, 1), polytomous data have multiple response options for each item: $j = 0, \dots, J$. The probability of selecting any given option is found using a series of

dichotomizations of the differences between two contiguous options. A dichotomization is a set of responses such that each response is specified $j \leq \text{integer}$, $j > \text{integer}$, and the set contains $n-1$ responses. Using Samejima's (1999; also see Ostini & Nering, 2006) Graded Response Model, Penfield and Algina give the following probability function.

$$P_{ij}(\theta, G) = \gamma_{ij-1}(\theta, G) - \gamma_{ij}(\theta, G) \quad (21)$$

where P_{ij} is the probability of selecting response j for item i given ability level θ and membership in group G ; γ_{ij} is the cumulative probability of selecting a response greater than j for item i given ability level θ and membership in group G . The cumulative probability is set to 1 where $j < 0$, and is set to 0 for $j = J$.

In the 1PL model, the probabilities are expressed as

$$\gamma_{ij}(\theta, G) = \frac{\exp(Da_i(\theta - b_i) + G\omega_{ij})}{1 + \exp(Da_i(\theta - b_i) + G\omega_{ij})} \quad (22)$$

where ω_{ij} is DIF for item i and response j . Everything else is as previously defined.

Calculating the variance for this expanded model requires use of the Liu-Agresti cumulative common odds ratio, expressed by

$$\hat{a}_{LAI} = \frac{\sum_{k=1}^K \sum_{j=0}^J A_{jk} D_{jk} / N_k}{\sum_{k=1}^K \sum_{j=0}^J B_{jk} C_{jk} / N_k} \quad (23)$$

Where K is an ability group, J is the number of dichotomizations of each level of J . The Mantel-Haenszel statistic (\hat{a}_{MH}) is, thus, a special case of the Liu-Agresti cumulative common odds ratio. Camilli and Penfield's (1997) variance estimators ($\hat{\tau}^2$) become

$$\hat{v}^2 = \frac{\sum_{i=1}^n [\log(\hat{a}_{LAI}) - \hat{\mu}]^2 - \sum_{i=1}^n s_i^2}{n} \quad (24)$$

unweighted, and

$$\hat{v}^2 = \frac{\sum_{i=1}^n [\log(\hat{a}_{LAI}) - \hat{\mu}]^2 - \sum_{i=1}^n w_i}{\sum_{i=1}^n w_i^2} \quad (25)$$

weighted, where s_i^2 is the estimated variance and $\hat{\mu}$ is the mean of the $\log(\hat{a}_{LAI})$, n is the number of items in the test, and w_i is s_i^{-2} .

Proposed effect sizes for the dichotomous case ($\ln(\hat{a}_{MH})$) are modeled after the ETS guidelines for small, medium and large DIF (which is the use of the -2.35 scaling factor introduced by Longford, et al., 1993; see also, Camilli & Penfield, 1997). Taking into account that when 25% or more of test items exhibit DF, then the effect size is large, a small effect size would be $\hat{\tau}^2 < .07$; a medium effect size $.07 \leq \hat{\tau}^2 \leq .14$; and a large effect size $> .14$. Because the $\ln(\hat{a}_{LAI})$ is the general case of the $\ln(\hat{a}_{MH})$, it also appears appropriate to use the same effect sizes for the polytomous case (Penfield & Algina, 2006).

Variance tests are most appropriate for use as an index of DF across a test. Because variance tests are developed from the Mantel-Haenszel/Liu-Agresti statistic, which has low power for detecting non-uniform DIF (Millsap & Everson, 1993), they may not be appropriate for data that match the two- or three-parameter logistic model (Camilli & Penfield, 1997). Neither are they appropriate for hypothesis testing, because the distribution of the estimators is not known (Penfield & Algina, 2006).

SIBTEST. SIBTEST (Simultaneous Item Bias Test) was developed by Shealy & Stout (1993a, 1993b) using a standardization model. As such it has much in common with the standardization models developed by Dorans and Kulick (1986) and by Wainer (1993), but was developed independently of them. I briefly review the Dorans and Kulick model before discussing SIBTEST.

Standardization, as a methodology, attempts to control for variable differences before making group comparisons. Group comparisons are made on examinees that have equal observed scores on the measurement instrument, the observed score being a surrogate for ability.

That is, given two related variables, group differences on one variable are controlled for before comparing the groups on the second variable. This is done for both items and group abilities (Dorans & Kulick, 1986; Zhou, et al., 2006).

The standardization model has two weighted indices of DF (Dorans & Kulick, 1986). Score weights for calculating the indices are calculated for each score group based on the reference group. These weights are first applied to individual difference scores, and then summed across score levels to calculate the indices. The first index is a standardized p-difference (D_{STD}). It permits DIF cancellation; that is, DIF of opposite signs may cancel out at the test level. D_{STD} is calculated as

$$D_{STD} = \frac{\sum_{s=1}^S K_s [P_{fs} - P_{rs}]}{\sum_{s=1}^S K_s} \quad (26)$$

where s is the ability level, P_{fs} is the probability of a correct answer by the focal group at the ability level, P_{rs} is the probability of a correct answer by the reference group at the ability level, and K_s is the number of examinees in the focal group at the ability level.

The second index is a root mean weighted squared difference (RMWSD). It does not allow for DIF cancellation, and requires relatively large sample sizes to avoid bias from sampling error. The RMWSD is calculated as the square root of the squared, standardized p-difference plus the variance of the difference between the probabilities of the studied groups.

$$RMWSD = [D_{STD}^2 + \text{VAR}(P_{fs} - P_{rs})]^{.5} \quad (27)$$

SIBTEST is an implementation of the standardization approach that is built on Shealy and Stout's (1993a) multidimensional IRT model of DF. The model holds that there are two classes of abilities that affect scores: target ability, which is intentionally measured, and nuisance determinants, which are inadvertently measured. DF comes from nuisance determinants having different levels of prevalence in different examinee groups.

SIBTEST uses an internal set of test items as the matching criterion. This requires a valid sub-test score

$$X = \sum_{i=1}^n U_i \quad (28)$$

and a studied sub-test score

$$Y = \sum_{i=n+1}^N U_i \quad (29)$$

where U_i is the set of answers. The difference scores are calculated as

$$\bar{Y}_{Rk} - \bar{Y}_{Fk}$$

where k is the test score level on the valid sub-test, R and F are the reference and focal groups respectively, and \bar{Y}_{Gk} is the average Y -score for all examinees in group G at score level k . If DF is not present, then $\bar{Y}_{Rk} - \bar{Y}_{Fk} \cong 0$ for all levels of k , and if DF is present then $\bar{Y}_{Rk} - \bar{Y}_{Fk} <> 0$, where the inequality indicates which group is favored by the DF ($>$ for Reference, and $<$ for Focal). Thus, defining the difference function as

$$B(\theta_k) \equiv \bar{Y}_{Rk} - \bar{Y}_{Fk} \quad (30)$$

Then the estimated test index of unidirectional DF is

$$\hat{\beta}_U = \sum_0^n \hat{p}_k (\bar{Y}_{Rk} - \bar{Y}_{Fk}) \quad (31)$$

where \hat{p}_k is the proportion of all examinees at valid sub-test score level $X = k$. Nandakumar (1993) proposes guidelines for estimating the effect size of the DF index ($\hat{\beta}_U$). A small effect size is $\hat{\beta}_U < .05$. A medium effect size is $.05 < \hat{\beta}_U < .1$, and a large effect size is $.1 < \hat{\beta}_U$.

The continuously weighted index of unidirectional DF (precisely, marginal item response functions, IRFs) is

$$\beta_U = \int_{\theta} B_{\theta} f_F(\theta) d\theta \quad (32)$$

where $f_F(\theta)$ is the probability density function of θ . The estimated test statistic ($\hat{\beta}_U$) is Dorans and Kulick's (1986) p -difference, D_{STD} , index. The β_U index can also be weighted by the

reference- or focal-groups, but the combined focal-reference group adheres more closely to nominal significance than using either group examinee population alone (Shealy & Stout, 1993b).

The statistic for testing $H_0: \bar{Y}_{Rk} - \bar{Y}_{Fk} = 0$ can be shown to be the ratio of the test index to the standard error.

$$B = \frac{\hat{\beta}_U}{\hat{\sigma}(\hat{\beta}_U)} \quad (33)$$

where $\hat{\sigma}(\hat{\beta}_U)$ is the standard error of the studied scores from group G at score level $X = k$.

SIBTEST is a non-parametric, multidimensional IRT model based, computationally non-intensive program. It was designed to detect DF at both the item and test level (Shealy & Stout, 1993a). It also performs DBF testing (Russell, 2005; Wipasillapa, n.d.). SIBTEST allows for DIF amplification and cancellation at the test level, and has a significance test. SIBTEST was originally designed to detect unidirectional, uniform differential functioning only (Shealy & Stout, 1993b). Li and Stout (1996) modified SIBTEST to handle non-uniform DF. Simulation studies (Shealy & Stout; Zhou, et al., 2006) have shown that SIBTEST adheres well to nominal significance levels with acceptable levels of Type 1 errors, and has good power (greater than 80%) comparable to the Mantel-Haenszel test. Moderate contamination with DF of the validation test reduces power, but not enough so that performance is seriously affected. When the proportion of DIF items is large (between 40 to 60%), SIBTEST performs adequately under balanced (DF items do not systematically favor either test group) conditions, but had inadequate performance under unbalanced (DF items systematically favor either the focal or reference group) conditions (Gierl, et al., 2004).

DFIT. IRT methods approach the problem of DF detection in two different ways: parameter comparison, and evaluation of the area between different IRFs (Kim, Cohen, & Park,

1995). Item parameters being a summary of the Item Response Function determine the shape of the Item Characteristic Curve (ICC). Comparing either the parameters or the ICCs are thus equivalent methods (Thissen, Steinberg, & Wainer, 1988). DF estimation by measuring the area between ICCs was in use in the late 1970s (Rudner, et al., 1980). Raju (1988) expanded on this early work by developing formulas for calculating the exact areas between ICCs for the one-, two- and three- parameter logistic models, and in the process demonstrating a major limitation of this method: when the guessing (“c”) parameters are not equal, the area between the ICCs is infinite. Another limitation of this method is the lack of weighting by examinee density: that is, sparsely populated areas between the ICCs contribute equally with heavily populated areas to setting the DF index (Oshima & Morris, 2008.)

Raju (1990) expanded on his earlier work by developing formulas for the signed area

$$SA_{kl} = \int_{-\infty}^{\infty} (\hat{F}_1 - \hat{F}_2) d\theta \quad (34)$$

and the unsigned area

$$SA_{kl} = \int_{-\infty}^{\infty} |\hat{F}_1 - \hat{F}_2| d\theta \quad (35)$$

between two ICCs: where k is the one-, two- or three-parameter (where the c -parameter is equal across groups) IRT models, l is an indicator for the signed or unsigned model, and F_1 is the IRF for group 1, and F_2 is the IRF for group 2. Raju also proposed significance tests for both the signed and unsigned areas to distinguish true difference from sampling error. The proposed significance test for the signed area is the normal Z -score:

$$Z = \frac{SA-0}{\sigma(SA)}$$

which is compared to the appropriate Z_{crit} score to determine significance at the selected α . As a cautionary note, Raju suggested that the Z_{crit} comparison be performed at the third standard

deviation (i.e., $3 * Z_{crit}$) because the standard deviation of the signed area depends on sample size, and IRT analysis requires large sample sizes.

The unsigned area cannot be measured with the standard Z-score, because the assumption of a normal distribution does not hold. However, an alternative exists in the formula developed to calculate the unsigned area.

$$H = \frac{2(\hat{a}_2 - \hat{a}_1)}{D\hat{a}_2\hat{a}_1} \ln \left\{ 1 + \exp \left[\frac{D\hat{a}_2\hat{a}_1(\hat{b}_2 - \hat{b}_1)}{\hat{a}_2 - \hat{a}_1} \right] \right\} - (\hat{b}_2 - \hat{b}_1) \quad (36)$$

where a_x and b_x are the discrimination and difficulty parameters and D is the adjustment factor of the 2PL model. Based on the fact that $|H|$ has a half-normal distribution, it seems appropriate to use H to calculate a Z-score. Thus,

$$Z = \frac{H-0}{\sigma(H)} \quad (37)$$

In this case also, Raju (1990) suggested that the Z_{crit} comparison for these scores be performed at the third standard deviation (i.e., $3 * Z_{crit}$) because the standard deviation of the signed area depends on sample size, and IRT analysis requires large sample sizes.

Raju and colleagues (1995) proposed a method within the IRT frame of DF measurement that would have an additive measure of item DF. This would allow the identification of the effect of individual items (added to or removed from a test) on differential functioning at the test level. They describe the model beginning with DTF. Assuming that each examinee has a true score (T_s , or expected proportion correct), for both the Focal and Reference groups, then

$$T_s = \sum_{i=1}^n P_i(\theta_s) \quad (38)$$

where P_i is the probability of a correct answer for item i , and θ_s is the ability level for examinee s .

DTF exists where the examinees' scores in each group are not equal. Thus, DTF is the squared difference of the expected value of each true score:

$$DTF = E(T_{sF} - T_{sR})^2 = E(D_s^2) \quad (39)$$

or

$$DTF = \int_{\theta} D_s^2 f_F(\theta) d\theta = \sigma_D^2 + (\mu_{TF} - \mu_{TR})^2 = \sigma_D^2 + \mu_D^2 \quad (40)$$

where $f_F(\theta)$ is the density function of θ in the focal group and μ_{Tx} is the mean true score in each group. Because DTF is also the expected squared sum of the difference scores, it may also be written as

$$DTF = E[(\sum_{i=1}^n d_{is})^2] \quad (41)$$

where $d_{is} = (P_{iF}(\theta) - P_{iR}(\theta))$ is as explained above. By taking into account the covariance between the difference in probabilities for each item (d_i) and for the test (D), DIF can be defined as the expected value of the covariance of the item and test differences plus the mean item differences times the mean test difference.

$$DIF_i = E(d_i D) = Cov(d_i D) + \mu_{d_i} \mu_D \quad (42)$$

This is called Compensatory DIF (CDIF), because offsetting positive and negative values cancel out at the test level. CDIF shows the amount that each item contributes to DTF (Oshima, Raju, & Flowers, 1993). Non-Compensatory DIF (NCDIF) is a special case of CDIF that has the assumption that no item except for the studied item exhibits DIF. Because it is a squared value, NCDIF cannot offset DIF among items.

Faced with the sensitivity of χ^2 to sample size, and the high false positive rate of Fleer's .006 Monte Carlo estimated cutoff score, Oshima, Raju and Nanda (2006) developed a method of calculating individualized cutoff scores for each item to assess the significance of DF for individual items. This consists of four steps: determining the item parameters for each item; replicating these parameters as many times as needed; determining the NCDIF distribution indices for each item; calculating a cutoff value for each item at the desired significance level. This method of significance testing was implemented for dichotomous items in the DIFCUT

procedure (Nanda, Oshima, & Gagné, 2006), and was adapted to use with polytomous items by Raju et al. (2009).

