Essays on Risk Management of Insurance Companies

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BY

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ACCEPTANCE

This dissertation was prepared under the direction of the Qianlong Liu Dissertation Committee. It has been approved and accepted by all members of that committee, and it has been accepted in partial fulfillment of the requirements for the degree of Doctor of Philosophy in Business Administration in the J. Mack Robinson College of Business of Georgia State University.

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ABSTRACT

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This dissertation is devoted to studying the risk management of insurance companies. It focuses on risk management behaviors such as investment, portfolio management, and derivative use. The risk involved in the study includes the risk caused by adverse selection, investment risk, and interest rate risk. It also compares the risk-taking attitude among insurers with different ownership statuses.

The first chapter studies competitive equilibria of an insurance market with adverse selection where insurers' investments are risky. It characterizes the incentive-efficient allocations of the economy. Competitive equilibria are not incentive-efficient because of the externality imposed by agents' incentive constraints that cannot be completely internalized in the Walrasian economy. Specifically, the equilibrium allocations achieved in the market can only be ones with restricted subsidization although incentive-efficient allocations feature unrestricted subsidization when the proportion of low-risk insurees is large. Finally, it demonstrates incentive-efficient allocations in

the new economy with investment risk can be successfully decentralized in the Walrasian market affiliated with consumption rights that internalize the externality imposed by agents' incentive constraints.

The second chapter studies duration-matching behavior by life insurers in response to interest rate movements. Previous literature finds the evidence that the duration gap between assets and liabilities is negatively correlated to interest rates, which is interpreted as ex post duration adjustment. It further hypothesizes that value-maximizing life insurers have incentives to make pre-emptive matching in anticipation of future interest rates. It uses the term spread of the yield curve as the indicator of expected change in interest rates and show that U.S. life insurers actively shorten the durations of their asset portfolios reacting to an increasing in both the interest rate level and expected future interest rates proxized by the term spread. To identify the causal relation, it exploits the monetary policy shock associated with 2013 taper tantrum in the difference-in-differences analysis. Consistent with the hypotheses, there is a negative impact of the tape tantrum on the asset duration of highly exposed life insurers comparing with insurers with low interest rate risk exposure.

The last chapter is derived from the study about duration matching behaviors. It investigates the role that the ownership status of life insurers plays in the attitude of risk-taking. Mutual ownership is of interest because of the dramatic evolution of the mutual market and its large market share. Following previous studies of capital advantage in different ownership types, the paper suggests that mutual insurers are more conservative in taking interest rate risk relative to stock insurers. Evidence from the U.S. life insurance industry shows that U.S. life insurers maintain a smaller duration gap and react more intensively to interest rate risk. After considering different corporate structures, the argument remains consistent in that mutual corporations take less interest rate risk than two types of stock corporations, mutual holding corporations and stock corporations.

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This work is dedicated to my parents and brother, to people who I care or care about me, and to Atlanta.

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1.1 Introduction

The question of how adverse selection affects competitive equilibria of insurance markets has been widely studied since Rothschild and Stiglitz [RS] (1976) formed the fundamental model to describe an insurance economy with adverse selection. RS show that a competitive equilibrium may not exist and, if it does, may not be incentive efficient. Subsequent studies revisit RS's model by suggesting different equilibrium concepts that incorporate the strategic behavior of insurers (Wilson (1977), Miyazaki (1977) and Spence (1978)). Bisin and Gottardi [BG] (2006)). BG adopt the approach of Prescott and Townsend (1984), and show that the introduction of a market for "consumption rights" internalizes the externality imposed by agents' incentive constraints in the RS economy and, thereby, achieves incentive efficiency. All the aforementioned studies pioneered by RS focus on the liability side of insurers' balance sheets and, in particular, abstract away from insurers' *asset risk*. Because insurers' assets are risk-free in the above frameworks, insurer default plays no role in the analyses.

We extend the analysis of Bisin and Gottardi (2006) to incorporate insurers' asset risk. We examine how the possibility of insurer default affects competitive equilibria, the incentive efficient allocations of the economy, and the decentralization of the allocations with the introduction of new markets that internalize the externalities in the original insurance economy. In particular, we show that incentive-efficient allocations could feature unrestricted subsidization. We then demonstrate that the Externality-Prescott-Townsend (EPT) economy, in the terminology of BG, can only partially internalize the externality imposed by incentive compatibility constraints because it can only decentralize allocations with restricted subsidization. Finally, we show that the Arrow-Lindahl-Prescott-Townsend (ALPT) economy can completely solve the externality issue by introducing consumption rights in the market.

The presence of asset risk for insurers changes the indemnity structure from a fixed payment, as in the fundamental model proposed by RS, to a *payment schedule* that is contingent on the realization of asset shocks. The incentive-efficient allocations of insurees, thereby, have some new characteristics that we

[2] This charpter is a joint work with Dr. Ajay Subramanian.

explore by analyzing equilibrium allocations in the EPT economy. We show that the subsidization between different types of insurees has unrestricted form. More precisely, at any incentive-efficient allocation the expected value of the subsidy is non-zero when the fraction of low-risk insurees is low. The EPT economy only partially solves the externality problem by decentralizing a special RS separated allocation with the restricted subsidy, the expected value of which is zero. Hence, when the unrestricted subsidization is a necessity for incentive-efficient allocations, the EPT economy is not an ideal decentralization mechanism, while the ALPT economy can completely internalize the externality by trading consumption rights in the market. The consumption right in the ALPT economy actually functions as a channel through which insurees who impose externality on others have to make a compensation in the form of trading such rights. We extend the primary results about the ALPT equilibrium in Bisin and Gottardi (2006) to our scenario where investment activities are considered. We finally show that the ALPT equilibrium converges to the Miyazaki-Wilson [MW] allocation if all initial endowment of consumption rights are allocated to low-risk insurees.

The setup of the economy, including the characteristics of insurees, the insurance policy structure and information symmetry, is consistent with those in the model by RS. We introduce investment risk in the model through the presence of a risky asset in which insurance companies can invest the capital they raise via insurance premia. Since the investment return of insurers varies with the return of the risky asset, the indemnity payment becomes contingent on the investment risk. Specifically, in the model the risky asset has two possible returns: "high" and "low". The incorporation of asset risk complicates the nature of possible subsidization between different types of insurees as a result of which the expected value of the subsidy is not enough to parametrize the subsidization. Because contracts must now be contingent on insurers' asset returns, we require a two-dimensional subsidy vector to characterize the subsidization in our scenario.

We first demonstrate that, given a subsidy level, in any incentive-efficient allocation, the high-risk insuree is optimally insured at the subsidy level and the low-risk insuree is sub-optimally insured at the subsidy level conditional on the incentive constraint of the high-risk type. We then examine the effects of altering the subsidy so that it becomes increasingly favorable for high-risk insurees on the efficiency of the corresponding allocation. The RS separating and pooling allocations, and the MW allocation are still important to demarcate the set of incentive-efficient allocations even in the presence of asset risk. The RS separating allocation is the equilibrium allocation when there is no subsidization. But the RS separating allocation is not efficient when the fraction of low-risk insurees is large. In that case, the presence of the subsidy that is favorable to high-risk insurees can support more efficient allocations. The incentive-efficient allocation with the lowest level of subsidy is referred as to the MW allocation where low-risk insurees obtain their maximal utility among all feasible and incentive-compatible allocations. As the subsidy becomes more and more favorable to high-risk insurees, thereby tightening the incentive constraint of low-risk insurees, the incentive-efficient allocation at the point corresponds to the one with the highest level of subsidy, which could be the RS pooling equilibrium[3].

We then decentralize incentive-efficient allocations in the EPT economy and, thereby, highlight the nature of the cross-subsidization in incentive efficient allocations. Since the EPT equilibrium allocations have to be *fair*, there are only two possibilities for the nature of subsidization in equilibrium. The first type of equilibrium entails complete non-subsidization among insurees and the second is cross-subsidization with zero expected value that we refer to as non-subsidization in expected value. The equilibrium allocation with no cross-subsidization has the form of the RS separated allocation. The subsidization with zero expected value—restricted subsidization—could improve the equilibrium efficiency compared with the no subsidy case. In particular, cross-subsidization with zero expected value here represents the situation where high-risk insurees in the high-return state offer a positive subsidy to, and in the low-return state acquire positive subsidy from, the low-risk insurees. Despite having a zero expected value, this cross-subsidization smoothes the difference between the indemnities of high-risk insurees in the high- and the low-return states. It, thereby, improves the welfare of the high-risk type. Low-risk insurees then have chance to improve welfare by purchasing more insurance since the incentive constraint of the high-risk insurees is relaxed. But it is only possible when the proportion of low-risk insurees is large enough. This is because it is less costly for each of the low-risk insurees in a larger group to subsidize a smaller group of high-risk insurees. When the cost for low-risk insurees to subsidize high-risk insurees whose welfare thereby is improved is low enough, low-risk insurees are able to improve the welfare by increasing their insurance coverage. As for the implementation of the two types of equilibrium allocations just discussed, we accordingly suggest two types of insurance

[3]It details the situation when the RS pooling allocation is the equilibrium with the highest level of subsidy and when it is not in the section 3

contracts to decentralize them. The equilibrium allocation in the case of complete non-subsidization can be decentralized by trading the insurance policy with the fixed ratio of indemnities in the EPT economy. The insurance contract with the free ratio of indemnities can be employed to achieve equilibrium allocation with restricted subsidy.

A further question is whether the subsidy with non-zero expected value (unrestricted subsidization) can support more efficient allocations than restricted subsidization. We add a testing constraint that the expected subsidy of low-risk insurees is less and equal to zero into the MW programming to verify this issue. Consistent with the similar intuition underlying the fact that restricted subsidy could improve the RS separating allocation, it is proved that unrestricted subsidy is necessary to support incentive-efficient allocations when the proportion of low-risk insurees is large enough. After completing the characterization of subsidization, we apply the ALPT economy into our new scenario to decentralize incentive-efficient allocations. An equilibrium price vector is shown to exist, under which the first and second welfare theorem hold in the ALPT economy.

The remainder of the paper is organized as following. Section 2 introduces the setup of the economy and the insurance model with investment risk. Section 3 defines the incentive-efficient allocation and demonstrates its property and association with subsidization. Section 4 employs the EPT economy to decentralize incentive-efficient allocations and decomposes the subsidization structure between high- and low-risk insurees in the context of the new insurance model. Section 5 analyzes the failure of the EPT economy as a way of decentralization and further explores the subsidy structure in incentive-efficient allocations. Finally, in section 6 incentive-efficient allocations are successfully decentralized in the ALPT economy where consumption rights serve the key role to internalize the externality. Section 7 concludes.

1.2 Setup and a Benchmark Model

This is a one-period model of a competitive insurance market. At $t = 0$, insurers sell insurance to insurees and make investments with the fund from equity capital and premium. At $t = 1$, damage occurs to insurees, and insurers pay indemnity to insurees.

Insurers are assumed to be risk-neutral. They invest premium collected from insurees at $t = 1$ into a risky asset with a state-contingent rate of return $\tilde{R}$. The rate of return $\tilde{R}$ is equal to r in the high-return state and d in the low-return state with $r > 1$ and $d < 1$. The low-return state happens with the probability q. And the expected return of the risky asset $E\tilde{R} = (1-q)r + qd$ is greater than 1 that is assumed to be the risk-free rate. Because of the uncertainty in the investment return, it is possible that the available assets, including the investment return owned by insurers at $t = 1$ (before paying indemnity), are not able to cover all the claims they are faced with. Insurers have limited liability.

Insurees owning the initial wealth w at $t = 0$ are subjected to only one type of risk of the loss l at $t = 1$ with the probability p. Insurees purchase insurance against their loss risk. Assume there are a finite number of insurees in the market and the number of insurees who incur the loss satisfies the Law of Large Number. Insurees have von Neumann–Morgenstern expected utility function. Utility functions satisfy Inada conditions.

In the economy, it is populated by a continuum of insurees with two types, indexed by g and b. The fractions of different insurees are denoted by ξ^i with $i = \{b, g\}$. They differ in the probability that loss may incur, which is assumed to be $p^g < p^b$. They are privately informed of their types. Insurers seek to screen insurees by offering them exclusive contracts that are designed specifically for each type of insurees. The insurance contract indexed by the type of insurees can be characterized by (P^i, c_H^i, c_L^i) where for the insuree i, P^i is premium, and (c_H^i, c_L^i) are indemnities in the high-return and the low-return state.

So there are three possible states for insurees according to the realization of investment return and the occurrence of the loss. They are the state when no damage occurs, the state when damage occurs with the realization of the high-return on the risky asset and the state when losses occur with the realization of the low return from the risky asset. The set of states is denoted by S. It has $s \in S \equiv \{ND, DH, DL\}$ for an arbitrary state. Let π_s^i be the probability that an individual state s is realized for an insuree of type i. The net transfers in different states for insurees after purchasing insurance can be summarized in the table below.

State s	Probability π_s^i	Net transfer z_s^i
ND	$\pi_{ND}^i = 1 - p^i$	$z_{ND}^i = -P^i$
DH	$\pi_{DH}^i = p^i(1-q)$	$z_{DH}^i = -P^i + c_H^i$
DL	$\pi_{DL}^i = p^i q$	$z_{DL}^i = -P^i + c_L^i$

An arbitrary insurance contract $(P^i, c^i_H, c^i_L) \in R^3$ of an insuree of type i can be equivalently represented by a net transfer vector $\mathbf{z}^i = (z^i_{ND,} z^i_{DH}, z^i_{DL}) \in R^3$.

1.3 Incentive-efficient allocations:

Feasible allocations Feasible allocations: It is optimal that the premium collected is invested in the risky asset since the expected return from the risky asset exceeds the risk-free return. All return on the investment of premium and equity capital is used to pay indemnity to insurees. Equity capital requires a rate of return equal to the expected rate of return of the risky asset $E\tilde{R}$. All these leaves insurers are earning zero expected profit in the competitive market. Therefore, the investment return from investing premium is equal to the expected indemnity in the equilibrium.

As competitive insurers earning zero expected-profit, a feasible allocation requires that the expected value of net transfers of all insurees is less and equal to zero. A feasible allocation described by net transfers of the both types $\{\mathbf{z}^g, \mathbf{z}^b\}$ satisfies the following resource feasibility constraint:

$$\sum_{i \in \{g,b\}} \xi^i [\pi^i_{ND}(-P^i E\tilde{R}) + \pi^i_{DH}(-P^i r + c^i_{DH}) + \pi^i_{DL}(-P^i d + c^i_{DL})] \leq 0.$$

Denoting by net transfer vector $\{\mathbf{z}^g, \mathbf{z}^b\}$, it has

$$\sum_{i \in \{g,b\}} \xi^i [(E\tilde{R} - \pi^i_{DH} - \pi^i_{DL}) z^i_{ND} + \pi^i_{DH} z^i_{DH} + \pi^i_{DL} z^i_{DL}] \leq 0; \tag{1.1}$$

and

$$E\tilde{R} - \pi^i_{DH} - \pi^i_{DL} = (1-q)(r - p^i) + q(d - p^i).$$

The ***Expected value of subsidy*** for an insuree i receiving (offering) can be represented by VT^i defined as:

$$VT^i(\mathbf{z}^i) = (E\tilde{R} - \pi^i_{DH} - \pi^i_{DL}) z^i_{ND} + \pi^i_{DH} z^i_{DH} + \pi^i_{DL} z^i_{DL}.$$

Note that a binding feasible allocation implies $VT^i = -VT^j$ for $i, j \in \{g, d\}$ and $i \neq j$.

No equity capital case: When there is sufficient equity capital initially held by insurers, insurees are allowed to freely choose arbitrary allocations in the three states as long as the feasibility constraint satisfies the condition of zero expected profit with respect to all states. Insurers do not necessarily earn zero profit when risky asset realizes the high return. The same goes for the case when the risky asset realizes the low return. This is because the existence of equity capital can transfer value between the case of high investment return and the case of low investment return.

Now, if there is no equity capital case, the zero-profit condition with respect to all states still is required to hold. But more feasibility constraints should be added because of the absence of equity capital. The investment return of premium, which is contingent on the realization of the risk asset, should be able to exactly cover the expected claims, which may also differ. Specifically, when the high (or low) investment return is realized, the investment return should be able to pay the expected indemnity in the corresponding state. In other words, for an insurer keeping the total premium unchanged, it is impossible to increase the indemnity paid in the state DH (or DL) at the expense of decreasing the indemnity paid in the state DL (or DH) . The value transfer channel between the two states is not accessible without equity capital. The two feasibility constraints just described are as follows.

$$\sum_{i\in\{g,b\}} \xi^i[p^i(z^i_{DH} - z^i_{ND}) + z^i_{ND}r] \leq 0, \tag{1.2}$$

and

$$\sum_{i\in\{g,b\}} \xi^i[p^i(z^i_{DL} - z^i_{ND}) + z^i_{ND}d] \leq 0. \tag{1.3}$$

Note that with the two in equations holding, the zero-profit condition (1.1) holds as well.

Now start to formally characterize efficient allocations involved in adverse selection.

Definition 1. An incentive-efficient allocation is an allocation $\{\mathbf{z}^g, \mathbf{z}^b\}$ that satisfies the following conditions: (a) it is feasible; (b) it is incentive compatible; (c) there does not exist another feasible and incentive-compatible allocation $\{\hat{\mathbf{z}}^g, \hat{\mathbf{z}}^b\}$ such that $\{\hat{\mathbf{z}}^g, \hat{\mathbf{z}}^b\}$ is an Pareto improvement for $\{\mathbf{z}^g, \mathbf{z}^b\}$, that is, $U^b(\hat{\mathbf{z}}^b) \geq U^b(\mathbf{z}^b)$ and $U^g(\hat{\mathbf{z}}^g) \geq U^g(\mathbf{z}^g)$, with at least one inequality being strict.

Characterization of subsidizations The set of incentive efficient allocations can be parametrized by the per-capita subsidy that insurees of one type pay to insurees of the other type. Let $IT^i_{s'}$ be the per-capita subsidy transferred from insurees of type i to the insurees of type j in the state s' where $i, j \in \{g, d\}$ and $j \neq i$, and $s' \in \{DH, DL\}$. Formally, $IT^i_{s'}$ is defined by the following resource feasibility constraints:

$$p^i(z^i_{DH} - z^i_{ND} + IT^i_{DH}) + z^i_{ND} r = 0$$

and

$$p^i(z^i_{DH} - z^i_{ND} + IT^i_{DL}) + z^i_{ND} d = 0.$$

The feasibility constraint of subsidy can be derived from the feasibility constraints of net transfers (1.2) and (1.3):

$$\sum_i \xi^i p^i IT^i_{s'} \geq 0,$$

for $s' = \{DH, DL\}$.

Therefore, an arbitrary subsidization can be specified by the subsidy vector $IT^i \equiv (IT^i_{DH}, IT^i_{DL})$, for $i \in \{g, b\}$.

Proposition 2. *At an arbitrary incentive-efficient allocation which corresponds to the subsidy vector* IT^i*, at least one of the two types of insurees is optimally insured at the subsidy level. The other type of insurees is* **sub-optimally insured** *at the subsidy level conditional on the incentive constraint of the former type.*

Proof. See Appendix. □

Building on the two claims above, two classes of incentive-efficient allocations can be classified according to which type of insurees have the optimal net transfers at the subsidy vector IT^i. Then, at an arbitrary incentive-efficient allocation where the insuree i is optimally insured, the insuree j is required to maximize his or her utility at the corresponding subsidy vector conditional on the incentive constraint of the insuree i being binding. For convenience, we refer to insuree j as being ***sub-optimally insured***. The analysis in the paper focuses on the case that it is insurees of the type b who is optimally insured. It will be explained the other class of incentive-efficient allocation is counterfactual.

The property of incentive-efficient allocations in the claim 2 actually supplies a method to solve out incentive-efficient allocation at a given IT^i. Take an example of an arbitrary subsidy vector IT^i supporting an incentive-efficient allocation. The insuree b is optimally insured at IT^i so z^b in the allocation is solved from:

$$\max_{\{z^b\}} : U^b(\mathbf{z}^b) = \sum_{s\in S} \pi_s^b u(\omega_s + z_s^b)$$
$$s.t. : \begin{cases} p^b(z_{DH}^b - z_{ND}^b + IT_{DH}^b) + z_{ND}^b r \leq 0, \\ p^b(z_{DH}^b - z_{ND}^b + IT_{DL}^b) + z_{ND}^b d \leq 0. \end{cases}$$

The insuree g is sub-optimally insured at IT^i conditional on the incentive constraint of the insuree b being binding. It gives:

$$\max_{\{z^g\}} : U^g(\mathbf{z}^g) = \sum_{s\in S} \pi_s^g u(\omega_s + z_s^g)$$
$$s.t. : \begin{cases} p^g(z_{DH}^g - z_{ND}^g + IT_{DH}^g) + z_{ND}^g r \leq 0, \\ p^g(z_{DL}^g - z_{ND}^g + IT_{DL}^g) + z_{ND}^g d \leq 0, \\ U^b(z^g) = U^b(z^b). \end{cases}$$

The correspondence between incentive-efficient allocations and subsidy vectors, it is convenient to use IT^i to parameterize incentive-efficient allocations. In the following paragraphs, we will elaborate upon how the incentive-efficiency of allocations changes with the level of subsidization. Three important allocations that have been widely cited will be introduced. They are the Rothschild-Stiglitz (RS) separating allocation, the Miyazaki-Wilson (MW) allocation, and the RS pooling allocation.

(1) Start from the case where there is no cross-subsidization between the two types of insurees. The insuree g cannot get optimal insured as the insuree b does. This is because the insuree g utility level is limited by the incentive constraint of the insurees b that require the type b insurees prefer $\mathbf{z}^b$ to $\mathbf{z}^g$. Instead, the insuree g maximizes his or her utility conditional on the incentive constraint of the insuree b being binding. The optimal allocation in this case has the form of RS separating allocation. From previous work, whether the RS separating allocation is incentive-efficient depends on the ratio between the two types of insurees. A high proportion of type g insuree in the population makes it possible to improve both utilities of the two types by subsidizing type b insurees.

(2) Then, consider that type g agent make positive subsidy to type b agent. The utility of the insuree b, $U^b(\mathbf{z}^b)$, can be increased accordingly, thereby relaxing the incentive constraint of the insuree b. This means the incentive compatibility constraint no longer prohibits type b insuree from obtaining a higher utility. However, whether $U^g(\mathbf{z}^g)$ is able to increase also depends on ξ^g the proportion of insurees of type g. This is because, on the one hand, the positive subsidy from the insuree g to the insuree b relaxes the incentive constraint of the insuree b. On the other hand, the subsidization tightens the budget constraint of the insuree g. If ξ^g is high enough, the increasing of $U^b(\mathbf{z}^b)$ is possible with the increasing of $U^g(\mathbf{z}^g)$. It means that in this case the RS separating allocation is not incentive efficient. With a small enough positive subsidy from the insuree g to the insuree b there exists another set of $(U^g(\hat{\mathbf{z}}^g), U^b(\hat{\mathbf{z}}^b))$ which is a Pareto improvement for the RS separating allocation.

(3) As the subsidy from type g insurees to type b insuree increases, the utility of the insuree g achieve its maximum. At this point it corresponds to the MW allocation. The programming to solve out the MW allocation could be helpful to generate more intuition. In the context of this paper, the MW programming has the following form:

$$\max_{\{z^g, z^b\}} : U^g(\mathbf{z}^g) = \sum_{s \in S} \pi_s^g u(\omega_s + z_s^g) \tag{1.4}$$

It is subjected to the constraints:

$$\sum_{i \in \{g,b\}} \xi^i [p^i(z_{DH}^i - z_{ND}^i) + z_{ND}^i r] \leq 0, \tag{1.5}$$

$$\sum_{i \in \{g,b\}} \xi^i [p^i(z_{DL}^i - z_{ND}^i) + z_{ND}^i d] \leq 0, \tag{1.6}$$

$$U^b(\mathbf{z}^g) \leq U^b(\mathbf{z}^b). \tag{1.7}$$

The constraint 1.5 and (1.6) are general feasibility constraints for the market. The constraint (1.7) is the incentive constraint of the insuree b. (The incentive constraint of the insuree g is ignored since it is not binding at the optimum.) It is necessary to emphasize that the MW allocation coincides with the RS separated allocation when the proportion of type g insurees is not large enough for such subsidy to support a Pareto-improvement on the RS separated allocation.

Now it is ready to analyze the incentive-efficiency of allocations parameterized by subsidy levels which is illustrated in the Figure 1. If the RS separated allocation is incentive efficient as in panel (a), as the subsidy level increases from zero, the utility of the insuree g decreases and the utility of the insuree b increase. So the allocations generated by such subsidy are still incentive efficient until the incentive constraint of the insuree g becomes binding. After that, type g insurees would deviate to the allocation of type b insurees. If the RS separated allocation is not incentive efficient as in the panel (b), the subsidy level increasing from zero is not able to support an incentive-efficient allocation until it reaches to the level corresponding to the MW allocation. Afterwards, the utility of the insuree g starts to increase and the utility of the insuree b keeps deceasing, which implies the incentive-efficient allocation starts to exist after the subsidy achieves to the MW level.

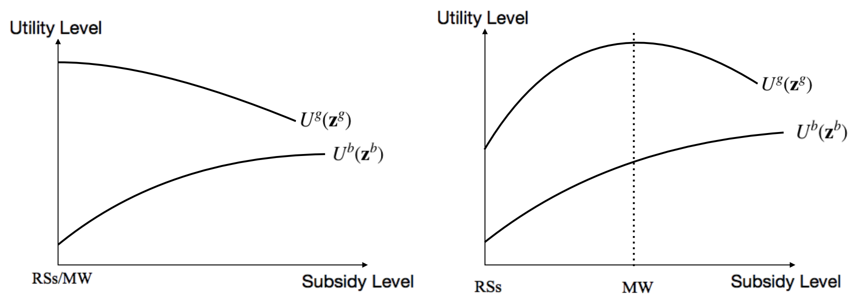
(4) As the subsidy becomes more and more favorable to type b insuree, the utility of them increases accordingly. There exists a level of subsidy such that the incentive constraint of type b insuree is becoming non-binding.

The other case of incentive-efficient allocations requires type g insuree is optimally insured. Type b insuree is sub-optimally insured subjected to the binding constraint of type g insuree. In order to separate the two types of insurees, type b insurees have to be overinsured so that they have higher consumption in the states of losses. Otherwise, they would deviate to type g's allocation. It is counterfactual, so we focus on the former case in the paper.

1.4 EPT Equilibrium

The concept of competitive equilibrium uses the idea of Walrasian equilibrium in which insurees and insurance companies act as price takers. At the quoted price, they choose optimal consumption and production allocations so that utility and profit are maximized. In the equilibrium, markets achieve clearing. However, the concept of Walrasian equilibrium here also has its special feature because of the environment of adverse selection. To separate the different types of insurees the contracts offered in the markets have to be incentive

Figure 1.1: Incentive efficient allocations and subsidization



(a) RSs and MW coinciding

(b) RSs and MW not coinciding

Figure 1.—Incentive efficiency of allocations around the level of subsidization from type g insurees to type b insurees. RSs denotes the RS separated allocation and MW denotes the MW allocation.

compatible. Specifically, the trade that one type of insuree can make is subjected to the incentive constraint conditional on the trade that the other of type insurees make. So the restriction of incentive compatibility imposes an extra constraint on the consumption set of insurees.

Now we consider such Walrasian equilibrium to be associated with the economy structure proposed by Prescott and Townsend (1984), which is referred to as EPT equilibrium.

1.4.1 EPT Economy

Admissible allocation The section will describe and then formalize the admissible net transfers that insurees can make in the Walrasian equilibria economy. At first, in the market, there are two types of contracts indexed by the type of insurees. They are designed respectively for the two corresponding types of insurees. After insurees declare their types, they can only trade in the market designed for the type. So each insuree would either trade net transfers designed for the insuree g or ones designed for the insuree b. They cannot participate in the traders for the two types at the same time. Secondly, their allocations have to be incentive compatible. For instance, given the allocation of the insuree who claims to be type b, the admissible allocation for the insuree g should satisfy the incentive constraint of the market for type b. That is, insuree b would not deviate from the allocation it chooses to the allocation designed for the insuree g. Finally, non-negativity of consumption is assumed.

Let the allocation of each insuree chooses be $\mathbf{z} \equiv \{z_g, z_b\} \in R^6$ where $z_i \equiv \{z_{i,ND}, z_{i,DH}, z_{i,DL}\}$ for $i, j \in \{g, d\}$. $\mathbf{z}$ represents the amount of the six commodities of net transfers an insuree can trade. $\bar{\mathbf{z}}_g$ and $\bar{\mathbf{z}}_b$ represent the net transfers made by the insurees who claim to be type g and b, respectively. Accordingly, $\bar{\mathbf{z}}_g = (\bar{z}_g, 0)$ and $\bar{\mathbf{z}}_b = (0, \bar{z}_b)$.

Definition 3. Given the net transfers $\bar{z}_g$ and $\bar{z}_b$ made in the two markets, the set of admissible net transfers for each type of insurees, $Z(\bar{\mathbf{z}}_g, \bar{\mathbf{z}}_b)$, is represented by $\mathbf{z} \equiv \{z_g, z_b\}$ that satisfies following conditions: (a) $z_i \neq 0$ implies $z_j = 0$ for all $i = \{g, b\}$ and $i \neq j$; (b) $\mathbf{z} + (\omega, \omega) \gg \mathbf{0}$ where $w = (w_{ND}, w_{DH}, w_{DL})$; and (c) incentive-compatible constraints,

$$\sum_{s \in S} \pi_s^i u(\omega_s + \bar{z}_s^i) \geq \sum_{s \in S} \pi_s^i u(\omega_s + z_s^j). \tag{1.8}$$

The admissible set of allocations just defined gives the all possible traders insurees can make in the economy. Taking a price vector $\mathbf{q} \equiv \{q_g, q_b\}$ where $q_i = \{q_{i,ND}, q_{i,DH}, q_{i,DL}\}$ as given, insurees make their choices within the admissible set to optimize utility. For the insuree i,

$$\begin{aligned} \underset{z_i \in Z(\bar{z}_g, \bar{z}_b)}{max:} \quad & U^i(z_i) = \sum_{s \in S} \pi_s^i u(\omega_s + z_s^i) \\ s.t.: \quad & \mathbf{z}^i \cdot \mathbf{q} \leq \mathbf{0}. \end{aligned}$$

Insurer's technology: In the framework, insurers that offer insurance contracts are described as supplying net transfers contingent on the insuree's types and the realizations of uncertainty as to the damage. The technology that specifies the relation between input and output of the six contingent commodities of net transfers is consistent with the feasibility constraints of insurees. This is because feasible allocations for insurees are defined to be accessible allocations of net transfers in the sense of resources.

As there is no equity capital, the premium collected by insurers directly determines the maximum indemnity they can pay in the state DH. The same goes for the state DL. In other words, it is impossible, for instance, to increase the indemnity of the state DH by reducing the indemnity of the state DL without the change of premium. Let $\mathbf{y} \equiv \{y_g, y_b\}$ denote the supply vector of net transfers of contingent commodities on a per capita basis, where $y_i = (y_{i,ND}, y_{i,DH}, y_{i,DL})$ for $i = \{g, b\}$. The technology can be characterized by Y, a set of supply vector $\mathbf{y}$, such that $\mathbf{y}$ satisfies:

$$\sum_{i \in \{g,b\}} [p^i(y^i_{DH} - y^i_{ND}) + y^i_{ND} r] \leq 0,$$

and

$$\sum_{i \in \{g,b\}} [p^i(y^i_{DL} - y^i_{ND}) + y^i_{ND} d] \leq 0.$$

Under the technology, the insurers choose the optimal supply to earn maximal profit taking a certain set of prices of contingent commodities as given:

$$\max_{\mathbf{y} \in Y} : \mathbf{y} \cdot \mathbf{q}.$$

Now it is ready to define Walrasian equilibrium in the economy. The equilibrium here is referred to as EPT equilibrium following Bisin and Gottardi (2006). Assume insurees with the same type choose the symmetric allocations.

Definition 4. An EPT equilibrium is characterized by the net transfers of the two types of insurees choose, $(\mathbf{z}^g, \mathbf{z}^b)$, the supply of net transfer of insurers, $\mathbf{y}$, and a price vector $\mathbf{q}$ such that given the net transfers $(\bar{\mathbf{z}}_g, \bar{\mathbf{z}}_b)$ made by insurees who claim to be the two types of insurees:

- $\mathbf{z}^g$ and $\mathbf{z}^b$ is the optimal choices of the insuree g and b, respectively, for the given $(\mathbf{q}, \bar{\mathbf{z}}_g, \bar{\mathbf{z}}_b)$;
- $\mathbf{y}$ solves the maximization problem of insurers given $\mathbf{q}$;
- Markets clear under the choices of all insurees and insurers. $\sum_i \xi^i \mathbf{z}^i \leq \mathbf{y}$;
- the given net transfers $(\bar{\mathbf{z}}_g, \bar{\mathbf{z}}_b)$ is consistent with the two types of insurees' optimal choices $(\mathbf{z}^g, \mathbf{z}^b)$, namely, $\bar{\mathbf{z}}_g = \mathbf{z}^g$ and $\bar{\mathbf{z}}_b = \mathbf{z}^b$.

If insurers can make a positive profit by trading with only one type of insurees, then, they have the incentive to supply infinite large amount of the insurance to such type of insurees. That is because it would give insurers unbounded positive profit under the fact of constant return to scale technology. Therefore, in the equilibrium both of two types contracts designed for the two types of insurees have actuarial fairness price. Combining this point with the property of competitive market, it gives the condition of EPT equilibrium.

Lemma 5. *At an EPT equilibrium, the premium of each insurance contract has to be actuarially fair*

$$(E\tilde{R} - \pi^i_{DH} - \pi^i_{DL})z^i_{ND} + \pi^i_{DH}z^i_{DH} + \pi^i_{DL}z^i_{DL} = 0, \tag{1.9}$$

for $i = \{g, b\}$*. It follows the prices of the contingent commodities:*

$$q^i = (E\tilde{R} - \pi^i_{DH} - \pi^i_{DL}, \pi^i_{DH}, \pi^i_{DL}),$$

for $i = \{g, b\}$*.*

The fair premium here means the premium of each contract is equal to the expected value of indemnities in all possible states. This result also implies the expected value of subsidy for insuree i, VT^i has to be zero in equilibrium for $i = \{g, b\}$. The following analysis will further explore the nature of the EPT equilibrium building on the condition of fair premium. We will state that according to the state of subsidization, two cases can be divided given the fair premium condition

1.4.2 Complete non-subsidization

One case of fair premium is the two types of contracts can be offered by insurers separately. It means the premiums received from the contracts designed for type g insurees is only used to pay indemnities for type g insurees. The same goes for the insuree b. That is, there is no indemnity transferred at all between the two types of contracts in any possible states. Here we refer to the case as ***complete non-subsidization*** for conveniency. In this case the subsidization vector $IT^i \equiv (0, 0)$ for $i \in \{g, b\}$.