DFIT works effectively with polytomous and dichotomous data, with either a unidimensional or multidimensional model (Flowers, Oshima, Raju, 1999). It performs tests of DIF, Differential Bundle Functioning (DBF) and DTF (Oshima, Raju, & Domaleski, 2006). DFIT can analyze DF at adjacent and non-adjacent ability levels, and can bundle examinees as well as test items (Oshima, et al.). Oshima, Raju, and Nanda (2006) developed an effective test of significance for DFIT for dichotomous data, which has been extended to polytomous data (Raju, et al., 2009).

DFIT is calculation intensive. It works well when parameter calibration and linking are accurate, and when the data fit the IRT model. DFIT requires a large sample for accurate parameter estimation. Linking requires a DF free anchor set of test items, so iterative linking is necessary. The first linking uses all test items as the anchor set. Items with large DF are removed, and a second linking is performed with the remaining, non-DIF items as the anchor set (Oshima & Morris, 2008; Oshima, et al., 1997). DTF, in particular, cannot be interpreted before linking (Oshima, et al.).

Comparative Studies Among the Methods

While there are many studies comparing the different methods of evaluating DF, the great majority of them compare the methods at the item level (Finch, 2005; Navas-Ara, & Gómez-Benito, 2002; Roussos & Stout, 1996; Wanichthanom, 2001; Zwick, 1990). There are few comparative studies that look at DTF across methods (Wipasillapa, n.d.). Some studies look at DTF in one method but look at DIF or DBF for another (Russell, 2005).

In the context of evaluating a translated test, Zumbo (2003) compared both Exploratory Factor Analysis and Confirmatory Factor Analysis at the test level to both the Mantel-Haenszel procedure and to logistic regression at the item level. He simulated 5000 responses using a 3PL model with only uniform DIF, and using a 2 x 4 design and one test per design condition. DIF ranged from about 3% to 42%. The Exploratory Factor Analysis produced a single-factor model that matched the original model under all conditions of DIF, and the Confirmatory Factor Analysis indicated statistical equivalence of the original and the translated models at all DIF levels.

Pae and Park (2006) tested performance on a 33 item dichotomous subset of a standardized test of English using 15,000 Korean college students equally divided among males and females, and among humanities and sciences students. Logistic Regression was used to evaluate DIF. Logistic Regression indicated the presence of DIF in 22 of the items. Following this, separate Confirmatory Factor Analyses were run on a set of four nested data models comparing males to females. Each model was based on a single latent factor and differed in the variance constraints imposed on the data. In all cases model fit indices were acceptable, however, Chi-square difference statistics between each model and the unconstrained model indicated in all cases that the factor matrices were different for men and women. Looking specifically for a DIF cancellation effect, Pae and Park took 5 items favoring men and 5 items favoring women, each set having the same cumulative amount of DIF, and checked them with CFA. Chi-square tests of the differences in factor loading, error variances and factor variances indicated that there was no DIF cancellation.

Four dissertations have compared DTF using different methods. Fesq (1995) compared Marginal Maximum Likelihood (MML) with the Random Effects Model (REM; Longford, et al.,

1993) and with the Summed Chi-square Statistic (SCS; Camilli & Penfield, 1997). She found the MML and REM methods to be comparable. All methods controlled Type I error well for the 1PL model but not for the 3PL model. DTF estimates were poor in the presence of embedded DIF. Petroski (2005) compared DTF for DFIT with several bootstrap methods. He found very high Type I error rates for DFIT, but acceptable Type I error rates and power for the bootstrap methods. Russell (2005) compared SIBTEST for DBF (differential bundle functioning) with DFIT for DTF, noting that the two methods “are used for different practical applications and at different levels of analysis” (p. 3). He found that both methods had inflated Type I error, and that DFIT had low power while SIBTEST had adequate power. Russell’s choice of test levels to compare is confusing because SIBTEST also calculates DTF (Shealy & Stout, 1993a), and DFIT also calculates DBF (Oshima, Raju, Flowers, & Slinde, 1998). Wipasillapa (n.d.) compared SIBTEST and DFIT for DIF, DBF, and DTF. For DTF, she found that for 50 item tests with 1000 or fewer students SIBTEST had greater power than DFIT.

Theoretical comparisons include Takane and de Leeuw (1987) who presented a formal proof of the equivalence of Factor Analysis and IRT, and Raju, Lafitte, and Byrne (2002), who presented a theoretical comparison of CFA and IRT approaches. Raju, et al. noted five similarities and six differences and called for further research into the relative advantages of each approach and into their comparability. The similarities are 1) both look at how a theoretical construct is related to a set of measurements; 2) both look at true score similarities between persons from different groups who have been matched on the construct; 3) neither implies equal population distributions of scores on the construct; 4) both can identify the source of measurement nonequivalence; and 5) both can make use of graphs of the item response functions. The differences between the approaches are 1) CFA assumes a linear relationship

between the construct and the true score, while IRT assumes a non-linear relationship; 2) logistic regression is better for expressing relationships between continuous constructs and dichotomous scores, so IRT is better in this case, but the advantage decreases as the number of score points increase; 3) CFA can easily handle several latent constructs, while IRT has only begun to look at multidimensionality; 4) a strict form of CFA requires equivalent item error variance, while IRT does not consider it because it can vary as a function of the construct; 5) IRT is explicit about the probabilities of answer selection at a given value of the construct, which is difficult to obtain in CFA; and 6) DIF amplification/cancellation is explicit in IRT, but mostly ignored in CFA.

Discussion

Each of the methods discussed have different strengths and weaknesses. The computational simplicity and the ability to use relatively small sample sizes are shared by CFA, Variance Analysis and SIBTEST. DFIT on the other hand is computationally intensive and requires (in most cases) sample sizes greater than 500 (Oshima & Morris, 2008; Rudner, et al., 1980). Yet the extra effort required to compute the item parameters enables DFIT to keep item difficulty and discrimination from being confounded with group differences (Raju & Ellis, 2003). CFA, DFIT and SIBTEST have tests of significance, where Variance Analysis depends on the experience of the researcher to decide the importance of the computed DTF (Camilli & Penfield, 1997). CFA significance tests are many and the subject of much controversy, primarily because of the influence that sample size has on Chi-square tests (Lance & Vandenberg, 2002). Because of its base in Mantel-Haenszel DIF analysis, Variance Analysis is mostly restricted to analysis of tests with uniform DF. CFA, DFIT and SIBTEST have no restriction on the kinds of DF that can be effectively analyzed.

The importance of a DF measure depends on its intended use. Being able to select the most appropriate measure is an advantage of having all four measures of differential functioning available (Raju, et al., 1995). The importance of a method of analyzing DF, as seen in the above synopsis, similarly depends on the circumstances of its application. Having a precise knowledge of the capabilities of each of these methods will enable the researcher and practitioner to select the best method for the task at hand.

CHAPTER 3

METHOD

Study Design

This study is a simulation to evaluate and compare two DTF detection methods: Variance (using DIFAS 4.0; Penfield, 2007), and DFIT (using DIFCUT; Nanda, Oshima, & Gagne, 2006). Simulation studies are common in quantitative literature, especially in situations where statistical theory is weak or non-existent or where the assumptions for theory have been violated (Fan, Felsölvályi, Sivo, & Keenan, 2001). Such situations commonly occur both for studying different conditions within a method and for comparing different methods (Fidalgo, Hashimoto, Bartram & Muñoz, 2007; Meade, Lautenschlager, and Johnson, 2007; Nandakumar, 1993; Raju, et al., 1995).

Conditions of Study

Fixed Factors

Data Generation. Simulated data were generated using the three parameter logistic (3PL) model with a fixed guessing parameter for dichotomous items using Lord's (1968) item parameters² from the SAT-Verbal test (Table 2.) Parameters for 80 items are available. The guessing parameter for the DF items was set to .20, which is commonly used in the literature (Güler & Penfield, 2009; Monahan & Ankenmann, 2005) and represents the expected random guessing parameter for an item with five answer options.

² Parameters were provided by Ratna Nandakumar, personal communication, April 4, 2009.

Table 2
Lord's (1968) item parameters from the SAT-Verbal test

Item Number ^a	Items 1 to 20			Item Number ^a	Items 21 to 40		
	a	b	c ^b		a	b	c ^a
1	1.1	0.7	0.20	21	0.7	0.5	0.20
2	0.7	0.6	0.20	22	1.2	0.3	0.20
3	0.9	0.4	0.20	23	0.9	0.2	0.20
4	1.4	0.1	0.20	24	0.7	0.4	0.20
5	0.9	0.9	0.16	25	0.6	0.2	0.20
6	1.2	0.7	0.12	26	1.0	0.7	0.15
7	0.9	0.3	0.20	27	0.6	1.2	0.12
8	0.4	0.8	0.20	28	1.6	1.1	0.12
9	1.6	1.1	0.06	29	1.1	2.0	0.16
10	2.0	1.1	0.05	30	1.1	2.4	0.09
11	0.9	1.5	0.20	31	2.0	1.4	0.11
12	1.4	0.4	0.20	32	1.7	1.3	0.17
13	1.6	0.1	0.16	33	0.9	1.0	0.15
14	1.2	0.5	0.20	34	0.5	0.4	0.20
15	1.2	1.4	0.11	35	0.5	0.6	0.20
16	1.8	1.4	0.12	36	0.9	1.6	0.11
17	2.0	1.6	0.16	37	1.3	0.4	0.18
18	1.0	1.6	0.13	38	1.3	1.4	0.06
19	1.5	1.7	0.09	39	1.1	1.2	0.05
20	1.2	1.6	0.09	40	1.2	1.1	0.05

^a DIF was embedded in the last n items of each test.

^b The *c*-parameter are fixed at .20 for all simulations

Replications. All simulation conditions were replicated 100 times to increase the likelihood of stable results (Nandakumar, 1993). This number of replications is the predominant number used in all simulation studies published in the *Journal of Educational Measurement* between 1998 and 2008 (Hunter & Oshima, 2010).

Test Length. Test lengths were set at 20, 30 and 40 items as being representative of realistically sized tests. Monahan and Ankenmann (2005) noted seven studies published between 1993 and 1996 that used test lengths of 10 to 50 items. A review of articles published in the

Journal of Educational Measurement between 1998 and 2008 (Hunter & Oshima, 2010) found that simulation studies dealing with DIF used 20 to 360 items. Güler & Penfield (2009) chose 60 items as being realistic in respect to the number of items on standardized, multiple-choice tests. Swaminathan and Rogers (1990) used test sizes of 40, 60 and 80 items.

Type of DIF. Embedded DIF was uniform, because contingency table methods (e.g., Mantel-Haenszel, on which DIFAS is built) are known for being insensitive to non-uniform DIF (Millsap & Everson, 1993).

Varied Factors

Ability. The ability (θ) parameters were randomly assigned, $\sim N(0,1)$, for each simulee for assessing the effect of the different levels of DIF. The sensitivity of the methods to different levels of ability between the groups (i.e., impact) was assessed using both mean differences, and differences in the standard deviations. Holding the standard deviation at 1, group mean differences were varied by 0, .5, and 1.0 to simulate no, moderate and large differences respectively (Russell, 2005).

While the manipulation of mean ability is common in simulation studies of DF (Garrett, 2009; Oshima, et al., 1997; Penfield & Algina, 2006; Russell, 2005), standard deviation has rarely been considered outside of studies investigating the ability distribution (Monahan & Ankenmann, 2005; Woods, 2007, 2008). Woods suggests looking at different ability distributions from the ones she studied to see whether DF conditions would be consistent with previous research. Monahan and Ankenmann found that having both unequal means and variance ratios increased the Type I error rate. Holding the mean differences at 0, and the standard deviation in the Reference group at 1, standard deviation in the focal group is set to 1.0, 1.5, and 2.0, to investigate the effect of the variation of sample distribution.

Percent DIF. Four DIF conditions were created with DIF embedded in 0, 5, 10 and 20 percent of the items. The last items of each test were used as the DIF items. Using the first or the last test items to embed DIF, instead of scattering DIF, throughout the test is relatively common (Oshima, et al., 1997; Snow & Oshima, 2009; Güler & Penfield, 2009; Penfield, 2007). This procedure has the advantage of making the DIF items easy to locate during analysis.

Magnitude of DIF. Recent studies have used DIF magnitude ranging from small to large (.10, .25, .40, .50, .75, and 1.0). Those authors chose these values both to be able to compare their findings with other studies and to investigate the effect of values of magnitude outside of the range previously studied (Garrett, 2009; Raiford-Ross, 2008; Stephens-Bonty, 2008; Thurman, 2009). Because I was looking at the effects of DIF amplification to produce significant DTF, and at the effects of DIF cancellation to eliminate DTF, I used very small to medium magnitudes of DIF: specifically .02, .20 and .40. I simulated uniform DF by adding the magnitude constant to the b -parameter (Raju, Fortmann-Johnson, et al., 2009). I did not simulate non-uniform DF because of the insensitivity of contingency table methods to non-uniform DF (Millsap & Everson, 1993).

Balanced and Unbalanced DIF. Balanced DIF was assigned equally favoring the reference and focal groups for each of the three DIF percentage conditions (5, 10, and 20) defined above. Different sets of items were used for the reference and focal groups (Nandakumar, 1993.) For example, a 20-item test with four DF items, would have items 17 and 18 with DF in the reference group, and items 19 and 20 with DF in the focal group (see Nandakumar, 1993). Unbalanced DIF was assigned favoring the Reference group for each of the three DIF percentage conditions (5, 10, and 20) defined above using the last items of each test.

Sample Size. Studies in the JEM between 1998 and 2008 set the number of simulated participants ranged from 1 to 200,000. The four most common numbers were 500 (33 studies), 1000 (25 studies), 2000 (12 studies), and 3000 (13 studies). Over half of the studies used a single group size (Hunter & Oshima, 2010). In this study, I used two different sample sizes (1000 and 3000) with each program to compare DTF statistics, and power variations. Larger sample sizes are chosen because DFIT requires sample sizes greater than 500 (Oshima & Morris, 2008; Rudner, et al., 1980), and because small sample sizes used with Mantel-Haenszel methods yield unstable results (Zwick, Ye, & Isham, 2012).

Sample sizes for the reference and focal group sample sizes were set to 1000:1000 and to 3000:3000 for assessing the effect of sample size on impact and balanced DF. Blitz and Morris (2011) found that the IPR method of DIF detection used in DFIT is sensitive to unequal sample sizes, producing error rates much lower than the nominal rate when the reference group is larger than the focal group, and much higher than the nominal rates when the focal group is larger. To assess whether this effect carries over into DTF, unequal sample sizes were set to 3000:1000 (reference:focal) to assess the effect of large differences in sample sizes.

For the impact condition this resulted in a 1 X 3 X 3 (mean x standard deviation x sample size; varying the standard deviation for a constant mean) plus a 3 X 1 X 3 (mean x standard deviation x sample size, varying the mean for a constant standard deviation) design for 15 conditions. The DF conditions had a 3 X 3 X 2 X 3 (% DF x DF magnitude x direction x sample size) design. Table 3 lists the fixed factors examined in the study. Varied factors appear in Table 4.

Table 3
Fixed Factors in the Study Design

<i>Factor Category</i>	<i>Factor</i>
Model	Three PL with Fixed c-parameter
Number of Replications	100
Type of DIF	Uniform

Table 4
Varied Factors in the Study Design

<i>Factor Category</i>	<i>Factor</i>
Test Length	20
	30
	40
% Items with DIF	0
	5
	10
	20
Magnitude of DIF	.02
	.20
	.40
Direction of DIF	Balanced
	Unbalanced
Ability Distribution - Mean	0.0
	.5
	1.0
Ability Distribution – Standard Deviation	1.0
	1.5
	2.0
Sample Size	1000 : 1000
	3000 : 3000
	3000 : 1000

Programming Structure

Three SAS shell programs ran a series of SAS macro programs. The first shell program imported the set of item parameters from an EXCEL[®] spreadsheet, and invoked four macros: (1)

to manipulate the parameters to create three sets of parameters with 20, 30 and 40 parameters each, (2) to add the appropriate amount of DF to the appropriate parameters for each input file, (3) to create files with balanced DF, and (4) to create files with unbalanced DF.

The second shell program generated 100 files of simulated data for each set of study conditions. It ran two macros. The first macro invoked program IRTGEN (Whittaker, Fitzpatrick, Williams, & Dodd, 2003), which had been modified to run as a SAS macro, to generate data for the balanced and unbalanced conditions. The second macro invoked program IRTGEN, which had been modified to run as a SAS macro and to apply differences to the mean and sample deviation of selected files to simulate differences in sample distribution, for impact conditions.

The third shell program invoked five SAS macros that performed item calibration, linking and analysis. The first macro formatted the SAS data files as text files with a “.dat” extension, which is the format required by BILOG-MG3 (Zimowski, Muraki, Mislevy, & Bock; 2003). The second macro created a BILOG command file to process each data file, and invoked BILOG-MG3³ to perform item calibration. The third macro created an R (R Core Team; 2014) command file for plink (Weeks, 2010) to process each data file, and invoked plink to perform file linking. The fourth macro invoked DIFCUT (Nanda, Oshima, & Gagne, 2006), which is a SAS implementation of the DFIT method. DIFCUT had been modified to (1) run as a SAS macro program, and (2) produce a file listing linking items identified as free of DIF for use in second stage linking, and a file containing the DTF value and significance level for each file. The fifth macro read the linking items file created a new command file to process each data file,

³ In order to run BILOG from the command line using SAS, BILOG must be installed using a perpetual license. It will not run using a temporary license. (L. Stam, personal communication, August 4, 2011).

and invoked `plink` to perform second stage file linking. The macro to invoke `DIFCUT` was then invoked a second time to complete the analysis.

Data Generation

Data for both analysis streams (`DFIT` and `MH/LA`) was generated using a SAS 9.2 macro program to invoke the SAS macro program `IRTGEN` (Whittaker, Fitzpatrick, Williams, & Dodd, 2003) to generate data for all of the different combinations of test conditions. Seed values for each condition entered to `IRTGEN` were selected using the `randbetween` function of Microsoft Excel 2007® with arbitrarily selected limit values. All seed numbers generated were unique.

Where ability (θ) parameters with other than a normal distribution are needed, Whittaker and colleagues (2003) suggest commenting out the ability parameter generator within `IRTGEN`, and entering the parameters on a separate file. As an alternative to this, I modified `IRTGEN` using the formulas in Fan et al. (2001) to adjust the parameters created by `IRTGEN` before the item responses were generated. Additionally, to meet a requirement of `BILOG-MG3` (Zimowski et al., 2003), I added code to create an examinee (case) number for each record.

DFIT Analysis

Calibration, Equating and Linking

Another SAS macro program automated the processing of the IRT parameter estimation (Gagné, Furlow, & Ross, 2009) by running `BILOG-MG 3.0` (Zimowski et al., 2003) for each reference and focal simulated dataset. Separate calibration estimation was done for each set of conditions and replications. While there is some evidence that concurrent estimation performs better than separate estimation, it is not strong enough to warrant always using it (Hanson & Béguin, 2002). On the contrary, other studies have found that separate estimation generally gives

better results than concurrent estimation, except where all examinee groups are from the same population (Lee & Ban, 2010).

Item parameters to enter to DIFCUT (Nanda, Oshima, & Gagne, 2006) were generated from each file using linking between pairs of reference and focal files. Linking was done with plink (Weeks, 2010), an R (R Core Team, 2014) package using the Stocking and Lord Test Characteristic Curve method, which links files based on the sum of squared differences between the TCCs at a given θ . The Stocking and Lord method is similar to the Haebara method, a characteristic curve method that for each item sums the squared differences between the ICCs over the examinees at a given θ (Kolen & Brennan, 2004). Studies indicate that both methods give similar results (Hanson & Béguin, 2002; Lee & Ban, 2010; Oshima, Davey, & Lee, 2000). Kolen and Brennan reviewed studies that compared different kinds of equating, and concluded that “separate estimation using the test characteristic curve methods seems to be safest” (p. 174). Also, Oshima, Davey, and Lee suggest that the purpose of the linking determine the method used. Because this study looks at DTF, the Stocking and Lord Method, which looks at TCCs, seems preferable theoretically to the Haebara method. Linking was performed iteratively, as suggested by Kim and Cohen (1992) and Stark, Chernyshenko and Drasgow (2004). Only two iterations were performed, as indicated in Oshima and Morris (2008). All items were used to calculate the linking constant in the first DIFCUT run. Items identified as DIF-free were then entered to a second run of plink to generate a second linking constant for each replication and set of conditions.

Variance Analysis

Variance analysis was performed using DIFAS 4.0 (Penfield, 2007). The matching variable to estimate the ability levels was the equal interval method. This reduces the Type I

error rate of finding DIF (Donoghue & Allen, 1993). Only a single estimation was used for this analysis because DIFAS uses a Mantel-Haenszel estimation method. Research indicates that more accurate results are obtained by not removing items identified as having DF, and reanalyzing the data without them (Zwick, Ye, & Isham, 2012).

Evaluation of Results

Type I errors are the misidentification of non-DF items as containing DF. Power analysis is the ability of a method to adequately detect the existence of DF where it actually exists. Both of these evaluative methods are typically used in simulation studies of DF detection. Type I error of detecting DTF was assessed using Bradley's (1978) liberal rate of 7.5 percent for each condition, and compared across the two methods of DTF analysis. Similarly, a power analysis was performed for each condition for both methods of DTF analysis.

While Type I error analysis and power analysis are both important indicators of the effectiveness of a measurement instrument, they can also indicate that a small or trivial effect is statistically significant, possibly giving the impression that the results are practically important. Because the statistical significance of DTF depends on sample size, the interpretation of significance is difficult (Stark et al., 2004). This difficulty increases when the sample sizes are large and even more likely when power is high (Monahan, McHorney, Stump, & Perkins, 2007). An analysis of effect size can be used to estimate the practical importance of the results, and may help to reduce the acceptance of Type I and Type II errors (Hidalgo & López-Pina, 2004). This is especially important because of the large sample sizes (1000 and greater) used in this study.

A number of different effect size measures have been proposed for the Mantel-Haenszel test and for logistic-regression (Hidalgo & López-Pina, 2004; Monahan, et al., 2007; Penny & Johnson, 1999), but few have been proposed for DFIT. Stark and his colleagues (2004) proposed

three effect size indices for DFIT: DTFR, a measure, in raw score units, based on the differences in the TCCs between the reference and focal groups; d_{DTF} , the DTFR standardized using the standard deviation of the observed focal group scores; and the RSR which compares the proportions of examinees from the reference and focal groups at a given cut score. Wright (2011) investigated the effect size for NCDIF in DFIT comparing it with the effect size for the Mantel-Haenszel statistic. However, I find no investigation of comparative effect sizes between Mantel-Haenszel and DFIT for DTF.

CHAPTER 4

RESULTS

The results are organized into three main groupings: the distribution of τ^2 , correlations between DFIT and MH/LA DTF indicators, and significance testing. A primary area of interest is the distribution of τ^2 . Because τ^2 is a variance measure, it is bound from zero to positive infinity, and is presumably not normally distributed. Therefore, its usefulness as an indicator for testing the null hypothesis is in question (Penfield & Algina, 2006). Determining empirical distributions may provide further information on this issue. Strong correlations between the DFIT- DTF indicator and MH/LA τ^2 would provide support for the usefulness of the MH/LA DTF indicator. Significance testing examines the obtained amounts of significant DTF for DFIT and τ^2 , looking at τ^2 using an empirically derived cutoff method of determining the rate of statistical significance. Results obtained from DFIT and MH/LA are compared.

Distributions⁴

One of the noted problems of using the MH/LA variance method to determine the presence of DTF is that the distribution of its output statistic (τ^2) is unknown (Penfield & Algina, 2006), leaving in doubt the appropriate method to use to determine significance. Penfield (2005)

⁴ Because DFIT already has a proven method of identifying significance (IPR; Oshima, Raju & Nanda, 2006), it is not necessary to determine the distribution of its outputs.

states that the distribution of τ^2/SE is not normal⁵, so the variance should not be used to test significance. However, Camilli & Penfield (1997) indicate that the normality assumption could hold, and, if it does, then a distribution of the log odds ratio (on which τ^2 is based) could be defined. To investigate the distribution, I plotted the outputs from DIFAS for the null condition (mean 0, SD 1, no embedded DF) by condition group and over all for the group sizes. Based on a comparison of normal and kernel density curves, for the equal sample sizes, the graphs most commonly resemble an F-distribution, while unequal sample sizes appear to have normal distribution, at least over the range of scores calculated for this study, (see Figures 1 – 3.)

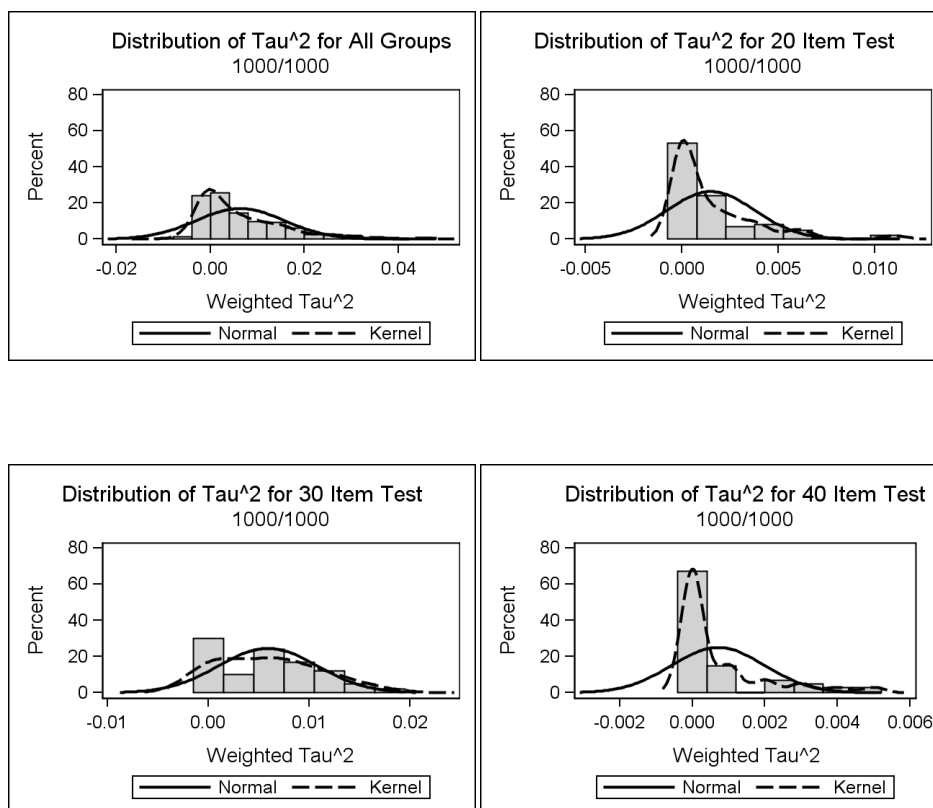


Figure 1. Null Condition 1000/1000 Simulee Group Sizes – MH/LA Method

⁵ This is based on the logic that variances range from 0 through positive infinity, a non-normal distribution. The ratio of a variance to its standard error, having the same range, also cannot be normally distributed. (R. Penfield, personal communication, March 31, 2014.)