The insurance contract with fair premium corresponding to this case satisfies the conditions represented by the notation (P^i, c^i_H, c^i_L) :

$$E\tilde{R}P^i = (1-q)p^i c^i_H + qc^i_L,$$

and

$$rP^i = p^i c^i_H, dP^i = p^i c^i_L,$$

for $i = \{g, b\}$. The first condition derives from the fair premium. The second one implies there is no cross-subsidization by indicating the indemnity for each type of insurees comes from the investment return of premium collected from the insurees of this type. It immediately follows the ratio of indemnities in the two states is fixed, equal to $\frac{r}{d}$. For one unit of premium paid to purchase such contract, the indemnity payment guaranteed is $(\frac{r}{p^i}, \frac{d}{p^i})$ for the two states. Such contracts that are consistent with complete non-subsidization are referred as to ***insurance with a fixed ratio of indemnities.***

EPT equilibrium under the insurance with the fixed ratio of indemnities Now we demonstrate that the EPT equilibrium generated accordingly has the form of the RS separating equilibrium. Offered such contract, the insuree b chooses the optimal allocation at the actuarially fair price.

$$\max_{\{z^b\}} : U^b(\mathbf{z}^b) = \sum_{s \in S} \pi^b_s u(\omega_s + z^b_s)$$
$$s.t. : \begin{cases} (E\tilde{R} - \pi^b_{DH} - \pi^b_{DL}) z^b_{ND} + \pi^b_{DH} z^b_{DH} + \pi^b_{DL} z^b_{DL} \leq 0, \\ d(z^b_{DH} - z^b_{ND}) = r(z^b_{DL} - z^b_{ND}). \end{cases}$$

The optimal allocation satisfies:

$$\begin{cases} (1-p^b)u'(\omega_{ND} + z^b_{ND}) = (1-q)(r-p^b)u'(\omega_{DH} + z^b_{DH}) + q(d-p^b)u'(\omega_{DL} + z^b_{DL}), \\ (d-p^b) z^b_{DH} = (r-p^b) z^b_{DL}. \end{cases}$$

When the damage occurs, the insuree b's consumption levels satisfies $\omega_{DL} + z^b_{DL} < \omega_{DH} + z^b_{DH}$ ($\omega_{DH} = \omega_{DL}$ since the amount of loss is a constant). This is the result of the uncertainty of the return from the risky asset and the absence of equity capital. Therefore, if the insurers are subjected to complete non-subsidization, the insuree b has to bear some risk themselves.

Then, type g insurees would choose the allocation such that the insuree g would not deviate from his or her own allocation. So type g insurees can only get sub-optimally insured conditional on the incentive constraint of type g insurees being binding. And they also bear some risk themselves when the damage occurs. The optimal contract of the insuree g can be solved from:

$$\max_{\{z^g,z^b\}} : U^g(\mathbf{z}^g) = \sum_{s\in S} \pi_s^g u(\omega_s + z_s^g)$$
$$s.t.: \begin{cases} U^b(z^g) = \sum_{s\in S} \pi_s^b u(\omega_s + z_s^g) \leq U^b(z^b), \\ (E\tilde{R} - \pi_{DH}^g - \pi_{DL}^g) z_{ND}^g + \pi_{DH}^g z_{DH}^g + \pi_{DL}^g z_{DL}^g \leq 0, \\ d(z_{DH}^g - z_{ND}^g) = r(z_{DL}^g - z_{ND}^g). \end{cases}$$

Now we argue the optimal allocation $\{\mathbf{z}^g, \mathbf{z}^b\}$ obtained above and the set of fair prices of net transfers $q^i = (E\tilde{R} - \pi_{DH}^i - \pi_{DL}^i, \pi_{DH}^i, \pi_{DL}^i)$ constitute an EPT equilibrium. Since the insurance contracts purchased by both types of insurees have fair premium, insurers earn expected zero-profit. Markets of contingent commodities clear because contracts have a fair price and there is complete no subsidy between different types of them. Moreover, the two programmings above guarantee that both of types of insurees maximize their utilities given the choices of the others. So all of these together indicates that the price vectors $\mathbf{q} = (q^g, q^b)$ is a Walrasian equilibrium and $\{\mathbf{z}^g, \mathbf{z}^b\}$ is the equilibrium allocation.

Therefore, if insurance policies offered in the market are ones with a fixed ratio $\frac{r}{d}$ of indemnities, the Walrasian equilibrium has the form of the RS separating allocation. Both of the insuree g and b bear some risk themselves in the equilibrium.

1.4.3 Non-subsidization in expected value

The other case of fair premium is that indemnity transferring is allowed between the two types of contracts but conditional on the expected value of subsidy VT^i being zero. For instance, in the state DH the contract designed for the insuree g subsidizes the contract b while in the state DL the contract b subsidizes the contact g. However, the expected value of the net transfer VT^i is still equal to zero. Thus, subsidy vector IT^i is subjected to for $i = \{g, b\}$:

$$(1-q)IT^i_{DH} + qIT^i_{DL} = 0.$$

We refer to this case as ***non-subsidization in expected value***. We can see complete non-subsidization is a special case of non-subsidization in expected value. We will refer to $IT^i_{s'}$ as the ***indemnity subsidy*** from i to j in the state s' in the following content.

Then, we demonstrate whether and when such cross-subsidization can support an allocation that is a Pareto improvement on the case of complete non-subsidization. If there exits such allocation, how an EPT equilibrium can implement it.

As the increasing of indemnity subsidization IT^g_{DL} from zero, the utility that insuree b can obtain would keep increasing until the two contingent consumptions reach the same level, namely, $w_{DH} + z^b_{DH} = w_{DL} + z^b_{DL}$. This is because indemnity subsidy in the state DL for type b insurees can smooth the difference of consumption levels between state DH and DL, which supports a higher level of utility. When the consumptions of the state DH and DL reach the same level, the utility of the insuree b achieves its maximum.

The change of the optimal utility of the insuree g as the increasing of IT^g_{DL} is not so evident as that of the insuree b. On the one hand, the incentive constraint of type b insurees is relaxed because they obtain a higher level of utility. It follows that the insuree g can obtain a higher utility by purchasing more insurance. On the other hand, the indemnity subsidization in the bad investment state from the insuree g to b worsens the indemnity distribution of the insuree g between the good and bad investment states. It makes the difference of indemnities of the insuree g in the two states becomes larger, thereby undermining the maximal utility the insuree g can obtain. If the optimal utility of the insuree g gets improved because of the indemnity subsidy, the new pair of allocations after such subsidy indeed is a Pareto improvement on the case of complete non-subsidization.

An intuitive conjecture is that the insuree g makes a positive subsidy in the state DL to the insuree b at the maximum of the utility when the ratio of different types of insurees satisfies some condition. To verify this conjecture, we solve the following problem:

$$\max_{\{z^g, z^b\}} : U^g(\mathbf{z}^g) = \sum_{s \in S} \pi_s^g u(\omega_s + z_s^g) \ . \tag{1.10}$$

It is subjected to the constraints:

$$\sum_{i \in \{g,b\}} \xi^i [p^i (z_{DH}^i - z_{ND}^i) + z_{ND}^i r] \leq 0, \tag{1.11}$$

$$\sum_{i \in \{g,b\}} \xi^i [p^i (z_{DL}^i - z_{ND}^i) + z_{ND}^i d] \leq 0, \tag{1.12}$$

$$U^b(\mathbf{z}^g) \leq U^b(\mathbf{z}^b), \tag{1.13}$$

$$(E\tilde{R} - \pi_{DH}^i - \pi_{DL}^i) z_{ND}^i + \pi_{DH}^i z_{DH}^i + \pi_{DL}^i z_{DL}^i = 0, \text{ for } i = \{g, b\}, \tag{1.14}$$

$$IT_{DL}^g \geq 0. \tag{1.15}$$

The constraints (1.11) and (1.12) are general feasibility constraints for the market. The constraint (1.14) guarantees that the two types of contracts have fair premium. The constraint (1.13) is the incentive constraint of the insuree b. (The incentive constraint of the insuree g is ignored since it is not binding at the optimum.) The inequality (1.15) is a test constraint. If it is slack at optimum, it implies there is a positive subsidy in the state DL from the insuree g to the insuree b at the optimal allocations.

Lemma 6. *At the optimum of the problem (1.10), feasibility constraint (1.11) and (1.12), and the incentive constraint of the high-risk insuree (1.13) are binding. The indemnity subsidy constraint (1.15) is slack when the proportion of type g insurees is large enough.*

Proof. See Appendix. □

When the proportion of type g insurees is large enough, a proper subsidy with expected value of zero as just described can make a Pareto improvement on the case of complete non-subsidization. Type b insurees have a more smooth allocation in different states because of such subsidy from type g insurees. It follows that type g insurees are able to increase their insurance coverage without violating the incentive constraint of the insuree b.

The extreme case appears that the insuree b bears no risk when the proportion of the insuree g is large enough to a certain extent. That is, the insuree b obtains the same indemnity level at the state DH and DL. At that point, the insuree b achieves the maximal level of utility she or he can obtain by purchasing a contract with fair premium. Then, we demonstrate that with a contract with the free ratio of indemnities in the two states an EPT equilibrium can be implemented that corresponding to the extreme case.

The ***insurance contract with the free ratio of indemnities*** (P^i, c_H^i, c_L^i) satisfies the condition:

$$E[\tilde{R}]P^i = (1-q)p^i c_H^i + qc_L^i,$$

for $i = \{g, b\}$. Compared with the contract with the fixed ratio, this kind of contact only requires to have an fair premium.

EPT equilibrium under contacts with the free ratio of indemnities Offered such contacts, the insuree b can choose the indemnities level in the two states freely. The optimization problem they faced with now is subjected to a less strict constraint than that imposed by the contract with the fixed ratio of indemnities.

$$\begin{aligned} &\max_{\{z^b\}} : U^b(z^b) = \sum_{s \in S} \pi_s^b u(\omega_s + z_s^b) \\ &s.t. : (E\tilde{R} - \pi_{DH}^b - \pi_{DL}^b) z_{ND}^b + \pi_{DH}^b z_{DH}^b + \pi_{DL}^b z_{DL}^b \leq 0. \end{aligned} \tag{1.16}$$

Because of the less strict constraint, we can conclude safely the optimal utility in this case is higher than (or equal to) the case where insurees are offered contracts with the fixed ratio of indemnities. Solving the problem the first order condition gives the specific result:

$$\begin{cases} \pi_{ND}^b u'(\omega_{ND} + z_{ND}^b) = (E\tilde{R} - \pi_{DH}^b - \pi_{DL}^b) u'(w_{DH} + z_{DH}^b), \\ z_{DH}^b = z_{DL}^b. \end{cases}$$

where the indemnities in the state DH and DL reach the same level. In this case, the indemnity that the insuree b receives in the bad investment state is subsidized by the insuree g , namely, IT^g_{DL}, which is equal to $(z^b_{DL} - z^b_{ND}) + z^b_{ND} d/p^b$.

The optimal choice of the insuree g now is chosen from the admissible set which is subjected to the choice of the insuree g.

$$\begin{aligned} &\max_{z_g \in Z(\cdot, \bar{z}_b)} : U^g(z^g) = \sum_{s \in S} \pi^g_s u(\omega_s + z^g_s) \\ &s.t. : (E\tilde{R} - \pi^g_{DH} - \pi^g_{DL}) z^g_{ND} + \pi^g_{DH} z^g_{DH} + \pi^g_{DL} z^g_{DL} \leq 0. \end{aligned} \tag{1.17}$$

The admissible set actually requires that the utility of the insuree g be upper-bounded by the incentive constraint of the insuree b. So equivalently, the two constrains below gives the same result with the problem of the insuree g.

$$\begin{cases} U^b(z^g) = \sum_{s \in S} \pi^b_s u(\omega_s + z^g_s) = U^b(z^b), \\ (E\tilde{R} - \pi^g_{DH} - \pi^g_{DL}) z^g_{ND} + \pi^g_{DH} z^g_{DH} + \pi^g_{DL} z^g_{DL} = 0. \end{cases}$$

The EPT equilibrium under contracts with the free ratio of indemnities is also the one that has the form of the RS separating equilibrium. Type b insurees obtain the same indemnity at the state DH and DL as the result of the subsidy. They bear no risk. Type g insurees also benefit from the subsidy because they can access a larger set if admissible allocations after the subsidization.

1.4.4 Welfare properties of the EPT equilibria:

The second welfare theorem holds for the EPT economy, which is to say all incentive-efficient allocation can be implemented in the EPT economy with a feasible transfer. Let $\mathbf{t}^i = (t^i_{DH}, t^i_{DL})$ denote the transfer vector that the insuree i receives for $i \in \{g, b\}$. A feasible transfer satisfies $t^g_{s'} + t^b_{s'} = 0$ for $s' \in \{DH, DL\}$.

Proposition 7. *An arbitrary incentive-efficient allocation* $(\mathbf{z}^g, \mathbf{z}^b)$ *can be obtained at the EPT equilibrium under insurance contracts with the fixed ratio of indemnities if insurees receive a transfer* $\mathbf{t}^i$ *that is consistent with the subsidy vector* IT^i *at* $(\mathbf{z}^g, \mathbf{z}^b)$*, namely,* $\mathbf{t}^i = -IT^i$.

The process to find and verify an EPT equilibrium for an arbitrary incentive-efficient allocation $(\mathbf{z}^g, \mathbf{z}^b)$

is as follows. From the property of incentive-efficient allocations, we know that the insuree b is optimally insured and the insuree g is sub-optimally insured at subsidy vector IT^i. There is an statement that is equivalent to the property. That is, receiving a transfer consistent to the subsidy vector IT^i, the optimal allocation for the insuree b who is offered the fair contract with the fixed ratio of indemnities is $\mathbf{z}^b$. Conditional the choice of the insuree b, the optimal choice for the insuree g is $\mathbf{z}^g$ if the same type of contract designed for them is offered. As $(\mathbf{z}^g, \mathbf{z}^b)$ is feasible, markets clear in the EPT economy. Therefor, the EPT economy reaches the equilibrium in which the actuarial-fairness price is the equilibrium price and $(\mathbf{z}^g, \mathbf{z}^b)$ is the equilibrium allocation.

On the contrary, the first welfare theorem does not hold. This is, the equilibrium derived from the EPT economy is not always incentive efficient. We will use the next chapter to explain where the externality that results in the inefficiency comes from, why the EPT economy cannot internalized such externality.

1.5 Externality and Efficiency

The externality in the economy is rooted in the incentive constraint imposed on the allocations of low-risk insurees. They cannot get optimally insured because it would make high-risk insurees deviate from their own allocations by announcing to be the low type. So the consumption of type b insurees actually have an externality that would affect the amount of consumption type g insurees can access. However, such externality is not able to be internalized in the EPT economy. This is because in the EPT economy, both types of insurees are only offered the contracts with fair premium at the equilibrium. This effect of the externality caused by the incentive constraint is not able to get compensated through the trading of contingent commodities with fair prices in the EPT economy. So it is not surprise that competitive Walrasian equilibria obtained in the last chapter may not be efficient.

We have seen allowing the subsidy with zero expected value can improve the welfare of the both types of insurees who consume allocations without any subsidization. Now consider to get rid of the restricted condition of subsidization that VT^i is equal to zero. (This implies the premium of contracts does not have to be fair.) For convenience, we call the cross-subsidization that is only required to be feasible ***unrestricted subsidization***. We will argue in this chapter that efficient allocations may rely on the unrestricted subsidization, especially when the proportion of the low-risk insurees is large. In this case, the EPT economy in which unrestricted subsidization is not accessible at the equilibrium cannot support the efficient allocation.

In the Chapter of Incentive-Efficient allocations, we have stated the general idea that cross-subsidization may improve the RS separating allocation which implies subsidization is necessary to support incentive-efficient allocation in some cases. Now we will explicitly specify how the different types of cross-subsidization can affect the efficiency of the allocation.

We have seen in the last chapter relaxing the subsidization constraint from complete non-subsidization to non-subsidization in expected value may improve the efficiency. Despite having a zero expected value, the subsidy for the insuree b smoothes the difference of indemnities between the good and bad investment state. So the utility of the insuree b becomes be able to increase. It follows the incentive constraint of the insuree b is relaxed, thereby offering the chance for insuree g to obtain a higher utility level. Whether $U^g(\mathbf{z}^g)$ is able to increase under such restricted subsidization depends on ξ^g the proportion of insurees of type g. It has showed that the welfare of both types of insurees can be improved when the proportion of insuree g is large compared with the case of complete non-subsidization.

Then, move on to unrestricted subsidization. In order to check whether the unrestricted subsidization is necessary to realize incentive-efficient allocations, we employ the approach referred to as Miyazaki-Wilson programming. The approach gives the incentive-efficient allocation in which insuree g obtains the highest utility among all incentive-efficient allocations:

$$\underset{\{z^g,z^b\}}{max}: U^g(\mathbf{z}^g) = \sum_{s\in S} \pi_s^g u(\omega_s + z_s^g) \ . \tag{1.18}$$

It is subjected to the constraints:

$$\sum_{i\in\{g,b\}} \xi^i [p^i(z_{DH}^i - z_{ND}^i) + z_{ND}^i r] \leq 0, \tag{1.19}$$

$$\sum_{i\in\{g,b\}} \xi^i [p^i(z_{DL}^i - z_{ND}^i) + z_{ND}^i d] \leq 0, \tag{1.20}$$

$$U^b(\mathbf{z}^g) \le U^b(\mathbf{z}^b), \tag{1.21}$$

$$VT^g \le 0. \tag{1.22}$$

The constraint (1.19) and (1.20) are general feasibility constraints for the market. The constraint (1.21) is the incentive constraint of the insuree b. (The incentive constraint of the insuree g is ignored since it cannot be binding at the optimum.) The constraint (1.22) is used to check whether the expected value of subsidy from insuree g to b is greater than zero. If this testing constraint is not binding at optimum, it implies the incentive-efficient allocation needs supporting by unrestricted subsidization.

Lemma 8. *At the optimum, feasibility constraint* (1.19) *and* (1.20) *and the incentive constraint of the high-risk insuree* (1.21) *are binding. The indemnity subsidization constraint* (1.22) *is slack when the proportion of insuree* g *is large.*

Proof. See Appendix. □

If ξ^g is larger enough, the subsidy with VT^i being zero as the case of non-subsidization in expected value is not able to support an incentive-efficient allocation. Unrestricted subsidization needs to get involved.

The incentive-efficient allocation that the WM problem (1.18) solves out is the one in which insuree g has the highest utility. The subsidy vector at the optimum corresponds to the subsidy with the lowest expected value from insuree g to b that can support an incentive-efficient allocation. As the increasing of the expected value of the subsidy from such level, the welfare of insuree g will go down and comparatively the welfare of the insuree b will go up. The allocations obtained in this way are still incentive efficient until the incentive constraint of the insuree g becomes binding. After that, more subsidy to insurees b will make insurees g deviate from their own allocation.

1.6 ALPT Equilibrium

In the EPT economy, the insurance policies offered to insurees are only ones with fair premium. So it is impossible to decentralize incentive-efficient allocations in the EPT economy when the unrestricted subsidization which may result in the premium is not actuarially fair exists in such allocations Bisin and Gottardi (2006) suggested introducing a special commodity–consumption right–in to the EPT economy in order to enlarge the admissible set of insurance commodities.

1.6.1 ALPT Economy

Incentive compatible constraints and consumption rights: Since the consumption of insurance commodities of one type of insuree can impose an externality on the consumption of the other type through the incentive-compatible constraint, it is necessary to compensate the insurees who are negatively affected by such externality should we try to implement efficient allocations in the market involved in adverse selection. So as markets for pollution rights can internalize environmental externalities, consumption rights introduced here play the same role to internalize the externality caused by the consumption through the incentive constraint.

We first introduce the economy incorporating consumption rights, referred to as the ALPT economy by Bisin and Gottardi (2006). Then, we demonstrate that by bringing the new commodity into the market the admissible set of insurees are enlarged so that incentive-efficient allocations are accessible for them in the ALPT economy.

The market structure that includes consumption rights can be described by three main components. 1) the choice set of insurees which includes how much consumption rights are held; 2) the specification of the initial endowments allocation and the producing technology of consumption rights; 3) the enforcement mechanism specifying how much consumption rights needed to hold for a given amount of the consumption of contingent insurance transfers.

Besides consuming the contingent insurance transfers, insurees now can choose as well how much consumption right to hold so as to guarantee their consumption of the contingent transfers. Assume there are initially total $\omega\xi^i$ units of consumption i in the market. Each insurees i holds $(1-\alpha)\omega$ units of consumption right i consumption rights and $(\xi^j/\xi^i)\alpha\omega$ units of consumption right j. They can trade their endowment in the market of consumption rights.

The trade of insurance commodities can produce more consumption rights that can be traded in the market. Specifically, for the units z^i of insurance that one insuree i trades, it produces an additional $(\xi^j/\xi^i)z_i$ units of consumption right j for the insuree. As a result, for each of insurees j there is z_i more units of the right j that are available to purchase in the market.

Let $\zeta^i \equiv \{\zeta^i(g), \zeta^i(b)\}$ denote the net trade of these two types of consumption rights for the insuree i. So the amount of consumption rights i that the type i insurees hold, represented by $\chi^i(i) \equiv \{\chi_{ND}(i), \chi_{DH}(i), \chi_{DL}(i)\} \in R^3_+$, has the relations:

$$\chi^i(i) = \zeta^i(i) + (1-\alpha)\omega,$$

and

$$\chi^i(j) = \zeta^i(j) + \frac{\xi^j}{\xi^i}\alpha\omega + \frac{\xi^j}{\xi^i}z^i.$$

The enforcement mechanism requires that in order to consume the insurance commodities of the amount $\omega + z^i$, the insuree i has to hold the amount of consumption rights that is subjected to:

$$\sum_{s\in S}\pi^j_s u(\chi_s(i)) \geq \sum_{s\in S}\pi^j_s u(\omega_s + z^i_s). \tag{1.23}$$

The admissible set which now includes the choice of consumption rights as well is defined as following.

Definition 9. The set of admissible trades of every insuree Z is given by the vectors $\mathbf{z} \equiv \{z_g, z_b\} \in R^6$ and the vector $\zeta \equiv \{\zeta(g), \zeta(b)\} \in R^6$ such that:

- $\mathbf{z} + \omega \gg \mathbf{0}$;
- for $i, j \in \{g, b\}$ and $i \neq j$, if $z_i \neq 0$, then $z_j = 0$. For i such that $z_i \neq 0$, the following condition holds:
  - $(1-\alpha)\omega + \zeta(i) \geq \mathbf{0}$, $\zeta(j) + (\xi^j/\xi^i)(\alpha\omega + z_i) \geq 0$;
  - $\sum_{s\in S}\pi^j_s u(\zeta^i_s(i) + (1-\alpha)\omega_s) \geq \sum_{s\in S}\pi^j_s u(\omega_s + z_{i,s})$.

Additional attentions need to be paid on the first condition in terms of net transfer of consumption rights. It requires that the amount of holding of rights be non-negative. The rest of conditions of the admissible set is similar to ones in the EPT economy.

The insuree's problem in the ALPT economy now becomes to maximize utility by choosing the allocation of insurance transfers and as well of consumption rights from the admissible set.

$$\begin{aligned} \underset{(\mathbf{z}^i,\zeta^i)\in Z(\bar{z}_g,\bar{z}_b)}{max:} \quad & U^i(z_i) = \sum_{s\in S} \pi_s^i u(\omega_s + z_s^i) \\ s.t.: \quad & \mathbf{z}^i \cdot \mathbf{q} + \zeta^i \cdot \mathbf{p} \leq \mathbf{0}, \end{aligned}$$

where $\mathbf{p} \equiv \{p(g), p(b)\}$ in which $p(i) \equiv \{p_{ND}(i), p_{DH}(i), p_{DL}(i)\}$, for $i = \{g, b\}$, is the price vector of consumption rights.

The insurer's problem and technology is the same as before. At the equilibrium of the new economy, consumption rights is market clear besides other equilibrium conditions. The corresponding equilibrium defined by Bisin and Gottardi (2006) is as following.

Definition 10. An ALPT equilibrium is characterized by the net transfers of insurance commodities and consumption rights $(\mathbf{z}^i, \zeta^i)$ for each type of insuree, the product vector $\mathbf{y}$ for insurers, the price vectors $(\mathbf{q}, \mathbf{p})$ for insurance commodities and consumption rights and trades level of insurance commodities in the market $\bar{z}_i$ for $i \in \{g, b\}$ and $i \neq j$ such that:

- $(\mathbf{z}^i, \zeta^i)$ is the optimal choices of the insuree i for the given $(\mathbf{q}, \mathbf{p}, \bar{\mathbf{z}}_j)$;
- $\mathbf{y}$ solves the maximization problem of insurers given $\mathbf{q}$;
- markets clear under the choices of all insurees and insurers for insurance commodities and consumption rights, namely $\sum_i \xi^i \mathbf{z}^i \leq \mathbf{y}$ and $\sum_i \xi^i \zeta^i \leq \mathbf{0}$;
- the trade level of insurance in the market, $\bar{z}_i$ is consistent with the two types of insurees' optimal choices z^i, namely, $\bar{z}_i = z^i$.

The most obvious difference about the setup between the EPT economy and the ALPT economy focuses on the consumption rights which, therefore, to changes the enforcement method of the incentive compatibility.

However, allocations at equilibrium in the ALPT economy actually satisfy the same incentive-compatible constraints as in the EPT economy. For instance, the maximization of the utility of the insuree b, together with the market-clearing conditions at equilibrium, implies that the holding of consumption rights g for the insuree g, $\chi_s(g)$, is equal to the consumption of insurance commodities for the insuree b, $\omega + z^b$. The same goes for the insuree b. It, then, immediately follows that at equilibrium the enforcement condition of consumption rights (1.23) is actually equivalent to the incentive compatible constraints (1.8).

On the other hand, the set of possible equilibrium allocation is enlarged compared with the one in the EPT economy where equilibrium allocations are only limited to ones with zero expected subsidy, $VT^i = 0$. In the ALPT economy, it is the existence of consumption rights that offers a channel to carry out subsidization between the different types of insurees., as a result of which possible equilibrium allocations can be arbitrage ones as long as they are incentive compatible and budget feasible.

On this basis, the ALPT equilibrium allocations are the same as the ones obtained when insurees are free to choose any incentive compatible and budget feasible bundle in the market designated for their own type. In the early version in the year 2000 and 2004 of Bisin and Gottardi (2006), they verify this argument and use it to prove the existence of the ALPT equilibrium. Following the similar reasoning, we have the existence of the ALPT equilibrium in our context:

Theorem 11. *There is always an ALPT equilibrium in which equilibrium prices are as follows:*

$$q_{i,ND} = (E\tilde{R} - \pi^i_{DH} - \pi^i_{DL}), q_{i,s'} = \pi^i_{s'}, \tag{1.24}$$

and

$$p_{ND}(g) = \beta(\alpha)(E\tilde{R} - \pi^b_{DH} - \pi^b_{DL}), p_{s\prime}(g) = \beta(\alpha)\pi^b_s, p_s(b) = 0, \tag{1.25}$$

where $i \in \{g,b\}$, for $s' \in \{DH,DL\}$ and $s \in \{ND,DH,DL\}$, and $\beta(\alpha) \in (0, \xi^b/\xi^g)$. (In fact, $p_s(g) = \beta(\alpha)q_{b,s}$.)

Proof. See Appendix. □

The prices of insurance commodities is the same as the equilibrium price in the EPT economy. This is, insurees still trade insurance commodities the same way as they trade the insurance with fair premium. The price of consumption rights b being zero comes from the fact that type b insurees are not restricted by the incentive constraint because they have no incentive to deviate to the allocation designed for type g insurees. On the contrary, type g insurees have to pay for consuming insurance commodities g, which is resulted from the need to the internalization of the externality imposed by the incentive constraint of type g insurees. Finally, the price of consumption rights is associated with the amount of initial endowment allocated to insurees. It varies with different the allocation plan of consumption rights. However, the parameter β is supposed to be less than ξ^b/ξ^g. Otherwise, it would be the case that type b insurees are able to free to any insurance allocation. This is because for type b insurees the gain from trading consumption rights b produced is large enough to self finance the expenditure on insurance commodities.

Since the MW allocation is more of the interest, we care about how to achieve the MW allocations in the ALPT economy. It is natural to consider the scenario where the initial endowment of consumption rights g to the insuree b, α, is zero. Because this is the case that the subsidy from type b insurees to type g insurees is minimal.

Proposition 12. *Under the price vector (1.24) and (1.25), the ALPT equilibrium allocations approaches to the MW allocation as α goes to $\mathbf{0}$. Namely, $lim_{\alpha \to 0} \mathbf{z}^i(\alpha) = \mathbf{z}^i_{MW}$ for $i \in \{g, b\}$.*

Proof. See Appendix. □

1.6.2 The implementation approach

We will give a simplified approach to implement ALPT equilibrium and then, demonstrate incentive-efficient allocations can be decentralized by the ALPT equilibrium.

- The enforcement mechanism only requires low-insurees to hold certain amount consumption right to guarantee their consumption of insurance transfers, not high-risk insurees. This is a valid simplification because only the incentive compatible constraint of high-risk insurees is binding at equilibrium. That is, only the consumption of low-risk insurees is restricted by the externality imposed by incentive constraints. So in the market of consumption rights only rights for insurance g will be considered to trade.

- All initial endowment of consumption rights, which is consumption rights for insurance g, are owned by type g insurees. Each of them has w unit of such rights so that they are able to consume their initial wealth without purchasing extra consumption rights.

- Consumption rights are required to purchase by insurance firms to back the sale of insurance contracts, instead of the case that insurees trade consumption rights among themselves.

1.6.3 Welfare properties of ALPT Equilibria

The first welfare theorem holds well in the ALPT economy. The proof of the proposition can use the standard form of proof that is used to prove the Walrasian equilibrium allocation is Pareto-efficient in the general equilibrium framework. Here is he idea behind the proof.

Proposition 13. *All ALPT equilibria are incentive efficient.*

Proof. See Appendix. □

Since any incentive-efficient allocation can be decentralized in the EPT economy with an initial transfer allocation. It is straightforward to extend the second welfare theorem in the EPT economy scenario to the ALPT economy.

Proposition 14. *An arbitrary incentive-efficient allocation* $(\mathbf{z}^g, \mathbf{z}^b)$ *can be obtained at the ALPT equilibrium under insurance contracts with the fixed ratio of indemnities if insurees receive a transfer* $\mathbf{t}^i$ *that is consistent with the subsidy vector* IT^i *at* $(\mathbf{z}^g, \mathbf{z}^b)$*, namely,* $\mathbf{t}^i = -IT^i$*, and* $\alpha = 0$*.*

The process to verify the existence of the ALPT equilibrium for incentive-efficient allocations is similar as that in the EPT economy. For any incentive-efficient allocation $(\mathbf{z}^g, \mathbf{z}^b)$, $\mathbf{z}^i$ is still the best possible choice for the insuree i in the ALPT equilibrium if the insuree is offered transfer $\mathbf{t}^i$. With the insuree i choosing the allocation $\mathbf{z}^i$, the market clears for insurance commodities and consumption rights. Therefore, $(\mathbf{z}^g, \mathbf{z}^b)$ is also an ALPT equilibrium.

1.7 Conclusion

We have extended the study of the competitive insurance market with adverse selection to incorporating investment activities. With the new setup of our model, we explore the structure of incentive-efficient allocations, in particular, with respect to the subsidization structure. We then analyze how the EPT economy and the ALPT economy behave as to the decentralization of incentive-efficient allocations. The EPT economy fails to completely internalize the externality stemming from the incentive-compatible constraint, but the decentralized allocations obtained in the EPT economy are helpful for revealing the property of subsidization of incentive-efficient allocation. We show that unrestricted subsidy is necessary to support incentive-efficient allocations when the proportion of low-risk insurees is very large. Finally, we demonstrate that in the ALPT economy, unrestricted subsidy can be decentralized through the trading of consumption rights. Moreover, the first and second welfare theorems hold for the ALPT economy.

We can improve the study from two aspects in the future. First, equity capital can be included in the insurance model so that the investment activity of insurers would be more completely described. It will surely add complexity to the analysis of incentive allocations and their decentralization, but it seems worthwhile to make such a tradeoff to make the model more intuitive. The second aspect is to simplify or improve the mechanism of consumption rights in the ALPT economy.

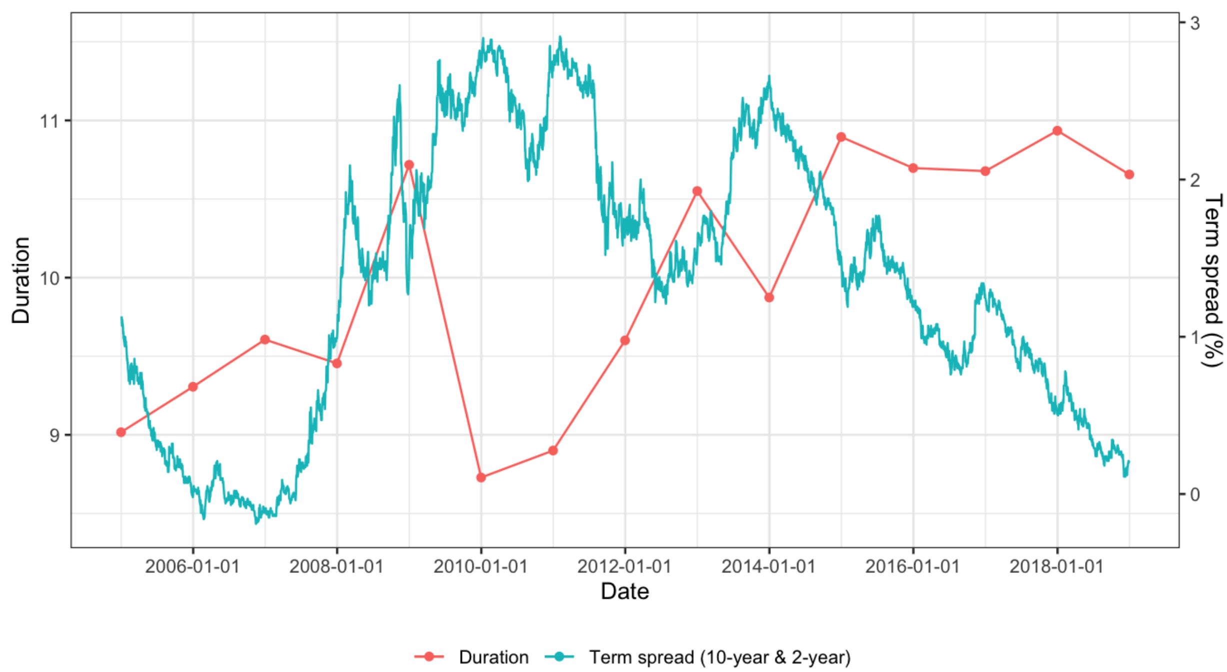
2 Interest Rates and the Duration Matching of Life Insurance Companies

2.1 Introduction

Life insurers are exposed to interest rate risk because of the duration mismatch between assets and liabilities. Interest rate risk management aimed at bridging the duration gap can potentially immunize their portfolios against fluctuations in interest rates. Regulators and rating agencies also exert pressure on insurers to manage their risk exposures. The National Association of Insurance Commissioners (NAIC) ties risk-based capital (RBC) surcharges to interest rate risk. The rating agency, A.M. Best considers the surplus value of insurers as an important indicator of financial strength. To hedge the interest rate risk exposure, life insurers strategically manage asset portfolios, which primarily consist of bonds, to align the durations of their assets with liabilities. Sophisticated life insurers also trade interest rate derivatives to appropriately manage their asset risk exposures.

Figure 2.1: Average duration of industrial bond holdings

The figures illustrates the evolution of the aggregate duration of life insurance companies with interest rate movements. The red dots indicates the average duration of the bond holding of the life insurance industry. The blue curve is the term spread of between 10-year and 2-year treasury yields.



Given the asset-liability portfolios of life insurers are typically mismatched[4], how life insurers' matching behavior reacts to interest rate movements is important and of interest. Previous literature finds the evidence that asset duration is negatively associated with interest rates. Domanski et al. (2017) documents that German life companies hunted for the longer duration assets when long-term interest rates in Europe fell sharply in 2014 to historically low levels. Ozdagli and Wang (2019) show the same hedging pattern in the U.S. life insurance industry. The aforementioned studies interpret the matching pattern negatively associated with interest rates as *ex post* duration adjustment. However, life insurers still sustain surplus loss upon downward interest rates although *ex post* matching duration can the enlarged duration gap afterward. Such surplus loss caused by mismatched duration can avoid or reduce if life insurers are able to close the duration gap beforehand in anticipation of *future* interest rates. Therefore, we hypothesize that in addition to *ex post* duration adjustment life insurers also have the incentive to match duration *ex ante* according to their expectations of future interest rates. Based on expectations theory and its supportive literature[5], we use the term spread of the treasure yield curve as the market sign that life insurers perceive to form their expectations of future interest rates and investigate its relation with the matching behavior of life insurers. The preliminary evidence in Figure 2.1 shows a negative correlation between the term spread and the aggregate duration of life insurance industry, which suggests a systematic matching pattern associated with the expectation of future interest rates.

Motivated by this evidence, this paper disentangles different impacts of interest rate on life insurers and proposes the two channels underlying the duration-matching mechanism, *ex post* adjustment responding to the interest rate level and *ex ante* adjustment responding to the market signal of future rates, the term spread. *Ex post* adjustment is to rematch the duration gap which varies with interest rate movements because of the convexity difference between the asset and the liability portfolio[6]. *Ex ante* matching is able to alleviate surplus loss (exploit surplus gain) from downward (upward) interest rate movements by making pre-emptive adjustment according to the market signal of future rates. Despite the difference of two types of adjustment

[4]The argument of life insurers mismatching is mentioned in CGFS (2011), EIOPA (2014), Domanski et al, (2015), Paulson and Rosen (2016) and Koijen and Yogo (2017).

[5]The argument of validation of expectations theory (and rational expectations theory) is unsettled. The justification of using the term spread to proxize the market signal of future interest rates is detailed in Section 2.2.2.

[6]For life insurance company, the duration of the liability portfolio is more sensitive to interest rates than the duration of the asset portfolio. See Domanski et al. (2017) and Gilbert (2017).

behavior, they are both driven by the incentive that life insurers choose an optimal duration gap to maximize their surplus value. Moreover, financially constrained insurers expose themselves to a large interest rate risk by choosing a relatively large duration gap relative to non-constrained insurers. Such role that the financial constraint play in terms of risk-taking is verified empirically and also consistent with the finding of Rampini et al. (2019) in bank holding companies.

We verify the hypothesized channels of duration matching in three steps. First, we construct a representative asset-liability portfolio for life insurers, and show that they are, indeed, prompted to systematically adjust duration reacting to interest rate movements. Then, we verify the stylized facts of matching behavior testing on the U.S. life insurance industry during the period 2004-2018. Finally, we exploit the monetary policy shock, 2013 taper tantrum, as the identification strategy to copy with potential reversal relation between bond trading and interest rate movements.

To confirm the incentive of life insurers to match duration, we study how the duration gap varies with interest rates without active duration adjustment. To get around the lack of liability information for individual companies, we construct an asset-liability portfolio using industry aggregate data from several different sources and use it as a representative portfolio for individual insurers. Using the maturity information of assets and insurance policies, we project its cash flow so as to evaluate the effective duration gap with the actual interest rates of treasury securities in 2015. We show that the effective duration gap is negatively correlated to the level of the yield curve. This provides the rationale to maintain a proper duration gap with the change in the interest rate level. Similarly, we shows the negative association between the term spread and the future duration gap up to night-month ahead. It is long enough to incentivize life insurers to make pre-emptive duration adjustment.

The primary challenge of examining the hypothesized matching behavior reacting to interest rate movements is to compute an appropriate measure of duration adjustment. Because life insurance companies are primarily liability-driven, we consider the duration adjustment to be from the asset side and control the interest rate exposure in liabilities, instead of using a vague measure of the duration gap caused by the lack of liability information. We also incorporate the derivative trading into duration adjustment by combined with detailed derivative holding information with the computation function of the Bloomberg Swap Manager.

The hypothesized negative association of the asset duration with the interest rate level and the term spread is verified using the new asset duration measure. Moreover, we show that the life insurers actively increase asset duration on average by 0.22 year in response to the 1% decrease in the interest rate level and 0.19 year in response to the 1% decrease in the term spread. The pre-emptive duration adjustment results more from the change in the long-end of yield curve. Cross-sectionally, we find that the derivative users adjust duration more intensively compared with non-derivative users, which implies that trading derivatives could be more efficient to manage the interest rate risk than structuring bond portfolios.

Further, an identification strategy could be of value, because, for instance, reversal causality could exist. Although duration matching behavior reacts to interest rate movements, it could also affect interest rates given that insurers have significant holdings in corporate and treasury bonds[7]. For identification, we employ the 2013 taper tantrum as the identification strategy and further test the hypothesized association with a DID identification. The interest rate shock was caused by the sudden news from Bernanke that Federal Open Market Committee (FOMC) was considering a potential plan of quantitative easing. It led to the rapid increasing of treasury yields and the market's anticipation of upward future yields. In the DID analysis, we first compare the treatment impact of the taper tantrum on highly exposed life insurers to the interest rate risk with that on the life insurers with relatively low interest rate risk exposure. We also compare the group of life insurers with the group of property & causality (P&C) insurers. P&C insurance companies is the valid control group because the duration of their liabilities is much shorter than that of life insurance companies so that they are much less exposed to large fluctuations in the durations of their asset-liability portfolios in response to interest rate movements. With a continuous DID specification (Acemoglu et al., 2004) we show that that the increasing of highly exposed life insurers in duration exposure after shock is indeed less than that of mildly exposed life insurers. Such reduction on duration exposure is positively correlated with the proportion of long-term liabilities and also with the term spread. The same DID results appears in the comparison between life insurers and P&C insurers as well.

Structure of the paper: In the following content, Section 2.2 introduces the framework of duration matching and the predictive function of the term spread verified by previous literature. Section 2.3 speci-

[7]Murray and Nikolova (2020) demonstrates that insurers' investment demand for corporate bonds affect their price. Greenwood and Vayanos (2018) document a strong effect of pension and insurance company assets on the long end of the yield curve.

fies two channels of duration matching and develops main hypotheses of the association between duration adjustment and interest rate movement. Section 2.4 describes the data and constructs duration measures. Section 2.5 verifies the causality underlying matching channels, and Section 2.6 exhibits the empirical testing of main hypotheses. Section 2.7 provides the identification strategy investigating taper tantrum 2013. Section 2.8 concludes.

2.2 Preliminary

2.2.1 The framework of duration matching

Life insurers feature long-term liability portfolio including life insurance[8] (whole life and universal life policies) and annuities which provide long-term and fixed rate payments. The liability duration for a life insurance company is typically more than 20 years. Therefore, they would expose to severe duration risk if these long-term and fixed liability payments are not well matched by assets or interest rate derivatives. The marginal present value change of the assets caused by the change in the interest rate will be considerably different from that in the liabilities. In other words, the surplus value is exposed to the interest rate fluctuation if the interest rate risk exposure in the liability is not well hedged by the asset and derivatives. Other reasons to match the duration come from the aspects of the rating agency and the regulator. Well-matched duration avoid the volatility of surplus so that enhance life insurers' financial strength, which rating agencies, for instance, A.M. Best[9], monitor closely. National Association of Insurance Commissioners (NAIC) ties risk-based capital (RBC) surcharges to interest rate risk, so high interest rate interest risk contributes to a higher RBC, which pushes insurers close to required minimal RBC ratio[10] (Lombardi, 2006). Moreover, hedging interest rate risk can ensure life insurers have sufficient internal funds available to take advantage of attractive investment opportunities, especially when external sources of finance are more costly than internally generated funds (Froot et al., 1993).

To match the dollar duration of asset and liability portfolios, life insurers have to actively hold a large

[8] Most term life insurance policies last 10, 20 or 30 years, but many companies offer additional five- or 10-year increments, some up to 35 or 40 year terms.

[9] AM Best is the only global credit rating agency with a unique focus on the insurance industry. Best's Credit Ratings, which are issued through A.M. Best Rating Services, Inc., are a recognized indicator of insurer financial strength and creditworthiness.

[10] RBC ratio is defined as total adjusted capital divided by risk-based capital.

proportion of long-term bonds in their asset portfolios, such as long-term treasury bonds. Life insurers can also employ the off-balance-sheet approach, using interest rate derivatives, to match the long duration of the liability. For instance, fixed payments contracted in insurance policies can be exchanged for floating cash flows using receive-fixed and pay-floating swaps or interest rate floors, thereby being immunized against the fluctuation in the interest rate.

If duration is perfectly matched, or within some tolerance, the change in the value of the assets on some basis (economic, market value or book value) will be equal to that in the liabilities with the fluctuation of the interest rate. However, this is not an easy task. First, duration matching is costly. The long-term nature of life insurance and annuities requires assets with long maturity to match duration while long-term bonds (main assets of life insurers) with the favorable return is not always available. Ozdagli and Wang (2019) also propose a model of duration-matching under adjustment costs to explain the mechanism behind duration adjustment reacting to interest rate changes. They suggest German life insurers balance the extent of duration matching against the adjustment cost from bond acquisitions and disposals. Moreover, other than hedging duration risk, there are other ALM objectives that insurers need to comply with, such as maximizing investment income. Because of these reasons, life insurers might choose to take some level of interest rate risk and live with a duration gap to some extent.

Based on the framework of duration matching, this paper proposes the two channels through which life insurers are impacted to the movement of the interest rate and adjust the matching behavior accordingly. The first one explains life insurers' incentive to adjust the duration gap based on the current interest rate level. Because of the convexity difference between the asset and liability portfolio, the interest rate fluctuation causes life insurers to deviate from the optimal level of duration matching. As a result, they are exposed to more or less the interest rate risk than what they choose to bear. So they would like to restore the duration-matching to the optimal level. The second channel specifies the process that life insurers adjust duration ex ante driven by their beliefs of the future interest rate. Life insurers are subject to the interest rate risk is because the present value of surplus is exposed to the interest rate fluctuation in the existent of the duration gap. If life insurers can foresee the change in the future interest rates, by adjusting the duration gap ex ante they have the opportunity to mitigate the surplus loss during the adverse interest rate fluctuation (or enhance surplus value during the favorable interest rate fluctuation). For instance, anticipating the decline of the interest rate they can close the duration gap ex ante, which would otherwise reduce the value of surplus.

2.2.2 The term spread as the indicator of the expected interest rates

It is well accepted that the term spread contains the predictive information of the future economy. For instance, the flat (sometimes inverted) yield curve implies the subsequent economic recessions. The rational expectations theory takes a further steps by hypothesizing that the term spread, which reflects the market expectations of forward rates, is an efficient forecast of the future interest rates. However, the validation of rational expectational theory is bewildering with some supportive evidence among others reject its predicative value. The argument needed by this paper to build up hypotheses is that life insurers as market participants form their expectation of future interest rates based on the term spread. In other words, this paper takes the stand that the slope of the yield curve is a valid market indicator of the change in future interest rates. In the following paragraph it is going to review relevant literature and derive the desire conjecture.

Among the evidence that supports the expectations theory, Cook and Hahn (1990) found the evidence of forecasting power for short-rates. Their testing shows that yield curve from one to five years can forecast the one-year rate over the following three or four years. The finding in Campbell and Shiller (1991) also confirmed the forecasting power for the short-term rates(maturities less than one year). They prove that when yield spreads between the long-term and the short-term bonds is relatively high, short-term rates tend to rise over the lifetime of the long-term bonds. The supportive evidence about long-term rates is provided by Froot (1989), where it uses the survey data by Goldsmith-Nagan that reports the expectations of financial-market participants[11]. It finds that at long maturities the surveyed expectation of interest rates is consistent with the yield curve spread. In addition, Longstaff (2000) argues the expectations hypothesis cannot be ruled out theoretically by showing that the hypothesis can be consistent with the absence of arbitrage if markets are incomplete. Therefore, with the literature just listed it is safe to claim that the term spread reflects the expectation about the future rates, which would be realized in actual interest rates if the expectation is rational.

[11] The expectation of interest rates surveyed by Goldsmith-Nagan includes three-month treasury bills, three-month Eurodollar deposits, twelve-month treasury bills, the Bond Buyer index, and the thirty-year mortgage.

With the literature just listed, it aims to justify using the term spread as a market indicator of the expectation of future rates, which would be consistent with actual interest rates if the expectation is rational. However, it cannot claim that the proxy is ideal given that the coexistence of the literature that debunk the expectations theory. In Section 2.5, it will further demonstrate the comovement of the term spread and the duration gap, which makes it convincing that life insurers react to the varying of the term spread.The following two subsections elaborate how life insurers are affected to the interest rate movement through the two channels and hypothesize how they adjust their duration-matching accordingly.

2.3 Duration adjustment channels

2.3.1 Duration matching, the interest rate level and convexity

The difference of convexity between the asset and the liability portfolio plays the key role of driving the calibration of portfolio duration responding to the interest rate change. Because of the convexity difference, even if an insurer' duration is already matched to a proper extent, the change of the interest rate level would change the duration of the asset and the liability portfolio differently, thereby enlarging or reducing the duration gap. The varying duration gap with the interest rate level makes it necessary for life insurers to adjust duration dynamically. For instance, the duration gap between assets and liability (liability duration being larger) is widened as interest rates decline (Domanski et al., 2017 and Gilbert, 2017). Specifically, this is because the duration of liabilities increases by a greater amount than the duration of assets. The policyholder's behavior amplifies this impact because the likelihood of surrender and lapse reacts to the interest rate changes[12]. In a word, the decline of the interest rate exposes life insurers to higher interest rate risk because of the convexity of the asset-liability portfolio and also the policyholder's behavior. To alleviate interest rate risk resulted from a widened duration gap, life insurers should either increase dollar duration in bond portfolios or purchase the interest rate derivative. In contrast, if the interest rate continue rising, the duration of liabilities decreases by a greater amount than the duration of assets, which reduces

[12]For instance, in the environment of the declining interest rates, policyholders are more likely to hold on to their insurance polices because there is lack of more profitable investment opportunities in the market. Ozdagli and Wang (2019) finds the positive relationship between interest rates and surrender/lapse rates. Kubitza et al. (2020) also provide empirical evidence that an increase in interest rates leads to more policy surrender activity, and then estimate the magnitude of forced asset sales and its sensitivity in a calibrated model.

the duration gap. In other word, because of the convexity difference between assets and liabilities the rising of the interest rate level mitigates duration mismatching without rebalancing portfolios. So life insurers are less constrained to maintain the level of duration in the asset portfolio and are able to cater for other ALM objectives.

In a word, the adverse interest rate fluctuation causes life insurers to derivate from the extent of duration-matching to which they would like to choose because of the different convexity nature of the asset and liability portfolio. To dynamically rematch their duration, life insurer adjust the duration in the asset side in a negative pattern correlated to interest rates, the evidence of which is found in both Germany and U.S. life industry by Domanski et al. (2017) and Ozdagli and Wang (2019). We summarize the matching behavior attributed to *ex post* adjustment in Hypothesis 2.3.1.

> Hypothesis 1: As the increasing (decreasing) of the interest rate level, life insurers decrease (increase) the dollar duration in assets by trading bonds or/and interest rate derivatives.

2.3.2 Ex-ante duration matching and term structure factors

Driven by their beliefs of the future change in the interest rate life insurers might have the incentive to adjust their asset duration ex-ante, either through structuring asset portfolios or trading interest rate derivatives. The reason that prompts the ex-ante adjustment is that life insures can avoid the suppressing of surplus in the case of adverse interest rate changes or make use of favorable interest rate changes to enhance their surplus. For instance, it is already discussed that the declining events of the interest rate can enlarge or generate[13] the duration gap because of the convexity difference between the asset and the liability portfolio. The declined interest rate also causes surplus to be suppressed because of the longer duration of the liability. Specifically, the larger dollar duration of the liability portfolio than that of the asset portfolio means that the present value of the liability increases by a large amount than that of the asset as a result of the declined interest rate[14]. However, if life insures, observing the term structure, can perceive the possible decline tendency of the

[13]Even if there are some perfectly matched life insures, which means the duration gap is zero. The declining interest rate would generate the duration gap (larger duration in the side of liabilities) immediately because of the difference in convexity.

[14]In the practice, such conceptual process corresponds to the situation: life insurers have to re-invest expired bonds at a lower yield in the environment of declining interest rates while contractural case outflows of insurance policies stay fixed, in which the capital is eroded as a result of reduced investment income.

future interest rate, and if they increase ex ante the dollar duration in the asset side before the falling of the interest rate, the surplus loss from mismatching as the decline the interest rate can be reduced or avoided. So this gives them the incentive to adjust the duration ex-ante observing the sign of the future interest rate change. (The symmetric reasoning can be made for the case of the increasing of the interest rate.)

The empirical implication in Ozdagli and Wang (2019) also provide the motivation to explore of the relationship between the duration adjustment and the term factors that contain predictive information of the future interest rate rather than only the current level of rates. Because their testing result shows that the interest rate level alone does not significantly affect the duration adjustment, which is against the common sense that life insurers tend to lengthen their asset duration when the interest rate declines as the case study in Domanski et al. (2017).

Based on the literature about the term structure previously discussed, the future interest rate level is positively associated with the slope of the term structure and negatively associated with the curvature of the term structure. Observing these factors of the term structure, which implies the future change of the interest rate level, life insurers might adjust their interest rate risk exposure accordingly.

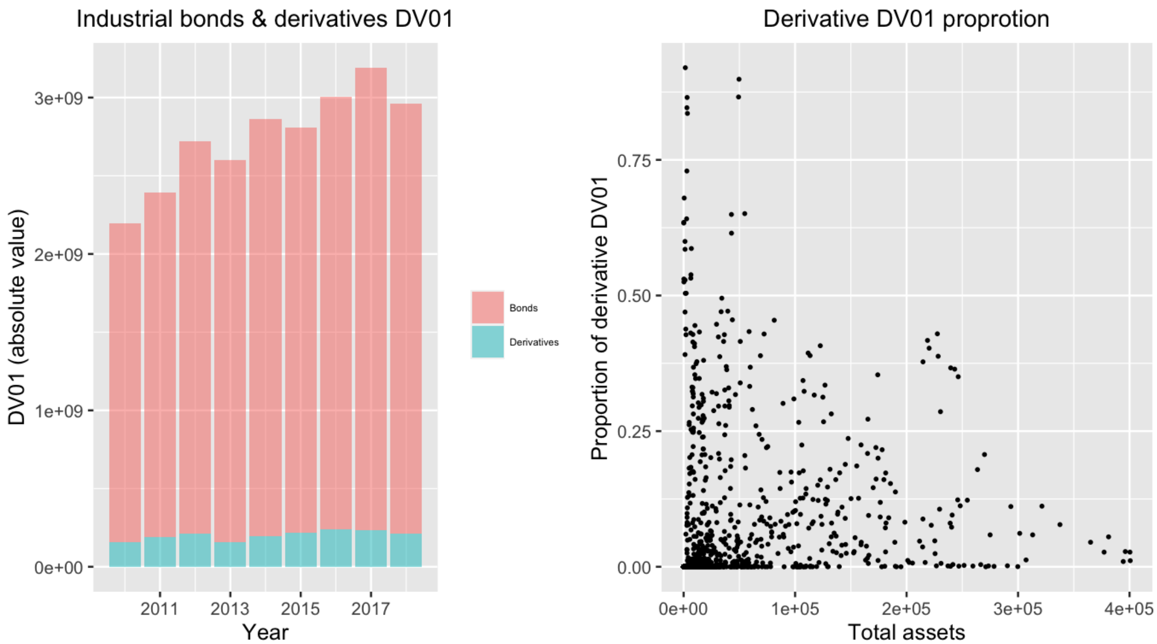
Hypothesis 2: As the increasing (decreasing) of the term spread, life insurers decrease (increase) the dollar duration in assets by trading bonds or/and interest rate derivatives.

2.4 Data and measurement

The primary data source is NAIC statutory annual report compiled by S&P global market intelligence (acquired SNL financial at 2015). The bond holding information is drawn from Schedule D which provide the detailed security-level holding. The derivative holding information is drawn from Schedule DB which is the holding data at the instrument-level. All insurers' characteristics used in this paper is also pull out from the same data platform, except one of financial constraint measures, A.M. Best financial strength. Here acknowledge that A.M. Best company generally shares financial strength ratings of U.S. life insurance companies for time period from year 2010 to year 2018.

Figure 2.2: Duration exposure of bonds and derivatives

The derivative duration exposure illustrated in the both panels are the absolute value of $DV01$ as the $DV01$ of derivative could be negative. It is easier to use absolute value instead of original value to provide a meaningful comparison of duration exposure of bonds and derivatives. The combined $DV01$ in the right panel is the summation of the absolute $DV01$ of bonds and derivative as well. The unit of total assets in the right panel is million dollars. The dot in the right panel are year-insurer observations.



2.4.1 Duration measure of the bond portfolio

The interest rate can affect the duration of the bond portfolio through two channels. First, as in the duration formula, duration is a function of the interest rate, which means the interest rate changes the duration even holding the bond and derivative portfolios unchanged. (Specifically, the duration of bonds with coupon decreases with yield to maturity.) This is how the duration is defined. Second, the interest rate might also change the duration because insures might react to the interest rate change by restructuring bond portfolios, which is what the paper wants to test. So a duration measure required should be able to isolate the second channel from the first one. Common duration measures, such as Macaulay duration or modified duration, can't satisfy this requirement since they are a function of the interest rate.

A revised duration that are invariant to the change of the interest rate level and include all information of the cash flows of bonds. This measure is designed to capture the adjustment of insurers' bond holding, so it incorporates the amount of coupon and principal and the frequency of coupon payment and maturity.

To suit this requirement, the duration is computed this way. For a bond, calculate its duration at different effective interest rate rates according to the duration formula (ex. modified duration). Then, take the average of the duration values at different interest rate rates. The average duration is invariant to the interest rate level but able to take into consideration all rest of information of bonds. See Appendix for the detail of computation.

2.4.2 Duration measure of interest rate derivatives

In Swap Manager function, Bloomberg provides the calculation of DV01 for interest rate floors, caps and swaptions contracts with almost all customized contract attributes such as notional value, strike price, effective and maturity date, floating index and payment frequency. The valuation setting also provide the option to choose the yield curve date that discounted rates are based on. Basically, with the swap manager function, DV01 at any given date for any interest rate derivative contract can be evaluated. Therefore, in order to obtain to DV01 for interest rate derivatives held by life insurers, it only needs to input the holding information of derivatives in the statutory report.

To obtain a measure of $DV01$ that only reflects the holding of derivatives not the interest rate level, it evaluates $DV01$ using a swap curve averaged over all swap curves at NAIC report dates. That is, the averaged swap curve is compose of swap rates that are the average value of rates with the same term on all swap curves.

The strike price of caps and floors are not able to be obtain from Schedule DB of the annual report because many insurers report the strike price and the floating rate together without specifying which is which. The strike price of these interest rate derivatives are, therefore, simplified as the swap rates implied by the average swap yield curve as well. In Schedule DB, the effective date of the derivative contract is not reported. It uses the report date as the effective date and the time difference between report date and the expiration date as the maturity of the contract. Other contact information that are specific for different types of interest rate derivatives are list in Appendix B.1.

The $DV01$ of caps can be exported directly from Bloomberg Terminal. Then, the $DV01$ of interest rate floors can be obtained from the put-call parity. That is, short a cap option and long a floor option with the same strike price and payment structure is equivalent to long a swap contract that pays the floating-rate and receive the fixed-rate equal to the strike price of the cap and the floor. Thus, the $DV01$ of long a floor are the result of the $DV01$ of the long position of a received-fixed swap minus a the $DV01$ of the short position of a cap.

Like swaps, caps and floors, Bloomberg also provides the calculation of $DV01$ for swaption contracts. However, it has to ignore the duration exposure of swaption here because the information to calculate $DV01$ is far from complete. Only partial of companies report the tenor of underlying swaps which has a linear relation with the value of $DV01$. A proximation of the tenor is not possible as the tenor could vary from 1 year up to 30 year. The reason that the impact of ignorance of swaption duration is negligible is that the notional exposure of swaptions is very limited. For the same reason it ignore the duration exposure of forwards, futures and other interest rate derivatives.

Figure 2.2 illustrates the comparison of duration exposure at the industrial level and the individual insurer level. At the industrial level as shown in the left panel of Figure, the dominant duration exposure exits in the bond portfolios and the derivative duration exposure is comparably very limited in all years of the sample period. However, for individual insurers the bond the duration exposure does not necessarily cluster in the bond portfolios as in the right panel. There are quite a few observations where derivative duration accounts for a large chunk of total combined duration exposure. Such case is common for small-size insurers while for the large insurers whose total assets are greater than $300,000$ million dollars the more than 87.5% of duration exposure still exists in bond holdings.

2.4.3 Measures of Interest rate movements

To measure the level of interest rates, two measures are derived from the yield curve. One is the simple average of a sequence of spot rates[15] on the treasury yield curve. The other level measure is the average of the same spot rates weighted by the holding information of bonds. For instance, the weight of 10-year spot rate is determined by the proportion of 10-year bonds held by the life insurance industry within the sample period. The measure of the term spread is using the difference between the 10-year and the 3-month

[15] The spot rates include those with maturities being 1-month, 3-month, 6-month, 1-year, 2-year, 5-year, 10-year, 20-year and 30-year.

treasury rate for the short end of the yield curve and the difference between the 30-year and 10-year for the long end of yield curve. The 10-year and 2-year term spread is also computed to compared with the 10-year and 3-month term spread as another common term spread for the short end of yield curve.

Appendix Table B.4 shows the correlation matrix of the interest rate factors. It is designated to exclude the possibility of collinearity. Namely, the measures of the interest rate level might be highly correlated with the measures of the term spread. From the table, the weighted level of interest rate is less linearly correlated with the measures of the term spread, which shows that it is worthy testing on the weighed level in order to assure of the valid estimation of the effect of the interest rates factors.

2.4.4 Other cross-sectional attributes and their measures

The duration for the liability portfolio is an important variable to determine the duration matching because it affect the duration gap directly. However, it have been a difficulty to calculate duration for the liability portfolio of insurers because of the lack of relevant product information. Kirti (2017) simply assumes average modified duration of these liabilities is 15 years and Domanski et al (2017) consider roughly the aggregate cash flow of the liability portfolio to be a deterministic value with a fixed decay rate. These proxies are far from the accurate measurement of the liability duration. Following the idea of the mismatch measure first suggested by Flannery and James (1984) and then applied by Colquitt and Hoyt (1997) into the life insurance industry. This paper use the amount of long-term liabilities to proxize the interest rate risk exposure in the liability. Long-term liabilities here include individual and group annuity reserves, ordinary life insurance reserves and supplementary contract reserves. Additionally, leverage is also used as another measure of interest rate risk exposure in the liability. Computed as the liability value relative to that of the surplus, the leverage represents the amount of liability per unit of the surplus. The larger the leverage is, the higher interest rate risk exposure in the liability it can be conditional on the surplus.

Regulatory constraint is measured by the company action level risk based capital (RBC) ratio. Then, use dummy variables to indicate insurers as four categories according to their RBC ratio percentiles. The surcharge of the interest rate risk in the RBC ratio reflect that the regulatory requirement of the capital sufficiency (the numerator of the RBC ratio formula) relative to the risk-taking (the denominator, RBC). It is consistent with the economic incentive to hedge the interest rate risk, which is conceptualized in the paper as the exposure of the interest rate risk and the value of surplus. Although the RBC ratio requirement is not strict in the sense that it is far from being binding for many insurers and the adjustment frequency required is not very high, it does reflects the necessity of duration-mating on the economic basis. The concern of using RBC ratio as an attribute of life insurers is the endogeneity issue, the reversal casualty. The extent of being regulatorily constrained indeed affects the duration-matching while at the same time the duration adjustment through structuring bond portfolio affects the RBC and, therefore, the RBC ratio as well. In Section 2.6, financial strength ratings evaluated by A.M. Best company is employed to examine the role that financial constraint plays in duration matching.

Table 2.1: Summary statistics

The sample period of summary statistics is from 2004 to 2018. Panel A reports the summary statistics for the observations of all life insurers in the time period. “Combined asset duration” is the dollar duration in the bond portfolio plus the dollar duration of interest rate derivatives, the unit of which is dollar per basis point. The unit of “total assets” is thousand dollars. “Long-term liability proportion” is measured by the value of long-term liabilities over that of total liabilities. Panel B reports summary statistics for annul treasury yields, the units of which is percentage. “Interest rate level” is the simple average of treasury yields with selected maturities. “Interest rate level (weighted)” is the average weighted by the holding information of bonds. “Term spread (10Y-3M)” is the difference between the 10-year treasury yield and the 3-month treasury yield.

Statistic	N	Mean	St. Dev.	25%	Median	75%
Panel A: Life Insurer Observations						
Combined asset duration	9,107	4,227,932	16,742,319	22,572	228,086	1,609,559
Total assets	9,107	7,351,011	28,095,211	46,999	356,236	2,701,602
Long-term liability proportion	9,107	0.512	0.328	0.201	0.569	0.819
CAL RBC ratio	9,107	1,110.168	3,420.612	336.605	476.720	763.750
Panel B: Interest Rates of treasury						
Interest rate level	3,753	2.213	1.259	1.224	1.711	2.837
Interest rate level (weigted)	3,753	2.693	1.137	1.797	2.355	3.607
Term spread (10Y-3M)	3,750	1.844	1.051	1.160	1.935	2.610
Term spread (20Y-10Y)	3,753	0.537	0.258	0.320	0.540	0.760
Term spread (30Y-20Y)	3,227	0.170	0.189	0.020	0.240	0.310
Term spread (30Y-10Y)	3,753	0.684	0.348	0.400	0.740	0.940

2.5 Incentives of duration matching

This section is to construct a static industrial asset-liability portfolio of life insurers. It is used as a representative asset-liability portfolio in the life industry. With the representative portfolio, it can then derive the corresponding change in the duration gap with the change of interest rates. The goal is to justify the incentive of insurers to match duration with the interest rate fluctuation. For instance, as the interest rate decreases, the duration gap of the representative portfolio would get enlarged, which would incentivize life insurers to lengthen the asset duration.

2.5.1 Simulate a representative asset-product portfolio

Liability portfolio The representative liability portfolio is constructed using the industrial aggregate information of assets and liabilities. The asset portfolio is aggregated from long-term bond holding in Schedule D. The information of the liability by types of products is difficult to obtain as the categorization of insurance and annuity products is very rough in the statutory report. To determines a proximate value proportion of products of different types, several data sources is used. The proportion of life insurance and annuities of the industry is in reference to the reserve value of them in ACOLI (2018). Within life insurance products, the fact amount of different insurance products calculated in LIMRA and SOA (2019) is used to determine their value proportion. The value proportion of term life insurance by terms is collected from SOA and LIMRA (2018). The annuity proportion by types is from IRI (2015). Appendix Figure B.2 summarizes the age profiles and the calculation of risk exposure.

To project cash flow of insurance policies over the time, the exposure information cross the age of policyholders for each type of insurance policies is also needed. Referring to survey reports, Shaughnessy and Tewksbury (2019) and Drinkwater et al. (2018) carried out by SOA and LIMRA, the exposure information falling into different age cohort is summarized in Appendix Table B.1.

Proportion of the asset and the liability exposure Long-term bonds and liabilities exits both in the general account and the separate account[16]. Only the aggregate value in the general account is used as the

[16]The asset risk exposure predominantly exists in the general account. As of 2014, there are 144 of 688 operating life insurers filling separate accounts statement with NAIC. And 90 of them have bond holding in their separate accounts. The aggregate bond

gross value of the representative asset-liability portfolio. The rationale is that the products in the separate account are mostly investment-linked contracts whose payoff depends on the performance of underlying assets which are designated separate account assets. This means that theoretical there should be no interest rate risk existing in the separate account as the payoff of policies in the separate account is consistent with the investment income of general account assets. The total face value of long-term bonds reported in Schedule D Part 1A of the general account is $5,370,201,984$ \$000s and the total value of life insurance and annuity reserve are $2,031,864,712$ \$000s and $2,731,094,781$ \$000s, respectively[17].

2.5.2 Project cash flows of the asset-liability portfolio

In order to study how the duration gap changes with interest rate movement, the cash flows of the representative asset-liability portfolio need to be projected across time. The asset-liability portfolio constructed to calculate the duration gap is the industrial portfolio of the year end of 2014. The representative portfolio uses the information of security-level bond holding and insurer-level of product reserve of general accounts. Appendix Table B.3 lists the value of bonds and life insurance and annuity reserve in the two accounts which are the primary interest rate rate risk exposure on the balance-sheet.

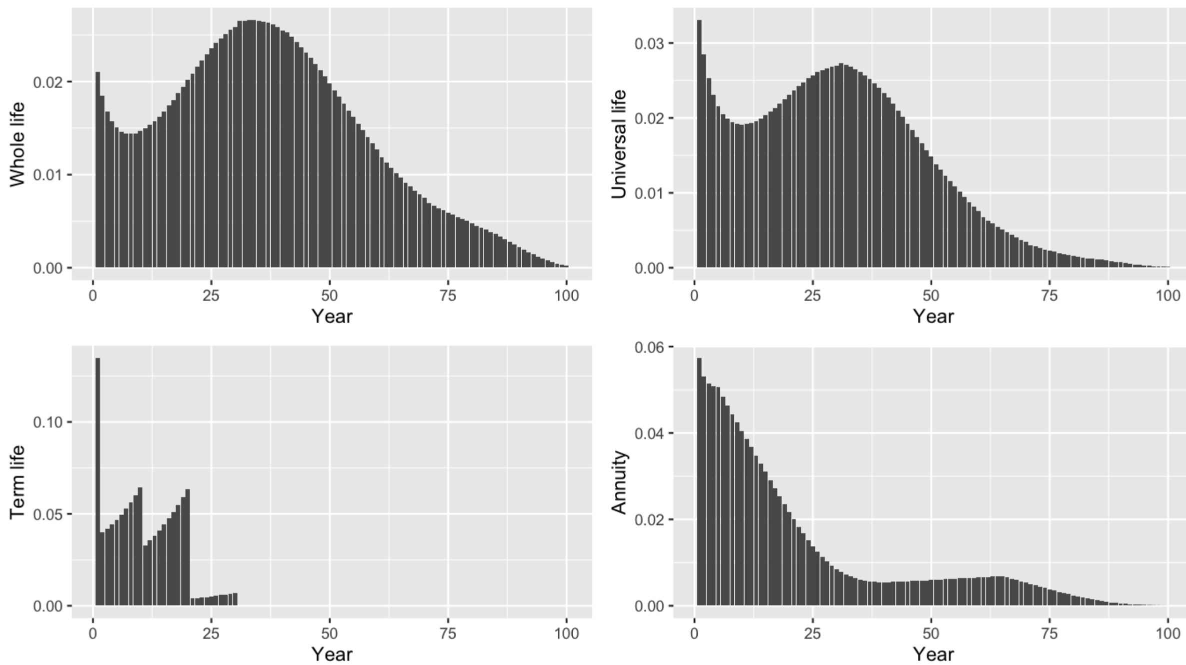
Cash flows of the asset portfolio The interest rate risk exposure in the asset portfolio dominantly exists in bond holding. Using security-level holding data from investment detail (For the general account, the detailed holding information is reported in Schedule D Part 1A.), it projects the cash flows of bonds, coupon payment of each term and principal when they expire. Since the goal is to build a represent portfolio for the life industry, it aggregate cash flows of all bond holding reported by all operating life insurers in 2014. The coupon is standardized as annual payments and the all coupon and expiring principal payment are considered to be paid at the beginning / ending of each year closest to their actual pay-date.

Cash flows of the liability portfolio The exposure of liabilities by types of products and by age cohorts has been obtained as in Table B.2 and Table B.1. Within each age cohort, assume the exposure is

holding in the separate accounts only account for 6.1% of bond holding in the general accounts.