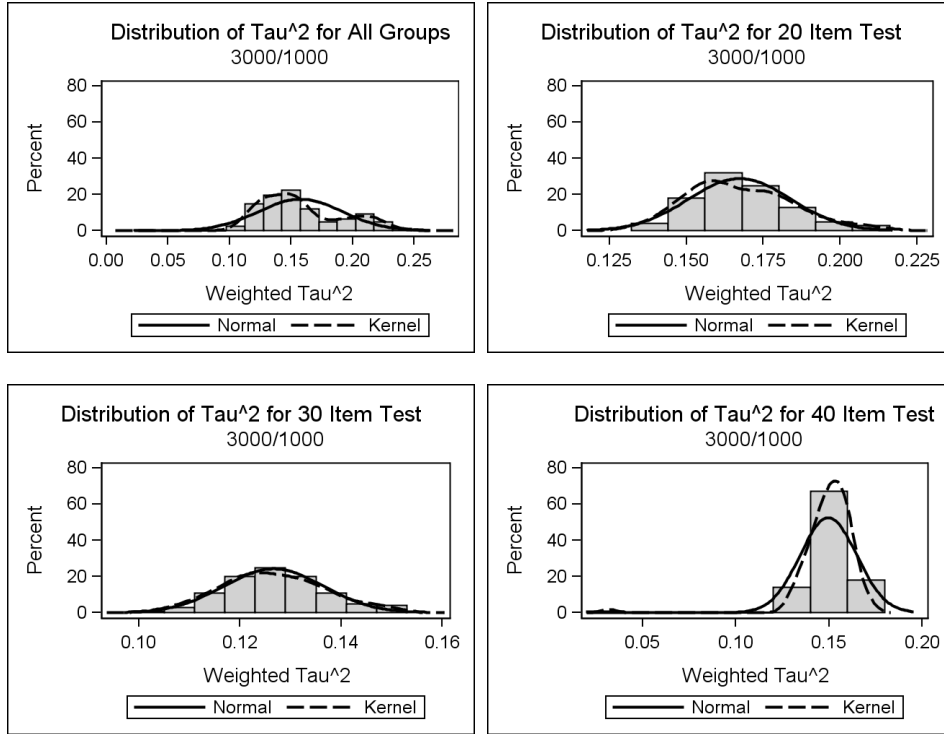


Figure 2. Null Condition 3000/1000 Simulee Group Sizes — MH/LA Method

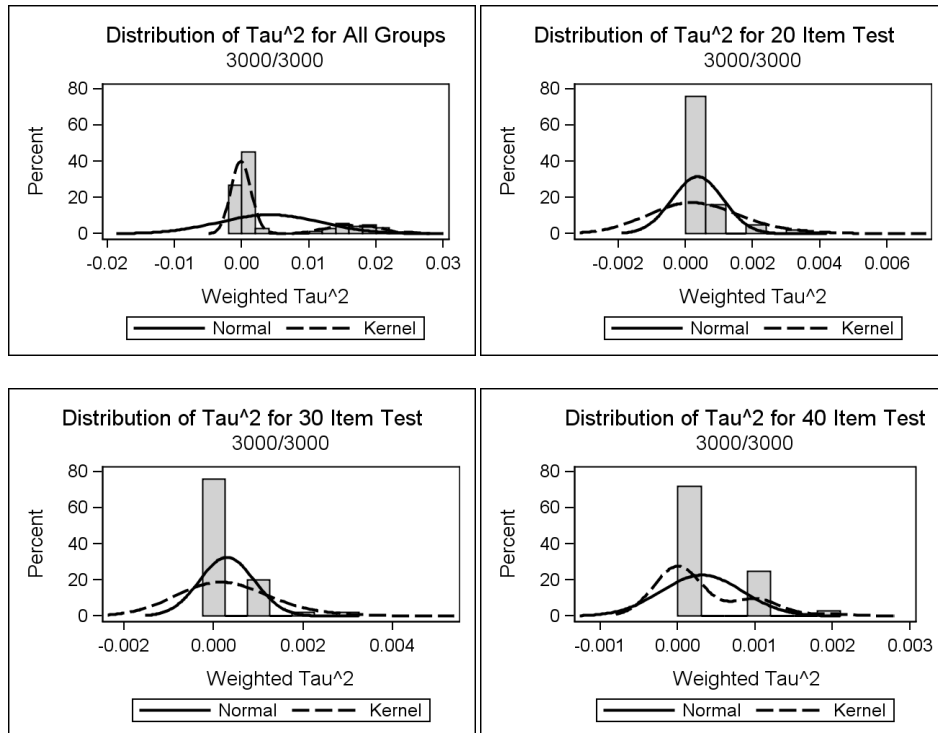


Figure 3. Null Condition 3000/3000 Simulee Group Sizes – MH/LA Method

Correlations

To investigate the relationship between the DFIT-DTF significance indicator and the τ^2 values, I computed correlations on matched pairs of τ^2 and DTF values for each condition. Of 256 pairs, only 190 (74%) produced results. Correlations of τ^2 and DFIT-DTF ranged from negligible to moderate. The Pearson correlations were both positive and negative, ranging between approximately -.3 and .3, with the great majority falling between $\pm .20$. See Figures 4 through 6 for a sample correlation plot for each test condition by test size.

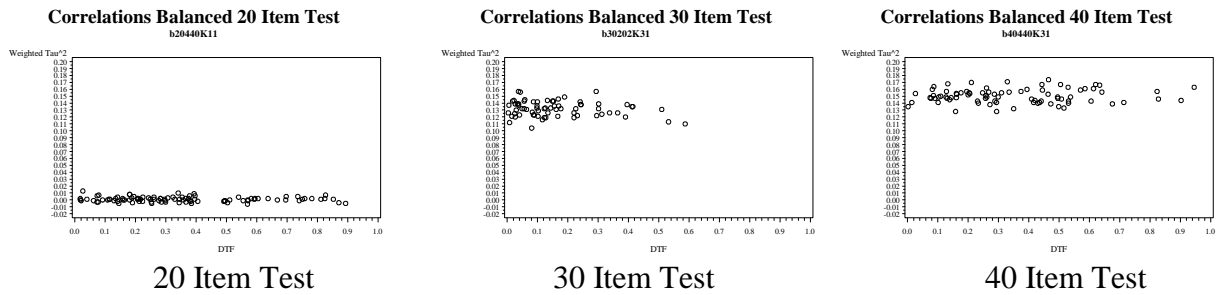


Figure 4. Sample Correlation Plots for the Balanced Condition

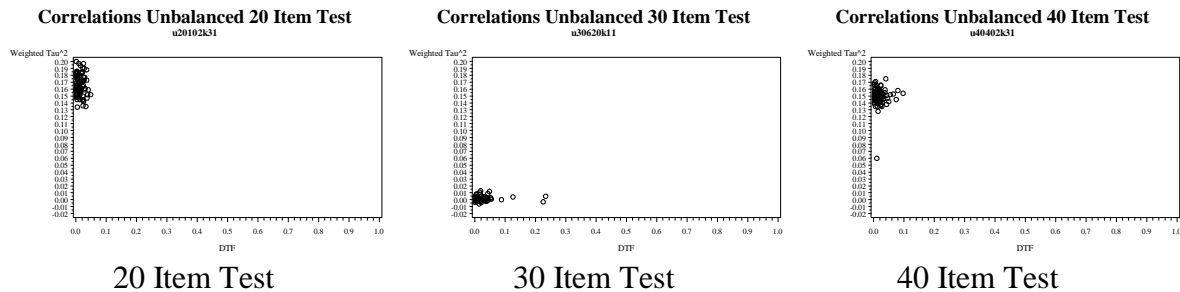


Figure 5. Sample Correlation Plots for the Unbalanced Condition

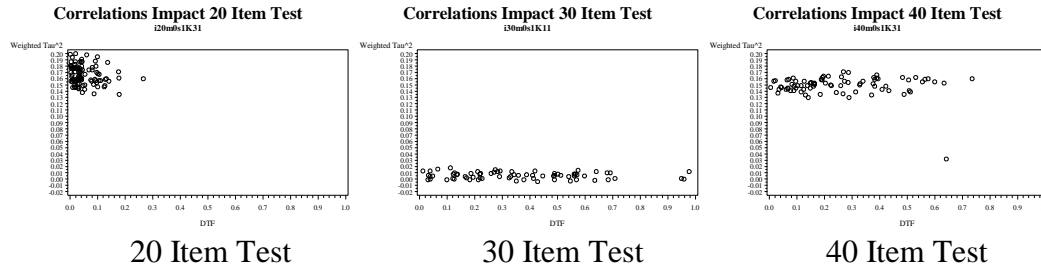


Figure 6. Sample Correlation Plots for the Impact Condition

Significance Testing

DFIT has its own method of significance testing (Oshima et al., 2006). However, the distribution of the τ^2 statistic is unknown, making the appropriateness of using common methods of significance testing uncertain. Bradley's liberal criterion (1978) is used in all of the following analyses to evaluate the size of the Type I error rate for the nominal alpha of .05, and alpha levels within the range of .025 to .075 are accepted as robust.

DFIT. In calculating the item parameters for DFIT, BILOG-MG3 had over 3100 tests that did not converge (Table 5). These failures generated empty or incomplete files, which DIFCUT read, producing a DTF statistic equal to zero but with a significance level of .0001. Over all, approximately six percent of the files failed to converge. The unbalanced condition had the most (1477) files with convergence errors, followed by the balanced condition (1218). The impact

Table 5
Count of Files that Did Not Converge in BiLOG-MG3

Count of Files with Non-Convergence by Items and Sample Size				
Items				
Sample Size	Balanced	Impact	Unbalanced	Grand Total
20	89	144	15	248
1000:1000	64	71	13	148
3000:1000	25	71	2	98
3000:3000	0	2	0	2
30	544	751	215	1510
1000:1000	312	405	153	870
3000:1000	174	271	43	488
3000:3000	58	75	19	152
40	585	582	193	1360
1000:1000	337	341	142	820
3000:1000	194	183	29	406
3000:3000	54	58	22	134
Grand Total	1218	1477	423	3118

Count of Files with Non-Convergence by Sample Size and Items				
Sample Size				
Items	Balanced	Unbalanced	Impact	Grand Total
1000:1000	713	817	308	1838
20	64	71	13	148
30	312	405	153	870
40	337	341	142	820
3000:1000	393	525	74	992
20	25	71	2	98
30	174	271	43	488
40	194	183	29	406
3000:3000	112	135	41	288
20	0	2	0	2
30	58	75	19	152
40	54	58	22	134
Grand Total	1218	1477	423	3118

condition had 423. Twenty-item tests had the fewest failures (248), while 30-item tests had 1510 failures, and 40-item tests had 1360 failures. From the perspective of sample sizes, almost 60%

of the lack of convergence was for the 1000/1000 group sizes, with another 30% for the 3000/1000 group sizes, and only nine percent for the 3000/3000 group sizes. These files were removed from later analysis.

The null condition had data generated using a standard normal distribution, and calibrated separately for the Focal and Reference groups for each condition. No DF was embedded in any item. Results for these tests showed that all but two conditions exhibited Type I error within the nominal .05 acceptable rate. Thirty-item tests for group sizes 1000/1000 had a .16 error rate, and 40-item tests for group sizes 3000/3000 had a .10 error rate (Table 6.)

Table 6
DFIT: Number of Tests with Significant DTF – Null Condition (Type I Error)¹

Test Items	Mean	Standard Deviation	Number of Simulees					
			1000/1000	%	3000/1000	%	3000/3000	%
20	0	1.00	1 (99) ²	1%	0 (98)	0%	0 (100)	0%
30	0	1.00	12 (77)	16%	3 (80)	4%	0 (99)	0%
40	0	1.00	2 (72)	3%	3 (84)	4%	10 (98)	10%

¹ Significance was tested using Bradley's (1978) liberal criterion (.075).

²Number of tests with significant DTF (total number of tests with results.)

For the Balanced group of conditions, all conditions had DF embedded in some of the last items of the test. For example, a 20-item test with four DF items, would have items 17 and 18 with DF favoring the focal group, and items 19 and 20 with DF favoring the reference group. DF is balanced in the sense that each group had one-half of the DIF items. However, because the characteristics of the items in each group were different, DTF would be minimized, but not necessarily netted to zero. Thus, in a loose, but not the strictest, sense significant DTF may be defined as Type I error.

For the Balanced DIF condition and 20-item test, all conditions except two exhibited less than Bradley's 7.5 percent detection rates. For tests with four DF items and group sizes 1000/1000, where DF magnitude was .20, eight tests exhibited significant DTF, and where test magnitude was .40, 26 tests exhibited significant DTF, 8 and 27 percent respectively. For 30-item tests, all conditions for the 1000/1000 group sizes exhibited detection rates greater than 7.5 percent. Additionally, for the 3000/3000 group sizes where DF magnitude was large (.40), both the two DF item and 6 DF item conditions exhibited detection rates greater than Bradley's liberal criterion of 7.5 percent. For 40 item tests, all 3000/3000 group sizes conditions had less than 7.5 percent significant DTF, and none had a detection rate greater than 7.5 percent for the 3000/1000 group sizes. However, for the 1000/1000 group sizes, detection rates greater than 7.5 percent were found at the moderate DF magnitude (.20) for the two DF item condition, at .20 and .40 levels of magnitude for the four DF item condition, and at all levels of magnitude for both the four and eight DF item conditions. For the 3000/1000 group sizes detection rates less than 7.5 percent occurred for all conditions. For all test sizes, group sizes of 1000/1000 showed unexpected amounts of DTF in most cases, especially as the number of items with DF increased and as the magnitude of DF increased (Table 7). Overall for the balanced condition, the 1000/1000 group sizes had the most conditions with identified DTF greater than 7.5 percent (15 out of 21 cases, or approximately 70%.)

All Unbalanced conditions had DF embedded in some items of the Focal group. So, detection of DTF was expected, and detection rates may be considered a surrogate for power. For the Unbalanced DIF condition and 20-item tests, only two combination of conditions for group sizes 1000/1000 detected significant DTF: one test for one DF item with .02 magnitude, and four

tests for four DF items with .20 magnitude. All other conditions showed no DTF detection. For 30-item tests, significant DTF was detected for all group sizes. The 1000/1000 group sizes showed the most cases of DTF over all. There was a modest increase of detection as the number of DF items and magnitude increased for the 1000/1000 group sizes, but not for the 3000/1000 or the 3000/3000 group sizes. The 40-item tests showed a similar pattern as 30-item tests, but significant DTF was detected for the 1000/1000 group sizes at higher rates. For all combinations of conditions, only three for the 40-item tests with 1000/1000 group sizes exceeded Bradley's liberal criterion (Table 8).

For the Impact condition, which had no embedded DF, but which varied the group mean or standard deviation in four combinations over three test sizes and three group size combinations, DTF did not occur in any condition (Table 9). This is probably an effect of the effective linking process.

Table 7

DFIT: Number of Tests with Significant DTF - Balanced

Test Items	DF Items	Magnitude	Number of Simulees						
			1000/1000	%	3000/1000	%	3000/3000	%	
20	2	0.02	0 (94) ¹	0%	0 (99)	0%	0 (100)	0%	
		0.20	0 (93)	0%	2 (97)	2%	0 (100)	0%	
		0.40	3 (87)	3%	2 (94)	2%	1 (100)	1%	
	4	0.02	2 (99)	2%	5 (98)	5%	0 (100)	0%	
		0.20	8 (96)	8%	0 (90)	0%	1 (100)	1%	
		0.40	26 (98)	27%	0 (95)	0%	0 (100)	0%	
	30	2	0.02	8 (75)	11%	0 (69)	0%	0 (99)	0%
			0.20	20 (81)	25%	2 (74)	3%	1 (96)	1%
			0.40	19 (74)	26%	5 (77)	6%	11 (98)	11%
6		0.02	9 (62)	15%	2 (82)	2%	2 (96)	2%	
		0.20	7 (87)	8%	1 (80)	1%	3 (98)	3%	
		0.40	16 (75)	21%	5 (78)	6%	10 (95)	11%	
40		2	0.02	2 (79)	3%	1 (94)	1%	2 (100)	2%
			0.20	51 (80)	64%	1 (70)	1%	4 (96)	4%
			0.40	4 (91)	4%	4 (87)	5%	2 (96)	2%
	4	0.02	6 (87)	7%	3 (91)	3%	2 (97)	2%	
		0.20	9 (81)	11%	1 (72)	1%	2 (96)	2%	
		0.40	15 (88)	17%	6 (87)	7%	1 (96)	1%	
	8	0.02	20 (69)	29%	1 (80)	1%	3 (99)	3%	
		0.20	8 (87)	9%	1 (77)	1%	4 (98)	4%	
		0.40	18 (80)	23%	3 (75)	4%	1 (100)	1%	

¹ Number of tests with significant DTF (total number of tests with results.)