[17] The detailed value of assets and liabilities assigned in the general account and the separate account is listed in Table B.3 in Appendix.

Figure 2.3: Cash flows of life industry products



Note: the value of y-axis is the relative value. The gross exposure of each type of products is normalized to unit 1.

evenly distributed among the policyholders with different ages. After determining the cash flow of a single insurance police for all types and all ages of policyholders, it sums up cash flows of all individual polices to derive the aggregate cash flow of the liability portfolio.

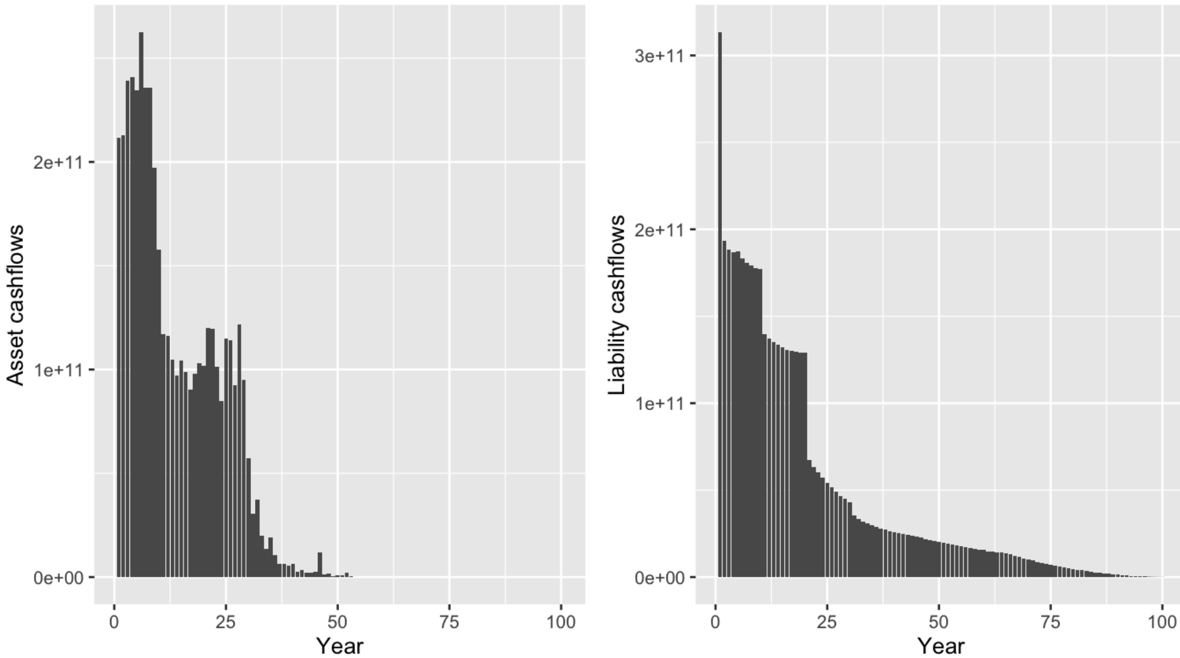
We simplifies the payment of cash flows of different life insurance industry products into two primary types. For life insurance products, the cash flow of them is considered as a lump-sum payment of death benefit at death, which is applicable to life insurance including term life, whole life and universal life insurance. The cash flow of annuities is simplified as periodical payments. The premium of annuity policy is annualized payments until death or the expiration of the contract applicable to annuities.

The death and living benefit payments is determined by discounting the value of polices by the mortality rate. The mortality rate references to United States national life tables in 2015 (Arias and Xu, 2018). As for life insurance polices, it use age 100 as the terminal age, which means the death benefit would be paid to police holders when they reach 100. As for annuities, it assumes that the living benefit is paid until age 100 even if policyholders can live beyond 100 years old. More specifically, it assumes a unified discount rate when insurers set the death and living benefits. The benefit payments is derived based on the principle of pricing: actuarial value of the benefit payment being equal to premium. For universal and whole life insurance, it assumes half of premium or reserve is set to pay death benefits (the other half is the saving component which is paid upon surrender). Therefore, for those types of insurance contracts only 50% of the amount of gross premium or reserve is used as their exposure measure. The average annualization age of annuities is assumed to be 65. That is, the first payment of living benefit is paid once policyholders reach age 65.

It is worthy mentioning that withdrawal and surrender activities of policyholders are not taken into considerations when calculating interest rate risk exposure. First, it is because that withdrawal and surrender activities does not affect the calculation of duration by definition. Withdrawal /surrender is policyholder's behavior that would change the structure of payment cash-flow of insurers. Nevertheless, the interest rate risk is about the present value change of a scheduled cash-flow caused by the interest rate change. The change in payment cash-flows caused by contract-holder behavior (such as lapses, withdrawals, transfers, recurring deposits, benefit utilization, option election, etc) are unscheduled. In this sense, surrender / withdrawal cannot be incorporated in the concept of duration. Secondly, withdrawal /surrender should not impose extra loss on insurers related to the interest rate. Because the Withdrawal /surrender policy is designed more friendly to insurers than policyholders. Although, withdrawal/ surrender behavior caused by the change in interest rate can affect the value of cash-flows, such policyholders' behavior is difficult to predict accurately. In practice, in the absence of relevant and fully credible empirical data, the actuary company set behavior assumptions and the variance in behavior assumptions can significantly affect the results (NAIC VM, 2020). Such difficulty in predicting policyholders' behavior makes it impossible to incorporate the behavioral reasons (rather than the term of payments) into the concept of duration.

The projection of the payment cash flow of different insurance polices is shown in Figure.(2.3). The cash flows of whole life and universal life insurance policies have the similar pattern with the summit value of the cash-flow around year 25. The distribution exhibits a relative heavy tail. In contrast, the exposure of annuities heavily distributed among the early years, from year 1 to year 10, proximately. The non-smooth cash-flow shape of term insurance policy is due to the several fixed-term of contracts. In the sample deriving

Figure 2.4: Asset and liability cash-flows



this figure, term insurance are composite of yearly renew term, 10-year level premium term, 20-year level premium term and 30-year level premium term. The spike in the first year is explained by the exposure of policies with yearly renew term.

Aggregating the cash flow of individual policies, the projection of cash flows of the industrial portfolios for assets and liability is as Figure 2.4. The spike of the liability cash-flow in the year is due to the exposure of yearly renew term insurance. The liability cash-flow has a heavier tail than the asset cash-flow. This is because the maximal maturity of majority treasury bonds are 30-year while the future life of policyholders can up to much more than 30 years. The longer and heavier tail of the cash flows of liabilities relative to the ones of assets can explain the different duration and convexity nature of the asset and the liability portfolio. That is, both the duration and convexity of the liability portfolio is larger than those of the liability portfolio.

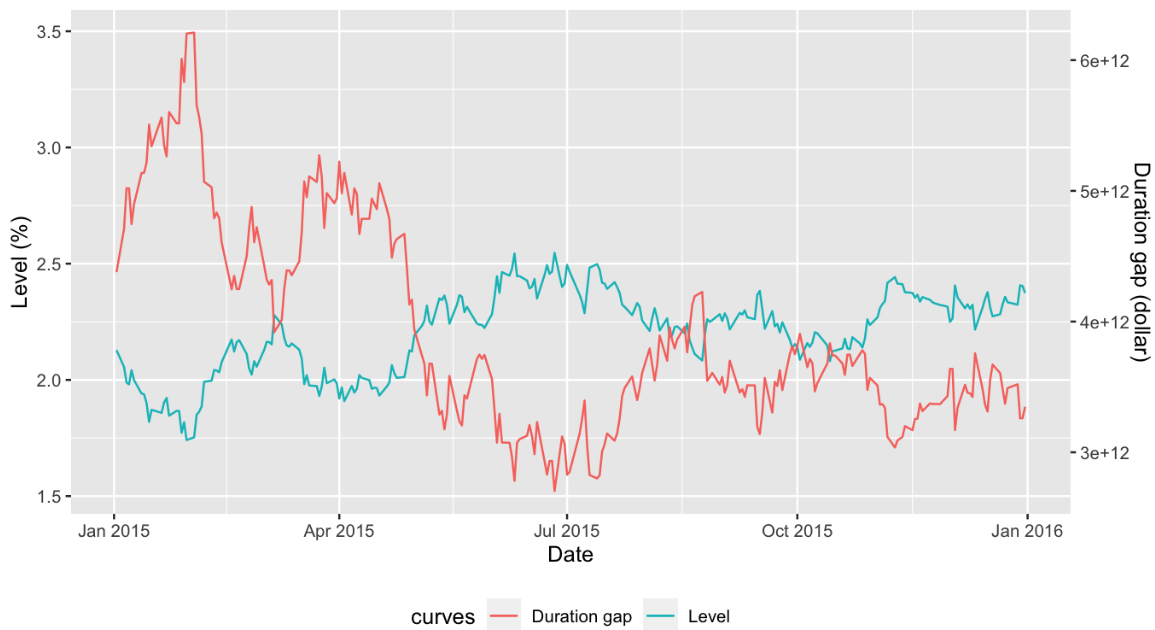
2.5.3 The incentives of duration adjustment

Interest rate risk exposure exists in both the general account and the separate account. To figure out the duration gap between the asset and the liability portfolio, it is necessary to take into consideration the risk exposure in both accounts.

The discount rates used to calculate duration are treasure yields. The time window investigated is from 2010 and 2018, during which periods it collects daily yields for 1-year, 2-year, 3-year, 5-year, 7-year, 10-year, 20-year and 30-year treasure notes or bonds.

To obtain the entire yield curve (yearly) from one-year to thirty-year, it first calculates the forward rates based on the internal interest rates of these treasure bonds. The implied forward rates in-between two types of treasure bonds that have the closest term is assumed to be equal. (Give a numeric example here.) Using these yearly forward rates from year one to year 30, it, then, derivates all internal interest rates corresponding to all 30 year maturities. Appendix Figure B.2 illustrates the interpolated yield curves.

Figure 2.5: The evolution of the duration gap and the interest rate level during year 2015



Duration gap and term structure factors This section is devoted to justify the incentive of life insurers to adjust their duration reacting to the fluctuation in the interest rate. Specifically, it aims to show that if the level of the interest rate declines, the duration gap would get enlarged. Also, the reduced the slope of the term structure predicting the decline of the future interest rate is another signal that would enlarge the duration gap. Therefore, the worsened mismatching would incentivize the adjustment of the duration.

The duration measure employed is the effective duration where the discount rate is used the internal rate of each year interpolated from treasury yield curves. Since the maximal maturity of treasury yield curve is only up to 30 years, for that cash flows that are beyond 30 years, it assumes the discount rate is equal to the yield of 30-year treasury bonds. The level of the term structure is computed as the average of the internal interest rates of treasury bonds from one-year to 30-year. The slope of the term structure is measured by the difference between 10-year and 3-month treasury bonds.

The first conjecture specifies the relation of the duration gap and the interest rate level that motivates matching behavior. In particular, the reduced level of interest rates enlarges the dollar duration gap of a typical asset-liability portfolio of life insurers, which incentives life insurers to close the gap.

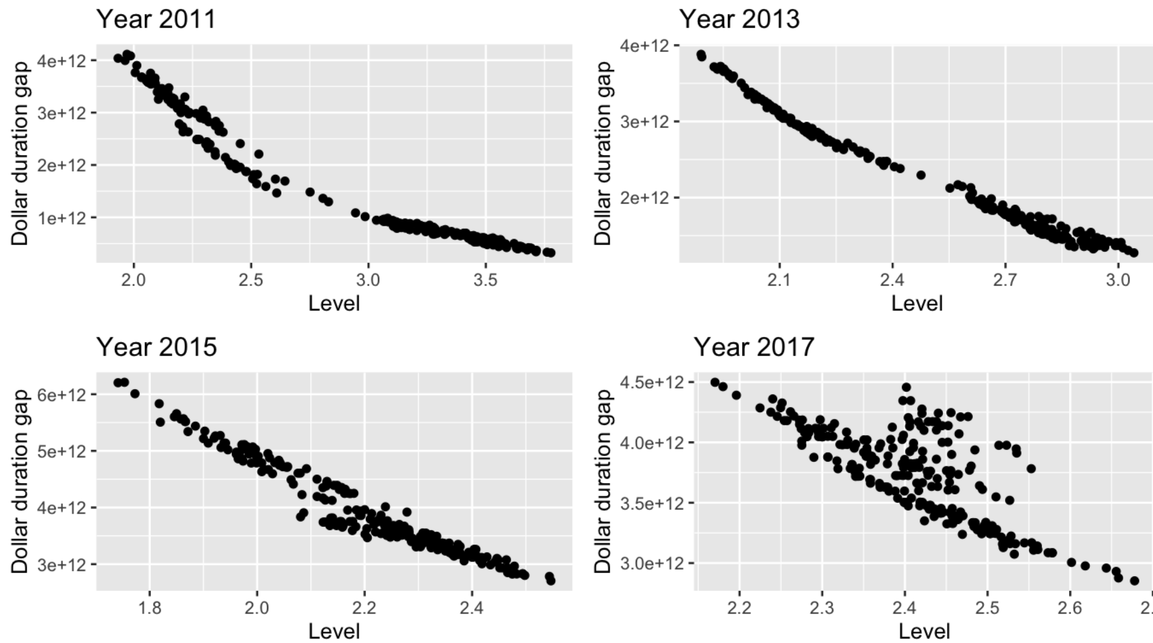
Conjecture 15. *The dollar duration gap of a representative asset-liability portfolio of life insurers is negatively associated with the level of interest rates.*

Figure 2.5 illustrates the change of the duration gap of the representative portfolio of year-end 2014 with the evolution of the interest rate level of year 2015. It shows the clear negative association between the interest rate level and the dollar duration gap. The reason of the negative pattern is rooted in the fact that the cash flow of the liability portfolio has a much longer tail as show in the Figure 2.4. Therefore, the convexity of liability duration is larger than that of asset duration, which means as the interest rate decreases, the liability duration increases by a large amount that the asset duration.

Figure 2.6 plots the daily levels of the term structure against the duration gap of the asset-liability portfolio during year 2011, 2013, 2015 and 2017. As presented in the figure, there are negative correlation between the level and the duration gap in all the four panels. For one value of the interest rate level it could correspond to multiple values of the duration gap, especially, as in the right-bottom panel. The reason is that each of the level could be derived from different yield curves from which different values of duration gap can be evaluated.

The second conjecture is employed to reason why life insurers would adjust duration with the change in the slope of the term structure. It proposes that the reduced (increasing) slope would enlarge (close) the future duration gap so that motivates adjustment behavior.

Figure 2.6: The interest rate level and the dollar duration gap



Conjecture 16. *Controlling the interest rate level, the future dollar duration gap is negatively associated with the slope of the term structure.*

The necessity for controlling the interest rate level is due to the fact that the dollar duration gap is (negatively) correlated to the interest rate as shown in the conjecture 1. The current interest rate level would still have an impact on the future duration gap as the sudden change of the yield curve is very uncommon. Therefore, in the follow regression in Table 2.2 it controlling the current the interest rate level investigates the correlation between the current slope value and the future dollar duration gap.

The main information that this table conveys is the negative association between the slope and the future duration gap. The negative sign of slope coefficients persists from 2 weeks ahead up to 9 months ahead, as in columns from (1) to (9). The negative correlation disappears after one year, as in column (6), (7) and (8). If the duration gap becomes worse, the surplus value would be suppressed without duration-matching. Economically, the surplus loss is due to the fact that the expired investment (mainly, bonds) would receive a lower return after reinvestment in general while the liabilities with longer terms still pay out the same contractual yield to policyholders. That is, the investment income would be not able to cover the payment at the liability side as well as before because of the reinvestment risk, which causes the gradual erosion of the surplus value. According to the first five columns in Table 2.2, a decreasing in the slope would subject life insurers to a constant surplus loss for the next night months if they do not rematching the duration. Such surplus loss would not stop until next year as indicated in column (6).

Table 2.2: The slope of the term structure and the future duration gap

	Dependent variable: future dollar duration gap (in trillion)							
	2-week after	1-month after	3-month after	6-month after	9-month after	1-year after	1.5-year after	2-year after
	OLS	*OLS*	*OLS*	*OLS*	*OLS*	*OLS*	*OLS*	*OLS*
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
level	-1.588***	-1.521***	-1.414***	-1.186***	-1.027***	-1.348***	-1.489***	-2.220***
	(0.029)	(0.034)	(0.043)	(0.058)	(0.071)	(0.108)	(0.137)	(0.219)
slope (10y&3m)	-0.747***	-0.711***	-0.614***	-0.405***	-0.203***	0.635***	1.418***	2.160***
	(0.021)	(0.024)	(0.031)	(0.043)	(0.053)	(0.086)	(0.118)	(0.197)
Constant	8.496***	8.265***	7.821***	6.861***	6.082***	5.169***	3.913***	4.048***
	(0.060)	(0.070)	(0.084)	(0.107)	(0.120)	(0.136)	(0.126)	(0.139)
Observations	2,241	2,230	2,209	2,168	2,126	2,001	1,876	1,751
R^2	0.820	0.760	0.644	0.417	0.256	0.100	0.072	0.069
Adjusted R^2	0.820	0.760	0.644	0.417	0.255	0.099	0.071	0.068
Residual Std. Error	0.578 (df = 2238)	0.664 (df = 2227)	0.799 (df = 2206)	0.995 (df = 2165)	1.106 (df = 2123)	1.166 (df = 1998)	1.029 (df = 1873)	1.010 (df = 1748)
F Statistic	5,109.420*** (df = 2; 2238)	3,522.405*** (df = 2; 2227)	1,994.872*** (df = 2; 2206)	775.602*** (df = 2; 2165)	365.103*** (df = 2; 2123)	111.036*** (df = 2; 1998)	72.829*** (df = 2; 1873)	64.623*** (df = 2; 1748)
Note:								*p<0.1; **p<0.05; ***p<0.01

The magnitude of the coefficient of the slope is stronger for the duration gap of the nearer future. It means the impact of the slope on the duration gap gets weaker and weaker over the time. The current interest rate level is negatively related to the future duration gap, which is as expected. The reasoning is: the high (or low) current interest rate level implies the future interest rate level is more likely to be high (or low) as the sudden change of the yield curve is very rear. The high (or low) future interest rate level leads to the low (or high) duration gap, the negative relation implied from which has been verified from Conjecture 1.

2.6 Stylized facts of matching behaviors

This section verifies the stylized facts derived from the hypotheses between duration adjustment in assets and interest rate movements. The first challenge is to compute a reasonably accurate measure of duration adjustment. Kirti (2017) and Ozdagli and Wang (2019) uses the duration gap between the asset and the liability portfolio, which is most natural measure of duration adjustment. However, the computation of the measure is very rough for the lack of insurance product information while the asset duration can be precisely calculated. As pointed out by Kirti (2017) and Ozdagli and Wang (2019), their calculation of the duration at the liability side is very rough. That is, as the adjustment variable the duration gap can only be vaguely measured because of the lack of information from the liability side. To get around the issue, we instead use duration exposure in assets (including derivatives) as the independent variable to measure the duration adjustment and control the liability risk exposure using the proportion of long-term liabilities at the right-hand-side of regression specifications, for instance, as in Equation 2.1. Therefore, the imprecise measurement of liability risk exposure would not distort the precision of asset duration. The implement of duration matching implied by this approach is more aligned with the reality for two reasons. First, life insurance are primarily liability-driven business[18], which means assets are chosen to match with liabilities in a risk efficient manner (Gilbert, 2016). Secondly, even with complete information of their liability portfolios, it is still challenge for actuaries to precisely calculate liability duration because of the complicity of various insurance products. Not like asset duration, there seems no unified rule to calculate liability duration.

The using of derivative is also considered to study the duration adjustment behavior of life insurers. As shown in 2.2, there are quite a few insurers holding substantial duration exposure in their derivative portfolios. It is reasonable to incorporate derivative duration in studying the duration matching, especially for those large holders of interest rate derivatives. Therefore, the asset duration measure used in this paper is the asset duration combined with derivatives, the total duration in the bond and the interest rate derivative portfolio.

Additionally, to study the portfolio adjustment behavior, namely, how life insurers actively balancing their portfolios out of considerations of matching duration, we follow the approach used by Ozdagli and Wang (2019), where they identify such active adjustment behavior by isolating the duration change cause the active adjustment from duration change caused by in interest rates. Next subsections will test the stylized facts of duration matching for both overall duration adjustment and active duration adjustment.

2.6.1 Overall duration adjustment

Equation 2.1 below gives the specification that explores the association of asset duration with interest rate movements:

[18] See Berends et. al. (2014) and Bragt and Kort (2010).

Table 2.3: Asset duration and interest rate movements

This table reports the regression result of overall duration adjustment. The sample period is from 2004 to 2018. For the time period from 2004 to 2009, as lack of derivative data to compute its duration exposure, only non-derivative users are included in the sample. Non-derivative users are determined as those who are below the top 5% percentile in terms of the absolute book value of derivatives. Column (1) to (4) uses fixed effect model and column (5) uses the OLS model. The dependent variable "average duration" is defined as the sum of bonds and interest rate derivatives duration divided by total assets. Long-term liability proportion is the percentage of long-term reserves in total liabilities. The dummy variable of RBC ratios indicates the insures whose RBC ratios are below 10% percentile.

	Dependent variable: Average duration				
	(1)	(2)	(3)	(4)	(5)
Level	-0.186^{***}	-0.334^{***}	-0.347^{***}	-0.316^{***}	-0.421^{***}
	(0.025)	(0.041)	(0.051)	(0.049)	(0.047)
Term spread (10Y-3M)		-0.270^{***}	-0.256^{***}		-0.268^{***}
		(0.039)	(0.033)		(0.065)
Term spread (30Y-10Y)			-0.098	-0.611^{***}	-0.395
			(0.142)	(0.158)	(0.242)
Long-term liability proportion	1.875^{***}	1.968^{***}	1.970^{***}	1.918^{***}	3.377^{***}
	(0.433)	(0.436)	(0.436)	(0.434)	(0.111)
Log[total assets] (in thousand)	0.605^{***}	0.504^{***}	0.499^{***}	0.542^{***}	0.185^{***}
	(0.134)	(0.136)	(0.138)	(0.137)	(0.014)
Dummy: RBC ratio < 10%	-0.137	-0.131	-0.131	-0.135	0.187
	(0.136)	(0.137)	(0.137)	(0.136)	(0.122)
Constant					4.341^{***}
					(0.323)
Fixed effects	*Firm*	*Firm*	*Firm*	*Firm*	–
Observations	9,107	9,107	9,107	9,107	9,107
R^2	0.806	0.808	0.808	0.807	0.116
Adjusted R^2	0.786	0.789	0.789	0.787	0.116
Residual Std. Error	1.717	1.707	1.708	1.714	3.492
Note:					*p<0.1; **p<0.05; ***p<0.01

$$d_{it} = \alpha_i + \beta_1 \mathbb{E}[\text{level}]_{[t-1:t]} + \beta_2 \mathbb{E}[\text{term spread}]_{[t-1:t]} + \Gamma X_{it} + \varepsilon_{it}. \tag{2.1}$$

In the equation d_{it} is the average duration of the asset portfolio defined as $\frac{D_{it}}{\text{Asset}_{it}}$. D_{it} indicates the sum of Macaulay duration of bond portfolios and interest-rate derivatives divided by the value of total assets. $\mathbb{E}[\text{level}]_{[t-1:t]}$ and $\mathbb{E}[\text{term spread}]_{[t-1:t]}$ are the average interest rate level and term spread from $t-1$ to t, respectively. X denotes attributes of insurers including total assets, the financial constraint and the interest rate risk exposure in liabilities measured by the proportion of long-term liabilities in total liabilities. The dummy variable of RBC ratios indicates the insures whose RBC ratios are below 10% percentile.

Table 2.3 shows that U.S. life insurers increase asset duration on average 0.3 year reacting to the 1% decreasing of the interest rate level during the period 2004-2018. Both the changes in the short-end and the long-end of the term spread (10Y-3M term spread and 30Y-10Y term spread, respectively) significantly affect the duration adjustment. The change in the long-term liability proportion is positively significant, which means that the more risk exposure in the liability, the more dollar duration change relative to the asset increases because of duration-matching. The size effect measured by the logarithm value of total asset is positively significant, which means the extent of duration adjustment is larger for the insurers with more assets. The most constrained insurers, those whose RBC ratios fall below 10% percentile, appear to bear more interest rate risk, indicated by the negative sign of the dummy variable of low RBC ratios. But the coefficients are not statistically significant.

Appendix Table B.5 compares the duration adjustment across derivative users and non-derivative users. The sign and significance of the coefficients keep consistent cross-sectionally. Derivative derivative usurers react more intensively towards interest rate fluctuation than non-derivative users. This suggests derivative hedging against hedge interest rate risk could be more intensive than hedging by structuring bond portfolios.

Appendix Table B.9 provides the robustness test in terms of alternative duration measures. Columns (1), (2) and (3) use the simple average of spot rates on the treasury yield curve as the measure of the interest rate level. Columns (4), (5) and (6) use the average of sport rates weighted by the aggregate holding information of bonds as the measure of the interest rate level. All coefficients of interest rate factors variables indicate the expected effects on the average duration consistent with the hypothesis. Moreover, the term spread of the long-end of yield curve more substantially impacts the asset duration than that of the short-term.

2.6.2 Active portfolio adjustment

In practice, there are three approaches to actively adjust the duration of the asset portfolio, balancing current bond portfolios, trading interest rate derivatives and increasing bond holding using external funding from issuing debt or equity or the capital transferring from other affiliates. It is of interest to investigate how these aspect operations of life insurers react to interest rate movement. Although the result in Table 2.3 verifies the hypothesized association between interest rate movements and duration adjustment, it is not able to show that such duration adjustment comes from the active portfolio (including trading bonds and derivatives) structuring, because the portfolio duration also affects the change in interest rates other than portfolio structuring.

Specifically, as in the duration formula, duration is a function of the interest rate, which means the interest rate changes the duration even with the bond and derivative portfolios unchanged. (Specifically, the duration of bonds with coupon decreases with yield to maturity.) This is how the duration is defined. Second, the interest rate might also change the duration because insures might react to the interest rate change by restructuring bond portfolios, which is what the paper wants to test. So a duration measure required should be able to isolate the second channel from the first one. Common duration measures, such as Macaulay duration or modified duration, cannot satisfy this requirement since they are a function of the interest rate. To overcome the difficulty, this paper employs the duration measure of active duration adjustment proposed by Ozdagli and Wang (2019) and test whether the hypothesized duration matching pattern is indeed driven by portfolio structuring. The active duration measures the duration adjustment between a current time and a historic time. It hypothesizes that no adjustment was made on the assets in the historic time (legacy assets) and then calculates the duration of legacy assets at the current time. The difference between the duration of current assets and the duration of legacy assets is defined as active duration adjustment. It captures the duration change caused by asset adjustment, not changes in interest rates. Specifically, the active duration measure is defined as

$$AD_t = D_t - LD_{t-1}(t), \tag{2.2}$$

where D_t is the combined asset dollar duration at time t. $LD_{t-1}(t)$ represents the dollar duration evaluated at time t for the legacy asset of time $t-1$. That is, $LD_{t-1}(t)$ is the hypothesized duration of the asset portfolio of time $t-1$ when the time moves to time t. So the interest rates used to evaluate the hypothesized duration is the interest rates of time t.

The basic specification of active duration adjustment is

$$Ad_{it} = \beta_1 \Delta\mathbb{E}[\text{level}]_{[t-1:t]} + \beta_2 \Delta\mathbb{E}[\text{term spread}]_{[t-1:t]} + \Gamma\Delta X_{it} + \varepsilon_{it}, \tag{2.3}$$

where Ad_{it} is the average active duration defined as active duration divided by total assets, $\frac{AD_{it}}{V(\text{Asset})_{it}}$. $\mathbb{E}[\text{level}]_{[t-1:t]}$ and $\mathbb{E}[\text{term spread}]_{[t-1:t]}$ are the average interest rate level and term spread from $t-1$ to t, respectively. X denotes attributes of insurers including total assets, the financial constraint and the interest rate risk exposure in liabilities measured by the proportion of long-term liabilities in total liabilities. The dummy variable of RBC ratios indicates the insures whose RBC ratios are below 10% percentile.

Table 2.4 tests the association of active duration adjustment and the interest rate factors, parallel to the analysis regarding overall duration adjustment in Table 2.3. The negatively significant signs of the interest rate level and the term spread implies that life insurers indeed active structure portfolios (bonds and/or interest rate derivatives) with the movements of interest rates. Specifically, the average of asset duration increases around 0.1 year with 1% decreasing of the interest rate level, and increase around 0.5 year with 1% decreasing of the 30Y-10Y term spread. The 10Y-3M term spread also has the negative impact on the asset duration, although the average effect is much less than the term spread of the long-end of yield curve, the 30Y-10Y term spread. Columns (1) and (2) use the simple average of spot rates on the treasury yield curve as the measure of the interest rate level. For robustness, Columns (3), (4) and (5) use the average of sport rates weighted by the aggregate holding information of bonds as the measure of the interest rate level. Table 2.5 compares the cross-sectional difference between derivative users and non-derivative users, which confirms the finding of duration adjustment in Appendix Table B.5 that the asset duration of derivative users responds more intensively to interest rate movements. This may suggest using interest rate derivative can help insurers better match duration.

Apart from testing the impact of interest rates on matching behavior, the role of the financial constraint, measured by RBC ratio is also investigated. From Table 2.3 and Table 2.4, the dummy variable indicating the life insurers with low RBC ratios, so that most constrained, generally negatively affect the asset duration. It suggests that most constrained life insurers would have a worse matching thereby exposing themselves to larger interest rate risk. The finding here is consistent with Rampini et al. (2019) in that poorly capitalized

Table 2.4: Active duration adjustment: Interest rate movements

The sample period is from 2010 to 2018. For the time period from 2004 to 2009, as lack of derivative data to compute its duration exposure, only non-derivative users are included in the sample. Non-derivative users are determined as those who are below the top 5% percentile in terms of the absolute book value of derivatives. The dependent variable is the active duration adjustment measured by active duration (defined in Equation 2.2) divided by total assets. Covariant "Level (simple average)" is the simple average of a sequence of spot rates on the treasury yield curve. Covariant "Level (weighted average)" is the average of the same spot rates weighted by the holding information of bonds with the corresponding maturity. The dummy variable of RBC ratios indicates the insures whose RBC ratios are below 10% percentile.

	Dependent variable: Active duration adjustment							
	Interest rate level				Weighted interest rate level			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Δ Level	−0.019 (0.025)	−0.074** (0.031)	−0.131*** (0.037)	−0.127*** (0.031)				
Δ Level (weighted average)					−0.012 (0.028)	−0.127*** (0.041)	−0.128*** (0.041)	−0.129*** (0.035)
Δ Term spread (10Y-3M)	−0.073*** (0.024)		−0.070*** (0.024)	−0.073*** (0.021)	−0.063*** (0.018)		−0.026 (0.020)	−0.030* (0.018)
Δ Term spread (30Y-10Y)		−0.496*** (0.115)	−0.486*** (0.115)	−0.464*** (0.090)		−0.520*** (0.102)	−0.450*** (0.116)	−0.440*** (0.091)
Δ Long-term liability proportion	0.798*** (0.164)	0.773*** (0.164)	0.773*** (0.164)	0.762*** (0.294)	0.799*** (0.164)	0.769*** (0.164)	0.774*** (0.164)	0.763*** (0.294)
Δ Log[total assets] (in thousand)	1.925*** (0.058)	1.931*** (0.058)	1.932*** (0.058)	1.618*** (0.134)	1.924*** (0.058)	1.932*** (0.058)	1.931*** (0.058)	1.617*** (0.134)
Dummy: RBC ratio < 10%	−0.087** (0.043)	−0.087** (0.043)	−0.087** (0.043)	0.007 (0.070)	−0.087** (0.043)	−0.087** (0.043)	−0.087** (0.043)	0.008 (0.070)
Constant	0.575*** (0.014)	0.564*** (0.015)	0.553*** (0.015)		0.576*** (0.015)	0.555*** (0.016)	0.555*** (0.016)	
Fixed effects	-	-	-	Firm	-	-	-	Firm
Observations	8,194	8,194	8,194	8,194	8,194	8,194	8,194	8,194
R^2	0.122	0.123	0.124	0.264	0.122	0.124	0.124	0.263
Adjusted R^2	0.122	0.123	0.124	0.183	0.122	0.123	0.123	0.183
Residual Std. Error	1.183	1.182	1.182	1.141	1.183	1.182	1.182	1.141

Note: *p<0.1; **p<0.05; ***p<0.01

financial institutions hedge less against interest rate risk and foreign exchange risk. The negative effect of financial constrained hold cross-sectionally within derivative users and non-derivative users as in Table 2.5. Be curious about matching status of the least constrained insurers, Appendix Table B.6 includes the dummy indicating insurers with highest RBC ratios, although no systematical pattern is found there.

2.6.3 Robustness to other interest rates and insurer groups

Different benchmark interest rates: Corporate bond yields The bond portfolios of life insurers consist of not only industrial bonds but also treasury bonds. The ratio of the two types of bonds being

Table 2.5: Active duration adjustment: Derivative users and non-derivative users

This table reports regression analysis of the reaction of the overall asset duration to interest rate movements. The full sample includes observations from 2004 to 2018. For the time period from 2004 to 2009 where only non-derivative users are included because the detailed derivative data is not available. Non-derivative users from 2004 to 2009 are identified as those who are below the top 5% percentile in terms of the absolute book value of derivatives. Derivative users in the time period from 2010 and 2018 are are life insurers whose $DV01$ of interest rate derivatives is greater than 5% of $DV01$ of bond portfolios. The dependent variable is the active duration adjustment measured by active duration (defined in Equation 2.2) divided by total assets. Long-term liability proportion is the value percentage of long-term reserves in total liabilities. The dummy variable of RBC ratios indicates the insures whose RBC ratios are below 10% percentile.

	Dependent variable: Active duration adjustment					
	All life insurers		Non-derivative users		Derivative users	
	(1)	(2)	(3)	(4)	(5)	(6)
ΔLevel	-0.131^{***}	-0.127^{***}	-0.129^{***}	-0.121^{***}	0.072	0.166
	(0.037)	(0.031)	(0.038)	(0.032)	(0.188)	(0.233)
ΔTerm spread (10Y-3M)	-0.070^{***}	-0.073^{***}	-0.062^{**}	-0.061^{***}	-0.369^{***}	-0.413^{***}
	(0.024)	(0.021)	(0.025)	(0.022)	(0.127)	(0.135)
ΔTerm spread (30Y-10Y)	-0.486^{***}	-0.464^{***}	-0.514^{***}	-0.487^{***}	0.021	0.188
	(0.115)	(0.090)	(0.118)	(0.093)	(0.444)	(0.472)
ΔLong-term liability proportion	0.773^{***}	0.762^{***}	0.812^{***}	0.815^{***}	−1.600	-1.701^{*}
	(0.164)	(0.294)	(0.167)	(0.294)	(1.017)	(0.984)
ΔLog[total assets] (in thousand)	1.932^{***}	1.618^{***}	1.924^{***}	1.610^{***}	2.122^{***}	2.360^{***}
	(0.058)	(0.134)	(0.059)	(0.135)	(0.494)	(0.806)
Dummy: RBC ratio < 10%	-0.087^{**}	0.007	-0.091^{**}	0.012	−0.208	-0.454^{**}
	(0.043)	(0.070)	(0.044)	(0.071)	(0.292)	(0.176)
Constant	0.553^{***}		0.561^{***}		0.302^{***}	
	(0.015)		(0.016)		(0.072)	
Fixed effects	-	Firm	-	Firm	-	Firm
Observations	8,194	8,194	7,908	7,908	286	286
R^2	0.124	0.264	0.125	0.266	0.122	0.308
Adjusted R^2	0.124	0.183	0.125	0.183	0.103	0.119
Residual Std. Error	1.182	1.141	1.190	1.150	0.896	0.888
Note:						*p<0.1; **p<0.05; ***p<0.01

around 3 : 1[19]. It is reasonable to examine the association of duration matching behavior with both treasury yields and corporate bond yields.

The corporate bond yields used in the test are in reference to high quality market (HQM) corporate bonds yields. The HQM yield curve pertains to the high quality corporate bond market, i.e., corporate bonds rated AAA, AA, or A in S&P's ratings. The regression methodology of the HQM curve blends AAA, AA, and

[19]The industrial and miscellaneous accounts for 75.7% and government and revenue bonds accounts for 23.2% of total investment. (From Insurance fact book 2019)

A bonds into a single yield curve that represents the market-weighted average quality of high quality bonds. The HQM methodology also projects yields beyond 30 years maturity out through 100 years maturity.

Interest rates tested in Column (1) and (2) are treasury yields where column (1) is from the original specification from Appendix Table B.7. The term spread used in Column (2) is the difference between the 10-year and the 2-year treasury yield, which is another commonly used term spread. In column (3) and (4), HQM corporate bonds yields are tested. The mixed yields in column (5) is the weighted average of treasury yields and HQM corporate bond yields with the ratio of two types of yields being 3 to 1. The ratio is approximately equal to the ratio of government bonds and corporate bonds in the asset portfolio of life insurers.

As shown in column (2), the term spread between the ten-year and the two-year treasury bonds exhibits the same significance as ten-year and three-month spreads, but the magnititute of it is less pronounced than the latter. It might imply that ten-year and the three-month spread is more reliable indicator for life insurers in terms of forecasting the future interest rate change. Using HQM corporate bond yields in column (3) and (4) generates the same association between duration matching behavior and interest rate movements as treasury bond yields. Both the level and the term spread of corporate bond yields are negatively correlated with the asset duration. The weighted average yields of treasury bonds and corporate bonds tested in column (5) remains robust. It provides a more convince evidence than only using treasury yields for the matching pattern of duration with interest rate movements. Because corporate bonds also constitute around 30% of total assets of life insurance companies.

Insurance company groups To test the robustness of the impact of interest rate movement on duration matching, it studies the size-effect of life insurers at the group level. Two types of group-level samples, SNL group and NAIC group, are employed as shown in Table B.8. As in column (3) and (4), the coefficients of the interest level and the slope are consistent with those in column (1) and (2). In the group-level analysis, the sign and the significance of liability risk exposure are consistent with the result using individual company data. Additionally, in column (1) the size-effect of companies measured by the value of total asset is not significant using individual company data. After replace the continuous size measure with a dummy variable of large insurers, the dummy variable is positively significant, which means large insurers hedge more against the interest rate risk. In both group standards as in column (3) and (4), the size-effect is not significant in the SNL company group and the NAIC company group.

2.7 Identification

The section devotes to further verifying the causal effect of interest rate movements on duration matching behavior by providing an identification strategy. One may be suspicous that the correlation obtained from Section 2.6 is due to a revesal causal relationship. It is well-documented that the investment behavior of insurers (and, more generally, institutional investors) significantly affects asset prices[20]. In particular, Domanski and Shin (2017) suggests that the decline of long-term interest rates in Europe in 2014 cause life insurance companies and pension funds hunt for long-term bonds which further downward pressure on interest rates. Moreover, the difference-in-differences analysis employed in this section can also rule out the other possible factors that affect asset duration but are not included into empirical specifications.

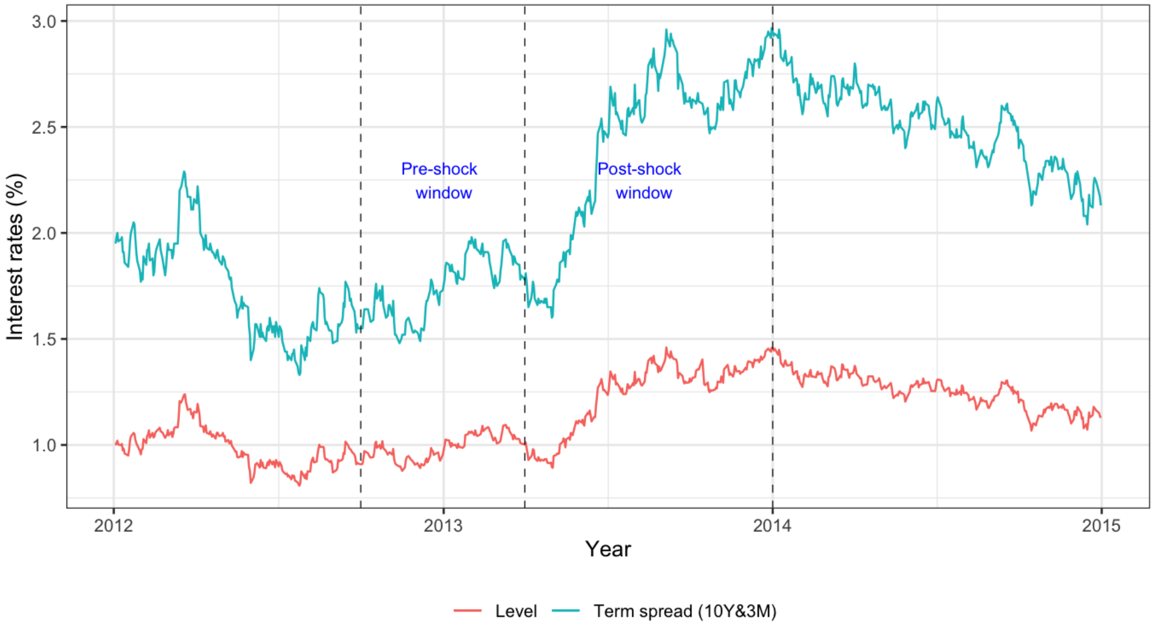
2.7.1 2013 taper tantrum

The accidental interest rate event, 2013 taper tantrum is employed as an exogenous shock on the treasury yield market. On May 22 congressional appearance Bernanke first revealed the Federal Open Market committee's (FOMC) thinking on a taper of its quantitative easing program in answering a lawmaker's question. The market reacted to the news dramatically: bond yields rocketed higher and stock prices dropped; financial conditions over the ensuing months tightened, which surprised the Fed. As shown in Figure 2.7, the interest rate level rose rapidly shortly after the second quarter and the increasing tendency does not disappear until the end of 2013. Simultaneously, the 10-year and 3-month term spread also kept moving upwards almost for the rest of year, which reflected the increasing future rates of the market. According to the hypotheses, facing both the increase of interest rate level and the term spread, life insurers would have less incentive to lengthen asset duration to match up with liability duration. That is, the effect of interest rate shock on the average duration that results from the consideration of duration matching is presumably negative.

[20] See Ellul et al. (2011), Greenwood and Vissing-Jorgensen (2018), Girardi et al. (2018) and Koijen and Yogo (2019)

Figure 2.7: Interest rate movements around 2013 taper tantrum

The shock of the taper tantrum happened in 2013Q2, which is trigged by Bernanke's speaking in congressional appearance on May 22. The pre-shock window including 2 quarters is from 2012Q4 to 2013Q1. The post-shock windows including three quarters is from 2013Q2 to 2013Q4.



2.7.2 Difference-in-differences analysis

To isolate the effect of interest rate movements on life insurers from other factors, we employ a DID analysis where two sets of comparison are tested. First, we test how the life insurers who are highly exposed to the interest rate risk differentiate from ones who have relatively low interest rate risk exposure in terms of duration adjustment. Secondly, we consider P&C insurance companies as the control group, which are compared with life insurers. Choosing P&C insurers as the control group is valid because they have short-term of liabilities so that do not need to match duration. Traditionally, P&C companies generally have much shorter term of liability than life insurers. They have less exposure to interest rate risk. Duration mismatching is not a consideration in their asset-liability management. In terms of NAIC accounting, the account rule of reserve is not interest-rate-sensitive. The pre-shock period is chosen as the two quarters from 2012Q4 to 2013Q1 because the it is a time period right before the shock where the interest rate movements are relatively stable. The subsequent three months, from 2013Q2 to 2013Q4, are chosen as the post-shock window covers the upwards tendency of both the interest rate level and the term spread. A two month post-shock window until 2013Q3 is also examined for robustness.

To accurately capture adjustment behavior around the taper tantrum, we aggregate the quarterly data from the daily bond transactions which is reported in Part 3, Part 4 and Part 5 of the annual NAIC filling. The observations of the life insurers which have significant derivative holdings are excluded from the sample, because the derivative transaction is not available in the statutory data so that quarterly derivative data cannot be obtained. The data used ranges from Jau 1st 2013 to Dec 31st 2014, two complete years. The quarterly data of asset and long-term liabilities holding is computed from annual records by assuming the even quarterly change within each year. Baseline specification is as Equation 2.4. The dummy variable "After shock" takes the value for the time t after the shock happened. Life insurer$_i$ is the dummy indicating life insurers. X_{it} denotes other insurer attributes.

$$Ad_{it} = \beta_1 \text{After shock}_t + \beta_2 \text{Treated insurer}_i + \gamma \text{After shock}_t \times \text{Treated insurer}_i + \Gamma \Delta X_{it} + \varepsilon_{it} \quad . \tag{2.4}$$

A continuous DID specification (Acemoglu, et al, 2004) where the continuous measure of interest rate risk exposure replaces the dummy indicator of the treatment group, life insurers. The interest rate risk exposure in the liability portfolio can be measured continuously by the proportion of long-term liabilities. In specification 2.5 below, LLP_{it} denoting the long-term liability proportion takes value zero for P&C insurers.

$$Ad_{it} = \beta_1 \text{After shock}_t + \beta_2 \text{LLP}_{it} + \gamma \text{After shock}_t \times \text{LLP}_{it} + \Gamma \Delta X_{it} + \varepsilon_{it} \quad . \tag{2.5}$$

Table 2.6 summarizes the DID estimates comparing life insurers with high exposure of interest rate risk. As shown with the positive sign of the post-shock dummy, life insurers on average increase their asset duration after the revealing of the possible taper. The significant positive sign of interaction term indicates that the treatment group, life insurers with more long-term liabilities increases less average duration relative to ones with low risk exposure. This is consistent with the hypothesized effect of the taper shock on the asset duration: both the increasing of the interest rate level and the term spread reduces the incentive of life insurers to match up the asset duration with liability duration. This implies that the taper tantrum imposes a negative impact on those life insurers who are more exposure to the interest rate risk, which is verified by the negative significant coefficient of the interaction. To measure the treatment rate, the treatment dummy is replaced with the proportion of long-term liabilities, which is to proxize the interest rate risk exposure of liabilities. The coefficients of the new interaction remain negative, which are significant in the post-shock window of three quarters, but the window of two quarters. As a robustness test, Table 2.7 uses the P&C insurers, which have no demand to match duration, as the control group. The same correlation as in Table 2.6 is derived. It restates that life insurers add less duration into their bond portfolio relative to P&C insurers after the tape tantrum, which is due to the consideration of duration matching.

Table 2.6: Difference-in-differences: High interest rate risk exposure v.s. low interest rate risk exposure
This table reports the DID result comparing life insurers with high exposure of interest rate risk. The shock of the taper tantrum happened in 2013Q2. The pre-shock period including 2 quarters is from 2012Q4 to 2013Q1. Two different post-shock windows are tested, which are from 2013Q2 to 2013Q3 (two quarters) and from 2013Q2 to 2013Q4 (two quarters), respectively. The treatment group is those life insurers whose long-term liabilities account for more 50% of total liabilities. The rest of life insurers (long-term liability proportion $< 50\%$) are considered as the control group. All specifications are estimated with the OLS model and tested on quarterly data.

	Dependent variable: Active duration adjustment					
	Post-shock window: Two quarters			Post-shock window: Three quarters		
	(1)	(2)	(3)	(4)	(5)	(6)
Post-shock	0.223***	0.220***	0.199***	0.136***	0.132***	0.109***
	(0.018)	(0.018)	(0.022)	(0.015)	(0.015)	(0.018)
Highly exposed life insurers	0.101***	0.098***		0.103***	0.100***	
	(0.015)	(0.015)		(0.016)	(0.016)	
ΔLong-term liability proportion			0.137***			0.139***
			(0.019)			(0.019)
ΔLog[total assets] (in thousand)		0.195***	0.191***		0.175***	0.174***
		(0.051)	(0.051)		(0.037)	(0.036)
Post-shock × Highly exposed life insurers	−0.067**	−0.063**		−0.076***	−0.072***	
	(0.028)	(0.028)		(0.025)	(0.025)	
Post shock × Long-term liability proportion			−0.060			−0.066*
			(0.040)			(0.035)
Observations	2,164	2,164	2,164	2,704	2,704	2,704
R^2	0.028	0.034	0.042	-0.002	0.006	0.012
Adjusted R^2	0.027	0.033	0.040	-0.004	0.005	0.011
Residual Std. Error	0.383 (df = 2161)	0.382 (df = 2160)	0.380 (df = 2160)	0.393 (df = 2701)	0.391 (df = 2700)	0.390 (df = 2700)
Note:						*p<0.1; **p<0.05; ***p<0.01

Column (1) use two dummy variables to indicate the time period after shock and life insurers. Then, in column (2) and (3) the after-shock-dummy is further replaced with the interest rate factors, the level and the term spread, which are continuous so that can more precisely specify the magnititute of the shock.

Table 2.7: Difference-in-differences: impact of taper tantrum on life insurers

The shock of the taper tantrum happened in 2013Q2. The pre-shock period including 2 quarters is from 2012Q4 to 2013Q1. Two different post-shock windows are tested, which are from 2013Q2 to 2013Q3 (two quarters) and from 2013Q2 to 2013Q4 (three quarters), respectively. The treatment group is life insurers and the control is P&C insurers. All specifications are the OLS model and tested on quarterly data.

	Dependent variable: Active duration adjustment					
	Post-shock window: Two quarters			Post-shock window: Three quarters		
	(1)	(2)	(3)	(4)	(5)	(6)
Dummy: Post-shock	0.171***	0.168***	0.170***	0.098***	0.096***	0.097***
	(0.005)	(0.005)	(0.005)	(0.004)	(0.004)	(0.004)
Dummy: Life insurer	0.085***	0.082***		0.086***	0.083***	
	(0.010)	(0.010)		(0.011)	(0.010)	
ΔLong-term liability proportion			0.134***			0.136***
			(0.018)			(0.019)
ΔLog[total assets] (in thousand)		0.341***	0.341***		0.317***	0.316***
		(0.020)	(0.020)		(0.017)	(0.018)
Post-shock × Life insurer	−0.029*	−0.027*		−0.036**	−0.036***	
	(0.015)	(0.015)		(0.014)	(0.014)	
Post-shock × Long-term liability proportion			−0.019			−0.048*
			(0.027)			(0.025)
Observations	12,824	12,818	12,263	15,930	15,924	15,254
R^2	0.019	0.041	0.041	-0.001	0.019	0.019
Adjusted R^2	0.019	0.040	0.041	-0.001	0.019	0.018
Residual Std. Error	0.377 (df = 12821)	0.373 (df = 12814)	0.373 (df = 12259)	0.386 (df = 15927)	0.382 (df = 15920)	0.383 (df = 15250)

Note: *p<0.1; **p<0.05; ***p<0.01

Furthermore, since the essential difference of life insurers from P&C insurers in terms to duration-matching is resulted from their long-term liability exposure. In Column (5) and (6) it uses the proportion of long-term liability to replace life insurer dummy variable, which is a more precise specification. All intersection terms are negatively significant: after taper tantrum life insurers have relatively lower duration exposure in their assets compared with P&C insurers. Specifically, the high interest rate level or term spread after the taper tantrum causes life insurers to response with a lower the duration exposure relative to P&C insurers.

2.8 Conclusion

This paper provides an understanding of duration matching behavior by disentangling the two different impact of interest rate movement on life insurers. Life insurers have the incentive to adjust the duration gap based on the current interest rate level because of the convexity difference portfolios causes life insurers to deviate from the optimal level of duration matching with the interest rate fluctuation. So they would like to restore the duration-matching to the optimal level that what they choose to bear. The ex ante adjustment is driven by life insurers' anticipation of the future interest rate. If life insurers can foresee the change in the future interest rates, by adjusting the duration gap ex ante they have the opportunity to mitigate the surplus loss during the adverse interest rate fluctuation (or enhance surplus value during the favorable interest rate fluctuation).

The empirical investigation on the U.S. life insurance industry for the time period from 2010 to 2018 confirms the negative association of the interest rate level and slope with combined asset duration. It further shows that the combined dollar duration in the asset and derivative portfolios is negatively associated with the level and slope of the term structure. Exploiting 2013 taper tantrum as the identification strategy, the DID analysis using P&C insurers as the control group shows that life insurers increase less duration exposure through bond trading after shock relative to P&C insurers. The reduction of increasing duration exposure persists for three quarters, which is positively correlated with the proportion of long-term liabilities and also with the term spread.

3 Ownership Structure, Capital Cost and Risk Taking

3.1 Introduction

Mutual ownership can eliminate the conflict of interest between investor-owners and policyholder-customers, which incentivizes life insurers to choose a mutual organizational structure facing managerial discretion, as suggested by Mayers and Smith (1981). On the other hand, mutual ownership is subject to the disadvantage of accessing the external capital market. The cost of external financing of mutual companies is much higher than the ones with the stock corporate structure. The decline wave of mutual ownership in the insurance industry at the end of the last century is attributed to such capital disadvantage by scholars. Harrington and Niehaus (2002) find that mutual insurers maintain higher capital ratios, and their capital ratios are more sensitive to income shocks than stock insurers. Viswanathan and Cummins (2003) examine the evidence around demutualization. They find that before demutualization converting property & casualty (P&C) insurers exhibit lower capital ratios and converting life insurers owner a smaller proportion of liquid assets, which is consistent with the capital access hypothesis. Following the streaming of literature, we further study how the differential in capital accessibility affects the attitude of risk-taking.

The central argument of this paper is straightforward: insurers with mutual ownership, which are restricted to access the capital market, are more conservative towards taking interest rate risk compared with insurers with stock ownership. Theoretically, this argument is consistent with the tradeoff between external financing and risk management characterized by Froot, Scharfstein, and Stein (1993). They propose that when the cost of external financing is not comparatively high, it would be more favorable for corporations to generate funds by more actively managing risk. In the context of the life insurance industry, we study the heterogeneous attitude of life insurers with respect to interest rate risk, which directly affects the profitability and the volatility on the balance sheet. We expect mutual life insurers take less interest rate risk relative to stock insurers because of the disadvantage of access to the capital market. Specifically, we hypothesize that mutual insurers maintain a smaller duration gap, so that expose themselves to less interest rate risk than stock insurers. Also, compared with stock insurers, mutual insurers are expected to adjust duration more intensively as they should be more sensitive to the fluctuation in interest rates.

The motivation to classify insurers with respect to corporate structures is from the fact that capital transferring among the parent company and affiliates is prevailing in the insurance industry. The individual insurers' ability to raise external capital depends on their own ownership status and the ownership of their affiliates or parent companies. For instance, the insurer subsidiary may receive capital transferred from its parent company who can access the capital market. So a stock subsidiary beneath a stock parent company has a larger capital advantage than a mutual parent company. So it is more reasonable to compare the capital advantage of insurers across different corporate structures.

Besides external financing, another critical way in the insurance industry to obtain capital is through the internal capital market (ICM)[21]. Parent companies and affiliates can transfer capital within corporations. So the ownership status of corporations, which determines the capital accessibility of other insurers within one corporation, matters when evaluating the risk-taking attitude of one individual insurer. For instance, a stock insurer in a mutual corporation has less capital advantage compared with a tock insurer in a stock corporation. Therefore, we classify individual insurers according to the ownership status of their corporations and further examine the role of ownership playing in risk-taking. Three types of corporations are identified, mutual corporations, stock corporations, and mutual holding corporations[22]. Individual insurers within mutual corporations are expected to more risk-averse towards interest rate risk than those within the corporations of the other two types.

Examining life insurers' management of interest rate risk follows the approach in the previous chapter, where it focuses on the duration adjustment of the asset sides by controlling the interest rate risk of liabilities. Using the same life insurer-year sample (2004 to 2018), we find that mutual insurers, on average, maintain a duration gap of around 3-year less than stock insurers. Upon changes in interest rates, they also react more intensively than stock insurers by increasing duration by 0.15-year through actively trading bonds and interest rate derivatives. Insurers in mutual corporations exhibit a similar pattern in that they both have a smaller duration gap and adjust more intensively in response to interest rate movements relative to insurers in stock corporations or in mutual holding corporations. We also compare the risk-taking attitude

[21] Cummins and Weiss (2016) find that P&C insurers are heavy users of internal capital markets (ICMs) and those ICM transactions are shown to be efficient.

[22] mutual holding corporations is a hybrid of mutual corporations and stock corporations, which are generated from mutual holding company conversion, a non-fully demutualization.

between insurers in stock corporations and those in mutual holding corporations. Interestingly, there is no consistent evidence showing that insurers in mutual holding corporations are more risk-averse, although they are restricted to control at least 50 of voting rights when issuing stocks, which is required for stock corporations.

We conclude this paper with a discussion of the mutual holding structures. Two main points are addressed. First, we provide the possible explanation for the risk-taking of mutual holding corporations. By analyzing the special structure of mutual holding corporations, it suggests mutual holding corporations have a hybrid corporate structure of the mutual structure and the stock structure. The riskier behaviors of mutual holding companies could be due to overexploiting capital ability after conversion. We then discuss a debate between full demutualization and MHC conversion, which focuses on whether policyholders under MHC conversion are fairly treated. The debate is rooted in whether the future policy dividend received from MHC conversion is comparable with the one-time surplus allocation in full demutualization. Using existing cases of demutualization to obtain the approximation of benefit that policyholders can receive for the two scenarios could provide insight for the debate.

Structure of the paper The rest of paper is arranged as following. Section 3.2.1 develops the main argument that mutual insurers are more risk averse in terms of interest rate risk. Section 3.2.2 extends the hypothesis by interacting ownership status with corporate structures. Data and empirical testing are presented in Section 3.2.3 and Section 3.2.4. A discussion about mutual holding company conversion is provided in Section 3.2.5. And Section 3.2.6 concludes.

3.2 Hypothesis developement

We can frame the interaction between ownership structure and risk-taking following the model of Froot, Scharfstein, and Stein (1993). In their model, one decision for value-maximizing firms to make is to tradeoff the two capital raising methods, risk management, and external financing. To achieve maximal profit, firms are required to choose the level of risk management such that the marginal cost of risk management is equal to that of external financing. Otherwise, firms can always rebalance the use of two methods, thereby raising the same amount of capital but at a low cost.

In the context of this study, the ownership status plays a role in risking-taking because mutual insurers are not able to access the capital market while stock insurers can. They only have the limited ability of external financing, such as issuing surplus notes. So for mutual insurers, the marginal cost of risk management is relatively small compared with external financing; they are more likely to rely more on risk management to raise capital. Based on the reasoning, the central argument we make is mutual insurers tend to take less interest rate risk than stock insurers.

3.2.1 Ownership status and interest rate risk taking

Although there are only than 80 mutual life insurance companies in the U.S. as of 2020 (compared with over 600 stock life insurers), mutual life insurers account for over one-quarter of the industry's assets, insurance in force, premiums, and also benefits paid (Gleen Dailly, overview). Because the policyholders of mutual companies are the owner, there is no conflict of interest between shareholders and policyholders. The primary advantage of mutual ownership is that policyholder ownership eliminates the conflict of interest between investor-owners and policyholder-customers[23].

Companies have some financial disadvantages compared with stock companies. The most critical one is they have limited flexibility to raise capital and to merge with or acquire other companies because they cannot issue stock. Financial reporting is less flexible because all transactions are reflected on the parent's books. Non-insurance subsidiaries may receive a valuation penalty for being associated with a heavily regulated parent. A mutual company's identification as a life insurance company may hinder efforts to provide comprehensive financial services. Mutual companies may also have a higher tax burden than stock companies, although this has been a subject of debate.

Mutual insurers have a higher cost to raise external capital, which leads to the difference in capital structure from stock insurers. For instance, Mayers and Smith (1981) explore the incentive to take the different ownership structures of life insurance companies, stock ownership versus mutual ownership. Furthermore, Harrington and Niehaus (2002) find that other things being equal, the higher costs of raising capital should cause mutual insurers to have higher ex-ante target capital to liability ratios than stock insurers. They also

[23] See Hansmann (1996) and Mayer and Smith (1981).

find that mutual insurers' capital ratios also should be more sensitive to income than those of stock insurers. This is because, in the favorable event of income, mutual insurers tend to maintain the higher capital ratio as a buffer against future negative capital shock as it is more costly for them to raise external capital. In the adverse event of income, it takes longer for mutual to restore their target capital-liability ratio because of the same reason.

We explore the heterogeneity of the attitudes of two types of insurers towards interest rate risk. As the income of life insurers, which largely depends on the difference of the investment and the policy payouts, are highly interest-rate sensitive, interest rate risk is the key to asset-liability management in the life insurance industry. Because of higher capital costs for mutual life insurers, they are expected to be more conservative towards interest rate risk than stock life insurers. Theoretically, this argument is consistent with Froot, Scharfstein, and Stein (1993). They suggest that when external sources of finance are more costly, hedging is incentivized because it can generate funds internally to take advantage of attractive investment opportunities. Because raising external capital is much more difficult so that costly for them, mutual life insurers presumably engage in more hedging against interest rate risk relative to stock insures. This implies mutuals could maintain a smaller duration gap between assets and liabilities than stock insurers. Besides, because of the same reason, mutuals are presumably more sensitive to interest rate movements as well as described in Conjecture 17.

Conjecture 17. *Mutual life insurers are more conservative towards duration matching. Ceteris paribus,*

- *mutual life insurers maintain a smaller duration gap than stock insurers,*
- *and (or) mutual life insurers adjust asset duration more intensively reacting to interest rate movements, relative to stock life insurers.*

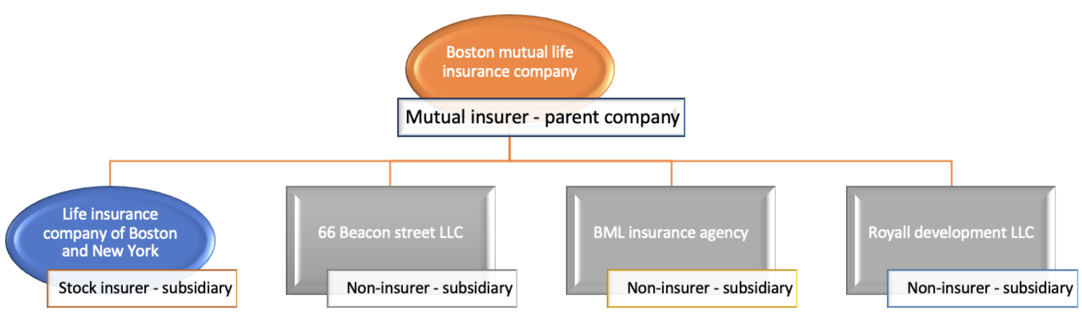
3.2.2 Ownership status in different corporate structures

To explore the role of ownership status, the motivation to classify insurers with respect to corporate structures is from the fact that capital transferring among the parent company and affiliates is prevailing in the insurance industry[24]. The individual insurers' ability to raise external capital depends on the ownership status of not only itself but also its affiliates or parent company. For instance, the insurer subsidiary may receive capital transferred from its parent company who can access the capital market. So a stock subsidiary beneath a stock parent company has a larger capital advantage than that of a mutual parent company. So it is more reasonable to compare the capital advantage of insurers across different corporate structures.

The following paragraphs of this section first analyze the capital advantages of different corporate structures, particularly the mutual holding company structure. Then, it will compare the risk-taking of individual insurers within these corporate structures.

Mutual corporate structure and demutualization The mutual corporate structure can only include a standalone mutual insurer without any downstream subsidiaries. It can also be composed of a mutual parent insurer with direct or indirect subsidiaries underneath the mutual parent company. As an example shown in Figure 3.1, the mutual parent company, Boston mutual life insurance company, owns four direct subsidiaries. One is a stock insurer, a life insurance company of Boston and New York. The other three are non-insurer subsidiaries. The individual insurers in the mutual corporate structure can either be a mutual company or a stock subsidiary company of a mutual parent insurer.

Figure 3.1: Mutual corporate structure



Insurers in the mutual corporate structure are least accessible to the capital market because of the nature of mutuality. Not like the insurers in a stock structure, they can raise external capital from selling shares to the public. The raised capital can be transferred among parent companies and subsidiaries. The capital

[24]Cummins and Weiss (2016) documents that P&C insurers are heavy users of internal capital markets (ICMs) and those ICM transactions are shown to be efficient

advantage of the stock structure is the primary motivation of mutuals to demutualize. Viswanathan and Cummins (2013) find that in the years before demutualization, converting property-liability mutuals exhibit significantly lower surplus-to-asset ratios. This capital constraint eases after demutualization. Converting life-health mutuals hold a significantly lower proportion of liquid assets; in addition, they have a higher proportion of separate accounts under management. This liquidity constraint and increased focus on a higher managerial discretion activity drive the demutualization decision. At the end of the last century, the U.S. insurance market has witnessed a wave of demutualization. In 1965, there were approximately 152 mutual life companies in the U.S., and in 1993 the number went down to there were around 102. As of 2020, there are only 28 individual mutual life insurers.

Insurers in the other two corporate structures can access the capital market by selling shares, which should have the capital advantage over insurers in a mutual structure. Therefore, we hypothesize that insurers in the mutual structure are more conservative towards the interest rate risk than insurers in the other two corporate structures.

There are two types of demutualization: fully demutualization and mutual holding company (MHC) conversion. In the first type of conversion, a former mutual company completely converts to a stock company. The ownership of policyholders of the former company is exchanged for one-time compensation in the form of cash, policy credits, or shares in the newly formed stock company. The conversion process is complicated and lasts long because the previous aggregate surplus is needed to be allocated among policyholders.

The MHC conversation is a process where a mutual insurance company generates two companies: a stock subsidiary and a mutual holding company that holds the stock company. In the mutual MHC conversion, the policyholders lose their ownership of the company but remain the right to receive policyholder dividends. The shares of the stock company are issued to the public or transferred to its original policyholders depending on the type of demutualization. In the traditional demutualization, the original policyholders are compensated in the form of stock, cash, or policy credits for losing their previous membership interest. In more modern cases, policyholders are offered subscription rights, thereby being able to purchase the shares of the converted stock company. The MHC conversion is much shorter because there is no surplus allocation to policyholders.

Figure 3.2: Corporate structure of a mutual holding company

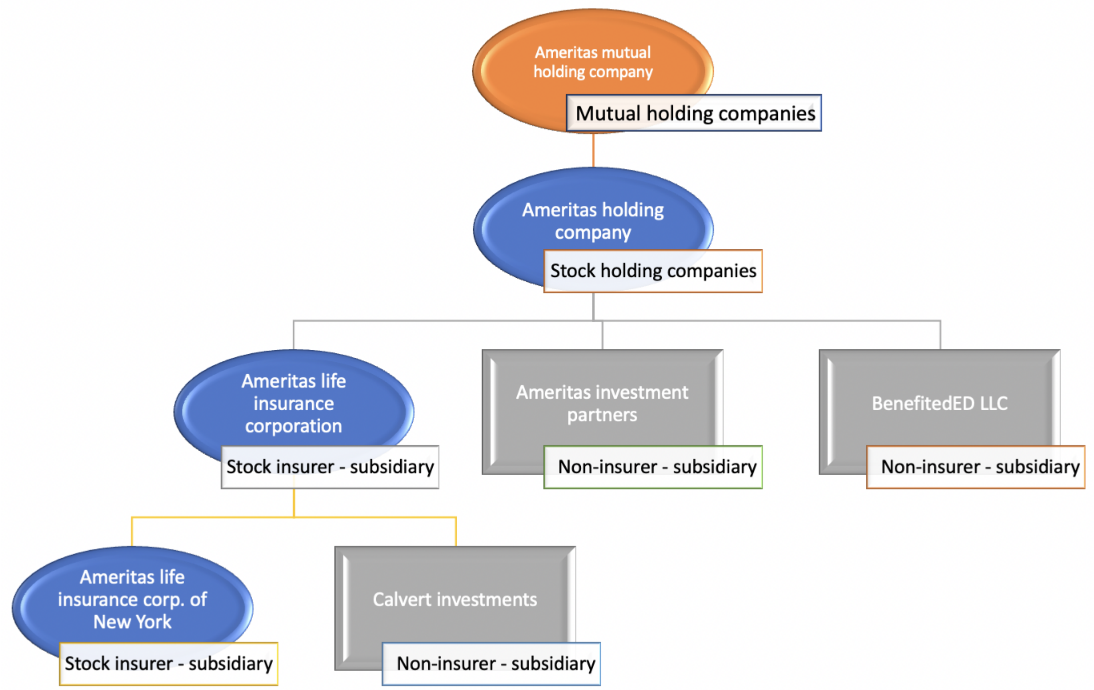


Figure 3.2 is an example of the MHC corporate structure. The MHC, Ameritas mutual holding company, on the top, has a directly controlled stock holding company, Ameritas holding company. Underneath the stock holding company, it is the stock life insurance company, Ameritas life insurance corporation, which sustain previous insurance before demutualization. Sometimes, there is no such stock holding company interposing between the MHC and its stock insurance subsidiaries.

Internal capital markets Besides external financing, insurers can receive capital through the internal capital market. A subsidiary can receive capital from its parent company when it incurs an income shock, and the capital transferring in the opposite direction is also very common. Cummins and Weiss (2016) find that P&C insurers are heavy users of internal capital markets (ICMs), and those ICM transactions are shown to be efficient. So when studying the capital advantage, it is reasonable to classify insurers by the types of their corporate structures. The capital ability of the individual insurers within one corporation is transferable to the corporation.

Insurers in the mutual corporate structures are restricted to the external market because of the mutuality nature of the mutual parent companies. In contrast, insurers in the stock corporation or in the mutual holding corporation (the MHC corporate structure) are able to access the public capital market, so they have the capital advantage over the insurers in the pure mutual corporation. In Conjecture 18, we compare the interest risk-taking across different corporate structures and hypothesize that insurers in the pure mutual structure are more conservative towards interest rate risk relative to insurers with the other two corporate structures.

Before elaborating the argument in Conjecture 18, it is worth summarizing the different types of individual insurers within different corporations. The pure mutual corporation consists of mutual parent insurers and stock subsidiary insurers. It can as well be one standalone mutual company. The stock corporation consists of stock parent insurers and stock subsidiary insurers. Similar to the mutual corporation, it can be a standalone stock insurer. The mutual holding corporation consists of a mutual holding company and its stock subsidiaries. As all insurance businesses are transferred to stock subsidiaries in the mutual holding corporation structure, the individual insurers in the mutual holding corporation are all stock companies.

Conjecture 18. *Life insurers in the mutual corporate structure are more conservative towards interest rate risk than insurers in the stock corporate structure or the MHC structure. Ceteris paribus, relative to life insurers in the other two corporate structures,*

- *1) life insurers in the mutual corporate structure maintain a smaller duration gap,*
- *2) or mutual life insurers adjust asset duration more intensively, reacting to interest rate movements.*

3.2.3 Data

Identify the ownership status of corporations Each individual insurer reports their ownership status to NAIC in the annual statutory report. So it is easy to identify the ownership status for individual insurers. The identification of corporate structures is not that straightforward. As in both the mutual structure and the MHC structure, there are mutual parent insurers. One can start with their corporate structures and identify their stock insurer subsidiaries[25]. The identification of the MHC from mutual insurers uses two sources of

[25] The information is from Schedule Y of annual statutory report. One can easily access the same information from the company information in S&P Global market intelligence.

information. First, the company database of S&P Global market intelligence provides the classification of mutual holding companies under ownership status. But it does not exhaust all of them. We refer to Gleen Daily as a supplement where he lists most demutualization history of life insurers. Apart from these stock insurers who are subsidiaries of either a mutual company or an MHC, the rest of the individual stock insurers belong to the stock corporate structure.