Table 8
DFIT: Number of Tests with Significant DTF – Unbalanced

Test Items	DF Items	Magnitude	Number of Simulees					
			1000/1000	%	3000/1000	%	3000/3000	%
20	1	0.02	1 (100) ¹	1%	0 (97)	0%	0 (100)	0%
		0.20	0 (98)	0%	0 (100)	0%	0 (100)	0%
		0.40	0 (94)	0%	0 (100)	0%	0 (100)	0%
	2	0.02	0 (98)	0%	0 (100)	0%	0 (100)	0%
		0.20	0 (98)	0%	0 (94)	0%	0 (99)	0%
		0.40	0 (95)	0%	0 (79)	0%	0 (100)	0%
	4	0.02	0 (98)	0%	0 (100)	0%	0 (100)	0%
		0.20	4 (93)	4%	0 (91)	0%	0 (100)	0%
		0.40	0 (77)	0%	0 (66)	0%	0 (99)	0%
30	2	0.02	1 (80)	1%	1 (74)	1%	2 (98)	2%
		0.20	5 (69)	7%	1 (63)	2%	4 (96)	4%
		0.40	4 (66)	6%	4 (70)	6%	1 (95)	1%
	3	0.02	7 (76)	9%	5 (89)	6%	1 (98)	1%
		0.20	6 (75)	8%	1 (81)	1%	4 (96)	4%
		0.40	7 (91)	8%	0 (70)	0%	1 (95)	1%
	6	0.02	7 (75)	9%	0 (83)	0%	3 (100)	3%
		0.20	3 (71)	4%	5 (84)	6%	2 (91)	2%
		0.40	2 (75)	3%	1 (49)	2%	0 (88)	0%
40	2	0.02	6 (80)	8%	1 (80)	1%	3 (99)	3%
		0.20	2 (77)	3%	1 (76)	0%	2 (94)	2%
		0.40	14 (76)	18%	0 (77)	0%	0 (98)	0%
	4	0.02	12 (85)	14%	1 (78)	1%	1 (97)	1%
		0.20	4 (78)	5%	3 (87)	3%	2 (99)	2%
		0.40	5 (86)	6%	1 (99)	1%	2 (92)	2%
	8	0.02	10 (86)	11%	2 (87)	2%	1 (97)	1%
		0.20	7 (79)	9%	0 (79)	0%	5 (99)	5%
		0.40	8 (82)	10%	2 (77)	3%	3 (96)	3%

¹ Number of tests with significant DTF (total number of tests with results.)

Table 9
DFIT: Number of Tests with Significant DTF – Impact

Test Items	Mean	Standard Deviation	Number of Simulees					
			1000/1000	%	3000/1000	%	3000/3000	%
20	0	2	0 (100)	0%	0 (100)	0%	0 (100)	0%
		1.50	0 (100)	0%	0 (100)	0%	0 (100)	0%
	1	1	0 (100)	0%	0 (100)	0%	0 (100)	0%
30	0	2	0 (100)	0%	0 (100)	0%	0 (100)	0%
		1.50	0 (99)	0%	0 (100)	0%	0 (100)	0%
	1	1	0 (99)	0%	0 (99)	0%	0 (100)	0%
40	0	2	0 (100)	0%	0 (100)	0%	0 (100)	0%
		1.50	0 (100)	0%	0 (100)	0%	0 (100)	0%
	1	1	0 (100)	0%	0 (99)	0%	0 (100)	0%
	0.5	1	0 (96)	0%	0 (100)	0%	0 (100)	0%

¹ Number of tests with significant DTF (total number of tests with results.)

MH/LA Variance Method.

The variance method produced varying numbers of negative τ^2 values across the different test conditions. Because variances are squared values, it does not make sense that there should be negative values. However, Camilli and Penfield (1997) noted that negative values for estimates do occur, and that common practice is to set such negative values to zero. Following their suggestion, I set all negative values of τ^2 to zero for the main analysis. Only 477 files in the impact condition had negative τ^2 . Of these about 70% were for group sizes 1000/1000. There were none for unequal group sizes or for the balanced or unbalanced conditions. (See Table 10).

Table 10
Counts of Negative τ^2

Group Size	Number of Items			Total
	20	30	40	
1000/1000	118	97	120	335
3000/3000	68	38	36	142
Total	186	135	156	477

MH/LA Variance Method with empirical cutoffs. Null empirical cutoff values for an approximate five percent error rate were calculated based on all τ^2 values within each group size (Table 11).

Table 11
MH/LA: Number of Tests with Significant DTF using Empirical Cutoff– Null Condition (Type I Error)¹

Test Items	Mean	Standard Deviation	Number of Simulees					
			1000/1000	%	3000/1000	%	3000/3000	%
20	0	1	8(100) ²	8%	5(100)	5%	8(100)	9%
30	0	1	5(100)	5%	5(100)	5%	4(100)	4%
40	0	1	6(100)	6%	5(100)	5%	3(100)	3%

¹ Significance was tested using Bradley's (1978) liberal criterion (.075).

²Number of tests with significant DTF (total number of tests with results.)

Because the cumulative frequencies did not match the .95/.05 cut point in about half of the conditions, the τ^2 value chosen as the cutoff value for these conditions was either slightly less than or greater than the ideal value depending on the frequency values for the condition. The cutoff value was selected to give the closest proportion to .05 as possible (Tables 12 and 13).

Table 12
Empirical Cutoffs for τ^2

Test Items	Mean	Standard Deviation	Cutoff Values by Test Items X Group Size		
			Number of Simulees		
			1000/1000	3000/1000	3000/3000
20	0	1	0.0055	0.198	0.0015
30	0	1	0.014	0.145	0.0005
40	0	1	0.0035	0.164	0.0005

Table 13
Frequency Counts to Determine Empirical Cutoffs for τ^2 : Test Size 20 Sample Sizes 3000_3000

Weighted Tau ²					
Weighted_Tau_2	Frequency	Percent	Cumulative Frequency	Cumulative Percent	
0	76	76.00	76	76.00	
0.001	16	16.00	92	92.00	
0.002	5	5.00	97	97.00	
0.003	2	2.00	99	99.00	
0.004	1	1.00	100	100.00	

In the balanced condition, using the empirical cutoff detected DTF at rates less than 7.5 percent for most levels of magnitude for 20- and 30-item tests with 3000/1000 group sizes. With 40-item tests, detection rates were 10 percent and above for tests with two or eight DF items, and less than 10 percent with four DF items and magnitude of .40. The 1000/1000 and 3000/3000 group sizes had more conditions with significant DTF, at the higher rates using empirical cutoff values (Table 14). With two exceptions, the empirical cutoff method exhibited the expected DF

Table 14

MH/LA: Number of Tests with Significant DTF using Empirical Cutoff – Balanced

Test Items	DF Items	Magnitude	Number of Simulees						
			1000/1000	%	3000/1000	%	3000/3000	%	
20	2	0.02	10(100) ¹	10%	2(100)	2%	6(100)	6%	
		0.20	5(100)	5%	1(100)	1%	10(100)	10%	
		0.40	7(99)	7%	1(100)	1%	17(100)	18%	
	4	0.02	10(100)	10%	0(100)	0%	7(98)	7%	
		0.20	14(100)	14%	3(100)	3%	11(100)	11%	
		0.40	10(100)	10%	6(100)	6%	27(99)	27%	
	30	2	0.02	0(100)	0%	5(100)	5%	5(100)	5%
			0.20	0(100)	0%	5(100)	5%	7(99)	7%
			0.40	0(100)	0%	8(100)	8%	5(100)	5%
6		0.02	0(99)	0%	4(99)	4%	4(100)	4%	
		0.20	0(99)	0%	3(100)	3%	34(100)	34%	
		0.40	1(100)	1%	5(100)	5%	95(100)	95%	
40		2	0.02	4(100)	4%	17(100)	17%	8(100)	9%
			0.20	9(100)	9%	12(100)	12%	4(100)	4%
			0.40	14(100)	14%	10(100)	10%	10(100)	10%
	4	0.02	7(100)	7%	6(100)	6%	3(100)	3%	
		0.20	17(100)	17%	3(100)	3%	14(100)	14%	
		0.40	49(100)	49%	9(100)	9%	83(100)	83%	
	8	0.02	7(100)	7%	10(100)	10%	4(100)	4%	
		0.20	25(100)	25%	12(100)	12%	51(100)	51%	
		0.40	80(100)	80%	24(100)	24%	100(100)	100%	

¹ Number of tests with significant DTF (total number of tests with results.)

amplification effect across group sizes, test sizes and DF magnitude. The first exception was the 30-item test for group sizes 1000/1000 which showed DTF detection rates of zero to one percent

for all conditions. These rates were much smaller than the comparable rates for 20-item tests. The second exception was that twenty-item tests showed unexpectedly large amounts of DTF for magnitude .02.

For the unbalanced condition, using the empirical cutoff method detected DTF for the 3000/1000 group sizes exceeding the liberal criterion of 7.5 percent in only three conditions, all for 40-item tests. These rates overall were smaller than those detected for the 1000/1000 and the 3000/3000 group sizes. The detection rates for the 1000/1000 and the 3000/3000 group sizes were similar to those of the balanced condition. The 30-item tests for group sizes 1000/1000 also presented the anomalous pattern of negligible DTF detection rates (Table 15).

Using the empirical cutoff values with the impact condition produced Type I error rates at unacceptably high levels except for three cases for the 30-item tests with group sizes 1000/1000, and for six cases in the 3000/1000 group sizes for the zero mean difference in each test size (Table 16).

Table 15
 MH/LA: Number of Tests with Significant DTF using Empirical Cutoff – Unbalanced

Test Items	DF Items	Magnitude	Number of Simulees					
			1000/1000	%	3000/1000	%	3000/3000	%
20	1	0.02	8(100) ¹	8%	1(100)	1%	5(100)	5%
		0.20	10(100)	10%	3(100)	3%	8(100)	8%
		0.40	5(100)	5%	2(100)	2%	15(100)	15%
	2	0.02	10(100)	10%	4(100)	4%	8(100)	8%
		0.20	8(100)	8%	0(99)	0%	14(100)	14%
		0.40	9(100)	9%	2(100)	2%	20(100)	20%
	4	0.02	7(100)	7%	3(100)	3%	16(100)	16%
		0.20	7(100)	7%	0(100)	0%	14(100)	14%
		0.40	10(100)	10%	0(100)	0%	31(100)	32%
30	2	0.02	0(100)	0%	7(100)	7%	7(100)	8%
		0.20	0(100)	0%	4(100)	4%	6(100)	6%
		0.40	0(99)	0%	2(100)	2%	8(100)	8%
	3	0.02	0(100)	0%	4(100)	4%	5(100)	5%
		0.20	0(100)	0%	7(100)	7%	10(100)	11%
		0.40	0(100)	0%	5(100)	5%	17(100)	17%
	6	0.02	0(100)	0%	4(100)	4%	5(99)	5%
		0.20	0(100)	0%	5(100)	5%	34(99)	34%
		0.40	0(99)	0%	0(100)	0%	91(98)	93%
40	2	0.02	6(100)	6%	4(100)	4%	6(100)	7%
		0.20	10(100)	10%	8(100)	8%	7(100)	7%
		0.40	12(100)	12%	4(100)	4%	28(99)	29%
	4	0.02	2(100)	2%	14(100)	14%	2(100)	2%
		0.20	17(100)	17%	3(100)	3%	17(97)	18%
		0.40	51(99)	52%	2(100)	2%	90(100)	90%
	8	0.02	7(100)	7%	13(99)	13%	1(100)	1%
		0.20	13(100)	13%	5(100)	5%	39(100)	39%
		0.40	65(100)	65%	0(100)	0%	99(100)	99%

¹ Number of tests with significant DTF (total number of tests with results.)

Table 16
MH/LA: Number of Tests with Significant DTF using Empirical Cutoff – Impact

Test Items	Mean	Standard Deviation	Number of Simulees					
			1000/1000	%	3000/1000	%	3000/3000	%
20	0	2	27(100) ¹	27%	2(100)	2%	73(100)	73%
		1.5	15(100)	15%	0(100)	0%	33(99)	33%
	1	1	92(100)	92%	70(100)	70%	100(100)	100%
	0.5	1	26(100)	26%	21(100)	21%	91(100)	91%
30	0	2	1(100)	1%	3(100)	3%	73(100)	73%
		1.5	0(100)	0%	3(100)	3%	22(100)	22%
	1	1	8(100)	8%	68(100)	68%	99(100)	99%
	0.5	1	0(100)	0%	68(100)	68%	68(100)	68%
40	0	2	37(100)	37%	2(100)	2%	52(100)	52%
		1.5	9(100)	9%	5(100)	5%	16(100)	16%
	1	1	70(100)	70%	100(100)	100%	100(100)	100%
	0.5	1	18(100)	18%	86(100)	86%	52(100)	52%

¹ Number of tests with significant DTF (total number of tests with results.)

DFIT vs. MH/LA Variance Method

Balanced conditions. For the 20-item tests with 1000/1000 group sizes, the MH/LA Variance method using empirical cutoffs showed moderate to large DTF rates, while DFIT showed small to moderate amounts of DTF. With group sizes of 3000/1000, DFIT showed no or small DTF, while the Variance method using empirical cutoffs showed similar rates to DFIT for 20- and 30-item tests, but larger rates for 40-item tests. For 30-item tests, DFIT had higher DTF

detection rates than the MH/LA Variance method for the 1000/1000 group sizes, but much lower rates for the 3000/1000 and 3000/3000 group sizes. Results for the 40-item tests were similar to those for the 30 item tests.

Unbalanced conditions. DFIT had smaller percentages of significant DTF for all conditions of 20-item tests when compared to the variance method. For 30-item tests DFIT had larger rates of DTF with group sizes 1000/1000 than the variance method, and similar rates to the variance method with group sizes 3000/1000. DFIT had smaller rates with group sizes 3000/3000 than the variance method. For 40-item tests, DFIT produced smaller rates of DTF for all cases of group sizes 3000/1000 and 3000/3000 than the variance method with either type of cutoff. DFIT produced similar rates as the variance method using empirical cutoffs for group sizes 1000/1000.

Impact conditions. DFIT produced smaller percentages of DTF across all conditions for all test sizes than the variance method. This indicates that equating is resolving the distributional differences between the reference and focal groups for DFIT, while the MH/LA variance method identifies them as DTF.

CHAPTER 5

DISCUSSION

Comparing different methods of detecting DF under different conditions is a commonly used technique (Drasgow, 1987; Li & Stout, 1996; Raju, Drasgow, & Slinde, 1993; Wipasillapa, n.d.; Zumbo, 2003) that permits benchmarking different methods to see whether any method has a relative advantage under the conditions tested. Additionally, this helps verify whether a new method performs at least as well as existing methods. Knowledge of the strengths and weaknesses of different detection methods helps practitioners to select the most appropriate one for their data analyses.

The Mantel-Haenszel/Liu-Agresti (MH/LA) variance method has not been compared with any other method of calculating DTF. This study, comparing it with an established method, helps to situate it among the existing methods of DF detection. This comparison also helps to understand variance methods of evaluating differential functioning, and aids in understanding whether, and to what extent, these methods may be used to verify a null hypothesis of DTF not being present (Penfield & Algina, 2006).

Three major conditions were modeled in this study: (1) items in which DF was divided equally between the reference and focal groups; (2) items in which DF was inserted only in the focal group, and (3) items without DF, but in which the reference and focal groups differed on their distribution characteristics (mean and standard deviation). Data for these three conditions were processed using two different methods of DTF detection: DFIT (Raju, van der Linden, &

Fleer, 1995) as implemented in the DIFCUT program (Nanda, Oshima, & Gagne, 2006) and a variance method based on the MH/LA contingency table model (Camilli & Penfield, 1997; Penfield & Algina, 2006) as implemented in DIFAS 5.0 (Penfield, 2005). Results derived from these two methods were expected to differ because of their differing approaches to modeling DTF. DFIT models DTF as a sum of the signed DIF for each item. The variance method models DTF as a sum of the unsigned DIF variances (Camilli & Penfield, 1997). Thus, Penfield and Algina indicate that small amounts of DIF may add up to indicate large effects in overall DTF, but cancellation cannot exist.

DFIT. In DFIT, the null condition which had no embedded DIF, and the same distribution, $\sim N(0, 1)$ showed DTF in excess of Bradley's liberal 7.5 percent criterion in two conditions. This supports Petroski's (2005) finding of high Type I error rates for DFIT. However, the fact that only two out of nine conditions showed Type I error suggests that these present results may come from the randomness of the data generation process.

The impact condition had no DF added into the data. Therefore, the DTF indices should not exhibit DTF, and none was found for the impact condition in DFIT. The probable cause for no significant DTF's being detected (i.e., desirable results) would be that the linking process was very successful in putting the focal and reference groups on the same scale. Another possible cause is that DFIT may have low sensitivity to the distributional differences included in this study.

The balanced conditions showed a general increase of DTF detections from smaller levels of magnitude to greater and from tests with fewer DF items to more DF items. This was as expected given that there is a greater likelihood for the occurrence of amplification with higher levels of DF and more items over which to accumulate it. Similar patterns were found for the

unbalanced condition. An unexpected finding was that group sizes of 1000/1000 had much higher rates of DTF detection than both group sizes of 3000/1000 and of 3000/3000. Greater amounts of DTF would be expected in the larger group sizes, because there is more opportunity for DF to occur. This finding could be related to DFIT's known inaccuracies with small sample sizes. However, all group sizes were greater than recommendations in the literature, which are for greater than 200 examinees per group with the 1PL model, greater than 500 examinees with the 2PL model and greater than 1,000 examinees with the 3PL model (Oshima & Morris, 2008).

A broader comparison of the balanced and unbalanced conditions shows several unexpected patterns. Contrary to expectations, the balanced condition had higher rates of identified DTF than the unbalanced condition. In about half of the cases, the balanced condition produced rates over twice as large as the unbalanced condition. Generated parameters were verified to ensure that DIF magnitude was correctly assigned to both the reference and focal groups. This indicates that DF amplification and cancellation may not be functioning as expected in DFIT.

Group sizes 1000/1000 were problematic. For most conditions, group sizes 1000/1000 exhibited over twice as much DTF as the other two group sizes. Similarly, trivial DF magnitude (.02) showed unexpectedly high levels of DTF. These levels were frequently greater than those for magnitude .20, and occasionally greater than those for magnitude .40. Almost all of this was within group sizes 1000/1000. I can find no explanation for these two occurrences.

For both the balanced and the unbalanced conditions with unequal sample sizes, rates of DTF were all within Bradley's liberal rate of 7.5 percent. Blitz and Morris (2011), investigating the Item Parameter Replication (IPR) method, the method that DFIT uses to detect significance, found that for a reference group larger than the focal group under conditions of no DIF the Type

I error rate was lower than expected. This characteristic should under identify DTF when the reference group is larger than the focal group. This appears to be what happened in the current study with 3000 reference group size and 1000 focal group size. Thus, it is reasonable to expect that the observed rate of DTF would have been larger had separately estimated covariance files for both the focal and reference groups been used.

MH/LA Variance. An empirical cutoff method of determining significance for the MH/LA variance method was evaluated. In determining the cutoff points for the empirical cutoff method, both sample sizes and test sizes should be considered. Cutoff points were calculated in two ways: 1) using sample sizes only, and 2) using test sizes crossed with sample sizes. Data was processed using both methods. A comparison of the two results showed that using cutoffs calculated using sample sizes only can put all of the detected DTF within a single test size. Whereas cutoffs calculated using test size crossed with sample size put approximately five percent of DTF in each test x sample size group. Once appropriate cutoffs have been determined, the empirical cutoff method seems to work reasonably well as a method of evaluating the relative importance of the results of the different methods. The empirical cutoff method of significance testing showed consistent DTF detection rates among all three sample size groups. Still, there is sensitivity to sample size, with more DTF being shown for those conditions with more simulees. Rates of DTF detection were consistently high for samples with distributional differences. Only eight out of 36 conditions showed DTF at less than the liberal 7.5% rate.

Overall, the MH/LA method showed elevated rates of DTF detection. This is not unexpected in that it deals only with positive values (the τ^2 variances), which allows for DIF amplification, but not for DIF cancellation (Camilli & Penfield, 1997). It is also sensitive to

distributional differences between the studied groups. Because of this, comparisons of the score distributions for each of the studied groups need to be done to ensure their comparability before using the MH/LA method.

Comparison.

DFIT and the MH/LA variance method measure DTF in different ways, so are not strictly comparable, but are useful for different objectives. As such they may be considered complementary. Because the MH/LA variance method cannot offset DF for the Reference group against DF for the Focal group, it is not able to provide a picture of whether a test is fair overall to examinees, but only an indication of how much DF exists for all groups. By doing this, it can serve as a preliminary indicator of whether an in depth DIF study needs to be conducted, especially for a newly developed instrument, or one used for a new purpose, such as a test normed on a national population being used for immigrants with limited cultural or language familiarity to those of the norm group.

DFIT is a summative method of DTF evaluation. Where DIF favoring one group may have positive values, DIF favoring the other group would have negative values. By combining the positive and the negative values, DFIT is able to derive an estimate of whether the instrument as a whole favors one group over the other. Where no group has an advantage at the test level, the need for a DIF study is obviated.

Practical Implications

A concern among applied researchers is “Which method should I use?” Both DFIT and the Variance method as implemented in DIFAS (Penfield, 2005) have been used in applied studies across different content areas and in cross-cultural studies. Both methods have been used to assess translated versions of different instruments (Ellis, & Mead, 2000; Guerrero 2001; He &

Wolf, 2010; Morales, Flowers, Gutierrez, Kleinman, & Teresi, 2006; Price and Oshima, 1998; van den Broeck Bastiaansen, Rossi, Dierckx, and De Clercq, 2013; and Wang, and Russell, 2005). In the area of industrial and organizational psychology, Baker, Caison, and Meade (2007) used DFIT to study a test for predicting college student attrition. (Also see Braddy, Meade, & Johnson, 2006; Maurer, Raju, & Collins, 1998; Meriac, Poling, & Woehr, 2009; Meriac, Woehr, & Banister, 2010; O'Brien, 2007; O'Brien & LaHuis, 2011; and Wang, & Russell, 2005.) In the area of mental health, Cameron, Crawford, Lawton, & Reid, 2013; Kim, Pilkonis, Frank, Thase, and Reynolds (2002); van den Broeck Bastiaansen, Rossi, Dierckx, and De Clercq (2013) have made contributions. In other areas, Donovan and Drasgow (1999) used DFIT to look at a survey to measure experiences of sexual harassment; Wipasillapa (n.d.) used DFIT to assess DTF in a mathematics test; and using DIFAS, Baer, Samuel, & Lykins,(2011), and Van Dam, Earleywine, and Danoff-Burg (2009) studied a questionnaire to assess mindfulness.

Because either method is suited for such a wide range of content areas, selecting a method becomes a question of the conditions of the research and the data. DFIT requires a large number of observations (at least 200 per group for a 1PL model; Oshima & Morris, 2008). The variance method, implemented in DIFAS 4.0 (Penfield, 2007) will work with small sample sizes (Ferne & Rupp, 2007; Lai, Teresi, & Gershon, 2005). Baer and her colleagues (2011) had a total sample of 230 participants. Van Dam (Van Dam, et al., 2009) and van den Broeck (van den Broeck, et al., 2013) both had sample sizes of less than 500. The model fitted to the data is also a concern. Like other contingency table models, the variance method works with 1PL, and 2PL without crossing DIF models. It will not handle data with crossing DF (Millsap & Everson, 1993), 3-PL models, or multidimensional data. However, DFIT can handle those kinds of data

(Oshima, et al., 1997), but requires large sample sizes for adequate parameter estimation (Oshima & Morris, 2008).

Penfield and Algina (2006) state that testing for significance is not appropriate with the variance method. However, Baer (Baer, et al., 2011), apparently ignoring Penfield and Algina's guidance, affirm that, for a subset of the instrument, the DTF indicator was "nonsignificant" (p. 7). In lieu of effect size, Penfield and Algina have suggested an indicator of DTF magnitude based on the number of items exhibiting moderate to large DIF as described by the ETS scale, where 25% or more of the items in an assessment exhibit moderate or large DIF, then assessment as a whole exhibits a large amount of DIF. Determination of this effect size requires that a DIF analysis be run in addition to the DTF analysis. DIFAS can process small sample sizes, although results may be unstable (Zwick, et al., 2012). DFIT has an established test of statistical significance (Oshima, et al., 2006) and an effect size for NCDIF, but needs an effect size indicator for CDIF and DTF (Wright, 2011).

Because DFIT and DIFAS analyze data differently, and provide answers to different questions, the requirements of the two programs should not be a concern in selecting which one to use. However, given that the requirements of the study have been defined, knowledge of the requirements of the selected program is needed. DIFAS is a Windows® based application, available from author without charge. It has a user friendly, point and click GUI interface. Data input files require simple formatting before entry. DIFAS runs very quickly (seconds for a single file), but running many files requires much operator intervention, because, DIFAS cannot be automated to process large numbers of files, as in a simulation study.

As a program, DFIT exists in three forms: DFIT8 (Raju, Oshima, & Wolach, 2009), R package DFIT (Cervantes, 2014) available from CRAN, and the SAS® program DIFCUT (the

form used in this study; Nanda, et al., 2006) available from the author. DFIT requires preprocessing to obtain item parameters and to link files from different groups to a common scale, before it can be run. Once DFIT has run, the linking process and DFIT processing must be run a second time to ensure correct estimation of DF (Oshima & Morris, 1998).

DFIT is computationally intensive because of the IPR method (Oshima, et al., 2006) that calculates the cutoff levels used to determine statistical significance. Because of this, DFIT takes minutes to run a file that requires seconds to run by DIFAS. In situations where many files need to be run sequentially, the required time can be weeks depending on the number of items in the assessment, and the sample size being processed. DIFCUT, which is written in SAS/IML[®], is easily modifiable to test extensions of the DFIT model, and is easy to run as a macro program where many files need to be processed in sequence.

Limitations/Problems

As with all studies, this one has its limitations. The first is that the distribution of the variance estimator (τ^2) is not known. Until the distribution is determined, standard tests of significance cannot be used. While the empirical cutoff methods for determining significant DTF offers a potential method for determining significance, the number of replications used to calculate the cutoff was small, 900 for each set of group sizes.

The type and manner of embedding DTF in the data is the next limitation. DIF was added to the last items of each set of parameters. This was based on the assumption that DTF resulted from multiple DIF items. Other methods of DTF generation could have been used by differing the structure of the test: for example, generating data using parameters based on a multidimensional model (Ackerman, 1989; Oshima, & Miller, 1992).

The large convergence problems in BILOG-MG3 caused the loss of a large amount of data. Greater precision and reduced error rates may have been expected had all test data been available. The version of DIFCUT used did not incorporate the suggestions of Blitz and Morris (2011) to use separate covariance files for the reference and focal groups. Use of covariance files generated for each group, instead of assuming that the covariances for the reference group are the same as those for the focal group, increases the accuracy of the Type I error when the sample sizes are unequal. This may have changed the error rate for the 3000/1000 sample size groups that were noted in this study.

Future research

Future research in this area would include large simulation studies to evaluate the distribution of τ^2 . Although the data from the current study indicated that τ^2 may have an approximate normal distribution, the uncertainty about the distribution expressed by Penfield and Algina (2006), and the theoretical assertion by Penfield (2005) that τ^2 is not normally distributed warrant a detailed investigation into the distribution. In this regard, investigation into the distribution of the log odds ratio (Camilli & Penfield, 1997) needs to be done. Once the distribution is known, more exact and appropriate methods of evaluating significance⁶ can be determined, as well as more precise methods of determining effect size (see Penfield & Algina). Beyond distribution studies, investigation into the setting of empirical cutoff values of significance using a large simulation (perhaps a Markov Chain Monte Carlo study) would be useful. Studies comparing results of data created using different methods of embedding DF (such as simple multidimensional , or multidimensional where the dimensions differ in strength or quality among the different groups; see Sinharay & Holland, 2007), or where the data does not

⁶ Perhaps similar logic to the half-normal distribution that Raju (1990) proposed for area measures could be used to determine a Z-score for MH/LA variance

match the model could inform researchers decisions on appropriate methods of analysis when a particular kind of DF is suspected by shedding light on how these analysis methods react to different kinds of DF and to misfit data (such as 3-PL data which has non-zero asymptotes with a 1-PL model which expects asymptotes to be equal to zero, Rogers & Swaminathan, 1993).

Penfield and Algina (2006) offered a suggestion for effect size measures based on the assumption of a normal distribution of τ^2 . However, these suggestions are tentative pending a determination of the actual distribution of τ^2 . Wright (2011) has begun investigation of effect size in DFIT, but to date this is limited to non-compensatory DIF, and needs to be expanded to DTF. A comparison of effect sizes between these methods must wait for these two lines of research.

While DFIT handles the three models (1PL, 2PL and 3PL) of dichotomous data, plus models of polytomous data, the MH/LA contingency table is strongest in processing 1PL data, and works with 2PL data where there is no interaction of the discrimination parameter (θ) with examinee skill level, that is, there are no crossing ICCs. The current study used a 3PL model with the guessing parameter fixed—basically a 2PL model. In addition, no skill level (θ) interaction was incorporated into the discrimination parameter. While this meets the minimum requirements of the MH/LA method, it would be worthwhile to repeat this study using data that fits the 1PL model and to compare it with data that fits the 2PL model without crossing.