Variables Financial Strength Rating (FSR) assigned by AM Best[26] to insurers is an independent opinion of an insurer's financial strength and ability to meet its ongoing insurance policy and contract obligations. So it is another common proxy of financial constraints for insurers used by scholars besides RBC ratios. FSR has seven rating categories from the highest level, Superior, indicated by alphabet symbols A++ and A, to the lowest level, Poor, indicated by D[27].

Corresponding to the letter ratings, AM Best also assigns two digital numeric rating codes, with A++ being 10 and D being 42. The complete conversion chart between letter ratings and numeric ratings is as in Appendix Figure C.1. If insurers are assigned two different ratings within one year, the average numeric rating is used to determine the corresponding letter ratings. If an insurer has multiple branches and each of the branches is evaluated separately, the simple average numeric ratings of all its branches are used to determine the letter ratings for the insurer. The number of insurer-year observations is listed in Appendix Table C.1.

Summary statistics The sample includes 845 individual life insurers in total, where 37 of them are mutual companies and the rest are stock companies. Table 3.1 reports the corporation distribution of all these individual insurers. The majority are taking the stock corporate structure, and few of them are taking the mutual structure or the mutual holding company structure. In addition, it reflects the fact of corporate structures that mutual individual insurers only appear in mutual corporations as the standalone insurer or as the parent insurer. In contrast, stock insurers can be the subsidiaries of mutual corporations and MHC

[26]Here to acknowledge that A.M. Best generally shares financial strength ratings of U.S. life insurance company from 2004 to 2019.

[27]The complete categories of AM Best financial strength rating are Superior (A++, A+), Excellent (A, A-), Good (B++, B+), Fair (B, B-), Marginal (C++, C+), Weak (C, C-) and Poor (D). It also includes four non-rating designations, E, F, S and NR, to represent the status of not rated by AM Best because of in violation of regulatory requirements, insolvency or other reasons.

corporations besides appearing in stock corporations.

Table 3.1: Numbers of individual insurers with different ownership status and corporate structures
The table summarizes the numbers of insurers with different types of ownership status and corporate structures. All insurers are individual insurers who own a unique NAIC company code. Mutual corporations is short for insurance corporations with a pure mutual corporate structure. The similar definition goes for stock corporations which is short for a pure stock corporate structure. MHC corporations is short for the corporate structure associated with a mutual holding company.

	Mutual insurers	Stock insurers	Sum
Mutual corporations	37	26	**63**
Stock corporations	0	746	**746**
MHC corporations	0	36	**36**
Sum	**37**	**808**	**845**

Table 3.2 reports the summary statistics of the U.S. life insurance companies. It covers all insurer-year observations during the period 2004 to 2018. Panel A reports the result of the entire sample. The average duration is 6.9 and the annual active duration adjustment is 0.55 . Around 51% of liabilities are long-term liabilities. The mean of company action level (CAL) risk-based capital (RBC) ratios is extraordinarily large, which might not be a good reference for the average level. The median of CAL RBC ratios is around 480%. The same situation goes for the value of total assets: the median size of life insurers is 373 millions while the average size is 7409 millions. The divergence of the size and the level of RBC ratios suggest that there are a few gigantic companies in the life insurance industry.

In Panel B and C, the sample is separated by the ownership status of insurers. There are 335 mutual observations (insurer-year) and the stock observations are in the majority, with 7879 observations. The size and long-term proportion of mutual insurers are relatively larger than those of stock insurers. The average size of mutual insurers is more than twice that of stock insurers. The financial constraint measure, RBC ratios, does not exhibit a clear divergence between the two types of insurers. The duration and duration adjustment of mutual insurers are larger than those of stock insurers, which is preliminary evidence suggesting that mutual life insurers may be more safe players relative to stock insurers in terms of interest rate risk.

3.2.4 Empirical testing

This section investigates the role of organizational forms and the associated corporate structures in the taking of interest rate risk by life insurers. It begins with the choices of empirical specifications used to examine

Table 3.2: Summary Statistics

This table reports summary statistics of the sample used by this paper. Observations are on the year-insurer basis including all U.S. life insurers and spanning from 2004 to 2018. Variable "Active duration adjustment" represents the annul duration adjustment through portfolio structuring, which excludes the effect of interest rate varying on duration. Variable "CAL RBC ratios" is short for the risk-based capital ratios of company action levels. All summary statistics are calculated before winsorization. The summary statistics in Panel A includes all life insurers. Panel B and Panel C reports the summary statistics of mutual insurers and stock insurers, respectively.

Statistic	N	Mean	St. Dev.	25%	Median	75%
Panel A: All life insurers						
Dummy: Mutual insurers	8,214	0.041	0.198	0	0	0
Duration	8,214	6.856	4.131	3.783	6.672	9.407
Active duration adjustment	8,214	0.549	3.533	0.000	0.517	1.261
Total assets (in million)	8,214	7,408.910	28,406.010	48.484	372.510	2,813.487
Long-term liability proportions	8,214	0.514	0.328	0.206	0.571	0.819
CAL RBC ratios (%)	8,214	1,091.925	3,272.156	339.832	481.085	763.938
Panel B: Mutual life insurers						
Duration	335	8.075	3.078	6.077	7.688	9.913
Active duration adjustment	335	0.701	1.831	0.171	0.702	1.410
Total assets (in million)	335	17,670.430	51,922.360	197.368	842.053	2,706.760
Long-term liability proportions	335	0.635	0.266	0.476	0.742	0.846
CAL RBC ratios (%)	335	1,469.126	7,743.670	354.340	459.750	615.175
Panel C: Stock life insurers						
Duration	7,879	6.804	4.162	3.653	6.590	9.382
Active duration adjustment	7,879	0.542	3.587	-0.00002	0.509	1.256
Total assets (in million)	7,879	6,972.610	26,874.670	47.307	358.386	2,838.592
Long-term liability proportions	7,879	0.509	0.330	0.198	0.563	0.817
CAL RBC ratios (%)	7,879	1,075.887	2,934.913	339.455	482.120	773.380

the hypotheses. It then provides the test results and their implications. The sample of empirical tests is all individual life insurers within the period from 2004 to 2018. The observation is on an annual basis.

Empirical specifications The Baseline specification is as Equation 3.1 below, where the duration adjustment is regressed on the ownership types (of interest), interest rate factors, and insurers attributes. The roles of ownership forms in both duration adjustment and portfolio adjustment are examined in separate specifications, as in Equation 3.2 and Equation 3.3 below, where the former is tested on the level basis while the latter is tested on the changing basis.

$$Duration_{it} = \alpha_1 \mathbb{E}[\text{level}]_{[t-1:t]} + \alpha_2 \mathbb{E}[\text{term spread}]_{[t-1:t]} + \beta \text{Ownership}_i + \Gamma X_{it} + \varepsilon_{it}. \quad (3.1)$$

In the context of duration matching, this suggests that mutual insurers could maintain a smaller duration gap, or they respond more intensively to interest rate movement relative to stock insurers. To test the first part of Hypothesis 17, a dummy variable indicating mutual insurers is included in the specification (3.2), the coefficient of which is hypothesized to be positive. To test the second part of Hypothesis 17, the interaction of interest rate factors and the mutual dummy variable is included in the specification (3.2). If mutual life insurers indeed react to interest rate movements more intensively than stock life insurers, the interaction item is expected to negatively impact the duration adjustment in assets. Both the mutual dummy variable and the interaction of the mutual variable and interest rates in specification (3.3) are used to compare the adjustment intensity because of the different form of the dependent and independent variables.

$$d_{it} = C + \beta_1 Mutual_i + \beta_2 Mutual_i \times IR_t + \beta_3 IR_t + BX_{it} + \varepsilon_{it}, \quad (3.2)$$

$$Ad_{it} = C + \gamma_1 Mutual_i + \gamma_2 Mutual_i \times \Delta IR_t + \gamma_3 IR_t + \Gamma \Delta X_{it} + \varepsilon_{it}. \quad (3.3)$$

Specifically, in specification (3.2), β_1 estimates the duration gap between assets and liabilities. β_2 and β_3 measure the extra adjustment of mutual insurers relative to stock insurers. For instance, with 1% decline of the interest rate level, mutual insurers decrease β_2 year in asset duration. Different from β_1 in specification (3.2), γ_1 in specification (3.3) does not measure the duration gap because of the difference form of the specification. Instead, together with γ_2 and γ_3, it measures the extra intensity that mutual insurers react to interest rate movement than stock insurers. For instance, with 1% decline of the interest rate level, mutual insurers decrease $\gamma_1 + 2\gamma_2$ year in asset duration. So the hypothesis expects a positive value for the estimate of γ_1 and a negative value of γ_2 if mutual insurers are indeed more risk-adverse than stock insurers. The interpretation of positive γ_1 adjustment is that mutual insurers increase more or reduce less duration in adjustment. The interpretation of negative γ_2 is that mutual insurers adjust duration more intensively with the change in interest rates. It can see that the interpretation of specification (3.3) is not very straightforward. As an alternative test to compare the active duration adjustment, the sample of mutual insurers and the sample of stock insurers are tested separately as in specification (3.1). The coefficients of interest rate factors are expected to be larger for mutual insurers than stock insurers.

Test results: Mutual insurers and stock insurers The test of Hypothesis 17 in terms of duration adjustment is reported in Table 3.3. Columns from (1) to (4) compare the duration gap of the two types of insurers by including a dummy variable of mutual insurers. The significant and positive dummy coefficients show that mutuals maintain a smaller duration gap than stock insurers, which implies that they are more conservative towards interest rate risk. Then, to examine the second part of the hypothesis, the interactions of the mutual dummy variable with interest rate factors are included in columns (2), (3), and (4). The interactions with interest rate levels all have a negative and significant coefficient, which implies that mutual life insurers adjust asset duration more intensively relative to stock insurers. The coefficients of the interactions with the term spread between 10-year and 3-month are only significant in column (3), and the interactions with the term spread between 30-year and 10-year do not exhibit a significant impact on asset duration. That is, there is only mere evidence that mutual insurers react more intensively to the predictive signal of future interest rates.

The parallel tests to Table 3.3 is also implemented on the active adjustment of portfolios as in 3.4. Generally, the active duration adjustment through the portfolio structuring bears out the same conclusion about the role of ownership status. Mutuals' asset duration is adjusted more intensively relative to stock insurers. The dummy variable indicating mutuals from Columns (1) and (4) are positive and significant. Note that the two specifications are not the same as the specification (1) and (2) in Table 3.3 in the sense that here the dummy tests the change of duration instead of the magnitude of the duration gap. After adding interaction terms in Columns (2), (3), and (4), only the interaction of the interest rate level and the mutual dummy variable exhibits marginal significance. Taking a specific example, suppose that the interest rate level declines 1%, the decrease of asset duration for mutual insurers would be around 0.4 year, which is calculated by 0.15 (the coefficient of the dummy variable) + 0.25 (the coefficient of the interaction with the interest rate level).

Test results: Ownership status of corporations Table 3.5 and Table 3.6 verify the argument that the mutual structure is a more conservative corporate structure compared with the stock structure. Table tab: owner structure - two types - overall - mixed compares duration adjustment between insurers with a mutual structure with the rest of insurers, either with a pure stock structure or mutual holding company structure. In all specifications, the dummy variable indicating the mutual structure is positively significant, which implies the larger duration gap for insurers with the mutual corporate structure. The difference in duration gaps are around 2.7 years. Moreover, the interaction items of the mutual structure dummy variable with the interest rate level are negatively significant, and those with the term spread are marginally significant. That is, insurers in the mutual structure adjust duration more intensively, reacting to interest rate movements. So life insurers in the mutual structure are more conservative towards interest rate risk.

3.6 examines the argument from the perspective of active duration adjustment. The hypothesized association exists in the dummy variable of the mutual structure, although not exists in the corresponding interaction items. This is enough to verify the larger active duration adjustment of mutual structure insurers. On average, life insurers in the mutual structure increase more or reduce less duration for 0.095 years than those in the stock structure. In addition, Appendix Table C.4 employs the second approach to examine the second part of the hypothesis. It assumes the same specification for the two groups of insurers and compares their coefficients of interest rate factors. The test results show that the portfolio adjustment of insurers with the mutual structure is more sensitive to the three interest rate factors than that of insures with the non-mutual structure. The larger coefficients of interest rate factors for mutual structure insurers exhaust in both OLS estimations and panel fixed effect estimations.

Table 3.7 further explores the two non-mutual corporate structures, the pure stock structure, and the MHC structure. Both of the two dummy variables indicating the insurers of the two structures show negative estimates of their coefficients, which is consistent with previous tests in that mutuals maintain a relatively larger duration gap. What is worthy of attention is that the duration gap of insurers with the MHC structure is even larger than that of insurers with the stock structure. A similar result exhibits on the interaction items. This may suggest that MHC corporations could bear more interest rate risk than stock corporations. As the interest rate factors have a negative impact on asset duration, the positive coefficients of interactions mean that insurers with the stock or MHC structure adjust duration more intensively than those with the mutual structure. Interestingly, the estimates of interaction coefficients associated with the MHC structure are larger than those associated with the stock structure, which suggests that MHC corporations react more intensively to the interest rate fluctuation than stock corporations. This contradicts the evidence just mentioned on the duration gap, where it shows MHC corporations maintain a smaller duration gap relative to stock corporations. A further discussion of the ownership structure of MHC corporations will be provided in the next section.

The parallel examination based on the active adjustment of duration is reported in Appendix Table C.5. Only the dummy variable indicating the insurers in the stock corporation exhibits hypothesized coefficient estimates. The dummy variable associated with the MHC corporation and all interactions is not statistically significant. That is sufficient to argue the insurers in the stock corporation are more risk preferred than the insurers in the other two corporations, but no more evidence about the insurers of the MHC corporation can be derived from the set of tests.

To explore more evidence of the difference in duration adjustment through active structuring portfolios, Table 3.8 employs the different approach, using the separate specifications for insurers in different types of corporations. In this set of tests, the variable proxies the financial constraint in this table employs RBC ratios instead of A.M. Best's ratings. Because there are quite a few insurers not having the Best's designation of ratings, using Best's ratings as the measure of financial constraints would lead to the sample size very small for insurers in MHC corporations and mutual corporations. By comparing the estimates of interest rate factors of the three groups, it shows that insurers in mutual corporations actively adjust more strongly, reacting to interest rate changes, which is consistent with the argument that mutual corporations are more sensitive to the interest rate. The discrepancy between MHC corporations and stock corporations in terms of risk-taking is still not ambiguous. There is no dominant evidence showing which of the two types of corporations are more risk-averse. The difference between the estimates of them is minor.

3.2.5 Discussion of mutual holding company conversion

Mutual holding company conversion is interesting because of its complexity. In this section, we first analyze the special ownership status of the mutual holding corporations and its implication for its capital accessibility and its attitude of risk-taking. We then discuss the debates about mutual holding company conversion versus full demutualization.

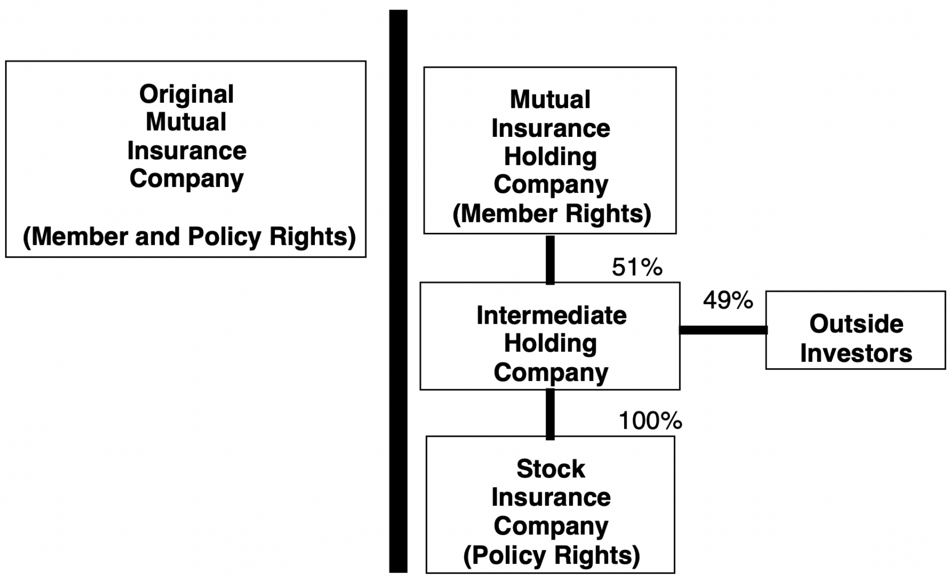
As the result of demutualization, the mutual holding company can issue stock shares to the market to raise external capital. The capital advantage of the mutual holding corporation should be comparable to normal stock corporations. On the other hand, the capital advantage created by MHC conversion is restricted by the regulation rule on MHCs. MHCs are required to retain the majority (50%) of the voting right of their stock subsidiaries, as illustrated in Figure 3.3. That means they can only sell stock shares that account for not exceeding 50% of the voting right. So the ability to raise external capital of insurers in the MHC structure is partially restrained by the requirement which is not imposed on stock corporations.

Because of such constrained stock issuing ability, the capital advantage of mutual holding corporations should be weaker than that of stock corporations. Nevertheless, the evidence in Table 3.7 and Table 3.8 from the last section supports that mutual holding companies are riskier in terms of taking interest rate risk. The possible explanation of the inconsistency could be drawn from the psychological aspect. Mutual insurers are long subjected to the constraint of external capital financing, and once the constraint is gone, they might go too risky and end up taking more risk relative to stock corporations.

In full demutualization, eligible policyholders receive compensation for giving up their ownership in the mutual company in the form of stocks of the newly created stock holding company, cash, or policy credits. Policyholders of mutual holding company conversion, on the other hand, do not fully give up their member rights and can receive policyholder dividends. There are arguments that mutual holding company conversion is more favorable for the manager than policyholders. For instance, Rambeck (2001) discusses the MHC conversion of Liberty mutual. He suggests that policyholders get an interest in the newly created MHC and no shares in the newly created stock insurer subsidiary. But interest in an MHC is like the air under a pea-less shell. In contrast, Koeppel (1998) argues that policyholders can benefit from the increased policy dividend resulting from a more flexible financing environment after conversion.

A potential approach to determine which two types of demutualization are more favorable for policyholders can be carried out by comparing the value that policyholders can receive in the two scenarios. The latest MHC conversion happened at the beginning of this century, so the time period that can be used to track the policy dividend received by policyholders after conversion is long enough. One can compare the total dividend (a flow of payments) received in the cases of MHC conversion with the one-time compensation from full demutualization. The heterogeneity across insurance companies and the discounted effect of cash flows would bring complexity to the comparison. Nevertheless, an approximate result by aggregating and averaging is still able to shed light on the debate on which types of demutualization.

Figure 3.3: Mutual holding company conversion



3.2.6 Concluding remarks

This paper furthers the investigation of the effect of different ownership statuses. Mutual insurers having difficulty in accessing the external capital market have relatively larger capital ratios and are more sensitive to income shock than stock insurers. This paper proposes that because of the same reason, mutual life insurers are more risk-averse than life insurers with stock ownership. Using the data from the U.S. life insurance industry, this paper compares the interest rate risk-taking between insurers with the two types of ownership. It finds that mutual insurers expose themselves to a smaller exposure of interest rate risk by maintaining a smaller duration gap. They are also more sensitive to interest rate movements than stock insurers in that the duration adjustment is more intensive. Because of the popularity of the internal capital market in the insurance industry, insurers are able to access capital through the internal network within corporations. This paper further compares the attitude of risk-taking among insurers with different types of corporations. Mutual corporations are still most risk-averse, which is consistent with the main argument. Mutual holding corporations and normal stock corporations, both of which can raise capital from the external market, are taking more interest rate risk. The comparison result between the two types of corporations is ambiguous, though.

This paper provides insight into the risk management of financial institutions. Rauh (2008) and Rampini, Viswanathan, and Vuillemey (2019) show that Financial constraint plays an important role in the extent of hedging. This paper suggests that the ownership status, which determines the capital ability of firms, matters as well in risk management. Risk management (including hedging) as an approach to enhancing financial strength enable firms to make use of investment opportunities. And there might exist a tradeoff between risk management and external financing, which is determined by the relative cost of the two approaches (Froot, Scharfstein, and Stein, 1993). This paper exhibits a real scenario supporting their argument. Mutual insurers, which are more costly to raise external capital, rely more on risk management.

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A Appendix for Section 1

A.1 Proof

Proof. Lemma 2.

Part 1:

Suppose there is an incentive-efficient allocation at which neither of the two types of insurees are optimally insured. According to whether the two incentive constraints of the allocation are binding, two cases will be analyzed.

$$U^g(\mathbf{z}^g) \geq U^g(\mathbf{z}^b),$$

$$U^b(\mathbf{z}^b) \geq U^b(\mathbf{z}^g).$$

Case 1: at least one of the two incentive constraints of the allocation is not binding. For instance, the incentive constraint of g type is not binding. The following analysis is to look for an Pareto improvement allocation. As the insuree b is not optimally insured, keeping the level of VT^b unchanged it is possible to increase $U^b(\mathbf{z}^b)$ a little bit (for instance, increase ε) by adjusting $\mathbf{z}^b$ properly. Accordingly, $U^g(\mathbf{z}^b)$ would increase or decrease. As long as the increasing of $U^b(\mathbf{z}^b)$, ε, is small enough, the change of $U^g(\mathbf{z}^b)$ would be less than a very small positive value so that the incentive constraint of the insuree g would not be violated. So there is another allocation supporting a utility level ($U^b(\mathbf{z}^b)$,$U^b(\mathbf{z}^b)$+ε) which is corresponding to an Pareto improvement.

Case 2: both of the two incentive constraints are binding. To look for an Pareto improvement allocation, $\mathbf{z}^g$ (or $\mathbf{z}^b$) is adjusted conditional on the level of VT^g unchanged. Therefore, for the insuree g there are only two variables of net transfers that can be freely adjusted instead of three because of condition of the fixed VT^b. For instance, the two variables are (z^g_{DH}, z^g_{DL}) for the insuree g. We want to find an adjustment vector $(\Delta z^g_{DH}, \Delta z^g_{DL})$ such that $U^g(\mathbf{z}^g)$ would increase and $U^b(\mathbf{z}^g)$ would decrease. As the gradients of $U^g(\mathbf{z}^g)$ and

$U^b(\mathbf{z}^g)$ with respect to (z^g_{DH}, z^g_{DL}) are not directly proportional,

$$\frac{\partial U^g(\mathbf{z}^g)}{\partial z^g_{DH}} \frac{\partial U^b(\mathbf{z}^g)}{\partial z^g_{DL}} \neq \frac{\partial U^g(\mathbf{z}^g)}{\partial z^g_{DL}} \frac{\partial U^b(\mathbf{z}^g)}{\partial z^g_{DH}},$$

such $(\Delta z^g_{DH}, \Delta z^g_{DL})$ does exists. Under such adjustment, both of the incentive constraints still hold. But the allocation corresponding to the adjustment is a Pareto improvement allocation

Part 2:

Here prove Lemma 2 in the case that the insuree b is optimally insured. Suppose there is an incentive-efficient allocation $\{\mathbf{z}^g, \mathbf{z}^b\}$ where the incentive constraint of the insuree b is not binding. That is, $U^b(\mathbf{z}^g) = C$, where C is a constant which is less that $U^b(\mathbf{z}^b)$. The incentive-efficient allocation $\mathbf{z}^g$, therefore, is the solution of the following optimization.

$$\begin{aligned} \underset{\{\mathbf{z}^g\}}{max}: &\quad U^g(\mathbf{z}^g) \\ s.t.: &\quad \begin{cases} U^b(\mathbf{z}^g) = C, \\ VT^g(\mathbf{z}^g) = vt^g, \end{cases} \end{aligned}$$

where vt^g is the subsidization level of the incentive-efficient allocation $\{\mathbf{z}^g, \mathbf{z}^b\}$.

When $U^g(\mathbf{z}^g)$ is at its maximal value, $U^b(\mathbf{z}^g)$ is not at its maximal value. So it is possible to increase the value of C at the optimum of $U^g(\mathbf{z}^g)$. In order to find out how the maximal value of $U^g(\mathbf{z}^g)$ would change with the change of C, the envelop theorem is employed.

$$\left.\frac{\partial U^g(\mathbf{z}^g)}{\partial C}\right|_{\mathbf{z}^g = \mathbf{z}^{g(*)}} = \lambda,$$

where λ is the Lagrangian multiplier of the constraint $U^b(\mathbf{z}^g) = C$. So the sign of λ is needed to be find out. Solve out λ from the optimization problem and then, eliminate z^g_{ND} using the condition $VT^g(\mathbf{z}^g) = vt^g$. The first order condition with respect to z^g_{DH} is:

$$\frac{\partial L}{\partial z^g_{DH}} = [\pi^b_{DH} u'_{DH} - \frac{\pi^b_{ND}\pi^g_{DH}}{E\tilde{R} - \pi^i_{DH} - \pi^i_{DL}} u'_{ND}] - \lambda[\pi^g_{DH} u'_{DH} - \frac{\pi^g_{ND}\pi^g_{DH}}{E\tilde{R} - \pi^i_{DH} - \pi^i_{DL}} u'_{ND}] = 0.$$

Since $\pi^b_{DH} > \pi^g_{DH}$ and $\pi^g_{ND} > \pi^b_{ND}$, it has $\lambda > 1$. Therefore, as the increasing of C, the maximal value of $U^g(\mathbf{z}^g)$ would increase. In another word, by making the incentive constraint of the insuree b tighter, the allocation corresponding to an Pareto improvement can be obtained. □

Proof. Lemma 6.

(1) The binding of feasibility constraints (1.11) and (1.12) is very intuitive. For instance, suppose the feasibility constraint for the good state of investment is bot binding. An infinitesimal increase of the value of z^g_{DH} can lead to a larger $U^g(\mathbf{z}^g)$.

(2) Suppose the incentive constraint (1.13) is not binding. So it is always possible ,without violating the incentive constraint, to make positive subsidization from type b insurees to type g insurees such that the utility of the insuree g increases. For instance, $IT^b = (\varepsilon, 0)$ and $IT^g = (-\frac{\xi^b p^b}{\xi^g p^g}\varepsilon, 0)$ where ε is a positive infinitesimal number. Under this subsidy, the consumption of the two types of insurees now becomes:

$$(\omega + z^b_{ND}, \omega + z^b_{DH} - \varepsilon, \omega + z^b_{DL}),$$

and

$$(\omega + z^g_{ND}, \omega + z^b_{DH} + \frac{\xi^b p^b}{\xi^g p^g}\varepsilon, \omega + z^b_{DL}),$$

which give a higher utility of the insuree g. The incentive constraint of the insuree b still holds as long as ε is a small enough positive number. Therefore, it can be concluded that the incentive constraint (1.13) is binding since the opposite way cause the contradiction.

(3) The main result of the claim is the existence of cross-subsidization with $VT^i = 0$ at optimum. Suppose there is no subsidization at the optimum. It implies that the RS separating allocation is the solution of the programming. Here let the solution be $\{z^g, z^b\}$. Then we show that there exists another allocation in which type g insuree has the higher utility than in the RS separating allocation. And besides, the subsidy of such allocation satisfies $IT^g_{DL} > 0$ and $IT^g_{DH} < 0$. Therefore, we can claim the existence of subsidization at the optimum. Specifically, it is the subsidy from the insuree g to the insuree b in the state DL and the subsidy from the insuree b to the insuree g in the state DH with $VT^i = 0$.

Consider the subsidization $IT^g_{DL} = \varepsilon$ and $IT^g_{DH} = \frac{\pi^b_{DL}}{\pi^b_{DH}}\varepsilon$ (ε is a positive infinitesimal real number) such that the fair premium constraint still hold for the both type. Imposed on the subsidy the consumption of type insures b, $c^b = (c^b_{ND}, c^b_{DH}, c^b_{DL})$, at the three states becomes:

$$(\omega + z^b_{ND}, \omega + z^b_{DH} - \frac{\pi_{DL}}{\pi_{DH}}\varepsilon, \omega + z^b_{DL} + \varepsilon).$$

The consumption of type g insuree, $c^g = (c^g_{ND}, c^g_{DH}, c^g_{DL})$, after subsidization is:

$$(\omega + z^g_{ND}, \omega + z^b_{DH} + \frac{\xi^b p^b \pi_{DL}}{\xi^g p^g \pi_{DH}}\varepsilon, \omega + z^b_{DL} - \frac{\xi^b p^b}{\xi^g p^g}\varepsilon).$$

At the RS separating result, type g insurees are sub-optimally insured conditional on the incentive constraint of type b insurees being binding. As the subsidy imposed is infinitesimal, the incentive constraint can still be able to bind with the increase of η units of insurance that the insuree g choose to buy. Therefore, it is available that we increase the premium of type b insuree such that the incentive constraint keeps binding after the subsidy ε. Specifically, $U^b(z^g)$ is not at maximum since $U^g(z^g)$ reaches its maximum. So there exists the change η that can keep the following incentive constraint binding.

$$U^b(z^b, \varepsilon) = U^b(z^g, \varepsilon, \eta).$$

Accordingly, the change of consumption in the state DH and DL are $\frac{r-p^g}{p^g}\eta$ and $\frac{d-p^g}{p^g}\eta$, respectively. The incentive constraint now becomes:

$$U^b(z^b, \varepsilon) = \pi^b_{ND} u(\omega + z^b_{ND}) + \pi^b_{DH} u(\omega + z^b_{DH} - \frac{\pi_{DL}}{\pi_{DH}}\varepsilon) + \pi^b_{DL} u(\omega + z^b_{DL} + \varepsilon), \tag{A.1}$$

and

$$U^b(z^g, \varepsilon, \eta) = \pi^b_{ND} u(\omega + z^g_{ND} - \eta) + \pi^b_{DH} u(\omega + z^g_{DH} + \frac{\xi^b p^b \pi_{DL}}{\xi^g p^g \pi_{DH}}\varepsilon + \frac{r - p^g}{p^g}\eta) + \pi^b_{DL} u(\omega + z^g_{DL} + \frac{\xi^b p^b}{\xi^g p^g}\varepsilon + \frac{d - p^g}{p^g}\eta).$$

In order to examine the change of $U^g(z^g)$ after the subsidization, differentiate $U^b(z^g, \varepsilon, \eta)$ with respect to ε. Note that here η is the function of ε subjected to the incentive constraint of type b insuree. The calculation is processed as following. First differentiate the both side of equation (A.1) with respect to ε, to solve out the expression of $\frac{d\eta}{d\varepsilon}$. Then, plugging into the expression differentiate $U^g(z^g,\varepsilon,\eta)$ with respect to ε. It gives the following result. For convenience, here use $u'_s = \frac{\partial u(x)}{x}\,\big| x = c^g_s$ for $s \in \{ND, DH, DL\}$.

$$\frac{\partial U^g(z^g,\varepsilon,\eta)}{\partial \varepsilon} = \frac{Y}{X}A + \frac{\xi^b}{\xi^g}B.$$

The notation $X = -\pi^b_{ND}u'_{ND} + \frac{r-p^g}{p^g}\pi^b_{DH}u'_{DH} + \frac{d-p^g}{p^g}\pi^b_{DL}u'_{DL}$ and $Y = -\pi^g_{ND}u'_{ND} + \frac{r-p^g}{p^g}\pi^g_{DH}u'_{DH} + \frac{d-p^g}{p^g}\pi^g_{DL}u'_{DL}$, and $A = u'(\omega + z^b_{DL} + \varepsilon) - u'(\omega + z^b_{DH} - \frac{\pi_{DL}}{\pi_{DH}}\varepsilon)$ and $B = \frac{p^b\pi^g_{DL}}{p^g\pi^g_{DH}}(1-q)(p^b - p^g)u'_{ND}(u'_{DH} - u'_{DL})$.

Now it is time to examine the sign of $\frac{\partial U^g(z^g,\varepsilon,\eta)}{\partial \varepsilon}$. Observe that $X = \frac{\partial U^b(z^g,\varepsilon,\eta)}{\partial \eta}$ is positive since the change of units of insurance purchased η increases the value of $U^b(z^g,\varepsilon,\eta)$ such that $U^b(z^b,\varepsilon) = U^b(z^g,\varepsilon,\eta)$. $Y = \frac{\partial U^g(z^g,\varepsilon,\eta)}{\partial \eta}$ is also positive when $\varepsilon$is infinitesimal. This is because at the allocation z^g type g insurees are sub-optimally insured conditional on the incentive constraint be binding. They could have got higher utility by purchase more insurance if they are subjected to the incentive constraint. So the positive change of amount of insurance purchased which is η would increase the utility of type g insurees. Moreover, A is positive and B is negative when ε is small. Finally, it can be concluded that the sign of $\frac{\partial U^g(z^g,\varepsilon,\eta)}{\partial \varepsilon}$ is positive when ξ^g is large enough. In this case, the subsidization corresponding to ε does support a higher utility for type g insuree than the RS separating allocation. It follows that the subsidization does exists at optimum. The subsidization constraint (1.15) is slack at optimum. □

Proof. Theorem 11.

The first step is to show the claim: at an ALPT equilibrium the choice of any insuree is the same as the one obtained when the insuree is free to choose any incentive compatible and budget feasibility constraint allocations in the EPT economy.

This is equivalently to show that the set of ALPL equilibrium allocations is same as the set of incentive compatible and budget feasibility constraint allocations. There are two differences between the two sets. One is the budget constraint in the ALPT economy contains the trade of consumption rights while the one in the EPT economy does not. But we have show in the Proposition 14 that the budget constraint of ALPT economy does not impose extra restriction other than that in the EPT economy. The other difference is the enforcement condition of consumption rights in the ALPT economy. It is actually equivalent to incentive constraints at equilibrium. The reasoning is as following. To maximize the utility, each of insurees would sell out the rights that they own but are designed for the other type of insurees. For instance, the insuree b holds $\frac{\xi_g}{\xi_b} z_b + \alpha\omega$ units of rights g where $\frac{\xi_g}{\xi_b} z_b$ units come from the producing mechanism and $\alpha\omega$ units come from the endowment. Since the insuree b sells out all rights g at equilibrium, each of insurees g at equilibrium holds $\omega + z_b$ units of rights g which is equal to the consumption of insurance of the insuree b. A symmetric argument holds for the insuree b. So the enforcement condition is equivalent to incentive constraints at equilibrium. Now the claim in question can be concluded.

Based on the first step, then we show the existence of equilibrium price vector. In particular, we show there exists β such that the price vector (1.24) and (1.25) defines an equilibrium.

Because the objective function is quasi-concave as $u(\cdot)$ is assumed to satisfy Inada condition, the demand of each type of insurees is single-valued that is parameterized by the β. For an arbitrary β, solve out for the two types of insurees the optimal value of ζ_s^i of which β is the parameter. Then, combining the condition of market clearing for rights, the value of β is well determined. □

Proof. Proposition 12.