The DFIT data needs to be rerun using separate covariance files for the reference and focal groups. These were both estimated in the current study, so only modifications to DIFCUT (Nanda et al., 2006) to read both files would need to be made, the data reprocessed, and the outputs compared with the current study's outputs. This would permit seeing the effect of the new method on DTF.

Finally, more empirical studies using collected data need to be performed. These would help to evaluate the usefulness of both methods of DTF detection in situations that actually occur.

Summary. This study contributes to the literature in several ways. First, kernel plot graphs of distributions of the τ^2 DTF indicator from the MH/LA variance test give indications that for equal group sizes this statistic may have an F-distribution, while for unequal group sizes the kernel plots indicate a tendency towards normal. This indicates that τ^2 may have a distribution such that standard tests of significance may be used. Second, correlations of DTF values between matched pairs of variance and DFIT results show at the most a weak correlation, indicating that the two methods measure different things. They are, thus, not comparable, and may be used in conjunction to gain greater insights into the data.

For the null condition with no embedded DF, DFIT produced Type I error within the nominal rate for seven out of nine conditions. Smaller samples (1000 simulees in each of the reference and focal groups) showed unexpectedly greater levels of DTF than the larger sample sizes. Other than this, DFIT produced increasing amounts of DTF for tests with more items and larger sample sizes. DFIT unexpectedly showed very low rates of DTF for the Impact condition, where there was no embedded DF, but the distributions (mean and standard deviation) varied between the reference and focal groups. This is probably because of the linking process.

The MH/LA variance method, like DFIT, generally produced less DTF at smaller test sizes and sample sizes, with increasing rates as both test and sample sizes increased. This method produced lower DTF detection rates for unequal sample sizes (3000 reference group, and 1000 focal group) than for the equal sample sizes. This was unexpected in that the variance method allows only DF amplification. For the unbalanced condition where all DF was in the focal

(smaller) group, DTF rates similar to those of the 1000//1000 group sizes were expected, and for the balanced condition where DF was embedded in both reference and focal groups, rates between the two equal group sizes were expected. The variance method proved exceptionally sensitive to the impact condition.

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APPENDICES

Appendix A PROGRAM CREATE PARAMETERS^{7, 8}

```

OPTIONS linesize=80;
libname prmtrs "F:\Dissertation Data\Parms" ;

PROC IMPORT
    out= params
    datafile= 'F:\Dissertation Data\ITEMPARAMS.xls'
    dbms= EXCEL replace ;
    getnames= yes;
    *    mixed = yes;
run;
/*          Macro GenParams takes a set of input parameters from an EXCEL spreadsheet and
creates
4 SAS data sets with 20 30 40 and 80 parameters by reading the 1st n records from the input
spreadsheet.
*/

%macro GenParams ;

/* i represents the number of items
   This generates parameter datasets with 20, 30, 40 and 80 items */
%Do i = 1 %to 4 ;
    %if    &i = 1 %then %let numi = 20  ;
    %else %if &i = 2 %then %let numi = 30  ;
    %else %if &i = 3 %then %let numi = 40  ;
    %else          %let numi = 80  ;

Data prmtrs.Parm&numi ;
    keep A B C;
    set params (obs = &numi) ;

%end;          /* end DO i= */

%mend GenParams ;

/*    macro createDIFparams

```

⁷ PROC IML code from P. Gagne, personal communication, February 21, 2011. Macro coding was written by author.

⁸ Most of the macro calls at the end of the program have been deleted to conserve space.

reads the 4 SAS parameter files and adds the appropriate amount of DF to the appropriate records for each input file

```
*/
```

```
%macro createUDIFparams (numa=,dfi=,mgdf=,mag=);
```

```
PROC IML ;
```

```
parmnames = { a b c } ;
```

```
    USE prmtrs.PARM&numa;
```

```
    READ ALL INTO PARAMETERS ;
```

```
    Parameters [&numa-&dfi+1 : &numa,2] = parameters[&numa-&dfi+1 :  
&numa,2]+ &mgdf ;
```

```
    a= parameters [,1] ;
```

```
    b= parameters [,2] ;
```

```
    c= parameters [,3] ;
```

```
    create aparm from a ;
```

```
    append from a ;
```

```
    create bparm from b ;
```

```
    append from b ;
```

```
    create cparm from c ;
```

```
    append from c ;
```

```
*      print parameters;
```

```
*      print &numa &dfi &mgdf ;
```

```
create prmtrs.ParmUF&numa&dfi&mag from PARAMETERS [colname= parmnames] ;  
append from parameters;
```

```
QUIT ;
```

```
%mend createUDIFparams ;
```

```
%macro createBDIFparams (numa=,dfi=,mgdf=,mag=);
```

```
PROC IML ;
```

```
parmnames = { a b c } ;
```

```

USE prmtrs.PARM&numa;
  READ ALL INTO PARAMETERSREF ;

  ParametersREF [&numa-(&dfi/2)+1 : &numa,2] = parametersREF[&numa-
(&dfi/2)+1 : &numa,2]+ &mgdf ;
  a= parametersref [,1] ;
  b= parametersref [,2] ;
  c= parametersref [,3] ;

  create aparmr from a ;
  append from a ;
  create bparmr from b ;
  append from b ;
  create cparmr from c ;
  append from c ;

create prmtrs.ParmBR&numa&dfi&mag from PARAMETERSREF [colname= parmnames] ;
append from parametersREF;

USE prmtrs.PARM&numa;
  READ ALL INTO PARAMETERSFOC ;

  ParametersFOC [&numa-&dfi+1 : &numa-(&dfi/2),2] = parametersFOC[&numa-
&dfi+1 : &numa-(&dfi/2),2]+ &mgdf ;
  a= parametersfoc [,1] ;
  b= parametersfoc [,2] ;
  c= parametersfoc [,3] ;

  create aparmf from a ;
  append from a ;
  create bparmf from b ;
  append from b ;
  create cparmf from c ;
  append from c ;

create prmtrs.ParmBF&numa&dfi&mag from PARAMETERSFOC [colname= parmnames] ;
append from parametersFOC;

QUIT ;

%mend createBDIFparams ;

%GenParams ;          /* Create 4 parm files */

/* The next 36 lines invoke Macro createUDIFparams to create

```

the Focal group parameter files that exhibit unbalanced DF for the normally distributed files. */

```
%createUDIFparams(numa=30,dfi=2,mgdf=.02,mag=02) ;
%createUDIFparams(numa=30,dfi=2,mgdf=.20,mag=20) ;
%createUDIFparams(numa=30,dfi=2,mgdf=.40,mag=40) ;
%createUDIFparams(numa=30,dfi=3,mgdf=.02,mag=02) ;
%createUDIFparams(numa=30,dfi=3,mgdf=.20,mag=20) ;
%createUDIFparams(numa=30,dfi=3,mgdf=.40,mag=40) ;
%createUDIFparams(numa=30,dfi=6,mgdf=.02,mag=02) ;
%createUDIFparams(numa=30,dfi=6,mgdf=.20,mag=20) ;
%createUDIFparams(numa=30,dfi=6,mgdf=.40,mag=40) ;
%createUDIFparams(numa=40,dfi=2,mgdf=.02,mag=02) ;
%createUDIFparams(numa=40,dfi=2,mgdf=.20,mag=20) ;
%createUDIFparams(numa=40,dfi=2,mgdf=.40,mag=40) ;
%createUDIFparams(numa=40,dfi=4,mgdf=.02,mag=02) ;
%createUDIFparams(numa=40,dfi=4,mgdf=.20,mag=20) ;
%createUDIFparams(numa=40,dfi=4,mgdf=.40,mag=40) ;
%createUDIFparams(numa=40,dfi=8,mgdf=.02,mag=02) ;
%createUDIFparams(numa=40,dfi=8,mgdf=.20,mag=20) ;
%createUDIFparams(numa=40,dfi=8,mgdf=.40,mag=40) ;
```

/* The next 30 lines invoke Macro createBDIFparams to create the Reference & Focal groups parameter files that exhibit balanced DF for the normally distributed files.

There are 6 fewer invocation than for unbalanced DF because conditions with odd numbers of items were dropped-- you can't have 1/2 item DF on file. */

```
%createBDIFparams(numa=20,dfi=2,mgdf=.02,mag=02) ;
%createBDIFparams(numa=20,dfi=2,mgdf=.20,mag=20) ;
%createBDIFparams(numa=20,dfi=2,mgdf=.40,mag=40) ;
%createBDIFparams(numa=20,dfi=4,mgdf=.02,mag=02) ;
%createBDIFparams(numa=20,dfi=4,mgdf=.20,mag=20) ;
%createBDIFparams(numa=20,dfi=4,mgdf=.40,mag=40) ;
%createBDIFparams(numa=30,dfi=2,mgdf=.02,mag=02) ;
%createBDIFparams(numa=30,dfi=2,mgdf=.20,mag=20) ;
%createBDIFparams(numa=30,dfi=2,mgdf=.40,mag=40) ;
%createBDIFparams(numa=30,dfi=6,mgdf=.02,mag=02) ;
%createBDIFparams(numa=30,dfi=6,mgdf=.20,mag=20) ;
%createBDIFparams(numa=30,dfi=6,mgdf=.40,mag=40) ;
```

run;

Appendix B PROGRAM SIMULATE DATA⁹

```

OPTIONS linesize=80 SPOOL;
PROC PRINTTO LOG="G:\Dissertation\SIMLOG.LOG" ;

libname prmtrs "G:\Dissertation\Parms" ;
libname results "G:\Dissertation\RESULTS" ;
libname files "G:\Dissertation\files" ;

%let reps = 100 ;          ***** the number of replications to perform ;

run;
/*          macro DissGenData generates simulated data for
              reference and focal groups
              30 balanced conditions
              36 unbalanced conditions      */
%macro DissGenData (SEED=, DataIn= ,DataOut= ,Items= ,Examinees= ) ;

/* r is the number of replications
   Each combination of test condidtions is replicated 100 times */
%Do r = 1 %to &reps ;

    /* The %INCLUDE gives the file path for finding the IRTGEN macro.
       IRTGEN is the macro (by Whittaker et al.) that takes the input item
       parameters and generates both a set of theta values for the specified
       number of examinees (NE), and a set of item responses for the number of
       test items (NI) for each examinee.
    */
    %INCLUDE 'G:\Dissertation\IRTGENcase.SAS' ;
    %IRTGEN(MODEL=L3, DATA=&DataIn, out=&DataOut , NI=&Items ,
NE=&Examinees );

%end;          /* end DO r= */

%mend DissGenData;

/*          macro GenDistData generates simulated data for
              reference and focal groups
              Focal Mean = 0, .5, 1.0

```

⁹ Most of the macro calls at the end of the program have been deleted to conserve space.

```

Focal St D = 1, 1.5, 2.0
Reference Mean, St Dev = (0,1)
Number Items = 20, 30, 40, 80
*/
%macro GenDistData (SEED=, DataIn= ,DataOut= ,Items= ,Examinees= ,M= ,SD= );

/* r is the number of replications
   Each combination of test condidtions is replicated 100 times */
%Do r = 1 %to &reps ;

    /* The %INCLUDE gives the file path for finding the IRTGEN macro.
       IRTGEN is the macro (by Whittaker et al.) that takes the input item parameters
and
       generates both a set of theta values for the specified number of examinees
(NE), and
       a set of item responses for the number of test items (NI) for each
examinee.
       ModIRTGEN replaces the following line of code in IRTGEN
           ELSE THETA=RAND(&DIST);
with
           ELSE THETA=&Mx + (RAND(&DIST)*&SDx);

This allows modifying the Mean &/or SD of Theta
(See Fan, et. al. (2003). "SAS for Monte Carlo Studies." Cary,NC. SAS
Institute. p. 60.)
which is needed to test for the effect of impact resulting from a different ability level or
distributional variance among the Ref & Foc groups.
*/
%INCLUDE 'G:\Dissertation\ModIRTGENcase.SAS' ;
%IRTGEN(MODEL=L3,DATA=&DataIn, out=&DataOut, NI=&Items,
NE=&Examinees, Mx=&M, SDx=&SD );

%end;          /* end DO r= */

%mend GenDistData ;

*
Input          Interpret file names as follows:
parm           = parameter file
u/b           = unbalanced / balanced DF
f/r           = focal / reference
20/30/40/80   = the number of test items on the file
1/2/3/4/6/8/16 = the number of test items with DF (the last n items on the file)
02/20/40      = the magnitude of DF (.02, .20, .40) in each item that has
DF

```

[NB - parm20 parm30 parm40 parm80, denotes files with no embedded

DF]

Output

u = unbalanced DF
 f/r = focal / reference
 20/30/40/80 = the number of test items on the file
 1/2/3/4/6/8/16 = the number of test items with DF
 02/20/40 = the magnitude of DF in each item
 k33/k11/k31 = the number of simulees in the reference/focal files for
 these conditions
 &r = the number of the replication the generated this file.

Matching the file names between the reference and focal groups controls the comparison of like test conditions. */

/* The following 216 lines invoke IRTGEN for each combination of unbalanced parameters for the focal & reference files. */

*UNBALANCED FOCAL 3000:3000 ;

%DissGenData (SEED=1265133,
 DataIn=prmtrs.parmuf20102,DataOut=results.uf20102k33&r.,Items=20,Examinees=3000);
 %DissGenData (SEED=3169540,
 DataIn=prmtrs.parmuf20120,DataOut=results.uf20120k33&r.,Items=20,Examinees=3000);
 %DissGenData (SEED=3210497,
 DataIn=prmtrs.parmuf20140,DataOut=results.uf20140k33&r.,Items=20,Examinees=3000);
 %DissGenData (SEED=2861243,
 DataIn=prmtrs.parmuf20202,DataOut=results.uf20202k33&r.,Items=20,Examinees=3000);

run ;

Appendix C PROGRAM COMBINE DATA¹⁰

```

OPTIONS linesize=80;
PROC PRINTTO      LOG= "E:\Dissertation\COMBLOG.LOG" ;

libname combined "E:\Dissertation\Combined\" ;
libname combdat  "E:\Dissertation\Combdat\" ;
libname results  "E:\Dissertation\RESULTS\" ;
run;
/*      macro CombineData combines simulated data for
           reference and focal groups
           for like conditions
           into one data set
*/

%macro CombineData (DSNref= , DSNfoc= , DataOut=output ,Items= ) ;

/* r is the number of replications from the data generation.
   Each combination of test condidtions was replicated 100 times */

%Do r = 1 %to 100 ;

Data combined.&DataOut.&r ;
  set results.&DSNfoc.&r (in=focref)
      results.&DSNref.&r      ;
  input

      If focref then group = 1;                *if from foc file set group = 1
= foc ;
      else group = 0;                          *if from ref file set group = 0 = ref ;
*      Theta2 = round(theta*100);             *fix theta for DIFAS - 2
integers ;
*      keep group case Theta2 theta r1-r&Items ;

      keep group case r1-r&Items ;

Data _NULL_ ;
  SET "E:\Dissertation\Combined\&DataOut.&r" ;
  FILE "E:\Dissertation\Combdat\&DataOut.&r..dat" ;
  PUT   @1 group   @3 case @11 r1-r&Items;

%end;          /* end DO r= */
%mend CombineData ;

```

¹⁰ Most of the macro calls at the end of the program have been deleted to conserve space.

/* Interpret file names as follows:

Input

u	= unbalanced DF
b	= balanced DF
i	= impact
f/r	= focal / reference
20/30/40/80	= the number of test items on the file
1/2/3/4/6/8/16	= the number of test items with DF
02/20/40	= the magnitude of DF in each item
k33/k11/k31	= the number of simulees in the reference/focal files

for these conditions

&r	= the number of the replication the generated this file.
----	--

Output

u	= unbalanced DF
b	= balanced DF
i	= impact
20/30/40/80	= the number of test items on the file
1/2/3/4/6/8/16	= the number of test items with DF
02/20/40	= the magnitude of DF in each item
k33/k11/k31	= the number of simulees in the reference/focal files

for these conditions

&r	= the number of the replication the generated this file.
----	--

The following lines invoke the macro that combines
the focal & reference files with like test conditions. ;

```
* UNBALANCED 3000:3000 ;
%CombineData(DSNfoc= uf20102k33 , DSNref= ur20102k33 ,
Dataout=u20102k33 , Items=20 );
%CombineData(DSNfoc= uf20120k33 , DSNref= ur20120k33 ,
Dataout=u20120k33 , Items=20 );
%CombineData(DSNfoc= uf20140k33 , DSNref= ur20140k33 ,
Dataout=u20140k33 , Items=20 );
%CombineData(DSNfoc= uf20202k33 , DSNref= ur20202k33 ,
Dataout=u20202k33 , Items=20 );
run ;
```

Appendix D PROGRAM CALIBRATE-LINK-DFIT¹¹

```

OPTIONS SPOOL linesize=80 ;
**** The next line writes the log to a data file instead of to the screen.
**** It prevents screen cache overflows which will halt the program.    ;
PROC PRINTTO LOG= "E:\Dissertation\SIMLOG.LOG" ;
PROC PRINTTO Print= "E:\Dissertation\SIMPRINT.txt" ; * save print output ;

libname prmtrs "E:\Dissertation\Parms" ;
libname results "E:\Dissertation\RESULTS" ;
libname DATFILES "E:\Dissertation\DATFILES" ;
libname BLGfiles "E:\Dissertation\BILOGfiles" ;
libname SLLinks "E:\Dissertation\LinkFiles" ;
libname ComItems "E:\Dissertation\CommonItems" ;
libname DTF "E:\Dissertation\DTF" ;

%let reps = 100 ;          ***** the number of replications to perform ;
%let tic=%str(%) ;

%put The value of tic is &tic ;

run;
/*****
**** Read SAS data file & make them .dat file for BILOG to read.
**** */
%macro MakeDatFiles (DataIn= ,Items= ) ;
/* r is the number of replications from the data generation.
   Each combination of test condidtions was replicated 100 times */

%Do r = 1 %to &reps ;

Data _NULL_ ;
    SET results.&DataIn&r ;
    FILE "E:\Dissertation\DATFILES\&DataIn&r..dat" ;
    PUT @1 case
        @5 theta
        @17 (r1-r&Items) (1.) ;

run ;

%end;          /* end DO r= */

```

¹¹ Most of the macro calls at the end of the program have been deleted to conserve space.

```

%mend MakeDatFiles ;

%macro runBILOG (Root= ,items= ) ;

/* r is the number of replications
   Each combination of test condidtions is replicated 100 times */
%Do r = 1 %to &reps ;

***** Following section modified from Jas. Algina - personal communication,
***** May 29, 2011
***** ;

data program ;
    file 'C:\Program Files\BILOGMG\dissertation.blm';
put
%str(">TITLE  logistic simulated data &root&r ") /
"    two title lines required      " /
%str(">GLOBAL DFName=&tic.E:\Dissertation\DATFILES\&root&r..dat&tic,
NPArm=3, SAVE; ") /
%str(">SAVE  SCORE=&tic.E:\Dissertation\BILOGfiles\&root&r..sco&tic ,      ") /
%str("covariance=&tic.E:\Dissertation\BILOGfiles\&root&r..cov&tic , ") /
%str("PARM=&tic.E:\Dissertation\BILOGfiles\&root&r..par&tic ; ") /
%str(">LENGTH nitems = &Items; ") /
%str(">INPUT NTOtal = &Items, NALt = 5, NIDch = 4 ; ") /
%str(">Items INUMBERS=(1(1)&Items), INAMES=(I1(1)I&Items);" ) /
%str(">TEST1 TName = itms-&r ; ") /
%str("4A1,12x,&Items.A1 ") /
%str(">CALIB cycles =100; ") /
%str(">score method=2; ") ;

***** End Algina                      ***** ;

***** Call Bilog                      ***** ;

dm " x 'E:\Dissertation\runBILOG" ;

***** End BILOG-MG call                ***** ;

run ;
%end;                                /* end DO r= */

```

```

%mend runBILOG ;

%macro runPLINK (ref= , foc= , items= ) ;

/* r is the number of replications
   Each combination of test condidtions is replicated 100 times */
%Do r = 1 %to &reps ;

***** Following section creates file to run R and plink ***** ;
*****
***** Each line of the R code does the following:
*****

***** library(lattice)                # loads lattice which plink requires ;
***** library(plink)                   # loads plink ;
***** setwd('E:/Dissertation/BILOGfiles') # set directory to read input ;
***** gr.pars <- read.bilog("&ref&r..par") # reads reference file ;
***** gf.pars <- read.bilog("&foc&r..par") # reads focal file ;
***** common <- matrix (c(1:&items,1:&items), &items, 2) # specify the common
items ;
***** #           where 1:&items = the items in common in reference & focal files
                   &items = the number of items in the
                   2 = the number of groups ;
***** pars <- combine.pars(list(gr.pars, gf.pars), common) # combine the elements
;
***** #           put parameters onto scale of first group (This is the default of "base.grp".
See p. 15 JSS article) ;
***** out <- plink(pars, rescale = 'SL', method = 'SL', ;
***** weights.t = as.weight(30, normal.wt = TRUE) ) # call to plink ;
***** conname <- link.con(out)                # extract linking constants ;
***** setwd("E:/Dissertation/LinkFiles")      # set directory to save output file
;
***** write.csv(conname,"L&foc&r..csv")      # save the
linking constants to output file ;
***** q()                                     # close R ;
***** n                                       # does not save R work space ;
***** ;

data _null_ ;
   file 'C:\Program Files\R\R-2.15.2\bin\runlink.r';
   PUT " library(lattice)"                ;
   PUT " library(plink)"                  ;
   PUT " setwd('E:/Dissertation/BILOGfiles') " ;
   PUT " gf.pars <- read.bilog('&foc&r..par') " ;
   PUT " gr.pars <- read.bilog('&ref&r..par') " ;

```

```

PUT " common <- matrix (c(1:&items,1:&items), &items, 2)" ;
PUT " pars <- combine.pars(list(gr.pars, gf.pars), common)" ;
PUT " out <- plink(pars, rescale = 'SL', method = 'SL', " ;
PUT "      weights.t = as.weight(30, normal.wt = TRUE))" ;
PUT " conname <- link.con(out)" ;
PUT " setwd('E:/Dissertation/LinkFiles') " ;
PUT " write.csv(conname,'L&foc&r..csv') " ;
PUT "q() " ;
PUT "n" ;

***** End create file to run plink ***** ;

***** run plink ***** ;
OPTIONS XWAIT XSYNC ;
X 'E:\R\R-2.15.2\bin\r.exe" CMD BATCH --vanilla --slave
  "E:\R\R-2.15.2\bin\runlink.r" ' ;

run ;
%end;          /* end DO r= */

%mend runPLINK ;

%macro runDIFCUT (focal= IF20M0S1HK11 , reference= IR20M0S1HK11 , link=
Lif20m0s1hK11 , items= , last= ) ;

/* r is the number of replications
   Each combination of test condidtions is replicated 100 times */

%Do r = 1 %to &reps ;

%INCLUDE 'E:\macroDIFCUT2stage.sas' ;
  %macroDIFCUT(foc= &focal , ref= &reference , lnk= &link ) ;

data ComItems.CI&focal&r ;
  set sigitem ;
  Itmcnt = 0 ;
  *initialize count number of non-DIF items ;
  Array sigitems (&items) $ Col1 - Col&items ;
  *read file from DIFCUT with significance codes ;
  Array linkitems (&items) $3. Link1 - Link&items ;
  *distinguish significant fm Non-significant items ;
  DO i = 1 to &items ;
    IF sigitems(i) = 'ns' or sigitems(i) = '*'

```

```

        THEN DO ;
                                linkitems(i) = i ;
                                Itmcnt = Itmcnt + 1 ;
                                END ;
        ELSE sigitems(i) = '' ;

        END ;
        DROP i Col1 - Col&items ;

        Data DTF.D&focal&r ;
                                * file with DTF values ;
        Set DTF_file ;

%end;                                /* end DO r= */

%mend runDIFCUT ;
%macro runPLINK2 (ref= , foc= , items= ) ;

/* r is the number of replications
   Each combination of test condidtions is replicated 100 times */
%Do r = 1 %to &reps ;

***** read the file containing item numbers for non-DIF items and blank for DIF items ;
data link ;
    set ComItems.CI&foc&r ;
    LENGTH linkvar1 300 ;                                ***** unless length is
set, CATX function

    sets length = 200 which is too short for

    80 character tests (23MR2014) ;
    CALL SYMPUT ('itemcount',Itmcnt) ;                    ***** create macro variable w
# items to link ;

    ***** concatenate all input variables, removing extra blanks, into a single variable
[linkvars] ;
    linkvar1 = CATX(", ", of Link1 - Link&items ) ;

    ***** create a macro variable from the non-DIF items ;
    CALL SYMPUT ('links',linkvar1) ;

run ;

***** Following section creates file to run R and plink ***** ;
*****

```

***** Each line of the R code does the following:

```
***** library(lattice)           # loads lattice which plink requires ;
***** library(plink)             # loads plink ;
***** setwd('E:/Dissertation/BILOGfiles') # set directory to read input ;
***** gr.pars <- read.bilog("&ref&r..par") # reads reference file ;
***** gf.pars <- read.bilog("&foc&r..par") # reads focal file ;
***** common <- matrix (c(&links), &itemcount, 2) # specify the common items
& number of common items ;
***** #           where &links = the items in common in reference & focal files
                &items = the number of items in the
                2 = the number of groups
                his information to format the matrix for plink to calculate linking constants;
***** pars <- combine.pars(list(gr.pars, gf.pars), common) # combine the elements
;
***** #           put parameters onto scale of first group (This is the default of <base.grp>.
See p. 15 JSS article) ;
***** out <- plink(pars, rescale = 'SL', method = 'SL',           ;
*****           weights.t = as.weight(30, normal.wt = TRUE) ) # call to plink --SL =
Stocking-Lord method ;
***** conname <- link.con(out) # extract linking constants ;
***** setwd("E:/Dissertation/LinkFiles") # set directory to save output file
;
***** write.csv(conname,"L&foc&r..csv") # save the
linking constants to output file ;
***** q() # close R ;
***** n # does not save R work space ;
***** ;
```

```
data _null_ ;
      file 'E:\R\R-2.15.2\bin\runlink.r';
PUT " library(lattice)"           ;
PUT " library(plink)"             ;
PUT " setwd('E:/Dissertation/BILOGfiles') " ;
PUT " gf.pars <- read.bilog('&foc&r..par') " ;
PUT " gr.pars <- read.bilog('&ref&r..par') " ;
PUT " common <- matrix (c(&links), &itemcount, 2)" ;
PUT " pars <- combine.pars(list(gr.pars, gf.pars), common)" ;
PUT " out <- plink(pars, rescale = 'SL', method = 'SL', " ;
PUT "           weights.t = as.weight(30, normal.wt = TRUE) )" ;
PUT " conname <- link.con(out)" ;
PUT " setwd('E:/Dissertation/LinkFiles') " ;
PUT " write.csv(conname,'L&foc&r..csv') " ;
PUT "q() " ;
PUT "n" ;
```

```
***** End create file to run plink ***** ;
run ;
```

```
***** run plink ***** ;
```

```
OPTIONS XWAIT XSYNC ;
X ' "E:\R\R-2.15.2\bin\r.exe" CMD BATCH --vanilla --slave
  "E:\R\R-2.15.2\bin\runlink.r" ' ;
```

```
run ;
```

```
%end; /* end DO r= */
```

```
%mend runPLINK2 ;
```

```
*****
The following code are comments describing system of naming files, and
the invocations of the above macros
*****;
```

```
* Interpret file names as follows:
```

```
DF
  L = when present indicates the file containing
the Stocking-Lord Linking constants
  u/b = unbalanced / balanced DF
  f/r = focal / reference
  20/30/40/80 = the number of test items on the file
  1/2/3/4/6/8/16 = the number of test items with DF (the last n items on the
file)
  02/20/40 = the magnitude of DF (.02, .20, .40) in each item
that has DF
  k33/k11/k31 = the number of simulees in the reference/focal files
for these conditions
  &r = the number of the replication that
generated this file.
```

```
IMPACT
```

```
Dist N(0,1) N(0,1)
      N(.5,1)
      N(1,1)
      N(0,1.5)
      N(0,2.0)
```

```

/*          Interpret file names as follows:
           i          = impact
           f/r        = focal / reference
           20/30/40/80 = the number of test items on the file
           m          = mean by the above distributions,
where h = 0.5
           s          = standard deviation by the above
distributions, where h = 0.5
           k33/k11/k31 = the number of simulees in the
reference/focal files for these conditions
           k33/k11/k31 = the number of simulees in the
reference/focal files for these conditions
           &r         = the number of the replication the
generated this file.

```

Matching the file names between the reference and focal groups controls the comparison of like test conditions.

```
*/
```

```

*****
*****

```

The following 516 lines call the macro that converts SAS files into .dat file for entry into BILOG-MG.

First are the 216 lines for the unbalanced DIF conditions

```

*****
*****

```

```
*UNBALANCED FOCAL 3000:3000 ;
```

```
%MakeDatFiles (Datain=uf20102k33,Items=20 );
```

```
%MakeDatFiles (Datain=uf20120k33,Items=20 );
```

```
%MakeDatFiles (Datain=uf20140k33,Items=20 );
```

```
%MakeDatFiles (Datain=uf20202k33,Items=20 );
```

```
%MakeDatFiles (Datain=uf20220k33,Items=20 );
```

```
%MakeDatFiles (Datain=uf20240k33,Items=20 );
```

```
%MakeDatFiles (Datain=uf20402k33,Items=20 );
```

```
run
```