The problem of the insuree b is as:

$$\max_{\{\mathbf{z}^g, \zeta^g\}} : U^g(\mathbf{z}^g) = \sum_{s \in S} \pi_s^g u(\omega_s + z_s^g)$$
$$s.t. : \begin{cases} \mathbf{z}^g \cdot \mathbf{q}^g + \beta(\alpha)\zeta^g \cdot \mathbf{q}^b \leq \mathbf{0}, \\ \sum_s \pi_s^b u((1-\alpha)\omega_s + \zeta_s^g) \leq \sum_s \pi_s^b u(\omega + z_s^g). \end{cases}$$

The problem of the insuree g is as:

$$\max_{\{\mathbf{z}^b\}} : U^b(\mathbf{z}^b) = \sum_{s \in S} \pi_s^b u(\omega_s + z_s^b)$$
$$s.t. : \begin{cases} \mathbf{z}^b \cdot \mathbf{q}^b + \beta(\alpha)\zeta^b \cdot \mathbf{q}^b \leq \mathbf{0}, \\ \zeta^b(g) + (\xi^g/\xi^b)(\alpha\omega + \mathbf{z}^b) \geq \mathbf{0}. \end{cases}$$

In order to maximize $U^b(\mathbf{z}^b)$, $\mathbf{z}^b \cdot \mathbf{q}^b$ needs to be maximized. It follows that $\zeta^b(g) = -(\xi^g/\xi^b)(\alpha\omega + \mathbf{z}^b)$. That is, the insuree b sells out all right g owned and supply the minimal amount of the right g required by consuming the insurance b. The constraints, then, implies the expected subsidy of the insuree b:

$$VT^b = \frac{(\xi^g/\xi^b)\beta(\alpha)\alpha}{1-(\xi^g/\xi^b)\beta(\alpha)} \sum_s \pi_s^b \omega_s.$$

Denote VT^b at the level of the MW allocation as VT^b_{MW}. If we can find a function $\beta(\alpha)$ that can satisfy the following two conditions , it can conclude $\lim_{\alpha\to 0} \mathbf{z}^i = \mathbf{z}^i_{MW}$. The conditions are

$$\lim_{\alpha\to 0} \beta(\alpha) = (\xi^b/\xi^g),$$

and

$$\lim_{\alpha\to 0} VT^b = VT^b_{MW}.$$

Specifically, when α goes to 0 the problem solves out the MW allocation for the insuree g if the first condition holds. The second condition guarantees the optimal choice of the insuree b coincides with the MW allocation as well.

So now the goal is to find function $\beta(\alpha)$. Imposing L'Hôpital's rule on the second condition, it gives

$$\lim_{\alpha\to 0} VT^b = \frac{\beta(\alpha)}{-\beta'(\alpha)} \sum_s \pi_s^b \omega_s = VT^b_{MW}.$$

Since VT^b_{MW} is a determined non-negative number, $\beta(\alpha)$ constrained by the two conditions exits and not unique. Thus we can conclude as α converges to 0, the ALPT equilibrium allocation converges to the MW allocation at the price vector $\beta(\alpha)$. □

Proof. Proposition 13.

Suppose there is an ALPT equilibrium allocation $(\mathbf{z}^g, \mathbf{z}^b)$ that is not incentive-efficient. That is, there is an incentive-compatible and feasible allocation $(\hat{\mathbf{z}}^g, \hat{\mathbf{z}}^b)$ that is a Pareto-improvement on $(\mathbf{z}^g, \mathbf{z}^b)$ since ALPT equilibrium allocations satisfy incentive-compatible and feasible conditions. So at least one inequality being strict for the following two inequalities that are budget constraints of the two types of insurees:

$$\hat{\mathbf{z}}^g \cdot \mathbf{q} + \hat{\zeta}^g \cdot \mathbf{p} \geq \mathbf{0},$$

and

$$\hat{\mathbf{z}}^b \cdot \mathbf{q} + \hat{\zeta}^g \cdot \mathbf{p} \geq \mathbf{0}.$$

Multiple the two inequities indexed by two types with the proportion of the types and then add them two up. It obtains

$$(\sum_i \xi^i \hat{\mathbf{z}}^i) \cdot \mathbf{q} > 0,$$

which means the feasible constrain is violated. □

Proof. Proposition 14.

First solve out the incentive-efficient allocation w.r.t. IT^i. Then, show such optimal choices can be implemented by one ALPT equilibrium when IT^i is given.

The incentive-efficient allocation w.r.t. IT^i can be solved out according to claim 2. At an arbitrage IT^i, the optimal of choice that type b insuree is optimally insured:

$$\begin{aligned} &\max_{\{z^b\}} : U^b(\mathbf{z}^b) = \sum_{s \in S} \pi_s^b u(\omega_s + z_s^b) \\ &s.t.: \begin{cases} p^b(z_{DH}^b - z_{ND}^b + IT_{DH}^b) + z_{ND}^b r \leq 0, \\ p^b(z_{DH}^b - z_{ND}^b + IT_{DL}^b) + z_{ND}^b d \leq 0, \end{cases} \end{aligned} \tag{A.2}$$

Given the choice of type b insuree, type g insuree is sub-optimally insured at IT^i:

$$\begin{aligned} &\max_{\{z^g\}} : U^g(\mathbf{z}^g) = \sum_{s \in S} \pi_s^g u(\omega_s + z_s^g) \\ &s.t.: \begin{cases} p^g(z_{DH}^g - z_{ND}^g + IT_{DH}^g) + z_{ND}^g r \leq 0, \\ p^b(z_{DL}^g - z_{ND}^g + IT_{DL}^g) + z_{ND}^g d \leq 0, \\ U^b(z^g) \leq U^b(z^b). \end{cases} \end{aligned} \tag{A.3}$$

From Claim 2, all feasibility constraints and the incentive compatible constraint of the insuree b are binding at optimum. Combining the feasible condition of the subsidy of indemnities $\xi^g p^g IT^g_{s'} + \xi^b p^b IT^b_{s'} = 0$, it follows that the feasibility constraints of insuree g in the problem above is equivalent to the feasibility constraint of the market (1.2) and (1.3) at equilibrium. The same goes for the insuree b.

Given subsidization IT^b, the type b insuree in the ALPT economy is faced with problem:

$$\begin{aligned} \max_{\{\mathbf{z}^g,\zeta^g\}\in Z(z^b,\zeta^b)}: \quad & U^g(z^g) = \sum_{s\in S} \pi^g_s u(\omega_s + z^g_s) \\ s.t.: \quad & \mathbf{z}^g \cdot \mathbf{q} + \zeta^g \cdot \mathbf{p} \leq \mathbf{0}, \end{aligned}$$

where Z is admissible set depending on the choice of type b insuree. So for the given choice of type b insuree, the admissible set can be represented by the market feasibility constraint and the incentive constraint:

$$\max_{\{\mathbf{z}^g\}}: U^g(z^g) = \sum_{s\in S} \pi^g_s u(\omega_s + z^g_s)$$
$$s.t.: \begin{cases} \mathbf{z}^g \cdot \mathbf{q} + \zeta^g \cdot \mathbf{p} \leq \mathbf{0}, \\ \sum_{i\in\{g,b\}} \xi^i [p^i(z^i_{DH} - z^i_{ND}) + z^i_{ND} r] \leq 0, \\ \sum_{i\in\{g,b\}} \xi^i [p^i(z^i_{DL} - z^i_{ND}) + z^i_{ND} d] \leq 0, \\ U^b(z^g) \leq U^b(z^b). \end{cases} \tag{A.4}$$

Accordingly, given subsidization IT^b type b insuree's problem in the ALPT economy is:

$$\begin{aligned} \max_{\{\mathbf{z}^b,\zeta^b\}\in Z(z^g,\zeta^g)}: \quad & U^b(z^b) = \sum_{s\in S} \pi^b_s u(\omega_s + z^b_s) \\ s.t.: \quad & \mathbf{z}^b \cdot \mathbf{q} + \zeta^b \cdot \mathbf{p} \leq \mathbf{0}, \end{aligned}$$

which can represented by

$$\max_{\{\mathbf{z}^b\}}: U^b(\mathbf{z}^b) = \sum_{s\in S} \pi^b_s u(\omega_s + z^b_s)$$
$$s.t.: \begin{cases} \mathbf{z}^b \cdot \mathbf{q} + \zeta^b \cdot \mathbf{p} \leq \mathbf{0}, \\ \sum_{i\in\{g,b\}} \xi^i [p^i(z^i_{DH} - z^i_{ND}) + z^i_{ND} r] \leq 0, \\ \sum_{i\in\{g,b\}} \xi^i [p^i(z^i_{DL} - z^i_{ND}) + z^i_{ND} d] \leq 0, \\ U^b(z^g) \leq U^b(z^b). \end{cases} \tag{A.5}$$

Compare the problems to find out the incentive-efficient allocation w.r.t. IT^i (A.2) and (A.3) with the problems in the ALPT economy (A.4) and (A.5). The main difference is there are additional budget constraints in the problems of the ALPT economy.

As for budget constraint $\mathbf{z}^b \cdot \mathbf{q} + \zeta^b \cdot \mathbf{p} \leq \mathbf{0}$, it always holds at the ALPT equilibrium since $\zeta^b \cdot \mathbf{p} = -\mathbf{z}^b \cdot \mathbf{q}$ from market clear condition. As for budget constraint $\mathbf{z}^g \cdot \mathbf{q} + \zeta^g \cdot \mathbf{p} \leq \mathbf{0}$, it is actually the zero-profit constraint of the market (1.1) at equilibrium. With the feasibility constraints of the market (1.2) and (1.3) holding, the zero-profit constraint holds as well. Therefore, the each of budget constraints does not narrow the constraint set of the programming of the ALTP problem. We can claim any incentive-efficient allocation w.r.t. IT^i can be decentralized by an ALPT equilibrium. □

B Appendix for Section 2

B.1 Data

Treasure rates We use ten years' treasury rate as a presentative rate that life insurers refer to when they make investment decision. The monthly data of 10 year's treasury rate is widely available. The time period from January 2003 to November 2018 is chosen as the window of observation. In particular, the monthly rate is usually the treasury rate calculated on the rest day of each month.

Derivative data The derivative information is obtained from Schedule DB of the NAIC regulatory report where it includes transactions and holding data of each derivatives. During the sample period from 2010 to 2018, the holding information of derivatives is reported in Part A Section 1. It records all options, caps, floors, collars, swaps and forwards holding information as of the report date (December 31) of the year. S&P Global market intelligence (previous SNL financial) aggregates the information of Schedule DB for all each life insurer year by year. This paper export the aggregated data from this data platform.

To server the purpose of this paper, it needs to identify derivatives that are used to hedge the interest rate risk. First, the derivatives of interest are those used to hedge the interest rate risk, not other types of risk. Then, the recognition[28] of derivatives should be hedging. Finally, because it is not able to compute the

[28] Other categories of derivative recognition are replication, income generation and "other" category.

duration for forward derivatives using information available in Schedule DB and also the risk exposure of forward contracts are very limited, this paper does not include the forward holding in the sample.

Since 2010 insurers has been required to specify the type of risk that each derivative position hedges against, such as interest rate risk, default credit default risk and foreign exchange risk. This paper focus on derivatives to hedge against the interest rate risk[29]. The formate of risk specification is not completely uniform. In particular, some insurers use single letter or numbers instead of complete words to indicate different types of risks. For these insures, it needs to dig into the Schedule BD of their annual statutory report to check the definition of these notations.

Financial constraints Two types of data are used to measure the extent of being financial constrained. One is the RBC ratio, which is used by Ellul etc. (2011). In this paper, life insurers are categorized into three groups according to their company action level RBC ratios. The least and the more constrained constrained group are those whose RBC ratios falls into the top and the bottom 25 percentile; the rest of insurers are the middle group.

Financial strength ratings evaluated by A.M. Best[30] is another measure of financial strength used in the paper. In general, AM Best assigns one financial strength rating to each company identified by one unique NAIC number. However, it exists the case where life insurance entity identified by one NAIC number but owning multiple subsidiaries are evaluated with respect to each subsidiary so that they could be assigned to different ratings for different subsidiaries. In this case, an average ratings of all subsidiaries is computed to be associated with the life insurance company who are assigned different ratings for its subsidiaries.

Bond transaction Data The insurance statutory data in Part 3, Part 4 and Part 5 of Schedule D fills transaction records of long-term bonds and stock. Part 3 reports all bond holdings acquired but not disposed yet in the current filling year and Part 4 reports all bond holdings disposed but not acquired in the current filling year. All bond holdings that are both acquired and disposed in the current filling year are report in

[29]The risk hedged by each derivative position is not available in Schedule DB before year 2010, which is also the reason the sample period of this paper starts from year 2010.

[30]Here to acknowledge that A.M. Best generally shares financial strength ratings of U.S. life insurance company from 2010 to 2019.

Part 5.

The type of transactions in terms of security types can be identified by the line numbers, which is specified by SNL team. If exporting data from Screener, bond transactions and stock transactions are already separated.

The bond issue information in Part 3, Part 4 and Part 5 of Schedule D is not as complete as that in Part 1, long-term bond holding records (only Part 4 reports the maturity of each bond transacted). But the full CUSIP is reported, which can be used to look up the maturity, coupon rates, and coupon payment frequency. For instance, using CUSIP one can link transaction data with bond issue information in Fixed Income Securities Database (FISD). Alternatively, we linked the bond issue information exploiting the Screener function of S&P Global market intelligence where bond transactions and stock transactions are separated and bond issue information can be directly added through field selector. There are two drawback of this approach. First, it is that the coupon frequency is not available in this way; a simplified assumption of the frequency is needed (assume semi-annually paid). Second, there is date exporting limit so that it needs export data manually for multiple times.

B.2 Measure construction

B.2.1 Asset duration

To calculate asset duration, the information of each insurer' investment portfolio is required. Insurance investment holding data in S&P Global Market Intelligence (SNL) provides the annually bonds holding information. It includes the holding data of all U.S. life insurers and the time range is from 2004 to 2018.

The idea of a relatively simply measure of the asset duration is combining the carrying value and the time to maturity to form an approximate of duration. The duration of an insurer in a year is the average year to maturity weighted by carrying value of the bond portfolio. The annual carrying value of holding bonds and the maturity of them are exported from the section of the insurance investment.

Construct the duration measure of the bond portfolio For a bond with coupon payment and the principal being C and F. Its annual effective interest rate is i and the nominal interest rate in one coupon payment period is y, so it has $(1+y)^m = 1+i$, where m is the coupon frequency per year. N is the total number of payment periods of coupon in the rest life of the bond. The duration following directly from the definition is the first order derivative of price with respect to yield divided by price. The price and the duration of the bond are calculated as:

$$P = \frac{C}{1+y} + \frac{C}{(1+y)^2} + \cdots + \frac{C+F}{(1+y)^N} = \frac{C[1-(1+y)^{-N}]}{y} + \frac{F}{(1+y)^N};$$

$$\frac{dP}{di} = \frac{dP}{dy}\frac{dy}{di} = \{-\frac{C[1-(1+y)^{-N}]}{y^2} + \frac{NC}{y(1+y)^{N+1}} - \frac{NF}{(1+y)^{N+1}}\} \times \frac{1}{m}(1+i)^{1/m-1};$$

$$\text{duration} = -\frac{dP}{di} \times \frac{1}{P}.$$

The special case is the bonds without coupons or whose coupon is only paid at maturity, the duration is reduced as the time to maturity in years. The portfolio duration is the average duration of bonds in the portfolio weighted by their fair value.

B.2.2 Derivative duration

The blow list the assumptions that simplifying the calculation of $DV01$ of interest rate derivatives.

Swaps

- The payment frequency of the fixed leg is assumed to be semi-annual. The fixed rate is in reference to the swap yield curve at statutory report dates.
- The float rate is in reference to the 3 month LIBOR as it is most common floating index in the derivative holding information reported to NAIC. The payment frequency of the float leg is, therefore, assumed to be quarterly.
- The discount rates is in reference to the U.S. treasury yield curve in order to keep consistent with the discount rate used to compute bond duration.
- The time to maturity from the reporting date to the expiration date is taken as the maturity of the swap.

Caps and floors

- The information of payment frequency is incomplete in the NAIC report. Here it assumes that the payments are twice per year since the interest rate derivatives are typically paid semi-annually.
- The float rate of cap is in reference to the 3 month LIBOR as it is most common floating index in the derivative holding information reported to NAIC. The payment frequency of the float leg is assumed to be quarterly.
- The discount rates is in reference to the U.S. treasury yield curve in order to keep consistent with the discount rate used to compute bond duration.
- The time to maturity from the reporting date to the expiration date is taken as the maturity of the swap.
- The $DV01$ of caps is exported from Bloomberg Terminal. Then, the $DV01$ of interest rate floors can be obtained from the put-call parity.

Relation of $DV01$ and $\frac{\partial P}{\partial r}$ the $DV01$ measures the change in present value caused by one basis point of the interest rate, namely, $DV01 = P(r+0.01\%) - P(r)$. Applying the Taylor expansion to the expression, it gives

$$DV01 = \frac{\partial P(r)}{\partial r} \times 0.01\% + \frac{\partial^2 P(r)}{\partial r^2} \times (0.01\%)^2 + \cdots .$$

Thus, $DV01$ can be proximately equal to $\frac{\partial P(r)}{\partial r} \times 0.01\%$.

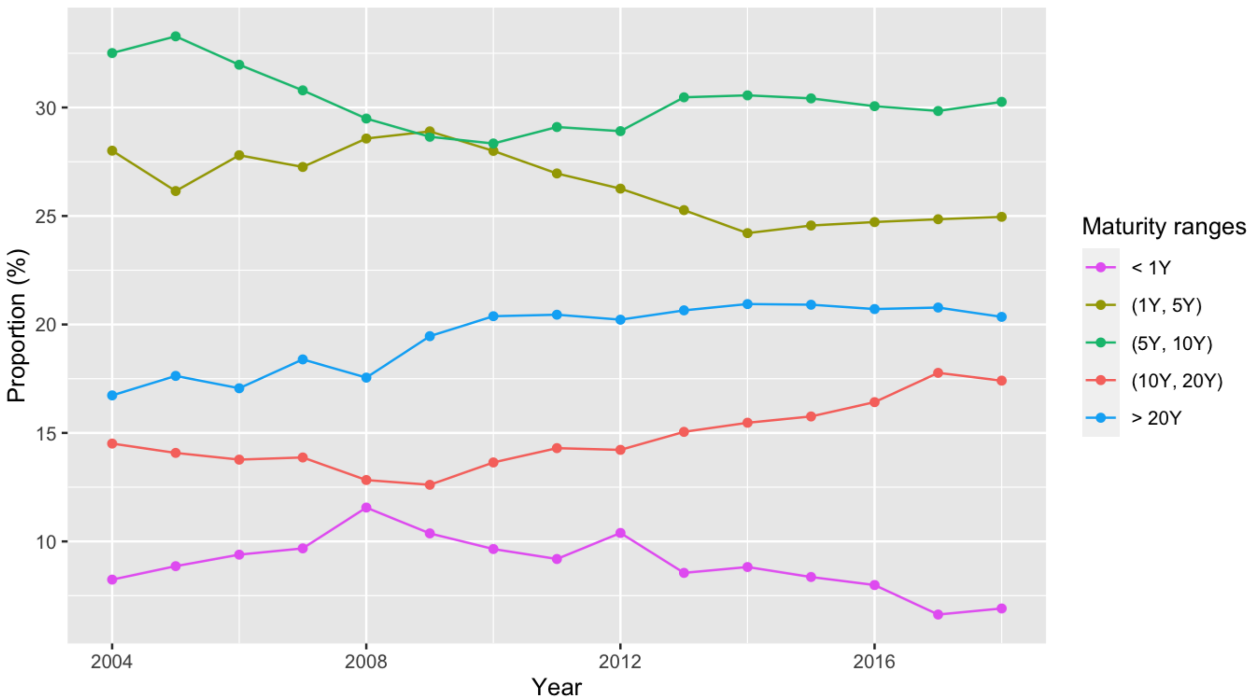
B.2.3 Measure of interest rate movement

The holding information of bonds is from Schedule D Part 1A Section 1 of the annual statutory report. As showed in the table below held bonds are divided into 5 maturity ranges. First, for each maturity range, use the average of corresponding spot rates of treasury bonds within the range as the average rate of this range. Then, calculate the weighted average of these rates for these maturity ranges, which is the measure of the weighted interest rate level. The weights of each maturity ranges is determined by the value proportion of bonds falling into that maturity range. The value used is the aggregate carrying value of the life insurance industry in the entire sample period. Figure B.1 shows that the value proportion of bonds in each maturity range does not have substantial change during the sample period.

Maturity ranges	Spot rates averaged for each range
1 year or less	Average rate of 1-month, 3-month, 6-month, 1-year
over 1 year through 5 years	Average rates of 1-year, 2-year, 3-year, 5-year
over 5 years through 10 years	Average rates of 5-year, 7-year, 10-year
over 10 years through 20 years	Average rates of 10-year, 20-year
over 20 years	Average rates of 20-year, 30-year

B.3 Figures and tables

Figure B.1: Maturity distribution of the bond holding of the life insurance industry
The figure illustrate the proportion of the held bonds by the life insurance industry that fall into each maturity bucket. The proportion is calculated based on the carrying value of bonds at the statutory reporting date, December 31th of each year. The data is from Schedule D Part 1A Section 1 of the annual statutory report.



C Appendix for Section 3

C.1 Figure and tables

Figure B.2: Interpolate yield curves

In this figure it interpolates the complete treasury yield curves at three dates. The complete yield curves are interpolated by assuming the implied forward rates between two known spots rates with closest terms to be equal. The red and green curves are examples of upward yield curve and the blue one is more of flat yield curve.

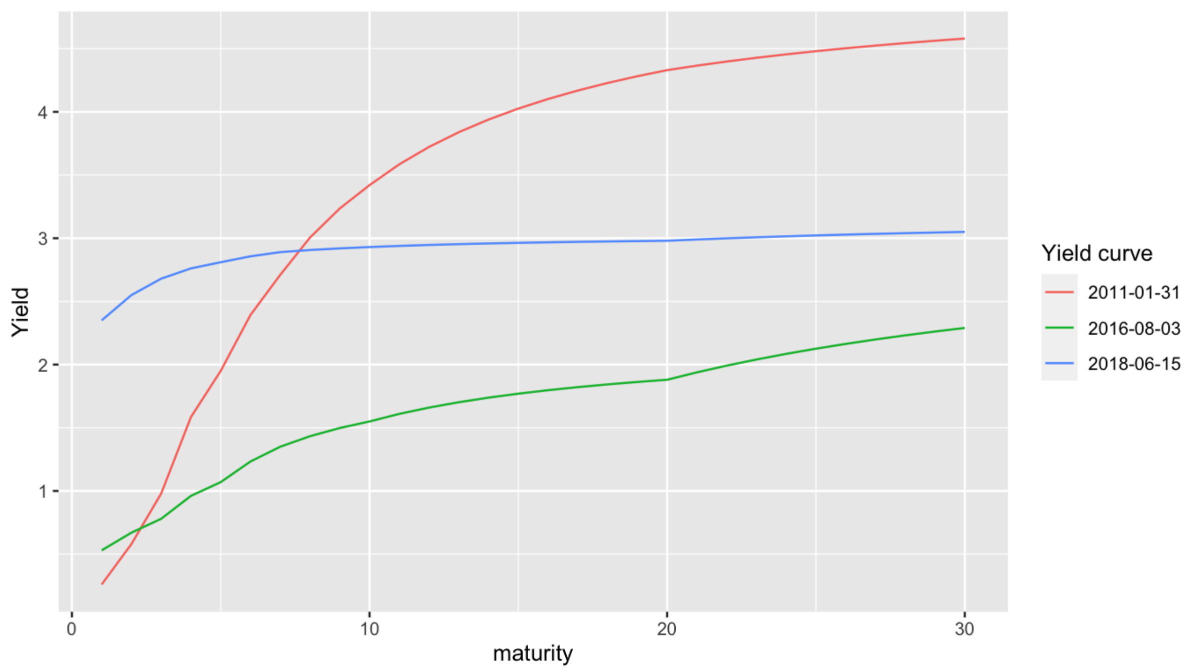


Table 3.3: Ownership status and interest rate risk-taking

The table reports the duration adjustment with a focus on the role of organizational forms. The observations of all tests are U.S. life insurers from 2004 to 2018. Mutual dummy variable is to indicate mutual companies, and the other dummy variable "AMB rating below B-" is to indicate life insurers with A.M. Best rating being lower than B-, which are considered as financially constrained insurers. All specification are estimated by OLS.

	Dependent variable: Asset duration			
	(1)	(2)	(3)	(4)
Level	-0.587^{***}	-0.559^{***}	-0.477^{***}	-0.538^{***}
	(0.051)	(0.053)	(0.041)	(0.052)
Term spread (10Y-3M)	-0.347^{***}	-0.330^{***}	-0.435^{***}	
	(0.070)	(0.072)	(0.059)	
Term spread (30Y-10Y)	-0.699^{***}	-0.677^{**}		-1.392^{***}
	(0.263)	(0.269)		(0.220)
Long-term liability proportion	3.836^{***}	3.843^{***}	3.841^{***}	3.829^{***}
	(0.124)	(0.124)	(0.124)	(0.124)
Log[total assets] (in thousand)	−0.008	−0.010	−0.008	−0.005
	(0.017)	(0.017)	(0.017)	(0.017)
Dummy: AMB rating below B-	-0.685^{**}	-0.692^{**}	-0.697^{**}	-0.740^{**}
	(0.349)	(0.349)	(0.349)	(0.349)
Dummy: Mutual insurer	0.741^{***}	3.306^{***}	2.997^{***}	3.142^{**}
	(0.193)	(1.282)	(0.931)	(1.275)
Mutual insurer × Level		-0.608^{**}	-0.554^{***}	-0.589^{**}
		(0.243)	(0.190)	(0.243)
Mutual insurer × Term spread (10Y-3M)		−0.383	-0.457^{*}	
		(0.341)	(0.274)	
Mutual insurer × Term spread (30Y-10Y)		−0.455		−1.312
		(1.274)		(1.028)
Constant	7.758^{***}	7.659^{***}	7.166^{***}	7.429^{***}
	(0.376)	(0.380)	(0.326)	(0.378)
Observations	6,845	6,845	6,845	6,845
R^2	0.143	0.144	0.143	0.141
Adjusted R^2	0.142	0.143	0.142	0.140
Residual Std. Error	3.295	3.294	3.295	3.300

Note: *p<0.1; **p<0.05; ***p<0.01

Table 3.4: Ownership status and active duration adjustment

The table reports the active duration adjustment with a focus on the role of organizational forms. The observations of all tests are U.S. life insurers from 2004 to 2018. Mutual dummy variable is to indicate mutual companies, and the other dummy variable "AMB rating below B-" is to indicate life insurers with A.M. Best rating being lower than B-, which are considered as financially constrained insurers. All specification are estimated by OLS.

	Dependent variable: Active duration adjustment			
	(1)	(2)	(3)	(4)
ΔLevel	−0.171***	−0.163***	−0.160***	−0.170***
	(0.040)	(0.041)	(0.041)	(0.041)
ΔTerm spread (10Y-3M)	−0.096***	−0.087***	−0.087***	−0.096***
	(0.026)	(0.027)	(0.027)	(0.026)
ΔTerm spread (30Y-10Y)	−0.572***	−0.585***	−0.572***	−0.583***
	(0.127)	(0.130)	(0.127)	(0.130)
ΔLong-term liability proportion	1.015***	1.018***	1.020***	1.019***
	(0.200)	(0.200)	(0.200)	(0.200)
ΔLog[total assets] (in thousand)	2.043***	2.040***	2.041***	2.041***
	(0.066)	(0.066)	(0.066)	(0.066)
Dummy: AMB rating below B-	−0.398***	−0.401***	−0.399***	−0.401***
	(0.129)	(0.129)	(0.129)	(0.129)
Dummy: Mutual insurer	0.156**	0.143*	0.130*	0.172**
	(0.070)	(0.078)	(0.073)	(0.076)
Mutual insurer × ΔLevel		−0.182	−0.249*	−0.032
		(0.197)	(0.134)	(0.168)
Mutual insurer × ΔTerm spread (10Y-3M)		−0.184	−0.182	
		(0.126)	(0.126)	
Mutual insurer × ΔTerm spread (30Y-10Y)		0.290		0.259
		(0.623)		(0.623)
Constant	0.561***	0.562***	0.562***	0.560***
	(0.017)	(0.017)	(0.017)	(0.017)
Observations	6,202	6,202	6,202	6,202
R^2	0.145	0.146	0.146	0.145
Adjusted R^2	0.144	0.144	0.144	0.144
Residual Std. Error	1.136	1.136	1.136	1.136

Note: *p<0.1; **p<0.05; ***p<0.01

Table 3.5: Ownership status of corporations and interest rate risk-taking

The table examines the role of ownership structures in duration adjustment. The dummy variable, "mutual structure", indicates individual life insurers that are in a mutual corporate structure, which including mutual parent insurers and their stock subsidiaries. All specification are estimated by OLS.

	Dependent variable: Asset duration			
	(1)	(2)	(3)	(4)
Level	-0.590^{***}	-0.540^{***}	-0.550^{***}	-0.542^{***}
	(0.052)	(0.054)	(0.053)	(0.054)
Term spread (10Y-3M)	-0.347^{***}	-0.321^{***}	-0.308^{***}	-0.347^{***}
	(0.071)	(0.074)	(0.073)	(0.071)
Term spread (30Y-10Y)	-0.703^{***}	-0.620^{**}	-0.703^{***}	-0.562^{**}
	(0.264)	(0.277)	(0.264)	(0.272)
Long-term liability proportion	3.859^{***}	3.871^{***}	3.870^{***}	3.870^{***}
	(0.125)	(0.125)	(0.125)	(0.125)
Log[total assets] (in thousand)	-0.005	-0.005	-0.005	-0.005
	(0.017)	(0.017)	(0.017)	(0.017)
Dummy: AMB rating below B-	-0.648^{*}	-0.629^{*}	-0.637^{*}	-0.629^{*}
	(0.349)	(0.349)	(0.349)	(0.349)
Dummy: Mutual corporation	0.365^{***}	2.917^{***}	2.276^{***}	2.764^{***}
	(0.140)	(0.938)	(0.679)	(0.929)
Mutual corporation × Level		-0.547^{***}	-0.436^{***}	-0.529^{***}
		(0.178)	(0.138)	(0.177)
Mutual corporation × Term spread (10Y-3M)		-0.293	-0.434^{**}	
		(0.245)	(0.199)	
Mutual corporation × Term spread (30Y-10Y)		-0.909		-1.547^{**}
		(0.917)		(0.748)
Constant	7.709^{***}	7.470^{***}	7.529^{***}	7.483^{***}
	(0.377)	(0.387)	(0.382)	(0.387)
Observations	6,809	6,809	6,809	6,809
R^2	0.143	0.144	0.144	0.144
Adjusted R^2	0.142	0.143	0.143	0.143
Residual Std. Error	3.297	3.295	3.295	3.295

Note: $^{*}p<0.1$; $^{**}p<0.05$; $^{***}p<0.01$

Table 3.6: Corporation types and active duration adjustment

The table examines the active duration adjustment of life insurers with different ownership structure. Specifications in odd columns are estimated by OLS and those in even columns are fitted into the fixed-effect panel regression model.

	Dependent variable: Active duration adjustment			
	(1)	(2)	(3)	(4)
Level	-0.175^{***}	-0.176^{***}	-0.175^{***}	-0.175^{***}
	(0.040)	(0.041)	(0.040)	(0.040)
Term spread (10Y-3M)	-0.093^{***}	-0.093^{***}	-0.091^{***}	-0.093^{***}
	(0.026)	(0.026)	(0.027)	(0.026)
Term spread (30Y-10Y)	-0.597^{***}	-0.597^{***}	-0.597^{***}	-0.599^{***}
	(0.127)	(0.128)	(0.128)	(0.129)
Long-term liability proportion	1.015^{***}	1.015^{***}	1.014^{***}	1.015^{***}
	(0.200)	(0.200)	(0.200)	(0.200)
Log[total assets] (in thousand)	2.101^{***}	2.101^{***}	2.102^{***}	2.101^{***}
	(0.067)	(0.067)	(0.067)	(0.067)
Dummy: AMB rating below B-	-0.389^{***}	-0.389^{***}	-0.389^{***}	-0.389^{***}
	(0.129)	(0.129)	(0.129)	(0.129)
Dummy: Mutual corporation	0.096^{*}	0.096^{*}	0.093^{*}	0.097^{*}
	(0.050)	(0.050)	(0.051)	(0.051)
Mutual corporation × Level		0.009		
		(0.057)		
Mutual corporation × Term spread (10Y-3M)			−0.022	
			(0.054)	
Mutual corporation × Term spread (30Y-10Y)				0.017
				(0.212)
Constant	0.557^{***}	0.557^{***}	0.557^{***}	0.557^{***}
	(0.017)	(0.017)	(0.017)	(0.017)
Observations	6,168	6,168	6,168	6,168
R^2	0.148	0.148	0.148	0.148
Adjusted R^2	0.147	0.147	0.147	0.147
Residual Std. Error	1.133	1.133	1.133	1.133
Note:				*p<0.1; **p<0.05; ***p<0.01

Table 3.7: Corporation types and interest rate risk-taking

The table examines the role of three ownership structures in duration adjustment. The individual insurers of the stock structure or the mutual holding company (MHC) structure are indicated by two dummies variables. All specification are estimated by OLS.

	Dependent variable: Asset duration			
	(1)	(2)	(3)	(4)
Level	-0.593^{***}	-0.793^{***}	-0.984^{***}	-1.069^{***}
	(0.052)	(0.103)	(0.136)	(0.169)
Term spread (10Y-3M)	-0.349^{***}	-0.350^{***}	-0.740^{***}	-0.350^{***}
	(0.071)	(0.070)	(0.194)	(0.070)
Term spread (30Y-10Y)	-0.704^{***}	-0.697^{***}	-0.701^{***}	-2.096^{***}
	(0.264)	(0.264)	(0.264)	(0.729)
Long-term liability proportion	3.860^{***}	3.866^{***}	3.871^{***}	3.872^{***}
	(0.125)	(0.125)	(0.125)	(0.125)
Log[total assets] (in thousand)	0.002	0.002	0.002	0.002
	(0.017)	(0.017)	(0.017)	(0.017)
Dummy: AMB rating below B-	-0.659^{*}	-0.653^{*}	-0.637^{*}	-0.632^{*}
	(0.349)	(0.349)	(0.349)	(0.349)
Dummy: Stock corporation	-0.320^{**}	-0.843^{***}	-2.117^{***}	-2.616^{***}
	(0.140)	(0.287)	(0.681)	(0.932)
Dummy: MHC corporation	-0.913^{***}	-2.017^{***}	-3.985^{***}	-4.349^{***}
	(0.206)	(0.421)	(0.995)	(1.339)
Stock corporation × Level		0.205^{**}	0.407^{***}	0.501^{***}
		(0.098)	(0.139)	(0.177)
MHC corporation × Level		0.448^{***}	0.759^{***}	0.838^{***}
		(0.149)	(0.206)	(0.261)
Stock corporation × Term spread (10Y-3M)			0.413^{**}	
			(0.200)	
MHC corporation × Term spread (10Y-3M)			0.644^{**}	
			(0.297)	
Stock corporation × Term spread (30Y-10M)				1.500^{**}
				(0.750)
MHC corporation × Term spread (30Y-10Y)				1.974^{*}
				(1.077)
Constant	7.995^{***}	8.497^{***}	9.702^{***}	10.146^{***}
	(0.405)	(0.463)	(0.726)	(0.926)
Observations	6,809	6,809	6,809	6,809
R^2	0.144	0.145	0.146	0.146
Adjusted R^2	0.143	0.144	0.145	0.144
Residual Std. Error	3.294 (df = 6800)	3.292 (df = 6798)	3.292 (df = 6796)	3.292 (df = 6796)

Note: *p<0.1; **p<0.05; ***p<0.01

Table 3.8: Corporation type and active duration adjustment

The table examines the active duration adjustment of life insurers with different corporate structure, the mutual corporate structure, the mutual holding company (MHC) structure, and the stock corporate structure. Specifications in odd columns are estimated by OLS and those in even columns are fitted into the fixed-effect panel regression model. The variable proxizes the financial constraint in this table employs RBC ratios instead of A.M. Best's ratings. It is to avoid insufficiency of observations for the group of insurers with the MHC structure and the mutual structure. All insurers are required to report RBC ratios but only part of insurers own rating designation from A.M. Best.

	Dependent variable: Active duration adjustment					
	Mutual corporations		MHC corporations		Stock corporations	
	(1)	(2)	(3)	(4)	(5)	(6)
ΔLevel	−0.199*	−0.212*	−0.117	−0.127	−0.128***	−0.122***
	(0.120)	(0.111)	(0.132)	(0.105)	(0.040)	(0.034)
ΔTerm spread (10Y-3M)	−0.119	−0.115	−0.069	−0.064	−0.065**	−0.068***
	(0.078)	(0.084)	(0.083)	(0.055)	(0.026)	(0.023)
ΔTerm spread (30Y-10Y)	−0.634*	−0.654**	−0.638	−0.696**	−0.481***	−0.458***
	(0.377)	(0.259)	(0.416)	(0.278)	(0.126)	(0.100)
ΔLong-term liability proportion	1.368*	1.162	3.383***	3.008**	0.711***	0.724**
	(0.719)	(1.105)	(1.083)	(1.171)	(0.172)	(0.307)
ΔLog[total assets] (in thousand)	2.439***	2.148***	1.652***	1.650***	1.965***	1.609***
	(0.246)	(0.504)	(0.208)	(0.198)	(0.064)	(0.151)
Dummy: RBC ratio < 10%	0.079	0.324	−0.418	−0.440**	−0.084*	−0.021
	(0.168)	(0.277)	(0.494)	(0.193)	(0.046)	(0.073)
Constant	0.615***		0.624***		0.541***	
	(0.050)		(0.054)		(0.017)	
Fixed effects	–	Firm	–	Firm	–	Firm
Observations	643	643	440	440	7,037	7,037
R^2	0.141	0.288	0.176	0.341	0.125	0.260
Adjusted R^2	0.133	0.207	0.165	0.275	0.124	0.177
Residual Std. Error	1.073	1.026	0.983	0.915	1.199	1.162

Note: *p<0.1; **p<0.05; ***p<0.01

Table B.1: Insurance exposure by policyholders' ages

This table summarize the exposure of insurance and annuity products by type. In addition, the age cohort of each exposure data is also specified. The exposure of each type products in each of age cohorts is measured by two variables. One is the percentage of polices by their number within the same type of polices. The other exposure variable is the average face value / premium of a certain type of police in each age cohort.

<table>
<tr><th></th><th colspan="8">term life insurance</th><th colspan="2">whole life</th><th colspan="2">universal life</th><th colspan="2">variable annuity</th></tr>
<tr><th>Age cohort</th><th>exposure of policies (YRT)</th><th>average exposure in face value (YRT)</th><th>exposure of policies (10-Year LPT)</th><th>average exposure in face value (10-Year LPT)</th><th>exposure of policies (20-Year LPT)</th><th>average exposure in face value (20-Year LPT)</th><th>exposure of policies (30-Year LPT)</th><th>average exposure in face value (30-Year LPT)</th><th>exposure of policies</th><th>average exposure in face value</th><th>exposure of policies</th><th>average exposure in face value</th><th>exposure of policies</th><th>average premium</th></tr>
<tr><td>0-4</td><td rowspan="4">4%</td><td rowspan="4">$34,000</td><td rowspan="4">1%</td><td rowspan="4">$142,000</td><td rowspan="4">0</td><td rowspan="4">$172,000</td><td rowspan="4">0</td><td rowspan="4">$206,000.00</td><td rowspan="4">7%</td><td rowspan="4">$80,000</td><td rowspan="4">7%</td><td rowspan="4">$49,000</td><td rowspan="12">24%</td><td rowspan="12">$132,026</td></tr>
<tr><td>5-9</td></tr>
<tr><td>10-14</td></tr>
<tr><td>15-19</td></tr>
<tr><td>20-24</td><td rowspan="2">24%</td><td rowspan="2">$233,000</td><td rowspan="2">9%</td><td rowspan="2">$419,000</td><td rowspan="2">10%</td><td rowspan="2">$404,000</td><td rowspan="2">16%</td><td rowspan="2">$421,000.00</td><td rowspan="2">6%</td><td rowspan="2">$81,000</td><td rowspan="2">9%</td><td rowspan="2">$64,000</td></tr>
<tr><td>25-29</td></tr>
<tr><td>30-34</td><td rowspan="2">50%</td><td rowspan="2">$349,000</td><td rowspan="2">25%</td><td rowspan="2">$591,000</td><td rowspan="2">43%</td><td rowspan="2">$599,000</td><td rowspan="2">58%</td><td rowspan="2">$616,000.00</td><td rowspan="2">9%</td><td rowspan="2">$116,000</td><td rowspan="2">10%</td><td rowspan="2">$132,000</td></tr>
<tr><td>35-39</td></tr>
<tr><td>40-44</td><td rowspan="2">18%</td><td rowspan="2">$389,000</td><td rowspan="2">32%</td><td rowspan="2">$684,000</td><td rowspan="2">34%</td><td rowspan="2">$600,000</td><td rowspan="2">24%</td><td rowspan="2">$575,000.00</td><td rowspan="2">13%</td><td rowspan="2">$120,000</td><td rowspan="2">16%</td><td rowspan="2">$170,000</td></tr>
<tr><td>45-49</td></tr>
<tr><td>50-54</td><td rowspan="2">4%</td><td rowspan="2">$408,000</td><td rowspan="2">25%</td><td rowspan="2">$662,000</td><td rowspan="2">12%</td><td rowspan="2">$484,000</td><td rowspan="2">2%</td><td rowspan="2">$545,000.00</td><td rowspan="2">19%</td><td rowspan="2">$90,000</td><td rowspan="2">24%</td><td rowspan="2">$156,000</td></tr>
<tr><td>55-59</td></tr>
<tr><td>60-64</td><td rowspan="8"></td><td rowspan="8">$430,000</td><td rowspan="8">8%</td><td rowspan="8">$675,000</td><td rowspan="8">1%</td><td rowspan="8">$473,000</td><td rowspan="8">0%</td><td rowspan="8">$0.00</td><td rowspan="2">19%</td><td rowspan="2">$60,000</td><td rowspan="2">20%</td><td rowspan="2">$138,000</td><td>22%</td><td>$151,014</td></tr>
<tr><td>65-69</td><td>24%</td><td>$149,345</td></tr>
<tr><td>70-74</td><td rowspan="6">27%</td><td rowspan="6">$30,000</td><td rowspan="6">14%</td><td rowspan="6">$196,000</td><td>16%</td><td>$144,806</td></tr>
<tr><td>75-79</td><td>9%</td><td>$143,654</td></tr>
<tr><td>80-84</td><td rowspan="4">6%</td><td rowspan="4">$150,283</td></tr>
<tr><td>85-89</td></tr>
<tr><td>90-94</td></tr>
<tr><td>95-99</td></tr>
</table>

Table B.2: Exposure proportion by types of life insurance industry products

The table reports the exposure proportion of insurance and annuity products by types.

Life insurance	29.69%		
	Whole life	6.30%	
	Universal life	7.20%	
	Term life	16.19%	
		Yearly renew term	1.55%
		10-year level premium term	3.26%
		15-year level premium term	1.38%
		20-year level premium term	8.79%
		30-year level premium term	1.22%
Annunity	70.31%		
	Fixed annutiy	19.18%	
	Variable annuity	51.13%	

Table B.3: Industrial holding in the general account and separate account (As of 2014)

This table reports the aggregate holding of the life insurance industry in terms of bond portfolios and insurance and annuity reserve. The unit of the amount in the table is thousand dollar.

	Bonds	Life insurance reserve	Annuity reserve
General account	5370201984	2031864712	2731094781
Separate account	326039764	279522598	1855471244

Table B.4: Correlation matrix of interest rate factors

The table shows the correlation matrix of interest rate factors. The sample period to calculate Pearson coefficients is from 2004 to 2018. Variable "Level" is the simple average of a sequence of spot rates on the treasury yields, including those of 1-month, 3-month, 6-month, 1-year, 2-year, 5-year, 10-year, 20-year and 30-year treasury bonds. Variable "Level (weighted)" is the average of the same spot rates weighted by the holding information of bonds with the corresponding maturity.

	Level (simple average)	Level (weighted average)	Term spread (10Y&3M)	Term spread (10Y&2Y)	Term spread (30Y&10Y)
Level (simple average)	1	0.975	-0.639	-0.665	-0.789
Level (weighted average)	0.975	1	-0.460	-0.514	-0.719
Term spread (10Y&3M)	-0.639	-0.460	1	0.954	0.715
Term spread (10Y&2Y)	-0.665	-0.514	0.954	1	0.817
Term spread (30Y&10Y)	-0.789	-0.719	0.715	0.817	1

Table B.5: Overall duration adjustment: derivative users and non-derivative users

The table compares duration adjustment across interest rate derivative users and non-derivative users. The sample period is from 2010 to 2018. For the time period from 2004 to 2009, as lack of derivative data to compute its duration exposure, only non-derivative users are included in the sample. Non-derivative users are determined as those who are below the top 5% percentile in terms of the absolute book value of derivatives. The dependent variable "average duration" is defined as the combined asset dollar duration divided by total assets. "Level (spot rates)" is the average of a sequence of spot rates on the treasury yield curve. "Level (forward rates)" is the average of the forward rates of the yield curve.Long-term liability proportion is the value percentage of long-term reserves in total liabilities. The dummy variable of RBC ratios indicates the insures whose RBC ratios are below 10% percentile.

	Dependent variable: Average duration						
	All life insurers			Non-derivative users		Derivative users	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Level	-0.186***	-0.334***	-0.373***	-0.323***	-0.407***	-0.341**	-0.845**
	(0.025)	(0.041)	(0.037)	(0.041)	(0.037)	(0.150)	(0.349)
Term spread (10y&3m)		-0.270***	-0.329***	-0.258***	-0.344***	-0.384**	-0.761***
		(0.039)	(0.052)	(0.040)	(0.053)	(0.157)	(0.262)
Long-term liability proportion	1.875***	1.968***	3.377***	2.150***	3.201***	-4.367	7.249***
	(0.433)	(0.436)	(0.111)	(0.434)	(0.112)	(3.879)	(0.811)
Log[total assets] (in thousand)	0.605***	0.504***	0.186***	0.495***	0.236***	-0.182	-0.136
	(0.134)	(0.136)	(0.014)	(0.138)	(0.015)	(0.663)	(0.118)
Dummy: RBC ratio < 10%	-0.137	-0.131	0.187	-0.128	0.123	-0.495	3.160***
	(0.136)	(0.137)	(0.122)	(0.138)	(0.123)	(0.544)	(0.853)
Constant			4.059***		3.694***		8.062***
			(0.273)		(0.276)		(2.333)
Fixed effects	Firm	Firm	-	Firm	-	Firm	-
Observations	9,107	9,107	9,107	8,802	8,802	305	305
R^2	0.806	0.808	0.116	0.809	0.122	0.893	0.300
Adjusted R^2	0.786	0.789	0.116	0.789	0.121	0.866	0.289
Residual Std. Error	1.717	1.707	3.492	1.711	3.488	1.186	2.736

Note: *p<0.1; **p<0.05; ***p<0.01

Table B.6: Active duration adjustment: RBC ratios

This table reports regression analysis of the reaction of the overall asset duration to interest rate movements. The full sample includes observations from 2004 to 2018. For the time period from 2004 to 2009 where only non-derivative users are included because the detailed derivative data is not available. Non-derivative users from 2004 to 2009 are identified as those who are below the top 5% percentile in terms of the absolute book value of derivatives. Derivative users in the time period from 2010 and 2018 are are life insurers whose $DV01$ of interest rate derivatives is greater than 5% of $DV01$ of bond portfolios. The dependent variable is the active duration adjustment measured by active duration (defined in Equation 2.2) divided by total assets. Long-term liability proportion is the value percentage of long-term reserves in total liabilities. The dummy variables of RBC ratios indicates the insures whose RBC ratios are below 10% percentile and above 90% percentile, respectively.

	Dependent variable: Active duration adjustment					
	(1)	(2)	(3)	(4)	(5)	(6)
*Delta*Level	-0.131^{***}	-0.131^{***}	-0.131^{***}	-0.127^{***}	-0.127^{***}	-0.127^{***}
	(0.037)	(0.037)	(0.037)	(0.031)	(0.031)	(0.031)
*Delta*Term spread (10Y-3M)	-0.070^{***}	-0.070^{***}	-0.070^{***}	-0.073^{***}	-0.073^{***}	-0.073^{***}
	(0.024)	(0.024)	(0.024)	(0.021)	(0.021)	(0.021)
*Delta*Term spread (30Y-10Y)	-0.486^{***}	-0.487^{***}	-0.486^{***}	-0.464^{***}	-0.464^{***}	-0.464^{***}
	(0.115)	(0.115)	(0.115)	(0.090)	(0.091)	(0.091)
Delta Long-term liability proportion	0.773^{***}	0.769^{***}	0.774^{***}	0.762^{***}	0.763^{***}	0.763^{***}
	(0.164)	(0.164)	(0.164)	(0.294)	(0.294)	(0.294)
*Delta*Log[total assets] (in thousand)	1.932^{***}	1.930^{***}	1.930^{***}	1.618^{***}	1.619^{***}	1.619^{***}
	(0.058)	(0.058)	(0.058)	(0.134)	(0.133)	(0.134)
Dummy: RBC ratio < 10%	-0.087^{**}		-0.090^{**}	0.007		0.007
	(0.043)		(0.043)	(0.070)		(0.070)
Dummy: RBC ratio > 90%		-0.047	-0.056		0.034	0.034
		(0.061)	(0.062)		(0.117)	(0.117)
Constant	0.553^{***}	0.547^{***}	0.556^{***}			
	(0.015)	(0.015)	(0.016)			
Fixed effects	–	–	–	*Firm*	*Firm*	*Firm*
Observations	8,194	8,194	8,194	8,194	8,194	8,194
R^2	0.124	0.124	0.124	0.264	0.264	0.264
Adjusted R^2	0.124	0.123	0.124	0.183	0.183	0.183
Residual Std. Error	1.182	1.182	1.182	1.141	1.141	1.141

Note: *p<0.1; **p<0.05; ***p<0.01

Table B.7: Duration matching and corporate bond yields

This table tests the association between corporate bond yields and duration adjustment. The corporate bond yields are in reference to high quality market (HQM) corporate bonds yields. The mixed yields in column (5) is the weighted average of treasury yields and HQM corporate bond yields with the ratio of two types of yields being 3 to 1. The ratio is approximately equal to the ratio of government bonds and corporate bonds in the asset portfolio of life insurers. Long-term liability proportion is the value percentage of long-term reserves in total liabilities.

	Dependent variable: Combined DV01 / total assets (%)				
	Treasury yields		HQM corporate yields		Mixed yields
	(1)	(2)	(3)	(4)	(5)
Level: treasury bonds	-0.269***	-0.267***			
	(0.037)	(0.038)			
Term spread (10y&3m): treasury bonds	-0.114***				
	(0.025)				
Term spread (10y&2y): treasury bonds		-0.086***			
		(0.026)			
Level: corporate bonds			-0.292***	-0.288***	
			(0.037)	(0.037)	
Term spread (10y&6m): corporate bonds			-0.056***		
			(0.020)		
Term spread (10y&2y): corporate bonds				-0.058***	
				(0.022)	
Level: mixed yields					-0.281***
					(0.039)
Term spread (10y&2y): mixed yields					-0.082***
					(0.025)
Long-term liabilities proportion	0.015***	0.015***	0.015***	0.015***	0.015***
	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)
Log[total assets] (in billion)	0.004***	0.004***	0.004***	0.004***	0.004***
	(0.0005)	(0.0005)	(0.0005)	(0.0005)	(0.0005)
Dummy: RBC ratio < 10%	-0.001	-0.001	-0.001	-0.001	-0.001
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
Fixed effects	Firm	Firm	Firm	Firm	Firm
Observations	5,318	5,318	5,318	5,318	5,318
R^2	0.050	0.048	0.050	0.050	0.049
Adjusted R^2	-0.088	-0.090	-0.087	-0.088	-0.089
F Statistic (df = 5; 4643)	48.519***	46.595***	49.321***	49.070***	47.529***
Note:			*p<0.1; **p<0.05; ***p<0.01		

Table B.8: Duration adjustment and interest rates (Robustness test: company group level)

This table focus on the size-effect testing on the individual-level and the group-level data. Observations in column (1) and (2) are individual life insurance companies. Observations in column (3) and (4) are SNL groups and NAIC groups, respectively.

	Dependent variable: Combined DV01 / total asset			
	Individual insurers (1)	Individual insurers (2)	SNL insurer groups (3)	NAIC insurer groups (4)
Level	-0.026***	-0.026***	-0.026***	-0.032***
	(0.004)	(0.004)	(0.005)	(0.005)
Slope (10y&3m)	-0.017***	-0.016***	-0.013***	-0.010***
	(0.003)	(0.002)	(0.004)	(0.004)
Total assets (in billion)	0.00000		0.0002	0.0002
	(0.0003)		(0.0003)	(0.0003)
Dummy: large insurer		0.046***		
		(0.015)		
Saled long-term liabilities	0.260***	0.260***	0.156***	0.188***
	(0.021)	(0.021)	(0.033)	(0.036)
Dummy: RBC ratio < 10%	-0.008	-0.008	-0.004	-0.005
	(0.008)	(0.008)	(0.011)	(0.010)
Fixed effects	Individual	Individual	Individual	Individual
Observations	5,318	5,318	2,678	2,625
R^2	0.045	0.047	0.021	0.028
Adjusted R^2	-0.094	-0.092	-0.116	-0.107
F Statistic	43.741*** (df = 5; 4643)	45.614*** (df = 5; 4643)	10.166*** (df = 5; 2347)	13.286*** (df = 5; 2303)

Table B.9: Duration adjustment and interest rate movements (alternative measures)

The sample period is from 2010 to 2018. For the time period from 2004 to 2009, as lack of derivative data to compute its duration exposure, only non-derivative users are included in the sample. Non-derivative users are determined as those who are below the top 5% percentile in terms of the absolute book value of derivatives. Covariant "Level (simple average)" is the simple average of a sequence of spot rates on the treasury yield curve. Covariant "Level (weighted average)" is the average of the same spot rates weighted by the holding information of bonds with the corresponding maturity. The dummy variable of RBC ratios indicates the insures whose RBC ratios are below 10% percentile.

	Dependent variable: Average duration (asset dollar duration / total assets)					
	Level (simple average)			Level (weighted average)		
	(1)	(2)	(3)	(4)	(5)	(6)
Level (simple average)	-0.225***	-0.383***	-0.330***			
	(0.029)	(0.047)	(0.052)			
Term spread (10y&3m)		-0.290***			-0.173***	
		(0.042)			(0.030)	
Term spread (30y&10y)			-0.497***			-0.465***
			(0.174)			(0.143)
Level (weighted average)				-0.334***	-0.430***	-0.451***
				(0.041)	(0.050)	(0.061)
Long-term liability proportion	1.864***	1.963***	1.899***	1.903***	1.965***	1.944***
	(0.470)	(0.474)	(0.471)	(0.473)	(0.475)	(0.475)
Log[total assets] (in thousand)	0.585***	0.477***	0.534***	0.529***	0.468***	0.475***
	(0.148)	(0.149)	(0.151)	(0.148)	(0.149)	(0.152)
Dummy: RBC ratio < 10%	-0.178	-0.171	-0.177	-0.176	-0.172	-0.175
	(0.153)	(0.156)	(0.154)	(0.154)	(0.156)	(0.155)
Fixed effects	Firm	Firm	Firm	Firm	Firm	Firm
Observations	9,107	9,107	9,107	9,107	9,107	9,107
R^2	0.781	0.783	0.782	0.783	0.784	0.783
Adjusted R^2	0.759	0.761	0.759	0.761	0.762	0.761
Residual Std. Error	2.002	1.992	2.000	1.994	1.989	1.992

Note: *p<0.1; **p<0.05; ***p<0.01

Table B.10: Active duration adjustment and interest rate movements (alternative measures)

The sample period is from 2010 to 2018. For the time period from 2004 to 2009, as lack of derivative data to compute its duration exposure, only non-derivative users are included in the sample. Non-derivative users are determined as those who are below the top 5% percentile in terms of the absolute book value of derivatives. Covariant "Level (simple average)" is the simple average of a sequence of spot rates on the treasury yield curve. Covariant "Level (weighted average)" is the average of the same spot rates weighted by the holding information of bonds with the corresponding maturity.The dummy variable of RBC ratios indicates the insures whose RBC ratios are below 10% percentile.

	Dependent variable: Active duration adjustment								
	Level (simple average)				Level (weighted average)				
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
ΔLevel (simple average)	-0.019	-0.074**	-0.131***	-0.127***					
	(0.025)	(0.031)	(0.037)	(0.031)					
ΔTerm spread (10y&3m)	-0.073***		-0.070***	-0.073***		-0.063***		-0.026	-0.030*
	(0.024)		(0.024)	(0.021)		(0.018)		(0.020)	(0.018)
ΔTerm spread (30y&10y)		-0.496***	-0.486***	-0.464***			-0.520***	-0.450***	-0.440***
		(0.115)	(0.115)	(0.090)			(0.102)	(0.116)	(0.091)
ΔLevel (weighted average)					0.049**	-0.012	-0.127***	-0.128***	-0.129***
					(0.022)	(0.028)	(0.041)	(0.041)	(0.035)
ΔLong-term liability proportion	0.798***	0.773***	0.773***	0.762***	0.794***	0.799***	0.769***	0.774***	0.763***
	(0.164)	(0.164)	(0.164)	(0.294)	(0.164)	(0.164)	(0.164)	(0.164)	(0.294)
ΔLog[total assets] (in thousand)	1.925***	1.931***	1.932***	1.618***	1.924***	1.924***	1.932***	1.931***	1.617***
	(0.058)	(0.058)	(0.058)	(0.134)	(0.058)	(0.058)	(0.058)	(0.058)	(0.134)
Dummy: RBC ratio < 10%	-0.087**	-0.087**	-0.087**	0.007	-0.088**	-0.087**	-0.087**	-0.087**	0.008
	(0.043)	(0.043)	(0.043)	(0.070)	(0.043)	(0.043)	(0.043)	(0.043)	(0.070)
Constant	0.575***	0.564***	0.553***		0.590***	0.576***	0.555***	0.555***	
	(0.014)	(0.015)	(0.015)		(0.014)	(0.015)	(0.016)	(0.016)	
Fixed effects	-	-	-	Firm	-	-	-	-	Firm
Observations	8,194	8,194	8,194	8,194	8,194	8,194	8,194	8,194	8,194
R^2	0.122	0.123	0.124	0.264	0.121	0.122	0.124	0.124	0.263
Adjusted R^2	0.122	0.123	0.124	0.183	0.121	0.122	0.123	0.123	0.183
Residual Std. Error	1.183	1.182	1.182	1.141	1.184	1.183	1.182	1.182	1.141
Note:						*p<0.1; **p<0.05; ***p<0.01			

Table C.1: Number of insurer-year observations in each A.M. Best's rating category

The table provides the number of insurer-year observations falling into each level of A.M. Best's ratings. The sample of observations span from 2004 to 2018 covering all individual life insurers.

Financial strength ratings	Rating Scales													Non-rating designations	
	Superior		Excellent		Good		Fair		Marginal		Weak		Poor	E	F
	A++	A+	A	A-	B++	B+	B	B-	C++	C+	C	C-	D		
No. of insurer-year observations	1756	1815	1762	424	299	115	802	1000	14	16	25	55	16	19	12

Figure C.1: A.M. Best Financial Strength Ratings: letter ratings v.s. numeric ratings

Digits 1 & 2

Secure Ratings *Introduced in 1994*

10	=	A++	(Superior) *Introduced in 1992*
11	=	A+	(Superior)
12	=	A	(Excellent)
13	=	A-	(Excellent)
20	=	B++	(Good) *Introduced in 1992 Termed "Very Good" prior to 2008*
21	=	B+	(Good) *Introduced in 1992 Termed "Very Good" prior to 2008*

Vulnerable Ratings *Introduced in 1994*

22	=	B	(Fair) *Termed "Adequate" prior to 1997*
23	=	B-	(Fair) *Termed "Adequate" prior to 1997*
30	=	C++	(Marginal) *Introduced in 1992/ Termed "Fair" prior to 1997*
31	=	C+	(Marginal) *Termed "Fair" prior to 1997*
32	=	C	(Weak) *Termed "Marginal" prior to 1997*
33	=	C-	(Weak) *Termed "Marginal" prior to 1997*
42	=	D	(Poor) *Termed "Very Vulnerable" prior to 1997*
46	=	E	(Under Regulatory Supervision) *Introduced in 1992; replaces NA-10/ Termed "Under State Supervision" prior to 1997*
48	=	F	(In Liquidation) *Introduced in 1992; replaces NA-10/ Termed "Liquidation" prior to 1997*
90	=	S	(Rating Suspended) *Introduced in 1996*

Not Rated Categories (NR) *Introduced in 1996*

Assigned to companies reported on by A.M. Best, but not assigned a **Best's Rating**.

75	=	NR	Not Rated

Table C.2: Robustness: ownership status and interest rate risk-taking

The table reports the duration adjustment with a focus on the role of organizational forms. The observations of all tests are U.S. life insurers from 2004 to 2018 excluding those which use interest rate derivatives. Mutual dummy variable is to indicate mutual companies, and the other dummy variable "AMB rating below B-" is to indicate life insurers with A.M. Best rating being lower than B-, which are considered as financially constrained insurers. All specification are estimated by OLS.

	Dependent variable: Asset duration			
	(1)	(2)	(3)	(4)
Level	-0.601^{***}	-0.569^{***}	-0.501^{***}	-0.547^{***}
	(0.053)	(0.054)	(0.042)	(0.054)
Term spread (10Y-3M)	-0.373^{***}	-0.354^{***}	-0.439^{***}	
	(0.072)	(0.073)	(0.060)	
Term spread (30Y-10Y)	-0.593^{**}	-0.561^{**}		-1.339^{***}
	(0.273)	(0.279)		(0.228)
Long-term liability proportion	3.605^{***}	3.614^{***}	3.611^{***}	3.604^{***}
	(0.126)	(0.126)	(0.126)	(0.126)
Log[total assets] (in thousand)	0.051^{***}	0.049^{***}	0.051^{***}	0.052^{***}
	(0.019)	(0.019)	(0.019)	(0.019)
Dummy: AMB rating below B-	-0.564	-0.573	-0.578^{*}	-0.628^{*}
	(0.350)	(0.350)	(0.350)	(0.350)
Dummy: Mutual insurer	0.789^{***}	3.857^{***}	3.340^{***}	3.728^{***}
	(0.198)	(1.338)	(0.954)	(1.333)
Mutual insurer × Level		-0.701^{***}	-0.613^{***}	-0.689^{***}
		(0.251)	(0.193)	(0.251)
Mutual insurer × Term spread (10Y-3M)		-0.413	-0.526^{*}	
		(0.346)	(0.279)	
Mutual insurer × Term spread (30Y-10Y)		-0.733		-1.697
		(1.331)		(1.073)
Constant	7.140^{***}	7.021^{***}	6.608^{***}	6.804^{***}
	(0.388)	(0.392)	(0.334)	(0.390)
Observations	6,549	6,549	6,549	6,549
R^2	0.140	0.141	0.140	0.137
Adjusted R^2	0.139	0.140	0.139	0.136
Residual Std. Error	3.301	3.299	3.300	3.305
Note:				*p<0.1; **p<0.05; ***p<0.01

Table C.3: Robustness: Active duration adjustment and organizational forms

The table reports the active duration adjustment with a focus on the role of organizational forms. he observations of all tests are U.S. life insurers from 2004 to 2018 excluding those which use interest rate derivatives. Mutual dummy variable is to indicate mutual companies, and the other dummy variable "AMB rating below B-" is to indicate life insurers with A.M. Best rating being lower than B-, which are considered as financially constrained insurers. All specification are estimated by OLS.

	Dependent variable: Active duration adjustment			
	(1)	(2)	(3)	(4)
ΔLevel	−0.174***	−0.162***	−0.161***	−0.170***
	(0.042)	(0.043)	(0.042)	(0.043)
ΔTerm spread (10Y-3M)	−0.087***	−0.078***	−0.078***	−0.087***
	(0.028)	(0.028)	(0.028)	(0.028)
ΔTerm spread (30Y-10Y)	−0.622***	−0.625***	−0.622***	−0.624***
	(0.132)	(0.135)	(0.132)	(0.135)
ΔLong-term liability proportion	0.997***	1.001***	1.002***	1.002***
	(0.203)	(0.203)	(0.203)	(0.203)
ΔLog[total assets] (in thousand)	2.038***	2.035***	2.035***	2.037***
	(0.067)	(0.067)	(0.067)	(0.067)
Dummy: AMB rating below B-	−0.408***	−0.410***	−0.410***	−0.411***
	(0.130)	(0.130)	(0.130)	(0.131)
Dummy: Mutual insurer	0.166**	0.137*	0.134*	0.171**
	(0.072)	(0.081)	(0.076)	(0.078)
Mutual insurer × ΔLevel		−0.263	−0.281**	−0.100
		(0.204)	(0.140)	(0.174)
Mutual insurer × ΔTerm spread (10Y-3M)		−0.203	−0.202	
		(0.132)	(0.132)	
Mutual insurer × ΔTerm spread (30Y-10Y)		0.077		0.025
		(0.651)		(0.650)
Constant	0.569***	0.571***	0.571***	0.569***
	(0.017)	(0.017)	(0.017)	(0.017)
Observations	5,924	5,924	5,924	5,924
R^2	0.147	0.148	0.148	0.147
Adjusted R^2	0.146	0.146	0.146	0.146
Residual Std. Error	1.146	1.146	1.146	1.146

Note: *p<0.1; **p<0.05; ***p<0.01

Table C.4: Corporation type and interest rate risk-taking

The table compares the active duration adjustment of life insurers with mutual corporate structure with that of life insurers with non-mutual corporate structure. The non-mutual corporate structure includes mutual holding company (MHC) structure and the pure stock corporate structure. Specifications in odd columns are estimated by OLS and those in even columns are fitted into the fixed-effect panel regression model. The variable proxizing the financial constraint in this table employs RBC ratios instead of A.M. Best's ratings. It is to avoid insufficiency of observations for the group of insurers with the mutual structure. All insurers are required to report RBC ratios but only part of insurers own rating designation from A.M. Best.

	Dependent variable: Active duration adjustment			
	Mutual corporation		Non-mutual corporation	
	(1)	(2)	(3)	(4)
ΔLevel	−0.199*	−0.212*	−0.131***	−0.124***
	(0.120)	(0.111)	(0.039)	(0.033)
ΔTerm spread (10Y-3M)	−0.119	−0.115	−0.065***	−0.067***
	(0.078)	(0.084)	(0.025)	(0.022)
ΔTerm spread (30Y-10Y)	−0.634*	−0.654**	−0.501***	−0.477***
	(0.377)	(0.259)	(0.121)	(0.095)
ΔLong-term liability proportion	1.368*	1.162	0.751***	0.756**
	(0.719)	(1.105)	(0.169)	(0.304)
ΔLog[total assets] (in thousand)	2.439***	2.148***	1.952***	1.616***
	(0.246)	(0.504)	(0.061)	(0.141)
Dummy: RBC ratio < 10%	0.079	0.324	−0.090**	−0.025
	(0.168)	(0.277)	(0.046)	(0.073)
Constant	0.615***		0.546***	
	(0.050)		(0.016)	
Fixed effects	Firm	–	Firm	–
Observations	643	643	7,477	7,477
R^2	0.141	0.288	0.126	0.263
Adjusted R^2	0.133	0.207	0.125	0.182
Residual Std. Error	1.073	1.026	1.188	1.149

Note: *p<0.1; **p<0.05; ***p<0.01

Table C.5: Corporation type and active duration adjustment

The table examines the active duration adjustment of life insurers with different corporate structure, the mutual corporate structure, the mutual holding company (MHC) structure, and the stock corporate structure. Two dummy variables are used to indicate insurers with the three different corporate structures. The variable proxizing the financial constraint employs A.M. Best's ratings. The insurers with ratings below B- are considered as financially constrained. All insurers are required to report RBC ratios but only part of insurers own rating designation from A.M. Best.

	Dependent variable: Active duration adjustment					
	(1)	(2)	(3)	(4)	(5)	(6)
ΔLevel	−0.175***	−0.167**	−0.175***	−0.175***	−0.203**	−0.161
	(0.040)	(0.066)	(0.040)	(0.040)	(0.097)	(0.133)
ΔTerm spread (10Y-3M)	−0.093***	−0.093***	−0.113**	−0.093***	−0.135	−0.136
	(0.026)	(0.026)	(0.055)	(0.026)	(0.087)	(0.087)
ΔTerm spread (30Y-10Y)	−0.596***	−0.596***	−0.596***	−0.581**	−0.596***	−0.412
	(0.128)	(0.128)	(0.128)	(0.231)	(0.128)	(0.421)
ΔLong-term liability proportion	1.016***	1.015***	1.014***	1.016***	1.015***	1.012***
	(0.200)	(0.200)	(0.200)	(0.200)	(0.200)	(0.200)
ΔLog[total assets] (in thousand)	2.101***	2.101***	2.101***	2.101***	2.101***	2.102***
	(0.067)	(0.067)	(0.067)	(0.067)	(0.067)	(0.067)
Dummy: AMB rating below B-	−0.387***	−0.387***	−0.387***	−0.388***	−0.388***	−0.389***
	(0.129)	(0.129)	(0.129)	(0.129)	(0.129)	(0.129)
Dummy: Stock corporation	−0.099**	−0.099*	−0.095*	−0.099*	−0.091*	−0.099*
	(0.050)	(0.050)	(0.051)	(0.051)	(0.053)	(0.056)
Dummy: MHC corporation	−0.064	−0.064	−0.062	−0.069	−0.058	−0.078
	(0.074)	(0.074)	(0.075)	(0.075)	(0.077)	(0.083)
Stock corporation × Level		−0.009			0.031	−0.011
		(0.057)			(0.097)	(0.140)
MHC corporation × Level		−0.004			0.029	−0.071
		(0.086)			(0.140)	(0.205)
Stock corporation × Term spread (10Y-3M)			0.023		0.046	0.048
			(0.054)		(0.091)	(0.092)
MHC corporation × Term spread (10Y-3M)			0.016		0.038	0.040
			(0.082)		(0.132)	(0.132)
Stock corporation × Term spread (30Y-10M)				−0.011		−0.184
				(0.213)		(0.443)
MHC corporation × Term spread (30Y-10Y)				−0.091		−0.434
				(0.318)		(0.647)
Constant	0.653***	0.653***	0.650***	0.654***	0.646***	0.654***
	(0.049)	(0.049)	(0.049)	(0.050)	(0.050)	(0.054)
Observations	6,168	6,168	6,168	6,168	6,168	6,168
R^2	0.148	0.148	0.148	0.148	0.148	0.148
Adjusted R^2	0.147	0.146	0.146	0.146	0.146	0.146
Residual Std. Error	1.133 (df = 6159)	1.133 (df = 6157)	1.133 (df = 6157)	1.133 (df = 6157)	1.133 (df = 6155)	1.133 (df = 6153)

Note: *p<0.1; **p<0.05; ***p<0.01